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**ESSAYS ON THE THEORY OF
THE LABOUR-MANAGED FIRM**

by

Jan Erik Askildsen

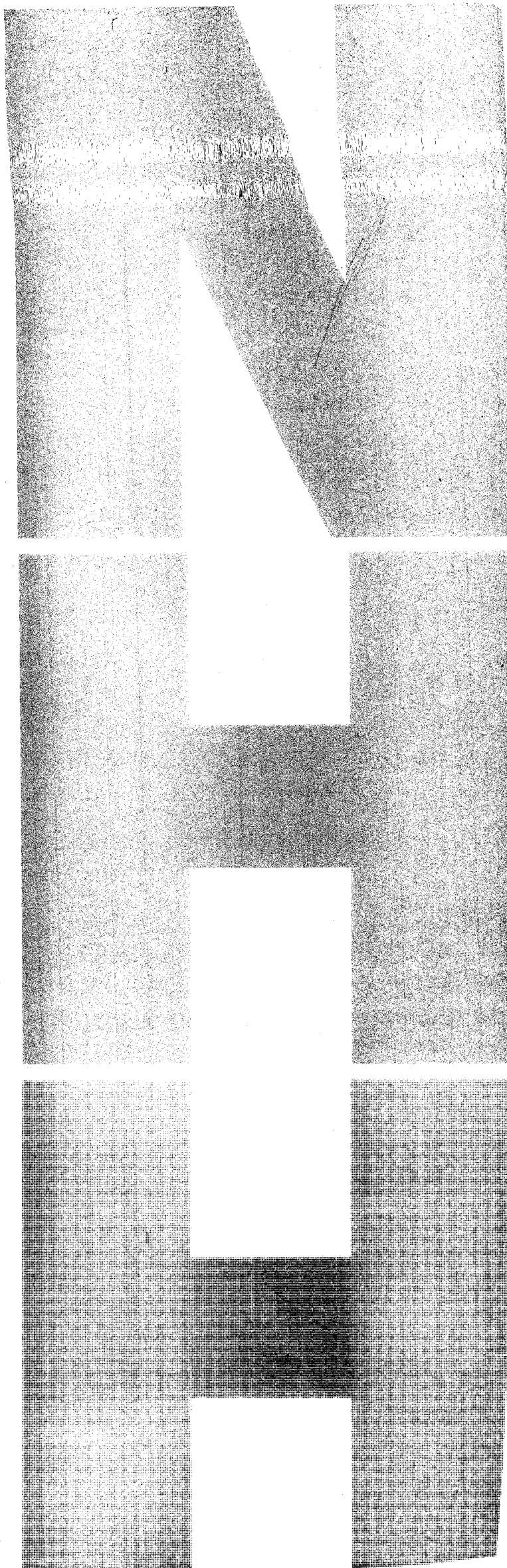


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ESSAYS ON THE THEORY OF THE LABOUR-MANAGED FIRM.



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ESSAYS ON THE THEORY OF THE LABOUR-MANAGED FIRM.

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INTRODUCTION

1. BACKGROUND.

During recent years there has been among academics a considerable interest in topics related to labour-management and labour-managed firms . Several theoretical and empirical studies have been carried through. Economists have analysed labour-managed firms with the intention of examining the effect on resource allocation in the economy when production activities are organised in this special manner. This thesis is intended to be a contribution to the study of some, in my opinion interesting, features of labour-managed firms. I think that the problems raised are relevant and of importance when discussing organisation of firms, and productive activity in general.

The special ground for my interest in studies of labour-managed firms is the idea that labour-management represents an interesting experiment in the search for the best way of organising production. Although several attempts to organise firms as labour-managed seem to have failed, the idea that the workers control their own working conditions should involve several desirable aspects which deserve a closer analysis. There should a priori be no reason to believe that a system where workers hire capital should do worse than a system where capital hires labour. Therefore it is of some interest to investigate why labour-management has so far carried relatively little success. Specifically I will aim at illuminating conditions for the emergence and survival of firms where at least some degree of labour-management constitutes an important feature, and investigate how such firms operate under different economic conditions. If there are special goals with respect to efficiency and income distribution, it is of interest to see how the firms can be controlled to meet these objectives. As implementation of

idealized models may create problems, it is important to see what kind of compromises which are viable and simultaneously compatible with a definition of labour-management. Then it may be the case that labour-managed firms are not as rare as it seems. I hope to make a contribution to the discussion of these problems.

I consider the idea of letting workers run the firm as an interesting social experiment. This is the main intention of writing a thesis on this subject. I do not consider the effect of labour-management on the extraction of human resources in a firm, although I think this is important. Several authors in the field of labour-management have drawn attention to the possibly better social relations within a firm when employees have a say in decisions. This may compensate for other problems arising when workers participate in decision making, which should be remembered if some postulates or results seem pessimistic. The analyses are strictly partial in the sense that only some relevant aspects are discussed, while other - possibly as least as important - matters are not considered.

2. Problems addressed.

My intention is to investigate labour-managed firms which are intended to operate also in a western capitalist economy. Much of the literature has been concerned with socialist labour-managed firms operating in a socialist economy. In many contexts it may be important to distinguish between firms in the different environments. Specifically, a labour-managed firm in a capitalist economy may meet problems which will not exist in a socialist economy, which will again affect the optimal way of organising the firm.

An important class of problems to be addressed relates to finance of labour-managed firms. I argue that establishment and operation of labour-managed firms will be impeded by a credit rationing. Credit rationing is interpreted as a situation where the firm is restricted in its choice of financial position. In particular the problem exists in capitalist economies where

labour-managed firms constitute the exception to the general way of organising production. Credit rationing will influence the optimal financial structure of the firm. As joint-stock firms dominate in capitalist economies, labour-managed firms will tend to choose the same mode of finance. A subject investigated is labour-managed firms financed by means of shares. The share financed labour-managed firm is interesting from another point of view also. If the workers own shares, the firm will find itself in a new situation with respect to distribution of the firm's income among the workers. I will try to characterize partial equilibria of share financed labour-managed firms. In particular I will be concerned with choice of employment level and different modes of finance, and possible conflicts between insiders and outsiders which may be capital suppliers and/or potentially new workers. The idea of investigating this specific problem has its root in a local event. In 1985 the workers at a shipyard in Bergen took over the firm and established Solheimsviken AS as a labour-managed share financed firm. Three articles in chapter 5 are devoted to analyses of share financed labour-managed firms. One of the articles is written together with Norman J. Ireland and Peter J. Law at the University of Warwick, Coventry, England.

Taxation of labour-managed firms is another field of interest. Existing theory concentrates on efficiency improving taxation. It aims at solving problems of finding an optimal size of the firm, and an optimal supply of labour from the workers who are members of a collective. I concentrate on the latter efficiency problem. The taxation rules derived in the literature on labour-managed firms make use of unrealistic assumptions with respect to information availability. When the workers are heterogeneous, incentive-incompatibility of the first-best taxation system causes problems as the tax rates will vary over individuals. The same objections are valid which are made against lump-sum taxation. Therefore I have chosen to analyse the taxation of workers in labour-managed firms as a second-best taxation problem. This enables me to focus also on distributional aspects of taxation in economies consisting in labour-managed firms. In addition to a normative analysis, I wish also to make

a more systematic positive analysis of the effect of different tax rules on the decisions of the firm and the workers. I want to focus on the close relation which exists between the taxation of workers and firms. Two articles in chapter 4 deal with taxation problems.

In the optimal taxation literature indirect objective functions are widely used to characterize optima and investigate effects of taxation. This framework is not widely used in the theory of labour-managed firms. I consider it appropriate to carry through an analysis of the labour-managed firm and its workers by means of indirect objective functions. Hitherto existing analyses using the duality approach have concentrated on the firm's optimization problem. I will characterize the individual workers' choice of labour supply by using expenditure functions and indirect utility functions. The article on indirect objective functions constitute chapter 3.

As a background for my analyses, I consider it valuable and necessary to conduct an overview over and discussion of existing theory and literature. I have chosen to divide these surveys into two parts. The first one is to some degree rather general, but with some concentration on employment and labour supply decisions. In the other survey article I concentrate on literature on the finance of labour-managed firms. This division is chosen for two reasons. Firstly the articles become shorter, and the different models and conclusions are more clearly set out. Secondly it enables me to concentrate on and draw attention to some problems which will be particularly dealt with in other parts of my thesis. The literature review articles are found in chapter 2.

3. Acknowledgements.

I am grateful to several persons and institutions for different kinds of support during my work with the thesis. The main part of the work has been done during my three years as a research fellow at the Norwegian School of Economics and Business Adminis-

tration (NSEBA), Department of Economics. I had the pleasure of spending one of these years at the Department of Economics, University of Warwick, Coventry, England. Apart from NSEBA, I received financial support during this year from the Norwegian Council for Scientific Research (NAVF) and the Bank of Norway. The thesis has been completed simultaneously with my job as a researcher at Center for Applied Research (SAF), and I am grateful to SAF for granting me time to do this finishing work.

Many persons have read all or part of my papers and given me valuable comments and ideas. With the danger of not being complete, I would like to thank Bjørn Sandvik and Morten Berg for comments on articles on indirect objective functions and taxation respectively. Besides from giving comments on major part of my thesis, Kjell Erik Lommerud has been an important discussion partner and source of inspiration. During the year I stayed at University of Warwick I had the pleasure of learning Norman J. Ireland and Peter J. Law to know. In addition to their hospitality, I have benefitted from their considerable knowledge of the theory of labour-managed firms, interesting discussions and comments on major parts of my work. One of the articles in my thesis is a joint paper with Ireland and Law. Kåre Petter Hagen and Karl Ove Moene have read through all of my thesis, and commented upon details as well as substantial problems which I have hopefully been able to give a better treatment in this final version. Agnar Sandmo has been my tutor in the doctoral programme. He suggested the subject to me, and has given me a lot of comments, advice, inspiration and practical help. I am very grateful to Sandmo for his positive attitude and willingness to discuss small and large problems of any kind. Highly valued typing assistance has been offered me by several persons. Peter Hansteen at NSEBA, Vibeke Farestvedt at SAF and Mandy Broom at University of Warwick have assisted at different stages and in typing different parts. I thank all of the above-mentioned, and all those who have on different occasions listened to and discussed my ideas. Lastly I would like to thank my family for general support, and patience and understanding in periods with long hours of work.

**AN OVERVIEW OF SOME LITERATURE ON ALLOCATION OF LABOUR
IN ECONOMIES CONSISTING OF LABOUR-MANAGED FIRMS.**

by

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1. INTRODUCTION

The literature on labour-managed firms (LM-firms) and economies consisting of LM-firms (LM-economy) has grown rapidly during recent years. The seminal paper by Ward (1958) examined a model of the Yugoslav LM-firm - termed the Illyrian firm¹⁾ - the hitherto largest experiment on labour-management. Subsequent important contributions considered producer cooperatives in the Soviet Union (Domar (1966)) as well as China (Sen (1966)), and during the latest years a lot of empirical studies have been made on Yugoslav LM-firms as well as LM-firms in Western countries. The basic model is, however, the one developed by Ward.

In this survey I will present what I consider the main contributions to the theory of LM-firms. The literature on allocation of capital will however not be reviewed in this article. The starting point is the Illyrian firm. But I will be much concerned with extensions and modifications of the model, and discuss how these will affect efficiency results as well as comparative statics. Comparisons with profit-maximizing firms (PM-firms) and a capitalist economy consisting of PM-firms (PM-economy) play an important role in the existing literature, and therefore similarities and discrepancies will be commented upon.

In a subsequent chapter of this thesis I will be concerned with LM-firms operating in a predominantly PM-economy. Specifically I argue that such a firm may have problems in obtaining finance, which may result in the necessity of giving some concessions to capital suppliers. As a point of reference I will therefore in this article present some of the contributions to the theory of

how an LM-firm operates under first-best conditions and in a predominantly LM-economy. My main interest is examining decisions regarding production and employment of labour. Allocation of capital and the finance of an LM-firm is discussed in a separate literature survey. Individual labour supply decisions will be discussed, although not given an extensive treatment, as the subject constitutes a major part of a subsequent chapter of this thesis.

I start out by defining an LM-firm, and thereafter I discuss the Illyrian model. Several extensions are then discussed, allowing for different ways of taking the utility level of employment into consideration, as well as individual labour supply. The effect on employment and production of introducing uncertainty will be examined also. Most of the analysis is partial and related to the representative firm. But some general equilibrium results are presented, and the possibilities of obtaining a Pareto optimal allocation of resources underlies much of the analysis.

2. DEFINITION

I will define an LM-firm as a firm where its workers own and control the firm in which they work. All important decisions regarding production, investment and employment are taken by the workers collectively. In particular they have agreed upon rules under which workers are to be recruited and dismissed. The objective of the firm is to produce one or more products in such a way that the utility of the worker-members is maximized. In most models to be discussed this is interpreted as the maximization of dividend (wage, payment) per work unit, e.g. per worker or per hour of work. Generally, of course, the workers are free to choose any maximand to which a majority can agree. Also, the workers are free to delegate some decision-making to an elected management, which is then supposed responsible to the workforce.

The definition given here is somewhat weaker than the one given by e.g. Putterman (1984), who points out that the definition should not presuppose "additional things" like egalitarianism in income distribution or direct democracy in decision making. But he assumes that the political process should be egalitarian, which is an assumption I find unnecessarily strong, as there may in several occasions be good reasons to have a differing number of votes per worker. The main point is whether the ultimate decisions rest with worker-members of the firm. But then, of course, it becomes more difficult to qualify what are the precise properties of an LM-firm. Thus, we may formulate models in which it is allowed to hire workers on short-term contracts without giving these workers voting rights, see e.g. Domar (1966) and Miyazaki (1984), or the firm may be assumed to operate under conditions where external suppliers of funds influence the firm's maximand, see e.g. McCain (1977). Also membership can be traded, as discussed by e.g. Sertel (1982, ch. 2).

Although some of these qualifications may involve a dilution of the pure idea of workers' self-management, I think they should be included among acceptable concessions. There are two reasons for

this. Firstly, the phenomena will most probably be present in an LM-economy, unless the economic system is organized in a particularly favourable and ideal manner. Such an economy may be useful as a point of reference. But the theory of the LM-firm and the LM-economy should also have an applied perspective in the sense that our models capture reasonable characteristics of a "real" firm. Secondly, the deviations from the pure model of an LM-firm are not necessarily as serious as they may seem at first glance. Existence of hired workers may simply be the result of an inter-firm labour market for the exchange of labour-services among self-managed firms, and an inegalitarian political structure within a firm - unequal votes per worker - may reflect seniority which will always exist. Therefore, I find it reasonable to let the characteristics mentioned be part of a definition of an LM-firm. However, we may define a subset of LM-firms which we call "pure LM-firms". Most of the literature reviewed in this article belongs to the theory of the pure LM-firm, which also reflects the state of the art. What is commonly denoted the Illyrian firm, is such a pure model.

3. THE ILLYRIAN FIRM.

As it was formulated in the seminal work of Ward (1958), and further analysed by Domar (1966), Vanek (1970) and Meade (1972), the LM-firm is supposed to maximize sales minus non-labour costs per worker. The time perspective is short run, and each worker is assumed to supply a given (equal) amount of labour. The level of employment (membership) is the single variable input, and it alone will determine the level of production. Thus, the Illyrian firm is assumed to

$$(1) \quad \text{Max}_N \quad y = \frac{PX - rK}{N},$$

where y is dividend paid to each worker, P is price, X is production, where we write $X = f(\bar{K}, N)$, rK is non-labour costs which are assumed fixed in the short run, and N is number of workers (members) of the firm. With usual restrictions on production possibilities, involving positive and decreasing marginal productivities, the LM-firm's optimal allocation of labour is given by equality between dividend per worker and value marginal product of labour (VMP_N), i.e.

$$(2) \quad y = VMP_N = P \cdot \frac{\partial f(\bar{K}, N)}{\partial N} = Pf'_N.$$

The optimal choice of employment can be shown in a diagram, see Figure 1.

This gives rise to the well known negatively sloping supply curve of LM-firms. Doing comparative statics on the first order conditions yields

$$(3) \quad \frac{\partial N}{\partial P} < 0 \Rightarrow \frac{\partial X}{\partial P} < 0 \text{ when } rK = \bar{r}\bar{K}.$$

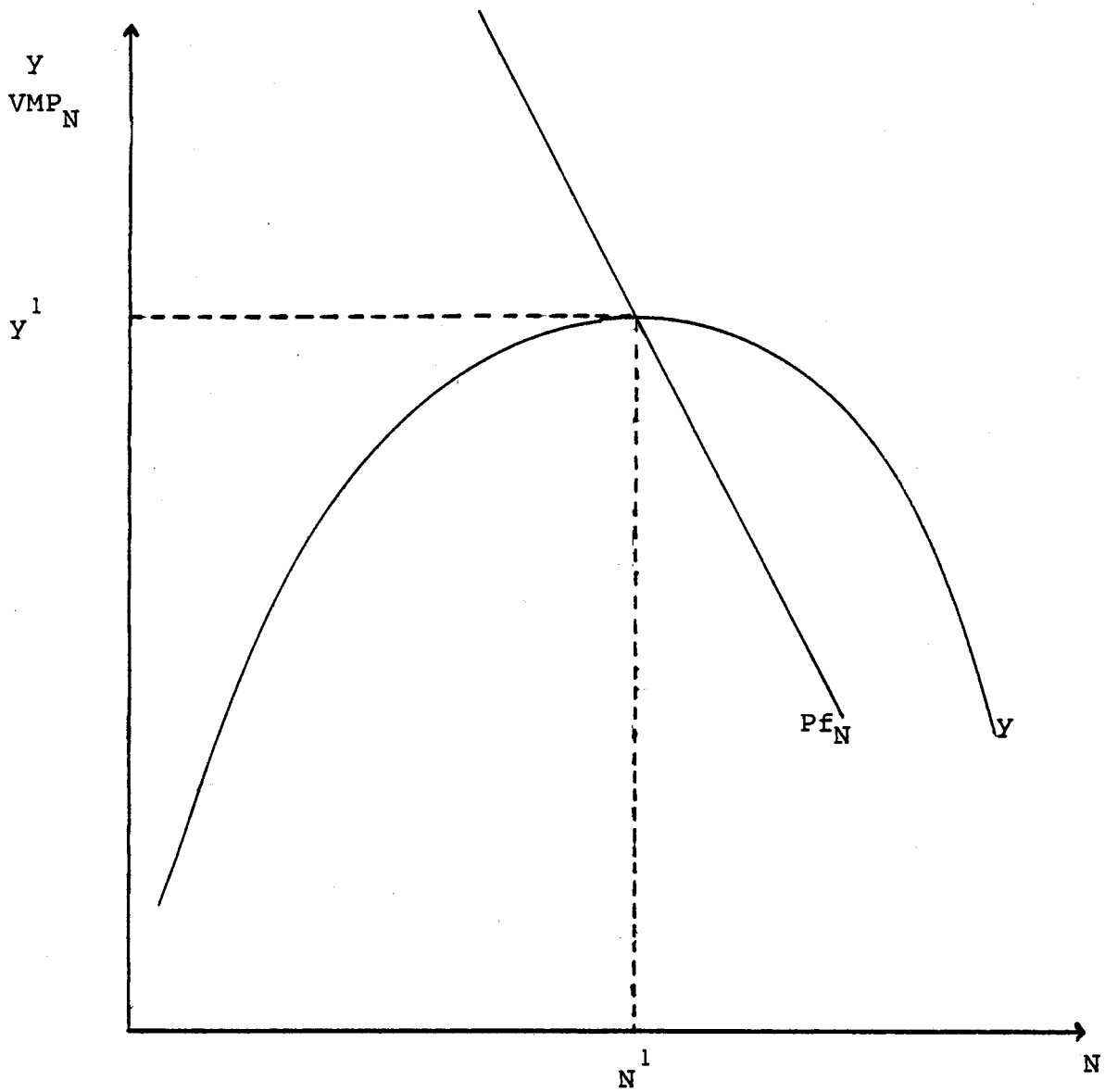


Figure 1: Optimal employment level of the Illyrian firm.

We show this in a diagram similar to Fig. 1, see Figure 2.

Although the result does not seem unambiguous by referring to Figure 2, it should be noted that it will hold whenever $\frac{X}{N} > \frac{\partial f(K, N)}{\partial N}$, which is met for the maximand (1). The explanation is simple. As long as non-labour factors are given, and labour productivity is decreasing, it will always pay remaining workers to reduce the size of the firm and capture economic rent, or value due theoretically to the fixed factors. However, if the costs for the use of factors are strictly positive (fixed costs, lump-sum taxes), the burden of these costs per worker increases as membership contracts until at some stage it will not pay to decrease the level of employment anymore. But if price increases, each worker can tolerate an increased fixed cost burden, which makes a further reduction of the employment level profitable.

This line of reasoning indicates, as it is pointed out by Greenwald (1979) and Miyazaki and Neary (1983), that the reaction of the LM-firm to a price change can be decomposed into a Slutsky-equation, showing a pure price effect for a given compensated utility (income) and a fixed cost effect, given price. According to Figure 2, the latter dominates, and the two effects can be shown as in Figure 3 by noting that the utility indifference curves are horizontal as employment does not matter.

The movement from N^0 to N^1 represents the price effect, which is positive, while the employment reduction, N^1 to N^2 , is explained by the fixed cost effect. \tilde{y}^1 is the fixed cost compensated dividend curve. Obviously, by using this way of reasoning, the effect of an increased fixed cost burden is to increase employment. When marginal productivity of labour is decreasing (the production function is strictly concave), the fixed cost effect will dominate the price effect.

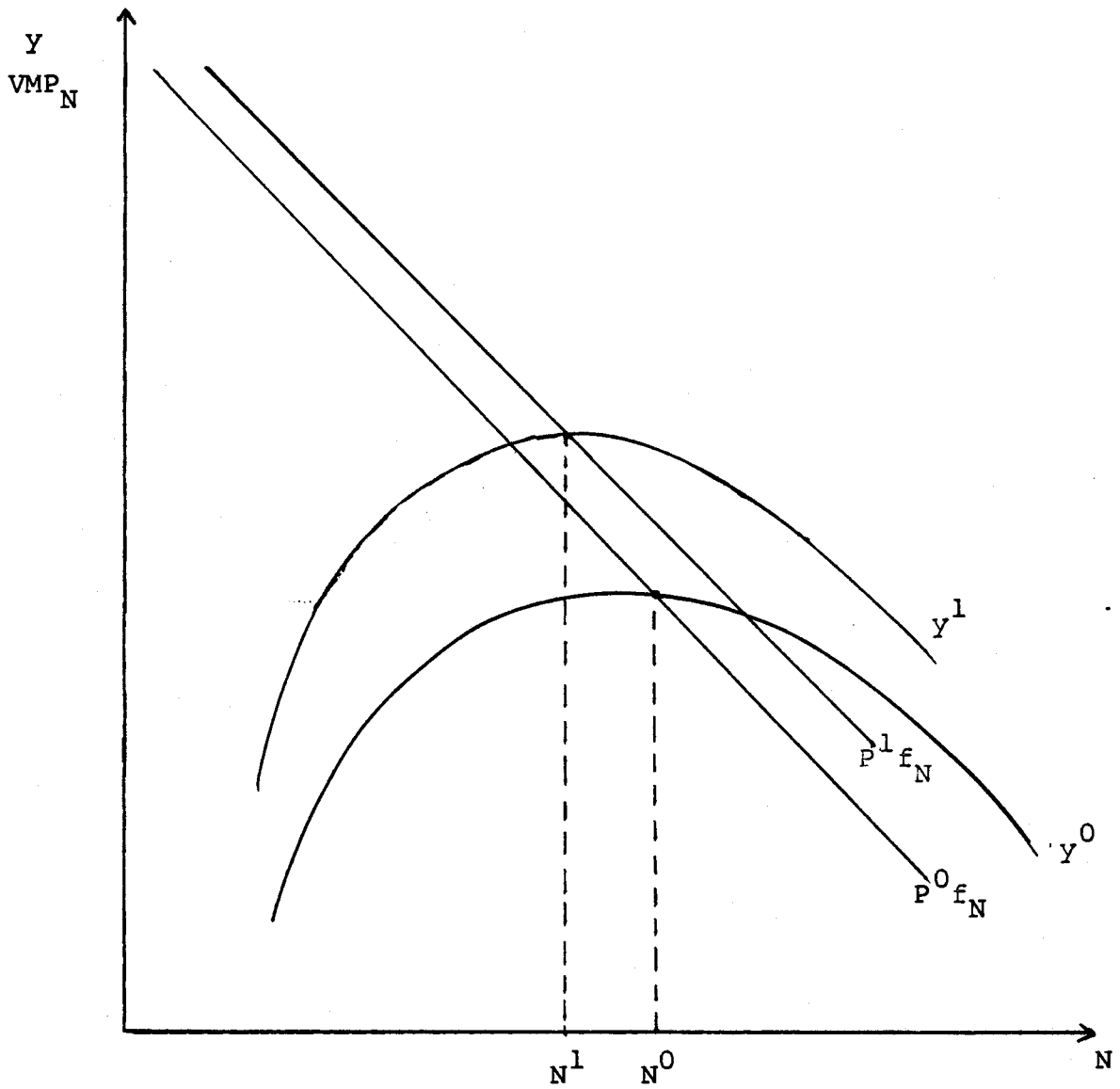


Figure 2: Employment levels of the Illyrian firm, with price levels P^0 and P^1 , $P^1 > P^0$.

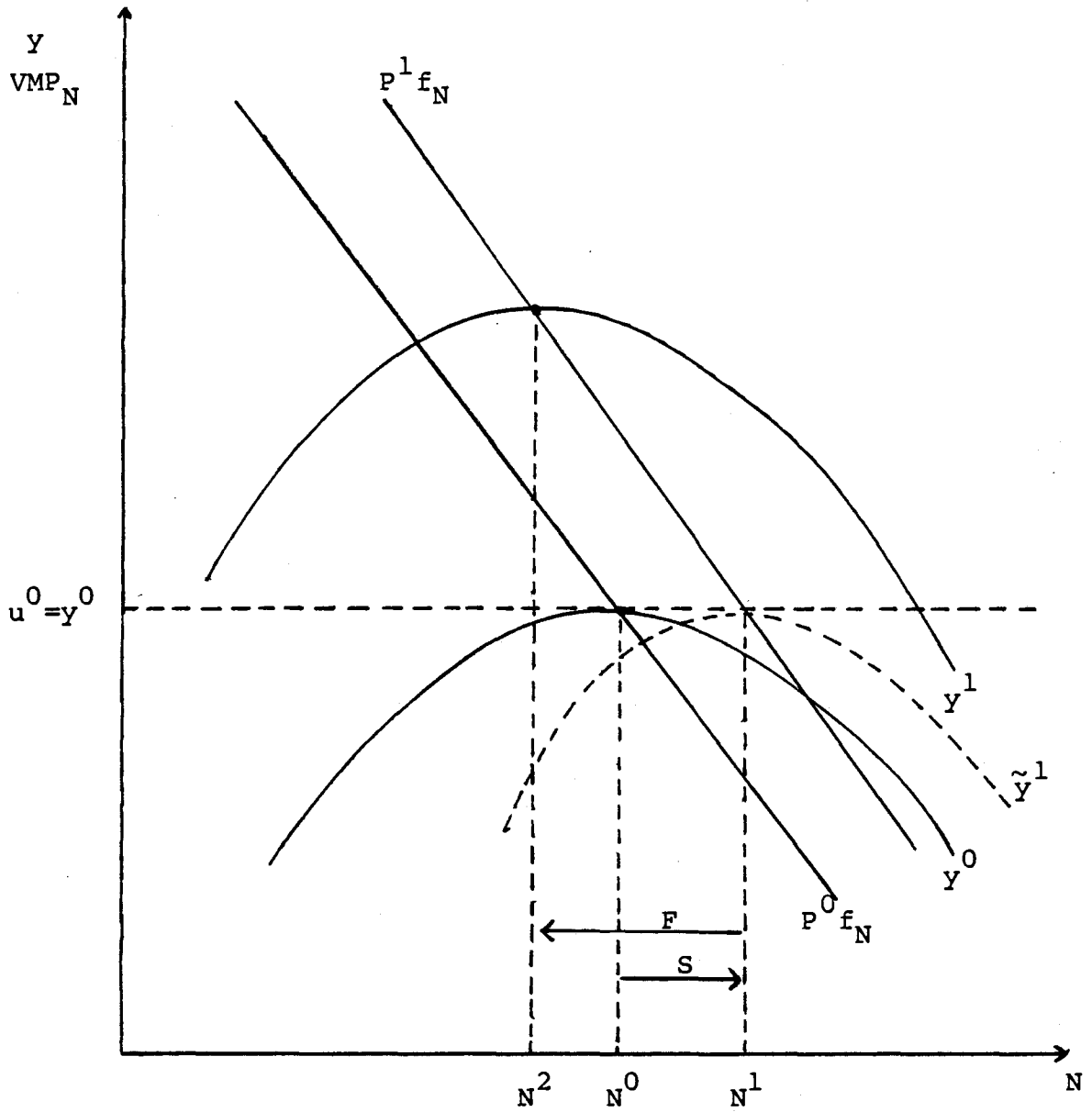


Figure 3: Decomposition of a price change.

$N^0 - N^1$ represents a pure price effect (S), while
 $N^1 - N^2$ is the fixed cost effect (F).

Vanek (1970) argues, on the other hand, that this negative supply response may vanish when multi-product firms are analyzed. Given some substitution possibilities in production, supply elasticities will be higher in multi-product than in single-product firms, arising from the fact that production within a firm of the various products will be adjusted to relative prices. Furthermore, the effect on employment is no longer unambiguous when joint production is studied.

The model is intended to resemble a socialist LM-firm. Capital is collectively owned, and income distribution is egalitarian. The time-perspective is short-run in the sense that capital level and firm structure are given. Evaluated by the definition given above, the assumptions under which the firm is supposed to operate are fairly restrictive. Thus, it will not necessarily give a good representation of how an LM-firm will operate. But it points to one important characteristic of an LM-firm, viz. the fixed cost effect. Employment level is changed only if this results in increased income per worker. Thus, the Illyrian firm's problem is the maximization of an economic surplus, and fixed costs are what give incentives to employing additional workers sharing in the burden. This effect has to be taken into consideration when analysing LM-firms.

4. SOME EXTENSIONS OF THE ILLYRIAN MODEL.

There is an obvious extension of the utility approach presented above. In effect the Illyrian firm is maximizing the income (utility) of the remaining workers, irrespective of whom and how many this affects. Later in this article it will be shown how the comparative static responses will change if the utility of all worker-members (members at a particular point of time) is maximized. But we may go even further and say that the level of employment has a general interest, e.g. that the firms (the workers) are concerned with the employment of the total work-force, say in a given region. As a special case, the work-force of concern may be the workers employed in the firm.

It will be shown below that this view of the LM-firm is general in the sense that other models emerge as special cases (see also section 6). A formulation which takes employment level explicitly into account, seems reasonable also from an applied point of view. Trade-unions are often modeled as agents which are concerned with income and employment level. This may be so because this is the way trade-union members are thinking. But these workers are also those who run LM-firms. Furthermore, the introduction of employment considerations involves the introduction of a collectivistic attitude, while the Illyrian model is strictly individualistic.

Because of the generality of the formulation, where the Illyrian model emerges as a special case, I find it reasonable to include a comparative static analysis of the Illyrian firm's long run allocation of capital and labour in this section.

Law (1977) and Smith (1984) have analyzed an LM-firm with the utility function

$$(4) \quad v(y, N),$$

i.e. a firm where the level of employment matters as well. Obviously the employment level cannot fall short of the employment level of the Illyrian firm if a positive utility is attached to employment. Furthermore, as this model is more general than the Illyrian model, in the sense that the Illyrian model emerges when $v_N = \frac{\partial v}{\partial N} = 0$, we use (4) when explaining the LM- firm's long-run optimization. When $v(y,N)$ is maximized w.r.t. N and K , with the production function $X = f(K,N)$ and y given by (1), we obtain the first order conditions

$$(5a) \quad Pf_N = y - \frac{v_N}{v_Y} \cdot N$$

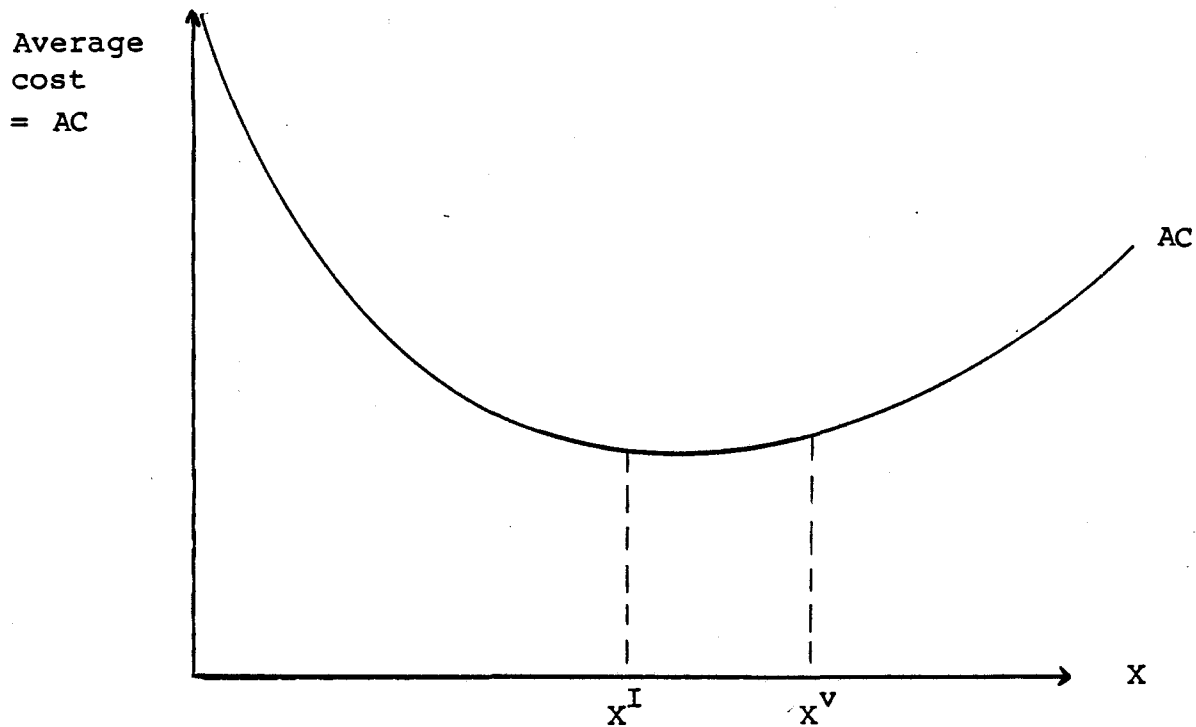
$$(5b) \quad Pf_K = r$$

Subscripts denote partial derivatives, which are all positive. Using the definition of y in (5a) and substituting for r from (5b) into (5a), we get

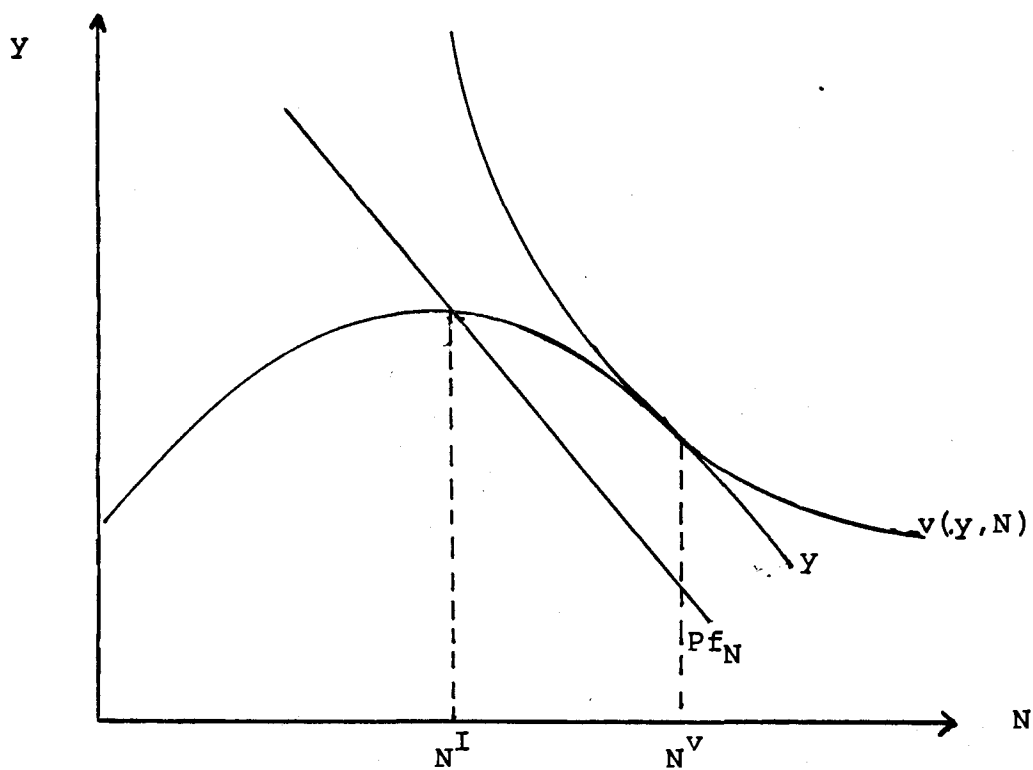
$$(6) \quad [f_N N + f_K K - X] + \frac{v_N}{v_Y} \frac{N^2}{P} = 0$$

Then we see immediately, by using Euler's theorem, that optimum requires production to take place at constant returns to scale when $v_N = 0$. If the equilibrium is unique, this presupposes a U-shaped cost- function. Furthermore, when $v_N > 0$, again using Euler's theorem, we see that production takes place at decreasing returns, and employment and production are higher than in the Illyrian firm with identical technology.²⁾ The equilibrium production and employment levels are sketched in Figure 4a, b.

In Figure 4, X^I and N^I refer to the Illyrian levels, whereas X^V and N^V are the preferred levels when $v_N > 0$. It should be



a) Production



b) Employment

Figure 4: Optimal production and employment levels of labour-managed firms.

noted that Figure 4a) holds when both factors are variable only, whereas Figure 4b) may represent short-run as well as long-run equilibrium.

When doing comparative statics on the first order conditions in (5) when $v_N > 0$, it is advantageous to apply a specific utility function. Thus, both Law (1977) and Smith (1984) use a Cobb-Douglas function. An interesting thing to note is that the short-run negative supply response prevails, although this result is not general in the sense that there may be functional forms giving other results. The marginal utility of additional employment in the range of operation is the vital factor.

The long-run responses are generally ambiguous. Ireland and Law (1984) have showed that the Illyrian firm, i.e. where $v_N = 0$, with a homothetic technology and a U-shaped average-cost function, will not respond to price changes or changes in the cost of non-labour factors. Generally the responses may be either, depending on whether the technology is (locally) K-biased or N-biased (see Ireland and Law (1985)). A K-biased technology can be defined as a technology where the capital/labour-ratio increases in the scale of operation. Then the output response to a price increase is negative, whereas the response is the "non-perverse" when technology is N-biased. In the latter case, the explanation can be given in terms of labour becoming relatively cheaper as scale of operation increases. A marginal price increase will then induce the workers to take on marginally more workers.

When $v_N > 0$, still these technical aspects matter. But the value of v_N plays a role as well. Thus, whereas the Illyrian firm with a homothetic technology and a U-shaped cost curve does not respond to price changes, the response is positive when $v_N > 0$, given the same homothetic technology.

Another interesting aspect emerging from (6) and Figure 4a) when $v_N = 0$, is the fact that the Illyrian firm will always produce

at a minimum of costs, assuming an equilibrium can be found where $N > 1$. This has lead Ireland and Law (1985) to formulate a minimum cost function

$$(7) \quad C(y,r) = \min yN + rK \quad \text{s.t } f(K,N) \geq \bar{X}$$

which shows the minimum cost of producing the production level \bar{X} (X^I in Figure 4a)). Corresponding to the cost function representing optimum in PM-firms, the properties of this cost function can be used to derive the comparative static results reported above. However, quite a lot of algebra is needed, and I will not go any further into that problem here.³⁾

Also Hey (1981a) defines a cost function to derive comparative static responses. He is interested in formulating a unified framework in which PM-firms as well as LM-firms can be studied. Thus, Hey defines the cost functions as minimum total costs consistent with producing \bar{X} and minimum capital costs consistent with producing $\frac{\bar{X}}{N}$ for PM- and LM-firms respectively. Using subscripts L to denote LM-firm and P to denote PM-firm, the two firms maximize

$$(8a) \quad Y_L = P \frac{X}{N} - C_L\left(\frac{X}{N}\right)$$

$$(8b) \quad Y_P = PX - C_P(X)$$

with respect to $\frac{X}{N}$ and X respectively. In both cases the first order condition is

$$(9) \quad P = \frac{\partial C_j}{\partial q_j}, \quad j = L, P; \quad q_j = \left(\frac{X}{N}\right)_L, X_P,$$

although the interpretation differs according to the definition of the cost-functions. According to second order conditions

$\frac{\partial^2 C}{\partial (q_j)^2} > 0$, and comparative static responses are obtained in a

straightforward manner. Specifically, $\frac{X}{N}$ as well as X increase when price increases. By noting that marginal productivity of labour is decreasing in N for given capital $K = \bar{K}$, it immediately follows that the LM-firm reduces production in the short run, which corresponds to Ward's initial result.

5. INDIVIDUAL LABOUR SUPPLY.

Now, in the above models, and in models to be discussed below, the individual workers supply an equal amount of work. This justifies the maximization of payout (dividend) per worker. On the other hand, it has been argued, see e.g. Ireland and Law (1982), that a producer cooperative is primarily concerned with the utility of its work-force, and that the proper maximand should be the maximization of a utility function with income and hours of work as arguments. This should represent the choice made by each individual worker, and it should be easily adopted when the work-force is fairly homogeneous.

Sen (1966) was the first to acknowledge the importance of individual labour supply decisions in the firm's optimization programme. He noted that the principle of paying every worker the same amount irrespective of work done corresponds to the Marxian concept "to each according to his needs". The obvious alternative is making payment "to each according to his work", and the two principles will generally not give the same result.

Although the problems related to individual labour supply will be further examined in subsequent chapters of this thesis, I find it illustrative to sketch the workers' choice of how long hours to work. Let $u(\tilde{y}, \ell)$ be a quasiconcave utility function where ℓ is the labour supply of the representative worker i , and assume that all workers are identical. We substitute into $u(\tilde{y}, \ell)$ for

$$\tilde{y} = \frac{PX - rK}{L} \cdot \ell,$$

where $L = \sum_{i=1}^N \ell^i = N\ell$ is total number of hours worked. The problem is

$$(10) \quad \text{Max}_{\ell} \quad u\left(\frac{PX - rK}{L} \cdot \ell, \ell\right)$$

which yields the first order condition

$$-\frac{u_y}{u_\lambda} = p \cdot \frac{\partial f(\bar{K}, L)}{\partial L} \frac{\partial L}{\partial \lambda} \frac{\lambda}{L} + \frac{PX - rK}{L} \left(1 - \frac{\partial L}{\partial \lambda} \frac{\lambda}{L}\right)$$

$u_y > 0$ and $u_\lambda < 0$ are partial derivatives w.r.t. income and leisure respectively. If there is full cooperation in fixing individual labour supply, i.e. $\frac{\partial L}{\partial \lambda} = N$, we see that the marginal rate of substitution between income and leisure equals the marginal productivity of labour, which is characterized by Sen (1966) as the condition for a socially optimal allocation of labour. The optimality condition holds when $VMP_L = y$ also. But as pointed out by Sen as well as Vanek (1970), and shown above, this is fulfilled only if the production function exhibits locally constant returns to scale. Thus, if there are decreasing returns, which can be represented as a free factor of production, or a scarce factor in collective supply, the workers will always benefit from reducing the employment level and increasing their individual labour supply, so as to capture as large a part of the economic rent as possible, and individual labour supply is too large compared to the social optimum.⁴⁾

However, if $\frac{\lambda}{L}$ were given, i.e. compensation is made by a fixed amount per worker irrespective of work done, then each worker receives only $\frac{1}{N}$ th of his value marginal product of labour. In the first order condition above the second term on the right hand side disappears, and only full cooperation, $\frac{\partial L}{\partial \lambda} = N$, will secure optimality. Labour supply will in other instances be too low because of undercompensation. Thus, in circumstances where full cooperation does not exist in deciding on individual labour supply, the two rules for making payment will induce respectively too high and too low labour supply compared to the social optimum.

Sen formulates an income distribution rule involving some part of dividend, γ , being paid according to work and the rest $(1-\gamma)$, being paid on a per capita basis. Furthermore, Sen assumes that the members of the collective may also take into account the welfare of the other members. This sympathy towards others ranges from 0 to 1, the last number involving "full" sympathy towards all other individuals.

In the case of full sympathy, the allocation of labour and production will be efficient irrespective of the value of γ . Otherwise efficiency requires $0 < \gamma < 1$ if the producer cooperative owns some factor other than labour, and $\gamma = 1$ if the production function exhibits constant returns to scale and all non-labour factors are hired. As pointed out by Browning (1982), in this context $(1-\gamma)$ acts as a Pigouvian tax on the use of a collective good to which the cooperative has free access, preventing the workers from supplying too much labour. It should be noted that Sen's analysis presupposes a Nash-Cournot reaction function when considering individual labour supply, and that labour is homogeneous. If the workers instead colluded in deciding on individual labour supply, they would always be able to reach an efficient allocation of labour, given the firm's employment level.

Browning (1982) argues, however, that the result is not robust. He shows that it is valid in the case of homogeneous labour only. If labour is heterogeneous, the allocation of labour resulting from a Nash-Cournot assumption can generally be improved upon, i.e. there exists no value of γ guaranteeing Pareto-optimality.

Like Sen Browning also assumes that factors of production other than labour may be available at zero cost. However, by assuming that all non-labour factors are hired at their market price, allocation of labour will be efficient when $\gamma = 1$. Letting l^i be labour supplied by worker i , $i=1, \dots, N$, this is seen by assuming $(PX-rK) = g\left(\sum_{i=1}^N l^i\right) < 0$ when $L = \sum_{i=1}^N l^i = 0$, i.e. there are fixed

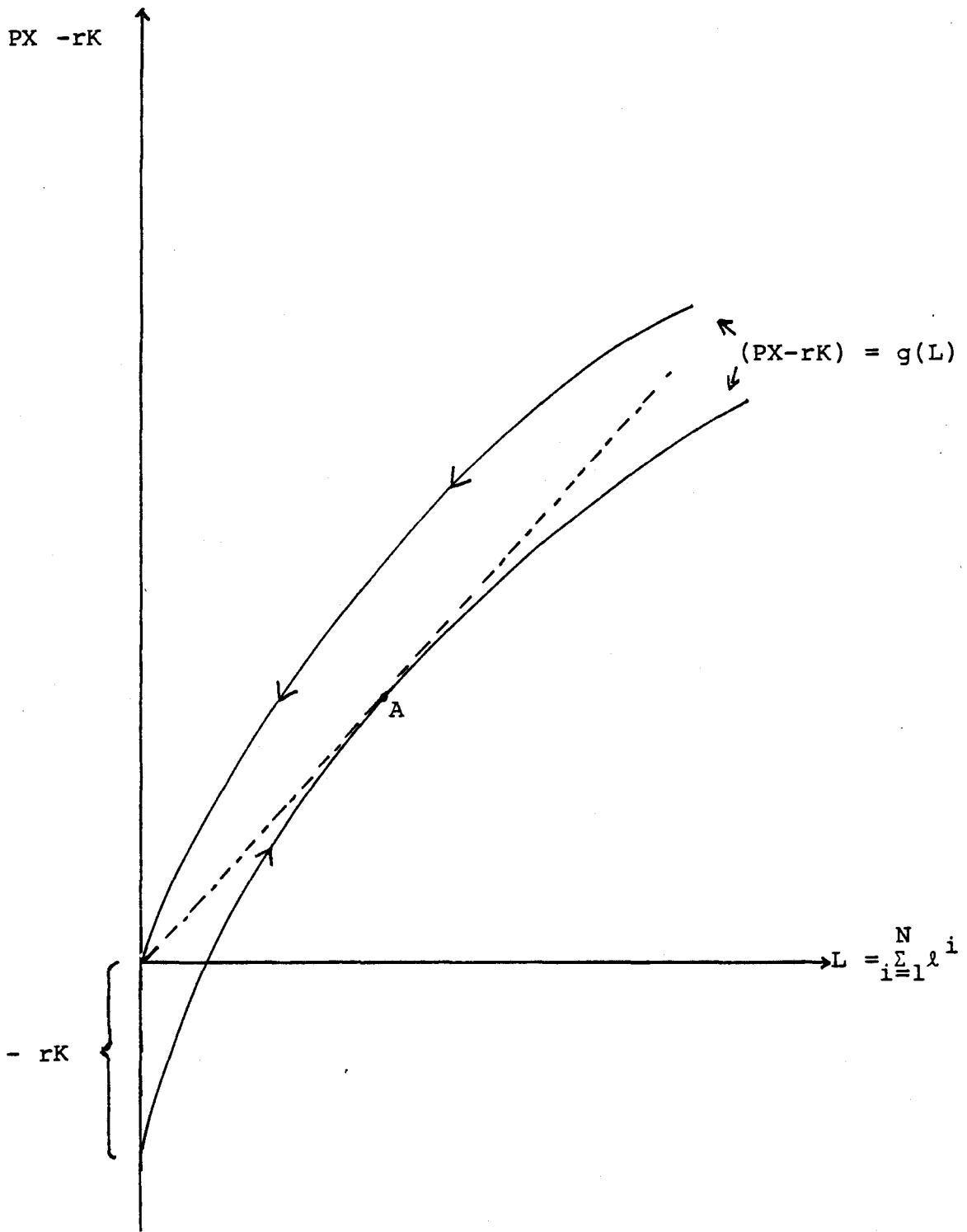


Figure 5: Optimal size of LM-firms with/without fixed costs, and when labour is the only variable factor.

costs. Alternatively, if $g(L) \leq 0$ when $L = 0$, we must have a production function with first increasing and then decreasing marginal productivities, see Ireland and Law (1984). The cases when $g(0) < 0$ and $g(0) = 0$ respectively are shown below (Figure 5). We assume full cooperation in fixing individual labour supply.

In A, the marginal reward $(\frac{PX - rK}{L})$ equals marginal productivity $(\frac{\partial g}{\partial L} = \frac{\partial X}{\partial L})$, i.e. an efficient amount of labour is supplied. With costless access to a factor of production, and non-homogeneous workers, the labour allocation will generally not be efficient. As it was shown by Sen (1966), a solution to the first of these problems - a free factor - requires a special reward function (see above). In the latter case - heterogeneity of the workforce - efficiency may be obtained with special reward functions if special restrictions on the profit (dividend) function are satisfied. The issue is further discussed by Browning (1982).

From Figure 5 we see that when $l^i = \bar{l}$ for all i , and only N is variable, there is no equilibrium size of the LM-firm involving more than 1 worker if $rK = 0$.

Berman (1977) looks at the various dividend sharing rules from a somewhat different viewpoint. He argues that in the short run it is not realistic to assume that the number of workers can be varied, i.e. he finds it unrealistic that LM-firms will dismiss workers so as to increase the income of the remaining workers. Instead he assumes, like Sen, that the number of hours to be worked is variable. By assuming that each worker can change his labour supply, we saw above that the allocation of labour will be efficient when there is perfect collusion between the workers in fixing the labour supply. In Bonin (1977) and Chinn (1979) the degree of cooperation in fixing hours of work is determined by the elasticity of total labour supply with respect to the individual supply, i.e. by

$$\eta = \frac{\partial L}{\partial l^i} \frac{l^i}{L}$$

Here $\frac{1}{N} \leq \eta \leq 1$, and $\eta = 1$ means perfect collusion. Ireland and Law (1981) summarize the views of Berman (1977), Bonin (1977), Chinn (1979) and Sen (1966), and examine the effect on labour supply and production of changes in prices and other parameters. In particular they show that the effect on labour supply of price changes will be similar, although not identical, to the ones which occur when wage changes in PM-firms, and the slope of the short-run supply curve of the LM-firm is ambiguous when N is given. Furthermore they indicate equivalence with respect to labour allocation between Sen's factor of sympathy in a model with Nash-Cournot reaction functions, and the labour supply elasticity (η). That is, complete sympathy towards fellow workers or complete collusion in fixing individual labour supply are sufficient for obtaining an efficient resource allocation.

It is probably a matter of discussion how important it is to allow for individual labour supply decisions in a model of a cooperative firm. Sen's and Chinn's analyses refer to Chinese agricultural cooperatives, while Bonin's discussion relates to Soviet cooperatives. The problems connected to a free factor of production seems relevant in the economies in question, as land may be provided at zero cost by the government, or at least not supposed to be paid its marginal value product. Whether the workers will in reality determine their labour supply individually or collectively is much of an empirical question, dependent among other things also on possible legal regulations. Generally it does not seem unrealistic to assume variability of labour supply in an LM-firm. It may take the form of overtime and adjusted holidays according to individual preferences. Then, considered as a positive analysis, it focuses on interesting incentive problems, which will be further discussed in another part of this thesis.

6. CONSTANT SHORT-RUN EMPLOYMENT LEVEL.

It is not unreasonable to assume that the number of hours to be worked can be varied in the short run. But according to several authors employment should be assumed fixed, contrary to the models formulated by Ward (1958) and Domar (1966). The reason for this may be that it is contrary to the idea of worker cooperatives to dismiss workers (cf. Berman 1977), or perhaps because of solidarity or difficulties in deciding who are to leave the cooperative when this is the optimal policy. Meade (1972) formulates some rules which he finds appropriate for an LM-firm. Although his model allows for a variable employment level, he lays down some conditions for termination of membership. These are "(a) that the partner concerned wishes to leave and (b) that he should obtain from the remaining partners permission to withdraw".⁵⁾ The justifications of the rules are on the one hand that no one can be forced to accept worse conditions than those of the members remaining in the collective. But at the same time no one can leave the collective and all its obligations to get better terms than those remaining without general agreement or compensation. This corresponds to the view taken by Robinson (1967), who argues that worker cooperatives will never dismiss workers with the sole objective of increasing the income of the (luckily) remaining workers.

It seems reasonable to assume that employment is fixed in the short run but that it can be varied in a medium and long-run time-perspective. Berman and Berman (1978) assume that in the long run LM-firms choose total number of hours to be worked. This is done by voting procedures where individual preferences concerning income, working hours and the firm's size (employment level) are taken into account. The optimal size of the firm, and responses to price changes, are as reported above, and it will be further discussed in another chapter of this thesis. We will

therefore turn now to an explanation of why the short-run employment level can be considered constant, or at least weakly responsive to price changes.

A model giving this result is formulated by Steinherr and Thisse (1979). They argue that the workers who vote for membership reduction will take into consideration the possibility they face themselves to be made redundant. Then, if redundancies are made randomly, so that all workers have the same possibility of being dismissed, and the workers are not risk-lovers, or if the workers maximize the utility of all workers so that redundant workers have to be compensated, then no dismissals will take place.

To illustrate the point, assume that the firm maximizes a utility function

$$(11) \quad \begin{array}{ll} \text{a) } V(N) = v(w) & \text{if } N = 0 \\ \text{b) } V(N) = v(y(N)) \cdot \frac{N}{N^0} + v(w) \frac{N^0 - N}{N^0} & \text{if } 0 < N < N^0 \\ \text{c) } V(N) = v(y(N)) & \text{if } N > N^0 \end{array}$$

where N^0 is the initial work-force, and w is the original dividend equal to the market wage rate and the value marginal product of labour. $\frac{N}{N^0}$ represents the probability of remaining with the firm, while $\frac{N^0 - N}{N^0}$ is the probability of being made redundant. $y(N)$, defined in (1), is dividend at employment level N . $v(w)$ and $v(y)$ are concave if the workers are risk-averse.

Assume that the price increases from P^0 to P^1 . We know from Figure 2 that the optimal employment level reduces to N^1 when income per remaining worker is maximized. Let us maximize (11b) w.r.t. N to see whether an income-maximizing employment level can be found. We get

$$(12) \quad \frac{\partial V}{\partial N} = 0 \Rightarrow v'(y) \cdot (Pf_N - y) + v(y) - v(w) = 0, \quad 0 < N < N^0$$

which we write as

$$Pf_N - w = 0, \quad 0 < N < N^0,$$

when the workers are risk-neutral. But as $w = y(N^0) = P^0 f_N$ and $P^1 f_N > P^0 f_N$ we see from Figure 2 that this expression is positive for $P^1 > P^0$ and $N \in [N^1, N^0]$, and no equilibrium employment level can be found with a reduction in the work-force. If the workers are risk-averse, this result is even further strengthened, while it may not hold if the workers are risk-lovers.

However, the level of membership will not be increased either, as we see by maximizing (11c) w.r.t. N :

$$(13) \quad \frac{\partial V}{\partial N} = Pf_N - \frac{P f(N, \bar{K}) - rK}{N} < 0, \quad N > N^0.$$

This result is easily confirmed by inspection of Figure 2. Thus, because of the way the marginal utility of a change in employment changes sign around the initial employment level, no utility increasing employment adjustment can take place. The results hold even though expected income may increase by a membership reduction. The reason for this somewhat peculiar outcome is the fact that the members are willing to employ all workers willing to accept a payout $w < P^1 f_N$. However, when membership exceeds the initial level N^0 , the workers become "ordinary" income maximizers without a specified opportunity cost. This makes the model somewhat questionable, as we see that adjustments may be made which will increase income, see Figure 2.

Steinherr and Thisse show that the same result is obtained also if the firm maximizes the welfare of all initial workers. If

e.g. dismissed workers have to be fully compensated by the remaining workers if they suffer an income loss, a price increase will never result in reduction of employment. Assume that the dismissed workers were to receive $y(N^0) - w$, where $y(N^0)$ is the income the workers would get if remaining with the firm. Then remaining workers will gain $(y(N^0) - P^1 f_N) < (y(N^0) - w)$ as $P^1 f_N > w$, and membership reduction is not profitable.⁶⁾

Smith (1984) has pointed out that the maximization of the utility of a pool of workers, i.e. the maximand (11b) without any restriction as to the level of N , will lead to a level of employment where $P f_N = w$, and income $y > w$ as shown in Figure 6 below.

With \bar{w} as the reservation wage, the equilibrium contains an implicit definition of a utility function $v(y, N)$. This corresponds to the result in (4) obtained when the utility function was explicitly defined as $v(y, N)$. Thus, with a proper definition of an opportunity cost of labour, and a restricted pool of workers, the result emerging from Figure 6 below shows the same willingness to pay for increasing employment, compared to the Illyrian firm, as may be attained by formulating the utility of employment explicitly. It should be noted also that this represents an efficient production decision, whereas efficiency does not necessarily hold in the Steinherr-Thisse model, which we see from the fact that first order conditions may not be met by equality. This is due to the shift in the maximand which does not allow for compensations for $N > N^0$.

