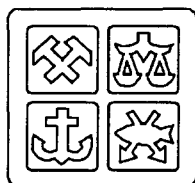


# FOUR ESSAYS ON ASYMMETRIC INFORMATION AND CONSUMER LOCK-IN

by

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A dissertation submitted for the degree of dr. oecon.



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To my parents

## PREFACE

Bringing this work through to its present stage would not have been possible without help from my supervisor, Terje Lensberg, to whom I am very much indebted. I am also grateful to Geir Asheim with whom I have collaborated with on one of the chapters in this thesis, and who throughout has given me more than a fair share of his time even though he never had any formal obligation in that respect. My sincere thanks also go to my committee members, Frøystein Gjesdal and Jon Vislie, for their many insightful remarks.

Apart for those already mentioned, I would like to thank: Tertit Hammer, Paul Klemperer, Kjell Erik Lommerud, Georg Nöldeke, Garth Saloner, Joseph Stiglitz, Gaute Torsvik, and Bent Vale for their part in improving the quality of this project. For all its short-comings, I have only myself to blame.

I would like to thank my colleagues – high and low – at the Institute of Economics for creating an excellent environment to work in and for keeping my spirits high when theorems tumbled. A special thank goes to the Library at the School, and its staff, for providing excellent service. I had the benefit of spending six months as a research fee student at the London School of Economics in 1988-89. A particular thank goes to my supervisor during that stay, John Sutton; the absence of empirical work in this thesis is definitely not him to blame.

At the outset, my main motivation for doing research in asymmetric information and consumer lock-in was the performance of the Norwegian credit market at that time (1987/88). Although, in the end, there is not much to learn about the credit market from reading this thesis, I feel obliged to thank all those Norwegian bankers who, through their collective efforts, influenced my actions.

Finally, I would like to thank my family, Tertit and Ludvig, for being so patient, and for teaching me that there is more to life than what the academic world can provide. I gratefully dedicate the thesis to my parents.

Bergen, October 10th, 1991

Tore Nilssen



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## INTRODUCTION<sup>1</sup>

This thesis is an attempt at combining the economics of information with the theory of industrial organization, by drawing on, respectively, the theory of competition in markets with asymmetric information, and the theory of consumer lock-in. In this way, we get a theory of consumer lock-in in markets with asymmetric information.

The motivation is the following: Suppose you observe in real life a particular market showing signs of firms fighting for market shares. A leading explanation for such fierce competition is that consumers, once they get attached to a particular supplier, are locked in. One reason for such lock-in is consumer switching costs, i.e., that direct costs are involved when a consumer switches from one supplier to another. Consumer switching costs may thus explain the market performance you observe. Suppose, however, that the market in question is a very special one: There is asymmetric information between suppliers and consumers, in the sense that consumers have private information that is vital to the profitability of selling to them. Could it be that asymmetric information alone, without the presence of consumer switching costs or any other lock-in mechanism, is sufficient to create consumer lock-in?

This question is addressed in Chapter 2 below, which is the key chapter of the thesis. There, the starting point is the Rothschild-Stiglitz (1976) model of an insurance market with asymmetric information, with consumers having private information on accident probabilities. It is necessary to extend their one-period model to a two-period framework. Only then are we able to understand how considerations about the future will affect today's competition and, hence, how consumer lock-in may arise in such a market.

How, then, could a consumer be locked in in a multi-period insurance market? It must be because his current insurer has learnt something about him that

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<sup>1</sup>Thanks to Tertit Hammer for comments that have greatly improved the exposition.



rival insurers do not know. This piece of information we take to be the consumer's accident experience: Each insurer is the only one having observed whether its old customers have had any accidents in the past or not. In plain language, it is argued that an insurance company's *customer files*, containing information that the company has on customers' *previous accidents*, are of value.

Public information on consumers' *previous contract choices* creates an obstacle to the potential benefit of an insurer's customer files: Suppose that high-risk consumers buy a contract today that differs from the one purchased by low-risk consumers; this is called *self-selection*. Such self-selection, however, has the consequence that any insurer, merely by observing a consumer's contract choice, can determine whether he is a high-risk or a low-risk. Any additional information about accident experience is of no value.<sup>2</sup>

Thus, for customer files to be of value in the second period, there cannot be self-selection in the first period. An equilibrium without initial self-selection is called a *pooling equilibrium*. In such an equilibrium, all consumers are offered the same first-period contract. Moreover, an insurer that has observed his customers' accident experience is able, later on, to adjust his subjective probability that any given old customer is a low-risk; this is because a low-risk consumer has a higher probability of having a good accident experience than a high-risk. The insurer is also able to discriminate among customers according their accident histories. Consider customers with, say, a good accident history. The insurer may find it profitable to cross-subsidize among them in such a way that expected losses on the relatively few high-risks in this group are more than compensated by expected profits on the relatively many low-risks. The reason why it is possible to lock in old customers and earn an expected profit on them is the informational advantage stemming from the customer files.

The existence of a pooling equilibrium (i.e., no initial self-selection) with consumer lock-in is the main result in Chapter 2. We are, thus, able to answer the question we started with in the affirmative: Customer files may have a value

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<sup>2</sup>There are indeed good reasons to argue that previous contract choices, in contrast to the content of the customer files, belongs to the public domain. Suppose that a consumer would be able to switch insurer if only the new insurer knew which insurance contracts he has bought in the past. Then, it is in the interest of the consumer to disclose this information by producing his own copies of previous contracts. In light of this, it is only reasonable to assume contract choices being directly observable by everybody at any time.

in themselves and may create consumer lock-in even when no other lock-in mechanism is present.

The research on consumer lock-in with asymmetric information invites further work to be done in many directions. Chapters 3 and 4 explore two ideas that may seem very different from each other. Each of them has, however, a firm connection to the analysis of Chapter 2.

In Chapter 3, which is co-authored by Geir B. Asheim, the possibility of *renegotiation* is studied. Like in Chapter 2, we are within the framework of the Rothschild-Stiglitz insurance model, and again, we analyse one particular consequence of making insurers able to take advantage of information received on consumers.

The idea is the following: In a set of contracts eliciting self-selection, the contract intended for low-risks is a partial-insurance contract in order to make it unattractive to high-risks. After contract choices are made, an insurer will have incentives to approach the consumers that have chosen the partial-insurance contract, inviting them to renegotiate, *i.e.*, to replace their initial contract with a full-insurance contract which is at least as good for them and more profitable for the insurer. The high-risk consumers will, however, anticipate all this when they make their contract choices and will therefore find it beneficial to simulate low-risk by also choosing the partial-insurance contract. Thus, with renegotiation, it seems difficult to get high-risks and low-risks to buy different contracts.

Having solved the problem of how to introduce renegotiation in a meaningful way into a one-period insurance model, we are able to report quite remarkable results from our exercise of comparing the Rothschild-Stiglitz insurance model with and without renegotiation. We find, in contrast to the original model, that, with renegotiation, there always exists an equilibrium in pure strategies. Moreover, for a considerable part of the parameter space, including all parameter combinations leading to non-existence of a pure-strategy equilibrium in the original model, there is pooling (*i.e.*, no self-selection) in equilibrium. For the rest of the parameter space, there is incomplete self-selection in the sense that one of the two offered contracts is chosen by all the low-risks and some of the high-risks, while the other contract is chosen by the rest of the high-risks. This contrasts with the original model, in which there is always complete self-selection. In fact, the result contrasts with all previous models following Rothschild and Stiglitz

(1976). Even though previous attempts have succeeded in obtaining existence of a pure-strategy equilibrium by some respecification of the original model, this is the first variation, to our knowledge, that reports pooling even for some cases where the original model features a pure-strategy self-selection equilibrium.

Working on consumer lock-in, one might soon come to the idea that the extent to which a consumer is locked in may depend in a very particular way on what his alternatives to staying with the current supplier are. Take the insurance market as an example, and suppose there are three different insurers in the market. When a certain consumer considers leaving firm A, his current insurer, he may have to take into account the fact that firm B is an insurer he has never been attached to before, while firm C, on the other hand, was his insurer until last year, when he switched to firm A. Clearly, if customer files have any value, firm C, with its file on our consumer, is able to offer him an insurance contract that the uninformed firm B is unable to match. In fact, with all its information on this consumer, firm C may be a serious threat to his current insurer, firm A. It seems, therefore, that a consumer who switches between suppliers of insurance over time has something to gain by a reduction of the lock-in effect. On the other hand, the fact that a consumer that comes switching from another insurer is more difficult to lock in than a completely fresh one should make firms hesitant to compete fiercely for such a consumer.

Unfortunately, Chapter 2 shows us that even a two-period model of the insurance market is very difficult to work with. And to make the point we outlined in the previous paragraph, we would need at least three periods. However, the problem is of interest also outside the insurance market. In [Chapter 4](#), therefore, a multi-period duopoly market with homogeneous products, with complete information, and with consumer switching costs is analysed. In fact, we are able to contribute to the theory of consumer switching costs by insisting on a difference between two kinds of such switching costs: One is called *transaction costs*, and it is incurred every time a consumer switches between suppliers. The other is called *learning costs* and is incurred only when a consumer switches to a supplier he has never been in contact with before. Thus, the higher is the fraction of learning costs out of total switching costs, the higher is the reduction in lock-in the consumer obtains by making one switch and, therefore, the less profit there is to be earned on a consumer that has switched from another firm. A high frac-

tion of learning costs seems to be a good parallel to a high value of customer files in the insurance market context of the previous paragraph.

In analysing this model, we find that, subject to a restriction on the demand function, an increase in the fraction of learning costs out of total consumer switching costs leads to an increase in welfare. We also find that such an increase lowers firms' introductory (first-period) price.

One should be careful with applying this insight in the insurance market, though. It seems tempting, based on Chapter 4 and the analogy noted above, to argue that valuable customer files are beneficial to society. Such a conclusion is, however, premature. One should wait until a proper analysis of the three-period insurance market has been carried out. Considering the complexity of Chapter 2, such an analysis may prove to be far more than a straightforward exercise.

A key ingredient of the results in Chapters 2 and 3 was the phenomenon of pooling: privately informed, heterogeneous agents behaving identically, in the sense that they all buy the same insurance contract. A natural question to ask, then, is: When, except in these two instances, will a model with asymmetric information feature pooling? There does not seem to be any readily available survey of the literature strictly directed at this question. Chapter 1, therefore, is written to fill this gap. A reader with a textbook knowledge of the economics of asymmetric information may want to answer the question straightforwardly as follows: Pooling occurs when the single-crossing condition on the informed agents' preferences does not hold. In my view, there is more to it than that. It is to be hoped that, by the end of that chapter, the readers will agree.

## Reference

Rothschild, Michael, and Joseph Stiglitz (1976), "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information", *Quarterly Journal of Economics* 90(4), 629-649.



## CHAPTER ONE:

**WHEN POOLING IS THE RULE:  
ASYMMETRIC-INFORMATION MODELS  
WITH INCOMPLETE SEPARATION<sup>1</sup>**

**Abstract:** This chapter contains an extensive survey of the literature on models of asymmetric information, exploring reasons for there being less than complete revelation of information in the equilibria of some of these models. Starting with one-period models, we find such reasons pertaining either to the incentives of the informed agent, to the incentives of the uninformed principal, or to the medium through which the parties communicate. We then move on to multi-period models and cover reasons pertaining to renegotiation and to lack of commitment.

## 1. INTRODUCTION

Asymmetric information is a characteristic of many instances of interacting individuals or firms. Prominent examples are the labour market, where a worker may be thought to have superior information on his own abilities; the credit market, with borrowers knowing more about characteristics of their projects than lenders do; and the insurance market, where a consumer is considered to know privately his or her inherent risk. Other examples include product markets where producers have superior knowledge on the quality of their products; bargaining situations with the adversaries not knowing each other's reservation prices; and elections, with the electorate having incomplete knowledge about the candidates' abilities and intentions.

Problems of asymmetric information may be solved by noting that, in many

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<sup>1</sup>A first version of this chapter was completed in July 1991. I am indebted to a number of people for discussing with me topics related to the occurrence of pooling in asymmetric-information models of various kinds; among them are: Geir Asheim, Georg Nöldeke, Joseph Stiglitz, and Gaute Torsvik. I would also like to thank Jon Vislie for his comments on the first version, and Greg LeBlanc for his written correspondence.

situations, the uninformed parties may draw inferences from the informed parties' actions, leading the good types among the informed to seek actions to take that would be unprofitable for the bad types to mimic. This idea of *self-selection* dates back to the work of Mirrlees (1971) and Spence (1973). As a solution to the problem of asymmetric information, it is partial in (at least) two ways. First, self-selection is obtained only at the cost of leaving the first-best outcome. Second, and the object of our present concerns, it does not always work. Often, agents of different types mimic each other, a phenomenon regularly known as *pooling*.

This chapter is not an introduction to the economics of asymmetric information; rather, some basic knowledge will be assumed. For introductory material, the reader is referred to the recent microeconomics textbook by Kreps (1990a) or surveys on optimum regulatory policy under asymmetric information by Besanko and Sappington (1987), Caillaud et al. (1988), and Baron (1989).

The organization of the chapter is as follows. In Section 2, some preliminaries, such as notation and terminology, are collected. We describe a general set-up, the principal-agent model, together with two important variations, the market screening model and the signalling model. Sections 3 through 6 constitute the main body of the chapter. In these Sections, we go through a number of reasons for the occurrence of pooling. In Section 3, we have collected reasons related to the agent's incentives, while we follow up in Section 4 with the principal's incentives. In Section 5, we are concerned with the medium through which messages are sent from the agent to the principal. In Section 6, finally, we treat multi-period situations. Section 7 concludes with some reasons for pooling that do not fit elsewhere and with some final words on both theory and empiricism.

## 2. PRELIMINARIES

Consider an economy consisting of a set  $A$  ("Agents") of informed agents and a set  $P$  ("Principals") of uninformed agents. Each member of  $A$  has information about one or more of his characteristics, information that is not available *ex ante* to the members of  $P$ . An informed agent's private information is an element in a set  $T$  ("Types"). These characteristics are such that they cannot be changed. The uninformed agents share beliefs about the informed agents' private information,

represented by a probability distribution function  $\rho$  defined over  $T$ .

The members of the economy interact with each other, and this interaction has interesting properties arising from the asymmetry of information. In this interaction, each informed agent in  $A$  has available actions from the set  $M$  ("Messages"), and each uninformed agent in  $P$  has available actions from the set  $C$  ("Contracts"). Out of the interaction, informed and uninformed agents earn payoffs represented by a mapping  $\Pi$  from  $T \times M \times C$  to  $\mathbb{R}^{\#A + \#P}$ , where  $\mathbb{R}$  is the real line and " $\#$ " denotes cardinality.

Below, we present the three main variations analysed in the literature. They are, respectively, the principal-agent model, the market screening model, and the signalling model.<sup>2</sup>

## 2.1 The principal-agent model: a single uninformed moving first

The three variations have many properties in common. We start with describing the principal-agent model, which constitutes our basic set-up.<sup>3</sup> This set-up is as follows:

- (i) The sets  $P$  and  $A$  are singletons, with a single principal facing a single agent.<sup>4</sup>
- (ii) At the outset, the agent  $A$  gets to know whether he is "Bad" or "Good":  $T = \{B, G\}$ . The principal  $P$ 's *ex ante* subjective probability that  $A$  is good is  $\rho_G \in (0, 1)$ .
- (iii) The message set  $M$  is the interval  $[\underline{m}, \bar{m}]$  on the real line.
- (iv) The principal  $P$  makes the first move, specifying the payment to the

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<sup>2</sup>Given the abundance of fascinating examples of all three models that is found later on in this chapter, the following three subsections are presented without any illustrative examples at all. Readers are asked to be patient.

<sup>3</sup>Early work on this kind of model is by Stiglitz (1977) on an insurance monopoly and Baron and Myerson (1982) on regulation. More by way of introduction can be found in Kreps (1990a, ch. 18).

<sup>4</sup>With no expression of prejudice intended, we let principals and agents belong to different genders in this paper, with principals being females and agents males throughout. Thus, we subscribe in a way to what may be called the "Osborne-Rubinstein compromise", see Osborne and Rubinstein (1990, p. x).



agent contingent on the message, *i.e.*,  $C$  consists of mappings  $s:M \rightarrow \mathbb{R}$ .

(v) The agent  $A$  moves second, incurring a cost  $c$  when sending his message  $m \in M$ ; this cost depends on the message and the agent's private information, or type,  $t \in T$ , *i.e.*,  $c = c(m, t)$ , such that, for all  $m, t$ : (a)  $c \geq 0$ , (b)  $\partial c / \partial m > 0$ , (c)  $c(m, G) < c(m, B)$ , (d)  $\partial c(m, G) / \partial m < \partial c(m, B) / \partial m$ , and (e)  $\partial^2 c / \partial m^2 \geq 0$ .

(vi) The principal  $P$ 's gross payoff  $r$  depends on the agent's message and type:  $r = r(m, t)$ , such that, for all  $m, t$ : (a)  $r(m, G) > r(m, B)$ , (b)  $r(\underline{m}, B) > c(\underline{m}, B)$ , (c)  $\partial r(\underline{m}, B) / \partial m \geq \partial c(\underline{m}, B) / \partial m$ , (d)  $\partial r(m, B) / \partial m < \partial r(m, G) / \partial m$ , and (e)  $\partial^2 r / \partial m^2 < 0$ .

(vii) Both  $P$  and  $A$  are risk neutral, and (net) payoffs  $\Pi$  are given by:  $\Pi = (\Pi^P, \Pi^A) = (r - s, s - c)$ .<sup>5</sup>

(viii) Both  $P$  and  $A$  have outside options, normalized to 0.

The crucial element in this set-up is property (d) of the informed agent's message cost function, implying that the cost difference between any two messages is lower for the good type than for the bad type. It goes under various names in the literature, like the single-crossing property, the sorting condition, or the Spence-Mirrlees condition.<sup>6</sup>

Note also property (a) of the principal's gross payoff function, giving the precise sense in which an agent of type  $G$  is good:  $r(m, G) > r(m, B)$ ; there is more for the principal to gain if the agent is good than if he is bad.<sup>7</sup>

We are looking for an *equilibrium* of this model. By equilibrium, we will in general mean a *Perfect Bayesian Equilibrium* (PBE). A PBE is a collection of strategies and beliefs such that, (i) given the beliefs, no player wants at any point to change strategy, and (ii) given the strategies, beliefs are given by subjective probabilities that are defined by Bayes' rule whenever it applies. For more on PBEa, see Tirole (1988, Sec. 11.5).

Let  $m(t)$  be the message chosen by an agent of type  $t$ . The principal seeks to

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<sup>5</sup>More generally, we could assume that there exists a transferable utility, *i.e.*, some good, such as money, which is evaluated equivalently by  $A$  and  $P$ . We then have, for  $i \in \{A, P\}$ , a utility function  $U^i(x, x')$  over a good  $x$ , the transferable utility, and a composite of other good,  $x'$ , such that:  $U^i(x, x') = x + V^i(x')$ . For more on this, consult, *e.g.*, Kaneko (1976).

<sup>6</sup>See, *e.g.*, Cooper (1984) for a discussion of the importance of this condition.

<sup>7</sup>In an insurance context, a good agent is a low-risk consumer; in a labour market context, he is a high-productivity worker.

construct a payment function  $s(m)$  that maximises her net expected payoff

$$\Pi^P = (1 - \rho_G)[r(m(B), B) - s(m(B))] + \rho_G[r(m(G), G) - s(m(G))],$$

subject to two sets of conditions: First, the *participation constraints* stipulate that no-one is left with less than the outside option:<sup>8</sup>

$$\Pi^P, \Pi^A \geq 0.$$

Second, the *incentive constraints* stipulate that an agent of type  $t$  does not profit from mimicking type  $t'$ :

$$s(m(t)) - c(m(t), t) \geq s(m(t')) - c(m(t'), t), (t, t') \in \{(G, B), (B, G)\}.$$

In solving this problem, the principal indirectly determines  $m(B)$  and  $m(G)$  through her choice of  $s(m)$ . Let a star (\*) denote equilibrium actions. To simplify notation, we introduce, for each  $t$ :  $m_t \equiv m^*(t)$ , and  $s_t \equiv s^*(m_t)$ . It turns out that, in the optimum solution, (a) the bad type sends his (first-best) efficient message:

$$\partial r(m_B, B)/\partial m = \partial c(m_B, B)/\partial m;$$

(b) the bad type's incentives to mimic the good type are exactly balanced:

$$s_G - s_B = c(m_G, B) - c(m_B, B);$$

and (c) the good type's payoff is kept down to his outside option:

$$s_G - c(m_G, G) = 0.$$

When the principal maximises  $\Pi^P$  using properties (b) and (c) above, we find that the good type's message  $m_G$  solves:

$$\partial c(m_G, G)/\partial m = \rho_G \partial r(m_G, G)/\partial m + (1 - \rho_G) \partial c(m_G, B)/\partial m.$$

Thus, the principal must weigh the efficiency of the good type's message against the bad type's incentives to mimic, with weights given by  $\rho_G$ , the probability that the agent is good. The higher probability that the agent is good, the less weight is put on the bad type's incentives. Since  $\partial c(m, G)/\partial m < \partial c(m, B)/\partial m$  by (v)(d) above, our condition on  $m_G$  implies that  $\partial r(m_G, G)/\partial m < \partial c(m_G, G)/\partial m$ ; thus, the good type sends a message that is too high, compared to the first best. The two

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<sup>8</sup>The participation constraint for the agent,  $\Pi^A \geq 0$ , must, of course, hold for each type. In a variation of the present set-up, denoted the case of *ex-ante contracting*, the agent obtains his private information only after the contract with the principal is written. There is now a single participation constraint for the agent, stating that his *expected* profit should be non-negative. With a risk-neutral agent, the principal's optimum solution is to design a two-part tariff that extracts all economic rent from the agent and leaves him with all the risk. This variation has been studied by Baron (1981) and Rey and Tirole (1986), among others. When necessary to keep the two regimes apart, the standard case will be denoted *interim contracting*.

types send different messages; *i.e.*, separation occurs. In fact,  $m_G > m_B$ , which is seen from noting that the first-best message is higher for the good type than for the bad type by our assumptions on  $r$  and  $c$ ; the second-best solution places a further wedge between the two messages.

## 2.2. The market screening model: competing uninformed moving first

One important twist to the basic set-up is to let there be *multiple principals* competing with each other. Thus,  $P$  is no more a singleton. The seminal work on market screening is by Rothschild and Stiglitz (1976). Competition in this model has a Bertrand-like feature driving profits to zero. Moreover, if the agent is separated, then it is not possible for a principal to cross-subsidize between types with a non-random payment function. For if she did so, then such a principal would be vulnerable to cream-skimming by other principals, being left with the loss-bringing type. Thus, the solution to the problem with separation and principals competing in non-random payment functions has the property that, for each  $t$ :

$$s_t = r(m_t, t).$$

As in the single-principal case, the bad type sends the first-best efficient message, *i.e.*,  $m_B$  satisfies:

$$\partial r(m_B, B) / \partial m = \partial c(m_B, B) / \partial m.$$

The last condition on equilibrium actions is that the bad type's incentives to mimic the good type is exactly balanced:

$$s_G - s_B = c(m_G, B) - c(m_B, B),$$

which, given the zero-profit constraints above, is equivalent to:

$$r(m_G, G) - r(m_B, B) = c(m_G, B) - c(m_B, B).$$

These conditions do not, however, ensure equilibrium in all circumstances. Specifically, suppose that all principals, except one, offer a function  $s$  according to the above. Would it now pay the last principal to deviate from this equilibrium and offer a cross-subsidizing payoff function? In an optimum deviation  $(s', m')$ , the deviator would receive the first-best message from the bad type, so that:  $m_B' = m_B$ . Moreover, she would exactly balance the bad type's incentives to mimic the good type, so that:  $s_G' - s_B' = c(m_G', B) - c(m_B', B)$ . Finally, she would exactly

balance the good type's incentives to stay with one of the other principals, so that:  $s_G' - s_G = c(m_G', G) - c(m_G, G)$ . If we differentiate her profit function subject to these conditions and evaluate at  $(s', m') = (s, m)$ , we find that it is optimum also for the last principal to offer  $(s, m)$  if and only if:

$$\partial c(m_G, G)/\partial m \leq \rho_G \partial r(m_G, G)/\partial m + (1 - \rho_G) \partial c(m_G, B)/\partial m.$$

When the above condition does not hold, there exists an equilibrium in mixed strategies, *i.e.*, where principals offer random payment functions. In this equilibrium, agent types are still separated. The proof of the existence of such a mixed-strategy equilibrium is provided by Dasgupta and Maskin (1986) in the general case and by Rosenthal and Weiss (1984) in an instructive special case (see also Asheim and Nilssen, 1991, Sec. 3).

The condition for existence of a pure-strategy equilibrium can be interpreted as an upper bound on  $\rho_G$ , *i.e.*, this equilibrium exists if the probability that the agent is of the good type is not too high, in particular if:

$$\rho_G \leq \frac{\frac{\partial c(m_G, B)}{\partial m} - \frac{\partial c(m_G, G)}{\partial m}}{\frac{\partial c(m_G, B)}{\partial m} - \frac{\partial r(m_G, G)}{\partial m}}.$$

### 2.3 The signalling model: informed moving first

A second important twist to the basic set-up is a change in the move sequence. Let us go back to the case of one principal and one agent. Suppose now that the agent moves first, sending a message  $m$  subject to the same cost function as before. The principal moves second, specifying the payment  $s(m)$  given the agent's message. A game with such a move sequence is called a *signalling game*, while a game with the original sequence, *i.e.*, with the uninformed moving first, is called a *screening game*; the terminology is due to Stiglitz and Weiss (1985). The seminal work on signalling is by Spence (1973).

A's optimum message depends on what he thinks about the inferences P will make from her observation of  $m$ . Suppose, in equilibrium, an agent of type  $t$  sends the message  $m(t)$ . Let P's revised beliefs, after observing  $m$ , be given by the

updated probability that the agent is good:  $\rho_G' = \rho_G'(m)$ . If we restrict the agent to using pure strategies, then, either:

(i)  $m(B) = m(G)$  and  $\rho_G'(m(B)) = \rho_G'(m(G)) = \rho_G$ , or:

(ii)  $m(B) \neq m(G)$ ,  $\rho_G'(m(B)) = 0$ , and  $\rho_G'(m(G)) = 1$ .

Thus, (i) is a case of pooling, with both types sending the same message, while (ii) is one of separation. Contrary to the models we have discussed so far, there typically exist multiple Perfect Bayesian Equilibria of the signalling game, both pooling and separating ones. If we allow the agent to use mixed strategies, there would potentially exist equilibria of a hybrid character.

One great task in the signalling literature has been to reduce the number of equilibria in reasonable ways. This has led to various *refinements* of the PBE concept. These refinements try to put restrictions on the uninformed agent's beliefs in case a message off the equilibrium path is observed. In such a case, the PBE concept has no bite, since Bayes' rule is inapplicable when the observed event happens with probability zero in equilibrium. For a review of the literature on equilibrium refinements in signalling games, see Kreps (1989).

To develop in detail the many equilibria of the present signalling game and how most of them could be eliminated through the use of equilibrium refinements would be a space-demanding task. It suffices here to recognize the existence of refinements that would eliminate any pooling or hybrid equilibrium. One such refinement is *equilibrium dominance*, due to Cho and Kreps (1987), and the argument is the following: In an equilibrium stipulating that the bad-type A sends the message  $m$ , he will not be believed by the principal to have sent an off-the-equilibrium-path message  $m' \neq m$  if the best response by P that could reasonably be expected from sending  $m'$  makes him worse off than if he sends the message  $m$ .

This implies restrictions on the principal's beliefs: Messages having the property of message  $m'$  above are now restricted to giving rise to beliefs putting probability 1 on the agent being good. This eliminates any pooling equilibrium; see Kreps (1989, Sec. 7) for details. Cho and Sobel (1990) show that a class of signalling games, to which ours belongs, has a unique equilibrium satisfying a refinement similar to the above one; this equilibrium is separating.

## 2.4. Concluding remarks on preliminaries

In this Section, we have presented the standard principal-agent model with adverse selection, together with two important twists: one with competing principals (*i.e.*, market screening), and one with signalling. We saw that all these models have the property that (reasonable) equilibria are separating. Before finalizing this preliminary Section, we would like to offer two remarks, one on cases of continuous type spaces, the other on the concept of pooling.

Throughout the Section, we have maintained the assumption that the type space is finite; in fact, we have dealt with the two-type case only. With a *continuous type space*, our conditions on  $c$  and  $r$  above would have to be reformulated; in particular, the Spence-Mirrlees condition in (v)(d) above would now read:

$$\partial^2 c(m, t) / \partial m \partial t < 0, \text{ for all } m, t.$$

Moreover, with a continuum of types,  $P$ 's prior beliefs are represented by a probability distribution defined over  $T = [\underline{t}, \bar{t}]$ . Let  $\rho: T \rightarrow \mathbb{R}_+$ , with  $\int_T \rho(t) = 1$ , be the density function of this distribution; here,  $\mathbb{R}_+$  is the positive part of the real line  $\mathbb{R}$ .

Equilibria are separating also with a continuous type space, with two caveats: First, restrictions on prior beliefs are sometimes necessary to ensure separation. One such restriction, following from Baron and Myerson's (1982) analysis of a principal-agent model, is the following monotonicity condition on  $\rho$ :

$$\partial \left( \frac{\int_{\underline{t}}^t \rho(\tau) d\tau}{\rho(t)} \right) / \partial t > 0, \text{ for all } t.$$

This condition is sometimes called the *monotonic hazard rate property*.

Second, theory is thin on market screening with a continuum of types. It is well-known that a pure-strategy equilibrium does not exist with a continuous type space; see Riley (1979). Whether a mixed-strategy equilibrium exists is an unresolved issue.

Regarding the concept of pooling, we can use the apparatus of this section to be more specific on its definition. In the case of a continuous type space  $T$ , we say that an equilibrium features *pooling* if there exists a subset  $T^P \subset T$ , where  $T^P$  is not a singleton, such that the message sent by the agent is the same whenever he is of a type in this subset:  $m(t) = m(t')$ ,  $\forall t, t' \in T^P$ . In the case of a finite type space,

this definition is extended to include cases where the agent, if he is of a type on the boundary of  $T^P$ , sends the pooling message with positive probability.

This is a very broad definition of pooling, in at least two senses. First, it covers everything except complete separation; *i.e.*, it encompasses what some other authors would denote partial pooling, semi-pooling, or semi-separation. These authors would restrict pooling to cover only cases where  $T^P = T$ , with no randomising on the boundary if  $T$  is finite.

Second, it does not necessarily imply that the agent may be lying. Some authors would restrict pooling to cover situations where the agent, when he is of one type, chooses to send the same message as that of another type, in order not to reveal to the principal(s) his true identity. But with our definition, in addition to such cases, pooling occurs also when the agent would like to completely reveal his identity but is unable to do so because of various circumstances.<sup>9</sup>

Even with this broad concept, we are not able to cover all interesting cases where an informed agent chooses to hold back his information. This is particularly so when it comes to our discussion of imperfect media in section 5.2 below. There, we have included cases of incomplete revelation of information that do not fit our definition of pooling but which nevertheless are so intriguing that they deserve mentioning.

With this as a background, we start analysing the key question in this chapter: *In what instances will there be pooling in equilibrium?* A quick answer to this question is: when the single-crossing property does not hold. As will be clear over the following pages, there is much more to say on the subject than that.

### 3. THE AGENT'S INCENTIVES

We start out with the broadest issue, pertaining to the incentives of the agent. In fact, this is the largest Section of the chapter. We begin with analysing the message cost function, discussing those deficiencies it may have that give rise to pooling. Second, we discuss cases where the agent has countervailing incentives, *i.e.*, he has reasons both to send a low message and to send a high message. Then, in

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<sup>9</sup>One instance where this is at work, is when there are restrictions on the message space; see subsection 3.3.1 below.

subsection 3.3., we collect a miscellany of other factors that may be behind an occurrence of pooling, each pertaining in some way or another to the agent's incentives.

### 3.1. The message costs

In this subsection, we record some arguments for pooling that can be directly related to properties of the message cost function. The first instance is that of costless messages, or cheap talk. The next one is a natural generalization to coincident message costs. Third, we explore the effects of discontinuities in the message cost function.

#### 3.1.1. Cheap talk

The simplest case to consider is that of costless messages, or cheap talk. This is the case where, for all  $m, t$ :

$$c(m, t) = 0.$$

One usually combines this with the message being unproductive, *i.e.*, for all  $m, t$ :

$$\partial r(m, t) / \partial m = 0.$$

With no message costs, it seems hard for the good type to distinguish himself from the bad type. And indeed, with cheap talk, there is always an equilibrium with complete pooling. In fact, the extraordinary with many cheap-talk models is that there still is room for some transmission of information, *i.e.*, there exist equilibria with semi-pooling.

The scope for less than complete pooling is due to the interplay of two assumptions: First, the principal still has preferences over the type space of the agent:

$$r(m, G) > r(m, B).$$

Second, the payment from the principal to the agent is no longer done with a transferable utility. Rather, the principal takes an action  $y$  having the property that its effect on the agent depends on the latter's type: The agent has a utility function  $U^A(y, t)$  depending on his type  $t$  and the principal's action  $y$ . This utility



function is assumed to satisfy:

$$\partial U^A(y, G)/\partial y > \partial U^A(y, B)/\partial y,$$

*i.e.*, the marginal benefit of an increase in  $y$  is stronger for a good agent than for a bad one. These two assumptions create a kind of endogenous message costs sufficient to create less than complete pooling.

The seminal paper on cheap talk is by Crawford and Sobel (1982). They consider a signalling game with costless messages; a continuum of types,  $T = [\underline{t}, \bar{t}]$ ; and a message space equal to the type space:  $M = T$ . Their main result is the following: In any equilibrium, there is a partition of the type space splitting  $T$  into  $N$  subsets delineated by elements of a vector  $a = (a_0, \dots, a_N)$ , where  $\underline{t} = a_0 < \dots < a_N = \bar{t}$ , such that, for each  $n \in \{1, \dots, N\}$ , an agent of type  $t \in (a_{n-1}, a_n)$  chooses any message in  $(a_{n-1}, a_n)$  with equal probability, and the principal takes the same action following any message in  $(a_{n-1}, a_n)$ . This kind of pooling equilibrium Crawford and Sobel call a *partition equilibrium*. They characterize an integer  $N^*$  such that, for any  $N \in [1, N^*]$ , an equilibrium like this exists.

Work on models with costless messages has continued in many directions since the Crawford-Sobel 1982 article. Matthews (1989) extends their model by adding a last stage where the agent makes a choice. The model, as Matthews sets it up, is now akin to one of ultimate bargaining:<sup>10</sup> First, the agent talks; second, the principal makes an offer; third, the agent comes back and either accepts or rejects. Matthews argues that this story fits well with U.S. politics, with the President as the agent and the Congress as the principal: The agent's cheap talk is now Presidential rhetorics and the agent's rejection is a Presidential veto.

Sobel (1985a) introduces a multi-period version of the original model in which the agent sends messages repeatedly to the principal. Sobel's model has been used by Bénabou and Laroque (1989) to analyze the effects of the presence of financial analysts in stock markets.

Farrell and Gibbons (1989a) consider the case where the agent sends his message to multiple principals; this case becomes interesting when the agent is allowed to choose whether to send his message to one principal in private or to send the message publicly.

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<sup>10</sup>*Ultimate bargaining* means that one party makes a take-it-or-leave-it offer to the other party who then either accepts or rejects.

Myerson (1988, 1989) considers cases where the agent and the principal are allowed to make use of a mediator or a noisy channel through which the message from the agent is sent. If the mediator is properly instructed, or the channel properly constructed, Pareto-improvements may be obtainable compared to the case of direct communication. A repeated version of this kind of model is analysed by Forges (1990). It should be noted that some authors, like Farrell and Gibbons (1989b), want to draw a line, among games with costless messages, between those with and without a mediator; only the latter is cheap talk, while the former, studied by Myerson, is a *mechanism*.

Given the multiplicity of equilibria in games with costless messages, as witnessed already in the work of Crawford and Sobel (1982), it seems only natural that there have been attempts to develop *equilibrium refinements* particularly tailored to apply to cheap-talk situations. Farrell (1988) requires that an equilibrium be *neologism-proof*; Rabin (1990) discusses *credible message* equilibria; Austen-Smith (1990a) argues in favour of *credible debate* equilibria; and Matthews, Okuno-Fujiwara, and Postlewaite (1990) define *announcement-proof* equilibria.

One area of research in which the idea of cheap talk has proved powerful is that of *preplay communication*. Take a game, and extend it by creating a stage in the beginning where the players of the game can communicate before the start of the original game. They may interchange messages on their intentions, what we may call *endogenous information*, or on their respective types, what we may call *exogenous information*.

Although the transmission of endogenous information is without effect in the original Crawford-Sobel model [see De Groote (1990)], there are instances where it helps. This is particularly so when the original game has multiple equilibria and a need for *coordination* arises. In Farrell (1987), two firms contemplate entering a new market and they would both prefer that only one of them enters; cheap talk in the beginning reduces inefficiency. In Farrell and Saloner (1988), firms in an industry face the problem of reaching agreement on compatibility standards for their products; cheap talk in the beginning, in the form of an industry committee, again reduces inefficiency. In both these papers, the authors allow for multiple rounds of cheap talk.

Preplay communication of players' types has been analysed in several contexts. Farrell and Gibbons (1989b) explore the force of cheap talk at the start of a

bargaining game, while Gibbons (1988) discusses cheap talk in the related issue of arbitration. In Farrell and Saloner (1985), like in their 1988 paper, the issue is compatibility standards, and the authors study the effect of preplay communication of private information. Austen-Smith (1990b) analyses cheap talk viewed as a debate in a legislature. The election models of Chilton (1990) and Harrington (1990) seem to have elements of both endogenous and exogenous information; however, there are one-to-one correspondences between types and intentions, so that the endogeneity is more apparent than real. Chilton analyses cheap talk among the electorate in the form of a pre-election poll, whereas Harrington analyses cheap talk among candidates during their campaigns.<sup>11</sup>

One interesting application of the Crawford-Sobel model is Stein's (1989) analysis of the U.S. Federal Reserve Board ("the Fed") and its problem in credibly announcing its preferences for its own future policies.<sup>12</sup> Quite in line with the predictions from Crawford and Sobel's analysis, Stein suggests that the Fed, in order to avoid problems of time inconsistency, makes imprecise announcements and only indicates in which range its true preferences are.

### 3.1.2. Coincident message costs

Suppose, contrary to the previous subsection, that a message is costly, but that  $c(m, G) = c(m, B) = c$ , for all  $m$ ; we might call it the case of "flat message costs". More generally, suppose that  $c(m, G) = c(m, B) = \hat{c}(m)$ , for all  $m$ ; we call it the case of *coincident message costs*. The latter obviously generalizes the former, which itself generalizes cheap talk. Such models should be expected to give predictions similar to a cheap-talk model, since the single-crossing property is broken in the same fashion.

One instance of coincident message costs studied in the literature is in Meurer (1989). Meurer considers the case of a patent holder whose patent may or may not be valid according to the law, and who has private information on the validity issue. The patent holder's message in this context is an offer to a potential

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<sup>11</sup>Cheap talk in elections is also considered by Ledyard (1989).

<sup>12</sup>See also Oh and Garfinkel (1990) and Garfinkel and Oh (1991) on this issue.

licensee about a particular sharing of industry profits among the two. The licensee responds by accepting or litigating. The patent holder's offer is a binding one and thus not an instance of cheap talk. However, as Meurer argues, the offer has the same impact on the patent holder's future profits whether his patent turns out to be a valid one or not. Thus, we have coincident message costs. The equilibrium in this model is either one of complete pooling or one of partial pooling, in which the bad type (the holder of an invalid patent) randomizes; Meurer calls the latter a bluffing equilibrium.

There are also cases of *partial coincidence* of message costs. This implies that the message space can be split into two subsets:  $M = M^1 \cup M^2$ , with  $M^1 = [\underline{m}, m']$  and  $M^2 = (m', \bar{m}]$ , for some  $m'$  in the interior of  $M$ . For  $m \in M^1$ , message costs are coincident:  $c(m, G) = c(m, B)$ . For  $m \in M^2$ , message costs have the standard properties of Section 2, including the single-crossing property. Moreover, the message cost function  $c$  is continuous at  $m'$  for each type.

Mester (1988) presents a market screening model with partially coincident message costs. She studies a two-period credit market where consumers may borrow in period 1 from banks that offer combinations of interest rate and collateral requirement. Consumers differ in the probability of a high period-2 income making them able to repay the loan; each consumer knows his probability, but banks do not. A collateral requirement means that the consumer has to keep a corresponding part of his period-1 income as escrow. There is thus a natural upper limit on a consumer's borrowing: He cannot collateralize more than his period-1 income. For combinations of interest rate and collateral requirement for which this collateral restriction is not binding, message costs are coincident: Interest rate and collateral requirement are perfect substitutes and a small change in a bank's contract affects a consumer's expected utility independently of his type. When the collateral restriction *is* binding, the consumer uses all his period-1 income as collateral and consumes in that period out of the borrowed amount. In this case, interest rate and collateral requirement are imperfect substitutes and a consumer's preference for the two depends on his type; the single-crossing property holds. Mester (1988) finds, for the two-type case, that the equilibrium is a pooling one if  $\rho_G$  is sufficiently high and  $B$  is close to  $G$ . In a typical pooling equilibrium, all consumers borrow without any collateral requirements, a situation Mes-

ter interprets as credit-card borrowing.

In the market for term-insured annuities, consumers are offered combinations of annuity payment and the term (or length) of the annuity. If the term is short, then a consumer will save part of the payment, implying that term and annuity payment are perfect substitutes. However, when the term is above some critical length, then all of the payment is consumed and a consumer's preferences across payment and term depend on the probability distribution of his possible lifespan. Thus, if consumers differ in life-expectancy, there is a partial coincidence of message costs. Townley and Boadway (1988) consider the case where consumers have private information on this probability distribution: Some consumers are long-lived and some are short-lived; in the eyes of the insurers, it is the long-lived ones that constitute the high risks and thus are bad. In a market screening model, Townley and Boadway find that a pooling equilibrium may exist in which an annuity is offered with a term such that the long-lived are exactly indifferent between saving and consuming the last dollar of the first annuity installment.<sup>13</sup>

### 3.1.3. Discontinuous message costs

Suppose that there exists some message  $m'$  in the interior of  $M$  such that, for each  $t$ , either:

$$c(m', t) \neq \lim_{m \uparrow m'} c(m, t), \text{ or}$$

$$c(m', t) \neq \lim_{m \downarrow m'} c(m, t).$$

*I.e.*, the message cost function is discontinuous at  $m'$ . This will potentially create problems for self-selection, in particular with a continuous type space.

To be specific, suppose that we can write  $c(m, t)$  as the sum of two components:

$$c(m, t) = c^1(m, t) + c^2(m),$$

where  $c^1(m, t)$  satisfies all conditions outlined in Section 2, while  $c^2(m)$  is given by:

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<sup>13</sup>Strictly speaking, what we have here is partially coincident indifference curves only, implying that  $\partial c(m, G)/\partial m = \partial c(m, B)/\partial m$  for  $m \in M^1$ . This, thus, further generalizes the specification given above.

$$c^2(m) = \begin{cases} 0, & \text{if } m < m' \\ C, & \text{if } m \geq m' \end{cases}$$

for some  $m'$ . Suppose first that  $C = 0$ , and let  $m(t)$  be the equilibrium message sent by a type- $t$  agent. Suppose, furthermore, that there exists a type  $t'$  such that  $m(t') = m'$ .

If now  $C$  is increased, so that  $C > 0$ , then an agent of type  $t'$ , or even higher than that, might not want to pay the price necessary to be distinguished from an agent sending a lower message. We may get an equilibrium where agent types between  $t^1$  and  $t^2$  all send the same message  $m''$ , with  $t^1 < t' < t^2$ , and  $m'' < m'$ .

One model where message costs are discontinuous in this way is the entry deterrence model of Harrington (1986). Here,  $A$  is an incumbent firm that has private information on production costs in the industry. By its production decision today, the incumbent sends a message to a potential entrant (outsider) about costs. Let now  $T = [\underline{t}, \bar{t}]$  be the type space, with  $t = 1/c$ , where  $c$  is marginal production costs. There is a critical  $t' \in T$  such that the outsider, in case of complete information, would want to enter if and only if  $t > t'$ . Thus, there is a discontinuity in the incumbent's profit; this discontinuity occurs at the production level  $m'$  that is the equilibrium production level under complete information when  $t = t'$ . Harrington shows that we indeed get the equilibrium suggested above: Whenever the true type is between some  $t_1$  and  $t_2$  (*i.e.*, true costs are between  $1/t_2$  and  $1/t_1$ ), the incumbent produces the same output level  $m''$ , with  $t^1 < t' < t^2$ , and  $m'' < m'$ .

Harrington (1987) extends his 1986 analysis to the case of multiple incumbents, with the restriction that the outsider only observes the aggregate message, *i.e.*, the incumbents' total production. He gets basically the same result as before. It should, however, be noted that the potential entrant's inability to observe each incumbent's choice is crucial for this result. According to Bagwell and Ramey (1989), if the outsider *can* observe each incumbent's action, then the scope for existence of pooling equilibria is reduced.<sup>14</sup>

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<sup>14</sup>Bagwell and Ramey prove the existence of separating equilibria without distortions, relative to the complete-information case. This is because the incumbent firms are unable to coordinate deception: If one incumbent deviates while the others play separating strategies, the outsider nevertheless makes the correct inference. Therefore, it does not pay to deviate from a non-distortive separating equilibrium.

An entry-deterrence model similar to Harrington (1986) is studied by Green and Laffont (1990) and they find an equilibrium with the same features. But the main concern of Green and Laffont is to consider the effect of extending this model to the case where entry by the outsider is to be deterred simultaneously on many fronts, where one front may be a particular product or location. Each front's type is known to the incumbent but not to his rival. When the outsider now takes her decision on whether to enter at a particular front, she may base this decision on the incumbent's behaviour on all fronts. This creates a change in the equilibrium, although still a pooling one. Let, as before,  $t'$  denote the critical type. Now there are some  $t_1^F$  and  $t_2^F$ , not necessarily equal to  $t_1$  and  $t_2$  above, but with  $t_1^F < t' < t_2^F$ , such that each  $\hat{t} \in (t_1^F, t')$  is pooled with a  $\tilde{t} \in (t', t_2^F)$ , in the sense that  $m(\hat{t}) = m(\tilde{t})$ .

Another version of discontinuous message costs would have the above message cost function be multiplicative rather than additive, *i.e.*:

$$c(m, t) = c^1(m, t)c^2(m),$$

with  $c^1$  and  $c^2$  each having properties as above. With  $C > 0$ , we might now expect that, in equilibrium, all agent types below some  $t' > \underline{t}$  send the same message.

One signalling model featuring something like the multiplicative version of discontinuous message costs is provided by Banks (1990a). There, two candidates compete to win an election, and each candidate has private information on his true preferred policy, which belongs to a continuum from the left to the right. The electorate constitutes the uninformed parties, the principals. A candidate may, in his pre-election campaign, announce a policy that differs from his true preferred policy, but only at a cost. The larger the distance between announced and true policies, the higher is this costs, and the cost function also satisfies the Spence-Mirrlees condition. However, these costs will only be suffered in office, *i.e.*, in case the candidate wins the election. Given the action of the other candidate, a candidate's probability of winning depends on his announced policy, his message. It turns out that, in equilibrium, there is a band of moderate candidate types that all announces the same ("average") policy, whereas extremist types on the left and on the right are separated out. The higher is  $C$ , the wider is the band

of pooled types.<sup>15</sup>

### 3.2. Countervailing incentives

Suppose that the agent has incentives both to send a high message and to send a low message. He may have one reason for creating the impression that he is good and another reason for creating the opposite impression. These two reasons obviously are in conflict, a conflict that invariably makes it impossible to sustain complete separation, and pooling occurs. In this Section, we record some results on situations featuring this phenomenon. To start with, we consider cases where the conflict stems from characteristics of the message cost function. Thereafter, we consider cases where the conflict arises due to multiple audiences. Finally, other reasons for the conflict are collected.

