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**Market Structure, Freight Rates and Assets
in Bulk Shipping**

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Contents

Preface

v

Ch. 1;	
<i>Valuation of VLCCs under income uncertainty</i>	1
Abstract	1
Introduction.....	1
The stochastic nature of income.....	3
The value of a VLCC	12
Summary and conclusions	19
Acknowledgements	19
References.....	20
Ch. 2;	
<i>Spot versus Time Charter Markets - The Case of VLCCs</i>	21
Abstract	21
Introduction.....	21
Equilibrium in the spot and time charter markets.....	26
Demand uncertainty and the time charter equilibrium	31
Supply uncertainty and the time charter equilibrium	42
Summary and main conclusions	47
Acknowledgements	48
Appendix.....	49
References.....	50
Ch. 3;	
<i>A model of the short run freight rate formation in the VLCC market</i>	53
Abstract	53
A short introduction to the VLCC market.....	53
Part one, An assignment model	58
Part Two, Simulations and empirical findings.....	69
Conclusions	77
Acknowledgements	77
Appendix	78
References.....	82
Ch. 4;	
<i>The BFI and the BIFFEX - Stochastic Properties and Valuation</i>	83
Abstract	83
Introduction	83
Part one, The dynamic structure of the BFI.....	84
Part two, A futures contract and a European call option on a futures.....	99
Part three, Forecasting the BFI, futures prices vs. the MRAS	111
Summary and conclusions	116
Acknowledgements	116
Appendix.....	117
References.....	126

Ch. 5;	
<i>The Structure of the Freight Rate,</i>	
<i>A Stochastic Partial Equilibrium Model for the VLCC Market</i>	129
Abstract	129
Introduction	129
Short term equilibrium in the tanker market.....	131
Dynamic equilibrium in the tanker market	134
The structure of the freight rate	145
Summary and conclusions	147
Acknowledgements	148
Appendix.....	149
References.....	153

Ch. 6;	
<i>The Stochastic Partial Equilibrium Model for the VLCC Market -</i>	
<i>Extensions and Applications</i>	155
Abstract	155
Introduction	155
Short term equilibrium	156
An extension of the SPE model	161
The dynamics of the VLCC market.....	166
The to tier version of the SPE model - simulations	175
Concluding remarks.....	182
Acknowledgements	182
References.....	183

Ch. 7;	
Note one;	
<i>The Stochastic Partial Equilibrium Model for the VLCC Market -</i>	
<i>Characteristics of the Shipbuilding Market</i>	187
Abstract	187
Introduction	187
Supply in the shipbuilding market	187
A multi-tier version of the SPE model	189
The SPE model with order book.....	192
Summary and concluding remarks	195
References.....	196

Note two;	
<i>Stochastic Continuous Time Markov Models with "Time to Build"</i>	197
Abstract	197
Introduction	197
The basic problem	197
Concluding remarks.....	203
Acknowledgements	204
References.....	205

Preface

This dissertation in maritime economics focuses on the bulk shipping markets. The main emphasis is on the freight rate formation and on the valuation of shipping assets. The market for Very Large Crude Carriers (VLCCs) is, in most of this study, used as a basis for discussion. However, many of our findings carry over to other bulk shipping markets directly or with only minor modifications. Others are of a more unique character to the VLCC market. In chapter four also the dry bulk markets are studied.

The approach to maritime economics taken here is thematically traditional. Most current studies, including this one, are heavily inspired by the works of Tinbergen and Koopmans from the 1930s. Even though maritime economists have a common interest in the shipping industry, they may not belong to the same theoretical school. Hence, researchers in this field have always felt free to choose whichever technical tool or theoretical approach that appears to be the most appropriate. I too, have tried to let the problem at hand decide what approach to use. Nevertheless, in this study of the bulk shipping freight rates and asset valuation, I use a somewhat restricted number of techniques, most of them related to the modelling of a dynamic environment under uncertainty. In doing so, I often follow in the footsteps of Mossin (1968) and Bjerksund & Ekern (1995).

One obvious reason why the shipping industry has attracted the interest of numerous economists and historians, is the huge amount of detailed statistics available. Although this study is mainly theoretical, I have indicated in all chapters, in one way or another, how the models fit the observations.

I hope that some part of this study may be of interest, not only to economists, but to practitioners in the shipping industry as well. I think there are two main lessons to be learned from this thesis. The first one is about the understanding of the dynamics of the bulk shipping markets. The second is about the valuation of shipping assets. All chapters are concerned with market structure and freight rate dynamics, whereas especially chapters one, two and four are concerned with asset valuation.

Overview

This thesis consists of seven chapters. Chapter one, "*Valuation of VLCCs under income uncertainty*", discusses some properties of the freight rate dynamics in the VLCC market. We suggest to describe the time charter equivalent spot rate by a geometric stochastic process with mean reverting properties. Given this freight rate dynamic, we derive the value of a VLCC given the option, in addition to operation, of laying up the vessel in periods of low freight rates, or terminating the project by selling the vessel for demolition.

Chapter two, "*Spot versus time charter markets - The Case of VLCCs*", is concerned with the relation between the freight rates earned in the spot markets and the time charter freight rates. We construct a partial equilibrium model with risk averse shipowners and cargo owners, and study the effects of both demand and supply uncertainty on volumes of time charter contracts to spot contracts, and the spot rate level to the time charter rate level.