Other results may be derived by changing some of the underlying assumptions. One of these assumptions seems important. It is assumed that all workers are treated in the same way, irrespective of seniority and other factors determining the workers' status in the firm. Also new workers are given equal status. In reality we would probably find arrangements favouring

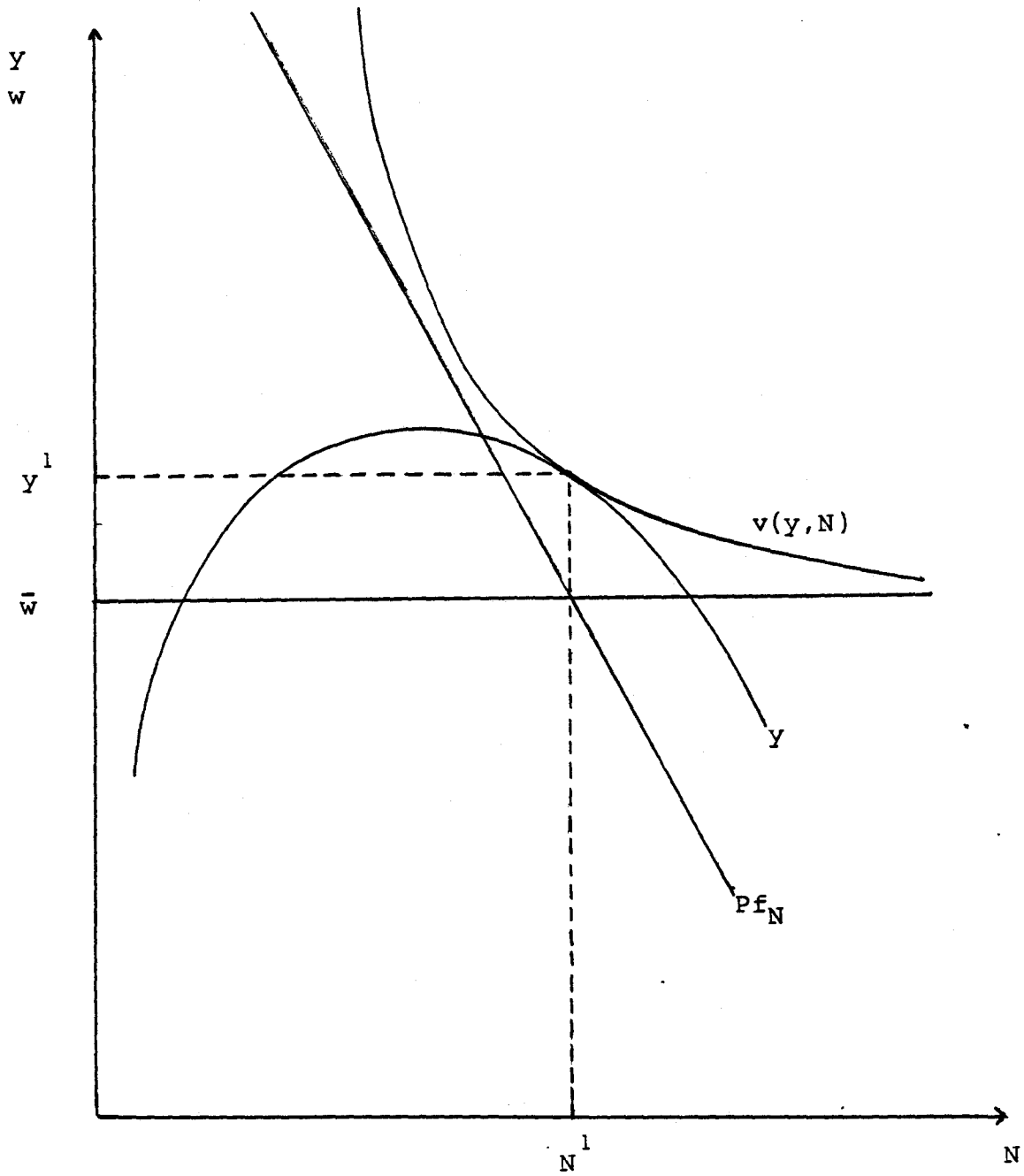


Figure 6: Employment level of an LM-firm with an implicit utility function defined over income and employment.

special groups of workers. It would be reasonable to take into account formation of coalitions within the firm. Surplus would then be divided according to the outcome of negotiations between distinct groups. On the other hand, the model constitutes an important contribution by taking into consideration more than income when investigating an LM-firm. As such it represents a progress compared to the Illyrian model, although the political process is probably much more complicated than the model indicates.

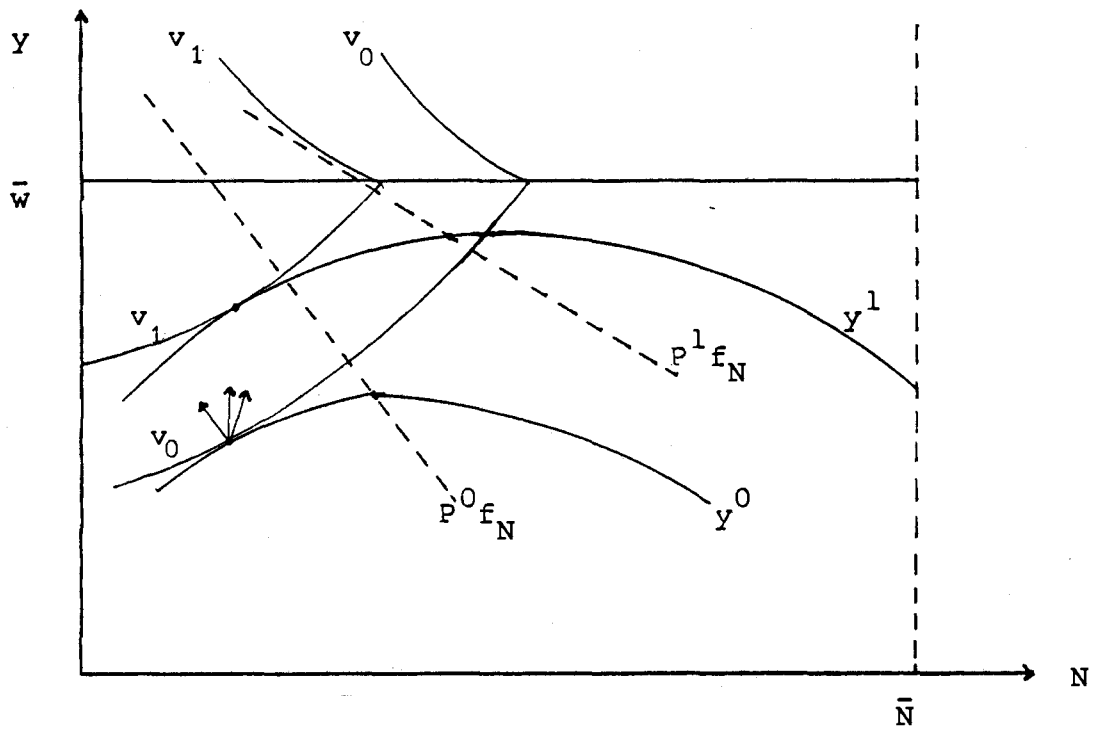
What I will denote a further sophistication in modelling the LM-firm is undertaken by Miyazaki and Neary (1983) who model the LM-firm as a contract-based production coalition of workers. They assume that the firm faces uncertain market prospects (price), and the contract specifies state-contingent employment levels and dividends. Their main target is to investigate responses to different states of nature ex post, and for different assumptions as to how risk is carried. The key to the analysis is the decomposition of price changes into a price effect and a fixed cost effect (Slutsky-equation), and the assumption that the firm maximizes the expected mean utility of income. The general result is that layoffs occur only if the value marginal product of labour falls short of the exogenously given opportunity cost of the workers, which corresponds to Steinherr and Thisse's result given an initial equilibrium. This means that price responses occur only in "bad" states of nature. Assume firstly that no compensation schemes exist. Although expected utility is high enough to induce the workers not to shut the firm down, states of nature may occur in which the ex post wage is below the reservation wage. The workers will then prefer not to work but some of them will have to do so because of the firm's fixed cost constraint (i.e. to avoid bankruptcy). Now, whereas the price effect in the Slutsky equation is always positive, the fixed cost effect is negative in these bad states of nature when workers prefer not working⁷⁾. On the other hand, the sign of the fixed cost effect is reversed when the take-home wage exceeds the opportunity cost of work. Then the total effect of a price increase (better state of nature) is ambiguous in the worst states and positive (non-perverse) in the better states.

The two outcomes can be illustrated as in the figures 7 a) and b). \bar{w} is the reservation wage, and we follow the approach used by Smith (1984).

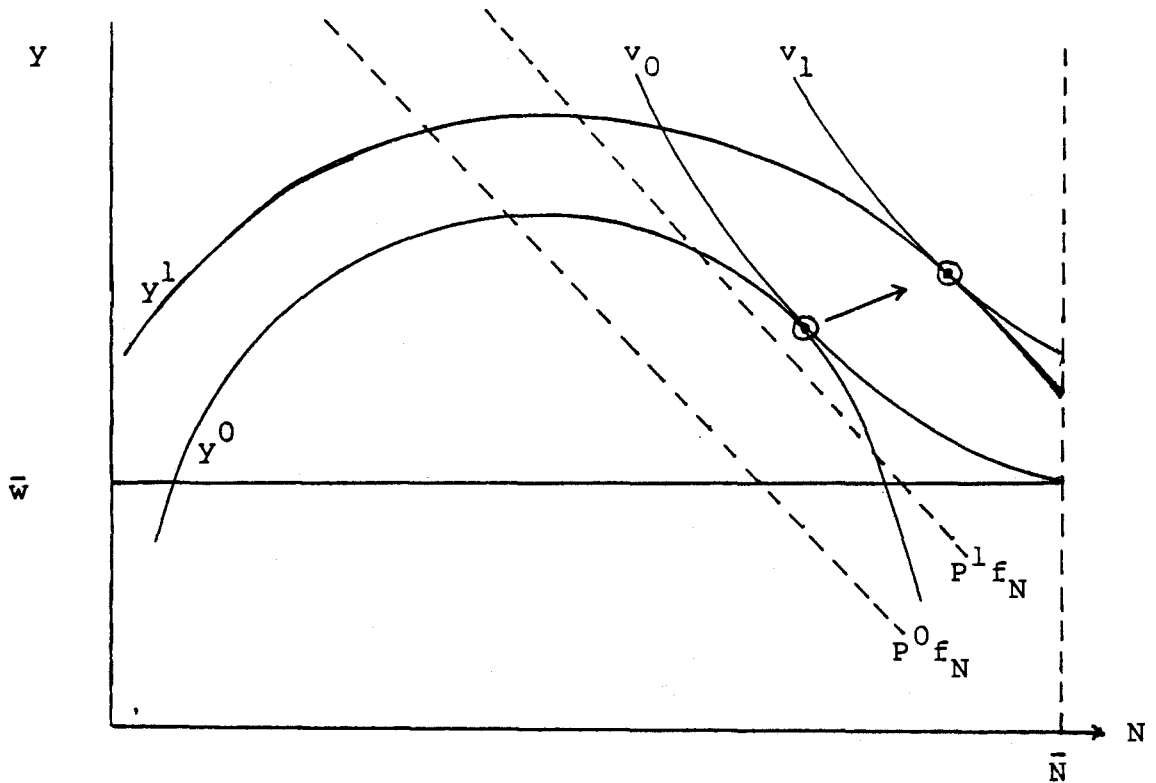
v_0 and v_1 represent the implicitly defined utility functions. Note that they make a kink at \bar{w} , as reduced employment is desired for $y < \bar{w}$. \bar{N} represents the size of the pool of workers.

Whereas for bad states of nature there exist segments of an upward as well as downward sloping supply curve when there are no insurance or transfer arrangements in existence, Miyazaki and Neary show that the ambiguity disappears if the workers can insure themselves completely against income fluctuations, either by risk shifting which insures debt obligations, or by compensation schemes operating between employed and non-employed workers. Then the fixed cost effect disappears, and the supply curve is upward sloping in states of nature where $N < \bar{N}$ is optimal.

Some of the objections raised against the Steinherr and Thisse-model may be raised against the approach taken by Miyazaki and Neary as well. Specifically they do not consider formation of subcoalitions and discrimination of some workers. But their contractual approach seems an important progress. It implies that the firm is considered a coalition among workers with constant membership size in the short run. Then the firm is more than just a production unit. Long term interests can be separated in contracts from short term interests. In this way the employment level in the short run depends on what alternatives are available to the workers. One important aspect, which is not explicitly discussed, is the affect of a governmental dole to laid-off workers on the work-incentives of LM-firms. Thus, the self-insurance arrangements of LM-firms, which the authors discuss, may be important for the possibilities of attaining an efficient resource allocation in an LM-economy.



a) $\frac{dN}{dP} = ?$



b) $\frac{dN}{dP} > 0$.

Figure 7: Employment in different states of nature, when the LM-workers are assumed to maximize the expected ex-ante mean utility of income.

7. UNCERTAINTY.

On the background of the discussion above, I consider it fruitful to explore the theory of the LM-firm under uncertainty. Apparently the introduction of uncertainty alters the optimal behaviour of the firm, as happens when PM-firms are concerned. The all-important factor here is the firm's attitude to risk. Sandmo (1971) has investigated the functioning of the PM-firm which maximizes the expected utility of profit. If price is the uncertain parameter, the risk-neutral firm facing an expected price \bar{P} will make the same production and employment decisions as the firm facing the price \bar{P} with certainty. However, if the firm is risk-averse, i.e. its utility function is concave, assuming it can be defined, then the firm facing \bar{P} will produce less if \bar{P} is an expected price with known distribution than if \bar{P} is a market price known with certainty.

A question is whether this result carries over to the LM-firm's optimal behaviour. Using the short-run framework developed by Sandmo, the problem is analysed by Muzondo (1979), Hawawini and Michel (1979), Ramachandran, Russel and Seo (1979), Bonin (1980) and Hey and Suckling (1980) when price uncertainty is considered, while Hawawini and Michel (1983) consider production uncertainty. The firm is assumed to maximize the expected utility of dividend, y , i.e.

$$(14) \quad \text{Max}_N V = E[u(y)] \quad \text{where } y = \frac{PX-F}{N} ,$$

and F is fixed costs (non-labour costs). The first order condition is

$$(15) \quad E[u'(y) (Pf_N - y)] = 0$$

which may be written, using laws of variance, as

$$(16) \quad E[Pf_N] = E[Y] - \frac{\text{Cov}[u'(y), (Pf_N - y)]}{E[u'(y)]}$$

The effect on production of uncertainty and risk aversion depends on the covariance term. It is zero when the firm is risk-neutral, leaving the firm with the same employment and production as if the firm were facing the price $E(P)$ with certainty. But if the firm is risk averse, by noting that both elements in the covariance term are negatively correlated to price since $u''(y) < 0$ and $f_N < \frac{X}{N}$, we find that

$$(17) \quad \text{Cov}[u'(y), (Pf_N - y)] > 0 \Rightarrow E[Pf_N] < E[Y].$$

As marginal productivities are decreasing, this means that a slight increase in risk aversion, when the firm is initially risk-neutral, results in increased employment and production, contrary to what is the optimal reaction of the PM-firm. The result holds true irrespective of whether the uncertainty applies to the market prospects (P) or the production activity (f_N).

The explanation of this result is quite simple. In a certain environment the workers of an LM-firm will vote for reduction in employment ($f_{NN} < 0$) until the burden of fixed costs makes further employment reductions unprofitable. However, uncertainty represents for the risk averse workers a burden similar to fixed costs which it is advantageous to share with more risk-takers. On the other hand, if forward markets exist the workers can hedge so as to avoid the risk, and the firm produces the same output as under certainty if the forward price equals the certainty price, see Hey (1981b). Also, as pointed out by Ireland and Law (1982, ch. 7.3), the LM-firm can reduce its risk by participating in several markets (multi-product firm).

Several comparative static responses have been investigated in the literature. Of most interest is the effect of changes in fixed costs (or lump-sum taxes), expected price, and different tax rates (unit-labour tax, ad valorem tax). Assuming decreasing absolute risk aversion, it turns out that the responses are

generally opposite to those of the PM-firm (except for a change in ad valorem tax, in which case the response is ambiguous), and for changes in fixed costs and expected price the responses correspond to the respective changes when the LM-firm faces no uncertainty⁸⁾ (see Muzondo (1979), Bonin (1980)).

Now, the alternative framework presented by Hey (1981a), which we have discussed previously, is useful under uncertainty as well, see Hey and Suckling (1980) also. Let us define q_j as X_p and $(\frac{X}{N})_L$ respectively. Write the cost functions as $C(q_j)$. Each firm chooses q such that

$$(18) \quad E[v(Y)] = E[v(Pq - C(q))]$$

is maximized. The first order condition is

$$(19) \quad E[v'(Y)(P - C'(q))] = 0,$$

and the results obtained by Sandmo (1971) applies to the LM-firm as well. The short run results derived by Muzondo (1979) and Bonin (1980) are then easily derived by assuming capital fixed. E.g., as a change from a fixed price to a random price with the same mean induces reductions in X_p and $(\frac{X}{N})_L$, assuming risk aversion, the latter implies an increase in employment and production as marginal productivity of labour is decreasing, given a level of non-labour factors.⁹⁾

Introducing variable individual work effort does not significantly alter the results reported above. The question is examined by Bonin (1977) and Ireland (1981). Whereas Ireland (1981) compares the effect of price uncertainty on employment and individual labour supply in LM- and PM-firms, Bonin (1977) investigates the allocation of labour between a private and collective firm facing price and productive uncertainty respectively. The additional results refer to the individual labour supply decisions. Because of income effects they are generally ambiguous. However, by using utility functions where the degree of risk

aversion is independent of labour supply, e.g. a utility function which is additive in income and leisure, it is shown that risk averse workers try to avoid risk by reducing labour supply to the risky project.

8. EFFICIENCY AND GENERAL EQUILIBRIUM.

In the literature reviewed so far, LM-firms are assumed to exist within a market economy, which may be a pure LM-economy or an economy consisting of different types of firms, also involving PM-firms. The market and technological restrictions which exist, are the same as those known generally from pure PM-economies. The individual agents are assumed to behave according to the same objectives, independent of economic system, i.e. the analyses consider utility maximizing workers choosing between levels of leisure and consumption. Thus, any firm faces the same exogenous market restriction. Such similarities in modelling PM- and LM-economies are advantageous in the sense that comparisons between the two systems and types of firms are easily made.

Indeed, many papers have been concerned with such comparisons. These can be grouped into two categories. Some aim to establishing equivalences between the two systems, i.e. pointing out similarities and differences arising from a given set of assumptions. Thus, Drèze (1974, 1976, 1985) studies properties of market equilibrium in the two types of economies, while e.g. Meade (1972) and Ireland and Law (1981) compare the comparative statics of PM- and LM-firms in a partial equilibrium setting.

The other approach in comparing the systems deals with the overall functioning of the two systems. The two types of firms can be subject to different working conditions. These conditions can favour one enterprise organization, and they will typically differ between economic systems. But then introduction of such assumptions and restrictions may imply that one type of firms will be superior to the other one when measured by efficiency. On the one hand e.g. Vanek (1970), Ireland (1981) and Reich and Devine (1981) compare PM- and LM-economies when conditions like disutility of work and need for supervision of workers differ. As labour-management is supposed to have favourable effects on such variables, an LM-firm can obtain better results than PM-firms. This is, however, contrasted by e.g. Alchian and Demsetz

(1972), Furubotn (1976), Jensen and Meckling (1979) and Williamson (1980) who assert, for different reasons, that management is better performed in hierarchical structures than egalitarian ones. Because of property rights problems and the incentives of the management and supervisors, the PM-firm will have the better possibilities of obtaining finance and lowering costs, particularly those related to supervision and monitoring, and thus allocate resources better than LM-firms. These problems relating to the Yugoslav LM-firm are discussed in Furubotn and Pejovich (1970).

Of course, what kind of firms are really the best ones, is hard to ascertain. It depends probably strongly on the environment in which firms have to operate. This environment may involve imperfections which favour one type of firm. Thus, it is not unreasonable to assume that in an economic system where one way of organizing firms is dominating, it will be problematic for other types of firms to be accepted and function properly. There may be several reasons for this. It can be considered a matter of discrimination, see e.g. Lommerud (1987). Financiers, raw-material suppliers and customers may be sceptical to worker-managers. Of course, an LM-firm may employ a fully professional management which is supposed to represent and negotiate the firm's interests. This is compatible with our definition of an LM-firm. But the workers may choose instead to do the practical management themselves. This may create problems if social and professional background differ from their counterparts. Another problem which may arise, concerns the optimal financial structure of the firm. It is advantageous for an LM-firm with collectively owned capital to be 100% externally financed (see e.g. Furubotn (1976)), but credit suppliers may be unwilling to contribute with 100% finance because of the risk connected to it. This calls for special financial arrangements which are hardly present in capitalist economies. As a result of this, the workers may be forced to choose a capitalist way of organizing the firm.

Stated more formally, the views of Ireland et. al. and Williamson et al., can be interpreted as partial equilibrium results in a context where a general equilibrium setting would be appropriate. Drèze (1976) points out that choice of working conditions is a problem involving nonconvexities because of its public good aspect. Comparisons can be drawn to the choice of product quality, where the existence of nonconvexities makes efficiency results only locally valid. Putterman (1984) and Lommerud (1987) use the same approach when discussing works by Williamson et al. They argue that the economic environment lays down conditions which favour conventional forms of organization. This means that the equilibrium studied is not a global one, as there is no natural way of leaving one (local) equilibrium position to attain another one. If this is so, arguments like "social revealed preference" to explain existence of firms by a revealed preference of workers as to choice of production organization, and "economic Darwinism" to explain existence of firms by survival through efficiency, are not good explanations of why one firm structure dominates at a given point of time. Instead, this structure is established as a result of a local optimization, given an economic environment. Then this local optimization argument is important both when explaining firm structure and working conditions within firms to which workers have to adjust.

Thus, although LM-firms are rare outside Yugoslavia, barring cases like Mondragon in Spain, plywood cooperatives in USA and perhaps the kibbutz system in Israel, the relatively extensive literature on labour-management during recent years should be justifiable. When situated in systems of overall "traditional" profit-maximizing enterprises, conditions under which competition takes place are probably unfavourable to LM-firms. But nevertheless it is important to explain how LM-firms behave under perfect conditions. Knowing optimal reactions of agents to changes in economic and other variables under first best conditions are necessary both when systems are to be compared, and when the effect of various imperfections are to be analyzed. But first and foremost we need to know whether the system in ques-

tion is such that the economy is pulled toward a position where resources are rationally allocated. To be able to do so, we need a general equilibrium framework.

The above analysis and discussion of partial equilibrium results have indicated that obtaining an efficient resource allocation may not be a trivial matter in an LM-economy. However, Drèze (1974, 1985) shows that the set of Pareto-optimal allocations in an LM-economy may be identical to those of a PM-economy with the same restrictions on technology and resources, and assuming also that the utility functions of the households are identical.

The derivation of the equivalence and efficiency result rests upon a proper definition of the economy. The special feature of the labour-managed economy is that labour is a non-traded good, and thereby it has no market price. However, skills and amount of work done can be assessed in the same manner as in a traditional capitalist economy, and thereby enable the establishment of weights for payment for different kinds of work in terms of quantity as well as quality. Generally there will exist initial assets also, to which the workers (the firms) will have access. We have mentioned above the suboptimality that will result from free access to a factor of production (Sen (1966)). Drèze assumes that this problem is solved by charging rents for using these assets. The ownership of the assets, and the distribution of the rent, can be assumed given exogenously. What is important is that the rent is levied in a way that enables no firm to obtain a yield which is denied other groups of workers. Lastly, it is crucial that potential firms rather than existing firms are considered, i.e. the number of firms is endogenously determined, so that the workers are free to costlessly set up new firms and close down old ones as responses to changes in relative profitability between lines of business. The importance of this is seen easily by looking at partial equilibrium analyses of the representative LM-firm. We showed above that in the short run the Illyrian LM-firm will react "perversely" to price changes. These wrong reactions to price signals may be present also in the long run and for quite sophisticated specifications of the LM-firm's maximand. Changes in prices mean that production should be changed. From PM-economies we know that this can

happen either through establishing or closing down enterprises, or by changes in level of production in existing firms. But in an LM-economy the change in the level of production may go in the wrong direction. Then we will have to rely on entry and exit of firms and mobility of labour to attain a market equilibrium. Unless this happens, income differentials will be present, i.e. marginal products of labour will differ between lines of business for workers of same ability. Then there is not equality between demand and supply in the labour market. This disequilibrium condition will, however, vanish through entry and exit of firms, so that supply and demand are equalized to establish a Walrasian equilibrium. In this equilibrium the market shadow price of labour is equal to marginal productivity, and equalized among firms and lines of business.

Ichiishi (1977) explains the formation of firms by looking at the productivity of different coalitions. A firm in an LM-economy is considered a coalition of the ultimate consumers, where each coalition is assigned a production possibility set. As any coalition being a subset of the set of all economic agents is possible, including the single member coalition and the grand coalition of all agents, at each point of time a number of potential firms exist. The firms in operation will be only those producing at lowest costs, given the assumptions made on technology and preferences, which lead to the establishment of a competitive equilibrium price vector. Possibilities of different sizes of firms are contained in this formulation by allowing for segments of increasing and decreasing returns to coalitions.

A coalition production economy model is used by Greenberg (1979) also. He shows that the efficiency results obtained by Drèze may be sensitive to the definition of what kind of coalitions are allowed in the economy. In particular, by assuming that workers are not allowed to form subcoalitions within existing firms, share the proceeds from the productive activity of the subcoalition among those who are members of it, and simultaneously receive a share in the original firm, Greenberg finds that an

equilibrium allocation exists. Not all equilibrium allocations are Pareto-optimal, due to the restriction on the formation of coalitions. But by defining an embracing technology, the resulting equilibrium may be Pareto-optimal. This technology represents a weaker restriction on the formation of subcoalitions. The workers are allowed to form subcoalitions and share the proceeds from these subcoalitions among the members. Furthermore, in the replica economy there exists an efficient price system supporting this equilibrium, coinciding with the equilibrium in a PM-economy.

Greenwald (1979) argues that the comparison between the labour-managed economy and the idealized capitalist economy may not be the most relevant one. Instead the labour-managed firm should be compared to a strongly unionized capitalist firm, defined as a firm where the union is strong enough so as to capture the whole of the economic rent (profit), i.e. the union is restricted by a bankruptcy constraint only. In this case the responses to parameter changes are identical between a labour-managed firm and the unionized firm. As the profit (economic rent) can be considered the return to a scarce factor in free supply, the solution to the problem of existence of an equilibrium will depend on the ability of the economic system to establish an equilibrium "rent" vector. Drèze (1974) assumed that these "free" factors of production were owned by the households, and that these ownership rights were given initially ("historically"). However, Greenwald argues that fixing the rents may create problems, so that they may be fixed either too high or too low, causing problems in defining an equilibrium allocation. This is related to the problem of inducing worker-managers to reveal their true productivity by means of incentive compatible mechanisms, cf. Guesnerie and Laffont (1984). It must be noted that in Greenwald's scenario the workers have alternative employment prospects in a self-employed sector, which may imply that for some level of rent all workers will leave the cooperative sector, causing non-continuities in the supply functions. However, if an equilibrium is defined, appropriate use of rent changes when price parameter changes, will induce the labour-managed

firm to undertake the same response as the idealized capitalist firm, cf. the Slutsky-equation approach examined above, assuming that the first-best solution can be implemented.

The assumptions made concerning entry and exit of firms may be quite strong ones, even in the long run. There are probably large costs involved in establishing and closing down enterprises, both to entrepreneurs and society. Especially in a shorter time perspective, these assumptions turn out to be quite unrealistic. Then, if the possibility of attaining a competitive equilibrium depends crucially on entry and exit of firms, we must admit that obtaining efficiency in an LM-economy may create problems. (Of course, entry and exit assumptions cause the same problems in a PM-economy.)

However, suggestions have been made on how to cope with the allocational problem, which in particular relate to the short-run allocational decisions. Vanek, Pienkos and Steinherr (1977) show that using price controls and lump-sum taxes as allocational instruments removes the problem of a backward bending supply curve, and the problem of a possible instability and wrong reactions to price signals also. Although their discussion refers to imperfect competition, it has general interest as means of controlling and directing allocation of labour, and thereby level of production. Another suggestion aimed at influencing the incentives of the firms is put forward by Ireland and Law (1978). They suggest the establishment of an Enterprise Incentive Fund, whose objective is to tax and subsidize firms depending on their allocational decisions. The firms make payments, respectively receive money transfers, depending on whether the workers' dividend exceeds or falls short of a market shadow wage. Then efficiency may be obtained through firms acting according to their self interests.

A problem with these suggestions is that they require some knowledge of parameters which the governing body will have to obtain from the firms involved. Guesnerie and Laffont (1984) have examined the possibility of implementing incentive correcting

mechanisms, and they show that the firms will have incentives to reveal true values in special cases only. This will be further discussed in a subsequent paper dealing with optimal taxation.

On the other hand, the allocational problems may diminish, or even vanish, if the firm is allowed to deviate from the relatively strict definition of the LM-firm as it is formulated by Ward (1958). Some modifications are discussed above, and others should be mentioned also, in particular as elements of these ideas will play an important role in the papers on finance incorporated in this thesis. Thus, Meade (1972) suggests that experimenting with the incentive structure may be useful for reaching a better resource allocation than that of the Illyrian firm. In particular this may be obtained through the inegalitarian cooperative, which allows for members having unequal shares in the firm's value added. These may be determined e.g. according to seniority. A model for trade in membership rights is discussed in Sertel (1982).

Furthermore, if the LM-firm is allowed to employ non-members on short-term contracts, the firm's production decision may be favourably affected, as shown by Domar (1966). However, then the possible dilution of the firm as labour-managed becomes immediate, as it may be found advantageous by remaining workers always to hire new workers instead of giving them full membership rights. This is discussed by Miyazaki (1984). He argues that this is an important explanation of the relatively rare successful experiments with labour-management in capitalist economies. In his model he assumes that the LM-firm is able to insure its debt obligations by making them state contingent. In addition there may exist self-insurance schemes so that temporarily furloughed member will receive the same constant income stream as the non-furloughed workers. Uncertainty is caused by a random market price, and the workers, who maximize the expected income from participation, are assumed risk averse. The hired workers receive the market wage rate. If the workers outside the firm cannot completely insure their wage, it may pay workers of a

bankrupt PM-firm to transform the firm into an LM-firm, although in some states they will receive a payment lower than the going market rate. However, if the constant income stream received by all members exceeds the expected market wage, it will always pay to substitute hired workers for leaving workers, and the firm will turn into a capitalist firm. Only market conditions under which the workers do not receive a payout above the market wage rate are consistent with a pure LM-firm, although the size of the optimal membership may be infinite. But if capital markets are not perfect, an LM-firm with a finite, unique membership size may exist.

9. CONCLUDING REMARKS.

The results discussed so far refer mainly to LM-firms operating under first-best market conditions. The only imperfection touched so far is the possibility of imperfect capital markets and insurance arrangements under uncertainty. Although many other imperfections could be considered, e.g. different kinds and degrees of monopolization in the labour as well as product markets, I will in later chapters of this thesis concentrate on two important problems which affect the possibility of reaching a first-best optimum. One is the lack of a perfect credit market, and the other is the existence of distributional goals which may call for the introduction of efficiency disturbing tax functions. The theory outlined above, combined with an overview of the literature on the finance of LM-firms, is intended to serve as a background for these analyses.

FOOTNOTES

- 1) Illyria is an old Latin term for the Adriatic coast (Yugoslavia, Albania).
- 2) The properties are local only. The existence of non-convexities of the cost-function may make the solution non-unique, and the statement is not globally valid. See also Ireland and Law (1985).
- 3) See Ireland and Law (1985) for a complete treatment.
- 4) The desire to reduce the level of employment is discussed above, and it is explained slightly more formally in the subsequent survey-article on the finance of LM-firms. The individual over-supply of labour is shown formally in the article on optimal taxation of LM-workers and -firms.
- 5) Meade (1972), p. 421.
- 6) Steinherr and Thisse (1979) assume that dismissals are carried through randomly. However, other principles of membership reduction may change the result. Brewer and Browning (1982) argue that a principle of last-in-first-out may make membership reductions involving up to 50 % of the work-force profitable.
- 7) Some algebra is needed to show this formally, and this will not be done here as the analysis is carried through by Miyazaki and Neary. A reasonable explanation can be given, however, by arguing that a price increase will increase the relative price of "staying at home", which is the alternative which faces the marginal worker.
- 8) Some of the comparisons between certainty and uncertainty scenarios may change if, as pointed out by Hawawini (1984), the certainty case is defined as the decision made after

the price (or the productive environment) is revealed, rather than the case where the firm faces the expected price.

- 9) A complete treatment of certainty and uncertainty scenarios is given in Hey (1981a).

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**THE FINANCE OF LABOUR-MANAGED FIRMS
- A REVIEW OF THE LITERATURE**

by

Jan Erik Askildsen

THE FINANCE OF LABOUR-MANAGED FIRMS
- A REVIEW OF THE LITERATURE

1. INTRODUCTION

The finance of the labour-managed firm has been a subject of continuous interest. Several problems have been addressed, ranging from appropriate formulation of the firm's maximand to the optimality of different ways of obtaining finance. A general concensus exists throughout the literature that in an economy consisting of labour-managed firms the main problem is the provision of internal finance. Therefore most interest has been devoted to assess the profitability of internal versus external finance, and to the establishment of proper financial institutions. In this overview of the literature I will examine the main contributions.

I will follow the literature in emphasizing the problems of finance in a labour-managed firm with the property rights structure which is found in the Yugoslav (Illyrian) firm. This will be explained further in the next section, where I will draw attention also to some alternative models. In Section 3 I discuss the internal finance of a labour-managed firm with collective ownership of capital, the Illyrian firm, while I broaden the scope in Section 4 to allow for external finance at a fixed rate of interest. Then in Section 5 some suggestions are discussed, which are aimed at solving problems of external finance under uncertainty. Lastly in Section 6 these suggestions are considered within the wider context of an optimal financial environment of a labour-managed firm.

2. THE MODEL AND SOME BASIC PROBLEMS

I will define a labour-managed firm as a firm where the workers own the means of production and make the decisions regarding production, employment and investment. The firm will maximize the utility of the workers or of a dominating coalition of workers, see Furubotn (1976). A utility index can be established, where present as well as future consumption and non-pecuniary benefits are arguments. It is, however, common to assume that the objection of the firm is per period wage maximization, or the maximization of sales minus non-labour costs per worker (dividend), due to Ward (1958). The model is intended to represent the Yugoslav (Illyrian) firm. In the discussion we will concentrate on this variant of a labour-managed firm, with a proper modification to allow for intertemporal optimization.

The importance of this modification is pointed out by Furubotn and Pejovich (1970) and Furubotn (1971). They argue that the maximization of per period dividend will result in no internally financed investment being undertaken. However, the Yugoslav firm is by no means completely externally financed. Indeed, in each period the workers decide on the allocation of a gross surplus, "profit" after payment of non-labour costs and taxes, to a wage fund and to an investment fund. According to the Ward hypothesis, the latter should be zero, which corresponds to his assumption that the firm is entirely externally financed. On the other hand, it is not difficult to find reasons why the allocation to investment should be non-zero. The government may require it as a condition for being willing to supply credit. If there is a professional management of the firm, its utility may be increasing in their ability to generate internal finance, as this may influence their future job prospects. But it may also be rational from the workers' point of view not to maximize wage payment per period. If they are concerned with present and future wealth, they will maximize an intertemporal utility function. The cost of sacrificing consumption today will be mitigated by the enhanced

future consumption prospects. Thus, if the representative worker expects to remain with the firm for T periods, the proper maximand, see Furubotn and Pejovich (1970), is

$$(1) \quad \text{Max } u(C_1, C_2, \dots, C_T)$$

where C_t , $t = 1, \dots, T$, is consumption in period t . Then the comparison of the marginal rate of substitution between present and future consumption to the yield of investment in capital equipment will give the optimal consumption pattern over time.¹⁾ An example of a situation where positive investment is optimal is shown in figure 1 below:

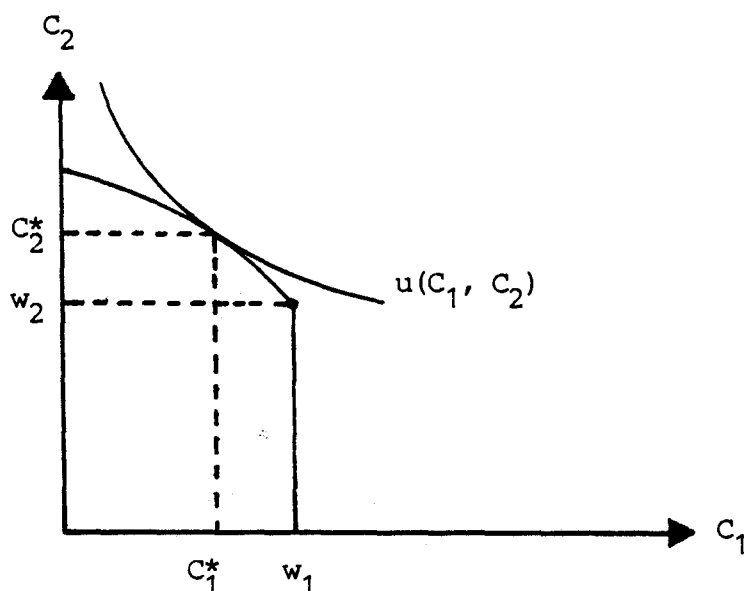


Figure 1: Optimal consumption and investment over time. Two periods. w_t , $t = 1, 2$, is maximum income in the two periods when investment is zero. (See Furubotn (1971)).

In figure 1 the workers can choose between consumption today (C_1) and consumption in the future (C_2). w_1 is maximum income in period 1, resulting if total income is consumed. Then w_2 will be the period 2 income. However, part of w_1 , say $(w_1 - C_1^*)$ may be withheld for investment, and thereby increase income and consumption in period 2. When investment is zero, income and consumption in period 2 is w_2 . Investment (saving) may however increase consumption to a point like C_2^* . The curvature AB reflects the yield of the investment, which together with the intertemporal utility function $u(C_1, C_2)$ will give an optimal level of investment and consumption over time.

Now, to proceed from here, we must explain the property rights structure under which the firm is assumed to produce. The main feature of the Illyrian firm is that the workers have no individual claims on the receipts of an investment. Those will reap the benefits who are employed at the time an investment pays off. This causes two main problems that the workers will consider before voting for an investment project to be undertaken, viz. the horizon problem and the common property problem, cf. Jensen and Meckling (1979). The first problem relates to the tenure period of the initial workers compared to the life-time of an investment project, which may for long-lived projects result in truncated flows as a leaving member of the collective will have no right to share in the income after his departure. The latter problem points to the fact that potential new workers will share in the proceeds of an investment on equal terms with those workers who made the sacrifice by undertaking the investment. Both aspects will affect the profitability of an internally financed investment, although I will in the discussion to follow concentrate on the horizon problem.

It should be noted, however, that the property rights structure of the Illyrian firm is not the only conceivable one. A possible way out of the problem is to follow Ward (1958) and Vanek (1970) and assume external finance only. But as remarked by Jensen and Meckling (1979), a pure rental firm, i.e. a firm financed 100% externally at a fixed capital cost (interest rate), is impossible, mainly because of the need to make investments in intangibles. This is recognized by Vanek (1977c) and McCain (1977), who suggest the introduction of different varieties of variable-income bond finance without voting rights to bond holders. Sertel (1982) proposes on the other hand that membership rights should be traded, which would be a substitute for the poor functioning capital market. Lastly, as pointed out by Gui (1984), the "Basque" labour-managed firm - the firms in the Mondragon cooperative system in the Basque provinces of Spain - may represent a more efficient solution to the property rights problem, compared to the

Illyrian model. Here membership rights cannot be traded. But each worker possesses a personal account to which he has to allocate funds when joining a firm. As a member the worker receives work payment as well as interest on his privately provided capital, and he is allowed to withdraw his funds when retiring. The problems discussed in this article refer to the labour-managed firm with collectively owned capital, as this is the model most widely analysed in the literature.

3. INTERNAL FINANCE

Let us assume that the labour-managed firm consists of N workers, and that this level of membership is constant over the time horizon we consider. This means that we consider the yields due to the initial worker-members. Excluding the common property problem - new workers share in the proceeds on equal terms with old workers - is not necessarily an unreasonable simplification, as the initial workers have the possibility to refuse membership to new workers.

Firstly we have to establish the planning horizon T of the initial workers. According to Furubotn (1979) it does not need to be lengthy. At any cross section of time t , it is given by the formula $T_t = T_0 - t$, $t = 1, 2, \dots, T_0$, where T_t is the planning horizon and T_0 is the total tenure period. Furubotn discusses how the horizon will depend on the political process of decision making within the firm, notably the formation of a dominating coalition. If such a coalition is stable and holds the power over time, the horizon of the decision making majority will continuously diminish. On the other hand, we may as well assume that there is a median worker who holds the balance of power, and whose remaining tenure period is an average of that of the total workforce. Alternatively we may consider the problem as a bargaining problem, in which case the horizon will be some weighted average of the horizon of all workers.

Now, as there exist no tradable claims on the residual of the firm, and we have also to take into consideration that all workers have the same claim on the yield from an investment project, independent on past effort and on how long they have stayed with the firm, each worker voting for an investment to be undertaken will be sure to have his part of the investment expense recuperated during his tenure period with the firm. I.e. the yield from the project must repay the principal and secure an increase in future income above the best alternative available, which we assume is a deposit in a privately owned bank account.

Thus, if the yearly return on an investment is r , we must have for a marginal project worth \$1 to be undertaken, that

$$(2) \quad \sum_{t=1}^T \frac{r}{(1+i)^t} = 1,$$

where i may be interpreted as the rate of time preference or the opportunity cost of financial capital in the economy, i.e. the interest rate on savings accounts. We can solve for r to get

$$(3) \quad r^* = \frac{i(1+i)^T}{(1+i)^T - 1}$$

which is obviously higher than i for a finite T . We can calculate r for different time horizons, T , and opportunity costs, i , to obtain the required rate of return. We see that r is decreasing in T , and it will be high for short horizons. Now, as pointed out by Zafiris (1982), this formula is in itself not special for a labour-managed firm, as a capitalist firm will also require the principal recuperated. The difference arises only when the lifetime of the investment project exceeds the horizon of the workers. The residual claimant in a capitalist firm will simply cash in his expected future claims by selling his shares, while a worker-member of a cooperative loses all claims when retiring. This effect, that the capital costs will increase if the lifetime of the capital equipment exceeds the tenure period of the representative worker, is termed the Furubotn-Pejovich effect (McCain (1977)).

This is however not the only explanation of the alleged increase in capital costs of labour-managed firms. A labour-managed firm will often be subject to what Bonin (1985) refers to as a strong capital maintenance rule. Such a rule is in operation in Yugoslavia (Furubotn and Pejovich (1970)).

According to this rule, the book value of the stock of capital has to be kept intact in perpetuity, i.e. disinvestment cannot take place. There may be economic efficiency reasons (see below) as well as social reasons for having this rule. An important social consideration is the desire to prevent capital from being converted into individualized consumption. However, its effect is to increase capital costs compared to the capitalist firm, and Zafiris (1982) argues that the capital maintenance rule is the main explanation of the high capital costs. While the repayment of the principal should be compared to the depreciation of the capital equipment in capitalist firms, and thereby results in the same capital cost in the two firms if the horizon of investments and owners are the same, the capital maintenance rule is unique and will call for a double counting. Mathematically we can still use (3) to represent the required rate of return. But it should be noted that r is now the required return after allowance has been made for depreciation. Therefore in (3) r^* represents a gross hurdle rate without the capital maintenance rule, while it is a net rate when this rule is in operation, i.e. calculated after depreciation has been made. In both cases r^* is the required rate of return.

As mentioned above, there may be efficiency reasons for having a capital maintenance rule, in addition to the social one mentioned. Vanek (1977a) showed that there are underlying economic forces which tend to reduce the size of the firm over time, by Vanek termed the self-extinction forces. How strongly these forces work, depends on the technology of the firm. Firstly, if there are globally decreasing returns to scale, it will always pay to reduce the size of the firm, and an equilibrium can be reached only with a one-man firm. But even constant returns cause problems. This can be explained, following Vanek (1977a), by assuming that the firm is initially in a position where the value marginal product of capital equals the time preference.²⁾ Write the production function as

$$X = N \cdot f\left(\frac{K}{N}\right),$$

where X is production, K is capital and N is labour (employment level). $f(\frac{K}{N})$ is increasing in $\frac{K}{N}$ at a decreasing rate, and $f(0) = 0$. All investments are internally financed. The workers will maximize payment by maximizing production per worker, e.g. by not replacing members who retire, and thereby increase the K/N -ratio. However, assuming technical complementarity, the reduction in the labour force reduces value marginal product of capital below the equilibrium level giving equality between the marginal productivity of capital and the time preference. Then disinvestment takes place until the firm eventually turns into a one-man firm. The reason for this is the workers' strive to capture the yield due to capital. If the workers have to pay fixed costs, e.g. to the government, there will be a minimum size of the firm which will turn out to be the unique equilibrium size, see figure 2 below:

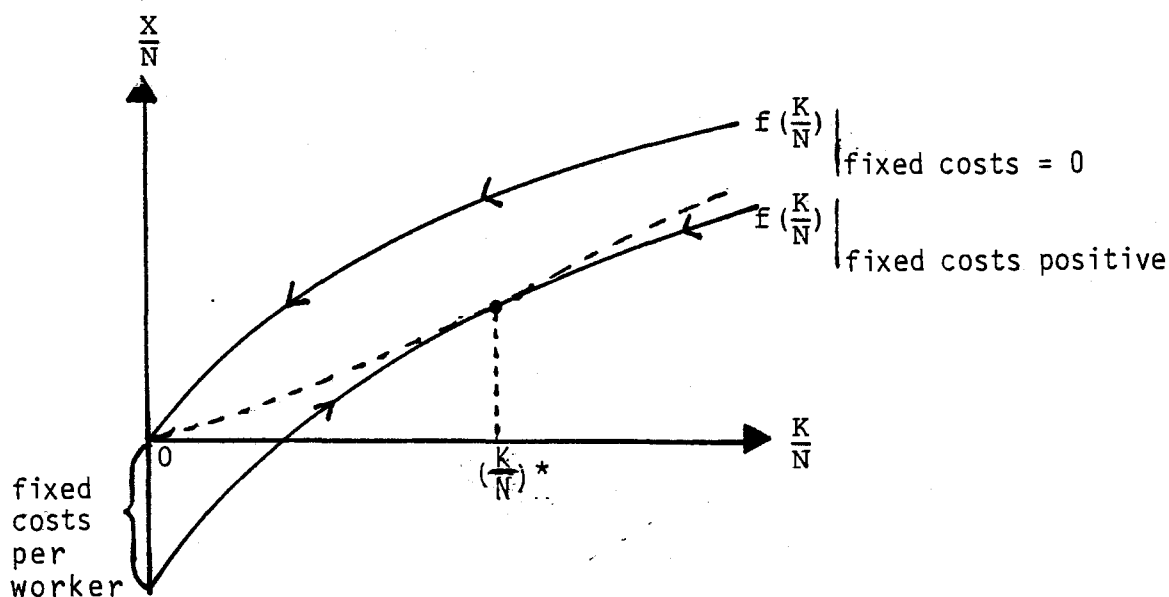


Figure 2. Optimum firm size under constant returns to scale and social ownership of capital, with and without fixed costs

In figure 2, equilibrium positions are 0 and $(\frac{K}{N})^*$ respectively, where there is equality between the marginal efficiency

of capital, the slope of the $f(\frac{K}{N})$ -curve, and the cost per unit of capital. This condition is found from the maximization of $[\frac{X}{N} - \frac{F}{N}]$ with respect to N , where F is fixed costs and $X = N \cdot f(\frac{K}{N})$.

If the firm has a technology with increasing returns to scale at low production levels, and then decreasing returns, see e.g. Vanek (1970, 1977a) and Ireland and Law (1984), an equilibrium is reached where $N > 1$ because of the diseconomies of too low a production. But if the firm is internally financed with a hurdle rate $r^1 > i$, production is lower than in the pure rental firm, or the profit-maximizing firm, see figure 3 and Vanek (1977a).

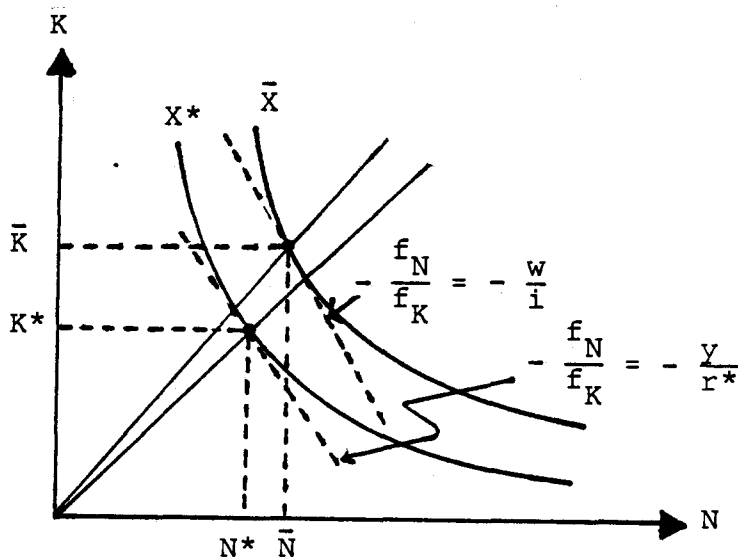


Figure 3. Production and capital-labour ratio in an internally financed labour-managed firm.

In figure 3, at point (\bar{N}, \bar{K}) , there are locally constant returns to scale. In this position capital and labour are paid the market remunerations i and w respectively. However, in the self-financed labour-managed firm self extinction forces are under operation, reducing N and K , until payout per worker is

maximized at (N^*, K^*) where $X^* < \bar{X}$ and $\frac{K^*}{N^*} < \frac{\bar{K}}{\bar{N}}$. This is exactly what was shown in the preceding figure also. But now an internal solution is secured by inefficiency of a too low scale of production.

The effect discussed above is termed the Ward-Vanek effect (McCain (1977)).

On the background of the setting outlined above, Furubotn (1976) discusses the long run efficiency of the self-financed labour-managed firm. He lays down a number of restrictions within which the firm has to operate, and he assumes that the firm maximizes a utility function defined over consumption and several non-pecuniary environmental factors. The solution to the constrained maximization problem shows how the firm chooses between consumption now and in the future, i.e. between payout and investment in productive assets. Now, assume that the workers have the opportunity to choose between investment in individually owned savings accounts or in non-owned (collectively owned) capital equipment. The rate of return has to be corrected for the property rights so as to be able to compare the yield from an investment in non-owned assets to the yield from individual savings. By this correction we require that the return from investment in the firm, $r(I)$, repays the workers the principal and an annual compensation equal to the bank rate of interest, after allowances for depreciation. If the marginal productivity of capital is falling, given employment, the marginal yield of an investment is

$$(4) \quad \frac{\partial r^*(I) \cdot I}{\partial I} = r^* + \frac{\partial r^*}{\partial I} I,$$

where I is investment spending, and r^* is the (property-rights corrected) return per period, and $\frac{\partial r^*}{\partial I} < 0$. An investment will be undertaken if

$$(5) \quad r^* + \frac{\partial r^*}{\partial I} \geq i,$$

where i is the return on owned deposits in savings accounts. Now, we know that the corrected rate of return is lower than the uncorrected return, \tilde{r} , which includes allowances for depreciation, so that

$$(6) \quad r^* + \frac{\partial r^*}{\partial I} < \tilde{r} + \frac{\partial \tilde{r}}{\partial I}.$$

Pareto-optimality requires investments to be carried through until there is equality between the uncorrected rate of return, r , and the market opportunity cost of capital i . But it is obvious from (5) and (6) that the firm's investment decision will exclude socially profitable projects, and a socially optimal investment level cannot be obtained, unless, as pointed out by Furubotn, the savings rate is fixed below the opportunity cost of capital.

It must be noted that Furubotn's analysis refers to the Yugoslav labour-managed firm. However, as pointed out by others, see e.g. Bonin (1985), a strong capital maintenance rule will increase capital costs in addition to the increase in costs caused by the horizon and common property problems.

Berman and Berman (1978), on the other hand, argue that the assumptions taken by Furubotn are the sole explanations of the inefficiency of his labour-managed firm. In particular they find the capital maintenance rule inappropriate in a long term perspective, as it will preclude the liquidation of non-profitable investments. Furthermore, they find the reliance on self-finance unrealistic, and they see no reason why the workers should not be allowed to allocate their savings to the firm by means of owned investments. Furubotn also assumes that the employment level cannot shrink over time, and at the same

time that hours of work per worker is fixed. According to Berman and Berman, also a profit-maximizing firm facing such restrictions would fail to allocate its resources efficiently.

Other aspects pointed out by Berman and Berman are the lack of a capital market and no entry and exit of firms in Furubotn's model. These will strongly influence the allocation in the economy. In addition we could argue that as there exist individual savings deposits with the banks, there should be no reason why the banks should not lend funds to the firms, in particular when taking into consideration the high rate of return on the marginal project when internal finance is the alternative source of finance.

Furubotn (1978) argues that entry of firms will probably be low in the labour-managed economy, due to the lack of incentives to entrepreneurship. Thereby this will not be a means by which resources are moved to more productive uses. Hiring capital equipment is often held to be an alternative to internal investment. This will, however, according to Furubotn give reduced incentives to household the capital stock properly. This increases agency costs, see Jensen and Meckling (1976, 1979), and it will make investments in hired capital more expensive than investments in owned resources. However, Furubotn does not consider this in a general equilibrium context, where incentives may exist to household the hired capital equipment properly. In particular we should expect to find that the owner of the equipment would demand a rental reflecting the real costs, which would directly depend on the way the equipment is being used.

On the other hand, there are assets which it is hard to imagine can be hired, and both Furubotn (1978) and Jensen and Meckling (1979) point to the need for investments in intangibles like education and organisational development. It may also be difficult to obtain bank loans for such projects. This calls for at least some internal investment, and the problems raised by Furubotn and Jensen and Meckling may exist. Simultaneously, however, in such cases there will be strong personal interests

which will affect the decisions. It may e.g. be the case that "intangible projects" are less price elastic than investments in capital equipment. In addition, as these projects may be considered public goods, there may be scope for an efficiency-improving governmental intervention (subsidised loans).

We can, however, conclude that self-finance of collectively owned capital reduces the level of investment. Let us demonstrate the result in a diagram. If SS is the savings-schedule for owned assets and II the marginal efficiency of investment, after allowances for depreciation if the capital maintenance rule is in operation, the effect of the Yugoslav property rights structure can be shown as in figure 4; adapted from Furubotn (1974).

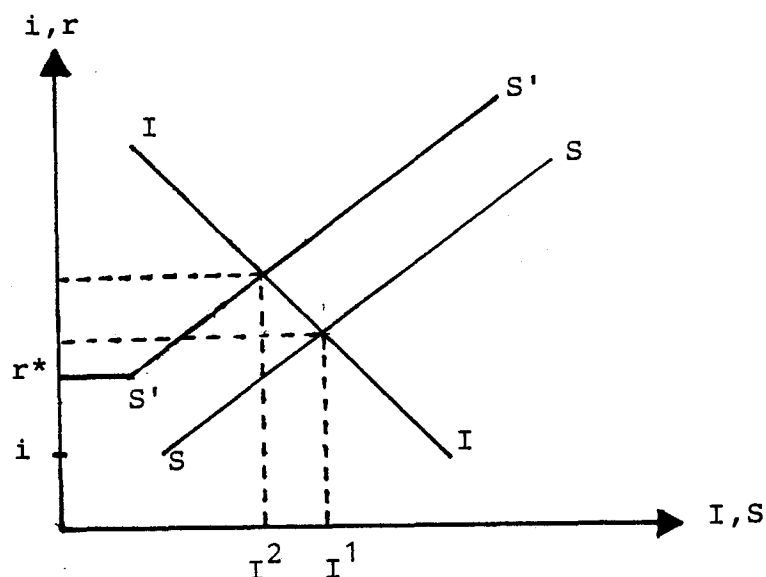


Figure 4. Level of investment in the labour-managed firm with the Yugoslav property rights structure.

r^* is the minimum acceptable return on investments (cf. (3)), and the property-rights corrected savings-schedule will be $r^*S'S'$. The vertical distance between SS and $S'S'$ reflects the increase in return required when going from owned (SS) to non-owned ($S'S'$) assets. We may think of SS as the supply of funds to savings accounts, or to individually owned shares.

With investment in non-owned assets, the level of investment is reduced to I^2 , from the level I^1 when investment takes place in owned assets.

4. FIXED INTEREST RATE BANK FINANCE

Up till now we have restricted attention to the internally financed labour-managed firm. But as pointed out at the end of the preceding section, and argued by Berman and Berman (1978), the assumption is unreasonable that the firm does not use any external sources to finance its activity. E.g. Vanek argues that internal finance, apart from being inefficient, is also ideologically unacceptable.³⁾

Nevertheless, the optimal way to finance an investment in labour-managed firms has been a subject of fierce discussion in a lot of articles. On the one side Vanek (1970, 1977a-d) and Furubotn and Pejovich, see e.g. Furubotn and Pejovich (1970), Furubotn (1971, 1974, 1976, 1980a,b) and Pejovich (1976), have argued that self-finance is always inefficient. The labour-managed firm should be completely externally financed as long as it has to operate within the property rights structure of the Yugoslav firm, or a firm with a capital maintenance rule. This is mainly the view taken by Gui (1981) also, although he concentrates on establishing the conditions under which an investment project is more profitable in a capitalist firm than in a labour-managed firm, given some degree of external finance.

On the other side, Stephen (1978, 1979, 1980), Stephen and Smith (1975), Zafiris (1982) and Bonin (1985) argue that some degree of self-finance will normally be optimal, and that in some cases 100% internal finance may be the optimal policy. The crucial factors in determining the financial structure are the remaining tenure period with the firm of the initial workers, the lifetime of the assets in question, the period of a possible loan, and the interest rates of deposits with the banks and borrowings from the banks.

The factors mentioned above will determine the hurdle rate of profitable investments financed by borrowings or by internally provided funds. The centrepiece of the discussion is whether

the critical rate of internally financed investments will exceed or fall short of the critical rate of externally financed projects. The authors seem to disagree on this according to the line indicated above. In addition there seems to be some disagreement as to the application of the capital maintenance rule on externally financed investments. In earlier papers, e.g. Furubotn (1974), the author assumes that loans can be repaid from the depreciation funds, so that no capital maintenance has to be carried through during the repayment period of the loan. Stephen (1979) argues that this implies treating assets differently depending on their source of finance, which will of course affect the relative profitability of internal and external finance. However, in a later paper, when discussing the optimality of external finance, Furubotn assumes explicitly that the capital maintenance rule applies for bank financed investments also (Furubotn (1980b)). This, in turn, according to Furubotn, will imply that also the externally financed labour-managed firm will operate with a capital level lower than the socially optimal level indicated by equality between the opportunity cost of capital (deposit rate) and the marginal rate of return on capital (cf. (5) and (6)).

Now, assume that the capital maintenance rule is in operation. Then (3) gives the hurdle rate for an externally financed project with a finite loan period also. Note that also this hurdle rate is calculated net of depreciation allowances. r^* is obviously decreasing in T . If the repayment period can be extended beyond the horizon of the initial work-force, the hurdle rate of external finance is lower than that of internal finance since not all of the principal need to be repaid, while the opposite is the case when the repayment period is shorter than the horizon of the work-force. Using this, Bonin (1985) shows, when the bank lending rate equals the deposit rate of interest, that external finance will be used exclusively if and only if the repayment period exceeds the horizon of the (initial) workforce.⁴⁾ In other instances there will generally be a mixture of internal and external finance, and the gap

between the bank deposit rate and the lending rate is crucial. Furubotn (e.g. 1980a) tends to argue that this gap will be small, favouring external finance, while Stephen (1979) tends to argue that it may be large because the capital maintenance rule affects the effective lending rate in the same manner as the required return on internally financed investments is affected. This may favour a segment of internal finance before the firm starts borrowing.