#### 3.2.1. A decomposed message cost function

Let the message cost function  $c(m, t)$  be split into two elements:

$$c(m, t) = c^1(m, t) + c^2(m, t),$$

such that the two elements have conflicting properties: The first element,  $c^1$ , behaves like a standard message cost function; in particular:  $\partial c^1(m, t)/\partial m > 0$  and  $\partial c^1(m, G)/\partial m < \partial c^1(m, B)/\partial m$ . The second element,  $c^2$ , has the opposite properties, in the sense that the above inequalities are changed:  $\partial c^2(m, t)/\partial m < 0$  and  $\partial c^2(m, G)/\partial m > \partial c^2(m, B)/\partial m$ .<sup>16</sup> With such a message cost function, it may be impossible to obtain self-selection, so that pooling entails.

The theory of principal-agent models featuring such a decomposed message cost function, instead of the standard one, is developed by Tracy Lewis and David

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<sup>15</sup>Models related to Banks (1990a) are found in Reed (1989) and Hess (1991). They, too, discuss elections under asymmetric information about candidate preferences. Reed's model is very similar to that of Banks, while Hess presents a somewhat richer model which still shares the basic feature: Your pre-election message expenditures matter only if you win the election.

<sup>16</sup>The other three properties of a message cost function, listed in item (v) in section 2.1 above, are the same for  $c^1$  and  $c^2$ .



Sappington in a series of papers. In fact, they are the ones that have coined the term "countervailing incentives". They use it, however, to cover only cases relating to the message cost function. Here, we would like to make the point that countervailing incentives, understood as the phenomenon that an agent may have one reason for wanting to be perceived as good and another reason for wanting to be perceived as bad, are of a more general nature than a reading of Lewis and Sappington's work suggests.

Lewis and Sappington, in their papers, assume a continuum of types,  $T = [\underline{t}, \bar{t}]$ . Their general finding is that, in equilibrium, there are  $t^1, t^2 \in T$ ,  $\underline{t} < t^1 < t^2 < \bar{t}$ , such that agent types between  $t^1$  and  $t^2$  are pooled. For types above  $t^2$ , the first element of the cost function dominates and higher types send higher messages. For types below  $t^1$ , the second element dominates and lower types send lower messages.

In Lewis and Sappington (1989a), they reconsider, for the case of countervailing incentives, the problem of optimal regulatory policy formulated by Baron and Myerson (1982). Here, the message is the regulated firm's report on its production costs, and the principal's action is, for each reported cost level, a combination of price and quantity. The countervailing incentives occur because of a negative relation between a regulated firm's marginal costs and its fixed costs. The authors find that agent types in a medium range are treated identically.

The same happens in the variation reported in Lewis and Sappington (1989b, 1989c), where the principal, in addition to the price and quantity functions, specifies the regulated firm's production capacity and allows the firm to sell unrestrictedly its production in excess of what the principal demands. The regulated firm may increase its outside earnings by increasing its outside price. But this reduces the compensation it receives from the regulator. Thus, countervailing incentives occur.

In Lewis, Perry, and Sappington (1989), the setting is a supplier and a buyer who earlier have gone into a contractual relationship. Today, when the supplier has obtained private knowledge of his opportunity cost  $c$ , *i.e.*, the price he gets for sales elsewhere, the parties renegotiate the contract. In this renegotiation, the buyer makes a take-it-or-leave-it offer to the supplier, in the form of a price-quantity combination conditioned on the supplier's report of his  $c$ . If the supplier

rejects this offer, the old contract prevails. Again, the supplier has countervailing incentives for the same reasons as in the previous paragraph: When  $c$  is low, outside earnings are low and it is more profitable for the supplier to engage in a continuing relationship with the buyer. On the other hand, when  $c$  is high, the buyer must increase its compensation to the supplier to get him to supply. The buyer's optimum offer has the following properties: In a medium range of reported  $c$ , the initial contract is continued. Outside this range, the supplier is offered an increased price. For low values of reported  $c$ , this is accompanied with an increase in contracted quantity; for high values, there is a reduction in quantity.

Lewis, Feenstra, and Ware (1989) study a situation where a government wants to eliminate price support from an industry in which the workers have private information on their skills and income opportunities in other industries. The government offers a combination of transfer and required output conditioned on a worker's report of skill level. Upon the problem is imposed a political-feasibility restriction that the new scheme gain support from a given fraction of the workers. This creates countervailing incentives: A worker would like to report low skills in order to be "bribed" to give his support; he would like to report high skills in order to receive the government's compensations necessary to retain high-skilled workers. In the optimum solution, there is a medium range of worker skills that are pooled.

In line with the political-economy theme of the previous paragraph, Feenstra and Lewis (1987) consider two countries, Home and Foreign, renegotiating a trade agreement between them. The Home economy is a two-good one with the importable good being labour intensive. A trade agreement is a pair of exchanged quantities of importables and exportables. The capital endowment of Home's median voter is known to the Home government but not to the Foreign one. There are countervailing incentives here, since, in addition to the standard welfare gains from trade, there are political costs for the Home government of agreeing on a particular trade level. These political costs depend on the capital endowment of Home's median voter. The less capital this voter is endowed with, the higher are the political costs. This is so because there is a component of welfare loss attached to imports: An increase in imports lowers the price of the labour-intensive importable and shifts income from labour to capital; thus, a voter low on capital does not like high imports. Therefore, Home would like to report a

low capital endowment of its median voter to protect itself from these political costs, and to report a high endowment to reap the gains from trade. The negotiations are handled by a third party putting weights on both countries' welfare. It turns out that, if the weight put on Home is the smaller of the two, then there is a medium range of values of Home's reported capital endowment all leading to the *status quo* agreement being upheld. Outside this range, an increase in reported endowment implies increased trade. For low values, trade decreases, while for high values, trade increases, relative to the *status quo* agreement.

In concluding this subsection, consider the extreme case where the decomposed message cost function is degenerate, in the sense that  $c^1$ , the element in the cost function behaving normally, equals zero. Thus, the total message cost function has opposite properties, compared to the standard case, throughout the whole type space. A case in point is the regulation of a labour-managed firm; see Thomson (1982) and Guesnerie and Laffont (1984a, 1984b).<sup>17</sup> A well-known result from the theory of labour-managed firms is that such a firm, by maximising per-capita value-added rather than profit, responds to a decrease in price by expanding, rather than contracting, output. This pathological behaviour creates countervailing incentives in the present degenerate sense: The regulator wants the firm to send a high message, but the labour-managed firm has inherent incentives to send a low message. Applying the principal-agent model of Baron and Myerson (1982) to the regulation of such a labour-managed firm, Guesnerie and Laffont (1984a, 1984b) find that the optimum contract involves complete pooling.<sup>18</sup>

### 3.2.2. Multiple audiences

Suppose that the set of principals can be partitioned into two subsets that do not interact with each other; however, they do, of course, all interact with the agent

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<sup>17</sup>The result of Guesnerie and Laffont is nicely reviewed in Caillaud et al. (1988, Sec. 5).

<sup>18</sup>Interestingly, this does not carry over to other specifications of the regulation problem. Laffont and Tirole (1986) suggest a model of regulation that differs from Baron and Myerson (1982) mainly by the inclusion of moral hazard. This moral hazard is, however, analytically trivial in the sense that it can be handled through the agent's first-order condition. Askildsen (1990) shows that applying a model of the Laffont-Tirole kind, instead of the Baron-Myerson one, to the regulation of a labour-managed firm makes complete separation feasible.

A. Thus, we write  $P = P_1UP_2$ . Now, A may want members of  $P_1$  to believe that he is a bad type at the same time as he would like members in  $P_2$  to believe that he is a good type. If this is the case, we have an instance of countervailing incentives, and there will be some pooling in equilibrium.

There are some nice examples in the literature of models where an agent sends his message to more than one audience.<sup>19</sup> One particularly convincing case is discussed by Gertner, Gibbons, and Scharfstein (1988). They study a firm which has private information on its profitability and whose choice of debt-equity structure is observed by both investors in the capital market and a potential entrant into the firm's product market. The firm would like to pretend being a high-profit firm in order to attract investors but would, at the same time, like to pretend being a low-profit firm in order to discourage the outsider from entering. Thus, the firm has countervailing incentives. It turns out that the form of equilibrium (pooling or separating) is the one that maximises the firm's *ex ante* expected profit in the product market; thus, conditions in the product market are decisive in shaping the firm's financing decisions.

Caillaud (1990) is very much related to the previous one, since, again, the "second audience" is potential entrants. But instead of a firm signalling to financial investors, Caillaud discusses a version of Baron and Myerson's (1982) principal-agent model in which the agent is not a regulated monopoly, but rather a regulated incumbent who is threatened by entry from firms that are beyond the reach of the regulating body. Again, countervailing incentives arise: The firm gets compensated by the principal for reporting low costs. At the same time, it wants the outsiders to believe that its industry is a high-cost one.<sup>20</sup> In equilibrium, there may be several non-empty intervals of types that are pooled within each such interval.

Engelbrecht-Wiggans and Kahn (1991) consider an oral contract auction in which the winning bidder proceeds to negotiate with its employees on wage terms during work on the contract. Of course, bidders would want to bid high in

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<sup>19</sup>In addition to the ones mentioned here, we also discuss one such model, by Farrell and Gibbons (1989a), in subsection 3.1.1 on cheap talk, and also two models, by Darrough and Stoughton (1990) and Wagenhofer (1990), in subsection 3.3.4 on verifiable disclosure.

<sup>20</sup>Thus, Caillaud's model may also be considered a two-audience extension of Harrington's (1986) entry-deterrence model, with the additional audience being the regulator.

order to win the contract. On the other hand, since the oral auction reveals information about the winner that is of value to its employees in the following negotiations, bidders have countervailing incentives as long as the winning bid is observed by employees. The outcome of this oral auction is, therefore, an equilibrium which may be pooling, partially pooling, or separating, depending on parameter values. On the other hand, in a sealed-bid second-price auction, although the identity of the winner is known, the winning bid is not. Therefore, such an auction, by protecting the winner from exploitation by third parties, reduces the scope for pooling relative to an oral auction.

### 3.2.3. Conflicting roles – and more

Consider an agent who owns an asset and has private information on the asset's quality. He may sell some of it on the market at a price and keep the rest for his own consumption. If the price on the market increases with the quality he reports, he may want to report a higher quality than the true one. On the other hand, if the true quality is high, he wants to keep most of the asset for himself, which makes him want to report a low quality in order to keep the demand down. Thus, the agent's role as a seller of the asset conflicts with his role as a consumer of the same asset. The outcome of this conflict is that the equilibrium will be a pooling one.

The situation described above has been analysed by Jean-Jacques Laffont and Eric Maskin in a series of papers.<sup>21</sup> In Laffont and Maskin (1987, 1989a), the agent is a monopoly seller of a good. In Laffont and Maskin (1989b), the case is complicated by introducing several firms selling a product whose quality all firms know, but not the consumers. In Laffont and Maskin (1990), there is again only one informed agent, but this time he is an insider trader on the stock market and thus a would-be buyer rather than seller. Related to these papers is Kumar (1988), who study the relation between an inside entrepreneur and his outside shareholders. When the entrepreneur decides on his message, how much dividends to pay the outsiders, his desires to provide the latter with incentives to reinvest are in con-

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<sup>21</sup>See also Stiglitz (1982) and Nöldeke (1991).

flict with his desires to be left with as much as possible when the firm is liquidated. The pooling equilibrium in Kumar's model is of the partition kind described in subsection 3.1.1 when discussing the Crawford-Sobel cheap-talk model.

Two other papers exist in which this conflict of roles is described but where the authors discuss how to implement the first-best rather than whether the game typically features a pooling equilibrium or not.<sup>22</sup> Cramton and Palfrey (1990) analyze a problem of explicit collusion with side payments in which each firm has private cost information. In this case, firms may wish to overstate or understate their costs, depending on whether they are asked by the cartel organizers to expand or contract output. Cramton, Gibbons, and Klemperer (1987) study a situation where a partnership is to be dissolved among associates with different, privately known valuations of an indivisible asset. A partner would like to overstate or understate his own valuation depending on whether he is going to sell or buy stakes in the asset.

We conclude the discussion of countervailing incentives with two pieces of literature serving to illustrate the diversity of situation in which the phenomenon arises. Consider first Hendricks and Kovenock (1989). They discuss ultimate bargaining between two oil companies on the transfer of drilling leases in the case where the firms have private information on the amount of oil in the area. Firm 1 has a longer lease than firm 2 and makes an all-or-nothing offer to it. If firm 2 accepts, it gets the appropriate amount from firm 1, transfers the lease, and reveals its private information. If firm 2 rejects, it decides whether to drill or not and firm 1 acts optimally to this decision. Before its offer, firm 1 is allowed to report a subset of  $T$  in which its true type belongs.<sup>23</sup> Models with this kind of message space go in the literature under the name of *games of persuasion*. Such games are also studied by Milgrom (1981), S. Grossman (1981), and Milgrom and

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<sup>22</sup>The fact that these two papers are cited by Lewis and Sappington (1989a, 1989b) as examples, outside their own work, of countervailing incentives, indicates that Lewis and Sappington themselves perceive this phenomenon as one not necessarily relating to the message cost function.

<sup>23</sup>An agent of type  $t \in T$  is allowed to report any set  $m \subset T$  such that  $t \in m$ ; thus, the message space is a subset of the power set of  $T$ . The interpretation is that the agent cannot lie about his type, but he does not have to tell the whole truth.

Roberts (1986).<sup>24</sup> These authors find equilibria to be fully separating. Hendricks and Kovenock (1989), on the other hand, find that there is some degree of pooling in any equilibrium, including one with complete pooling. This is because a low-type firm 1, *i.e.*, one with little oil, would want firm 2 to reject and drill, while a high-type firm 1 would want firm 2 to accept its offer. This lack of unanimity among types creates pooling.

Finally, Poitevin (1989) analyzes an entrepreneur who needs outside capital and who must decide on how much of this capital to raise through debt and how much through equity. After the financial structure is determined, the success of his project depends on his effort, unobservable to investors. This *ex-post* moral hazard problem makes the entrepreneur want *ex ante* to issue debt, since debt provides him with more discipline than does equity. At the same time, however, the entrepreneur has private information on the productivity of his project. He may indicate high productivity by issuing so much equity that a bad type would not want to mimic. It turns out that, in equilibrium, there is pooling; moreover, the *ex-post* moral hazard effect dominates, so that both types choose full debt financing.

### 3.3. Agent's incentives: other factors

The four issues we treat in this subsection all pertain, in some sense or another, to the agent's incentives, without being directly connected to the previous issues. First, we discuss fallacies of the message space. Second, we discuss risk aversion in the case of *ex-ante* contracting. Third, moral-hazard issues are briefly considered. And finally, we discuss the case of verifiable disclosure of information.

#### 3.3.1. The message space

Restrictions on the message space may cause pooling to occur in equilibrium. The basic reason for this is that such restrictions may make it difficult for the

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<sup>24</sup>Leland (1981) explains how games of persuasion may be viewed as a special class of signaling games.

good type to distinguish himself from the bad type. These restrictions are of two kinds in which the message space is finite or bounded. We treat these in turn. Then, we remark on the related issue of models with non-contingent contracts.

*A finite message space.* Suppose the message space now consists of two distinct messages only:  $M = \{\underline{m}, \overline{m}\}$ . Whether a pooling equilibrium exists or not depends on the types' costs and benefits of mimicking rather than self-selecting. And in the principal-agent model, the principal's payoff from pooling versus separation also matters. The reason pooling may be the outcome is the inflexibility created by the finite message space. Obviously, if the number of types is greater than the number of messages, there will always be some pooling.

In a sense, a finite message space is a rather trivial cause for pooling. Nevertheless, instances of this is reported in the literature. Flannery (1986) analyzes whether a firm with private information about its quality can signal this information to investors through its choice of debt maturity.<sup>25</sup> Bad firms prefer long-term debt, and thus a good firm may signal its type by issuing short-term debt. However, with only two maturity levels to choose from (short and long), there exist pooling equilibria for large ranges of the parameters. The pooling may be at either a short-term or a long-term debt structure.

Hughes and Schwartz (1988) analyze whether a firm can signal its quality by its choice of method for inventory valuation. Bad firms prefer the LIFO method, so that a good firm may signal its type by sticking to the FIFO method. Again, pooling equilibria are shown to exist and may be at either LIFO or FIFO.

Of the two papers discussed above, the latter by Hughes and Schwartz (1988) seems to have the greater appeal, since their restriction of the message space is a natural one: There are indeed only two methods of valuing inventory. Their prediction of pooling should nevertheless be interpreted with care: As Fellingham (1988), in discussing this paper, remarks, the scope for pooling is reduced if the model is enriched by, *e.g.*, allowing firms additional ways of signalling quality.

A variation of the theme of finite message space is found in papers on macroeconomic policy games by Backus and Driffill (1985) and Barro (1986). They model privately informed governments such that the good type has only one

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<sup>25</sup>See also Diamond (1991).



message to send: The "dry" government is restricted to a zero-inflation policy.<sup>26</sup> Thus, the only question is whether the bad type (the "wet" government) will mimic this policy or not. A comparison of these papers with Vickers (1986), in which the message space is more standard, makes it clear that restricting the dry government this way increases the scope for pooling considerably.<sup>27</sup>

*A bounded message space.* Several authors impose upper bounds on the message space with the effect that the good type, even if he sends the highest feasible message  $\bar{m}$ , is unable to distinguish himself from the bad type, with the result that they both send the message  $\bar{m}$  in equilibrium.

Kennedy (1989), for example, analyses a product market where consumers are uncertain about product quality and where high-quality producers may signal their type by a high first-period quantity, entailing a low introductory price. The scope for signalling is limited by the non-negative price constraint. Thus, there may exist cases with several types pooling at price zero. Note, though, that for this to occur, consumer preferences must feature satiation, such that demand is finite at price zero. Otherwise, with infinite demand at price zero, profits will tend to minus infinity at this price, thus making signalling prohibitively costly for firms of any type. Thus, non-satiation restores complete separation.

In analyzing a macroeconomic policy game like the ones discussed above, Hoshi (1988) argues that there is a lowest conceivable inflation rate: If low inflation is obtained by a low money supply, there is a limit on how little money there can be in circulation in the economy. On the other hand, many macroeconomists would argue that there are other ways of obtaining a low inflation, such as by regulating the central bank's lending rate.

Giammarino and Lewis (1989) present a model in which a firm negotiates equity financing with an underwriter. The firm gives a take-it-or-leave-it offer of the number of new shares issued. The firm, which has private information on its value, may want to offer many shares in order to prove itself of high value. However, there is a certain number of new shares, call it  $m'$ , beyond which the underwriter will accept the offer with probability 1 because, with that many shares, her

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<sup>26</sup>This is also how Milgrom and Roberts (1982) treat the strong monopolist in their incomplete-information analysis of the chain-store paradox.

<sup>27</sup>The crucial difference between Vickers and the others is also noted by Driffill (1989).

payoff will be non-negative irrespective of the firm's type. Giammarino and Lewis choose to restrict the message space at  $m'$ , thus making it impossible for the good firm to distance itself from the bad type. This, apparently, is the main reason for pooling, in their paper, for a huge range of parameter values.

But there are also models which provide definitive bounds on the message space. Consider, for example, Boyer et al. (1990). They study two firms sequentially entering a spatial market, with the first firm having private information on its costs. While a high-cost firm would like a peripheral location, thus serving only a small part of the total market, a low-cost firm would like to locate in the centre. The scope for the low-cost firm to signal by exaggerating its message is nil: Which-ever direction it moves away from the preferred central location, it moves towards the periphery. The implication is that, for a huge range of parameters, the two types are pooled at the centre: The high-cost firm mimics the low-cost firm, which has nowhere else to go. Thus, Boyer et al. (1990) have provided us with a model with a very natural constraint on the message space.

So, too, has Reinganum (1988), analysing a particular model of ultimate bargaining with private information on both sides. The case she considers is that of plea bargaining, with the prosecutor having private information on the strength of her case, and the defendant having private information on whether he is guilty or not. Based on the strength of her case, measured by the probability  $t \in T = [0, 1]$  that the defendant will be found guilty at trial, the prosecutor offers him a sentence  $m(t)$ . If this offer is rejected, the case goes to court. In this model, there is a critical  $t'$  such that, if  $t < t'$ , then  $m(t) = 0$ ; *i.e.*, if the case is sufficiently weak, then it is dismissed. Thus, pooling occurs because a prosecutor with a weak case (a good type in the eyes of the defendant) can go no further than to a complete dismissal.<sup>28</sup>

Finally, we should mention that bounds on the message space may be im-

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<sup>28</sup>Three things should be noted here. First, dismissals are not done to save costs of trial in this model, since  $t'$  is independent of such costs. Second, not all systems of criminal justice work like Reinganum's model, which is probably meant to model the U.S. system. In Norway, although it is at the prosecutor's discretion to dismiss a case on grounds that it is too weak, she is not allowed to negotiate by guaranteeing a lighter sentence if the defendant pleads guilty. Third, with private information on both sides, Reinganum's model can be viewed as exhibiting an *informed principal* (the prosecutor). Although information on the part of the principal may lead to pooling, see subsection 4.2, this is, however, not the reason for pooling in the present model, as Reinganum's model satisfies Maskin and Tirole's (1990b) common-value condition for separation.

posed by the principal for reasons of fairness. One case in point is income maintenance programmes, where the government (the principal) self-imposes the restriction that no individual shall be left with income below some minimum. When designing its welfare policy under such a constraint, it leaves individuals in the poor end of the type space with no possibilities to accept lower income than others in return for higher contributions from the government. Thus, minimum income levels create pooling. Besley and Coate (1991) is a recent contribution to this literature.

*Non-contingent contracts.* There is a class of asymmetric-information models, not otherwise considered in this survey, in which principals are not allowed to offer contingent contracts. When left with non-contingent contracts, one cannot obtain any screening. Thus, even though these models depict situations of asymmetric information, they are designed so as to make the pooling question irrelevant, in the sense that pooling is assumed, rather than derived, in these models. Prominent papers are in this category, including Stiglitz and Weiss (1981) on credit rationing. Authors introducing contingent contracts into the Stiglitz-Weiss model have found that this may eliminate credit rationing, because now self-selection can be upheld; see, e.g., Bester (1985), whose contracts are contingent on collateral, and Terlizzese (1989), who avoids the use of collateral by employing more complex contracts.

Related to this is the issue of whether self-selection may deteriorate welfare. If this is the case, then society prefers pooling and the optimum public policy is to make contingent contracts illegal. For example, in the market screening model, restricting the competing principals to offering pooling contracts is welfare enhancing if and only if a pure-strategy equilibrium does not exist. The principals earn zero expected profit in any case, and the bad type always prefers pooling to separation, so the question boils down to when the good type prefers pooling, which is exactly when a pure-strategy equilibrium cannot be upheld. The welfare effects of this kind of restrictions in a signalling model are discussed by Aghion and Hermalin (1990), while Fishman and Hagerty (1990) show that a related kind of restriction may be welfare-enhancing in a game of persuasion. Reinganum (1988), in her model of plea bargaining discussed above, finds that, if the fraction of guilty among all arrested is sufficiently high, then one increases welfare by limiting the prosecutor to giving pooling sentence offers.

### 3.3.2. Risk aversion and *ex-ante* contracting

The agent's attitude towards risk is generally not of particular concern, apart from the fact that risk aversion may be necessary in order to make a model interesting.<sup>29</sup> Whether the agent is risk neutral or risk averse does not affect the scope for pooling, except in one case. This is the case of *ex-ante* contracting, *i.e.*, the variation of the principal-agent model in which the contract between the principal and the agent is signed before the agent gets to know his type.

As mentioned in Section 2.1, *ex-ante* contracting implies that there is only one participation constraint for the agent, based on his expected payoff from entering into the contract, where the expectation is taken with regard to the probability distribution  $\rho$  over agent types. Suppose that there is a continuum of types:  $T = [\underline{t}, \bar{t}]$ . Intuitively, when the agent is risk averse, he will have problems *ex ante* with entering into a contract implying that bad types are severely punished when sending low messages, since there is a probability that he will turn out to be such a bad type. The optimum contract for the principal is one where, for some  $t' \in (\underline{t}, \bar{t})$ , depending on the degree of risk aversion, agent types in the lower range  $[\underline{t}, t']$  are pooled. By thus cutting the lower tail for the agent, the principal obtains a contract that is an improvement for her in case of a good type. However, when the agent is very risk averse, his only concern is with the lower tail, and the scope is reduced for the principal to obtain any benefits from higher types through an *ex-ante* contract with pooling in the lower tail. In the limit, when the agent is infinitely risk averse, thus caring only for his payoff in the worst outcome, the principal can do no trade-off like this, and there is again complete separation.<sup>30</sup> The results in this paragraph are due to Salanié (1988, 1990).

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<sup>29</sup>For example, when either the principal-agent model or the market screening model is applied to insurance.

<sup>30</sup>It may be worthwhile to note that the participation constraint in case of *ex-ante* contracting with an infinitely risk averse agent, written:

$$\min_t \Pi^A(m(t), t) \geq 0,$$

is formally equivalent to the participation constraint in the standard principal-agent model:

$$\Pi^A(m(t), t) \geq 0, \forall t.$$

A situation related to the above one is the following, studied by Holmström and Ricart i Costa (1986). Suppose that, at the time of contracting, the principal and the agent are symmetrically informed. Suppose, furthermore, that the decision on whether to improve the parties' information about the agent's type is left to the discretion of the agent. If now the latter is risk averse, he may prefer not to take any action at all, out of fear that he thus would disclose that he is a bad type. In the paper by Holmström and Ricart i Costa, the two parties are a risk averse manager and his superior, and the action is an investment decision. They find the second-best contract in this case, exhibiting under-investment compared to the first-best. This under-investment may be suitably interpreted as pooling caused by the combination of a risk averse agent and *ex-ante* contracting.

### 3.3.3. Moral hazard

Moral hazard is the term used in the literature to describe situations where the agent, by some action, may affect his type, and where this action is unobservable to the principal. Although an important topic, and a major one in the literature, we will only touch upon it here, since we are concentrating in this survey on adverse-selection type phenomena.

Some authors have analysed models with both moral hazard and adverse selection. The basic complication created by the introduction of moral hazard is that the agent's indifference curves typically are no longer convex, thus making the single-crossing condition fail to hold.<sup>31</sup> Stiglitz and Weiss (1986) consider market screening in the credit market with the above set-up and find that, even though contracts are contingent on collateral, there may exist a pooling equilibrium and, thus, credit rationing. A similar result is obtained by Aron (1987), studying market screening in a labour market where both adverse selection and moral

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<sup>31</sup>On the non-convexity of indifference curves under moral hazard, see Arnott and Stiglitz (1988).

hazard are present.<sup>32</sup>

### 3.3.4. Verifiable, but costly, disclosure

A particular branch of the literature studies situations in which the agent does not communicate by sending some message; rather, he is able to state his type and verify that he is telling the truth.<sup>33</sup> Then, of course, it pays for everybody, except the worst, to report his type. However, this is not so if there are some costs associated with stating one's type. Typically, with a continuous type space  $T = [\underline{t}, \bar{t}]$ , there exists some  $t' \in (\underline{t}, \bar{t})$  such that an agent of a type in the lower range  $[\underline{t}, t']$  does not disclose his type, and thus agent types in this range are pooled. The higher are the disclosure costs, the higher is the critical type  $t'$ .

An early statement of the above result is in Jovanovic (1982). In Sobel (1985b), the model is enriched by assuming there are two agents and a principal; the two agents are two bargainers who each decides whether or not to report his claim to a disputed item to a judge, the principal, who is to decide who shall have the right to the item.

One case where the assumption of verifiable, but costly, disclosure seems particularly appropriate is that of financial statements. Information may be withheld, but whenever it is included in the financial statement, the fact that the statement is audited makes the information verifiable. This case was first studied by Verrecchia (1983). Dye (1986) goes on to assume that the private information of the reporting firm is multi-dimensional. He is thus able to consider cases of *partial disclosure*, in which the firm reports its type along some, but not all, dimensions.

In many of the models discussed above, the agent's private information is,

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<sup>32</sup>On the other hand, if contracts are non-contingent but *ex-post* testing is feasible, then the addition of moral hazard may create scope for self-selection, with an equilibrium in which a bad agent shirks while a good one does not, and the principal monitoring *ex post* and not paying an agent that has been found shirking; see Bar-Ilan (1991). Pooling may recur, however, if the principal is unable to precommit to performing the test; more on this is in subsection 6.2.

<sup>33</sup>This situation differs from a game of persuasion, a concept due to Milgrom (1981) and introduced in Sec. 3.2 above. Here, truth-telling implies reporting one's type, whereas in a game of persuasion, truth-telling implies reporting any subset of the type space that includes one's type.

in fact, some signal that is only imperfectly correlated with his true type. Our discussion simplifies by making the correlation perfect. This simplification does not affect the intuition. In Verrecchia (1990a), however, the discussion centres around this imperfect correlation between the agent's information and his true type. Verrecchia shows that an increase in the precision of the agent's information decreases the range of pooling types. He also finds that, if the agent can himself choose the correlation *ex ante*, he would prefer that his information is completely uncorrelated with his true type, so that this information is never disclosed, disclosure costs are saved, and complete pooling results.

Two papers on verifiable disclosure towards multiple audiences relate to our discussion in subsection 3.2.2 on this topic. Quite in line with the discussion there, Darrough and Stoughton (1990) and Wagenhofer (1990) find that the presence of a potential entrant, in addition to the investors, makes a firm more willing to disclose bad news, thus reducing the scope for pooling.<sup>34</sup>

In a variation on the theme of verifiable disclosure, there is no disclosure costs, but there is a positive probability that the agent does not have any private information at all. In this case, an agent of a bad type may hide behind this uncertainty and, by not disclosing his type, pretend to be without private information; see Farrell (1986) and Shavell (1989a). As Farrell shows, even a small such probability produces a lot of pooling in the lower type range. See also Dye (1985) and Jung and Kwon (1988), discussing the contents of firms' financial statements under such circumstances.

#### 4. THE PRINCIPAL'S INCENTIVES

This section complements the previous Section, in that we here treat reasons for pooling that are found on the principal's side of the table. First, we consider the

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<sup>34</sup>Verrecchia (1990b) provides a discussion of Darrough and Stoughton (1990). In Bernhardt and LeBlanc (1991), the firm's message is again observed by both competitors and investors. The firm can choose between (a) costless, verifiable information and (b) costly signals. The reason for considering a type (b) message is that it merely contains some summary information, like profitability, which is useful for investors but on which competitors are unable to base an entry decision. The type (a) message, on the other hand, contains technical production information that is vital to competitors. In equilibrium, good types either disclose more verifiable information than bad types or disclose nothing at all.

gross payoff function, an issue that parallels our discussions of various aspects of the message cost function in subsections 3.1 and 3.2.1 above. Second, we discuss the case where the principal herself has incentives to hide information. Third, we look at the case where the principal may audit the agent to find out about his type, but only at some cost; this complements the case of agents' disclosure costs discussed in subsection 3.3.4. And finally, we look at restrictions on the principal's actions, in parallel to the discussion of restrictions on the agent's message space in subsection 3.3.1 above.

#### 4.1. The gross payoffs

The principal's gross payoff function  $r$  measures her benefit, gross of the payment to the agent, from an agent of type  $t$  sending the message  $m$ :  $r = r(m, t)$ . We have assumed so far that  $\partial r(m, G)/\partial m > \partial r(m, B)/\partial m$  for every  $m$ , or, in the case of a continuous type space:  $\partial^2 r(m, t)/\partial m \partial t > 0$ , for every  $m$  and  $t$ ; in words, the principal benefits more from a marginal increase in the agent's message, the higher is the type of the agent. Suppose now that this is no longer true. In particular, suppose that, with a continuous type space, there exists some  $t' \in (\underline{t}, \bar{t})$  such that:

$$\partial^2 r(m, t)/\partial m \partial t < 0, \text{ for } t < t', \text{ and}$$

$$\partial^2 r(m, t)/\partial m \partial t > 0, \text{ for } t > t'.$$

One might say that, with this gross payoff function, it is the principal, rather than the agent (see subsection 3.2), that has countervailing incentives.<sup>35</sup>

A useful discussion of this case is provided by Greenwood and McAfee (1991). They argue that type-dependent externalities may cause society, the principal, to have countervailing incentives, and they quote several examples. To consider just one of these, take the classical case of education. It may be true that the marginal social benefit from an increase in a child's education is higher, the brighter the child is. However, it may also be true that less intelligent people need to be taught things that are automatically understood by others, so that there may also be reasons for society to intensify education among low-ability

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<sup>35</sup>One may even formulate the present case through a decomposed gross payoff function, to further emphasize the connection to subsection 3.2.1.



children, and increasingly so the lower the ability is.

Referring to the above modified gross payoff function, Greenwood and McAfee (1991) find that the optimum contract in circumstances like this implies pooling all agent types below some  $t''$ , where  $t'' > t'$ ; *i.e.*, the range of pooling is broader than the range of "perverse" gross payoffs.

A special case occurs if the principal's incentives are of the opposite kind throughout the whole type space, *i.e.*:  $\partial^2 r(m, t)/\partial m \partial t < 0, \forall t \in T$ . Situations where this is the case are also discussed by Greenwood and McAfee (1991).<sup>36</sup> The solution for society in such a situation is pooling across all types, and Greenwood and McAfee show that regulation by a quantity limit now performs just as well as price regulation.<sup>37</sup>

A signalling model with countervailing principal incentives is analysed by Banks (1990b).<sup>38</sup> He considers a situation where the agent has the power to give the principal a take-it-or-leave-it offer; this offer serves thus as the message. Moreover, the agent has private information on the outcome in case the principal rejects; the type space is a subset of the message space. While the agent has preferences represented by a utility function that is monotonic across the message space (and the type space), the principal has single-peaked preferences with a preferred outcome,  $t'$ , in the interior of the type space. In Banks's interpretation, the agent is a monopoly agenda setter and the principal is a (median) voter. Like in the corresponding principal-agent set-up, we obtain a pooling equilibrium in which an agent of any type below some  $t'' > t'$  gives the same offer.

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<sup>36</sup>In their own words, "the individuals most desiring the commodity are the ones society least wants to have it" [Greenwood and McAfee (1991, p. 114)].

<sup>37</sup>Compare this with the case of degenerate countervailing incentives, such as the regulation of a labour-managed firm; see Guesnerie and Laffont (1984a, 1984b) and our discussion at the end of subsection 3.2.1. The two cases share the feature that the gross payoff and message cost functions are in conflict throughout the type space. Moreover, they also have in common a solution with complete pooling. It may be a matter of convention under which heading to put any given example. (It may, for instance, not always be so clear on which side the "pathology" is.)

<sup>38</sup>A related, although more complex, model is analysed by Ferejohn (1990).

## 4.2. The information-protecting principal

The information that the principal elicits from the agent through a carefully chosen contract proposal may be observable to outside parties with whom the principal interacts. In such a case, the principal must weigh the benefit of becoming informed about the agent's type against the benefit of keeping the outsiders uninformed about it. This case is discussed by Caillaud and Hermalin (1989) and is shown to entail extensive pooling. These authors also consider the case where the outside parties can observe the contract chosen by the agent only, rather than the whole menu. Even in this case, there is pooling.<sup>39</sup>

A related case of an information-protecting principal occurs when the principal herself has private information that may be revealed to the agent through her contract proposal. The general treatment of this case is by Maskin and Tirole (1990a, 1990b). They find that the principal herself having private information has no effect on the possibility of obtaining self-selection from the agent. However, they delineate a particular case in which the principal chooses not to disclose her information. Thus, we get some pooling regarding the principal's (contract proposer's) private information but separation regarding that of the agent. The case in point is when the agent's payoff depends only on the contract the principal offers and not on the information she has; this is what Maskin and Tirole call the private-value case.<sup>40</sup>

Maskin and Tirole, in their papers, do not consider moral hazard. Suppose now that the principal has private information on the agent's quality, or will obtain so in the future. The agent does not have any private information but is able to keep secret his choice of effort. In order to encourage the agent to put in the right amount of effort, the principal may or may not want to reveal her infor-

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<sup>39</sup>This theme is related to the multiple-audience literature discussed in subsection 3.2.2 above. The important difference is that the decision whether to reveal information to the second audience now is with the first audience, the principal, rather than with the agent.

<sup>40</sup>There is a considerable literature, apart from the Maskin-Tirole papers, on the informed principal, starting with Myerson (1983). Note also that, by appropriately renaming the players, we may regard the informed-principal case as a particular signalling game: First, the "agent" sends his message, which is a possibly complex contract proposal. Second, the "principal" observes this message and takes some action, such as accepting or rejecting one of the contracts proposed. The private-value case is when the "principal"'s payoff does not depend on the "agent"'s private information, which amounts to assuming, in our notation, that  $\partial r(m, t)/\partial t = 0$  for all  $m$  and  $t$ .

mation to the agent. Thus, moral hazard with an informed principal may lead to pooling in the sense that the principal holds back her information; see Prendergast (1991). But this seems to hold only when skills cannot be contracted upon; compare Prendergast's analysis with that of Beaudry (1989), where no pooling equilibria survive a standard refinement argument.

### 4.3. Disclosure through costly auditing

Suppose that the agent first decides which message to send to the principal, and that there are coincident message costs (see subsection 3.1.2). The principal next decides whether or not to audit the agent. During an audit, the agent's type is disclosed with certainty to the principal. However, the principal incurs a cost of auditing. Therefore, if the value of information is too small, the principal will prefer not to audit. Realizing this, the agent, when taking his decision on which message to send, will not have incentives to separate himself from bad types if knowledge of his type is of sufficiently small value to the principal. Thus, we might expect that, with a continuous type space  $T = [\underline{t}, \bar{t}]$ , there exists some type  $t' > \underline{t}$  such that any type  $t \in [\underline{t}, t')$  sends the same message  $m(t) = m(\underline{t})$ , and that the equilibrium message makes a jump at  $t'$ :  $m(t') > m(\underline{t})$ , and  $\partial m(t)/\partial t > 0$ , for  $t > t'$ . Models with this feature is found in the literature on strategic auditing, see Chatterjee and Morton (1989).<sup>41</sup>

A similar model is discussed by Shavell (1989b). He models litigation, where the agent is the plaintiff and the principal is the defendant. The plaintiff's mes-

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<sup>41</sup>Suppose that an audit will disclose the agent's type only with some probability less than 1. Suppose, furthermore, that the quality of the audit, as measured by this probability, is at the discretion of the principal, with the cost of an audit being a function of its quality. Now, the principal's decision is no longer a binary one of whether or not to audit. Thus, when the agent's message indicates a low value of information for the principal, she may choose to respond with a low-quality, cheap audit rather than none at all. This reduces the scope for pooling, as seen from comparing the Chatterjee-Morton paper with Reinganum and Wilde (1986), where only complete separation is reported. This is, in effect, a demonstration of pooling caused by restrictions on the principal's response, a subject treated in subsection 4.4 below.

sage is a verifiable statement about his true claim,<sup>42</sup> while auditing in this case is tantamount to taking the dispute to court. Both parties incur costs of trial, and the scope for pooling therefore depends on both these cost elements.

#### 4.4 A restricted response

Consider a signalling model where the principal's response is one of two, say Yes or No. If the principal is restricted to playing a pure strategy when responding, this greatly affects the equilibrium of the game and may lead to pooling. This is most clearly demonstrated in a paper by Shleifer and Vishny (1986) on corporate takeovers. Here, the informed A is a large minority shareholder of a firm who has private information on the firm's true value and who is facing a fringe of small, uninformed shareholders, P. Shleifer and Vishny (1986) find conditions under which the large shareholder offers to buy shares in order to bring him in a majority position. The equilibrium in their model is a complete pooling one. However, as Hirshleifer and Titman (1990) subsequently have pointed out, this pooling result is due to the seemingly innocuous assumption made by Shleifer and Vishny, that a small shareholder, when indifferent between tendering and not tendering her shares in response to the large shareholder's offer, always chooses to tender. Hirshleifer and Titman find that, by allowing an indifferent small shareholder to randomize, the equilibrium becomes completely separating. The intuition is basically that forcing the small shareholders to play pure strategies creates a discontinuity in A's payoff.<sup>43</sup> This discontinuity is smoothed by allowing mixed strategies.

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<sup>42</sup>Thus, Shavell's analysis fits in with the models of verifiable disclosure discussed in subsection 3.3.4. But here, pooling occurs because of costs of auditing, rather than costs of disclosure. Auditing costs are, however, shared between the parties in this case.

<sup>43</sup>The story here is thus related to the one in subsection 3.1.3 on discontinuous message cost functions.

## 5. THE MEDIUM

So far, we have not had much to say about *how* the agent's message is sent. We have implicitly assumed that the medium that the agent uses is neutral.<sup>44</sup> We now turn to cases in which this is not the case. We consider in particular two instances of non-neutrality. The first is when the medium, or rather the mediator, is itself a strategic party with its own interests to cater for. The second is when the medium does not transmit the agent's message perfectly, with the agent having better access to the medium than does the principal, either in terms of observing how it works or in terms of having control over it.

### 5.1. The strategic mediator

Suppose that the agent, in addition to giving an unverifiable statement about his type, may at a fixed fee hire an auditor that with probability  $z$  detects any discrepancy between the agent's statement and his true type. The auditor, on the other hand, may choose to shirk, in which case it never finds any discrepancy. The principal may bring the auditor to court if she believes that it has shirked. The court finds with probability  $y$  any discrepancy between the auditor's report and the agent's true type, but cannot tell if it is due to shirking or bad luck; if a discrepancy is detected, the auditor must pay damage awards to the principal. Such a model has been analysed by Melumad and Thoman (1990).

There cannot be complete separation in this model, because, in such a case, the principal will never sue the auditor which then would not have any incentives to work hard, thus making the incentives to hire an auditor the same for both bad and good agents. Therefore, allowing the auditor to behave strategically, in the sense that he is allowed to shirk, creates a scope for pooling. Melumad and

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<sup>44</sup>One exception is in subsection 3.1.1, where we briefly mention the work by Myerson (1988) and others on *mechanism design*. There, the issue is how the agent and principal together can optimally construct a medium. This relates particularly to the case of an imperfect medium (subsection 5.2 below), since the question is to find the ideal imperfection for the medium.

Thoman find that, when the auditor fee is low and damage awards are high, there is an equilibrium with complete pooling in which both agent types hire an auditor and claim to be good and the principal sues the auditor whenever the latter agrees with the agent's claim.

## 5.2. An imperfect medium

The two cases to be considered here relate to situations where the agent's message is received imperfectly by the principal. To be specific, suppose that, if the agent sends the message  $m$ , the principal receives  $\mu$ , where:

$$\mu = m + \eta.$$

Here,  $\eta$  is a random variable, called the noise parameter, typically with mean 0 and variance  $\sigma^2 > 0$ .<sup>45</sup> The two cases differ in the following way. In the first case, the agent observes the realization of  $\eta$  before he decides which message  $m$  to send. In the second case, the realization of  $\eta$  is unobservable to the agent; instead, he is given the power to influence the precision of the medium through the choice of  $\sigma^2$ . As will be clear immediately, we are now in the catchy-phrase department.

### 5.2.1. Signal-jamming

Consider first the case where the agent observes *ex ante* the outcome of the noise parameter  $\eta$ . A model with this feature was first analysed by Holmström (1982), while the term "signal-jamming" seems to appear for the first time with Fudenberg and Tirole (1986). Neither of these two models exhibit pooling equilibria,

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<sup>45</sup>Relating to the previous footnote, mechanism design in this set-up implies that the agent and the principal are allowed to come together before the real game starts and determine the probability distribution of the random variable  $\eta$  in the optimum way. In this subsection, however, we consider cases only where the principal has no access whatsoever to the noise parameter. The interest stems from the assumption that the agent, on the other hand, does have such access.

though.<sup>46</sup> Rather than delineate exactly under what conditions pooling does and does not occur, we report on some models of signal-jamming that do exhibit pooling and try to track down the reasons for pooling in these models.

One instance of signal-jamming with pooling is found in models of stock markets with noise trading. Suppose there is one big trader who privately observes the amount of noise trading, the latter being trading for liquidity or life-cycle motives rather than for speculative reasons. The big trader may want all the small speculative traders not to know how much he has traded. For example, he may have information to the effect that a currently low-priced firm is underpriced and is worth a take-over if sufficiently many shares can be bought at the current market price. In order not to disclose his intentions, the big trader may want to hide behind the noise trading. That is, he takes advantage of situations where noise traders sell out so that he can buy large quantities without being detected. This situation is analysed by Kyle and Vila (1989). The related case of a futures market, where the noise traders are hedgers with an inelastic demand for insurance, is discussed by Vila (1988). A simple variant is analysed in Vila (1989), in which there may or may not be a noise trader present (*i.e.*, the noise parameter is, in a sense, binary), and the big trader – the "manipulator" – is the only one to know whether it is there or not; his decision is whether to enter the market himself. In equilibrium, there is partial pooling: The big trader plays a mixed strategy to keep the other speculative traders – who are unable to distinguish a manipulator from a noise trader – from gaining complete information on the presence of the noise trader.

One feature of the pooling equilibria in the situations described in the previous paragraph should be noted: Pooling occurs in the sense that the *received* message is constant across types. Formally, if the agent observing the realisation  $h$  of  $\eta$  sends the message  $m(h)$ , then there is some set  $H$  of realisations of  $\eta$  such that  $\mu(h) = m(h) + h$  is constant across  $H$ . Of course, to obtain pooling in this sense, *sent* messages differ within the set  $H$ .

An example of signal-jamming with pooling in the standard sense, *i.e.* such that different agent types send the same message, is found in an analysis of herd

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<sup>46</sup>Berlin and Calem (1987) present a pooling equilibrium in a signal-jamming model similar to the one in Fudenberg and Tirole (1986). However, their result is probably due to very special assumptions regarding the noise parameter.

behaviour among managers by Scharfstein and Stein (1990). There are two types of managers in their model: "Smart" ones receive informative signals about the value of an investment, while "dumb" ones receive uninformative signals. A manager does not himself know whether he is smart or dumb. However, he may use the signal he receives, which is his private information, to make an inference about his own smartness. There are two managers making investment decisions in sequence. The interest centres around the second one, called Mr. 2. Scharfstein and Stein find that the only equilibria in their model are such that Mr. 2 makes his decision, not according to the signal he receives but according to what Mr. 1 did before him. Thus, there is pooling, in that Mr. 2's message is constant across his set of private information.

### 5.2.2. Ambiguity

The second case of an imperfect medium is when the agent has control over how noisy the medium is. Recall the characterization of an imperfect medium:

$$\mu = m + \eta,$$

where the random variable  $\eta$  has a variance  $\sigma^2$ . Suppose that there is a minimum feasible variance  $\underline{\sigma}^2 > 0$ , but that the agent, before he learns his type, is allowed to set the variance higher than this. This may be interpreted as pooling, in the sense that, by making the received message noisier, the agent reduces the informational value of the message he sends, thus creating an effect similar to mimicking. An election model with this feature is studied by Alesina and Cukierman (1990). It is from them we have taken the term "ambiguity" to describe this case. A politician, they find, may choose to be ambiguous in his first term in office because ambiguity affects voters' expectations about his second-term policy in a way that increases his chances of being reelected.

If the principal can observe the agent's choice of precision, and if, furthermore, precision is costly, then, not surprisingly, pooling may disappear. See, e.g., Titman and Trueman (1986) on this in the context of a firm choosing among auditors with known quality whose reports, therefore, have known precision.

Another way to look at the issue of producing precision is found in the elec-



tion model of Harrington (1991). Here, voters (the principals), in addition to not knowing the preferences of the incumbent politician (the agent), are uncertain *ex ante* about which of two feasible policies they prefer. The incumbent first chooses his policy. Then the voters, observing his policy and learning how it performs, decide whether to re-elect him. Harrington finds that a range of types for the incumbent exists such that he chooses his less preferred policy today in order to confound the voters in their learning, thus securing his own re-election.

## 6. THE FUTURE

When an agent and a principal are involved in an enduring relationship, problems may arise for the achievement of self-selection at the very start of it. In particular, the agent may ask himself whether it may cost him dearly in the future if he reveals himself today, or whether it may benefit him in the future to take the cost of mimicking another type today. Concerns for the future among informed parties is a very important reason for the occurrence of pooling at early stages.