In chapter three, "*A model of the short term freight rate formation in the VLCC market*", we leave for a moment the focus on uncertainty at an aggregated level, and try to disclose the micro structure of the freight rate dynamics of the VLCC market. We apply a "matching & bargaining" approach to describe the short run freight rate formation in the Persian Gulf. We assume price competition among the agents of the market, and show how the cost of waiting, for both shipowners and cargo owners, may influence the freight rate dynamics.

In chapter four, "*The BFI and the BIFFEX - Stochastic properties and valuation*", we turn to the dry bulk markets. We study the Baltic Freight Index and the futures written on this index, the BIFFEX futures. We discuss the stochastic nature of the index and the risk attitude of the agents in the market, and use this knowledge to price a futures contract. Then we derive a formula for the valuation of a European option on a futures.

We return to the VLCC market in chapter five, "*The structure of the freight rate - A stochastic partial equilibrium model of the VLCC market*". Here we construct a stochastic partial equilibrium model, where demand is assumed to be uncertain. In a short term perspective supply can only to a limited extent be adjusted. But, in the long run, new vessels can be built to meet

any increase in demand, and depreciation of the fleet will gradually adjust supply downwards if demand decreases. In addition to demand and supply for shipping services, the model must, consequently, also include a shipbuilding industry. Finally, we derive a stochastic process for the freight rates in the model.

In chapter six, "*The stochastic partial equilibrium model of the VLCC market - Extensions and applications*", we develop the model introduced in chapter five a bit further. We also relate our model to the maritime economic literature and market characteristics. Then we estimate parameter values and run a simple version of the model.

Chapter seven contains two notes, "*The stochastic partial equilibrium model of the VLCC market - Characteristics of the shipbuilding market*" and "*Stochastic continuous time markov models with "time to build" - Formulation and a sketch of a possible solution*", which both present possible extensions to the model in chapters five and six, as regards the representation of the shipbuilding industry. First, we focus on the switching between different production levels and second, we investigate the possibility of modelling construction time in a continuous stochastic setting.

All seven chapters are related thematically, and most of them also theoretically. However, they are all self contained, written as individual papers, and may be read separately. At the start of each chapter, there is an abstract describing the approach used and giving the main findings. References are listed at the end of each chapter.

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During my work on this dissertation I have benefited greatly from all I learnt as part of the Strategy of Improved Shipping Analysis (SIS) IV group from 1991 to 1993. I want to take this opportunity to express my gratitude to Olav Bruåsdal, Frank Isachsen, Ellen Aarvaag Martinussen, Arne Osmundsvaag, Bjarte Stokseth, Ann-Elisabeth Svendsen and Pernille Østensjø for fruitful cooperation and an enjoyable time. A special thanks to Tor Wergeland for his admirable work with organising SIS and for all help over many years.

Many practitioners in the shipping industry have been very generous and provided me with valuable comments on my writings as well as with empirical observations. It has also been of utmost importance for me to have had the opportunity of working part time with the shipowners AS J. Ludwig Mowinckels Rederi during my studies. It has been great fun, and I hope the staff's inspiration have given this thesis a bit of a practitioner's touch. I would especially like to thank Øyvind Holte and Lars Edvard Høgestøl, from whom I have learnt a lot.

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Ch. 1;

Valuation of VLCCs under income uncertainty

Abstract

In this paper two alternative ways of modelling the stochastic nature of the time charter equivalent spot rate in the market for Very Large Crude Carriers are presented. Bjerk Sund and Ekern (1995) propose that the freight rate follows an Ornstein-Uhlenbeck process. We follow up this approach of relating uncertainty directly to the rate process itself, by suggesting a geometric mean reversion process. Empirical findings are presented. Then we address the question of valuing a VLCC. Due to the presence of uncertainty, flexibility to choose operation policy influences the value. We focus on lay up and scrapping as alternatives to spot operation. The option to lay up is relatively more important for a new vessel than for an old one, whereas the option to scrap becomes relatively more valuable as the vessel gets older.

Introduction

Valuation of shipping assets motivates our search for a proper description of the stochastic nature of the freight rate. In this respect, shipping assets include the value of contracts of affreightment, time charters and bare boat agreements of different duration, as well as forwards and options written on these contracts. Further, shipping assets naturally include value of ownership and new building contracts and options on these. Knowledge of the nature of the risk associated with the income stream is vital, not only because risk adverse investors will demand an extra premium in order to take on the high degree of risk of most shipping investments, but also because uncertainty itself influences the value of an asset if any kind of option is involved.

The profitability of operating Very Large Crude Carriers (VLCCs) has been very volatile. VLCCs, i.e., vessels of above 200,000 dead-weight tonnes (dwt.) are mainly used for transporting crude oil out of the Persian Gulf to North America, West Europe and the Far East. Usually, the VLCCs return to the Gulf in ballast. Thus, demand for the transportation service offered by these vessels mainly depends on the volumes of crude oil moved out of the Persian Gulf area. The main variations in demand stem from shifts in overall oil consumption and from changes in the importance of the different oil supplying regions. These factors are closely related to the price of oil. A low oil price increases total demand, but in addition, the relative importance of the Persian Gulf area as an oil supplier increases since, in general, the marginal cost of producing oil in this region is lower than in the rest of the world. As the

oil price falls, consumers increase consumption, producers close down marginal wells, especially in the US, and thereby imports to Europe, the Far East and the US increase. This means more demand for oil tankers. In short, changes in oil demand and trading patterns influence the demand for VLCCs.

