FOUR ESSAYS ON ECONOMETRIC SPECIFICATION

by

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CHAPTER 1

INTRODUCTION

1. INTRODUCTION

1.1 Preliminary remarks

The purpose of this research is to analyze specification of econometric models, with emphasis on dynamics.

The work presented consists of four essays. Two of the essays investigate different aspects of parameterizations of econometric models from a theoretical point of view, while the remaining two consider monetary policy questions using techiniques of dynamic econometric modelling.

1.2 The theoretical part

The first paper considers a simplification of the computation of long–run multipliers in dynamic models. It demonstrates that a simple reparameterization of a dynamic linear regression model gives the long–run coefficients as the ratio of two coefficients. A simplified formula for the computation of the standard errors is also presented.

The notion of long–run multipliers is traditonally associated with weakly stationary series. Recently a lot of literature has considered estimation and inference when the series are non–stationary. If a linear combination of two such series are stationary, the series are said to be cointegrated. It turns out that the estimator of the long–run coefficients in the stationary environment is the full information maximum likelihood estimator of the cointegration parameters when there is only one such relationship. The long–run notion of relationships thus generalizes nicely.

The role of reparameterizations is also the focus of the second paper of the theoretical part of the thesis. It considers how the problem of collinearity relates to different parameterizations of the same statistical model. The paper demonstrates that the interesting aspect of collinearity is not correlation between variables, but the precision of estimates of the parameters of interest. It is well known that although individual coefficients might be estimated with low precision, linear combinations of the same coefficients might be estimated with high precision. If these combinations are the parameters of interest, collinearity is not a problem. However, all existing measures of collinearity focus on variables, not parameters. A measure is therefore proposed that evaluates collinearity relative to parameters.

2.2 The applied part

The applied part of the thesis consists of two papers on monetary economics, both of which rely heavily on parameterizations and long–run relationships.

The third paper of the thesis, jointly written with Jan Tore Klovland, relates the concept of cointegration to the policy problem of GDP targeting. The problem investigated is which of money and credit provides the best indicator of the development in GDP. Given a large shock in terms of the deregulation of Norwegian credit markets, which affected both money and credit, it should be possible to identify the monetary variable, if any, still tracking income or GDP. This problem is ideally suited to cointegration techniques. Just substitute "tracking" with "being cointegrated with" above. It turns out that the monetary aggregates perform much better than measures of credit in this respect.

The last paper picks up where the previous one left off. A demand function for narrow money is estimated that is constant across the credit deregulation mentioned above. In addition the model has parameters that are both weakly and super exogenous. The parameters are therefore invariant to the kinds of monetary and fiscal policy taking place over the sample, and the model is not vulnerable to the Lucas critique for the class of interventions that has taken place. This means that a wide range of policy experiments can be conducted with the model.

To be specific, the demand for nominal money growth per quarter depends negatively upon the money market rate and the quarterly change in the spread between own yield and the alternative yield on time deposits. There is an immediate positive effect from growth of prices and real expenditure, while there is a smaller adjustment to changes in the money–income ratio in the previous quarter. Finally there is an adjustment to deviations from the long–run desired relation between real money, real income, the own yield and the maximum alternative yield for long term investments. So in the short–run agents speculate in the money market and change their money holdings between demand and savings deposits, while in the long–run the the portfolio is adjusted between money and bonds.

The implications of the analysis are that money is endogenously determined by prices, real expenditure and interest rates, and that these determinants can be varied for a wide class of policy analyses.

CHAPTER 2

ESTIMATION OF LONG RUN COEFFICIENTS IN ERROR CORRECTION MODELS

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Estimation of Long Run Coefficients in Error Correction Models

Gunnar Bårdsen

I. BACKGROUND

The Autoregressive-Distributed lag model (AD) offers a flexible framework for dynamic modelling, a drawback being the need for additional computations in order to assess the long-run properties of the model.

Wickens and Breusch (1988) propose a method, based on Bewley (1979), which gives point estimates of both the long-run coefficients and their variances by means of reformulations of the AD, but the method requires use of instrumental variables.

Wickens and Breusch also argue that their reformulations of the AD are preferable to the Error Correction Model (ECM), the latter model class being considered. '...not a particularly convenient form for estimation — especially of θ [the long-run coefficient]...'.[1]

The present note argues that the ECM approach provides an efficient research strategy, since the ECM gives estimates of the long-run coefficients by means of ordinary least squares (OLS) — the only additional computation being required is the ratio of two parameters.

II. DEFINITIONS

An AD with k exogenous variables $x_j, j=1,\ldots,k$, is written[2]

$$y_t = a_0 + \sum_{i=1}^{m} \alpha_i y_{t-i} + \sum_{j=1}^{k} \sum_{i=0}^{n} \beta_{ji} x_{jt-i} + u_t, \tag{1}$$

or in matrix notation:

$$y_t = a_0 + \mathbf{y}_{-1}\boldsymbol{\alpha} + \sum_{j=1}^{k} \mathbf{x}_j \boldsymbol{\beta}_j + u_t, \tag{1'}$$

‡I would like to thank Knut Aase, David F. Hendry, Jan Tore Klovland and a referee for helpful comments.

[1] See Wickens and Breusch (1988) p. 193.

[2] Hendry *et al.* (1984) provides an extensive survey of dynamic models, including AD and ECM.

where $\mathbf{y}_{-1}=[y_{t-1}y_{t-2}\dots y_{t-m}]$, $\boldsymbol{\alpha}=[\alpha_1\alpha_2\dots\alpha_m]'$, $\mathbf{x}_j=[x_{jt}x_{jt-1}\dots x_{jt-n}]$ and $\boldsymbol{\beta}_j=[\beta_{j0}\beta_{j1}\dots\beta_{jn}]'$.

The coefficient a_0 represents the constant term, but could of course also be a vector including other deterministic components such as seasonal dummies and trend. The number of lags on all x_j are made equal for ease of exposition.

The long-run coefficient θ_j is derived from (1) by the formula

$$\theta_j = -\beta^*_{jn}/\alpha^*_m, \tag{2}$$

where α^*_m and β^*_{jn} are defined as

$$\alpha^*_m = \sum_{i=1}^{m} \alpha_i - 1 \tag{3}$$

and

$$\beta^*_{jn} = \sum_{i=0}^{n} \beta_{ji}. \tag{4}$$

Armed with these definitions, we can turn to the problem of estimating θ_j and the associated short run dynamics.

III. ESTIMATING LONG-RUN COEFFICIENTS

The standard method of obtaining $\hat{\theta}_j$ has been to estimate (1) by means of OLS, and thereafter compute $\hat{\theta}_j$, using (2). The variance may be computed according to the general approximation formula:[3]

$$\hat{\text{var}}(f) \approx \sum_h \left(\frac{\partial f}{\partial \gamma_h}\right)^2 \hat{\text{var}}(\hat{\gamma}_h) + 2\sum_{g<h}\left(\frac{\partial f}{\partial \gamma_g}\right)\left(\frac{\partial f}{\partial \gamma_h}\right)\hat{\text{cov}}(\hat{\gamma}_g, \hat{\gamma}_h), \tag{5}$$

where $f=f(\hat{\gamma}_1, \hat{\gamma}_2, \dots, \hat{\gamma}_H)$.

In our model $f=\hat{\theta}_j$ and $\hat{\gamma}_h=\hat{\alpha}_1,\dots,\hat{\alpha}_m,\hat{\beta}_{j0},\dots,\hat{\beta}_{jn}$, so only the variance part of the formula will involve $m+n+1$ summations. With many lags this method may become a bit cumbersome.[4]

As noted by Wickens and Breusch, more convenient approaches are desirable. But although the reformulations of (1) provided by Wickens and Breusch give point estimates of θ_j, their use of instrumental variables seems an unnecessarily complicated approach. The same end can be achieved by reformulating the AD as an ECM and obtain $\hat{\theta}_j$ from the OLS estimates.

To see this, first note that the AD in (1′) can trivially be rewritten as[5]

[3] The formula can be found in Kmenta (1986), p. 486.

[4] The econometrics program PC-GIVE has the computations of long run coefficients with standard errors as an option.

[5] For an elaboration upon this result, see Spanos (1986) p. 386.

$$\Delta y_t = a_0 + \mathbf{y}_{-1}\boldsymbol{\alpha}_{-1} + \sum_{j=1}^{k} \mathbf{x}_j\boldsymbol{\beta}_j + u_t, \tag{6}$$

where $\Delta y_t = y_t - y_{t-1}$ and $\boldsymbol{\alpha}_{-1} = [(\alpha_1 - 1)\,\alpha_2 \ldots \alpha_m]'$. Next, define the square transformation matrices **M** and **N**, which only differ in being of order m and $n+1$:

$$\underset{(m\times m)}{\mathbf{M}} = \begin{bmatrix} 1 & 0 & \cdots & & 0 \\ -1 & 1 & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix} \quad \text{and } \mathbf{M}^{-1} = \begin{bmatrix} 1 & 0 & \cdots & & 0 \\ \vdots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \\ \vdots & & & \ddots & 0 \\ 1 & \cdots & \cdots & \cdots & 1 \end{bmatrix}.$$

Then (6) can be reformulated as

$$\Delta y_t = a_0 + \mathbf{y}_{-1}\mathbf{M}\mathbf{M}^{-1}\boldsymbol{\alpha}_{-1} + \sum_{j=1}^{k} \mathbf{x}_j\mathbf{N}\mathbf{N}^{-1}\boldsymbol{\beta}_j + u_t, \tag{7}$$

or equivalently:

$$\Delta y_t = a_0 + \mathbf{y}^*_{-1}\boldsymbol{\alpha}^* + \sum_{j=1}^{k} \mathbf{x}^*_j\boldsymbol{\beta}^*_j + u_t, \tag{8}$$

where $\mathbf{y}^*_{-1} = \mathbf{y}_{-1}\mathbf{M} = [\Delta y_{t-1}\Delta y_{t-2} \ldots \Delta y_{t-m+1} y_{t-m}]$, $\boldsymbol{\alpha}^* = \mathbf{M}^{-1}\boldsymbol{\alpha}_{-1} = [\alpha^*_1\alpha^*_2 \ldots \alpha^*_m]'$, $\mathbf{x}^*_j = \mathbf{x}_j\mathbf{N} = [\Delta x_{jt}\Delta x_{jt-1} \ldots \Delta x_{jt-n+1} x_{jt-n}]$ and $\boldsymbol{\beta}^*_j = \mathbf{N}^{-1}\boldsymbol{\beta}_j = [\beta^*_{j0}\beta^*_{j1} \ldots \beta^*_{jn}]'$.

Since our main interest is the coefficients of the regressors expressed in levels, it will be most convenient to write (8) out, isolating these terms:

$$\Delta y_t = a_0 + \sum_{i=1}^{m-1} \alpha^*_i \Delta y_{t-i} + \sum_{j=1}^{k}\sum_{i=0}^{n-1} \beta^*_{ji}\Delta x_{jt-i} + \alpha^*_m y_{t-m} + \sum_{j=1}^{k} \beta^*_{jn} x_{jt-n} + u_t. \tag{9}$$

No restrictions are imposed upon the model, hence estimating (1) and (9) will give identical results. But more is implied. In (9) the short run dynamics are explicit in the differenced terms, and the long-run coefficients are found as ratios of the levels coefficients by using (2).

Since linear models are invariant to linear transformations, as demonstrated, it is a matter of convenience whether (1) or (9) is estimated. A more interesting aspect of the transformation in (9) is the simplification in the computation of the variance it implies.

Given $\hat{\theta}_j = -\hat{\beta}^*_{jn}/\hat{\alpha}^*_m$, the large sample variance of $\hat{\theta}_j$ is found from (5):

$$\widehat{\text{var}}(\hat{\theta}_j) = \left(\frac{\partial\hat{\theta}_j}{\partial\beta^*_{jn}}\right)^2 \widehat{\text{var}}(\hat{\beta}^*_{jn}) + \left(\frac{\partial\hat{\theta}_j}{\partial\alpha^*_m}\right)^2 \widehat{\text{var}}(\hat{\alpha}^*_m) + 2\left(\frac{\partial\hat{\theta}_j}{\partial\beta^*_{jn}}\right)\left(\frac{\partial\hat{\theta}_j}{\partial\alpha^*_m}\right) \widehat{\text{cov}}(\hat{\beta}^*_{jn}, \hat{\alpha}^*_m), \tag{10}$$

which can be expressed as:

$$\hat{\text{var}}(\hat{\theta}_j)=(\hat{\alpha}^*_m)^{-2}[\hat{\text{var}}(\hat{\beta}^*_{jn})+(\hat{\theta}_j)^2\,\hat{\text{var}}(\hat{\alpha}^*_m)+2\hat{\theta}_j\hat{\text{cov}}(\hat{\beta}^*_{jn},\hat{\alpha}^*_m)]. \tag{11}$$

By this approach the same formula applies regardless of the number of lags involved, unlike the formula used to compute $\hat{\text{var}}(\hat{\theta}_j)$ in (5). All parameters required in the computation of (11) are provided by the OLS estimation of (9). Of course, the estimates of $\hat{\theta}_j$ and $\hat{\text{var}}(\hat{\theta}_j)$ from (1) and (9) are identical.

Equation (9) is simply an ECM. This is seen by rewriting the equation as:

$$\Delta y_t=a+\sum_{i=1}^{m-1}\alpha_i^*\Delta_{y-i}+\sum_{j=1}^{k}\sum_{i=0}^{n-1}\beta_{ji}^*\Delta x_{t-i}+\alpha_m^*ECT+u_t \tag{12}$$

where ECT (Error Correction Term) $=[y_{t-m}-\Sigma_{j=1}^{k}\theta_j x_{jt-n}]$. The ECT is constructed by means of (2) and (9). Still, no restrictions are imposed upon the model.

Since the residuals are unaffected by the transformations from (9) to (12), $\mathbf{u}'\mathbf{u}$ is the same in the two models,[6] where $\mathbf{u}'=[u_1\ldots u_T]$. But although estimation of (12), after imposing $\hat{\theta}_j$ from (9), will replicate[7] the coefficient estimates from (9) the estimated standard errors will be smaller, as will the standard error of the regression. This is because the computer program will fail to correct for the k degrees of freedom lost in imposing $\hat{\theta}_j$, $j=1,\ldots,k$. Hence, the estimated standard error of the regression from (12), $\hat{\sigma}_{\text{ecm}}$, will be computed smaller than the equivalent from (9), $\hat{\sigma}_{\text{ad}}$:

$$\hat{\sigma}_{\text{ecm}}=\sqrt{\frac{\hat{\mathbf{u}}'\hat{\mathbf{u}}}{T-[1+m+nk]}}<\hat{\sigma}_{\text{ad}}=\sqrt{\frac{\hat{\mathbf{u}}'\hat{\mathbf{u}}}{T-[1+m+(n+1)k]}}. \tag{13}$$

Of course, this correction should be undertaken, after which the standard errors for each parameter will be identical in (9) and (12).

Another way of applying (12) could be to impose the long-run coefficients upon the variables without lags, forming the ECT, and then use the ECT at the lag determined by the data.

IV. ASPECTS OF INFERENCE

When estimating (9) as the general model, it is important to have in mind that successive lags of the same variable are not independent. Hence, as Wickens and Breusch note, the lag length should be chosen on the basis of variability between successive lags, that is: another lag must provide new information. Accordingly, the appropriate hypothesis when testing lag lengths chosen too small is

$$H_0\colon \beta^*_{jn}=\beta^*_{jn-1}, j=1,\ldots,k, \text{ and } \alpha^*_m=\alpha^*_{m-1}$$

[6] Minor discrepancies might arise because of rounding errors.
[7] See footnote 6.

against

$$H_1: \beta^*_{jm} \neq \beta^*_{jm-1}, j = 1, \ldots, k, \text{ or } \alpha^*_m \neq \alpha^*_{m-1}.$$

The statistics from testing H_0 on (9) will coincide exactly with the results from testing lag lengths in the AD, as discussed in Spanos (1986) pp. 540–41.

In the case of a single coefficient this can easily be demonstrated. Under these circumstances

$$H_0: \beta^*_{jm} = \beta^*_{jm-1}$$

and

$$H_1: \beta^*_{jm} \neq \beta^*_{jm-1}.$$

The appropriate t-statistic takes the form

$$t = \frac{\beta^*_{jm} - \beta^*_{jm-1}}{\sqrt{\hat{\text{var}}(\hat{\beta}^*_{jm} - \hat{\beta}^*_{jm-1})}} \sim t(T-1-m-(n+1)k).$$

But since $\hat{\beta}^*_{jm} = \hat{\beta}^*_{jm-1} + \hat{\beta}_{jm}$ from (4):

$$t = \frac{\hat{\beta}_{jm}}{\sqrt{\hat{\text{var}}(\hat{\beta}_{jm})}},$$

which is the ordinary t-test, suitable in testing individual coefficients being zero in the AD.

The complications of evaluating the statistical significance of individual coefficients with ordinary t-tests arise because the variances of successive lags are also interdependent.

The variance of $\hat{\beta}^*_{jn}$ is

$$\hat{\text{var}}(\hat{\beta}^*_{jn}) = \sum_{i=0}^{n} \hat{\text{var}}(\hat{\beta}_{ji}) + 2 \sum_{i=0}^{n-1} \sum_{g=i+1}^{n} \hat{\text{cov}}(\hat{\beta}_{ji}, \hat{\beta}_{jg}), \tag{14}$$

which can be reformulated to

$$\hat{\text{var}}(\hat{\beta}^*_{jn}) = \hat{\text{var}}(\hat{\beta}^*_{jn-1}) + \hat{\text{var}}(\hat{\beta}_{jn}) + 2 \sum_{i=0}^{n-1} \hat{\text{cov}}(\hat{\beta}_{ji}, \hat{\beta}_{jn}) \tag{15}$$

So if $\hat{\beta}^*_{jn-1}$ is significantly different from zero, it can in fact induce significance of $\hat{\beta}^*_{jn}$, even if $\hat{\beta}_{jn}$ itself has a high variance. The requirement is simply that $\Sigma_{i=0}^{n-1} \hat{\text{cov}}(\hat{\beta}_{ji}, \hat{\beta}_{jn})$ is sufficiently negative in magnitude.

Hence, while t-tests of individual coefficients being zero under the null hypothesis are to be avoided, the appropriate testing procedure in the general model is the methodology proposed earlier: the testing of indifference between successive lags, as Wickens and Breusch propose.

But during the search for a parsimonious model ordinary t-tests with the coefficient(s) being zero under the null hypothesis can be appropriate, since

CHAPTER 3

COLLINEARITY:
MEASURES AND PARAMETERIZATIONS.

COLLINEARITY: MEASURES AND PARAMETERIZATIONS

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Abstract

The role of reparameterizations in analyzing multicollinearity is the focus of the paper. Existing measures are vulnerable to different parameterizations of the same statistical model because collinearity is measured as correlation between variables and not the degree of precision of estimated parameters. A measure that evaluates collinearity relative to parameters is proposed.

KEYWORDS: Multicollinearity, reparameterizations, specifications.

COLLINEARITY: MEASURES AND PARAMETERIZATIONS

By Gunnar Bårdsen[1]

Multicollinearity is a problem of obtaining good parameter estimates. It is therefore better regarded as lacking variability within a sample than as high correlation between variables. This means that a reparameterization of the model can give good estimates of some of the new coefficients. The problem of multicollinearity, in the following called collinearity, is therefore finding a parameterization of a statistical model with economically meaningful and precisely estimated coefficients.

Leamer (1983), Maddala (1988, chap. 7), and Spanos (1986, ch. 20.6) all emphasize this point, but it is probably most explicitly stated by Hendry (1989, p. 97):

> "Thus the important issue in a model is NOT the degree of correlation between the variables,(...) but the precision with which the *parameters of interest* (...) can be determined."

The objective is to analyze collinearity and parameterizations, and to propose a collinearity measure that is

– easy to compute
– robust against reparameterizations
– intuitive.

1. DEVELOPING THE MEASURE

Some well known results will serve as a backdrop:

In the model

(1) $$\boldsymbol{y} = X\boldsymbol{\beta} + \boldsymbol{u},$$

where $X = [\boldsymbol{x}_1 \cdots \boldsymbol{x}_h \cdots \boldsymbol{x}_k]$ is a $(T \times k)$ matrix, $\boldsymbol{\beta} = [\beta_1 \cdots \beta_k]'$, and where $\boldsymbol{u}$ is the vector of residuals, collinearity between the variables can be expressed as:

(2) $$X\boldsymbol{c} + \boldsymbol{v} = \boldsymbol{0},$$

where perfect collinearity means $\boldsymbol{v} = \boldsymbol{0}$.

Let subscript $-h$, where $h = 1,\dots,k$, mean the deletion of element h when it is applied to a vector; it means the deletion of column h when applied to a matrix: $X_{-h} = [x_1 \cdots x_{(h-1)}\ \ x_{(h+1)} \cdots x_k]$; and it means the deletion of row h when applied to an inverted matrix.

The variable of interest is x_h. Given $c_h \neq 0$, equation (2) is rewritten

$$x_h = X_{-h}c_{-h}(-c_h)^{-1} + v(-c_h)^{-1} = X_{-h}d_{-h} + v_h, \tag{3}$$

with $c_{-h} = [c_1 \cdots c_{(h-1)}\ \ c_{(h+1)} \cdots c_k]'$.

The variances and covariances of the OLS estimates of (1) can be expressed as

$$var(\hat\beta_h) = \sigma^2(\hat v_h{}'\hat v_h)^{-1} \tag{4}$$

and

$$cov(\hat\beta_h, \hat\beta_{-h}) = -\sigma^2(\hat v_h{}'\hat v_h)^{-1}\hat d_{-h},$$

with $\hat\beta_{-h} = [\hat\beta_1 \cdots \hat\beta_{(h-1)}\ \ \hat\beta_{(h+1)} \cdots \hat\beta_k]'$, and where $\hat v_h$ and $\hat d_{-h}$ are OLS estimates of (3).[2]

A collinearity measure based on $\hat v_h{}'\hat v_h$ seems a natural next step, but using $\hat v_h{}'\hat v_h$ directly is meaningless since it depends upon the units of measurement of x_h. One solution is to scale x_h to unit length, as recommended by Belsley *et al.* (1980), that is $\|\tilde x_h\| \equiv (\tilde x_h{}'\tilde x_h)^{\frac12} = 1$, where $\tilde x_h = x_h s_h$ and $s_h = (x_h{}'x_h)^{-\frac12}$.

The scaling matrix S:

$$\underset{(k\times k)}{S} = \begin{bmatrix} s_1 & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & & & \vdots \\ \vdots & & s_h & & \vdots \\ \vdots & & & s_{(h+1)} & \vdots \\ \vdots & & & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & s_k \end{bmatrix}, \tag{5}$$

is therefore used to rescale equations (2) and (3) as

$$XSS^{-1}c + v = \tilde X\tilde c + v = 0$$

and

$$\tilde x_h = XS_{-h}S_{-h}^{-1}c(-s_h^{-1}c_h)^{-1} + v(-s_h^{-1}c_h)^{-1} = \tilde X_{-h}\tilde c_{-h}(-\tilde c_h)^{-1} + v_h s_h = \tilde X_{-h}\tilde d_{-h} + \tilde v_h, \tag{6}$$

with

$$\underset{(k-1 \times k)}{S_{-h}^{-1}} = \begin{bmatrix} s_1^{-1} & 0 & & 0 & \cdots & 0 \\ 0 & \ddots & & & & \vdots \\ \vdots & & s_{(h-1)}^{-1} & 0 & & \vdots \\ \vdots & & 0 & s_{(h+1)}^{-1} & & \vdots \\ \vdots & & & & \ddots & 0 \\ 0 & \cdots & 0 & & 0 & s_k^{-1} \end{bmatrix}.$$

The residuals from the OLS regression of (6) are

$$\hat{\tilde{v}}_h = \tilde{x}_h - \tilde{X}_{-h}\hat{\tilde{d}}_{-h}$$

and the scaled residual sum of squares $R\tilde{S}S_h$ follows:

$$R\tilde{S}S_h = \hat{\tilde{v}}_h{}'\hat{\tilde{v}}_h.$$

Note that $0 \leq R\tilde{S}S_h \leq 1$ since the total sum of squares from zero now equals unity:

(7) $$\tilde{x}_h{}'\tilde{x}_h = 1 = \hat{\tilde{d}}_{-h}{}'\tilde{X}_{-h}{}'\tilde{X}_{-h}\hat{\tilde{d}}_{-h} + \hat{\tilde{v}}_h{}'\hat{\tilde{v}}_h.$$

Hence $R\tilde{S}S_h$ should provide an objective measure of lack of data variability.

This derivation simplifies since $\hat{\tilde{v}}_h{}'\hat{\tilde{v}}_h = \hat{v}_h{}'\hat{v}_h(x_h{}'x_h)^{-1}$, so (7) can be written

(8) $$1 = \hat{d}_{-h}{}'X_{-h}{}'X_{-h}\hat{d}_{-h}(x_h{}'x_h)^{-1} + \hat{v}_h{}'\hat{v}_h(x_h{}'x_h)^{-1}.$$

Another way of expressing (8) is $1 = \tilde{R}_h^2 + R\tilde{S}S_h$, where $\tilde{R}_h^2$ is the uncentered coefficient of determination.

It is not necessary to run the auxiliary regression or transform any variables since $R\tilde{S}S_h = \sigma^2[x_h{}'x_h \cdot var\ (\hat{\beta}_h)]^{-1}$ from (4). The actual measure is the sample equivalent $C(\hat{\beta}_h)$:

(9) $$C(\hat{\beta}_h) = \hat{\sigma}^2[x_h{}'x_h \cdot v\hat{a}r(\hat{\beta}_h)]^{-1}.$$

It is easily computed since both $\hat{\sigma}^2$ and $v\hat{a}r(\hat{\beta}_h)$ are standard output of any regression – the only auxiliary computation is the inner product $x_h{}'x_h$.

Four properties of $C(\hat{\beta}_h)$ are evident. First, no scaling is necessary – only the computation of the scaling factors $x_h{}'x_h$. Second, $C(\hat{\beta}_h) = 1$ if x_h is orthogonal to X_{-h}; and $C(\hat{\beta}_h) = 0$ if $x_h = X_{-h}d_{-h}$. Third, $C(\hat{\beta}_h) = 1 - \tilde{R}_h^2$. And fourth, taking the square root produces Leamer's (1978, p. 179) $c_2(\beta_h)$. So what is the point? The point is that many things might change if the model is rewritten in a different form.

2 PARAMETERIZATIONS

Reparameterizations are sometimes regarded as "solutions" to the original collinearity problem in (1). The purpose of this section is to analyze the consequence of different parameterizations and to show that collinearity measures based on correlation between variables can give wrong conclusions.

Let any non–singular reparameterization matrix be P with columns a_i, inverse P^{-1} with rows b_i', so $P_{-h} = [a_1 \cdots a_{(h-1)} \ a_{(h+1)} \cdots a_k]$ and $P_{-h}^{-1} = [b_1 \cdots b_{(h-1)} \ b_{(h+1)} \cdots b_k]'$, $i = 1,...,k$.[3] Then (1), (2) and (3) can be expressed as

$$(10) \qquad y = XPP^{-1}\beta + u = [Xa_1 ... Xa_k][b_1'\beta ... b_k'\beta]' + u = X^*\beta^* + u,$$

$$(11) \qquad XPP^{-1}c + v = [Xa_1 ... Xa_k][b_1'c ... b_k'c]' + v = X^*c^* + v = 0,$$

and

$$(12) \qquad x_h^* = X_{-h}^* d_{-h}^* + v_h^*,$$

with

$$x_h^* = Xa_h,\ X_{-h}^* = XP_{-h} = [Xa_1 \cdots Xa_{(h-1)} \ Xa_{(h+1)} \cdots Xa_k],$$
$$d_{-h}^* = P_{-h}^{-1}c(-b_h'c)^{-1} = [b_1'c \cdots b_{(h-1)}'c \ b_{(h+1)}'c \cdots b_k'c]'(-b_h'c)^{-1},$$

and

$$v_h^* = v\,(-b_h'c)^{-1}.$$

It is evident that (1) and (10) are different parameterizations of the same statistical model.

Weakness of existing measures be illustrated by rewriting (4) in a familiar way:

$$(13) \qquad var(\hat\beta_h) = \sigma^2(x_h'x_h R\tilde{S}S_h)^{-1} = \sigma^2[TSS_h(1 - R_h^2)]^{-1},$$

where $TSS_h = x_h'x_h - T \cdot \bar{x}_h^2$ is total sum of squares from mean, while $R_h^2 = 1 - (\hat{v}_h'\hat{v}_h / TSS_h)$ denotes the coefficient of determination.

Assuming (1) is a correct specification – and so disregarding σ^2 – the source of a high $var(\hat\beta_h)$ must be either lack of variation in x_h, represented by TSS_h, or near linear dependence with the other variables, as expressed by R_h^2.

The source of lacking variability is irrelevant for the imprecision of $\hat\beta_h$. Most collinearity measures, including the condition number of Belsley *et al.* (1980), will only register high correlation between variables.

For example: Any reparameterization that transforms x_h to x_h^* will also change TSS_h. This renders the coefficient of determination, R_h^2, totally uninteresting as a measure of collinearity since it is a function of TSS_h and therefore dependent upon the parameterization of the model.[4]

Analysis of collinearity in the reparameterized model should fulfill two requirements. First, the measurement of collinearity should be made relative to the transformed model. Accordingly, as (10) is the model of interest, it seems natural to scale the model after the transformation and not before. And second, the analysis should be made relative to parameters and not variables: If a variable in the reparameterized model is changed while its coefficient is the same, the collinearity measured for that coefficient should remain constant. For this to hold true the scaling matrix must be defined in terms of the original variables, so S is still defined as in (5). But the relationship between $C(\hat{\beta}_h)$ and $\tilde{R}_h^2$ breaks down when the model is reparameterized, since $\tilde{R}_h^{2^*} = 1 - \hat{\tilde{v}}_h^{*\prime}\hat{\tilde{v}}_h^*(\tilde{x}_h^{*\prime}\tilde{x}_h^*)^{-1} \neq 1 - R\tilde{S}S_h^*$, where $R\tilde{S}S_h^* = \hat{\tilde{v}}_h^{*\prime}\hat{\tilde{v}}_h^*(x_h'x_h)^{-1}$.

$R\tilde{S}S_h^*$ follows since (11) and (12) are now replaced by

$$XPSS^{-1}P^{-1}c + v = [Xa_1s_1 \ldots Xa_ks_k][s_1^{-1}b_1'c \ldots s_k^{-1}b_k'c]' + v = \tilde{X}^*\tilde{c}^* + v = 0,$$

and

$$\tilde{x}_h^* = \tilde{X}_{-h}^*\tilde{d}_{-h}^* + \tilde{v}_h^*,$$

with

$$\tilde{x}_h^* = Xa_hs_h,$$

$$\tilde{X}_{-h}^* = XP_{-h}S_{-h} = [Xa_1s_1 \cdots Xa_{(h-1)}s_{(h-1)} \;\; Xa_{(h+1)}s_{(h+1)} \cdots Xa_ks_k],$$

$$\tilde{d}_{-h}^* = S_{-h}^{-1}P_{-h}^{-1}c(-s_h^{-1}b_h'c)^{-1}$$

$$= [s_1^{-1}b_1'c \cdots s_{(h-1)}^{-1}b_{(h-1)}'c \;\; s_{(h+1)}^{-1}b_{(h+1)}'c \cdots s_k^{-1}b_k'c]'(-s_h^{-1}b_h'c)^{-1},$$

and

$$\tilde{v}_h^* = v(-s_h^{-1}b_h'c)^{-1}.$$

The total sum of squares from zero of the estimated auxiliary regression simplifies to: $\tilde{x}_h^{*\prime}\tilde{x}_h^* = x_h^{*\prime}x_h^*(x_h'x_h)^{-1} = \hat{d}_{-h}^{*\prime}X_{-h}^{*\prime}X_{-h}^*\hat{d}_{-h}^*(x_h'x_h)^{-1} + \hat{v}_h^{*\prime}\hat{v}_h^*(x_h'x_h)^{-1}$. And so $C(\hat{\beta}_h^*)$ is given as the original in (9):

$$C(\hat{\beta}_h^*) = \hat{\sigma}^2[x_h'x_h \cdot v\hat{a}r(\hat{\beta}_h^*)]^{-1}. \tag{14}$$

Another way of writing (14) is[5]

$$C(\hat{\beta}_h^*) = (\hat{\sigma}^2/\bar{\sigma}^2)[v\bar{a}r(\bar{\beta}_h)/v\hat{a}r(\hat{\beta}_h^*)], \tag{15}$$

where a bar over a variable refers to estimates from the regression $y = x_h\beta_h + u_h$. Equation (15) shows that the measure is given as the ratio of the variance of β_h estimated singly without transformation to the variance of the estimate of the reparameterized β_h^* estimated jointly with the other coefficients, corrected for variation in goodness of fit. The trivial point is that β_h^* can equal β_h even if $x_h^* \neq x_h$. It is the parameters that matters – not the variables.