If the principal and the agent are allowed to agree, at the start of the game, to a contract to which they both are committed, then, in the principal-agent model, it is possible for the principal to design a menu of multi-period contracts such that self-selection is obtained at the initial, contract-offer stage. Baron and Besanko (1984), *e.g.*, study the multi-period extension of Baron and Myerson's (1982) model. Basically, they find that the offered multi-period contract consists of the single-period one being repeated in each period in the case when the agent's type does not change over time. Below, we limit ourselves to this constant-type case. Moreover, we let time develop in discrete steps, with stages indexed by  $\tau \in \{1, 2, \dots, \bar{\tau}\}$ .

There are basically two reasons why Baron-Besanko style results of initial complete separation may not hold. These are, first, that the contract may be renegotiable at a later stage and, second, that the two parties, particularly the principal, may not be able to commit to future behaviour through a long-term contract. We treat these in turn. The Section is concluded with a discussion of sequential bar-

gaining under asymmetric information.

### 6.1. Renegotiation

Suppose that the parties are able to commit to a long-term contract. There may still be problems, though, if they are allowed, at some future date, to replace the initial contract with a new one that they both prefer. The latter act is what goes under the name of *renegotiation*.

The literature on renegotiation has grown vastly in recent years. The idea has been applied to complete-information situations for purposes of equilibrium refinement in repeated games; it has been applied to symmetric-information situations to study contract incompleteness due to unforeseen contingencies; and it has been applied to asymmetric-information situations, which we discuss presently.<sup>47</sup>

In a multi-period asymmetric-information model with interim contracting, the basic reason for there being scope for renegotiation at some future date is that the principal, in the meantime, has received some information that she did not have at the time the initial contract was signed. If the initial contract is separating, then the agent's initial choice of contract is, in itself, information that places the principal in a new position and that may prompt her to offer the agent a new contract that they both prefer to the initial one. The incentive for the principal to renegotiate is stronger, the better the agent, through the initial self-selection, proves himself to be. This, on the other hand, provides an agent of a bad type with incentives to mimic a good type at the initial stage, so that he can harvest the benefit of the future renegotiation. To counter these latter incentives, the initial contract will, in general, feature some pooling.

In the case of *ex-ante* contracting, it is the agent who receives new information after the initial contract is signed. Even if the principal does not know the character of this information, she knows that it has arrived, and there is thus a possible scope for renegotiation. For a survey of the literature on renegotiation with asymmetric information, see Dewatripont and Maskin (1990).

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<sup>47</sup>Examples from the first two branches of the renegotiation literature are Farrell and Maskin (1989) and Hart and Moore (1988), respectively.

The seminal article is by Dewatripont (1989).<sup>48</sup> He considers a multi-period ( $\bar{\tau} > 2$ ) model with a finite set of types,  $T = \{\underline{t}, \dots, \bar{t}\}$ , where the agent is a risk-averse firm and the principal is a risk-neutral worker. Since the two are symmetrically informed when signing the initial contract at  $\tau = 1$ , this is a model of *ex-ante* contracting. Between  $\tau = 1$  and  $\tau = 2$ , the firm (the agent) privately observes the price of its product. Dewatripont assumes that the initial contract offer (at stage  $\tau = 1$ ) is made by the agent but that any renegotiating offer (at a stage  $\tau > 1$ ) is made by the principal. He finds that there is never complete pooling, that lower types pool initially, and that this pooled set is peeled off from the top over time: If  $t'$  is the top type in the pooled set in period  $\tau'$ , then an agent of type  $t'$  is fully separated from time  $\tau' + 1$  on. Thus, there is always at least one type that is revealed at any time, so that it takes at most  $\#T - 1$  periods until there is complete separation.

In discussing a bargaining situation with renegotiation, Hart and Tirole (1988) allow for mixed strategies, in contrast to Dewatripont. On the other hand, they restrict themselves to the two-type case. They consider both interim and *ex-ante* contracting. Their results are generally comparable to those of Dewatripont.

Laffont and Tirole (1990) present a very nice treatment of a two-type, two-period principal-agent model with long-term contracts that are renegotiable, and with interim contracting. The principal makes the offers in both periods. In equilibrium, the principal offers the agent two contracts to choose from. One contract is for the good type; if the agent chooses this one, he reveals his good type and first-best is obtained in both periods. The other contract is a pooling contract which is followed in the next period by the principal's optimum contract conditional on her revised beliefs. Laffont and Tirole find that, as the discount factor increases, the good type chooses the pooling contract with non-decreasing probability. In the case of a continuous type space, they find that, although a complete-separation contract is feasible for the principal, it is not the optimum one.

Two papers with close relations to those discussed so far are the following:

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<sup>48</sup>"Seminal" may be a somewhat too strong characterization, given the apparent simultaneity in the work in this area, and the actual preemption of Dewatripont's paper by at least one other article. However, there seems to be some consensus that he pioneered this literature. His work was part of a Ph.D. dissertation submitted in 1986. It should be added, though, that a predecessor to the renegotiation literature is found in Holmström and Myerson (1983), discussing contract durability much in the same spirit as this more recent literature.

First, Bester (1990) considers debt renegotiation. Like Dewatripont (1989), he assumes *ex-ante* contracting. But the contract he considers has some idiosyncratic features, most important of which is that it is incomplete. Therefore, he finds, contrary to the above papers, that allowing renegotiation increases the agent's *ex-ante* expected utility. Second, Lewis and Sappington (1990) study renegotiation in a long-term relationship between a regulatory agency and a regulated firm. Their analysis follows closely Laffont and Tirole (1990).

Motivated by the slow revelation of information caused by renegotiation, as discussed above, Dewatripont and Maskin (1989) ask whether it might be preferable for the two parties to restrict P's ability to receive any new information after the initial contract has been signed. They consider the case of *ex-ante* contracting when the message space is two-dimensional and A sends his two messages in sequence. Thus,  $M = M_1 \times M_2$ ; after the initial contract is signed, A learns his type; then he chooses some  $m_1 \in M_1$ , followed by some  $m_2 \in M_2$ . If P observes  $m_1$ , she may want to renegotiate before A chooses  $m_2$ . Dewatripont and Maskin discuss whether the two parties would be better off if P could observe only one of  $m_1$  and  $m_2$ . They also discuss criteria for choosing which message to make unobservable.

Hosios and Peters (1989a) distinguish themselves from the authors mentioned so far by introducing renegotiation in a one-period principal-agent model, *i.e.*, a model where contracts first are signed and then are executed. They show that renegotiation-proofness, in the sense of robustness with regard to the introduction of a bargaining game after the signing of the contract, can be represented as an extra constraint, in addition to participation and incentive-compatibility constraints, to the original problem. Like in the other models, this extra constraint entails a pooling equilibrium.

Seidmann (1990) is also related to this literature. He presents a multi-period monopoly pricing problem that is very close in structure to renegotiation with *ex-ante* contracting. The setting is the following: A is an upstream firm that is the monopoly supplier to a downstream industry. At time 1, when no-one knows A's costs of production, A writes a contract stating the price of his product conditioned on the (as yet unknown) production costs:  $p(t)$ ,  $t \in T = [\underline{t}, \bar{t}]$ ,  $t = 1/c$ , where  $c$  = production costs. At time 2, the firms in the downstream industry make relationship-specific investments that enable them to make use of A's product. At

the same time, A gets to know his true costs. At time 3, A announces his type and sets a price  $p \leq p(t)$ . Finally, the downstream firms report their demand to A at this price. The crucial feature of this model, making it one of renegotiation, is that A is allowed to change his price after he has learnt his type, as long as he does not make the downstream firms worse off.<sup>49</sup> *I.e.*, he must either stick to his "list price" or introduce a discount. Seidmann finds that the optimal contract for the upstream firm is one where, for some  $t' > t$ ,  $p(t) = \bar{p}$ ,  $\forall t \in [t, t']$ , while for  $t \in [t', \bar{t}]$ , A sets, for each type  $t$ , his preferred price given the amount of investment among buyers that this contract  $p(\cdot)$  induces. Thus, we get pooling among bad (high-cost) types.

Dewatripont and Roland (1990, 1991) apply the ideas of renegotiation to the topical question of whether reforms in Eastern Europe should be full or gradual. The government tries to restructure the industrial sector, in which each worker has private information on his productivity. The notion of renegotiation appears in this framework by the government's ability to implement its policy at any time only with unanimous approval from the workers affected by the reform; thus, the policy may change over time, but only such that no worker type is hurt. Dewatripont and Roland find conditions under which gradual reform is preferable. A gradual reform implies gradual revelation of workers' productivity, *i.e.*, pooling.<sup>50</sup>

Market screening with renegotiation has been considered, for the two-type case, by Asheim and Nilssen (1991). The equilibrium is never completely separating and is completely pooling for a sufficiently high prior  $p_G$  that A is good. They also find that, contrary to (most) principal-agent models, allowing renegotiation in market screening may leave all agent types better off in equilibrium; this hap-

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<sup>49</sup>Use of the term "renegotiation" may seem somewhat misplaced. Perhaps it is better to use the term *reselection* to cover instances like this one. This would also be consistent with language used elsewhere; see Dekel and Farrell (1990).

<sup>50</sup>Dewatripont and Roland also discuss the effects of weakening the political constraint so that the government may change its policy as long as it has majority approval, as opposed to unanimity. This variation may be considered a case intermediate between renegotiation and no long-term commitment (see the next subsection): The government may now adjust its policy when new information arrives, as long as the workers that are hurt by the new policy constitute a minority.

pens only if the equilibrium is a completely pooling one.<sup>51</sup>

We conclude this subsection by recording three interesting variations on the theme of renegotiation. The first variation features *ex-ante* contracting and, after the initial contract is signed, A's type being determined *endogenously*, rather than exogenously as above. Specifically, A makes a decision that is unobservable to P and that determines his type. Consider the two-type case and, to save on notation, suppose A can choose  $t \in \{B, G\}$  after the initial contract is signed. In deciding his type, A makes a commitment that creates incentives for P to renegotiate, even if she does not know the agent's choice. In equilibrium, A plays a non-degenerate mixed strategy when choosing type, *i.e.*, he chooses G with probability  $\rho_G \in (0, 1)$ . This equilibrium may be suitably interpreted as one of pooling, since the principal is incompletely informed when renegotiation starts. In fact, the renegotiation stage is equal to a single-period standard principal-agent model where  $\rho_G$ , as usual, is P's subjective probability that A is good. For analyses of this case, see Fudenberg and Tirole (1990) and, especially, Ma (1991).<sup>52</sup>

The second variation is the case of *recontracting*. Now, there are multiple principals:  $P = \{P_1, \dots\}$ . The agent, rather than returning to the first principal for renegotiations, turns to others to sign additional contracts. If each principal can at any time observe the contracts already signed by the agent with other principals, problems of information revelation, similar to those of renegotiation, occur. A thorough analysis of this case is done by Beaudry and Poitevin (1990). Gale (1989) and Gale and Stiglitz (1989), in discussing multi-period trading on a stock market, consider problems with a similar structure: A privately informed shareholder, when selling some of his shares, may be unable to commit not to sell more shares in the future.<sup>53</sup>

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<sup>51</sup>Market screening with renegotiation is also considered by Hillas (1987), Mori (1989), and Dionne and Doherty (1991).

<sup>52</sup>Since they include unobservable actions, these models feature moral hazard and are thus related to those cited in subsection 3.3.3, particular to Aron (1987), who also considers a multi-period model.

<sup>53</sup>Branco and Mello (1991) present a variation on this theme, wherein the seller is the government privatising a firm and the uncertainty is not on whether also the remaining part will be privatized but on whether the government will interfere with the firm's business after privatisa-

In the third variation, the novelty is the importance of multiple agents. Suppose there are several agents with whom the principal signs an *ex-ante* long-term contract. The agents receive new private information in each period. Any communication among the agents and the principal is publicly observable. Rather than the potential use by the principal of information sent by the agents, the problem now is that one agent may take advantage of information sent by other agents. This complicates the contract design problem of the principal. Even though she herself may be able to commit to the contract, she cannot prevent any misuse of information among agents. Thus, the revelation principle does not hold; see Burguet (1990a).<sup>54</sup> Burguet (1990b) studies a model involving an owner of a durable good designing a contract to lease the good to a set of potential users, each of whom privately learns his valuation of the good only after leasing it for a period.

## 6.2. No long-term contracts

Suppose that the principal and the agent are involved in a relationship stretching over two periods, but that the principal at the start is unable to commit herself to any restrictions on her second-period action. In such a case, the agent must presume that any information revealed in the first period will be fully exploited by the principal in the second one. Thus, the good type, instead of being exploited, may choose to mimic the bad type during the first period. This leads inevitably to pooling, since a good agent, rather than sending a message that distances himself from the bad type, has incentives to send the same message as the latter.

Thorough analyses of a two-period principal-agent model without long-

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tion.

<sup>54</sup>The revelation principle says that, when the principal can commit to her contract, she may restrict attentions to payoff functions  $s(\cdot)$  such that an agent of type  $t \in T$  finds it optimum to send  $m(t)$ , thus receiving  $s(m(t))$ ; see, e.g. Myerson (1979). This does not exclude pooling, since the same message may be optimum for several types for a given payoff function  $s(\cdot)$ . In the present case, the principle is weakened, so that the principal restricts attention to payoff functions  $s(\cdot)$  such that an agent of type  $t \in T$  finds it optimum to send  $m(S)$ , where  $t \in S$ ,  $S \in \Gamma$ , and  $\Gamma$  is some partition of the type space  $T$ .

term contracts are provided by Laffont and Tirole (1987, 1988).<sup>55</sup> Their first paper considers the two-type case, while the second one considers the case of a continuous type space. They find that, if the discount factor is high enough, so that sufficient weight is put on the future, then there cannot be complete separation in equilibrium. They find that, in the two-type case, the incentive-compatibility constraint, which is binding only for the bad type in the one-period model, may be binding for either type, or for both types.<sup>56</sup>

An early treatment of this problem is by K. Roberts (1984), who considers optimum taxation when the government is unable to commit to future tax rates. In his model, there is an infinite time horizon and no discounting, and the equilibrium features complete pooling. J. Roberts (1987) shows that an iterative planning procedure developed in the earlier literature, called the MDP procedure, is infeasible when the central planner is unable to commit herself, unless the agents are myopic. Milgrom (1987) treats, for the case of no long-term contracting, a situation where the agent, by some action after the first period, determines his type;<sup>57</sup> Milgrom finds that the agent will never choose type with a pure strategy, so that the principal will never be fully informed at the start of the second period.

Baron and Besanko (1987) extend the principal-agent model of Baron and Myerson (1982) to the case of multiple periods with no long-term contracts. They restrict, however, the principal to offering linear contracts, *i.e.*, payment is restricted to be an affine function of the message. Freixas, Guesnerie, and Tirole (1985) preempt the Laffont-Tirole papers with a two-period model similar to theirs, but they, too, consider linear contracts only. Skillman (1986) extends Freixas-Guesnerie-Tirole to a general finite-horizon model; however, he restricts the agent to using pure strategies, thus excluding any gradual revelation of information.

Gradual revelation through the agent's play of a mixed strategy is, on the other hand, a regular phenomenon in the so-called *reputation* models; see, *e.g.*,

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<sup>55</sup>The results in these papers are reviewed in Laffont (1987, Sec. IV).

<sup>56</sup>Comparisons of the present case of no long-term contracting with the case above of renegotiation are found in Laffont and Tirole (1990) and Lewis and Sappington (1990).

<sup>57</sup>Thus, Milgrom's analysis complements those of Fudenberg and Tirole (1990) and Ma (1991) for the renegotiation case; see the previous subsection.



the seminal contributions by Kreps and Wilson (1982) and Milgrom and Roberts (1982). These models are, however, special in other respects, notably in that the good type is typically "stupid" and often modelled with a degenerate message space.<sup>58</sup> Reputation models consider how a smart informed agent, when playing against uninformed parties that are either myopic or non-strategic, can hold back his true identity over a long period by mimicking stupidity, *i.e.*, keeping a reputation for being weird. An account of theoretical results in this literature is included in Fudenberg (1990).

Multi-period market screening without long-term contracts has been considered by Nilssen (1991). He finds that the introduction of competing principals, compared to the case of a single principal, has two effects. First, one principal's offer of a pooling contract can be exploited by other principals to obtain a cheap separation. This greatly reduces the scope for pooling. Second, competition drives profits to zero. This creates limits on the possibilities for such a cheap separation. Pooling may still occur, then, but is not as prevalent as in the single-principal case.

Dynamic models without long-term contracting have been applied in a variety of contexts. Hosios and Peters (1989b), for example, discuss the case of an insurance monopoly. In the labour-market area, Gibbons (1987) explains why piece-rate compensation schemes for workers are no longer optimum when there is no commitment to future action. In this model, it is the workers that are privately informed. On the other hand, Giammarino and Nosal (1990), in a model where a firm has private information on the quality of its management, consider a situation where it pays bad firms to mimic good firms during an economic downturn. This pooling equilibrium thus explains wage smoothing over the business cycle, the authors argue.

Hillas (1987) and Jost (1988) consider models with *testing*. Suppose the principal may offer to inspect the agent by performing a specified test. If the principal can commit to the test specifications and offers a range of different tests, the agent reveals his type through the choice of test. However, now the test has served its purpose and it is no longer necessary. Therefore, if the principal is not committed to performing the test, it will not be done. Realising this, the agent will not self-

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<sup>58</sup>This has already been commented on in subsection 3.3.1 when discussing finite message spaces in general.

select. In this case, therefore, there will be pooling.

Several papers discuss cases within industrial organization. LeBlanc (1991) considers entry deterrence when the incumbent may choose between limit pricing, which is signalling its costs before the rival's entry decision in order to prevent entry, and predatory pricing, which is signalling its costs after the rival has entered in order to induce exit. LeBlanc finds that the incumbent may choose to abstain from limit pricing and rather go for predatory pricing if entry occurs. With this strategy, the incumbent leaves the entrant with no information on its costs at the time of the entry decision. Thus, no limit pricing is tantamount to first-period pooling. Sharpe (1986), Bagwell (1990), and Kennedy (1990) consider multi-period models where the quality of a firm's product is initially unknown to consumers; these authors find conditions under which firms of different types set the same price in the first period. Finally, Blume and Easley (1987) consider a two-period Cournot model where only a subset of the firms in the industry know the production costs. They find that, if the number of informed firms is small enough, there will be pooling in equilibrium. In particular, if there is only one informed firm, an equilibrium with complete pooling always exists.

In Tauman and Weiss (1990),  $P$  is an innovator who possesses some technology that will lower fixed costs of production in an industry where there at present is one incumbent producer. The incumbent,  $A$ , has private information on its variable production costs.  $P$  makes an offer to  $A$  of licensing the new technology. However,  $P$  cannot commit herself from thereafter licensing the same technology to potential entrants to the industry, thereby lowering the value of the license to the incumbent. With this lack of commitment, it is not possible for  $P$ , through her initial licensing offer, to elicit information from  $A$  on the true variable production costs, and pooling entails.

In government procurement, there are often problems of cost overruns. When the government and the contracting firm cannot write contracts on how to react to future instances of cost overrun, it is difficult for the government to get the firm to reveal at the outset private information that is relevant to the probability that cost overruns will occur in the future. Two models which are quite different but which nevertheless share this view of cost overruns are presented by Lewis (1986) and Spulber (1990). Lewis emphasizes that high-cost firms may work hard early on to keep cost overruns from occurring until the govern-

ment is so locked in with the project that it wants it completed even with high costs. Spulber argues that a contract auction will not elicit any information on firms' costs when firms are allowed to breach in the event of a cost overrun. If firms are allowed to breach only when paying a compensation to the government, then, with a continuous type space, there exists some critical type  $t' \in T$  such that, in the contract auction, firm types below  $t'$  are pooled.

With pooling following from the parties being unable to commit to any future action at all, one may ask how much commitment power is sufficient to obtain separation. We conclude this subsection by reporting on two papers with answers to this. Baron and Besanko (1987), in their analysis of the two-period Baron-Myerson model, find that, if the principal is restricted to offering fair contracts in period 2 and the agent has to participate in that period if he participated in the first, then there is complete separation in period 1. Their notion of fairness implies that an agent is not exploited in period 2 if he reports truthfully his type in period 1. Fairness thus gives the principal a limited amount of commitment power that turns out to be sufficient to obtain separation. The other paper is by Anton and Yao (1987). They consider a two-period procurement situation where period 1 is the development phase and period 2 the production phase for some governmental equipment. They avoid pooling in the development contract of period 1 because the principal, although she cannot commit exactly to future behaviour, is able to commit to the production contract in period 2 being auctioned out. This bit of commitment turns out to be sufficient to obtain first-period separation.

### 6.3. Sequential bargaining

Consider a seller and a buyer who are bargaining over the price of an asset, with the buyer having private information about his valuation of the asset. Thus, the seller is  $P$  and the buyer is  $A$ . To be specific, we let the bargaining proceed in the following fashion: Time develops in discrete steps,  $\tau \in \{1, \dots\}$ . At  $\tau$  odd,  $P$  offers a price. At  $\tau$  even,  $A$  decides whether to accept or reject this price. In case of acceptance, the asset is transferred at this price, and the game ends. In case of rejection,

the game proceeds to P's next offer.

This is admittedly a very special structure, chosen to serve our limited ambitions in this subsection.<sup>59</sup> In particular, this is a game where the uninformed seller is the only one making offers. Thus, the game has the flavour of a repeated principal-agent model. There are two important differences, though. First, the end of the game is here determined endogenously, at A's discretion. Second, P has no contingent contracts at her disposal during the bargaining, only a single-valued price.

The present bargaining situation has been analysed, among others, by Fudenberg, Levine, and Tirole (1985). They find an equilibrium of a character close to what we have recorded earlier in this Section. Let  $s$  be seller P's valuation of the asset, and let buyer A's valuation equal his type  $t \in T = [\underline{t}, \bar{t}]$ . If  $\underline{t} > s$ , then the equilibrium is described by a list  $\{a_0, \dots, a_N\}$ , with  $\bar{t} = a_0 > \dots > a_N = \underline{t}$ , such that A accepts at time  $\tau = 2n$  if he is of type  $t \in [a_n, a_{n-1}]$  and rejects if he is of type  $t \in [a_N, a_n)$ . P's price offers decline over time: At times  $\tau = 2n - 1$  and  $\tau = 2n + 1$ , they are such that type  $a_n$  is indifferent between accepting and rejecting at  $\tau = 2n$ ; at  $\tau = 2N - 1$ , the price offer equals  $\underline{t}$ . Thus, there is pooling at the start of the bargaining, with types being peeled off from the top of the pooled set over time.

Note that P will never want to offer a price (less than or) equal to her valuation  $s$ . Thus, if  $\underline{t} \leq s$ , then bargaining may go on for an infinitely long time, with the price offer approaching  $s$  asymptotically. Correspondingly, A's acceptance is delayed for low types, with the integer  $N$  above no longer finite, and  $a_n$  approaching  $s$  as  $n$  goes to infinity.

The greatest impact of this particular bargaining model came through the work of Gul and Sonnenschein (1988), who showed that, as the time between offers goes to zero, the time until the lowest-type buyer accepts also goes to zero in the above described equilibrium. Thus, the pooling interpretation of the above equilibrium hinges on there being discrete time. Because this model in its structure is similar to one of intertemporal price discrimination by a durable-goods monopolist, the Gul-Sonnenschein result implies that such a monopolist is

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<sup>59</sup>Those who are interested in the theory of sequential bargaining in general or bargaining under asymmetric information in particular, are wise to consult Osborne and Rubinstein (1990) and Kennan and Wilson (1990), respectively.

unable to set prices above the lowest valuation among consumers; *i.e.*, if  $\underline{t} \leq s$ , we have that price equals marginal costs. This proposition, known as the Coase conjecture, received its formal vindication from Gul, Sonnenschein and Wilson (1986). However, the story does not stop here. The above equilibrium is the unique one only if we impose upon A's acceptance decision at any time the condition that it depend solely on P's beliefs and current offer.<sup>60</sup> If this condition is lifted, and if  $\underline{t} \leq s$ , we will find equilibria with pooling, even if the time between offers approaches zero. In the durable-goods interpretation, this implies that the Coase conjecture is reversed. See Ausubel and Deneckere (1989).

The market-screening parallel to the above story has several sellers confronting a privately informed buyer, with the sellers making simultaneous offers at each  $\tau$  odd. This case is discussed by Vincent (1990), who calls it a *dynamic auction*. Related is also the durable-goods oligopoly models of Ausubel and Deneckere (1987) and Gul (1987). The general report from these papers is that pooling results are strengthened by the introduction of multiple principals.<sup>61</sup> Phrased within the durable-goods analogy, the introduction of competing producers raises price above marginal costs, even if we restrict the informed buyer's strategy in the above described manner. However, a comparison of Vincent's paper with Nöldeke and van Damme (1990a) suggests that the result may hinge on the choice of equilibrium refinement.

## 7. CONCLUDING REMARKS

This Section starts out with a discussion of two items of pooling occurrence that seem to fail to be naturally fitted in with any of the above. These are the type space and the organization of market screening. We continue with two items that may have bearing on the importance of our writings above. The first, on equilibrium refinements, is a theoretical point mostly concerned with signalling

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<sup>60</sup>Conditions like this one are known as *Markov conditions*, and the idea is that a strategy should be conditioned only on that part of the history that is in some sense relevant.

<sup>61</sup>Compare this with market screening without long-term contracting, studied by Nilssen (1991) and discussed in the previous subsection. In that case, the introduction of multiple principals reduces the scope for pooling.

models. The second one addresses empirical work on the incidence of pooling.

### 7.1. The type space

In subsection 3.3.1 above, we found that restrictions on the message space may lead to the existence of pooling in equilibrium. With regard to the type space, quite on the contrary, a *lack* of restrictions may give the same effect. Whereas we with the message space were concerned with the good types' end of the space, the concern here is with the bad types' end.

Hellwig (1990) has shown that, if the type space is unbounded in the lower end, then it may be impossible to separate among lower types. In particular, he constructs an example in which  $T = (-\infty, \bar{t}]$  and finds that a critical type  $t' > -\infty$  exists such that all types  $t < t'$  are pooled. This holds in both the signalling model and the market screening model. The basic reason for this result is that, with no limit on  $\underline{t}$ , this lowest type cannot serve as a starting point from which to construct a separating equilibrium from incentive-compatibility considerations.

As an example from the literature, Hellwig cites Glosten (1989), where the uninformed parties' prior beliefs are represented by the normal distribution. Thus, since this distribution has support on the whole real line, there is no lower bound on the worst type, and a separating equilibrium fails to exist.

### 7.2. The organization of market screening

As noted in subsection 2.2, the market screening model may fail to exhibit a pure-strategy equilibrium. This was the basis, in the literature following Rothschild and Stiglitz's (1976) seminal piece, for exploring the effects of other ways of organizing a market with asymmetric information. These efforts aimed at constructions such that an equilibrium in pure strategies always exists. Below, we will try to give a brief account of these developments. Every suggestion has an equilibrium that coincides with the pure-strategy separating equilibrium, described in subsection 2.2, when the latter exists. The interest, therefore, centres on the equilibrium features in case there is no pure-strategy equilibrium in the base model.

Reference is made to this case only in the following.

Wilson (1977) suggests that the competing principals interact in sequence such that, after the initial contract offers, each principal is allowed to react by withdrawing any offer that is unprofitable in light of rivals' initial offers; see Fernandez and Rasmusen (1989) for a game-theoretic formulation of this suggestion. The outcome of this market organization is a pooling equilibrium in which the good type's utility is maximized subject to the zero-profit constraint (on a pooling offer) on principals; call this the *Wilson outcome*. It turns out that Wilson's result is sensitive to his provision that each principal offers one contract only. If this is relaxed, so that each principal may offer contracts to more than one type, then the equilibrium is separating and may entail cross-subsidization from the good type to the bad; see Miyazaki (1977) and Spence (1978). However, in order to avoid a chain of reactions, Mattesini (1990) argues, the case of multiple contracts per principal requires the specification, by each principal, of a fall-back contract to which he returns at a withdrawal; introduction of such a fall-back contract partly restores Wilson's pooling result.<sup>62</sup>

Another variation is suggested by H. Grossman (1979). Here, the principals' initial contract offers are followed by the agent applying for a contract. Confronted with an applicant, a principal has then the right to accept or reject. This twist leads again to the Wilson outcome with pooling. Game-theoretic formulations are suggested by Hellwig (1987) and Desruelle (1989).

### 7.3. More on equilibrium refinements

As we saw in subsection 2.3. above, standard refinements leave us with a separating equilibrium in the signalling model. However, it is fair to say that the refinement literature in general is not met with equal enthusiasm in all quarters, not

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<sup>62</sup>Reactions of the opposite kind of those suggested by Wilson, i.e., where each principal is allowed to react to current offers on the market by *adding* new offers to its portfolio (rather than withdrawing current ones), is showed by Riley (1979) to lead to separation. Game-theoretic formulations of the Riley outcome seem, however, hard to obtain without running into problems of multiple equilibria; see, e.g., Kreps (1990a, p. 650).

even within the game-theorist profession.<sup>63</sup> And more to the point, refinements on which the conclusion is based that the signalling equilibrium is separating have received particular criticism. The so-called "Stiglitz critique", for example, questions the way pooling equilibria are deleted by these refinements.<sup>64</sup> Some authors have also put forward alternative refinement concepts that, in some signalling games at least, lead to a pooling equilibrium; see, *e.g.*, Overgaard (1990).

Closely related to the refinement of equilibria is the issue of equilibrium *selection*,<sup>65</sup> recently associated largely with the work of Harsanyi and Selten (1988). Güth and van Damme (1989) have used their theory to arrive at the Wilson outcome, defined in the previous subsection, in a signalling model with competing principals.

Finally, we should put forward some caution about the effect of refinements in dynamic games. A widely used notion is that, if P at some time  $\tau'$  is certain that A is not of type  $t$ , then she continues to be certain of this at all times  $\tau > \tau'$ . This is an effective restriction on P's beliefs, known as "Support Restriction" or "Never Dissuaded Once Convinced".<sup>66</sup> Some of the results reported in Sec. 6 above make use of this idea. However, its critics have shown that it is in conflict with other refinement arguments, and that it generally overstates the occurrence of pooling, relative to these other refinements. More on this is, *e.g.*, in Nöldeke and van Damme (1990b).

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<sup>63</sup>Even "refiners" feel the need to express their doubts once in a while; see, *e.g.*, Kreps (1990b, Ch. 5).

<sup>64</sup>The name stems from a reference to Joseph Stiglitz in Cho and Kreps (1987, p. 203), where it is first stated. The critique goes something like this. Consider a pooling equilibrium in which both types send the same message  $m$ . This equilibrium is dismissed by the equilibrium-dominance refinement (see subsection 2.3) because there exists a message  $m'$  that the good type may deviate to since, if P reacts to a deviation to  $m'$  as if it was indeed a message sent by the good type, then the bad type would not want to send  $m'$  instead of  $m$ . According to the critique, one should not stop here, however. For if the good type is expected to deviate to  $m'$ , then P infers that anyone sending  $m$  is bad and acts accordingly when  $m$  is sent. Realizing this in the first place, the bad type may find that it pays to mimic the good at  $m'$  rather than being revealed at  $m$ . Thus, it may not be possible for the good type to deviate to any message  $m'$  with the desired property, and pooling at  $m$  is in equilibrium after all. See also Kreps (1989, pp. 38-39) and van Damme (1991, p. 39).

<sup>65</sup>While equilibrium *refinement* aims at deleting unreasonable equilibria, so that the number of equilibria that are left after the exercise may vary from case to case, equilibrium *selection* aims at picking one equilibrium, thus obtaining uniqueness by definition.

<sup>66</sup>The latter name is from Osborne and Rubinstein (1990, ch. 5), who count among the proponents.



#### 7.4. Empirical work on pooling

Models of asymmetric information are inherently difficult to test empirically. The basic problem for the researcher is that he cannot measure the type of an agent; if he could, then also the principal could, a case in which there is no asymmetric information after all. Successful empirical work, therefore, has confined itself to testing equilibrium outcomes. For example, in testing whether a real-life situation features a separating equilibrium, one may test whether agents' behaviour differs. If it does, even after controlling for all observable effects, then one may have support for the conclusion that self-selection takes place, which implies that there was asymmetric information in the first place. The problem with this line of research, for us at least, is that a rejection of a self-selection hypothesis is unable to distinguish a symmetric-information hypothesis from one of asymmetric information and pooling. Therefore, while asymmetric information in itself is hard to ascertain, the occurrence of pooling is harder still.

Thus, while empirical evidence of self-selective signalling at present is quite substantive, the literature is almost silent on the pooling issue. But there are some exceptions. Kennan and Wilson (1989) give a nice overview of the empirical work that has been done to test strike incidents in union-firm negotiations as outcomes of pooling equilibria in bargaining games with various sorts of private information. Another interesting study is by Slovin, Sushka, and Polonchek (1991). In a multi-period situation, pooling may be traced by checking whether agents performing identically in early stages differ in later stages. This late-stage behaviour is a sign of self-selection and thus of asymmetric information, so that the early-stage behaviour must be a sign of pooling, rather than symmetric information. Their analysis of multi-stage sales of shares on the stock market provides empirical support for the model of Gale and Stiglitz (1989), cited above in subsection 6.1 as an instance of recontracting.<sup>67</sup>

Since the measurement problems are so serious when doing empirical work on real-life problems, there seems to be particularly good reasons to do *experi-*

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<sup>67</sup>Mention should also be made of empirical studies of reputation effects, such as Wolfson (1985) on oil and gas drilling partnerships.

*mental* research on asymmetric-information models in general and the pooling question in particular. One study of the latter sort is by Cadsby, Frank, and Maksimovic (1990). In testing a signalling model of a capital market, they find that, in cases where the theory predicts multiple Perfect Bayesian Equilibria, including one that is separating and one that is pooling, and where refinements tend to select the separating one, the subjects in their experimental tests most often end up in a pooling equilibrium. King and Wallin (1991) perform a test of a model with voluntary disclosure when the agent may not have the information (see the end of subsection 3.3.4).<sup>68</sup>

In concluding this tour of the literature on pooling equilibria, a retrospect indicates that we have been too long on theory compared to empirical work – on the basis of a page count, the ratio is something like 57:2! This suggests that the marginal social value of the next paper to be published on the subject is higher if it is empirical than if it is theoretical. There are indeed good reasons to subscribe to the views expressed by Roth (1991), writing on the future of game theory, that one should put more emphasis in the future on empirical work, experimental and otherwise, than on theory – particularly such empirical work that is directed primarily at testing and informing theory.

There should be ample possibilities on the pages above to find a theoretical explanation for pooling that so far has not been put to an empirical test. The empirically inclined out there are hereby invited to help themselves.

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## CHAPTER TWO:

CONSUMER LOCK-IN WITH ASYMMETRIC INFORMATION<sup>1</sup>

**Abstract:** A two-period version of the Rothschild-Stiglitz insurance market model is presented. Its features include: repeat purchase; limited commitment among both insurers and consumers; and private information on a customer's accident history. It is shown that an equilibrium may exist exhibiting first-period pooling and second-period lock-in of consumers.

## 1. INTRODUCTION

Industrial organization is a field with much progress during the last decade. Applications of its various ideas of strategic behaviour has found their ways to many different industries. In a somewhat glaring contrast to this development, some industries have not caught the interest of industrial economists, even if they obviously feature oligopolies. One such industry is insurance.

A reason for this apparent lack of interest may be found in Milgrom and Roberts (1987). They state, slightly paraphrased, that the play of a game with "informational incompleteness, but no informational asymmetries, [while] useful ... for studying such issues as insurance, ... would not generate any interesting forms of strategic behavior" (pp. 184-185). The idea presented in this chapter of getting around this state of affairs is to introduce a dynamic model of competition in an insurance market in which informational asymmetries are allowed to develop over time, thus creating strategic behaviour that is indeed of significant interest.

This chapter builds on the seminal work by Rothschild and Stiglitz (1976) who consider an insurance market with asymmetric information. In a one-

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<sup>1</sup>The first version of this chapter was completed in August 1989. I would like to thank Geir Asheim and Terje Lensberg for all their help throughout the work on this chapter; Frøystein Gjesdal and Jon Vislie for valuable comments on earlier versions; and Georg Nöldeke, Joseph Stiglitz, and Bent Vale for fruitful conversations. Thanks also to participants at seminars in Bergen, Warsaw, Barcelona and Lisbon, for their many useful reactions.



period, two-type model, they show that the adverse-selection problem created by the asymmetric information can be partially alleviated: In equilibrium, high-risk consumers get full insurance, while low-risk consumers get only partial insurance in order to create disincentives for the high-risks to buy the low-risk insurance contract. Each firm earns zero expected profit.

Below is an attempt at extending their model to a multi-period framework. What makes the present extension interesting, is the combination of two plausible assumptions: first, that an individual's accident history is known only to him and his insurer; and, secondly, that neither firms nor consumers are able to commit to long-term (multi-period) insurance contracts.

The first of these assumptions creates asymmetry of information *between firms*, in addition to the consumer-firm asymmetry. But whereas accidents are private information, it is assumed that all contract purchases are publicly observable. This means that the informational asymmetry among firms will have no consequences unless consumers pool in the first period, in the sense that consumers of different types buy the same contract in that period. When this happens in equilibrium, we will denote it a *pooling equilibrium*. Rothschild and Stiglitz (1976) show that such an equilibrium cannot exist in the one-period model. A similarly strong result is not obtained in the two-period case. On the contrary, it is proved here that, for some reasonable parameter values, a pooling equilibrium does exist.

The first-period pooling in this equilibrium creates a possibility for each firm to take advantage in the second period of its information on its old customers' accident histories. When this informational advantage can be turned into a positive profit, we say that *consumer lock-in* entails in the sense that a consumer's previous insurer is able to outbid outside firms and still earn a profit. We establish in here the existence of a pooling equilibrium featuring such consumer lock-in.

Apart from the existence of a pooling equilibrium with consumer lock-in, the present work informs the debate on insurance markets in two respects. First, we find an equilibrium in *pure strategies* in the two-period model for parameter values that only allow a mixed-strategy equilibrium in the one-period one. Second, we find that, in the first period of our equilibrium, risk averse consumers may actually be *selling* insurance to risk neutral firms.

Readers may want to object to the assumption that accident information is private. In particular, one may argue that, by regulation, public records of accidents are kept such that any insurer has free access to this information.<sup>2</sup> However, these systems seldom work perfectly; whenever they don't, there is room for doubt on a particular consumer's true accident record. Evidence from the U.S. suggests that such records are indeed of limited value: In one survey, only 47% of a sample of more than 27,000 car accidents known to insurers and meeting statutory requirements were reported to the public records.<sup>3</sup> Our assumption, although extreme, catches the essential feature of any imperfectly functioning public record.

In the present model, the informational asymmetry between firms is endogenously determined. In this sense, the present model is unique in the insurance literature. But in studies of other markets, notably credit and labour markets, models are found having this feature.<sup>4</sup> These models can be split into two groups.

Both Greenbaum, Kanatas and Venezia (1989) and Sharpe (1990) present credit market models where the information on a consumer's type is *ex ante* symmetric among consumers and firms. Thus, self-selection is not an issue, since consumers have no superior knowledge. These authors conclude, as we do here, that consumers get captured because of the informational asymmetry between firms developing over time.

Greenwald (1986)<sup>5</sup> studies a multi-period labour market where the wage contracts are non-contingent and short-term, and where information about a worker is completely revealed to his current employer after one period. Thus, Greenwald's model and the present one differ in two important respects: First,

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<sup>2</sup>The author would like to record his indebtedness to various commentators whose willingness to describe the various kinds of information they were required to disclose when applying for their car insurances has prompted this paragraph.

<sup>3</sup>AIRAC (1984), as quoted by D'Arcy and Doherty (1990, p. 152).

<sup>4</sup>See also the model in Blume and Easley (1987) of endogenous information acquisition in a two-period Cournot market.

<sup>5</sup>Related labour market models are in Waldman (1984, 1990), Lazear (1986), Milgrom and Oster (1987), Riordan and Staiger (1988), Gibbons and Katz (1989), and Novos (1990a, 1990b). A credit market model with essentially the same structure is found in Fischer (1990).

the exclusion of contingent contracts makes separation impossible in Greenwald's model.<sup>6</sup> Again, this breaks with one of the key features of the present model – individuals' choice between pooling and separating contracts. Second, in the present model, a firm's information on old customers is not perfect.<sup>7</sup> Although its information is superior to that of its rivals, each firm still has to provide its old customers with incentives for truth-telling. Again, this constitutes an essential difference. Still, there is one important similarity in the results of the two models: Both predict that new customers/employees are given better offers, relative to expected riskiness/ability, than old ones.

Multi-period insurance models so far have considered either the case of monopoly or the case of long-term contracts, or both. The only one that is really connected to the present model, is Cooper and Hayes (1987).<sup>8</sup> In their model, competing firms are allowed to sign long-term contracts. This makes it possible to separate consumers in the following way: In equilibrium, two contracts are offered in the first of two periods; one contract covering only period one, and another contract covering both periods. This latter contract is constructed such that only low-risks will want to buy it; this is because of the way a first-period

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<sup>6</sup>This holds also for all the papers of the previous footnote. Dewatripont and Maskin (1990) and Kanemoto and MacLeod (1990), on the other hand, present two-period models with contingent contracts. But Dewatripont and Maskin also allow for long-term contracts and for renegotiation after the first period, while Kanemoto and MacLeod assume that the agent's first-period contract choice is observed by the contract offeror only.

<sup>7</sup>This feature is also found in labour market papers by Lazear (1986), O'Flaherty and Siow (1989), and Waldman (1990). Lazear's model is very different from ours with regard to the move structure. In O'Flaherty and Siow's model, there is no worker-firm asymmetry of information. Waldman's model focuses on limited, as opposed to no, commitment; he studies up-or-out (tenure) contracts in which workers agree to quit after one period in case of a bad outcome. All these authors, like Greenwald, restrict attention to non-contingent contracts.

<sup>8</sup>In Berkovitch and Venezia (1986), analyzing the market for life insurance, firms are allowed to offer long-term contracts, just like in Cooper and Hayes (1987), but contracts are non-contingent. In Kunreuther and Pauly (1985), consumers are completely myopic and contracts are non-contingent. In Haneda (1986) and Malueg (1988), only long-term contracts are allowed. In Palfrey and Spatt (1985) and Cotter and Jensen (1989), there is neither consumer-firm nor firm-firm asymmetry of information. The models of Wilson (1977), Myerson (1988), and Winter (1989) are multi-period but without consumer repurchases. Single-period models with a dynamic structure allowing firms to make reactions are discussed, e.g., in Cave (1985). Multi-period models of insurance monopoly include Dionne (1983), Dionne and Lasserre (1987), Gal and Landsberger (1988), and Hosios and Peters (1989); the model of Hosios and Peters is of particular interest here, since their monopolist is unable to commit to long-term contracts. For a survey emphasizing adverse selection in multi-period models of the insurance market, see Dionne and Doherty (1991a).

accident is punished in the second period.<sup>9</sup>

Since the Cooper-Hayes model is very close to the present one, except for the commitment issue, and since the two models give opposite predictions regarding the timing of firms' profits, it should be possible to perform an empirical test of whether firms in fact can commit to long-term contracts: According to Cooper and Hayes, firms' expected profit earned on a consumer is higher in the start of the relationship than in the end.<sup>10</sup> According to the present model, it is the opposite: Firms compete for their customers without the possibility to give credible promises on future contracts. This and the informational rent earned on old customers bring down the offer given to new customers. Given the work of D'Arcy and Doherty (1990) based on data from U.S. insurance firms, it is tempting to argue that the empirical test has been performed already. Their results reject the Cooper-Hayes model with regard to the timing of firms' profits, thus lending support to any model with the opposite prediction.<sup>11</sup>

As already mentioned, it is a crucial feature of the present model that insurance contracts are assumed to be short-term ones. A rationale for this assumption may be given along the following lines: In reality, there are more than two periods in a relationship and, most probably, any contract encompasses several periods. But as long as contracts cannot be sustained for the whole duration of the relationship, a two-period model without commitment may very well be a good approximation. With this assumption, the present model is related to the literature on the principal-agent problem under no commitment; see Roberts (1984), Freixas, Guesnerie, and Tirole (1985), Laffont and Tirole (1987, 1988), Gibbons (1987), Baron and Besanko (1987), Hosios and Peters (1989) (on an insurance monopoly), and Lewis and Sappington (1990). A general theme of these papers is that pooling is the likely outcome.

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<sup>9</sup>Note that, without the ability to commit, the firm would not have incentives to carry out such a punishment.

<sup>10</sup>A very recent paper by Dionne and Doherty (1991b) extends the Cooper-Hayes analysis to the case with renegotiation; they verify the same pattern of profit in this variation.

<sup>11</sup>In another paper, D'Arcy and Doherty (1989) discuss practical implications of this pattern of profits for an insurance company's pricing decisions. It should be noted that the empirical evidence reported in Dionne and Doherty (1991b) is more mixed than is that of D'Arcy and Doherty (1990).

One interpretation of the present work is as an extension of the above works to the case of competing principals. It turns out that pooling can occur in this case, too. But the possibilities for pooling in a situation of competition is limited by one firm's incentives to take advantage of other firms' offering a pooling contract in order to get itself a cheap separation; this is an effect that is not caught in the usual principal-agent framework. On the other hand, the possibilities for such a cheap separation are limited by the competing principals' zero-expected-profit constraints; this is, again, a new aspect relative to the single-principal case. On balance, pooling occurs in cases where the profit effect makes itself particularly felt relative to the separation effect.

The chapter is organized as follows. In the next Section, the model is presented in detail. In Section 3, we establish the existence of one particularly interesting equilibrium – a pooling equilibrium with consumer lock-in. In Section 4, other issues are discussed, in particular the existence of a separating equilibrium in pure strategies, the reasonableness of firms' beliefs, and the sequential nature of moves. Section 5 concludes.

## 2. THE MODEL

An individual will in each of two periods either have an accident or avoid it. Denote his income without accident and with accident  $W_1$  and  $W_2$ , respectively. Without insurance,  $(W_1, W_2) = (W, W - D)$ , where  $W$  is positive and finite and  $D \in (0, W)$  is the cost of a damage.<sup>12</sup> In Figure 1, a consumer's income is illustrated by points in  $(W_1, W_2)$ -space. His income without insurance is thus located at point E. We define the contract space as follows:

$$\mathbb{A} \equiv \{(\alpha_1, \alpha_2) \in \mathbb{R}^2 \mid \alpha_1 + \alpha_2 \leq D, \alpha_1 \geq -g, \text{ and } \alpha_2 \geq D - W\},$$

where  $g$  is finite, positive, and constant, and  $\mathbb{R}$  is the real line. A one-period *insurance contract* is now  $\alpha = (\alpha_1, \alpha_2) \in \mathbb{A}$ . One may interpret this as the individual paying to an insurance company a premium  $\alpha_1$  in return for which he gets  $\alpha_1 +$

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<sup>12</sup>By assumption, there is neither saving nor borrowing between the two periods. For our purpose,  $W$  is just a lump-sum income received by each individual at the start of each period. The residual after insurance purchase and possibly an accident is consumed before next period starts.

$\alpha_2$  if an accident occurs in that period;  $\alpha_2$  is the insurer's (net) liability. Thus, *with* insurance, the consumer's income is  $(W_1, W_2) = (W - \alpha_1, W - D + \alpha_2)$ . To simplify illustration, we let throughout a contract  $\alpha$  be depicted in  $(W_1, W_2)$ -space at the point  $(W - \alpha_1, W - D + \alpha_2)$ . In Figure 1, the contract space  $\mathbb{A}$  is thus delineated by the triangle OGB.