It is usual to assume that demand only to a very small extent depends on the freight rates. Due to the large scale operation, the cost of transportation at sea is a minor share of the total oil price, and therefore demand is supposed to be inelastic to freight rates.

In the short run, supply is quite inelastic when there are no idle vessels available. Speed and efficiency in loading and discharging can only to a limited degree be increased. However, in the case when freight rates are very low many vessels may be laid up. Then, short-run supply can readily be increased by re-entering of mothballed vessels.

Since demand is inelastic to changes in freight rates and because there is a short-term upper limit to supply, freight rates can be very high at times. High rates, or rather anticipation of high rates, will trigger shipowners to order new vessels. However, there will be a lag of about a year and sometimes even longer, from a ship is ordered to delivery from the yard. Thus, sky-high rates will not be a persistent situation. On the contrary, the market usually clears at a low rate level that is seldom sufficient to cover investment costs of a new vessel. Hence, investing in a VLCC is a gamble. The reward is high for those in possession of a vessel if capacity becomes scarce and rates rocket, but it is often too late to order vessels when the market is strong because the freight rates will probably be back to the normal low level before the vessels are delivered. The first drawers will be the winners of the game of the short-term rate peaks, but high rates occur only occasionally and uncertainty about when demand will hit the short-term supply limit is high. Therefore, everyone who orders new vessels will be losers if the anticipation of high rates fails, and the winners on the supply side will then be the patient ones who did not order. In the low market caused by over capacity, second hand prices will be depressed, especially so since many shipowners are forced to sell due to liquidity shortage.

The spot freight rate in the VLCC market is quoted in World Scale points. This is an index developed by the World Scale Association in London in order to compare the profitability of different trades. Given the cargo size and the

WS rate, the income from each journey is determined. The income is supposed to cover all costs and includes fixed capital costs and operational costs. If the vessel is laid up, operational costs will be removed. Often the charterer hires the vessel for a certain period of time and not for a specific journey, in which case the time charter contract is used. The time charter freight rate is quoted in USD per day and is supposed to cover all expenses except costs directly related to where the vessel is used, i.e., the charterer must cover bunker costs and channel and harbour charges himself. Fuel is a major cost, and historically, the price has been volatile.

Bjerksund & Ekern postulate that the spot rate follows an Ornstein-Uhlenbeck process and that costs are constant. In the applications of Andersen (1992) and Stray (1992) uncertainty of the bunker price is taken account of by estimating a "time charter equivalent spot rate". In short, the spot rate income less bunker costs and charges on a daily basis. This time charter equivalent spot rate is then assumed to follow an Ornstein-Uhlenbeck process. The main argument for suggesting that the freight rate has a mean reverting nature is capacity adjustments. High profitability triggers new ordering and low profitability makes shipowners lay up their vessels. If the market prospects are very poor shipowners may decide to sell their vessels for demolition. The option of keeping vessels idle puts a floor to the rate level in the medium run. If operational costs are not covered the shipowner will be better off by laying up his vessel. However, there will be costs related to taking a vessel off the market, keeping it idle and re-enter it later on. Therefore, one might experience slightly negative time charter equivalent spot rates for short periods.

The stochastic nature of income

Intuitively, the Ornstein-Uhlenbeck process does not give a very realistic description of the spot freight rates since the Ornstein-Uhlenbeck process is not downward restricted. The spot freight rates will never be negative, but since the Ornstein-Uhlenbeck process is normally distributed around a given mean, the Ornstein-Uhlenbeck process often gives negative values if volatility is high. During short intervals the spot rates may become so low that the estimated time charter equivalent spot rate will be negative. However, this does not occur very frequently, and the rate will only be slightly negative. The time charter equivalent spot rate is negative if the voyage income is less than the total of fuel consumption and harbour and channel costs. If this is the case, the shipowner will obviously be better off by laying

up the vessel than by keeping it in operation. Thus, the market almost always clears at a positive time charter equivalent spot rate. Therefore, it may be useful to try a process that is downwards restricted in order to describe both the spot rate and the time charter equivalent spot rate. Hence, we suggest that the freight rate can be appropriately described by a Geometric Mean Reversion (GMR) process. Let the increment of the process be given by

$$dX_t = \kappa(\alpha - \ln X_t)X_t dt + \sigma X_t dZ_t \quad (1)$$

The parameter κ is a constant that governs the degree of mean reversion of the process. A high κ implies a strong reversion of the process, and vice versa. The log of the process is reverted toward the level given by α . The instantaneous standard deviation of the relative change in the freight rate is given by σ . Further, we have that Z_t is a one dimensional standard Brownian motion, i.e., $dZ_t \sim N[0, dt]$. Let \mathcal{F}_t be a sigma field generated by $\{Z_s, 0 \leq s \leq t\}$, i.e., \mathcal{F}_t represents all information generated by the Brownian motion, which is available at time t .