This derivation also shows why the condition number $\kappa(X) \equiv (\lambda_{max}/\lambda_{min})^{\frac{1}{2}}$, where the λ's are the eigenvalues of $\tilde{X}'\tilde{X}$, goes wrong. Collinearity in a reparameterized model will be diagnosed from scaling $X^{*\prime}X^*$ and the eigenvalues are not invariant to this, while $C(\hat{\beta}_h)$ is.[6]

3. EXAMPLES

3.1. *A simple derivation*[7]

Take the simplest model:

$$y = x_1\beta_1 + x_2\beta_2 + u .$$

If x_1 and x_2 are collinear a suggested reparameterization in the literature has been

$$y = (x_1 - x_2)\beta_1 + x_2(\beta_1 + \beta_2) + u,$$

which implies a P given by

$$P = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}.$$

Now if $x_1 \approx \gamma x_2 \Leftrightarrow (x_1 - x_2) \approx (\gamma - 1)x_2$, but let us continue:

The collinearity equation for the first variable is

$$x_1^* = (x_1 - x_2) = x_2\hat{d}_2^* + \hat{v}_1,\ \hat{d}_2^* = [-(\hat{c}_1 + \hat{c}_2)/\hat{c}_1].$$

But since $\hat{v}_1$ is unchanged from the original specification the variance of $\hat{\beta}_1^* = \hat{\beta}_1$ is unchanged. Accordingly, $C(\hat{\beta}_1^*)$ will equal $C(\hat{\beta}_1)$.

What about the variance of $\hat{\beta}_2^* = \widehat{(\beta_1+\beta_2)}$? The collinearity equation is

$$\boldsymbol{x}_2 = \boldsymbol{x}_1^* \hat{d}_1^* + \hat{\boldsymbol{v}}_2^*,\ \hat{d}_1^* = -\hat{c}_1/(\hat{c}_1+\hat{c}_2), \text{ and } \hat{\boldsymbol{v}}_2^* = -\boldsymbol{v}(\hat{c}_1+\hat{c}_2)^{-1}.$$

Using (3) and (4) the variance follows:

$$var\widehat{(\beta_1+\beta_2)} = (\sigma^2/\hat{\boldsymbol{v}}'\hat{\boldsymbol{v}})[-(\hat{c}_1+\hat{c}_2)]^2 = (\sigma^2/\hat{\boldsymbol{v}}'\hat{\boldsymbol{v}})[(-\hat{c}_1)^2 + (-\hat{c}_2)^2 + 2\hat{c}_1\hat{c}_2]$$
$$= var(\hat{\beta}_1) + var(\hat{\beta}_2) + 2cov(\hat{\beta}_1, \hat{\beta}_2).$$

So, assuming $\hat{\beta}_2^* = \widehat{(\beta_1+\beta_2)}$ is an interesting estimate, the success of the reparameterization will depend on the covariance between the coefficients being negative.[8]

3.2. *Estimating long–run coefficients: cointegration*[9]

Parameterizations along the lines of the previous example can be of interest in a dynamic linear regression model:

(16) $$y_t = \boldsymbol{x}_t\boldsymbol{\beta}_0 + \sum_{j=1}^{p}(y_{t-j}\alpha_j + \boldsymbol{x}_{t-j}\boldsymbol{\beta}_j) + \boldsymbol{d}_t\boldsymbol{\phi} + u_t,$$

where $\boldsymbol{x}_t = [x_{1t}\ldots x_{kt}]$ and $\boldsymbol{d}_t\boldsymbol{\phi}$ represents deterministic components. All lags are made equal for ease of exposition. The reparameterized model is:

(17) $$\Delta y_t = \Delta\boldsymbol{x}_t\boldsymbol{\beta}_0 + \Sigma_{i=1}^{p-1}(\Delta y_{t-i}\alpha_i^* + \Delta\boldsymbol{x}_{t-i}\boldsymbol{\beta}_i^*) + y_{t-p}\alpha_p^* + \boldsymbol{x}_{t-p}\boldsymbol{\beta}_p^* + \boldsymbol{d}_t\boldsymbol{\phi} + u_t,$$

where

$$\begin{cases} \alpha_i^* = \Sigma_{j=1}^{i}\alpha_j - 1, \\ \boldsymbol{\beta}_i^* = \Sigma_{j=1}^{i}\boldsymbol{\beta}_j \end{cases}$$

and $\Delta w_t \equiv w_t - w_{t-1}$, $w_t = y_t, \boldsymbol{x}_t$.

Equation (17) is also an error correction model:

(18) $$\Delta y_t = \Delta\boldsymbol{x}_t\boldsymbol{\beta}_0 + \sum_{i=1}^{p-1}(\Delta y_{t-i}\alpha_i^* + \Delta\boldsymbol{x}_{t-i}\boldsymbol{\beta}_i^*) + \alpha_p^*(y_{t-p} - \boldsymbol{x}_{t-p}\boldsymbol{\theta}) + \boldsymbol{d}_t\boldsymbol{\phi} + u_t,$$

where the cointegrating vector, which coincides with the vector of long–run coefficients, is defined as

$$\boldsymbol{\theta} = -\boldsymbol{\beta}_p^*/\alpha_p^*.$$

And if the estimates of the long–run coefficients are:[10]

$$\hat{\theta}_h = -\hat{\beta}^*_{hp}/\hat{\alpha}^*_p,\ h = 1,\dots,k,$$

the large sample variance of $\hat{\theta}_h$ can be estimated by

$$\text{(19)}\qquad \hat{\text{var}}(\hat{\theta}_h) = (\hat{\alpha}^*_p)^{-2}[(\hat{\theta}_h)^2\hat{\text{var}}(\hat{\alpha}^*_p) + \hat{\text{var}}(\hat{\beta}^*_{hp}) + 2\hat{\theta}_h\hat{\text{cov}}(\hat{\alpha}^*_p, \hat{\beta}^*_{hp})].$$

Any collinearity in the dynamic linear regression model will only be absent in the error correction model if parameter sums are more precisely estimated than individual parameters; collinearity is a property of the chosen parameterization.

Let me elaborate. $\hat{\text{Var}}(\hat{\beta}^*_{hi})$ is the variance of a sum of parameters, so the standard formula applies:

$$\hat{\text{var}}(\hat{\beta}^*_{hi}) = \Sigma^{i}_{j=0}\hat{\text{var}}(\hat{\beta}_{hj}) + 2\Sigma^{i-1}_{j=0}\Sigma^{i}_{g=j+1}\hat{\text{cov}}(\hat{\beta}_{hj}, \hat{\beta}_{hg}),\ i = 1,\dots,p,$$

but it can also be written

$$\hat{\text{var}}(\hat{\beta}^*_{hi}) = \hat{\text{var}}(\hat{\beta}^*_{h(i-1)}) + \hat{\text{var}}(\hat{\beta}_{hi}) + 2\Sigma^{i-1}_{j=0}\hat{\text{cov}}(\hat{\beta}_{hj}, \hat{\beta}_{hi}).$$

If $\hat{\text{cov}}(x_{h(t-j)}, x_{h(t-i)}) > 0$, which is likely with $I(1)$ series, $\hat{\text{cov}}(\hat{\beta}_{hj}, \hat{\beta}_{hi}) < 0$.[11] Accordingly $\hat{\text{var}}(\hat{\beta}^*_{hi})$ is adjusted for collinearity between individual variables.

The same line of reasoning applies to $\hat{\text{var}}(\hat{\theta}_h)$ in equation (19) since $sign\{\hat{\text{cov}}(y_{t-p}, x_{h(t-p)} \mid y_0, x_{h0})\} = (-1)\cdot sign\{\hat{\text{cov}}(\hat{\alpha}^*_p, \hat{\beta}^*_{hp})\} = sign\{\hat{\theta}_h\} \Leftrightarrow 2\hat{\theta}_h\hat{\text{cov}}(\hat{\beta}^*_{hp}, \hat{\alpha}^*_p) < 0$. So the covariance term will always be negative. This is an illustration of cointegration: as $\hat{\text{cov}}(y_{t-p}, x_{h(t-p)} \mid y_0, x_{h0})$ goes to infinity, $\hat{\text{var}}(\hat{\theta}_h)$ goes to zero.

3.3. *A numerical illustration*

An artificial dataset taken from Belsley (1984) can serve as a final example. The data generating process is

$$\boldsymbol{y} = \beta_1\boldsymbol{x}_1 + \beta_2\boldsymbol{x}_2 + \beta_3\boldsymbol{x}_3 + \epsilon,$$

with $\beta_1 = 3$, $\beta_2 = 0.6$, $\beta_3 = -0.9$ and $\epsilon \sim NIID(0,\sigma^2)$. The constant term is $\boldsymbol{x}_1$.

A regression produces

$$\underset{}{\boldsymbol{y}} = \underset{(0.784)}{3.192\boldsymbol{x}_1} + \underset{(0.555)}{0.810\boldsymbol{x}_2} - \underset{(0.555)}{1.302\boldsymbol{x}_3}$$

$$R^2 = 0.31,\ \hat{\sigma}^2 = 0.308\cdot 10^{-4}.$$

As Belsley notes, the data are extremely collinear. Regressing x_3 on x_1 and x_2 gives $\tilde{R}_h^2 = 0.99999$ while the condition number $\kappa(X) = 1342.$[12] The collinearity of each parameter is assessed to be $C(\hat{\beta}_1) = 2.5\cdot 10^{-6}$, $C(\hat{\beta}_2) = 5\cdot 10^{-6}$ and $C(\hat{\beta}_3) = 5\cdot 10^{-6}$. So at this point all measures reach the same conclusion.

A reparameterization along the lines of the earlier examples could be

$$y = (\beta_1+\beta_2+\beta_3)x_1 + (\beta_2+\beta_3)(x_2-x_1) + \beta_3(x_3-x_2) + \epsilon,$$

which implies

$$P = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}.$$

The regression gives

$$\begin{gathered} y = \underset{(0.001)}{2.699x_1} - \underset{(0.784)}{0.493(x_2-x_1)} - \underset{(0.555)}{1.302(x_3-x_2)} \\ R^2 = 0.31,\ \hat{\sigma}^2 = 0.308\cdot 10^{-4}, \end{gathered}$$

but now $\kappa(X) = 2.42$. This is because the condition number only considers correlation between variables, while $C(\hat{\beta}_h^*)$ analyzes collinearity relative to the parameters.

The collinearity of each new coefficient is found to be $C(\widehat{\beta_1+\beta_2+\beta_3}) = 1$, $C(\widehat{\beta_2+\beta_3}) = 2.5\cdot 10^{-6}$ and $C(\hat{\beta}_3) = 5\cdot 10^{-6}$. So the sum of the parameters are estimated with extremely high precision ($\beta_1+\beta_2+\beta_3 \equiv 2.7$), while the other coefficients are badly determined.

4. CONCLUDING REMARKS

The subject has been collinearity and different parameterizations of a model. The main point is that collinearity is not well defined as correlation between variables, since a different parameterization can produce variables with different correlation, while inference for some of the parameters remain unchanged. Regarding collinearity as a problem of obtaining precise estimates of the parameters of interest seems more sensible. Since no measures exist that focus cleanly on this aspect, such a measure has been proposed.

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FOOTNOTES

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[2] Hill (1987) gives a good derivation of these results. The original exposition can be found in Theil (1971, p. 166).

[3] The matrix P must be non–diagonal; otherwise the elements of P^{-1} will be the resiprocals of P, which means that the t–values will be unchanged:

$$t(\hat{\beta}_i^*) = \hat{\beta}_i^* \cdot [\hat{var}(\hat{\beta}_i^*)]^{-\frac{1}{2}} = (a_{ii})^{-1}\hat{\beta}_i \cdot [(a_{ii})^{-2} \cdot \hat{var}(\hat{\beta}_i)]^{-\frac{1}{2}} = t(\hat{\beta}_i).$$

[4] See also Spanos (1986, p. 386).

[5] This was suggested to me by David F. Hendry.

[6] See the discussion in Belsley et al. (1980, pp. 177 – 183).

[7] A simular example is analyzed by Bacon(1988, pp. 311) and Belsley et al. (1980, pp. 177 – 180).

[8] Contrast this derivation with the erroneous conclusion reached by Bacon (1988, p. 311).

[9] Introductions to integration and cointegration can be found in Hendry (1986) and Granger (1986).

[10] This is the nonlinear least squares estimator investegated by Stock (1987). An independent derivation can be found in Bårdsen (1989) together with the variance formula given below. See also Johansen and Juselius (1990) and Johansen (1990).

[11] The notation $I(1)$ means "integrated of order 1". See footnote 9.

[12] Any $\kappa(x) \geq 30$ is considered "harmful" by Belsley et al. (1980).

CHAPTER 4

FINDING THE RIGHT NOMINAL ANCHOR: THE COINTEGRATION OF MONEY, CREDIT AND NOMINAL INCOME IN NORWAY.

WITH JAN TORE KLOVLAND.

FINDING THE RIGHT NOMINAL ANCHOR: THE COINTEGRATION OF MONEY, CREDIT AND NOMINAL INCOME IN NORWAY*

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ABSTRACT

Using cointegration techniques this paper presents an empirical analysis of the relationship between nominal GDP or domestic expenditure on the one hand and money and credit variables on the other. The main findings are: (1) In the period from 1966 to 1983 there is a relatively firm relationship between the nominal income variables and credit, which subsequently breaks down completely during the ensuing period of credit market deregulation; (2) Nominal income and the broad money stock, M2, are cointegrated throughout the period 1966 to 1989 within a model augmented by the own rate of interest on M2 and a bond yield. Thus M2, adjusted for the effects of interest rates affecting the demand for money, seems to provide the most reliable long–run anchor for nominal income in Norway in the period considered here.

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1. INTRODUCTION

Which financial quantity variable – money or credit – does provide the most reliable information about the ultimate effects of monetary policy on nominal income? Is it credit, which for decades has been the monetary authorities' main target variable in Norway, or is 'the quantity of money ... "all that matters" for the long–run determination of nominal income'?[1] This is a crucial question for monetary authorities everywhere, irrespective of the design of financial markets or the choice of exchange rate system.

This issue has always been regarded as a fundamental one in monetary theory. Following the significant changes in the conduct of monetary policy in the 1980s, empirical research on this issue has also been intensified in recent years, especially in the United States. Before reviewing briefly some relevant theoretical and empirical aspects of this literature (section 2), we add some further remarks on the specific issues addressed in this paper and their relation to the peculiar institutional features of financial markets and policy formulation in Norway.

The empirical analysis undertaken here is not based on an assumption that either money or credit should serve as a short–run target for monetary authorities in Norway in a rigid sense. Neither money nor credit bears a sufficiently tight relationship with nominal income in the short run to warrant targeting these financial aggregates on a monthly or maybe even quarterly basis. Our concern is to examine which financial quantity variable performs best as a 'policy guide' or 'information variable'[2] with respect to the desired long run path of nominal income.

Contrary to contemporary official statements it appears in retrospect that neither money nor credit aggregates have been taken seriously as intermediate targets of monetary policy in Norway until very recently. In official policy statements sectoral credit aggregates, particularly bank credit, used to play a significant role. It became increasingly clear during the past two decades, however, that the instruments used to control credit growth were grossly inadequate, as realized growth rates persistently surpassed the target levels by whopping figures.[3] The ineffectiveness of monetary

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[1] Friedman and Schwartz (1982, p. 57).

[2] See B.Friedman (1983a, 1988a) for a discussion of the role of money and credit as information variables.

[3] Between 1967 and 1987 there was an overshooting of original targets for credit growth as

policy largely stemmed from the overriding goal of interest rate smoothing, which in practice implied keeping nominal interest rates lower than the market–clearing level. Thus in practice monetary policy was conducted without a financial quantity variable to anchor the path of nominal income.

With the exchange rate taking priority over nominal interest rates as from the 1986 devaluation, it may be argued that some form of nominal anchor now has been imposed on the economy. However, purchasing power parity is only assumed to reflect nominal disturbances, and imperfectly so in anything but the long run. Real shocks (to for example productivity or terms of trade) may affect the real exchange rate perrmanently, being of particular importance to the resource based Norwegian economy.[4] Consequently, pegging the exchange rate is no panacea for achieving the desired long–run course of nominal income. Finding a financial aggregate which is closely linked with nominal income is still an important issue.

2. THE TRANSMISSION OF MONETARY IMPULSES

The proposition that changes in the stock of money has a long run effect upon nominal income is hardly controversial, although there is still little consensus concerning which of David Hume's (1752) 'one hundred canals' actually carry the bulk of monetary impulses. To cite just one example from the vast literature on the macroeconomics of monetary influences on nominal income, none is more appropriate than Milton Friedman's (1956) restatement of the quantity theory of money. In this approach the importance of money for the course of nominal income follows from the existence of a stable and well–defined demand–for–money function coupled with a supply function depending on at least some important factors which do not affect the demand side as well. While most other macroeconomic models yield qualitatively the same results in the long run, it is well known that there are differences of opinion as to the stability of this relationship. The really controversial issue regards the short run, whether the business cycle is 'a dance of the dollar', as Irving Fisher (1923) and his successors

formulated in annual National Budgets in 20 out of 21 years, cf. the Report of the committee on monetary policy (NOU 1989:1, *Penger og kreditt i en omstillingstid, Oslo, 1989*), p. 59.

[4] For an evaluation of the empirical evidence and limitations of purchasing power parity, see Dornbusch (1987). Edison and Klovland (1987) found that the effects of real factors were quite important in testing for PPP relationships between Norway and the United Kingdom over the past century.

maintained. This issue is, however, beyond the scope of the present paper, in which the main focus is on long–run relationships.

The proposition that credit may play a role in the monetary transmission mechanism is also widely recognized, but again this is more a question of relative importance rather than either money or credit.[5] In an economy characterized by highly segmented credit markets and enforced rationing of intermediated credit to large borrower groups, as was more or less the case in Norway until the end of 1983, there is, of course, no lack of arguments for linking credit with nominal income or expenditure. In addition, recent theoretical developments have shown that, even in an economy without disequilibrium credit rationing, there are several routes through which credit markets interfere with the monetary transmission mechanism.[6] In the model developed by Stiglitz and Weiss (1981) the loan supply curve may bend backwards due to informational asymmetries, causing a form of credit rationing by banks. Bernanke (1983) and Blinder and Stiglitz (1983) stressed the special role played by bank credit in an economy where important sectors of borrowers do not have easy access to non–intermediated forms of credit. Disruptions of financial flows to such sectors are highlighted in periods such as the Great Depression of the 1930s, when increased riskiness of loans and shrinkage of borrowers' collateral caused by worsening of their balance sheet position made these sectors highly dependent on the sustained credit creation ability of the banking system. But even in more normal periods many economies exhibit institutional features of credit market segmentation which enhance the role of bank credit.

Bernanke and Blinder (1988) have developed a very simple model of aggregate demand which in general allows for both money and credit. There is a separate role for the credit market if bank loans and other forms of customer–market credit are not considered as perfect substitutes for auction–market credit (or bonds) by either borrowers or lenders. Similarly, there is a role for money as long as money and bonds are not perfect substitutes. The fuzziness of the distinction between money and bonds has been a preoccupation in much of Tobin's work,[7] but whether the process of

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[5] Friedman and Schwartz (1963, p.32) are inclined to 'casting the "credit" market as one of the supporting players rather than a star performer'. In the macroeconomic models summarized in Brunner and Meltzer (1988) the transmission of monetary impulses to output depends on the operation and properties of the credit market.

[6] Gertler (1988) contains a survey of the literature on the links between the financial system and aggregate economic behaviour.

[7] Cf. Tobin (1969, p.334): 'The essential characteristic — the only distinction of money from securities that matters...— is that the interest rate on money is exogenously fixed by law or convention, while the rate of return on securities is endogenous, market determined'.

financial innovation eventually creates new money substitutes that completely blur the distinction between the two types of assets is in the end an empirical question.

In this framework the crucial condition which determines whether money or credit is the variable to target is the relative magnitude of money–demand and credit–demand shocks. We are thus led to examine the relative stability of the long run demand function for money and for credit.

The apparent breakdown in the early 1980s of the demand function for M1, the money stock definition monitored most closely by the monetary authorities in the United States, has led some economists to suggest that credit aggregates may bear a more stable relationship to nominal income than does money.[8] On balance, though, the empirical evidence from the US, where most of the studies have been made, is mixed.

Bernanke (1988, p.11), drawing on the results in Bernanke and Blinder (1988), concluded that 'credit demand has been more stable than money demand since the deregulation process began in 1980'.[9] On the other hand, the cointegration tests presented in B. Friedman (1988a) show that neither monetary aggregates (monetary base, M1, M2) nor credit were cointegrated with nominal income in samples ending in 1987.[10] Indeed, Benjamin Friedman (1988b, p.63), who was one of the leading proponents of targeting credit (in addition to money) in the early 1980s concluded that 'the movement of credit during the post–1982 period bore no more relation to income or prices than did any of the monetary aggregates'. Moreover, the results in Mehra (1989), who used data from 1952 through 1988, indicate that M2, nominal GNP and the commercial paper rate form a cointegrating vector. Thus under less stringent conditions, allowing the money stock to adjust to interest rate movements, the broad monetary aggregate may still seem to be a candidate for the role as a policy guide.

The evidence from the United States so far thus gives little or no indication as to whether money or credit bears the most stable relationship to nominal income. The evidence for the United Kingdom surveyed by Goodhart (1989) gives a similar impression. We therefore proceed to the empirical analysis on Norwegian data with no firm preconceptions, either on theoretical or empirical grounds, as to the most likely

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[8] See e.g. B. Friedman (1983a,1983b).

[9] Similar conclusions can be found in Fackler (1988) and Lown (1988).

[10] In the paper introducing the cointegration approach Engle and Granger (1987) found that no monetary aggregate, except possibly M2, was cointegrated with nominal GNP.

outcome. It should also be noted that the organization of financial markets and the design of monetary policy in Norway differ quite much from these countries, particularly with respect to the attention given to credit growth by the authorities. The interesting question is then whether this fact may tip the balance in favour of the credit aggregates.

3. THE DATA AND THE DEREGULATION OF CREDIT MARKETS IN NORWAY

In the empirical analysis on Norwegian quarterly data we report the outcome of testing for cointegration between different money or credit aggregates on the one hand and income or expenditure and interest rates on the other. We are focusing on four financial quantity variables:

M1 = narrow money stock
M2 = broad money stock
KA = total domestic credit
KB = domestic bank credit.

The main difference between M1 and M2 is the inclusion of time and saving deposits in the latter. KA is a comprehensive measure of domestic credit extended to the private sector and local governments from all private and public banks and financial intermediaries.[11] KB is limited to ordinary loans from commercial and savings banks only, being included because of the long–standing preoccupation with bank credit by the monetary authorities. All data are seasonally unadjusted.[12] Further details on the data can be found in Appendix 1.

Most attention will be given to the broad aggregates, M2 and KA, which are the variables now regularly monitored by the monetary authorities. Figure 1 shows the four–quarter growth rates of these two variables over the period 1967 Q1 to 1989 Q1. Figure 2 presents the same curves for M1 and KB. Our main concern here is to examine the long run behaviour of these series in relation to nominal income or expenditure, but a comparison of the short run movements is of some interest in light

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[11] See BØ (1988) for a description of this aggregate. The data used before 1983 reflect a slightly narrower definition due to data availability. See Appendix 1 for further details.

[12] M2 and KA have been adjusted for distortions to the published banking statistics figures in 1986 and 1987. Such adjustments were of less relevance to M1, but here substantial changes in the definition of demand deposits employed in the banking statistics have made this series suspect after 1986.

of the deregulation of financial markets and the significant changes in monetary policy in the 1980s.

Table 1 gives a summary statement of some main events in the process of deregulation of financial markets in Norway. At the end of 1983 all regulations specified here were in operation. The only important form of intermediated credit not subject to quantitative restrictions was credit granted by loan associations to large real capital investment projects in manufacturing industries. The developments in 1984 and 1985 implied a drastic relaxation of credit rationing with regard to borrowers who did not have access to auction–market credit, households and small businesses in particular. The surge in credit growth beginning about 1984 is clearly visible in the growth rates of KA and KB in Figures 1 and 2. The temporary reversal to direct credit controls in 1986 and part of 1987 turned out not to be particularly effective. The financial institutions were to a large extent able to channel credit flows to their customers through new financial instruments, evading the existing regulations. A major factor which finally helped to bring an end to the credit boom was probably the move towards a more flexible interest rate policy in December 1986.[13]

A comparison of growth rates of M2 and KA as shown in Figure 1 reveals that prior to 1983 these two financial aggregates expanded at a similar rate in the long run, although M2 growth was somewhat more volatile in the short run. As from 1983 the growth rates began to differ markedly. Credit growth largely outstripped the rate of increase of the money stock. This came about as banks, in particular, were able to fund their loan expansion from sources other than deposit liabilities, chiefly by being given the opportunity to borrow from the central bank on a large scale and attracting funds from abroad. Accordingly, M2 and KA bear roughly the same long–run relationship to nominal income until 1983; thereafter, the trends are diverging.

These empirical relationships are highlighted in Figures 3 to 6, which show the (logarithm of) the ratio of the four financial aggregates to nominal expenditure (see definition below). The solid lines show actual values, while the dotted lines represent four–quarter moving averages. All ratios hover around a roughly constant level up to 1983, exhibiting relatively mild cyclical fluctuations. Thereafter the credit–expenditure ratios start rising in an unprecedented manner, signalizing a break in the previously relatively stable relationships. This contrasts with the seemingly

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[13] Steffensen and Steigum (1990) and NOU 1989:1 contain an analysis of the financial deregulation process.

Table 1. Credit market regulations in Norway, 1983 – 1989.

Type of regulation	Dates when abolished (A) or reintroduced (R)				
	BANKS	FINANCE COMP.	LOAN ASS.	LIFE INSUR.	NON–LIFE INSUR.
Direct loan controls[1]	A1984Q1 R1986Q1 A1987Q3	A1988Q3	A1988Q3		A1988Q3
Primary reserve req.	A1987Q2	A1987Q3		A1987Q2	
Bond investment quota[2]	A1984Q1			A1985Q1	
Loan guarantee limits	A1984Q3 R1986Q1 A1988Q3	A1984Q3 R1986Q1 A1988Q3	A1984Q3 R1986Q1 A1988Q3	A1984Q3 R1986Q1 A1988Q3	A1984Q3 R1986Q1 A1988Q3
Max int. rate on loans	A1985Q3			A1985Q3	

1) Credit extended by the finance companies in the form of factoring and leasing contracts was exempted as from 1984Q3. The regulations concerning mortgage loan associations only applied to loans to households and selected industries.
2) The dates refer to the point in time when the required percentage of growth was set equal to zero, viz. net additions to the bond portfolio were no longer required. The regulation was completely removed in 1985 Q1 for banks and in 1985 Q3 for life insurance companies.
General notes. If no date is specified, no regulation applies. In all other cases the regulation was in operation at the end of 1983. The information is compiled from *Annual reports of the Norges Bank* 1984 — 1988 and various issues of *Penger og Kreditt* in the same period.

normal behaviour of the ratio of M2 to expenditure. It thus appears that the process of deregulation and rapid financial innovations which gained momentum around 1983/1984 fundamentally changed the relationships between credit and income. The role of credit as a useful information variable can still be rescued, however, if there are other variables which can account for this changing relationship.

In order to simplify the exposition we will be using 'nominal income' rather vaguely when referring to the aggregate nominal measure of economic activity (production or expenditure) which is assumed to be the variable on which the authorities are focusing. In the empirical analysis below we employ whichever of the following variables yielding the closest relationship with the financial aggregates:

Y = nominal gross domestic product

X = nominal gross domestic expenditure, excluding investment in oil and gas, pipeline transport, ships and oil platforms.

The petroleum and shipping sectors are excluded from X since prices and economic activity generated in these sectors are determined by forces largely exogenous to domestic monetary policy. We include both a production and an expenditure measure since both are of concern to the authorities' policy goals; Y has a direct bearing on internal balance (production and employment) while X, being a measure of aggregate demand, is the variable most directly influenced by monetary and fiscal policy. The course of these variables may differ to some extent in the short and intermediate run in an open economy – and more so in Norway than in most other countries – but the choice between them should matter less in the analysis of long run behaviour. Several other income variables were examined, including GDP minus oil and shipping, but we have chosen to report the results only from the specifications that proved to be most stable empirically.

Finally, the interest rates employed in the money and credit equations are:

RD1 = average rate of interest on demand deposits

RD2 = average rate of interest on time and savings deposits

RL = yield on long–term bonds issued by private mortgage loan associations

RB = average interest rate on bank loans.

4. THE INDIVIDUAL TIME SERIES PROPERTIES

4.1 *Motivation*

The cointegration technique developed by Granger (1986) and Engle and Granger (1987) lends itself in a natural way to assessing the robustness of the long–run relationships between nominal income (GDP or domestic expenditure) on the one hand and money and credit on the other. If no stable long–run relationships exist between two variables Y and M, the residuals ψ_t from the cointegrating regression

$$(1) \qquad \psi_t = Y_t - \alpha M_t$$

(where α_t is the estimated cointegrating parameter) will tend to drift apart over time. The results from applying such tests are reported in section 5.1. An alternative procedure proposed by Johansen (1988) is employed in section 5.2.

Before testing for cointegration can be performed it must be verified that the variables involved are integrated of the same order. A variable Z is said to be integrated of order d $\left[Z \sim I(d)\right]$ if it has a stationary, invertible non–deterministic ARMA representation after differencing d times. Accordingly, we first proceed to an examination of this aspect of the time series, paying special attention to the seasonality of the data used here.

4.2 *Testing for seasonal unit roots*

Most macroeconomic time series are found to be integrated of order one,[14] i.e. there is a unit root in the autoregressive representation of the levels of the variables. Testing for unit roots with data that are appropriately seasonally adjusted, or in cases where no seasonality is present, is conducted within the framework developed by Dickey and Fuller (1979, 1981).[15] This type of tests assumes that the root of interest is at the zero or annual frequency and that there are no other unit roots at other (seasonal) frequencies.

This assumption is no longer a priori plausible when the sample consists of seasonally unadjusted data. In a recent paper Hylleberg, Engle, Granger and Yoo (1990), hereafter referred to as HEGY, have proposed a test for unit roots in a univariate time series which explicitly tests for roots at seasonal frequencies as well.[16]

This procedure may be briefly outlined as follows: The time series Z_t is assumed to be generated by a general autoregression

$$\varphi(L)Z_t = \varepsilon_t$$

[14] Nelson and Plosser (1982), Schwert (1987).

[15] Extensions of the Dickey–Fuller tests to deal with various forms of non–white residuals or structural change have been suggested by Said and Dickey (1985), Phillips (1987) and Perron (1989).

[16] Osborn *et al.* (1988) present a comparison of the hypotheses embedded in different unit root tests i a seasonal framework, also suggesting a new test which may distinguish between seasonal and non–seasonal unit roots.

where $\varphi(L)$ is a polynomial in the lag operator L defined by $L^j Z_t = Z_{t-j}$ (j=1,2,...) and ε_t is a serially uncorrelated stochastic variable with mean zero and constant variance. HEGY show that testing for unit roots at all frequencies with quarterly data can be derived from the ordinary least squares regression of $\Delta_4 Z_t = Z_t - Z_{t-4}$ on lagged values of Z_t and a deterministic part μ_t [intercept (I), seasonal dummies (SD), linear time trend (TR)]

$$\Delta_4 Z_t = \pi_1 Y_{1,t-1} + \pi_2 Y_{2,t-1} + \pi_3 Y_{3,t-2} + \pi_4 Y_{3,t-1} + \sum_{i=1}^{p} \gamma_i \Delta_4 Z_{t-i} + \mu_t + \varepsilon_t$$

where

$$Y_{1t} = (1 + L + L^2 + L^3)Z_t$$
$$Y_{2t} = -(1 - L + L^2 - L^3)Z_t$$
$$Y_{3t} = -(1 - L^2)Z_t$$

There will be no seasonal unit roots if π_2 (bi–annual cycle) and either π_3 and π_4 (annual cycle) are different from zero. And $\pi_1 \neq 0$ corresponds to no unit root at the long–run or zero frequency. Tests on individual π's are based on *t*–tests. The joint test for π_3 and π_4 is an *F*–test whose critical values are given in HEGY. The specification of the deterministic component, μ_t, may include none, some or all of the variables I, SD and TR defined above, depending on which alternative is considered most appropriate.

Table 2 reports the results from applying the HEGY seasonal unit root tests to quarterly data on money, credit, nominal income variables as well as interest rates. The sample starts in 1967 Q2 or later, depending on the maximum lag p on Z_{t-i} that is required to whiten the residuals. The last observation used is 1989 Q1, except for M1, for which data are not available after 1986 Q4. All variables, except interest rates, are in logs. Results are reported for five different combinations of I, SD and TR. In view of the seemingly strong seasonality exhibited by the money, credit and income series in Figures 1 and 2 most attention is given to the equations where the seasonal dummies are included in the case of these variables.

For each variable the joint hypothesis that $\pi_3 = \pi_4 = 0$ is rejected, in most cases individual tests for either $\pi_3 = 0$ or $\pi_4 = 0$ are also rejected. With the exception of (the logarithm of) KA (LKA) the hypothesis $\pi_2 = 0$ is also rejected. Accordingly, with the possible exception of KA, there does not appear to be unit roots at the seasonal frequencies.

Turning now to testing for a unit root at the long–run frequency, using the t_1 statistic, all variables are found to be $I(1)$, i. e. integrated of order one.