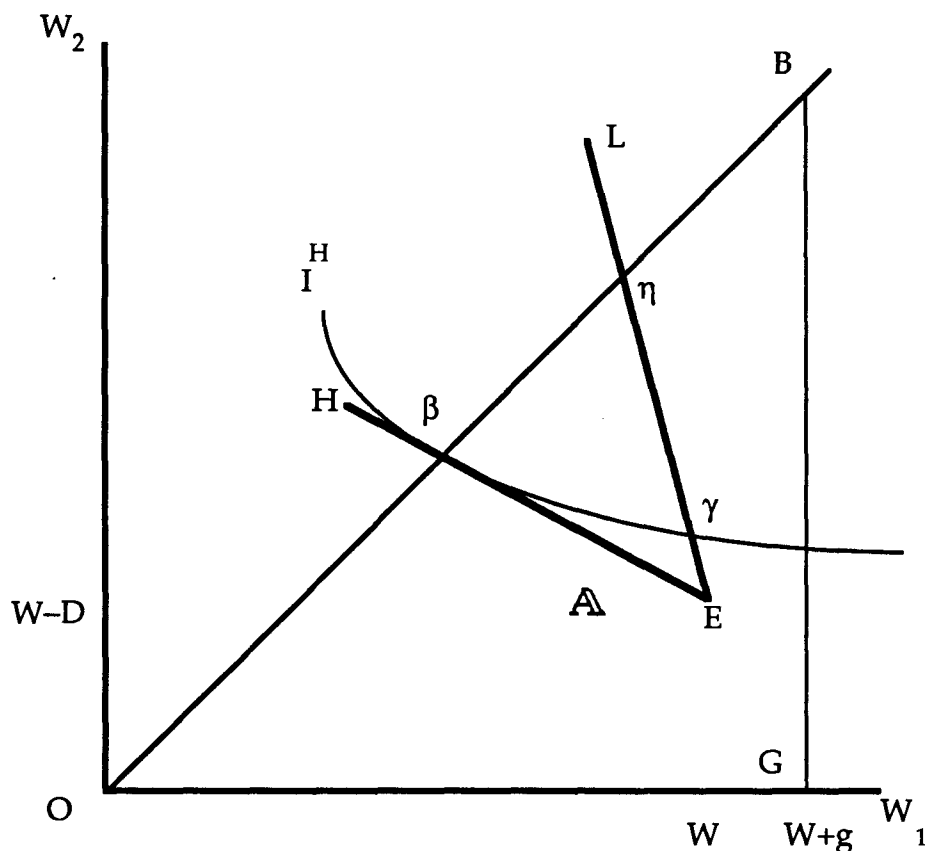


Figure 1.

**The demand for insurance.** There is a continuum of consumers; we assume they are spread uniformly over the unit line  $[0, 1]$  with density 1. A generic consumer is denoted by  $j \in [0, 1]$ . Consumers are identical except for the probability of an accident. In particular, the set of consumer types is  $\{L, H\}$ . The accident probability of an L (low-risk) type is  $p_L$ , that of an H (high-risk) type is  $p_H$ ; these probabilities satisfy:

$$0 < p_L < p_H < 1.$$

The fraction of low-risks in the population is  $\lambda \in (0, 1)$ . The (one-period) *expected*

utility from buying the contract  $\alpha$  for a consumer of type  $k \in \{H, L\}$  is:<sup>13</sup>

$$V(p_k, \alpha) = (1 - p_k)U(W - \alpha_1) + p_kU(W - D + \alpha_2), \quad (2.1)$$

where  $U: [0, \infty) \rightarrow \mathbb{R}$  is the utility of money income. We make the following standard assumptions on  $U$ : (i)  $U' > 0$ ; and (ii)  $U'' < 0$  (risk aversion).

It follows from  $p_H > p_L$ , (2.1), and our assumptions on  $U$ , that a low-risk consumer's indifference curve is steeper than a high-risk consumer's indifference curve everywhere in  $(W_1, W_2)$ -space; see Figure 2. This is the present model's version of the Spence-Mirrlees sorting condition: Since his accident probability is lower, a low-risk consumer requires a less increase in no-accident income ( $W_1$ ) to compensate for a given decrease in accident income ( $W_2$ ).

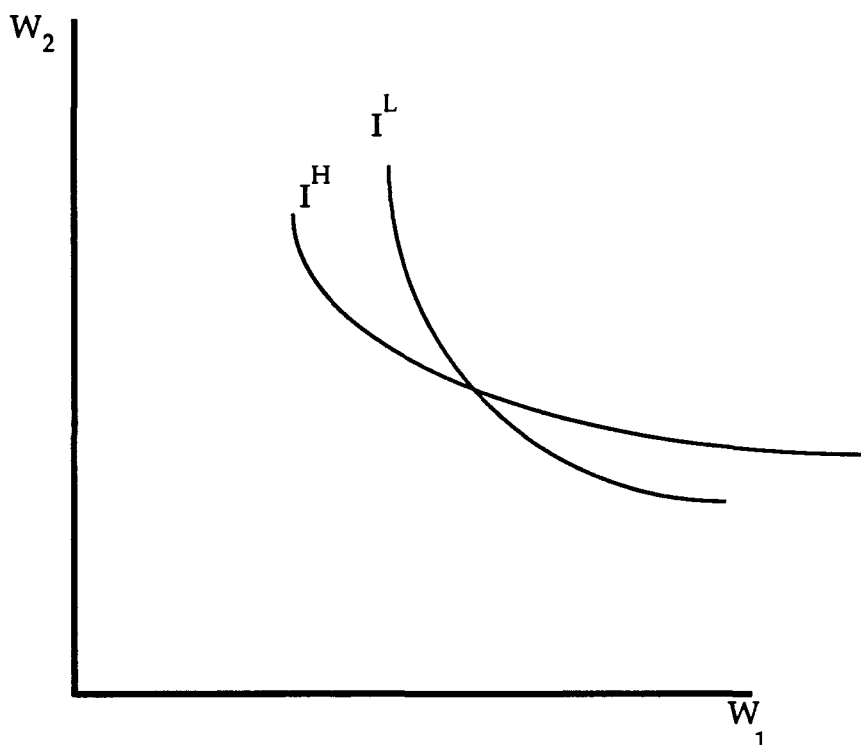


Figure 2. High-risk and low-risk indifference curves.

Each individual discounts the future with a discount factor  $\delta \in (0, 1]$ ; expected utility over both periods is denoted *overall expected utility*. It is assumed that no consumer comes new to the market in the second period and that consumers at any time observe all available offers without incurring search costs. No indivi-

<sup>13</sup>Denote the righthand side of (2.1) expected utility on *extensive form*.

dual can by any action affect his accident probability; i.e., there is no moral-hazard problem. A consumer is restricted to buying insurance from only one firm in each period.<sup>14</sup>

**The supply of insurance.** The supply side of the market consists of a finite number of firms, indexed by  $i \in N \equiv \{1, \dots, n\}$ .<sup>15</sup> Each firm is assumed to be a risk-neutral maximiser of its total discounted expected profit, with the same discount factor  $\delta$  as consumers, and to have financial resources sufficient to supply any number of insurance contracts within the set  $\mathbb{A}$ .<sup>16</sup> The expected profit from selling the contract  $\alpha$  to an individual who is believed to be low-risk with probability  $b$ , is:

$$\begin{aligned}\pi(\alpha, b) &= (1 - b)[(1 - p_H)\alpha_1 - p_H\alpha_2] + b[(1 - p_L)\alpha_1 - p_L\alpha_2] \\ &= [(1 - b)(1 - p_H) + b(1 - p_L)]\alpha_1 - [(1 - b)p_H + bp_L]\alpha_2\end{aligned}\quad (2.2)$$

In analysing the second period of our model, it is useful to distinguish between a consumer's previous insurer, which we denote the *informed firm* relative to this consumer, and all others, which we denote the *uninformed firms*.

Based on the above descriptions of consumers and firms, we can make the observation that, along an indifference curve, profit is higher with more insurance. This is stated in Lemma 1 below, complete with a proof; it should be noted, however, that it is a standard result, related to the first-best efficiency of full insurance; see, e.g., Borch (1990). Define first a contract  $\alpha$ 's *coverage* as the ratio  $\alpha_2/(D - \alpha_1)$ , which varies from zero at no insurance to one at full insurance. Along an indifference curve, the coverage increases towards full insurance.

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<sup>14</sup>Suppose, e.g., that insurers reimburse consumers only in exchange for a document, such as a receipt, verifying expenses. This, in effect, makes it impossible to get a damage covered from more than one firm. This argument may not hold in the presence of non-market institutions providing insurance; see Arnott and Stiglitz (1991), who prove such institutions to be harmful in a moral-hazard context. However, given the atomistic character of consumers, it seems natural to assume that no mutual-insurance arrangements exist.

<sup>15</sup>By specifying a finite number of firms and, later on, allowing each firm to offer more than one contract each period, this paper follows a version of the Rothschild-Stiglitz model that Dasgupta and Maskin (1986) attribute to Hahn (1977).

<sup>16</sup>The assumption of unlimited insurance capacity is not necessarily an innocuous one; see, e.g., Winter (1988, 1989).



**Lemma 1:** Consider two different contracts on the same high-risk [resp., low-risk] indifference curve. The contract, of the two, with the higher  $\pi(\cdot, 0)$  [resp.,  $\pi(\cdot, 1)$ ] is the one with the higher coverage, i.e. the one which is closer to full insurance.

**Proof:** We do it here for high-risks only; it is identical for low-risks. For any contract  $\alpha^H$  which is not a full-insurance one, so that  $\alpha_1^H + \alpha_2^H < D$ , a firm's isoprofit curve is steeper than a high-risk consumer's indifference curve:

$$\frac{d\alpha_1^H}{d\alpha_2^H} \Big|_{d\pi(\alpha^H, 0)=0} = \frac{p_H}{1-p_H} > \frac{p_H U'(W - \alpha_1^H)}{(1-p_H)U'(W - D + \alpha_2^H)} = \frac{d\alpha_1^H}{d\alpha_2^H} \Big|_{dV(p_H \alpha^H)=0}$$

where the inequality follows from  $U'' < 0$ . Thus, the firm improves its expected profit by moving  $\alpha^H$  towards full insurance along the high-risk indifference curve. QED.

**The structure of the game.** In the present model, there are two periods, but the number of stages in the game is higher. There are three stages in which firms make their moves. And inbetween, consumers make their contract choices and any accident occurs. All in all, the game consists of the following stages:

*Period 1:*

*Stage 1.1:* Firms make simultaneous first-period offers to all consumers.

These offers are observed by everybody.

*Stage 1.2:* Consumers choose among the offers from stage 1.1. Everybody observes their choices.

*Stage 1.3:* Every consumer and his insurer observe whether any accident occurs. First-period contracts are fulfilled.

*Period 2:*

*Stage 2:* Firms make simultaneous second-period offers to consumers on whom they have no accident information. These offers are observed by everybody.

*Stage 3.1:* Firms make simultaneous second-period offers to consumers on whom they do have accident information, i.e., to old customers.

*Stage 3.2:* Consumers choose among the offers from stages 2 and 3.1.

*Stage 3.3:* Accidents happen and second-period contracts are fulfilled. The game ends.<sup>17</sup>

This move sequence implies that, in the second period, for a given consumer, all uninformed firms move before the informed one. Put another way, each firm gives offers first to all consumers on which it is uninformed and then to those on which it is informed. Apart from its analytical tractability, this structure makes precise the notion that a firm's customers don't leave to a rival firm before they have received a final offer from it.<sup>18</sup>

Two remarks should be made regarding the content of the stages specified above. Firstly, the information a firm obtains about whether its customers had an accident in the first period is, by assumption, *not* shared with its rivals. Secondly, it is assumed that any accident is observed by both the consumer having the accident and his insurer. We abstract thus from the possibility for the consumer not to report an accident in the first period in order to gain in the second-period offers.<sup>19</sup>

Before proceeding, we emphasize the two sources of asymmetry of information in this model. One is the assumption that, before the start of period one, each individual knows his probability of accident, while firms don't. The other is the assumption that, in period one, any accident is observed only by the consumer having the accident together with his first-period insurer. At the start of the second period, each firm has learnt something from the first period – *viz.* the accident histories of its old customers – so that there is asymmetry of information also among firms: Each firm knows more about its old customers than about those of the other firms, and more than the other firms know; hence the distinction, for each consumer, between the informed firm and the uninformed ones.

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<sup>17</sup>When we later on use the term "stage 1", this should be thought of as substages 1.1 through 1.3 taken together. The same holds true for stage 3.

<sup>18</sup> Referring back to the discussion of related literature in Sec. 1, this is the move sequence used by Greenwald (1986) and subsequent labour market papers. Greenbaum, Kanatas, and Venezia (1989) and Sharpe (1990) have the opposite sequence, with the informed firm moving first, while in Fischer (1990), the parties move simultaneously.

<sup>19</sup>The effects of allowing this in the monopoly case are discussed by Hosios and Peters (1989).

**Firms' strategies.** In order to define the equilibrium, we must define the firms' strategies. To simplify our analysis, we start with imposing the following assumption:

*Assumption (A):* (i) A firm's offers to consumers on whom it has identical information are identical.

(ii) A firm's offer to a consumer is based on its information on this consumer only.

(iii) Accidents are distributed independently across consumers according to consumer type and the probabilities  $p_H$  and  $p_L$ .

(iv) Consumers are independently assigned to being high-risk or low-risk.

Assumption (A)(i) does not imply that consumers on whom a firm has identical information buy identical *contracts* from it; rather, it implies that these consumers are offered the same *set* of contracts to choose from.<sup>20</sup>

Note the similarity in spirit between Assumption (A)(ii) here and the "no-signaling-what-you-don't-know" condition of Fudenberg and Tirole (1991). The present assumption pertains to offers the consumer has received previously, his choice among these offers, and, if he is an old customer, his accident experience. In all these respects, the firm does not use information on any other consumer as basis for its offer to this consumer.

Assumptions (A)(iii) and (iv) are straightforward.

The consequence of Assumption (A) is that the identity of a consumer (i.e., his "home" on the unit line  $[0, 1]$ ) is not payoff-relevant information. A firm is not able to draw any inferences from information on one consumer (or group of consumers) about other consumers. Therefore, we can save on notation by concentrating on a single group of consumers, all of the same type and with the same contract choice, the same insurer, and the same accident experience.

What we are after in the formal development below, is the following: Consider a firm that, at some point in time, has collected (i) information on the histo-

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<sup>20</sup>Assumption (A)(i) relates to the literature on whether, in a credit market with asymmetric information, there will be any credit rationing: In the terminology used by Clemenz (1986), the present assumption eliminates Type II rationing, whereby "[s]ome loan applicants are denied a loan even though for the bank they are indistinguishable from accepted applicants" (p. 18).

ry of the game, denoted  $h$ , containing firms' previous offers, contractual relationships established in the past between consumers and firms, and what the firm knows about consumer accidents; and (ii) consumer-specific information, denoted  $I$ , containing a consumer's previous contract choice and insurer, as well as his accident experience if he is an old customer. A firm's strategy is a prescription of which offers, contingent on  $I$ , to make following any history  $h$ . Let  $\mathfrak{H}$  be the set of all histories and  $\mathfrak{I}$  the set of all consumer-specific information. A firm's strategy is then a list of mappings, one for each time the firm makes a move, from  $\mathfrak{H} \times \mathfrak{I}$  to the offer space  $S$ , which is defined more precisely below; it suffices here to note that  $S \subset \mathbb{A}^2 \cup \{\emptyset\}$ .

There are three different stages at which firms are called upon to make their moves in this game. For  $t \in \{1, 2, 3\}$  and  $i \in N$ , let  $\mathfrak{H}_i^t$  denote the set of histories for firm  $i$  at stage  $t$ , with  $h_i^t$  as a typical element in  $\mathfrak{H}_i^t$ ; and let  $\mathfrak{I}_i^t$  denote the set of firm  $i$ 's consumer-specific information at stage  $t$ , with  $I_i^t$  as a typical element in  $\mathfrak{I}_i^t$ .

At stage 1, the game has no history, and there is no consumer-specific information, so  $\mathfrak{H}_i^1 = \mathfrak{I}_i^1 = \{\emptyset\}$ ,  $i \in N$ . Thus, we may write firm  $i$ 's stage-1 strategy as  $s_i^1 \in S$ .

At stage 2, the history  $h_i^2 \in \mathfrak{H}_i^2$  of firm  $i$  contains: (i) all firms' offers at stage 1,  $s^1 \equiv (s_1^1, \dots, s_n^1) \in S^n$ ; (ii) contractual relationships established in period 1, described by the mapping  $B: [0, 1] \rightarrow \mathbb{A} \times N$ , where  $B(j) = (a_1, \iota_1)$  lists the contract  $(a_1)$  and supplier  $(\iota_1)$  chosen by consumer  $j$  in that period; and (iii) the accident histories observed by the firm, described by a mapping  $r_i: C_i \rightarrow \{0, 1\}$ , where  $C_i \subset [0, 1]$  is the set of firm  $i$ 's first-period customers, and where  $r_i(j) = 1$  if consumer  $j$  had an accident in period 1 and  $r_i(j) = 0$  otherwise. For  $s^1$  and  $B$  to be mutually consistent, we know that they must satisfy:  $a_1 \in A(s_i^1)$ , where we introduce the convention that, for any vector  $G$ ,  $A(G)$  denotes the set of elements of  $G$ .

Since a firm makes offers at stage 2 only to new customers, i.e., consumers it did not insure in the past, the relevant consumer-specific information that firm  $i$  has at this stage,  $I_i^2 \in \mathfrak{I}_i^2$ , is the pair  $(a_1, \iota_1)$ , with  $i \neq \iota_1$ . Thus,  $I_i^2 = B(j)$  for all consumers  $j$  who receive an offer from firm  $i$  at stage 2. Therefore, by Assumption (A), the only elements of  $h_i^2$  on which an offer to consumer  $j$  may depend are  $s^1$  and

$B(j) = I_i^2$ . Thus, given some  $I_i^2 \in \mathcal{I}_i^2$ , the mapping  $\tilde{\sigma}_i^2: \mathcal{H}_i^2 \times \mathcal{I}_i^2 \rightarrow S$ , describing firm  $i$ 's stage-2 strategy, is constant over  $\mathcal{H}_i^2$ , except for variations in  $s^1$  and  $B(j)$ . But  $B(j) = I_i^2$ ; therefore, there exists a function  $\sigma_i^2: S^n \times \mathbb{A} \times N \rightarrow S$  such that  $\tilde{\sigma}_i^2(h_i^2, I_i^2) = \sigma_i^2(s^1, I_i^2)$  whenever  $s^1$  is the stage-1 offers corresponding to  $h_i^2$ . We may then, without further loss of generality, represent the stage-2 part of firm  $i$ 's strategy by the function  $\sigma_i^2$  instead of by  $\tilde{\sigma}_i^2$ . Given the "reduced-form" history  $s^1$ ,  $\sigma_i^2(s^1, \cdot)$  details firm  $i$ 's offer to consumers at this stage conditioned on the consumer-specific information  $I_i^2$ . All firms' stage-2 strategies are listed in  $\sigma^2 \equiv (\sigma_1^2, \dots, \sigma_n^2)$ .

At stage 3, the history  $h_i^3 \in \mathcal{H}_i^3$  of firm  $i$  consists of  $h_i^2$  and all firms' stage-2 offers, given by  $\sigma^2(s^1, B(j))$ , where  $s^1 \in S^n$  are stage-1 offers and  $B(j)$  are consumer  $j$ 's choices in period 1. Let  $r: [0, 1] \rightarrow \{0, 1\}$  record consumers' first-period experience, where  $r = 0$  denotes no accident and  $r = 1$  denotes an accident. The relevant consumer-specific information at stage 3,  $I_i^3 \in \mathcal{I}_i^3$ , equals  $(a_1, i, r)$ , since now offers are made to a firm's old customers only, on whom the firm has accident information. Thus,  $I_i^3 = (B(j), r(j))$  for all consumers  $j$  who receive offers at stage 3. Therefore, by Assumption (A), the only elements of  $h_i^3$  on which an offer to consumer  $j$  may depend are  $s^1$ ,  $\sigma_{-i}^2(s^1, B(j))$ , and  $(B(j), r(j)) = I_i^3$ . Thus, given some  $I_i^3 \in \mathcal{I}_i^3$ , the mapping  $\tilde{\sigma}_i^3: \mathcal{H}_i^3 \times \mathcal{I}_i^3 \rightarrow S$ , describing firm  $i$ 's stage-3 strategy, is constant over  $\mathcal{H}_i^3$ , except for variations in  $s^1$ ,  $\sigma_{-i}^2(s^1, B(j))$ ,  $B(j)$ , and  $r(j)$ . But  $(B(j), r(j)) = I_i^3$ ; therefore, there exists a function  $\sigma_i^3: S^{n(n-1)} \times \mathbb{A} \times N \times \{0, 1\} \rightarrow S$  such that  $\tilde{\sigma}_i^3(h_i^3, I_i^3) = \sigma_i^3(s^1, s_{-i}^2, a_1, r)$ , where  $s_{-i}^2 = \sigma_{-i}^2(s^1, a_1, r) \in S^{n-1}$ , whenever  $s^1$  and  $s_{-i}^2$  are the stage-1 and stage-2 offers corresponding to  $h_i^3$ . We may then, without further loss of generality, represent the stage-3 part of firm  $i$ 's strategy by the function  $\sigma_i^3$  instead of by  $\tilde{\sigma}_i^3$ . Given the "reduced-form" history  $(s^1, s_{-i}^2)$ ,  $\sigma_i^3(s^1, s_{-i}^2, \cdot)$  details firm  $i$ 's offer to consumers at this stage conditioned on the consumer-specific information  $(B, r)$ . All firms' stage-3 strategies are listed in  $\sigma^3 \equiv (\sigma_1^3, \dots, \sigma_n^3)$ .

Thus, firm  $i$ 's strategy in this game is given by  $\sigma_i \equiv (s_i^1, \sigma_i^2, \sigma_i^3)$ ,  $i \in N$ , and the collection of all firms' strategies is  $\sigma \equiv (s^1, \sigma^2, \sigma^3)$ . Let  $\mathcal{A}$  be the set of all strategy collections.

With regard to the offer space  $S$ , we let each firm in each period offer con-

sumers a menu of maximum two contracts to choose from – one for each possible type.<sup>21</sup> In particular, let  $S \equiv S' \cup \{\emptyset\}$ , where:

$$S' \equiv \{(\alpha^H, \alpha^L) \in \mathbb{A}^2 \mid \text{either: } V(p_H, \alpha^H) > V(p_H, \alpha^L), \\ \text{or: } V(p_H, \alpha^H) = V(p_H, \alpha^L) \text{ and } V(p_L, \alpha^L) \geq V(p_L, \alpha^H)\}.$$

That is, a non-empty offer consists of two contracts that we label according to consumer preferences. Note that this allows also for single-contract offers: Whenever an offer is a singleton  $\alpha \in \mathbb{A}$ , we represent it as  $(\alpha, \alpha) \in S$ .

**Firms' beliefs.** We assume that firms with identical information hold identical beliefs. Thus, let  $\mu: S^n \times \mathbb{A} \rightarrow [0, 1]^3$  map stage-1 offers and consumer contract choice into firms' subjective probabilities, after period 1 but before period 2, that a consumer is low-risk. Here,  $\mu(s^1, a_1) = (\mu_U, \mu_0, \mu_1)$ , with  $\mu_U$  representing the beliefs of an uninformed firm and  $\mu_r$  the beliefs of an informed firm having observed the accident history  $r \in \{0, 1\}$ .

**Consumer behaviour.** Consumers are assumed to be rational and to have rational expectations. We allow consumers to condition their choices on firms' strategies and not only on the offers available.<sup>22</sup> To be precise, let  $a_1: \{H, L\} \times \mathcal{S} \rightarrow \mathbb{A}$  be a mapping from the consumer's type and firms' strategies into his first-period choice; and let  $a_2: \{H, L\} \times \mathbb{A} \times \{0, 1\} \times \mathcal{S} \rightarrow \mathbb{A}$  be a mapping from the consumer's type, his first-period choice, his accident history, and firms' strategies into his second-period choice. We impose on these two mappings that, for each  $(k, \sigma) \in \{H, L\} \times \mathcal{S}$ ,  $a_1(k, \sigma) \in B^1(k, \sigma)$  and, for each  $(k, a_1, r, \sigma) \in \{H, L\} \times \mathbb{A} \times \{0, 1\} \times \mathcal{S}$ ,  $a_2(k, a_1, r, \sigma) \in B^2(k, a_1, r, \sigma)$ , where:

$$B^2(k, a_1, r, \sigma) \equiv \operatorname{argmax}_{\alpha} V(p_k, \alpha), \\ \text{such that: } \alpha \in A(\sigma_{-i}^2(s^1, a_1, r), \sigma_i^3(s^1, s_{-i}^2, a_1, r)),$$

and:

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<sup>21</sup>An argument for why this is not restrictive is offered by Dasgupta and Maskin (1986, note 10) for the one-period model.

<sup>22</sup>This is done in order to concentrate on the strategic behaviour among firms. Moreover, each consumer is one out of uncountably many and thus of measure zero with no possibility to affect firms' profits or their beliefs.

$$B^1(k, \sigma) \equiv \operatorname{argmax}_{\alpha} V(p_k, \alpha) + \delta[p_k V^2(k, \alpha, 1, \sigma) + (1 - p_k) V^2(k, \alpha, 0, \sigma)],$$

such that:  $\alpha \in A(s^1)$ ,

with:  $V^2(k, \alpha, r, \sigma) \equiv V(p_k, \alpha')$  with  $\alpha' \in B^2(k, \alpha, r, \sigma)$ , for each  $(k, \alpha, r, \sigma)$ ; i.e.,  $V^2$  is the expected utility from contracts in  $B^2$ .

There are problems, though: The sets  $B^1$  and  $B^2$  may not be singletons, meaning that the consumer may be indifferent between contracts offered. This indifference creates two kinds of problems.<sup>23</sup>

First, consumer indifference may lead to situations where, say, a consumer, if he is a high-risk, mixes between two contracts whereas he, if he is a low-risk, buys one of those contracts with certainty.<sup>24</sup> In such a case, the probability that a consumer buying the latter contract is a low-risk, is somewhere in  $[\lambda, 1]$ , depending on how the high-risks randomize over the two contracts. In this analysis, on the other hand, we want to concentrate on one particular equilibrium in which consumers are pooled in the first period, with high-risks and low-risks buying the same contract without anyone of them randomizing. To clarify matters, we make the assumption that  $a_1$  and  $a_2$  are single-valued selections from  $B^1$  and  $B^2$ , respectively, thus ensuring that situations other than complete pooling and complete separation are avoided.

Second, we have to make sure that firms' best-reply correspondences exist. This problem is a standard one, and it can to some extent be solved by assuming that the consumer choice mappings select contracts that maximise profits.<sup>25</sup> We have, however, a particular problem here because of the model's sequential structure in period 2. We want to make sure that an informed firm at stage 3,

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<sup>23</sup>A third one would be related to the existence, in our model, of multiple informed agents. It has been argued – e.g. by Demski and Sappington (1984), Ma, Moore, and Turnbull (1988), and Ma (1988) within a single-principal multiple-agent framework – that tie-breaking rules maximising uninformed's payoff subject to informed's indifference may be vulnerable to collusion among the latter. We abstract safely from this problem here, since we have uncountably many informed agents, collusion among whom seems unlikely.

<sup>24</sup>Invoking the continuum of consumers, we do not need to allow a consumer to randomize to state this argument: We have here a situation where some high-risks buy one contract and the rest of the high-risks together with all the low-risks buy another contract because high-risks are indifferent between the two.

<sup>25</sup>A related tie-breaking rule, stipulating that the consumer go to the firm whose expected profit will be the greatest, is suggested by Cave (1985).

when matching offers from uninformed firms, does not attract a high-risk consumer if doing so is unprofitable for it. It follows from Lemma 1 that, absent any incentive constraints, the best contract for a firm to offer a consumer is the full-insurance one. Thus, if matching the uninformed firms' offers with a full-insurance contract earns negative expected profit, then any contract matching them is unprofitable. This motivates highlighting the contract  $\beta = (\beta_1, \beta_2)$ , the full-insurance contract earning zero expected profit when sold to a high-risk consumer, defined by:

$$\beta_1 + \beta_2 = D, \quad (2.3a)$$

$$\beta_1(1 - p_H) = \beta_2 p_H. \quad (2.3b)$$

Equation (2.3a) says that the contract offers full insurance, while equation (2.3b) says that it earns zero expected profit when sold to a high-risk. Solving this system of equations gives:  $\beta = (p_H D, (1 - p_H) D)$ . The contract  $\beta$  is depicted in Figure 1 above.

The above discussion motivates the following assumption on  $a_1$  and  $a_2$ :

*Assumption (B):* (a)  $\forall (k, \sigma) \in \{H, L\} \times \mathcal{A}$ ,  $a_1(k, \sigma)$  is a single-valued selection from  $B^1(k, \sigma)$ , maximising  $\pi(\alpha, b(k))$ , with  $b(H) = 0$  and  $b(L) = 1$ .

(b)  $\forall (k, a_1, r, \sigma) \in \{H, L\} \times \mathcal{A} \times \{0, 1\} \times \mathcal{A}$ ,  $a_2(k, a_1, r, \sigma)$  is a single-valued selection from  $B^2(k, a_1, r, \sigma)$  maximising  $\pi(\alpha, b(k))$ , with  $b(H) = 0$  and  $b(L) = 1$ , subject to:

(i) If a consumer is low-risk and  $B^2$  contains a contract that is offered by the informed firm, then he chooses the informed firm as his second-period insurer.

(ii) If a consumer is high-risk and  $B^2$  contains a contract that is offered by the informed firm, with an expected utility  $V^2(H, a_1, r, \sigma) \leq V(p_H, \beta)$ , then he chooses the informed firm as his second-period insurer.

(iii) If a consumer is high-risk and  $B^2$  contains a contract that is offered by an uninformed firm, with an expected utility  $V^2(H, a_1, r, \sigma) > V(p_H, \beta)$ , then he chooses an uninformed firm as his second-period insurer.

Since these mappings are single-valued, we write, given the consumer type



$k$  and firms' strategies  $\sigma$ ,  $a_1 = a_1(k, \sigma)$ ; and, given consumer type  $k$ , first-period contract choice  $a_1$ , accident history  $r$ , and firms' strategies  $\sigma$ ,  $a_2 = a_2(k, a_1, r, \sigma)$ .

**Definition of equilibrium.** Equilibrium strategies will be required to satisfy *sequential rationality*: For given beliefs, no player wants at any point to change his strategy. It is furthermore required that, for given strategies, beliefs are given by subjective probabilities that are defined by Bayes' rule whenever it applies. A collection of strategies and beliefs constitute a *perfect Bayesian equilibrium* (PBE) if the above requirements on strategies and beliefs are fulfilled; see, e.g., Tirole (1988, Sec. 11.5) or Fudenberg and Tirole (1991). To be precise, an assessment  $(\sigma^*, \mu^*)$  is a PBE for this game under the following conditions:

(i) *Sequential rationality*:

– Given stage-1 offers  $s^1$ , stage-2 offers  $s^2$ , beliefs  $\mu^*$  and consumer choices  $a_1$  and  $a_2$ , each firm maximises its expected profit on second-period offers to old customers; thus, we obtain  $\sigma^{3*}$ .

– Given stage-1 offers  $s^1$ , other firms' equilibrium stage-2 strategies  $\sigma_{-i}^{2*}$ , equilibrium stage-3 strategies  $\sigma^{3*}$ , beliefs  $\mu^*$ , and consumer choices  $a_1$  and  $a_2$ , each firm maximises its expected profit on second-period offers to new customers; thus, we obtain  $\sigma^{2*}$ .

– Given other firms' equilibrium stage-1 strategies  $s_{-i}^{1*}$ , equilibrium period-2 strategies  $(\sigma^{2*}, \sigma^{3*})$ , beliefs  $\mu^*$ , and consumer choices  $a_1$  and  $a_2$ , each firm maximises its total discounted expected profit on first-period offers to consumers; thus, we obtain  $s^{1*}$ .

(ii) *Bayes-consistent beliefs*. Beliefs are generated by Bayes' Rule whenever feasible: Given strategies  $\sigma^*$  and consumer choices  $a_1$  and  $a_2$ ,

– a firm's subjective probability that a *new* customer is low-risk equals the actual probability, according to  $a_1$ , of the consumer being low-risk given his first-period choice among the first-period offers:

$$\mu_U^*(s^{1*}, a_1) = 0, \text{ if } a_1 = a_1(H, \sigma^*) \neq a_1(L, \sigma^*);$$

$$\mu_U^*(s^{1*}, a_1) = 1, \text{ if } a_1 = a_1(L, \sigma^*) \neq a_1(H, \sigma^*);$$

$$\mu_U^*(s^{1*}, a_1) = \lambda, \text{ if } a_1 = a_1(H, \sigma^*) = a_1(L, \sigma^*);$$

– a firm's subjective probability that an *old* customer is low-risk equals the actual probability, according to  $a_1$ , of the consumer being low-risk given his first-period choice among the first-period offers, adjusted by use of Bayes' Rule for the firm's accident information:

$$\mu_0^* = \frac{\mu_U^*(1 - p_L)}{1 - p_H + \mu_U^*(p_H - p_L)}; \quad \mu_1^* = \frac{\mu_U^* p_L}{p_H - \mu_U^*(p_H - p_L)}.$$

Since consumer choices are single-valued, by Assumption (B), there are only three cases to distinguish regarding equilibrium beliefs. These are two cases of separation –  $(\mu_U, \mu_0, \mu_1) = (0, 0, 0)$  and  $(\mu_U, \mu_0, \mu_1) = (1, 1, 1)$  – and a case of pooling:

$$(\mu_U, \mu_0, \mu_1) = \left( \lambda, \frac{\lambda(1 - p_L)}{1 - p_H + \lambda(p_H - p_L)}, \frac{\lambda p_L}{p_H - \lambda(p_H - p_L)} \right), \quad (2.4)$$

where we have used the equilibrium definition above. We turn now to the analysis.

### 3. POOLING AND LOCK-IN

This section contains an exploration of whether, and when, consumers are locked in with their previous insurers in period two. The main steps are the following. First, we construct a pooling equilibrium of the game and provide non-primitive conditions for its existence; to obtain this, we characterise the second-period equilibria following separation and pooling, respectively, in period one. We then proceed to verify that there are primitives for which the pooling equilibrium exists that we have constructed. This is done through a numerical example. In this example, firms earn positive expected second-period profits. We thus have established that consumer lock-in is possible.

The pooling equilibrium we construct and the non-primitive conditions for its existence are provided in Proposition 1 below. Before we are ready to state this result, however, we need a lot of notation.

A useful reference is the contract pair  $(\beta, \gamma)$ . These contracts constitute the pure-strategy separating equilibrium of the Rothschild-Stiglitz one-period model

with symmetric information among firms (when such a pure-strategy equilibrium exists). The contract  $\beta$  is defined in (2.3) above. The contract  $\gamma$  is defined by:

$$U(W - p_H D) = (1 - p_H)U(W - \gamma_1) + p_H U(W - D + \gamma_2) \quad (3.1a)$$

$$\gamma_1(1 - p_L) = \gamma_2 p_L \quad (3.1b)$$

Equation (3.1a) says that a high-risk consumer is indifferent between the two contracts  $\beta$  and  $\gamma$ ; the lefthand side of (3.1a) equals  $V(p_H, \beta)$  with appropriate substitutions from (2.3). Equation (3.1b) says that the contract  $\gamma$  earns zero expected profit when sold to a low-risk consumer. The contract  $\gamma$  is depicted in Figure 1 above.

Let the contract  $\eta$  be the full-insurance zero-profit contract when sold to low-risks:

$$\eta_1 + \eta_2 = D \quad (3.2a)$$

$$\eta_1(1 - p_L) = \eta_2 p_L \quad (3.2b)$$

Equation (3.2a) says that the contract  $\eta$  offers full insurance. Equation (3.2b) says that it yields zero expected profit when sold to a low-risk. Solving the above system of equations, we get:  $\eta = (p_L D, (1 - p_L)D)$ . The contract  $\eta$  is depicted in Figure 1 above.

Let, for  $r \in \{0, 1\}$ , the contract pair  $\tau^r = (\tau^{rH}, \tau^{rL})$  be one that solves:

$$\max_{\alpha^H, \alpha^L} (1 - \mu_r)\pi(\alpha^H, 0) + \mu_r\pi(\alpha^L, 1), \text{ subject to:} \quad (3.3a)$$

$$\alpha_1^H + \alpha_2^H = D \quad (3.3b)$$

$$V(p_H, \alpha^H) = V(p_H, \alpha^L) \quad (3.3c)$$

$$V(p_L, \alpha^L) = V(p_L, \gamma), \quad (3.3d)$$

where  $\gamma$  is defined by (3.1). I.e., the contract pair  $\tau^r$  maximises the profit for the informed firm when the consumer has the first-period accident record  $r$ ,  $r \in \{0, 1\}$ , subject to the restrictions that the consumer, if he is high-risk, is offered full insurance and is indifferent between the two contracts, and, if he is low-risk, obtains the same expected utility as from the contract  $\gamma$ . With the help of this definition, we go on to define the contract pair  $v^r = (v^{rH}, v^{rL})$ ,  $r \in \{0, 1\}$ , as follows:

$$\text{If } \frac{1 - \lambda}{\lambda} < \frac{p_L^2(1 - p_L)}{p_H(p_H - p_L)}K, \text{ then: } v^r = \tau^r; \quad (3.4a)$$

$$\text{if } \frac{p_L^2(1-p_L)}{p_H(p_H-p_L)}K \leq \frac{1-\lambda}{\lambda} < \frac{p_L(1-p_L)^2}{(1-p_H)(p_H-p_L)}K, \text{ then: } v^0 = \tau^0, \text{ and } v^1 = (\beta, \gamma); \quad (3.4b)$$

$$\text{if } \frac{1-\lambda}{\lambda} \geq \frac{p_L(1-p_L)^2}{(1-p_H)(p_H-p_L)}K, \text{ then: } v^r = (\beta, \gamma); \quad (3.4c)$$

where:

$$K \equiv \frac{U'(W - p_H D)}{U'(W - \gamma_1)} - \frac{U'(W - p_H D)}{U'(W - D + \gamma_2)}, \quad (3.5)$$

with  $\gamma$  defined in (3.1).

Next, we define the contract  $\zeta$ , which is the first-period pooling contract of the equilibrium we construct below:

$$\begin{aligned} \zeta \equiv \operatorname{argmax}_{\alpha \in \mathbb{A}} V(p_L, \alpha), \text{ subject to: } & \pi(\alpha, \lambda) + \delta\{(1-\lambda)[p_H\pi(v^{1H}, 0) \\ & + (1-p_H)\pi(v^{0H}, 0)] + \lambda[p_L\pi(v^{1L}, 1) + (1-p_L)\pi(v^{0L}, 1)]\} \geq 0. \end{aligned} \quad (3.6)$$

Thus,  $\zeta$  maximises the expected utility of a low-risk consumer subject to a non-negative overall profit constraint.

In order to describe the belief structure of the equilibrium, we define the subsets  $\mathbb{A}_H$  and  $\mathbb{A}_L$  of the contract space  $\mathbb{A}$ . For  $k, h \in \{H, L\}, k \neq h$ ,

$$\mathbb{A}_k \equiv \{\alpha \in \mathbb{A} \mid \alpha \neq \zeta,$$

$$V(p_h, \alpha) + \delta V(p_h, \alpha^k) \leq V(p_h, \zeta) + \delta V(p_h, \alpha^h), \text{ and} \quad (3.7a)$$

$$V(p_k, \alpha) + \delta V(p_k, \alpha^k) \geq V(p_k, \zeta) + \delta[p_k V(p_k, v^{1k}) + (1-p_k)V(p_k, v^{0k})], \quad (3.7b)$$

where  $v^r$  and  $\zeta$  are defined in (3.4), and (3.6), respectively, and where  $\alpha^H \equiv \beta$ , defined in (2.3), and  $\alpha^L \equiv \eta$ , defined in (3.2). For each consumer type, this set is defined in terms of one restriction regarding the other type's preferences, (3.7a), and one restriction regarding its own preferences, (3.7b). The meaning of  $\mathbb{A}_H$  and  $\mathbb{A}_L$  is given after Proposition 1 below.

Finally, for each consumer type  $k \in \{H, L\}$ , define the contract set  $\mathbb{P}_k$ :

$$\mathbb{P}_k \equiv \{\alpha \in \mathbb{A} \mid \pi(\alpha, b(k)) \geq 0\}, \quad (3.8)$$

with  $b(H) = 0$  and  $b(L) = 1$ . Thus,  $\mathbb{P}_k$  consists of all contracts yielding a non-negative expected profit when sold to consumers of type  $k$ .

We are now ready to state:

**Proposition 1:** Given the above definitions, consider the following collection of strategies and beliefs:

*Strategies:*

Period 1:  $s^1 = (\zeta, \zeta)^n$ .

Period 2: (i) If  $\mu_U = 0$ , then:  $(s^2, s_{v_1}^3) = (\beta, \beta)^n$ ;

(ii) if  $\mu_U = 1$ , then:  $(s^2, s_{v_1}^3) = (\eta, \eta)^n$ ;

(iii) if  $\mu_U = \lambda$ , then:  $s^2 = (\beta, \gamma)^{n-1}$  and  $\sigma_{v_1}^3(a_1, r) = v^r, r \in \{0, 1\}$ .

*Beliefs:*

(i)  $\mu_U(s^1, a_1) = 0$ , if  $a_1 \in \mathbb{A}_H$ ;

(ii)  $\mu_U(s^1, a_1) = 1$ , if  $a_1 \in \mathbb{A}_L$ ; and

(iii)  $\mu_U(s^1, a_1) = \lambda$ , otherwise.

This collection of strategies and beliefs constitute an equilibrium if and only if, for each  $k \in \{H, L\}$ :

$$\mathbb{A}_k \cap \mathbb{P}_k = \emptyset. \quad (3.9)$$

Before we prove Proposition 1, some amount of explanation is required. In the equilibrium described in this Proposition, firms offer the contract  $\zeta$  in period one. In the (off-the-equilibrium-path) case of separation in period one, firms offer in period two the contract  $\beta$  if a consumer is believed to be high-risk and the contract  $\eta$  if he is believed to be low-risk. In the case of pooling in period one, uninformed firms offer the contract pair  $(\beta, \gamma)$  in period two. An informed firm offers the contract pair  $v^1$  in period two if the consumer had an accident in period one and the contract pair  $v^0$  if he did not have one.

The beliefs are, given these strategies, that a consumer is high-risk if he buys a first-period contract only a high-risk would want to buy (i.e., a contract in  $\mathbb{A}_H$ ), and that he is low-risk if he buys a first-period contract only a low-risk would want to buy (i.e., a contract in  $\mathbb{A}_L$ ). If neither of these applies, the firms' prior beliefs are not updated on the basis of a consumer's purchase of a first-period contract. The contract sets  $\mathbb{A}_H$  and  $\mathbb{A}_L$  merit some further comments. Let us

consider  $\mathbb{A}_L$  in detail; corresponding reasoning holds for  $\mathbb{A}_H$ .

If a high-risk consumer chooses to mimic a low-risk one by choosing  $\alpha \in \mathbb{A}_L$  in period one, then he is believed with certainty to be low-risk and is offered  $\alpha^L = \eta$  in period two.<sup>26</sup> If he chooses  $\zeta$  instead while low-risks choose  $\alpha \in \mathbb{A}_L$  then he has disclosed his type and is offered  $\alpha^H = \beta$  in period two. Condition (3.7a) states that a high-risk consumer's benefit of the latter must outweigh that of the former: Even if he is disclosed as a high-risk this way, the high-risk consumer chooses  $\zeta$  rather than some contract in  $\mathbb{A}_L$ ; otherwise, firms could not rationally attach probability of a low-risk equal to 1 to contracts in  $\mathbb{A}_L$ .

A low-risk consumer must weigh the value of being separated against the value of being pooled together with the high-risk consumers. The righthand side of (3.7b) is the overall expected utility for a low-risk consumer of buying the pooling contract  $\zeta$  in period one; note that the second-period expected utility is weighted by the consumer's true accident probability, since his accident history determines the offer he receives in period two in a pooling equilibrium. The lefthand side of (3.7b) is his overall expected utility from a choice of a first-period contract in  $\mathbb{A}_L$ , given that this makes firms certain he is a low-risk so that they offer him  $\alpha^L = \eta$  in period two. When (3.7b) is satisfied, a low-risk consumer prefers a contract in  $\mathbb{A}_L$  to  $\zeta$ ; otherwise, no consumers would prefer contracts in  $\mathbb{A}_L$  and they would be chosen by mistake only, implying that firms could not rationally attach probability of a low-risk equal to 1 to such contracts.

The belief structure of the equilibrium in Proposition 1 is illustrated in Figure 3. Let, again,  $k, h \in \{H, L\}$ ,  $k \neq h$ .  $I_\zeta^k$  is the  $k$ -type indifference curve through the first-period pooling contract  $\zeta$ . The set  $\mathbb{A}_k$  is below  $I_h^k$  by condition (3.7a), and above  $I_k^k$  by condition (3.7b).

Proposition 1 states that the collection of strategies and beliefs described

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<sup>26</sup>Any contract in  $\mathbb{A}_L$  is, by construction, able to separate low-risks from high-risks. A single, high-risk consumer deviating by choosing a contract in this set does not change firms' beliefs that a consumer choosing such a contract is a low-risk with certainty. This is because there are uncountably many high-risks, each one being of measure zero and, therefore, unable to affect beliefs. As is usual in this kind of analysis, concerted deviations are not considered.

therein constitute an equilibrium of the game if, given the first-period pooling contract  $\zeta$ , no other contract exists that both serves as a separating contract for a certain consumer type, relative to  $\zeta$ , and is profitable for a firm when sold to this consumer type. For each type  $k$ , inclusion in  $\mathbb{P}_k$  is a *profitability constraint* on a separating deviation from this pooling equilibrium, while inclusion in  $\mathbb{A}_k$  is a *self-selection constraint* on such a separating deviation. In case both these constraints cannot be satisfied for any type, a separating deviation is not feasible and the pooling equilibrium of Proposition 1 exists.<sup>27</sup>

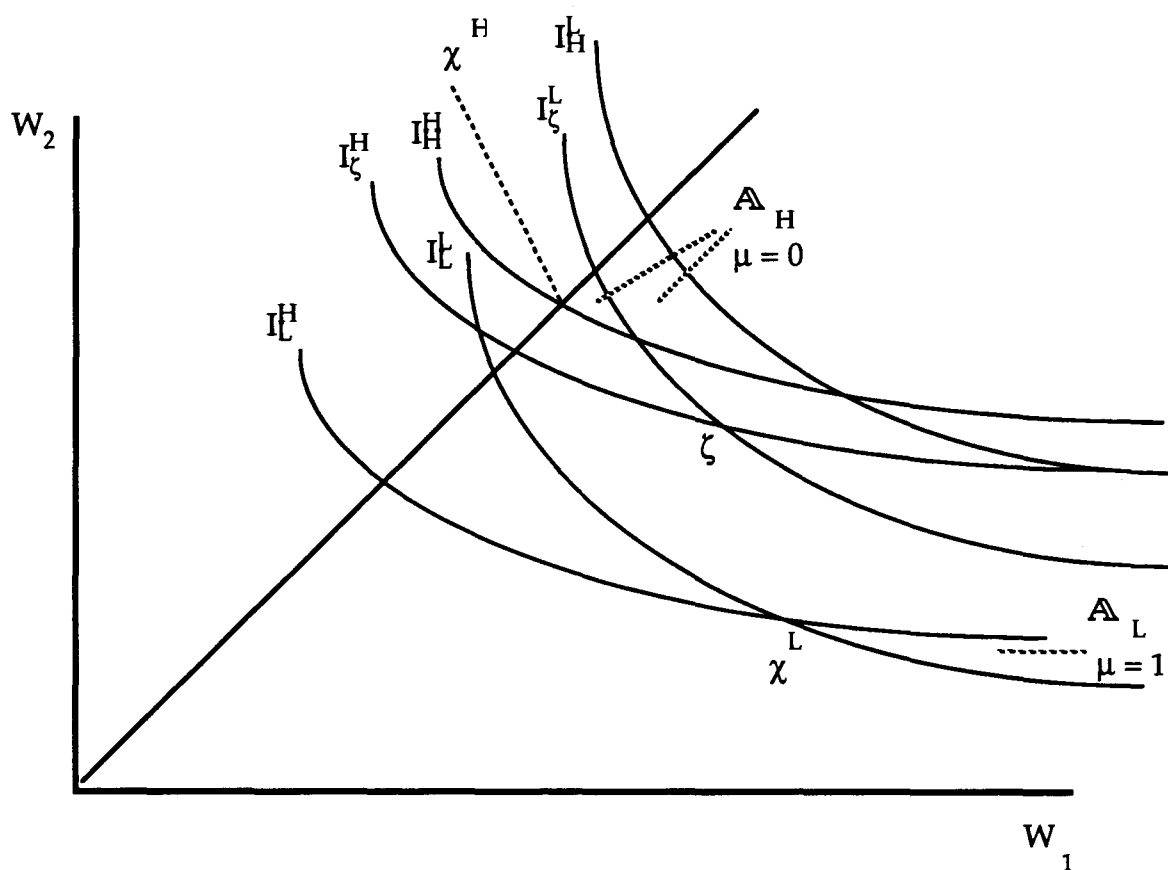


Figure 3.