Let us illustrate this in a diagram similar to figure 4, a procedure followed by Furubotn as well as Stephen:

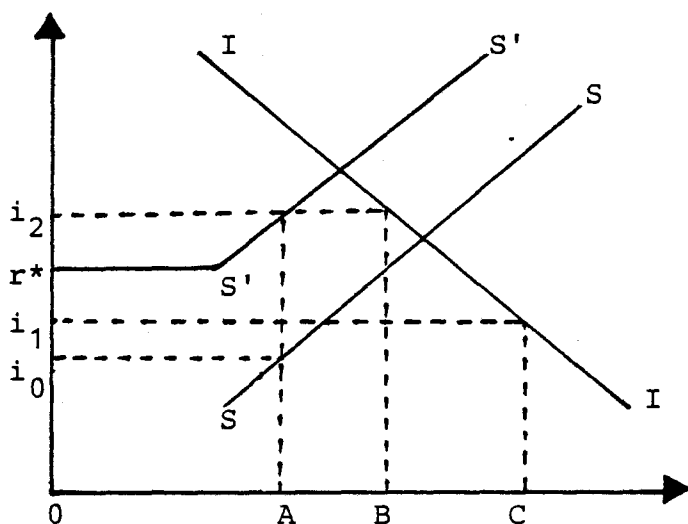


Figure 5. Level of investment under internal and external finance.

Still II represents the marginal efficiency of investment, and can be considered the demand for investment projects. SS is the supply of funds to owned assets, and $r^*S'S'$ represents the supply of funds to non-owned assets. Let i_0 be the bank deposit interest rate, and let i_1 be the lending rate. It is reasonable to assume, as does Furubotn, that this rate is below the hurdle rate of internal investments. If the loans have infinite maturity, i.e. the principal need never be repaid as long as the book value of the assets are kept intact, i_1 will

be the cost of borrowing, and the projects will be 100% externally financed. However, as the repayment period becomes finite, the critical rate for accepting borrowing will increase because of the capital maintenance rule (the required return according to (3) will increase). Thus, at some stage, depending on the repayment period, the critical lending rate may exceed r^* . Then, as argued by Stephen, the application of the cheapest-source-first rule⁵⁾ will imply some internal and some external finance. An example of this situation is given by the "effective" lending rate i_2 in figure 5. Total investment is OB, of which OA is internally financed and AB is borrowed at the constant "effective" (critical) rate i_2 . It should be noted that investment is reduced compared to the level OC when the "effective" rate stayed below r^* .

Gui (1981) is concerned with the comparison of investment decisions between labour-managed firms and capitalist firms. While increased external finance will make the capitalist better off only if the bank lending rate is below the owner's discount rate (time preference), the same will hold for the labour-managed firm provided the lending rate is not too much higher than the time preference. When comparing investment projects with a given level of external finance the capitalist firm is generally better off. Exceptions arise if membership is allowed to be reduced over time in the labour-managed firm, and if the unit labour costs are higher in the capitalist firm. Gui shows also that different forms of governmental subsidy will increase the relative profitability of investing in the labour-managed firm, which is justified on the grounds of the special property rights structure, and that governments often intervene with capital contributions to bankruptcy threatened capitalist firms.

The discussion above indicates clearly that the property rights structure and the capital maintenance rule result in a reduced investment level compared to that of the capitalist firm, or a labour-managed firm with individually owned capital. Although self-finance may take place, from other reasons than being forced upon the firm because of credit rationing, see Stephen

(1978), or resulting from the utility function of the management that may have special interests in arguing in favour of self finance, see Furubotn and Pejovich (1970), it is clear that self-finance is less attractive under labour-management with social ownership of capital than under regimes where ownership is individualised. This will tend to increase the demand for credit if a system of labour-management is to reach the same set of Pareto optimal allocations as an economy consisting of profit-maximizing firms. Indeed, if this is to occur, see Drèze (1976), the firm has to be able to obtain loans with infinite roll-over, or by other means be able to obtain 100% external finance at a market clearing rate. This is the assumption taken by Ward (1958) and Vanek (1970) also.

Vanek (1977a) puts it this way:

"It has always puzzled me how it could have been possible that a productive organisation based on co-operation, harmony of interests and the brotherhood of men, so appealing and desirable on moral and philosophical grounds, could have done so poorly when subject to a practical test. It seems to me that we now have both an explanation and a way of remedy".⁶⁾

The explanation is the inefficiency of internal finance. The remedy is introducing effective ways of obtaining external finance. This can be done by a system of large-scale renting of capital assets (hired capital equipment as mentioned above), e.g. organised by the government or some other formal institution (Vanek (1977b)). This has to be accompanied by formal legislation favouring the idea of self-management. Although fascinating, the idea unfortunately creates problems. Firstly there is the problem connected with investment in intangibles and problems of control, cf. Jensen and Meckling (1979). Secondly there are political problems of introducing the system in economies where other types of firms dominate.

Vanek seems to argue that such problems can be solved in a fully decentralized labour-managed economy by founding a "National Labor Management Agency". This body is assumed to supply funds as well as give advice in planning etc. In particular in a world of uncertainty its role may be crucial, as will become clear from the analysis to follow. That subject has so far not been mentioned. But it should not come as a surprise that the introduction of uncertainty creates new problems, which will strongly affect the optimal financial structure of a labour-managed firm.

5. UNCERTAINTY

Market and production uncertainty will affect the optimal way of financing the firm. Two interesting approaches to the problem can be considered, due to Vanek (1977c) and McCain (1977). Both authors take as their departure the idea that income uncertainty calls for the introduction of variable-income finance, or more precisely a financial instrument whose remuneration is dependent on how well the firm is doing. Simultaneously they try to find solutions which maintain self management of the firm and voting rights vested with the workers. The suggestions are intended to represent realistic ways of organising a financial environment. In a later chapter of this thesis I will question the possibility of combining such variable-income finance with complete self management and an egalitarian power structure within the firm.

The two suggestions differ in their practical formulations, and in what issues they aim at solving. It is taken for granted that internal finance is not profitable because of the Furubotn-Pejovich effect. Exclusive fixed interest rate external finance may however be undesirable from the workers' point of view. Vanek is concerned with finding a financial structure which results in an optimal risk-taking by the workers, while McCain argues that bankruptcy risk may make external finance at a high gearing ratio inordinately expensive. Both approaches call for supply of risk taking capital.

Vanek (1977c) introduces a system of share-cropping which implies that the financiers participate in risk taking by receiving a part of the firm's income. If both fixed-income debentures (risk free bonds) and variable-income debentures (risk participating bonds) are available, the firm should be able to choose the best combination of these bonds. The introduction of variable-income debentures will reduce the risk held by the workers by reducing the variance in income. But the

bond suppliers will demand an expected return above the interest rate on fixed-income debentures as a compensation for the risk they are supposed to share.

Now, assume that the firm can choose among different portfolios consisting of fixed- as well as variable-income debentures. The workers' utility is assumed increasing in income and decreasing in the variance of income (risk aversion). Increased variable-income finance can be obtained at an increasing cost only. Given an investment expense, there exists a most preferred portfolio mixture, as shown in figure 6.⁷⁾

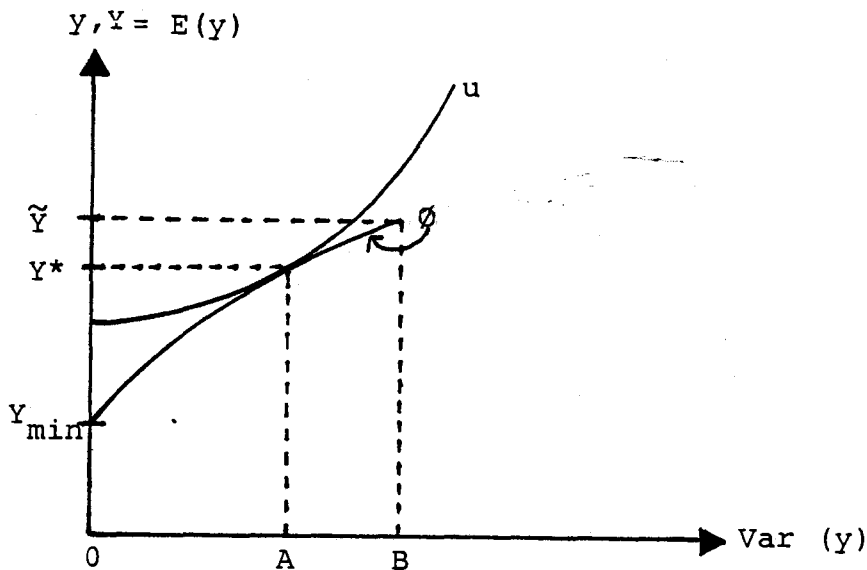


Figure 6. Optimal level of risk held by workers of a labour-managed firm.

Variable-income debentures share in risk. Therefore the bond-holders will demand an expected return which is at least as high as the return on risk free bonds. This will affect the income to be shared among the workers. Let $E(y)$ be expected dividend per worker (or increase in expected dividend by undertaking an investment project), and $\text{var}(y)$ its variance. y is income net of costs of finance (interest paid on bonds). The highest expected income obtainable is \tilde{Y} when all finance

is provided by means of fixed interest rate bonds. The workers will have to carry all risk, so that variance is represented by maximum variance B . The workers can reduce their risk by choosing portfolios consisting of variable-income debentures. At its most extreme we may imagine that all risk is shifted to bondholders, and the workers are receiving a risk free wage. This situation is represented by Y_{\min} . It is reasonable to assume that the financiers will demand an increasingly larger compensation as their part in risk taking increases. We must remember that bonds carry no voting rights. E.g. in $(0, Y_{\min})$ the workers have the exclusive voting rights, while bondholders carry all risk. This solution will induce high agency costs (see Jensen and Meckling (1976) and the discussion in part 6 of this paper relating to their article). Using this, we can draw a concave curve θ between the two extrema, (B, \bar{Y}) and $(0, Y_{\min})$, which may be interpreted as the boundary of the opportunity set giving portfolios of different combinations of risk-taking by outsiders. In the same figure we have drawn an indifference curve u reflecting the workers risk aversion. Then the optimal risk taking by the workers is found in (A, Y^*) . The workers may buy some of the variable-income debentures themselves. This will not reduce the total risk taken by the workers. But it may improve efficiency compared to exclusive fixed interest rate finance if the workforce is heterogeneous in terms of attitude to risk.

McCain (1977) investigates the optimal financial environment of a collective which has the option of issuing ordinary bonds, participation bonds or rely on self finance.⁸⁾ Due to the "law of increasing risk" (bankruptcy risk), Kalecki (1937), the interest rate on ordinary bonds will increase in the gearing ratio when uncertainty prevails. This means that fixed interest rate finance will not be possible under uncertainty.

The participation bonds may represent an optimal way of reducing capital costs. The bonds carry no voting rights. But they are assumed to share in risk. This, McCain argues, will

create problems if the suppliers of bonds do not have any guarantee that the workers' and the financiers' interests are the same. Instead of Vanek's share-cropping McCain therefore formulates a rule for income sharing which ties payout to holders of risk participating bonds to wage-dividend to the workers. When the firm maximizes payout per worker, it also maximizes the return to bonds.

McCain's argument is quite simple. Assume firstly no uncertainty. Then the interest rate on ordinary bonds is fixed as there is no bankruptcy risk, and the market rate of return on participation bonds will in equilibrium equal the risk free rate. The workers are therefore indifferent to mode of finance. Self-finance, however, will be ruled out because of the Furubotn-Pejovich effect (see above). Introducing uncertainty changes the investment decision, and possibly the financial structure, as capital costs increase. The return to ordinary bonds, r , is determined endogenously in each firm's optimization problem as

$$(8) \quad r = r\left(\frac{B}{K}\right), \quad r' > 0,$$

where B is ordinary bond finance and K is total capital in the firm. The expected return to participation bonds, r^{**} , is determined in the market, and it is thereby exogenous to the firm. Assume that the bond-holders value expected returns. To be able to obtain risk participating finance, the workers must determine a parameter z such that

$$(9) \quad r^{**} = z \cdot E(y),$$

where $E(y)$ is expected dividend to the workers. The equilibrium financial structure of the firm is found where the marginal costs of the three modes of finance (self finance included) are equal. Self finance will, however, not be used according to McCain. But the sustainability of this result hinges on some special assumptions made⁹⁾.

If internal finance is zero, the required return to internally financed investments does not affect the optimal solution, and the Furubotn-Pejovich effect does not play any role. Then, McCain argues, the labour-managed firm financed by risk participation bonds cannot do worse than the capitalist corporation financed by means of shares. Indeed, in some cases the labour-managed firm will even do better. This may occur when the capitalist corporation is partly self-financed and the capital markets are not perfect. Then there may be a divergence between the corporation's internal opportunity cost (time preference) and the market rate of return, as the capitalist corporation will typically make use of internal finance.

On the other hand, if participation bonds are not available, the labour-managed firm may rely on internal finance also. As pointed out by McCain, the Furubotn-Pejovich effect and the Ward-Vanek effect will be mitigated by the increasing cost of external finance, and conditions may be met for the firm to undertake internally financed investments.

The analysis by McCain is interesting in the sense that it broadens the set of financial instruments available to the firm beyond those considered by e.g. Furubotn, Pejovich and Stephen. We can illustrate McCain's results within the framework used by Furubotn et al. By referring to figure 5, certainty represents the case where the lending rate is i_1 , and no internal finance is undertaken. Introducing uncertainty means that the cost of borrowing increases due to the "law of increasing risk", and i_1 will approach r^* . If participation bonds are not available, the cost of borrowing will eventually supercede r^* , and some internal finance is profitable. However, the introduction of participation bonds may lower the capital costs, and i_1 will again fall below r^* .

A problem in interpreting McCain's results is the use of quite special assumptions. In particular this affects his conclusion that internal finance will be ruled out when participation bonds are available. On the other hand, the financiers' interests should be better taken care of than in Vanek's share-cropping system, as their return is tied to the workers' return. However, considerable control problems remain. The workers' utility may depend on more than income. Work-place consumption, which it is hard to measure, may be a good substitute for a high take-home wage. Then agency costs will be high. Alternatively there may be negotiations between workers and financiers in determining rules for payout. But then self-management, contrary to assumption, is no longer total. The solution may be either to accept outsiders' voting rights, and/or abolish collective ownership of capital and allow for the introduction of possibly vote-carrying share finance. We will return to the subject in section 6, and in a later chapter of this thesis.

6. THE OPTIMAL FINANCIAL ENVIRONMENT?

In the literature reviewed so far, there is a consensus of opinion that the finance of labour-managed firms is a subject pointing to a potentially serious problem for the system of labour-management. Although the labour-managed firm can be given a property rights structure different from the Illyrian firm's collective ownership of capital, see Gui (1984), the "socialist" structure represents properties many proponents of labour-management will find desirable.

Theoretically the problems involved can be solved internally by different systems of tradeable claims. Tradeable job rights, suggested by e.g. Sertel (1982), would be the precise analogue of shares in a capitalist economy, and common property problems and horizon problems should disappear as the wealth of the firm is capitalized in the claims. On the other hand, there is not a stock exchange continuously evaluating these claims, and correspondingly transaction costs and miscalculations will be larger than in a capitalist environment (Furubotn (1980c)). Schlicht and von Weizsäcker (1977) find the system not feasible, apart from practicable problems, mainly due to the incomplete separability of person and property right. They point, however, to another underlying feature which may affect the possibility of obtaining debt.

After having considered different modes of finance; internal finance, fixed interest external finance, variable-income finance (non-voting shares) and leasing, they argue that the main problem in financing labour-managed firms is the commitment problem. Outsiders will contribute with finance only if they believe that the decision-makers make "good" decisions. A good decision from a financier's point of view is one that maximizes the long-term wealth of the firm. Schlicht and von Weizsäcker argue that the degree to which the workers commit themselves to the firm will be an indication of the quality of the decisions made. A solution to the commitment problem is having a low mobility of labour between firms, which can be obtained through unemployment or other mechanisms which increase the mobility costs. Furubotn (1979) argues on the other hand that a low turnover of labour is not necessarily an indication of a

long planning horizon. Decisions may be taken by a dominating coalition, whose remaining tenure period may be short.

But whatever the solution is to the commitment problem, proper financial institutions will have to be established. Thus, Nutzinger (1975) discusses in a more general context different procedures which can be followed in financing labour-managed firms. He argues that capital can be supplied by means of shares held by the workers (owners), by the state or by any economic agent in cooperation with the workers. Unless shares are supplied by the workers only, the shares should bear no voting right. Nutzinger indicates toward the end of his paper that control problems may arise because of outside finance, which are probably hard to solve. He does not give any solution to the problems. But he argues that experimenting with different forms of finance may give interesting results of importance to the system of labour-management. Although labour-management may be favourable to working conditions and effort (see e.g. Vanek (1970) and McCain (1977, Section V)), the problem of finance has turned out difficult to handle. The suggestions put forward by Vanek and McCain represent possible paths to follow.

However, the propositions are, as indicated previously, not without problems. Drèze (1976) and Jensen and Meckling (1979) comment upon the suggestion put forward by Vanek (1977c). If both workers and outsiders hold shares, and the shares held by the workers are identical to those held by outsiders, the workers can always do better by issuing all shares in the market than by keeping a fraction of them themselves. But if risk is taken by outsiders and voting control remain with the workers, the solution will generally not be efficient as the workers are receiving a risk free wage giving no incentive to allocate the resources according to the preferences of the residual claimants. Thus, probably no-one will be willing to buy risk-sharing bonds without compensation through payment and/or voting control. The same applies when inside held shares/bonds carry voting right whereas the outside held shares do not. In addition that scenario implies an increase in the number of restrictions. On the other hand, as argued by

McCain, the restrictions result from the credit market, in reality facing all types of firms. Furthermore he argues that the non-voting bondholders are comparable to the minority shareholders in ordinary stock companies. To them the voting rights are generally of less importance, as long as they can be assured that the firm's maximand corresponds to theirs.

Jensen and Meckling (1976) consider the relation between the firm and its financiers in terms of agency costs. These include the control costs indicated above. They argue that the capitalist joint-stock firm does not eliminate agency costs, but that these are minimized by that property rights structure. Thus, in order to minimize agency costs, the labour-managed firm must convince the capital suppliers (outsiders) that they have common interests with the workers. Probably this is not as easy as convincing minority share-owners in stock companies, unless there exist institutions like the one suggested by Vanek (1977b, 1970, ch. 15).

Following these lines of reasoning, one might be tempted to believe that the cost of capital is minimized in the joint stock firm (Meade (1972)) maximizing payout per capital unit. Then all capital suppliers take part in risk and decisions (although the workers have no formal say, which may influence their productivity). But as attitude to risk differs among capital suppliers as well, different financial instruments should exist, including the possibility of obtaining fixed returns. A compromise may be the codetermined firm, which maximizes (a utility function of) payout to capital suppliers and workers. The bargaining power of the different parties plays a crucial role in determining the relative shares in the return.

The importance of this approach is that it throws light on the performance of workers as well as capital suppliers of a firm.

Thus, as pointed out by McCain (1982, p. 41), the inefficiency of labour-owned firms resulting from reduced portfolio diversification and ownership by risk averse workers, may be a source of increased monitoring and better performance in terms of X-efficiency. This indicates that there may be an optimal way of

organising the firm which gives some control to workers and some control to capital suppliers. (For an overview, see McCain (1982)). Or, as it is formulated by Drèze:

"Tentative as this conclusion may still be, I regard it as providing theoretical justification for the participation of both labor and capital - whether it be publicly or privately owned - in decisions affecting the future of the firm and hence of its workers and capital owners."¹⁰⁾

FOOTNOTES

- 1) For the time being we are excluding alternative investments in financial assets.
- 2) The time preference should be corrected for the property rights structure, resulting in a higher hurdle rate if the life of the capital equipment exceeds the remaining tenure period of the workers.
- 3) See Vanek (1970), ch. 14.9 and Vanek (1977d).
- 4) Bonin (1985), proposition 5, p. 64.
- 5) Stephen (1979, 1980) argues that Furubotn is led into his error of recommending external supply only because of his concern with maximizing the level of investment, see p. 154 and p. 798 respectively.
- 6) Vanek (1977a), p. 187.
- 7) See Vanek (1977c), figure 11.4, p. 225.
- 8) Ordinary bonds and participation bonds should be comparable to Vanek's fixed-income debentures and variable-income debentures respectively.
- 9) Specifically these assumptions are: The minimal rate of return on internal finance is at least twice as high as r^{**} - the market rate of return on participation bonds, and that the workers are not allowed to reduce the level of the workforce below half the initial level.
- 10) Dréze (1976), p. 1137.

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ON THE OPTIMIZATION PROBLEM OF LABOUR-MANAGED FIRMS,
AND THE APPLICATION OF INDIRECT OBJECTIVE FUNCTIONS TO
REPRESENT THE EQUILIBRIUM

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1. INTRODUCTION

In this paper I will use indirect objective functions to represent the optimal choice of labour and other factors of production in a labour-managed firm. It will be shown that comparative static results are easily derived by using a dual approach, both when the firm's and its workers' individual optimization problems are considered. As will be shown, the results are valid for quite general formulations of the firm's objective function (maximand).

A labour-managed firm (LM-firm) is defined as a firm where the workers own and control the firm in which they work. All important decisions concerning production, investment and employment are made collectively or by an elected management expressing the interests of the workers. The objective of the firm is to produce quantities of one or more products in such a way that dividend (wage, payment) to the workers is maximized.

Characterization of the optimal decisions of the LM-firm concerning employment, use of capital and level of production is well known from the literature. Most theory refers to the Ill-yrrian (Yugoslav) LM-firm (Ward (1958)). But an important extension of the theory has received much attention. Ireland and Law (1982) argue that the utility of the workers is the prime concern of worker-cooperatives. This implies that individual labour supply decisions have to be taken into consideration.

I find it valuable to summarize some existing results on the optimal allocation of labour and non-labour factors, and choice of production, by means of indirect objective functions. But simultaneously I want to stress the importance of viewing the firm's optimization programme in two stages: the firm's collective decisions and the individual workers' individual optimization. All relevant information about the firm's equilibrium choice of factor and production combinations is contained in the LM-firm's dividend function, and each worker's indirect utility function, or its dual the expenditure function, represents the household's equilibrium labour supply decisions.

A dividend function is used by Ireland and Law (1982), ch. 2.10, and Brewer and Browning (1982) introduce a maximum revenue function to analyse the optimal employment of the LM-firm. In this paper I will use this approach when discussing the firm's long-run optimization problem. But since its dual, the cost function, has been used in some contexts, I shall give a brief outline of the firm's cost minimization problem.

It has been shown by Hey (1981) and Ireland and Law (1985) that the LM-firm's optimal production and factor employment decisions can be conveniently expressed by cost functions. In the general problem, where all factors of production are assumed variable, a proper definition of the firm's technology is required. Specifically, so as to be able to find a unique equilibrium, the technology must exhibit first increasing and then decreasing returns to scale. In that scenario the LM-firm's optimal production level is found where average costs are at their minimum, implying the existence of a U-shaped average cost curve. By choosing this cost-minimizing production level, the payout to the workers is maximized, assuming that prices of output and non-labour factors of production are given. Then there exists a minimum cost function in the parametric prices and the maximum attainable dividend, whose properties are equivalent to those of the non-restricted cost function representing the equilibrium of profit-maximizing firms (PM-firms), see e.g. Varian (1978). Ireland and Law (1985) use this cost function to do comparative

statics on the firm's equilibrium when prices of products and non-labour factors of production change.

Now, whereas this minimum unit cost function is defined when all factors of production are variable only, Hey (1981) shows that the cost-function approach is more generally applicable by defining a cost function over non-labour costs. For every level of production, the firm will always choose factor combinations so that the burden per worker of non-labour costs is minimized. Then a minimum cost function exists, which is equivalent to the PM-firm's cost function, and it can be defined with or without restrictions on the variability of some factors of production. But to be able to obtain this equivalence, the cost function must be defined in terms of production, factor use and costs per worker. But then the cost function can be used to study long-run as well as short-run changes when price parameter changes. On the other hand, we will not necessarily be able to tell how absolute levels of the variables change.

I shall consider a model which stresses the importance of considering the long-run and short-run optimization problems separately, which will be defined as the firm's and the household's problems respectively. A similar approach is taken by Ireland and Law (1981), and I find it useful to show that the main results can be confirmed by this dual exposition of the problem. The long run problem is, as seen from the point of view of the members of the LM-firm, mainly a problem of finding relative factor combinations that maximize the utility from participation. This justifies the use of dividend functions that define relative production and factor levels. In the short run, however, employment level as well as the use of non-labour factors should be considered constant, see e.g. Berman (1977) and Steinherr and Thisse (1979). Then the workers' problem becomes closer to the leisure-work decisions of workers in PM-firms, and a programme characterizing their labour supply decisions is of interest. The indirect utility function and the expenditure function offer an informative characterization of this short-run equilibrium.

The model is outlined in the next section. The management's problem, i.e. the long run problem, is discussed, and a dividend function is used to derive responses to price changes. Then the workers' optimal choice of labour supply, i.e. the short run problem, is characterized. In Section 3 I use indirect utility functions and expenditure functions to analyse the equilibrium of the households (the individual workers). Comparative static results are derived in Section 4, using the expenditure function to derive Slutsky-equations. The importance of how income is distributed among the workers is stressed, and the influence of this on the comparative statics is examined. In Section 5 I briefly discuss the optimization in a medium-term time perspective, and discuss some results relating to the comparative statics of a multiproduct firm. Some concluding remarks are given in a final section.

2. CHARACTERIZATION OF OPTIMAL EMPLOYMENT AND PRODUCTION DECISIONS

Model

It may be natural to consider the optimization of the LM-firms and their workers in two steps. In the long run all input factors, including labour, are considered variable. Level of employment, total number of hours to be worked and use of non-labour factors are freely chosen according to the behaviour rule, which is maximum dividend per unit worked. Let us assume that the firm's decisions are made by a democratically elected management, which expresses the interests of the workers. In the short run, however, the number of workers is fixed as it seems unrealistic that anyone can be forced to leave the cooperative, cf. Berman (1977) and Steinherr and Thisse (1979). Hours of work can on the other hand be varied, and each worker decides himself on how many hours to work. This decision can be made by collusion or under the assumption that each worker takes the labour supply of other workers as given (Nash-Cournot).

These two steps, short and long run, can be analysed separately. When management makes its decisions on long-run factor allocation and production, the labour supply of the individual workers is taken as given. Total number of hours to be worked, and thereby total number of workers, are to be determined. We assume that the decision is made according to the preferences of the workers. When the workers decide on their labour supply, the number of workers in the cooperative as well as the level of non-labour factors of production are taken as given exogenously.

We shall analyse a representative LM-firm producing one or more products, denoted by a vector X (vector of Q elements). Products are sold at market price P , where P is a vector of Q elements also. The number of workers in a firm is denoted by N_j , and each worker supplies l_j^i , $i=1\dots,N$, $j=1\dots,J$, units (hours) of work to firm j . The total number of working hours in a firm is $L_j = \sum_{i=1}^N l_j^i$. The enterprise makes use of non-labour factors also, denoted by a vector of S elements, K_j . An S -element vector r denotes the user price of non-labour factors. In the short run all elements of K_j are fixed, and non-labour costs are then for notational convenience denoted by $F_j = \sum_{s=1}^S r_{js} K_{js}$. Furthermore we assume that there are J firms. In much of the analysis to be made this is of little importance, so that notation indicating which firm we analyse, will be omitted when it is clear that we are referring to the representative firm j .

Long-term-optimization--The-management's-problem

The management chooses the levels of non-labour factors, employment and production in such a way that dividend per labour unit is maximized.

The unit of labour is to be interpreted as a worker, an hour of work or a standard efficiency unit. In this representation

tation we will use hours of work as the standard unit since it is general enough to be interpreted as any other unit, and at the same time it is a unit which is practically measurable.

Of course, the choice of unit will not be made arbitrarily, in particular as it affects the way surplus is distributed among the workers. Some aspects of this will be discussed in several contexts below, with special mention of how they relate to the allocation of labour and production. Here I will comment briefly on two general points. Firstly, I find the optimization problem of the Illyrian firm (Ward (1958)) inappropriate in several circumstances. The maximization of dividend per worker will probably be considered unfair in cases other than those where all workers have to work the same number of hours. Although this may sometimes be the case, it is certainly not a generally accepted assumption in the theory on labour supply, and the behaviour rule should take into consideration the incentives directing labour supply decisions. Secondly, we need a programme which is so formulated that its elements can be measured. This may be problematic if efficiency units are other than hours of work. Of course, some elements of productivity are easily measured, like e.g. special qualifying education. But measuring e.g. speed of work will require quite comprehensive supervision activity, and measuring quality of work may prove prohibitively costly. On the other hand, the variables indicated are not necessarily correlated with hours of work, although they are important in obtaining economic results. Thus, maximizing with respect to number of hours to be worked, does not necessarily yield the best result. If extensive cooperation between workers or strong solidarity between them were existent, choice of behaviour rule would probably be a minor problem. Then even maximization of dividend per worker might prove satisfactory. But it seems not realistic to start out with such an assumption, although it may well be an interesting special case.

There is another justification for using hours of work as a representative unit in the maximization problem. Other aspects which exert influence on quality of work, intensity of work etc., can be considered a social good, see Drèze (1976). Then we

have to draw attention to free-rider problems, and that the workers have incentives not to reveal their true productivity. But this may imply that these variables (quality, intensity) vary more among groups of workers (departments) than among individual workers. Therefore it may prove easier to find a distribution rule which takes into account group differences without affecting the firm's optimization problem as such, by varying payment among groups of workers.

Of course, this is not a perfect solution to the free-rider problem. But it represents a formulation which is comparable to the one used when PM-firms are studied. Furthermore, on an abstract level, it is easily generalized to allow for compensation according to any productivity measure. This is further discussed later in this section, and in footnote 5 where we show how dividend may be varied among groups of workers as well as among workers. As presented here, hours worked is assumed to be a good proxy for amount of work done, obtained by a minimum of information and cost.

Dividend, y , is defined as

$$(1) \quad y = \frac{PX - rK}{L} .$$

Maximum production associated with input levels L and K is given by a twice differentiable production function. As there may be more than one product, the efficient production plans are denoted by

$$(2) \quad f(X, L, K) = 0 .$$

The usual properties involving positive and decreasing marginal productivities are assumed.

Now, an LM-firm can produce one or more products, as indicated by the production function above. Here we are not primarily concerned with the choice of products to produce, so that it is

sufficient to assume that production facilities are optimally allocated between different activities. This does not mean that we disregard effects of joint-production, which may be of importance when studying LM-firms¹⁾. It is rather a result of our intention to concentrate on the allocation of input factors. Therefore it is convenient to assume that the various products are easily singled out as functions of input factors, i.e. for each product q we assume (K may still be interpreted as a vector) that

$$(2') \quad X_q = g(L_q, K_q), \quad q = 1, \dots, Q.$$

Thus, the derivatives $\frac{\partial(P X_q)}{\partial L_q} > 0$ and $\frac{\partial(P X_q)}{\partial K_q} > 0$ below relate to the g -function in (2'), and the same will apply to the second order derivatives, i.e. the assumption that marginal productivities are decreasing means that $\frac{\partial^2 g}{\partial(L_q)^2} < 0$ and $\frac{\partial^2 g}{\partial(K_q)^2} < 0$. In the one-product case, the equivalence between (2) and (2') is obvious as we have then that $X-g(L,K) = 0$.

Now we can write the management's optimization problem as

$$(3) \quad \begin{aligned} \text{Max}_{L,K} \quad Y &= \frac{PX-rK}{L} \\ \text{s.t.} \quad f(X,L,K) &= 0. \end{aligned}$$

According to (3), all workers share on equal terms in income from all production activities. First order conditions defining optimal factor combinations and production are, when marginal productivities relate to the g -function in (2') and we note that $\frac{\partial L}{\partial L_q} = 1$:

$$(4a) \quad \frac{\partial(P_q X_q)}{\partial L_q} = \frac{PX-rK}{L}, \quad q = 1, \dots, Q$$

$$(4b) \quad \frac{\partial(P_q X_q)}{\partial K_q} = r, \quad q = 1, \dots, Q.$$

$\frac{\partial(PX)}{\partial L}$ is the value marginal product of labour, which will be

denoted VMP_L .²⁾ We see that the value marginal products will be equalized over the production activities.

There may be different types of labour that are to be compensated at different rates. This will not alter the optimal solution as L may be viewed as a common efficiency unit. When dividend is paid according to type of work, productivity or hours of work, the individual payment is given as some ratio to this unit. Then the management's task is to maximize dividend per unit irrespective of its interpretation. See also footnote 5, and the discussion above.

Thus, X , N and K are chosen according to (4) to give the optimal position of the firm given P , r and the workers' individual labour supply. When the variables are at their optimal levels, the equilibrium may be characterized by a dividend function, Y' .³⁾ This maximum value function expresses maximum dividend per worker as a function of the price parameters P and r . When Z is the production possibilities set, we write the dividend function as

$$(5) \quad Y' = \text{Max } y \\ \text{s.t. } L, K, X \in Z \subseteq R^{1+S+Q} .$$

Y' is convex (not strict convexity) in P and $-r$, and the dividend function will be used to study effects on the endogenous variables L and K of shifts in price parameters. As the derivatives of Y' at prices P_q^0 and r_s^0 are respectively $\frac{X}{L^q}$ and $\frac{K}{L^s}$, $q=1, \dots, Q$, $s=1, \dots, S$, when conditions (4) are fulfilled, we can immediately find the changes in $\frac{X}{L^q}$ and $\frac{K}{L^s}$ resulting from parameter shifts. By using the convexity of Y' , which implies positive second order derivatives, we have

$$(6a) \quad \frac{\partial \frac{X}{L^q}}{\partial P_q} \geq 0, \quad q=1, \dots, Q ,$$

$$(6b) \quad \frac{\partial \bar{L}_s^K}{\partial r_s} \leq 0, \quad s=1, \dots, S.$$

Generally it is hard to determine the total effects on production and employment, as this is influenced by several cross effects, whose signs depend on technical relations in production. Substitution possibilities between factors as well as products matter. In the short run these may be quite small, but they are probably of more significance in a longer time perspective. As the discussion relates here to long-run decisions, we must take substitution possibilities into account. Then, from (6a) we find, if the relative price of product q increases, that the optimal production per worker of product q increases. But this also affects the production of other products through internal factor and product adjustments, provided substitution possibilities exist. I.e., if possible, production facilities will be transferred so as to increase production of product q . But this means that the production of product q can be increased without increasing employment.

Let us examine this adjustment process somewhat more closely. If the production functions (g -functions) are homogenous, equally proportionate changes in all factors do not affect relative marginal productivities. But dividend, y , changes when P_q does, implying that labour becomes relatively more expensive if P_q rises. This causes a shift in factor combinations towards non-labour inputs, which affects labour productivity positively or negatively depending on technical relations. Thus, the effect on employment (or more precisely, total number of hours to be worked) is ambiguous when the relative price of one product increases when the production function is homogenous, and the total effect on scale of production is ambiguous as well.

The above discussion relates to the general case with more than one factor of production. In the two-factor case it is possible to establish the results more precisely. When there are constant returns to scale, technical complementarity between

the factors of production follows, i.e. in (2'), $\frac{\partial^2 g(K,L)}{\partial K \partial L} = \frac{\partial^2 g(K,L)}{\partial L \partial K} > 0$. Furthermore, in the two-factor case the cross-effects have to be opposite to the direct ones. Then we have:

$$(6c) \quad \frac{\partial \frac{X}{L^q}}{\partial r} \leq 0,$$

$$(6d) \quad \frac{\partial \frac{K}{L^q}}{\partial P} \geq 0.$$

From (6d) we see that the $\frac{K}{L}$ ratio is non-decreasing. As technical complementarity means that labour productivity is increasing in the other factor of production (capital), we see that (6a) can be satisfied through either a reduction in employment or an increase in the use of capital. (Of course, if there is technical complementarity between all factors, the last point is generally valid. But to establish technical complementarity in the general case, further assumptions are required.)

Now, in the two-factor case, according to (6a) and (6d) the effects on total employment and production are ambiguous. The effects on L, K and X must be found by doing comparative statics on the first order conditions. When the production function is homogenous of the first degree, this yields the result that:

$$\frac{dL}{dP} = \frac{dK}{dP} = \frac{dX}{dP} = 0,$$

assuming an equilibrium can be found. Furthermore, also

$$\frac{dL}{dr} = \frac{dK}{dr} = \frac{dX}{dr} = 0.$$

The LM-firm cannot do better by changing factor mix when its production function is homogenous of degree one. This implies that the firm's dividend-function is linear in P and r.⁴⁾ However, we have problems in defining maximum when constant returns to scale prevail over some range. To define a unique equilibrium, the average cost curve should be u-shaped. But then the 0-responsiveness and linearity indicated above will generally not hold.

In the general case the effects are harder to trace out, particularly when input-output relations differ between lines of production depending on scale advantages. Net effects on employment, use of non-labour factors and production will depend on technical relations and transfer possibilities. To state ultimate results, we would need knowledge of these technical relations.

If all prices change with the same percentage, and scale advantages do not differ among lines of production, there is no reason to adjust product mix. Total effects on employment and production levels will depend on technical relations as optimal factor mix will change. The same occurs if prices of non-labour factors change. This may influence product mix also, as factor intensities may vary between lines of production, and thus affect the relative profitability of the products.

If there is no possibility of changing the product combination, the effect of a single price changing is identical to an equally proportionate change in all prices. On the other hand, if the scale of production of the product in question can be changed, and held fixed for other products, the result will be as in the one product case. These cases are, however, probably of little empirical interest, as some substitution possibilities will exist in the long run. In the short run, they might prove more interesting.

Some of the aspects mentioned above will be discussed also at the end of next section, although from a somewhat different viewpoint.

Short-term optimization - the households' problem.

Each worker chooses labour (hours of work) (l^i) and consumption (C^i) so as to maximize a utility function given the number of workers in the LM-firm(s):

$$(7) \quad u^i = u^i(c^i, l^i), \quad i=1, \dots, N.$$

$u^i(c^i, l^i)$ is individual i 's quasi-concave utility function, where $u_c^i = \frac{\partial u^i}{\partial c^i} > 0$ and $u_l^i = \frac{\partial u^i}{\partial l^i} < 0$. The household's budget is restricted by dividend (income) from work in the LM-firm(s). Income per unit worked is not constant, as the worker can himself/herself decide upon how many hours to work, which will affect the dividend. Furthermore, dividend may be distributed according to different rules, e.g. proportional to hours worked or on a per capita basis independent of hours worked by each worker. Whether all workers receive the same payment for a labour unit (hour, year etc.), plays a minor role in this context. The workers may belong to different pay categories also, characterized by productivity or type of work, which may affect the dividend received. Although these qualifications are not taken explicitly into consideration in all results to be developed, this is easily done without changing the way in which the analysis is carried through.^{5),6)}

Sen (1966) specifies an income distribution rule which contains both payment according to "work" (proportional to hours worked) and payment according to "needs" (equal amount per worker). He shows that some combination of payment according to "work" and payment according to "needs" can be found which secures a Pareto-optimal supply of labour, irrespective of collusion or "social consciousness" among the workers. Such a rule is easily incorporated into a general dividend formula, as it will be shown below.

Here we will as a starting point use a quite general dividend formula, allowing for payment according to work done, according to "needs" (Sen (1966)), and also distribution of dividend according to other productivity measures or type of work (task in production). Denote share of dividend due to worker i by D_j^i , $i=1, \dots, N$, $j=1, \dots, J$. As the share will vary among the workers, we use superscript i to indicate the different workers. A pro-

portion of dividend, γ , may be paid according to work done, and $(1-\gamma)$ according to "needs", where $0 \leq \gamma \leq 1$. Then the reason for different payment to workers in the same firm is due either to variation in individual hours of work (l^i), or different productivity or type of work. Payment according to type of work or productivity is in the general formula taken care of by the parameters δ_1^i and δ_2^i . The dividend sharing rule of firm j is:⁷⁾

$$(8) \quad D_j^i \equiv \delta_1^i \gamma_j \frac{l_j^i}{L_j} + \delta_2^i (1-\gamma_j) \frac{1}{N}, \quad i=1, \dots, N.$$

According to (8) dividend is composed of two elements. Both of these elements may be affected in different ways by the factors mentioned above which will influence payout (productivity, activity, etc). Therefore the two parts of D_j^i is assigned δ_1^i and δ_2^i respectively, and one of them may well be equal to unity while the other is not. Thus, we may also interpret (8) as a linear approximation of a non-linear dividend distribution (wage) system. Also, as the economy consists of several firms, dividend is only paid to workers who are working in ("member of") firm j , i.e. $D_j^i = 0$ for non-members.

Now, assume that the actual dividend distribution is given by some specification of (8). We are interested in finding the optimal labour supply of a representative worker. In most cases there will be need to distinguish between work in different firms. Although relative price changes leading to shifts in relative dividend among firms may lead to shifts of occupation or hours worked in each firm, we will concentrate on studying the actions of a representative worker employed in only one representative firm. In our analyses here, nothing is lost in terms of generality by assuming membership in one firm. When studying LM-firms this should not be considered a strong assumption as workers will, because of the firm's structure of codetermination, probably have loyalty and interests towards the firm beyond the pure economic ones. Thus, we will omit the

subscript j , and concentrate on a worker working in a representative firm.

The representative worker's optimization problem can be written as in (9) below. In this partial equilibrium analysis we have also omitted the superscript i , as the worker may be any employed in an LM-firm. This is however a notational simplification only, and not an assumption that all workers are identical. Superscript i will be used when it is needed in interpreting results. Thus, we write

$$(9) \quad \text{Max } u(C, l)$$
$$\text{s.t. } pC = (PX-F) \cdot D$$

C and p may be interpreted as vectors of consumption goods and prices of consumption goods respectively. First order conditions are:

$$(10a) \quad u_C - \lambda p = 0$$

$$(10b) \quad u_l + \lambda \left[\frac{\partial(PX)}{\partial L} \frac{\partial L}{\partial I} \cdot D + (PX-F) \frac{\partial D}{\partial I} \right] = 0$$

$$(10c) \quad (PX-F) \cdot D = pC$$

λ is a Lagrange multiplier. We assume an interior solution and monotonicity so that $\lambda > 0$. In order to have efficiency in Pareto sense, the marginal rate of substitution between consumption and work must equal the value marginal product of work. As it is seen from (10a) and (10b), this condition will be fulfilled only in certain circumstances. Degree of collusion and social consciousness and formulation of the dividend distribution rule are decisive in this context. This is discussed in several other articles as well, see e.g. Sen (1966), Bonin (1977), Berman (1977) and for a summary Ireland and Law (1981).

A few comments on the marginal rate of substitution, MRS, is appropriate. The MRS is, for some representative worker i , defined by using (10a) and (10b) as

$$(11) \quad - \frac{U_1^i}{U_C^i} = \frac{1}{P} \left[\frac{\partial(PX)}{\partial l^i} \cdot D^i + (PX-F) \frac{\partial D^i}{\partial l^i} \right].$$

It is of interest using a somewhat more general formulation of (11). As indicated in (8), payment can be made according to various units. In footnote 5 we showed that the production function can be written in terms of efficiency units, and that different work categories are easily taken into consideration. Then, instead of using L as input factor, let us use L_E , where L_E is number of efficiency units supplied to the LM-firm, given employment level. As a special case, L_E is number of hours worked. Furthermore, let α^i be a parameter transforming worker i 's labour supply into efficiency units. Then α^i reflects the productivity of worker i . Worker i 's supply of efficiency units is $l_E^i = \alpha^i l^i$. Of course, if L_E (and l_E^i) is number of hours, then $\alpha^i = 1$. Since it should not matter in this context to which work category worker i belongs, i.e. we do not analyse choice of occupation, we do not take into account that payment may differ among groups of workers (β_b , defined in footnote 5, is 1). Dividend to worker i will be, when $\delta_1^i = \delta_2^i = \delta^i = \frac{\alpha^i}{\sum_j \alpha^j}$ (see also footnote 5):

$$(8') \quad D^i = \gamma \frac{\alpha^i l^i}{\sum_i \alpha^i l^i} + (1-\gamma) \frac{\alpha^i}{\sum_i \alpha^i}$$

Furthermore, we have that

$$\frac{\partial(PX)}{\partial l^i} = \frac{\partial(PX)}{\partial L_E} \frac{\partial L_E}{\partial l^i} \frac{\partial l^i}{\partial l^i} = \alpha^i \frac{\partial(PX)}{\partial L_E} \cdot \frac{\partial L_E}{\partial l^i}.$$

Letting $\frac{\partial L_E}{\partial l_E^i} \frac{l_E^i}{L_E} = \eta^i = \eta$ (11) can be written:

$$(11') \quad -\frac{u_1}{u_C} = \frac{1}{p} \cdot \alpha^i \left\{ \gamma \left[\frac{\partial(PX)}{\partial L_E} \cdot \eta + \frac{PX-F}{L_E} (1-\eta) \right] \right. \\ \left. + (1-\gamma) \frac{\partial(PX)}{\partial L_E} \cdot \frac{\partial L_E}{\partial l_E^i} \frac{1}{\sum_i \alpha^i} \right\}$$

η is the elasticity of total number of efficiency units in the individual worker's supply of efficiency units. The interpretation of the r.h.s. of (11') is independent on what kind of units η measures. When it is not ordinary hours, α^i transforms the terms in bracket to the same units as $-\frac{u_1}{u_C}$. Conditions for Pareto-optimality, and comparative statics, can therefore be analysed irrespective of what kind of labour units which enter into the production function and the dividend sharing rule (cf. Vanek (1970), ch. 12). Particularly, when $\alpha^1 = 1$, i.e. efficiency units are number of hours worked, then $\eta = \frac{\delta L}{\delta l} \frac{l}{L}$, and the interpretation of (11') is straightforward.

As to η , it lies in the interval $\frac{1}{N} \leq \eta \leq 1$ if $l_E^i = \bar{l}_E$ for all i initially. In our comparative static analysis, we will assume $\eta \approx \frac{1}{N}$, i.e. a Nash-Cournot assumption for individual labour supply reactions. Then we know that labour allocation is Pareto-optimal when "social consciousness" is perfect or when some optimal value of γ is chosen (Sen (1966)). When $\gamma = 1$, the allocation is Pareto-optimal when the firms are in long run equilibrium, i.e. $\frac{\partial(PX)}{\partial L_E} = y_E$, see (1) and (4a).

However, in the analysis to follow, we assume $\alpha^i = 1$ for all i belonging to firm j . Furthermore, the analysis is independent of the work category to which the worker belongs, so that also $\beta^i = 1$ (see footnote 5). As we have argued above, this simpli-

fication does not seriously affect the generality of the results.

3. INDIRECT OBJECTIVE FUNCTIONS TO REPRESENT HOUSEHOLD EQUILIBRIUM.

We will now see how the individual workers' optimal choice of labor supply and consumption can be represented by indirect objective functions. These may be useful in other contexts also. Particularly when taxes are introduced, indirect objective functions have analytical advantages, partly because of the formulation's lucidity, and partly because the problem is formulated in terms of prices and income. Some effects of taxation will be examined in a subsequent paper.

Assume that the worker has chosen the optimal leisure and consumption levels according to (10). Then worker i 's indirect utility function can be written as:

$$(12) \quad v(p, P, F, l^{h\#i}, N, \gamma, m) = \max u(C, l)$$

$$\text{s.t. } pC - (PX-F) \cdot D \leq m.$$

Here $l^{h\#i}$ is a vector of labour supply from workers other than i working in the same firm, and m is minimum lump-sum income necessary to reach a given utility level. This indirect utility function, or its dual the expenditure function, is useful when deriving labour supply functions and studying the effect of price changes. The expenditure function is:

$$(13) \quad e(p, P, F, l^{h\#i}, N, \gamma, U_0) = \min pC - \left[\gamma \frac{PX-F}{L} \cdot l \right. \\ \left. + (1-\gamma) \frac{PX-F}{N} \right]$$

$$\text{s.t. } u(C, l) \geq u_0.$$

The expenditure function is to be interpreted as minimum lump-sum income, m , necessary to reach a given utility level, u_0 . We will make use of this function when doing comparative statics. By using Roy's identity, the indirect utility function could be used instead. Some remarks regarding information contained in the indirect utility function (and in the expenditure function as well) should however be made. We will show how the individual members of a producer collective like an LM-firm are affected by the labour supply decisions of other workers and the level of employment (membership) of the firm.

Let us investigate first how a representative worker's maximum attainable utility is influenced by the level of employment. This reflects the employment policy each member wants the management to follow, and the management is supposed to act in accordance with the workers' preferences.⁸⁾ In the long run we assume that each worker's labour supply is fixed and on average equal to \bar{l} . If γ is either 1 or 0, i.e. $D = \frac{1}{N}$ or $D = \frac{1}{L}$, differentiation of (12) with respect to N yields, assuming $\frac{1}{N} = \frac{1}{L}$:

$$(14) \quad \frac{\partial v(\cdot)}{\partial N} = \lambda \cdot \frac{1}{L} \left(\frac{\partial(PX)}{\partial L} \cdot \bar{l} - \frac{PX-F}{N} \right).$$

(For simplicity, the indirect utility function is denoted by $v(\cdot)$). λ is interpreted as marginal utility of money (income), and $\lambda > 0$. Thus, the sign of the effect on worker i 's maximum utility by changing the level of employment is dependent on how the firm has allocated its resources. In long term equilibrium, $\frac{\partial(PX)}{\partial L} \cdot \bar{l} = \frac{PX-F}{N} = y$ and the worker is indifferent between increasing or reducing employment. However, if $\frac{\partial(PX)}{\partial L} = VMP_L < \frac{y}{\bar{l}}$, which will be the immediate effect of a price increase, we see that utility is decreasing in the number of workers. Thus, each worker for whom $\lambda > 0$ will find it profitable to reduce the employment level, and accordingly vote for this to occur. Of course, all workers could agree instead upon a reduction in individual labour supply, thereby reducing the need for membership reduction. But as membership reduction can be carried through by

natural retirement etc., there should be no need to dismiss workers who do not want to leave. Then reduction in individual labour supply will take place only if the workers find it optimal to do so according to their individual leisure-work decision.

However, in the long run the firm may change the level of non-labour factors also, and K will be affected by changes in $L(N)$. We can write the effect on maximum utility from a marginal change in N , when non-labour factors are affected as well, as:

$$(14') \quad \frac{\partial v(\cdot)}{\partial N} = \lambda \cdot \bar{I} \left[\left(\frac{\partial(PX)}{\partial L} \cdot \bar{I} - \frac{PX-rK}{N} \right) + \sum_s \frac{\partial K_s}{\partial L} \bar{I} \left(\frac{\partial(PX)}{\partial K_s} - r_s \right) \right],$$

where $\frac{\partial L}{\partial N} = \bar{I}$. Again, assuming a price increase, we see that the effect on maximum utility of changing membership is no longer unambiguous when the firm is not in a long run equilibrium. If there is technical complementarity in production, i.e. $\frac{\partial K_s}{\partial L} > 0$, and if $\frac{\partial(PX)}{\partial K_s} > r_s$, $s=1, \dots, S$, it may be optimal to increase membership as the change in use of non labour factors offsets the effect discussed above. With technical substitutability, however, there will definitely be a reduction in membership. But an overall complementarity seems most realistic, and in the case with two factors of production and constant returns to scale, this is always the case. See also discussion in Section 2, under the heading "Long-term optimization".

In a shorter time perspective individual hours of work can be varied. Expelling members, and for the same reason engaging new members, is considered to be an unrealistic assumption in the short run, cf. the presentation of the model in Section 2 above. But the utility of worker i will be affected if the other workers change their individual labour supply. Suppose worker h , $h \neq i$

and $h \in N$, changes his labour supply l^h . Then the effect on individual i 's utility is, assuming $D^i = \frac{l^i}{L}$:⁹⁾

$$(15) \quad \frac{\partial v^i(\cdot)}{\partial l^h} = \lambda^i \left[\left(\frac{\partial(PX)}{\partial L} - \frac{PX-F}{L} \right) \frac{\partial L}{\partial l^h} \frac{l^i}{L} + \frac{PX-F}{L} \frac{\partial l^i}{\partial l^h} \right]$$

Here $0 \leq \frac{\partial l^i}{\partial l^h} \leq 1$. Furthermore, the value this derivative takes, has implications for the value of $\frac{\partial L}{\partial l^h}$. If $\frac{\partial l^i}{\partial l^h} = 0$, then $\frac{\partial L}{\partial l^h} = 1$. On the other hand, $\frac{\partial l^i}{\partial l^h} = 1$ implies $\frac{\partial L}{\partial l^h} = N$, assuming two arbitrarily chosen workers are being studied. Then, if $\frac{l^i}{L} \approx \frac{1}{N}$, we find that assuming $\frac{\partial l^i}{\partial l^h} = 1$ will yield

$$(15a) \quad \frac{\partial v^i(\cdot)}{\partial l^h} = \lambda^i \cdot \frac{\partial(PX)}{\partial L},$$

while the effect on maximum utility when $\frac{\partial l^i}{\partial l^h} = 0$ is

$$(15b) \quad \frac{\partial v^i(\cdot)}{\partial l^h} = \lambda^i \left[\left(\frac{\partial(PX)}{\partial L} - y \right) \frac{l^i}{L} \right].$$

Thus, the assumptions made concerning individual labour supply reactions are decisive for the outcome. When all workers change individual labour supply simultaneously, (15a), worker i 's indirect utility is always increasing in the labour supply of other workers. The reason is that the worker's marginal utility of income is positive, and a unanimous increase in labour supply will definitely increase the income of each worker. But assuming that worker i does not respond to the change in labour supply by worker h yields a result similar to the one discussed in (14), i.e., the effect on indirect utility is dependent on whether the firm is in long run equilibrium, when the effect is zero, or whether $VMP_L \geq y$. Particularly, when prices increase, and we

find that $VMP_L < y$, the increased labour supply from worker h will have negative influence on the maximum utility of worker i. This is so because worker h increases his share in total income, $\frac{l^h}{L}$, at the expense of the other workers, which means, given the new price level, a reduced income to worker i unless he retaliates by increasing his labour supply.

We can also find the relative strength of this effect between two workers by taking the same derivative of h's maximum utility, $v^h(\cdot)$, with respect to labour supply of worker i, l^i . Now, suppose $l^i = l^h$, $\frac{\partial L}{\partial l^h} = \frac{\partial L}{\partial l^i}$, and $\frac{\partial l^i}{\partial l^h} = \frac{\partial l^h}{\partial l^i}$ initially. Then

$$(15c) \quad \frac{\frac{\partial v^i(\cdot)}{\partial l^h}}{\frac{\partial v^h(\cdot)}{\partial l^i}} = \frac{\lambda^i}{\lambda^h}$$

Thus, differences in the effect of labour supply changes on maximum utility of workers in the same firm is due to differences in marginal utility of income. Allowing for interpersonal comparisons, i.e. assuming strict concavity, this can be interpreted as saying that the richest workers will care less than the poorer workers when someone changes his labour supply. With the assumption that the workers within a firm are identical, this indicates that the effect on maximum utility of labour supply changes may vary among firms if the overall prosperity of the workers differs. But then it is probably the case that the willingness to collude in fixing individual labour supply differs among firms also, in such a way that, ceteris paribus, the poorest firms will have the largest value of $\eta = \frac{\partial L}{\partial I} \frac{l}{L}$ (or $\frac{\partial l^i}{\partial l^h}$).

The terms "rich" and "poor" need to be commented upon. As used here, we think of "rich" and "poor" in terms of income differentials. This may however be of little interest without further

specification of reasons for the inequality in income. It may be due to differences in external income (wealth) sources or to unequal payment for the same number of hours of work, and the comparison above will be valid. But if consumption of leisure differs, i.e. the workers supply an unequal number of hours, differences in preferences can be the sole explanation of income differentials. Then there is no reason to assume systematic differences in marginal utility of income. We see from (15) that the variables within the bracket will differ among the workers, so that the explanations of why the effects on maximum utility differ when someone changes hours of work, are much more complicated. Therefore, "rich" and "poor", as they are used above, refer to total income of workers when their hours of work and preferences do not differ significantly.

4. COMPARATIVE STATICS

We have indicated above that the indirect utility function (12) as well as the expenditure function (13) are useful when doing comparative statics and deriving Slutsky equations. Here I will use the expenditure function. For notational convenience, let it be denoted $e(\cdot)$.

Since the specification of D , and in particular the value of γ , is important when analysing the effect of parameter changes, it is of interest allowing for payment according to Sen's (1966) rule. Compensation is therefore conveniently divided into a fixed part, y^f , and a variable part, y^v , where

$$\begin{aligned} y^f &= (1-\gamma) \frac{PX-F}{N}, \\ y^v &= \gamma \frac{PX-F}{L} \end{aligned}$$

But then we can rewrite (13) as

$$(13') \quad e(p, y^v, y^f, U_0) = \min pC - [y^v l + y^f] \text{ s.t. } u(C, l) \geq U_0$$

When the agent has made the optimal choices of labour supply and other endogenous variables, given the values of the parameters of the model, we know that other effects than those of first order may be disregarded when these parameters change, cf. Dixit

(1976). If e.g. prices change marginally, the existing number of hours worked continue to be an optimal choice. This means that we can find compensated (Hicksian) supply (and demand) functions directly from the expenditure function. Let us assume that the firm is in short run equilibrium so that $\frac{\partial y^v}{\partial I} = 0$. Then by differentiating $e(\cdot)$ in (13') with respect to y and noting that $y = y^f + y^v$, we get (cf. Shephard's lemma):

$$(16) \quad \frac{\partial e(\cdot)}{\partial y} = \frac{\partial e(\cdot)}{\partial y} \left(\frac{\partial y}{\partial y^f} + \frac{\partial y}{\partial y^v} \right) = - (1+\gamma).$$

This is the compensated labour supply function $l^k(p, y^v, y^f, u_0) \equiv l^k(p, P, N, l^{h \neq i}, F, \gamma, u_0)$, which for notational convenience is written $l^k(\cdot)$.

We see that it is composed of a constant and a variable element, which is due to the possible existence of a fixed as well as a variable work payment element (y^f and y^v respectively). If all payment is made according to work done ($\gamma=1$), (16) will become

$$(16a) \quad \frac{\partial e(\cdot)}{\partial y^v} = - 1,$$

and if $\gamma = 0$, that is all payment is fixed irrespective of work done, then

$$(16b) \quad \frac{\partial e(\cdot)}{\partial y^f} = - 1.$$

Now, it seems most reasonable to assume that payment is made in the short run according to work done, i.e. (16a) holds. The comparative statics will be done under this assumption, although we will also discuss solutions for cases where $0 \leq \gamma < 1$.