Like the Ornstein-Uhlenbeck process, the GMR process has mean reversion properties. In addition, it fulfils our requirement of being downwards restricted since zero is an absorbing level. Further, the process may prove to be a reasonable approximation to the fact that the rates often stay at a moderate level with low volatility for long periods followed by short periods of high rates and high volatility. The suggested process secures that reversion is strong as the rate is high, but reversion is weak if the rate is at a low level. Due to the geometric nature of the last term of (1), the process also relates high rates to high volatility and vice versa.

If the incremental change in the freight rate is given by the above geometric mean reversion relation, then the rate at time τ is given by

$$X_\tau = e^{e^{-\kappa(\tau-t)} \ln x_t + \left(\alpha - \frac{\sigma^2}{2\kappa}\right) (1 - e^{-\kappa(\tau-t)}) + e^{-\kappa(\tau-t)} \sigma \int_t^\tau e^{\kappa s} dZ_s} \quad (2)$$

given the rate level at time t , x_t , $t < \tau$. According to our hypothesis, the freight rate will be lognormally distributed with conditional mean given by

$$E[X_\tau | \mathcal{F}_t] = e^{e^{-\kappa(\tau-t)} \ln x_t + \left(\alpha - \frac{1}{2} \frac{\sigma^2}{\kappa}\right) (1 - e^{-\kappa(\tau-t)}) + \frac{1}{2} \sigma^2 \left[\frac{1 - e^{-2\kappa(\tau-t)}}{2\kappa} \right]} \quad (3)$$

and conditional variance

$$\text{Var}[X_\tau | \mathcal{F}_t] = e^{2e^{-\kappa(\tau-t)} \ln x_t + 2\left(\alpha - \frac{1}{2} \frac{\sigma^2}{\kappa}\right) (1 - e^{-\kappa(\tau-t)}) \left(e \frac{\sigma^2}{\kappa} (1 - e^{-2\kappa(\tau-t)}) - e \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa(\tau-t)}) \right)} \quad (4)$$

In order to estimate the parameters of the process, taking logs makes the logarithm of the freight rate a linear function of the logarithm of yesterday's freight rate and the increment of the white noise generator. Since $dZ_t \sim N[0, dt]$, the log of the freight rate is normally distributed too, with conditional mean

$$E[\ln X_\tau | \mathcal{F}_t] = e^{-\kappa(\tau-t)} \ln x_t + \left(\alpha - \frac{1}{2} \frac{\sigma^2}{\kappa}\right) (1 - e^{-\kappa(\tau-t)}) \quad (5)$$

and conditional variance

$$\text{Var}[\ln X_\tau | \mathcal{F}_t] = \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa(\tau-t)}) \quad (6)$$

Estimation of the parameters of the processes

To estimate the parameters of the processes we use quarterly time charter equivalent spot rates from 1969 to 1993. These observations cover almost the whole history of the VLCC market up until 1993, since very few VLCCs were in operation before 1969. These years include the extremely high rates of the early 1970s, the depressed market of the late 1970s and early 1980s, the period of optimism in the late 1980s and the disappointment of the early 1990s. Evidently, the market is quite young and a major part of the existing VLCC fleet was constructed in the 1970s. This short history leaves us with some problems when estimating the parameters of the freight rate process. In fact, we are only in possession of observations from a period of about the life span of a vessel. Due to over capacity during most of these years the rates have on the whole stayed at a too low level to justify any new-building. However, it may perhaps be reasonable to expect that long-term market clearance makes the freight rates converge towards a rate level that covers all costs, including capital costs. Estimating our process using available

observations will evidently not give a mean reversion level at such a high level. At this point, however, we use these observations for estimating the parameters, regardless of the above mentioned weaknesses.

The following discrete version of the log of the freight rate process was used to estimate the coefficients

$$\ln X_t = \beta_0 + \beta_1 \ln X_{t-1} + \varepsilon_t$$

whence $\beta_0 = \left(\alpha - \frac{1}{2} \frac{\sigma^2}{\kappa} \right) (1 - e^{-\kappa})$ and $\beta_1 = e^{-\kappa}$. We have that the error term is normally distributed, $\varepsilon_t \sim N \left[0, \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa}) \right]$, that is,

$$\text{Var}[\ln X_t | \ln X_{t-1}] = \text{Var}[\varepsilon_t] = \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa}) \approx \frac{\tilde{\varepsilon}^T \tilde{\varepsilon}}{N-1}$$

Using ordinary least squares we get the following estimates of the parameters of the GMR process:

Table 1; Estimated parameter values of the GMR process

	Estimated coeff.	Standard error	T- value
$\tilde{\beta}_0$	2.3342	0.62849	3.7139
$\tilde{\beta}_1$	0.74072	0.069092	10.721
$\tilde{\varepsilon}^T \tilde{\varepsilon} / (N-1)$	0.71		

Using standard t - tests, it follows that both β_0 and β_1 are different from zero at a 1% level of significance.