In order to prove Proposition 1, we go through a series of Lemmas. We start with analysing period two in the simplest cases:  $\mu_U = 0$ , and  $\mu_U = 1$ . If a consumer's first-period purchase makes firms certain of his type, then there is effectively

<sup>27</sup>We concentrate here on single-contract deviations from a pooling contract. This seems to run counter to firms being allowed to offer *two* contracts. However, for  $k, h \in \{H, L\}$ ,  $k \neq h$ , if  $\mathbb{A}_k \cap \mathbb{P}_k$  is empty, then the cheapest way to separate consumer of type  $h$  is clearly to offer a single contract: the most profitable contract in  $\mathbb{A}_h$ .

no asymmetry of information among firms at the start of period two, since the observation of the consumer's accident experience does not add, in any payoff-relevant way, to a firm's knowledge. We have:

**Lemma 2:** Suppose  $\mu_U \in \{0, 1\}$ . Let  $\beta$  and  $\eta$  be defined by (2.3) and (3.2), respectively. There exists an equilibrium of the second-period game in which each firm's action is:

(i) to offer the contract  $\beta$  if  $\mu_U = 0$ ,

(ii) to offer the contract  $\eta$  if  $\mu_U = 1$ ;

and its expected profit equals zero.

**Proof:** Although we here have a sequential move structure, this result of existence follows from arguments similar to the discussion of insurance markets under complete information in Rothschild and Stiglitz (1976, Sec. I.5). QED.

We now turn to the more complicated analysis of the pooling case, i.e., when  $\mu_U = \lambda$ . Instead of analysing the informed firm's equilibrium behaviour at stage 3 following any possible history, we delineate considerably the cases to consider with the help of Lemma 3 below. Define first, for each consumer type, the expected utility from a particular vector  $s^2$  of uninformed firms' offers in stage 2:

$$V_k(s^2) \equiv \max_{\alpha} V(p_k, \alpha) \text{ such that } \alpha \in A(s^2), k \in \{H, L\}. \quad (3.10)$$

**Lemma 3:** In equilibrium following any vector  $s^1 \in S^n$  of stage-1 offers, if such an equilibrium exists, uninformed firms' expected second-period profits equal zero.

**Proof:** Suppose the expected second-period profit of an uninformed firm is negative. Then, it would pay for the firm to change its offer to one that the consumer would not choose, since this would give zero profit. Therefore, its expected second-period profit cannot be negative in equilibrium.

Suppose next that expected second-period profit of an uninformed firm is



positive. This implies that the firm earns a positive expected profit on selling to at least one consumer type.

Suppose first that it offers a contract  $\alpha$  that earns a positive expected profit when sold to a high-risk consumer:  $\pi(\alpha, 0) > 0$ . Suppose also that  $\alpha$  is not dominated, in terms of the high-risk consumer's expected utility, by other uninformed firms' offers at stage 2:  $V(p_H, \alpha) = V_H$ , with  $V_H$  defined in (3.10). (If this firm's offer is dominated in this sense, consider instead another uninformed firm whose offer is undominated.) Consider now the contract  $\beta$ , defined in (2.3), providing full insurance such that  $\pi(\beta, 0) = 0$ . It follows from Lemma 1 that any contract  $\alpha'$  on the same high-risk indifference curve as  $\beta$  or higher earns a negative expected profit:  $V(p_H, \alpha') \geq V(p_H, \beta)$  implies  $\pi(\alpha', 0) \leq 0$ . Equivalently,  $\pi(\alpha, 0) > 0$  implies  $V(p_H, \alpha) < V(p_H, \beta)$ . Therefore, from Assumption (B)(b)(ii), the informed firm attracts the high-risk consumer if it matches the uninformed firm's offer. And since  $\pi(\alpha, 0) > 0$ , it is profitable for the informed firm to do so. Therefore, an uninformed firm cannot attract a high-risk consumer with a profitable offer.

Suppose next that the uninformed firm offers a contract  $\alpha$  that earns a positive expected profit when sold to a low-risk:  $\pi(\alpha, 1) > 0$ . Suppose also, again, that  $\alpha$  is undominated among uninformed firms' offers at stage 2:  $V(p_L, \alpha) = V_L$ , with  $V_L$  defined in (3.10). From Assumption (B)(b)(i), the informed firm attracts a low-risk consumer if it matches the uninformed firm's offer. If  $V(p_H, \alpha) > V(p_H, \beta)$ , then the informed firm can match  $\alpha$  without attracting high-risks even if  $V(p_H, \alpha) = V_H$ , by Assumption (B)(b)(iii). If  $V(p_H, \alpha) \leq V(p_H, \beta)$ , then there exists a full-insurance contract  $\alpha^H$  such that  $V(p_H, \alpha^H) = V(p_H, \alpha)$  and  $\pi(\alpha^H, 0) \geq 0$ , since  $\pi(\beta, 0) = 0$ . Thus, if  $\pi(\alpha, 1) > 0$ , then it is profitable for the informed firm to match it. Therefore, an uninformed firm cannot attract a low-risk with a profitable offer.

We have now excluded offers with negative expected profits and those with positive expected profits. Thus, uninformed firms' profits in equilibrium, if an equilibrium exists, must equal zero. QED.

Next, we resolve the existence issue left open in the previous Lemma:

**Lemma 4:** For any vector  $s^1 \in S^n$  of stage-1 offers, there exists an equilibrium in which each uninformed firm offers the contract pair  $(\beta, \gamma)$  at stage 2.

**Proof:** By (2.3b) and (3.1b), both  $\beta$  and  $\gamma$  earn zero expected profit:  $\pi(\beta, 0) = 0$ , and  $\pi(\gamma, 1) = 0$ . By Lemma 3, an uninformed firm cannot earn a higher expected profit with an alternative offer. Moreover, since  $V(p_H, \beta) = V(p_H, \gamma)$  by (3.1a), incentive constraints are satisfied. Thus, offering  $(\beta, \gamma)$  at stage 2 is an equilibrium strategy for an uninformed firm. QED.

We now turn to the details of stage 3, at which the informed firm makes its second-period offer. We first state a preliminary result:

**Lemma 5:** Suppose  $\mu_U = \lambda$ . Let  $V_L$  be defined by (3.10). If, at stage 3, the informed firm's optimum offer is such that it attracts both a high-risk consumer and a low-risk one, then this offer, denoted  $(\alpha^H, \alpha^L)$ , has the following properties:

$$\alpha_1^H + \alpha_2^H = D, \tag{3.11a}$$

$$V(p_H, \alpha^H) = V(p_H, \alpha^L), \tag{3.11b}$$

$$V(p_L, \alpha^L) = V_L. \tag{3.11c}$$

**Proof:** In (3.11b),  $V(p_H, \alpha^H) \geq V(p_H, \alpha^L)$  is necessary for a high-risk consumer to choose  $\alpha^H$ .  $V(p_H, \alpha^H) > V(p_H, \alpha^L)$  would imply that expected profit on the low-risk consumer could be improved, by Lemma 1. Thus,  $V(p_H, \alpha^H) = V(p_H, \alpha^L)$ . From Lemma 1, the firm maximises its profit on a high-risk consumer by offering full insurance subject to (3.11b); thus, we obtain (3.11a). In (3.11c),  $V(p_L, \alpha^L) \geq V_L$  is necessary to attract the low-risks, while  $V(p_L, \alpha^L) > V_L$  again would imply that expected profit could be improved; thus,  $V(p_L, \alpha^L) = V_L$ . QED.

We can now state more precisely what is the informed firm's equilibrium

behaviour at stage 3 in the case when uninformed firms offer  $(\beta, \gamma)$  at stage 2. A useful way of viewing the informed firm's problem is as one of a constrained monopolist. The uninformed firms' offers of  $(\beta, \gamma)$  serve as outside options more restrictive than the ones confronting a pure monopolist but otherwise parallel.<sup>28</sup> Lemma 6 below is in fact proved following Stiglitz' (1977) analysis of an insurance monopoly. Define:

$$\lambda^* \equiv \frac{1}{1 + \frac{p_L(1-p_L)^2}{(1-p_H)(p_H-p_L)}K}, \quad (3.12a)$$

and:

$$\lambda^{**} = \frac{1}{1 + \frac{p_L^2(1-p_L)}{p_H(p_H-p_L)}K} \quad (3.12b)$$

with  $K$  defined in (3.5). We see, since  $p_H > p_L$ , that  $\lambda^* < \lambda^{**}$ .

**Lemma 6:** Suppose  $\mu_U = \lambda$ . Let  $\beta, \gamma, v_r, \lambda^*$ , and  $\lambda^{**}$  be defined by (2.3), (3.1), (3.4), (3.12a), and (3.12b), respectively. If uninformed firms offer  $(\beta, \gamma)$  at stage 2, i.e.,  $s_{-i}^2 = (\beta, \gamma)^{n-1}$ , then the informed firm's optimal offer at stage 3 is given by:  $\sigma_{i_1}^3(a_1, r) = v_r, r \in \{0, 1\}$ . Furthermore, if  $\lambda > \lambda^*$ , then the informed firm's expected second-period profit is positive: If  $\lambda^* < \lambda < \lambda^{**}$ , the informed firm earns positive expected profit on old customers without first-period accidents only; if  $\lambda > \lambda^{**}$ , the informed firm earns positive expected profit on all old customers.

**Proof:** The contracts  $\beta$  and  $\gamma$ , offered by uninformed firms, earn zero expected profit each, by (2.3b) and (3.1b):  $\pi(\beta, 0) = 0$  and  $\pi(\gamma, 1) = 0$ . If the informed firm's strictly optimum offer differs from  $(\beta, \gamma)$ , then this offer attracts both types; a contract  $\alpha$  attracting high-risks only would earn negative expected profit, while a contract earning positive profit when sold to low-risks, attracts high-risks also if it attracts low-risks. Thus, by Lemma 5, the informed firm's optimum offer satis-

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<sup>28</sup>The individual-rationality conditions confronting a monopolist are given by self-insurance, which is strictly dominated by  $\beta$  and  $\gamma$  in terms of consumers' expected utility. Thus, our informed firm faces stricter conditions than a monopolist does.

fies (3.11). Denote the informed firm's offer  $\omega = (\omega^H, \omega^L) = ((\omega_1^H, \omega_2^H), (\omega_1^L, \omega_2^L))$ ; its average expected profit per contract is, for  $r \in \{0, 1\}$ :

$$\pi^r = (1 - \mu_r)[\omega_1^H(1 - p_H) - \omega_2^H p_H] + \mu_r[\omega_1^L(1 - p_L) - \omega_2^L p_L].$$

By differentiating this expression subject to the three conditions in (3.11), and evaluating the differentiation at  $\omega = (\beta, \gamma)$  for  $r = 0$  and  $r = 1$  separately, the conditions defining  $v^0$  and  $v^1$  in (3.4) are obtained. The differentiation and evaluation are done in the Appendix. The offer  $(\beta, \gamma)$  gives zero expected profit; if this offer is no longer optimum, then the optimum contract pair gives positive expected profit. Inspection of (3.4) shows that the condition for this is  $\lambda > \lambda^*$ , with  $\lambda^*$  defined in (3.12a), and that the optimum contract pairs differ from  $(\beta, \gamma)$  for both accident histories if and only if  $\lambda > \lambda^{**}$ , with  $\lambda^{**}$  defined in (3.12b). QED.

It follows from Lemma 6 and (3.4) that, when  $\lambda$  is sufficiently high, the informed firm's stage-3 offers differ from  $(\beta, \gamma)$  for an old customer both if he had an accident in period one and if he did not. What this might look like in  $(W_1, W_2)$ -space is illustrated in Figure 4. For a medium range of  $\lambda$ , the consumer is offered  $(\beta, \gamma)$  by the informed firm if he had an accident in period one and a different offer if not; in this case,  $v^{1H}$  and  $v^{1L}$  in Figure 4 coincide with  $\beta$  and  $\gamma$ , respectively. Finally, for low values of  $\lambda$ , an old customer is offered  $(\beta, \gamma)$ , irrespective of his accident experience; the informed firm can do no better than matching the uninformed firms' offer.

Note the relation between Lemma 6 and the analysis of the standard one-period Rothschild-Stiglitz model: In the latter, a pure-strategy equilibrium exists, with each firm offering  $(\beta, \gamma)$ , if and only if:

$$\lambda \leq \frac{1}{1 + \frac{p_L(1 - p_L)}{p_H - p_L} K}, \quad (3.13)$$

with  $K$  defined by (3.5). This is seen by carrying out the above analysis for the case  $\mu_0 = \mu_1 = \mu_U = \lambda$ .<sup>29</sup>

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<sup>29</sup>See also Chapter 3 below: the proof of Proposition 1 in Appendix 1.

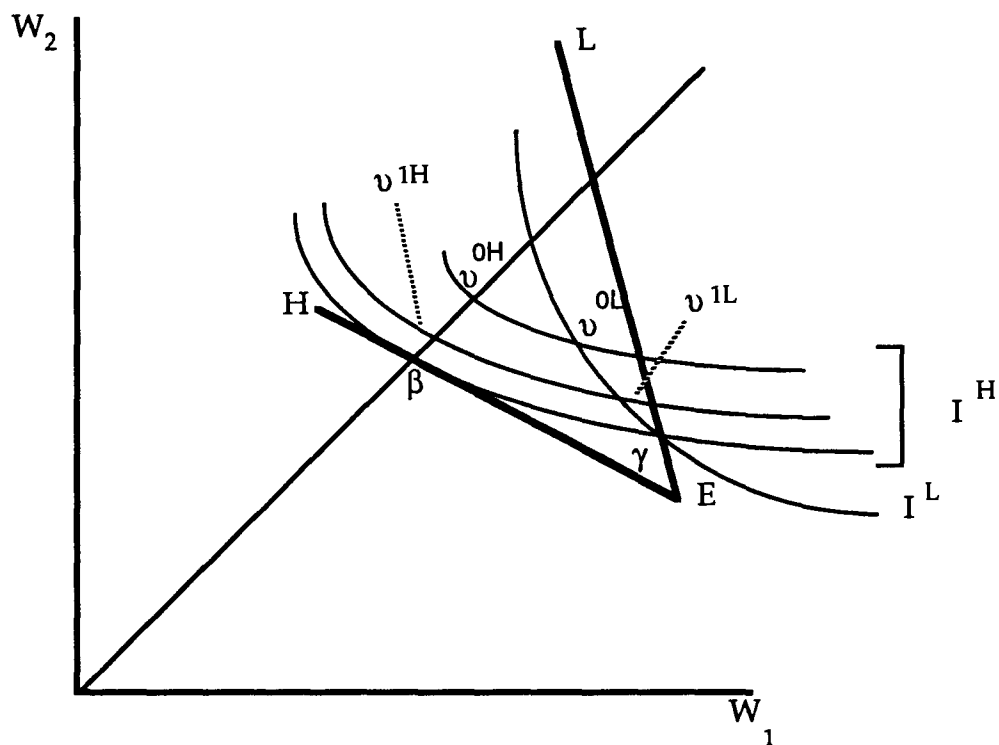


Figure 4. Second-period offers.

We are now ready to prove Proposition 1.

**Proof of Proposition 1:** Note first that, if all firms offer the contract  $\zeta$  in period 1, then the specified second-period strategies are in equilibrium by Lemmas 2, 4, and 6. The task is, therefore, to verify that there exists a strategy prescribing the stage-1 action  $(\zeta, \zeta)$  that is a best reply to itself under the given structure of beliefs. This is done in two steps: In step I, we show that no other contract giving rise to second-period beliefs such that  $\mu_U = \lambda$  (a *pooling deviation*) is included in a best reply to  $(\zeta, \zeta)$ . In step II, we show that no contract giving rise to second-period beliefs such that  $\mu_U \in \{0, 1\}$  (a *separating deviation*) is included in a best reply to  $(\zeta, \zeta)$  if and only if the condition in Proposition 1 is satisfied. Since this is a symmetric equilibrium, we maintain throughout the hypothesis that all firms but one offer the contract  $\zeta$  in the first period and we consider what is the one firm's best reply to this.

(I) We first check whether a strategy involving some pooling contract other

than  $\zeta$  is a best reply. Consider first a pooling contract  $\alpha$  such that  $V(p_L, \alpha) < V(p_L, \zeta)$ . This will not attract the consumer and is therefore not better than offering  $\zeta$ . Consider next some pooling contract  $\alpha \neq \zeta$  such that  $V(p_L, \alpha) \geq V(p_L, \zeta)$ . This contract, being a pooling one, will attract a low-risk consumer with probability  $\lambda$ ; if it would attract one consumer type only, then  $\zeta$  and  $\alpha$  would be a separating pair of contracts, a case that is treated below. From Lemma 6, we have that the firm will earn a second-period expected profit on each of these consumers equal to the term in curly brackets in (3.6), where the contract  $\zeta$  is defined. Clearly, the constraint in (3.6) is binding. And since  $V(p_L, \alpha) \geq V(p_L, \zeta)$ ,  $\alpha \neq \zeta$ ,  $U' > 0$ , and  $U'' < 0$ , it follows that total expected profit from offering  $\alpha$  is negative, which is worse than offering  $\zeta$ . This concludes the proof that  $\zeta$  is the best reply to itself among pooling contracts. It follows also from this that no other pooling contract has this property.

(II) Let  $k \in \{H, L\}$ . A separating contract in the set  $\mathbb{A}_k$ , defined in (3.7), is only attractive for a consumer of type  $k$ . There will be complete information among all firms about the type of a consumer buying a contract in this set, so that second-period expected profit is zero; thus, total expected profit from a deviating separating contract is negative, and such a deviation will not pay off, if and only if the separating contract is not in  $\mathbb{P}_k$ , defined in (3.8). QED.

By Proposition 1, we can replace the question, Does a pooling equilibrium exist? with the more specific one, Are there conditions simultaneously making  $\mathbb{A}_L$  disjoint from  $\mathbb{P}_L$  and  $\mathbb{A}_H$  disjoint from  $\mathbb{P}_H$ ? The answer is: Yes. But still, to prove elegant results at this point turns out to be a difficult problem. Thus, we resolve the existence question by the construction of a numerical example.

But first, we may increase our understanding of the model by analysing further the separating sets  $\mathbb{A}_H$  and  $\mathbb{A}_L$ . In particular, since we are interested in the profitability of contracts in these sets, we define, for each set, the most profitable contract in the set when sold to consumers of its type. I.e., we define, for  $k \in \{H, L\}$ , the contract  $\chi^k$ :

$$\chi^k \equiv \operatorname{argmax} \{ \pi(\alpha, b(k)) \mid \alpha \in \mathbb{A}_k \}, \quad (3.14)$$

where  $b(H) = 0$  and  $b(L) = 1$ . With this definition, we can restate the crucial condition (3.9) as:

$$\pi(\chi^k, b(k)) < 0,$$

for  $k \in \{H, L\}$ , with  $b(H) = 0$  and  $b(L) = 1$ . We have:

**Proposition 2:** Let  $\gamma$ ,  $v^r$ ,  $\zeta$ , and  $\chi^k$  be defined by (3.1), (3.4), (3.6), and (3.14), respectively.

(i) The contract  $\chi^H = (\chi_1^H, \chi_2^H)$  satisfies:

$$\chi_1^H + \chi_2^H = D, \text{ and} \quad (3.15a)$$

$$U(W - \chi_1^H) =$$

$$V(p_H, \zeta) + \delta[p_H U(W - v_1^{1H}) + (1 - p_H)U(W - v_1^{0H}) - U(W - p_H D)]. \quad (3.15b)$$

(ii) The contract  $\chi^L = (\chi_1^L, \chi_2^L)$  satisfies:

$$U(W - \chi_1^L) = U(W - \zeta_1) + \delta[U(W - \gamma_1) - U(W - p_L D)], \text{ and} \quad (3.16a)$$

$$U(W - D + \chi_2^L) =$$

$$U(W - D + \zeta_2) - \delta\left[\frac{1-p_H}{p_H}U(W - \gamma_1) + U(W - p_L D) - \frac{1}{p_H}U(W - p_H D)\right] \quad (3.16b)$$

**Proof:** Clearly, for each type, profit from selling to this type can be increased if the own-type restriction (3.7b) is not binding; therefore, it holds with equality for the profit-maximising contract.

(i) Since (3.7b) holds with equality, we get (3.15b) from inserting in (3.7b)  $k = H$ ,  $\alpha^H = \beta$ , and  $\beta_1 = p_H D$ , and from use of the fact that all the contracts  $\beta$ ,  $v^{0H}$ , and  $v^{1H}$  are full-insurance ones: It is easily seen from (2.1) that, if  $\alpha_1 + \alpha_2 = D$ , then  $V(p, \alpha) = U(W - \alpha_1)$ ,  $\forall p \in (0, 1)$ . By Lemma 1, profit increases towards full insurance; thus, we obtain (3.15a).<sup>30</sup>

(ii) Again, (3.7b) holds with equality. But now, (3.7a) constrains the profit-maximising contract; this follows from  $p_H > p_L$ , implying that low-risk indifference curves are the steeper, and from Lemma 1, implying that iso-profit lines are

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<sup>30</sup>In this case, condition (3.7a) restricts in a direction a firm would not want to go; therefore, it is not binding here.

steeper than indifference curves. Thus,  $\chi^L$  is the contract satisfying both (3.7a) and (3.7b) with equality, with  $k = L$  and  $h = H$ :

$$V(p_H, \chi^L) = V(p_H, \zeta) + \delta[V(p_H, \beta) - V(p_H, \eta)] \quad (3.17)$$

$$V(p_L, \chi^L) = V(p_L, \zeta) + \delta[V(p_L, \gamma) - V(p_L, \eta)] \quad (3.18)$$

In (3.18), we make use of the fact that, in the equilibrium of Proposition 1,  $V(p_L, v^{0L}) = V(p_L, v^{1L}) = V(p_L, \gamma)$ ; to see this, recall, in particular, condition (3.3d) above.

Now, we rewrite the two equations, using  $V(p_H, \beta) = U(W - p_H D)$  and  $V(p_H, \eta) = U(W - p_L D)$ . We also write the lefthand sides of (3.17) and (3.18) on their extensive forms, using (2.1), and rearrange to get:

$$U(W - D + \chi_2^L) = \frac{1}{p_H} [V(p_H, \zeta) + \delta[U(W - p_H D) - U(W - p_L D)] - (1 - p_H)U(W - \chi_1^L)] \quad (3.17')$$

$$U(W - D + \chi_2^L) = \frac{1}{p_L} [V(p_L, \zeta) + \delta[V(p_L, \gamma) - U(W - p_L D)] - (1 - p_L)U(W - \chi_1^L)] \quad (3.18')$$

Thus, the righthand side of (3.17') equals that of (3.18'). By using this, together with the extensive forms of  $V(p_H, \zeta)$ ,  $V(p_L, \zeta)$ , and  $V(p_L, \gamma)$  from (2.1), and rearranging, we get (3.16a). By substituting (3.16a) into (3.17') and rearranging, we get (3.16b). QED.

The content of Proposition 2 can be illustrated by going back to Figure 3, where we find  $\chi^H$  depicted as the full-insurance contract on the  $I_H^H$  indifference curve, while  $\chi^L$  is where the  $I_L^H$  and  $I_L^L$  indifference curves cross. We can now obtain, in Proposition 3 below, a quite specific condition on the emptiness of  $\mathbb{A}_H \cap \mathbb{P}_H$ :

**Proposition 3:** Let  $v^r$ ,  $\zeta$ , and  $\chi^H$  be defined by (3.4), (3.6), and (3.14), respectively. We have that  $\pi(\chi^H, 0) < 0$  or, equivalently,  $\mathbb{A}_H \cap \mathbb{P}_H = \emptyset$ , if and only if:

$$V(p_H, \zeta) + \delta[p_H U(W - v_1^{1H}) + (1 - p_H)U(W - v_1^{0H})] > (1 + \delta)U(W - p_H D) \quad (3.19)$$

**Proof:** By Proposition 2(i),  $\chi^H$  is a full-insurance contract. By (2.3),  $\pi(\beta, 0) = 0$ .



Thus,  $\pi(\chi^H, 0) < 0$  if and only if:  $U(W - \chi_1^H) = V(p_H, \chi^H) > V(p_H, \beta) = U(W - p_H D)$ . Insertion from (3.15b) now gives (3.19). QED.

Essentially, condition (3.19) says that a high-risk consumer is better off in our pooling equilibrium than with twice the fair and full-insurance  $\beta$  contract. It is, thus, a direct extension of equivalent condition in the one-period model:  $V(p_H, \zeta) > V(p_H, \beta) = U(W - p_H D)$ . However, from the lefthand-side of the condition, we see clearly the importance of firms' second-period lock-in possibilities. If firms could not exploit their old customers profitably, then we would have  $v^{0H} = v^{1H} = \beta$ , also when  $\lambda > \lambda^*$ . This would reduce condition (3.19) to the one we have in the one-period model. In Figure 3, this amounts to the two high-risk indifference curves  $I_{\beta}^H$  and  $I_{\zeta}^H$  coinciding. Consumer lock-in, however, is to the benefit of high-risks, at least in expectation, and strictly so when  $\lambda > \lambda^*$ . Therefore, the self-selection constraint with regard to high-risks is sharpened as we move from one to two periods ( $I_{\beta}^H$  is strictly above  $I_{\zeta}^H$  in Figure 3), thus making room for a pooling equilibrium.

Contrary to the high-risk case, there seems to be no way to verify the emptiness of  $\mathbb{A}_L \cap \mathbb{P}_L$  in general terms, without calculating  $\pi(\chi^L, 1)$ . Some insight may nevertheless be gained from relating the two contracts  $\zeta$  and  $\chi^L$  to each other:

**Proposition 4:** Let  $\zeta$  and  $\chi^L$  be defined by (3.6) and (3.14), respectively. We have:

- (i)  $\chi_1^L < \zeta_1$ ; and
- (ii)  $\chi_2^L < \zeta_2$ .

**Proof:** (i) Note, from (3.1b) and (3.2b), that both  $\gamma$  and  $\eta$  yield zero expected profit when sold to low-risks. Since  $\eta$  is a full-insurance contract and  $\gamma$  is not, and since  $p_L > 0$ , we have that:

$$\gamma_1 < \eta_1 = p_L D. \tag{3.20}$$

Now, since  $U' > 0$  and  $\delta > 0$ , the claim follows from (3.16a).

(ii) We rewrite (3.16b) as:

$$U(W - D + \chi_2^L) = U(W - D + \zeta_2) - \frac{\delta}{p_H}[U(W - p_L D) - U(W - p_H D)] \\ - \frac{\delta(1 - p_H)}{p_H}\{U(W - \gamma_1) - U(W - p_L D)\}$$

Here, the term in square brackets is positive, since  $p_H > p_L$  and  $U' > 0$ . Since, again,  $U' > 0$ , the term in curly brackets is also positive, by (3.20). Now, since  $U' > 0$ ,  $\delta > 0$ , and  $0 < p_H < 1$ , the claim follows. QED.

According to Proposition 4,  $\chi^L$  is southeast of  $\zeta$  in  $(W_1, W_2)$ -space, and strictly so; this is also seen in the illustration in Figure 3. This contrasts with the one-period model, in which the two contracts coincide:  $\chi^L = \zeta$ . Thus, when we move from one to two periods, the cheapest contract that separates low-risks moves down and to the right in  $(W_1, W_2)$ -space. Although not as clear-cut as the high-risk case, this should make it feasible that  $\pi(\chi^L, 1)$  be negative, thus making room for pooling.

Closer to general principles than this seems hard to get. We, therefore, complete this analysis with proving existence of a pooling equilibrium with lock-in by way of a numerical example. We have the following main result:

**Theorem:** *There is an open set of primitives of our model such that a symmetric equilibrium exists which is characterised by:*

(i) *in the first period, all firms offering one and the same contract and earning negative expected profits; and*

(ii) *in the second period, each firm earning a positive expected profit on its old customers.*

**Proof:** The proof is by way of a numerical example. Let:

$$W = 10, D = 9, p_H = 0.7, p_L = 0.05, \lambda = 0.97, \text{ and } \delta = 1.$$

Moreover, let the utility function be:

$$U(y) = \sqrt{y}.$$

Thus, consumers exhibit decreasing absolute risk aversion and constant relative

risk aversion.<sup>31</sup> In this case, we have the following Rothschild-Stiglitz contracts:

$$\beta = (6.3, 2.7), \text{ and } \gamma = (0.0499, 0.949).$$

If  $\mu_U = \lambda = 0.97$ , then:

$$\mu_0 = 0.990 \text{ and } \mu_1 = 0.698.$$

Moreover,  $K = 0.914$ , so that  $\lambda^* = 0.825$  and  $\lambda^{**} = 0.995$ . Thus, since  $\lambda^* < \lambda < \lambda^{**}$ , a firm earns positive average expected profit in period two on an old customer who did not have an accident in period one, if a pooling equilibrium exists. The optimum offer to such an old customer is  $v^0 = (v^{0H}, v^{0L})$ , where:

$$v^{0H} = (2.04, 6.96) \text{ and } v^{0L} = (0.481, 6.33).$$

The average expected profit thus earned on this old customer in period two equals 0.0975. On the other hand,  $v^1 = (\beta, \gamma)$ , yielding zero expected profit. The first-period contract in a pooling equilibrium is:

$$\zeta = (0.192, 3.87).$$

We now find that:

$$V(p_H, \zeta) + \delta[p_H U(W - v_1^{1H}) + (1 - p_H)U(W - v_1^{0H})] = 4.68.$$

Thus, recalling (3.19), there is no profitable contract separating high-risks from  $\zeta$ , since:

$$(1 + \delta)U(W - p_H D) = 3.85 < 4.68.$$

Moreover, we find that:

$$\chi^L = (-0.214, -0.737),$$

and the profit earned on  $\chi^L$  is negative:

$$\pi(\chi^L, 1) = -0.166 < 0.$$

Thus, both  $\mathbb{A}_H \cap \mathbb{P}_H$  and  $\mathbb{A}_L \cap \mathbb{P}_L$  are empty in this case, and the pooling equilibrium exists.

We now appeal to continuity considerations: The second-period equilibria described above are all continuous in the primitives, since the  $V$  and  $\pi$  functions are continuous. Therefore,  $\zeta$  is continuous in the primitives. Thus, for primi-

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<sup>31</sup>To see this, recall that the Arrow-Pratt measure of absolute risk aversion is defined as:  $\rho_A(y) = -U''(y)/U'(y)$ , while the measure of relative risk aversion is:  $\rho_R(y) = -yU''(y)/U'(y)$ ; see, e.g., Laffont (1989, Sec 2.1). With  $U(y) = \sqrt{y}$ , we have:  $U'(y) = 1/(2\sqrt{y}) > 0$ , and  $U''(y) = -1/(4y\sqrt{y}) < 0$ . Thus,  $\rho_A(y) = 1/(2y)$ , and  $\rho_A'(y) = -1/(2y^2) < 0$ . Furthermore,  $\rho_R(y) = 1/2$ , so that  $\rho_R'(y) = 0$ .

tives close to the ones used in this example, we will still have  $\pi(\chi^H, 0)$  and  $\pi(\chi^L, 1)$  negative and the existence of a pooling equilibrium. QED.

Our numerical example in the proof above makes use of rather reasonable parameter values. In particular, we have a low  $p_L$  and a high  $\lambda$ . Thus, the example covers what is perhaps the most prevalent situation – a market in which the huge majority of consumers constitute very minor risks but which is "infected" by a small fraction of truly bad risks.

With its high  $\lambda$ , the numerical example also shows that a pure-strategy equilibrium may exist in a multi-period insurance market in a case with so many low-risks that only a mixed-strategy equilibrium exists in the one-period model.<sup>32</sup>

The equilibrium first-period pooling contract  $\zeta$  may, in the present model, be of a perverse kind. To quote one case, put  $D = 5$ , instead of 9, in the above numerical example. Now, a pooling equilibrium exists with  $\zeta = (-0.0143, -0.0275)$ .<sup>33</sup> With both elements of  $\zeta$  negative, this constitutes a case where, in fact, consumers sell insurance to firms. But note that there is nothing in the definition of  $\zeta$ , in (3.6), that excludes such perverse contracts; this is true even if we restrict  $\zeta$  to being a zero-profit contract. In the one-period model, such a perverse pooling contract would be vulnerable to a separation by high-risks. In our two-period model, however, high-risks buy the perverse first-period contract because of the benefit they gain from the second-period lock-in. Thus, the present analysis highlights a new aspect of the Rothschild-Stiglitz model: *A competitive pooling contract may imply consumers selling insurance to firms.*

One may want to object to such perverse equilibrium contracts, with consumers selling insurance to firms, on grounds that they are not expected to be observed in reality. This point is well taken and it can easily be taken into account in our model by restricting the contract space  $\mathbb{A}$ . This, however, implies a restriction also on feasible separating deviations, making pooling easier to sustain. Al-

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<sup>32</sup>In the numerical example above,  $\lambda = 0.97$ , whereas, from (3.13), the critical value in the one-period model with the same parameters is 0.937.

<sup>33</sup>Some key figures for this example:  $\beta = (3.5, 1.5)$ ;  $\gamma = (0.0123, 0.234)$ ;  $v^0 = ((1.78, 3.22), (0.166, 2.58))$ ;  $v^1 = (\beta, \gamma)$ ;  $\chi^H = (3.21, 1.79)$ ,  $\pi(\chi^H, 1) = -0.289$ ;  $\chi^L = (-0.233, -2.869)$ ,  $\pi(\chi^L, 0) = -0.0669$ .

though strange results may occur, the present specification of the contract space is chosen to give separating deviations the best possible chance. And still, pooling equilibria with lock-in are proven to exist.

Another reason for not restricting the contract space is that any such restriction will be somewhat ad hoc: Where should one draw the line, and why? One restriction that cannot so easily be disputed, though, is that a consumer never be left with a negative wealth.<sup>34</sup> And cases are certainly conceivable where this restriction helps throwing separating deviations out of the contract space.<sup>35</sup>

#### 4. DISCUSSION

All of the previous Section was devoted to establishing the existence of one particularly interesting equilibrium of the game. The discussion in this Section centres particularly on the existence, in this model, of other equilibria.

##### 4.1 NON-POOLING EQUILIBRIA

**A separating equilibrium.** Let a *separating equilibrium* be one in which consumers are separated on the basis of their first-period purchases. Define the contract  $\varphi$  by:

$$(1 + \delta)U(W - p_H D) = (1 - p_H)U(W - \varphi_1) + p_H U(W - D + \varphi_2) + \delta U(W - p_L D) \quad (4.1a)$$

$$\varphi_1(1 - p_L) = \varphi_2 p_L \quad (4.1b)$$

Equation (4.1a) says that a high-risk consumer is indifferent between receiving the contract  $\beta$  in both periods and receiving  $\varphi$  in period one and  $\eta$  in period two. Equation (4.1b) says that  $\varphi$  yields zero expected profit when sold to a low-risk. We have:

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<sup>34</sup>Even this restriction would be disputable if we allow for borrowing and saving. But allowing this would raise a whole series of new questions.

<sup>35</sup>Try the numerical example above, with  $D$  even closer to  $W = 10$ .

**Proposition 5:** Let  $\beta$ ,  $\eta$ , and  $\varphi$  be defined by (2.3), (3.2), and (4.1), respectively.

Consider the following collection of strategies and beliefs:

*Strategies:*

Period 1:  $s^1 = (\beta, \varphi)^n$ ;

Period 2: (i) If  $\mu_U = 0$ , then  $(s^2, s_{l_1}^3) = (\beta, \beta)^n$ ;

(ii) if  $\mu_U = 1$ , then  $(s^2, s_{l_1}^3) = (\eta, \eta)^n$ .

*Beliefs:*

(i)  $\mu_U(s^1, a_1) = 1$ , if:

$$a_1 \in \{\alpha \in \mathbb{A} \mid V(p_H, \alpha) \leq (1 + \delta)U(W - p_H D) - \delta U(W - p_L D)\};$$

(ii)  $\mu_U(s^1, a_1) = 0$ , otherwise.

This collection of strategies and beliefs constitutes an equilibrium if and only if:

$$\lambda \leq \frac{1}{1 + \frac{p_L(1 - p_L)}{p_H - p_L} \left[ \frac{U'(W - p_H D)}{U'(W - \varphi_1)} - \frac{U'(W - p_H D)}{U'(W - D + \varphi_2)} \right]}. \quad (4.2)$$

**Proof:** The condition (4.2) is derived the same way as the conditions in Lemma 6 above (see the Appendix for the procedure). The second-period part of the equilibrium strategies follows from Lemma 2. In this equilibrium, second-period profits are zero, implying zero first-period profits. Thus, the offer to high-risks is  $\beta$ , unaffected by the existence of a second period. The first-period offer to low-risks is subject to the incentive constraint formalised in the condition defining the beliefs. And again due to competition, the first-period offer to low-risks must also satisfy a zero-profit constraint. Thus, low-risks are offered  $\varphi$  in period one. **QED.**

According to Proposition 5, there exists a separating equilibrium in pure strategies if  $\lambda$  is sufficiently low. At stage 1 of this equilibrium, firms offer the contract pair  $(\beta, \varphi)$ . Following this, high-risks choose  $\beta$  and low-risks choose  $\varphi$ , thus totally disclosing their respective types. Therefore, in period two, firms are certain of consumers' types and offer  $\beta$  to high-risks and  $\eta$  to low-risks.

Proposition 5 is a direct extension of the result of Rothschild and Stiglitz (1976): Put  $\delta = 0$  in the above, so that we have complete myopia, and we are back to their theorem, since, with  $\delta = 0$ ,  $\varphi$  coincides with  $\gamma$ . From this Proposition, it follows that a separating pure-strategy equilibrium exists for a smaller range of values of  $\lambda$  in the two-period model than in the one-period model, and that separation is even dearer for low-risk consumers when we move from one to two periods.

What about the existence of a separating equilibrium for high values of  $\lambda$ ? On one hand, it would be natural to try an extension to this two-period model of Dasgupta and Maskin's (1986) existence result for a separating mixed-strategy equilibrium in the one-period case. On the other hand, it is hard to establish, without having a complete characterization of the mixed-strategy equilibrium, whether and when such an equilibrium is vulnerable to pooling deviations. We leave this for future work.<sup>36</sup>

**Hybrid equilibria.** A hybrid equilibrium is one in which consumers are partially separated by type in period 1, e.g. because some contract is bought only by some of the high-risks and another contract is bought by the low-risks together with the rest of the high-risks. These equilibria are ruled out of the present analysis by Assumption (B). On the other hand, any such equilibrium would lead to a potential for information on customers to have a value, since the informed firm's posterior belief still would be more precise than the uninformed firms' ones.

## 4.2. BELIEFS

There are belief structures, other than the one specified in Proposition 1, that also would be able to support the same strategies, thereby creating other Perfect Bayesian equilibria with pooling. A general theme in the literature on signaling games

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<sup>36</sup>One remark is offered, though: A candidate separating equilibrium, like the one in Proposition 5, will apparently be one with no "pooling beliefs", i.e., an equilibrium where there is no  $a_1 \in \mathbb{A}$  such that  $\mu_U = \lambda$ . However, since pooling deviations are conceivable, it is necessary with a discussion of reasonable beliefs, a theme touched upon in Section 4.2 below.

is that the set of equilibria can be reduced by imposing reasonable restrictions on beliefs off the equilibrium path; see Kreps (1989) for a survey. Contrary to signalling games, however, the strategic players of the present game are on the uninformed side of the market. When sending a message to the uninformed parties, a consumer is bound to choose among the contracts offered. But something about this game can be said which is akin to equilibrium refinements of signalling games. First, define the following subset of the contract space  $\mathbb{A}$ :

$$\mathbb{A}_{HM} \equiv \{\alpha \in \mathbb{A} \mid V(p_L, \alpha) < V(p_L, \zeta) \text{ and} \quad (4.3a)$$

$$V(p_H, \alpha) + \delta U(W - p_H D) \geq V(p_H, \zeta) + \delta [p_H V(p_H, v^{1H}) + (1 - p_H) V(p_H, v^{0H})]\}, \quad (4.3b)$$

with  $v^r$  and  $\zeta$  defined in (3.4) and (3.6), respectively. Here, (4.3b) coincides with (3.7b), while (4.3a) is stricter than (3.7a), with  $k = H$  in (3.7). Therefore,  $\mathbb{A}_{HM} \subset \mathbb{A}_H$ . Moreover,  $\mathbb{A}_{HM}$  clearly makes a cut in the unprofitable end of  $\mathbb{A}_H$ , while it coincides in the profitable end:  $\operatorname{argmax} \{\pi(\alpha, 0) \mid \alpha \in \mathbb{A}_{HM}\} = \operatorname{argmax} \{\pi(\alpha, 0) \mid \alpha \in \mathbb{A}_H\} = \chi^H$ . In terms of Figure 3 above,  $\mathbb{A}_{HM}$  is that part of  $\mathbb{A}_H$  which is below  $I_{\zeta}^L$ . We have:

**Proposition 6:** Let the contract  $\zeta$  be defined by (3.6), and let the subsets  $\mathbb{A}_L$  and  $\mathbb{A}_{HM}$  of the contract space  $\mathbb{A}$  be defined by (3.7) and (4.3), respectively. In any pooling equilibrium where  $\zeta$  is offered in the first period,  $\mathbb{A}_{HM}$  is the minimum set giving rise to beliefs that a consumer is high-risk, and  $\mathbb{A}_L$  is the maximum set giving rise to beliefs that a consumer is low-risk.

Formally, let  $(\sigma, \mu)$  be a collection of strategies and beliefs, with  $\mu = (\mu_U, \mu_0, \mu_1)$ . If  $(\sigma, \mu)$  is a (Perfect Bayesian) pooling equilibrium with  $s^1 = (\zeta, \zeta)^n$ , then:

- (i)  $\{a_1 \in \mathbb{A} \mid \mu_U(s^1, a_1) = 0\} \supseteq \mathbb{A}_{HM}$ , and
- (ii)  $\{a_1 \in \mathbb{A} \mid \mu_U(s^1, a_1) = 1\} \subseteq \mathbb{A}_L$ .

**Proof:** (i) Suppose, to the contrary, that there exists some  $a_1 \in \mathbb{A}_{HM}$  such that



$\mu_U(s^1, a_1) \in \{\lambda, 1\}$ .<sup>37</sup> Thus, a high-risk consumer could choose this contract in period one (if it is offered in that period), instead of  $\zeta$ , without revealing his type; and this he may want to do, since  $V(p_H, a_1) \geq V(p_H, \zeta)$  by (4.3b). Suppose first that  $\mu_U(s^1, a_1) = 1$ . This belief is not consistent with a high-risk choosing  $a_1$ ; thus,  $\mu_U(s^1, a_1) \neq 1$  in a Perfect Bayesian Equilibrium (PBE). Suppose next that  $\mu_U(s^1, a_1) = \lambda$ . If this belief is consistent with consumer behaviour, then  $a_1$  is chosen by a low-risk consumer, too, instead of  $\zeta$ . But  $V(p_L, a_1) < V(p_L, \zeta)$  by (4.3a); thus,  $\mu_U(s^1, a_1) \neq \lambda$  in a PBE. This proves the claim.

(ii) Suppose  $a_1 \in \mathbb{A} \setminus \mathbb{A}_L$ . By (3.7), with  $k = L$ , there are two cases to consider:

(a)  $V(p_H, a_1) + \delta V(p_H, \eta) > V(p_H, \zeta) + \delta V(p_H, \beta)$ . In this case, it would pay a high-risk consumer to choose  $a_1$  in the first period (if it is offered in that period), instead of  $\zeta$ , if doing so makes firms believe with certainty that he is a low-risk and thus offers him  $\eta$  in the second period. Thus,  $\mu_U(s^1, a_1) \neq 1$  in a PBE.

(b)  $V(p_L, a_1) + \delta V(p_L, \eta) < V(p_L, \zeta) + \delta V(p_L, \gamma)$  (where we have made use of the condition (3.3d) on  $v^{0L}$  and  $v^{1L}$ ). In this case, it would *not* pay a low-risk consumer to choose  $a_1$  in the first period (if it is offered in that period), instead of  $\zeta$ , even if doing so makes firms believe with certainty that he is a low-risk. Thus,  $\mu_U(s^1, a_1) \neq 1$  in a PBE. QED.

With the help of Proposition 6, we may eliminate most pooling equilibria, other than the one of Proposition 1, with arguments close in spirit to equilibrium refinements of the signalling literature, particularly those based on "conscious signals"; see Kreps (1989, Sec. 6). Since they have the same profit-maximising contract, we could have substituted  $\mathbb{A}_{HM}$  for  $\mathbb{A}_H$  throughout Section 3 without changing results. We have chosen to concentrate on  $\mathbb{A}_H$  because of its symmetry with  $\mathbb{A}_L$ .

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<sup>37</sup>Here, we make use of Assumption (B) and its implication that  $\mu_U(s^1, a_1) \in \{0, \lambda, 1\}$ . The proof would also go through in the general case where  $\mu_U(s^1, a_1) \in [0, 1]$ .

### 4.3 SEQUENTIAL MOVES

Due to the sequential-move structure of period two of this game, there is a continuum of equilibria of a subgame following any first-period history. This is so because, in period two, an uninformed firm is able to increase the informed firm's second-period expected profit and decrease the second-period expected utility of the latter's old customers, or vice versa, by varying its second-period offer in a way that has no effect on its own expected profit.

Consider first cases where consumers' first-period purchases are separating, so that all firms know these consumers' types by period two. There is a continuum of equilibria at this point, in all of which the "uninformed" firms are outbid by the "informed" firm and are left with zero profits.<sup>38</sup> These equilibria are characterised by all "uninformed" firms offering contracts that are weakly dominated, in terms of the consumer's expected utility, by the contract  $\beta$  in the case of high-risks and by the contract  $\eta$  in the case of low-risks. Whenever the dominance is strict for all "uninformed" firms' offers, the last-moving "informed" firm earns a positive expected profit in period two. A rationale for nevertheless concentrating on the  $\beta$  and  $\eta$  equilibria is the following:

Suppose there is a small chance that the "informed" firm will not make an offer. However small the probability for this to happen is, the "uninformed" firms now have incentives to care about their outcome in this unlikely, but still possible, case. Competition leads the "uninformed" firms to offering  $\beta$ , respectively  $\eta$ .

Two remarks to this argument are warranted: First, this is similar to trembling-hand perfection (Selten, 1975); however, the existence of a trembling-hand perfect equilibrium is not secured in a game with infinite action spaces like the

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<sup>38</sup>The quotation marks are to emphasize that there is no real asymmetry of information among firms here. But the move sequence prevails, making the distinction necessary.

present one.<sup>39</sup> Second, the above argument requires that there be competition among "uninformed" firms, i.e.,  $n - 1 \geq 2$ , or  $n \geq 3$ .

Consider next cases where first-period contracts are pooling so that there is a difference in beliefs between uninformed firms and the informed firm. Again, we are confronted with a plethora of equilibria due to the fact that an uninformed firm is left with zero profit whenever it pays for the informed firm to outbid it.