We know that the compensated and uncompensated labour supply will be identical in equilibrium. Let $l^u(p, P, F, l^{h \neq i}, N, \gamma, m) = l^u(\cdot)$ denote the uncompensated labour supply. Then we can write:

$$l^k(\cdot) \equiv l^u(\cdot).$$

As $e(\cdot)$ is concave in y (or P and $-F$), we know that

$$(17) \quad \frac{\partial l^k(\cdot)}{\partial y} = - \frac{\partial^2 e(\cdot)}{\partial y^2} > 0.$$

This is the substitution term in the labour supply Slutsky equation, and we see that it is positive in P as

$$\frac{\partial y}{\partial P} = \frac{\chi}{\Gamma} > 0.$$

The complete Slutsky equation is found by differentiating the identity $l^k(\cdot) \equiv l^u(\cdot)$ with respect to y . This yields:

$$\frac{\partial l^k(\cdot)}{\partial y} = \frac{\partial l^u(\cdot)}{\partial y} + \frac{\partial l^u(\cdot)}{\partial m} \cdot \frac{\partial m}{\partial y}.$$

Then, using the equilibrium identity $m \equiv e(\cdot)$ and the fact that $\frac{\partial e}{\partial y} = -1$, cf. (16a), this can be written as

$$\frac{\partial l^k(\cdot)}{\partial y} = \frac{\partial l^u(\cdot)}{\partial y} - 1 \frac{\partial l^u(\cdot)}{\partial m}.$$

The effect on the labour supply of price changes is found by differentiating y with respect to P to get the Slutsky equation:

$$(18a) \quad \frac{\partial l^u(\cdot)}{\partial P} = \frac{\chi}{\Gamma} \left(\frac{\partial l^k(\cdot)}{\partial y} + 1 \cdot \frac{\partial l^u(\cdot)}{\partial m} \right).$$

We assume that leisure is a normal good, so that $\frac{\partial l^u}{\partial m} < 0$, and the income effect is negative. As the substitution term (see (17)) is positive, the total effect on individual labour supply of a change in P , the selling price of the LM-firm's product(s), is ambiguous. This corresponds to what we know about reactions of workers in PM-firms when wage changes.

The Slutsky equation in (18a) should be compared to the one we find when we assume that $\gamma = 0$. Then, as

$$\frac{\partial e(\cdot)}{\partial y^F} = -1,$$

i.e. a constant, and as the substitution term is given as the second order derivative of $e(\cdot)$ with respect to y , the substitution term is zero. Thus, the compensated labour supply is constant whatever is y (or P). Of course, there will still be an income effect, which we find in the same manner as we did for $\gamma = 1$. The Slutsky equation is:

$$(18b) \quad \frac{\partial l^U(\cdot)}{\partial P} = 1 \cdot \frac{X}{N} \cdot \frac{\partial l^U(\cdot)}{\partial m} < 0.$$

In (18b) the substitution term is omitted because it is zero.

Furthermore, if $0 < \gamma < 1$, we derive the general Slutsky equation, assuming that $\frac{1}{\Gamma} = \frac{1}{N}$, i.e. all workers supply the same amount of work initially:

$$(18c) \quad \frac{\partial l^U(\cdot)}{\partial P} = \gamma \cdot \frac{X}{L} \cdot \frac{\partial l^k(\cdot)}{\partial y^V} + \frac{X}{N} \cdot \frac{\partial l^U(\cdot)}{\partial m}.$$

The signs are as in (18a), i.e. the total effect is ambiguous. The relative importance of the substitution term is dependent on the value of γ .

In the same way we can also find the effects of changes in non-labour costs (fixed costs). When $\gamma = 1$, the Slutsky equation is:

$$(19) \quad \frac{\partial l^U(\cdot)}{\partial F} = - \left(\frac{\partial l^U(\cdot)}{\partial y} + 1 \frac{\partial l^U(\cdot)}{\partial m} \right) \cdot \frac{1}{\Gamma}$$

The effect of a change in F corresponds to a price change, although the sign is opposite. Again we can derive the results for

different values of γ . But these are similar to those in (18b) and (18c), noting that the signs are opposite. In particular, an increase in F will definitely lead to more work when $\gamma = 0$, cf. (18b).

These results should be compared to those obtained by Bonin (1977) and Ireland and Law (1981). Bonin assumed perfect collusion between the workers, and found that increases in fixed costs would definitely lead to increased labour supply when payment was made according to work done. This result was confirmed by Ireland and Law (1981) when their elasticity of labour supply, $\eta = \frac{\partial L}{\partial I} \frac{1}{\Gamma}$, was equal to one, i.e. complete collusion. The reason for the ambiguous sign in our model when the same specification is made with regard to dividend distribution, is our implicit Nash-Cournot assumption in individual labour supply decisions. This is seen from (13) where $l^{h \neq i}$ is a given parameter in the individual workers' labour and consumption allocation. The effect of price changes corresponds to what Bonin and Ireland and Law have found. The magnitude of the changes may differ, however, because of the different assumptions on how the labour supply of other workers enters into each worker's maximization programme.

Ireland and Law (1981) discuss the effect on labour supply of changing the relative importance of the two elements in D also. They find that labour supply will be reduced if the weight of the fixed payment element is increased when $\eta < 1$. This is confirmed by our analysis, which we see by deriving the Slutsky equation for a change in γ . When γ changes, both y^v and y^f are affected. If we assume $\frac{1}{N} = \frac{1}{\Gamma}$, the income effects of changes in y^f and y^v will, however, outweigh each other, and there remains a pure substitution effect originating from a change in y^v . An increased γ means that more payment is made according to work, and it is easily seen that the effect on labour supply of increased γ (reduced per capita payment) is, as expected:

$$\frac{\partial l^u(\cdot)}{\partial \gamma} = \frac{\partial l^k(\cdot)}{\partial \gamma} \cdot \frac{\partial y}{\partial \gamma} = \frac{\partial l^k(\cdot)}{\partial y^v} \cdot \frac{PX-F}{L} > 0$$

(Cf. also Sen (1966)).

5. A NOTE ON THE CHOICE OF EMPLOYMENT LEVEL WHEN MEMBERSHIP IS VARIABLE AND NON-LABOUR COSTS ARE FIXED

It may be of interest to analyse what happens if the number of workers can be varied, and non-labour costs are fixed. We have seen previously, cf. (14), what conditions that have to be fulfilled if membership level is to be changed in an LM-firm. The optimal adjustments of employment when e.g. a price changes, can be seen directly from the value of $\frac{\partial v^i}{\partial N}$. The ultimate effect is, however, dependent on the individual labour supply reactions also. For instance, when the price of a product increases, then $\frac{\partial(PX)}{\partial L} < \gamma$, and the derivative of $v^i(\cdot)$ with respect to L is negative. Then maximum utility can be increased by reducing total number of hours worked. But we know from (18a) that the workers will increase or reduce their labour supply dependent on the relative strength of income and substitution effects. If the substitution effect dominates, the individual labour supply is increased, and if this is an overall reaction, it is definitely profitable to reduce the level of membership (N). In the opposite case, when $\frac{\partial l^u}{\partial P}$ is negative (the income effect dominates), the effect on membership adjustment is dependent on the sum of each worker's individual labour supply reactions and the totally optimal change in working hours.¹⁰⁾

It is worth noting that it is possible to use the dividend function to find the firm's optimal hours of work (L) also, as seen from the viewpoint of worker i . As non-labour costs are fixed, we write (5) as:

$$(5') \quad Y'' = \text{Max} \frac{PX-F}{L}$$

$$\text{s.t. } L, K, X \in \underline{Z} R^{1+S+Q}$$

The properties of Y'' are as explained previously, and particularly we find that¹¹⁾

$$\frac{\frac{\partial X}{\partial L}}{\partial P} > 0.$$

A price increase will lead to an increase in production per hour worked. But as non labour factors are fixed, hours of work have to be changed. Marginal productivity of labour is positive and diminishing, so that an increase in $\frac{X}{L}$ is obtained by a reduction in hours worked. Thus we get unambiguously that

$$(20) \quad \frac{\partial L}{\partial P} < 0$$

when non-labour factors are fixed. This corresponds to the result we got when discussing the effect of price changes by using the individual indirect utility functions. The total effect on employment will be as explained above, i.e. it depends on the individual adjustments of hours of work.

The results above may relate to an overall price change if there is joint production, or to a situation where the firm produces one good only. If it is the price of good q that changes, then still

$$\frac{\frac{\partial \frac{X_q}{L}}{\partial P_q}}{\partial P_q} > 0$$

from (5'). But now several possibilities emerge. From (4a) we see that value marginal product of labour differs among lines of production. If L is constant, this calls for some reallocation of labour. Then production of good q will increase, assuming this reallocation of labour is technically possible. Of course,

the production of the other products will decrease. We can also consider another interesting case. Assume that no reallocation of resources is possible within the firm. Then production per worker of good q can be increased through a reduction or an increase in L . The reason is that all workers share in the income from all production activities on equal terms.

It can be shown that the effect on employment (or number of hours worked) of a price change on product q depends on whether good q has a large or small part of total sales. This is so because sales of other products are now to be considered as negative fixed costs. By differentiating the first order condition (4a) with respect to P_q and L and noting that $\frac{dL}{dL} = 1$, we find:

$$\frac{dL}{dP_q} = \frac{\frac{X_q}{L} - \frac{\partial X_q}{\partial L}}{P_q \frac{\partial^2 X_q}{\partial L^2}} .$$

The denominator is negative, while the sign of the numerator depends on whether $\frac{X_q}{L} \gtrless \frac{\partial X_q}{\partial L}$. This sign can be found from the first order condition (4a), so that

$$(20') \quad \frac{\partial L}{\partial P_q} \gtrless 0 \quad \text{as} \quad \sum_{q' \neq q} P_{q'} X_{q'} \gtrless F .$$

Thus, price changes on the main products will tend to have a negative influence on employment, and thus production of the product, while the opposite tends to occur when prices of by-products change.

Generally degree of substitution possibilities and recruitment policy will determine the final outcome.

6. CONCLUDING REMARKS

The intention of this paper has been threefold. Firstly I wanted to represent some main results from the theory of LM-firms in a first best environment by means of indirect objective functions. This is important because these functions have proven particularly useful in second best tax analysis, which will be the subject of a subsequent paper. Secondly we saw that using an indirect utility function to represent the workers' equilibrium choice gave us some interesting information on how the individual workers' decisions are interrelated. We were also able to relate the labour supply decisions to differences in wealth and marginal utility of income. Besides, we found that the workers' labour supply functions may be rather special, as they may consist of different elements depending on the formulation of the firm's income distribution rule. Lastly I wanted to stress the importance of the different stages in the total optimization programme of an LM-firm. Because labour is not a traded good, we have to consider the workers' contributions in a wider context. In particular I find it important to focus on how individual properties of the workers affect the firm's equilibrium, as this will be the subject of subsequent papers of this thesis.

FOOTNOTES

¹ I thank Agnar Sandmo, Kåre Petter Hagen and Bjørn Sandvik for comments and helpful discussions.

1. See Vanek (1970), chapter 3.5.

2. The complete formulation of the maximization problem is

$$(i) \quad \text{Max}_{L, K, X} \quad Y = \frac{\sum_{q=1}^Q p_q X_q - \sum_{s=1}^S r_s K_s}{L}$$

s.t. $f(X, L, K) = 0.$

The Lagrange function is

$$(ii) \quad \Lambda = \frac{\sum_{q=1}^Q p_q X_q - \sum_{s=1}^S r_s K_s}{L} - \lambda f(X, L, K).$$

This gives first order conditions

$$(iii) \quad \frac{\partial \Lambda}{\partial X_q} = 0 \Rightarrow \frac{p_q}{L} - \lambda \cdot \frac{\partial f}{\partial X_q} = 0, \quad q = 1, \dots, Q$$

$$(iv) \quad \frac{\partial \Lambda}{\partial L} = 0 \Rightarrow - \frac{\sum_{q=1}^Q p_q X_q - \sum_{s=1}^S r_s K_s}{L^2} - \lambda \cdot \frac{\partial f}{\partial L} = 0$$

$$(v) \quad \frac{\partial \Lambda}{\partial K_s} = 0 \Rightarrow - \frac{r_s}{L} - \lambda \cdot \frac{\partial f}{\partial K_s} = 0, \quad s = 1, \dots, S.$$

From (iii) we have that

$$(vi) \quad L = \frac{p_q}{\frac{\partial f}{\partial X_q}}, \quad q = 1, \dots, Q,$$

which we insert into (iv) and (v) to get

$$(vii) \quad \frac{\sum_q P_q X_q - \sum_s r_s K_s}{L} = P_q \cdot \left(- \frac{\frac{\partial f}{\partial L}}{\frac{\partial f}{\partial X_q}} \right), \quad q = 1, \dots, Q,$$

$$(viii) \quad r_s = - \frac{\frac{\partial f}{\partial K_s}}{\frac{\partial f}{\partial X_q}}, \quad s = 1, \dots, S, \quad q = 1, \dots, Q,$$

The right-hand side of (vii) and (viii) are both positive as $\frac{\partial f}{\partial X_q} < 0$, $\frac{\partial f}{\partial L} < 0$, $\frac{\partial f}{\partial K_s} < 0$, and they are value marginal products of labour and marginal product of non-labour factors respectively, i.e., by differentiating (2) we find that:

$$\frac{dX_q}{dL} = - \frac{\frac{\partial f}{\partial L}}{\frac{\partial f}{\partial X_q}}, \quad q = 1, \dots, Q$$

and

$$\frac{dX_q}{dK_s} = - \frac{\frac{\partial f}{\partial K_s}}{\frac{\partial f}{\partial X_q}}, \quad q = 1, \dots, Q, \quad s = 1, \dots, S.$$

Thus, (4a) and (4b) correspond to (vii) and (viii). In (4a-b), however, notation is simplified.

3. See Ireland and Law (1982), chapter 2.10.

This function can be compared to the PM-firm's profit function, and it is the dual to the cost function discussed by Hey (1981).

4. We see that (6a) and (6d) also explain the short-run equilibrium of the Illyrian firm when there is only one variable input and one output. As K is fixed, a price increase will by (6d) lead to a reduction in L (and N). Then production

is definitely reduced, although proportionately less than the reduction in employment, as seen from (6a).

5. A dividend taking into consideration different productivity of workers as well as workers belonging to different categories as to type of work and payment, is easily formulated by a normalization to a common unit.

Suppose firstly that payment to the workers is being made according to their productivity. A vector α with elements $\alpha^i > 0$, $i=1, \dots, N$, transforms hours of work of each worker to efficiency units. Denote efficiency units supplied by worker i by l_E^i . Total supply of efficiency units, L_E , is then: $L_E = \sum_i l_E^i = \sum_i \alpha^i l^i$.

Then the LM-firm's maximand is:

$$(i) \quad \text{Max}_{K, L_E} \frac{PX - rK}{L_E} \quad \text{s.t.} \quad f(X, K, L_E) = 0.$$

Solving this with respect to efficiency units yields:

$$(ii) \quad \frac{\partial(PX)}{\partial L_E} = \frac{PX - rK}{L_E} = y_E^i$$

If compensation is made according to efficiency and hours of work, each worker will receive $y_E^i \cdot \alpha^i \cdot l^i$, $i=1, \dots, N$.

Similarly, employment and compensation can be found when there are, say B , categories of workers who are to be unequally compensated because of different tasks in production. Letting β be a B -vector of relative compensation (relative valuation of one hour of work), the standard maximization problem is, when each worker belongs to one of B categories, denoted L_b , $b=1, \dots, B$, and L_B is a vector of hours worked by different categories of workers:

$$(iii) \quad \text{Max}_{L_B, K} \quad \frac{PX - rK}{\beta L_B} \quad \text{s.t.} \quad f(X, L_B, K) = 0.$$

As one group can arbitrarily be chosen as a standard, in each category workers will be used until

$$(iv) \quad \frac{\partial(PX)}{\partial L_b} = \frac{PX - rK}{\beta L} \cdot \beta_b, \quad b=1, \dots, B.$$

Payment to the individual workers is then given by (iv), and it depends on the category to which each worker belongs.

In Furubotn and Pejovich (1973) such a system is demonstrated for the case of a Yugoslav labour-managed firm.

6. It is worth remarking that such a system of dividend distribution may be a solution to the free-rider problem. Any production unit will create its specific working conditions, to which workers will adapt themselves. Such working conditions may be considered a public good having the effect that no-one will have an incentive to elicit their true productivity unless payment is made according to this. Particularly in egalitarian LM-firms this free-rider problem may be great, as those with productivity above the general level will have no incentive to work harder than what is needed to get the fixed payment. This indicates the possible need for some kind of rewards to better-skilled workers, unless a well developed social consciousness exists.
7. As this formulation takes into consideration distributional as well as efficiency aspects, it corresponds to the problem of constructing an optimal linear income tax.
8. We assume that it is possible to formulate a welfare function for each firm. This is discussed by Sen (1966).
9. If $D = \frac{1}{N}$, then we get $\frac{\partial v(\cdot)}{\partial l^k} = \frac{\partial(PX)}{\partial L} \cdot \frac{\partial L}{\partial l^k} \cdot \frac{1}{N}$, which is always positive. (Each worker receives $\frac{1}{N}$ of worker k's extra

effort.)

10. Of course, the members do not necessarily have identical utility functions, so that some may increase and some may reduce hours of work. Anyway, the net effect will be as explained.

11. See footnote 4.

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LABOUR-MANAGED FIRMS AND TAXES

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1. Introduction

We can give three motives for taxation of firms and individuals. Firstly taxes are needed to finance governmental spending in an economy where the firms are privately owned. Secondly taxes are used as means for redistribution of income to achieve goals with respect to egalitarianism. Thirdly taxes may be used to improve upon resource allocation. In the literature on labour-managed firms taxation is discussed mainly in the latter context. Focus has been on two special problems, viz. the negative short-run supply response to price changes (negatively sloped supply curve), Domar (1966), and the workers' incentives to supply too much or too little work, Sen (1966). Little attention has been devoted to problems of income distribution. In an economy consisting of labour-managed firms, there are two important reasons for income differentials. The workers may be of unequal ability, and thereby be paid differently, and the firms may face market prospects and a productive environment that vary, which accordingly results in unequal possibilities of paying dividend (wage). Proper redistribution of income calls for a simultaneous view of taxation of firms and households. A discussion of how tax burden can be shifted from firms to individuals, or the other way round, is the main topic of this paper. In particular I am interested in analysing how this may affect individual labour supply. An additional problem to be addressed is taxation as a means to correct for externalities.

Taxation of labour-managed firms has been discussed in articles by Domar (1966), Sen (1966), Furubotn and Pejovich (1970), Suckling (1974) and Muzondo (1979). Sen's discussion is an optimal taxation analysis, in which he formulates an optimal rule for

distribution of income within a firm. The other analyses are positive in the sense that they discuss how employment decisions (Domar, Suckling and Muzondo) and investment decisions (Furubotn and Pejovich) are affected by different kinds of taxes levied on the firms. Taxation of dividend (payment to the workers) is neutral as to the effect on the marginal factor employment decisions. On the other hand, lump-sum taxation and ad-valorem taxation of output have positive effects on the labour employment decisions of the labour-managed firm.

In the next section of this paper I discuss taxation of labour-managed firms in a general context, and relate it briefly to existing theory, which is concerned mainly with taxation as a means of improving the allocational decisions of labour-managed firms. The model is presented in Section 3, and in Section 4 I examine the labour-managed firm's long run allocational decisions when it is subject to dividend taxation. I discuss some problems related to the correction for externalities. The individual labour supply decisions are analysed in Section 5. The main result is the close connection between the taxation of firms and workers. This gives the government a means for shifting tax burden from firms to workers without affecting the labour supply decisions. Some special tax problems are discussed in Section 6. I summarize the main findings in a concluding section. In an Appendix I show the effect of cash-flow taxation on the firm's investment decisions.

2. Some Grounds for Taxation in an LM-economy

Much of the literature on labour-managed firms (LM-firms) relates to analyses of efficiency and comparative static results. Market conditions are assumed to be perfectly competitive. As it was shown in the seminal works of Ward (1958), Domar (1966) and Sen (1966), efficiency problems may arise in an economy consisting of labour-managed firms (LM-economy). The short-run problems are an individual over-supply or under-supply of labour, Sen (1966), and the possibility of a backward-bending

supply curve for output, Ward (1958), Domar (1966), or at least low responsiveness to price signals, Steinherr and Thisse (1979). The problems may be reduced by using more sophisticated rules for distribution of income than the one used by Ward. But as pointed out by Bonin (1984), Ward's fundamental result seems quite robust. The long run problems are mainly the underinvestment and possible self-extinction of the self-financed LM-firm, see Vanek (1977). It can be shown, however, that the resource allocation in an economy consisting of externally financed LM-firms can be consistent with Pareto-optimum, Drèze (1976).

Suggestions have been put forward on how to cope with the allocational problems caused by the backward-bending supply curve of LM-firms. These include lump-sum taxation (Domar (1966)), the establishment of an Enterprise Incentive Fund (Ireland and Law (1978)), and ad valorem taxation of production (Suckling (1974), Muzondo (1979)). Sen (1966) suggests that the problems related to over- or under-supply of labour should be solved by formulating an optimal rule for distribution of income among the workers of the firm. His solution involves deviding the workers' income into a "needs" payment and a "work" payment. The former is a predetermined share to each worker in the firm's income net of non-labour costs. "Work" payment is on the other hand distributed according to work done. In the long run underinvestment and choice of inefficient capital-labour ratios cause problems. Furubotn and Pejovich (1970) show how tax discrimination in favour of surplus allocated to the Investment Fund, as opposed to surplus distributed as wage, will induce LM-firms to undertake more internally financed investment.

Distributional goals and general efficiency problems like externalities¹⁾ and imperfect competition constitute in themselves grounds for taxation. The former will be discussed in this paper as well as in a subsequent paper dealing with the normative aspects of taxation. Discussion of taxation as a means to correct for externalities is a natural part of the positive analysis presented here.

In this paper I will discuss distributional problems which it is particularly important to be aware of in an economy where LM-firms operate. First I trace out the allocational effects of different tax schedules. As lump-sum taxation must be excluded, distortive taxes cannot be avoided. If these taxes are introduced to achieve distributional aims, the distortion in prices and allocation is a loss incurred in the economy, which should be minimized. On the other hand, when trying to cope with efficiency problems, such allocational disturbances may be intended consequences of introducing taxes as a free market process does not result in an optimal resource allocation. Then we are interested in the sign of the deviation from the initial equilibrium.

3. The Model.

In the analysis below the following notation will be used:

- X : output (production) in quantity.
- P : price of product produced by the LM-firm.
- L : number of hours worked.
- N : number of workers employed by the LM-firm.
- l^i : supply of labour from worker i , $i=1, \dots, N$.
- K : non-labour input, capital.
- r : price of capital (market rate of interest).
- A : retained surplus.
- F : fixed costs (plus a possible retained surplus),
i.e. $F = rK + A$, $A \geq 0$, in the short run.
- C : consumption good (composite).
- y : dividend (wage) to the workers.
- t^C , t^I , t^P : tax rates.
- a, b : lump-sum elements in the tax functions.

We examine a firm operating in a perfectly competitive market economy, which may be considered a sector of the total economy. The firm takes prices of output and capital as given.

We shall consider the optimization of a representative firm and household (worker) respectively.

The firm is assumed to maximize payout per labour unit. In the short run we assume that capital as well as employment level are given, while these are variable in a longer time perspective. Then the backward bending supply curve will not necessarily exist, as production responses will depend entirely on how the individual workers respond to changes in price parameters. Thus, we assume that each worker maximizes a quasiconcave utility function

$$(1) \quad u(C,y)$$

In the long run, however, the management of the firm will consider individual labour supply as given, and their objective is the maximization of y , where we define y as

$$(2) \quad y = \frac{PX-rK}{L} .$$

Of course, when individual labour supply is given, this is the same as the maximization of payout per worker. The firm's production X is given by the production function

$$(3) \quad X = f(K,L) ,$$

where marginal productivities are assumed positive and decreasing.

We shall assume that taxes are levied on the firms as well as households.

In the short run we are interested in deriving the effect of taxes on individual labour supply, as this will determine the firm's production when employment and capital levels are fixed. The long run problem I have defined as the firm's problem. But if the management is assumed to represent the interest of the workers, they will be primarily concerned with the optimal capital and production per worker, and the effect taxes will have on these ratios. Then it is convenient to discuss the long run tax rate responses by means of a dividend function, which defines the optimal capital and production per worker.

4. Taxes Levied on LM-Firms

Firstly we will look at how the LM-firm is influenced by taxes. We can imagine several ways in which taxes can be imposed on the firm, e.g. on total sales, on sales minus non-labour costs (total dividend) or on retained surplus (dividend not distributed to the workers). Here I shall concentrate on total dividend as tax base. But the other possibilities will be considered also.

When the tax base is total dividend, the tax on LM-firms will be

$$t^c(PX-rK) + b .$$

Here t^c is the marginal dividend tax rate and b is a lump-sum element on the firm in the corporate tax function. This allows for the possibility that the tax system is progressive, which means that the average tax rate is increasing in the dividend. This is the case when $b > 0$. The LM-firm maximizes dividend per worker, and the after-tax dividend becomes

$$(4) \ y_T = \frac{(PX-rK)(1-t^c) + b}{L} .$$

The management will maximize (4) taking account of the technological restrictions given by the production function (3). We write $\frac{\partial f(K,L)}{\partial L} = f_L$ and $\frac{\partial f(K,L)}{\partial K} = f_K$. The first order conditions are:

$$(5a) \ Pf_L = \frac{PX-rK}{L} + \frac{b}{(1-t^c)L}$$

$$(5b) \ Pf_K = r.$$

Here we see that (5a) deviates from the corresponding first order condition in a first best economy if $b \neq 0$, but (5b) corresponds to the first best condition. That is, the price of non-labour factors (capital) is not directly influenced by this

tax structure. The "price" of labour is on the other hand dependent on the lump-sum element b . If $b = 0$, taxes will not influence the first order conditions, and taxes are neutral²⁾ with respect to the LM-firm's allocation of labour and other factors of production. This should be compared to a tax on pure profit in a PM- economy. As shown by Sandmo (1974), such a tax schedule is neutral with respect to factor use when full deductability of costs is allowed (both labour and non-labour costs).

A further comment on the conditions in (5) should be made. Taxes on LM-firms are neutral as far as first order conditions are considered when they are levied as a percentage of total dividend (surplus). But the allocation is distorted by the introduction of a lump-sum element, or a fixed element which does not vary with income, into the tax function by affecting the dividend ("price") to the workers. Inspection of (4) above indicates that this is not surprising. b is actually an income element which the workers receive/pay as members of a specific collective. Thus, we must look at it as part of "wage" (price of labour), so that when b is positive (or negative), the amount payable to the workers is directly influenced by a fixed element in the tax function. Furthermore, since b is tax-free, changes in t^c will affect the relative importance of different elements in the dividend; e.g. an increase in t^c will make b (transfers from the government to the firms) a relatively more important element of personal income. Instead, if also b were to be taxed, i.e. if the tax function were formulated as

$$t^c(PX - rK + b),$$

then changes in t^c would not influence first order conditions as all income elements are affected in the same manner. But still b would be part of the dividend, and thus the price of labour in this economy.

If taxes were imposed on sales as an ad valorem tax, however, the tax would not be neutral, irrespective of the value of b .

Taxes on a potential retained surplus will be neutral with respect to the price of non-labour factors, but labour "cost" will be affected.³⁾ Retained surplus is defined as $A = PX - rK - \sum_i y_i$, which yields a behaviour rule implying the maximization of

$$\frac{PX - rK - A(1+t) + b}{L} .$$

The conditions for neutrality of the dividend tax can be illustrated by constructing a maximum dividend function characterizing the equilibrium. Denote maximum dividend by $Y_T^!$. If the behaviour rule is formulated as in (4), we can write:⁴⁾

$$(6) \quad Y_T^! = \max_{L, K} \frac{(PX - rK)(1 - t^c) + b}{L}$$

$$\text{s.t. } X = f(K, L)$$

From (6) we see that for $b = 0$ a tax on dividend is a tax on the firm's maximum payout. Then the same choice of K and L should be made irrespective of t^c . With $b > 0$, however, taxes are the equivalent of fixed costs, which will affect resource allocation. Then changes in t^c will affect production, while this was not the case for $b = 0$.

I will not consider the comparative statics of tax rate changes in detail, as this has been investigated by Domar (1966), and later by Suckling (1974) and Muzondo (1979). But one allocational problem connected to taxation deserves a comment.

It is commonly argued that taxation of production (sales) may be used to correct for externalities. Assume that an ad valorem tax is levied on total sales (see footnote 3,a). Under the assumption that taxes do not affect the gross product price, i.e. P , a tax rate change will affect the firm in the same manner as a change in P . As shown in Askildsen (1986), the production and employment effects are generally ambiguous when such a change occurs. We can use the properties of the dividend function Y_T^1 to derive the effects of a change in the ad valorem tax rate, or equivalently a change in P of same magnitude but of reversed sign. It can be shown that⁵⁾

$$\frac{\partial \frac{X}{L}}{\partial t^c} \leq 0 \quad \text{and} \quad \frac{\partial \frac{K}{L}}{\partial t^c} \leq 0.$$

In the short run, where non-labour costs are fixed, a price increase, or equivalently a tax reduction, will definitely lead to reduced production. These effects will also exist if demand is not perfectly elastic, i.e. when generally

$$\frac{\partial P}{\partial t^c} \geq 0, \quad \text{and} \quad \frac{\partial P(1-t^c)}{\partial t^c} < 0.$$

If demand is perfectly inelastic, however, the consumers will bear the tax burden.

These effects are important to bear in mind if taxes are to be used to reallocate resources when there are externalities (see footnote 1). The traditional use of taxes and subsidies is not necessarily adequate in an LM-economy as effects are ambiguous when the tax rate is changed. It is only in the short run that resource allocation can be unambiguously directed as desired. But then the tax policy has to be reversed compared to its implementation in PM-economies, i.e. we are to subsidize the polluting firms. To give a brief explanation of this, consider the polluting activity as a factor of production to which the firm has costless access. In the short run the firm cannot

change production technology. Taxation will therefore imply demand for more labour to share in the cost burden, while subsidization is a compensation for reducing the exploitation of the free factor, and it is optimal to have fewer workers share in the compensation. In the long run, this policy is likely to create problems. New firms will probably enter into the subsidized industry which may lead instead to totally increased production. If ad valorem taxation were introduced, the existing firms will expand production in the short run, while the effect is ambiguous in the long run. It is on the other hand probable that this taxation will lead to exit of firms as relative profitability is changed between lines of business. Thus, ultimately the price correcting taxation may give the desired result, although this cannot be confirmed in the partial context used here. The industry's production is reduced if the remaining firms totally reduce their production. If they end up with an increased production, total production can be reduced through exit of a sufficient number of firms. As a policy instrument, however, ad valorem taxation is generally not sufficient to handle externalities.

Before discussing taxation of households, I will draw some comparisons between taxes on PM-firms and LM-firms. It has been stated above that taxes levied on pure profits in PM-firms are neutral with respect to factor use and production. Then taxes are

$$t^P \cdot \Pi = t^P(PX - WN - rK)$$

and the profit function is

$$(7) \quad \text{Max } (PX - WN - rK)(1-t^P) + b$$
$$K, N$$
$$\text{s.t. } X = f(K, L)$$

Here t^P is the tax rate on the PM-firms' profit, Π is profit of a representative firm, W is wage-rate, and the other symbols are

as defined above. (7) should be compared to (6). The criterion for the tax to be neutral is that deductability is allowed for all real costs, including true depreciation and costs of holding financial and real assets. Neutrality implies that a project is profitable irrespective of the introduction of taxes. The factors of production will be used to the same extent and intensity. It is worth remarking that the introduction of a lump-sum element does not alter neutrality qualities of the profit tax system as we saw happened when the tax levied on the dividend of LM-firms included a lump-sum element b . The reason is that no decision variables are affected by this transfer from the government to the owners of the firm. With respect to LM-firms, we observed that such a lump-sum element affected directly the internal "market price" of labour (wage), (see (5a)).

For both types of firms the neutrality result hinges on a criterion of full deductability of all costs. Thus, if the tax authorities do not allow for deductability of true capital costs, taxes are no longer neutral neither in PM- nor in LM-firms⁶⁾. As to labour-costs, things may be somewhat different. In the dividend tax schedule of LM-firms as indicated here, it is not necessary to determine the labour costs so as to find the correct tax base, as these "costs" are taxed together with a possible retained surplus (or pure profit when comparing with PM-firms). When establishing the neutral tax schedule of PM-firms, we assume that it is possible to single out and deduct true labour costs. It may be advantageous in terms of calculating the tax liability that the tax base is as broad as possible. Dividend in LM-firms is definitely a broader tax base than profit in PM-firms. However, if taxes are to be imposed on retained surplus in LM-firms, it may be hard to solve problems of defining and calculating true costs.

There is one more aspect according to which the systems differ. Taxes on LM-firms' dividend directly affect the optimization of the firms' workers. In a PM-economy the workers are affected by profit taxation only indirectly, through the effect of the tax-

ation on relative market prices. Therefore we will now turn to studying the effect on labour supply when taxes are levied on firms as well as households, and show that taxation of dividend may have a largely different effect from a tax on profits of PM-firms. However, it is worth mentioning that this is not surprising, as taxes may play an entirely different role in LM-firms than taxes levied on PM-firms. This is however a subject to which I will return later in this paper.

5. Taxes on Households and Firms: Individual Labour Supply.

Let us look now at how the individual labour supply is affected when taxes are introduced. For the sake of completeness, and also because it may be of importance in itself, we suppose that taxes are levied on firms (dividend taxation) as well as on households (income tax). The income tax on households (individual workers) is made progressive by introducing a fixed element a , $a \geq 0$, into the tax function as we did when examining taxation of LM-firms only. Letting y^i be maximum individual income, i.e. y^i does not change as a result of a marginal change in individual labour supply, and t^I the income tax rate, $0 \leq t^I < 1$, individual i 's after tax income, y_T^i , is:

$$y_T^i = y^i(1-t^I) + a .$$

We can write worker i 's budget restriction as:

$$(8) \quad C^i = \frac{1}{L^i} [(PX-F)(1-t^C) + b](1-t^I) + a, \quad i=1, \dots, N.$$

Price on consumption is assumed equal to one. Observe that F is not necessarily equal to rK . It may also include retained surplus, i.e. $F = rK + A$, $A \geq 0$. Remember that we consider a short-run time perspective.

We assume that all income is distributed according to hours of work. Other rules could have been applied as well, e.g. letting a proportion γ be distributed according to work done, while $(1-\gamma)$ is distributed on a per capita basis.⁷⁾ Dividend could be distributed according to other productivity measures than hours of work also. But qualitatively the comparative static results are not affected by excluding these possibilities. On the other hand, if the tax function were a non-linear one, the results would have become more complicated. Then we should have considered the correlation between the marginal tax rate and the productivity measure to be used, as the marginal tax rate would become a function of the productivity also. In the linear tax function, the marginal tax rate, t^I , is constant.

Each worker (household) maximizes the utility function (1) given the budget restriction (8). Omitting the superscript i , the first order conditions are (8) and:

$$(9a) \quad u_C - \lambda = 0$$

$$(9b) \quad u_L + \lambda \left\{ [P f_L \cdot \frac{1}{L} + (RX-F) \frac{1}{L} (1 - \frac{\partial L}{\partial I} \frac{1}{L})] (1-t^C) \right. \\ \left. + b \cdot \frac{1}{L} (1 - \frac{\partial L}{\partial I} \frac{1}{L}) \right\} (1-t^I) = 0$$

λ is a Lagrange-multiplier, where $\lambda > 0$, assuming monotonicity, and $u_C = \frac{\partial u}{\partial C} > 0$ and $u_L = \frac{\partial u}{\partial L} < 0$.

(9a) and (9b) can be compared to the first order conditions when taxes are not present, and we see that the tax levied on the LM-firm as well as the individual income tax affect the allocation of labour and consumption.

In order to see how labour supply is influenced by taxes, we introduce the expenditure function. The expenditure function shows minimum lump-sum income, m , necessary to reach a given utility U_0 . It can be written:

$$(10) \quad e[y(P, t^c, t^I, b, F, N), a, U_0] = e(\cdot) = \min C - \{[PX-F](1-t^c) + b\} \frac{1}{L} (1-t^I) + a \} \text{ s.t. } u(C, l) \geq U_0.$$

$e(\cdot)$ is concave in y . The compensated labour supply function $l^k(\cdot)$ is found by taking the derivative of $e(\cdot)$ with respect to dividend, y , and we can easily find the Slutsky equation showing the effect of an income tax change by differentiating the identity between compensated and uncompensated labour supply ($l^u(\cdot)$):

$$\frac{\partial l^k(\cdot)}{\partial y} \frac{\partial y}{\partial t^I} = \frac{\partial l^u(\cdot)}{\partial y} \frac{\partial y}{\partial t^I} + \frac{\partial l^u(\cdot)}{\partial m} \frac{\partial m}{\partial y} \frac{\partial y}{\partial t^I}$$

This gives the Slutsky equation by rearranging terms. We use the identity $m \equiv e(\cdot)$, and find:

$$(11) \quad \frac{\partial l^u}{\partial t^I} = - \frac{(PX-F)(1-t^c) + b}{L} \left(\frac{\partial l^k}{\partial y} + 1 \frac{\partial l^u}{\partial m} \right).$$

When the income tax rate, t^I , changes, the labour supply reaction is ambiguous. The substitution term, $\frac{\partial l^k}{\partial y}$, is positive since $e(\cdot)$ is concave in y , while the last term in the parenthesis, the income effect, is negative if leisure is a normal good.

(11) also shows the effect of tax rate changes when all income is taxed on the households, i.e. when $t^c = b = 0$. In that case, both distributed and retained surplus are taxed. (This system may, of course, lead to liquidity problems if retained surplus is not an inconsiderable amount.)

The effect can be compared to a price change when taxes are not present. The signs are opposite to each other, and the magnitude of the changes will differ because of the fraction outside the parenthesis. Thus, if the price of the firm's product changes, assuming $t^c = t^I = 0$, we can derive the following expression:

$$\frac{\partial l^u}{\partial P} = \frac{\lambda}{L} \left(\frac{\partial l^k(\cdot)}{\partial y} + 1 \cdot \frac{\partial l^u(\cdot)}{\partial m} \right).$$

We can also find the effect of a change in the firm's dividend tax rate on labour supply. The Slutsky equation is:

$$(12) \quad \frac{\partial l^u(\cdot)}{\partial t^c} = - \frac{(PX-F)(1-t^I)}{L} \left(\frac{\partial l^k(\cdot)}{\partial y} + 1 \cdot \frac{\partial l^u(\cdot)}{\partial m} \right).$$

Here the signs are the same as in (11). Furthermore we see that the effect on labour supply is identical in the two cases when $b = 0$ and the tax rates are of the same value, i.e. $t^I = t^c$. We see also that in both instances, the magnitude of the labour supply reactions is dependent on the value of the other tax rate, i.e. a high dividend tax rate reduces the effect of a change in t^I , and the other way round. Furthermore, it is always possible to find values of t^c and t^I leading to identical effects on labour supply of tax changes also when $b \neq 0$. This value is given by

$$(13) \quad t^c = t^I + \frac{b}{PX-F}.$$

But this means that the government will in some circumstances possess the possibility to affect income distribution at the same time as labour supply reactions are minimized. As we see from (11), (12) and (13), this is done by changing the two tax rates in opposite directions, and in such a way that the effects shown there outweigh each other. The condition can be written as

$$\frac{\partial l^u}{\partial t} = \frac{\partial l^u}{\partial t^c} + \frac{\partial l^u}{\partial t^I} = \left(\frac{\partial l^k}{\partial y} + 1 \frac{\partial l^u}{\partial m} \right) \left[\frac{-(PX-F)(1-t^c) - b - (PX-F)(1-t^I)}{L} \right]$$

If (13) holds initially, this expression is zero for $b = 0$ if $dt^C = -dt^I$. Generally, by using (13), we find that $\frac{\partial l^U}{\partial t} = 0$ if

$$dt^C = -dt^I \frac{1 + \alpha \frac{\partial L}{\partial t^I}}{1 - \alpha \frac{\partial L}{\partial t^C}}, \text{ where } \alpha = - \frac{(1-t^C)+b}{(PX-F)^2 (1-t^C)} b. \text{ Note that } \frac{\partial L}{\partial t^C}$$

and $\frac{\partial L}{\partial t^I}$ are the sums of all individual responses. Thus, the tax burden can be shifted from individuals to firms or the other way round.

We see however that the neutral tax rate changes are not necessarily independent on which individual we study. If there were only one firm, there would be no problem. In reality this is not the case, and we see that the outweighing tax rate changes will differ among workers in different firms unless $b = 0$ or $\frac{b}{PX-F}$ are the same for all firms. If these assumptions are not fulfilled, it will be possible to find tax rate changes that outweigh each other on average only. But the labour supply of some workers will be affected. A further complicating element would be the introduction of non-linear tax functions. Then the marginal tax rate depends on the individual income, and constant labour supply can be attained only when all individuals and firms are identical (except when the tax functions are individually formulated). If the tax function is non-linear so as to redistribute income, the homogeneity assumption is not realistic. But still it should be possible to formulate rules which minimize the average deviations from the initial chosen positions.

This "double" taxation possibility may be of importance, as income (wealth) disparities can be due to either different profitability possibilities of the firms or differences in the individuals' possibilities to earn income from other sources. If wealth disparities are due mainly to one of these sources, the indicated tax rate changes may be useful if the objective is to influence income distribution.

We have used here a constant marginal tax rate, which is of course a simplification. Nevertheless the analysis throws some light on how certain kinds of income disparities can be handled in an LM-economy. Although it may be difficult to carry through the exact calculations, we can conclude thus far that the analysis throws light on the policy tools which the authorities can use to control personal income distribution. Of course, not all kinds of wealth differences can be handled in this manner. E.g., it may well be the case that the overall richest workers are employed in the overall richest firms. But then we possess the possibility of reducing the importance of one source of unequal income between households in a way that reduces the loss of efficiency by choosing to tax at the heaviest rate the source where the disturbances are at their lowest.

6. Some Issues Regarding Taxation when LM-Firms are Present

From the analysis above, we see that taxes affect directly the individual workers even though all tax may be levied on the firm. This is somewhat different from a PM-economy, where the households have to own shares in the firms so as to be directly affected by profit taxation. All other effects on factor allocation and income are indirect through the market. This may have some significance if it is the case that membership rights are more stable than ownership of shares, which seems to be a reasonable assumption. But then both fiscally and as a means of income equalization, profit taxation will probably play a less important role than dividend taxation of LM-firms. The reason for this is that the tax base is broader, that more persons (households) are affected by dividend taxation than by profit taxation and that ownership rights are more stable than shareholdings (less elastic supply and demand).

However, a reservation may be appropriate. We are comparing two tax systems which are comparable from a theoretical point of view in the sense that under certain assumptions they do not affect the firms' optimizing behaviour. Thus, the above conclusion is of the greatest interest if this neutrality of the tax systems is a desirable quality. But these taxes are not necess-

arily the only corporate taxes that are in operation. There may be different kinds of taxes levied on sales and factor use also, which will affect wages of the workers in PM-firms as well as dividend in LM-firms. The ultimate effects of such taxes are generally hard to calculate, as the incidence will depend on elasticities of supply and demand in the product markets as well as in the labour market. Thus, although it is not trivial to trace out the ultimate effects of taxation, it seems reasonable to argue that the public sector has at least as good possibilities to raise income and influence personal income distribution through the corporate sector in an LM-economy as in a PM-economy. The justification for the statement is the close connection between the earnings of the firms and the personal income, and the fact that if a tax can be levied on a PM-firm, it can be levied on an LM-firm as well. However, there may be practical problems in defining a tax base. It is not the scope for this paper to consider these.

We have argued that there are reasons to tax LM-firms and households at different rates. Given the assumption of a perfectly competitive product market, the households will bear the burden of the tax⁸⁾. This is independent of whether the tax is formally levied on the firm or on the households. But this means that the two tax rates, t^c and t^I , can be considered constituting a composite tax rate t , where

$$t = t(t^c, t^I)$$

Using the composite tax rate t has obvious analytical advantages if we aim to find optimal tax rates, which is the subject of a subsequent paper. I find it of importance, however, to discuss how the workers' optimizing behaviour makes the taxation of both units, the firm and the households, necessary.

Now, if the economy in question is one consisting of both PM- and LM-firms, we will have to face the problem of establishing

tax systems that do not discriminate between types of firms. Then we should obviously either abolish all corporate taxation, and thereby only tax households, or non-discriminating taxes should be imposed on firms, in such a way that the same tax rate is used on PM-firms' profit, on LM-firms' dividend and also for income taxation of the workers of PM-firms. The latter may create problems particularly if one aims to use the tax system to redistribute income, as the distribution of ownership rights to the proceeds from an LM-firm is generally more egalitarian than the distribution of shares in a PM-firm. The former may on the other hand create problems related to tax evasion.

This view becomes more clear when assuming that the LM-firm does not maximize income only. Rather the workers are maximizing their utility from participation, which e.g. Furubotn (1976) has expressed by a utility function

$$(14) \quad v = \phi(C, Q),$$

where C is ordinary consumption as defined above, while Q is a vector of aspects affecting the utility of work (working conditions). These may be environmental factors as well as direct consumption on the work place, several of which may be hard to assess quantitatively. (14) may also express the utility of the owners as well as the workers of PM-firms. In both scenarios it may be considered advantageous to avoid taxation by increasing work-place consumption. In particular in an LM-firm, which is owned by its workers, and where the workers can decide themselves on working conditions, the possibilities for realizing such a policy seem good. Then two distributional problems arise immediately, viz. among LM-firms of different wealth, and between the workers of a PM-firm and an LM-firm, where the former by assumption have smaller possibilities to change their working conditions.

Thus, as long as the public sector possesses limited possibilities to supervise the firms, there have to be some taxation of LM-firms, and for the same reason also of PM-firms. On the other

hand, it should be obvious that surplus taxation of firms only (profit, dividend) is insufficient if the economy consists of both PM-firms and LM-firms.

There is another more fundamental problem that should be mentioned. Furubotn and Pejovich (1970) argue that wealth maximization, rather than per-period dividend maximization, should be the maximand of the LM-firm. This is so because investments add to productive capacity in future periods only. But, as opposed to PM-firms, the workers of (Illyrian) LM-firms are allowed to enjoy the usufruct of capital only, i.e. they have no individual property rights. This will increase the price of an internally financed investment project (Furubotn-Pejovich effect). But it is the "property-rights adjusted" capital cost which has to be used in the per-period income maximization if this is to be compatible with wealth maximization. Then a serious problem when defining the tax base is that this capital cost is hard to calculate as it depends, in addition to the factors determining the user price of capital in PM-firms, on the time horizon of the existing work force.

In an Appendix to this paper I derive the user price of capital in LM-firms more formally. There I show also how a cash-flow tax may be used to encourage LM-firms to undertake investments. This may be of special significance in LM-firms since their capital cost will exceed that of PM-firms when investments are internally financed. In addition it will not be necessary to assess the "property-rights adjusted" capital cost for calculating the tax liability.

7. Concluding Remarks.

I have argued that tax problems constitute an important issue in an economy where LM-firms are present. Deliberately I have not defined a pure LM-economy, as I think that the taxation problems relate at least as much to an economy where PM- and LM-firms coexist. In addition, a mixed economy will create some special problems which have to be considered, notably how the tax system

can be formulated so as not to discriminate between the ways of organizing firms.

One major point I have made is the close connection between the taxation of the firms and the households. This relates in particular to the short run problems, when capital as well as employment level should be considered fixed. As the workers are members of a collective because this gives them the highest utility in terms of income and work enjoyment, it is correspondingly their contribution, the labour supply decisions, which it is important to investigate so as to trace out the comparative statics of tax rate changes. But this means that the workers are directly affected by taxation irrespective of whether the tax is levied on the firm or on the households. In the long run, however, it seems reasonable to assume that a management can decide on optimal factor mix and production, taking the individual labour supply decisions as given.

We have analysed different tax schedules, although we have concentrated on the taxation of the equivalent of profit in PM-firms, viz. the dividend. The dividend tax is neutral only when the tax function is linear with no lump-sum element. Then the long run decisions as to factor mix and employment are unaffected by the tax, although we can split the total effect into outweighing price and capital cost effects. However, when there is a lump-sum element in the tax function, neutrality does not hold, and factor mix and production are affected by taxation. When a tax rate increases, production per worker and capital-labour ratio tend to decrease because of the price effect which reduces the internal "market price" of labour, while the cost effect, through the deductability of capital costs, tends to increase the ratios. Then the effects are generally ambiguous, and the specific technology will determine the outcome. A problem this result raises is how to establish rules for taxation/subsidization to correct for external effects. This problem becomes particularly urgent in a short run Illyrian model where the level of employment is variable.

In the short-run stage, when the decision unit is the individual worker (household), we found that changes in dividend taxation as well as ordinary income taxation will affect labour supply decisions. The result corresponds, of course, to the effect of taxation of workers in PM-firms. An important difference is, however, that in the LM-scenario the government has got in effect two tax rates, and we showed that these can be changed in a way that do not affect labour supply decisions. This may be of some importance for distributional purposes, e.g. if the government finds it optimal to shift the tax burden from firms to individuals, or the other way round, according to where the most important source of income inequalities can be found.

Lastly we adressed the problem of defining the tax base. This is particularly important in a mixed economy if the workers are to be treated in a non-discriminating way irrespective of employment in PM- or LM-firms. One problem is the incentives the workers may have to avoid taxation by transferring consumption from households to work-place when tax rates differ between the units. The possibility for doing so may differ between workers in PM- and LM-firms. Another special problem, which is basic for the neutrality of the dividend tax, is that the capital cost used when calculating the tax liability must take the special property rights structure of the Illyrian firm into consideration.

Footnotes

* I am indebted to Agnar Sandmo, Kåre Petter Hagen, Karl Ove Moene and Morten Berg for helpful discussions and comments on previous drafts.

1. Externalities can be handled in different ways. From the theory dealing with economies consisting of traditional firms, we know that price correcting subsidies and taxes (Pigou-taxes) may lead to the desired results with a minimum of cost and knowledge of technical relations within the firms (see Baumol (1972)).

2. We define a neutral tax as a tax which does not affect the first order conditions, i.e. the introduction of taxes do not distort marginal decisions concerning allocation of labour and other factors of production, as compared to the case without any taxation (first-best economy).

3. First order conditions are in the two cases respectively:

$$a) \quad \begin{array}{l} \text{Max} \\ N, K \end{array} \quad y_T = \frac{PX(1-t^c) - rK}{L} \quad \text{s.t. } X = f(K, L)$$

$$\Rightarrow \quad (i) \quad Pf_L = \frac{PX(1-t^c) - rK}{L} + \frac{b}{(1-t^c)L}$$

$$(ii) \quad Pf_K = \frac{r}{1-t^c},$$

and

$$b) \quad \begin{array}{l} \text{Max} \\ N, K \end{array} \quad y_T = \frac{PX - rK - A(1+t^c)}{L} \quad \text{s.t. } X = f(K, L)$$

$$\Rightarrow \quad (i) \quad Pf_L = \frac{PX - rK - A(1+t^c)}{L} + \frac{b}{L}$$

$$(ii) \quad Pf_K = r$$

4. See Ireland and Law (1982), ch. 2.10, and Askildsen (1986) for a discussion of the dividend function.
5. See Askildsen (1986) for a derivation of this result. We use the property that the dividend function is convex in P (and $-t^C$). Then the comparative static result follows directly by noting that resources are optimally allocated, and no increase in income can be obtained through reallocation of resources.
6. See e.g. Sandmo (1974).
7. The possibility of reaching a socially optimal allocation of resources is dependent on the value of γ , see Sen (1966). But γ does not influence the sign of the deviation from an initial equilibrium position following upon a tax rate change in a way which is qualitatively different from that following upon a price change. The significance of γ for the comparative static results is discussed in Askildsen (1986).
8. This is a partial equilibrium result. In a more general context it is unreasonable to argue that all other prices than the implicit price of labour are fixed.

APPENDIX

Cash-Flow Taxation and Internal Investment in LM-Firms.

Because of the horizon problem (Furubotn-Pejovich-effect), the LM-firm is well known for its reluctance to undertake internal investment (see e.g. Furubotn (1976)). The reason is that the work-force at the point of time when an investment is undertaken will demand the investment expense recouped during their tenure with the firm, which may fall short of the life-time of the project. In addition there exists often a requirement that the capital stock have to be kept intact in perpetuity, by Bonin (1985) called a strong capital maintenance rule.

In this Appendix I will derive a tax function which will give equality between the internal cost of capital and the market rate of interest. I will use a cash-flow tax schedule for this purpose, as this is a tax function which is directly related to the treatment of investments. It has also attracted some interest concerning taxation of PM-firms, see e.g. Sandmo (1979).

I will assume firstly that the tax function is linear, and we find that this tax function is neutral if and only if the tax rate is constant over time (Sandmo (1979)). But in that case the tax function fails to give equality between the internal capital cost and the market rate of interest, which is a desired property. The latter can be obtained by using a non-linear tax function.

I will assume that the tax is payable by the households. In practice, part of the tax may be paid by the firm. If the tax schedule is linear, the same percentage will be paid in tax irrespective of the point of time when the workers take out their surplus. A marginal investment project will be examined, and it is assumed that any tax free thresholds are superceded without taking this project into consideration.

Now, with a cash-flow tax the investment expense is deductible. Every year the firm (the workers) pay tax out of income minus depreciation of the capital stock. The latter is assumed to be allocated to the Investment Fund (strong capital maintenance rule). The Investment Fund earns an interest equal to the market interest rate. The proceeds therefrom are excluded from the analysis. A linear income tax will give the workers the opportunity to postpone payment of tax by investing today for future income. This amounts to a tax credit but not to a reduced tax on the total. However, if annual income is highly fluctuating, this may be a means of smoothing income profiles over time. If the marginal utility of income in some years is low enough, this may prove profitable. It depends also on how the tax rates develop over time.

Assume that the workers are homogenous, and that surplus is distributed on a per capita basis. Level of employment, \bar{N} , is considered given. Let us see whether a linear tax will influence the desire to invest through a tax deductible retained surplus allocated to the Investment Fund. T is the lifetime of the project, K the amount invested, t^τ the tax rate in year τ , δ depreciation rate and $f(K, \bar{N})$ a production function with usual properties. The production function is the same over time, and price P is also constant. The workers have a concave utility function $u(y)$, where y is the change in income resulting from the marginal project. The workers choose K so as to maximize the present value of the cash flow V , where the discount factor is the market rate of interest i . ρ is the internal rate of return of the investment. Then we can write the maximization problem as:

$$(A1) \quad \begin{aligned} \text{Max}_{K_1} V = & - u^0 \left[\frac{K_1}{N} (1-t^0) \right] \\ & + \sum_{\tau=1}^T u^\tau \left\{ \frac{[P f(K_1 (1-\delta)^{\tau-1}, \bar{N}) - \delta K_1 (1-\delta)^{\tau-1}] (1-t^\tau)}{N(1+i)^\tau} \right\} \end{aligned}$$

The first order condition is

$$(A2) - \frac{\partial u^0}{\partial y^0} \cdot \frac{1}{N}(1-t^0) + \sum_{\tau=1}^T \frac{\partial u^{\tau}}{\partial y^{\tau}} \frac{1}{N(1+i)^{\tau}} \left(P \frac{\partial f}{\partial K_1} \delta \right) (1-\delta)^{\tau-1} (1-t^{\tau}) = 0$$

Here we see that the marginal investment decision will be unaffected by t^{τ} if and only if the tax rate is constant over time, i.e. if $t^{\tau} = \bar{t} \quad \forall \tau$. This corresponds to Sandmo's (1979) result when investigating cash-flow taxation of PM-firms.

Let us reformulate the condition (A2) as

$$(A3) \quad Pf_K = \delta + \frac{\frac{\partial u^0}{\partial y^0} (1-t^0)}{\sum_{\tau} \frac{\partial u^{\tau}}{\partial y^{\tau}} \frac{1}{(1+i)^{\tau}} (1-\delta)^{\tau-1} (1-t^{\tau})}$$

which reduces to

$$(A4) \quad Pf_K = \delta + \frac{(1-t^0)}{\sum_{\tau} \frac{(1-\delta)^{\tau-1}}{(1+i)^{\tau}} (1-t^{\tau})} = \delta + \rho.$$

when $u(y^t)$ is linear $\forall \tau$.

This can be used to illustrate the Furubotn-Pejovich effect (horizon problem). Assume t^{τ} is constant over time. If $T = 5$, $\delta = 0,1$ and $i = 0,1$ (the discount factor equal to the market interest rate), we find that in order to invest from retained surplus, we must have

$$Pf_K = 0,1 + 1,64 = \delta + \rho > \delta + i = 0,1 + 0,1.$$

However, note that the average capital cost is reduced by the introduction of taxation, as the rate ρ relates to the part of the investment which the workers have to finance themselves, i.e. $K(1-t)$. Then, returning to the case where t^{τ} fluctuates, we see that the cost of internal finance decreases if $t^{\tau} < t^0$,

$\tau = 1, \dots, T$, i.e. if the tax rate is reduced over time. This is not a completely unrealistic case. In order to encourage internal investment, we observe often that the government gives special treatment to capital gains by reducing taxes on these. There is no reason why this should not be done in an LM-economy as well. Favourable tax treatment does, however, imply certain demands on the accountancy of the firms to prevent the workers from manipulation of the tax base. These demands relate to the registration of activity, costs and income. This problem is, however, not special for an LM-economy.

Lastly I will look at how taxes may influence internal investment if the tax function is non-linear. Tax payable in each period is $t(y)$. With similar assumptions as above, we write the maximization problem as

$$(A5) \quad \text{Max}_{K_1} V = -\frac{K}{N}(1-t(\frac{K}{N})) + \frac{1}{N} \int_1^T [Pf_K(K_1(1-\delta)^{\tau-1}, \bar{N}) - \delta K_1(1-\delta)^{\tau-1}] \\ \cdot [1-t(\frac{Pf(K_1(1-\delta)^{\tau-1}, \bar{N}) - \delta K_1(1-\delta)^{\tau-1}}{N})] e^{-\tau i} d\tau$$

The first order condition is

$$(A6) \quad \frac{\partial V}{\partial K_1} = - [1-t(\frac{K}{N}) - \frac{K}{N} t'(\frac{K}{N})] \frac{1}{N} \\ + \frac{1}{N} \int_1^T (Pf_K - \delta)(1-\delta)^{\tau-1} [(1-t(y^\tau)) - t'(y^\tau) \cdot y^\tau] e^{-\tau i} d\tau = 0$$

$$\text{where } y^\tau = \frac{1}{N} [Pf(K_1(1-\delta)^{\tau-1}, \bar{N}) - \delta K_1(1-\delta)^{\tau-1}]$$

We can rewrite the condition (A6) as

$$(A7) \quad Pf_K = \delta + \frac{[1-t(\frac{K}{N}) - \frac{K}{N} t'(\frac{K}{N})]}{\int_1^T (1-\delta)^{\tau-1} [(1-t(y^\tau)) - t'(y^\tau) y^\tau] e^{-\tau i} d\tau} = \delta + \rho$$

According to Bonin (1985, Corollary to Proposition 4), the source of finance is irrelevant if the internal rate of return equals the interest rate. Thus, if there exists a tax function $t(\frac{K}{N})$ and $t(y^\tau)$, where $y^\tau = g(\frac{K}{N})$, which makes ρ , the fraction in the expression above, equal to i , the (after tax) interest rate, then the firm is indifferent between internally and externally financed investment.

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**OPTIMAL TAXATION OF LABOUR-MANAGED FIRMS
AND ITS WORKERS: SOME ISSUES**

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Jan Erik Askildsen

1. Introduction.

Much interest has been devoted in the literature to the efficiency of labour-managed firms. Although it can be shown that under certain assumptions the labour-managed firm will make the same allocational decisions as the comparable profit-maximizing firm, Drèze (1976), it is well recognized that situations exist where the labour-managed firm does not allocate its resources efficiently. Two examples of efficiency problems are the scale of operation of a self-financed firm, see Vanek (1977), and the labour supply decisions of the individual workers when the cooperative has access to a free factor of production, see Sen (1966). In this paper I will show how lump-sum and income taxation can be used to cope with these problems, and to obtain a fair distribution of income in a way which minimizes the loss of efficiency.