From $\tilde{\beta}_1$ we derive an estimate of κ of 0.003289. By combining the value of $\tilde{\beta}_0$ and the variance of the dependent variable we get an estimate of α of 10.58 and an estimate of the variability coefficient, σ , of 0.1007. This gives the following estimated relation for the incremental change in the time charter equivalent spot rate

$$dX_t = 0.003289(10.58 - \ln X_t)X_t dt + 0.1007 X_t dZ_t.$$

The estimation of the parameters of the geometric mean reversion process gives the following sample statistics:

$$\begin{aligned}\hat{R}^2 &= 54\% \\ \text{Durbin Watson statistic} &= 2.0 \\ \text{Durbin } h \text{ statistic} &= 0.05\end{aligned}$$

Since the Durbin h statistic indicates lack of autocorrelation, the estimated coefficients seem to be efficient, that is, the coefficients have the lowest possible variance.

If the specified model is a good description of the stochastic nature of the freight rates, as it seems to be, we ought to be able to reject the hypothesis that the freight rate follows a random walk. For this purpose we apply the Dickey-Fuller unit root test and receive a Dickey-Fuller F-value of 9.3. The critical value at a 5% level of significance is 6.5. Thus, there is a fairly significant indication that the freight rate does not follow a random walk.

For comparison, we also estimate the parameters of the Ornstein-Uhlenbeck process. The incremental change in the freight level is in this case given by

$$dX_t = \kappa(\alpha - X_t)dt + \sigma dZ_t \quad (7)$$

Then the freight rate level at time τ , given the rate level at time t , x_t , is given by

$$X_\tau = e^{-\kappa(\tau-t)}x_t + \alpha(1 - e^{-\kappa(\tau-t)}) + e^{-\kappa(\tau-t)}\sigma \int_t^\tau e^{\kappa s} dZ_s \quad (8)$$

A freight rate following an Ornstein-Uhlenbeck process is normally distributed with conditional mean

$$E[X_\tau | \mathcal{F}_t] = e^{-\kappa(\tau-t)}x_t + \alpha(1 - e^{-\kappa(\tau-t)}) \quad (9)$$

and conditional variance

$$\text{Var}[X_\tau | \mathcal{F}_t] = \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa(\tau-t)}) \quad (10)$$

The discrete counterpart to the Ornstein-Uhlenbeck process, used for estimating the parameters, is

$$X_t = \beta_0 + \beta_1 X_{t-1} + \varepsilon_t$$

where $\beta_0 = \alpha(1 - e^{-\kappa})$ and $\beta_1 = e^{-\kappa}$. As above the error term is normally distributed $\varepsilon_t \sim N\left[0, \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa})\right]$ and hence,

$$\text{Var}[X_t | X_{t-1}] = \text{Var}[\varepsilon_t] = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa}) \approx \frac{\tilde{\varepsilon}^T \tilde{\varepsilon}}{N-1}$$

We estimate the parameters of the process using ordinary least squares and get the following results;

Table 2; Estimated parameter values of the Ornstein-Uhlenbeck process

	Estimated coeff.	Standard error	T- value
$\tilde{\beta}_0$	2915.1	1355.5	2.1506
$\tilde{\beta}_1$	0.80022	0.061769	12.955
$\tilde{\varepsilon}^T \tilde{\varepsilon} / (N-1)$	0.95947E+08		

From these parameter values we derive an estimate of the mean reversion level, α , of USD 14,592 per day, an estimate of the standard deviation of the increment of the process, σ , of 1,142 and of the degree of convergence, κ , of 0.00244.

The estimation of parameters of the Ornstein-Uhlenbeck process gives us the following sample statistics:

$$\hat{R}^2 = 64\%$$

$$\text{Durbin Watson statistic} = 1.5$$

$$\text{Durbin } h \text{ statistic} = 2.8$$

There is a strong degree of autocorrelation in the residuals. Trying to estimate the parameters by using the first difference of the freight rates, also gives a *Durbin h* of 2.8, and offers hardly any improvement as far as efficiency is concerned.

In order to compare the Ornstein-Uhlenbeck and the geometric mean reversion specifications, we estimate the non-linear model of Box and Cox;

$$\frac{X_t^\lambda - 1}{\lambda} = \beta_0 + \beta_1 \frac{X_{t-1}^\lambda - 1}{\lambda} + \varepsilon_t$$

In the case that $\lambda = 1$, the above relation reduces to the linear specification of the Ornstein-Uhlenbeck process, though each observation value is reduced by 1. If $\lambda = 0$, then the relation is equal to the equation used for estimating the parameters of the geometric mean reversion process.

We use maximum likelihood techniques to estimate the parameters of the Box-Cox specification. Let the maximum value of the log-likelihood function in this unrestricted case, be given by \mathcal{L}_{ur} . Then we estimate the parameters given the restrictions that $\lambda = 1$ and $\lambda = 0$, and receive the values of the maximum likelihood functions \mathcal{L}_r^c and \mathcal{L}_r^{GMR} , respectively. It follows that for large samples $-2(\mathcal{L}_r - \mathcal{L}_{ur}) \sim \chi_k^2$ where k is the number of restrictions, which in our case is equal to one. From the likelihood ratio test, we know that if χ_k^2 , for a given significance level, is above the critical value, then we can reject the hypothesis that the restriction does not apply.