Our choice of the contract pair  $(\beta, \gamma)$  as the uninformed firms' equilibrium offers is consistent with our choice in the case of symmetric information, argued for above. The two contracts  $\beta$  and  $\gamma$  also serve, as mentioned above, as benchmarks, constituting the pure-strategy equilibrium in the one-period case, and it might be argued informally that they have virtues as a focal point in the sense of Schelling (1960). However, the most important reason for not worrying too much about the many equilibria of this subgame is that all of them have properties parallel to those outlined in Lemma 6 for the chosen equilibrium, with the informed firm earning a positive expected second-period profit if  $\lambda$  is sufficiently high. One should, therefore, believe that also Proposition 1 has parallels. Note, however, that the argument put forward above in favour of  $\beta$  and  $\eta$  is not applicable to  $\beta$  and  $\gamma$  here.<sup>40</sup>

It should be noted that other authors record similar problems of multiplicity of equilibria in models with sequential moves. See, in particular, the multi-period model of the labour market by Waldman (1990); the language in his note 9 is very close to the above one. See also the consumer lock-in paper by Banerjee and Summers (1987) and the "no money-losing offers" restriction discussed by

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<sup>39</sup>Strictly speaking, trembling-hand perfection is not even defined for games with infinite action spaces. However, the weaker requirement of admissibility is. But even though it is weaker, it does not secure existence; see, e.g., Harrington (1989).

<sup>40</sup>For high values of  $\lambda$ , uninformed firms will play a mixed-strategy equilibrium if the informed firm stays out with probability one; when the latter probability is strictly between zero and one, it is not clear what the equilibrium strategy of an uninformed firm is like, since the possibility of the informed firm making its move influences the support of any mixed-strategy equilibrium. There may also be other effects. Some effects of restrictions on the support of a mixed-strategy equilibrium are analysed by Fershtman and Fishman (1991) in the context of a search market with price dispersion.

Hart and Tirole (1990, Appendix B) in a sequential Bertrand duopoly.<sup>41</sup> An exact discussion of how it affects the set of equilibrium outcomes to take the indifference seriously in a model with sequential moves is provided by Dewatripont (1987) in a spatial context.

Note also the following paradox: In Section 2, we argue strongly for detailed tie-breaking rules to resolve consumer indifference, since otherwise we would have problems with equilibrium existence. Here, we observe that our way of resolving indifference among consumers causes problems for equilibrium uniqueness. It thus seems difficult to obtain existence and uniqueness at the same time.

## 5. CONCLUSION

With its prediction that today's insurance purchase by a consumer determines a long-term relation with one insurer, the present model establishes a source of consumer lock-in, characteristic of markets with asymmetric information, which is different from other sources of consumer lock-in, such as transaction costs, learning costs, search costs, or artificially imposed costs.<sup>42</sup> This chapter introduces the concept of *informational consumer lock-in*.<sup>43</sup>

Would the results of this chapter be applicable to other markets than insurance? An essential assumption here is that a consumer's accident history is not verifiable to other parties than his previous insurer. In the insurance context, this makes sense (in the absence of perfectly functioning public records), since there is nothing to show before you in case you have gone through without any accident – the good outcome is a "nothing". In the *credit market*, on the other hand, the good outcome is a "something"; if an entrepreneur has had success

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<sup>41</sup>One may even wish to consult Appendix B to Chapter 4 below.

<sup>42</sup>The recent literature on consumer lock-in in oligopolies counts a large number of contributions. The one which is closest in spirit to the present model is Klemperer (1987). See Farrell (1987) for a review of the issues involved. Chapter 4 below is a contribution to this literature.

<sup>43</sup>A similar effect has already been noted by Kunreuther and Pauly (1985), but under assumptions that seem much too restrictive. The models of Greenbaum, Kanatas, and Venezia (1989) and Sharpe (1990), producing the same effect, are without any consumer-firm asymmetry of information.

with his latest project, this success is verifiable (auditable sales figures, etc.). Thus, it does not seem appropriate to predict informational lock-in in a credit market.<sup>44</sup> In *labour markets*, the verifiability of an employee's work will depend on the type of work. While the good outcome of an executive's efforts is substantially a "something" (just like an entrepreneur's success), it is more like a "nothing" in the case of a factory worker.

Something can be said about the need for further work. First of all, it is necessary to develop a multi-period insurance market without any informational asymmetry among firms, i.e., where accidents are observed by all insurers. This way, one would be able to analyse the occurrence of pooling in a context where consumer lock-in is not an issue. Preferably, such a model should have simultaneous moves in period two.

Second, note that, in the particular equilibrium we have discussed here, firms keep *all* their old customers. It can be argued that it is more sensible with a situation where firms keep old customers with a good accident history, earning positive expected profits on them, while terminating their relations to old customers with a bad accident history. It is conjectured, based on work in progress, that outcomes like this are obtained with alternative move structures, i.e., in cases where the informed firm is not the last to move.

Third, it would be of interest to see how things change as we increase the number of periods. It is conjectured that such an increase will strengthen the presence of a pooling equilibrium and of consumer lock-in. It is, however, not clear how strong the effect will be.

Finally, it may be argued that consumers incur regular switching costs also in the insurance market. How will this affect the result of this chapter? Clearly, the consumer lock-in gets aggravated in a direct way. An unsettled question, however, is how the chances of getting a pooling equilibrium are affected by the

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<sup>44</sup>The view expressed here seems to contrast with views expressed elsewhere to the effect that a bank does have better information on its own customers than its rivals have; see, e.g., Fama (1985). However, in the credit market, there exists information that one more naturally think of as private to the bank than one does with the borrower's success, such as information on the customer's deposit account and on his efforts in making his project a success. Thus, there may be informational asymmetries among banks, but it is of a kind which is perhaps best modelled by differences in monitoring costs. One attempt at analysing lock-in in a credit market based on private information on customers' deposit accounts is in Vale (1991). Vale argues that such lock-in possibilities may serve as a basis for economies of scope in banking: A bank's deposit activities lower the costs of its credit activities.

introduction of such costs.

In view of these remarks, the work reported in this chapter should be considered a first step in the analysis of multi-period competition in insurance markets.

## APPENDIX

In this Appendix, we complete the proof of Lemma 6. The contract pair  $(\beta, \gamma)$  is a pure-strategy best response for the informed firm to the uninformed firms' offers of  $(\beta, \gamma)$  if and only if it is unable to increase its profits by choosing otherwise. If the offer  $\omega = (\omega^H, \omega^L) = ((\omega_1^H, \omega_2^H), (\omega_1^L, \omega_2^L))$  is a best response, it fulfils three requirements, by Lemma 5. First, from (3.11a):

$$\omega_1^H = D - \omega_2^H, \quad (\text{A.1})$$

implying:

$$V(p_H, \omega^H) = U(W - \omega_1^H). \quad (\text{A.2})$$

Second, from (3.11b), using (A.2) and (2.1):

$$U(W - \omega_1^H) = p_H U(W - D + \omega_2^L) + (1 - p_H) U(W - \omega_1^L), \quad (\text{A.3})$$

Third, from (3.11c), with  $V_L = V(p_L, \gamma)$  from (3.10):

$$p_L U(W - D + \omega_2^L) + (1 - p_L) U(W - \omega_1^L) = p_L U(W - D + \gamma_2) + (1 - p_L) U(W - \gamma_1) \quad (\text{A.4})$$

Average expected profit per contract is given by:

$$\pi^r = (1 - \mu_r)[\omega_1^H(1 - p_H) - \omega_2^H p_H] + \mu_r[\omega_1^L(1 - p_L) - \omega_2^L p_L] \quad (\text{A.5})$$

where  $r \in \{0, 1\}$ . We want to find the effect on the profit (A.5) of changing the contract offers subject to constraints (A.1)–(A.4). The procedure is the same as one used by Stiglitz (1977). Note first that, by total differentiation in (A.4):

$$\frac{d\omega_1^L}{d\omega_2^L} = \frac{p_L U'(W - D + \omega_2^L)}{(1 - p_L) U'(W - \omega_1^L)}. \quad (\text{A.6})$$

Note also that, from (A.1):

$$\frac{d\omega_1^H}{d\omega_2^H} = -1. \quad (\text{A.7})$$

Next, by total differentiation in (A.3), using (A.6) to substitute for  $(d\omega_1^L/d\omega_2^L)$  and (A.7) to substitute for  $(d\omega_1^H/d\omega_2^H)$ :

$$\frac{d\omega_2^L}{d\omega_2^H} = \frac{(1 - p_L)U'(W - \omega_1^H)}{(p_H - p_L)U'(W - D + \omega_2^L)} \quad (\text{A.8})$$

Total differentiation of the profit function (A.5) gives, using first (A.7) and then (A.6) and (A.8):

$$\begin{aligned} \frac{d\pi^r}{d\omega_2^H} &= -(1 - \mu_r) + \mu_r \left[ (1 - p_L) \frac{d\omega_1^L}{d\omega_2^L} - p_L \right] \frac{d\omega_2^L}{d\omega_2^H} \\ &= -(1 - \mu_r) + \frac{\mu_r p_L (1 - p_L)}{p_H - p_L} \left[ \frac{U'(W - \omega_1^H)}{U'(W - \omega_1^L)} - \frac{U'(W - \omega_1^H)}{U'(W - D + \omega_2^L)} \right]. \end{aligned} \quad (\text{A.9})$$

In order to derive conclusions from local properties of the profit function, we must check that it is globally well-behaved, in particular that  $d^2\pi/(d\omega_2^H)^2 < 0$  for all relevant values of  $\omega_2^H$ . We have that:

$$\begin{aligned} \frac{d^2\pi^r}{(d\omega_2^H)^2} &= \frac{\mu_r p_L (1 - p_L)}{p_H - p_L} \{ U''(W - \omega_1^H) \left[ \frac{1}{U'(W - \omega_1^L)} - \frac{1}{U'(W - D + \omega_2^L)} \right] \right. \\ &\quad \left. + \frac{p_L (U'(W - \omega_1^H))^2 U''(W - \omega_1^L)}{(p_H - p_L) (U'(W - \omega_1^L))^3} + \frac{(1 - p_L) (U'(W - \omega_1^H))^2 U''(W - D + \omega_2^L)}{(p_H - p_L) (U'(W - D + \omega_2^L))^3} \right\}. \end{aligned}$$

Since  $U' > 0$  and  $U'' < 0$ ,  $d^2\pi/(d\omega_2^H)^2 < 0$  if the term in square brackets is non-negative. And, since  $U'' < 0$ , this holds if and only if  $\omega_1^L + \omega_2^L \leq D$ , which is one of the inequalities that define the contract space  $\mathbb{A}$ . Thus, the profit function is well-behaved with regard to the contract parameters. Therefore, we can state that  $(\beta, \gamma)$  is a best response if and only if marginal profit (A.9) is non-positive at  $(\omega^H = \beta, \omega^L = \gamma)$ . The conditions in (3.4) are obtained by substituting  $(\beta, \gamma)$  for  $(\omega^H, \omega^L)$  in (A.9); using that  $V(p_H, \beta) = U(W - p_H D)$  by (2.3); and substituting for  $\mu_r$  from (2.4), first  $\mu_0$  and then  $\mu_1$ .

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## CHAPTER THREE:

RENEGOTIATION IN AN INSURANCE MARKET<sup>1</sup>

*(Co-author: Geir B. Asheim)*

**Abstract:** Renegotiation is the act of replacing an existing contract with new ones that are beneficial for the contracting parties in light of new information. In the present work, we extend the theory of markets with asymmetric information to the case where a firm is allowed to renegotiate with consumers the terms of a contract once it is signed. In a simplified version of the Rothschild-Stiglitz model of an insurance market, we characterize equilibrium when renegotiation is allowed and compare it with equilibrium in the case without renegotiation. We find that, with renegotiation, the equilibrium is never fully separating, and it is pooling when the fraction of low-risk consumers is high. Also, for a high fraction of low-risk consumers, both types of consumers are strictly better off compared to the case without renegotiation.

## 1. INTRODUCTION

Renegotiation is the act of replacing an existing contract with new ones that are beneficial for the contracting parties in light of new information. The present analysis is about such renegotiation and it has a double motivation.

On one hand, some authors suggest that the optimum renegotiation-proof contract signed between a principal and an agent tends to reveal less information about the agent than the optimum contract when renegotiation is not feasible; see, e.g., Dewatripont (1989), Hart and Tirole (1988), Laffont and Tirole (1990), and the survey by Dewatripont and Maskin (1990a). Here, we extend this kind of analysis to a simple situation in which several competing principals are present, so that a renegotiation-proof contract must also be an equilibrium contract on a

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<sup>1</sup>The first version of this chapter was completed in November 1990. I am, of course, indebted to Geir Asheim for his generous collaboration on this chapter. Thanks to Georg Nöldeke for discussions; and to participants at seminars in Oslo, Bergen, Helsinki, and Tilburg for comments.

competitive market with asymmetric information.<sup>2</sup> In particular, we find that a result valid in the case when the principal is a monopolist, *viz.* that allowing for renegotiation cannot lead to a Pareto improvement, does not generally hold in the case with competing principals.

On the other hand, several authors have analysed competition in markets with asymmetric information, *i.e.* where consumers are better informed than firms on some payoff-relevant parameter. A typical example is the Rothschild-Stiglitz study of the insurance market. In that model, firms offer insurance and the asymmetry of information concerns consumers' accident probabilities. Rothschild and Stiglitz (1976) show that, in equilibrium, high-risk consumers are offered fair (*i.e.* zero-profit), full-insurance contracts while low-risk consumers are offered fair contracts with partial insurance, the latter being due to a binding truth-telling constraint on the high-risks. When allowing for mixed strategies, the basic feature of the equilibrium of separating consumers by type holds in all circumstances, as shown by Dasgupta and Maskin (1986). In this chapter, we pose the following question: How is this analysis affected by allowing a consumer and a firm to renegotiate after a contract is signed? The firm can infer information on the consumer based on the signing of the contract. For example, if consumers are completely separated on the basis of the signing of contracts, any partial-insurance contract between a firm and a low-risk consumer can be profitably renegotiated to a full-insurance one. However, this implies in turn that any high-risk consumer who anticipates such renegotiation prefers to disguise by purchasing the partial-insurance contract.<sup>3</sup> We find, as one would expect, that allowing for renegotiation reduces the possibilities for separation.

Accepting the idea of renegotiation, there is still the question of how the notion should be incorporated into a formal model. We will argue that renegotiation should be allowed to go on indefinitely; otherwise, there would be a last

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<sup>2</sup>The idea of competition with renegotiation is also present elsewhere in the literature. The analysis of Caillaud, Jullien, and Picard (1990) differs from ours in that they study competing agencies, *i.e.*, principals that compete through the actions of their respective agents. The analyses of Mori (1989), Dewatripont and Maskin (1990b), and Dionne and Doherty (1991) differ from ours in that they study two-period models where renegotiation is not allowed to occur immediately after the signing of contract but only after one period has elapsed.

<sup>3</sup>One readable discussion of this, cast within an insurance monopoly, is found in the recent textbook by Kreps (1990), on pp. 677-679.

stage of renegotiation at which no further renegotiation could take place, the existence of which would greatly influence the play of the whole game. We model this indefinite renegotiation atemporally. Thus contrasting with the recent literature on non-cooperative, sequential bargaining, our approach relates closer with cooperative game theory. Invoking Occam's Razor, our view is that renegotiation should be introduced into the insurance market model in the simplest possible manner. In particular, duplicating our results as an equilibrium of an extensive-form game will be left for future research. The main tool of our analysis of renegotiation is the *theory of social situations* developed by Greenberg (1990). The application of this theory to a problem of asymmetric information seems to be a novelty.

Like Rothschild and Stiglitz (1976) and most of the papers that followed, we maintain the assumption that a consumer is unable to sign an insurance contract with more than one firm. To us, this is a reasonable assumption.<sup>4</sup> And it makes it possible to dichotomize competition and renegotiation. In our analysis, we first characterise contracts that are renegotiation-proof, independent of the market structure. Thereafter, we study competition with firms restricted to offering such renegotiation-proof contracts.<sup>5</sup>

This chapter has a predecessor in Hillas (1987), who presents major steps towards the present analysis. However, his analysis of renegotiation in a competitive insurance market shows that the Rothschild-Stiglitz model, as it stands, is intractable for this kind of study. The problem is that no renegotiation-proof contracts exist; see Hillas (1987, Sec. 5.5). Here we resolve the problem by simplifying the original model in one respect that Hillas did not try, *viz.* with regard to consumer preferences. In the present chapter, consumers' indifference curves are piecewise linear. On one hand, this linearization of the model ensures the exist-

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<sup>4</sup> Such behaviour can at any rate be imposed by including clauses in the contracts to the effect that an insurer has no obligations to cover a damage unless it is the single coverer. The firm may enforce such a clause simply by requiring verification in the form of a receipt in original in turn for paying out liabilities.

<sup>5</sup>Note the distinction between renegotiation and *recontracting*. The latter term covers instances where a consumer, after signing a contract with one firm, is allowed to sign additional contracts with other firms. Clearly, if later insurers are informed about the contracts that are already signed, problems arise about truthful revelation similar to those, outlined above, arising under renegotiation. A recent and important analysis of recontracting in this sense is by Beaudry and Poitevin (1990). There are earlier, related analyses by Jaynes (1978) and Hellwig (1988).

ence of renegotiation-proof contracts. On the other hand, preferences are no longer founded on expected utility theory. At present, we feel that the benefit of the former outweighs the costs of the latter.

Our specification of consumer preferences resembles the one suggested in an unpublished paper by Rosenthal and Weiss (1982).<sup>6</sup> As they show, this kind of specification also makes it possible to obtain a complete characterisation of a mixed-strategy equilibrium for parameter values for which no pure-strategy equilibrium exists.

In Section 2, we introduce some notation and terminology. In Section 3, we solve for a mixed-strategy equilibrium in the case without renegotiation, which serves as a bench-mark case for the subsequent analysis. Next, we allow for renegotiation by imposing the restriction that firms can only offer renegotiation-proof contracts. Therefore, in Section 4, we define the concept of renegotiation and characterise the set of renegotiation-proof contracts. Then, in Section 5, we characterise the market equilibrium when contracts are restricted to be renegotiation-proof. Section 6 sums up the conclusions of the analysis by making a comparison with the bench-mark case.

## 2. PRELIMINARIES

Consider a continuum of individuals uniformly distributed on the unit line  $[0, 1]$  with density 1. Each individual faces two possible states in the future: In state 1, no accident is experienced and his terminal wealth is  $W_1$ . In state 2, an accident is experienced and his terminal wealth is  $W_2$ . Without insurance,  $(W_1, W_2) = (W, W - D)$ , where  $W$  is positive and finite and  $D \in (0, W]$  is the cost of a damage. A typical insurance contract is  $c = (c_1, c_2)$ , where  $c_1$  denotes the premium and  $c_2$  denotes the benefit, net of premium, in the case of an accident. With the insurance contract  $c$ , the individual's state-contingent wealth becomes  $(W_1, W_2) = (W - c_1, W - D + c_2)$ .

Throughout this chapter, we will focus on the polar case where  $D = W$ , so

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<sup>6</sup>The unpublished work of Rosenthal and Weiss (1982) consists of two parts, one on the insurance market and one on the labour market. A version of the latter was published as Rosenthal and Weiss (1984).

that, without insurance, we write  $(W_1, W_2) = (D, 0)$ , and with insurance, we write  $(W_1, W_2) = (D - c_1, c_2)$ .

The individuals are identical except for the probability of an accident. There are two types of individuals: The low-risk (L) type has probability  $p^L$ , the high-risk (H) type probability  $p^H$ ,  $0 < p^L < p^H < 1$ . No individual can by any action affect his accident probability. The fraction of the L type in the population is  $\lambda \in (0, 1)$ . The ratio of low-risks to high-risks is defined as:

$$r \equiv \frac{\lambda}{1 - \lambda}$$

An individual of type  $K$ ,  $K \in \{H, L\}$ , has preferences over  $W_1$  and  $W_2$  representable by a homothetic utility function  $U^K(W_1, W_2)$  with piecewise linear indifference curves. In a representation homogeneous of degree 1,<sup>7</sup> indifference curves are linear with slope  $-(1 - p^K)/(a + p^K)$  for  $W_1 \geq W_2 \geq bW_1$ , and linear with slope  $-(a + 1 - p^K)/p^K$  for  $bW_2 \leq W_1 \leq W_2$ . Here,  $a > 0$  is a risk aversion parameter: The higher  $a$  is, the more risk averse are individuals. For later purposes, we define a critical level of risk aversion,  $a^*$ :

$$a^* \equiv \frac{p^H - p^L}{1 - p^H}$$

For  $W_1 < bW_2$ , indifference curves are parallel with the  $W_2$ -axis. For  $W_2 < bW_1$ , indifference curves are parallel with the  $W_1$ -axis. A pair of indifference curves, one for each consumer type, is depicted in Figure 1. The parameter  $b$  is assumed to be strictly positive but not so high that its value directly influences the equilibria of the model; in particular, we assume:

$$0 < b \leq \frac{a(1 - p^H)(1 - p^L)}{(a + p^H)(1 - p^L) - p^L(1 - p^H)(a + 1)}$$

A *feasible* contract is one where  $c_1, c_2 \geq 0$  and  $c_1 + c_2 \leq D$ . We introduce two different partitions of the set of feasible contracts. First, a feasible contract is a *full-insurance* one if  $c_1 + c_2 = D$  (i.e.,  $W_1 = D - c_1 = c_2 = W_2$ ); otherwise, it is a *partial-*

<sup>7</sup>One such representation is the following. For  $bW_1 \leq W_2 \leq W_1/b$ ,

$$\begin{aligned} U^K(W_1, W_2) = \max \quad & \xi_0 + \xi_1 + \xi_2 \\ \text{subject to:} \quad & \xi_0 + \frac{a+1}{1-p^K} \xi_1 \leq W_1 \\ & \xi_0 + \frac{a+1}{p^K} \xi_2 \leq W_2 \end{aligned}$$

For  $W_2 < bW_1$ ,  $U^K(W_1, W_2) = U^K(W_2/b, W_2)$ . For  $W_2 > W_1/b$ ,  $U^K(W_1, W_2) = U^K(W_1, W_1/b)$ .



insurance contract. Second, a feasible contract is *adequate* if  $c_1 + c_2/b \geq D$  (i.e.,  $bW_1 = b(D - c_1) \leq c_2 = W_2$ ); otherwise, it is *inadequate*.

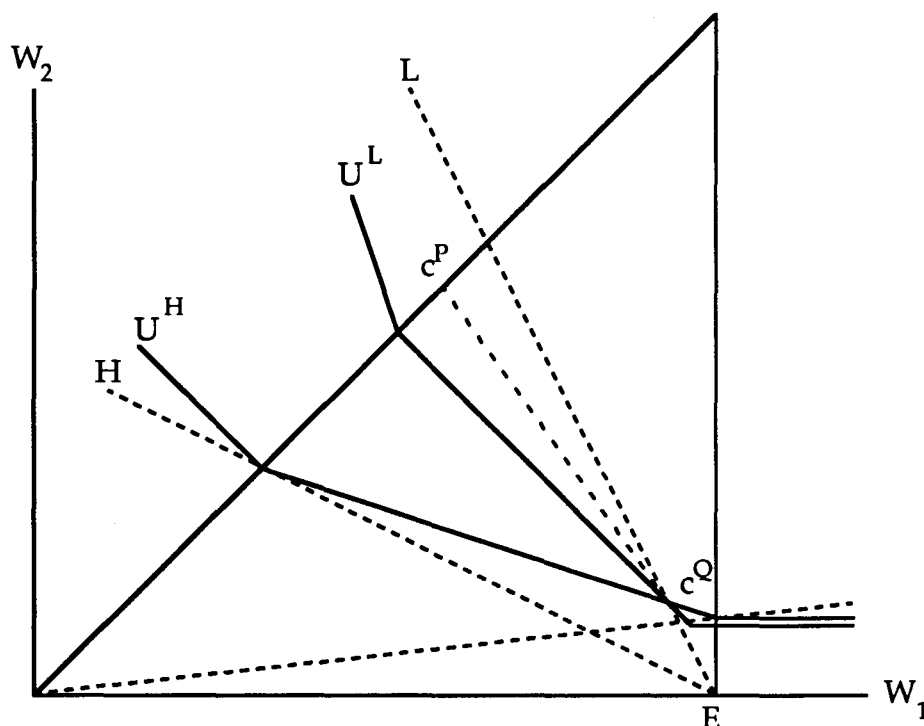


Figure 1

Readers may feel that our restriction to the case  $D = W$  is strong and that the introduction of the parameter  $b$  is somewhat arbitrary; a comment may therefore be warranted. These assumptions together ensure the convenient feature that consumers always (weakly) prefer any feasible contract to being self-insured. The former assumption can, however, be relaxed somewhat without changing the subsequent analysis.

For  $K \in \{H, L\}$ , define  $u^K(c)$  as the expected utility for consumers of type  $K$  from buying the contract  $c$ :

$$u^K(c) \equiv U^K(D - c_1, c_2).$$

Thus, consumers have piecewise linear indifference curves in contract space, or  $(c_1, c_2)$ -space, with slopes as follows: When  $c$  is adequate, they have slopes  $(1 - p^K)/(a + p^K)$ ; when  $c$  is inadequate, they are parallel with the  $c_1$ -axis.

Note that, since  $p^H > p^L$ , indifference curves have the so-called single-crossing property; in particular, on the set of adequate contracts, we have:

$$\frac{1 - p^H}{a + p^H} < \frac{1 - p^L}{a + p^L}.$$

On the supply side of the market, there are two firms.<sup>8</sup> Each of them is a risk-neutral profit-maximiser with financial resources sufficient to supply any number of insurance contracts. The expected profit from selling contract  $c$  to consumers of type  $K$  is:

$$\pi^K(c) = (1 - p^K)c_1 - p^K c_2.$$

Thus, firms' isoprofit curves in  $(c_1, c_2)$ -space, when selling to type- $K$  consumers,  $K \in \{H, L\}$ , have a slope such that the feasible contract preferred by type- $K$  consumers at a fixed expected profit provides full insurance:

$$\frac{1 - p^K}{p^K} > \frac{1 - p^K}{a + p^K}.$$

In Figure 1, EH and EL are iso-profit – in fact, zero-profit – curves, drawn in  $(W_1, W_2)$ -space, for contracts sold to high-risks and low-risks, respectively.

In order to analyse renegotiation in this model, we need some more notation and terminology. The terminology employed here concurs with Greenberg (1990). Let, for  $K \in \{H, L\}$ ,  $\mu^K$  measure consumers of type  $K$ , with  $0 \leq \mu^H \leq 1 - \lambda$  and  $0 \leq \mu^L \leq \lambda$ . A *type profile*  $\mu$  is a pair  $\mu \equiv (\mu^H, \mu^L)$ . We define a *population*  $F$  as a combination of a type profile and a fall-back contract:  $F \equiv (\mu, c)$ . The terminology reflects the fact that  $F$  contains all relevant information on a set of consumers. The fall-back contract  $c$  of the population  $F$  is the contract these consumers already are signed up with; throughout, we assume that  $c$  is a feasible contract. A *position*  $G$  for a firm is a pair  $G \equiv (F, \pi)$  of a population  $F$  and profits  $\pi$  earned on customers outside  $F$ . Let  $\Gamma$  be the set of positions.

A *feasible outcome* for a firm in position  $G = (F, \pi) = (\mu, c, \pi)$  is a pair of (sub)populations  $(F_h, F_\ell) \equiv ((\mu_h, c_h), (\mu_\ell, c_\ell))$  such that:

(i) (Exhaustiveness)  $F_h$  and  $F_\ell$  exhaust  $F$ ; i.e., their type profiles sum to the type profile of  $F$ :

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<sup>8</sup>Extending the analysis to the general case of  $n$  firms would imply (i) a minor change in the equilibrium strategies of Proposition 2 below for the case without renegotiation, and (ii) a more cumbersome notation than the one in Appendix 2 below in order to achieve the desired game formulation for the case with renegotiation. Since no new insights would be gained this way, we limit our analysis to the two-firm case.

$$\mu_h + \mu_\ell \equiv (\mu_h^H, \mu_h^L) + (\mu_\ell^H, \mu_\ell^L) = (\mu^H, \mu^L) \equiv \mu.$$

(ii) (No Dominance)  $c_h$  does not dominate  $c_\ell$  or vice versa; i.e.,  $c_h$  and  $c_\ell$  may be ordered such that:

$$u^H(c_h) \geq u^H(c_\ell), \text{ and}$$

$$u^L(c_\ell) \geq u^L(c_h).$$

(iii) (Incentive Compatibility) High-risk consumers choose  $c_\ell$ , and low-risk consumers  $c_h$ , only if indifferent between  $c_h$  and  $c_\ell$ , i.e.:

$$\mu_h^H = \mu^H, \text{ if } u^H(c_h) > u^H(c_\ell), \text{ and}$$

$$\mu_\ell^L = \mu^L, \text{ if } u^L(c_\ell) > u^L(c_h).$$

(iv) (Individual Rationality) The contracts of  $F_h$  and  $F_\ell$  are at least as good for consumers as the fall-back contract of  $G$ :

$$u^H(c_h) \geq u^H(c), \text{ and}$$

$$u^L(c_\ell) \geq u^L(c).$$

A feasible outcome  $(F_h, F_\ell) = ((\mu_h, c_h), (\mu_\ell, c_\ell))$  is *degenerate* if:  $\mu_h = 0$ , or  $\mu_\ell = 0$ , or  $c_h = c_\ell$ . A degenerate outcome is simply written  $F_d \equiv (\mu_d, c_d)$ , where  $\mu_d = \mu_h + \mu_\ell$ , and where  $c_d = c_h$  if  $\mu_h > 0$ , and  $c_d = c_\ell$  if  $\mu_\ell > 0$ .

The set of all feasible outcomes in a position  $G$  is denoted  $X(G)$ . The profit a firm earns from a feasible outcome  $(F_h, F_\ell)$  in position  $G = (F, \pi)$  is defined as:

$$\Pi(G)(F_h, F_\ell) \equiv \sum_{k=h, \ell} \sum_{K=H, L} \mu_k^K \pi^K(c_k) + \pi$$

This concludes the preliminaries. We now turn to the case without renegotiation.

### 3. AN INSURANCE MARKET WITHOUT RENEGOTIATION

Throughout, market competition is modelled by firms offering consumers a pair of utility levels, as opposed to a pair of contracts, which is standard in previous literature. We do this because it simplifies the analysis of renegotiation. Below, we show that this has no effect on the analysis of the case without renegotiation. Let  $(\underline{u}^H, \underline{u}^L)$  be a pair of utility levels. Provided that a firm's offer to the one type does not dominate the firm's offer to the other type, it follows from the single-

crossing property of indifference curves that there is a unique adequate contract at which high-risks have utility  $\underline{u}^H$  and low-risks have utility  $\underline{u}^L$ . This implies that we unambiguously can represent an offer of  $(\underline{u}^H, \underline{u}^L)$  by the adequate contract  $c$  such that  $u^H(c) = \underline{u}^H$  and  $u^L(c) = \underline{u}^L$ .

Let the two offers from firms 1 and 2 be  $c$  and  $c'$ . In the case without renegotiation, we impose the following tie-breaking rule: If some consumers are indifferent between  $c$  and  $c'$ , then they split evenly among the two firms. Now, let  $G^0 \equiv (\mu, 0, 0)$  be the initial position of a firm;  $c = 0 = (0, 0)$  – self-insurance – is the initial fall-back "contract", and  $\pi = 0$ , since the firm has no customers outside the population  $(\mu, 0)$ . The tie-breaking rule above implies that there are nine possible initial positions, because the initial type profile has  $\mu^H \in \{0, (1 - \lambda)/2, (1 - \lambda)\}$  and  $\mu^L \in \{0, \lambda/2, \lambda\}$ , depending on whether the firm's offer dominates, matches, or is dominated by the other firm's offer, in terms of the expected utility for each consumer type.

**Lemma 1:** Given an adequate offer  $c$  (i.e.,  $c_1 \geq 0$ ,  $c_1 + c_2 \leq D$ ,  $c_1 + c_2/b \geq D$ ), a firm maximises its profit over  $X(G^0)$ ,  $G^0 = (\mu, 0, 0)$ , subject to the constraint that  $(u^H(c_h), u^L(c_\ell)) = (u^H(c), u^L(c))$ , by choosing the outcome  $(F_h, F_\ell) = ((\mu_h, c_h), (\mu_\ell, c_\ell))$ , with:  $\mu_h = (\mu^H, 0)$ ;  $\mu_\ell = (0, \mu^L)$ ;  $c_\ell = c$ ;  $c_{h1} + c_{h2} = D$ ; and  $u^H(c_h) = u^H(c)$ .

**Proof:** See Appendix 1.

Lemma 1 says that, with an offer  $c$ , the firm maximises its profit by giving the contract  $c$  to all the low-risks and the full-insurance contract on the same high-risk indifference curve as  $c$ , to all the high-risks. This result makes clear that our framework corresponds exactly to the more standard framework of Rosenthal and Weiss (1982) and others. It also highlights the fact that, without renegotiation, there will always be separation unless all consumers are offered full insurance.

By means of the tie-breaking rule and Lemma 1 above, a mapping from a pair of offers from the firms to a feasible outcome in an initial position for each firm has been constructed. By the definition of  $\Pi$  in Section 2, we have, for each

$\lambda$ , determined firms' profits as a function of their offers in the case where renegotiation is not allowed, and thus, for each  $\lambda$ , constructed a simultaneous-move game with each firm's strategy set being the set of adequate offers. Proposition 1 and 2 characterise, for each  $\lambda$ , a Nash equilibrium of this game. First, define:

$$r^* \equiv \frac{p^H - p^L}{a(1 - p^L)} = \frac{a^*(1 - p^H)}{a(1 - p^L)}$$

The merit of this definition becomes clear immediately. We have:

**Proposition 1:** There exists a unique symmetric equilibrium in pure strategies where each firm's offer,  $c = c^Q$ , is given by:

$$(c_1^Q, c_2^Q) \equiv \left( \frac{p^L(1 - p^H)}{(1 - p^L)(1 + r^*)}D, \frac{1 - p^H}{1 + r^*}D \right),$$

if and only if  $r \leq r^*$ .

**Proof:** See Appendix 1.

This Proposition parallels the result of Rothschild and Stiglitz (1976): When the fraction  $\lambda$  of low-risks is low, there exists a separating equilibrium in pure strategies. But when  $\lambda$  gets high enough, it is profitable to deviate from the strategy of Proposition 1. This happens when there are so many low-risks relative to high-risks that it pays to subsidize the latter in order to earn a profit on the former. The offer  $c^Q$  of Proposition 1 is depicted in Figure 1.

When no pure-strategy equilibrium exists, i.e., when  $r > r^*$ , there exists a symmetric equilibrium in mixed strategies, as shown by Dasgupta and Maskin (1986). The characterisation of this equilibrium is provided in Proposition 2. Let the contract  $c^P$  be given by:

$$(c_1^P, c_2^P) \equiv ([ (1 - \lambda)p^H + \lambda p^L ]D, [ (1 - \lambda)(1 - p^H) + \lambda(1 - p^L) ]D);$$

i.e.,  $c^P$  is the full-insurance contract earning zero profit when sold to all consumers of both types. In Figure 1,  $c^P$  is depicted for a case where  $r > r^*$ . Given the linearity of this model, the locus of offers earning zero profit is the set of convex combinations of  $c^P$  and  $c^Q$ , i.e., the straight dotted line between  $c^P$  and  $c^Q$  in Figure 1. Let  $PQ$  denote this locus:

$$PQ \equiv \{c \mid c = \alpha c^P + (1 - \alpha)c^Q, 0 \leq \alpha \leq 1\}.$$

This set is the support of each firm's equilibrium mixed strategy. We describe this strategy by the cumulative distribution function  $\Phi(c_2)$  of the liability part  $c_2$  of the contract as it traces out values between  $c_2^Q$  and  $c_2^P$ .

**Proposition 2:** If  $r > r^*$ , then there exists a symmetric equilibrium in mixed strategies where each firm makes its offer on the support PQ according to the cumulative distribution function  $\Phi(c_2)$  given by:

$$\Phi(c_2) = \begin{cases} 0, & \text{for } c_2 < c_2^Q; \\ \left( \frac{c_2 - c_2^Q}{c_2^P - c_2^Q} \right)^{\frac{r}{r^*} - 1} & \text{for } c_2^Q \leq c_2 < c_2^P; \\ 1, & \text{for } c_2 \geq c_2^P. \end{cases}$$

We are not in a position to claim that this is the unique symmetric equilibrium, nor are we able to come up with another one. With the caveat that uniqueness is still an open question for  $r > r^*$ , we will use the above equilibrium, described in Propositions 1 and 2 for various parameter values, for comparative studies in Section 6. Note that Proposition 2 holds for the general case of  $n$  firms if  $\Phi(\cdot)$  is replaced by  $\Phi^{n-1}(\cdot)$ .

The proof of Proposition 2 goes as follows: Any offer in the support of the equilibrium strategy earns zero profit. We thus have to show that a firm, when playing against  $\Phi(\cdot)$ , will earn a non-positive profit by choosing any offer outside this support. To this end, let  $x_2$  be a reparameterization of  $c_2$  as follows:

$$x_2 = \frac{c_2 - c_2^Q}{c_2^P - c_2^Q},$$

and let

$$\Phi^*(x_2) \equiv \Phi(c_2) = x_2^{\frac{r}{r^*} - 1}.$$

Instead of offering a contract  $c \in PQ$ , the firm considers offering the contract  $c^T$  with the properties that it attracts  $\Phi^*(x_2 + t)$  of the low-risks and  $\Phi^*(x_2)$  of the high-risks;  $c$  and  $c^T$  are thus on the same high-risk indifference curve. Let the profit earned on the low-risks from the offer  $c^T$  be denoted  $H(t)$ ; the (negative) profit earned on high-risks is irrelevant, since the fraction of high-risks attracted

by this offer is independent of  $t$ . Thus,  $H(t) \equiv \lambda\Phi^*(x_2 + t)\pi^L(c^T)$ . We have:

**Lemma 2:** The profit earned on low-risks from a deviating offer as described above can be written:

$$H(t) = \lambda(1 - \lambda)D(p^H - p^L)(x_2 + t)^{\frac{r}{r^*} - 1} \left[ x_2 - \left( \frac{r}{r^*} - 1 \right) t \right]$$

**Proof:** See Appendix 1.

With Lemma 2 at hand, we can prove Proposition 2. The point is that, if any deviation off PQ yields less profit than the contract in PQ having the same high-risk expected utility, then any deviation off PQ yields negative profit. By differentiation in the expression in Lemma 2, it is easily verified that  $H'(0) = 0$ ,  $H'(t) > 0$  for  $t \in (-x_2, 0)$ , and  $H'(t) < 0$  for  $t \in (0, 1 - x_2)$ , if  $r > r^*$ . Thus, it does not pay to deviate from the equilibrium strategy of Proposition 2, so the Proposition is proved.

#### 4. RENEGOTIATION-PROOF CONTRACTS

The purpose of this section is to define renegotiation-proofness and characterize the set of renegotiation-proof contracts for an insurance firm. The intuition is summarized as follows: A contract is renegotiation-proof if and only if the firm cannot profit by offering the consumers who were signed up for this contract a new pair of renegotiation-proof contracts which are at least as good for the consumers and strictly better for the firm.

Readers may notice that our definition of renegotiation-proofness is in terms of *stability* in the sense of von Neumann and Morgenstern (1947). This notion is the unifying solution concept in Greenberg's (1990) *theory of social situations*. The definitions and subsequent analysis will be cast within the framework of this theory.

In Section 2, the concepts of a position and a feasible outcome in a position have already been introduced. A position characterizes a population by its type profile and fall-back utility levels, and gives the outside profit opportunities of

the firm. Formally,  $G = (F, \pi)$  is a position,  $F = (\mu, c)$  is a population,  $\pi$  is outside profit,  $\mu = (\mu^H, \mu^L)$  is a type profile, and  $c$  is a contract defining fall-back utility levels. A feasible outcome in a position is a pair of contracts with associated type profiles satisfying incentive and individual rationality constraints. As explained in Section 2, this implies that a feasible outcome is a pair of subpopulations  $(F_h, F_\ell)$ .

Renegotiation involves offering one of these subpopulations  $F_k = (\mu_k, c_k)$ ,  $k \in \{h, \ell\}$ , a new pair of contracts which are at least as good as its existing contract  $c$ . We assume that the firm has the power to distribute the existing type-profile among the new pair of contracts only constrained by incentive constraints. Renegotiation is profitable for the firm if its profit increases by replacing the subpopulation's existing contract (with associated type profile) with the new pair of contracts (with associated type profiles).

Formally, renegotiation is captured by the *inducement* of a new position given the outcome in the existing position, and the selection of a new feasible outcome in the new position. What can be induced is given by the *inducement correspondence*,  $\gamma$ , which determines what new positions can be induced from the outcome in the existing position. The following inducement correspondence states that the firm can enter into renegotiation with one of its subpopulations. Let  $G = (F, \pi)$ . If  $(F_h, F_\ell) \in X(G)$  is nondegenerate, then:

$$\gamma(G, (F_h, F_\ell)) = \{G_h, G_\ell\} = \{(F_h, \pi_h), (F_\ell, \pi_\ell)\},$$

where  $\pi_h = \sum_{K=H,L} \mu_\ell^K \pi^K(c_\ell) + \pi$ , and  $\pi_\ell = \sum_{K=H,L} \mu_h^K \pi^K(c_h) + \pi$ ; if  $(F_h, F_\ell) \in X(G)$  is degenerate, then we write  $(F_h, F_\ell) = F_d$ , and:

$$\gamma(G, F_d) = \{G_d\} = \{(F_d, \pi)\}.$$

By this notation, it follows that, if  $(F_h, F_\ell) \in X(G)$ ,  $G' \in \gamma(G, (F_h, F_\ell))$ , and  $(F_{h'}, F_{\ell'}) \in X(G')$ , then such renegotiation is profitable for the firm if and only if  $\Pi(G')(F_{h'}, F_{\ell'}) > \Pi(G)(F_h, F_\ell)$ . Recall that  $\Gamma$  denotes the set of positions; we call the pair  $(\gamma, \Gamma)$  a *situation*.

Given the outcome  $(F_h, F_\ell)$  in position  $G$ , the firm would like to induce a position  $G'$  if the outcome that results from the renegotiation,  $(F_{h'}, F_{\ell'}) \in X(G')$ ,



yields the firm a higher profit, i.e.  $\Pi(G')(F_h', F_\ell') > \Pi(G)(F_h, F_\ell)$ . Renegotiation will, however, only lead to an outcome from which no further renegotiation will take place, since otherwise, we can replace the original outcome with one that includes the subsequent renegotiation.<sup>9</sup> It is thus natural to argue that an outcome is renegotiation-proof if it is *not* profitable for the firm to renegotiate to any renegotiation-proof outcome in a position that the firm can induce, while an outcome is not renegotiation-proof if it *is* profitable for the firm to renegotiate to some renegotiation-proof outcome in a position that the firm can induce.

This line of thought leads to the following: For each  $G$ , let  $\sigma(G) \subseteq X(G)$  denote the set of renegotiation-proof outcomes in the position  $G$ . We call  $\sigma$  a standard of behaviour (SB). Require (in accordance with the argument above) that such an SB is *optimistic internally stable* in the sense that

IS  $(F_h, F_\ell) \in \sigma(G) \Rightarrow$  there do not exist  $G' \in \gamma(G, (F_h, F_\ell))$  and  $(F_h', F_\ell') \in \sigma(G')$  such that:  $\Pi(G')(F_h', F_\ell') > \Pi(G)(F_h, F_\ell)$ ,

and that such an SB is *optimistic externally stable* in the sense that

ES  $(F_h, F_\ell) \in X(G) \setminus \sigma(G) \Rightarrow$  there exist  $G' \in \gamma(G, (F_h, F_\ell))$  and  $(F_h', F_\ell') \in \sigma(G')$  such that:  $\Pi(G')(F_h', F_\ell') > \Pi(G)(F_h, F_\ell)$ .

We say that  $\sigma$  is an *optimistic stable standard of behaviour* (OSSB) if it satisfies both IS and ES.<sup>10</sup> Note that the internal stability explains why an outcome is renegotiation-proof by the property that it cannot be profitably renegotiated to another renegotiation-proof outcome, while the external stability explains why an outcome is not renegotiation-proof by the property that it *can* be renegotiated

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<sup>9</sup>Our model is an atemporal one. But as the language in the text suggests, an interpretation is that it allows for indefinite renegotiation.

<sup>10</sup>Greenberg (1990) distinguishes between optimistic and conservative SSBs. In our context, the SSB is optimistic if the firm is able to choose among the elements of  $\sigma$ . In a conservative SSB, the firm would expect the worst element of  $\sigma$  to be realized. The OSSB seems the more suitable concept here and we do not explore the properties of conservative SSBs.

to another renegotiation-proof outcome.

Defining renegotiation-proofness by the imposition of internal and external stability may appear circular. Still, the problem has a hierarchical structure which allows us to uniquely determine the set of renegotiation-proof outcomes. In fact, the main result of this section establishes the existence and uniqueness of an OSSB  $\sigma$  for the situation  $(\gamma, \Gamma)$ . Furthermore, this OSSB is fully characterized. Define:

$$r^{**} \equiv \frac{p^H - p^L - a(1 - p^H)}{a(1 - p^L)} = r^* - \frac{1 - p^H}{1 - p^L} = \left(\frac{a^*}{a} - 1\right) \frac{1 - p^H}{1 - p^L}.$$

Notice that  $r^{**} > 0$  if and only if  $a < a^*$ .

**Proposition 3:** There exists a unique OSSB  $\sigma$  for the situation  $(\gamma, \Gamma)$ . This OSSB is characterized as follows:  $(F_h, F_\ell) \in \sigma(G)$  if and only if  $(F_h, F_\ell) \in X(G)$  and one of the following two conditions are satisfied:

- (i)  $(F_h, F_\ell)$  is a degenerate full-insurance outcome.
- (ii)  $(F_h, F_\ell) = ((\mu_h, c_h), (\mu_\ell, c_\ell))$  satisfies:  $\mu_h = (\mu_h^H, 0)$ ,  $\mu_\ell = (\mu_\ell^H, r^{**}\mu_\ell^H)$ ,  $c_{h1} + c_{h2} = D$ ,  $c_{\ell 1} + c_{\ell 2}/b \geq D$ , and  $u^H(c_h) = u^H(c_\ell)$ , with  $r^{**} > 0$ .

In order to establish this result, the following Lemma is useful. Informally, it states that, in a nondegenerate renegotiation-proof outcome, both the offered contracts are renegotiation-proof.

**Lemma 3:** Consider a nondegenerate outcome  $(F_h, F_\ell) \in X(G)$ , with  $\gamma(G, (F_h, F_\ell)) = \{G_h, G_\ell\}$ . Then,  $(F_h, F_\ell) \in \sigma(G)$  if and only if (the degenerate outcomes)  $F_h$  and  $F_\ell$  satisfy  $F_h \in \sigma(G_h)$  and  $F_\ell \in \sigma(G_\ell)$ .

**Proof:** Suppose  $F_k \notin \sigma(G_k)$ ,  $k = h$  or  $\ell$ . By ES, there exist  $G' \in \gamma(G_k, F_k)$  and  $(F_h', F_\ell') \in \sigma(G')$  such that  $\Pi(G')(F_h', F_\ell') > \Pi(G_k)(F_k)$ . However, since  $G' = G_k \in \gamma(G, (F_h, F_\ell))$  and  $\Pi(G_k)(F_k) = \Pi(G)(F_h, F_\ell)$ , it follows by IS that  $(F_h, F_\ell) \notin \sigma(G)$ . Conversely, if  $(F_h, F_\ell) \notin \sigma(G)$ , there exist, by ES,  $G' \in \gamma(G, (F_h, F_\ell))$ , and  $(F_h', F_\ell') \in \sigma(G')$  such that

$\Pi(G')(F_h', F_\ell') > \Pi(G)(F_h, F_\ell)$ . Since  $G' = G_k \in \gamma(G_k, F_k)$  and  $\Pi(G)(F_h, F_\ell) = \Pi(G_k)(F_k)$ ,  $k = h$  or  $\ell$ , it follows by IS that  $F_k \notin \sigma(G_k)$ ,  $k = h$  or  $\ell$ . QED.

The proof of Proposition 3 is based on the observation that a degenerate full insurance outcome cannot be profitably renegotiated to any feasible (and, hence, not to any renegotiation-proof) outcome. By ES, such an outcome is renegotiation-proof. This allows for an inductive argument on a hierarchical structure.