Sen (1966) was the first to acknowledge the relevance of the individual labour supply decisions to the optimal allocation of labour within the firm, and his contribution has later been further developed by e.g. Bonin (1977), Chinn (1979) and Ireland and Law (1981). The problem addressed is how the workers can be induced to supply a socially optimal amount of labour. E.g., if a labour-managed firm has access to a free factor of production, the workers will over-supply labour in their strive to capture the rent accruing to this factor. The problem may be solved through cooperation among the workers in fixing labour supply. On the other hand, when cooperation is ruled out, Sen showed that the optimal labour supply can be brought about by distri-

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buting part of the net value added according to hours worked, and the remaining part on a per capita basis. The procedure is generalized by Bennet (1984) who examines the optimal payout rule when the workers are heterogeneous and for different formulations of the social welfare function. However, Bennet, and also Guesnerie and Laffont (1984) in a more general framework, points to the problems of implementing the optimal policy because of incentive incompatibility.

Both these analyses are within the framework of optimal taxation theory. But they assume a high degree of information held by the planner, as he is supposed to impose individualized taxation. On the other hand, we know from the literature on the optimal taxation of income that second-best tax formulae can be derived that require a minimum of information about the characteristics of the tax payers, see Dixit and Sandmo (1977), Atkinson and Stiglitz (1980) and Sandmo (1983). Thus, by applying the approach used in the optimal income taxation literature, the implementability problems should be reduced. The derivation of such optimal income taxation formulae is the main scope of this paper.

In the next section I discuss some efficiency problems that may exist in an economy consisting of labour-managed firms, and with reference to Domar (1966) I show how lump-sum taxation can be used in influencing the scale of operation. This part is a reiteration of Domar's result, saying implicitly that I assume that firms can be controlled by means of lump-sum taxation, while this is not possible for the control of individual worker's labour supply. The remaining part of the paper is devoted to the control of individual labour supply, and in Section 3 I show how the efficiency result outlined by Sen (1966) can be established by means of a non-compensating tax schedule. I compare the tax rate found to one which taxes away all profit. In Section 4 I derive formulae for the optimal income taxation of workers in labour-managed firms, following Dixit and Sandmo (1977). On the purpose of doing comparisons, I derive also the corresponding formulae when the workers are employed by a profit-maximizing firm. Some issues concerning the implementation of the optimal policy are discussed in Section 5, while the main results are summarized and some general remarks are made in the concluding section.

2. Lump-Sum Taxation and the Optimal Scale of Production.

The labour-managed firm is in its most original setting assumed to maximize dividend, y , defined as sales (net of materials cost), PX , minus non-labour costs, F , per worker. In the short run F can be considered given, while the non-labour costs, notably capital expenses, are generally variable in the long run. Assume that capital, K , is the only non-labour factor. User cost of capital is denoted r . We can write the production as

$$(1) \quad X = f(K, N)$$

where marginal productivities are positive and decreasing. The labour-managed firm's maximand is

$$(2) \quad y = \frac{PX - rK}{N} .$$

First order conditions become:

$$(3a) \quad Pf_N = y$$

$$(3b) \quad Pf_K = r ,$$

where $f_N = \frac{\partial f}{\partial N}$ and $f_K = \frac{\partial f}{\partial K}$. By using the definition of y in (3a), and inserting for r from (3b) into (3a), we find that

$$(4) \quad X = f_N \cdot N + f_K \cdot K ,$$

which implies from Euler's theorem that there are constant returns to scale at the point of operation. If the average cost function is U-shaped, this means that production takes place at a minimum of costs. But certainly it adds restrictions also to the appropriate formulation of the production function, ruling out globally decreasing returns to scale as this will result in a one-man firm. As shown by Vanek (1970) and Ireland and Law (1984), however, a production function exhibiting first increasing and then decreasing returns will fulfil the condition given by (4), and production will occur at the cost minimizing level.

However, a further assumption has to be met if the firm is to produce at this efficiency level. The firm's capital is generally assumed collectively owned. According to Furubotn and Pejovich (1970) and Vanek (1977) this increases the cost of internally financed investments, as the workers at any point of time will demand the principal invested recuperated during their period of tenure with the firm (Furubotn-Pejovich effect). (In addition there may also be a requirement that the firm is not allowed to disinvest.) Then, given a level of capital, the workers will always have an incentive to reduce the level of membership and increase the productivity of labour, and thereby pay-out. But this reduces the productivity of capital below its former optimal level, and disinvestment will take place, provided it is possible (Ward-Vanek effect). According to Vanek, and Furubotn and Pejovich, this can be solved only with 100 % external finance, assuming that the property rights structure cannot be changed (socialist labour-managed firm). However, a capitalist labour-managed firm should be free to allow individual ownership of capital, and thus solve the problem of high costs of internal finance.

Thus, the completely externally financed (socialist) labour-managed firm with an appropriate technology should be able to produce efficiently, and the economy consisting of these firms should be able to reach the same Pareto-optimal allocation of resources as the capitalist economy (Drèze (1976)). But under internal finance and collective ownership of capital, the scale of operation will be too low, and under decreasing returns to scale no equilibrium is reached in which the firms have more than one worker. On the other hand, given the existence of an efficient production level, e.g. by assuming a homothetic technology as examined by Ireland and Law (1984), another problem is that production tends to be constant when prices change. Then, unless exit and entry of firms can take place at non-prohibitive costs, the labour-managed economy will not react correctly to demand changes in the sense that resources will not be transferred to the most preferred activity.

However, if it is the case that the production in some lines of business is too low (high) because of restricted entry (exit), this means that there is in effect an economic rent captured by the producers. But in that case the government can increase (decrease) production by introducing a lump-sum tax. Then the firm will maximize

$$(2') y = \frac{PX - rK - S}{N}$$

where S is a lump-sum tax. First order conditions are as in (3a,b). But now (4) becomes

$$(4') X = f_N N + f_K K - \frac{S}{N},$$

which implies that production will take place at decreasing (increasing) returns if S is positive (negative).

This applies correspondingly to the short run optimization, assuming that the individual labour supply is given. Let $K = \bar{K}$, and write $rK = F$. The first order condition for allocation of labour is still (3a), and we find easily that

$$\frac{dN}{dP} < 0 \quad \text{and} \quad \frac{dN}{dF} > 0.$$

Again, the scale of the firm will be reduced to a level where it will not pay the remaining workers to reduce the employment level anymore because of the fixed cost burden. But then obviously a lump-sum tax has the same effect as increased fixed costs, and it can be used to obtain the desired level of operation.

The arguments above can be illustrated by figure 1 and 2 below, showing the long-run and short-run responses respectively to lump-sum taxation, assuming in the long run that $X = Q(f(K,N))$ is a homothetic production function, see Ireland and Law (1984).

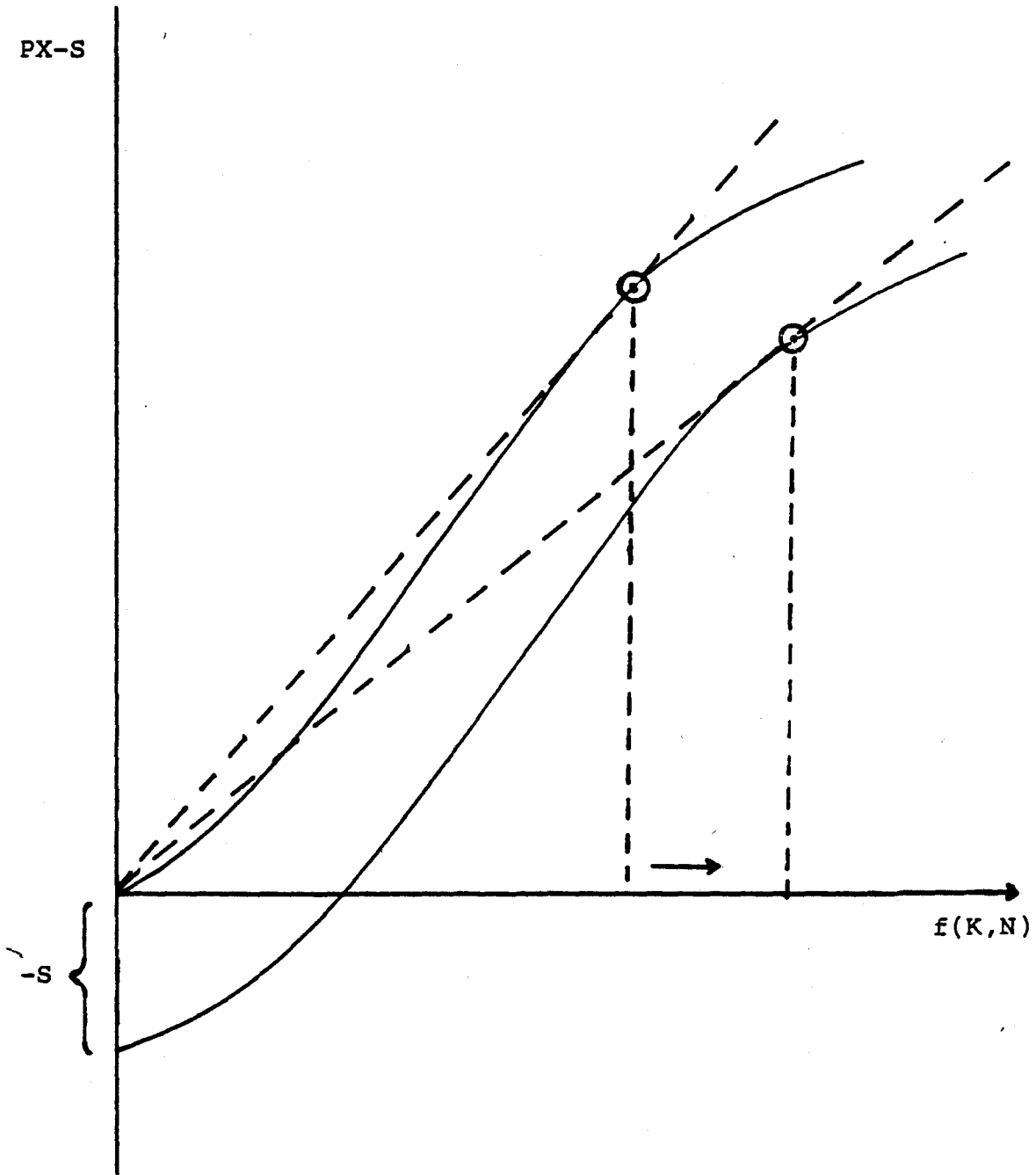


Figure 1. Long run equilibrium with homothetic technology, and with/without lump-sum taxation. The arrow indicates the response to the introduction of a lump-sum tax, S .

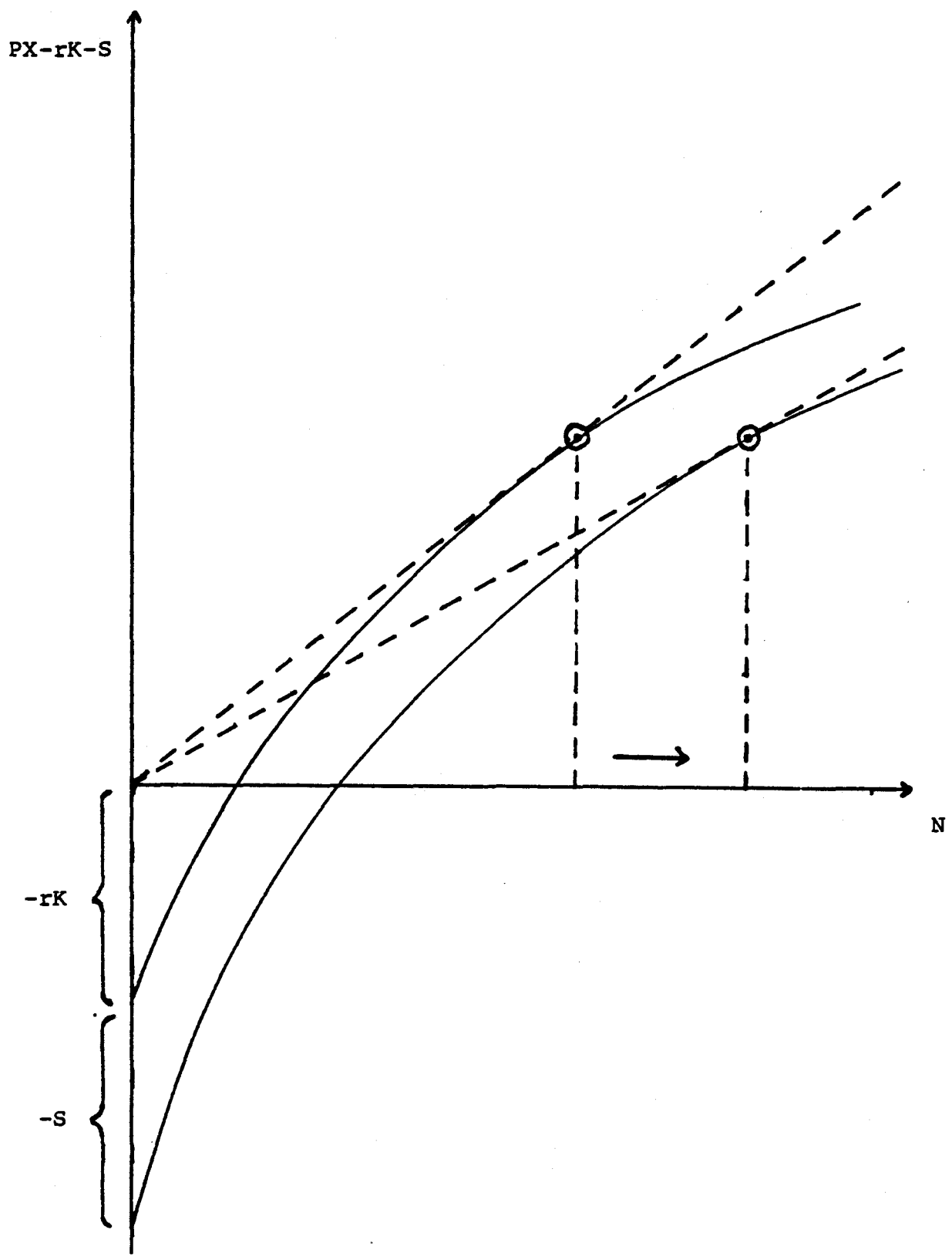


Figure 2. Short run effect of lump-sum taxation. The arrow indicates the employment response.

Thus, the problem of finding the correct scale of operation in an economy consisting of labour-managed firms can be solved by means of lump-sum taxes. The lump-sum tax corresponds to the rent proposed by Domar (1966), which should be imposed on firms according to location and different natural conditions affecting the firms' productivities. However, we have assumed so far that the individual workers' labour supply is constant. This is probably not a realistic assumption, and we shall see that loosening this assumption will drastically alter the optimal policy towards producer cooperatives.

3. Sen's Taxation Rule vs. a Non-Compensating Tax Schedule.

A major difference between a classical profit-maximizing firm and a labour-managed firm is the status of the workers. The profit-maximizing firm can choose to buy any amount of labour it desires. The labour-managed firm is, on the other hand, concerned mainly with the welfare of its workforce. This should incorporate the freedom of each worker to make decisions on the individual labour supply (hours of work). But this will make total work done a variable it is hard to control, unless the collective is able to make compensating changes in the number of workers. In particular in the short run the latter can hardly be fulfilled, and the (short run) production level is very much dependent on the individual labour supply decisions. The first one to recognize this was Sen (1966). He assumes that each worker maximizes a utility function defined over income and leisure, i.e.

$$(5) \quad u(y, 1-l) ,$$

where u is concave, and increasing in y and decreasing in l , the number of hours worked by each worker. I have normalized time so that $0 \leq l \leq 1$. Income is given by

$$(6) \quad y = \frac{PX - F}{L} \cdot l ,$$

where $L = \sum_{i=1}^N l^i$ is the total number of hours worked by all workers. F is the sum of all non-labour costs.

Sen was concerned with finding the socially optimal allocation of work effort. Whether an efficient solution is obtained or not depends, according to Sen, on the rule for the distribution of income from the cooperative and the degree of sympathy towards fellow-workers, which may vary from none to full sympathy, the latter being a sufficient condition for attaining a socially optimal allocation of work.

However, the same aspect can be approached from a somewhat different viewpoint. E.g. Bonin (1977) and Chinn (1979) are concerned with the degree of collusion among the workers of a cooperative in stipulating individual labour supply. The degree of collusion may range from none to full cooperation. The former may be interpreted as a Nash-Cournot assumption, where each worker decides on his labour supply under the assumption that fellow-worker's labour supply remains unchanged. Ireland and Law (1981) have showed that the two approaches can be expressed in the same framework with a parameter $\theta \in (\frac{1}{N}, N)$, expressing the elasticity of total labour supply in each worker's individual supply. Thus, a $\theta = \frac{1}{N}$ is the same as a Nash-Cournot reaction function, or in terms of Sen's terminology no sympathy towards fellow-workers. With full sympathy or cooperation, $\theta = N$, and a socially optimal allocation of labour-input is obtained.

When "social consciousness" is not complete, Sen showed that there is a way of distributing income which will result in an optimal labour allocation. This will come about if a proportion α , $0 < \alpha < 1$, of income is distributed on a per capita basis, while $1 - \alpha$ is the proportion of income distributed according to effort (hours of work). The procedure has later been generalized by Bennet (1982, 1984), see Section 5.

Sen's α is determined so that

$$(1-\alpha) = \frac{\eta}{\beta}$$

where

$$\eta = f_L \frac{L}{X}, \quad \beta = \frac{PX - F}{X}.$$

This choice of α will lead to equality between the value marginal product of labour and the marginal rate of substitution between income and leisure.

Let us find the socially optimal allocation of labour, assuming, as did Sen, that the workers are homogeneous and that every worker takes the other workers' supply of labour as given when deciding on their individual labour supply (Nash-Cournot).

Each worker maximizes (5) w.r.t. λ , given (6) and (1) with $K = \bar{K}$ and L substituted for N . The first order condition, where $u_Y = \frac{\partial u}{\partial Y}$ and $u_\lambda = -\frac{\partial u}{\partial (1-\lambda)}$, may be written as

$$(7) \quad \frac{u_\lambda}{u_Y} = MRS = Pf_L \left(\frac{\lambda}{L} + \frac{PX-F}{X} \cdot \frac{1}{Pf_L \cdot X} \cdot \frac{L-\lambda}{L} \right) = Pf_L \left(\frac{\lambda}{L} + \frac{L-\lambda}{L} \frac{\beta}{P\eta} \right)$$

Now, $MRS = Pf_L$ if and only if $\left(\frac{\lambda}{L} + \frac{L-\lambda}{L} \frac{\beta}{P\eta} \right) = 1$. This implies $\beta = P\eta$, or, noting that

$$\beta = P\eta \Leftrightarrow \frac{PX-F}{X} = Pf_L \frac{L}{X},$$

that

$$(8) \quad PX - rK = Pf_L L,$$

where as noted previously $F = rK$, assuming that capital is the only non-labour (hired) factor.

In Section 2 we saw that a labour-managed firm without fixed costs or access to a free factor of production ("land") will produce always at a minimum of costs, i.e. at a point where constant returns to scale prevail. (8) shows that this condition is sufficient for a socially optimal allocation of labour in the

labour-managed firm with variable labour supply, provided that the non-labour factors of production are optimally allocated and that F reflects fully the associated costs.

Our results so far serve as an illustration of the similarities between the results derived by Domar (1966) and Sen (1966).

Thus, the allocation of individual labour, choice of employment level and use of other factors of production in the labour-managed firm are socially optimal if

- (i) There is full cooperation in determining individual labour supply, or all workers have complete sympathy towards their fellow-workers,

or,

- (ii) there are constant returns to scale at least over some range, and that production takes place within this range,

which implies that

- (iii) there are no free factors of production ("land") and fixed costs.

Barring these cases, the welfare of the workers and the economy's resource allocation can be improved upon by taxation. It should be noted also that the possible inefficiency accounted for in Section 2 may be further strengthened by the possibility of too low a labour supply when the labour supply decisions are made individually. Assume that the firm's technology is such that there is first a range of increasing returns to scale, and then decreasing returns. Then Vanek (1977) argues that the (partly) self-financed labour-managed firm will produce somewhere in the increasing-returns range. As it will be shown below, this is the range where individual labour supply will be too low. In addition the workers reduce employment so as to increase productivity of labour, and accordingly payout. This will reduce the productivity of capital which induces

disinvestment. Then the firm will move into areas where labour supply is continuously being reduced compared to the efficiency level, cf. (8). Assuming technical complementarity between the factors of production, this means a further reduction in the productivity of capital. But this implies also that Sen's α , or the tax rate to be developed below, can be used to offset the self-extinction forces considered by Vanek. In particular, using a second-best approach, we can argue that this tax policy may be used to induce an oversupply of labour if the problems connected with the scale of operation are the most serious efficiency problems.

An important objection against Sen's taxation rule is its informational requirements when workers are heterogeneous. Let us therefore investigate whether the same result can be obtained by a general taxation of the firm and its workers. Then we see whether second best taxation can be used to control labour supply in an efficient manner. A tax on the firm's surplus after the deduction for non-labour costs is neutral when considering the firm's allocation of labour and capital. But this taxation will affect each worker's individual supply of labour. This is an intended distortion, and it should be considered a good property that the tax schedule does not affect the long run allocational decisions of the firm. If t is the tax rate, and there is no lump-sum compensation, we can write each worker's maximization problem when subject to taxation as

$$(9) \quad \text{Max}_l u(y, 1-l)$$
$$\text{s.t. } y = \frac{PX-F}{L} \cdot l \cdot (1-t)$$

It does not matter whether the tax is technically levied on the firm or on the workers individually. The first-order condition is

$$(10) \quad \frac{u_l}{u_y} = \text{MRS} = \text{Pf}_L \left(\frac{l}{L} + \frac{L-l}{L} \frac{PX-F}{\text{Pf}_L \cdot L} \right) (1-t)$$

MRS = Pf_L if t is fixed so that

$$(11) \quad t^* = \frac{(L-\lambda) [(PX-F) - Pf_L \cdot L]}{Pf_L \cdot L \cdot \lambda + (L-\lambda)(PX-F)}$$

We find easily from (11) that

$$t^* \begin{cases} > \\ < \end{cases} 0 \text{ as } (PX-F) \begin{cases} > \\ < \end{cases} Pf_L \cdot L.$$

Assume that F reflects the market valuation of marketed non-labour factors of production. Then equality in the conditions above is equivalent to constant returns to scale in production, cf. (8). When $t^* > 0$ we have, using (8), that

$$X > f_K K + f_L L$$

where we assume $f_K = \frac{r}{p}$ from first order conditions for optimal allocation of marketed non-labour factors of production. Then, if $f(K, L)$ is homogeneous, it follows from Euler's law that the production function is homogeneous of a degree $k < 1$. The subsidization condition $t^* < 0$ can be explained in a similar manner. We interpret decreasing returns to scale, $k < 1$, as the existence of a free factor of production to which the firm has costless access, i.e. a non-marketed factor. If $k > 1$, on the other hand, there are fixed costs or taxes levied as a fixed amount on the firm, which induces the firm to increase the level of operation, Domar (1966).

Then we can state the following rule for taxation of a labour-managed firm and its workers:

- (i) $t^* > 0$ if there are decreasing returns to scale, or a free factor of production ("land").

(ii) $t^* < 0$ if there are increasing returns to scale, or fixed costs.

This is equivalent to the rule established by Pigou (1924). He argued that taxation should be used in industries with decreasing returns to scale. The difference from ours is that in Pigou's case it was related to the optimal production of an industry. E.g., with decreasing returns, each firm produces too much in their effort to capture as large a part as possible of the economic rent. Here it is the workers' strive to capture the economic rent of the firm that is the argument for a tax.

It should be noted also, that if the workers are heterogeneous, λ will differ among them. Then the value of t^* is not necessarily correct from an efficiency point of view. But heterogeneity among the workers does not affect the choice between taxation and subsidisation. These problems are however better analysed by using the traditional framework to derive optimal tax rules, to which we will proceed after having made a comparison between the taxation of profits in profit maximizing firms and the incentive correcting taxation of labour-managed firms.

Let us assume that the labour-managed firm produces under decreasing returns to scale. Although an equilibrium cannot be found in the Illyrian model in the range of decreasing returns, the assumption is not unrealistic. The reason why the size of the firm is not being reduced can be explained in terms of solidarity, utility of having a given level of employment etc.

The oversupply of labour from the individual workers is explained by the workers' aim at capturing the economic rent. This calls for an efficiency correcting tax rate $t^* > 0$.

The profit-maximizing capitalist firm operating under the same conditions will make a profit π given by

$$(12) \pi = PX - rK - wL ,$$

where w is the market wage rate. We know that a profit tax, allowing for the deduction of the true costs of production, is neutral with respect to the firm's allocational decisions, see e.g. Sandmo (1974). Then the profit-maximizing firm's profit tax may be set at a level arbitrarily close to 100 %. The labour-managed firm will make the same profit, distributed as dividend among its workers. If there were a tax rate, t' , which taxed away all this profit, this tax rate would be given by

$$t' = \frac{\pi}{PX-rK} = \frac{PX-rK-wL}{PX-rK} = 1 - \frac{wL}{PX-rK} = 1 - \frac{w}{\frac{PX-rK}{L}} = 1 - \frac{w}{\bar{y}}$$

Thus, if $w = \bar{y}$, and thereby $\pi = 0$, we have $t' = t^* = 0$, whereas $t', t^* > 0$ if $w < \bar{y}$, which represents the case where a part of the workers' dividend is profit.

t' and t^* have some similar properties. t' is defined as a tax rate that taxes away all profit, while t^* is a tax rate which gives incentives to a correct labour supply. When $\pi > 0$, and accordingly $t' > 0$, this profit can be considered the return to a free factor, whose existence causes decreasing returns to scale. The oversupply of labour results from the workers' strive to capture this profit - or return to a scarce factor, and $t^* > 0$ is a substitute for a market compensation. Thereby, if profit were taxed away by t' , no efficiency correcting taxation would be necessary. To define this profit would however not be an easy task.

Now, assume that $w = Pf_L$, i.e. the wage of the workers in an identical profit maximizing firm reflects the value marginal product of labour. Then we may write

$$(13) t' = 1 - \frac{w}{\bar{y}} = 1 - \frac{Pf_L}{\bar{y}}$$

Now, having developed two tax rates, t^* and t' , represented by (11) and (13) respectively, we can compare them to find that

$$t' = t^* \text{ if and only if } \frac{PX-rK}{L} = Pf_L$$

which implies $\pi = 0$ and $t^* = 0$. In particular we find that

$$t' > t^* \text{ if } \left[1 - \frac{Pf_L \cdot L}{PX-rK} \right] > \frac{(L-\lambda) [(PX-rK) - Pf_L \cdot L]}{Pf_L \cdot L \cdot \lambda + (L-\lambda)(PX-rK)} \Leftrightarrow Pf_L < \frac{PX-rK}{L}$$

which holds if $t^* > 0$. Thus, if $t^* > 0$, the optimal tax rate is lower than a rate which taxes away all profit. We can give an intuitive explanation of this result. The 100 % profit tax is identical to Sen's tax. But Sen's rule imply a compensation in the sense that the profit is redistributed on a per capita basis, which neutralizes the income effect. A pure substitution effect remains. The income taxed away by t^* is not redistributed. Then the income effect of taxation will result in a higher optimal labour supply, and hours of work should be reduced to a smaller degree.

4. Formulae for Optimal Income Taxation of Workers in Labour-Managed and Profit-Maximizing Firms.

I will now, within the framework used in the optimal income taxation literature, derive rules for optimal taxation in an economy consisting of labour-managed firms, in which the workers may be either homogeneous or heterogeneous. The tax rates will be compared to those which prevail in a similar economy consisting of profit-maximizing firms.

(i) Model

In the LM-economy, an economy consisting of labour-managed firms, LM-firms, we assume that there is a tax rate t_L^I on total

income, levied on each household. The corresponding tax rate in the PM-economy, an economy consisting of profit-maximizing firms, PM-firms, is denoted t_p^I . In addition there may be a tax on profit which we term t^π . Lastly there is a lump-sum element $a > 0$, equal for all households, in the income tax functions. In both scenarios we assume that N workers are employed by a representative firm. But whereas each worker in the LM-firm gets a share of total dividend equal to $\gamma^i \frac{\ell^i}{L}$, where $L = \sum_{i=1}^N \ell^i$ and γ^i reflects the ability (productivity) of worker i , each worker in the PM-firm receives w^i reflecting his ability, and a share α^i in profit. We can write income of the workers in the two types of firms as

$$(14) \text{ PM: } Z^i = \ell^i w^i + \alpha^i \pi, \quad 0 \leq \alpha^i \leq 1, \quad \sum_{i=1}^N \alpha^i = 1$$

$$(15) \text{ LM: } Y^i = \ell^i \gamma^i Y = \frac{PX-rK}{L} \ell^i \gamma^i$$

We abstract from income from other sources. The workers will maximize the utility functions

$$(16) u^i (Z^i, 1-\ell^i)$$

$$(17) u^i (Y^i, 1-\ell^i)$$

subject to (14) and (15) respectively.

Generally utility functions will differ among individuals. In the optimal taxation literature it is common, however, to assume heterogeneity in terms of ability, rather than in terms of how workers evaluate income and leisure. We will follow this route here as well, and assume that the form of u does not differ among individuals. I will, however, retain the index i on the

utility functions. Another vital assumption concerns the workers' choice of labour supply. We assume that every worker can predict the labour supply of their fellow workers. This assumption should not be too restrictive in view of the identical form of the workers' utility functions, provided all workers know the distribution of abilities. Specifically we assume a Nash-Cournot reaction function.

The government is assumed to require a given tax income \bar{T} . In the two scenarios, governmental tax income will be T^P and T^L respectively:

$$(18) \quad T^P = \sum_i \lambda^i w^i t_P^I - Na + t^\pi \pi$$

$$(19) \quad T^L = \sum_i \lambda^i Y^i t_L^I - Na$$

We assume that the government is concerned with finding welfare maximizing tax rates. Therefore we can formulate the government's problem as maximizing a welfare function, W , subject to the tax requirements:

$$(20) \quad \text{Max } U(u^1, u^2, \dots, u^N)$$

$$\text{s.t. } T = \bar{T}.$$

We can express the workers' (and capitalists') utility by means of indirect utility functions, which will facilitate the analysis:

$$(21) \quad v_P^i (w^i (1-t_P^I), \alpha^i \pi (1-t^\pi), a), \quad i = 1, \dots, N$$

$$(22) \quad v_L^i (Y^i (1-t_L^I), a), \quad i = 1, \dots, N$$

Then the social indirect welfare function becomes

$$(23) \quad W = W(v^1, v^2, \dots, v^N)$$

and the government's problems are respectively:

$$(24) \text{ PM: } \begin{aligned} & \text{Max} && W(v_p^1, \dots, v_p^N) \\ & t_p^I, t^\pi, a \end{aligned}$$

$$\text{s.t. } \sum_i \lambda^i w^i t_p^I - Na + t^\pi \pi = \bar{T}$$

$$(25) \text{ LM: } \begin{aligned} & \text{Max} && W(v_L^1, \dots, v_L^N) \\ & t_L^I, a \end{aligned}$$

$$\text{s.t. } \sum_i \lambda^i y^i t_L^I - Na = \bar{T}$$

The problem in (24) can be somewhat simplified by making an assumption concerning the taxation of profits. As pointed out before, a profit tax is neutral w.r.t. the firm's allocational decisions. But the tax will obviously affect each worker's decision on labour-supply through an income effect and ownership part (α^i) if taxation affects relative profitability of different portfolios. In that respect it does not matter whether the tax is levied on the firm or on the households. As a profit tax below 100% will not affect the firm's allocational decisions, it seems to be a reasonable assumption that taxation should not discriminate against different types of income. This leads us to assume that the tax rate on profit equals the rate paid on income from labour services, i.e. $t^\pi = t_p^I$, and each PM-worker's tax liability is, when we drop the superscript I:

$$(26) \quad t_p \cdot (w^i \lambda^i + \alpha^i \pi), \quad i = 1, \dots, N, \quad 0 \leq \alpha^i \leq 1$$

We must consider also whether α^i will be affected by economic factors like changes in the tax rates. α^i may be given by external factors which are not likely to be affected by these. Then a tax on profit will have a pure income effect. However, generally it would be a reasonable assumption that these ownership rights may be traded, and that each household's optimal α^i will depend on economic factors like the tax rates. Thus, with the restriction that we must always have $\sum_i \alpha^i = 1$, this will imply that we will have a Slutsky-equation for property shares as well as for labour supply.

With the problems formulated as in (24) and (25), they can be solved by using Lagrange' method, see also Dixit and Sandmo (1977), Atkinson and Stiglitz (1980, ch. 13) and Sandmo (1983). The Lagrange expressions and the resulting first order conditions when solving for t_L , t_p and a are reported in the Appendix.

(ii) Homogeneous Workers

Let us first assume that all workers are identical (equal ability). Then we may carry through the analysis as though the economy consists of one worker (household). Obviously $\frac{\partial \alpha^i}{\partial t_p}$ and $\frac{\partial \alpha^i}{\partial a}$ are both zero, i.e. taxation does not influence the distribution of ownership rights in the PM-economy.

In equilibrium we can use the properties of the indirect utility function, saying that

$$\frac{\partial v_P^i}{\partial w^i(1-t_p)} = \lambda^i \lambda_P^i, \quad \frac{\partial v_L^i}{\partial Y^i(1-t_L)} = \lambda^i \lambda_L^i, \quad \frac{\partial v_P^i}{\partial \pi(1-t_p)} = \lambda^i \alpha^i, \quad \frac{\partial v^i}{\partial a} = \lambda^i.$$

Using this in the first order conditions, we can derive (see Appendix) the following well known rule for optimal taxation of PM-workers:

$$(27) \theta_p w t_p (-S_w^P + \frac{\partial \lambda}{\partial a} \alpha \pi) = 0.$$

θ_p is the Lagrange multiplier in the welfare maximization problem (24), to be interpreted as the shadow price on governmental spending, and

$$S_w^P = - \left(\frac{\partial \lambda}{\partial t_p} + \frac{\partial \lambda}{\partial a} w \lambda \right)$$

is the substitution term of the Slutsky-equation for changes in t_p . Of special interest to us is the latter element in the paranthesis in (27). According to (27) the efficiency loss of an income tax is offset by the taxation of profits, which has a pure income effect. On the other hand it is quite obvious that this does not necessarily hold if α changes following a change in the tax rate. But, as argued above, this can hardly take place when the workers are homogeneous without violating the restriction $\sum_i \alpha^i = 1$, so that we abstract from that possibility. Nevertheless, the optimal tax rate will only by a mere coincidence deviate from 0, as compensated labour supply is generally affected by the tax. However, if $t_p^I = 0$ and $t^\pi > 0$, and also if $\frac{\partial \lambda}{\partial t^\pi} = 0$, it can easily be verified that equilibrium requires $\theta_p = \lambda$, and the optimal allocation is not responsive to the profit tax, which may then be fixed according to the government's tax requirement.

We now turn to the taxation of the homogeneous LM-workers. We can set $\gamma^i = 1$ since all workers are equally productive.

We define

$$\varepsilon = \frac{\partial Y}{\partial \lambda} \frac{\lambda}{Y}$$

and θ_L as the shadow price on governmental spending in the LM-economy. We use the Slutsky-equation where

$$S_{Y}^L = - \left(\frac{\partial \lambda}{\partial t_L} + \frac{\partial \lambda}{\partial a} \lambda Y \right)$$

is the substitution term accounting for the compensated labour supply to obtain the optimal tax rate t_L^* :

$$(28) \quad t_L^* = \frac{-\lambda \varepsilon}{\theta_L + \varepsilon(\theta_L - \lambda)}$$

We note that when $\varepsilon = 0$, we have $t_L^* = 0$. As $\varepsilon = 0$ when $\pi = 0$ (see (29) below), this corresponds to the optimal income tax rate in the PM-economy. However, whenever $\varepsilon \neq 0$, and $\pi \neq 0$, the optimal tax rate is non-zero, while this is the case in the PM-economy only if a profit tax does not affect labour supply. But in the PM-scenario there is no efficiency argument favouring a given profit tax rate; it may indeed take on any value since the tax has an income effect only. In the LM-scenario efficiency requires $t_L^* \neq 0$.

As it will be shown below, Sen's taxation rule emerges as a special case of (28). Before proceeding to interpreting the formula further, it might be useful to consider implementation of the tax schedule, or more precisely what kind of information is needed, and how it can be extracted. The extraction of information from the workers causes no problems as they are assumed identical. All workers know that the taxation improves welfare of all workers, and they have no individual characteristics to be concealed. Larger problems may arise if the firms in the economy are not identical. Then information

is needed about each firm's production and use of factors of production. This problem, relevant for the "heterogeneous-workers-case" as well, will be further discussed in Section 5. In addition to information about the production environment, we must know the shadow price on governmental spending and costs of taxation as reflected by θ_L . This problem, is general for all taxation. It will be shortly commented upon below.

Let us examine the term ε further. Assuming $\frac{\partial L}{\partial \lambda} = 1$, we can elaborate on it to get

$$(29) \quad \varepsilon = \frac{\lambda}{L} \left(\frac{Pf_L \cdot L}{PX - rK} - 1 \right),$$

i.e. $\varepsilon \neq 0$ if there are fixed costs or a free factor ("land"), and the result corresponds to that obtained previously.

We note also that in the general tax formula, it is not sufficient to consider only the efficiency correction in production. In that sense this tax rate differs from the one derived by Sen (1966) and t^* in section 2 of this paper. There are costs of taxation involved, and the cost of government spending, θ_L , may differ from the private utility of money. (Both parameters are positive.) This will affect the value of the optimal tax rate in one direction or the other.

It may be of interest to consider a special case. Assume that $\theta_L = \lambda$. Then the utility of money is the same in the government and private sectors. The optimal tax rule becomes

$$(28') \quad t_L^* = -\varepsilon = \frac{\lambda}{L} \left(1 - \frac{Pf_L \cdot L}{PX - rK} \right),$$

which we remember is the tax rate that taxes away all profit, see (13). This also corresponds to Sen's α , which is that part of income which should be distributed on a per capita basis. The reason why this tax rate deviates from the tax rate developed in

section 3, which are both claimed to be optimal, is exactly the one we gave there. What we have got here, is a situation where the government can compensate the workers by using the lump-sum element a , and it will then be able to induce the workers to supply the optimal amount of labour, simultaneously as all profit is taxed away and distributed on a per capita basis.

More generally, the optimal tax rule (28) says that the government may be able to obtain a given tax revenue simultaneously as the allocation of the resources in the economy is improved upon. This depends on the values of θ_L and λ . Thus, if θ_L is very high, which indicates high costs of government spending, the optimal tax rate is low. In the PM-economy we have a situation where the government is under some circumstances able to get a tax income without loss of efficiency.

(iii) Heterogeneous Workers

I will now proceed to considering the optimal tax rate when the workers are heterogeneous, and derive optimal tax formulae in the PM- and LM-scenarios.

The first order conditions are as reported in Appendix. We derive the optimal income tax when the workers are employed by PM-firms, and the same workers also share in the residual according to ownership rights. After lengthy manipulations of first order conditions (see Appendix), and using the definition of covariances, we find that t_p should be set so that:

$$(30) \quad t_p^* = \frac{1}{\theta_p A} [\text{Cov}(W_i \lambda^i, \lambda^i w^i) + \text{Cov}(W_i \lambda^i, \alpha^i \pi)]$$

where

$$A = \overline{S_w^i w^i} + \overline{S_\alpha^i \pi} - \text{Cov}\left(\frac{\partial \ell^i}{\partial a} w^i, w^i \ell^i\right) - \text{Cov}\left(\frac{\partial \alpha^i}{\partial a} \pi, \alpha^i \pi\right) \\ + \frac{\partial \ell^i}{\partial a} w^i \alpha^i \pi + \frac{\partial \alpha^i}{\partial a} \pi \ell^i w^i$$

S_w^i and S_α^i are substitution terms of the Slutsky-equations for labour supply and ownership rights respectively. A bar denotes average values. If ownership rights change, we assume that this occurs in a zero-sum way. When α^i is given, e.g. from historical reasons, the terms involving changes in α^i are zero.

In a similar manner we find the expression for the optimal income tax in the LM-economy:

$$(31) \quad t_L^{**} = \frac{\text{Cov}\left[W_i \lambda^i (1 + \gamma^i \varepsilon^i Y \frac{\partial \ell^i}{\partial a}), \gamma^i \ell^i\right] - \overline{W_i \lambda^i \gamma^i \varepsilon^i S_Y^i}}{\overline{S_Y^i \gamma^i} [\varepsilon^i (\theta_L - W_i \lambda^i) + \theta_L] - \text{Cov}\left[\gamma^i \frac{\partial \ell^i}{\partial a} (\varepsilon^i (\theta_L - W_i \lambda^i) + \theta_L), \gamma^i \ell^i\right]}$$

ε^i is defined as above. But it relates here to each worker's elasticity of income in hours of work. Observe that if $\pi = 0$, and consequently $\varepsilon = 0 \forall i$, (31) converts into (30), and the taxation rules are identical in the PM- and LM-economies, provided costs of taxation do not differ.

From the expressions (30) and (31) above, we see that the optimal tax rates are of any sign, and they are positive if and only if numerator and denominator are of the same sign. In the denominator of (31) we recognize the terms determining the optimal tax rate in the one-consumer case (homogeneous workers),

see (28) and (A5) in Appendix. Thus, it is reasonable to interpret the denominator as taking care of the efficiency aspects, while the numerator captures the distributional aspects.

The informational requirements are quite considerable when an optimal tax rate is to be formulated, which is supposed to take into consideration efficiency aspects as well as distributional aspects. This holds true in any economic system, and constitutes as such no special problems for the formulation of optimal tax rates in LM-economies. What is needed is knowledge of the distribution of abilities. But as utility functions are the same, we do not need to know which individuals who have got the specific abilities. There is one additional informational requirement in the LM-scenario which needs a comment. It relates to comparisons with optimal taxation in a PM-economy and when comparing with the formulations of tax functions in Sen (1966) and Bennet (1984). The planner has got to know the productive conditions of the different firms, assuming these differ. In Bennet's formulation this information is held by the manager of the firm, in addition to the information he has about each worker's ability. It is a matter of judgement whether it is easier to extract information about firms' productive environment than individuals' abilities.

The optimal tax rule in the PM-scenario is discussed elsewhere in the literature, see e.g. Sandmo (1983). Here it will be used for comparison purposes only. But before doing these comparisons, some comments should be made on the expression for optimal income taxation of LM-workers. The term $W_i \lambda^i$ is the social marginal utility of consumer i's consumption. In the covariance term in the numerator of (31), we see that the social marginal utility of consumption is multiplied by a term showing the effect of lump-sum income on labour supply and thereby on profit, which will again affect the government's income. This makes it reasonable to interpret the term

$$W_i \lambda^i (1 + \gamma^i \epsilon^i_y \frac{\partial \lambda^i}{\partial a})$$

as the social marginal utility of i 's income, cf. Diamond (1975). Then the tax rate depends on the covariance between the individual marginal utility of income and the supply of efficiency units of labour, which corresponds again to Diamond's prescription for taxation in a many-person economy. The sign of this expression is ambiguous. It may be reasonable to assume that $\text{Cov}(W_i \lambda^i, \gamma^i \ell^i) < 0$, in particular if W is utilitarian and $W_i = 1 \forall i$. It is harder to assess the additional effect caused by lump-sum income affecting labour supply and profit. We should therefore limit ourselves to note that the characterization of the optimal tax rate is affected in either way by this term.

It may be illustrative to give a tentative comparison between the two tax rates t_p^* and t_L^{**} . The two tax rates will be identical if $\varepsilon = 0 \forall i$, which depends on the technology, and in which case, as mentioned, $\pi = 0$. We will use this as a benchmark position. Specifically, we assume $t_p^* = t_L^{**} > 0$ initially. This can be justified by assuming $\text{Cov}(W_i \lambda^i, \ell_w^i) < 0$ and $\text{Cov}(\frac{\partial \ell^i}{\partial a} w^i, w^i \ell^i) \geq 0$. We know that $S_w^i < 0$. Of course the corresponding terms explaining t_L^{**} will be determined accordingly, so that $\text{Cov}(W_i \lambda^i, \gamma^i \ell^i) < 0$ and $\text{Cov}(\gamma^i \frac{\partial \ell^i}{\partial a}, \gamma^i \ell^i) \geq 0$, while obviously $S_Y^i < 0$.

Assume that profit increases marginally above zero, e.g. by a slight increase in production, so that this takes place on the increasing part of the average cost curve. In both scenarios the tax rates may be increased or decreased. But it is of some interest to examine the efficiency and distributional effects separately.

The distributional argument for a tax rate change in the PM-scenario (the numerator of (30)) depends on the term $\text{Cov}(W_i \lambda^i, \alpha^i \pi)$. It is reasonable to assume that it is negative, i.e. the higher an individual's share in profit is, the lower

the social marginal utility of consumption will be (or marginal utility of money if W is utilitarian). Then the tax rate should be increased according to distributional arguments. The efficiency arguments are generally ambiguous. But if $\frac{\partial \lambda^i}{\partial a} < 0$, $\frac{\partial \alpha^i}{\partial a} < 0$ and $\text{Cov}(\frac{\partial \alpha^i}{\partial a} \pi, \alpha^i \pi) \geq 0$, the tax rate should be reduced to better the allocation of the resources in the economy. The total effect will depend, of course, on the relative strength of the distributional and efficiency considerations.

In the LM-scenario the distributional considerations are ambiguous, as we have to assess the covariance between marginal utility of income and individual labour supply. The other term in the denominator calls for an increased tax rate as $\epsilon^i < 0$ and $S_Y^i < 0$. Then a necessary, but not a sufficient, condition for arguing in favour of a reduced tax rate, using distributional arguments, is that

$$\text{Cov}(W_i \lambda^i (1 + \gamma^i \epsilon^i Y \frac{\partial \lambda^i}{\partial a}), \gamma^i \lambda^i) > \text{Cov}(W_i \lambda^i, \gamma^i \lambda^i)$$

This condition is met if the social marginal utility of income tends to vary less negatively with the supply of efficiency units of labour than does the social marginal utility of consumption. This will depend on the effect of a lump-sum income on individual labour supply and "profit", i.e. ϵ^i . It is not unlikely that the condition above will be met; cf. the covariance term when e.g. $\frac{\partial \lambda^i}{\partial a}$ is uniform and note that ϵ^i is increasing in λ^i . The efficiency considerations are examined previously, and the same argument applies. Note however that the net effect is ambiguous, in particular because of the efficiency of government spending, i.e. the term $(\theta_L - W_i \lambda^i)$.

Thus, although it is impossible to reach any clearcut results, the above discussion points to the difference in the recommendations concerning an optimal income tax. In the LM-scenario the optimal rate depends on the distribution of productivity ("skill"), i.e. γ^i , and the hours worked. This explains all income differences. In the PM-scenario, however, the distribution of the ownership rights is an additional argument to be

considered. These may of course outweigh the effect of the productivity distribution. But there is no reason to believe that this is more likely than the opposite possibility.

5. Some Comments on the Implementability of Optimal Tax Rules.

Some comments should be made on the implementation of an optimal tax schedule in an LM-economy. Different classes of problems may arise. On the one hand there are activities aimed at avoiding taxation. On the other hand we have the problems of incentive compatibility.

Some of the problems relating to the first category are discussed in Askildsen (1985). Most important are the incentives which may exist to disguise income as consumption within the firm if the main burden of taxation is levied on the households, and the problems of obtaining satisfactorily distributional effects when firms are the main object of taxation. However, it can be shown also that there exists a tax levied on the households which has the same effect as a tax levied on the firm. This means that a part of the tax may be paid by the firm, while the households make up for the distributional aspects. Thus, we can define a function (or a correspondence)

$$t^I = t(t^H, t^F)$$

where t^H and t^F are taxes levied on households and firms respectively. For every pair of (t^H, t^F) there is at least one t^I which will give the same resource allocation as would exist if t^H and t^F were used separately. The "general" tax rate t^I is the one investigated in this paper. But the technical composition of the tax schedule may cover a sophisticated system of double taxation as well as deductability on households for taxes paid by the firm. What is of ultimate interest in our context is the actual percentage of income paid in tax. But having said this,

it should be noted that a neutral tax rate will be incompatible with a tax rate taking distributional considerations.

The other category of problems create more difficulties. The problems connected to the implementation of the first best rule are discussed by Bennet (1982, 1984) and Guesnerie and Laffont (1984). In order to be incentive compatible, the relative remuneration should be positively related to productivity, i.e. the more able workers should receive the highest payment per hour worked. However, if there is a planner, e.g. the management of a firm, who is assumed to maximize the welfare of the workforce, Bennet (1984) shows that for different formulations of the welfare function welfare is maximized if the most able workers receive the lowest work-related payment, which is equivalent to a high individual tax rate. This is incentive incompatible as it will induce the workers not to reveal their true productivity. However, if the firm is large, and the elasticity of substitution between income and leisure exceeds unity, Bennet (1982) finds that the incompatibility disappears for a utilitarian welfare function. Bennet assumes in his models that the firm's management knows the preferences of all workers, and will give each heterogeneous worker a welfare maximizing work-related payment. This amounts to implementing an individually based tax schedule. The individual tax rates are chosen so that the most productive workers supply the highest amount of labour. This is the cheapest way of increasing the welfare of all workers. Through the complex income formula of a revenue sharing firm, where income of each worker depends on production and his marginal productivity in addition to his and other workers' work-related payment, this is obtained via a reduced work-related payment to the most able workers. Guesnerie and Laffont (1984) discuss the implementation of the first best rule in an optimal taxation setting also. They assume that the government has incomplete information about the productivity of a firm, to which it is to allocate labour from a pool of workers. The government maximizes a utilitarian welfare function. They find that an allocation can be implemented if and only if the decision variable, the amount of labour to be allocated to the firm, is negatively correlated to the unknown

productivity parameter. Then the demand for labour has to decrease in productivity if the first best rule is to be implemented. The firm will have no incentive to reveal the true productivity if higher productivity implies more workers to share in the income.

The above-mentioned studies point to the problems of obtaining a socially efficient allocation of labour when households and firms are taxed separately. Because of huge problems with incentive incompatibility, the prospects do not seem promising to implement first best taxation schemes. In our approach some problems vanish, as we do not need any knowledge of each worker's characteristic. He will be taxed (or subsidized) according to his income, not according to his ability. Indeed, this is also the general argument against the lump-sum taxation approach taken by Bennet. If the government knew all the properties of the taxpayers, there is no need for using second-best rules. But incentives are large to conceal individual properties which affect a person's tax liability.

An intended contribution of this paper is to reduce the incentive problems resulting from Bennet's individual tax functions. However, problems remain, which need comments. Firstly there are limitations as to what kind of heterogeneity with which we manage to cope. We have limited attention to differences in ability, and thereby income. But utility functions are assumed identical. If this were not the case, knowledge would be needed of distribution of ability and the associated utility functions.

A second, and probably more serious problem, concerns the issue raised by Guesnerie and Laffont (1984). We have seen that the optimal tax rates require information about production conditions, see (28) and (31). In a one-firm setting this does not cause problems. But an economy consists generally of several non-identical firms. Then the planner must decide on how much to tax each firm by fixing a firm specific \bar{T} , which may give the firms incentives to, on behalf of its workforce, to conceal

information about the productivity conditions so as to affect tax burden. This problem is probably not very significant when the workers are homogeneous, as they know that taxation is a means of increasing welfare of all workers. Thus it is a substitute for a rule requiring all workers to work equally long hours (full cooperation in fixing labour supply). When the workers are heterogeneous, however, problems arise in extracting information about distribution of skills. In addition the planner must know under what productive conditions each worker operates. The solution to the problem may be to levy this part of the "efficiency tax" on firms according to information about the conditions under which production takes place, while the remaining part of the taxation problem is solved in the ordinary way by trading off distributional considerations as against efficiency aspects through taxation on the individual households. Thereby we are left with the problem of extracting information about each firm's productive environment, which we will face also when the workers are identical.

6. Concluding Remarks.

The model and the problems we have examined above have their roots in the theory of the labour-managed firm as well as in the optimal taxation literature. When deciding on an optimal income tax, there are three considerations to be taken. Firstly, there is the problem of obtaining an optimal allocation of labour because the workers employed by a labour-managed firm have in some instances an incentive to over-supply labour, in other instances incentives to under-supply labour. This problem is specific for a labour-managed economy. Secondly there are distributional problems, i.e. how to obtain a fair distribution of welfare. Thirdly we may face problems with incentive-compatibility of the tax system.

Our main results are that a governmental body can use taxes to improve upon the allocation of labour in a labour-managed economy where the workers decide individually on labour supply.

Of course, in the most general setting, where the workers are heterogeneous, this is only part of the story, as there is a trade-off between efficiency and distributional considerations. Two different efficiency correcting tax schedules were developed. One of them did not allow for any redistribution of income, and we showed that this implied that not all potential profit should be taxed away. On the other hand, if the taxation were combined with a poll tax, the tax function is identical to the income distribution rule developed by Sen (1966) if taxation can be carried through costlessly. This means that all profit should be taxed away and redistributed on a per-capita basis. In contrast to these results we know that in the profit-maximizing economy profits can be taxed without worsening the firm's resource allocation. However, profit taxation may influence each worker's capital and labour supply, and neutrality no longer holds. When the workers are heterogeneous, the maximization of a welfare function will generally call for some redistribution of income. In the profit-maximizing economy the ability (productivity) of the workers and ownership rights in the residual of the firm are the main reasons for income inequality.

In a labour-managed economy any residual is distributed according to supply of efficiency units of labour, which is then the sole explanation of inequalities. If we introduce taxation to redistribute income, the strive to capture economic rent will influence the distributional arguments for a given tax rate in one way or another. Lastly, this second-best way of influencing the allocation and redistribution of income does not suffer from the incentive-incompatibility which is the problem of individualized tax schedules based on information about the workers' productivity.

It should be noted that our analysis is short-term in its character. We have simply assumed that capital and other non-labour factors are optimally allocated. This may be a reasonable simplification if the firm is completely externally financed at a given market price of capital. However, if some internal finance would have to be undertaken, the Furubotn-Pejovich effect will result in a self-extinction of the firm, see Vanek (1977). Then

of course the comparison with the profit-maximizing firm is no longer valid. On the other hand, it should be noted that the profitability of internal investment may be influenced by taxation. This would hold in particular if the firm were subject to cash-flow taxation, and the individual tax rates were progressive. Then tax policy could be used to encourage internal investment in the labour-managed firm.

APPENDIX

Following the procedure used by e.g. Dixit and Sandmo (1977), Atkinson and Stiglitz (1980, ch. 13) and Sandmo (1983), we formulate the government's maximization problems by means of Lagrange expressions, where θ_P and θ_L are Lagrange multipliers:

$$(A1) \text{ PM: } L^P = W[v_P^1(w^1(1-t_P), \pi(1-t_P), a), \dots, v_P^N(\cdot)] \\ + \theta_P [\sum_i (\lambda^i w^i + \alpha^i \pi) t_P - Na - \bar{T}]$$

$$(A2) \text{ LM: } L^L = W[v_L^1(Y(1-t_L)\gamma^1, a), \dots, v_L^N(\cdot)] \\ + \theta_L [\sum_i \lambda^i \gamma^i Y t_L - Na - \bar{T}]$$

Using the property $\frac{\partial \pi}{\partial t_P} = (Pf_L - w) \frac{\partial L}{\partial t_P} = 0$ when the PM-firm allocates labour optimally, we can write the first order conditions when (A1) and (A2) are maximized w.r.t. t_L , t_P and a as:

$$(A3a) \quad \frac{\partial L^P}{\partial t_P} = - \sum_i W_i \frac{\partial v_P^i}{\partial w^i(1-t_P)} w^i - \sum_i W_i \frac{\partial v_P^i}{\partial \pi(1-t_P)} \pi + \theta_P [\sum_i \frac{\partial \lambda^i}{\partial t_P} w^i t_P \\ + \sum_i \lambda^i w^i + \sum_i \frac{\partial \alpha^i}{\partial t_P} \pi t_P + \sum_i \alpha^i \pi] = 0$$

$$(A3b) \quad \frac{\partial L^P}{\partial a} = \sum_i W_i \frac{\partial v_P^i}{\partial a} + \theta_P [\sum_i \frac{\partial \lambda^i}{\partial a} w^i t_P + \sum_i \frac{\partial \alpha^i}{\partial a} \pi t_P - N] = 0$$

$$(A4a) \quad \frac{\partial L^L}{\partial t_L} = \sum_i W_i \frac{\partial v_L^i}{\partial Y(1-t_L)\gamma^i} (-\gamma^i Y + \gamma^i(1-t_L) \frac{\partial Y}{\partial \lambda^i} \frac{\partial \lambda^i}{\partial t_L}) \\ + \theta_L [\sum_i \frac{\partial \lambda^i}{\partial t_L} \gamma^i Y t_L + \sum_i \lambda^i \gamma^i \frac{\partial Y}{\partial \lambda^i} \frac{\partial \lambda^i}{\partial t_L} t_L + \sum_i \lambda^i \gamma^i Y] = 0$$

$$(A4b) \quad \frac{\partial L^L}{\partial a} = \sum_i W_i (\frac{\partial v_L^i}{\partial Y(1-t_L)\gamma^i} \frac{\partial Y}{\partial \lambda^i} \frac{\partial \lambda^i}{\partial a} \gamma^i(1-t_L) + \frac{\partial v_L^i}{\partial a}) \\ + \theta_L [\sum_i \frac{\partial \lambda^i}{\partial a} \gamma^i Y t_L + \sum_i \lambda^i \gamma^i t_L \frac{\partial Y}{\partial \lambda^i} \frac{\partial \lambda^i}{\partial a} - N] = 0$$

In (A3) and (A4) $W_i = \frac{\partial W}{\partial v^i}$ reflects each worker's weight in the social welfare function. Note that y is endogenous, so that any change in ℓ may change y as well. However, when there is no profit, i.e. $\pi = 0$, and (locally) constant returns to scale prevail, we find that $\frac{\partial y}{\partial \ell^i} = 0 \forall i$, and the two sets of equations turn out to be identical.

(i) Homogeneous Workers

a) PM-worker

We rewrite (A3a, b) as

$$(A3a') - \lambda \ell w - \lambda \alpha \pi + \theta \left(\frac{\partial \ell}{\partial t_p} w t_p + \ell w + \alpha \pi \right) = 0$$

$$(A3b') \lambda + \theta_p \left(\frac{\partial \ell}{\partial a} w t_p - 1 \right) = 0$$

Note that $\frac{\partial \alpha}{\partial t_p} = \frac{\partial \alpha}{\partial a} = 0$ when the workers are identical.

By multiplying (A3b') with $(\ell w + \alpha \pi)$ and adding it to (A3a'), we obtain

$$\theta_p \left(\frac{\partial \ell}{\partial t_p} w t_p + \frac{\partial \ell}{\partial a} w t_p \ell w + w t_p \frac{\partial \ell}{\partial a} \alpha \pi \right) = 0$$

Rewriting this we get (27).

b) LM-worker

Set $\gamma^i = 1$, and rewrite (A4a,b) as

$$(A4a') \lambda \ell (-Y + (1-t_L) \frac{\partial Y}{\partial \ell} \frac{\partial \ell}{\partial t_L}) + \theta_L \left(\frac{\partial \ell}{\partial t_L} y t_L + \ell \frac{\partial Y}{\partial \ell} \frac{\partial \ell}{\partial t_L} t_L + \ell Y \right) = 0$$

$$(A4b') \lambda (1-t_L) \frac{\partial Y}{\partial \ell} \frac{\partial \ell}{\partial a} + \lambda + \theta_L \left(\frac{\partial \ell}{\partial a} y t_L + \ell t_L \frac{\partial Y}{\partial \ell} \frac{\partial \ell}{\partial a} - 1 \right) = 0$$

We multiply (A4b') by λy and add to (A4a').