5 TESTING FOR COINTEGRATION BETWEEN NOMINAL INCOME AND MONEY OR CREDIT

5.1 *The Engle–Granger cointegrating regression*

When we consider a p–component vector of series $x_t \sim I(d)$, a linear combination of these series

$$\beta' x_t = z_t \tag{2}$$

will also in general be integrated of order d. If however $z_t \sim I(d-b)$, $b > 0$, then x_t is said to be cointegrated of order (d, b): $x_t \sim CI(d, b)$, still following Engle and Granger (1987). Note that the number of cointegrating vectors are given by the number of columns, or the rank, $0 \leq r < p$ of β.

Stock (1987) established the important result that if the series are cointegrated and $r = 1$, a super–consistent estimate of β is provided by the OLS–regression of (2), choosing x_{1t}, say, as the dependent variable. This is the method advocated by Engle and Granger (1987).

The success of this approach depends upon all variables being stationary at the seasonal frequencies, as the estimates might otherwise not be unique, as argued in Hylleberg *et al.* (1988). But according to the tests above, cointegrating regression should be a valid procedure for most of the variables in the present data set. It follows naturally that testing for cointegration in this original set–up implies establishing whether the residuals from the cointegrating regression represent a stationary series. This is easily done by applying the standard Dickey–Fuller (DF and ADF) and

Sargan–Bhargava (CRDW) tests.[17]

It might be conjectured that there can be some problems with this approach if the data contain deterministic components, as in the present case. First, since the critical values of Engle and Granger (1987) and Engle and Yoo (1987) are derived under the assumptions of no deterministic components, these values will not be appropriate if any deterministic components are not corrected for in using the tests. It also follows that the CRDW and DF tests are more likely to exhibit upward bias than the ADF test, since the latter will correct for the induced autoregression in the residuals through the augmentation. Secondly, if any deterministic terms do not cancel out, it is to be expected that this induces a bias in the estimate of the cointegrating vector from that part of the deterministic components that is not picked up by any corresponding representation in the regression. The problem hinges on the fact that such effects will not 'go away' by expanding the sample size.

The natural solution to these problems would be to correct for any deterministic components present, which in this context might be to remove the seasonal means. This is the solution adopted in testing for unit roots in individual series, not only by Hylleberg *et al.* (1988), but also by Dickey, Hasza and Fuller (1984), Dickey, Bell and Miller (1986) and Osborn *et al.* (1988). Following Lovell (1963), theorem 4.1, this is equivalent to include seasonal dummies in the cointegrating regression. But what if no seasonal effects are present? A small simulation study indicated that in this case the critical values of the DF–statistic will be smaller than if seasonal dummies had been excluded. A conservative strategy should therefore be to adopt the usual critical values and include seasonal dummies in the regression.

5.1.1 *The models*

Our testing procedure for cointegration between (the logs of) nominal income and financial aggregates is in three steps of increasing model generality.

(a) *The constant–velocity model.* Here we test whether money (or credit) is cointegrated with nominal income and with cointegrating parameter $\alpha_1 = 1$ in the model

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17 See Engle and Granger (1987) for details.

$$(3) \quad LM_t = \alpha_1 LY_t + \mu_t + \psi_{1t}$$

where as before μ_t is the deterministic part.[18] The case where α_1 is unity is of particular importance since in this case (with no time trend) nominal income and money grow in exact proportion over time, i.e. the long–run income elasticity of money demand is equal to one. Accordingly, to achieve an x per cent growth in nominal income the trend growth of money should also be x per cent.

(b) *The simple velocity–drift model.* This model is represented by (2) but with the cointegrating parameter α_1 freely estimated. Thus if α_1 is less than unity the authorities must allow for an upward drift in velocity over time; to achieve an x per cent growth of nominal income money must grow by less than x per cent.

(c) *The interest rate augmented model.* In this model a vector of interest rates R_t, assumed to affect money (credit) demand or supply, is added to (3),

$$(4) \quad LM_t = \alpha_2 LY_t + \beta R_t + \mu_t + \psi_{2t}.$$

If LM, LY and R form a cointegrating vector, money (or credit) would be useful for monetary authorities as an information variable with respect to nominal income after being adjusted for the influence of R.

In the case of the money stock equations the interest rate vector R consists of the own rate (the bank deposit rates RD1 or RD2), which is assumed to take on a positive coefficient, and the bond yield, RL, which represents the rate of return on substitute assets. These money demand equations are broadly consistent with recent results from dynamic modelling of the demand for M1 and M2 on the same data set.[19]

The specification of credit demand equations is less obvious, since there is little or no recent empirical evidence on such equations with Norwegian data. Here we adopt a simple loan demand and supply model, similar to the one employed by King (1986), in

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[18] We have chosen to normalise on the financial aggregates rather than on nominal income since this facilitates a direct comparison with standard money demand functions.

[19] See Bårdsen (1992) and Klovland (1990) for the modelling of M1 and M2, respectively. The mone market rate, RS, was never of any importance in the cointegration tests, and is therefore not included in the analysis. However, this variable played a useful role in the dynamic modelling of the demand–for–money functions.

which the bank lending rate, RB, and the yield on mortgage loan association bonds, RL, are candidates in the demand and (possibly also) supply functions for credit. The cointegrating regressions must be viewed as reduced–form equations, which makes the signs of the RB and RL coefficients theoretically indeterminate.

5.1.2 *The empirical results*[20]

(a) *The constant–velocity model.* When α_1 is set equal to one a priori in (2), the HEGY procedure used in section 4.1 can be used to test for cointegration as well, since this restriction is equivalent to testing whether the variable (LM–LY) is $I(0)$ or not. The outcome of such tests is reported in Table 3. A separate test is conducted on the pre–deregulation sample ending in 1983 Q4. It turned out that the income variables that performed 'best', in the sense of being nearest to forming a cointegrating vector, was nominal expenditure, X, for M1 and M2 and nominal GDP, denoted by Y, in the case of KA and KB. Only these combinations of variables are reported here; other variants were of no particular interest.

It follows from the results in Table 2 that all income and financial variables have a unit root at the same (zero) frequency; hence cointegration between these variables is possible. On the other hand, since none of the variables, except possibly KA at the bi–annual cycle, has a unit root at the seasonal frequencies, these variables cannot be seasonally cointegrated.

There are only two cases where there is some evidence of cointegration in the period up to 1983 Q4. The strongest evidence is for cointegration between Y and KA. In this equation a unit root at the zero frequency is rejected in favour of stationarity at the 1 per cent significance level. A similar conclusion seems to be warranted at the 5 per cent level in the case of X and M1, after removal of deterministic seasonality in the auxiliary regression. The most important result, however, is that over the full sample there is no evidence of cointegration between any of these variables. In conclusion, whereas the authorities may have had some confidence in monitoring M1 with a view to assessing the long–run movements of nominal domestic expenditure[21] and likewise

[20] The results in this section were obtained using the recursive least squares option of PC–GIVE, version 6.0.

[21] Note once again that the performance of M1 is restricted to a sample ending in 1986 Q4.

monitoring KA in the case of nominal GDP prior to 1984,[22] these results indicate that the foundation of such guidelines subsequently disappeared. Accordingly, the constant–velocity model provides no role for either money or credit as information variables.

(*b*) *The simple velocity–drift model.* Relaxing the restriction $\alpha_1 = 1$ in (2) implies that the HEGY procedure can no longer be used. Instead we report the Durbin–Watson statistic from the cointegrating regression (2), CRDW, and the augmented Dickey–Fuller statistic (ADF). Including the seasonal dummy variables S1, S2, S3 in the cointegrating regression is natural considering the strong evidence of deterministic seasonality in Tables 4 and 5 (also compare the reduction in residual standard error between 1 and 2, 3 and 4 in these tables). However, the critical values of these statistics derived by Sargan and Bhargava (1983) and Engle and Granger (1987) are not tabulated for equations containing deterministic seasonals. Noting that the values of the test statistics are always lower with the seasonal dummies included, a conservative procedure is to rely primarily on this specification, applying the ordinary critical values in this case as well.

The results given in Table 4 show that in the sample ending in 1983 Q4 the estimated values of the cointegrating parameter α_1 are only slightly above unity in the case of M1, M2 and KA. Hence the difference from the constant–velocity model appears to be rather small. But, in contrast to that model, cointegration is no longer rejected at the 1 per cent level for M1, KA and KB (in the latter case the estimate of α_1 is 0.94). Consequently, all variables, except M2, seem to perform well before 1984.

Extending the sample to 1989 Q1 results in an upward drift in the estimates of α_1. Using recursive least squares, the time path of the parameter estimates can be traced, as shown in Figures 8, 10, 12 and 14. In the case of the credit variables there is a dramatic deterioration in the goodness of fit – the residual standard error increases by a factor of 3 in the case of KA, while it is more than 4 times higher in the KB equation when the sample is extended from 1983 Q4 to 1989 Q1. Figures 11 and 13 visualize the complete breakdown after 1983 of the previously relatively firm relationships between GDP and credit. It stands to reason that neither KA nor KB can be cointegrated with Y in the full sample, a conclusion which is evident from Table 4.

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[22] On the other hand, KA does not appear to be cointegrated with Q (GDP minus oil and shipping) before 1984. Q is probably the nominal income variable most closely monitored by the authorities in Norway.

Figure 9 shows that the M2 equation is only slightly affected by extending the sample, but equation B4 of Table 4 shows that it still does not pass the cointegration test. The cointegration between M1 and X is still accepted at the 5 per cent level on data up to 1986 Q4.

In conclusion, generalizing the constant–velocity model to allow for a possible drift in income–money (credit) ratios over time does not produce a cointegrating vector that can withstand the deluge of credit market deregulation after 1983. The credit equations collapsed completely, whereas the M2 equation turned out to be far more robust without passing the formal tests.

(*c*) *The interest rate augmented model.* The estimation results are given in Table 5, one–step residuals and recursive estimates of α_2 in Figures 15 to 22. The major difference between the results from the augmented model compared with the previous simple models is in the money stock equations, particularly M2. Augmenting the model with a view to reflecting a standard money demand specification yields a cointegrating vector consisting of LX, LM2, RD2 and RL. This model passes the test for cointegration at the 1 per cent level over the full sample. A similar result is obtained with M1 in the sample ending in 1986. The signs and magnitudes of the coefficient estimates are consistent with the range of estimates established in the money demand literature. Figures 15 through 18 show that the one–step residuals from these equations are relatively satisfactory even in the difficult post–1983 period, although some instability is discernible in the parameter estimates.[23]

The augmented credit equations again break down completely after 1983. This is clear from the test statistics in Table 5, vividly illustrated by the exploding residual errors in Figures 19 and 21 and the wave–like path of coefficient estimates in Figures 20 and 22. Thus, in contrast to the money stock equations, augmenting the model in order to take into account the effect of interest rate movements on credit growth does not lead to a cointegrating vector.

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[23] In Klovland (1990) it is found that a wealth–constrained money demand model is superior to the specification used here in terms of parameter stability and predictive performance.

5.2 *The Johansen procedure*

5.2.1 *Motivation*

Even in a small problem like ours it is quite possible that several long–run relationships exist. The problem of establishing the number of cointegrating vectors in a given set of variables has been solved by Johansen (1988).[24] Although this apparatus is quite impressive in terms of its complexity, the intuition behind the approach is rather simple.

Briefly described, the method relies upon the concept of canonical correlations from the theory of multivariate analysis. The data are divided into a differenced and a levels part. Under the assumption of *I(1)* processes the differenced data are stationary. The technique of canonical correlations is used to find linear combinations of the data in levels which are as highly correlated as possible with the differences. It follows that these linear combinations must be stationary, or cointegrated.

Another appealing aspect of the Johansen approach is its completeness in the sense that it provides tests of linear restrictions on the cointegrating vectors as well as estimates of its elements and information about its rank. Finally, this method also takes account of the short–run dynamics and any simultaneity in the estimation process.

5.2.2 *The procedure*

To be a bit more specific, the assumption is that x_t is generated by

$$(5) \qquad x_t = \Sigma_{i=1}^{k} \pi_i x_{t-i} + \mu i + \Phi D_t + \epsilon_t,$$

rewritten as

$$(6) \qquad \Delta x_t = \Sigma_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} - \pi x_{t-k} + \mu i + \Phi D_t + \epsilon_t,\ \epsilon_t \sim NIID(0, \Omega),$$

[24] The procedure is further developed in Johansen and Juselius (1990), which also contains some applications.

with $\Gamma_m = -I + \Sigma_{i=1}^{m}\pi_i$, $m = 1,...,k-1$; $\pi = I - \Sigma_{i=1}^{k}\pi_i = \alpha\beta'$ and i and D_t being intercept and seasonal dummies, respectively.

Equation (6) is the interim multiplier representation of (5).[25]

The estimate of β is then found in two steps. The first step consists of correcting the differences and the levels for the short–run and the deterministic components. This amounts to running the regressions

(7)
$$\begin{Bmatrix} \Delta x_t = \Sigma_{i=1}^{k-1}\Gamma_i\Delta x_{t-i} + \mu i + \Phi D_t + r_{0t} \\ x_{t-k} = \Sigma_{i=1}^{k-1}\Gamma_i\Delta x_{t-i} + \mu i + \Phi D_t + r_{kt} \end{Bmatrix}.$$

Next, the covariance matrix S of r_{0t} and r_{kt} is partitioned as

(8)
$$S = \begin{bmatrix} S_{00} & S_{0k} \\ S_{k0} & S_{kk} \end{bmatrix}.$$

A result from multivariate analysis then states that the r linear combinations $\hat{\beta}' r_{kk}$ maximizing the correlation with r_{00} are given by the r largest of the eigenvectors $\hat{\beta} = [\hat{\beta}_1 ... \hat{\beta}_r]$ corresponding to the p eigenvalues $\hat{\lambda}_1 > ... > \hat{\lambda}_p$ from solving[26]

(9)
$$|\lambda S_{kk} - S_{k0}S_{00}^{-1}S_{0k}| = 0.$$

In the present setting the eigenvectors are normalized to $\hat{\beta}' S_{kk}\hat{\beta} = I$.

The result also follows from obtaining estimates of α and Ω from the regression of r_{0t} on $\beta' r_{kt}$, which gives the concentrated likelihood function proportional to

(10)
$$L(\beta) = |\hat{\Omega}(\beta)|^{-T/2} = |s_{00} - s_{0k}\beta(\beta' s_{kk}\beta)^{-1}\beta' s_{k0}|^{-T/2}$$

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[25] See Hylleberg and Mizon (1989) for an extensive survey of different representations of cointegrated systems, including the interim multiplier form.

[26] A good introduction can be found in Krzanowski (1988, pp. 432 — 445).

or

$$(11) \qquad L^{-2/T}(\beta) = |s_{00}|\,|\beta'(s_{kk} - s_{k0}s_{00}^{-1}s_{0k})\beta|/|\beta' s_{kk}\beta|.$$

Equation (11) is minimized by the choice of $\hat{\beta} = [\hat{\beta}_1 ... \hat{\beta}_r]$ from (9) with the solution

$$(12) \qquad (L_{max})^{-2/T} = |s_{00}| \prod_{i=1}^{r} (1 - \lambda_i).$$

Since this result is derived under the hypothesis of $\pi = \alpha\beta'$, and the unconstrained function would be (12) with $r = p$, the likelihood ratio test for 'at most r cointegrating vectors', becomes

$$(13) \qquad -2ln(Q\,;\, r \mid p) = -T\,\Sigma_{i=r+1}^{p}(1 - \hat{\lambda}_i),$$

and a test of the relevance of column $r+1$ in β is obtained by computing

$$(14) \qquad -2ln(Q\,;\, r \mid r+1) = -T\,(1 - \hat{\lambda}_{r+1}).$$

Equation (13) is what is referred to as the 'trace' test in the tables, while (14) is denoted 'λ_{max}'.[27]

5.2.3 *The results*

The results of applying the Johansen procedure to the present information set can be seen in Tables 6 to 9.[28] In each case panel A refers to the sample period ending in 1983 Q4, while panel B reports the statistics obtained when the sample is extended to 1986 Q4 for LM1 and to 1989 Q1 for LM2, LKA and LKB.

The main result is that according to these tests stationary relationships exist between money, income and interest rates as well as credit, income, and interest rates – also after the credit market liberalization. Taken as such these findings are at odds with

[27]Johansen (1988, 1989) and Johansen and Juselius (1989) give further details on these tests.

[28] The results were obtained using a RATS—program written by SØren Johansen, Katarina Juselius and Henrik Hansen, which was kindly made available to us by Kenneth F. Wallis.

the conclusions obtained in the previous section.

But in our setting the issue of parameter stability is also crucial. Taking a closer look at the estimated cointegrating vectors, a familiar distinction between the different models readily appears. While the money models appear relatively unaffected by the credit deregulation, the estimates of the cointegrating vectors go 'all over the place' in the case of the credit equations.

This is especially evident for the parameters of the income variables. The long–run elasticities of income in the money models show only mild fluctuations between the two samples, but the corresponding estimates in the credit models are much more volatile. Regarding the interest rates, more unstable estimates are obtained in general. But again the credit models fare the worst – RB even changing sign in the KA model, while the M2 model appears to be basically unaffected by the extension to the deregulation period.

For the purpose of targeting a basic requirement is a model with stable coefficients. Given this requirement, the obvious candidate is the M2 model, thus reinforcing the conclusion reached in the previous section.

6. SOME CONCLUDING REMARKS

Using cointegration techniques this paper has presented the results of an empirical analysis on Norwegian data of the long–run relationship between nominal GDP or domestic expenditure on the one hand and money and credit variables on the other. The main findings are: (1) In the period from 1966 to 1983 there is a relatively firm relationship between the nominal income variables and credit, which subsequently breaks down completely during the ensuing period of credit market deregulation; (2) Nominal income and the broad money stock, M2, are cointegrated throughout the period 1966 to 1989 within a model augmented by the own rate of interest on M2 and a bond yield. Thus M2, adjusted for the effects of interest rates on the demand for money, seems to provide the most reliable long–run information on the course of nominal income in Norway in the period considered here.

The main reason why the augmented credit models fail to pass the Engle–Granger cointegration tests over the full sample, while the stability of the M2 equation is not much affected by the credit market deregulation process, is evidently that no simple and stable model of the credit market has been uncovered yet. It may be that after a transition period, in which credit markets adjust to a more market–oriented environment, the previously firm relationships between nominal GDP and credit aggregates will reemerge. It is also conceivable that further research may be able to model the credit market in a more satisfactory manner. Until such results materialize, however, the long–run stability of relatively simple demand–for–money functions speaks in favour of using money as the anchor for the long–run course of nominal income.

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APPENDIX A1

THE DATA

DEFINITIONS AND SOURCES OF THE DATA IN TABLE A1.

(1) **M1** = Coin and currency notes, unutilised bank overdrafts and building loans and demand deposits held by the domestic non–bank public. The bank deposits included in this aggregate comprise deposits in domestic and foreign currency with domestic commercial and savings banks and postal institutions, excluding all deposits held by non–residents. The data listed here have been adjusted for changes in the definition of deposits after 1986 Q4 by imposing a growth rate between 1986 Q4 and 1987 Q1 equal to the average of the previous three years. However, due to the continued changes in the coverage of the demand deposit item in the banking statistics data on M1 after 1986 Q4 are not used in the empirical analysis. Source: Norges Bank and own calculations.

(2) **M2** = M1 plus all time and savings deposits (except savings accounts with tax allowance). The published fugures on M2 from 1984 Q2 to 1988 Q1 have been adjusted for underreporting of deposit figures in the banking statistics (cf. *Penger og Kreditt,* 1987/4, pp. 195–205) as follows (in 1000 millions of NOK):

YEAR	Q1	Q2	Q3	Q4
1984	0.0	0.3	1.0	1.3
1985	2.0	2.3	3.0	3.8
1986	8.8	15.7	22.1	22.8
1987	22.3	21.8	13.0	3.5
1988	0.2	0.0	0.0	0.0

Source: As for M1.

(3) **KA** = Loans to the non–financial private sector and municipalities from all domestic private and public banks, private finance companies, loan associations, insurance companies and pension funds. Beginning 1983 Q1 this series was spliced with the Norges Bank's credit indicator by multiplying the former series by the ratio (1.12) between the two variables in December 1982. The coverage of the credit indicator is slightly broader, also comprising market loans through private intermediaries as well as bonds and loan certificates issued by certain sectors. (See BØ (1988) for further details). Sources: Compiled from various issues of *Credit Market Statistics*; as from 1983 Q1 data obtained from the Norges Bank.

(4) **KB** = Loans to the non–financial sector and municipalities from domestic commercial and savings banks.

(5) **Y** = Nominal gross domestic product. Source: Various issues of Quarterly National Accounts.

(6) **X** = Nominal gross domestic expenditure, excluding investment in the following sectors: petroleum and natural gas, pipeline transport, oil platforms and ships. Source: As for Y.

(7) **RD1** = Average interest rate paid on banks' demand deposits. Quarterly data prior to 1978 are

obtained by linear interpolation between end–of–year figures. Between 1978 Q1 and 1985 Q3 the series is a weighted average of lowest (weight = 1/3) and highest (weight = 2/3) interest rates paid on demand deposits by commercial and savings banks. As from 1985 Q4 properly averaged data compiled by the Norges Bank. Sources: Various issues of *Credit Market Statistics* and *Penger og Kreditt*.

(8) **RD2** = Average interest rate paid on banks' total deposits denominated in domestic currency (NOK). Methods of calculation and sources as for RD1.

(9) **RL** = Yield to average life of long term bonds (more than 6 years to expected maturity date) issued by private mortgage loan associations. Source: Own yield calculations based on bond prices quoted at the Oslo Stock Exchange.

(10) **RB** = Average effective interest rate (including commissions) on advances in NOK by commercial and savings banks. As from 1985 Q3 data compiled by the Norges Bank. Earlier end–of–year data from *Credit Market Statistics* crudely adjusted by multiplicative factors in order to avoid obvious breaks in the levels of the series. As a consequence the level of this interest rate series may be subject to a considerable margin of error, particularly before 1980. Quarterly movements prior to 1977 were estimated from changes in the discount rate of the Norges Bank, otherwise linearly interpolated between end–of–year figures. Between 1977 Q1 and 1985 Q3 quarterly data were interpolated using the highest figures on bank lending rates in the interest rate statistics published in *Penger og Kreditt*. Source: As for RD1.

GENERAL NOTES

Data on M1, M2, KA, KB and RL are quarterly averages of end–of–month data. The figures for RD1, RD2 and RB tabulated in A1 are end–of–quarter estimates; in the empirical analysis reported in the paper data for period t are constructed as averages of end–of–quarter figures for period t and $t-1$..

TABLE A1. QUARTERLY DATA 1966 - 1989. 1966-1983

YEAR	I	M1 *(1)*	M2 *(2)*	KA *(3)*	KB *(4)*	Y *(5)*	X *(6)*	RD1 *(7)*	RD2 *(8)*	RL *(9)*	RB *(10)*
1966 Q1	I	12.31	29.74	44.06	17.75	12.74	11.59	0.09	2.58	5.37	6.20
Q2	I	12.35	29.92	45.36	18.52	13.08	12.77	0.10	2.59	5.40	6.20
Q3	I	13.02	30.92	46.20	18.85	14.60	13.40	0.11	2.60	5.56	6.20
Q4	I	13.81	31.89	46.95	19.03	14.15	14.37	0.12	2.61	5.55	6.30
1967 Q1	I	13.64	32.54	48.20	19.50	13.51	12.75	0.12	2.64	5.75	6.40
Q2	I	13.69	32.92	49.52	20.25	14.40	14.26	0.12	2.67	5.77	6.40
Q3	I	14.05	33.77	50.33	20.54	15.82	14.51	0.13	2.70	5.65	6.40
Q4	I	14.76	34.85	51.24	20.72	15.97	15.32	0.13	2.73	5.59	6.50
1968 Q1	I	14.43	35.10	52.33	20.99	15.19	13.59	0.18	2.75	5.48	6.60
Q2	I	14.61	35.69	53.79	21.74	15.42	14.69	0.23	2.77	5.37	6.60
Q3	I	15.28	36.82	54.79	22.10	17.01	14.99	0.28	2.79	5.33	6.60
Q4	I	16.36	38.48	56.17	22.50	16.13	15.89	0.34	2.81	5.34	6.60
1969 Q1	I	16.45	39.64	57.65	23.11	16.04	15.03	0.38	2.83	5.35	6.60
Q2	I	16.57	40.13	59.62	24.34	16.40	15.97	0.42	2.85	5.37	6.60
Q3	I	17.44	41.32	61.05	24.97	18.40	16.59	0.46	2.87	5.75	7.60
Q4	I	18.38	42.61	62.95	25.58	18.58	18.99	0.50	3.47	6.70	7.60
1970 Q1	I	18.36	43.47	64.63	26.06	17.71	17.09	0.56	3.57	6.39	7.70
Q2	I	19.02	44.92	66.95	27.17	19.63	19.16	0.63	3.66	6.40	7.70
Q3	I	19.90	46.57	68.50	27.69	21.02	19.50	0.70	3.75	6.42	7.70
Q4	I	20.98	48.40	70.61	28.14	21.51	21.78	0.77	3.84	6.43	7.80
1971 Q1	I	21.33	50.59	72.54	28.58	21.11	19.34	0.81	3.84	6.44	7.80
Q2	I	21.70	51.23	75.30	30.03	21.83	21.33	0.84	3.84	6.44	7.80
Q3	I	22.71	52.74	77.12	30.69	23.49	21.86	0.87	3.83	6.41	7.80
Q4	I	23.77	54.71	79.39	31.31	22.68	23.79	0.90	3.83	6.41	7.80
1972 Q1	I	23.94	56.52	81.57	31.87	22.91	21.18	0.92	3.83	6.47	7.90
Q2	I	24.34	57.42	84.38	33.29	23.96	22.81	0.93	3.82	6.45	7.90
Q3	I	25.52	59.14	86.38	34.01	25.48	23.45	0.95	3.82	6.35	7.90
Q4	I	26.51	60.55	89.10	34.96	26.06	25.56	0.97	3.81	6.32	7.90
1973 Q1	I	26.51	62.60	91.87	35.83	25.78	23.29	0.96	3.86	6.37	8.00
Q2	I	26.98	64.20	94.97	37.28	26.48	25.43	0.95	3.92	6.42	8.00
Q3	I	28.04	66.00	97.12	37.82	29.47	26.36	0.95	3.97	6.39	8.00
Q4	I	29.16	67.47	100.55	39.03	30.13	29.20	0.94	4.03	6.39	8.00
1974 Q1	I	29.79	70.31	103.57	40.21	30.40	26.83	1.00	4.08	6.40	8.00
Q2	I	29.70	70.70	107.75	42.48	31.00	30.32	1.04	4.14	7.40	9.00
Q3	I	30.75	72.46	110.31	43.37	33.65	31.50	1.13	4.20	7.48	9.00
Q4	I	33.08	75.36	113.36	44.13	34.68	34.66	1.20	4.26	7.48	9.00
1975 Q1	I	33.79	77.62	116.79	45.46	33.86	31.54	1.26	4.32	7.56	9.10
Q2	I	34.59	79.00	121.06	47.58	36.97	35.69	1.33	4.38	7.56	9.20
Q3	I	37.30	83.61	124.52	48.67	38.58	36.15	1.39	4.44	7.62	9.30
Q4	I	39.49	86.78	129.97	50.77	39.29	40.39	1.46	4.50	7.50	9.40
1976 Q1	I	41.28	90.90	135.12	52.37	39.74	36.65	1.50	4.60	7.45	9.50
Q2	I	41.90	93.05	141.77	55.38	40.50	40.58	1.53	4.71	7.44	9.50
Q3	I	43.81	96.71	146.82	57.11	44.46	42.08	1.57	4.81	7.48	9.50
Q4	I	46.39	101.06	152.40	58.74	46.01	46.28	1.60	4.92	7.50	9.70
1977 Q1	I	48.00	106.56	158.32	60.86	45.85	42.26	1.65	5.00	7.52	9.70
Q2	I	47.73	108.17	165.73	64.53	45.50	47.33	1.71	5.07	7.58	9.80
Q3	I	50.00	113.36	170.99	66.12	49.50	48.51	1.76	5.15	7.59	9.80
Q4	I	52.55	118.04	177.37	67.84	50.68	53.06	1.82	5.22	7.96	10.00
1978 Q1	I	52.50	121.64	184.62	70.24	49.90	46.92	2.00	6.22	8.65	11.30
Q2	I	51.71	122.29	190.73	72.36	51.35	49.87	2.20	6.37	8.62	11.50
Q3	I	53.91	125.98	195.69	72.75	54.50	50.70	2.30	6.30	8.64	12.00
Q4	I	56.48	130.58	202.08	73.93	57.32	56.33	2.40	6.33	8.55	12.00
1979 Q1	I	56.67	135.80	208.98	75.62	55.01	49.28	2.20	6.22	8.45	12.00
Q2	I	57.33	137.71	216.84	79.02	56.76	53.47	2.50	6.15	8.44	12.00
Q3	I	61.29	143.42	222.79	80.97	61.24	53.99	2.60	6.50	8.44	12.00
Q4	I	63.12	148.50	229.42	82.50	65.67	60.63	2.60	6.88	9.19	12.10
1980 Q1	I	63.31	153.64	235.90	84.10	68.27	57.61	2.80	7.05	10.68	12.10
Q2	I	63.48	154.23	242.78	87.08	69.58	60.87	3.10	7.17	10.70	12.20
Q3	I	66.02	159.63	249.15	89.40	71.15	63.79	3.30	7.25	11.03	13.10
Q4	I	69.42	165.92	255.58	90.78	76.05	71.78	3.50	7.32	10.85	13.30
1981 Q1	I	70.25	173.40	262.49	92.97	77.06	63.30	3.80	7.33	11.74	13.50
Q2	I	70.84	174.12	271.42	97.33	78.77	68.47	4.00	7.33	12.71	13.70
Q3	I	75.21	180.94	278.38	99.35	84.47	70.75	3.60	7.38	13.13	13.70
Q4	I	77.48	186.12	286.32	101.87	87.38	79.42	3.40	7.72	13.13	13.80
1982 Q1	I	78.90	194.38	294.95	105.52	86.15	72.37	3.70	7.87	14.02	13.80
Q2	I	79.79	194.65	304.45	109.89	87.14	77.87	3.90	7.97	14.16	14.00
Q3	I	82.93	199.26	312.51	112.26	91.30	80.10	4.10	8.02	14.18	14.20
Q4	I	86.11	203.89	321.34	114.20	97.68	87.04	4.40	8.37	14.14	14.30
1983 Q1	I	86.77	212.06	333.19	119.26	96.47	78.65	4.50	8.27	14.02	14.30
Q2	I	87.13	210.07	343.21	125.67	98.50	82.07	4.90	8.30	13.96	14.40
Q3	I	91.62	217.70	354.90	128.74	100.40	85.36	4.70	8.20	13.81	14.20
Q4	I	95.48	223.48	366.13	128.28	106.82	91.78	4.80	8.30	13.46	14.10

TABLE A1. QUARTERLY DATA 1966 - 1989. 1984-1989

YEAR		I	M1 *(1)*	M2 *(2)*	KA *(3)*	KB *(4)*	Y *(5)*	X *(6)*	RD1 *(7)*	RD2 *(8)*	RL *(9)*	RB *(10)*
1984	Q1	I	96.34	234.30	379.27	138.59	106.27	86.48	4.90	8.47	13.27	14.10
	Q2	I	98.65	235.79	392.39	146.23	109.60	92.00	5.10	8.33	13.20	13.80
	Q3	I	105.03	246.60	407.79	152.71	114.78	95.49	5.10	8.58	13.19	13.50
	Q4	I	113.10	262.73	423.18	159.71	121.86	104.65	5.00	8.57	13.15	13.30
1985	Q1	I	117.69	276.97	438.57	169.05	122.34	98.15	5.50	8.65	13.14	13.20
	Q2	I	120.45	275.67	457.42	180.95	119.92	103.98	5.70	8.60	13.33	13.30
	Q3	I	129.29	290.83	482.32	193.50	123.89	108.73	5.60	8.73	13.66	13.40
	Q4	I	138.98	306.73	509.91	208.48	134.05	120.02	6.50	8.90	13.89	13.41
1986	Q1	I	139.59	321.63	534.51	222.83	122.97	112.24	6.60	9.05	14.65	14.00
	Q2	I	146.99	326.63	561.57	235.62	123.42	123.87	6.80	9.20	14.52	14.70
	Q3	I	150.57	337.60	590.16	245.50	128.96	126.12	7.00	9.50	14.51	15.10
	Q4	I	155.87	349.97	615.57	251.89	138.37	140.76	8.10	10.40	14.83	16.00
1987	Q1	I	158.72	362.43	646.85	270.39	135.10	125.33	8.80	10.90	15.12	16.10
	Q2	I	167.37	363.20	672.96	283.10	133.97	131.31	8.80	10.80	14.73	16.27
	Q3	I	169.60	364.30	694.60	301.33	142.02	135.84	9.10	11.00	14.50	16.65
	Q4	I	181.03	379.87	721.70	322.94	151.84	150.98	9.70	11.30	14.40	16.78
1988	Q1	I	186.00	399.54	742.23	338.13	145.56	132.90	10.00	11.40	14.66	16.87
	Q2	I	183.97	392.81	755.97	345.81	145.86	136.35	10.00	11.20	14.20	16.61
	Q3	I	195.36	395.35	779.83	340.03	149.09	137.73	9.70	10.90	14.20	16.49
	Q4	I	213.97	403.49	787.80	343.39	153.74	147.29	9.80	10.70	13.50	16.44
1989	Q1	I	220.94	413.30	788.30	347.90	151.00	133.41	8.80	9.70	12.20	15.64
	Q2	I	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Q3	I	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Q4	I	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 2. Tests for seasonal unit roots in money, credit, nominal income and interest rate variables 1967 Q1 – 1989 Q1.