**Proof of Proposition 3:** Let  $G = (F, \pi)$  and let  $\sigma$  be any OSSB for the situation  $(\gamma, \Gamma)$ .

(a) If  $F_d = (\mu_d, c_d) \in X(G)$  is a degenerate full insurance outcome (i.e.,  $c_{d1} + c_{d2} = D$ ), then, by ES,  $F_d \in \sigma(G)$ .

(b) Let  $F_d = (\mu_d, c_d) \in X(G)$  be a degenerate outcome with  $\mu_d^L > \max\{0, r^{**}\mu_d^H\}$  and  $c_{d1} + c_{d2} < D$ . Consider  $F_d' = (\mu_d', c_d')$  with  $\mu_d' = \mu_d$ ,  $u^L(c_d') = u^L(c_d)$ , and  $c_{d1}' + c_{d2}' = D$ . By (a), we have that  $G_d = (F_d, \pi) \in \gamma(G, F_d)$ ,  $F_d' \in \sigma(G_d)$ , and  $\Pi(G_d)(F_d') > \Pi(G)(F_d)$ . By IS,  $F_d \notin \sigma(G)$ .

(c') Let  $F_d = (\mu_d, c_d) \in X(G)$  be a degenerate outcome with  $\mu_d^L = r^{**}\mu_d^H > 0$ ,  $c_{d1} + c_{d2} < D$ , and  $c_{d1} + c_{d2}/b \geq D$ . The firm's profit can only be increased by separating out some high-risks. By (b), the rest must then move along the low-risk indifference curve and be given full insurance. But this means that no high-risks would be willing to be separated out. By ES,  $F_d \in \sigma(G)$ .

(c'') Let  $F_d = (\mu_d, c_d) \in X(G)$  be a degenerate outcome with  $\mu_d^L = r^{**}\mu_d^H > 0$  and  $c_{d1} + c_{d2}/b < D$ . Consider  $F_d' = (\mu_d', c_d')$ , with  $\mu_d' = \mu_d$ ,  $u^L(c_d') = u^L(c_d)$ , and  $c_{d1}' + c_{d2}'/b = D$ . By (c'), we have that  $G_d = (F_d, \pi_d) \in \gamma(G, F_d)$ ,  $F_d' \in \sigma(G_d)$ , and  $\Pi(G)(F_d') > \Pi(G)(F_d)$ . By IS,  $F_d \notin \sigma(G)$ .

(d') Let  $F_d = (\mu_d, c_d) \in X(G)$  be a degenerate outcome with  $0 \leq \mu_d^L < r^{**}\mu_d^H$ ,  $c_{d1} + c_{d2} < D$ , and  $c_{d1} + c_{d2}/b \geq D$ . Consider  $(F_h, F_\ell) = ((\mu_h, c_h), (\mu_\ell, c_\ell))$  with  $\mu_h = (\mu_h^H, 0)$ ,  $\mu_\ell = (\mu_\ell^H, r^{**}\mu_\ell^H)$ ,  $\mu_h + \mu_\ell = \mu_d$ ,  $c_{h1} + c_{h2} = D$ ,  $u^H(c_h) = u^H(c_\ell)$ , and  $c_\ell = c_d$ . By Lemma

3, (a), and (c'), we have that  $G_d = (F_d, \pi) \in \gamma(G, F_d)$ ,  $(F_h, F_\ell) \in \sigma(G_d)$ , and  $\Pi(G_d)(F_h, F_\ell) > \Pi(G)(F_d)$ . By IS,  $F_d \notin \sigma(G)$ .

(d'') Let  $F_d = (\mu_d, c_d) \in X(G)$  be a degenerate outcome with  $0 \leq \mu_d^L < r^{**} \mu_d^H$  and  $c_{d1} + c_{d2}/b < D$ . Consider  $(F_h, F_\ell) = ((\mu_h, c_h), (\mu_\ell, c_\ell))$  with  $\mu_h = (\mu_h^H, 0)$ ,  $\mu_\ell = (\mu_\ell^H, r^{**} \mu_\ell^H)$ ,  $\mu_h + \mu_\ell = \mu_d$ ,  $c_{h1} + c_{h2} = D$ ,  $c_{\ell1} + c_{\ell2}/b = D$ ,  $u^H(c_h) = u^H(c_\ell)$ , and  $u^L(c_\ell) = u^L(c_d)$ . By Lemma 3, (a), and (c'), we have that  $G_d = (F_d, \pi) \in \gamma(G, F_d)$ ,  $(F_h, F_\ell) \in \sigma(G_d)$ , and  $\Pi(G_d)(F_h, F_\ell) > \Pi(G)(F_d)$ . By IS,  $F_d \notin \sigma(G)$ .

By Lemma 3 and (a)–(d) above,  $(F_h, F_\ell) \in \sigma(G)$  for any  $(F_h, F_\ell) \in X(G)$  satisfying condition (ii) of Proposition 3. Conversely, it can be checked that there is no nondegenerate outcome  $(F_h, F_\ell) \in X(G)$ , with  $\gamma(G, (F_h, F_\ell)) = \{G_h, G_\ell\}$  and where  $F_h \in \sigma(G_h)$  and  $F_\ell \in \sigma(G_\ell)$  (according to (a)–(d)), that does not satisfy condition (ii) of Proposition 3. Hence,  $(F_h, F_\ell) \notin \sigma(G)$  for any nondegenerate  $(F_h, F_\ell) \in X(G)$  not satisfying condition (ii) of Proposition 3.

This establishes that, if there exists an OSSB for the situation  $(\gamma, \Gamma)$ , it is characterized by (i) and (ii) of Proposition 3. To establish that the SB characterized by (i) and (ii) of Proposition 3 satisfies IS and ES, invoke Lemma 3 and repeat the inductive argument above. **QED.**

Note that a contract offering partial but adequate insurance is renegotiation-proof if the type profile is such that the firm's iso-profit curves are parallel to the low-risk indifference curves. One can argue that renegotiation would take place even in this case since, if some high-risks accepted a full-insurance contract yielding them a slightly higher utility level, then the remaining insurees would get their original contract renegotiated along the low-risk indifference curve, yielding the rest of the high-risk an even higher utility level. Hence, such renegotiation brings all the high-risks onto higher utility levels, keeps the low-risks on their original utility level, and can be constructed so that the firm profits. The reason why such renegotiation would not take place in the present framework is that it involves a pair of contracts where the one dominates the other. Hence, having any insurees choose the poorer contract would violate incentive con-

straints.

Our definition of renegotiation-proofness is valid also in the case of nonlinear indifference curves. Establishing the existence of an OSSB in this case would, however, be a complex task, partly for reasons discussed by Hillas (1987, Secs. 5.4-5.6).

## 5. AN INSURANCE MARKET WITH RENEGOTIATION

Again, the two firms are modelled as offering the consumers a pair of utility levels  $(\underline{u}^H, \underline{u}^L)$  which can be unambiguously represented by the adequate contract  $c$ , with  $u^H(c) = \underline{u}^H$  and  $u^L(c) = \underline{u}^L$ . However, in this section we will impose that each firm's outcome from the competition in the insurance market be renegotiation-proof; else, renegotiation between the firm and its insurees would take place.

Formally, we need to construct, for each  $r$ , a simultaneous-move game with each firm's strategy set being the set of adequate offers. This is achieved by constructing a mapping from a pair of offers from the firms to a renegotiation-proof outcome in an initial position for each firm. Such a mapping is presented in Appendix 2. Here, it is more instructive to prove the following main result.

**Proposition 4:** For  $r \neq r^{**}$ , any (pure-strategy) Nash equilibrium of the game defined in Appendix 2 yields a unique outcome for each firm.

(a) If  $0 < r < r^{**}$ , then each firm's (nondegenerate) outcome is given by:

$$(F_H, F_L) = \left( \left( \frac{1}{2} \left( \frac{\lambda}{r} - \frac{\lambda}{r^{**}} \right), 0 \right), c_H \right), \left( \frac{1}{2} \left( \frac{\lambda}{r^{**}}, \lambda \right), c^R \right),$$

where  $c_H$  satisfies:  $c_{H1} + c_{H2} = D$  and  $\pi^H(c_H) = 0$ , and  $c^R$  satisfies:  $u^H(c_H) = u^H(c^R)$  and  $(\lambda/r^{**})\pi^H(c^R) + \lambda\pi^L(c^R) = 0$ , i.e.:

$$c^R = (c_1^R, c_2^R) \equiv \left( \frac{(a + p^L)(1 - p^H)}{(1 - p^L)(1 + r^{**})} D, \frac{1 - p^H}{1 + r^{**}} D \right).$$

(b) If  $r > \max \{0, r^{**}\}$ , then each firm's (degenerate) outcome is given by:

$$F_d = \left( \frac{1}{2} \left( \frac{\lambda}{r}, \lambda \right), c^P \right),$$

where  $c^P$  is the full-insurance zero-profit pooling contract defined in Section 3.

**Remark:** The contract  $c^R$  is depicted in Figure 2. This contract has the characteristic that it is situated on the zero-profit curve for a contract that is sold to a set of consumers with a type profile such that  $\mu^L/\mu^H = r^{**}$ . By construction, this zero-profit curve is parallel with low-risk indifference curves for adequate contracts, i.e., its slope is  $-(1 - p^L)/(a + p^L)$ .

**Proof:** (a) For  $r < r^{**}$ , it is sufficient to show the existence of a unique pure-strategy Nash equilibrium in which each firm offers  $c^R$ , since symmetry and Proposition 3 then imply that each firm's nondegenerate outcome is as stated in Proposition 4, yielding each firm zero profit.

**Existence.** Assume firm 2 offers  $c^R$ . If firm 1 also offers  $c^R$ , then it obtains zero profit. We need to establish that firm 1 cannot obtain a positive profit by any alternative feasible offer. Consider Figure 2.

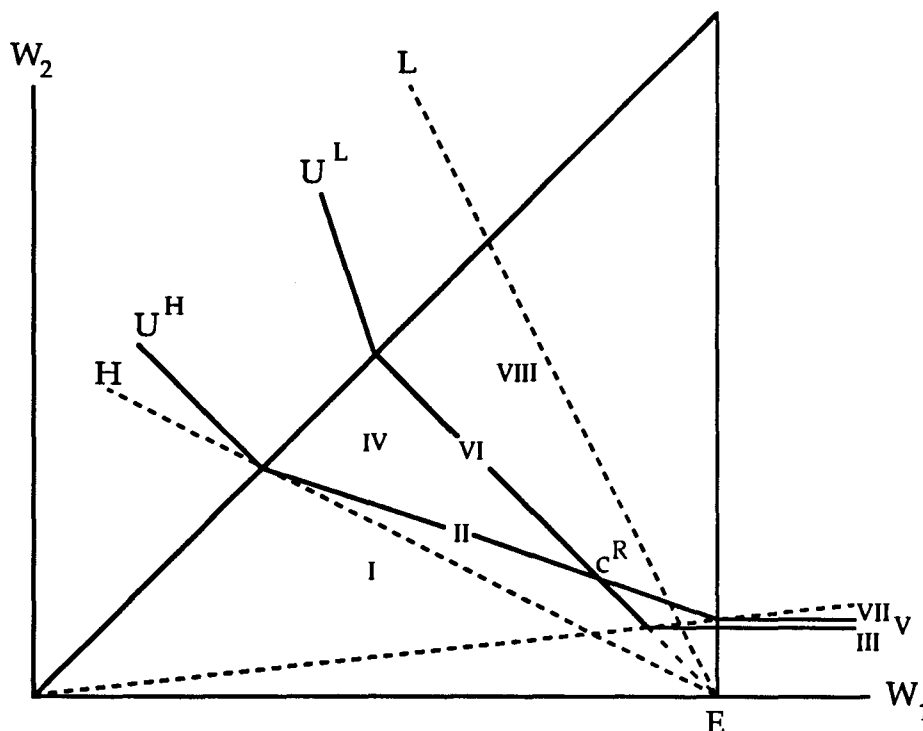


Figure 2

If firm 1 offers  $c$  in area I such that  $u^L(c) < u^L(c^R)$  and  $u^H(c) < u^H(c^R)$ , then firm 1 attracts no customers and obtains zero profit.

If firm 1 offers  $c$  in area II such that  $u^L(c) < u^L(c^R)$  and  $u^H(c) = u^H(c^R)$ , then

firm 1 attracts half the high-risks not needed to make  $c^R$  renegotiation-proof for firm 2, i.e., firm 1's type profile becomes  $(\lambda/r - \lambda/r^{**}, 0)/2$ . Only  $c'$ , defined by  $c_1' + c_2' = D$  and  $u^H(c') = u^H(c)$ , is renegotiation-proof, and the resulting degenerate outcome yields firm 1 zero profit.

If firm 1 offers  $c (\geq 0)$  in area III such that  $u^L(c) = u^L(c^R)$  and  $u^H(c) < u^H(c^R)$ , then firm 1 attracts half the low-risks. Suppose insurees are actually signed up with  $c$ . If the proportion of low-risks to high-risks exceeds  $r^{**}$ , renegotiation along the low-risk indifference curve would occur, making the resulting contract attractive to all high-risks. If the proportion of low-risks to high-risks does not exceed  $r^{**}$ , then, even with renegotiation (if any), the resulting contract would be unattractable to any high-risk. Hence, signing insurees up with  $c$  is not compatible with equilibrium beliefs for high-risks. The same argument goes for any  $c'$  with  $u^L(c') = u^L(c^R)$  and  $u^H(c') < u^H(c^R)$ , while  $c'$  with  $u^L(c') = u^L(c^R)$  and  $u^H(c') > u^H(c^R)$  is not compatible with the firm's profit-maximizing behaviour. Therefore, we follow the convention that insurees are signed up with  $c^R$ , with firm 1 attracting the type profile  $(\lambda/r^{**}, \lambda)/2$ . Again, the resulting degenerate outcome yields firm 1 zero profit.

If firm 1 offers  $c$  in area IV such that  $u^L(c) < u^L(c^R)$  and  $u^H(c) > u^H(c^R)$ , then firm 2 (by the argument above for area III) signs up insurees with the contract  $c'$  defined by  $u^L(c') = u^L(c^R)$  and  $u^H(c') = u^H(c)$ . Firm 1 attracts the high-risks not needed to make  $c'$  renegotiation-proof for firm 2, and it attracts no low-risks; i.e., firm 1's type profile becomes  $(\lambda/r - \lambda/r^{**}, 0)$ . Only  $c''$ , defined by  $c_1'' + c_2'' = D$  and  $u^H(c'') = u^H(c)$ , is renegotiation-proof, and the resulting degenerate outcome yields firm 1 a negative profit.

If firm 1 offers  $c (\geq 0)$  in area V such that  $u^L(c) > u^L(c^R)$  and  $u^H(c) < u^H(c^R)$ , then firm 1 attracts all low-risks, who (by the argument above for area III) must be signed up with the contract  $c'$ , defined by  $u^L(c') = u^L(c)$  and  $u^H(c') = u^H(c^R)$ . Symmetry with area IV implies that firm 1's type profile becomes  $(\lambda/r^{**}, \lambda)$ . Again, the resulting degenerate outcome yields firm 1 a negative profit.

If firm 1 offers  $c$  in area VI such that  $u^L(c) = u^L(c^R)$  and  $u^H(c) > u^H(c^R)$ , then symmetry with area III implies that firm 1 attracts the type profile  $(\lambda/r - \lambda/2r^{**}, \lambda/2)$ . Again, the resulting outcome (degenerate if and only if  $c_1 + c_2 = D$ ) yields

firm 1 a negative profit.

If firm 1 offers  $c (\geq 0)$  in area VII such that  $u^L(c) > u^L(c^R)$  and  $u^H(c) = u^H(c^R)$ , then symmetry with area II implies that firm I attracts the type profile  $((\lambda/r + \lambda/r^{**})/2, \lambda)$ . Again, the resulting nondegenerate outcome yields firm 1 a negative profit.

If firm 1 offers  $c$  in area VIII such that  $u^L(c) > u^L(c^R)$  and  $u^H(c) > u^H(c^R)$ , then firm 1 attracts all customers and the resulting outcome (degenerate if and only if  $c_1 + c_2 = D$ ) yields firm 1 a negative profit.

**Uniqueness.** Again, consider Figure 2. If firm 2 offers  $c'$  in area I, then firm 1 would want to offer  $c$  such that  $u^L(c) > u^L(c')$  and  $u^H(c) > u^H(c')$ . However, no best response exists. If firm 2 offers  $c'$  in area II, then firm 1 would want to offer  $c$  such that  $u^L(c) > u^L(c')$  and  $u^H(c) \leq u^H(c')$ . Again, no best response exists. If firm 2 offers  $c'$  in area III, then firm 1 would want to offer  $c$  such that  $u^L(c) \leq u^L(c')$  and  $u^H(c) > u^H(c')$ . Again, no best response exists. If firm 2 offers  $c'$  in area IV, then firm 1 would want to offer  $c$  such that  $u^L(c) > u^L(c')$  and  $u^H(c) < u^H(c')$ . Again, no best response exists. If firm 2 offers  $c'$  in area V, then firm 1 would want to offer  $c$  such that  $u^L(c) < u^L(c')$  and  $u^H(c) > u^H(c')$ . Again, no best response exists. If firm 2 offers  $c'$  in areas VI, VII, or VIII, then any best response for firm 1 yields firm 1 zero profit. For any best response  $c$  for firm 1,  $c'$  is not a best response to  $c$  for firm 2. This establishes that each firm offering  $c^R$  is the unique Nash equilibrium in pure strategies.

(b) For  $r > \max \{0, r^{**}\}$ , it is sufficient to show that  $(c, c')$  is a Nash equilibrium if and only if the firms' offers  $c$  and  $c'$  are in area II of Figure 3, i.e.,  $u^L(c) = u^L(c') = u^L(c^P)$ . This is because, by symmetry and Proposition 3, through renegotiation, each firm's degenerate outcome is as stated in Proposition 4, yielding each firm zero profit.

**Sufficiency.** Consider Figure 3. Assume firm 2 offers  $c'$  in area II. If firm 1 also makes an offer  $c$  in area II, then it obtains zero profit. We need to establish that firm 1 cannot obtain a positive profit by any offer in areas I and III of Figure 3.

If firm 1 offers  $c$  in area I such that  $u^L(c) < u^L(c') = u^L(c^P)$ , then firm 1 attracts no customers and obtains zero profit. (The reason why high-risks are not attracted even if  $(u^H(c^P) >) u^H(c) > u^H(c')$ , is that they realize that all low-risks will sign



up with firm 2 and will through renegotiation obtain  $c^P$ .)

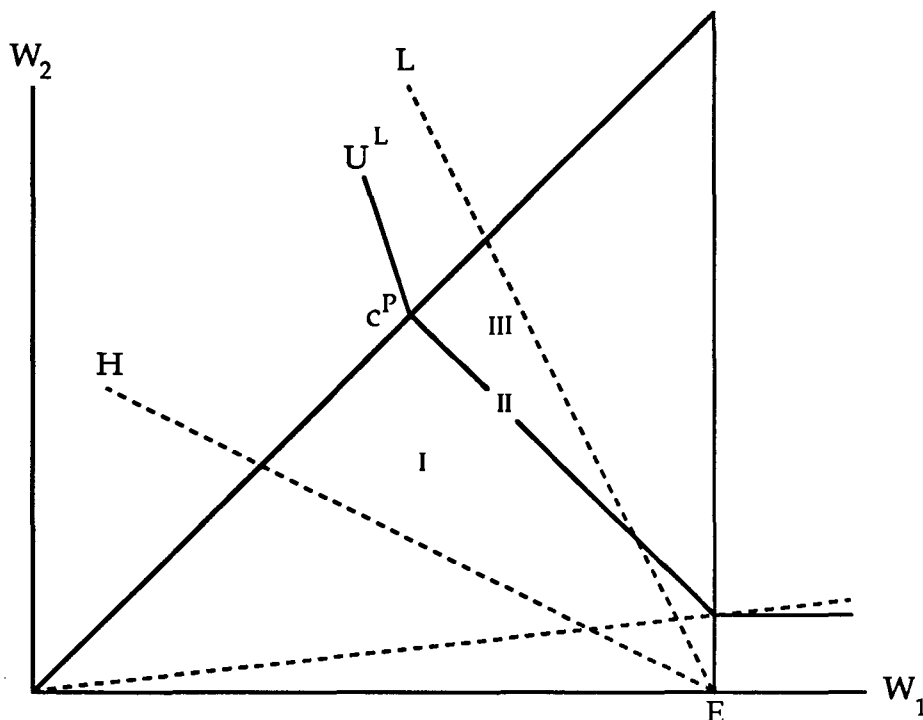


Figure 3

If firm 1 offers  $c$  in area III such that  $u^L(c) > u^L(c') = u^L(c^P)$ , then firm 1 attracts all low-risks, who through renegotiation obtain  $c''$ , defined by  $c_1'' + c_2'' = D$  and  $u^L(c'') = u^L(c)$ . Since  $u^H(c'') > u^H(c^P)$ , firm 1 will attract all high-risks as well. The resulting degenerate outcome yields firm 1 a negative profit.

Necessity. Again, consider Figure 3. If firm 2 offers  $c'$  in area I, then firm 1 would want to offer  $c$  such that  $u^L(c) > u^L(c')$ . However, no best response exists. If firm 2 offers  $c'$  in area III, then any best response for firm 1 yields firm 1 zero profit. For any best response  $c$  for firm 1,  $c'$  is not a best response to  $c$  for firm 2. This establishes that there is no Nash equilibrium in pure strategies with one or both firms making offers in areas I or III. QED.

The task to show, for each  $r$ , that no mixed-strategy equilibrium exists is left for future research.

## 6. COMPARATIVE ANALYSIS

We are now in a position to compare market equilibria with and without renegotiation. From the analysis above, we have:

**Proposition 5:** Compared to the case without renegotiation, allowing for renegotiation has the following effect on consumers' expected utility:

- (i) If  $0 < r < r^{**}$ , then high-risks are equally well off, and low-risks are worse off.
- (ii) If  $\max\{0, r^{**}\} < r < r^*$ , then high-risks are better off, and low-risks are worse off.
- (iii) If  $r > r^*$ , then all consumers are better off.

**Proof:** Follows from Propositions 1, 2, and 4. Notice that: (i)  $u^H(c^R) = u^H(c^Q)$  and  $u^L(c^R) < u^L(c^Q)$ ; (ii)  $u^H(c^P) > u^H(c^Q)$  and  $u^L(c^P) < u^L(c^Q)$ , if  $r < r^*$ ; and (iii)  $u^K(c^P) > u^K(c)$ ,  $c \in PQ$ ,  $c \neq c^P$ ,  $K \in \{H, L\}$ , if  $r > r^*$ . QED.

Note that firms always earn zero profits in equilibrium; thus, the above Proposition sums up the comparative welfare analysis. Also note that a necessary and sufficient condition for the first part of Proposition 5 to be non-vacuous, is that  $a < a^*$ .

Proposition 5 implies that high-risks are always (weakly) better off in the market equilibrium with renegotiation. The reason is that renegotiation makes it harder for low-risks to be separated out (see Proposition 6(b) below), and high-risks gain from being pooled with low-risks. The more interesting part of Proposition 5, though, is part (c), which states that, with few high-risks, allowing for renegotiation leads to a strict Pareto improvement for the insurees, while keeping the firms at the zero-profit level.<sup>11</sup> This contrasts results on renegotiation-proof contracts in a monopoly setting, where allowing for renegotiation limits the set of available contracts and, hence, cannot lead to a Pareto improvement.

The reason why allowing for renegotiation leads to a Pareto improvement in the present context is the competition between the duopolists. Without renegotiation

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<sup>11</sup>A similar result is obtained by Beaudry and Poitevin (1990) in their analysis of a credit market with recontracting, and by Mori (1989) in his analysis of a labour market with renegotiable seniority rules.

tiation, pooling is vulnerable to "cream skimming" whereby the one firm offers a deviating contract in order to attract the profitable low-risks only. In the case with few high-risks, this leads to a mixed-strategy equilibrium in which any point in the support, but the full-insurance pooling one, makes both types of insurees worse off; in particular, with few high-risks, low-risks do not mind full-insurance pooling. With renegotiation, such cream skimming cannot occur, making full-insurance pooling viable.

Regarding the features of the equilibrium, we note differences in the cases without and with renegotiation along two dimensions: whether the equilibrium is in pure or mixed strategies, and to what extent the consumers are separated by type in equilibrium.

**Proposition 6:** (a) Without renegotiation, equilibrium is in pure strategies if and only if  $r \leq r^*$ ; otherwise, it is in mixed strategies. With renegotiation, there always exists an equilibrium in pure strategies.

(b) Without renegotiation, consumers are always fully separated. With renegotiation, consumers are partially separated if  $0 < r < r^{**}$ ; they are pooled if  $r > \max\{0, r^{**}\}$ .

**Proof:** Follows from Propositions 1, 2, and 4. QED.

Thus, our analysis has bearing on two important discussions in the literature. First, there is the debate on whether equilibria in mixed strategies are appropriate; for differences in view, see Binmore and Dasgupta (1986), who are in favour, and Rubinstein (1991), who is skeptical. Although our inclination is in favour of mixed-strategy equilibria, the present analysis clearly shows that one can get around the controversy – at least as far as (our linear version of) the insurance market goes – by allowing renegotiation.

Second, there is an extensive literature discussing whether, in a competitive insurance market, consumers will be separated or not. Essentially, the disagreement concerns how one should model in what sequence the agents move. Some specifications, such as those of Rothschild and Stiglitz (1976) and Riley (1979), lead to full separation, while others, such as those of Wilson (1977) and Grossman (1979), lead to pooling when the fraction of low-risks is high (a condition

equivalent to our  $r > r^*$ , since it coincides with the non-existence of a pure-strategy equilibrium in the Rothschild-Stiglitz specification). Our model can also be interpreted as a particular specification of moves, with renegotiation between a firm and its insurees being allowed indefinitely. We find that this model leads to pooling not only when the fraction of low-risks is high, but that pooling occurs for an even broader range of parameter values, since  $r^{**} < r^*$ . Table 1 sums up Propositions 5 and 6.

Table 1.

Ratio of low-risks to high-risks	Without renegotiation (n)	With renegotiation (r)
<u><math>0 &lt; r &lt; r^{**}</math></u>		
Equilibrium in ...	pure strategies	pure strategies
Types are ...	fully separated	partially separated
High-risks' utility	$U_n^H$	$U_r^H = U_n^H$
Low-risks' utility	$U_n^L$	$U_r^L < U_n^L$
<u><math>r^{**} &lt; r &lt; r^*</math></u>		
Equilibrium in ...	pure strategies	pure strategies
Types are ...	fully separated	pooled
High-risks' utility	$U_n^H$	$U_r^H > U_n^H$
Low-risks' utility	$U_n^L$	$U_r^L < U_n^L$
<u><math>r^* &lt; r &lt; \infty</math></u>		
Equilibrium in ...	mixed strategies	pure strategies
Types are ...	fully separated	pooled
High-risks' utility	$U_n^H$	$U_r^H > U_n^H$
Low-risks' utility	$U_n^L$	$U_r^L > U_n^L$

Among the various specifications of competitive insurance markets suggested in the previous literature, the present study has a particular relation to Gross-

man (1979) that we would like to stress.<sup>12</sup> After firms offering contracts and consumers selecting among contracts offered, Grossman suggests that a firm be allowed to accept or reject any consumer selecting one of this firm's contracts. Like renegotiation, this feature captures the idea of firms taking advantage of the information created by consumers' selection of contracts. And both specifications lead to pooling for high values of  $r$ . On the other hand, when  $r < r^*$ , such that the pure-strategy Rothschild-Stiglitz equilibrium exists, the Grossman equilibrium coincides with the latter, while the equilibrium with renegotiation still differs from, and may even be Pareto inferior to, the Rothschild-Stiglitz equilibrium.

Lastly, we note the effects on our results of changes in consumers' risk aversion. We have the following, which has a straightforward proof:

**Proposition 7:** The critical values of  $r$ ,  $r^*$  and  $r^{**}$ , decrease when the risk aversion parameter,  $a$ , increases:

$$\frac{dr^*}{da} = \frac{dr^{**}}{da} = -\frac{r^*}{a} < 0.$$

Thus, increasing consumers' risk aversion broadens the range where the equilibrium without renegotiation is in mixed strategies; it broadens the ranges where consumers benefit from the existence of an insurance market with renegotiation; and it broadens the range where the equilibrium with renegotiation is pooling.

## APPENDIX 1

**Proof of Lemma 1:** By construction of preferences, consumers prefer  $c$  to being self-insured. Hence,  $(F_h, F_\ell) \in X(G^0)$ , since both incentive and individual-rationality constraints are satisfied. The Lemma follows from the incentive constraint of the high-risks as well as the fact that, for each type, iso-profit curves are steeper than indifference curves. QED.

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<sup>12</sup>Game-theoretic foundations for the Grossman formulation are found in Hellwig (1987) and Desruelle (1989).

**Proof of Proposition 1:** By Lemma 1, the profit-maximising outcome consistent with  $c^Q$  has  $c_h = (p^H D, (1 - p^H)D)$  and  $c_l = c^Q$ . Thus,  $c_h$  is the full-insurance contract yielding zero expected profit when sold to high-risks, while  $c^Q$  is given by high-risk indifference between  $c_h$  and  $c^Q$  and zero expected profit when sold to a low-risk. From Rothschild and Stiglitz (1976), we know that  $(c_h, c^Q)$  is the only possible symmetric pure-strategy equilibrium of this model. The condition for its existence,  $r \leq r^*$ , is proved as follows.

The optimum deviation from  $c^Q$  for a firm whose rival offers  $c^Q$ , is an offer  $c^x$  with the property that  $u^L(c^x) = u^L(c^Q)$ ; this is because  $u^L(c^x) < u^L(c^Q)$  implies no low-risk customers, while  $u^L(c^x) > u^L(c^Q)$  implies that profit can be increased without losing low-risk customers. This and Lemma 1 imply that the two contracts,  $(c_h^x, c_l^x) = (c_h^x, c^x)$ , given to high-risks and low-risks, respectively, by an optimally deviating firm, have three properties. First,  $c_h^x$  provides full insurance:

$$c_{h1}^x = D - c_{h2}^x. \quad (A1)$$

Second, high-risks are indifferent between  $c_h^x$  and  $c^x$ :

$$\frac{c_{h2}^x - c_2^x}{c_{h1}^x - c_1^x} = \frac{1 - p^H}{a + p^H},$$

which, when using (A1), we may write:

$$c_2^x = \frac{a + 1}{a + p^H} c_{h2}^x - \frac{1 - p^H}{a + p^H} (D - c_1^x). \quad (A2)$$

Third, low-risks are indifferent between  $c^x$  and  $c^Q$ :

$$\frac{c_2^x - c_2^Q}{c_1^x - c_1^Q} = \frac{1 - p^L}{a + p^L},$$

which we write as:

$$c_1^x = c_1^Q + \frac{a + p^L}{1 - p^L} (c_2^x - c_2^Q). \quad (A3)$$

Let  $\pi^*$  be the deviating firm's average expected profit per contract. By deviating, a firm attracts all consumers, so that  $G^0 = ((1 - \lambda), \lambda, 0, 0)$  for the deviating firm. Thus:

$$\pi^* \equiv (1 - \lambda)\pi^H(c_h^x) + \lambda\pi^L(c^x) = (1 - \lambda)[(1 - p^H)c_{h1}^x - p^H c_{h2}^x] + \lambda[(1 - p^L)c_1^x - p^L c_2^x]. \quad (A4)$$

We want to find the effect on  $\pi^*$  of changing the contracts  $(c_h^x, c^x)$  subject to the constraints (A1–A3). The same procedure as below is used by Stiglitz (1977) for

similar purposes. From (A1) and (A3), respectively, we have:

$$\frac{dc_{h1}^x}{dc_{h2}^x} = -1, \quad (A5)$$

and

$$\frac{dc_1^x}{dc_2^x} = \frac{a + p^L}{1 - p^L}. \quad (A6)$$

By totally differentiating (A2) and using (A6), we get:

$$\frac{dc_2^x}{dc_{h2}^x} = \frac{1 - p^L}{p^H - p^L}. \quad (A7)$$

We now totally differentiate the profit function (A4), using first (A5), and then (A6) and (A7):

$$\frac{d\pi^*}{dc_{h2}^x} = -(1 - \lambda) + \lambda[(1 - p^L)\frac{dc_1^x}{dc_2^x} - p^L]\frac{dc_2^x}{dc_{h2}^x} = -(1 - \lambda) + \frac{\lambda a(1 - p^L)}{p^H - p^L}.$$

A deviation pays if and only if  $(d\pi^*/dc_{h2}^x) > 0$ . By applying the definitions of  $r$  and  $r^*$  to the above expression, we find that this happens if and only if  $r > r^*$ . QED.

**Proof of Lemma 2:** Note first that the profit from selling the contract  $c^T$  to low-risks is:

$$\begin{aligned} \pi^L(c^T) &= (1 - p^L)c_1^T - p^L c_2^T = (1 - p^L)c_1 - p^L c_2 - [(1 - p^L)(c_1 - c_1^T) - p^L(c_2 - c_2^T)] = \\ &= \pi^L(c) - [(1 - p^L)\frac{a + p^H}{1 - p^H} - p^L](c_2 - c_2^T), \end{aligned}$$

since  $u^H(c) = u^H(c^T)$ , with  $c \in PQ$ . From the definition of  $c^T$ , we know that it satisfies:

$$u^H(c^T) = u^H(c), \text{ and:}$$

$$u^L(c^T) = u^L(c'),$$

where  $c'$  is given by:

$$c' \in PQ, \text{ and: } c_2' = c_2 + t(c_2^P - c_2^Q).$$

These two conditions can be shown to be equivalent to:

$$c_2 - c_2^T = t(c_2^P - c_2^Q) \frac{1}{a^*(a + 1)} [a + p^L - (1 - p^L)\frac{dc_1}{dc_2} |_{PQ}].$$

Note that:

$$\begin{aligned} c_2^P - c_2^Q &= [(1 - \lambda)(1 - p^H) + \lambda(1 - p^L)]D - \frac{1 - p^H}{1 + r^*}D = \\ &= \frac{r^*D}{1 + r^*} [(1 - \lambda)(1 - p^H) + \lambda(1 - p^L)(a + 1)]. \end{aligned}$$

In order to get any further, we need more knowledge about the set PQ. Offers in this set are characterised as follows: There is a contract  $c_h$  such that the contract pair  $(c_h, c)$ ,  $c \in PQ$ , has the following properties:

–  $c_h$  yields full insurance:

$$c_{h1} + c_{h2} = D;$$

– high-risks are indifferent between  $c_h$  and  $c$ :  $u^H(c_h) = u^H(c)$ , or:

$$\frac{c_{h2} - c_2}{c_{h1} - c_1} = \frac{1 - p^H}{a + p^H};$$

–  $(c_h, c)$  yields zero profit when all high-risks buy  $c_h$  and all low-risks buy  $c$ :

$$(1 - \lambda)[(1 - p^H)c_{h1} - p^H c_{h2}] + \lambda[(1 - p^L)c_1 - p^L c_2] = 0.$$

These three conditions are equivalent to the following one, which thus characterises a generic member  $c = (c_1, c_2)$  of PQ:

$$[\lambda(1 - p^L) + (1 - \lambda)\frac{1 - p^H}{a + 1}]c_1 - [\lambda p^L + (1 - \lambda)\frac{a + p^H}{a + 1}]c_2 + \frac{a(1 - \lambda)(1 - p^H)}{a + 1}D = 0.$$

From this, we get what we need. First, the profit from a contract  $c \in PQ$  when sold to low-risks is:

$$\pi^L(c) = (1 - p^L)c_1 - p^L c_2 = (1 - \lambda)(p^H - p^L)Dx_2.$$

Second, the inverse slope of PQ is:

$$\frac{dc_1}{dc_2} \Big|_{PQ} = \frac{\lambda(a + 1)p^L + (1 - \lambda)(a + p^H)}{\lambda(a + 1)(1 - p^L) + (1 - \lambda)(1 - p^H)} = \frac{r^*(a + 1)D}{(1 + r^*)(c_2^P - c_2^Q)} - 1.$$

Thus, we can write:

$$\begin{aligned} & [(1 - p^L)\frac{a + p^H}{1 - p^H} - p^L](c_2 - c_2^T) = \\ & [(1 - p^L)\frac{a + p^H}{1 - p^H} - p^L]\frac{t(c_2^P - c_2^Q)}{a^*(a + 1)}[a + p^L - (1 - p^L)(\frac{r^*(a + 1)D}{(1 + r^*)(c_2^P - c_2^Q)} - 1)] = \\ & t(c_2^P - c_2^Q)\frac{1 + r^*}{r^*(a + 1)}[a + 1 - (1 - p^L)(\frac{r^*(a + 1)D}{(1 + r^*)(c_2^P - c_2^Q)})] = t[(c_2^P - c_2^Q)\frac{1 + r^*}{r^*} - (1 - p^L)D] = \\ & tD[(1 - \lambda)(1 - p^H) + \lambda(1 - p^L)(a + 1) - (1 - p^L)] = \\ & tD(1 - \lambda)[ra(1 - p^L) - (p^H - p^L)] = \\ & tD(1 - \lambda)(p^H - p^L)(\frac{r}{r^*} - 1). \end{aligned}$$

Inserting this and our expression for  $\pi^L(c)$  into the expression for  $\pi^L(c^T)$ , we ob-



tain:

$$\pi^L(c^T) = (1 - \lambda)(p^H - p^L)D[x_2 - t(\frac{r}{r^*} - 1)].$$

Inserting this expression in the definition of  $H(t)$  proves the Lemma. QED.

## APPENDIX 2

Below, a mapping from a pair of offers from the firms ( $c$  from the one firm,  $c'$  from the other firm) to a renegotiation-proof outcome in an initial position for each firm is constructed.

If  $0 < r = \lambda/(1-\lambda) \leq r^{**}$ , low risks are split between the firms according to the expected utilities they are offered. However, a partial-insurance renegotiation-proof contract  $c_k$  requires that  $\mu_k^L/\mu_k^H = r^{**}$ , by Proposition 3; thus,  $\lambda/r^{**}$  of the high-risks are needed in order to offer renegotiation-proof contracts to low-risk insurees. The competition for high-risk insurees determines the split for only the remaining  $(1 - \lambda) - \lambda/r^{**} = \lambda/r - \lambda/r^{**}$  high-risks, who are split between the firms according to the expected utilities they are offered. The following table summarizes the above discussion, giving the type profile of a firm that offers  $c$  when the other firm offers  $c'$ .

$u^L(c) > u^L(c'), u^H(c) > u^H(c')$	$\mu(c, c', r) = (\lambda/r, \lambda)$
$u^L(c) > u^L(c'), u^H(c) = u^H(c')$	$\mu(c, c', r) = ((\lambda/r + \lambda/r^{**})/2, \lambda)$
$u^L(c) > u^L(c'), u^H(c) < u^H(c')$	$\mu(c, c', r) = (\lambda/r^{**}, \lambda)$
$u^L(c) = u^L(c'), u^H(c) > u^H(c')$	$\mu(c, c', r) = (\lambda/r - \lambda/2r^{**}, \lambda/2)$
$u^L(c) = u^L(c'), u^H(c) = u^H(c')$	$\mu(c, c', r) = (\lambda/2r, \lambda/2)$
$u^L(c) = u^L(c'), u^H(c) < u^H(c')$	$\mu(c, c', r) = (\lambda/2r^{**}, \lambda/2)$
$u^L(c) < u^L(c'), u^H(c) > u^H(c')$	$\mu(c, c', r) = (\lambda/r - \lambda/r^{**}, 0)$
$u^L(c) < u^L(c'), u^H(c) = u^H(c')$	$\mu(c, c', r) = ((\lambda/r - \lambda/r^{**})/2, 0)$
$u^L(c) < u^L(c'), u^H(c) < u^H(c')$	$\mu(c, c', r) = (0, 0)$

This determines the initial position for each firm as a function of its own

offer  $c$  and the offer  $c'$  of the other firm,  $G = (\mu(c, c', r), 0, 0)$ , given that  $0 < r \leq r^{**}$ . Lemma 4 below establishes what renegotiation-proof outcome in the position  $G$  is realized given a firm's own offer  $c$  and the offer  $c'$  of the other firm.

**Lemma 4:** Let  $0 < r \leq r^{**}$ . If  $c$  and  $c'$  are adequate offers such that  $\mu(c, c', r) \neq (0, 0)$ , then there exists a unique outcome  $(F_h, F_\ell) \in \sigma(G)$ ,  $G = (\mu(c, c', r), 0, 0)$ , satisfying:  $u^H(c_h) = \max\{u^H(c), u^H(c')\}$  and  $u^L(c_\ell) = u^L(c)$ .  $(F_h, F_\ell) = ((\mu_h, c_h), (\mu_\ell, c_\ell))$  is characterized by:  $\mu_h = (\mu_h^H, 0)$ ;  $\mu_\ell = (\mu_\ell^H, r^{**}\mu_\ell^H)$ ;  $\mu_h + \mu_\ell = \mu(c, c', r)$ ;  $c_\ell = c$ ;  $c_{h1} + c_{h2} = D$ ; and  $u^H(c_h) = u^H(c_\ell)$ .

**Proof:** Follows from Proposition 3 by considering each of the first eight cases above. QED.

The rationale behind requiring  $u^H(c_h) = \max\{u^H(c), u^H(c')\}$  is that the high-risks needed in order to make  $c_\ell$  renegotiation-proof can only be attracted in the competition with the other firm, if the other firm's offer to high-risks is matched.

This completes the description of the mapping from a pair of offers from the two firms to a renegotiation-proof outcome in an initial position for each of the firms in the case where  $0 < r \leq r^{**}$ .

Now turn to the case with  $r = \lambda/(1 - \lambda) > \max\{0, r^{**}\}$ . Again, low risks are split between the firms according to the expected utility that they are offered, and again a partial-insurance renegotiation-proof contract  $c_k$  requires that  $\mu_k^L/\mu_k^H = r^{**}$ . Since this cannot be satisfied with  $r > \max\{0, r^{**}\}$ , we adopt the convention that high-risk insurees are split between the firms in proportion to the split of the low-risks. The following table summarizes the above discussion, giving the type profile of a firm that offers  $c$  when the other firm offers  $c'$ .

$u^L(c) > u^L(c')$	$\mu(c, c', r) = (\lambda/r, \lambda)$
$u^L(c) = u^L(c')$	$\mu(c, c', r) = (\lambda/2r, \lambda/2)$
$u^L(c) < u^L(c')$	$\mu(c, c', r) = (0, 0)$

This determines the initial position for each firm as a function of its own offer  $c$  and the offer  $c'$  of the other firm,  $G = (\mu(c, c', r), 0, 0)$ , given that  $r > \max\{0, r^{**}\}$ . Lemma 5 below establishes what renegotiation-proof outcome in the position  $G$  is realized given a firm's own offer  $c$  and the offer  $c'$  of the other firm.

**Lemma 5:** Let  $r > \max\{0, r^{**}\}$ . If  $c$  and  $c'$  are adequate offers such that  $\mu(c, c', r) \neq (0, 0)$ , then there exists a unique degenerate outcome  $F_d \in \sigma(G)$ ,  $G = (\mu(c, c', r), 0, 0)$ , satisfying:  $u^L(c_d) = u^L(c)$ .  $F_d = (\mu_d, c_d)$  is characterized by:  $\mu_d = \mu(c, c', r)$  and  $c_{d1} + c_{d2} = D$ .

**Proof:** Follows from Proposition 3 by considering each of the first two cases above. QED.

This completes the description of the mapping from a pair of offers from the two firms to a renegotiation-proof outcome in an initial position for each of the firms also in the case where  $r > \max\{0, r^{**}\}$ .

We have now, for each  $r$ , determined firms' profits as functions of their offers in the case where renegotiation is allowed: If the firms' offers are  $c$  and  $c'$ , then the profit of the firm offering  $c$  is given by  $\Pi(G)(F_h, F_\ell)$ , with  $G = (\mu(c, c', r), 0, 0)$ , and where  $(F_h, F_\ell) \in \sigma(G)$  satisfies Lemma 4 or 5 (depending on  $r$ ). Thereby, we have, for each  $r$ , constructed a simultaneous-move game with each firm's strategy set being the set of adequate offers.

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## CHAPTER FOUR:

TWO KINDS OF CONSUMER SWITCHING COSTS<sup>1</sup>

**Abstract:** In this chapter, a distinction is introduced between two kinds of consumer switching costs – "transaction costs" and "learning costs". While transaction costs are incurred by a consumer at every switch between suppliers, learning costs are incurred only at a switch to a supplier that is new to him. In a multi-period duopoly model, it is shown that both the introductory price and welfare decrease as the fraction of transaction costs out of total consumer switching costs decreases.

## 1. INTRODUCTION

Some industries have the characteristic that, even though products are functionally identical, consumers incur costs when switching from one producer to another. In this way, ex-ante homogeneous products become ex-post heterogeneous for a consumer after he has purchased one of them. The study of markets with consumer switching costs has been extensive in recent years. Perhaps the most important contributions have come from Paul Klemperer.<sup>2</sup>

Klemperer points out three sources of switching costs:<sup>3</sup> (i) *Transaction costs*. Examples are costs incurred when switching from one bank to another, involving the closing of one set of accounts and the opening of another set in the other bank, and costs incurred when switching from one supplier of rented equipment

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<sup>1</sup>The first version of this chapter was completed in June 1990. I would like to thank Geir Asheim, Tertit Hammer, Paul Klemperer, Terje Lensberg, Kjell Erik Lommerud, Garth Saloner, Jon Vislie, and a journal's referee for helpful comments on earlier versions. I am particularly indebted to Geir Asheim and the referee for suggesting generalisations of my results. Thanks also to seminar participants at the University of Bergen and at the Norwegian School of Economics and Business Administration.

<sup>2</sup>Klemperer (1987a-c, 1988, 1989); Beggs and Klemperer (1990). Interesting contributions have also come from other authors, such as Farrell and Shapiro (1988).

<sup>3</sup>See, e.g., Klemperer (1987a, Sec. I), on which this paragraph is more or less based.

to another, involving returning the equipment to the old supplier and getting equipment at the other one. (ii) *Learning costs*. An example is the costs incurred when getting to know a new word processing system that has the same functions as others on the market but also has the functions spread differently around on the keyboard and has a manual written in a different style. (iii) *Artificial switching costs*. These costs arise as results of firms' actions. Examples of such actions include "frequent-flyer programmes" and other coupon systems, but also decisions on whether or not to make products compatible with others.<sup>4</sup>

There is another distinction to be drawn, that between exogenous and endogenous switching costs. In their nature, artificial switching costs are endogenous. The two other kinds can be seen as either exogenous or endogenous or both. Like Klemperer in his work, we concentrate in this chapter on exogenous switching costs and thus on transaction and learning costs.

Contrary to Klemperer, however, we will argue that there is an important difference between these two kinds of consumer switching costs. Transaction costs are incurred every time a consumer switches between suppliers, whereas learning costs are incurred only when the consumer turns to a supplier for the first time.<sup>5</sup>

To understand how this distinction works, a multi-period model is presented below. The model is, apart from the number of periods, kept as simple as possible, with identical consumers each having a per-period individual demand curve. Each consumer is confronted with two kinds of switching costs, one that is incurred at every switch, and one that is incurred only when switching to a supplier that is new to the consumer. On the supply side, there is a price-setting duopoly.

We find that an increase in transaction costs' share of total consumer switching costs entails a decrease in welfare as well as a decrease in firms' introductory

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<sup>4</sup>An analysis of coupon systems like "frequent-flyer programmes" is found in Banerjee and Summers (1987). Issues related to compatibility and its companion, standardisation, are surveyed by Farrell and Saloner (1987).

<sup>5</sup>Klemperer (1987a, note 22) mentions in passing another possible difference, though. He argues that, while increases in transaction costs will affect first-period demand, this is not necessarily true for learning costs, since learning costs may not imply start-up costs for the consumers. In the present analysis, we abstract from this particular difference to consider the more far-reaching consequences of differences in the costs of reswitching.

price. These results are driven by the interaction of two key assumptions of our model. Firstly, firms are allowed to price discriminate according to consumers' purchase histories. Secondly, consumers are explicitly assumed to have rational expectations.