Define $\varepsilon = \frac{\partial y}{\partial \lambda} \frac{\lambda}{y}$ and use the Slutsky equation to obtain

$$(A5) - S_Y^L Y \{ \lambda \varepsilon + t_L [\theta_L + \varepsilon(\theta_L - \lambda)] \} = 0$$

From (A5) we get (28).

(ii) Heterogeneous Workers

a) PM-workers

In the first order conditions (A3a,b) we use the properties of the indirect objective function stated in the text to obtain

$$(A6a) - \sum_i W_i \lambda^i \ell^i w^i - \sum_i W_i \lambda^i \alpha^i \pi + \theta_p \left[\sum_i \frac{\partial \ell^i}{\partial t_p} w^i t_p + \sum_i \frac{\partial \alpha^i}{\partial t_p} \pi t_p \right. \\ \left. + \sum_i (\ell^i w^i + \alpha^i \pi) \right] = 0.$$

$$(A6b) \sum_i W_i \lambda^i + \theta_p \left(\sum_i \frac{\partial \ell^i}{\partial a} w^i t_p + \sum_i \frac{\partial \alpha^i}{\partial a} \pi t_p - N \right) = 0.$$

Multiply (A6a) by $\frac{1}{N}$ and (A6b) by $\frac{\sum_i (\ell^i w^i + \alpha^i \pi)}{N^2}$, and add the two equations. We rearrange and use the definition of covariances saying that

$$- \sum_i W_i \lambda^i \ell^i w^i \frac{1}{N} + \sum_i W_i \lambda^i \sum_i \ell^i w^i \frac{1}{N^2} = - \text{cov} (W_i \lambda^i, \ell^i w^i),$$

and likewise for $\text{cov} (W_i \lambda^i, \alpha^i)$ and $\text{cov} \left(\frac{\partial \ell^i}{\partial a} w^i, w^i \ell^i \right)$. We also use the Slutsky equation in the text to obtain an expression in terms of substitution effects and income effects. Then we can derive:

$$\begin{aligned}
 (A7) \quad & - \text{cov} (W_i \lambda^i, \ell^i w^i) - \pi \text{cov} (W_i \lambda^i, \alpha^i) + \theta_p t_p [\overline{S_w^i w^i} + \overline{S_\alpha^i \pi}] \\
 & - \text{cov} \left(\frac{\partial \ell^i}{\partial a} w^i, w^i \ell^i \right) - \text{cov} \left(\frac{\partial \alpha^i}{\partial a}, \alpha^i \right) \pi^2 + \frac{\partial \ell^i}{\partial a} w^i \cdot \overline{\alpha^i \pi} \\
 & + \frac{\partial \alpha^i}{\partial a} \pi \cdot \overline{\ell^i w^i} = 0,
 \end{aligned}$$

where

$\overline{S_w^i w^i} = \sum_i S_w^i w^i \frac{1}{N}$ is the average substitution effect of labour supply functions. Other average substitution and income effects, and average wage and share dividend, are defined in a similar manner.

Now (A7) can be solved for t_p to yield (30).

b) LM-workers

The condition (31) for optimal taxation of LM-workers is derived in the same manner. We multiply (A4a) by $\frac{1}{N}$, (A4b) by $\sum_i \ell^i \gamma^i Y \frac{1}{N^2}$ and add the two expressions. We rearrange terms and use the definitions of covariances and average substitution and income effects as illustrated above to obtain

$$\begin{aligned}
 (A8) \quad t_L^{**} &= \frac{1}{B} \left[\text{Cov}(W_i \lambda^i, \ell^i \gamma^i) - \overline{W_i \lambda^i \gamma^i \varepsilon^i S_Y^i} \right. \\
 &\quad \left. + \text{Cov}(W_i \lambda^i \gamma^i \varepsilon^i Y \frac{\partial \ell^i}{\partial a}, \gamma^i \ell^i) \right]
 \end{aligned}$$

where

$$\begin{aligned}
 B &= \text{Cov}(W_i \lambda^i \gamma^i \varepsilon^i \frac{\partial \ell^i}{\partial a}, \gamma^i \ell^i) Y - \overline{W_i \lambda^i \gamma^i \varepsilon^i S_Y^i} \\
 &+ \theta_L \left[\overline{\gamma^i S_Y^i} + \overline{\gamma^i \varepsilon^i S_Y^i} - \text{Cov}(\gamma^i \frac{\partial \ell^i}{\partial a}, \ell^i \gamma^i) Y - \text{Cov}(\gamma^i \varepsilon^i \frac{\partial \ell^i}{\partial a}, \gamma^i \ell^i) Y \right]
 \end{aligned}$$

which can be simplified to yield (31).

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THE FINANCE OF A LABOUR-MANAGED FIRM.
AN EXAMPLE OF CREDIT RATIONING.

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0. Introduction

In the seminal paper by Ward (1958), the author assumes that the labour-managed firm is completely externally financed. In later contributions, notably Vanek (1970, 1977a), it has been argued that this is a necessary as well as a desirable property of a firm where the workers are assumed to participate and manage the firm as workers rather than as capital suppliers. 100% external finance at a fixed capital cost (rate of interest) creates problems. Firstly it may induce a high demand for credit in the economy. Furthermore, the workers may consider it suboptimal to take on all risk themselves, Vanek (1977b), and bankruptcy risk will affect financiers' willingness to supply capital, McCain (1977). This calls for risk sharing finance from outsiders. Ideally this should occur without reducing the workers' control of the firm. However, because of control problems agency costs are likely to be high, see Jensen and Meckling (1976), and it is a question whether the workers will prefer to trade off control as against reduced capital costs. But then the workers accept vote carrying shares held by outsiders, as a means to obtain sufficient risk-sharing and fixed interest rate loan finance. The problem can be represented as optimization under credit rationing. The workers choose composition and degree of risk sharing finance. Then they are able to obtain a given, desired degree of fixed interest rate loan finance.

In addition the firm will often face an ordinary credit rationing due to lack of capital in the economy. Owing to high demand for debt finance, labour-managed firms may be particularly severely affected.

In this paper I will investigate a model of a labour-managed firm which is rationed in the capital market in the sense that it is restricted in choice of financial position. Exclusive fixed interest debt finance is ruled out. The model is intended to represent some features which may be important to a labour-owned firm operating in a classical capitalist environment.

In the next section I will establish how it is that credit rationing is an important issue when discussing the finance of a labour-managed firm. Possible solutions to the problem will be discussed. Then in Section 2 I present the model to be used. Its main feature is that the firm can issue shares, and that a minority of these shares can be held by outsiders. In Section 3 I analyse the allocation of capital under credit rationing. The firm can obtain a given level of risk sharing finance through internal investment in owned assets, external shareholding and recruitment of new workers. Degree of internal and external finance (shareholding) depend on the credit ration; the degree to which the workers are willing to relinquish control, and the workers' opportunity cost of capital. All workers in the firm own an equal number of shares. Therefore the employment decision will directly affect the capital level, and the employment level will generally differ from that of the Illyrian firm. Allocation of labor is discussed more generally in Section 4. With reference to Miyazaki (1984) it is shown that a possible dilution of the firm as labour-owned and labour-managed is affected by credit rationing. Section 5 is devoted to a discussion of how workers may choose degree of outsiders' participation, while I in section 6 briefly investigates how willingness to undertake internal finance is affected by the workers' attitude to risk. In Section 7 I discuss some aspects concerning the subsidization of a labour-managed firm. In particular I argue that the government can subsidize the firm by supplying capital at a rate below or equal to the market (risk free) interest rate. This should be considered in the light of the alternative employment prospects of the workers, by carrying through a cost benefit analysis.

1. CREDIT RATIONING OF LABOUR-MANAGED FIRMS.

There is a general consensus in the literature that proper financial institutions are essential if a labour managed firm shall be able to obtain an efficient allocation of its resources. The classical labour-managed firm, Ward (1958) and Vanek (1970), is assumed to be completely externally financed. If internally financed investments are undertaken, there is no private ownership of the capital. On the background of the social ownership of capital, and with further reference to special Yugoslav institutions, notably the requirement to maintain the book value of an investment in perpetuity, Furubotn and Pejovich (1970) and Vanek (1977a) show that the self-financed labour-managed firm suffer from cronical underinvestment, often termed the Furubotn-Pejovich effect. The answer to this problem is always to apply external finance, as the cost of borrowing to undertake an investment will generally fall below the cost of financing the project from internal sources. These problems concerning internal finance are more closely discussed in a literature review in another chapter of this thesis.

The possibility of obtaining 100 % external finance is however questioned by some authors, and Furubotn (1980) argues that except when loans are given in perpetuity, i.e. the principal need not be repaid as long as the book value of the capital stock is kept intact, the required internal rate of return will still exceed the market interest rate. Jensen and Meckling (1979) argue that pure rental is impossible mainly because there will be difficulties in obtaining funds for investment in intangibles (education, organizational development). The problems caused by uncertain market prospects are perhaps even more significant. Due to the "law of increasing risk", Kalecki (1937), i.e. the bankruptcy risk, borrowers will demand an increasing interest rate as the gearing ratio increases (McCain (1977)). Capitalist firms can avoid this problem by issuing shares, while this is more problematic in a system where firms are supposed to be self-managed. Furthermore, the workers may find it suboptimal to bear all risk through the fluctuation of wage income, which

is a reason to introduce outsiders who are willing to share in the risk (Vanek (1977b)).

This has led both Vanek and McCain to suggest systems of risk participating bonds. The bonds are issued to outsiders and/or insiders. But they carry no voting rights. By tying the return on the bonds to the workers' wage dividend, the maximand of the firm, it is assumed that the bond holders can be sure of receiving a rate of return reflecting the performance of the firm.

Vanek suggests that the bonds should be supplied by a "National Labor Management Agency". Using a similar idea, Putterman (1984) argues that a system of labour-managed production firms and financial institutions, devoid of the bureaucratic inefficiencies of the Yugoslav economy, will produce a capital market where returns are equalized and an equilibrium is established. Then the capital costs will reflect agency costs in the same manner as discussed by Jensen and Meckling (1976), and the obtained equilibrium cannot be denoted suboptimal.

Whatever the actual institutions look like, a labour management equilibrium seems to require a well established financial infrastructure. Then the problem remains how the existing Western financial system can be converted into a desired scenario. Attempts have been made, like the Mondragon in Spain (see Bradley and Gelb (1982)). But there seems to be no generally observable tendency of conversion of Western capitalist institutions. This means that the scattered labour-managed firms will have to operate within the framework of a traditionally capitalist structure. Thus, although problems of control and confidence which are the basis for this paper, may be present in a pure labour-managed economy as well, they will most likely cause problems for labour-managed firms operating in an economy consisting of traditional capitalist (profit-maximizing) institutions.

The confidence problem exists between insiders and outsiders, as insiders have got the exclusive right to make allocational decisions. McCain (1977) argues that there should be no reason for

outsiders to worry, as long as the firm is maximizing a payout measure to which the outsiders' return is coupled. However, the workers are not primarily concerned with what the maximand looks like. What matters is the utility (income) they get from their participation. In a sense the outsiders and insiders have opposing interests, and the workers may in different ways redefine the maximand if that is appropriate. On the other hand, this means that the existence of outside bond holders represents a restriction on the workers' behaviour if they have to stick to some given rule. Furthermore, it is reasonable to assume that this outside interference in defining the maximand will increase as their part of finance increases. But this is the same as saying that the outsiders own shares carrying voting rights, not only non-voting bonds. In this paper I shall model this voting right explicitly. But I assume that it is used merely to secure a satisfactory payout.

There is another source of outside interference which may be just as important. Following Stiglitz and Weiss (1981), we can argue that credit rationing may be used by the banks in order to screen and discipline their customers. By supplying credit at an interest rate below the market clearing rate to some borrowers, or using other instruments affecting the effective interest rate in a discriminating way, there will be a queue of borrowers, and the financiers can require certain conditions to be fulfilled for being willing to grant a credit (see Lommerud (1985), p. 123-4). Because of the labour-managed firms' high costs of internal finance, they may be willing to pay a high interest rate on borrowings. With imperfect (asymmetric) information, credit suppliers do not know what is the real risk associated with a project. The lenders may use the interest rate which borrowers is willing to pay, as a signal of riskiness of the project. Then willingness to pay is used as a screening mechanism in the credit market. Stiglitz and Weiss argue that the customers with the highest willingness to pay may be refused a loan as a result of this screening.

On the background of this, I argue that a labour-managed firm operating in a capitalist economy will be credit rationed. This will affect the performance of the firm, and measures have to be found, which can reduce the problem. One procedure to follow is

to let outsiders participate with risk sharing capital. The outsiders will, however, exercise some supervision and influence decisions. But then capital is supplied de facto by means of vote carrying shares.

I find it reasonable that the outsiders will use their influence to guarantee themselves a minimum share dividend. Thereby they get a preference in payment decisions which reduces their risk from participation. Simultaneously it reduces the workers' possibilities of increasing their payout at the expense of the outsiders by different forms of manipulations. As seen from the workers' point of view, it should be considered the price they will have to pay for a certain degree of self control.

Unless the outside interference is some governmental body granting a subsidy (see Section 5, and Bradley and Gelb (1980)), we will assume that the outsiders will maximize the yield on the capital invested. They will accept a state-contingent payment. But the workers, who have the voting majority, should take on a larger part of the risk. This happens when the workers are bound by a minimum share dividend. The minimum share dividend must of course be agreed upon in advance, as the point of time when the investment takes place is the only moment when the minority group can influence decisions by exploiting the fact that the workers need (want) their participation. This takes into consideration the fact that once the capital is invested (i.e. the shares are bought), the outsiders can sell their shares to other outsiders, but they cannot require the shares redeemed.

The existence of a minimum share dividend is well known from theory on the managerial firm, see e.g. Yarrow (1976). In that case the conflict is between wealth-maximizing shareholders and the utility maximizing management. Profit can be absorbed as managerial slack or emoluments. This may provoke intervention from the shareholders. But there is a minimum share dividend which will prevent them from intervening. The minimum share dividend is a choice variable in the management's optimization,

whereas it is here treated as exogenous, given a state of nature.

In addition to a minimum amount, the dividend on shares may, and probably it will, depend on wage-dividend paid to the worker-owners. Then McCain's (1977) and Vanek's (1977b) rules emerge as special cases. Such a rule is a possible candidate for solving the problem of confidence between the two opposing groups. The minimum dividend will generally exceed the market interest rate because of credit rationing and the fact that the workers are dependent on the outsiders. Or, stated slightly differently, if increased debt finance were to take place, this would take place at increasing borrowing costs. It should be noted, however, that it is profit after the deduction of labour costs valued at the market wage rate which matters for the outsiders when assessing profitability of the firm. On the other hand, this can easily be taken explicitly into account in our formula, as share dividend can be made increasing in the excess of wage dividend above the relevant market wage rate.

As shares are negotiable in the sense that insiders can sell to other insiders (e.g. newcomers) the Furubotn-Pejovich problem is not existent. Those workers will reap the advantage from an investment decision, who are engaged at the point of time when the decision is made. This is independent of how long a time they stay with the firm, as long as the shares correctly reflect the value of the firm. The informational problems connected with this are not trivial to solve. But it will always be possible to find a price which is acceptable when shares are transferred, e.g. from leaving members to newcomers, through use of correct incentive-compatible mechanisms.

We will assume that the firm can borrow proportionally to its share capital. This is reasonable, as capital consists of tangibles as well as intangibles. We conjecture that it is easier to borrow for investment in machinery and fixed equipment than for investment in knowledge and human capital. Furthermore, according to the "law of increasing risk", the interest rate will increase as debt increases in proportion to capital (gearing ra-

tio). This last point is analysed by McCain (1977). We assume that the exogenous gearing ratio is sufficient to enable the firm to obtain debt at a given interest rate (nothing of importance is lost by making the interest rate exogenous).

From the above, we see that the firm faces two constraints: that on external borrowing and that on external shareholding. However, as it will become clear later, the truly crucial constraint is that on borrowing. External shares are introduced to provide sufficient capital only. The cost of this external share finance is the possible excess dividend above the market interest rate that has to be paid to outsiders. The extent of the external ownership can be made endogenous instead, attaching utility too to the degree of shares held by the insiders (the workers). This will be briefly considered in subsection 3(i).

In the short run membership should be considered constant. It is difficult to recruit new members as these will have to raise some capital. Symmetrically it will be very difficult to dismiss workers who have invested some of their private funds in the firm. Furthermore, we know from the theory on the labour-managed firm that if the members are to be treated equally, i.e. dismissals are to be enforced randomly or dismissed workers are to be compensated, employment is constant if workers are not risk-lovers (Steinherr and Thisse (1979)). It seems unlikely also that the outsiders can force the majority to vote for changes in membership that it does not want, except in special situations.

The problem is, according to McCain (1977), that the pure rental labour-managed firm faces credit rationing unless reliable financial instruments can be created. This is however a difficult task if capital suppliers are not to be given any voting rights (Jensen and Meckling (1979)). On the other hand, as argued by Miyazaki (1984), it will pay in some circumstances to reorganize a bankrupt capitalist firm as a worker cooperative. The reason is that the workers' long term utility may be increased compared to the alternative employment prospects, or

even possible unemployment. Indeed, it is often in this case we observe the emergence of labour-managed firms. They are however in some cases not long lived. This may be due to the financial problems considered above. But Miyazaki shows that it may be optimal to reduce membership by substituting hired workers for worker-members if the firm does well. Then the firm will eventually convert into a capitalist firm. But as argued by Miyazaki imperfections in the capital market may affect the possible dilution of the firm as labour-managed.

Thus, in a capitalist market economy the optimal solution for a labour-managed firm may involve compromising with capital suppliers. In this paper I will discuss one possible way of compromising. It may be of interest when considering a socialist labour-managed economy as well.

2. THE MODEL

I shall investigate the functioning of a labour-owned and -managed firm, which is financed by shares and debt. The shares are procured from the workers as well as from outsiders. The workers have the voting majority, and we shall assume that the shares are equally distributed among the insiders. Outside shares must not exceed a certain limit (percentage of total number of shares). This limit is set by the workers themselves. Borrowing is restricted to a given percentage of share capital.

I shall consider different modes of financing the firm. The alternatives open to the workers are

- increase internal finance (retained surplus) by means of shares
- recruit new workers supplying vote-carrying share capital
- allow (more) outsiders supplying vote-carrying share capital
- increase borrowings.

The latter, however, is endogenously determined by an exogenously given gearing ratio.

The period of consideration - the production period - is the rest of the horizon as it is seen from the point of view of the initial workers, i.e. it is the remaining employment period for the workers who are employed at the beginning of the production period.

In the discussion in Section 4, I shall assume also that there is a spot labour market where the firm can hire workers at a given market wage rate.

The price of the firm's product is dependent on uncertain market conditions. It is thereby to be considered a random variable with a known distribution. The firm produces one product. The price of shares is assumed equal to unity, and there is no depreciation of the capital.

Notation:

- X : quantity of output.
P : price of output.
y : wage dividend to the worker-owners.
 π : share dividend (amount per share).
r : rate of interest on debt.
B : borrowings.
b : limit on loans as a percentage of total stock of shares.
K : capital.
 \bar{A} : initial total shareholding
A : share capital, number of shares at price 1 in the production period.
 a^I : number of shares bought by each worker at the beginning of the production period (retained surplus).
 a^E : number of shares invested by outsiders.
 a^N : number of shares bought by new worker-members (exogenous)
 \bar{M} : number of initial worker-members of the firm.
m : new workers employed, $m \geq 0$.
N : total number of full-time workers engaged by the firm.
n : number of hired workers.
w : market wage rate, to be paid to hired workers.
 α : internal discount factor.
 β : minimum share dividend.
 $\bar{\delta}$: maximum percentage of share capital that can be held by outsiders.

Additional notation will be defined in due course.

I shall present the initial workers' (the majority group's) optimisation problem. The influence of outsiders is introduced as a restriction on the payment of share dividend (π). The firm maximizes wage dividend per worker for any value of π chosen. The choice variables are a^I , m, a^E and π .

We can establish the subsequent basic relations and definitions:

$$(1) \quad y = \frac{PX - rB - \pi A}{\bar{M} + m}$$

This is the payout to every full-time worker in the production period. As we restrict attention to full-time workers, payment depends on membership only, and it is equalised among the permanent work-force.

According to (1) the workers will for every choice of π receive a wage dividend y . As there may be outsiders who possess shares, the workers are not indifferent to relative payout rates. Distributing surplus according to share holdings means that also outsiders receive a part of total value added which would otherwise have accrued to the workers. Therefore share dividend is a choice variable which will be fixed so that wage dividend is maximized. But, as we shall see below, there are restrictions on the choice of π , so that generally it is not zero.

If the initial workers are to invest more capital in the firm (retain surplus), they supply each an equal number of shares, a^I . In addition to the initial shareholding \bar{A} , whose distribution we do not consider, total share capital in the production period is made of capital supplied by initial workers, new workers and external financiers (outsiders). Risk taking capital, share capital A , is then:

$$(2) \quad A = \bar{A} + \bar{M}a^I + ma^N + a^E$$

Total capital consists of shares and borrowings, B . All capital invested is used in production, so that

$$(3) \quad K = A + B$$

The credit rationing, i.e. maximum possible, or desired, fixed interest rate debt finance, takes the form of an exogenously determined gearing ratio. We write this as

(4) $B \leq bA$.

We will assume that the restriction in (4) binds. The reason may be "classical" credit rationing. But it may, as discussed above, also reflect the workers' optimal choice of degree of fixed interest rate debt finance.

There is a lower limit on share dividend. The limit is fixed in bargaining between insiders (initial workers) and outsiders. Generally there may be any coalition among shareholders which may influence the minimum share dividend denoted β . In particular this is likely to happen if the workers' shares are unequally distributed. That problem, however, is relegated to another article of this thesis.

We do not consider the bargain to determine minimum share dividend β . It is taken for granted that the workers and the outsiders manage to agree on a rule for the distribution of surplus.

The insiders will reduce the outsiders' yield as much as possible. But the outsiders would not have participated, were it not for two reasons, viz. that the workers need them and that the expected outcome is acceptable given the investors' opportunity cost. We have argued previously that this is above r . Then the outsiders will demand $\beta > r$. Market conditions must however be taken into account as well. They are reflected by the random variable P . The higher P is, the better are the market conditions, and the higher dividend will be claimed. But wage dividend y is increasing in the product price, and its value will thereby reflect the market prospects. Although the workers may manipulate the wage dividend, a functional relationship which will satisfy the outsiders can be established in negotiations and by use of proper control mechanisms whose costs the workers have to cover. Thus we will assume that given the market interest rate and any realised state of nature, β (and π) may vary with y according to an agreed upon rule. One example of such a rule is given in McCain (1977). Another example is found in Bradley and Gelb (1980) who instead use a maximum wage divi-

dend. We will not stick to any fixed formula here. Instead we interpret the relationship between β and y , together with the minimum level of β , see below, as reflecting the bargaining power of the two parties.

Making β dependent on y means that its determination is endogenised. The workers are maximizing y for every value of π . The outsiders will demand a share in surplus. Their share is through negotiation given by the relationship between β and y . Thus, the workers decide on wage and share dividend, given the outsiders' reactions to different levels of wage dividend. If the outsiders feel that this does not give them a sufficient share, or that they are being exploited, they have got the right to renegotiate conditions or withholding their supply of capital. This takes care of the control aspect. We write the determination of share dividend as

$$\beta = \beta(y; r),$$

where $\beta_y = \frac{\delta\beta}{\delta y} \geq 0$. Since the outsiders participate only when their expected return, $E\pi$, exceeds the market interest rate, we will have that

$$\beta_{\min} > r$$

unless in very bad states of nature where π may drop below r .

We depict the relationship between y and β in Figure 1.

By making shares negotiable, capital costs will decrease as the principal - i.e. the original amount of capital invested - can be repayed at any time. Losses incurred by issuing negotiable shares can be grouped into two classes. If egalitarianism matters, the fulfilment of that goal may be impeded. That problem will not be considered here. The other class of problems arising relates to the discussion above concerning shares held by outsiders. The workers have to pay some of the firm's surplus to non-members. I consider the determination of β

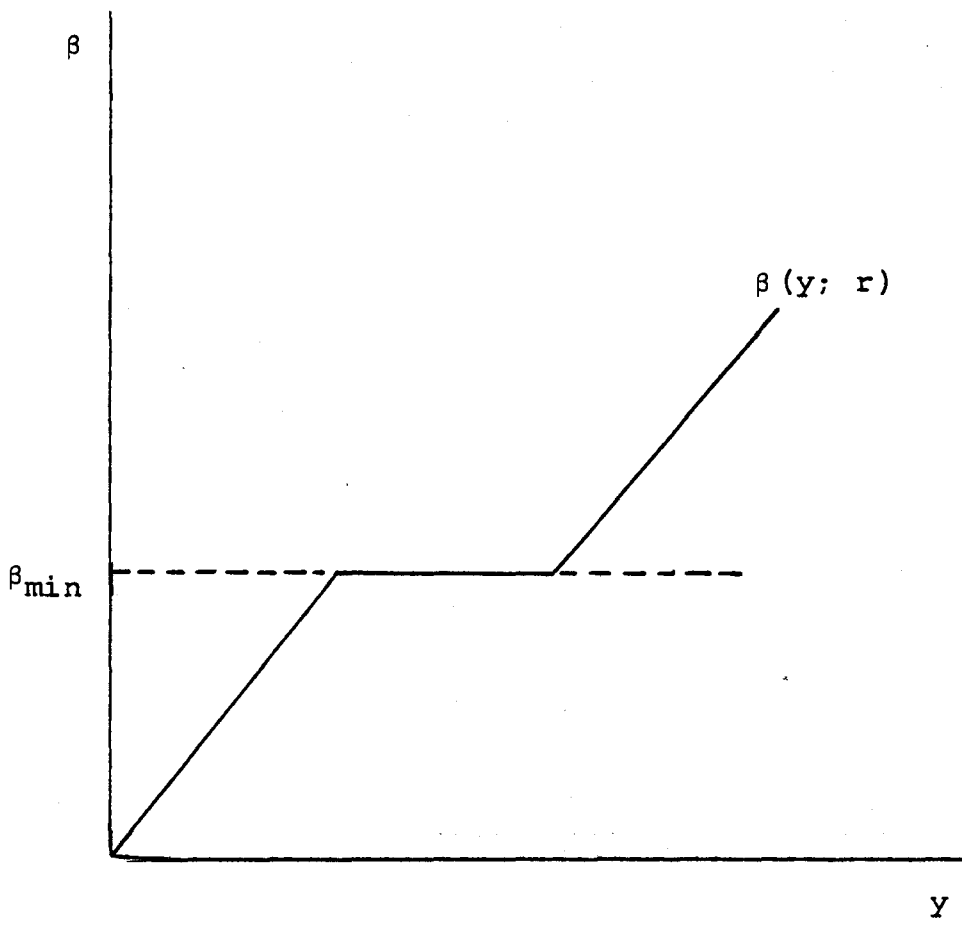


Figure 1: A possible relationship between the minimum dividend and wage dividend.

to depend on the degree of outsiders' participation. Thus, allowing outsiders to share in surplus, also means that they influence the allocational decisions of the firm. Therefore, the functional relationship between share dividend (π) and wage dividend (y) holds for a given degree of outsiders' control, as measured by the part of share capital they control. This maximum outside control, δ , is determined (exogenously) by the workers. Behind all this, the workers have traded off control as against income. This will be discussed more closely in Section 5.

Then we have that

$$(5) \quad \pi \geq \beta(y; \bar{\delta}, r)$$

It follows that a desired level of control by the workers is guaranteed by restricting the outside shares not to exceed a limit $\bar{\delta}A$ of total shares:

$$(6) \quad a^E \leq \bar{\delta}A$$

In addition to the behavioural restrictions, we have the following non-negativity restrictions:

$$(7a) \quad a^I \geq 0$$

$$(7b) \quad m \geq 0$$

$$(7c) \quad a^E \geq 0$$

Total employment level is

$$(8) \quad N = \bar{M} + m.$$

The relationship between the input factors labour and capital and the output is described by a production function $f(K,N)$ so that

$$(9) \quad X = f(K,N)$$

We assume positive marginal productivities i.e. $f_K = \frac{\delta f}{\delta K} > 0$ and $f_N = \frac{\delta f}{\delta N} > 0$, and a concave production function. It is assumed also that $f(0,N) = 0$

The workers have the voting majority. This is obtained through ownership of a majority of the shares. They may also enjoy codetermination (voting rights) as workers¹⁾.

We shall consider the workers' maximization problem as presented above. As the market conditions are uncertain, we must make assumptions concerning the workers' attitude to risk also. It is reasonable to assume that each worker is risk averse. This will affect his willingness to invest in the firm and his supply of labour. The individual attitude to risk will be the subject of a subsequent paper, and we consider here the workers' collective attitude to risk only.

I shall assume that the firm (i.e. the workers as a collective) is risk neutral. This will simplify the analysis, and enable us to concentrate on the main issue, which is to single out the sources of finance that the workers will find it optimal to use. Because of limited diversification possibilities of the owners of the firm compared to the owners of a capitalist firm, the assumption is not innocent. A justification may be that the risk is spread among all workers and that each project is considered marginal. Furthermore, the workers are able to spread the risk in deciding on their product range (see e.g. Ireland and Law (1982), ch. 7). If so, P and X should be considered vectors of prices and quantities. Also it is reasonable to believe that it is mainly through the individual supply decisions that the workers' possible risk aversion may play a crucial role (see section 3(i)). Lastly, if the maximization is carried through by a management, the management will generally not know the preferences of the workers. Therefore, the management may reasonably assume that any hedging is done individually, leaving it to maximize the expected income.

Then, if we assume risk neutrality, the management of the firm will maximize expected income per worker. Let the variables in the production period denote expected values, and we can write the maximand as

$$(10) \quad \text{Max} \left[(y + \pi a^I) \frac{1}{1+\alpha} - a^I \frac{\alpha}{1+\alpha} \right]$$

(10) is maximized w.r.t. a^I , a^E , m and π by using the definitions (1) - (3) and (8), the non-negativity restrictions in (7) and the restrictions (4) - (6) and (9).

We will assume that the credit rationing is binding. Then we may substitute bA for B in (4). The capital to be used may be written as:

$$(3') \quad K = (1+b) \left[\bar{A} + a^I \bar{M} + a^N m + a^E \right]$$

By making substitutions in (10), the Lagrange-function simplifies to:

$$(11) \quad L = \frac{1}{1+\alpha} (y + a^I \pi) - \frac{\alpha}{1+\alpha} a^I + \gamma [\pi - \beta(y; r, \bar{\delta})] - \theta (a^E - \bar{\delta}A)$$

γ and θ are Lagrange multipliers. γ reflects the restriction on share-dividend, imposed by outsiders. The workers' utility (payout) is decreasing in π (or β) whenever $a^E > 0$. This is easily seen from the definitions of wage and share dividend: A marginal increase in y gives each worker $\frac{1}{\bar{M}+m}$ th of the additional income.

By increasing π instead, each (homogeneous) worker receives $\frac{a^I}{A} < \frac{1}{\bar{M}+m}$ as $A < (\bar{M}+m)a^I$ when $a^E > 0$. However, note that this result does no longer hold if ownership rights are distributed in an inegalitarian manner. Thus, we will have $\pi = \beta$, and $\gamma \stackrel{>}{=} 0$ as the workers' income (utility) is decreasing in β .

θ measures how strongly the restriction on outside shareholding is binding. If $a^E < \bar{\delta}A$, θ is equal to zero. On the other hand, if the restriction binds, it means that the workers could increase their income by increasing δ . The reason why they do not enforce such a change is obviously the fact that they are willing to pay for a certain degree of control. But as to the maximization of income, obviously income is lower the lower is δ , as long as the credit ration binds. The sign of θ is discussed below.

(11) is maximized w.r.t. a^I , a^E , m and π . The Kuhn-Tucker first order conditions may be written as:

$$(12a) \quad \frac{\partial L}{\partial a^I} = 0 \Rightarrow \left\{ [Pf_K(1+b) - rb - \pi] \cdot \frac{1-\gamma\beta_Y(1+\alpha)}{(\bar{M}+m)(1+\alpha)} \cdot \bar{M} + \frac{\pi-\alpha}{1+\alpha} + \theta\bar{\delta}\bar{M} \right\} a^I = 0$$

$$(12b) \quad \frac{\partial L}{\partial m} = 0 \Rightarrow \left\{ [(Pf_N - Y) + (pf_K(1+b) - rb - \pi)a^I] \cdot \frac{1-\gamma\beta_Y(1+\alpha)}{(\bar{M}+m)(1+\alpha)} + \theta\bar{\delta}a^I \right\} m = 0$$

$$(12c) \quad \frac{\partial L}{\partial a^E} = 0 \Rightarrow \left\{ [Pf_K(1+b) - rb - \pi] \cdot \frac{1-\gamma\beta_Y(1+\alpha)}{(1+\alpha)(\bar{M}+m)} - \theta(1-\bar{\delta}) \right\} a^E = 0$$

$$(12d) \quad \frac{\partial L}{\partial \pi} = 0 \Rightarrow \left\{ -\frac{1-\gamma\beta_Y(1+\alpha)}{(\bar{M}+m)(1+\alpha)} A + \frac{a^I}{1+\alpha} + \gamma \right\} \pi = 0$$

The solution to the programme is given by (12a-d) together with (5) and (6). We assume that second order conditions are met.

Because of the nature of our problem (credit rationing), we can assume $\pi > 0$. Then (12d) holds by equality, and we find that

$$\gamma = \frac{A - a^I(\bar{M}+m)}{(1+\alpha)(A\beta_Y + \bar{M}+m)} \geq 0$$

$\gamma > 0$ if $a^E > 0$, and the restriction on share dividend will be binding. As the workers are homogeneous we will have $\gamma = 0$ when $a^E = 0$, since the workers are in that case indifferent as to the way payment is made. We can substitute for γ into the other first order conditions to obtain

$$\frac{1 - \gamma \beta_Y (1 + \alpha)}{(\bar{M} + m)(1 + \alpha)} = \frac{1 + a^I \beta_Y}{(1 + \alpha)(A \beta_Y + \bar{M} + m)} = C > 0,$$

which we shall use in our analysis.

We see that in addition to (the restricted) borrowings, the workers have three ways of financing their activity:

- (i) Retain surplus in the initial period (internal finance).
- (ii) Increase externally held shares.
- (iii) Increase the level of membership.

These means will again trigger off increased borrowings. In the next section I will investigate these three possibilities in turn. Afterwards I will look more closely at the conditions for changes in membership (Section 4) and choice of degree of outsiders' participation (Section 5).

3. ALLOCATION OF CAPITAL

(i) External Finance (a^E).

As noted above, we will assume that the credit rationing is binding. This amounts to saying that $Pf_K > r$ ²⁾. When $Pf_K = r$, the firm is able to obtain the credit it wants. As we assume that the workers prefer higher degrees of control to lower, they will not finance the firm by shares held by outsiders demanding a payout exceeding r (see below).

Let us now investigate the condition (12c) for allocation of externally held shares. The condition may be rewritten as

$$(12c') \left(Pf_K - \frac{\pi+rb}{1+b} \right) (1+b)C - \theta(1-\bar{\delta}) \leq 0, \quad a^E \geq 0, \quad \text{with at least one equality.}$$

Let us give an interpretation of θ . It may be termed the shadow price on outside held shares. It reflects what the workers are willing to pay for allowing outsiders share in the surplus of the firm. The sign of θ depends on degree of credit rationing and willingness to pay for the outsiders' financial contribution. Rewriting (12c') when $a^E = \bar{\delta}A$, i.e. outsiders' share finance is at its maximum, we find that

$$(13) \quad \theta = \left(Pf_K - \frac{\pi+rb}{1+b} \right) \frac{(1+b)}{(1-\bar{\delta})} \cdot C.$$

$\theta = 0$ if $0 \leq a^E < \bar{\delta}A$, otherwise non-zero. The shadow price is equal to the deviation of value marginal product of capital from the average price of capital multiplied by capital provided through increase in share supply and taking into consideration the relation between wage dividend and share dividend.

According to (13),

$$\theta \begin{matrix} > \\ < \end{matrix} 0 \text{ as } Pf_K \begin{matrix} > \\ < \end{matrix} \frac{\pi+rb}{1+b}.$$

Shadow price of outsiders' risk sharing contribution is positive, zero or negative as value marginal product of capital exceeds, is equal to or is smaller than an average price to be paid for each unit of money value of capital. This price is a weighted average of share dividend and the interest rate on borrowings, where the weight is $\frac{1}{1+b}$ for share capital and $\frac{b}{1+b}$ for borrowings. As the credit rationing becomes less severe, i.e. b increases, the average price of capital approaches r , the interest rate paid on borrowings:

$$\lim_{b \rightarrow \infty} \frac{\pi + br}{1+b} = r$$

With infinite b - no credit rationing - the price to be paid for a money value of capital is the market rate of interest. If outsiders demand a higher share dividend, there is no reason for the workers to issue shares to them.

Now, assume b finite and $r < Pf_K < \frac{\pi + br}{1+b}$, i.e. a binding ration. From (13) $\theta < 0$, which again implies $a^E = \bar{\delta}A$. There is a negative shadow price on shares, which means that the workers are interested in reducing outsiders' shareholding. We can interpret θ as the price the workers are willing to pay for a reduction in the maximum outside shareholding.

Given b , the credit rationing becomes more severe as Pf_K approaches the average price of capital $\frac{\pi + br}{1+b}$. When equality holds, $\theta = 0$, and $0 \leq a^E \leq \bar{\delta}A$. The restriction on outside shareholding is not binding, and the workers are indifferent to its level.

$\theta > 0$ if $Pf_K > \frac{\pi + br}{1+b}$, and the shadow price is thereby the price the workers are willing to pay for an increase in outsiders' share supply. The workers are able to increase income by increasing δ . If they do not do so, it means that this is a price they are willing to pay for the given degree of self management. In the objective function (11) the control aspect is

taken care of by the exogenous choice of δ , which represents a restriction the workers have chosen themselves. This choice will be further discussed in Section 5.

Outside share finance may be chosen as long as value marginal product of capital exceeds the market rate of interest on borrowings. Whether it will be chosen or not depends on the share dividend, or rather its excess over the rate of interest. When the marginal valuation of capital falls short of the weighted average capital price, the workers will consider a reduction in outsiders' maximum shareholding. In the reverse case they have a willingness to pay for its increase. When considering the valuation of share finance from outsiders, the borrowings made possible by changes in share capital and the outsiders' influence on distribution of surplus matter as well. Specifically, ceteris paribus we can show by partially differentiating the expression for θ in (13) with respect to a given β_y , i.e. how steep the β -function is in figure 1, that the shadow price is falling in β_y . We may interpret β_y as the influence exerted by the outsiders, or their aggressiveness in negotiations. θ is also decreasing in π .

(ii) Internal Investment (a^I)

The second source of increasing risk-sharing finance is internal investment in owned shares. It is equivalent to withholding surplus for investment in productive capital. The condition for this to occur is given by (12a). It may be rewritten as:

$$(12a') \left\{ (Pf_K - \frac{\pi+rb}{1+b}) (1+b) \bar{C}M + \frac{\pi-\alpha}{1+\alpha} + \theta \delta \bar{M} \right\} \leq 0,$$

$$a^I \geq 0, \text{ with at least one equality.}$$

If there is no credit rationing, b tends to infinity, and the average price of capital equals the interest rate paid on borrowings. The workers are indifferent to outside shareholding if the outsiders' accept a remuneration equal to the market rate of interest. Internal finance is dependent on the workers'

opportunity cost of capital, reflected by the value of α . If α exceeds π - the cost of providing capital is higher than the bank lending rate of interest, say because the workers are individually risk averse - then the workers will not choose internal finance. For $\alpha = \pi$ they are indifferent. We find that internal finance will be strictly positive for $\alpha < \pi$. Then the workers can provide capital cheaper than external financiers. In a situation without credit rationing this situation seems unlikely unless the workers are risklovers.

Let us investigate the conditions for increased finance from the initial workers more closely when the credit ration binds. We rewrite (12a') as

$$(14) \quad Pf_K \leq \frac{\pi+br}{1+b} - \frac{1}{c\bar{M}(1+b)} \left(\frac{\pi-\alpha}{1+\alpha} - \theta\delta\bar{M} \right)$$

$a^I = 0$ if inequality holds, whereas equality holds if $a^I > 0$.

Assume firstly no outside shareholding. Then $\theta = 0$. Let furthermore share dividend, π , be set equal to the opportunity cost of capital, α . If they exceed the market rate of interest, e.g. because of the workers' risk aversion with respect to their individual capital, then the average price of capital exceeds the borrowing rate of interest for a finite b (credit rationing). If so, internal finance will be undertaken until equality is brought about between value marginal product of capital and average price of capital. If the latter exceeds the former, no internal finance is undertaken.

It may be of interest to investigate how willingness to undertake internal finance depends on some parameters of the model, viz. the maximum outside shareholding (δ), the credit ration (b), the outsiders' share in surplus (β_Y -function) and internal opportunity cost of capital (α).

It is convenient to assume that $a^I = 0$ initially. Assume also that $m = 0$. The workers have considered whether external finance will be chosen. If the result of this consideration is that

value marginal product of capital exceeds the right hand side of (14), internal finance will be made. The r.h.s. of (14) represents the real capital costs. It is taken into consideration how internal finance affects the total financial position of the firm.

If an initial choice of $a^I = 0$ results in a value marginal product of capital above these real capital costs, income will be increased by choosing $a^I > 0$. Now, substitute for θ from (13), and denote the new right-hand side of (14) e. e will of course depend on all variables of the model. Then we can write (14) as:

$$(14') \quad Pf_K \leq \frac{\pi+rb}{1+b} - \frac{1-\bar{\delta}}{(1+b)c\bar{M}} \frac{\pi-\alpha}{1+\alpha} =$$

$$\frac{\pi+rb}{1+b} - e(\bar{\delta}, b, \beta_Y, \alpha, \dots) \equiv H$$

H reflects capital costs. Whether these exceed or fall short of what is previously denoted average price of capital, $\frac{\pi+b}{1+b}$, depends on the internal opportunity cost of capital, α . If it is initially below share dividend, i.e. $\pi > \alpha$, internal capital is cheaper than shares supplied by outsiders. Then, if the workers choose $a^E = 0$, π can be reduced to equality with α . The resulting capital cost is

$$\frac{\alpha+rb}{1+\alpha}$$

which exceeds r if b is finite and $\alpha > r$. If the initial choice of $a^I = 0$ gives a value marginal product of capital above this, internal finance will be undertaken before outside share finance is considered. See also subsection 3(iv).

However, the workers' opportunity cost for providing individually owned capital may be high. Investing in own firm limits diversification of (human and non-human) capital, while the

outsiders who invest probably are highly diversified. For $\alpha > \pi$ external share finance is cheaper than internal. The workers have already invested some capital, \bar{A} , in the firm. Will they increase their shareholding in this case, i.e. choose $a^I > 0$?

Obviously this depends on the capital shortage when a distribution over market prospects is known and any slack is exhausted. If $Pf_K > H$ with an initial choice of $a^I = 0$, then income is increased by investing more from owned resources. For given employment level, i.e. $m = 0$, decreasing marginal productivity of capital implies that capital level and production will be higher the lower is Pf_K . If initially $H < Pf_K$, internally financed investments will increase production and income accruing to the workers.

We can calculate how the value of H depends on the parameters mentioned by taking its partial derivatives. These are³⁾

$$\frac{\partial H}{\partial \delta} > 0, \quad \frac{\partial H}{\partial b} = ?, \quad \frac{\partial H}{\partial \beta_y} < 0, \quad \frac{\partial H}{\partial \alpha} > 0, \quad \text{if } \pi > \alpha$$

$$\frac{\partial H}{\partial \delta} = 0, \quad \frac{\partial H}{\partial b} < 0, \quad \frac{\partial H}{\partial \beta_y} = 0, \quad \frac{\partial H}{\partial \alpha} > 0, \quad \text{if } \pi = \alpha$$

$$\frac{\partial H}{\partial \delta} < 0, \quad \frac{\partial H}{\partial b} < 0, \quad \frac{\partial H}{\partial \beta_y} > 0, \quad \frac{\partial H}{\partial \alpha} > 0, \quad \text{if } \pi < \alpha.$$

It is assumed that $\pi > r$ and that $\beta_{yy} = 0$, cf. Figure 1.

I have made calculations for different values of π in relation to α . My comments are concentrated on the situation where internal discount factor exceeds share dividend rate.

The results reported above say that, for a maximum level of external share finance, capital level is more likely to be increased through internal finance - retained surplus - the higher is the accepted outsiders' participation and the credit

ration, and the lower is the opportunity cost of capital and the outsiders' influence on surplus distribution via the steepness of the minimum share dividend function. The results seem intuitively reasonable. High values of δ and b mean that the effects of internal finance will be large, as outsiders' share supply and borrowings can be increased accordingly to a high degree in a situation where capital shortage is urgent. Note that the sign of $\frac{\partial H}{\partial \delta}$ is reversed when share dividend rate exceeds the discount factor. This is so because we have assumed that the restriction on outsiders' shareholding binds. Increased internal finance also means increased external finance, which is less attractive when the price demanded by outsiders, π , exceeds the price the insiders will claim, α . Therefore, for $\alpha < \pi$, the restriction on maximum outside shareholding does not necessarily bind. For given employment level and when $\alpha > \pi$, this implies that production is increasing in δ , the maximum number of shares held by outsiders, and the credit ration b . It is decreasing in the internal opportunity cost of capital. Also, if the outsiders manage to negotiate a "tougher" relation between wage and share-dividend (a shift upwards in the β_y -function), the optimal capital level and production will be lower.

(iii) Recruitment of Members (m)

Let us now examine how the choice of membership level can affect the firm's financial position. The number of workers enjoying membership is assumed constant during the production period. Changes may occur at the beginning of the period. Recruitment will take place according to generally agreed upon procedures⁴⁾. In this subsection I will discuss how variations in membership may be used for affecting the financial position of the firm. In Section 4 I will investigate the optimal size of the firm and the composition of its work-force in a more general context. We rewrite (12b) as:

$$(12b') \left[(Pf_N - y) + (Pf_K - \frac{\pi+rb}{1+b}) (1+b)a^I \right] C + \theta \delta a^I \leq 0,$$

$m \geq 0$, with at least one equality.

The sign of θ depends on whether $Pf_K \stackrel{\geq}{<} \frac{\pi+rb}{1+b}$, see (12c'). Then (12b') implies that

$$(15) \quad \text{sgn} (Pf_N - y) = - \text{sgn} (Pf_K - \frac{\pi+rb}{1+b}) ,$$

provided there is an interior solution where $m > 0$, which we will assume for the time being. From (12c') we know that $Pf_K > \frac{\pi+rb}{1+b}$ if $\theta > 0$ and accordingly $a^E > 0$. This implies that the firm will recruit members to a point where the wage dividend, y , exceeds the value marginal product of labour. Of course this does not mean that the firm should employ members to a point where they extract more resources from the firm than they bring into it. Their total contribution and yield are dependent on the supply of capital as well as work. Thus, the workers will take into consideration the effect on finance also when deciding on the optimal level of membership. We see this more clearly by substituting for $\theta > 0$ from (12c') in (12b') and rearranging:

$$(16) \quad (Pf_N - y) + (Pf_K - \frac{\pi+br}{1+b}) a^I \frac{1}{1-\delta} (1+b) = 0$$

a^I is the number of shares that each worker contributes. Then $a^I \cdot \frac{1}{1-\delta}$ is the total increase in share capital made possible by an additional worker. Multiplying by $(1+b)$ gives total increase in capital from a change in the level of membership. We see that the excess of wage dividend above the value marginal product of labour is explained by the value of the marginal worker's induced capital contribution. The value of this contribution is again dependent on the credit ration the firm is able to obtain. Therefore, whether new members will be hired, depends, apart from the general market prospects, on how the credit market develops.

We note that the rule for the allocation of labour approaches the condition for labour allocation in the Illyrian firm as b increases. When the firm is no longer rationed in the credit market, the second term in (16) vanishes, and labour is allocat-

ed to a point where $Pf_N = y$. However, this presupposes that all capital can be borrowed, or that the workers are able to raise all funds internally at the market interest rate r . On the other hand, when the shares are negotiable, the horizon problem is not present. The reason why we may still find $\pi > r$ is risk-aversion, high individual costs of raising capital etc. From (16), when insiders own all shares, this situation will imply $Pf_N < y$. Thus, the capital-labour ratio will generally differ from that to be found in the 100 % externally financed Illyrian firm.

We cannot state unambiguously whether the level of membership will exceed or fall short of the level of membership in the Illyrian firm (see also section 4). This is so because changing the level of the permanent workforce is not the only financial instrument available. We see however, that if internal finance is given, and the excess of Pf_K above $\frac{\pi+rb}{1+b}$ is a measure of the credit rationing, the initial workers will apply more workers the more severe is the credit rationing. This means an increase in membership compared to that of the Illyrian firm with the same capital cost but no individual stakes in capital.

(iv) Comparisons

I will make a comparison among the financial instruments available. The instruments are issuing shares to outsiders, a^E , undertake internal investment, a^I or change the level of membership, m . The choice to be made concerning these variables will again affect the level of borrowing. Let us express the three conditions in a manner which makes comparisons more simple:

$$(17) \quad Pf_K \leq \frac{\pi+br}{1+b} + \frac{\theta(1-\bar{\delta})}{C(1+b)}, \quad a^E \geq 0$$

$$(18) \quad Pf_K \leq \frac{\pi+br}{1+b} + \frac{\theta\bar{\delta}}{C(1+b)} - \frac{1}{C(1+b)} \frac{\pi-\alpha}{1+\alpha}, \quad a^I \geq 0$$

$$(19) Pf_K \leq \frac{\pi+br}{1+b} - \frac{\theta\bar{\delta}}{C(1+b)} - \frac{1}{(1+b)} (Pf_N - y), m \geq 0$$

We will make a judgement as to which degree, and when, the three modes of finance are chosen. To be able to do so, some simplifying assumptions are helpful. Assume that the firm is initially in a position where external finance - from previous periods - is at its maximum. Given market prospects and a finite b , the firm is going to decide next period's optimal financial structure. Initially let $a^I = a^E = m = 0$. Which instruments will be chosen?

The three conditions (17), (18) and (19) require equality between marginal value product of capital and what I interpret as cost of internal finance, cost of external finance and cost of increased membership level respectively if the instruments are used (variables strictly positive). We see immediately that not all variables are necessarily positive.

Assume initially that $Pf_K > \frac{\pi+br}{1+b}$. Comparing (17) and (18), we find that equality in (18), which is a necessary condition for increasing internal finance, excludes external finance if $\bar{\delta} < 0,5$ (less than 50% ownership to outsiders) and $\alpha < \pi$. For $\alpha > \pi$, however, internal finance will not be chosen if the restriction on maximum external shareholding does not bind, i.e. unless $\theta > 0$. But the internal opportunity cost of capital may be so high that internal finance will not be undertaken at all. The result depends also on the accepted degree of outsiders' participation. If this is low, $0 < \bar{\delta} < 0,5$, internal finance may be undertaken for a discount rate exceeding the share dividend rate.

Thus, the workers will base their choice between internal and external share finance on the relation between their own required minimum return, α , and how much they have to compensate outsiders, π , and the maximum accepted ownership rights on the hands of outsiders.

The workers' required return must reflect their best alternative investment project, with the addition of a risk premium if investment in the firm is considered riskier and the workers are individually risk averse. If risk is small, and deposits in a savings bank to the market interest rate r represents the alternative, α should be close to r , and the workers prefer internal finance. Employment may be increased also, which again reduces the demand for external finance.

On the other hand, if the workers are incurred large costs in providing capital, because of riskiness or other problems in raising funds, we have $\alpha > \pi$, and internal finance may be excluded. Then the workers will choose a combination of external share finance and recruitment of new workers. If external share finance is initially at its maximum, obviously a necessary condition for increased external finance is an increase in the level of employment if internal opportunity cost of capital is so high that further finance from the initial workers is excluded.

4. EMPLOYMENT IN A CREDIT RATIONED LABOUR-MANAGED FIRM.

(i) Optimal Allocation of Labour.

I will now broaden the scope of analysis, and allow for a spot labour market where the firm can hire labour as non-members. Denote non-members by n , and let them receive a market wage rate w . The wage-dividend y which the worker-members receives, is then

$$(1') \quad y = \frac{PX - rB - wn - \pi A}{N-n}$$

where

$$(8') \quad N = \bar{M} + m + n$$

Substituting (1') instead of (1) into (10), and solving the maximization problem (11) as above, we get an additional first order condition as level of employment of hired workers is to be chosen as well. The condition for use of hired workers is:

$$(12e) \quad \frac{\partial L}{\partial n} = 0 \Rightarrow \left\{ (Pf_N - w) \frac{1 - \gamma\beta_Y(1+\alpha)}{(\bar{M}+m)(1+\alpha)} \right\} n = 0.$$

The total employment of the firm is given by the number of members of the collective $(\bar{M}+m)$ and the employment from hiring workers in the spot market. Although I do not make any specific mention of short-run behaviour, we may consider contracts which do not give membership rights as short-term contracts.

The condition (12e) for employment of hired workers may be rewritten as:

$$(12e') \quad (Pf_N - w)C \leq 0, \quad n \geq 0, \quad \text{with at least one equality.}$$

If $n > 0$, hired workers will be used to a point where value marginal product of labour equals the market wage rate w . On the other hand we will find $n = 0$ if the wage rate exceeds the value marginal product of labour.

The firm makes use of hired workers to the extent that the value marginal product of labour no longer exceeds the market wage rate. An apparent problem is that the initial workers may be interested under certain market conditions in transferring these short-run hiring contracts to long-run contracts without ownership rights. This is discussed by Miyazaki (1984), who finds that it will happen in the labour-managed firm who does well, i.e. when the utility of the representative worker-owner exceeds what could be attained elsewhere. This means that there is an economic rent, which has not been competed away. If so, retired workers will not be replaced, and the firm will convert into a one-man owned firm, using a number of hired workers on traditional contracts (a "capitalist" firm). In our model this would involve the possibility of an $m < 0$. We will investigate this problem more closely in subsection (4ii) by discussing the conditions under which it will be favourable to substitute hired workers for worker-owners.

Firstly, however, we will consider the optimal employment level somewhat more closely, and relate this to an efficient allocation of the factors of production.

I shall assume that $\pi > 0$, $a^I > 0$ and $a^E > 0$. Then we can substitute for γ from (12d) into (12a-c), for θ from (12c) into (12a-b), and lastly for $(Pf_K - \frac{\pi+rb}{1+b})$, assumed non-zero, from (12a) into (12b), to obtain, after some rearranging:

$$(20) \quad Pf_N \leq y + \frac{A\beta_Y + \bar{M}+m}{1+a^I\beta_Y} \cdot (\pi-\alpha) = y + gh$$

Here $g > 0$, while $h \gtrless 0$ as $\pi \gtrless \alpha$.

If $m > 0$, (20) holds with equality.

We investigate the relationship between wage-dividend and the market wage rate in Figure 2 below.

The two Pf_N schedules reflect $h \gtrless 0$ respectively or $\pi \gtrless \alpha$.

It will be advantageous for the workers in terms of income maximization to employ additional hired workers if $w < \bar{w}_1$ for $\alpha > \pi$, and if $w < \bar{w}_2$ for $\alpha < \pi$. On the other hand, if $\bar{w} > w_j$, $j = 1, 2$, we have $n = 0$ as payment to hired workers exceeds the maximum acceptable payment to non-members. Then we can infer nothing from the equilibrium as to how well the firm is doing compared to other firms where the workers receive w . But when hired workers are employed, we see that the worker-members receive a payout above the market wage rate if $\alpha > \pi$, whereas it may be optimal to employ hired workers for $w > y$ when $\alpha < \pi$. We remember that internal finance may not be chosen for a high internal opportunity cost of capital. Then the workers are inclined to pay surplus according to employment rather than capital ownership. It may be the other way round for a low α , as a high share dividend rate may increase the initial workers' share in surplus when $a^E > 0$. Note that the maximum acceptable wage is lower than \bar{w}_1 and \bar{w}_2 respectively if $m = 0$.

For $\pi = \alpha$, the conditions for choice of employment level are similar in this share financed labour-managed firm and the Illyrian labour-managed firm. Capital costs differ, however, so that the levels of employment will differ. Denote the condition the Illyrian employment level. If the internal (the workers') opportunity cost of capital exceeds the share dividend rate, the share financed labour-managed firm chooses employment below the Illyrian level. We remember from Section 3(iv) that the firm tended to prefer external to internal finance as α exceeds π . Thus, high internal capital costs give small incentives to internal investment, and the workers restrict membership size

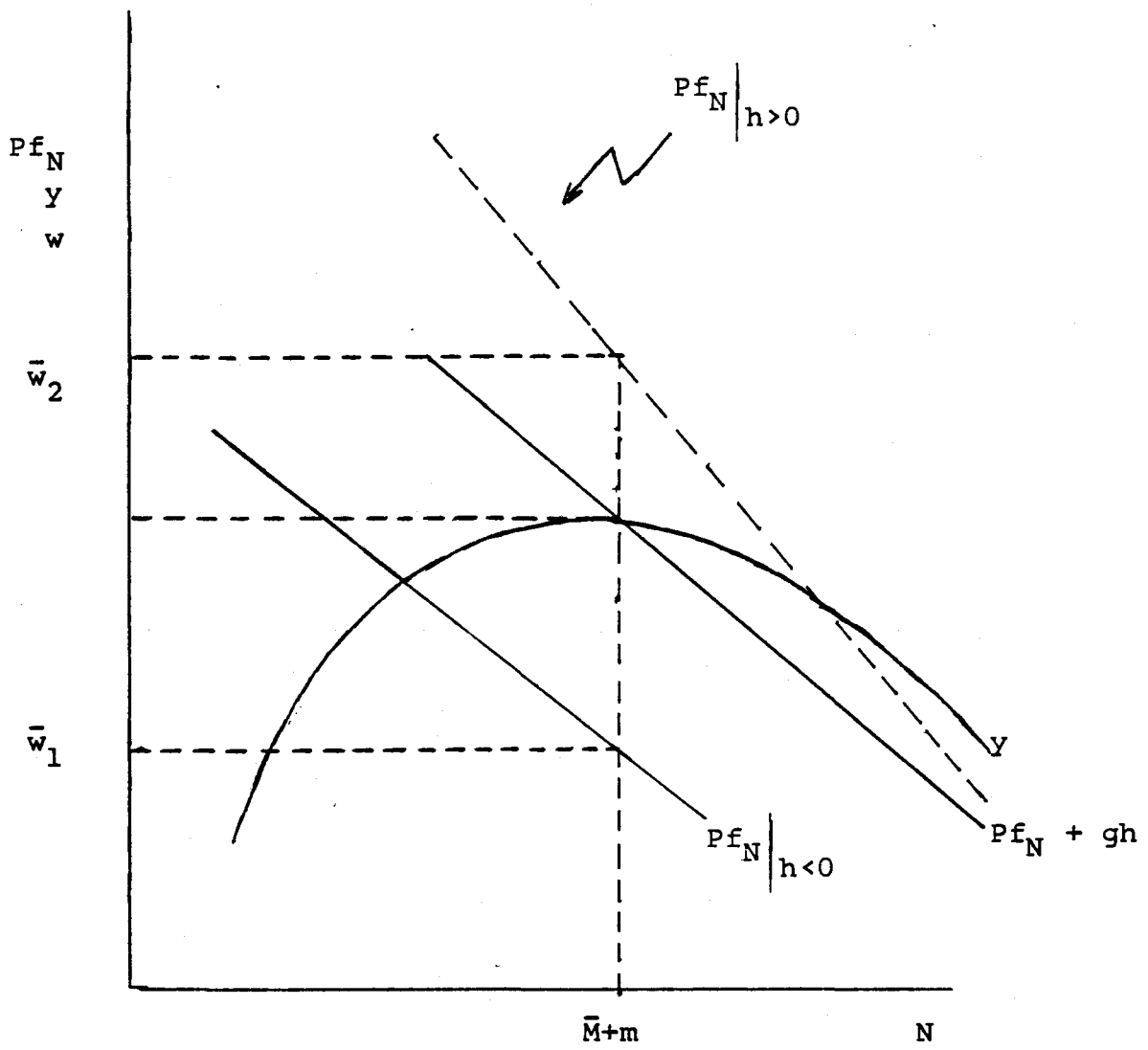


Figure 2: Employment and Credit Rationing.

as wage dividend is high. For a lower opportunity cost of capital, more internal finance may be undertaken, and the workers worry less for an increase in workforce to share in wage-remuneration as the initial workers' payout preferences are shifted in the direction of share dividend.