variable	μ_t	lags of $\Delta_4 Z_{t-i}$	zero frequency t_1 π_1	bi–annual t_2 π_2	annual frequency t_3 π_3	t_4 π_4	F $\pi_3 \cap \pi_4$
LM1		1,3–5	2.74	–1.81	–1.36	–0.83	1.30
	I	1,3–5	0.26	–1.75	–1.36	–0.83	1.30
	I,SD		0.29	–5.37**	–6.62**	–4.30**	58.73**
	I,TR	1,3–5	–2.99	–1.68	–1.38	–0.68	1.20
	I,SD,TR	1,4–5	–3.03	–3.04**	–4.25**	–0.27	9.47**
LM2		1–5	0.05	–0.46	0.33	–1.59	1.34
	I	1–5	–1.46	–0.48	0.18	–1.70	1.47
	I,SD	3–4	–1.18	–4.32**	–2.48	–6.16**	43.93**
	I,TR	1–5	–3.04	–0.45	0.11	–1.51	1.16
	I,SD,TR	1,4–5	–2.16	–4.25**	–2.78	–5.78**	43.20**
LKA		1–4	2.02	–1.24	–2.23*	–3.09**	7.93**
	I	1–4	0.23	–1.22	–2.34*	–2.96**	7.79**
	I,SD	1–4	0.15	–1.32	–2.81	–3.24**	10.68**
	I,TR	1–4	–2.50	–1.22	–2.40*	–2.90**	7.74**
	I,SD,TR	1–4	–2.48	–1.36	–2.91	–3.12**	10.57**
LKB		1,3,5	1.31	–3.08**	–1.62	–1.90	3.16*
	I	1,3,5	–0.13	–3.03**	–1.63	–1.83	3.04
	I,SD		–1.18	–4.54**	–3.47*	–8.02**	58.55**
	I,TR	1,3,5	–3.72*	–2.89**	–1.96*	–1.47	3.06*
	I,SD,TR		–1.55	–4.70**	–3.80*	–7.53*	58.50**
LX		1,2,4,5	1.44	–0.56	–0.32	1–18	0.76
	I	1,2,4	–1.38	–0.77	–0.55	–1.35	1.07
	I,SD		–1.11	–4.98**	–6.83**	–4.89**	57.96**
	I,TR	1,2,4,5	–1.20	–0.52	–0.47	–1.03	0.66
	I,SD,TR		–1.63	–5.11**	–7.06**	–4.80**	60.15**

variable	μ_t	lags of $\Delta_4 Z_{t-i}$	zero frequency t_1 π_1	bi-annual t_2 π_2	annual frequency t_3 π_3	t_4 π_4	F $\pi_3 \cap \pi_4$
LY		1,2,4,5	0.73	–1.93*	–1.65	–1.31	2.32
	I	1,2,4,5	–1.67	–1.95*	–1.79	–1.24	2.47
	I,SD	2	–1.24	–5.30**	–3.27	–2.61*	9.50**
	I,TR	1,2,4,5	–0.22	–1.93*	–1.77	–1.22	2.41
	I,SD,TR	2	–0.56	–5.29**	–3.25	–2.61*	9.44**
RD1		1–8,10	4.17	–7.13**	–3.62**	–2.46**	10.52**
	I	1–8,10	3.82	–7.20**	–3.77**	–2.41**	10.97**
	I,SD	1–8,10	3.76	–7.00**	–3.59**	–2.29	9.96**
	I,TR	1–8,10	1.48	–7.15**	–3.68**	–2.34*	10.33**
	I,SD,TR	1–8,10	1.46	–6.96**	–3.48*	–2.23	9.35**
RD2		1–5	2.01	–5.26**	–0.92	–3.80**	7.66**
	I	1–5	0.24	–5.24**	–0.95	–3.75**	7.50**
	I,SD	1,4–5	0.26	–7.67**	–1.10	–5.95**	18.03**
	I,TR	1–5	–2.18	–5.11**	–1.00	–3.59**	6.97**
	I,SD,TR	1,4–5	–2.19	–7.51**	–1.19	–5.65**	16.14**
RL			0.34	–5.99**	–3.70**	–6.05**	36.11**
	I		–1.14	–5.96**	–3.77**	–5.96**	35.88**
	I,SD		–1.18	–5.83**	–3.59*	–5.83**	33.78**
	I,TR		–1.49	–5.98**	–3.87**	–5.76**	35.33**
	I,SD,TR		–1.41	–5.85**	–3.68*	–5.60**	33.12**
RB		1–5	1.76	–6.25**	0.31	–4.07**	8.35**
	I	1–5	–0.80	–6.22**	0.25	–4.06**	8.29**
	I,SD	1–5	–0.79	–6.10**	0.22	–4.00**	8.02**
	I,TR	1–3	–2.33	–6.59**	1.03	–5.90**	18.03**
	I,SD,TR	1–3	–2.28	–6.46**	1.00	–5.78**	17.33**

Notes. The test procedures follow Hylleberg et al.[HEGY] (1990). The sample period starts in 1967 Q2; estimation begins in this or subsequent quarters depending on the number of lags of included. The end of the estimation period is 1989 Q1 for all variables except LM1, for which data end in 1986 Q4. For π_1,π_2,π_3 and $\pi_3 \cap \pi_4$ test statistics that are significantly different from zero at the 5 (1) per cent level are denoted by * (**); for π_4 the significance levels used are 2.5 (1) per cent.

Table 3. HEGY cointegration tests for variables in the 'constant velocity' model.

variable sample period	μ_t	lags of $\Delta_4 Z_{t-i}$	zero frequency t_1 π_1	bi-annual t_2 π_2	annual frequency t_3 π_3	 t_4 π_4	 F $\pi_3 \cap \pi_4$
LM1–		1,4	0.85	–0.49	–1.65	–0.76	1.64
LX	I	1,4	–2.12	–0.56	-1.92^{*}	–0.50	1.97
	I,SD	1	-3.08^{*}	-3.10^{*}	-4.94^{**}	–1.19	12.98^{**}
1967–	I,TR	1,3,4	–2.74	–0.41	–1.79	–0.80	1.95
1983	I,SD,TR	1	-3.76^{*}	-2.95^{*}	-5.12^{**}	–0.80	13.49^{**}
LM1–		1,4	1.41	–0.51	–1.78	–1.08	2.17
LX	I	1,4	–0.41	–0.52	–1.81	–1.04	2.19
	I,SD	1	–1.97	-3.00^{*}	-4.81^{**}	–1.68	13.02^{**}
1967–	I,TR	1,4,5	–1.92	–0.45	–1.61	–0.83	1.64
1986	I,SD,TR	1	–3.16	–2.84	–5.00	–1.27	13.35^{**}
LM2–		1,2,4	0.59	–0.51	–0.66	–0.68	0.45
LX	I	1,2,4,5	–1.74	–0.40	–0.54	–0.49	0.27
	I,SD		–2.12	-5.07^{**}	-6.74^{**}	-3.10^{**}	38.76^{**}
1967–	I,TR	1,2,4,5	–2.04	–0.40	–0.53	–0.49	0.26
1983	I,SD,TR		–2.40	-5.09^{**}	-6.78^{**}	-3.05^{**}	38.83^{**}
LM2–		1,2,4	1.34	–0.65	–0.65	–0.86	0.60
LX	I	1,2,4	0.14	–0.64	–0.64	–0.86	0.59
	I,SD		–0.63	-5.44^{**}	-6.67^{**}	-3.80^{**}	40.42^{**}
1967–	I,TR	1,4,5	–1.44	–0.46	–0.77	–0.47	0.41
1989	I,SD,TR		–1.82	-5.51^{**}	-6.84^{**}	-3.70^{**}	41.43^{**}

variable / sample period	μ_t	lags of $\Delta_4 Z_{t-i}$	zero frequency t_1 π_1	bi-annual t_2 π_2	annual frequency t_3 π_3	t_4 π_4	F $\pi_3 \cap \pi_4$
LKA–		2,8	–0.06	–5.60**	3.33	3.23	6.43**
LY	I	2,5	–4.22**	–4.87**	2.76	3.01	4.91**
	I,SD	2,5	–4.34**	0.56	1.50	1.15	1.15
1967–	I,TR	2,5	–4.41**	–4.88**	2.78	3.00	4.94**
1983	I,SD,TR	2,5	–4.15**	–4.36**	0.55	1.52	1.19
LKA–		1,2,4,5,8	1.13	–2.09*	1.22	1.05	0.77
LY	I	1,2,4,5,6	1.52	–2.13*	1.31	1.06	0.87
	I,SD	1,4,5,8	1.16	–1.80	–0.87	0.56	0.95
1967–	I,TR	1,2,4,5,8	0.39	2.18	1.36	1.13	0.94
1989	I,SD,TR	1,4,5,8	0.10	–1.84	–0.93	0.59	1.07
LKB–		2,8	–1.56	–5.14**	–1.23	–2.24*	3.49*
LY	I	2,8	–0.61	–5.10**	–1.19	–2.22*	3.37*
	I,SD		–1.07	–4.06**	–4.10**	–2.45*	13.82**
1967–	I,TR	2,8	–1.93	–5.11**	–1.24	–2.16*	3.31*
1983	I,SD,TR		–2.20	–4.08**	–4.27**	–2.33**	14.32**
LKB–		1,2,4,5,8	0.23	–1.63	–0.98	–1.54	1.72
LY	I	1,2,4,5	–2.63	–1.52	–1.32	–1.30	1.75
	I,SD	1,5	–2.81	–2.54	–4.62**	–1.88	12.52**
1967–	I,TR	1,2,4,5	–2.58	–1.51	–1.32	–1.33	1.78
1989	I,SD,TR	1,5	–2.77	–2.57	–4.60**	–1.91	1.63

See notes to Table 2 for explanation of statistics and significance levels.

Table 4. Tests for cointegration in the simple model.

A. Dependent variable: **LM1**

	1966 Q2 – 1983 Q4		1966 Q2 – 1986 Q4	
	(A1)	(A2)	(A3)	(A4)
LX	1.017	1.022	1.042	1.046
INTERCEPT	–0.02	–0.09	–0.10	–0.17
S1		0.111		0.109
S2		0.030		0.032
S3		0.048		0.049
SER	0.050	0.030	0.055	0.038
CRDW	2.34	0.65	1.91	0.43
ADF(k)	–5.05**	–4.01*	–4.86**	–3.47*
k	4	4	4	4
Q(m)	16.24	9.90	24.51	15.31
m	21	21	23	23

B. Dependent variable: **LM2**

	1966 Q2 – 1983 Q4		1966 Q2 – 1989 Q1	
	(B1)	(B2)	(B3)	(B4)
LX	1.017	1.024	1.052	1.055
INTERCEPT	0.83	0.74	0.72	0.64
S1		0.136		0.140
S2		0.059		0.062
S3		0.063		0.064
SER	0.061	0.039	0.066	0.044
CRDW	2.01	0.31	1.81	0.26
ADF(k)	–3.87*	–2.33	–4.11**	–2.79
k	4	0	4	4
Q(m)	18.12	10.16	23.62	6.26
m	21	22	23	23

C. Dependent variable: LKA

	1966 Q2 – 1983 Q4		1966 Q2 – 1989 Q1	
	(C1)	(C2)	(C3)	(C4)
LY	1.011	1.013	1.100	1.101
INTERCEPT	1.20	1.17	0.90	0.87
S1		0.042		0.049
S2		0.044		0.048
S3		–0.006		0.004
SER	0.041	0.034	0.102	0.101
CRDW	1.18	0.79	0.22	0.12
ADF(k)	–5.58**	–5.13**	–2.51	–2.91
k	8	8	8	8
Q(m)	21.00	15.27	25.86	22.27
m	19	19	23	23

D. Dependent variable: LKB

	1966 Q2 – 1983 Q4		1966 Q2 – 1989 Q1	
	(D1)	(D2)	(D3)	(D4)
LY	0.939	0.941	1.093	1.094
INTERCEPT	0.50	0.47	–0.01	–0.05
S1		0.038		0.052
S2		0.051		0.057
S3		0.002		0.009
SER	0.040	0.034	0.157	0.158
CRDW	1.12	0.74	0.11	0.06
ADF(k)	–5.37**	–4.88**	–3.58*	–2.82
k	8	8	8	8
Q(m)	19.14	12.36	17.02	16.15
m	19	19	23	23

Notes. The diagnostics are: SER = standard error of the cointegrating regression; CRDW = Durbin–Watson statistic from the cointegrating regression; Q(m) = Ljung–Box Q–statistic for autocorrelated residuals with m degrees of freedom.

The test statistic is: ADF(k) = Augmented Dickey–Fuller statistic with maximum lag equal to k, but with insignificant terms deleted. Test statistics that are significantly different from zero at the 5 (1) per cent level are denoted by * (**). The critical values are taken from Table 2 in Engle and Yoo (1987) – in order to minimise the type I error.

Table 5. Tests for cointegration in the augmented model.

A. Dependent variable: LM1

	1966 Q2 – 1983 Q4		1966 Q2 – 1986 Q4	
	(A1)	(A2)	(A3)	(A4)
LX	0.957	0.989	0.976	1.004
RD1	0.085	0.074	0.059	0.058
RL	–0.029	–0.030	–0.020	–0.026
INTERCEPT	0.28	0.15	0.19	0.09
S1		0.112		0.110
S2		0.030		0.032
S3		0.046		0.048
SER	0.047	0.024	0.047	0.025
CRDW	2.47	0.88	2.37	0.91
ADF(k)	–5.81**	–4.75**	–5.85**	–4.93**
k	4	4	4	4
Q(m)	13.88	11.55	23.07	21.32
m	21	21	23	23

B. Dependent variable: LM2

	1966 Q2 – 1983 Q4		1966 Q2 – 1989 Q1	
	(B1)	(B2)	(B3)	(B4)
LX	0.762	0.845	0.838	0.899
RD2	0.118	0.084	0.072	0.057
RL	–0.022	–0.016	–0.007	–0.009
INTERCEPT	1.33	1.11	1.16	0.98
S1		0.113		0.120
S2		0.046		0.050
S3		0.053		0.055
SER	0.048	0.028	0.052	0.031
CRDW	2.04	0.54	2.00	0.46
ADF(k)	–5.34**	–4.26*	–7.23**	–4.58*
k	5	0	4	4
Q(m)	11.83	11.58	21.08	10.35
m	20	24	23	23

C. Dependent variable: **LKA**

	1966 Q2 – 1983 Q4		1966 Q2 – 1989 Q1	
	(C1)	(C2)	(C3)	(C4)
LY	0.897	0.925	0.857	0.872
RB	0.053	0.046	0.082	0.079
RL	–0.026	–0.026	–0.022	–0.024
INTERCEPT	1.31	1.25	1.17	1.13
S1		0.041		0.042
S2		0.042		0.042
S3		–0.005		0.004
SER	0.035	0.027	0.090	0.089
CRDW	1.48**	1.13**	0.22	0.13
ADF(k)	–5.60**	–4.79**	–1.42	–1.63
k	8	0	8	8
Q(m)	16.97	15.89	19.36	17.03
m	19	19	23	23

D. Dependent variable: **LKB**

	1966 Q2 – 1983 Q4		1966 Q2 – 1989 Q1	
	(D1)	(D2)	(D3)	(D4)
LY	0.901	0.929	0.787	0.804
RB	0.030	0.024	0.099	0.096
RL	–0.022	–0.021	–0.025	–0.026
INTERCEPT	0.52	0.46	0.33	0.28
S1		0.039		0.042
S2		0.052		0.048
S3		0.003		0.009
SER	0.037	0.030	0.146	0.147
CRDW	1.36**	1.02**	0.10	0.06
ADF(k)	–5.57**	–4.51**	–1.21	–0.98
k	8	0	8	8
Q(m)	16.29	15.32	18.18	19.25
m	19	19	23	23

See notes to Table 4 for explanation of statistics and significance levels.

Table 6. The Johansen procedure for LM1.
VAR with 5 lags, constant and seasonal dummies included.

Panel A. Sample period 1967 Q3 – 1983 Q4, 66 observations.

The eigenvalues:				
	0.400	0.254	0.131	0.008

The test statistics:

	Testing the number of cointegrating vectors			
Test	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$
trace	62.839***	29.140*	9.777	0.524
λ_{max}	33.699***	19.362*	9.253	0.524

The eigenvectors:				
LM1	66.748	58.945	1.700	7.459
LX	–58.103	–59.363	0.434	–0.040
RD1	–16.318	–1.963	–0.040	–5.667
RL	5.528	0.711	0.122	0.846

Normalization by LM1 of the first eigenvector:
LM1 = 0.870LX + 0.244RD1 – 0.083RL

Panel B. Sample period 1967 Q3 – 1986 Q4, 78 observations.

The eigenvalues:				
	0.292	0.214	0.071	0.003

The test statistics:

	Testing the number of cointegrating vectors			
Test	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$
trace	51.625**	24.735	5.994	0.216
λ_{max}	26.890*	18.741	5.777	0.216

The eigenvectors:				
LM1	46.834	47.858	–5.026	–17.232
LX	–47.277	–47.201	8.325	13.418
RD1	–1.415	–4.759	–1.184	3.666
RL	0.774	1.764	–1.363	–1.285

Normalization by LM1 of the first eigenvector:
LM1 = 1.009LX + 0.030RD1 – 0.017RL

$$\begin{Bmatrix} * \\ ** \\ *** \end{Bmatrix} = \begin{Bmatrix} \text{greater than the 10 \% critical value;} \\ \text{greater than the 5 \% critical value;} \\ \text{greater than the 1 \% critical value.} \end{Bmatrix}$$

The critical values are taken from Johansen and Juselius (1990).

Table 7. The Johansen procedure for LM2.
VAR with 6 lags, constant and seasonal dummies included.

Panel A. Sample period 1967 Q4 – 1983 Q4, 65 observations.

The eigenvalues:				
	0.418	0.295	0.101	0.001

The test statistics:

	Testing the number of cointegrating vectors			
Test	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$
trace	64.795***	29.632*	6.933	0.030
λ_{max}	35.163***	22.699**	6.903	0.030

The eigenvectors:				
LM2	66.251	–30.342	–12.998	12.784
LX	–59.245	20.662	1.503	–12.842
RD2	–3.786	7.152	4.132	–1.216
RL	0.710	–2.393	–0.004	0.187

Normalization by LM2 of the first eigenvector:
LM2 = 0.894LX + 0.057RD2 – 0.011RL

Panel B. Sample period 1967 Q4 – 1989 Q1, 86 observations.

The eigenvalues:				
	0.299	0.146	0.042	0.023

The test statistics:

	Testing the number of cointegrating vectors			
Test	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$
trace	49.772**	19.246	5.687	2.014
λ_{max}	30.526**	13.559	3.672	2.014

The eigenvectors:				
LM2	47.546	33.213	4.174	–0.845
LX	–37.041	–31.388	–9.047	5.875
RD2	–5.486	0.045	0.654	–0.603
RL	1.153	–0.772	0.680	–0.306

Normalization by LM2 of the first eigenvector:
LM2 = 0.779LX + 0.115RD2 – 0.024RL

$\left\{\begin{matrix} * \\ ** \\ *** \end{matrix}\right\} = \left\{\begin{matrix}\text{greater than the 10 \% critical value;} \\ \text{greater than the 5 \% critical value;} \\ \text{greater than the 1 \% critical value.}\end{matrix}\right\}$

The critical values are taken from Johansen and Juselius (1990).

Table 8. The Johansen procedure for LKA.
VAR with 6 lags, constant and seasonal dummies included.

Panel A. Sample period 1967 Q4 – 1983 Q4, 65 observations.

The eigenvalues:				
	0.335	0.227	0.156	0.022

The test statistics:

	Testing the number of cointegrating vectors			
Test	$r=0$	$r\leq 1$	$r\leq 2$	$r\leq 3$
trace	55.688^{***}	29.205^{*}	12.445	1.415
λ_{max}	26.484^{*}	16.760	11.030	1.415

The eigenvectors:				
LKA	62.855	–9.680	–13.115	54.444
LY	–54.317	2.509	25.478	–49.386
RB	–3.160	3.561	–1.748	–3.495
RL	0.788	–1.551	–0.698	2.464

Normalization by LKA of the first eigenvector:
LKA = 0.864LY + 0.050RB – 0.013RL

Panel B. Sample period 1967 Q4 – 1989 Q1, 86 observations.

The eigenvalues:				
	0.288	0.174	0.136	0.020

The test statistics:

	Testing the number of cointegrating vectors			
Test	$r=0$	$r\leq 1$	$r\leq 2$	$r\leq 3$
trace	59.980^{***}	30.738^{*}	14.309	1.754
λ_{max}	29.242^{**}	16.429	12.555	1.754

The eigenvectors:				
LKA	5.045	–7.189	19.922	4.254
LY	–14.009	6.455	–14.614	5.908
RB	3.155	0.173	–2.267	–1.259
RL	–1.059	0.466	0.859	–0.846

Normalization by LKA of the first eigenvector:
LKA = 2.777LY – 0.625RB + 0.210RL

$$\begin{Bmatrix} * \\ ** \\ *** \end{Bmatrix} = \begin{Bmatrix} \text{greater than the 10 \% critical value;} \\ \text{greater than the 5 \% critical value;} \\ \text{greater than the 1 \% critical value.} \end{Bmatrix}$$

The critical values are taken from Johansen and Juselius (1990).

Table 9. **The Johansen procedure for LKB.**
VAR with 4 lags, constant and seasonal dummies included.

Panel A. Sample period 1967 Q2 – 1983 Q4, 67 observations.

The eigenvalues:	0.381	0.171	0.136	0.001
The test statistics:	Testing the number of cointegrating vectors			
Test	$r=0$	$r\leq 1$	$r\leq 2$	$r\leq 3$
trace	54.511**	22.357	9.801	0.041
λ_{max}	32.154**	12.557	9.759	0.041
The eigenvectors:				
LKB	44.971	–31.262	25.012	15.372
LY	–31.286	36.006	–26.382	–10.473
RB	–4.410	–0.480	1.419	–1.189
RL	1.819	–1.020	–0.751	0.746

Normalization by LKB of the first eigenvector:
LKB = 0.696LY + 0.098RB – 0.040RL

Panel B. Sample period 1967 Q2 – 1989 Q1, 88 observations.

The eigenvalues:	0.337	0.190	0.131	0.002
The test statistics:	Testing the number of cointegrating vectors			
Test	$r=0$	$r\leq 1$	$r\leq 2$	$r\leq 3$
trace	67.263***	31.091*	12.535	0.135
λ_{max}	36.172***	18.556	12.400	0.135
The eigenvectors:				
LKB	7.706	3.593	6.211	1.786
LY	–2.441	–9.683	–2.970	1.894
RB	–2.265	1.549	–0.831	0.179
RL	1.160	–0.191	–0.228	–0.815

Normalization by LKB of the first eigenvector:
LKB = 0.317LY + 0.294RB – 0.151RL

$$\begin{Bmatrix} * \\ ** \\ *** \end{Bmatrix} = \begin{Bmatrix} \text{greater than the 10 \% critical value;} \\ \text{greater than the 5 \% critical value;} \\ \text{greater than the 1 \% critical value.} \end{Bmatrix}$$

The critical values are taken from Johansen and Juselius (1990).

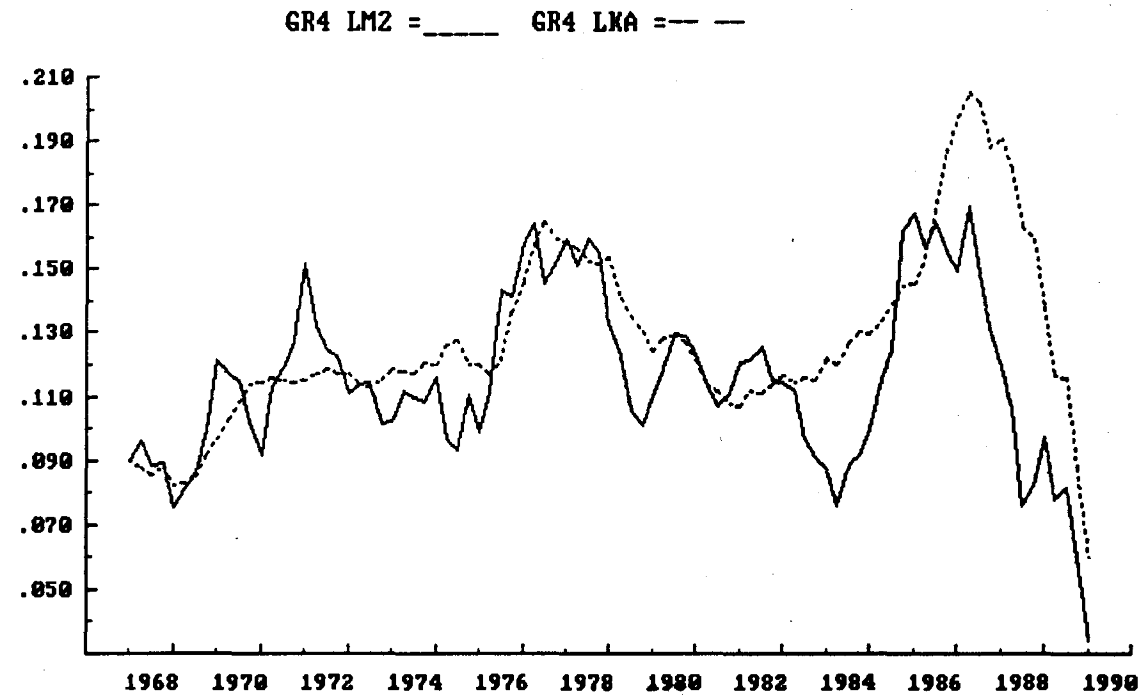

Figure 1. Four—quarter growth rates of the logs of M2 (solid line) and KA (dotted line).

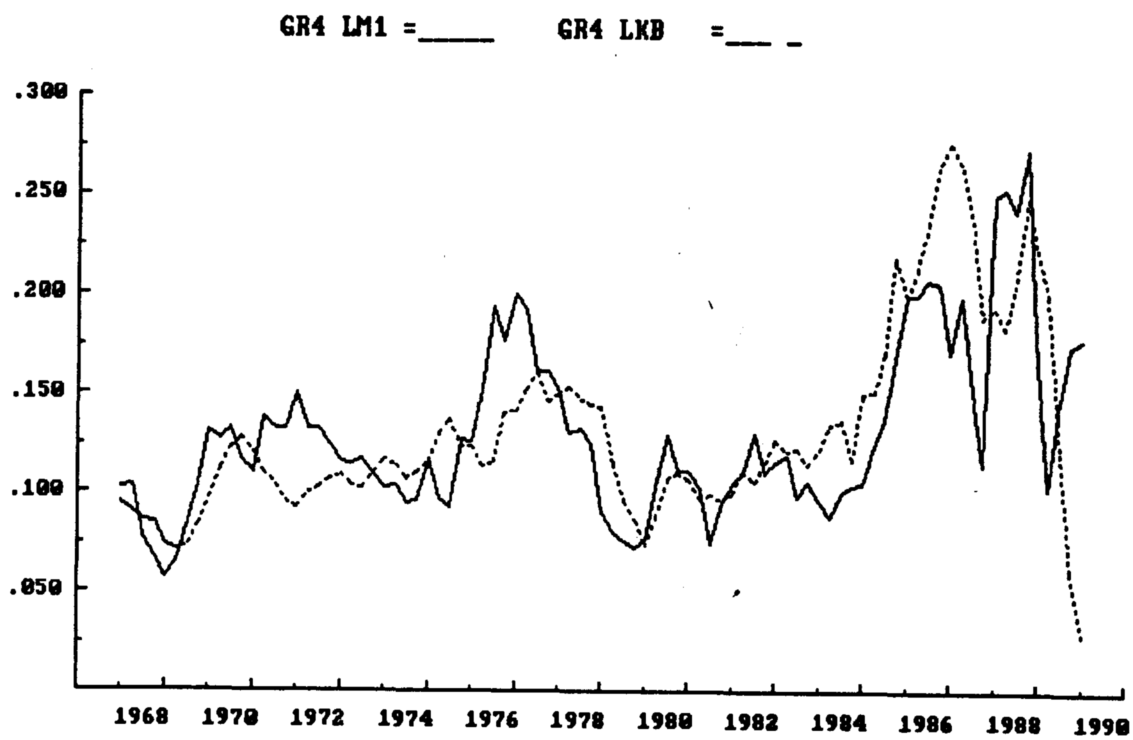

Figure 2. Four—quarter growth rates of the logs of M1 (solid line) and KB (dotted line).

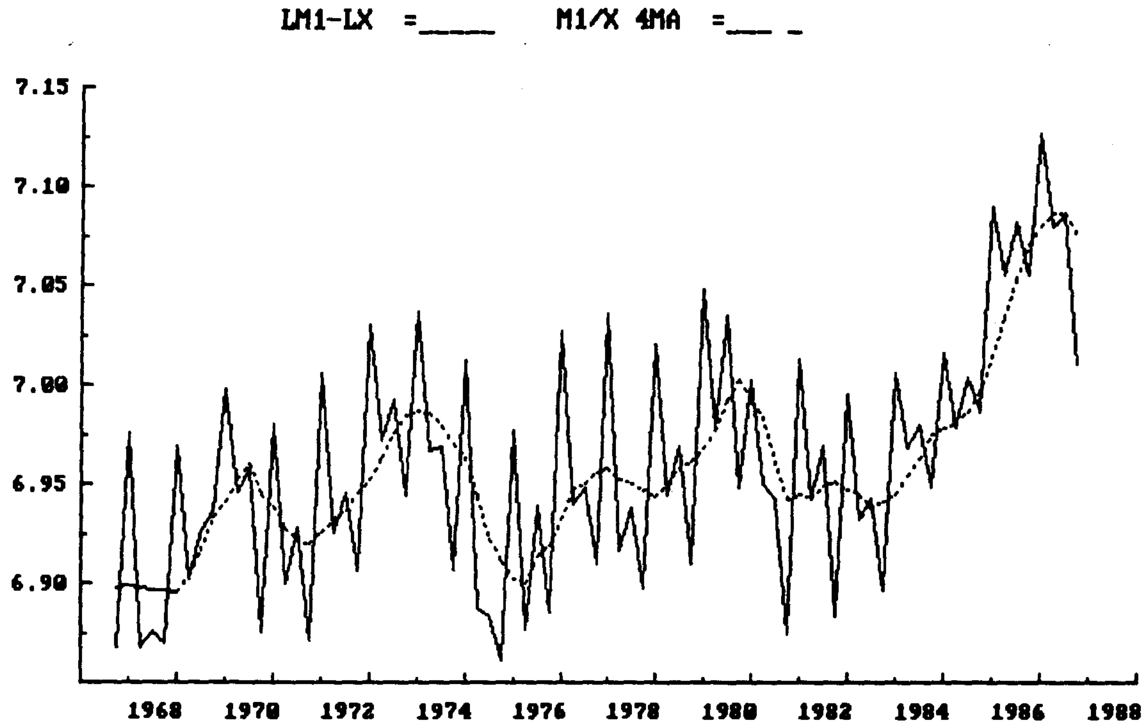

Figure 3. Actual and four–quarter moving average of the log of the ratio of M1 to X.

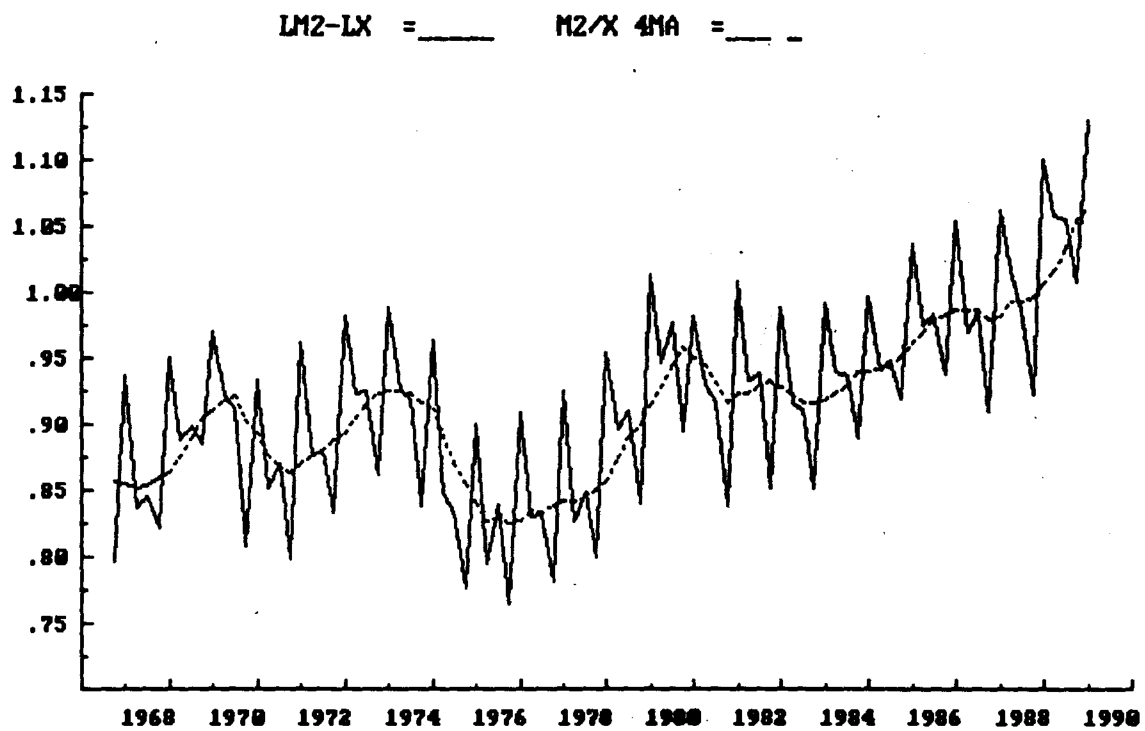

Figure 4. Actual and four–quarter moving average of the log of the ratio of M2 to X.