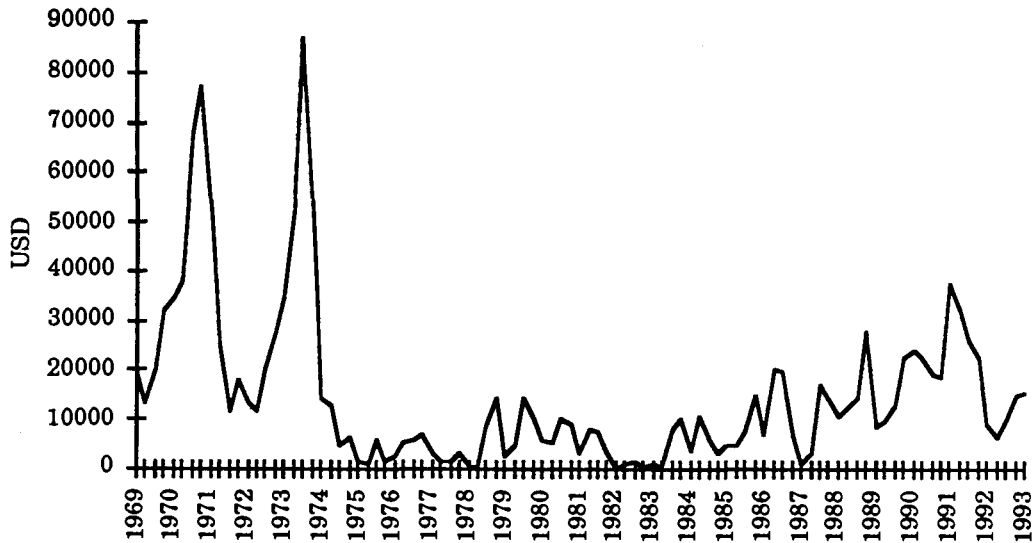
For our sample, we estimate the exponent λ to be equal to 0.32 with corresponding value of the maximum of the likelihood function $\mathcal{L}_{ur} = -969.5$. Further, in the restricted cases we have that $\mathcal{L}_r^{OU} = -1017.1$ and $\mathcal{L}_r^{GMR} = -982.7$. From these values, it follows that $\chi_k^2 = 95.2$ in the Ornstein-Uhlenbeck case and $\chi_k^2 = 26.6$ in the geometric mean reversion case. Evidently, at a 5% significance level, λ is different from both 0 and 1, but when selecting between our two models, the geometric mean reversion specification seems by far the best choice.

Freight rate simulations

We have argued that the Ornstein-Uhlenbeck representation of the freight rate has some obvious weaknesses. The graphs below may illustrate our points. The first graph shows the time charter equivalent spot rate in the VLCC market from 1969 to 1993. The other two graphs show simulations of an Ornstein-Uhlenbeck and a geometric mean reversion process, respectively. The random figures by which the graphs are generated are the same. The parameters of the processes are those estimated above. Each

graph consists of 4,000 points, i.e., it is equivalent to a period of approximately eleven years.