The resources are obviously not efficiently allocated when the firm faces a market restriction like a (binding) credit ration. The deviations from an optimal resource allocation can be found by inspection of (12e'), (14') and (16). Assume that all conditions hold by equality. Then we find

$$\frac{y}{f_N} \begin{matrix} > \\ < \end{matrix} P = \frac{w}{f_N} > \frac{r}{f_K}$$

This may alternatively be formulated as

$$\frac{y}{r} \begin{matrix} > \\ < \end{matrix} \frac{f_N}{f_K}$$

The production does generally not take place at a minimum of costs. Furthermore, the marginal rate of substitution in production deviates from the internal remuneration unless $Pf_K = \frac{\pi+br}{1+b}$ and $Pf_N = y$. The degree to which the resources are suboptimally allocated can be represented in a (K,N)-diagram (Figure 3), representing the case where $h < 0$ in Figure 2.

In point I in Figure 3, given the production \bar{X} , the $\frac{K}{N}$ -ratio is too low. This holds true irrespective of how the marginal productivities compare to market valuations. It is not surprising that a rationed firm produces inefficiently. But if a perfect functioning finance system cannot be created, the situation depicted in Figure 3 may well prevail.

On the other hand, given the workers' choice of $\bar{\delta}$, where $\bar{\delta} = 1$ is one extreme, the situation emerging after the choice of one specific δ cannot be termed suboptimal. Then the pivoting of the relative cost line from the first-best optimum represents the

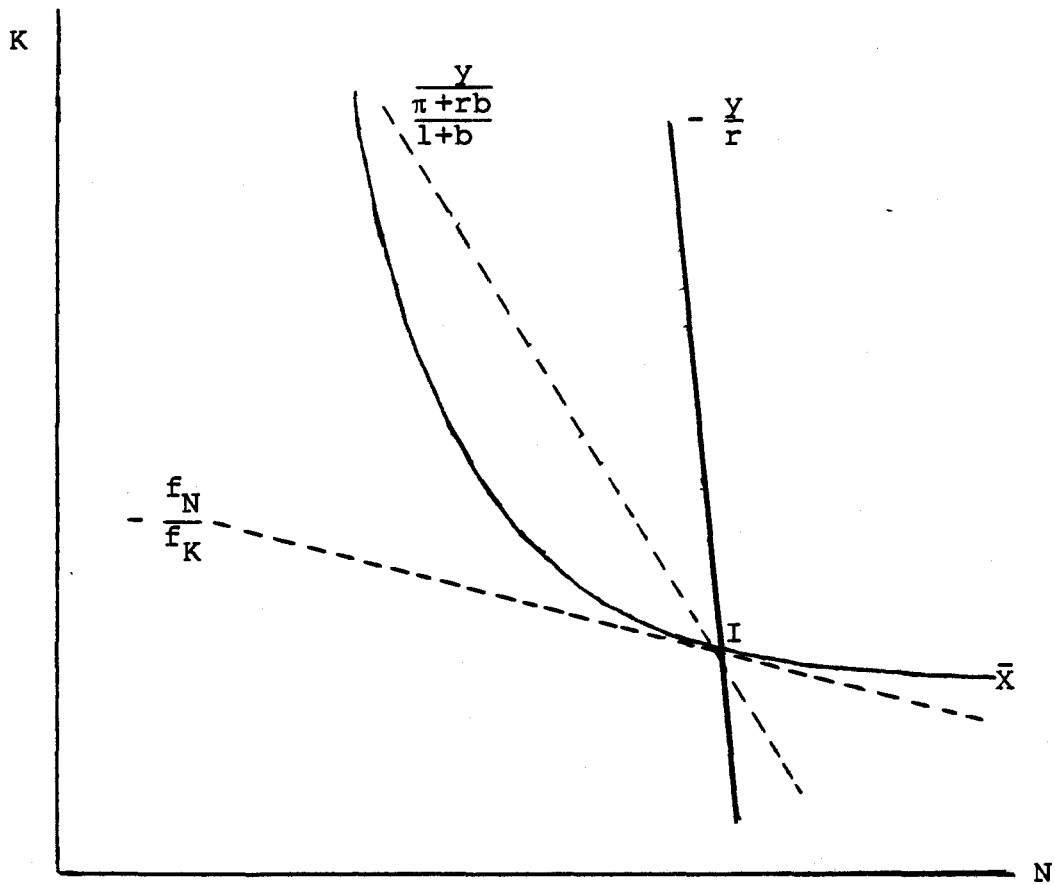


Figure 3: Capital-labour ratio of a credit rationed firm.

play a minor role. Then the remaining workers will capture an increasing part of economic rent. This is a variant obtained from the problem discussed by Miyazaki (1984). The economic rent corresponds to the difference between the utility obtained from being a worker-member of the firm and the utility from alternative employment. When the difference is positive, the firm may convert into a one-man-owned capitalist firm.

There are increasing returns to coalition if:

$$(21) \quad \int_1^{M+m} (a^I \pi + y)(\bar{M} + m) dm \geq (\hat{a}^I \hat{\pi} + \hat{y})(\bar{M} + m)$$

where $\hat{a}^I \hat{\pi}$ and \hat{y} are share-dividend and wage-dividend respectively when the workers are self-employed. The economic rent is the difference between the left-hand and right-hand side of (21). Condition (21) states that the workers will join a firm only if they receive at least the remuneration they would have earned as self-employment. The expression under the integral sign is payout received per worker-member in coalitions of differing sizes.

The condition can be illustrated in a diagram, see figur 4. OCB represents total payout when the workers incorporate, while OA is total income when the same workers operate as self-employed. On the segment OC there are increasing returns to coalition. The optimum size of the firm is $(\bar{M}+m)^{opt}$, and at this point there are locally constant returns to coalition. It will not pay to increase the coalition beyond this point, as the recruiting members will do better as self-employed or by forming a new firm⁶⁾. The economic rent is CE = OD, which is to be divided among the co-owners.

Now, assume that a member withdraws. A decision has to be made whether this worker is to be replaced. Previously, using the first order conditions (12), we discussed how this is dependent on financial aspects. If there are increasing returns to coalition, an additional problem is introduced. Let us assume

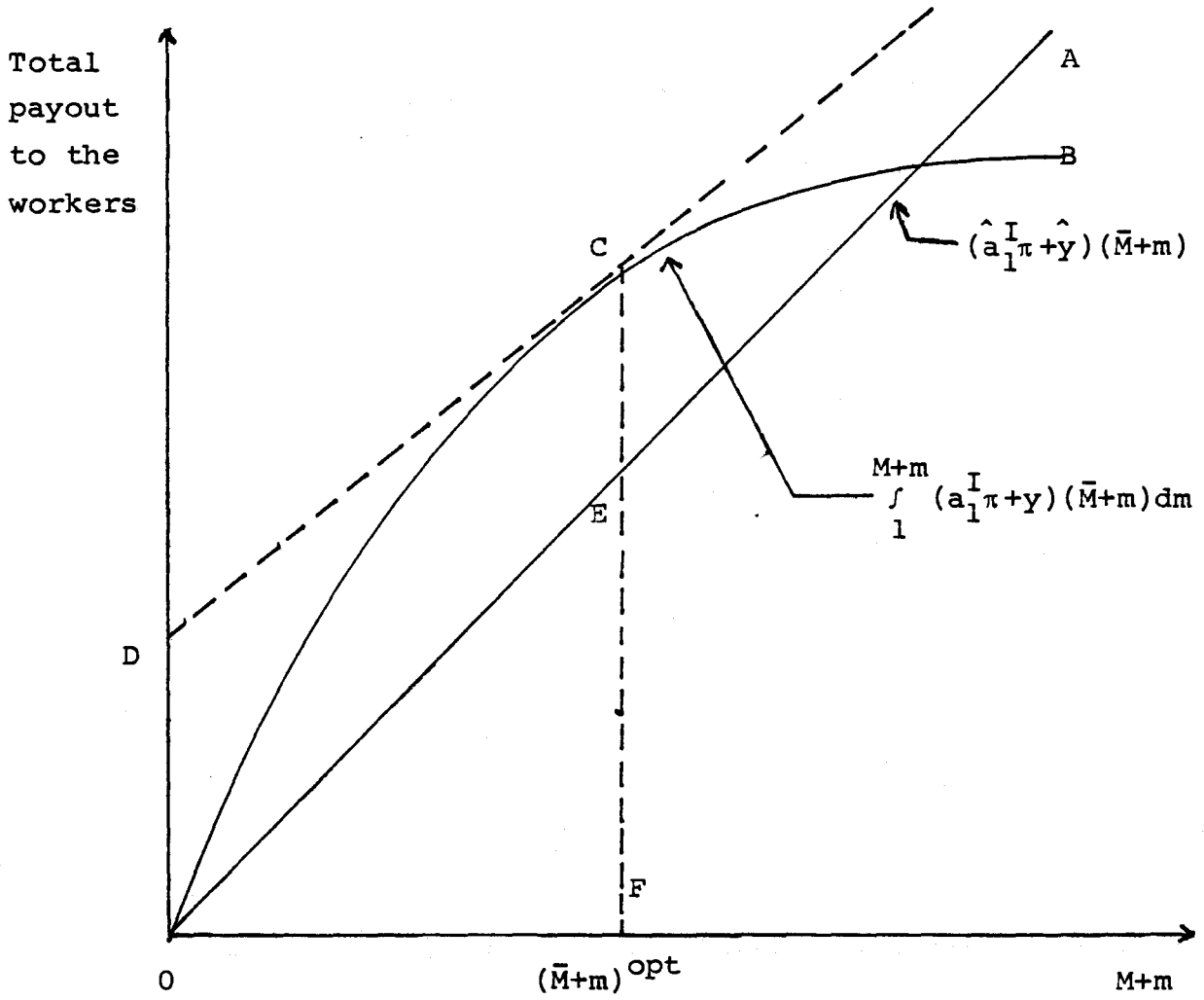


Figure 4: Constant and increasing/decreasing returns to coalition.

that the leaving member can be substituted by a worker of same ability. The remaining workers decide whether the new worker is to be a member (permanently employed), or one hired on a contractual wage w . If there are no financial considerations to take the solution is trivial. If the origin in Figure 4 is changed so as to start in $a^I r$, i.e. payout is $[a^I(\pi - r) + y]$, the curve OA should reflect the market wage rate. When membership is \bar{M} (F in figure 4), each member receives $\frac{DO}{\bar{M}}$ of economic rent. Each contractual worker receives $EF < CF$ where CF is total payout to worker-members. Then it is obvious that payout per worker-owner is increased when $m < 0$.

Now, the financial situation may create problems as discussed above. Here we assumed implicitly that the leaving members' shares could be converted into debt (or outside held shares, which would probably reduce the gain from replacing the leaving worker by a new member). It is not obvious that this is possible. The alternative may be to redeem the shares, or sell shares to remaining insiders or outsiders. The latter will increase the outsiders' control of the firm, and may result in an increase in π . Assume that the shares from leaving members have to be redeemed, and that the debt of the firm is at its maximum. If a leaving member is replaced by a hired worker, the change in the value of production is given by (cf. (2), (3), (4) and (6)):

$$\begin{aligned} & - Pf_N + Pf_N - Pf_K \cdot \frac{\partial K}{\partial m} = - Pf_K \left(\frac{\partial A}{\partial m} \frac{\partial K}{\partial m} + \frac{\partial K}{\partial B} \frac{\partial B}{\partial A} \right) \\ & = - Pf_K a^I \cdot \frac{1}{1-\delta} (1+b) \equiv S \end{aligned}$$

Note that the number of outside held shares is reduced also. Payout is reduced by

$$\begin{aligned} & y - w + a^I \pi + \bar{\delta} a^I \pi + (1+b)(1+\bar{\delta}) a^I r \\ & = y - w + a^I (1+\bar{\delta}) [\pi + (1+b)r] \equiv R. \end{aligned}$$

Membership is reduced if

$$R - S > 0$$

which we in terms of figure 5 can write as

$$(22) R - S = \frac{OD}{\bar{M}+m-1} - \frac{OD}{\bar{M}+m} > 0.$$

The contraction of membership takes place as long as the remaining members can gain by capturing an increasingly larger part of economic rent and substitute leaving worker-owners by hired workers⁷⁾.

5. CHOICE OF EXTERNAL PARTICIPATION

In the analysis above we have considered degree of outsiders' participation, δ , as exogenous. It is of interest to investigate how the optimal a^E and δ can be found if both variables are allowed to vary. To simplify I will assume that a^I , m and π are given. This means that the β -function is excluded from the analysis as we are to find optimal outsiders' participation for a given share dividend. Probably it would have been more realistic to assume that β varied (positively) with δ . Then also π would have to be determined.

However, this would result in some ambiguity and complexity, and I have excluded it in order to simplify the analysis.

Now, assume that the workers (the firm) have a welfare function defined over income, Y , and self-control $(1-\delta)$. Write the welfare function as

$$(23) \quad v(Y, 1-\delta)$$

where

$$Y = y + a_1^I \pi$$

and y is defined by (1). The partial derivatives, denoted v_y and $v_{(1-\delta)}$ respectively, are assumed positive. v is assumed concave.

By making the necessary substitutions from the definitions above, (23) is maximized w.r.t. a^E and δ , subject to (6). $\tilde{\theta}$ is the Lagrange multiplier, and we write the first order conditions as:

$$(24a) \quad v_y [Pf_K(1+b) - rb - \pi] \frac{1}{(\bar{M}+m)} - \tilde{\theta}(1-\delta) \leq 0, \quad a^E \geq 0,$$

with at least one equality

$$(24b) \quad -v_{(1-\delta)} + \tilde{\theta}A \stackrel{\leq}{=} 0, \quad \delta \stackrel{\geq}{=} 0, \text{ with at least one equality.}$$

δ is defined on $[0,1]$. We cannot rule out corner solutions. Nevertheless, assume that there is an interior solution. Then we substitute from (24b) into (24a) to obtain

$$(25) \quad \frac{v_{(1-\delta)}}{v_Y} = \frac{[Pf_K(1+b) - br - \pi]A}{(\bar{M}+m)(1-\delta)}$$

Assuming $0 < \delta < 1$, the expression on the right hand side of (25) is positive, zero or negative as

$$Pf_K - \frac{\pi+br}{1+b} \stackrel{\geq}{<} 0.$$

From the welfare function in (23), and by using the assumptions made concerning its partial derivatives, we know that

$$\frac{dY}{d\delta} = \frac{v_{(1-\delta)}}{v_Y} > 0.$$

We can represent the equilibrium in a diagram, see Figure 5.

Utility is defined so that $v^* > v^0$. In Y_0 , $a^E = 0$, and it represents the lowest utility the workers will get. If the right hand side of (25) is zero or negative, the possibility set is bounded by Y_0B and Y_0C respectively. Then utility is maximized by choosing $\delta = 0$. However, if Pf_K exceeds the weighted average price of capital (right hand side of (25) is positive), then it may be an optimal policy to pursue, as seen from the workers' point of view, to introduce outside shareholders. Y_0DA represents a possible boundary of the opportunity set. Optimum is found in D, resulting in $\delta^* > 0$ and $Y^* > Y_0$, and consequently $a^E > 0$.

Note that the boundaries indicated are examples only. The opportunity sets may take any form, and they are not necessarily as well shaped as indicated in Figure 5. E.g., convexity of the sets does not necessarily hold for $Pf_K > \frac{\pi+br}{1+b}$, which implies that we cannot rule out a non-unique solution. Also, in extreme cases, where e.g. the credit institutions are hostile to labour-managed firms, an equilibrium in A may be optimal. This means that the workers are unable to obtain finance at conditions which make labour-management profitable. Then complete conversion to a profit-maximizing firm owned by outsiders is the optimal strategy, and in terms of Figure 5, $v^* > v^0$ intersects $\delta = 1$ at A.

6. INTERNAL FINANCE AND ATTITUDE TO RISK

I will examine somewhat more closely the workers' willingness to supply capital. It will depend, apart from practical problems in raising funds, on their attitude to risk. Specifically, let us investigate the value of share dividend preferred in the absence of outside shareholders, i.e. for $a^E = 0$. This involves reasoning and discussion outside the model presented in Section 2.

If inside shares are evenly distributed, there is no reason to bother about the value of π compared to y (i.e. $\frac{\pi}{y}$). (The workers work equally long hours). But by investing out of their own savings, they increase their risk from participation. For risk-averse workers this is suboptimal compared to external finance at the market interest rate. Denote each worker's expected income I . We have

$$y + \pi a^I = I.$$

The net expected income, \bar{I} , is, after the deduction for the opportunity cost of capital:

$$\bar{I} = I - a^I r$$

By borrowing the amount $a^I(\bar{M}+m)$, the workers will receive \bar{I} . Then their gross income, I , is made up by an uncertain income, \bar{I} , and an amount $a^I r$ paid with certainty. Let us express the representative worker's utility in terms of gross income, denoted $u(I)$. $u(I)$ is each (identical) worker's individual utility function which is maximized w.r.t. hours of work and amount of capital invested in the firm. Let I^1 denote gross income when the firm is completely externally financed, and I^2 is income when there is some internal finance. Then, assuming that the representative worker is risk averse and labour supply is given,

$$u(I^1) > u(I^2).$$

In the absence of credit rationing, risk-averse workers will prefer external finance, while risk-neutral workers are indifferent. Note the assumption that shares are negotiable. If they were not, share dividend would have to include the principal as well. Then a risk neutral worker receiving $y + a_1^I r$ would definitely prefer external finance. The results correspond to those of Jensen and Meckling (1979) and McCain (1977).

But as pointed out earlier, the firm cannot rely on external fixed rental finance only. The workers may choose to supply the necessary capital themselves, e.g. on an equal basis. If they were risk neutral, they would demand a total payout of

$$I = y + \pi a^I \geq w + r a^I.$$

w and r are opportunity costs of labour and financial capital respectively. The composition of I (the relative size of y and π) does not matter. If the workers are risk averse, they require a payout of

$$I = y + \pi a^I > w + r a^I.$$

Again, the composition of I does not matter, assuming all workers manage to raise the funds privately on equal terms. If not, the composition of I may matter.

Assuming that pure rental has to be ruled out, we conclude that capital costs will exceed the market interest rate unless strict assumptions are fulfilled, even when the internal shares (bonds) are negotiable. In particular we note that all workers have to be risk-neutral and able to raise funds on equal terms for capital costs to be equal to the market interest rate. Therefore it will generally be optimal to issue all shares in the market as non-participating bonds, leaving the voting rights vested with the workers. However, I have argued that this may be difficult to put into effect in a capitalist market economy, see section 1, nor is it a trivial matter in a labour managed economy generally, see Jensen and Meckling (1979), pp. 486-488.

7. SUBSIDISING THE FIRM

We have so far assumed $\pi > r$. Suppose that $\pi \leq r$. If the workers are the sole owners π may take any value. What matters to the workers is the total payout, i.e. $\pi a^I + y$. Let us assume that the outside shareholder is some public authority (government, local authority). Under certain conditions they may be willing to accept a $\pi \leq r$ during a substantial period (remember that π is an expected value). Then the governmental capital supply plays the role of a subsidy granted to the firm. This may be considered a better result than leaving the workers unemployed for a shorter or longer time period. The case of the "Scottish Daily News" (SDN) may be considered a good example of this, see Bradley and Gelb (1980). The SDN was in 1974/75 taken over by the workers upon a threat of moving printing of the paper from Glasgow to Manchester. The British government intervened and supplied the workers with a "once and for all" governmental loan, provided the remaining capital could be raised elsewhere from sources aware of the risk. Part of risk taking capital was supplied by the workers. Bradley and Gelb argue that this way of supporting a firm is desirable. As the support is easily withdrawable, the long-run misallocation of resources is minimized. The workers managed to run the firm for half a year.

For example, firms threatened by bankruptcy may be taken over by the workers. But they may be short of capital. A private firm threatened by bankruptcy will hardly be able to raise capital in the free market. Either the government (or others) has to intervene, or the workers have to raise the capital themselves.

It is not obvious that 100 % state ownership is the best way of refinancing the firm. As seen from the workers, this means a new group of owners. The basis for conflict and attitude to choice between wage and employment are probably changed if the workers themselves own the firm. Thus, subsidising a labour-owned firm by co-ownership may be an optimal way to support firms when the aim is rescuing employment in a certain area. If risk averse workers are willing to invest in the firm, this shows that they

to some degree believe in it, and that they are willing to sacrifice something for its survival⁸⁾.

Now, in our model, if $\pi \leq r$, the workers will of course finance as much as possible by public shareholding. But there are limits to external finance if control is to be vested with the workers. Also the suppliers of capital, the government, will not be willing to supply an unlimited amount. Then from (11) and (12c) the restriction on outside shareholding is supposed to bind, and $\theta \geq 0$. Thereby, cf. (12c'), $(Pf_K - \frac{\pi+rb}{1+b}) \geq 0$, and $Pf_K > \pi$. As $\theta \geq 0$, a^I is chosen so that the government shares reach a maximum.

We see that the average (financial) capital cost, $\frac{\pi+br}{1+b}$, is equal to or below the value marginal product of capital, Pf_K . It will stay below the market interest rate if $\pi < r$, which would seemingly give the workers an incentive to increase shareholding beyond limits. This can take place through the recruiting of new members, or through retained surplus. Both these ways of finance involve a cost as seen from the existing worker-members, either in terms of delayed payment or that more members are to share the gain from subsidising. But of course the net yield may be everywhere positive, resulting in an optimum with infinite production. Thus some restrictions have to be imposed. A limited governmental capital supply is an obvious restriction which will solve the problem. Of course the workers will then take the maximum amount offered⁹⁾. Another remedy to the problem can be found in the technical restrictions. Thus, a technology with first increasing and then decreasing returns to scale will at some point make further growth unprofitable. We also note that β may be influenced by an increase in γ . Thus, π will probably approach r as the firm becomes more profitable for the workers. Therefore we assume that an optimum can be found¹⁰⁾.

The equilibrium should be found where the restriction on borrowing is binding, as there is no need for government subsidisation when the firm can obtain its funds in the credit market on equal terms with others. Thus, we have to assume that

$$Pf_K > r > \frac{\pi + br}{1+b}$$

An interesting feature of this subsidisation programme is that it affects directly the employment and investment decisions of the firm. Furthermore, by influencing the marginal value of capital of the firm in the direction of market remuneration, at the same time, given R , the remuneration of labour is driven in the direction of its marginal product valuation, see (16). (Pf_N is, however, not necessarily equal to the market valuation).

The case when $\pi < r$ is probably of most interest when the workers' alternative to employment in the labour-owned firm is unemployment. The government grants a subsidy by receiving $\pi < r$. But the alternative is paying unemployment compensation to the workers. A cost-benefit analysis will show which alternative is the best one. The fiscal effects as well as the utility of the workers should be taken into consideration.

Assume that M workers stay with the firm. If the firm leaves business, a fraction ξ of the workforce will be unemployed for a fraction τ of the planning horizon. Unemployment compensation is supposed to be $\phi\%$ of the market wage rate, w . Then the total unemployment compensation, D , which is to be paid to the workers in question is:

$$D = M\xi\phi\tau w$$

This amount is to be compared to the loss from buying shares giving a yield $\pi < r$. The fiscal effect of the subsidy is (excluding other periods than the production period):

$$- (\pi - r) a^E + D \stackrel{?}{<} 0$$

By taking over the firm, the workers receive Y^* , where

$$Y^* = (\pi + y) M$$

If the firm is closed down, the same workers will be paid \bar{Y} , where

$$\bar{Y} = D + (1 - \xi)(1 - \tau) wM$$

Then, although $\pi + y < w + r$, the total payout to the workers by running the firm may exceed what they will receive from the best alternative occupation. The net benefit by taking over the firm is:

$$W = Y^* - \bar{Y} - (\pi - r) a^E \stackrel{?}{\geq} 0.$$

We are here disregarding the personal non-pecuniary costs of unemployment. Different attitudes to risk should also be taken into consideration, as the variance of the values may be of importance to the parts involved. Thus, it points only to some aspects relevant to one way the government can subsidise bankruptcy threatened firms. It is not necessarily the best one, although it may have some advantages.

8. CONCLUDING REMARKS.

We have investigated a model of a labour-owned firm which is partly self-financed. If the workers own shares in the firm, and if these shares reflect perfectly the value of the firm, the efficiency problems vanish, which may result in self-extinction of the Illyrian firm. This could be handled by means of non-voting participation bonds as well. But I argue that it is doubtful whether outside investors will contribute capital without being given the right to codetermination, particularly if the firm operates in a capitalist market economy. Furthermore, I argue that the outsiders will use their influence on decisions to guarantee themselves a minimum income, which may be state contingent. Then the workers' willingness to rely on outside finance by means of shares is dependent firstly on the possibility to obtain debt, and secondly on the willingness to trade off control of the firm.

Individualistic ownership of the residual is needed to avoid problems incurred by social ownership. But we have to bear in mind, that relying on outsiders will result in reduced control over decisions by the workers. The aim was here to show one way in which this may occur. In a subsequent paper, I will discuss the problems of internal finance when the workers are heterogeneous.

APPENDIX

H is defined by (14') in the text. By using the definitions of C the complete expression is:

$$(A1) \quad H = \frac{\pi+br}{1+b} - e = \frac{\pi+br}{1+b} - \frac{(1-\bar{\delta}) [(\bar{A} + \bar{M}a^I + ma^N + a^E) \beta_Y + \bar{M} + m]}{(1+b)(1+a^I \beta_Y) \bar{M}} \cdot (\pi - \alpha)$$

Its partial derivatives w.r.t. $\bar{\delta}$, b , the β_Y -function and α are, when we assume that $\beta_{YY} = 0$, cf. Figure 1:

$$(A2) \quad \frac{\partial H}{\partial \bar{\delta}} = - \frac{\partial e}{\partial \bar{\delta}} = \frac{(A \beta_Y + \bar{M} + m)}{(1+b)(1+a^I \beta_Y) \bar{M}} (\pi - \alpha) \begin{matrix} \geq \\ < \end{matrix} 0 \text{ as } \pi \begin{matrix} \geq \\ < \end{matrix} \alpha$$

$$(A3) \quad \frac{\partial H}{\partial b} = \frac{r - \pi}{(1+b)^2} + \frac{(1-\bar{\delta})(A \beta_Y + \bar{M} + m)}{(1+b)^2 (1+a^I \beta_Y) \bar{M}} \cdot (\pi - \alpha)$$

We assume $\pi \geq r$. Then $\frac{\partial H}{\partial b} < 0$ if $\alpha > \pi$, while $\frac{\partial H}{\partial b} = ?$ if $\alpha < \pi$.

$$(A4) \quad \frac{\partial H}{\partial \beta_Y} = - \frac{\partial e}{\partial \beta_Y} = - (1-\bar{\delta})(\pi - \alpha) \frac{A - a^I (\bar{M} + m)}{(1+b)(1+a^I \beta_Y)^2 \bar{M}} \begin{matrix} \leq \\ > \end{matrix} 0 \text{ as } \pi \begin{matrix} \geq \\ < \end{matrix} \alpha.$$

$$(A5) \quad \frac{\partial H}{\partial \alpha} = - \frac{\partial e}{\partial \alpha} = \frac{(1-\bar{\delta})(A \beta_Y + \bar{M} + m)}{(1+b)(1+a^I \beta_Y) \bar{M}} > 0.$$

FOOTNOTES

- 1) In Scandinavian countries, as well as in West Germany, the workers are by law granted voting rights in firms larger than a certain minimum size, in FRG denoted "paritätische Mitbestimmung", indicating a 50 % participation in the boards of the firms.
- 2) Alternatively the credit rationing can be formulated as $r = r(B)$, or $r = r(\frac{B}{K})$, cf. McCain (1977). We find then that borrowing will be used to a point where $Pf_K = r(1 + \varepsilon_{\beta, r})$, where $\varepsilon_{\beta, r}$ is the elasticity of demand for debt in r .
- 3) See the Appendix for complete expressions of the derivatives.
- 4) There is a possibility for short-run variations in the "fixed" employed labour force by exploiting the unemployment security system. Here we rule this out.
- 5) See section 9, pp. 1136-7.
- 6) Note that in C, $\frac{\partial f(a^I \pi + \gamma)(\bar{M} + m) dm}{\partial m} = (\hat{a}^I \hat{\pi} + \hat{\gamma})$.
- 7) We disregard solidarity with hired workers. Furthermore, if the outside shares are held by the government or trade unions, their major claim may be that membership is not to be reduced.
- 8) Bradley and Gelb (1980) argue that the existence of individual capital stakes screens out pessimists, and a hard-working work-force will remain (see p. 670). The governmental intervention will affect this screening by its demands on individual participation from the workers. In Bradley and Gelb (1982) they discuss screening mechanisms with reference to Mondragon, and they argue that a capital requirement is a device which simultaneously take on the role of a locking-in mechanism.

- 9) In our programme (11) this would imply the introduction of an additional parameter (restriction).

- 10) In their analysis of the "Scottish Daily News", Bradley and Gelb (1980), the authors model the restriction by imposing a maximum wage the workers may receive, to avoid the workers pursuing a policy aimed at recuperating the capital invested as soon as possible.

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ALLOCATION OF CAPITAL AND LABOUR IN A LABOUR-OWNED FIRM
CONSISTING OF HETEROGENEOUS WORKERS

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ALLOCATION OF CAPITAL AND LABOUR IN A LABOUR-OWNED FIRM
CONSISTING OF HETEROGENEOUS WORKERS

1. Introduction

In this paper I will study the functioning of a firm which I will term a capitalist labour-managed firm. Three main features distinguish the model from the Illyrian (socialist) firm, viz. individual ownership of capital, an inegalitarian distribution of voting rights, and an inegalitarian distribution of income.

The latter of these is to some degree discussed in models of labour-managed firms where individual labour supply decisions are taken into consideration, see e.g. Ireland and Law (1981). But in most analyses, like the one referred to, the workers are assumed homogeneous. Models where ownership of capital is individualized are analysed by e.g. McCain (1977) and Vanek (1977b). They assume that the shares (bonds) carry no voting rights. Thereby the models fulfill a condition set by Putterman (1984) that a labour-managed firm should have an egalitarian power structure. On the other hand, Miyazaki (1984) has investigated a labour-managed firm which makes use of hired workers without giving them voting rights.

The background for most of the analyses referred to is the possible allocational inefficiency of the Illyrian labour-managed firm. The problems arising result from the strive to capture the economic rent, see e.g. Sen (1966) and Miyazaki (1984), the high cost of internally provided capital when capital is collectively owned, see e.g. Vanek (1977a), and a likely credit rationing facing labour-managed firms, see e.g. McCain (1977), Putterman (1984) and Askildsen (1986). I have argued elsewhere, Askildsen (1986), that these problems will be of special significance to a labour-managed firm operating in a capitalist economy. Therefore I will investigate here a model which is intended to resemble some features of such a firm.

I will assume that the labour-owned and -managed firm is financed by vote-carrying shares supplied by the firm's heterogeneous work-force. The internal finance is sufficient for the

firm to obtain the desired level of debt finance. The firm's optimization problem is solved by a Nash-Cournot arbitration formula with the individual workers' voting power as weights. However, as the workers supply capital as well as labour, the individual workers' relative capital and labour supply are determined by their utility maximization behaviour.

The intention of the paper is threefold. Firstly I want to characterize the firm's equilibrium choice of internal finance, employment and payout rates, and the individual workers' labour and capital supply. Secondly I am interested in investigating how these equilibrium choices are affected by some of the parameters of the model, like expected price and opportunity cost of capital. Finally I will discuss the possibilities for reaching unanimous decisions.

The model to be used in the analysis is outlined in Section 2. It will be assumed that the optimisation takes place in two stages, which for the matter of exposition will be dealt with separately. Uncertainty is introduced through the price of the firm's output. In Section 3 I characterize the optimum of the firm, and I investigate how the initial workers as members of a producer cooperative decide collusively on level of internal and external finance, and employment. I assume for the sake of simplicity that borrowings are given. But capital can be increased by internal investment (retained surplus) or by recruiting new workers supplying share capital (external finance). I show how the decisions depend on scale advantages and distribution of shares and hours of work. Conditions for unanimous decision making are discussed, and comparative static responses to changes in the price of output and opportunity cost of capital are reported. The coupling together of capital supply and employment level leaves most responses ambiguous, so that special cases have to be studied. Not surprisingly, the responses will differ markedly from those known from the Illyrian firm. The reason is that the workers possess here two instruments to capture the economic rent. In Section 4

I examine the individual workers' decisions on capital and labour supply. I focus on the importance of risk-aversion, time preference and disutility of work, and I compare the model with the Ekern-Wilson mean-variance model (Ekern and Wilson 1974) for unanimous decision making. Finally, in Section 5, some concluding remarks are made, in particular concerning the modelling of non-Illyrian variants of labour-managed firms. I argue that interesting features can be further analysed and discussed by using special functions and making assumptions enabling us to investigate special cases.

2. The Model

The reason why the workers supply capital by means of shares may be credit rationing. Here, however, I will not be particularly concerned with that problem, as it has been carefully examined in a preceding paper. Although the scenario considered may have its root in credit rationing, I will be concerned primarily with internal supply of capital from heterogeneous workers. This is equivalent to considering at one stage the individual labour and capital supply decisions, and at another stage to trace out conditions for a group of labour-owners to decide on changes in internal supply of capital and in the employment level (membership), which may affect the capital level as well. As we shall see, special conditions must be met for the decisions to be taken unanimously.

In the analysis to follow I will assume that the firm's optimisation problem is solved collusively by its workers. The decisions are not necessarily unanimous. I will assume that the firm (the workers as a collective) is risk neutral. But the workers are heterogeneous in the sense that their capital and labour supplies differ. Then it matters how payout is being made. Payout will be made according to shares and labour contributed, so that income will generally differ among the workers. Distribution of income is decided through bargaining. In a code-terminated firm it is common to use the Nash bargaining solution

to model how bargains between labour and capital are solved, see Svejnar (1982). I find it natural to use this formulation here as well. We can write the bargaining problem as

$$(1) \quad \text{Max } \prod_{i=1}^{N_0} (W_i - \bar{W}_i)^{\beta_i}$$

where each of the N workers receives income W_i and has a weight of β_i in the bargains (bargaining power). \bar{W}_i is each worker's reservation income, which may be the income he/she can obtain elsewhere in the economy. We can consider β_i as voting power. Formal (or informal) rules may give special rights to some groups. E.g., elder workers may have a larger say than younger, or large capital suppliers may have a larger weight in decisions than small capital suppliers.

Let us assume that a given number of workers, N_0 , has formed a firm. Each worker has initially supplied a_0^i units of capital (shares) at price 1. In addition they have borrowed some capital, \bar{B} . Then the total capital stock of the firm is the borrowings and the number of shares contributed by its workers. We denote capital by K . Together with labour input, L , where

$$(2) \quad L = \sum_{i=1}^N l^i,$$

and l^i , $i=1, \dots, N$, is each worker's individual labour supply, combinations of the two inputs are capable of producing quantities of a product X which are sold at an expected market price p . We write the production as

$$(3) \quad X = f(K, L)$$

Marginal productivities are assumed positive and decreasing, i.e.

$$\frac{\partial f}{\partial L} = f_L > 0, \quad \frac{\partial f}{\partial K} = f_K > 0, \quad \frac{\partial^2 f}{\partial L^2} = f_{LL} < 0 \quad \text{and} \quad \frac{\partial^2 f}{\partial K^2} = f_{KK} < 0.$$

We also assume technical complementarity, i.e.

$$\frac{\partial^2 f}{\partial L \partial K} = f_{LK} = \frac{\partial^2 f}{\partial K \partial L} = f_{KL} > 0.$$

The firm's stock of capital is

$$(4) \quad K = A + \bar{B} = \sum_{i=1}^N a^i + \bar{B}.$$

Now, assume \bar{B} is given, as well as the initial capital supply a_0^i . Assume also that membership level cannot contract, i.e. no workers can be dismissed without full compensation. We are interested in investigating conditions for utility increasing changes in internal finance and level of membership (permanent workforce). Increased internal finance means that the workers contribute with additional capital. First imagine that the workers consider increasing total share capital by the amount R , which may be e.g. a retained surplus from a previous period. In addition they consider whether the employment level should be increased. Individual labour and capital supply are assumed given. New shares are assumed distributed according to initial shareholdings. Afterwards I will consider the individual workers' incentives to supply labour and capital respectively. Then the level of the workforce (membership) will be assumed given.

The changes in membership will affect total share capital. The potentially new workers will buy some shares. It will be assumed that each new worker will supply on average an amount proportional to the average period of shareholding of the initial workers. If membership is constant, we assume that changes occur in total and in individual stock of shares if and only if internal finance is increased (retained surplus)¹⁾.

Given these assumptions, the stock of shares is

$$(5) \quad A = A_0 + R + g(N),$$

where R is internal finance (retained surplus) and

$$(6) \quad A_0 = \sum_{i=1}^{N_0} a_0^i$$

is the initial stock of shares, which is assumed given exogenously. We denote the new level of employment by N . In the forthcoming period, each initial worker owns shares given by

$$(7) \quad a^i = a_0^i \left(1 + \frac{R}{A_0}\right)$$

The g -function is defined by

$$(8) \quad g(N) = m(N - N_0), \quad N \geq N_0,$$

where

$$(9) \quad m = \frac{\sum_{i=1}^{N_0} a_0^i}{N_0} \quad s = \frac{A_0}{N_0} \quad s, \quad s \geq 0,$$

is an average of shares bought by each new worker. s is assumed given, and its role is to indicate how many shares new workers will buy. The workers will receive payout by means of wage dividend (y) and share dividend (π). Note that the product price p is an expected value with known distribution. Then, given a value of one of the means of payment, say π , the firm maximises the wage dividend. However, it is not arbitrary which weight is placed on the two means of making payment. Depending on relative participation by means of labour and capital, the marginal benefits of payment according to work and payment according to shares will differ among the workers. This will be further discussed below. Now, let us write wage dividend as²⁾

$$(10) \quad y = \frac{pf(K,L) - r\bar{B} - \pi A}{L}.$$

It should be clear from the discussion above that decisions are in reality taken in two stages, viz. by the workers as a collective, deciding on employment and total capital supply, and by the individual workers, who decide on hours of work and their

individual supply of capital. The individual decisions will again affect the collective's allocational decisions. To simplify things somewhat, I will look at the two stages separately. I will assume that the firm takes each worker's labour and initial capital supply as given. The individual workers consider the level of employment and borrowings as given.

Thus, the analysis is carried through as follows:

(i) Given a^i and l^i for all $i = 1, \dots, N$, the collective maximises collusively y for any value of π , by choosing R , N and π . As mentioned above, initial level of membership, N_0 , and stock of share capital, A_0 , are assumed given, as well as external finance \bar{B} .

(ii) The workers decide on capital supply and labour supply, given external finance and level of membership. We will assume a Nash-Cournot reaction function among the workers. Thus, what we will consider is how individual capital and labour supply depend on some parameters of the model.

A comment should be made on the particular model investigated. There may be several reasons why the firm is financed partly by individually owned shares. Firstly there may exist some underlying credit rationing, which may force the workers to contribute with some finance internally (see Askildsen (1986)). Secondly, some workers may find it profitable to invest in their own firm. In particular a controlling group of workers may then possess the possibility to affect means of payout ("wages") in a way which is favourable to them. Thirdly, the firm may have come into being by a take-over of a former capitalist firm by the workers, e.g. a bankrupt firm, or a buy-out of former capitalist owners. Fourthly, it may be found advantageous by the workers to form a firm which does not vary too much from the ordinary way of organising firms, assuming indirectly that the firm operates in a capitalist market economy. There may be legislative con-

siderations to be taken, as well as problems of discrimination against unfamiliar organisational forms. Lastly, the firm may be one in which the original shareholders have sold their shares to the workers as part of incentive schemes. In that case, however, it would probably not be reasonable to assume that the workers were to own all shares, or even a majority of the shares.

It is assumed that the firm does not employ hired workers, i.e. all workers have voting rights. Nevertheless, some effects of using hired workers are discussed briefly in Section 3.

When we investigate the optimisation of the firm, we assume that the individual workers are willing to supply the capital which they agree they should contribute. We assume also that there exist potential new workers that are willing to accept the terms given by the initial workers.

3. The Firm's Optimisation

The firm chooses R , N and relative payout rates so as to maximise income to the existing workforce for the remaining part of their tenure with the firm, which is assumed to be one period. We assume risk neutrality when considering the workers as a collective.

Each worker's income may be written as

$$(11) \quad W_i = \frac{1}{1+r} (y_1^i + \pi a^i) - \frac{r}{1+r} a_0^i \left(1 + \frac{R}{A_0}\right)$$

This formulation means indirectly that one option available to the workers is cashing in the shares and close down the firm. However, we assume they continue running it. The first element of (11) is the present value of future income, while the second term represents the cost of supplying shares to the firm. We assume that the common discount factor is the market rate of interest r , which is also the interest rate at which capital is borrowed.

The maximum welfare of the workforce is found by maximising the weighted product of the workers income. Taking the logarithmic transform of (1), the utility to be maximised is

$$(12) \quad U = \sum_i \beta_i \ln (W_i - \bar{W}_i).$$

When maximizing (12) there is a restriction that each worker receives at least the reservation income. However, the restriction will not bind. As shown by e.g. Roth (1977), a local maximum can be found where the product formula is maximized. All analysis will take place around this maximum where $W_i > \bar{W}_i$ for all i . Then we can exclude the reservation income from the discussion.

(i) First Order Conditions

By substituting from (4) - (9) into (10) and using (10) in (11) the problem in (12) can be solved as a Kuhn-Tucker programme where the welfare is maximised w.r.t. R , N and π . W_i is given from (11). The first order conditions are, after some simplifications:

$$(13a) \quad \sum_{i=1}^{N_0} \frac{\beta_i}{W_i} \left[\frac{l^i}{L} (pf_K - \pi) + \frac{a_0^i}{A_0} (\pi - r) \right] \frac{1}{1+r} \leq 0, \quad R \geq 0, \text{ with at}$$

least one equality.

$$(13b) \quad \sum_{i=1}^{N_0} \frac{\beta_i}{W_i} \left\{ \frac{l^i}{L} [(pf_L - y)\bar{l} + m(pf_K - \pi)] \right\} \frac{1}{1+r} \leq 0,$$

$(N - N_0) \geq 0$, with at least one equality.

$$(13c) \quad \sum_{i=1}^{N_0} \frac{\beta_i}{W_i} \left[-\frac{A}{L} l^i + a^i \right] \frac{1}{1+r} \leq 0, \quad \pi \geq 0, \text{ with at least one equality.}$$

If the level of membership changes, we have assumed that the recruiting members supply on average \bar{l} units of labour. This can be considered an approximation to the average labour supply of the workers, as there is a priori no reason to believe that there is any bias in the labour supply of workers entering the firm.

Note that $\frac{\partial g}{\partial N} = g' = m$ if $N > N_0$ • $g' = 0$ if $N = N_0$.

The first thing to be noted from (13) is the possibility of obtaining a unanimous decision. I will define unanimity as requiring $\frac{\partial W_i}{\partial R} = \frac{\partial W_i}{\partial N} = \frac{\partial W_i}{\partial \pi} = 0$ for all $i = 1, \dots, N_0$, or negative for all i , and we see that all decisions are unanimous when each worker supplies an equal proportion of shares and hours of work, i.e. $\frac{a^i}{A} = \frac{l^i}{L}$ for all $i = 1, \dots, N$. The investment decision is unanimous also whenever $pf_K = \pi = r$, while the condition (13b) for level of employment always fulfils the criterion for unanimity. Ekern and Wilson (1974) have analysed the unanimity problem in an economy with incomplete markets. The shareholders of a firm will be unanimous in their investment decisions if the marginal return of an investment in a firm can be expressed as a linear combination of the returns of other firms, i.e., the set of state-distributions of returns available does not change because of this firm's decision, or if the shareholders value only the mean and variance of their portfolios. When $pf_K = \pi = r$, the set of returns in the economy are not affected if r represents the market rate of interest.

When this equality does not hold, we do not know whether state-distributions available will change without further specification of the capital market outside the firm, i.e. the alternative portfolio selections available. As pointed out by Ekern and Wilson, each individual will acquire the same fraction of each firm if they care about mean and variance of their portfolios only. This would in our model imply $\frac{a^i}{A} = \frac{l^i}{L}$, which is the condition given above. This means that the workers differ in scale of participation only, and the result corresponds to decision making in the Illyrian firm with variable labour input across individuals. However, the workers of a labour-managed and

-owned firm will generally value properties other than mean and variance of their portfolios. This will be further discussed in Section 4.

We will not necessarily have an interior solution in (13). As to the level of employment, it should be noted that there are no hired workers. Thus, if $N = 0$ also $L = 0$, and production is zero.

But we see from (13b) that an interior solution is possible where $N > N_0$. This occurs, e.g., when $y < pf_L$ and $\pi > pf_K$, from (13b), implying from (13a) that $\pi > r$ as well. In this situation the insiders, i.e. the original workers, gain in income by increasing the return to capital above its opportunity cost. The increase in employment level may be profitable for two reasons. Firstly $pf_K > r$ if there is credit rationing, which may induce employment of new workers supplying capital. Secondly it may be the result of strategical considerations, owing to differing relative labour and capital supply between insiders and outsiders (new workers). This will be further discussed in the comparative statics section.

Let us examine now the determination of share dividend.

The share dividend to be paid, π , is determined by (13c). We observe that

$$\frac{a^i}{A} > \frac{l^i}{L}, \quad i = 1, \dots, N,$$

for those workers whose earnings are increasing in π/y . Now, assume firstly that β_i is uniform and that $W_i = \bar{W}$ for all i . This means equal bargaining strength among the workers, which can be compared to a voting rule where there is one vote per worker, and that all workers expect the same income. Then the left-hand side of (13c) is necessarily zero, and $\pi \geq 0$ as the

workers are indifferent to π/y . However, when $a_i \neq a_j$ and/or $l_i \neq l_j$, $i, j = 1, \dots, N$, $i \neq j$, each worker's relative supply of labour and capital is weighed by the inverse of his income (see 13c). We shall show that $\pi > 0$ when β_i is uniform, i.e. the equivalence of one vote per worker. Assume $\pi = 0$, and income will be lowest for those workers whose parts of shares exceed their parts of labour supply. But then (13c) requires $\pi > 0$, a contradiction. Furthermore, as bargaining power (voting rules) is changed in favour of large shareholders, i.e. β_i is increased in $\frac{a^i}{A}$, π will have to increase further so as to meet condition (13c). Thus we can conclude that the bargaining among heterogeneous workers results in $\pi > 0$, where each worker's relative power may constitute the equivalence of one vote per worker, one vote per share, or any rule in between these.

It is reasonable to assume that new workers do not supply more share-capital per worker than the initial workers. Therefore the next question to pose is whether $y > 0$. Also this question can be answered by inspection of (13c). We find that if the initial workers are fairly homogeneous, with respect to labour supply as well as capital supply, and if the possible new workers supply on average more labour than capital, then (13c) may be strictly positive since $\frac{a^i}{A} > \frac{l^i}{L}$ may occur for $i = 1, \dots, N_0$, while $\frac{a^i}{A} < \frac{l^i}{L}$ for new workers, and π is increased beyond limits. The problem may be solved by assuming that the distribution of shares among initial workers is sufficiently inegalitarian. Another remedy is to require a sufficiently high level of s . The latter requirement corresponds to a rule operating in Mondragon, see Bradley and Gelb (1983).

So much for the technical explanation, which is, of course, contingent upon the special bargaining model chosen. A more general economic explanation should be given in terms of the strive to capture the economic rent, which we assume is to be distributed according to hours of work (labour supply) and shareholding (capital supply). The bargaining strength between those who supply relatively much labour and those who supply relatively much capital will determine the final outcome. But the resulting relative payout rates will affect capital and labour supply. Thus, as everyone has got the right to withdraw, a threatpoint is established, which in our model may be considered reflected in the voting rules. If some workers were able to force upon the collective say a $\pi = 0$, this would imply that internal share supply is not needed, and consequently $a^i = 0$ for all i . A model where the individual labour and capital supply decisions are taken explicitly into consideration when determining relative payout rates, can be found in Askildsen, Ireland and Law (1987).

Inspection of (13a) shows that whether $R \geq 0$ depends on the relation between π , pf_K and r . The following possibilities can be singled out:

- (i) $pf_K < \pi < r \Rightarrow R = 0$
- (ii) $pf_K, r > \pi \Rightarrow R \geq 0$
- (iii) $pf_K = r = \pi \Rightarrow R \geq 0$
- (iv) $pf_K, r < \pi \Rightarrow R \geq 0$

According to (14i), the firm has been allocated too much capital, and further internal finance will not be desired, leaving $R = 0$. (14ii) may represent a symptom of credit rationing, which may require $R > 0$. According to (14iii) the workers will be indifferent as to ways of finance, while R may be strictly positive from (14iv) because of the capital remuneration exceeding its opportunity cost.

We can substitute

$$\sum_i \frac{\beta_i}{W_i} \frac{l^i}{L} = \sum_i \frac{\beta_i}{W_i} \frac{a^i}{A}$$

from (13c) into (13a-b) as $(pf_K - \pi)$ and $(pf_L - y)\bar{l}$ are independent of i . From (5) and (7) we find that

$$\frac{a_0^i}{A_0} \geq \frac{a_i}{A} \text{ as } N \geq N_0$$

Assume $N = N_0$, i.e. the employment level is constant over the production period. We will illustrate how the π/y -ratio may be determined by bargaining among the original workers. From (13b) $pf_L = y$ as $g' = 0$. (13a) reduces to

$$(pf_K - r) \sum_i \frac{\beta_i}{W_i} \frac{a^i}{A} = 0.$$

However, we may have $pf_K \stackrel{\geq}{\leq} \pi$, depending on bargaining strength among the workers. Thus, using (13c), a high β_i for those workers whose relative capital supply is high, will induce high values of π .

Now, let us assume that there is an interior solution, i.e. $R > 0$, $N > N_0$ and $\pi > 0$. We find by substituting from (13a) into (13b) that

$$(15) \quad \frac{pf_L - y}{pf_K - \pi} = - \frac{g'm \frac{1}{L} \sum_i \frac{\beta_i}{W_i} l^i}{\bar{l} \frac{1}{L} \sum_i \frac{\beta_i}{W_i} l^i} = - \frac{A_0}{N_0} \frac{s}{\bar{l}}, \quad N > N_0.$$

It follows that

$$\text{sign}(pf_L - y) = - \text{sign}(pf_K - \pi)$$

Also, if $pf_L = y$, then $pf_K = \pi = r$, and (15) have to be met by l'Hopital's rule, which will again imply that

$$\left. \begin{array}{l} \text{and} \quad pf_{LL} \bar{l} = - pf_{KL} m \\ \text{and} \quad pf_{LK} \bar{l} = - pf_{KK} m \\ \text{and} \quad \frac{A}{A_0} s = \frac{L}{N_0 \bar{l}} \end{array} \right\} \text{assuming } pf_L = y \text{ and } pf_K = \pi = r.$$

We see from the above discussion that for any given level of π (share-dividend), the level of employment will fall short of or exceed the employment level of the Illyrian firm, given by $pf_L = y$, depending on whether pf_K (value marginal product of capital) falls short of or exceeds π . Thus, if there is credit rationing, we may have $pf_K > \pi$, and the employment level may be increased so as to provide more capital. On the other hand, if share dividend is very high, there will be a reluctance to recruit members sharing in the high capital remuneration. However, if $s = 0$, then from (13b) workers will be hired until $pf_L = y$, which is the Illyrian employment level.

(ii) Second Order Conditions.

Let us assume that there is an interior solution, i.e. $R > 0$, $N > N_0$ and $\pi > 0$. To simplify notation we will write

$$\lambda_{\gamma\theta} = \frac{\partial^2 U}{\partial \gamma \partial \theta} = \sum_{i=1}^{N_0} \left[\frac{\beta_i}{W_i} \frac{\partial^2 W_i}{\partial \gamma \partial \theta} - \frac{\beta_i}{(W_i)^2} \frac{\partial W_i}{\partial \theta} \frac{\partial W_i}{\partial \gamma} \right];$$

$\gamma = R, N, \pi; \theta = R, N, \pi, r, p.$

The complete second order differentials are reported in the Appendix. Generally it is hard to determine the signs of the second order differentials. The individuals' income will differ as well as their favoured changes in the different variables, cf. the terms $\frac{\partial W_i}{\partial \gamma} \frac{\partial W_i}{\partial \theta}$ in the Appendix.

A negative definite matrix of second order differentials is a sufficient condition for a maximum. Thus, we require that

$$\begin{aligned} |H_1| &= \lambda_{RR} < 0 \\ |H_2| &= \lambda_{RR} \lambda_{NN} - \lambda_{RN} \lambda_{NR} > 0 \\ |H_3| &= \lambda_{RR} \lambda_{NN} \lambda_{\pi\pi} - \lambda_{\pi R} \lambda_{NN} \lambda_{R\pi} + \lambda_{RN} \lambda_{N\pi} \lambda_{\pi R} \\ &\quad - \lambda_{\pi N} \lambda_{N\pi} \lambda_{RR} + \lambda_{R\pi} \lambda_{NR} \lambda_{\pi N} - \lambda_{\pi\pi} \lambda_{NR} \lambda_{RN} < 0 \end{aligned}$$

Inspection of the second order differentials will show that a maximum may exist, in which $N > N_0$, $R > 0$ and $\pi > 0$. On the other hand, if $pf_K = \pi = r$ and $pf_L = y$, we find from (15) and using l'Hopital's rule, that $|H_2| = |H_3| = 0$.

This case should be further investigated. Using the results derived from (15), we find that

$$f_{LL} L + f_{LK} A = 0,$$

which implies, from Euler's theorem for homogeneous functions, that the production function exhibits (locally) constant returns to scale in L and K, remembering that $f_{LK} = f_{LA}$ from (4). The result corresponds to the dividend maximising choices of K and L made by the Illyrian firm, where the firm's production function exhibits first increasing and then decreasing returns to scale, cf. e.g. Ireland and Law (1985). The existence of fixed costs represents a right-ward shift of the minimum average costs when the cost-curve is U-shaped, and thereby a higher production level.

We have shown above that all decisions are unanimous when all workers supply the same proportion of labour and capital. Remember that this must hold as an average for new workers. But then from (13c) the workers are indifferent to the value of π , or more generally the π/y -ratio, which may be fixed at any ratio without affecting any worker's income (utility). Furthermore, we find that $\lambda_{\pi N} = \lambda_{\pi\pi} = 0$, and the second order conditions are met for $\lambda_{NN} = 0$ only, which implies constant returns to scale and $|H_2| = |H_3| = 0$. This is again the Illyrian property of the firm. The content of the property is that the workers will agree on surplus distribution rules when they differ in scale of participation only. Then $pf_K = \pi = r$ and $pf_L = y$ will hold as π may be set equal to r initially.

(iii) Comparative statics

The comparative statics can be presented in general terms. Thus, writing $\theta = r, p$ respectively, we find:

$$(16a) \quad \frac{dR}{d\theta} = \left| \frac{1}{H_3} \right| \left[-\lambda_{R\theta} (\lambda_{NN} \lambda_{\pi\pi} - \lambda_{\pi N} \lambda_{N\pi}) - \lambda_{\pi\theta} (\lambda_{RN} \lambda_{N\pi} - \lambda_{NN} \lambda_{R\pi}) - \lambda_{N\theta} (\lambda_{R\pi} \lambda_{\pi N} - \lambda_{\pi\pi} \lambda_{RN}) \right]$$

$$(16b) \quad \frac{dN}{d\theta} = \left| \frac{1}{H_3} \right| \left[-\lambda_{R\theta} (\lambda_{N\pi} \lambda_{\pi R} - \lambda_{\pi\pi} \lambda_{NR}) - \lambda_{\pi\theta} (\lambda_{R\pi} \lambda_{NR} - \lambda_{N\pi} \lambda_{RR}) - \lambda_{N\theta} (\lambda_{RR} \lambda_{\pi\pi} - \lambda_{\pi R} \lambda_{R\pi}) \right]$$

$$(16c) \quad \frac{d\pi}{d\theta} = \left| \frac{1}{H_3} \right| \left[-\lambda_{R\theta} (\lambda_{NR} \lambda_{\pi N} - \lambda_{\pi R} \lambda_{NN}) - \lambda_{\pi\theta} (\lambda_{RR} \lambda_{NN} - \lambda_{NR} \lambda_{RN}) - \lambda_{N\theta} (\lambda_{RN} \lambda_{\pi R} - \lambda_{\pi N} \lambda_{RR}) \right]$$

The responses in (16) are ambiguous, cf. Appendix. However, by making some assumptions and simplifications, the signs can be determined in special cases.

Let us assume that the firm consists of two groups of workers, so that within each the workers are identical³⁾. Without loss of generality we can assume that group one consists of the workers with the highest W . This implies, see Appendix, that the expression $\sum_i \frac{\partial W_i}{\partial \theta} \frac{\partial W_i}{\partial \gamma}$, where $\theta, \gamma = R, N, \pi$, is non-negative. Furthermore, let $l^1 = l^2$, i.e. everyone works the same number of hours. Then higher shareholding is the reason for higher income. We will assume also that the workers with the higher W_i vote for the highest N, R and π , and that

$$\frac{\partial W_1}{\partial \theta} \frac{\theta}{W_1} = \frac{\partial W_2}{\partial \theta} \frac{\theta}{W_2}, \quad \theta = r, p,$$

i.e. the elasticities of income with respect to r and p are the same for both groups.

Some more simplifying assumptions will be briefly commented upon below. Let $\pi \geq pf_K$, let the production function exhibit constant returns to scale in A and L , and let s be fixed so that new workers supply less shares than the initial workers. Lastly we assume that there is a negligible difference between the two groups of insiders measured in terms of shares they hold.

With these assumptions, the signs of the second order differentials can be determined as reported in the Appendix.

Then we can derive the comparative static results shown in the table, where the signs of the changes are indicated (a ? denotes an indeterminate sign):

	p	r^1	r^2	r
R	?	-	+	?
N	?	-	+	?
π	?	+	-	?

The effect of a change in r has been split into two effects: r^1 shows the effect of a change in the internal discount factor, and r^2 represents the effect of a change in the rate of interest on borrowings, which is initially assumed equal to the internal discount factor.

The effect of a price change is indeterminate. There are two opposing effects, and it is not clear which one will dominate. Let us consider the employment reaction. We know that the Illyrian labour-managed firm react perversely to price changes. The same tendency may exist in this model because of fixed costs and an economic rent, whose existence is the reason for the reluctance to hire new workers. On the other hand, the insiders own relatively more capital than the outsiders (new workers). We have assumed $\pi > pf_K$, implying $y < pf_L$ from (15). Then the insiders will gain from hiring additional workers supplying the same amount of work, but less capital, as this increases the insiders' share in the increased total surplus. But the ambiguity exists although $\bar{B} = 0$ (fixed costs are zero). The reason can be found by considering the internal finance response (ΔR) simultaneously. In the comparative static expressions, the same elements which tend to reduce N tend to increase R. Thus, after a price increase, and increase in employment and an increase in internal finance may be considered substitutes.