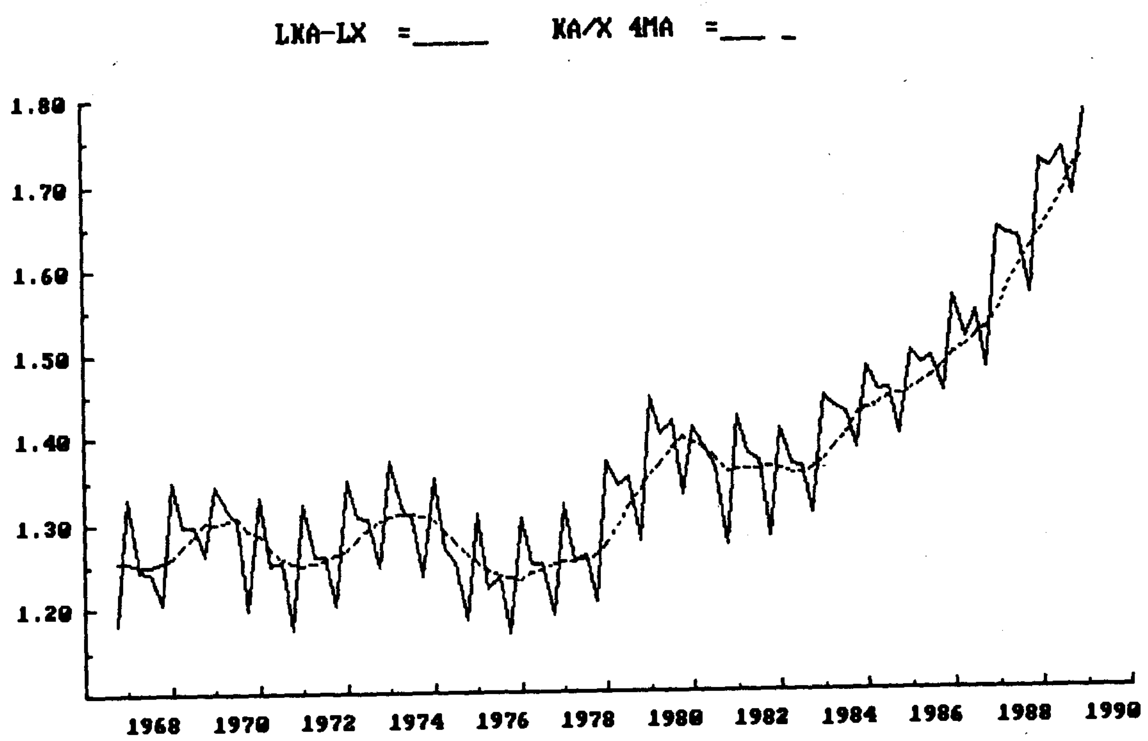

Figure 5. Actual and four–quarter moving average of the log of the ratio of KA to X.

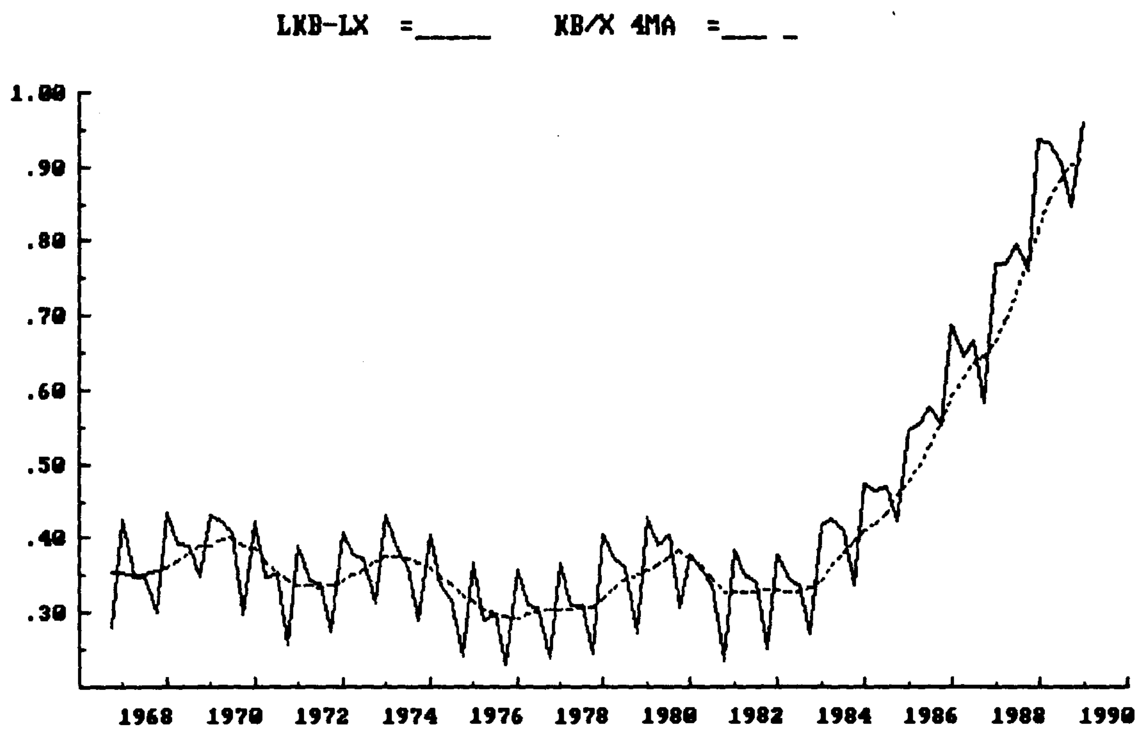

Figure 6. Actual and four–quarter moving average of the log of the ratio of KB to X.

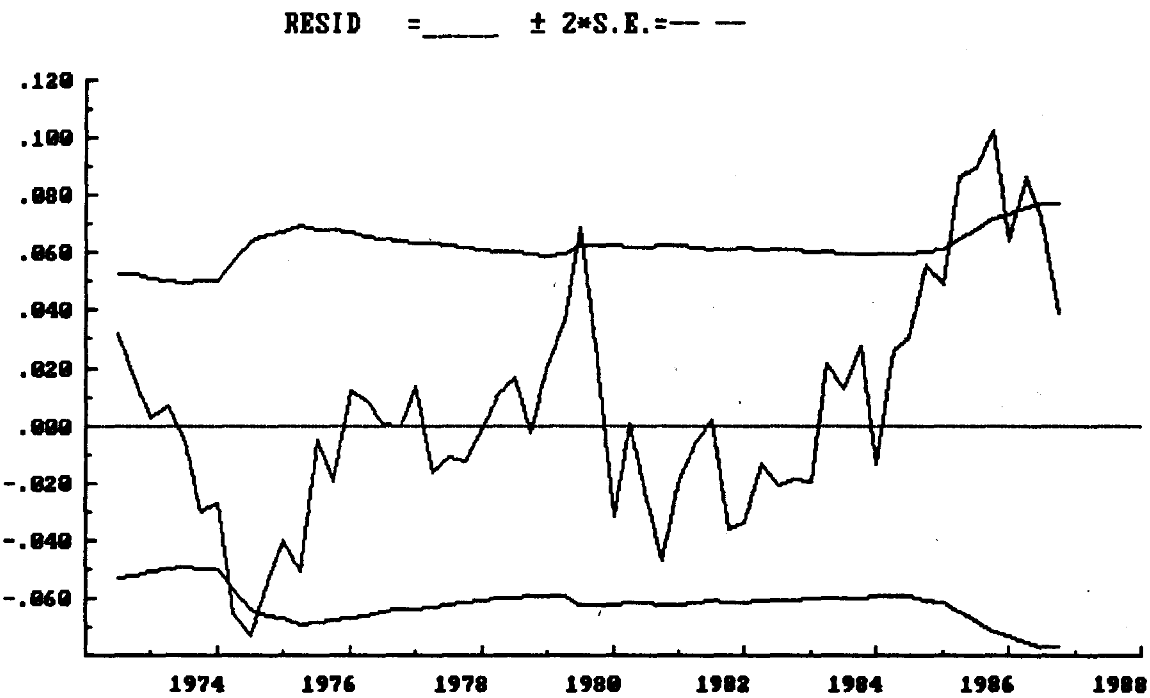

Figure 7. One–step residuals from the M1 equation in the simple model, Table 4 (A4).

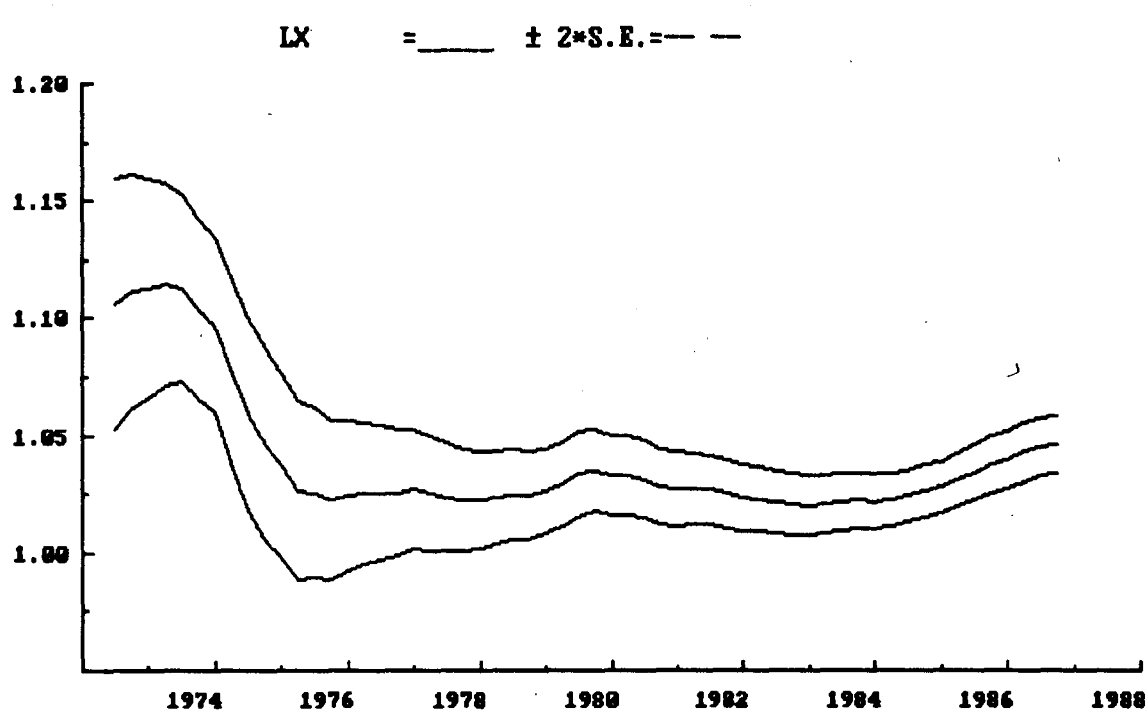

Figure 8. Recursive estimates of α_1 in the M1 equation in the simple model, Table 4 (A4).

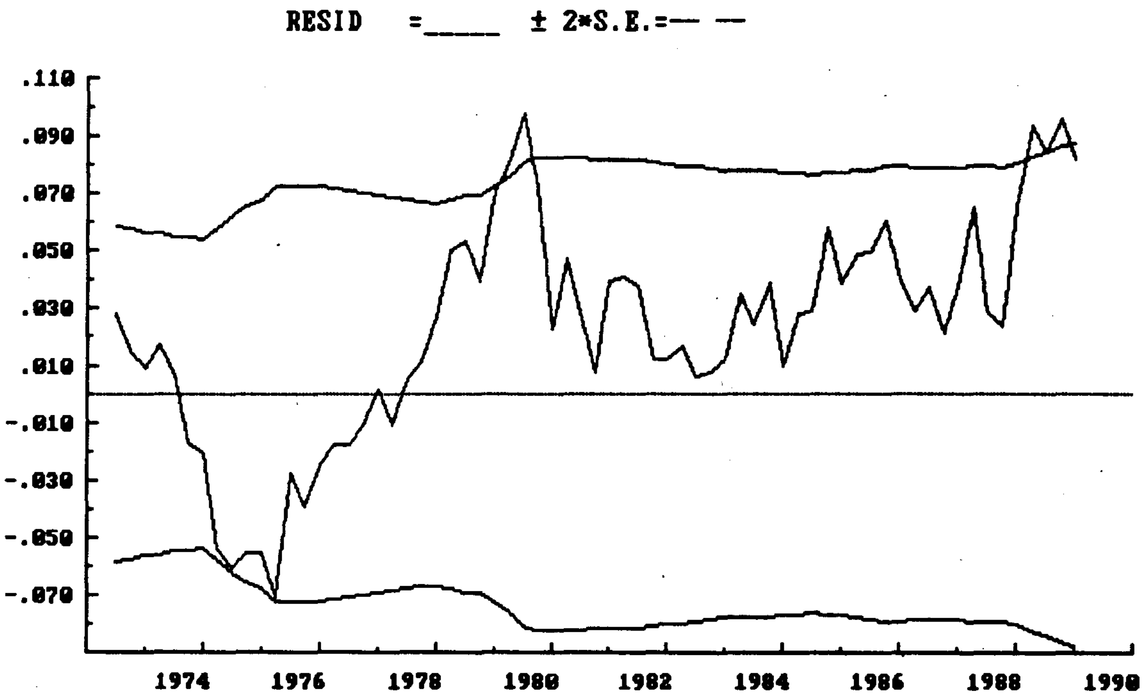

Figure 9. One–step residuals from the M2 equation in the simple model, Table 4 (B4).

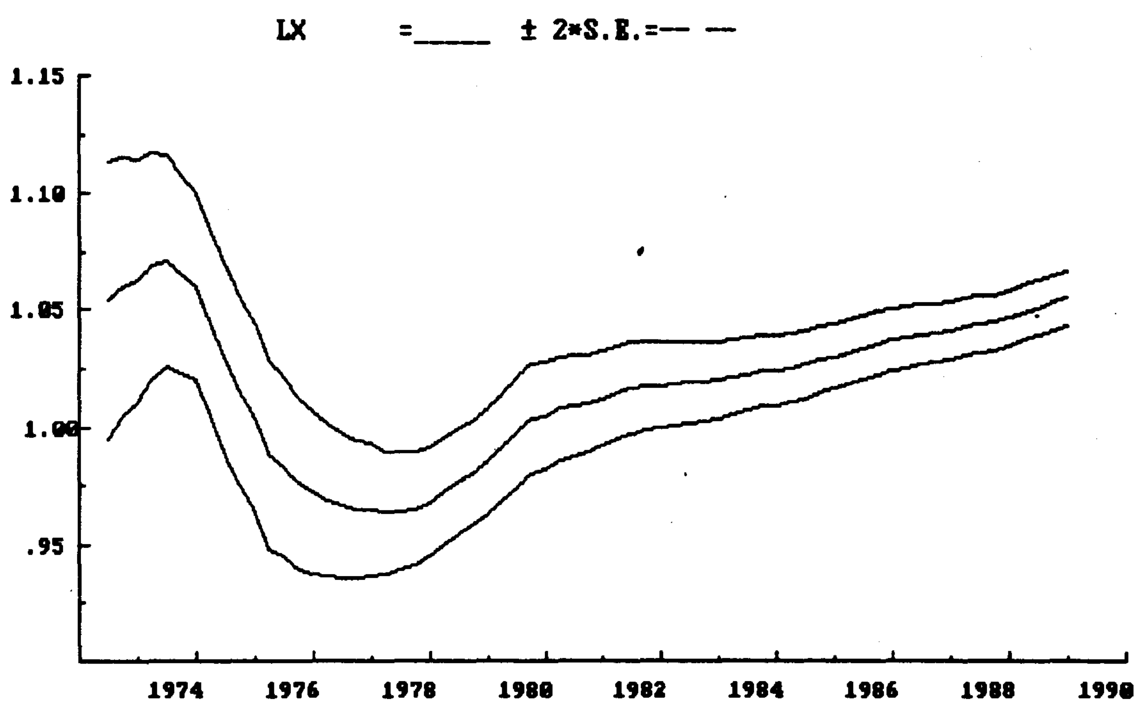

Figure 10. Recursive estimates of α_1 in the M2 equation in the simple model, Table 4 (B4).

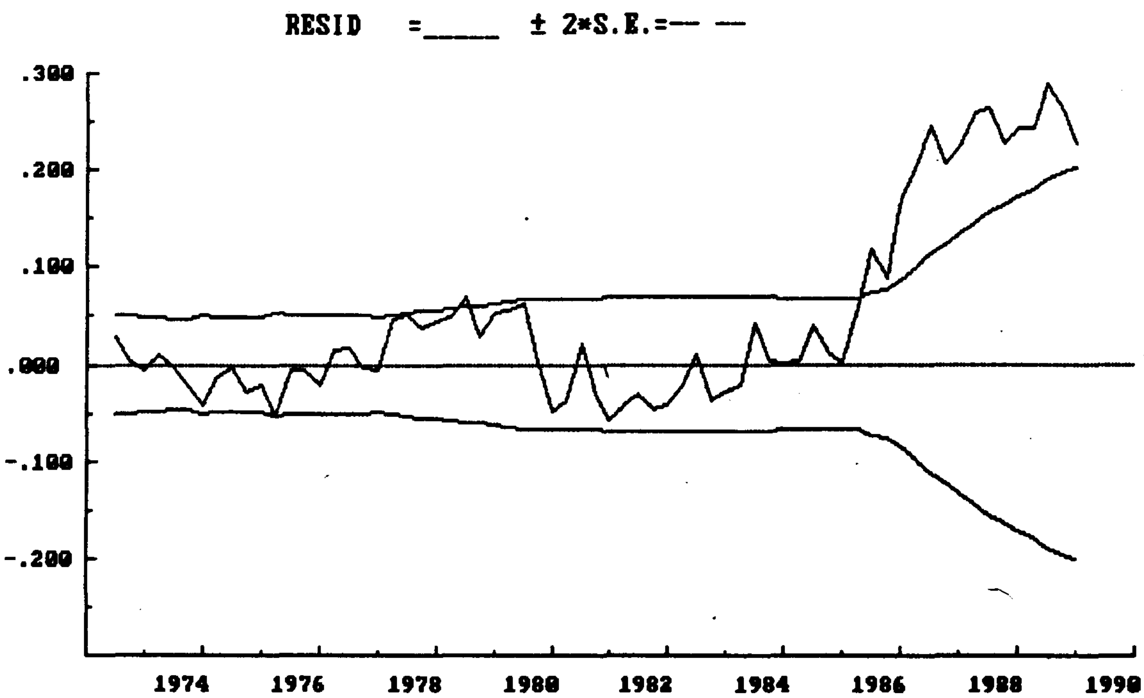

Figure 11. One–step residuals from the KA equation in the simple model, Table 4 (C4).

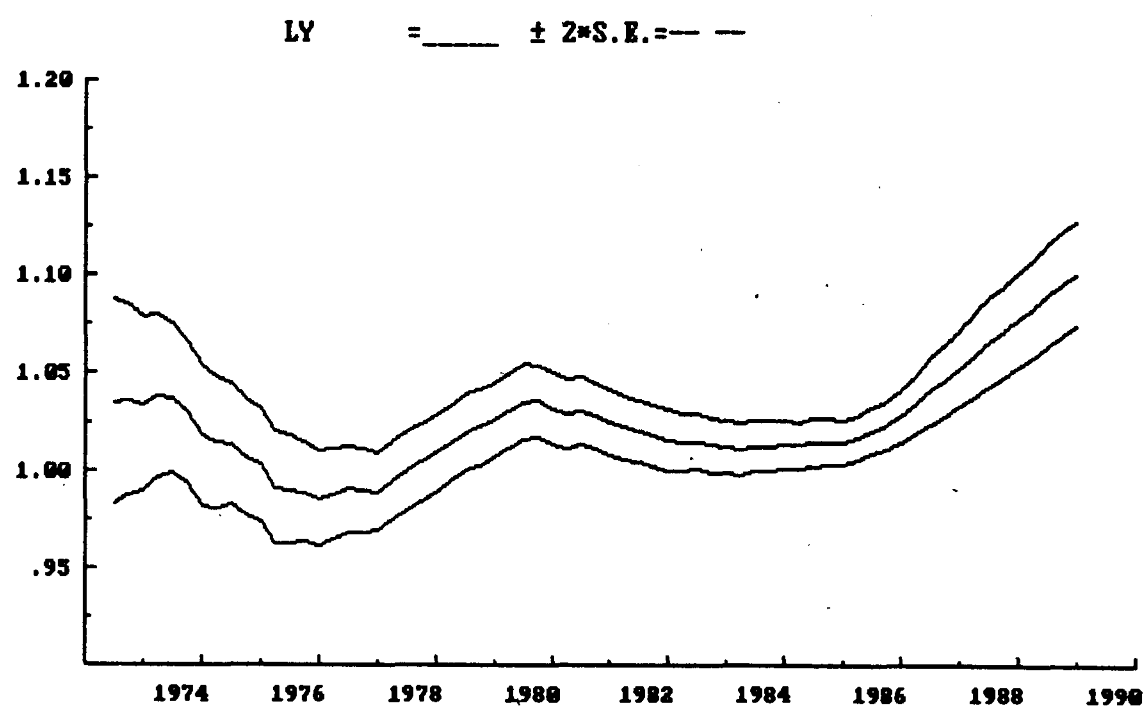

Figure 12. Recursive estimates of α_1 in the KA equation in the simple model, Table 4 (C4).

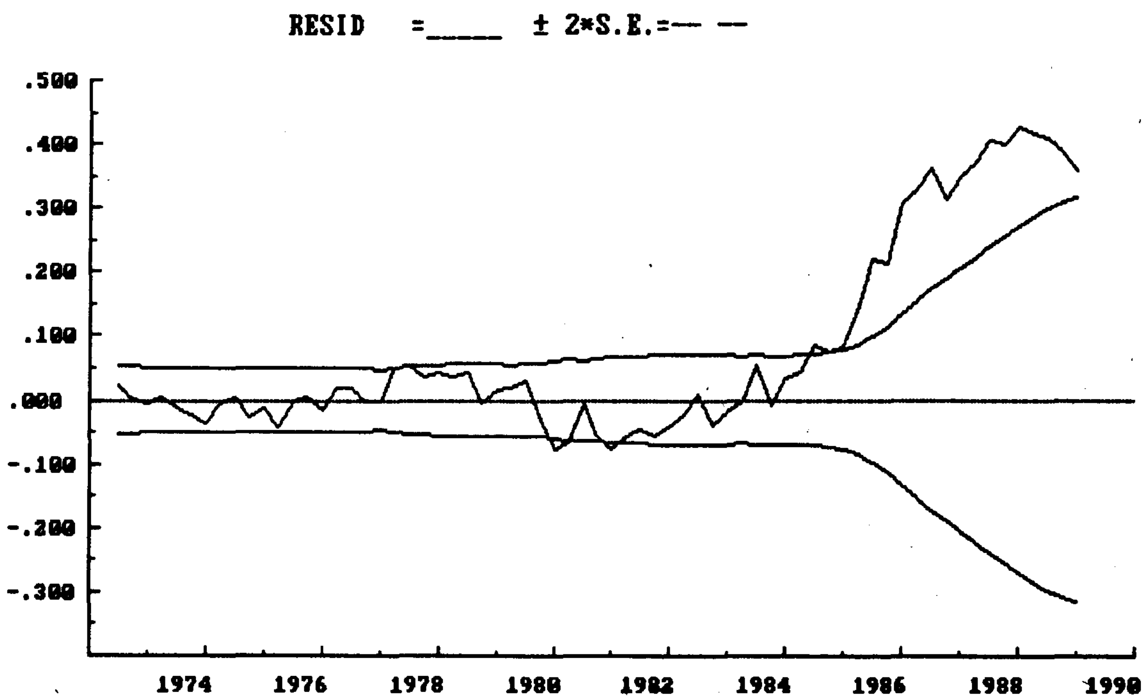

Figure 13. One-step residuals from the KB equation in the simple model, Table 4 (D4).

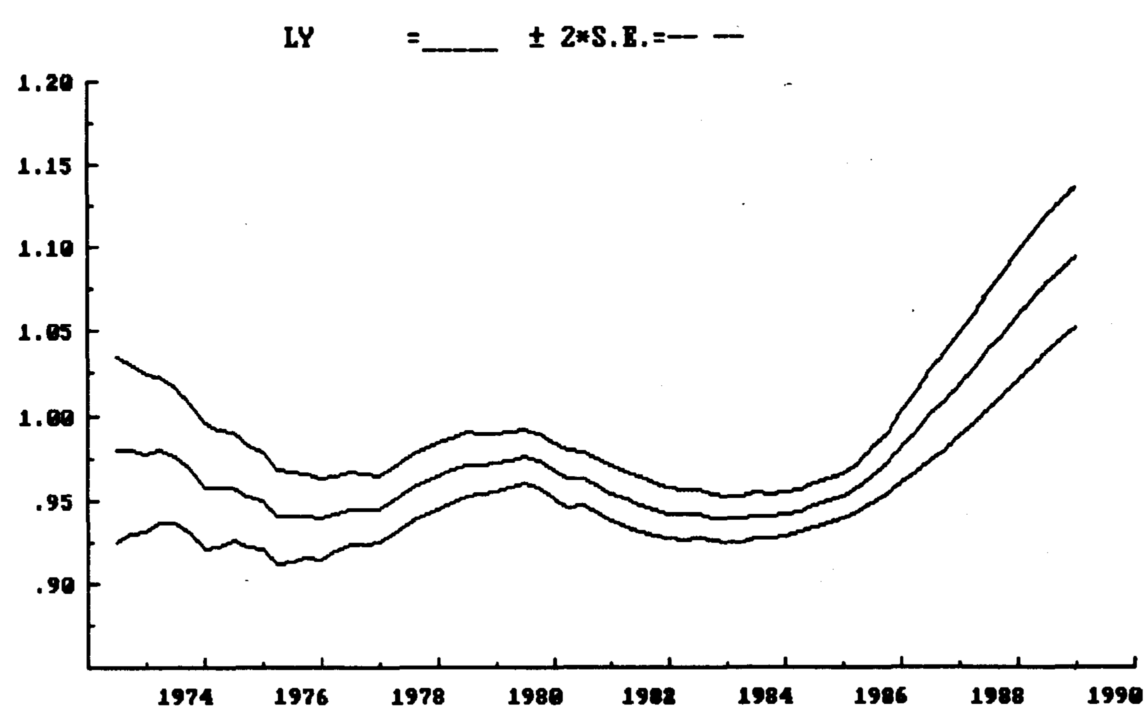

Figure 14. Recursive estimates of α_1 in the KB equation in the simple model, Table 4 (D4).

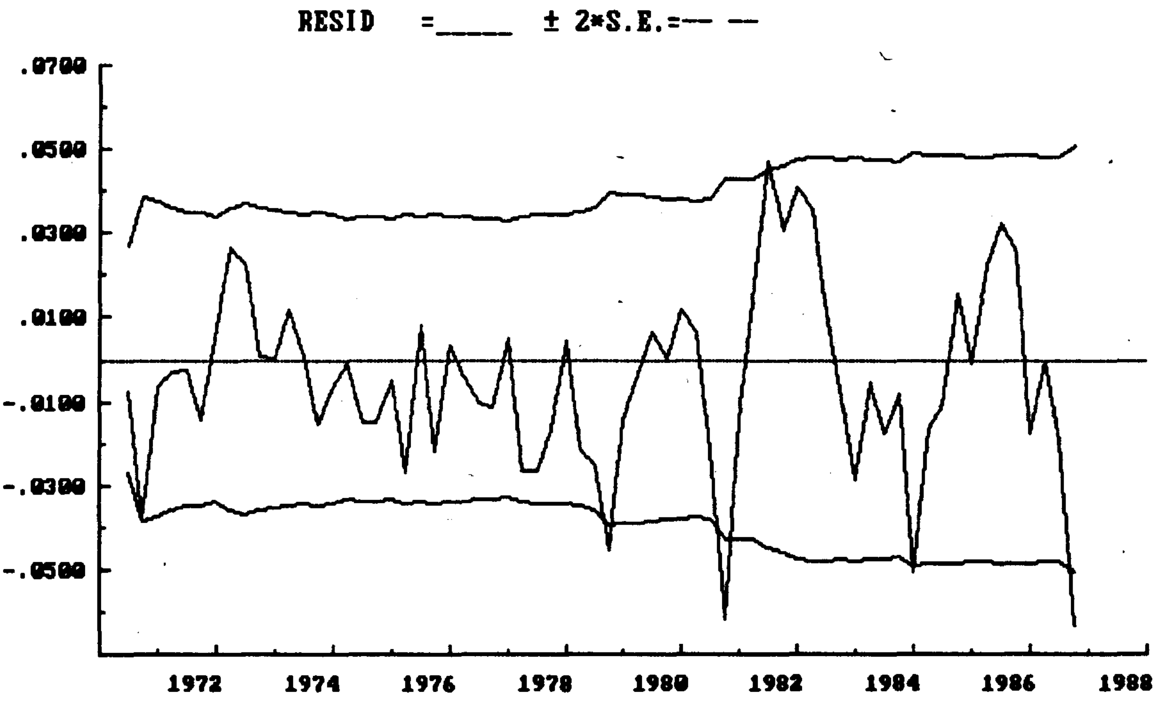

Figure 15. One–step residuals from the M1 equation in the augmented model, Table 5 (A4).

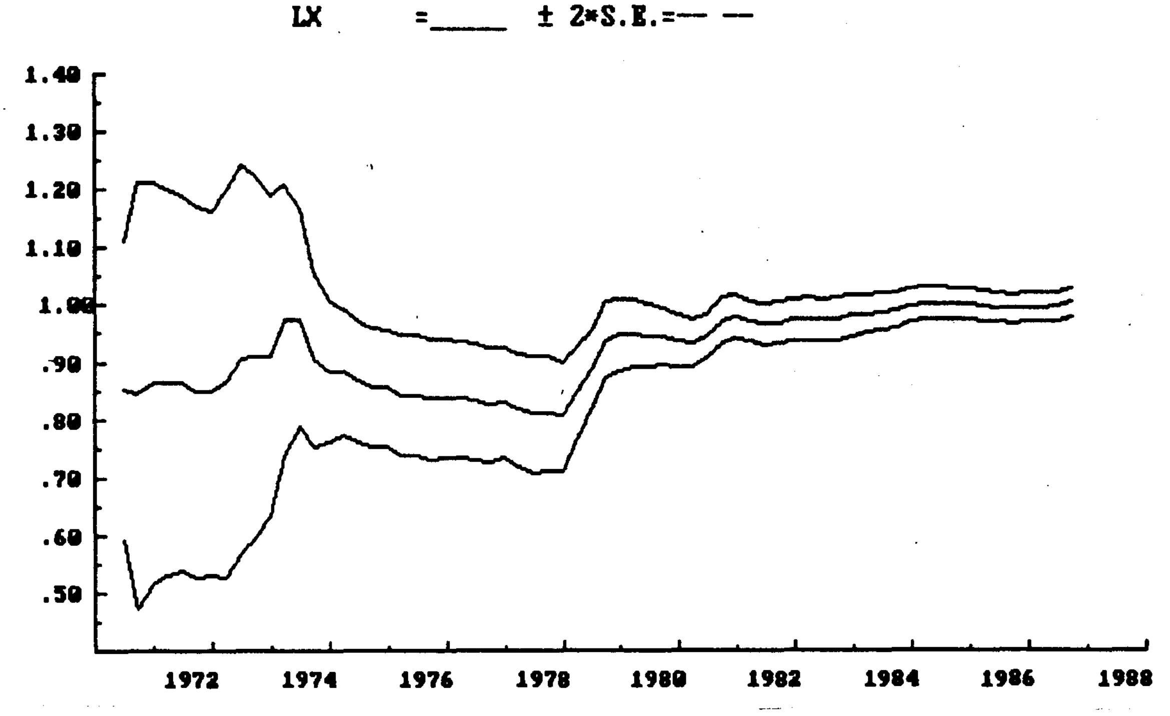

Figure 16. Recursive estimates of α_2 in the M1 equation in the augmented model, Table 5 (A4).