Figure 1; Time charter equivalent spot rates 1969 to 1993



Source: Fearnley's

Figure 2; Freight rate following an Ornstein-Uhlenbeck process

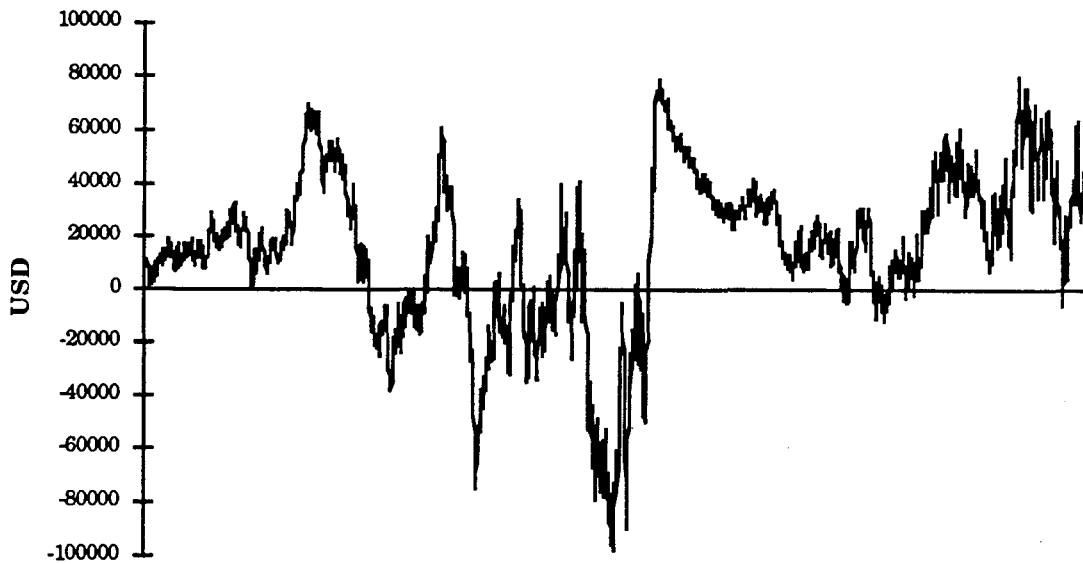
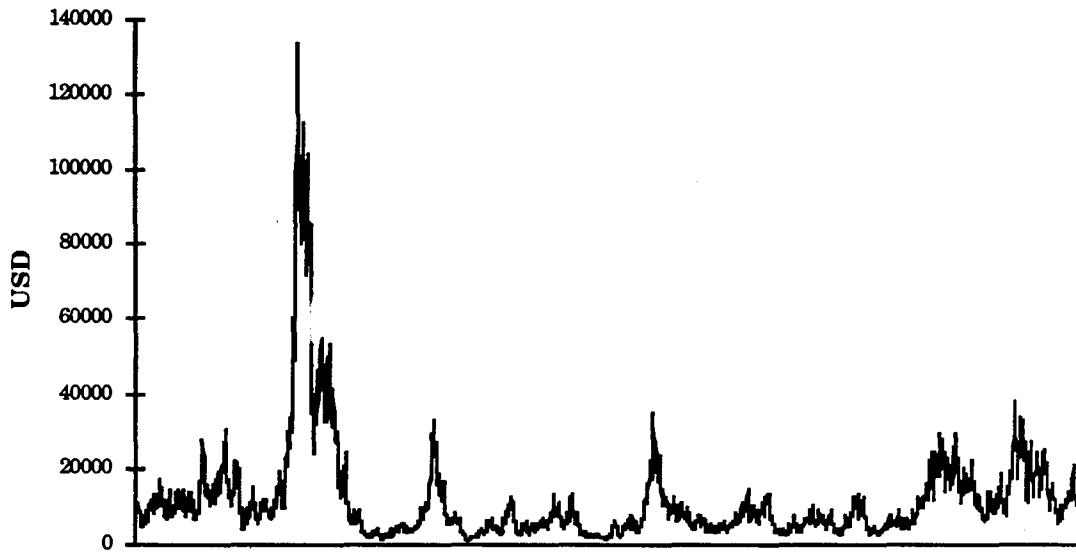


Figure 3; Freight rate following a geometric mean reversion

The average of the sample used for estimating the parameters, i.e. the freight rate observations plotted in the first graph, is USD 14,819 . The average of the simulated rates in the Ornstein-Uhlenbeck graph is USD 15,593, and the average of the rates in the geometric mean reversion graph is USD 11,493. However, for large simulated samples the averages approach the mean of the observed historical freight rates. After 36 million draws, i.e. equal to 98,000 years, the mean of the simulated Ornstein-Uhlenbeck process is USD 14,800, and the mean of the simulated geometric mean reversion process is USD 14,900.

The value of a VLCC

For nearly twenty years the freight rates in most bulk shipping segments have been too low to give a fair return on investments in new tonnage. Also today this is the prevailing situation in the crude oil tanker segment. Instead of building new vessels, investing in second hand tonnage has occasionally proved to be very profitable. However, in this part we will not discuss the possibility of earning profits from asset play, i.e., trying to beat the market by acquiring vessels at low prices and selling as second hand prices are high. Instead, we value a vessel as a going concern, i.e., as if the vessel was run by the owner until it is sold for demolition.

During the life of a vessel the shipowner has to make a number of decisions as regards the use of his vessel. As long as the vessel makes a nice profit and complies with quality and safety standards, the shipowner will obviously keep the vessel in operation. However, it may well happen that operation costs exceed the freight rate income. In these cases the shipowner may be better off by laying up his vessel. Hence, the option of laying up the vessel puts a floor to the potential losses from operation, although applying this flexibility entails costs. There are transaction costs related both to mothballing and re-entering.

If the future seems too grim, the shipowner can decide to terminate the project by scrapping his vessel. In addition, there are technical, and in many cases legislative limitations to the maximum age. When buying a vessel, one thereby receives a continuous option until the maximum age, to scrap the vessel and receive the value of the vessel as sold to a demolition yard. Thus, this flexibility has the structure of an American option.

An early paper that takes account of the option to lay up the vessel in periods of low rates is Mossin (1968). The lay up case has also been discussed in an unpublished report by Næss (1990) and in a book of Dixit and Pindyck (1994). The scrapping decision is discussed in Stray (1992).

The model

Our model is partly based on Martinussen (1993). We do not take into consideration the costs of laying up a vessel or re-entering into the market. We also ignore the costs related to reclassing a vessel. Approximately every fifth year a vessel goes through a major survey and may have to be upgraded

in order to comply with the standards of one of the classification societies. Further, we do not consider the effect of docking costs. Our model only focuses on the three alternatives; operation, lay up and scrapping.

The instantaneous cash flow from operation and lay up until the vessel is scrapped, is given by

$$C_t = (X_t - w)\chi_{X_t - w + m > 0} - m(1 - \chi_{X_t - w + m > 0}) \quad (11)$$

where X_t is the time charter equivalent spot freight rate, w is the operation costs except for voyage related costs, m is the cost of keeping the vessel mothballed and χ_A is an indicator function of the event A , where $A \in \{X_t - w + m > 0\}$.