The price discrimination in the present model is so-called third-degree.<sup>6</sup> A consumer's actual switching costs will differ according to whether he has switched in the past or not. Accordingly, we assume that a firm is able to distinguish consumers with a single supplier in the past (the loyal consumers) from the others (the disloyal consumers). With this assumption, the present work is related to the literature on third-degree price discrimination in oligopoly; see Borenstein (1985) and Holmes (1989).<sup>7</sup> These authors provide theoretical evidence that such discrimination may occur in a competitive market if products are heterogeneous and if there exists a way to categorize consumers which is correlated with consumer preferences across brands.

The present efforts may be seen as applying this theory to a market with consumer switching costs. Here, products are ex-ante homogeneous but become heterogeneous in the eyes of a consumer once he has made a purchase. We focus on the case where the categorization is perfectly correlated with preferences: A consumer has a high preference for (a high cost of switching away from) his old supplier if and only if he has been loyal to this supplier throughout the past; and firms are assumed to know a consumer's loyalty. Ours is a case where, in the terminology of Holmes (1989), the two groups of consumers, loyal and disloyal ones, are identical with regard to "industry elasticity" of demand but differ with regard to the "cross-elasticity" of demand among firms.

Empirical evidence that firms price discriminate according to differences in cross-elasticity is provided by Borenstein and Rose (1991) on the U.S. airline industry. More to the point is the empirical study by Borenstein (1989), showing that firms price discriminate on the basis of differences in consumer switching costs. His analysis of petrol retailing in the U.S. points in the direction that led

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<sup>6</sup>For an introduction to price discrimination, see Varian (1989).

<sup>7</sup>On the other hand, by assuming price discrimination, we pose ourselves in stark contrast to other papers in the switching-costs literature, where firms' inability to discriminate between old and new customers is the driving force; see, e.g., Farrell and Shapiro (1988) and Beggs and Klemperer (1990).



petrol is priced higher than unleaded because, since there are fewer stations selling it, buyers of leaded petrol face higher costs of switching from one station to another. Empirical support for the present assumption of price discrimination based on past switching is, regrettably, not available.<sup>8</sup> But it seems likely, at least in markets with individual treatment of customers, that firms possess information that correlates with consumer loyalty.<sup>9</sup>

The other key ingredient of our model is rational consumer expectations. Although a common assumption, the present analysis distinguishes itself by highlighting it as a crucial feature; if, instead, consumers were completely myopic, our main results would not go through. With rational expectations, consumers realize that switching now means a reduction in future switching costs, which again, because of the price discrimination, implies a reduction in future prices. This gives rise to the notion of *net consumer switching costs*, i.e., switching costs net of the future benefit of switching.

To see how our two key assumptions interact, consider a loyal consumer, i.e., one who has not switched in the past. Price discrimination implies that he is offered prices by the two firms that may differ from those offered a disloyal consumer. His previous supplier, denoted the incumbent, matches the rival's price such that the difference in consumer surplus from the two prices exactly equals the consumer's net switching costs.

An increase in transaction costs' share of (gross) consumer switching costs now has two effects. One effect comes from the consumer realizing that higher transaction costs imply a reduction in the future benefit from switching now, since reswitching later becomes more expensive. This effect drives the incumbent's price upwards, and we call it the *consumer effect* since it derives from the consumer's deliberations on whether to switch or not. Another effect comes from the rival firm realizing that higher transaction costs entail higher future

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<sup>8</sup>Nor does the present analysis predict that such discrimination should be observed, since we find that consumers do not switch in equilibrium.

<sup>9</sup>In his management textbook, Nagle (1987) observes that "[t]he retail price of an automobile is typically set by the salesperson who evaluates the buyer's willingness to pay. ... [T]he salesperson takes a personal interest in the customer, asking ... what kinds of cars he has bought before ... and whether he has looked at, or is planning to look at, other cars" (p. 158). In such a case, which Nagle calls "Segmenting by Salespeople", the firm seems able to distinguish loyal consumers from disloyal ones.

earnings if it manages to attract this loyal consumer, making him a disloyal, since, again, reswitching has become more expensive for the consumer. Higher future earnings imply that the rival can stand a lower price today. Therefore, this effect drives the incumbent's price downwards, and we call it the *rival effect* since it derives from the rival's deliberations on which price to set.

The strength of consumer rationality in this model is seen from the fact that the consumer effect described above dominates the rival effect. Therefore, an increase in transaction costs' share of consumer switching costs leads to an increase in the prices set by incumbents to loyal consumers.<sup>10</sup> This is so in all periods, except the last, when there is no future so that neither of the two effects is at work, and the first, when there is no incumbent. We find, moreover, that incumbents' prices are always positive in equilibrium. Thus, since the first-period price is determined by cut-throat competition in the usual Bertrand fashion, an increase in transaction costs' share implies a lower introductory price. And therefore, such an increase also leads all prices to move away from their efficient level at marginal costs, thus decreasing welfare.

The basic intuition behind the relative strength of the consumer effect can be seen from noting that a downward-sloping demand curve affects profit and consumer surplus asymmetrically. When price is above marginal costs, as is the case for the incumbents' prices in this model, a price change has a stronger effect on consumer surplus than on profit. On the other hand, when price is below marginal costs, as is the case with rivals' prices, a price change has a stronger effect on profit than on consumer surplus; this is so because profit is negative and, say, a price increase reduces both the loss-inducing demand and the loss on the demand left.

To be specific, let there be three periods with both consumers and firms using a discount factor equal to 1. Suppose that transaction costs' share of consumer switching costs increases such that transaction costs are increased, and learning costs decreased, with an amount equal to the area  $A + B$  in Figure 1. This implies

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<sup>10</sup>Actually, any rival effect may itself be decomposed into a consumer effect and a rival effect, but this time with regard to a disloyal consumer. This decomposition may be done recursively. The consumer effect is always the stronger in any period, thus reinforcing the rival effect of the previous period. Eventually, this also reinforces the rival effect on an incumbent's price to a loyal consumer. However, subject to a restriction on the shape of the demand curve at prices below costs, it is still dominated by the consumer effect.

that an incumbent's profit in period 3, the last one, increases with an amount equal to  $A - C$ . Thus, in period two, a rival firm can afford to incur an extra loss equal to this. Let the area  $D + E$  in Figure 1 be this extra loss, i.e.,  $D + E = A - C$ . This is the rival effect, but since it works through consumer surplus, it reduces to the area  $D = A - C - E$ . The consumer effect, on the other hand, amounts to the extra transaction costs  $A + B$ , without reductions. Thus, the total effect on the incumbent's price in period 2 is:  $(A + B) - D = (A + B) - (A - C - E) = B + C + E > 0$ .

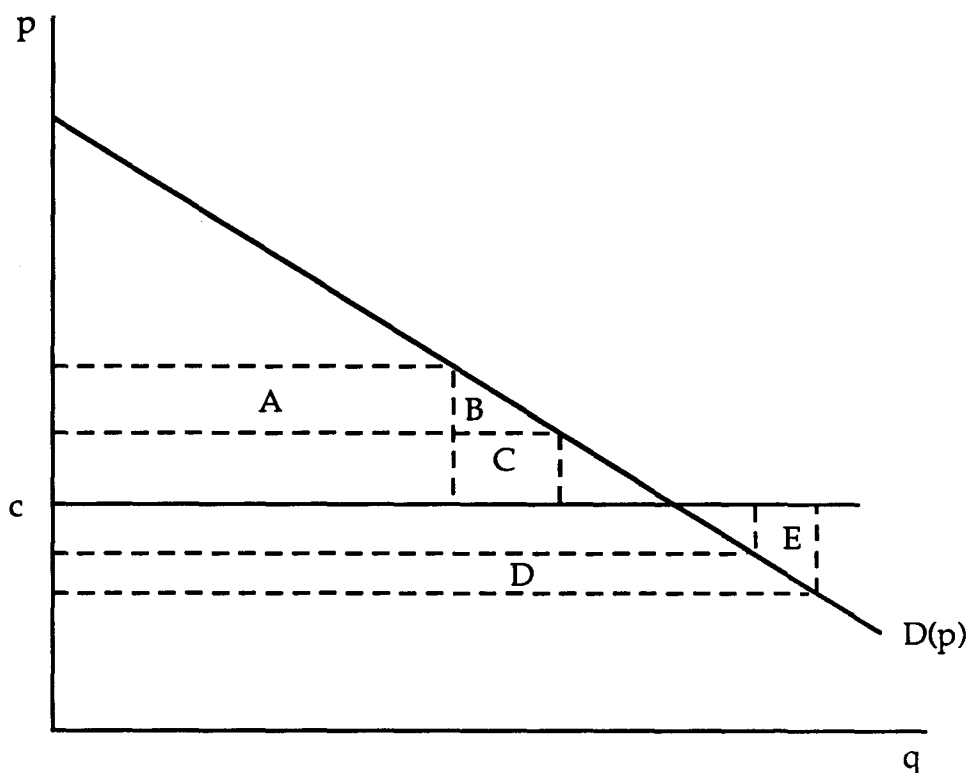


Figure 1

The chapter is organised as follows. In Sec. 2, the model is presented. In Sec. 3, the unique subgame-perfect equilibrium is described and characterised. In Sec. 4, this equilibrium is analysed with regard to the effects, discussed above, of a change in transaction costs' share of consumer switching costs. Sec. 5 concludes with a short discussion.

## 2. THE MODEL

Suppose all consumers are identical at the outset. As the market develops, they will differ with regard to their historical purchase patterns. But on this characteristic firms are assumed to be able to price discriminate. This means that we can keep the whole analysis on a per-capita basis, so that any consumer surpluses and profits mentioned in the sequel are measured per capita.

The market is open for  $T$  periods, with  $3 \leq T < \infty$ . The periods are identical in the sense that each of them starts with firms setting this period's prices simultaneously followed by consumers choosing which firm to buy from.

On the supply side, the market consists of two firms. Each of them produces a homogeneous product and is endowed with a technology exhibiting constant average costs  $c > 0$ , so that total costs of producing a quantity  $q$  are:  $C(q) = cq$ .

Each consumer in the economy is endowed with preferences over  $q$  and a composite of other goods,  $m$ , which also is the numeraire. Assuming that saving and borrowing is infeasible and that goods are non-storable, we represent these preferences with a periodical utility function  $U$  defined as:

$$U = V(q) + m \quad (2.1)$$

i.e., the composite good  $m$  is assumed to be a transferable utility. We make the following assumption on  $V$ :

*Assumption 1:* (i)  $V$  is twice differentiable; (ii)  $V' > 0$ ; (iii)  $V'' < 0$ .

Define  $p$  as the price of the product  $q$  and  $Y$  as periodical income; then, the budget constraint for a consumer is:  $pq + m = Y$ . Let  $\bar{p} \equiv V'(0)$ . When maximising  $U$ , in (2.1), subject to the budget constraint and to the non-negativity constraint  $q \geq 0$ , we obtain the individual periodical demand for  $q$  as a continuous function of the price  $p$ ,  $D(p)$ :

$$D(p) \equiv \begin{cases} (V')^{-1}(p), & \text{for } 0 \leq p < \bar{p}, \\ 0, & \text{for } p \geq \bar{p}. \end{cases} \quad (2.2)$$

From Assumptions 1(ii) and (iii), we have that  $D(p)$  approaches infinity as  $p$  goes to 0, and that, for  $p < \bar{p}$ :

$$D'(p) = 1/[V''(D(p))] < 0. \quad (2.3)$$

Consumers are assumed to have rational expectations in the sense that they foresee at any time the equilibrium of the rest of the game and behave according-

ly. The one-period consumer surplus at a price  $p$  is defined as:

$$S(p) \equiv V(D(p)) - V(0) - pD(p) = \int_p^{\bar{p}} D(x)dx \quad (2.4)$$

Consumers incur costs,  $s > 0$ , when switching between suppliers.<sup>11</sup> These costs are of two kinds. The first kind of switching costs is incurred by a consumer at every switch; these costs make up a fraction  $\alpha \in (0, 1)$  of total switching costs. The second kind is incurred by a consumer only when switching to a supplier who has not supplied him before; these costs make up the residual fraction  $(1 - \alpha)$ .

Common to both firms and consumers is the discount rate  $\delta \in (0, 1]$ .

Firms offer prices in each period from the feasible price range  $[0, \bar{p}]$ . If a firm offers a consumer a price  $p$ , then either the consumer buys from the other firm, or the firm earns a one-period profit on this consumer equal to:

$$\pi(p) \equiv (p - c)D(p). \quad (2.5)$$

The limit property of  $D(p)$  implies that  $\pi(p)$  becomes arbitrarily low as  $p$  goes to 0; thus, the lower bound on feasible prices, 0, will never be binding.

A very useful concept is the following:

$$R(p) \equiv \left| \frac{\pi'(p)}{S'(p)} \right| = 1 + \frac{(p - c)D'(p)}{D(p)}, \quad (2.6)$$

where the equality follows from noting that, by differentiation in (2.5) and (2.4), respectively, we get:  $\pi'(p) = D(p) + (p - c)D'(p)$ , and  $S'(p) = -D(p)$ . A price change has clearly opposite effects on profit and consumer surplus;  $R(p)$  measures the relative strength of these effects.

We now make three useful observations:

*Observation 1:*  $S(p) + \pi(p) \leq S(c), \forall p$ .

*Proof:* Clearly, social welfare is maximised when price equals marginal costs:  $S(c) + \pi(c) \geq S(p) + \pi(p), \forall p$ . Since  $\pi(c) = 0$ , the claim follows. *QED.*

*Observation 2:* If  $p \leq c$ , then  $R(p) \geq 1$ .

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<sup>11</sup>Consumers' start-up costs in period 1 are, without loss of generality, set to zero.

*Proof:* Follows from (2.6), since  $D'(p) < 0$ . QED.

*Observation 3:*  $R'(p) < 0, \forall p \in (0, c)$ , if the following Condition (C) holds,

$$D''(p) > \frac{(D'(p))^2}{D(p)} - \frac{D'(p)}{p-c}, \forall p \in (0, c). \quad (C)$$

*Proof:* Differentiating (2.6), we have:

$$R'(p) = (p-c) \left[ \frac{D''(p)}{D(p)} - \frac{(D'(p))^2}{(D(p))^2} \right] + \frac{D'(p)}{D(p)}.$$

By applying Condition (C), we find that this expression is negative for  $p \in (0, c)$ . QED.

Observation 1 is elementary welfare theory; we state it here, because it is the key to Proposition 2 below. Observation 2 states the asymmetry, discussed in the Introduction, in the effect of a price change on profit and consumer surplus at prices above and below costs. This Observation would do in the proofs of our comparative-statics Theorems for the case of  $T = 3$ . However, in the general  $T$ -period case, we need not only that  $R(p) > 1$  for  $p < c$ , but also that this ratio increases as  $p$  falls further below  $c$ . Because of this, and based on Observation 3, we impose:

*Assumption 2:* Condition (C) holds.

Condition (C), given in Observation 3 above, always holds as  $p$  approaches  $c$  or  $0$ ; this follows from inspection of (C), using the limit property of the demand function in the latter case. Thus, the bite of Assumption 2 is at intermediate prices. Condition (C) is a restriction on the curvature of the demand function; it requires that, for prices below costs, the demand curve be sufficiently convex or, at least, not too concave. The Condition is satisfied, e.g., if the demand function features a constant elasticity at prices below cost:  $D(p) = ap^b, a > 0, b < 0, \forall p \leq c$ .

Let  $p_m$  be the (lowest) price that maximizes a monopolist's one-period profit:  $p_m \equiv \min \operatorname{argmax}_p \pi(p)$ . Clearly,  $p < p_m$  implies  $R(p) > 0$ . For the sake of conveni-

ence, we assume that demand is sufficiently large, relative to switching costs and production costs, that any equilibrium price is strictly below  $p_m$ :

*Assumption 3:*

$$S(c) - S(p_m) = \int_c^{p_m} D(x)dx > s.$$

Finally, we make assumptions, standard in models of price-setting, asymmetric oligopoly, to ensure equilibrium existence and uniqueness. To ensure existence, we restrict an indifferent consumer to stay with the incumbent firm. To ensure uniqueness, we do not allow the rival firm to set a price that would incur a loss were the incumbent firm to set no price at all at the same time. (See Appendix B.)

### 3. THE EQUILIBRIUM

The two firms are allowed to set their prices according to a consumer's purchase history. There are two pieces of payoff-relevant information in such a purchase history in any period. One is which firm supplied the consumer in the previous period; denote this firm the *incumbent*, labeled  $i$ , and the other firm the *rival*, labeled  $r$ . The other payoff-relevant information is whether the consumer has been with the same firm throughout the past or whether he has been switching supplier one or more times previously; denote the consumer *loyal* in the former case, labeled  $\ell$ , and *disloyal* in the latter case, labeled  $d$ .

Now, let  $p_{ht}^j$  denote the price offered, in equilibrium, by firm  $j$  to an  $h$ -type consumer in period  $t$ , with  $j \in \{i, r\}$ ,  $h \in \{\ell, d\}$ , and  $t \in \{2, \dots, T\}$ . It turns out that a rival firm always offers the same price independent of a consumer's loyalty; thus, define  $p_i^r \equiv p_{\ell t}^r = p_{dt}^r$ , for  $t \in \{2, \dots, T\}$ . In period 1, there is no purchase history to condition the price on; thus, define  $p_1$  as the period-1 price offered by firms in equilibrium. The equilibrium is a subgame-perfect Nash equilibrium. We have:

*Proposition 1:* The unique subgame-perfect equilibrium of this game has the following prices:

$$p_t^f = \min \{p \mid \pi(p) + \sum_{\tau=t+1}^T \delta^{\tau-t} \pi(p_{d\tau}^i) = 0\}, t \in \{2, \dots, T\}; \quad (3.1)$$

$$p_{dt}^i = \{p \mid S(p) = S(p_t^f) - \alpha s\}, t \in \{2, \dots, T\}; \quad (3.2)$$

$$p_{\ell t}^i = \{p \mid S(p) = S(p_t^f) - s_t\}, t \in \{2, \dots, T\}, \quad (3.3)$$

with  $s_T = s$  and  $s_t = s[1 - \delta(1 - \alpha)]$ ,  $t \in \{2, \dots, T - 1\}$ ; and

$$p_1 = \min \{p \mid \pi(p) + \sum_{\tau=2}^T \delta^{\tau-1} \pi(p_{\ell\tau}^i) = 0\}. \quad (3.4)$$

The proof of Proposition 1 is in Appendix A. This equilibrium has several interesting features. First, as already mentioned, a rival firm's offer is the same to both consumer categories. This is because, independent of a consumer's purchase history so far, the rival's profit prospects are the same: If the consumer switches today, then he will be a disloyal in the future.

Second, a loyal consumer, in considering whether to switch and thus become disloyal, takes into account his benefit from reduced switching costs in the future. However, his *net switching costs*, i.e., his switching costs net of the discounted value of this future benefit, denoted  $s_t$ , is constant across time except for the last period, in which there is no future to consider. A disloyal consumer, on the other hand, can *not* reduce his future switching costs by switching further.

Third, Proposition 1, together with Observation 1, make it possible for us to establish how the equilibrium prices relate to each other. We have:

*Proposition 2:*

(i)  $p_2^f < \dots < p_T^f = c$ ;

(ii)  $c < p_{d2}^i < \dots < p_{dT}^i$ ;

(iii)  $c < p_{\ell 2}^i < \dots < p_{\ell T}^i$ ;

(iv)  $c < p_{dt}^i \leq p_{\ell t}^i$ ,  $t \in \{2, \dots, T\}$ , with the second inequality strict if and only if  $\delta < 1$  or  $t = T$ ;

(v)  $p_1 < p_2^f < c$ .

In the Proof, which is found in Appendix A, we apply Observation 1 to (3.1)



and (3.2) to prove the first inequality in (iv); the rest, then, follows easily. A direct consequence of parts (iii) and (v) of Proposition 2 is that price increases along the equilibrium path and is above costs, except in period 1:  $p_1 < c < p_{e2}^i < \dots < p_{eT}^i$ . Thus, being an incumbent is profitable right from the start.

We have so far considered the case of a finite  $T$ . We conclude this Section by reporting on the extensions of the two Propositions above to the case of an infinite horizon. In this case, prices are stationary and, with the obvious notation, we state the following Corollary, without proof:

*Corollary:* Suppose  $\delta < 1$ . In case of an infinite horizon,  $T = \infty$ , there exists an equilibrium of the model in which:

(i) prices are given by the following set of equations:

$$\pi(p^r) + \frac{\delta}{1 - \delta} \pi(p_d^i) = 0,$$

$$S(p_d^i) = S(p^r) - \alpha s,$$

$$S(p_\ell^i) = S(p^r) - s[1 - \delta(1 - \alpha)], \text{ and}$$

$$\pi(p_1) + \frac{\delta}{1 - \delta} \pi(p_\ell^i) = 0;$$

(ii) the prices are related in the following way:  $p_1 < p^r < c < p_d^i < p_\ell^i$ .

As is usual with an infinite horizon, this equilibrium is not unique; however, it has the virtue of being the limit of the unique finite-horizon equilibrium. The comparative-statics results in the next Section are proved for the case of  $3 \leq T < \infty$  only. However, they are true also for the equilibrium we have outlined for the case of  $T = \infty$ ; in fact, the proofs are simpler, and the results, like for the case of  $T = 3$ , do not hinge on Assumption 2. For more details on the  $T = \infty$  case, see Appendix C.

#### 4. ANALYSIS

In this section, we explore the consequences on prices and welfare of changes in the composition of consumer switching costs. With regard to prices, we find that

the introductory price  $p_1$  is lower, the higher is transaction costs' share of consumer switching costs:

*Theorem 1:* If the fraction of transaction costs out of total consumer switching costs increases, then the first-period price decreases; i.e.,  $dp_1/d\alpha < 0$ .

With regard to welfare, we find that society is better off the smaller is transaction costs' share of consumer switching costs. Define *total consumer surplus*, TS, as the discounted sum of the surpluses arising from each of the T prices consumers are subject to in equilibrium:

$$TS \equiv S(p_1) + \sum_{t=2}^T \delta^{t-1} S(p_{2t}^i) \quad (4.1)$$

Since firms' profits equal zero, welfare equals total consumer surplus. We state our welfare result as:

*Theorem 2:* If the fraction of transaction costs out of total consumer switching costs increases, then social welfare is reduced; i.e.  $dTS/d\alpha < 0$ .

The proofs of Theorems 1 and 2 are in Appendix A. But we record here the key step, also proved in Appendix A:

*Proposition 3:*

$$\frac{dp_{2t}^i}{d\alpha} > 0, t \in \{2, \dots, T-1\}.$$

In the introductory Section above, we argued that the positive effect on  $p_{2t}^i$  of an increase in  $\alpha$  is the sum of a negative rival effect and a positive consumer effect. We can, at this stage, be more specific and write, after differentiating (3.3):

$$\frac{dp_{2t}^i}{d\alpha} = \frac{1}{D(p_{2t}^i)} [D(p_t^f) \frac{dp_t^f}{d\alpha} + \delta s].$$

Here, the first term in square brackets is the rival effect, which is negative because  $dp_t^f/d\alpha$  is [see Lemma 1(i) in Appendix A]. The second term is the consumer effect, which is clearly positive. Proving that the latter dominates the former

involves, therefore, verifying a lower bound on  $dp_t^r/d\alpha$ . This is done in two steps. First, we establish that  $dp_{d,t+1}^i/d\alpha > 0$  and use this to get a lower bound on  $dp_{t+1}^r/d\alpha$  [see Lemma 1(ii) in Appendix A]. Second, by using Assumption 2 and Observation 3 [see (A.7) in Appendix A], we can apply this lower bound on  $dp_{t+1}^r/d\alpha$  in an expression for  $dp_{t+1}^i/d\alpha$  [(A.5) in Appendix A] to get a lower bound on  $dp_t^r/d\alpha$  which is stricter than the one already obtained on  $dp_{t+1}^r/d\alpha$  and which, moreover, is the desired one.

Let us conclude this Section with some discussion of Assumption 2. Its importance is not in the price dimension but rather in the time dimension. To see this, note that the rival effect may be decomposed into a further pair of consumer and rival effects [see (A.4) in Appendix A]. The latter effect, which derives from  $dp_{t+1}^r/d\alpha$ , is strengthened by an increase in  $\pi'(p_{t+1}^r)$  and weakened by an increase in  $\pi'(p_t^r)$ . Thus, we would like to have some lower bound on  $\pi'(p_t^r)/\pi'(p_{t+1}^r)$  at any time. On the other hand, any effect is stronger, the higher is  $D(p)$ , since it ultimately works through  $S'(p) = -D(p)$ . From Proposition 2(i), we know that  $p_t^r$  increases over time and, thus,  $D(p_{t+1}^r) > D(p_t^r)$ . This strengthens the effect from  $dp_{t+1}^r/d\alpha$ . To counterbalance this strengthening, we must set the required lower bound on  $\pi'(p_t^r)/\pi'(p_{t+1}^r)$  accordingly:  $\pi'(p_t^r)/\pi'(p_{t+1}^r) > D(p_t^r)/D(p_{t+1}^r)$ , or:  $R(p_t^r) > R(p_{t+1}^r)$ . By Proposition 2(i) and Observation 3, this is exactly what we obtain from Assumption 2.

## 5. DISCUSSION

We conclude this chapter with commenting on a couple of aspects of our analysis not touched upon earlier.

If learning costs are to be disregarded whenever a consumer switches back to a past supplier, independent of the time gone since he was last with this firm, then there can be no decay of learning. It can, however, be argued that such decay of learning takes place in reality. In the terminology of our model, this is tantamount to consumer switching costs not only depending on whether the consumer has switched or not but on the complete purchase history. This would make

the model more complex. It is also difficult to argue in favour of such an increasingly fine-tuned price discrimination. However, if we suppose that a part of the supplier-specific knowledge never decays, we may interpret learning costs in the present model as pertaining to this part.

We have assumed here that firms are perfectly able to distinguish loyal consumers from disloyal ones. It is, however, conceivable that this ability is not always perfect and, in particular, that it correlates with the fraction  $\alpha$  of transaction costs.<sup>12</sup> The reason for the latter might be that firms' information on consumers is more encompassing in markets such as markets for rentals, where also the fraction of transaction costs is high, whereas firms know less about consumers, including their past loyalty, in markets such as the market for word processors, where the fraction of learning costs is high. But note that, even putting this argument at one side for the moment, increasing the fraction  $\alpha$  increases the scope for price discrimination by strengthening a consumer's preferences for his incumbent supplier, whether the consumer is loyal or disloyal. Thus, differences in the ability to price discriminate will, if they are correlated with  $\alpha$  as indicated above, only strengthen the results of our model.

## APPENDIX A

*Proof of Proposition 1:* A consumer of type  $h$ ,  $h \in \{\ell, d\}$ , that is offered the two prices  $p_{ht}^r$  and  $p_{ht}^i$  in period  $t$  will switch from firm  $i$  to firm  $r$  if and only if the difference in consumer surplus between the two prices,  $S(p_{ht}^r) - S(p_{ht}^i)$ , is strictly greater than the (net) costs of switching. In equilibrium, the rival firm offers the lowest price it can afford without making (overall) losses, while the incumbent matches the rival's price so that the difference in consumer surplus exactly equals the consumer's costs of switching. The procedure for finding the unique equilibrium in each subgame is given in Appendix B.

As noted, the consumer stays with the incumbent in every subgame. If, however, a rival were to attract a consumer away from an incumbent in period  $t$ , the

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<sup>12</sup>Thanks to Garth Saloner for raising this issue.

rival will earn  $\pi(p_{dt}^i)$  in every period  $\tau$  thereafter, since after a switch the firm will be an incumbent with a disloyal consumer. This is true whether the consumer is loyal or not before this switch; thus,  $p_{dt}^i = p_{dt}^r = p_t^r$ . The non-negative profit constraint will be binding on the rival's offer. Thus, we get (3.1). Since a disloyal consumer has switching costs equal to  $\alpha s$  at any time, (3.2) follows.

Consider, next, the net switching costs  $s_t$  of a loyal consumer in period  $t$ . Clearly,  $s_T = s$ . We find  $s_t$ , for  $t \in \{2, \dots, T-1\}$ , by calculating the benefit, in terms of future increases in consumer surpluses, of switching:

$$\begin{aligned} s - s_t &= \sum_{\tau=t+1}^T \delta^{\tau-t} [S(p_{d\tau}^i) - S(p_{d\tau}^r)] = \sum_{\tau=t+1}^T \delta^{\tau-t} \{[S(p_\tau^r) - \alpha s] - [S(p_\tau^r) - s_\tau]\} \\ &= \sum_{\tau=t+1}^T \delta^{\tau-t} (s_\tau - \alpha s) = \delta(s_{t+1} - \alpha s) + \delta \left[ \sum_{\tau=t+2}^T \delta^{\tau-t-1} (s_\tau - \alpha s) \right] \\ &= \delta(s_{t+1} - \alpha s) + \delta(s - s_{t+1}) = \delta(1 - \alpha)s. \end{aligned}$$

Thus,  $s_t = s[1 - \delta(1 - \alpha)]$ ,  $t \in \{2, \dots, T-1\}$ , and we get (3.3). Finally, (3.4) follows from price competition in the standard way. *QED.*

*Proof of Proposition 2:* We start with part (iv). The second inequality here follows from a comparison of (3.2) and (3.3), noting that  $S' < 0$ , and that  $s_t > \alpha s$  if  $\delta < 1$  or  $t = T$ , with  $s_t = \alpha s$  otherwise. For the first inequality in part (iv), we have, for  $t \in \{2, \dots, T\}$ :

$$\begin{aligned} S(p_{dt}^i) &= S(p_t^r) - \alpha s, \text{ by (3.2) in Proposition 1;} \\ &\leq S(c) - \pi(p_t^r) - \alpha s, \text{ by Observation 1;} \\ &= S(c) + \left[ \sum_{\tau=t+1}^T \delta^{\tau-t} \pi(p_{d\tau}^i) \right] - \alpha s, \text{ by (3.1) in Proposition 1;} \\ &\leq S(c) + \left\{ \sum_{\tau=t+1}^T \delta^{\tau-t} [S(c) - S(p_{d\tau}^i)] \right\} - \alpha s, \text{ by Observation 1.} \end{aligned}$$

Thus, by rearranging:

$$\sum_{\tau=t}^T \delta^{\tau-t} S(p_{d\tau}^i) \leq \left[ \sum_{\tau=t}^T \delta^{\tau-t} \right] S(c) - \alpha s.$$

For this to be true for every  $t \in \{2, \dots, T\}$ , we must have that:

$$S(p_{dt}^i) < S(c), \forall t \in \{2, \dots, T\},$$

where the inequality is strict because  $\alpha s > 0$ . Now, since  $S' < 0$ , we have that  $p_{dt}^i > c$ ,  $t \in \{2, \dots, T\}$ , as claimed in part (iv).

In turn, this implies that  $\pi(p_{dt}^i) > 0$ . Thus, the summation in (3.1) is positive and decreases with  $t$  until it is zero at  $t = T$ . This implies that  $p_t^r$  increases with  $t$  until it equals  $c$  at  $t = T$ ; we have part (i). The first inequalities in parts (ii) and (iii) are already established. The rest of these inequalities follow from (3.2) and (3.3). (3.2) implies that  $p_{dt}^i$  is obtained from  $p_t^r$  through a time-invariant shift, for  $t \in \{2, \dots, T\}$ . (3.3) implies that the same is true for  $p_{dt}^j$ , for  $t \in \{2, \dots, T-1\}$ . Since  $s_{T-1} < s_T$ , we also have the last inequality in part (iii). Finally, the second inequality in part (v) follows from part (i). The first inequality in part (v) is obtained by a comparison of (3.1) and (3.4). Since, by part (iv), every term in the summation in (3.1), with  $t = 2$ , is positive but less than or equal to every term in (3.4) and, in addition, the summation in (3.4) has one extra positive term,  $\pi(p_{d2}^j)$ , the summation in (3.4) is greater than the one in (3.1). Thus,  $p_1 < p_2^r$ , as claimed. *QED.*

Before proving Proposition 3, we state and prove Lemma 1. Note that, in the proofs that follow, we make extensive use of  $R(p)$  as shorthand for  $\pi'(p)/D(p)$ .

*Lemma 1:*

- (i)  $\frac{dp_t^r}{d\alpha} < 0$ ,  $t \in \{2, \dots, T-1\}$ ,  $\frac{dp_T^r}{d\alpha} = 0$ ;
- (ii)  $\frac{dp_t^r}{d\alpha} > -\frac{s}{D(p_t^r)}$ ,  $t \in \{3, \dots, T\}$ .

*Proof:* (i) From differentiation in (3.2), using  $S'(p) = -D(p)$ , we have:

$$\frac{dp_{dt}^i}{d\alpha} = \frac{1}{D(p_{dt}^i)} [D(p_t^r) \frac{dp_t^r}{d\alpha} + s], t \in \{2, \dots, T\}. \quad (\text{A.3})$$

From differentiation in (3.1), we have, for  $t \in \{2, \dots, T-1\}$ , using (A.3):

$$\begin{aligned} \frac{dp_t^r}{d\alpha} &= -\frac{1}{\pi'(p_t^r)} \left\{ \sum_{\tau=t+1}^T \delta^{\tau-t} R(p_{d\tau}^i) [D(p_\tau^r) \frac{dp_\tau^r}{d\alpha} + s] \right\} \\ &= -\frac{\delta}{\pi'(p_t^r)} \left\{ R(p_{d,t+1}^i) [D(p_{t+1}^r) \frac{dp_{t+1}^r}{d\alpha} + s] - \pi'(p_{t+1}^r) \frac{dp_{t+1}^r}{d\alpha} \right\} \end{aligned}$$

$$= \frac{\delta}{\pi'(p_t^r)} \left\{ D(p_{t+1}^r) \frac{dp_{t+1}^r}{d\alpha} [R(p_{t+1}^r) - R(p_{d,t+1}^i)] - sR(p_{d,t+1}^i) \right\}. \quad (\text{A.4})$$

By Assumption 3 and Proposition 2(i) and (ii),  $p_{t+1}^r \leq c < p_{d,t+1}^i < p_m$ . By Observation 2,  $R(p_{t+1}^r) \geq 1 > R(p_{d,t+1}^i) > 0$ . Thus, the square-bracketed expression in (A.4) is positive and the second term in curly brackets is negative. This implies that:  $dp_t^r/d\alpha < 0$ , if  $dp_{t+1}^r/d\alpha \leq 0$ . The claim follows recursively by noting, from differentiation in (3.1) with  $t = T$ , that  $dp_T^r/d\alpha = 0$ .

(ii) Since  $dp_t^r/d\alpha < 0$  by part (i), we have, from differentiation in (3.1):

$$\sum_{\tau=t+1}^T \delta^{\tau-t} \frac{\pi'(p_{d\tau}^i)}{\pi'(p_t^r)} \frac{dp_{d\tau}^i}{d\alpha} > 0, t \in \{2, \dots, T-1\}.$$

Since this holds for every  $t \in \{2, \dots, T-1\}$ , we have that:

$$dp_{d\tau}^i/d\alpha > 0, t \in \{3, \dots, T\}.$$

We now have the claim by inserting from (A.3) and rearranging. *QED*.

*Proof of Proposition 3:* Differentiation in (3.3), using  $S'(p) = -D(p)$ , gives, for  $t \in \{2, \dots, T-1\}$ :

$$\frac{dp_{\ell t}^i}{d\alpha} = \frac{1}{D(p_{\ell t}^i)} \left[ D(p_t^r) \frac{dp_t^r}{d\alpha} + \delta s \right].$$

Insertion from (A.4) implies:

$$\begin{aligned} \frac{dp_{\ell t}^i}{d\alpha} &= \frac{\delta}{D(p_{\ell t}^i)} \left\{ \frac{1}{R(p_t^r)} \left[ D(p_{t+1}^r) \frac{dp_{t+1}^r}{d\alpha} (R(p_{t+1}^r) - R(p_{d,t+1}^i)) - sR(p_{d,t+1}^i) \right] + s \right\} \\ &= \frac{\delta}{D(p_{\ell t}^i)R(p_t^r)} \left\{ \left[ \pi'(p_{t+1}^r) \frac{dp_{t+1}^r}{d\alpha} + sR(p_t^r) \right] - R(p_{d,t+1}^i) \left[ D(p_{t+1}^r) \frac{dp_{t+1}^r}{d\alpha} + s \right] \right\}. \quad (\text{A.5}) \end{aligned}$$

The fraction outside curly brackets in (A.5) is positive. Moreover,  $R(p_{d,t+1}^i) < 1$ , by Observation 2, since  $p_{d,t+1}^i > c$  by Proposition 2(iv). Thus, the expression inside curly brackets in (A.5) is positive – and, therefore,  $dp_{\ell t}^i/d\alpha$  is positive – if the first square-bracketed expression in (A.5) is greater than the second one, or if:

$$\pi'(p_{t+1}^r) \frac{dp_{t+1}^r}{d\alpha} + sR(p_t^r) > D(p_{t+1}^r) \frac{dp_{t+1}^r}{d\alpha} + s. \quad (\text{A.6})$$

By Proposition 2(i),  $p_T^r = c$ , implying  $dp_T^r/d\alpha = 0$ . Thus, for  $t = T-1$ , (A.6) reduces to:  $R(p_{T-1}^r) > 1$ , which is true by Proposition 2(i) and Observation 2. For  $t < T-1$ ,  $R(p_{t+1}^r) > 1$  by Proposition 2(i) and Observation 2; thus, we may rewrite (A.6) as:

$$\frac{dp_{t+1}^r}{d\alpha} > -\frac{s}{D(p_{t+1}^r)} \frac{R(p_t^r) - 1}{R(p_{t+1}^r) - 1}. \quad (\text{A.7})$$

Since, for  $t + 1 \leq T - 1$ ,  $p_t^r < p_{t+1}^r < c$ , by Proposition 2(i), the second fraction on the righthand side of (A.6) is greater than 1, by Observations 2 and 3 and Assumption 2. Thus, by Lemma 1(ii), the inequality in (A.6) holds for  $t + 1 \in \{3, \dots, T - 1\}$ , i.e., for  $t \in \{2, \dots, T - 2\}$ .<sup>13</sup> Since the case of  $t = T - 1$  was done above, we are through. QED.

*Proof of Theorem 1:* Differentiation in (3.4) gives:

$$\frac{dp_1}{d\alpha} = -\frac{1}{\pi'(p_1)} \left[ \sum_{\tau=2}^T \delta^{\tau-1} \pi'(p_{\ell\tau}^j) \frac{dp_{\ell\tau}^j}{d\alpha} \right]. \quad (\text{A.8})$$

Since the summation is positive by Proposition 3, this expression is negative. QED.

*Proof of Theorem 2:* We differentiate in (4.1), using that  $S'(p) = -D(p)$ , to get:

$$\frac{dT S}{d\alpha} = -D(p_1) \frac{dp_1}{d\alpha} - \sum_{\tau=2}^T \delta^{\tau-1} D(p_{\ell\tau}^j) \frac{dp_{\ell\tau}^j}{d\alpha}.$$

Insertion from (A.8) gives:

$$\frac{dT S}{d\alpha} = \sum_{\tau=2}^T \delta^{\tau-1} D(p_{\ell\tau}^j) \frac{dp_{\ell\tau}^j}{d\alpha} \left[ \frac{R(p_{\ell\tau}^j)}{R(p_1)} - 1 \right]. \quad (\text{A.9})$$

By Proposition 2(iii) and (v),  $p_1 < c < p_{\ell\tau}^j$ ,  $\tau \in \{2, \dots, T\}$ . Thus, by Observation 2,  $R(p_1) > 1 > R(p_{\ell\tau}^j)$ ,  $\tau \in \{2, \dots, T\}$ . Therefore, the bracketed term in (A.9) is negative. Since  $dp_{\ell\tau}^j/d\alpha > 0$ ,  $\tau \in \{2, \dots, T - 1\}$ , by Proposition 3, and  $dp_{\ell T}^j/d\alpha = 0$ , the claim follows. QED.

## APPENDIX B

The following two assumptions, one resolving consumer indifference, the other restricting firms' strategies, are necessary to ensure existence and uniqueness, respectively, of the equilibrium.

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<sup>13</sup>This is the sole place where we make use of Assumption 2. As is clear from the text here, the argument making use of this Assumption is neither applicable nor necessary in the case of  $T = 3$ .



*Assumption 4:* If a consumer is indifferent<sup>14</sup> among the two offers in period  $t$ , then: (i) if  $t = 1$ , he chooses each firm with probability  $\frac{1}{2}$ ; (ii) if  $t \in \{2, \dots, T\}$ , he chooses with probability 1 the same firm that he chose in period  $t - 1$ .

*Assumption 5:* A firm will not offer a consumer a price that would induce a loss<sup>15</sup> if the other firm were to fail to offer the consumer a price at the same time.

Part (i) of Assumption 4 is straightforward. Part (ii) is needed in order for firms to maximise profit over a closed set. Without this, existence of an equilibrium would not be guaranteed. Assumption 5 excludes some, but not all, weakly dominated strategies. A similar formulation is found in Hart and Tirole (1990, Appendix B) in a model of price-setting duopolists with different marginal costs.<sup>16</sup>

To establish existence of a unique equilibrium for this game, one must establish existence and uniqueness in every subgame. It suffices, though, to look in detail at a single subgame, since the arguments are the same. Therefore, let  $h = \ell$  and  $t = T$ ; i.e., we consider last-period offers to loyal consumers. Since  $t = T$ , there is no future benefit from switching, and net switching costs are clearly  $s_T = s$ . Thus, the consumer switches if and only if:

$$S(p_{\ell T}^r) - S(p_{\ell T}^l) > s. \quad (\text{B.1})$$

There is, furthermore, no future profit for the rival accruing from attracting the consumer. We have:

*Claim:* The unique equilibrium prices offered in period  $T$  to loyal consumers are: (i)  $p_{\ell T}^r = c$ ; and (ii)  $p_{\ell T}^l = \{p \mid S(p) = S(c) - s\}$ .

<sup>14</sup>Indifferent, in the sense that both offers give him the same total discounted surplus provided equilibrium strategies are played in the rest of the game.

<sup>15</sup>Loss, in the sense that total discounted profit is negative provided equilibrium strategies are played in the rest of the game.

<sup>16</sup>It should be noted that, although Assumption 5 may seem sound, it is not completely satisfactory, since it leaves us with a unique equilibrium which itself includes weakly dominated strategies. A way around the problem, not explored here, would be to discretize the action space.

*Proof: Existence.* First, we show that  $p = p_{\ell T}^i$  is the incumbent's best response to  $p_{\ell T}^r = c$ . By (B.1), a price  $p > p_{\ell T}^i$  would imply the consumer switching, so that the incumbent earns zero profit. A price  $p < p_{\ell T}^i$  would imply that the incumbent could increase its profit by increasing its price towards  $p_{\ell T}^i$ . Second, we show that  $p = c$  is the rival's best response to  $p_{\ell T}^i = \{p \mid S(p) = S(c) - s\}$ . If the rival sets a price  $p > c$ , it would earn zero profit, since the consumer does not switch; this is the same profit as with  $p = c$ . A price  $p < c$  would entail a negative profit.

*Uniqueness.* Note first that, if the rival offers some  $p < c$  and the incumbent does not offer any price, then the rival incurs a negative profit. Thus, the rival offering  $p < c$  is excluded by Assumption 5. Let, therefore,  $p_{\ell T}^r \geq c$ . With:

$$p > S^{-1}(S(p_{\ell T}^r) - s), \text{ or } p \leq c,$$

the incumbent earns a non-positive profit. For:

$$p \in (c, S^{-1}(S(p_{\ell T}^r) - s)],$$

the incumbent's profit increases in  $p$ . Similar reasoning for the rival implies that, in equilibrium, the two prices must satisfy:

$$S(p_{\ell T}^r) - S(p_{\ell T}^i) = s. \tag{B.2}$$

Suppose that  $p_{\ell T}^r > c$  and that (B.2) holds. Then, the rival earns zero profit and could earn positive profit by decreasing its price slightly, so this cannot be an equilibrium price. Thus,  $p_{\ell T}^r = c$ . The unique equilibrium strategies are verified. *QED.*

## APPENDIX C

We discuss here the infinite horizon case, i.e.,  $T = \infty$ . Consider, first, net consumer switching costs for this case. It is clearly stationary, as are the equilibrium prices; thus, denote it  $s'$ . We determine  $s'$  from the following equation:

$$s - s' = \frac{\delta}{1 - \delta} (s' - \alpha s),$$

where the righthand side is the discounted value of the future benefit from switching. Solving the equation, we find:  $s' = s[1 - \delta(1 - \alpha)]$ . From this, part (i) of the Corollary is straightforward. Part (ii), like Proposition 2, follows easily if we

can show that  $p_d^i > c$ . By repeated use of part (i) and of Observation 1, we get:

$$\begin{aligned} S(p_d^i) &= S(p^r) - \alpha s \leq S(c) - \pi(p^r) - \alpha s = S(c) + [\delta/(1 - \delta)]\pi(p_d^i) - \alpha s \\ &\leq S(c) + [\delta/(1 - \delta)][S(c) - S(p_d^i)] - \alpha s \\ &= [1/(1 - \delta)]S(c) - [\delta/(1 - \delta)]S(p_d^i) - \alpha s. \end{aligned}$$

By rearranging, we find that:

$$S(p_d^i) \leq S(c) - (1 - \delta)\alpha s,$$

which, since the second term on the righthand side is strictly negative when  $\delta < 1$ , verifies that  $p_d^i > c$ . The steps to show the rest of part (ii) are the same as in the proof of Proposition 2, in Appendix A.

The equivalent to Lemma 1, in Appendix A, for the case of  $T = \infty$ , is:

$$-\frac{s}{D(p^r)} < \frac{dp^r}{d\alpha} < 0. \quad (\text{C.1})$$

To see that this is true, we differentiate the first two equations in the Corollary to get, after some labour (which recapitulates the proof of Lemma 1):

$$\frac{dp^r}{d\alpha} = -\frac{s}{D(p^r)} \frac{R(p_d^i)}{(1 - \delta)R(p^r) + R(p_d^i)}. \quad (\text{C.2})$$

Here, both fractions are positive; thus,  $dp^r/d\alpha < 0$ . Moreover, the second fraction is less than 1 (when  $\delta < 1$ ); thus,  $dp^r/d\alpha > -s/D(p^r)$ . This verifies (C.1).

Along the lines of the proof of Proposition 3, in Appendix A, we obtain, from the third equation in the Corollary and from (C.2):

$$\frac{dp_d^i}{d\alpha} = \frac{s\delta(1 - \delta)}{D(p_d^i)} \frac{R(p^r) - R(p_d^i)}{(1 - \delta)R(p^r) + \delta R(p_d^i)}. \quad (\text{C.3})$$

From part (ii) of the Corollary,  $p^r < c < p_d^i$ . Thus, from Observation 2,  $R(p^r) > 1 > R(p_d^i)$ . Applying this to (C.3) establishes that  $dp_d^i/d\alpha > 0$ .<sup>17</sup>

From the fourth equation in the Corollary, we get:

$$\frac{dp_1}{d\alpha} = -\frac{\delta}{1 - \delta} \frac{\pi'(p_d^i)}{\pi'(p_1)} \frac{dp_d^i}{d\alpha}. \quad (\text{C.4})$$

This is negative, since  $dp_d^i/d\alpha > 0$ . Thus, Theorem 1 holds for  $T = \infty$ .

The expression for total consumer surplus becomes, in this case:

$$TS = S(p_1) + [\delta/(1 - \delta)]S(p_d^i).$$

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<sup>17</sup>Notice that proving this result, in the present case, makes no use of Assumption 2.

When differentiating this, making use of (C.4), we obtain:

$$\frac{dT}{d\alpha} = \frac{\delta}{1-\delta} D(p_2^i) \frac{dp_2^i}{d\alpha} \left[ \frac{R(p_2^i)}{R(p_1)} - 1 \right].$$

Like in the proof of Theorem 2 in Appendix A, this expression is negative. Thus, also Theorem 2 holds for  $T = \infty$ .

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