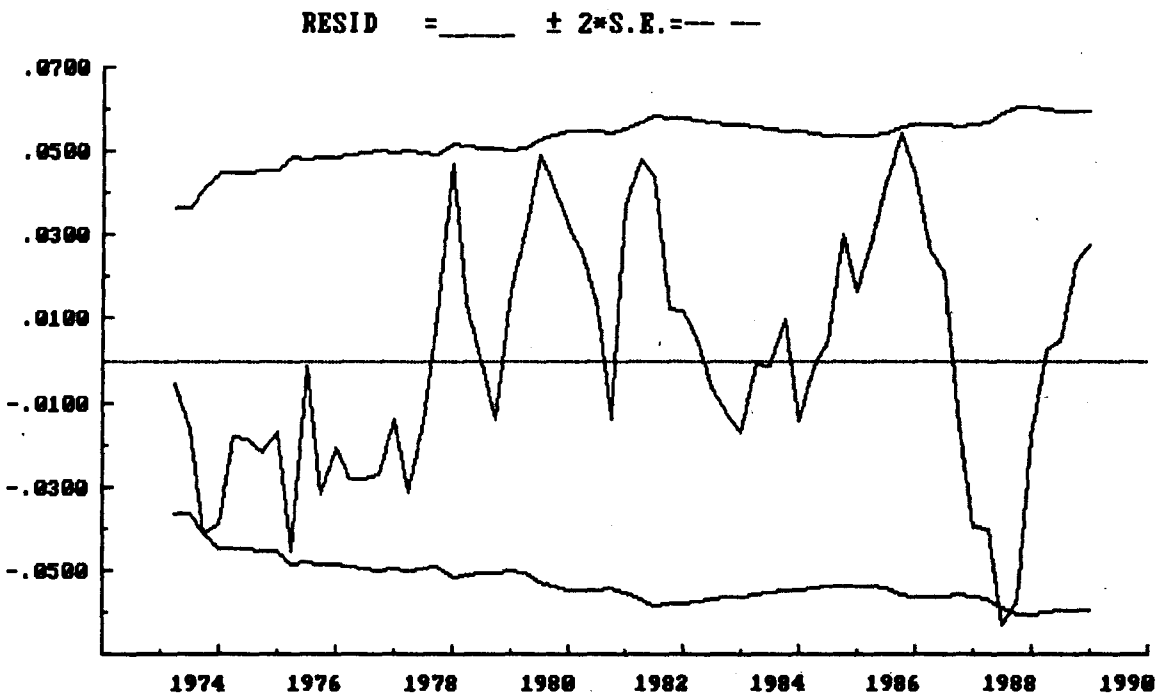

Figure 17. One–step residuals from the M2 equation in the augmented model, Table 5 (B4).

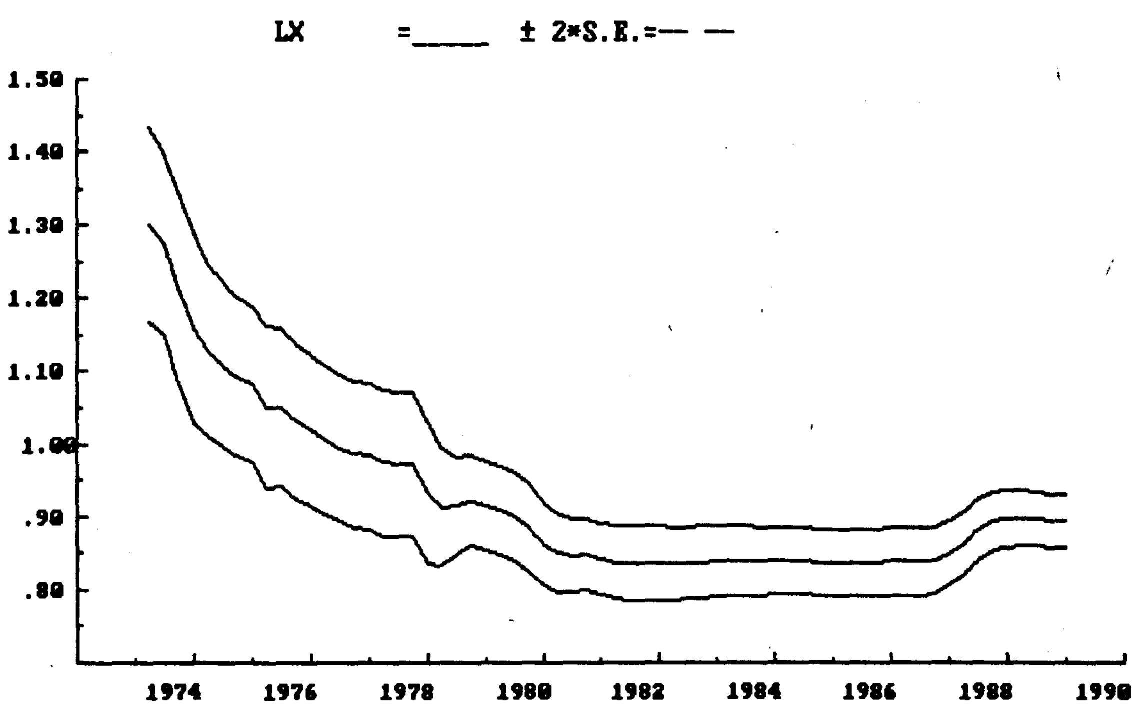

Figure 18. Recursive estimates of α_2 in the M2 equation in the augmented model, Table 5 (B4).

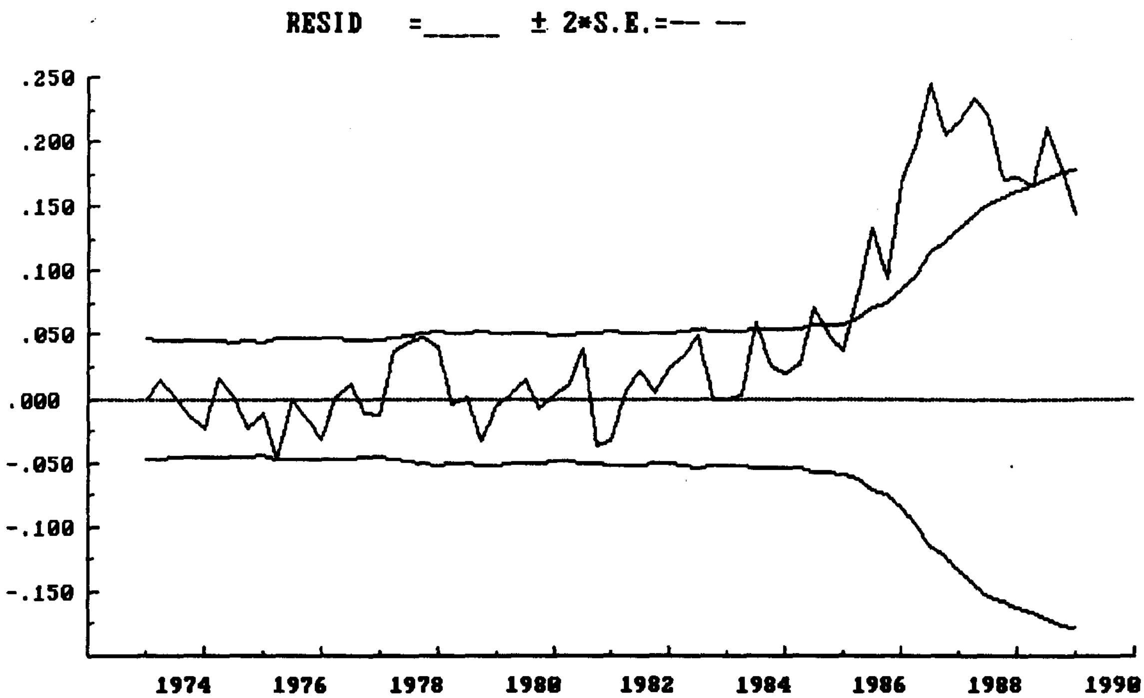

Figure 19. One–step residuals from the KA equation in the augmented model, Table 5 (C4).

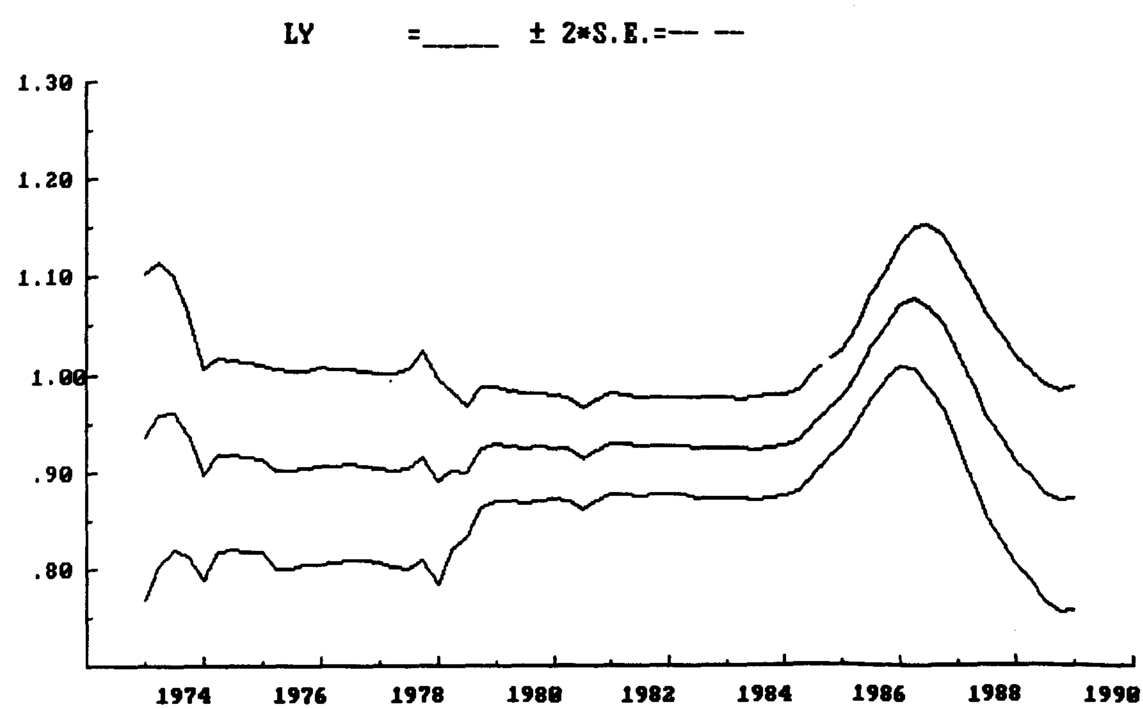

Figure 20. Recursive estimates of α_2 in the KA equation in the augmented model, Table 5 (C4).

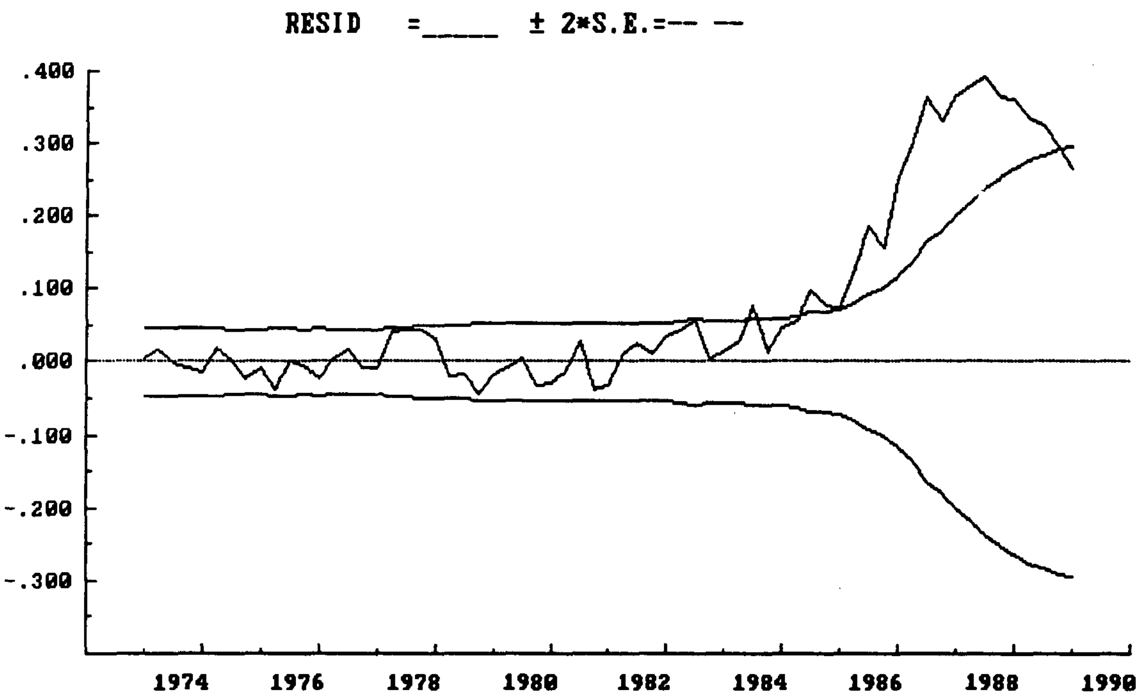

Figure 21. One–step residuals from the **KB equation** in the augmented model, Table 5 (D4).

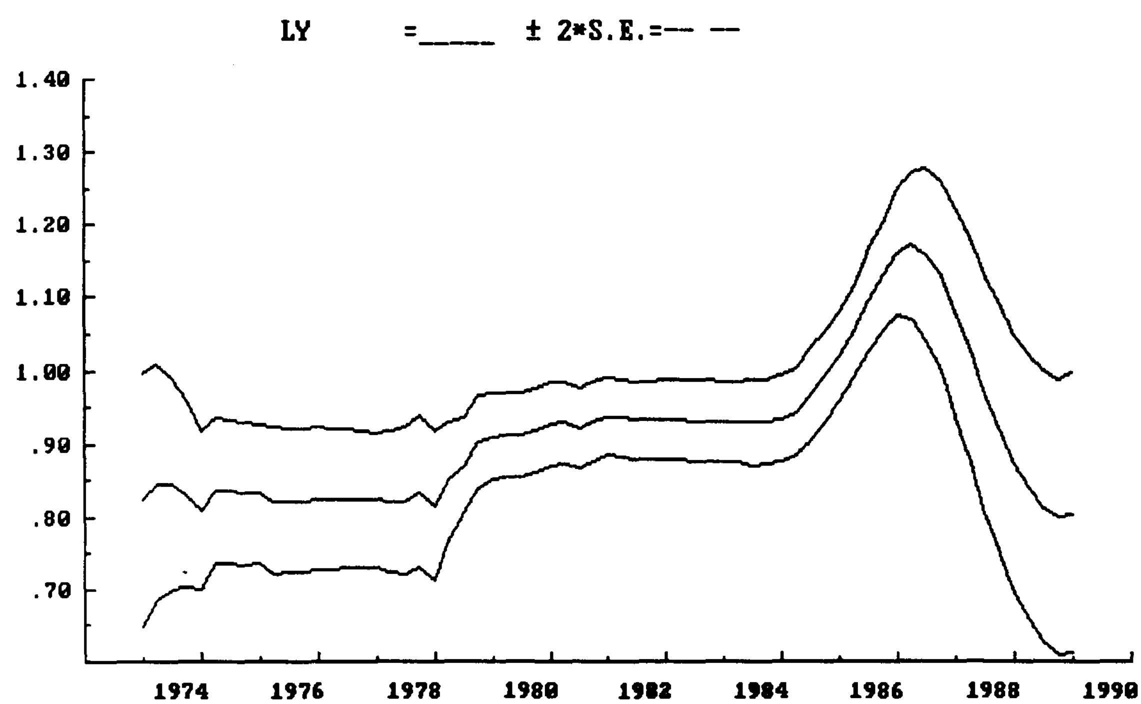

Figure 22. Recursive estimates of α_2 in the KB equation in the augmented model, Table 5 (D4).

CHAPTER 5

DYNAMIC MODELING OF THE DEMAND FOR NARROW MONEY IN NORWAY.

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DYNAMIC MODELING

OF

THE DEMAND FOR NARROW MONEY IN NORWAY.

by

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August 1991

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ABSTRACT

The role and stability of the demand for money are recurring issues in applied econometrics. Does a constant long–run demand for money function exist? If so, is money exogenous, and hence a policy variable, or endogenous?

The notion of cointegration provides a tool for identifying long–run relationships, to be embedded in dynamic error correction models with constant parameters, while the assumed exogeneity status of variables for the parameters of interest can be assessed by recently developed tests.

This paper derives a demand function for narrow money in Norway by applying these tools, starting out with a vector autoregressive representation that includes money (M1), prices, real expenditure and several interest rates.

Given a sample riddled with financial deregulation, changing monetary policy, and an economy switching its basis from industrial production towards oil exportation, one should not be surprised to find an unstable demand for money function. A conditional model with constant parameters is nevertheless established. A feature of the model is the crucial role played by the own yield, the interest rate on demand deposits. Finally tests for weak and super exogeneity are conducted. Prices, real expenditure and interest rates are super exogenous for the parameters of the demand for money. This means that simulation experiments can be conducted for the effects of monetary and fiscal policy on the demand for money.

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1. MOTIVATION

It is a widely held view that demand functions for narrow money with constant parameters do not exist; they are fragile econometric constructs, blown away with the changing policy regimes and financial innovations sweeping across the desks of econometricians at frequent intervals – especially during the last decade or so.[1]

Norway is an excellent testing field in this respect. Not only has the basis of the economy changed from industrial production to include a large oil exporting sector from the early 1970's on, but the monetary environment has been subject to numerous changes during the last twenty years. The changes relevant for the demand for money can be summarized as follows:

–While direct regulation had prevailed in the early part of the period considered here, the determination of interest rates was gradually left to market forces from the early 1980's.

–Targets for monetary policy changed from interest rates and credit volume to the exchange rate in 1986.

–A system of direct and selective controls of credit volume was replaced by a market–oriented policy in 1983.

The net result of these changes was a surge in bank lending, as well as in interest rates, creating a demand pressure and fueling inflation. The resulting pressure on the exchange rate caused a 10 % devaluation in May 1986. The "overreaction" from the private sector prompted a tight monetary policy in the form of higher required reserve ratios from the start of 1986 until credit regulations were totally abolished in 1988.

The story of credit market deregulation is told in Table 1.[2] All the regulations were in force at the end of 1983. Note in particular the reintroductions of loan controls and loan guarantee limits in 1986:I.

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[1] See Goldfeld and Sichel (1990) for international evidence on instability.

[2] Bårdsen and Klovland (1990) investigate the cointegration properties of money, credit and income in Norway.

The effect of a deregulation of credit rationing on money demand is uncertain.[3] During the regime of rationing, only a small number of liquid assets held by households could be converted quickly into money. Without rationing, many more assets – including human capital – could be converted quickly. Hence, larger precautionary money balances supposedly were held in the earlier period. Also, when considering loan applications, banks took previous saving into consideration, thus effectively reducing the demand for narrow money.

Everything said so far should suggest an unstable demand for money function for Norway, and this is indeed the conclusion reached by Fair (1987).

The present study presents a model of demand for narrow money in Norway with constant parameters estimated on data spanning the regime shifts described.

Section 2 discusses the choice of variables, presents the data and the econometric implementation of the demand function. Section 3 gives a brief methodological background for the estimation of the model in section 4, while section 5 investigates weak and super exogeneity and derives the consequences in the form of endogenous money and invariance of the parameters with respects to changes in the processes for prices, income and interest rates. Section 6 summarizes.

2. CHOOSING THE VARIABLES

A long–run money demand function sufficiently general to include most theoretical specifications is:

$$M = f(\underset{+}{P}, \underset{+}{X}, \underset{+}{RD1}, \underset{-}{\boldsymbol{R}}), \tag{1}$$

where M is demand for money; P is the price level; X is a measure of the volume of transactions, income, and/or wealth; $RD1$ is the own yield of holding money; and $\boldsymbol{R}$ is the vector of alternative costs to money holding. The expected signs of the coefficients, conditional on the demand equation being identified, are given below the variables. Whether a demand for money function is specified in real or nominal terms is irrelevant

[3] Goldfeld and Sichel (1990) identify financial deregulation as a source of narrow money demand functions breaking down.

as long as prices are allowed to enter the specification.

The choice of scale variable in a money demand function is usually between income, expenditure or wealth. Klovland (1990) finds a long–run wealth effect and a short–run influence from income in his study of Norwegian *M2* over 1968 – 1989.

The scale variable preferred in the present study is a measure of total final expenditure, or real absorption – i.e. real gross domestic expenditure, investment in ships and off–shore industry excluded; its implicit deflator is taken to be the price variable. The choice makes sense in modeling narrow money, since the impact from the oil sector would be more relevant using a broader definition of the money stock.

Narrow money has been interest bearing in Norway during the sample period and neglecting this fact would imply a potential misspecification. The own yield is represented by the interest rate on demand deposits.

As regards the opportunity cost of money holding, the literature is at least as diverse as for scale variables. In this study several interest rates are included, while the real–wage rate as a measure of the brokerage fee – following Laidler (1985, p. 68) – was discarded at an early stage. The long term bond yield is available together with the average rate on time deposits in banks. But considering Norway's position as a small open economy, an interest rate reflecting international influence is required – as stressed by Hamburger (1977). A natural candidate is the three month eurokrone rate. Although a surrogate measure for the earlier part of the period, the variable represents both a shadow price on credit in domestic markets as well as a covered yield on foreign assets.

Applied econometrics implies a choice of functional form. Here log–linearity is taken as a basis for an error correction model – building upon the work of Hendry (1979, 1985 and 1988) and Hendry and Ericsson (1991).

These assumptions specify equation (1) as

$$M = P^{\beta_1} \cdot X^{\beta_2} \cdot \exp\{\beta_3 \cdot RD1 + \beta_4 \cdot \mathrm{R}D2 + \beta_5 \cdot RL + \beta_6 \cdot RS\}, \tag{2}$$

so the long–run function to be estimated is

$$m = \beta_1 p + \beta_2 x + \beta_3 RD1 + \beta_4 RD2 + \beta_5 RL + \beta_6 RS, \tag{3}$$

where: M = narrow money; P = deflator of gross domestic expenditure X; X = real gross domestic expenditure; $RD1$ = interest rate on demand deposits; $RD2$ = interest rate on time deposits; RL = long–term private bond yield; RS = three–month eurokrone rate.[4]

Here and in the following, lower case letters of the regressors denote natural logarithms of the corresponding uppercase variables. The choice of interest rates in levels follows Trundle (1982).

The task at hand is amply illustrated in Figure 1, where the behavior of the inverse velocity, in logarithmic scale, is shown over time. The constant long run relationship between money and income – in other words: cointegration – falls totally apart after 1983. But such a changing trend is also evident for the interest rates in Figure 2. The own rate in particular starts growing toward the end of the sample period. Although narrow money has had a positive own yield throughout the sample period, the importance of the variable is clearly increasing after the credit deregulation. Consequently, money demand functions omitting this variable are likely to suffer a breakdown after 1983.

This preliminary examination suggests that the relationship between real money and real expenditure has to be augmented with other variables in order to obtain long–run money demand stability. The interest rates are natural candidates and the own yield in particular.

3. ECONOMETRIC APPROACH

3A. Systems Cointegration Analysis

Following Hendry and Mizon (1990), the starting point of estimation is a congruent statistical system of unrestricted reduced forms:

$$z_t = \sum_{j=1}^{p} \Pi_j z_{t-j} + \Phi D_t + v_t,\ v_t \sim IN(0, \Omega), \tag{4}$$

[4] Detailed definitions of the data are given in the appendix.

where z_t is a $(n \times 1)$ vector of $I(1)$ and/or $I(0)$ variables and D_t represents deterministic components.[5]

Utilizing $\Delta z_t \equiv z_t - z_{t-1}$, a convenient reparameterization of (4) is

$$\Delta z_t = \sum_{i=1}^{p-1} \Pi_i^* \Delta z_{t-j} + \Pi^* z_{t-p} + \Phi D_t + v_t, \tag{5}$$

with

$$\left\{\begin{matrix} \Pi_i^* & = \sum_{j=1}^{i} \Pi_j - I \\ \text{and} & \\ \Pi^* & = \sum_{j=1}^{p} \Pi_j - I \end{matrix}\right\}. \tag{6}$$

This is the VAR of Johansen (1988) and Johansen and Juselius (1990) used to investigate the cointegration properties of the system. Since v_t is stationary, the rank ϱ of the "long–run" matrix Π^* determines how many linear combinations of z_t are stationary. If $\varrho = n$ all z_t are stationary, while if $\varrho = 0$ so that $\Pi^* = 0$, Δz_t is stationary and all linear combinations of $z_t \sim I(1)$. For $0 < \varrho < n$, there exist ϱ cointegrating vectors, meaning ϱ stationary linear combinations of z_t. In that case Π^* can be factored as $\alpha\beta'$ with both α and β being $(n \times \varrho)$ matrices. The cointegrating vectors of β are the error correction mechanisms in the system, while α contains the adjustment parameters.[6] Johansen and Juselius (1990) provide a full procedure for estimation and testing within this framework.

3B. Exogeneity Concepts[7]

The joint distribution of (4) can be factorized into a conditional distribution for y_t given x_t and a marginal distribution for x_t. If the parameters of interest are only functions of the parameters of the conditional distribution, and if the parameters of the

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[5] The notation *I(1)* means "integrated of order 1". Introductions to integration, and cointegration which is encountered later on, can be found in Hendry (1986) and Granger (1986).

[6] This result is known as Granger's Representation Theorem. Engle and Granger (1987) gives the original result, while an extended version can be found in Hylleberg and Mizon (1989).

[7] See Engle, Hendry and Richard (1983) for the original exposition.

conditional and marginal distributions are variation free, then the parameters of interest are weakly exogenous.[8] Further, if $\boldsymbol{z}_t$ is weakly exogenous for the parameters of interest and there is no feedback from y_t to $\boldsymbol{z}_t$, then $\boldsymbol{z}_t$ is strongly exogenous for the parameters of interest. And finally, if $\boldsymbol{z}_t$ is weakly exogenous for the parameters of interest and they are invariant to changes in the parameters of the marginal distribution for $\boldsymbol{z}_t$, then $\boldsymbol{z}_t$ is super exogenous for the parameters of interest, to the class of changes occurring in the sample. A test of weak exogeneity for the elements of $\boldsymbol{\beta}$ is due to Johansen (1990) and illustrated in Johansen (1991). Weak exogeneity for all the conditional parameters can be tested via the tests of super exogeneity of Engle and Hendry (1990) presented in section 5.

3C. Conditional model analysis

In the case of a conditional model (4) reduces to the dynamic linear regression model:

$$y_t = \boldsymbol{\delta}_0'\boldsymbol{z}_t + \sum_{j=1}^{p}(\gamma_j y_{t-j} + \boldsymbol{\delta}_j'\boldsymbol{z}_{t-j}) + \boldsymbol{\varphi}'\boldsymbol{d}_t + u_t. \tag{7}$$

Some general considerations about dynamic specification, of equal validity in the individual equations in the systems analyzed above, can be illustrated by means of (7). The conditional model representation of (5) is:

$$\begin{aligned}\Delta y_t = \boldsymbol{\delta}_0'\boldsymbol{z}_t &+ \sum_{i=1}^{p-1}(\gamma_i^* \Delta y_{t-i} + \boldsymbol{\delta}_i^{*\prime}\Delta\boldsymbol{z}_{t-i}) \\ &+ \gamma^* y_{t-p} + \boldsymbol{\delta}^{*\prime}\boldsymbol{z}_{t-p} + \boldsymbol{\varphi}'\boldsymbol{d}_t + u_t,\end{aligned} \tag{8}$$

where

$$\left\{\begin{matrix}\gamma_i^* = \sum_{j=1}^{i}\gamma_j - 1 \\ \text{and} \\ \boldsymbol{\delta}_i^* = \sum_{j=0}^{i}\boldsymbol{\delta}_j\end{matrix}\right\}. \tag{9}$$

Equation (9) shows that the short run dynamics of (8) have a clear interpretation as

[8] Loosely speaking, "variation free" means that the parameters of the two distributions are free to take any value independently of each other.

the adjustment towards equilibrium; the parameters are implicit interim multipliers. See also Hylleberg and Mizon (1989). But (8) is also an error correction model:

$$\Delta y_t = \boldsymbol{\delta}_0'\Delta \boldsymbol{x}_t + \sum_{i=1}^{p-1}(\gamma_i^* \Delta y_{t-i} + \boldsymbol{\delta}_i^{*\prime}\Delta \boldsymbol{x}_{t-i})$$
$$+ \alpha\boldsymbol{\beta}'[y_{t-p}\ \boldsymbol{x}_{t-p}]' + \boldsymbol{\varphi}'\boldsymbol{d}_t + \boldsymbol{u}_t, \tag{10}$$

where $\alpha = \gamma_p^*$, and estimates of the elements in the cointegrating vector are:[9]

$$\hat{\beta}_k = \hat{\delta}_{kp}^*/\hat{\gamma}_p^*. \tag{11}$$

The large sample variance of $\hat{\beta}_k$ can be estimated by

$$\hat{\mathrm{var}}(\hat{\beta}_k) = (\hat{\gamma}_p^*)^{-2}[(\hat{\beta}_k)^2\hat{\mathrm{var}}(\hat{\gamma}_p^*) + \hat{\mathrm{var}}(\hat{\delta}_{kp}^*) + 2\hat{\beta}_k\hat{\mathrm{cov}}(\hat{\gamma}^*, \hat{\delta}_{kp}^*)]. \tag{12}$$

Details of this derivation can be found in Bårdsen (1989).

Equation (8) has two advantages over (7). First, it shows both the long–run solution of the model and the adjustment towards long–run equilibrium. And second, it may be a more efficient starting point for conducting a specification search for a parsimonious model under the null hypothesis of an error correction representation of the data generation process. But (8) and (10) can be inconvenient to use in the early stages of a simplification search since a natural first step is to restrict lag lengths, which means testing $\hat{\delta}_{kp}^* = \hat{\delta}_{k(p-1)}^*$, say, and could consequently lead to a lot of reformulations.

The testing of lag lengths is easier with the error correction term on the first lag, as in the usual exposition of error correction models: Rewrite equation (8) as

$$\Delta y_t = \boldsymbol{\delta}_0'\Delta \boldsymbol{x}_t + \sum_{i=1}^{p-1}(\gamma_i^\dagger \Delta y_{t-i} + \boldsymbol{\delta}_i^{\dagger\prime}\Delta \boldsymbol{x}_{t-i})$$
$$+ \gamma_p^* y_{t-1} + \boldsymbol{\delta}_p^{*\prime}\boldsymbol{x}_{t-1} + \boldsymbol{\varphi}'\boldsymbol{d}_t + u_t, \tag{13}$$

with

[9] This is the nonlinear least squares estimator investegated by Stock (1987). An independent derivation can be found in Bårdsen (1989) together with the variance formula given below. See also Johansen and Juselius (1990) and Johansen (1990).

$$\begin{bmatrix} \gamma_i^\dagger = -\sum\limits_{j=i+1}^{p} \gamma_j - 1\,, \\ \text{and} \\ \delta_i^\dagger = -\sum\limits_{j=i+1}^{p} \delta_j,\; i = 1,\ldots, p-1 \end{bmatrix}. \tag{14}$$

See also Harvey (1990, p. 281).

The consequence of moving the levels terms around in the unrestricted error correction model is clear from (9) and (14). In general, if the levels terms are on lag i, the coefficients of the differenced variables are increasing partial sums of the parameters of the dynamic linear regression model until lag $i-1$. From lag i onwards the coefficients of the differenced variables are decreasing partial sums multiplied by –1. The longest sum runs from lag $i+1$ to the final lag, while the shortest is the final lag. So in the form (13) sequential testing of maximum lag orders is straightforward. The disadvantage of this parameterization is that all the lagged short–run dynamics change sign compared to (8), so the implicit interim multiplier interpretation is lost.

Standard inference theory assumes weakly stationary data series but can be valid even if the series in equations (4) and (7) are $I(1)$. The first condition for this to hold, from Sims, Stock and Watson (1990), is formulated by Stock and West (1988, p. 86) as: "...the usual testing procedures are asymptotically valid if a regression can be rewritten so that the coefficients of interest are on stationary, zero mean regressors". The second result needed is due to Park and Phillips (1989, p. 117). Their theorem 5.3 ensures asymptotically normally distributed parameter estimates if the regressors are cointegrated.

The distribution of the cointegrating vectors is the final uncertainty. Phillips (1988, 1991), Phillips and Loretan (1991) and Johansen (1990) investigate inference in models such as (8). Their results show that the limit distribution of the long–run coefficients is a mixture of normals. A pilot simulation study in Bårdsen (1990) indicated that the normal distribution could be used for inference. Both the conditional and the VAR approach will be utilized in the next section.

As Belsley notes, the data are extremely collinear. Regressing $\boldsymbol{x}_3$ on $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ gives $\tilde{R}^2_{\mathrm{h}} = 0.99999$ while the condition number $\kappa(X) = 1342.$[12] The collinearity of each parameter is assessed to be $C(\hat{\beta}_1) = 2.5 \cdot 10^{-6}$, $C(\hat{\beta}_2) = 5 \cdot 10^{-6}$ and $C(\hat{\beta}_3) = 5 \cdot 10^{-6}$. So at this point all measures reach the same conclusion.

A reparameterization along the lines of the earlier examples could be

$$\boldsymbol{y} = (\beta_1+\beta_2+\beta_3)\boldsymbol{x}_1 + (\beta_2+\beta_3)(\boldsymbol{x}_2-\boldsymbol{x}_1) + \beta_3(\boldsymbol{x}_3-\boldsymbol{x}_2) + \epsilon,$$

which implies

$$P = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}.$$

The regression gives

$$\begin{gathered} \boldsymbol{y} = \underset{(0.001)}{2.699}\boldsymbol{x}_1 - \underset{(0.784)}{0.493}(\boldsymbol{x}_2-\boldsymbol{x}_1) - \underset{(0.555)}{1.302}(\boldsymbol{x}_3-\boldsymbol{x}_2) \\ R^2 = 0.31,\ \hat{\sigma}^2 = 0.308 \cdot 10^{-4}, \end{gathered}$$

but now $\kappa(X) = 2.42$. This is because the condition number only considers correlation between variables, while $C(\hat{\beta}^*_{\mathrm{h}})$ analyzes collinearity relative to the parameters.