As to the effect of a change in r , one comment should be made. An increase in the internal discount rate reduces the scale of operation, while an increase in fixed costs (r^2) increases the scale of operation. The latter corresponds to the effect of an increase in fixed costs in the Illyrian labour-managed firm. However, these signs may change if the outsiders are considerable capital suppliers, i.e. s is large.

In the comparative statics we have assumed a low s -value. It is probably reasonable to assume that new-comers supply less capital than those who have been with the firm for some time, as membership will generally imply some accumulation of capital (see Bradley and Gelb (1983) for a discussion of this in the Mondragon cooperatives). But remember that we may then run into problems as the π/y -ratio may be increased beyond limits. This problem can be avoided for a sufficiently smooth distribution of ownership rights, where the new workers own marginally fewer shares than the "poorest" insiders. Such a distribution of shares seems reasonable in a dynamical generation model, and it is probably not far from the reality in e.g. Mondragon (Bradley and Gelb (1983)). Also, if π tends to increase beyond limits, naturally it will be difficult to recruit new-comers supplying relatively few shares. Then there must be an upper limit for π , which in our model could be solved by fixing π in advance, so that $\pi > r$. But as noted above, the comparative static responses may be different for other values of s . This is due to the strategic considerations concerning employment of new workers, which can be shown more detailed by making different assumptions in the comparative statics.

Generally the comparative static responses depend on:

- $\pi \stackrel{>}{<} pf_K, r$.
- returns to scale, or a possible economic rent.
- distribution of shares, or in the special case where we have two groups of workers, degree of inequality between the groups.
- voting rules, i.e. distribution of β_i .
- labour and capital supply by new workers, i.e. the value of s .

However, what really matters, is how the insiders can make choices so as to capture as much as possible of the surplus, or an economic rent. This strive is being pursued along two dimensions: firstly the distribution among insiders, thereafter the consideration of advantages of recruiting more workers. In the special case when decisions are unanimous, we saw that no possibilities exist for exploiting relative differences as the workers differ from each other in scale of participation only. Therefore this firm is similar to the Illyrian firm with variable labour supply. The difference is that capital is here individually owned, rather than socially which is the case in the Illyrian firm. But this means that the model solves the problems connected with social ownership of capital, which it is well-known creates incentive problems.

Although we have in the preceding discussion been able to investigate closely rather special cases only, we have been able to draw attention to the close link between labour and capital supply, and in particular to the necessity of considering simultaneously relative payout rates, internal capital supply and possibilities of using outsiders of various categories to provide capital. The ultimate results depend on the functioning of the capital market and the strategic positions of the various groups within the firm.

4. The Individual Worker's Supply of Labour and Capital

Each worker chooses a^i and l^i so as to maximise utility. The individual labour and capital supply decisions will affect wage and share dividend in a manner which is well known from the literature on the internal labour market of cooperatives, see Sen (1966) and Ireland and Law (1981). Sen concentrated on the social optimality of a cooperative's allocation of labour, drawing attention particularly to the strategic decisions each worker will make because of his monopoly power. Thus, when deciding on labour supply, the worker will consider the marginal benefit of an extra hour of work as well as the effect this will

have on his total share in income. Then there will be equality between the value marginal product of labour and the marginal disutility of work only in special cases.

I will not at any length consider the efficiency aspects of the individual factor supply decisions. This is carefully discussed by e.g. Ireland and Law (1981) as far as the labour market is concerned. The monopoly aspects of the optimisation are valid correspondingly for the capital market. There is an efficient resource allocation only when there is full cooperation between the workers in fixing individual supply, or when an optimal payout rule is established, which requires some payment being made according to effort (contribution) and a share of total payout being distributed on a per capita basis, or when there are constant returns to scale.

I will assume that payout is made according to contribution (individual labour and capital supply), and that each worker determines his labour and capital supply. After briefly commenting on the efficiency properties of the optimal solution, I will concentrate on how attitude to risk, time preference and disutility of work affect labour and capital supply.

Each worker has a separable, additive utility function⁴⁾

$$(17) \quad u^i = u^i(Y^i + \tilde{a}^i) + v^i(l^i),$$

where Y^i is the present value of expected income and \tilde{a}^i is the net cost of buying shares. Marginal utility w.r.t. income and labour are respectively positive and negative. The utility function is strictly concave in $(Y^i + \tilde{a}^i)$ (risk aversion). Y^i and \tilde{a}^i are given by

$$(18) \quad Y^i = (\pi a^i + y l^i) \frac{1}{1+\alpha^i}$$

$$(19) \quad \tilde{a}^i = - \frac{\alpha^i}{1+\alpha^i} a^i$$

where we have assumed that the capital invested in the firm by each worker i can be provided from his/her own resources, with an opportunity cost α^i reflecting the time preference. We will later assume that α is identical for all i , and equal to a risk free market rate of interest. As the price is a random variable, the representative worker will maximise expected utility, i.e. choose a^i and l^i so that

$$Eu^i + v^i$$

is maximised.

Then the problem is

$$(20) \quad \text{Max}_{a^i, l^i} [Eu^i(Y^i + \tilde{a}^i) + v^i(l^i)]$$

given the definitions of Y^i and \tilde{a}^i in (18) and (19). π and y are defined and explained previously. Employment and borrowings are given exogenously. We assume a Nash-Cournot reaction function among the workers, i.e.

$$\frac{\partial A}{\partial a^i} = 1, \quad \frac{\partial \Sigma l^i}{\partial l^i} = 1, \quad \text{for all } i,$$

and

$$\frac{\partial A}{\partial l^i} = 0, \quad \frac{\partial \Sigma l^i}{\partial a^i} = 0, \quad \text{for all } i.$$

The first order conditions are, assuming that there is an interior solution so that $a^i > 0$, $l^i > 0$:

$$(21a) \quad Eu_Y^i \frac{\partial (Y^i + \tilde{a}^i)}{\partial a^i} = 0 \Rightarrow E \left\{ u_Y^i \left[\frac{l^i}{L} (pf_K - \pi) + \pi + \frac{\partial \pi}{\partial a^i} A \left(\frac{a^i}{A} - \frac{l^i}{L} \right) - \alpha^i \right] \right\} = 0$$

$$(21b) \quad E u_Y^i \frac{\partial Y^i}{\partial l^i} + v_1^i = 0 \Rightarrow E \left\{ u_Y^i \left[\frac{l^i}{L} (pf_L - y) + y + \frac{\partial \pi}{\partial l^i} A \left(\frac{a^i}{A} - \frac{l^i}{L} \right) \right] \frac{1}{1+\alpha^i} \right\} + v_1^i = 0$$

u_Y^i and v_1^i are the partial derivatives w.r.t. income and labour respectively. We assume $v_{11}^i < 0$, while $u_{1Y}^i = 0$.

From (21a-b) we see immediately that we will generally not have equality between value marginal products of capital and labour and the respective remunerations to capital (π) and labour (y). This indicates that the resource allocation is generally not efficient, cf. also Sen (1966). To illustrate, let us assume that each worker's utility function as well as the welfare function are risk neutral. Alternatively we can assume that all decisions are taken under certainty. Then resources are socially optimally allocated when $pf_K = \pi$, in which case $pf_K = \alpha^i$, and when $pf_L = y$, which implies equality between pf_L and $-v_1^i$. But this is exactly the case when the decisions of the firm are unanimous, see Section 3. We shall see that this is not a particularly likely situation in a firm consisting of heterogeneous workers bargaining to agree on payout rates.

Now, following Bonin (1977),⁵⁾ we can compare the allocation established by (21a,b) to its certainty equivalence. a^i and l^i , and thereby income ($Y^i + \tilde{a}^i$), are given by (21a,b). Write $\frac{\partial(Y^i + \tilde{a}^i)}{\partial z}$, $z = a^i, l^i$, as $g_z(a^i, l^i)$. The price that can be obtained is the source of the income uncertainty. The difference between the uncertain environment and the certainty equivalence is reflected by the covariance between marginal utility of income and the uncertain price. The sign of this covariance is dependent on the attitude to risk. We have assumed that worker i is risk averse, i.e. $u_{YY}^i < 0$. Then

$$(22) \quad \text{cov} (U_Y^i, p) < 0.$$

Using the laws of variance, we can write (21a,b) as

$$(23a) \quad E u_Y^i E g_a(a^i, l^i) + \text{cov} (u_Y^i, p) = 0$$

$$(23b) \quad Eu_Y^i Eg_1(a^i, l^i) + \text{cov}(u_Y, p) + v_1^i = 0$$

$\frac{\partial g_a}{\partial a^i} < 0$ and $\frac{\partial g_l}{\partial l^i} < 0$ from second order conditions. The signs of

$$\frac{\partial g_a}{\partial l^i} = \frac{\partial g_l}{\partial a^i} \text{ (Young's theorem) depend on whether}$$

$$(24) \quad \frac{l^i}{L} pf_{LK} + (pf_K - \pi) \frac{L - l^i}{L^2} + \frac{\partial^2 \pi}{\partial l^i \partial a^i} A \left(\frac{a^i}{A} - \frac{l^i}{L} \right) + \frac{\partial \pi}{\partial l^i} \\ \left(1 - \frac{l^i}{L} \right) \begin{matrix} > \\ < \end{matrix} 0$$

Note from (21) and (24) that the property $\frac{\partial g_a}{\partial l^i} = \frac{\partial g_l}{\partial a^i}$ implies $\frac{\partial \pi}{\partial a} = - \frac{\partial \pi}{\partial l^i} \frac{L}{A}$. In the unanimity scenario (24) reduces to $\frac{l^i}{L} Pf_{LK} > 0$.

Now, we can divide through by Eu_Y^i in (23a,b). Then we see that the higher is $\text{cov}(u_Y, p)$, the higher is Eg_z , $z = a^i, l^i$. Using the second order conditions, this means that l^i and a^i are reduced if there is a marginal increase in risk aversion, i.e., if we are comparing two different situations, where everything is identical except from the worker's attitude to risk, the more risk averse type will supply the lowest quantities of labour and capital. However, the magnitude of the changes in supplies depends on v_{11}^i and $\frac{\partial g_a}{\partial l^i}$. Firstly, the higher is the absolute value of v_{11}^i , the lower is the reduction needed in l^i to bring about equality in (23b) following upon an increase in risk aversion. Secondly, $\frac{\partial g_a}{\partial l^i}$ may become negative if e.g. π is sufficiently high, $\frac{l^i}{L} > \frac{a^i}{A}$ and/or $\frac{\partial \pi}{\partial l^i} < 0$. Then the reduction in a^i needed to bring about equality in (23a) is being reduced given that l^i is reduced from (23b), and likewise for l^i .

The effect of the disutility of work on labour supply should be well known. But capital supply may be affected as well from varying disutility of work, although we cannot a priori state in

which direction. This will depend on whether the two variables are complements or substitutes. In the former case, the tendency to reducing l^i will be somewhat offset by the reduction in a^i .

The effects on labour and capital supply of varying time preference, α^i , are found by doing comparative statics on the first order conditions in (21). To simplify, and avoid too much ambiguity, we assume that the worker is risk neutral. Then u_Y^i is constant across states of nature, and we do not need to discuss how the covariance term in (23) is affected by changes in the variables. We denote the differentials by $\lambda_{\beta, \tau}$; $\beta = a^i, l^i$; $\tau = a^i, l^i$. $\lambda_{\alpha\alpha}$ and λ_{ll} are negative from second order conditions. The signs of $\lambda_{al} = \lambda_{la}$ are briefly discussed above. $\lambda_{l\alpha}$ and $\lambda_{a\alpha}$ are negative.

The effects of changes in α^i are given by

$$(25) \quad \begin{bmatrix} \lambda_{aa} & \lambda_{al} \\ \lambda_{la} & \lambda_{ll} \end{bmatrix} \begin{bmatrix} da \\ dl \end{bmatrix} = \begin{bmatrix} -\lambda_{a\alpha} \\ -\lambda_{l\alpha} \end{bmatrix}$$

Then we find that

$$(26) \quad \left. \frac{da^i}{d\alpha^i} = \frac{-\lambda_{a\alpha}\lambda_{ll} + \lambda_{al}\lambda_{l\alpha}}{D} < 0 \right\} \text{ if } \begin{cases} (24) \text{ is non-negative} \\ (24) \text{ is negative} \end{cases}$$

where $D = \lambda_{aa}\lambda_{ll} - \lambda_{al}\lambda_{la} > 0$ from second order conditions for a maximum. $\frac{da^i}{d\alpha^i}$ is negative e.g. in the unanimity scenario. But when a^i and l^i are substitutes, a counteracting force exists, which makes the net result indeterminate. This is the case also for $\frac{dl^i}{d\alpha^i}$:

$$(27) \quad \left. \frac{dl^i}{d\alpha^i} = \frac{-\lambda_{aa}\lambda_{l\alpha} + \lambda_{a\alpha}\lambda_{la}}{D} < 0 \right\} \text{ if } \begin{cases} (24) \text{ is non-negative} \\ (24) \text{ is negative} \end{cases}$$

Before concluding, let us investigate the conditions for unanimous decision-making somewhat more closely. From Section 3 we remember that a sufficient condition for unanimity is $\frac{a^i}{A} = \frac{l^i}{L}$, which corresponds to Ekern and Wilson's (1974) result for unanimity among stockholders when their preferences can be represented by a mean-variance model. To see whether this condition may be met in our model, let us rewrite the first order conditions. Using (23), (24) and the definitions of $Eg_a(a^i, l^i)$ and $Eg_l(a^i, l^i)$ we can rewrite (21) as

$$(28a) \quad E\left[\frac{l^i}{L}(pf_K - \pi) + \pi - \frac{\partial \pi}{\partial l^i} L\left(\frac{a^i}{A} - \frac{l^i}{L}\right)\right] + \frac{\text{cov}(u_Y, p)}{Eu_Y} = E\alpha^i$$

$$(28b) \quad E\left[\frac{l^i}{L}(pf_L - y) + y + \frac{\partial \pi}{\partial l^i} A\left(\frac{a^i}{A} - \frac{l^i}{L}\right)\right] + \frac{\text{cov}(u_Y, p)}{Eu_Y} = \omega^i$$

where $\omega^i = -\frac{v_l^i}{Eu_Y} > 0$ is the marginal rate of substitution between leisure and expected income, i.e. the implicit price of leisure or opportunity cost of labour. It is reasonable to assume that α^i represents the return to a riskless asset which is available to all workers. Then $E\alpha^i = \bar{\alpha}$ for all i . We may assume that $\bar{\alpha} = r$.

Obviously $\frac{a^i}{A} = \frac{l^i}{L}$ represents one possible solution to (28). But these equilibrium ratios are not more likely than any other possible ratios where the equality condition is not met. There are two reasons for this. Firstly we see that ω^i will generally vary across individuals when they are heterogeneous, and the workers will have differing opportunity costs for "investment" in the "asset" labour. Secondly, we observe that the workers take "strategic" considerations as well (monopoly power), by valuing the effect their decisions have on the dividend rates π and y . Thus, our model cannot be considered a pure mean-variance model, and we should not expect the Ekern-Wilson result to hold.

On the other hand, $\frac{a^i}{A} = \frac{l^i}{L}$ may hold by coincidence if ω^i varies positively with $\frac{l^i}{L}$ across individuals. Apart from this situation the condition is met only when the workers are homogeneous.

5. Concluding Remarks

In the preceding sections I have investigated the functioning of a labour-managed and -owned firm consisting of heterogeneous workers supplying labour and capital. The model chosen was rather general, so as to be able to see how various assumptions may influence the results. The price to be paid for this generality is a lack of clearcut results in characterisations and in the comparative static analysis. However, by making certain simplifying assumptions, some special cases could be analysed. In particular we were able to draw attention to the close link which will exist between internal supply of capital and the demand for labour and capital services from potential new workers (outsiders). The outsiders are allowed into the firm in so far as they contribute to increase welfare of the initial work-force.

An analytically simplifying assumption made is the separation of the individual supply decisions and the firm's allocational decisions. I have assumed that the firm takes each worker's labour and capital supply as given, and optimises with respect to total labour and capital to be used. A slightly more sophisticated procedure would be to allow for individual adjustments depending on the collective decisions. If this were to be done in the general context chosen here, we would, however, impose further losses in terms of clearcut results. On the other hand, by making proper choices of the functional forms to be used, we may be able to consider several interesting cases. This is obviously the way to follow in further research in this field. For one such example, see Askildsen, Ireland and Law (1987).

I have chosen to investigate a collusive model of the firm. This can be justified by the long term perspective of the analysis. We could instead have assumed a competitive scenario where each worker casts his vote for the desired values of the endogenous variables according to his individual preferences. Then we would have to concentrate on defining the median worker, his optimal strategy and the restrictions facing him in his

decisions. We see from the simplified analysis of Section 4 that in this case the median worker's attitude to risk, time preference etc. will be of crucial importance to the firm's labour and capital allocation decisions.

The main purpose of investigating the specific model chosen was to represent a possible way of organising labour-managed firms in (Western) capitalist economies. In many lines of business we will find workers owning larger or smaller parts of the shares in their firms. This may develop into firms which can be denoted labour-managed. Of course, as the models to be investigated will be something in between the classical capitalist and the Illyrian labour-managed models of the firm, several formulations can be chosen, and different aspects discussed, which are all of interest. In that sense this model is only one special case. In particular the assumptions made concerning capital supply from initial workers and new workers will be crucial. I have assumed here that the workers act rather uniformly and mechanically when capital supply decisions are to be taken during the lifetime of the firm. The reason is that I have been concerned mainly with the collective decisions on the level of capital and employment, and that it is not unreasonable to assume that e.g. new workers must adjust to established rules. This is the case in Mondragon, where empirical observations seem to confirm that there is a queue of potential new workers allowing the insiders to put certain screening mechanisms into effect, see Bradley and Gelb (1983). The model highlights also the importance of the decisions taken concerning how to share the surplus of the firm. The efficiency aspect of this is additional to the problems caused by inefficient monitoring and the lack of proper incentive schemes in "traditional" labour-managed firms. However, some problems (the horizon problem) may be solved by the introduction of individual ownership of capital. Other problems may, however, develop, e.g. distributional problems.

It is important to note that the actual structure of a firm will depend on its historical background. Then I think it is of

importance to draw attention to heterogeneity in the workers' capital supply, because this is an aspect I believe is central to a labour-managed firm operating in a capitalist environment. As commented upon before, the reason for this may be credit rationing. There may be other reasons also why workers would like to invest in their own firm. Although risk-taking is being increased by investing in the same firm as the one where the worker's human capital is invested, the worker may find this profitable, and it may be advantageous for the collective to reduce its dependence on external financiers. It is a well known feature of different profit sharing schemes, and we often observe that the managers of firms own a considerable stock of shares in their own firm. But we see that the individual ownership of capital introduces several strategic as well as allocational considerations, which may differ significantly from problems facing capitalist profit-maximising firms and Illyrian labour-managed firms. My intention was to draw attention to some of these problems, and indicate also how these can be further investigated in more special models. The real problem at stake, however, is how to capture a possible economic rent, or generally how each participant can get as large a share as possible in the total surplus. In this respect the model investigated here does not deviate from models of traditional labour-managed firms, trade unions and joint-stock firms.

Footnotes

- 1) These are simplifications made for the matter of exposition. A number of different possibilities and stories could be considered. In particular, share capital and the distribution of shareholding may change even if membership is constant. This would happen if leaving workers were replaced by new workers with different time preference, attitude to risk etc., which might induce them to supply diverging amounts of capital (and labour). Furthermore, some members might leave without being replaced. However, in this case different assumptions as to the distribution of shares can be discussed within the framework of the model.
- 2) If π were to be the maximised for any value of y , this should be written symmetrically as

$$(10') \quad \pi = \frac{pf(K,L) - r\bar{B} - yL}{A}.$$

- 3) If there are two groups of workers, and the workers are homogeneous within each group, the last elements of the second order derivatives in A.1 - A.3 may be written as:

$$\sum_i \frac{\beta_i}{(w_i)^2} \frac{\partial w_i}{\partial \gamma} \frac{\partial w_i}{\partial \theta} = \left[\frac{\beta_1}{w_1} \frac{1}{w_1} \frac{\partial w_1}{\partial \gamma} \frac{\partial w_1}{\partial \theta} + \frac{\beta_2}{w_2} \frac{1}{w_2} \frac{\partial w_2}{\partial \gamma} \frac{\partial w_2}{\partial \theta} \right]$$

Now, from the first order conditions for an interior solution we know that

$$\frac{\beta_1}{w_1} \frac{\partial w_1}{\partial \gamma} = - \frac{\beta_2}{w_2} \frac{\partial w_2}{\partial \gamma}$$

Then we can write the expression above as

$$* \frac{\beta_1}{w_1} \frac{\partial w_1}{\partial \gamma} \left(\frac{\partial w_1}{\partial \theta} \frac{1}{w_1} - \frac{\partial w_2}{\partial \theta} \frac{1}{w_2} \right)$$

When $\theta = R, N, \pi$, we know that $\text{sign} \frac{\partial w_1}{\partial \theta} = - \text{sign} \frac{\partial w_2}{\partial \theta}$.

We can assume, without loss of generality, that group 1 consists of the "richest" workers (highest W). The sign of $*$ is dependent on which group of workers that wants the highest values of R , N and π , and on which group that is at most affected by changes in r and p . Assume e.g. that the workers with the highest W want the highest R , N and π . Then $*$ is positive when $\theta = R, N$ and π . On the other hand, when $\theta = r, p$, the sign of $*$ is given by

$$\frac{\partial W_1}{\partial \theta} \frac{\theta}{W_1} - \frac{\partial W_2}{\partial \theta} \frac{\theta}{W_2},$$

i.e. the difference in elasticity of income w.r.t. r and p will determine the sign.

- 4) The assumption of additive separability has been made so as to make the problem more tractable by eliminating the effect of labour supply decisions on the degree of risk aversion.
- 5) In Bonin (1977), Section 3, the author discusses the effect on labour allocation of price and production uncertainty on a collective farm where workers can allocate labour to the collective farm and a private plot. The main difference between our analysis and that of Bonin, is the introduction of a financial instrument provided by the workers themselves. An approach similar to Bonin's is taken by Ireland (1981) in his analysis of the disutility of supplying labour services. By explicitly modelling the Arrow-Pratt risk premium (Section V), the worker's labour supply is compared to the certainty case for the labour-managed firm as well as for the entrepreneurial firm. Both analyses conclude that individual labour supply (effort) is decreased when uncertainty is introduced, when the workers are risk averse.

Appendix

$\lambda_{\gamma\theta}$; $\gamma = R, N, \pi$ and $\theta = R, N, \pi, r, p$, is defined in the text.

The second order differentials are; where indication of signs is valid for the simplifying assumptions made in the text:

$$(A.1a) \quad \lambda_{RR} = \sum_i \left\{ \frac{\beta_i}{w_i} \frac{l^i}{L} pf_{KK} - \frac{\beta_i}{(w_i)^2} \left(\frac{\partial w_i}{\partial R} \right)^2 \right\} < 0 \quad (\text{second order conditions})$$

$$(A.1b) \quad \lambda_{RN} = \sum_i \left\{ \frac{\beta_i}{w_i} \frac{l^i}{L} [pf_{KK} m - \bar{l} (pf_K - \pi) + pf_{KL} \bar{l}] - \frac{\beta_i}{(w_i)^2} \frac{\partial w_i}{\partial R} \frac{\partial w_i}{\partial N} \right\} > 0.$$

$$(A.1c) \quad \lambda_{R\pi} = \sum_i \left\{ \frac{\beta_i}{w_i} \left(\frac{a_o^i}{A_o} - \frac{l^i}{L} \right) - \frac{\beta_i}{(w_i)^2} \frac{\partial w_i}{\partial R} \frac{\partial w_i}{\partial \pi} \right\} > 0.$$

$$(A.1d) \quad \lambda_{Rr} = \sum_i \left\{ - \frac{\beta_i}{w_i} \frac{a_o^i}{A_o} - \frac{\beta_i}{(w_i)^2} \frac{\partial w_i}{\partial R} \frac{\partial w_i}{\partial r} \right\} < 0$$

$$(A.1e) \quad \lambda_{Rp} = \sum_i \left\{ \frac{\beta_i}{w_i} \frac{l^i}{L} f_K - \frac{\beta_i}{(w_i)^2} \frac{\partial w_i}{\partial R} \frac{\partial w_i}{\partial p} \right\} > 0$$

$$(A.2a) \quad \lambda_{NR} = \sum_i \left\{ \frac{\beta_i}{w_i} \frac{l^i}{L} [pf_{LK} \bar{l} + m pf_{KK} - \bar{l} (pf_K - \pi)] - \frac{\beta_i}{(w_i)^2} \frac{\partial w_i}{\partial N} \frac{\partial w_i}{\partial R} \right\} > 0.$$

$$(A.2b) \quad \lambda_{NN} = \sum_i \left\{ \frac{\beta_i}{w_i} \left[\frac{l^i}{L} (pf_{LL} \bar{l} + m(pf_{LK} + m pf_{KK} + pf_{KL} \bar{l})) \right] - \frac{\beta_i}{(w_i)^2} \left(\frac{\partial w_i}{\partial N} \right)^2 \right\} < 0 \quad (\text{second order conditions}).$$

$$(A.2c) \quad \lambda_{N\pi} = \sum_i \left\{ \frac{\beta_i}{W_i} \left[\frac{l^i}{L} \left(\frac{A}{L} \bar{l} - m \right) \right] - \frac{\beta_i}{(W_i)^2} \frac{\partial W_i}{\partial N} \frac{\partial W_i}{\partial \pi} \right\} > 0.$$

$$(A.2d) \quad \lambda_{Nr} = \sum_i \left\{ \frac{\beta_i}{W_i} \frac{l^i}{L^2} \bar{l}B - \frac{\beta_i}{(W_i)^2} \frac{\partial W_i}{\partial N} \frac{\partial W_i}{\partial r} \right\} > 0.$$

$$(A.2e) \quad \lambda_{Np} = \sum_i \left\{ \frac{\beta_i}{W_i} \frac{l^i}{L} \left[f_L \bar{l} - \frac{x}{L} \bar{l} + m f_K \right] - \frac{\beta_i}{(W_i)^2} \frac{\partial W_i}{\partial N} \frac{\partial W_i}{\partial p} \right\} < 0$$

$$(A.3a) \quad \lambda_{\pi R} = \sum_i \left\{ \frac{\beta_i}{W_i} \left(\frac{a_0^i}{A_0} - \frac{l^i}{L} \right) - \frac{\beta_i}{(W_i)^2} \frac{\partial W_i}{\partial \pi} \frac{\partial W_i}{\partial R} \right\} > 0$$

$$(A.3b) \quad \lambda_{\pi N} = \sum_i \left\{ \frac{\beta_i}{W_i} \frac{l^i}{L} \left(\frac{A}{L} \bar{l} - m \right) - \frac{\beta_i}{(W_i)^2} \frac{\partial W_i}{\partial \pi} \frac{\partial W_i}{\partial N} \right\} > 0.$$

$$(A.3c) \quad \lambda_{\pi\pi} = - \sum_i \frac{\beta_i}{(W_i)^2} \left(\frac{\partial W_i}{\partial \pi} \right)^2 < 0.$$

$$(A.3d) \quad \lambda_{\pi r} = - \sum_i \frac{\beta_i}{(W_i)^2} \frac{\partial W_i}{\partial \pi} \frac{\partial W_i}{\partial r} = 0.$$

$$(A.3f) \quad \lambda_{\pi p} = - \sum_i \frac{\beta_i}{(W_i)^2} \frac{\partial W_i}{\partial \pi} \frac{\partial W_i}{\partial p} = 0.$$

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Some Consequences of Differential Shareholdings among Members

in a Labour-Managed and Labour-Owned Firm*

by

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I. INTRODUCTION

Institutional diversity among labour-managed and participatory firms is partly a reflection of differences in the economic environment when they were established and, in part, derives from differences in the motivations of the founders. Thus, for instance, a cooperative founded to provide an infrastructure for traditionally self-employed artisans could not be considered as the same kind of firm with the same market behaviour as a cooperative formed by the management and other employees "buying-out" the previous capitalist owners. Although such extreme examples are likely to differ in many aspects of behaviour, we shall only be concerned here with those arising from one structural factor. Our analysis will focus on the behaviour of a participatory firm which is partly financed by worker-members' shares. The structural factor in question is that of the distribution of the ownership of the equity of the firm among its members and the form of power or voting basis within the firm. We will concentrate on issues arising from these matters and attempt to exclude other factors as far as possible.^{1/} The firm's decision-making process is modelled as a sequential game and for reasons of tractability we are forced to adopt simple functional forms for utility and production functions.

A member's influence on his firm's behaviour may have much or little to do with the size of his shareholding. For instance, many traditional cooperatives formally allocate one vote to each member. Of course, actual influence may to some extent reflect seniority or skill level which may be related to shareholding. On the other hand, a cooperative formed from buying out the previous (capitalist) ownership may well operate as a private limited company and the management may act as agents for the members as shareholders rather than workers since members have votes according to the number of shares they hold. Many participatory

firms may adopt a voting system, de jure or de facto, which is intermediate between one-member-one-vote and one-share-one-vote.

Of course the voting system may not give significant power to any one member, but, in so far as majority viewpoints hold sway, so the voting system will determine firm behaviour. For this topic to be of interest it is necessary that members differ in their views. The origin of this diversity is assumed to lie in an unequal distribution of shareholdings among members. Where members differ in share ownership we may expect that divergent views on firm policy will arise and that such conflicts may have to be resolved by a voting process.

It has often been argued that cooperatives will prefer to raise capital from external sources rather than from members. The reasons suggested are either related to property rights problems (the horizon problem, see Jensen and Meckling (1979)), or to the concentration of risk involved in members having savings invested in their own firm. Nevertheless, external finance is unlikely to be available unless members' equity holdings provide both a buffer to reduce risk and a measure of the cooperative's self-confidence. It has been suggested that external capital may be provided if the return on this is linked to members' remuneration. Thus risk participation bonds (see Vanek, 1977, Ch.11 and McCain, 1977) provide incentives to members which are compatible with external financiers' interests. However, the problem arises of the direct influence of the financiers on the firm. This would be particularly apparent if there were just one or two financiers involved - a likely event if information acquisition by the financiers implied the existence of a fixed cost to capital supply. The tendency would be for the external financiers to exert influence on the firm's behaviour, perhaps prompting higher

employment or different work practices (see Ireland and Law, 1982, pp.51-3).

In this paper we assume that equity capital is all held by members and that this allows the firm to borrow from external sources up to a limiting gearing ratio. Beyond this point external finance becomes inordinately expensive. In order to acquire sufficient finance in total, an internal factor market exists to adjust the relative payment to shares and labour. For simplicity this relative payment is taken as fixed ex ante of the trading position of the firm being revealed.^{2/} The absolute payments reflect this trading position - in particular the revealed market price which was uncertain at the time of share purchase.

The internal factor market can be affected by changes in membership, since these will generally change the distribution of shareholding, particularly the mean propensity to purchase shares. Relative factor payments then respond to equilibrate the internal factor market. For example, if recruiting new members led to a greater positive skew in the distribution of shareholdings, since few new members were likely to make large share purchases, then capital would become relatively more scarce and the return to shares would increase relative to that paid to labour.

Thus a link exists between the behaviour of the firm (for example, the number of members), responses to a changing economic environment, (for example, a response to a demand change), and the power basis of decision-making, in terms of both the effects on, and the consequences of, the distribution of shareholdings. This paper is an initial attempt to investigate this link. Section II presents the model of the participatory firm and Section III shows how the firm's behaviour is

affected by the distribution of shareholding. Section IV comprises two extensions. The first explores the possibilities for cartels and other monopsonistic phenomena to arise within the internal factor market, and in the second the assumption of a limiting gearing ratio is relaxed. Section V presents conclusions and some speculation on future directions of research.

II. THE MODEL

We will assume that the firm is composed of N members who all work in the firm for a wage $\$w$ and supply capital to the firm, by buying dollar shares at the beginning of the production period for which they receive a value $\$v$ per share at the end of the production period. Members have no other residual interest in the firm. Thus, at the end of the production period, a member who has bought s_1 shares receives an income of $\$(w + vs_1)$, which he then consumes. At the beginning of the period he is endowed with a wealth of $\$m_1$, so that he has $\$(m_1 - s_1)$ for consumption during the production period. Both w and v are stochastic when viewed from the beginning of the production period as they will depend on demand conditions, represented by a parametric product price p which is a random variable with mean \bar{p} and variance σ^2 .

Workers are assumed to invest in the firm the amounts s_1 which maximise the expected value of their utilities.^{3/} The i th member's utility is assumed to take the specific form

$$U_1 = T\{C_1 C_2\}, \quad T'\{\cdot\} > 0, \quad T''\{\cdot\} \leq 0 \quad (1)$$

where C_1 is consumption during the production period, and C_2 is consumption after the production period. Thus

$$U_i = T\{(m_i - s_i)(w + vs_i)\} \quad (1')$$

which we can also write as

$$U_i = T\{(m_i - s_i)(1 + \psi s_i)w\} \quad (1'')$$

where $\psi = v/w$ is non-stochastic, representing the ratio of payments to shares and labour in the manner of a risk-participating bond, and w (and of course v) is stochastic, depending on the revelation of product price. Both w and v are assumed independent of member i 's decision over s_i , so that no one member has any monopoly power. Optimal s_i for member i is that number of shares which maximises expected utility where utility is given by (1''). Thus s_i satisfies

$$E\{T'\{\cdot\}w(-1 + \psi m_i - 2\psi s_i)\} = 0 \quad (2)$$

which implies, since ψ , m_i and s_i are not stochastic and $T'\{\cdot\} > 0$, $w > 0$, that^{4/}

$$s_i = \frac{1}{2}(m_i - 1/\psi) \quad (3)$$

The average internal supply of finance per member is then \bar{s} where

$$\bar{s} = \frac{1}{2}(\bar{m} - 1/\psi) \quad (4)$$

where \bar{m} is the arithmetic mean of the wealth levels of members.

Now consider the demand for members' capital. Suppose that the technology of the firm satisfies the following properties:

(i) the production function is $F(N, K)$ where N is membership and K is capital.

(ii) the production function can be written as $Q(Z)$ where $Z = \min(aN, bK)$.

(iii) $Q(0) = 0$, $Q'(Z) > 0$, $Q''(Z) \leq 0$ as $Z \leq Z_1$

(iv) $Q'(Z) = Q(Z)/Z$ at $Z = Z^*$.

Thus the production function is homothetic with zero elasticity of substitution. The required capital per man for full employment of both factors of production is thus a/b . Suppose that for every dollar of members' share capital raised, λ of loan from outside financiers can be obtained. Such loans are repaid at the end of the production period at a cost of r per dollar borrowed. Thus an average of k of equity capital must be raised from the members, where k satisfies:

$$(1 + \lambda)k = a/b \tag{5}$$

and the repayment for all capital raised by the firm (v per dollar share purchased, plus r per dollar of debt finance obtained, to provide a/b dollars of capital per member for N members) is

$$(v + \lambda r)kN$$

We are assuming that the firm wishes to take up all of its external

borrowing capacity up to the limiting gearing ratio of $\lambda.5/$ From (4), equating k and \bar{s} , yields

$$\psi = (\bar{m} - 2k)^{-1} \quad (6)$$

so that

$$s_i = \frac{1}{2}(m_i - \bar{m} + 2k) \quad (7)$$

$$m_i - s_i = \frac{1}{2}(m_i + \bar{m} - 2k) \quad (8)$$

and $1 + \psi s_i = \frac{1}{2}(m_i + \bar{m} - 2k)/(\bar{m} - 2k) \quad (9)$

Finally, consider the stochastic wage w . The factor returns, w and v , are defined by

$$w = pQ(aN)/N - (v + \lambda r)k \quad (10)$$

which simply states that the wage is equal to average revenue per member minus total capital cost per member. (10) can be rewritten, using (6), as

$$\begin{aligned} w &= pQ(aN)/N - \lambda r k - w k / (\bar{m} - 2k) \\ &= (pQ(aN)/N - \lambda r k) \frac{(\bar{m} - 2k)}{(\bar{m} - k)} \end{aligned} \quad (11)$$

Now define $Q(aN)/N = q(N)$ and note that $q'(N) \geq 0$ as $N \leq N^* = Z^*/a$ from (iv) above. Then, using (8), (9) and (11) in (1'') yields

$$U_i = T \left\{ \frac{1}{4} \frac{(m_i + \bar{m} - 2k)^2}{(\bar{m} - k)} (pq(N) - \lambda rk) \right\} \quad (12)$$

As is well known (see Ireland and Law, 1984), in the absence of any effects of varying membership on the distribution of m , each individual will prefer that membership level which maximises $q(N)$, i.e. where $q'(N) = 0$ and $N = N^*$. Membership is independent of price, and marginal changes in the mean or variance of price have no effect on optimal membership, capital and output levels. It is, however, highly likely that changing membership will have implications for the distribution of wealth among the members. The nature of these implications and their influence on firm behaviour are the subject of the rest of this paper.

III. SHARE DISTRIBUTION

It is to be expected that the distribution of the members' wealth (m_i) will depend on the size and nature of the firm, and can be influenced by changes in membership through recruitment or retirement of workers. We will assume that $d\bar{m}/dN < 0$, and hence consider the equilibrium level of firm membership. The assumption can be justified in several ways. If new members are being recruited, it is reasonable to assume that they have lower wealth than older workers who may have stayed with the firm for several periods. Thus, workers who retire are likely to have accumulated some capital during their tenure period with the firm, and the decision whether to replace them with a greater, equal, or smaller number of

new recruits will affect average share supply per worker. When considering the relation between \bar{m} and the size of the firm (N) in a static context we may assume that the firm has initially a given membership, but that adjustments can be made before production takes place. Only when membership and share purchases are in equilibrium will production occur.

The extent of membership reduction that can take place may be limited to not replacing any workers who have just retired, if members cannot be dismissed. In this case full adjustment to the membership equilibrium may not be possible. However, the unconstrained membership equilibrium is still relevant in that it indicates the direction of adjustment that will occur.

For simplicity, we will first take the case where individuals are not risk-averse ($T'[\cdot]$ is a constant) or, equivalently, where there is no risk ($\sigma^2 = 0$). Having analysed this case, risk and members' risk-aversion will then be introduced.

(1) The Certainty Case

Consider the i^{th} individual member with wealth parameter m_i . This member's utility is maximised if the membership level is N_i which satisfies

$$\frac{dU_i}{dN} = \frac{\partial U_i}{\partial \bar{m}} \frac{d\bar{m}}{dN} + \frac{\partial U_i}{\partial N} = 0 \quad (13)$$

Differentiating (12), given that $\sigma^2 = 0$ or $T'[\cdot] = 0$, implies that (13) requires

$$(\bar{m}-m_i)(pq(N) - \lambda rk) \frac{d\bar{m}}{dN} + (m_i + \bar{m} - 2k)(\bar{m} - k) pq'(N) = 0 \quad (14)$$

Now given that $pq(N) - \lambda rk > 0$ (else $w \leq 0$ from (11)), $d\bar{m}/dN < 0$ by assumption, $m_i + \bar{m} - 2k > 0$ (else $s_i \leq 0$), and $\bar{m} - k > 0$ (for $U_i > 0 \forall i$), $(\bar{m} - m_i)$ and $q'(N)$ must have the same sign. Also second-order conditions will hold if $q''(N)$ is sufficiently negative, whatever the form of $d^2\bar{m}/dN^2$.

We may also consider the comparative static responses of individual i 's optimal membership level N_i to changes in product price and the cost of outside capital (r). The signs of these responses are given by the derivatives of (14) with respect to p and r respectively. Collecting results, we have that:

$$\left. \begin{array}{ll} \text{if } m_i < \bar{m}, & N_i < N^*, \quad dN_i/dp < 0, \quad dN_i/dr > 0 \\ \text{if } m_i = \bar{m}, & N_i = N^*, \quad dN_i/dp = 0, \quad dN_i/dr = 0 \\ \text{if } m_i > \bar{m}, & N_i > N^*, \quad dN_i/dp > 0, \quad dN_i/dr < 0 \end{array} \right\} \quad (15)$$

These results reflect the membership level desired by the i th member. The question of the way in which actual decisions are made has not yet been considered. In this respect it is important to establish that N_i and m_i are co-monotonic, i.e. that if $m_i > m_j$ then $N_i > N_j$. To see this it is sufficient to show that $dN_i/dm_i > 0$. Using (14) again, it is clear that dN_i/dm_i has the same sign as

$$-(pq(N) - \lambda rk) \frac{d\bar{m}}{dN} + (\bar{m} - k) pq'(N) \quad (16)$$

and, substituting out $q'(N)$ in (16) from (14), this simplifies to

$$-(pq(N) - \lambda rk) \frac{d\bar{m}}{dN} + \frac{(m_i - \bar{m})}{(m_i + \bar{m} - 2k)} (pq(N) - \lambda rk) \frac{d\bar{m}}{dN}$$

which, since $d\bar{m}/dN < 0$, has the sign of

$$\frac{2(\bar{m} - k)}{(m_i + \bar{m} - 2k)} > 0$$

This proves that N_i and m_i are co-monotonic. Individuals with higher wealth parameters prefer higher membership levels. The intuition behind this result is very simple : an increase in membership which reduces \bar{m} leads to a relative scarcity of internal finance, so that ψ increases to persuade members to buy more shares. This is relatively more to the advantage of members who currently buy more shares.

The co-monotonicity implies that members can be ranked according to their wealth levels and this ranking also relates to their desired membership level (and capital and output levels) for the firm. Thus if the membership level decision was taken on the basis of a poll in which all members cast a single vote, only a level which the median member, in terms of wealth and shareholding, found ideal for him could be an equilibrium. Any lower level of membership would lead to the median member joining higher-wealth members to vote for an increase in membership, while any higher level would lead to a coalition of the median with lower-wealth members. The membership level adopted, and comparative static responses, thus depend crucially on whether decisions are taken on a one-man-one-vote basis, or a one-share-one-vote basis, or some alternative, and on the

relationship between the wealth of the critical member, who can sway the vote one way or the other, with that of the mean wealth level of the membership. If decisions are taken by a simple majority with one vote per member, then the median wealth individual has wealth \hat{m} generally less than, equal or greater than \bar{m} as the m -distribution is positively skewed, symmetric or negatively skewed. Thus if the m -distribution is

+ skewed	$\hat{m} < \bar{m}$	$N < N^*$	$dN/dp < 0$	$dN/dr > 0$
symmetric	$\hat{m} = \bar{m}$	$N = N^*$	$dN/dp = 0$	$dN/dr = 0$
- skewed	$\hat{m} > \bar{m}$	$N > N^*$	$dN/dp > 0$	$dN/dr < 0$

If the median worker has less wealth and shares than the average, the firm will tend to be smaller since the median worker will find it in his interest to reduce the scarcity of capital and increase the w/v ratio. An opposite argument holds if $\hat{m} > \bar{m}$. Thus, although there is a loss in efficiency, the median worker is better off as he can affect the relative payout rates in a favourable direction for himself.

If voting is on the basis of one vote per share then the critical wealth level would be above \hat{m} , say at \bar{m} . If voting is by some intermediate system, say one vote per share plus some additional common number of votes per member then the critical wealth level will be between \hat{m} and \bar{m} . Of course, as the membership level changes so \bar{m} , \hat{m} and \bar{m} may change. However, a sequence of marginal decisions can be envisaged leading to equilibrium membership and mean wealth levels. Whether or not the output response to a change in product price will be negative (as in the "Illyrian" model, when labour alone is variable; see Ward, 1958), will depend on the m -distribution and the voting system.

(ii) Uncertainty

If demand conditions are risky ($\sigma^2 > 0$) and members are risk averse ($T''\{\cdot\} < 0$) then the preferred membership level of the "critical" member and thus the policy of the firm will differ. Since (12) is linear in p and concave in N , the introduction of a small amount of uncertainty in product price leads (see Ireland, 1980, p.302-3) to a change in N_1 of the same direction as that arising from a reduction in product price under certainty. Thus, using (15), uncertainty causes a risk-averse membership to adopt a membership level nearer to N^* . If $m_1 < \bar{m}$ and $N_1 < N^*$, then N_1 will increase towards N^* ; if $m_1 > \bar{m}$ and $N_1 > N^*$ then N_1 will decrease towards N^* . This result can be confirmed by differentiating the expected value of (12):

$$\frac{dEU_1}{dN} = E[T'\{\cdot\}A((\bar{m}-m_1)(pq(N) - r\lambda k) \frac{d\bar{m}}{dN} + (m_1+\bar{m}-2k)(\bar{m}-k)pq'(N))] = 0 \quad (17)$$

$$\text{where } A = \frac{1}{4} \frac{(m_1+\bar{m}-2k)}{(\bar{m}-k)^2}$$

Equation (17) can be rearranged to obtain

$$G = (\bar{m}-m_1)(\bar{p}q(N) - r\lambda k) \frac{d\bar{m}}{dN} + (m_1+\bar{m}-2k)(\bar{m}-k) \bar{p}q'(N) =$$

$$-[(\bar{m}-m_1) q(N) \frac{d\bar{m}}{dN} + (m_1+\bar{m}-2k)(\bar{m}-k)q'(N)] \frac{E[T'\{\cdot\}](p-\bar{p})}{ET'\{\cdot\}} \quad (18)$$

Now the left-hand side of (18), denoted G , is zero if $\sigma^2 = 0$, and is decreasing in N (for second-order conditions). In the right-hand side, $ET'\{\cdot\} > 0$ since $T'\{\cdot\} > 0$ and $ET'\{\cdot\}(p-\bar{p}) < 0$ since marginal utility is decreasing as product price increases; thus this covariance between product price and marginal utility is negative.

Whether N is greater or less under uncertainty than with a certain price \bar{p} can be inferred from the sign of the left-hand side of (18).

Now (18) can be rewritten as

$$G(\bar{p}-\phi)/\bar{p} = \frac{\phi}{p} r\lambda k(\bar{m}-m_1) \frac{d\bar{m}}{dN} \quad (18')$$

where $\phi = -E[T'\{\cdot\}(p-\bar{p})]/ET'\{\cdot\} > 0$

Now $\bar{p} - \phi > 0$ since

$$(\bar{p}ET'\{\cdot\} + E[T'\{\cdot\}(p-\bar{p})])/ET'\{\cdot\} = \frac{ET'\{\cdot\}p}{ET'\{\cdot\}}$$

and $T'\{\cdot\}$, p are both always positive. Thus G has the sign of $-(\bar{m}-m_1)$, and the effect of uncertainty is to increase N beyond the certainty level if $m_1 < \bar{m}$ and to reduce N from the certainty level if $m_1 > \bar{m}$. These results are obviously related to those of Sandmo (1971), where it is shown that a risk-averse profit maximising firm produces less when faced with an uncertain price with expected value \bar{p} than when faced with a certain price \bar{p} , while Muzondo (1979), Hey (1981) and others show that a labour-managed firm with a fixed factor but variable labour produces more under uncertainty. Both scenarios suggest that commitments are reduced under uncertainty: the profit maximising firm hires less workers and the labour-managed firm spreads the fixed capital more by taking on more members. Our analysis here is rather different. If $N > N^*$, we know that $m_1 > \bar{m}$. In this case the behaviour of the firm corresponds to that of the risk averse profit maximising firm. The "critical worker" reduces his commitment by reducing his relative holding of risky shares. He votes to reduce membership, raising \bar{s} simultaneously as each individual's shareholding is reduced. The "price" to be paid is a reduction in v compared to w . When $N < N^*$, and $m_1 < \bar{m}$, relative commitment by the "critical" member is reduced by increasing the membership level, as in the labour managed firm with fixed costs. In this way the "critical" member reduces his share of labour supply and thereby the variance in income. But to obtain this, w is reduced relative to v . Note that as ψ increases, the worker's purchase of shares increases, although risk has increased!

The above argument demonstrates that divergence from a membership level of N^* arises from the heterogeneity of members and the ability of members to use recruitment decisions to manipulate the internal factor market. Such strategic behaviour becomes less worthwhile under uncertainty since factor prices are uncertain but loan repayment costs are not.

(iii) Some Possible Scenarios

We will contrast three likely scenarios. First, suppose that a previously capitalist firm is "bought out" by its management and workers. Shareholdings are extremely positively skewed reflecting large purchases by top management and small purchases by the shop floor workers. The underlying distribution of m may be as sketched in Figure 1. Here, one-man-one-vote would lead to $N < N^*$ and would be counter to the wishes of the largest shareholders. Thus if the management led in the buy-out operation, they would impose a one-share-one-vote system and the firm's "critical" member may well have $m_1 > \bar{m}$ so that $N > N^*$.

Contrast the above situation with that of a workers' cooperative which has been in existence for many years. There is an approximately even spread of wealth levels among members, reflecting the age distribution and other factors. A one-man-one-vote system is in operation and the median member has $\hat{m} = \bar{m}$. Thus membership is at N^* . One might envisage a steady-state situation where the model we have been considering is of a single typical production period. At the beginning of the period, some members with the highest wealth levels retire and are replaced by those with the next highest wealth level whose savings have been augmented by the previous production period's income. All members' wealth levels have increased and new members, with low wealth, are recruited to keep employment at N^* . The production period takes place, and retirements, recruitments and wealth adjustments are repeated.^{6/}

Yet another possibility is different forms of long term transfer of ownership rights to the workers. This may be part of the firm's incentive schemes ("profit-sharing"), or it may be a plan for workers' takeover of the firm through different formal organisations (e.g. "wage

earner funds"). In the first case, voting will obviously take place according to shares. In the latter case, however, it seems reasonable that each worker should be allocated one vote. But note that, in this case, there may be property rights problems to be solved, corresponding to those of the Illyrian firm.

IV. EXTENSIONS

(1) Members' Monopsony Power

In the analysis to date, the "critical" member's preferred policy prevailed but he was not able to do more for himself than adjust the wealth distribution, through changing membership level, in order to achieve a particular v/w ratio. His ability to benefit from this power was limited by the fact that all members could buy shares. Thus he is only setting a competitive relative price ratio; even if he has the funds, he cannot exert monopsonistic power in the market for shares.

Two alternative possibilities emerge. First a group of members may collude to restrict the demand for shares, and force up the v/w ratio. Secondly, the right to buy shares (i.e. membership) may be denied to some members. The latter is effectively a move away from full participation towards a labour-hiring firm. One strategy is to set a minimum shareholding so that many low wealth members choose to buy no shares at all.

The prospect of a cartel of members using monopsony power to force up relative payment to capital is subject to the usual problems of cartel instability. Thus if, say, the top $h\%$ wealth owners formed such a cartel, they would maximise the v/w ratio subject to the demand for

capital from the firm and the supply of capital from other members. However the cartel member with, say, the lowest wealth would have an incentive to leave the cartel and buy an unrestricted number of shares at the high return supported by the cartel's behaviour. Thus a cartel, at least in the case of a large firm with many members, is likely to be unstable, particularly if the distribution of wealth is reasonably continuous.

The alternative possibility is that a coalition is formed, not directly to restrict the supply of finance, but to change the rules of share sales so as to dissuade non-coalition members from purchasing shares. The institution of a minimum size of share purchase or high cost per share transaction may be strategies for this purpose. For example, suppose that there are just two types of member in the firm, those with high wealth m_h and those with low wealth m_l . If the minimum shareholding that can be purchased is set at a level such that the low-wealth members choose to buy none at all rather than purchase the minimum, even at the higher v/w ratio which results, then high-wealth members will obtain higher utility. This case is reminiscent of discussions relating to discrimination against new members; see, for example, Sapir (1980).^{7/} All the possibilities raised here reflect the power of a group of members to affect the rules or the performance of the internal factor markets.

(ii) Flexible Gearing

The assumption that the firm was faced by a binding constraint as to the amount it could borrow externally relative to internal finance through share sales, can be replaced by the existence of an increasing cost of external finance. Thus $a/b - \bar{s}$ is the amount per member of external finance required. Let

$$f(a/b-\bar{s}), \quad f(0) = 0, \quad f'(\cdot) > 0, \quad f''(\cdot) > 0$$

be the cost per member of acquiring this finance. The convexity of $f(\cdot)$ indicates the penal cost of borrowing in excess of a reasonable gearing ratio. Using

$$x = 1/\psi = w/v \tag{21}$$

and (3) and (4) yields

$$\begin{aligned} U_1 &= T \left\{ \frac{1}{4} (m_1 + x)^2 v \right\} \\ &= T \left\{ \frac{1}{2} (m_1 + x)^2 (pq - f(\cdot)) / (x + \bar{m}) \right\} \end{aligned} \tag{22}$$

since the firm's budget constraint can be written

$$xv = w = pq - f(\cdot) - v\bar{s}$$

so that

$$v = (pq - f(\cdot)) / (x + \bar{s}) = 2(pq - f(\cdot)) / (x + \bar{m})$$

using (4).

In the certainty case, maximising (22) with respect to x and N , remembering that \bar{m} and thus \bar{s} are dependent on N , yields after some simplification the respective first-order conditions:^{8/}

$$(pq-f(\cdot))\left(1 - \frac{m_1+x}{2(\bar{m}+x)}\right) - \frac{1}{4} f'(\cdot)(m_1+x) = 0 \quad (23)$$

$$pq'(N) + \frac{d\bar{m}}{dN} \left(\frac{1}{2}f'(\cdot) - \left(\frac{pq-f}{x+\bar{m}}\right)\right) = 0 \quad (24)$$

Using (23) in (24) implies

$$pq'(N) + \frac{d\bar{m}}{dN} (pq-f(\cdot)) \frac{2(\bar{m}-m_1)}{(m_1+x)(\bar{m}+x)} = 0 \quad (25)$$

so that $q'(N)$ has the sign of $\frac{d\bar{m}}{dN} (m_1-\bar{m})$, and if $\frac{d\bar{m}}{dN} < 0$

$$q'(N) \gtrless 0 \quad \text{and} \quad N^* \lessgtr N^* \quad \text{as} \quad m_1 \lessgtr \bar{m}.$$

This result confirms those obtained earlier, and exactly the same arguments can be used. Obviously, the simplification of a fixed-constraint gearing ratio was not crucial for the conclusions of Section 3.

Although further general analysis along these lines rapidly becomes complex, one particular development is worth pursuing. Suppose membership is fixed but that x is chosen to achieve the best (for 1) balance between internal and external finance. Then (23) defines the appropriate necessary first-order condition. Performing a comparative static exercise on (23) yields

$$\frac{dx^*}{dp} > 0 \quad \frac{dx^*}{dm_1} < 0 \quad \frac{dx^*}{d\bar{m}} > 0$$

Thus if product price increases, the firm adopts a higher wage to share payment ratio, leading to heavier reliance on external finance. The increased price will inevitably increase total payout to the workers of a fixed membership firm. Consumption will increase in both periods through an income effect. But the increased consumption in the initial period can only come about through reduced supply of internal finance, which again depends on the $\frac{v}{w}$ - ratio. The reduced internal finance is subsequently made up by an increased external finance, at an increased cost. By analogy, the introduction of uncertainty over the product price would lead to a lower wage to share payment ratio and less reliance on external finance. Note that increased average wealth leads to a higher w/v ratio, while, naturally, a member j with $m_j > m_i$ would desire a lower wage to share payment ratio.

V. CONCLUSION

Changing membership leads to repercussions in the internal factor market. Thus some members may benefit while others lose due to differences in factor endowments. In this paper, the members have been assumed to supply finance to the firm, and external finance is acquired complementary to this internal finance, through credit rationing by a fixed gearing limit, rather than as a substitute. It has been shown that members' potential to supply capital is co-monotonic with their preference for increases in firm size. How firms make their decisions is thus crucial since there is no "representative" member. Rather the "critical" member who holds the balance of power achieves the adoption of his ideal policy. The identification of the "critical" member, and thus the firm's chosen policy, depends therefore on both the allocation of power (for instance the

criteria for vote allocation among the members) and the distribution of members in relation to their capital supplies.

Our approach considered a sequential model of decision-making within the firm. Given the membership and the relative rewards for capital and labour provision, each member individually chooses his purchase of shares. Given this individualistic behaviour, the relative rewards are chosen so as to balance relative factor supplies with relative factor demands. Given this decision rule and individualistic share purchases, the current membership can choose to recruit new members or not replace those who leave. Equilibrium thus comprises equilibrium behaviour in all three stages of this sequential game. In the first stage, strategy is determined by a voting rule; in the second by an institutional efficiency rule, and in the third by individualistic Nash behaviour.

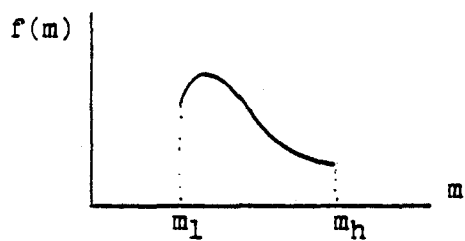
The firm behaviour of interest is that of membership adjustment. Technology is assumed as simple as possible. It was shown in Section III that "Illyrian" or orthodox behaviour would occur depending on whether the "critical" member has a lower m parameter and thus a lower shareholding than the member with the arithmetic mean \bar{m} . For example, with a one-man-one-vote system, a positively-skewed m -distribution would lead to Illyrian behaviour as the critical member would be a net user of internal finance. However, the introduction of uncertainty reduces the tendency for membership to diverge from the efficient output-per-unit-factor-maximising level N^* .

Finally, some extensions to the basic model have been considered. Specifically various ways in which monopoly power could be exercised by a cartel of members were examined. It was suggested that such a cartel might use its power to change the decision-making structure rather than to

influence the internal factor market by restricting supplies. The implications of relaxing the gearing constraint were also considered.

The analysis here is preliminary and reflects just one aspect of the important behavioural consequences of the internal organisation of a participatory firm. However, it serves to demonstrate the significance of members' heterogeneity and the problems of assuming that no conflict of interests arises within such firms.

Figure 1 : Share Distribution after a Management-Employee Buy-Out



Footnotes

- * This paper was written while Askildsen was visiting the University of Warwick.
- 1/ See, for example, Ireland and Law (1985b) for an analysis of labour supply under conditions of endogenous monitoring. In the present paper individual labour supply is held fixed and common to all members. Other structural factors which could be considered in the same way include the gradations of worker seniority and their skill levels.
- 2/ One might consider this a "fair returns firm", see Ireland and Law (1985a).
- 3/ Alternative forms of worker saving are ignored. Either workers have only relatively low returns from saving elsewhere so that no other form of saving is attractive, or moral pressure is exerted by the firm to ensure that all savings are channelled into shares.
- 4/ Problems of corner solutions have been ignored for simplicity. Thus restrictions on the levels of m_i , ψ and k necessary to achieve interior solutions are required. These are essentially that (7) and (8) below are positive for all i .
- 5/ The very reasonable basis for this assumption is that the membership prefers to borrow more from external financiers due to the high disutility it suffers from foregoing marginal current consumption.
- 6/ Of course our model does not incorporate a number of features of such a dynamic system. In particular, when taking current decisions, the median worker may take account of future decisions to be made by future median workers in future production periods. Thus what may well happen, is that the median worker at some stage votes for changing the voting rules to one-share-one-vote, so as to better his future position.
- 7/ Of course, there is also the possibility of the formation of a cartel of low-wealth members, establishing a maximum purchase of shares, so as to equalize income from participation. This would result in a shortage of capital, raising v/w , and thereby induce the low-wealth members to increase their supply of capital. This aspect may have some wider applications.
- 8/ Second-order conditions are assured if $q(\cdot)$ is sufficiently concave and $f(\cdot)$ sufficiently convex.

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