The collinearity of each new coefficient is found to be $C(\widehat{\beta_1+\beta_2+\beta_3}) = 1$, $C(\widehat{\beta_2+\beta_3}) = 2.5 \cdot 10^{-6}$ and $C(\hat{\beta}_3) = 5 \cdot 10^{-6}$. So the sum of the parameters are estimated with extremely high precision ($\beta_1+\beta_2+\beta_3 \equiv 2.7$), while the other coefficients are badly determined.

4. CONCLUDING REMARKS

The subject has been collinearity and different parameterizations of a model. The main point is that collinearity is not well defined as correlation between variables, since a different parameterization can produce variables with different correlation, while inference for some of the parameters remain unchanged. Regarding collinearity as a problem of obtaining precise estimates of the parameters of interest seems more sensible. Since no measures exist that focus cleanly on this aspect, such a measure has violated. Several of the equations exhibit autocorrelation, while the equations of the interest rates have non–normal residuals. This is not surprising given the many shocks to the system in the form of changing policy regimes and financial innovation described in section 1. These shocks can have altered the processes driving the variables.

Estimation with multivariate recursive least squares facilitates such stability analysis. Figures 3 to 6 provides a graphical account of "Break–point" F–tests: Chow–tests where the equations of the VAR at each period are tested for stability against the end period of 89:IV.[11] The horizontal line represents the critical values at the 5 % level. The test sequence fails to reject parameter stability of the money equation throughout the estimation period while the models for the interest rates are highly nonconstant.

Since the money demand equation passes all the diagnostics I choose to condition on the other variables instead of trying to model the shocks within the system. assuming those variables are weakly exogenous for the parameters of the money demand equation. The apparent nonconstancy of the marginal models will form the basis of tests of the validity of the assumption of weak exogeneity in section 5.

4B. Conditional Model Analysis

Table 3 displays the result from estimating the general model in the form of equation (13). "*RESET F(1,df2)*" is Ramsey's test for correct specification – performed by testing the relevance of adding the squared predicted values to the original model. The model is well determined according to the diagnostics, although there is some sign of autocorrelation.

The next step is to test and impose restrictions on the elements of cointegrating vector, which from equation (3) are:

$$m = \beta_1 p + \beta_2 x + \beta_3 RD1 + \beta_4 RD2 + \beta_5 RL + \beta_6 RS. \qquad (15)$$

A natural question at this point is what the testing sequence should be. A purely

[11] Only three lags of *RS* are used in these calculations, due to the limitations of 40 variables in PC–FIML.

statistical answer would probably be to impose all the restrictions and use an F–test. The strategy followed is a pragmatic mixture of statistical criteria and economic theory. First, homogeneity of real money with respect to real income is tested together with the exclusion of *RD2* and *RS* in the long run solution, and then the remaining long–run coefficients are estimated and imposed. The restrictions on the cointegrating vector are all easily accepted, as Table 4 shows. In the long run real money is homogeneous in real income. There appear to be considerable differences in the liquidity, and riskiness, between money and bonds, since a percentage point change in the own yield must be offset by at least three percentage points change in *RL*, the alternative yield on bonds, if no portfolio adjustment is to take place.

Next, the error correction mechanism is moved back to the longest lag in order to preserve the interim multiplier interpretation and this restricted model is the starting point of the "general–to–specific" search.[12] The building of the short run dynamics to obtain a parsimonious model with interpretable parameters is the most difficult part. The general approach is that of Hendry and Richard (1982, 1983), but the most important guidelines used are parsimony, robustness, and an economic interpretation. I have also opted for short lags and simple restrictions. The strive for parsimony has for instance resulted in the exclusion of seasonal dummies, since they are unnecessary – a striking result with seasonally unadjusted data.[13]

These considerations have reduced Table 3 to equation (16):

[12] I did try a simplification search with the error correction mechanism on the first lag, but it proved less successful.

[13] The F–test for adding three seasonal dummies yields $F(3,80) = 0.608$.

$$\begin{aligned}\Delta \hat{m}_t = & \underset{(0.146)}{0.540}\Delta p_t + \underset{(0.025)}{0.264}\Delta x_t - \underset{(0.024)}{0.058}\Delta (m-p-x)_{t-1} - \underset{(1.295)}{3.919}\Delta (RD2-RD1)_t \\ & - \underset{(0.047)}{0.117}RS_t - \underset{(0.004)}{0.290}(m-p-x-4.7RD1+1.5RL)_{t-5} - \underset{(0.011)}{0.044}\end{aligned} \tag{16}$$

$$R^2 = 0.60,\ \sigma = 0.0134,\ DW = 2.29,\ RSS = 0.0150,\ Normality\ \chi^2(2) = 2.87,$$
$$AR\ 1-5\ F(5,78) = 1.72,\ ARCH\ 4\ F(4,75) = 1.66,\ Hetero\ F(12,70) = 0.44,$$
$$RESET\ F(1,82) = 1.66,\ Func.\ form\ F(27,55) = 0.88.$$

"*Hetero F(df1,df2)*" is White's test for heteroskedasticity and tests the joint significance in a regression of the squared residuals on the regressors and their squares. The validity of the chosen functional form is assessed through the "*Func. form F(df1,df2)*" test due to White (1980): The squared residuals are regressed against all the squares and cross products of the regressors.

Note that whether one estimates real or nominal demand for money as long as inflation is included in the specification is a matter of indifference, the two are numerically and analytically equivalent. I will therefore continue to use the nominal version in the following.

According to equation (16), the demand for nominal money growth per quarter depends negatively upon the money market rate and the quarterly change in the spread between own yield and the alternative yield on time deposits.[14] There is an immediate positive effect from growth of prices and real expenditure, while there is a smaller adjustment to changes in the money–income ratio in the previous quarter. Finally there is the adjustment to deviations from the long–run desired relation between real money, real income, the own yield and the maximum alternative yield for long term investments. At least three times as large a yield on bonds over money is required before it is considered worthwhile to adjust the portfolio in the long–run. So in the short–run agents speculate in the money market and change their money holdings between demand and savings deposits, while in the long–run the the portfolio is adjusted between money and bonds.

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[14] The presence of *RS* is a bit puzzling considering the results of Table 4. One explanation could be that *RS* is stationary.

The "Break–point" F–tests, from recursive least squares estimation, in Figure 7 fail to reject parameter nonconstancy over the estimation period. And the standard error of the equation is virtually unchanged from 1976 on, as Figure 8 demonstrates. Consequently the model suggests that no structural change in agent behavior has taken place as a result of the credit liberalization. But this matter can be more thoroughly investigated by means of testing the invariance of the parameters.

5. TESTING EXOGENEITY[15]

The exogeneity status of the regressors in demand for money studies are always controversial. While Cooley and LeRoy (1981) argue that simultaneity is important, Laidler (1985) takes the opposite view. Bias due to failure of weak exogeneity is seldom significant when tested for – see for example Poloz (1980), Gregory and McAleer (1981), and Klovland (1983, 1990). But exogeneity has wider implications.

So far prices, expenditure and interest rates have been taken to be weakly exogenous; hence they can be conditioned upon for statistical analysis of the parameters of the demand equation. This is intuitively reasonable in the case of a small open economy with a fixed exchange rate. If this assumption is valid and the parameters are invariant to the class of interventions occurring during the sample period, the parameters are super exogenous – see Engle, Hendry and Richard (1983). This means that policy analysis can be performed by suitably changing the processes driving these variables. There are two ways to investigate this question, which are explained below.

5A. Testing Constancy of Marginal Models

If the conditional model has constant parameters, as shown, but the marginal models have nonconstant parameters, then the conditional model parameters could not depend upon the marginal model parameters. This is the test of Hendry (1988).

The following marginal models were estimated from univariate fifth–order

[15] The approach adopted in this section owes a lot to Hendry and Ericsson (1991).

autoregressive processes:[16]

$$\Delta\hat{p}_t = \underset{(0.098)}{0.361}\Delta p_{t-1} + \underset{(0.060)}{0.135}\Delta(p_{t-2}+p_{t-4}) + \underset{(0.007)}{0.030}[VAT(p)+FREEZE]_t$$
$$+ \underset{(0.003)}{0.006} + \underset{(0.003)}{0.006}Q_{t-1} - \underset{(0.003)}{0.004}Q_{t-2} - \underset{(0.003)}{0.001}Q_{t-3} \tag{17}$$

$R^2 = 0.39$, $\sigma = 0.0093$, $DW = 1.86$, $RSS = 0.0071$, *Normality* $\chi^2(2) = 0.22$,
AR 1–5 $F(5,78) = 0.21$, *ARCH 4* $F(4,75) = 0.37$, *Hetero* $F(8,74) = 0.63$,
RESET $F(1,82) = 0.32$, *Func. form* $F(16,66) = 0.32$.

$$\Delta\hat{x}_t = -\underset{(0.065)}{0.228}\Delta_3\Delta x_{t-1} + \underset{(0.014)}{0.071}[VAT(x)-0.5FREEZE]_t$$
$$-\underset{(0.008)}{0.094} + \underset{(0.019)}{0.120}Q_{t-1} + \underset{(0.006)}{0.121}Q_{t-2} + \underset{(0.008)}{0.176}Q_{t-3} \tag{18}$$

$R^2 = 0.95$, $\sigma = 0.0177$, $DW = 1.83$, $RSS = 0.0263$, *Normality* $\chi^2(2) = 1.43$,
AR 1–5 $F(5,79) = 1.33$, *ARCH 4* $F(4,76) = 0.58$, *Hetero* $F(7,76) = 1.33$,
RESET $F(1,83) = 0.68$, *Func. form* $F(11,72) = 0.93$.

$$\Delta\widehat{(RD2-RD1)}_t = \underset{(0.094)}{0.597}\Delta(RD2-RD1)_{t-1} - \underset{(0.098)}{0.184}\Delta^2(RD2-RD1)_{t-2} + \underset{(0.0002)}{0.001} \tag{19}$$

$R^2 = 0.32$, $\sigma = 0.0971$, $DW = 2.00$, $RSS = 0.8199$, *Normality* $\chi^2(2) = 87.96$,
AR 1–5 $F(5,82) = 3.26$, *ARCH 4* $F(4,79) = 0.15$, *Hetero* $F(4,82) = 0.12$,
RESET $F(1,86) = 0.01$, *Func. form* $F(5,81) = 0.10$.

$$\hat{RS}_t = \underset{(0.086)}{0.672}RS_{t-1} + \underset{(0.084)}{0.176}RS_{t-3} - \underset{(0.008)}{0.010} + \underset{(0.005)}{0.011}Q_{t-1} + \underset{(0.005)}{0.005}Q_{t-2} + \underset{(0.005)}{0.012}Q_{t-3} \tag{20}$$

$R^2 = 0.69$, $\sigma = 0.0179$, $DW = 1.95$, $RSS = 0.0270$, *Normality* $\chi^2(2) = 25.49$,
AR 1–5 $F(5,79) = 0.46$, *ARCH 4* $F(4,76) = 0.96$, *Hetero* $F(7,76) = 0.70$,
RESET $F(1,83) = 0.18$, *Func. form* $F(14,69) = 0.85$.

The dummy *VAT(p)* is unity in 1970:I and models the effect of the introduction of VAT on inflation, while *VAT(x)* is 1 in 1969:IV and –1 in 1970:I to capture the effect of the pre–announced introduction of VAT in 1970:I on expenditure demand. The

[16] The variable $\Delta(RD2-RD1)$ is multiplied by 100 in the following.

FREEZE dummy is unity in 1980:2 to represent the lifting of the wage and price freeze from 1979:I to 1980:I.

Figures 9 to 12 give sequential *F*–tests for the constancy of the parameters of the marginal models. The "Forecast" test for *RS* in Figure 12 evaluates model stability against an early period (72:III), while the "Break–point" tests in Figures 9 to 11 use the end of the sample (89:IV) as evaluation point. Constancy is easily rejected for all the models, implying the super exogeneity of the variables of the conditional model for the class of interventions occurring during the sample period.

5B. Testing Invariance

A different class of tests of weak and super exogeneity have been developed by Engle and Hendry (1990). If the marginal processes are constant, we can use Wu – Hausman tests for independence between the conditioning variables and the residuals. It implies testing the significance of the residuals from the marginal model, or reduced form, in the conditional model. And if the marginal processes have changed over the sample period, a test of invariance is to model the interventions in the marginal models, and test for the significance of this model part in the conditional model. If the parameters of the conditional model are invariant to the changes in the marginal processes, including these changes should have add no explanatory power.

We know by now that well specified marginal models are unavailable from the information set used so far. The set is therefore augmented with the following instruments: *REUR* = the eurodollar rate; *RBN* = the marginal lending rate of the central bank; *pimp* = prices of imports; *imp* = the volume of imports of the main trading partners of Norway weighted using the weights of the official currency basket; *cg* = real public expenditure; *ig* = real public investment.

Using this information, the following marginal models are obtained:

$$\begin{aligned}\Delta \hat{p}_t = &\underset{(0.042)}{0.326}\Delta(pimp-p)_{t-3} + \underset{(0.011)}{0.071}\Delta x_{t-2} + \underset{(0.013)}{0.105}\Delta x_{t-3} + \underset{(0.184)}{0.337}\Delta RL_{t-3} \\ &- \underset{(0.035)}{0.189}[p-0.35(m+x)-1.7RL-0.18pimp]_{t-4} \\ &+ 0.031[VAT(p)+0.5FREEZE+D76{:}II]_t - \underset{(0.300)}{1.595} \\ &+ \underset{(0.007)}{0.019}Q_{t-1} + \underset{(0.006)}{0.006}Q_{t-2} + \underset{(0.008)}{0.006}Q_{t-3} \end{aligned} \tag{21}$$

$R^2 = 0.71$, $\sigma = 0.0064$, $DW = 2.03$, $RSS = 0.0034$, *Normality* $\chi^2(2) = 0.85$, *AR 1–5* $F(5,78) = 1.02$, *ARCH 4* $F(4,75) = 0.45$, *Hetero* $F(12,70) = 0.60$, *RESET* $F(1,82) = 0.96$, *Func. form* $F(23,59) = 0.50$.

$$\begin{aligned}\Delta \hat{x}_t = &- \underset{(0.065)}{0.364}\Delta x_{t-1} + \underset{(0.016)}{0.067}\Delta ig_t + \underset{(0.041)}{0.147}\Delta imp_t \\ &- \underset{(0.008)}{0.053}[x-1.5(cg-ig+imp)+20RD2]_{t-2} \\ &+ \underset{(0.006)}{0.069}[VAT(x)_t+0.5(D78{:}I_t+D78{:}I_{t-1}-FREEZE_t)] \\ &+ \underset{(0.038)}{0.145} + \underset{(0.014)}{0.085}Q_{t-1} + \underset{(0.004)}{0.107}Q_{t-2} + \underset{(0.008)}{0.143}Q_{t-3} \end{aligned} \tag{22}$$

$R^2 = 0.98$, $\sigma = 0.0124$, $DW = 2.12$, $RSS = 0.0124$, *Normality* $\chi^2(2) = 1.77$, *AR 1–5* $F(5,76) = 0.64$, *ARCH 4* $F(4,73) = 0.36$, *Hetero* $F(13,67) = 0.65$, *RESET* $F(1,80) = 0.13$, *Func. form* $F(24,56) = 0.85$.

$$\begin{aligned}\Delta(\widehat{RD2-RD1})_t = &\underset{(0.066)}{0.498}\Delta(RD2-RD1)_{t-1} - \underset{(0.068)}{0.202}\Delta^2(RD2-RD1)_{t-2} - \underset{(0.235)}{0.839}RS_{t-1} \\ &- \underset{(0.930)}{3.857}\Delta RBN_{t-1} + \underset{(0.058)}{0.548}[D78{:}I+0.5(D69{:}IV+D81{:}IV)]_t + \underset{(0.024)}{0.067} \end{aligned} \tag{23}$$

$R^2 = 0.68$, $\sigma = 0.0667$, $DW = 2.21$, $RSS = 0.3741$, *Normality* $\chi^2(2) = 2.81$, *AR 1–5* $F(5,79) = 0.31$, *ARCH 4* $F(4,76) = 2.70$, *Hetero* $F(10,73) = 0.86$, *RESET* $F(1,83) = 0.26$, *Func. form* $F(18,65) = 0.90$.

$$\begin{aligned}\hat{RS}_t = &\underset{(0.074)}{0.485}RS_{t-1} + \underset{(0.162)}{1.056}RBN_t - \underset{(0.169)}{0.673}RBN_{t-1} + \underset{(0.044)}{0.099}REUR_t \\ &+ \underset{(0.010)}{0.069}(D77{:}IV+0.5D74{:}I\&II)_t + \underset{(0.005)}{0.012} \end{aligned} \tag{24}$$

$R^2 = 0.87$, $\sigma = 0.0119$, $DW = 1.99$, $RSS = 0.0118$, *Normality* $\chi^2(2) = 0.21$, *AR 1–5* $F(5,79) = 0.57$, *ARCH 4* $F(4,76) = 0.55$, *Hetero* $F(10,73) = 1.05$, *RESET* $F(1,83) = 4.30$, *Func. form* $F(17,66) = 1.45$.

The dummy *D74:I&II* takes the value –1 in 1974:1 and 1 in 1974:2. The rest are unity at the date indicated and zero elsewhere.

The "break–point" tests corresponding to the equations are shown in Figures 13 – 16. Only the equation for $\Delta(RD2-RD1)_t$ has nonconstant parameters, which is not surprising given the data generating process being one of political regulation over most of the sample.

From the results of the Johansen procedure, one could anticipate that the error correction term of the money demand equation would have some explanatory power, in the expenditure equation. This is not the case.

Testing for invariance is performed by adding the auxiliary variables in (21) – (24) to the conditional model to see if they affect the parameters of the model. The F–statistics for adding the intervention variables for Δp, Δx, $\Delta(RD2-RD1)$ and RS are: $F(3,80) = 0.82$; $F(4,79) = 1.59$; $F(2,81) = 0.15$ and $F(4,79) = 1.56$. None of the determinants of nonconstancy are significant, and the joint test that they are all zero is also accepted: $F(13,70) = 1.25$.[17]

For the case of $\Delta(RD2-RD1)$ the residuals from (23), or functions of them, could also represent interventions. For the other variables, testing the significance of the residuals is the Wu – Hausman test for weak exogeneity. The F–statistic for adding the residuals from (21) – (24) to the money demand equation yields $F(4,79) = 1.56$, which is not significant. Consequently, prices, real expenditure and interest rates can all be considered super exogenous for the parameters of the demand for narrow money in Norway.

6. CONCLUDING REMARKS

Given a sample riddled with changing policy regimes, the paper illustrates one way to go from a general statistical model to an interpretable and parsimonious representation of a demand for money function with constant parameters.

[17] As noted By Engle and Hendry (1990) and Hendry and Ericsson (1991), these tests appear to have considerable power. Misspecifying (16) by using lagged inflation and testing the incluson of the intervention variables of inflation resulted in $F(3,80) = 4.24$, which has a p–value of 0.0078.

The empirical model especially highlights the role of the own yield and the rate of return of alternative assets. In the long–run the own rate and the alternative yield on bonds reflect the considerable differences in the riskiness and liquidity between money and bonds. In the short–run agents respond to changes in the alternative yield represented by time deposits and the money market rate. The implications of the analysis are that money is endogenously determined by prices, real expenditure and interest rates, and that these determinants can be varied for a wide class of policy analyses.

APPENDIX: THE DATA

All data are seasonally unadjusted.

M = Coins and currency notes and demand deposits held by the domestic non–bank public. The bank deposits included in this aggregate comprise deposits in domestic and foreign currency with domestic commercial and savings banks and postal institutions, excluding all deposits held by non–residents. The data are rescaled to take account of a widening in the definition of demand deposits in 1987:1 and a break in the official data in 1987:2. Quarterly average of end–of–month data. Source: Bank of Norway.

X = Real gross domestic expenditure, excluding investment in the following sectors: petroleum and natural gas, pipeline transport, oil platforms, and ships. Sources: Various issues of Quarterly National Accounts.

P = Implicit deflator of X. Source: as for X.

$RD1$ = Average interest rate paid on banks' demand deposits. Quarterly data prior to 1978 are obtained by interpolation between end–of–year figures. Between 1978:I and 1985:III the series is a weighted average of lowest (weight = 1/3) and highest (weight = 2/3) interest rates paid on demand deposits by commercial and savings banks. As from 1985:IV properly averaged data is compiled by the Bank of Norway. End–of–quarter estimates averaged over periods t and $t-1$. Sources: Various issues of *Credit Market Statistics* and *Economic Bulletin of Norges Bank.*

RD2 = Average interest rate paid on banks' total deposits denominated in domestic currency (NOK). Methods of calculation and sources as for *RD1.*

RL = Yield to average life of long term bonds (more than six years to expected maturity date) issued by private mortgage loan associations. Quarterly average of end–of–month data. Source: Yield calculations based on bond prices quoted at the Oslo Stock Exchange.

RS = Three–month eurocurrency interest rate on NOK computed from the covered interest parity relationship using middle quotations on spot and three–month forward exchange rates (NOK against USD) and the three–month eurodollar interest rate. Quarterly average of end–of–month data. Source: Data on exchange rates and the eurodollar interest rate obtained from *International Financial Statistics* tapes and some private banks; as from 1988:I interest data as quoted in *Economic Bulletin of Norges Bank.*

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Table 1: Credit Market Regulations in Norway, 1983 – 1989

Type of regulation	Dates when abolished (A) or reintroduced (R)				
	BANKS	FINANCE COMP.	LOAN ASS.	LIFE INSUR.	NON–LIFE INSUR.
Direct loan controls[a]	A1984:I R1986:I A1987:III	A1988:III	A1988:III		A1988:III
Primary reserve req.	A1987:II	A1987:III		A1987:II	
Bond investment quota[b]	A1984:I			A1985:I	
Loan guarantee limits	A1984:III R1986:I A1988:III	A1984:III R1986:I A1988:III	A1984:III R1986:I A1988:III	A1984:III R1986:I A1988:III	A1984:III R1986:I A1988:III
Max int. rate on loans	A1985:III			A1985:III	

[a] Credit extended by the finance companies in the form of factoring and leasing contracts was exempted as from 1984:IV. The regulations concerning mortgage loan associations only applied to loans to households and selected industries.

[b] The dates refer to the point in time when the required percentage of growth was set equal to zero, viz. net additions to the bond portfolio were no longer required. The regulation was completely removed in 1985:I for banks and in 1985:III for life insurance companies.

General notes. If no date is specified, no regulation applies. In all other cases the regulation was in operation at the end of 1983. The information is compiled from *Annual reports of the Norges Bank* 1984 – 1988 and various issues of *Penger og Kreditt* in the same period. The table is taken from Bårdsen and Klovland (1990).

Table 2: The Johansen Procedure: VAR with 5 lags, constant and seasonal dummies. The sample is 1967:III to 1989:IV, 90 observations.

	The eigenvalues:						
	0.508	0.477	0.230	0.220	0.195	0.129	0.028
	The test statistics: Testing the number of cointegrating vectors						
Test type	$\varrho = 0$	$\varrho \leq 1$	$\varrho \leq 2$	$\varrho \leq 3$	$\varrho \leq 4$	$\varrho \leq 5$	$\varrho \leq 6$
trace	211.040*	147.187*	88.920*	56.833*	34.479*	15.000	2.556
λ_{max}	63.853*	58.267*	32.087	22.354	19.479	12.444	2.556
	The eigenvectors, β:						
m	1.000	–0.653	1.260	–0.012	0.034	0.066	0.329
p	–0.805	1.000	–2.805	–1.083	0.005	–0.388	1.133
x	–1.374	–6.816	1.000	2.128	–0.078	0.253	–2.061
RD1	–6.553	–67.551	0.312	1.000	–0.289	–0.321	10.737
RD2	1.544	194.563	7.255	–6.097	1.000	1.986	–19.163
RL	0.995	–58.492	2.696	2.275	–0.454	–1.000	–4.606
RS	0.097	14.552	3.281	2.368	–0.388	0.196	1.000
	The adjustment coefficients, α:						
m	–0.225	0.005	–0.028	0.017	–0.383	0.039	–0.026
p	–0.018	0.002	0.017	0.053	0.045	0.076	0.009
x	0.429	–0.005	–0.045	0.012	0.095	–0.203	–0.020
RD1	0.014	0.000	0.002	–0.003	–0.017	–0.008	–0.001
RD2	0.014	0.000	0.001	0.002	–0.029	–0.017	0.000
RL	0.052	0.005	0.002	0.002	0.056	–0.069	–0.002
RS	–0.062	–0.009	–0.002	0.018	0.593	–0.511	–0.007

Diagnostics:

	Normality $\chi^2(2)$	*AR 1–5 F(5,46)*	*ARCH 4 F(4,43)*
m	1.433	0.677	0.564
p	1.050	1.593	0.340
x	5.285	2.453*	0.335
RD1	0.011	2.422*	1.010
RD2	0.839	3.087*	0.518
RL	14.356*	0.762	0.366
RS	48.247*	0.597	0.012

A test statistic marked with "*" means that the relevant H_0 is rejected at the 5 % level. The critical values for the cointegration tests are taken from Osterwald–Lenum (1990).

Table 3: The General Model: Ordinary least squares estimates of the general model in the form of (13). The sample is 1967:III to 1989:IV; 90 observations, 45 parameters

	Differences: $(\gamma_i^{\dagger}, \delta_i^{\dagger})$					Levels: $(\gamma_p^{*}, \delta_p^{*})$
Lags:	0	1	2	3	4	1
m	–	−0.036 (0.136)	0.261 (0.142)	0.386 (0.146)	0.544 (0.136)	−0.360 (0.096)
p	0.615 (0.257)	−0.066 (0.263)	−0.363 (0.236)	−0.231 (0.276)	−0.207 (0.287)	0.335 (0.125)
x	0.376 (0.126)	−0.076 (0.209)	−0.134 (0.200)	−0.029 (0.194)	−0.077 (0.153)	0.356 (0.187)
RD1	1.884 (2.518)	1.175 (2.770)	−1.918 (2.999)	−2.944 (2.742)	0.046 (2.996)	1.249 (0.904)
RD2	−3.279 (3.053)	−3.159 (3.256)	1.678 (3.239)	−1.197 (3.267)	−2.644 (3.271)	1.194 (1.651)
RL	−1.002 (0.833)	0.224 (0.672)	−0.477 (0.724)	0.367 (0.757)	0.279 (0.836)	−0.841 (0.519)
RS	−0.099 (0.132)	−0.323 (0.178)	−0.143 (0.178)	−0.204 (0.149)	−0.022 (0.132)	0.178 (0.195)
d	−0.081 (1.700)	−0.031 (0.030)	−0.037 (0.024)	−0.022 (0.024)	–	–

Long run solution:

$$m = \underset{(0.257)}{0.931p} + \underset{(0.390)}{0.989x} + \underset{(1.979)}{3.470RD1} + \underset{(4.808)}{3.317RD2} - \underset{(1.485)}{2.336RL} + \underset{(0.576)}{0.496RS}$$

Diagnostics:

$R^2 = 0.97$, $\sigma = 0.0147$, $DW = 1.95$, $RSS = 0.0097$, *Normality* $\chi^2(2) = 0.41$,
AR 1–5 $F(5,40) = 2.35$, *ARCH 4* $F(4,37) = 0.58$, *RESET* $F(1,44) = 0.05$.

Test of lag lengths:

H_0: $\gamma_4^{\dagger} = \delta_4^{\dagger} = 0$; $F(7,45) = 2.60^{*}$.

Table 4: Testing Long–Run Restrictions: Restricting the model: $m = \beta_1 p + \beta_2 x + \beta_3 RD1 + \beta_4 RD2 + \beta_5 RL + \beta_6 RS$ from Table 3. The sample is 1967:III to 1989:IV, 90 observations

Panel A: Testing $\beta_1 = \beta_2 = 1$ and $\beta_4 = \beta_6 = 0$: $F(4,45) = 0.28$

Long run solution:

$$\underset{}{(m-p-x)} = -\underset{(0.257)}{0.069p} - \underset{(0.390)}{0.011x} + \underset{(1.980)}{3.470RD1} + \underset{(4.808)}{3.317RD2} - \underset{(1.485)}{2.336RL} + \underset{(0.576)}{0.496RS}$$

Panel B: Restricting $\beta_1 = \beta_2 = 1$ and $\beta_4 = \beta_6 = 0$:

Long run solution:

$$(m-p-x) = +\underset{(0.712)}{4.719RD1} - \underset{(0.408)}{1.520RL}$$

Diagnostics: 41 parameters:

$R^2 = 0.73$, $\sigma = 0.0142$, $DW = 1.93$, $RSS = 0.0099$, *Normality* $\chi^2(2) = 0.90$, *AR 1–5* $F(5,44) = 1.99$, *ARCH 5* $F(4,41) = 0.44$, *RESET* $F(1,48) = 0.01$.

Panel C: Testing: $m = p + x + 4.7RD1 - 1.5RL$: $F(6,45) = 0.19$.

Long run solution:

$$(m-p-x-4.7RD1+1.5RL) = -\underset{(0.476)}{0.069p} - \underset{(0.390)}{0.011x} - \underset{(1.980)}{1.230RD1} + \underset{(4.808)}{3.317RD2}$$

$$-\underset{(1.484)}{0.836RL} + \underset{(0.576)}{0.496RS}$$

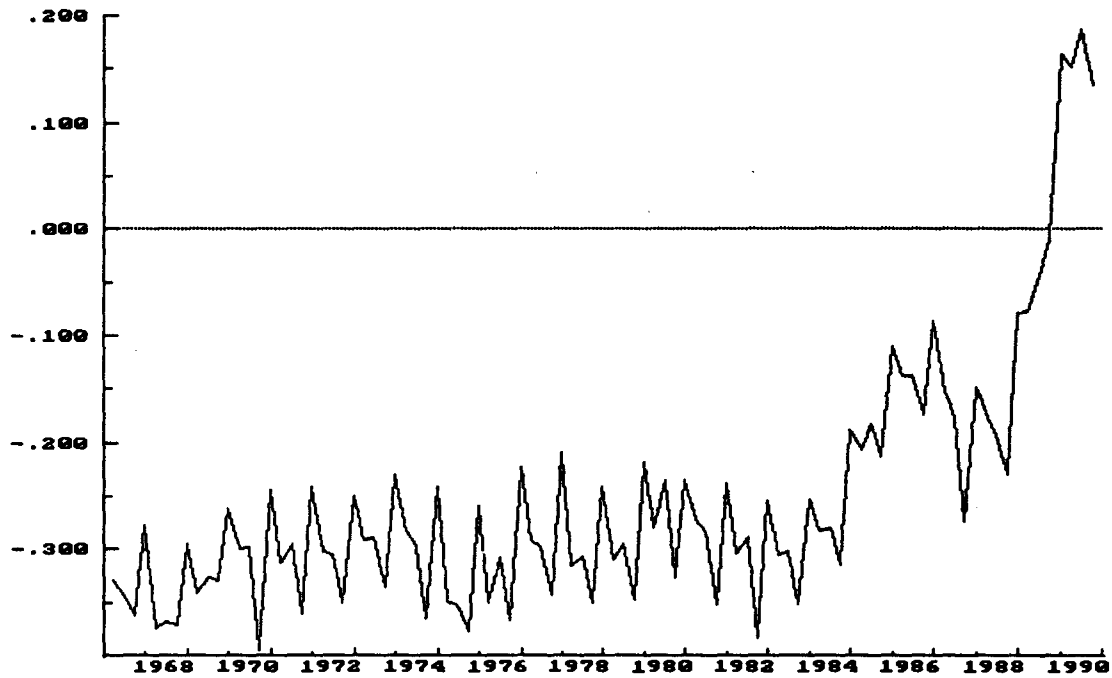

Figure 1: Inverse velocity in logarithmic scale: (m - p - x).

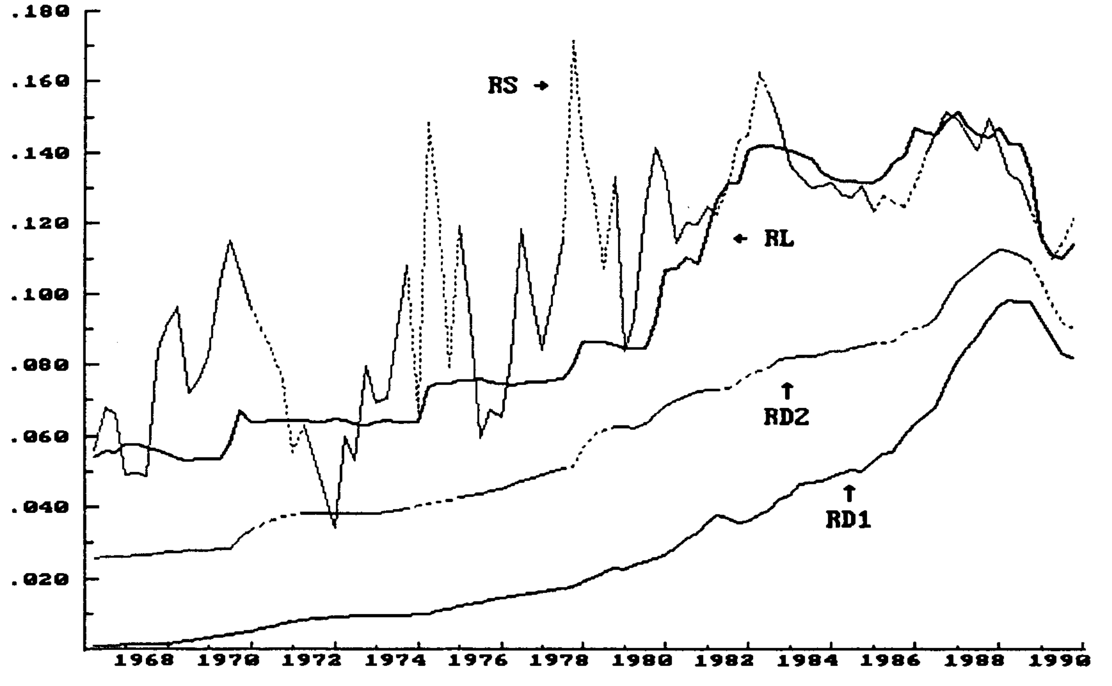

Figure 2: The interest rates.

Figure 3: "Break-point" F-tests for m in the VAR.

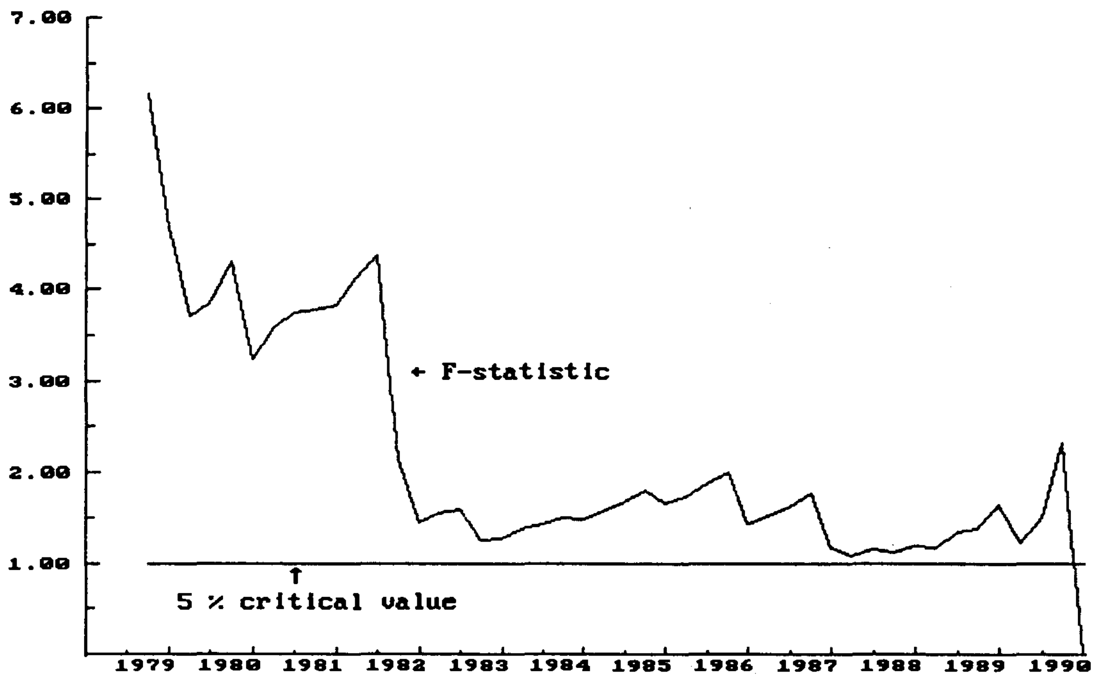

Figure 4: "Break-point" F-tests for RD1 in the VAR.

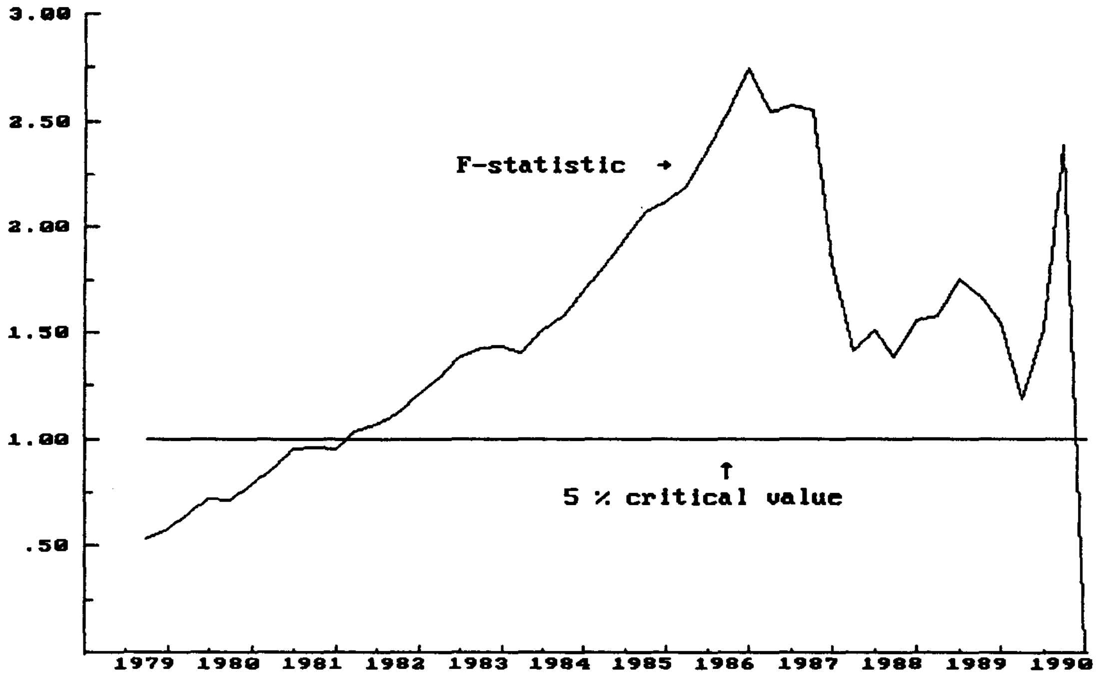

Figure 5: "Break-point" F-tests for RD2 in the VAR.

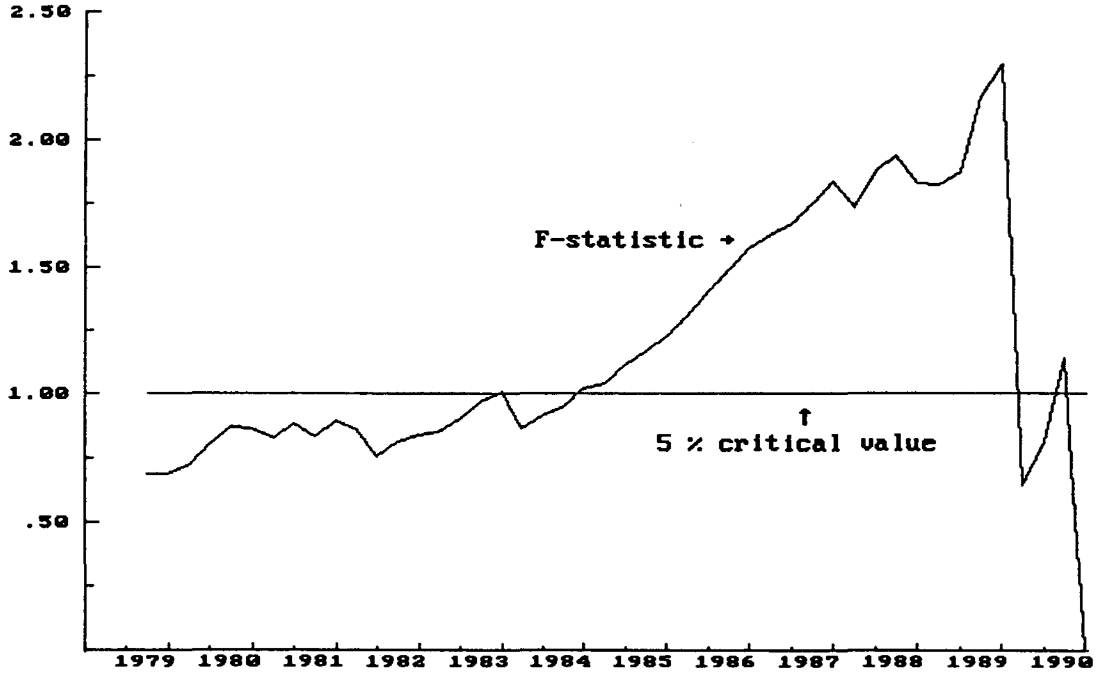

Figure 6: "Break-point" F-tests for RL in the VAR.

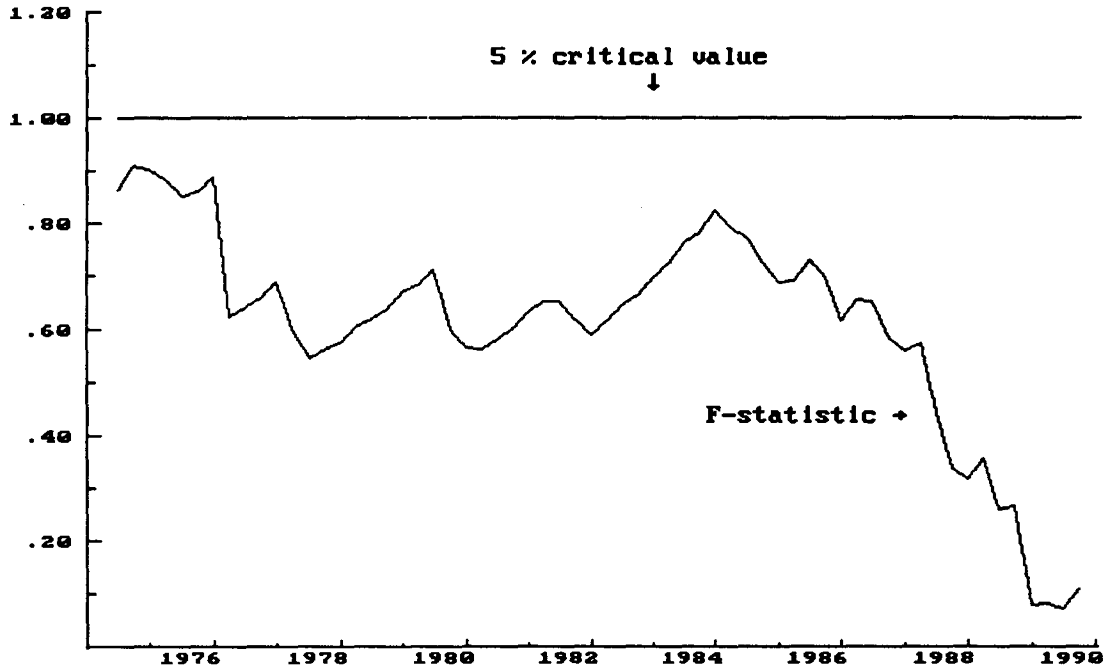

Figure 7: "Break-point" F-tests for the money demand equation.

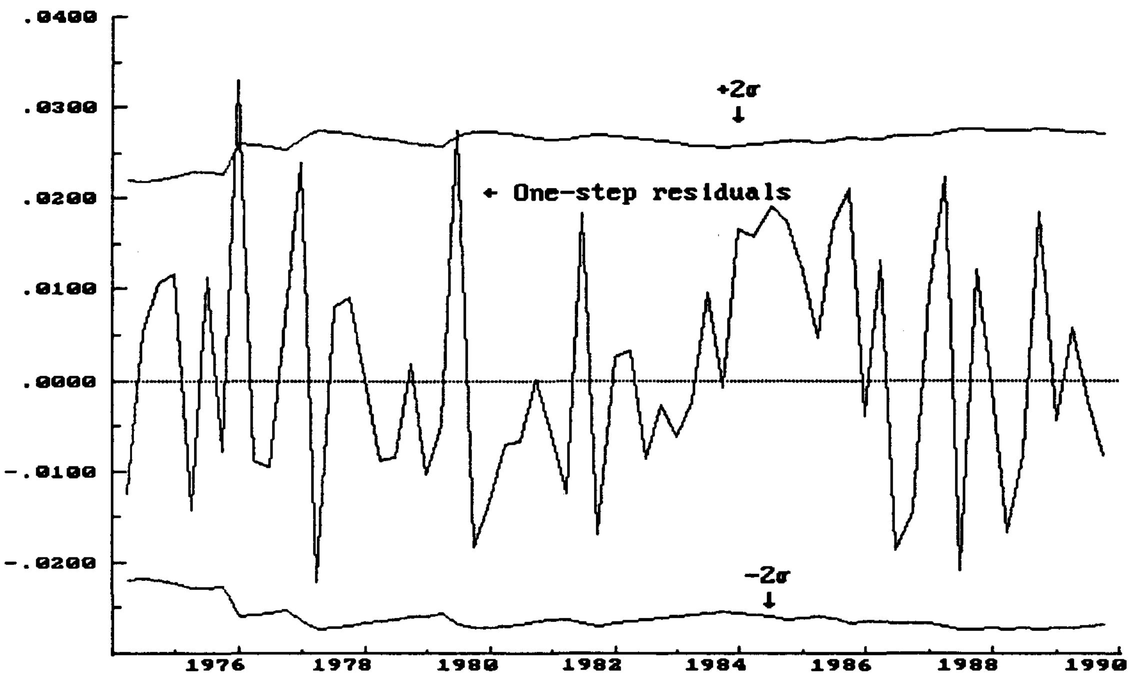

Figure 8: One-step residuals ± two standard errors of the money demand equation.

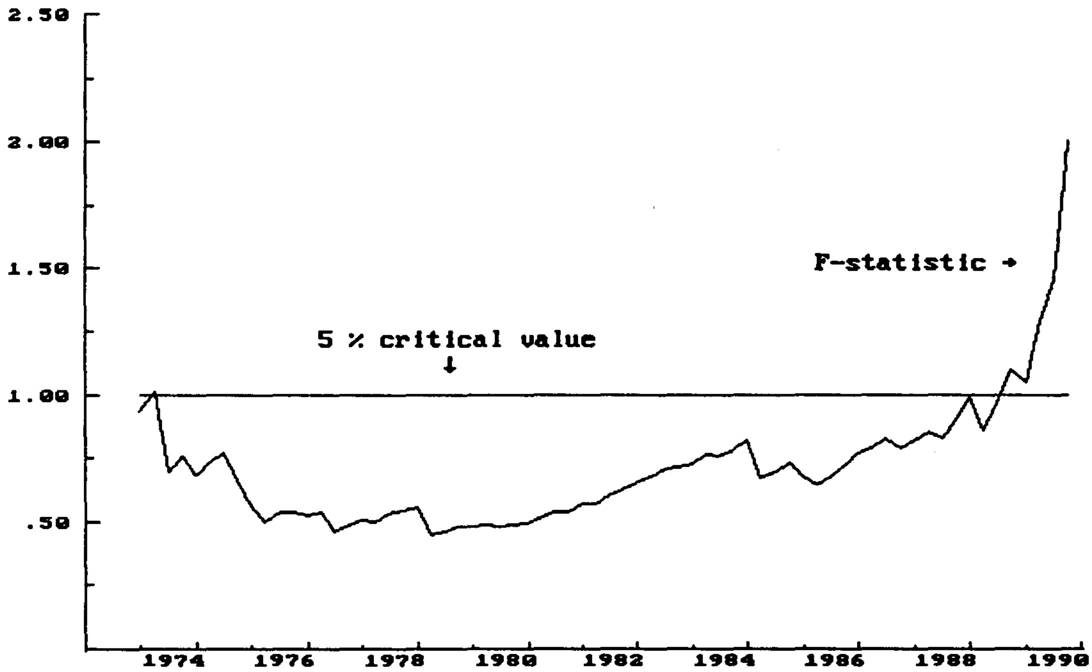

Figure 9: "Break-point" F-tests for the simple marginal model for Δp.

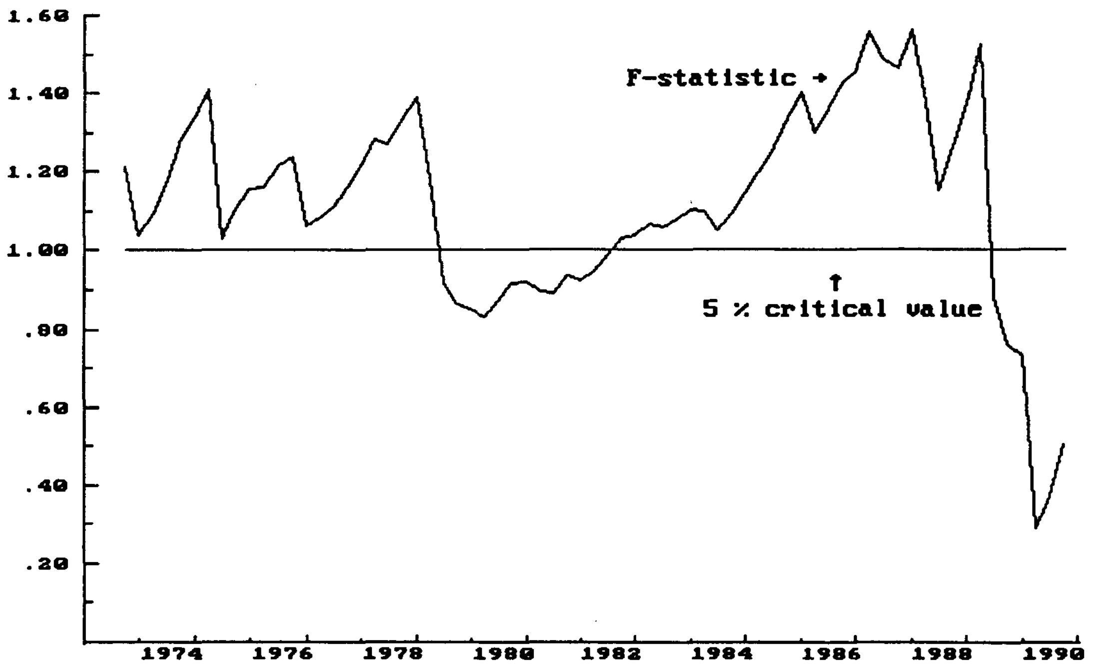

Figure 10: "Break-point" F-tests for the simple marginal model for Δx.

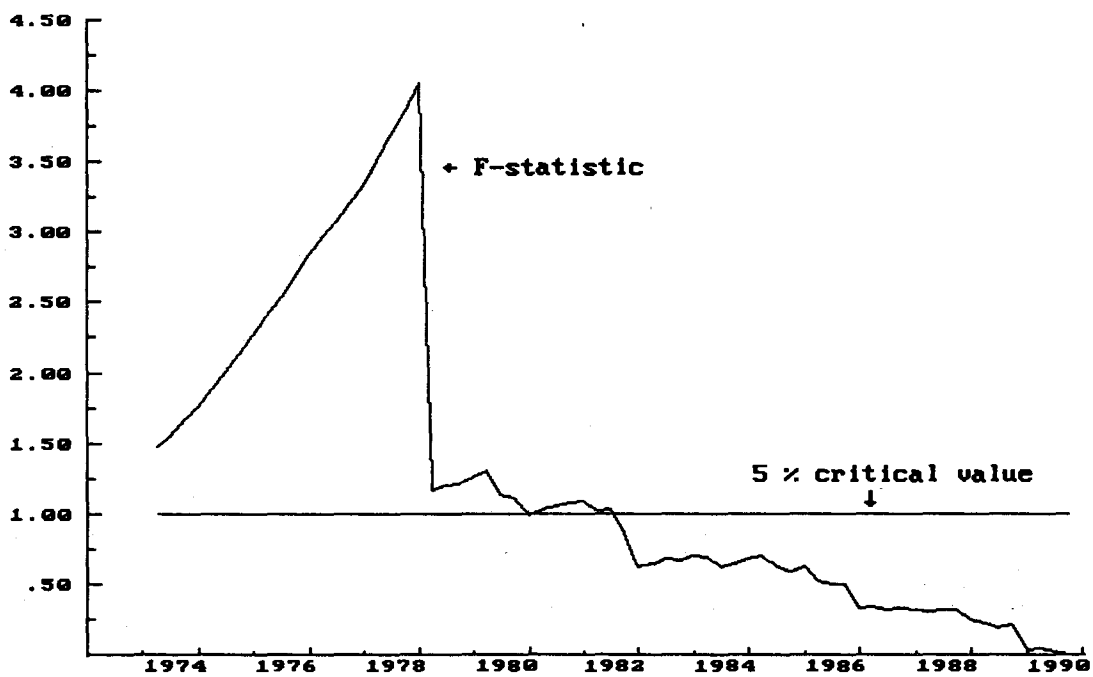

Figure 11: "Break-point" F-tests for the simple marginal model for Δ(RD2-RD1).

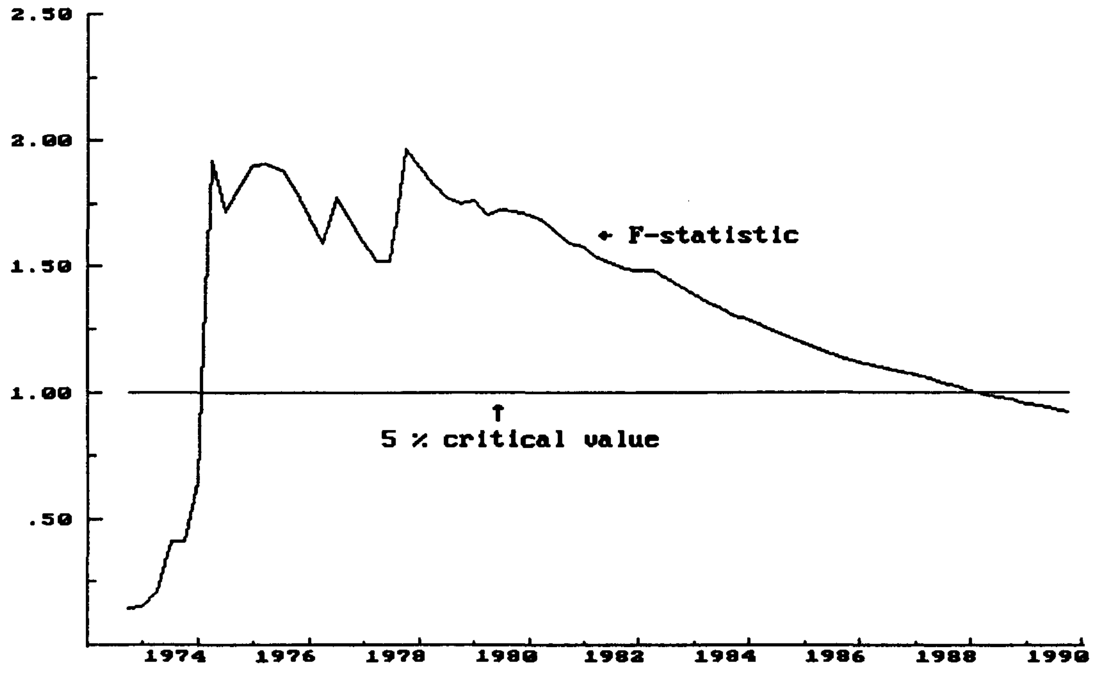

Figure 12: "Forecast" F-tests for the simple marginal model for RS.

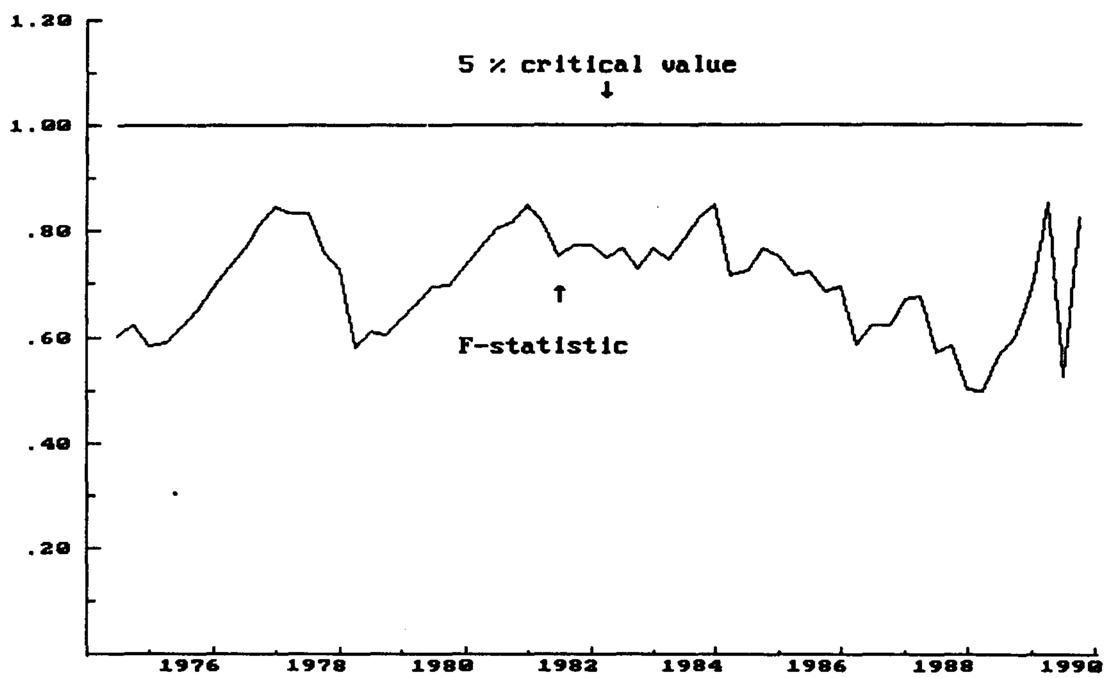

Figure 13: "Break-point" F-tests for the augmented marginal model for Δp.

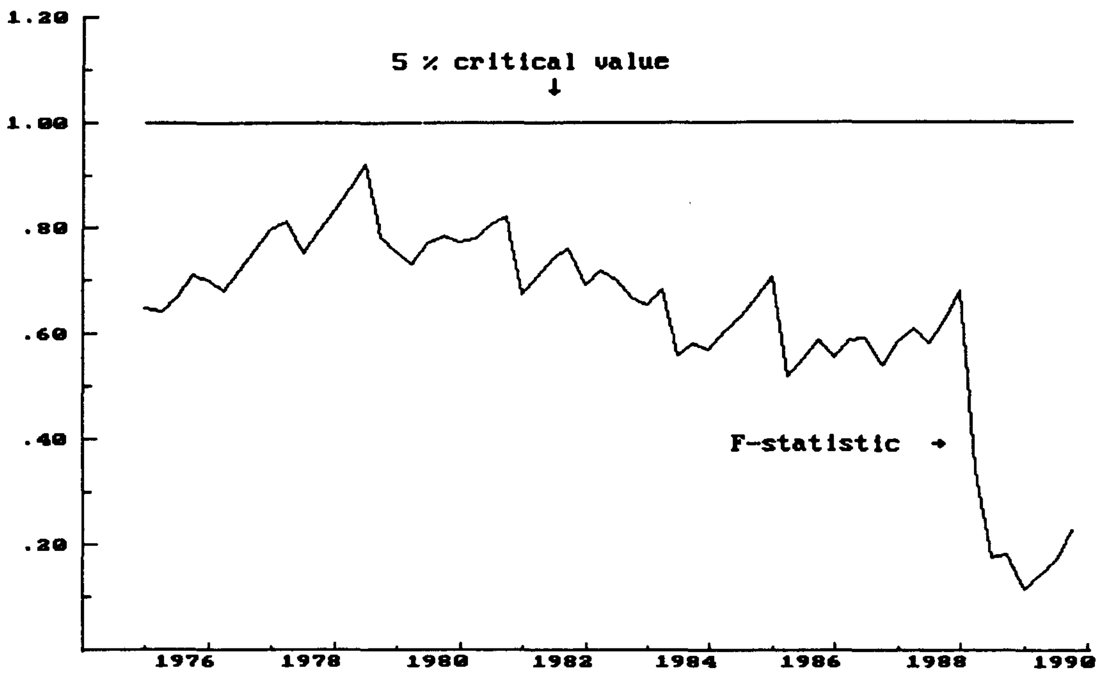

Figure 14: "Break-point" F-tests for the augmented marginal model for Δx.

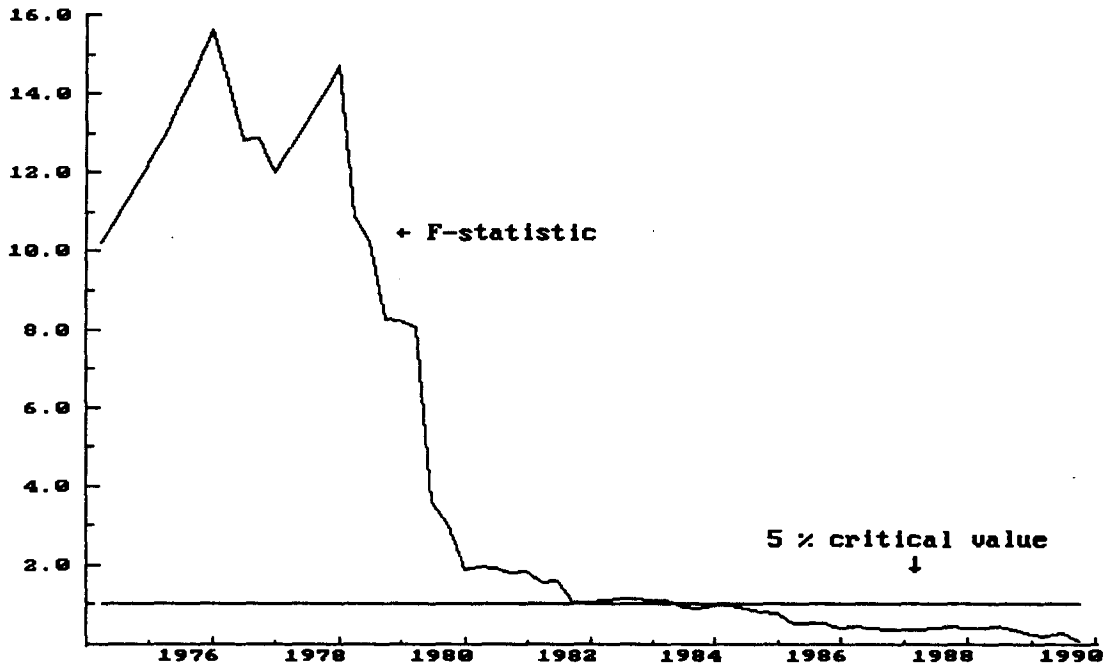

Figure 15: "Break-point" F-tests for the augmented marginal model for Δ(RD2-RD1)

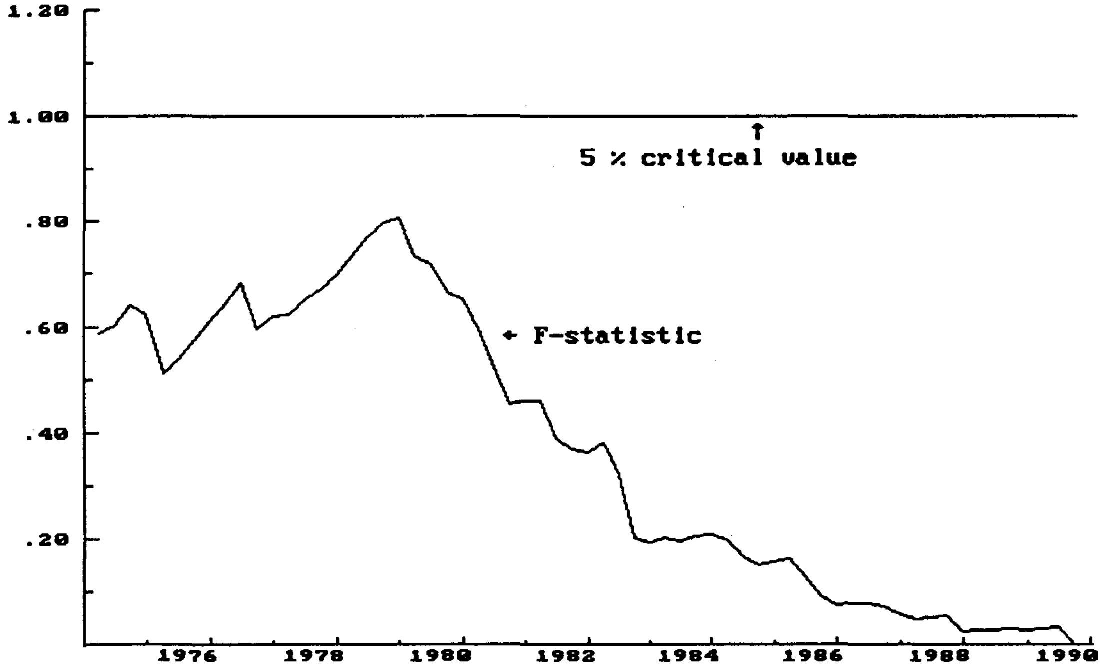

Figure 16: "Break-point" F-tests for the augmented marginal model for RS.