When the vessel reaches the maximum age at time T , its value must be equal to the value of the vessel as scrap, P_t . However, the vessel may be sold for demolition before the maximum age. If the value of a vessel as a going concern is less than the demolition value, then the vessel is scrapped. The termination date τ is equal to the stopping time given by

$$\tau = \inf\{0 < t \leq T; \Phi_{t,x} \leq P_t\} \quad (12)$$

The value of a vessel, i.e. the market value of the cash flow generated from time t to τ , is then given by

$$\Phi_{t,x} = E^Q \left[\int_t^\tau e^{-r(s-t)} C_s ds + e^{-r(\tau-t)} P_\tau | \mathcal{F}_t \right] \quad (13)$$

where Q is a certainty equivalent probability measure which depends on the true probability measure P and the risk attitude of the market agents.

In order to focus only on the effect of flexibility on the value of a vessel, we let $Q = P$ in the rest of this paper, i.e., we assume that the market agents are risk neutral.

Denote the state of the system by Y_t ,

$$Y_t = \begin{bmatrix} s+t \\ X_t \end{bmatrix} \quad (14)$$

Then it follows that the increment of Y_t is given by

$$dY_t = \begin{bmatrix} 1 \\ \mu_{X_t} \end{bmatrix} dt + \begin{bmatrix} 0 \\ \sigma_{X_t} \end{bmatrix} dZ_t \quad (15)$$

where, in the case the freight rate follows an Ornstein Uhlenbeck process, $\mu_{X_t} = \kappa(\alpha - X_t)$ and $\sigma_{X_t} = \sigma$ and in the case the freight rate follows a geometric mean reversion process, $\mu_{X_t} = \kappa(\alpha - \ln X_t)X_t$ and $\sigma_{X_t} = \alpha X_t$. For $t > 0$, the Ito diffusion Y_t has an infinitesimal generator \mathcal{A} which is given by

$$\mathcal{A} = \frac{\partial}{\partial t} + \mu_x \frac{\partial}{\partial x} + \sigma_x^2 \frac{\partial^2}{\partial x^2} \quad (16)$$

From (13) it follows that the value $\Phi_{0,x}$ is a solution to an optimal stopping problem¹. Then we have that the value function $\Phi_{t,x}$ must satisfy the two conditions given in (17) and (18) below.

$$\mathcal{A}\Phi_{t,x} = -e^{-rt}C_t \text{ given that } \Phi_{t,x} > \mathcal{P}_t \quad (17)$$

$$\lim_{t \rightarrow \tau} \Phi_{t,x} = e^{-r\tau}\mathcal{P}_\tau \text{ given that } \Phi_{t,x} \leq \mathcal{P}_t \quad (18)$$

We try a separated form of the value function given by $\Phi_{t,x} = e^{-rt}V_{t,x}$. Then it follows from (17) that the value function must satisfy the following partial differential equation for $\Phi_{t,x} > \mathcal{P}_t$

$$rV + \frac{\partial V}{\partial t} + \mu_x \frac{\partial V}{\partial x} + \frac{1}{2} \sigma_x^2 \frac{\partial^2 V}{\partial x^2} + C_t = 0 \quad (19)$$

¹ The problem has a combined Dirichlet and Poisson structure. See Øksendal (1992) for details on this formulation.

Simulation procedures

A closed form solution to our problem is not available. We have used the explicit finite difference method to approximate the value of $V_{t,x}$. An appropriate grid is constructed with values of t and X_t as follows

$$(0, \Delta t, 2\Delta t, \dots, i\Delta t, \dots, T)$$

$$(x_0, x_0 + \Delta x, x_0 + 2\Delta x, \dots, x_0 + j\Delta x, \dots, x_{\max})$$

Let V_{ij} be the value of $V_{t,x}$ at the node (i, j) , i.e. for $t = i\Delta t$ and $x = x_0 + j\Delta x$. Thus, the partial derivatives of $V_{t,x}$ at the node $(i-1, j)$ may be discretely approximated by

$$\frac{\partial V}{\partial x} = \frac{V_{i,j+1} - V_{i,j-1}}{2\Delta x}$$

$$\frac{\partial^2 V}{\partial x^2} = \frac{\frac{V_{i,j+1} - V_{i,j}}{\Delta x} - \frac{V_{i,j} - V_{i,j-1}}{\Delta x}}{\Delta x} = \frac{V_{i,j+1} + V_{i,j-1} - 2V_{i,j}}{(\Delta x)^2}$$

$$\frac{\partial V}{\partial t} = \frac{V_{i,j} - V_{i-1,j}}{\Delta t}$$

Then we have the value of $V_{i-1,j}$ given by

$$V_{i-1,j} = BV_{i,j-1} + CV_{i,j} + DV_{i,j+1} + F$$

B , C and D depend on the specified underlying process. F is given by

$$F = \frac{1}{1+r\Delta t} \left(\text{Max} \left[(x_j - w)\Delta t, m\Delta t \right] \right)$$

If we assume that the freight rate follows the Ornstein Uhlenbeck process we have that

$$B = \frac{1}{1+r\Delta t} \left[- \left(\kappa(\alpha - x_j) \frac{\Delta t}{2\Delta x} \right) + \frac{1}{2} \sigma^2 \frac{\Delta t}{(\Delta x)^2} \right]$$

$$C = \frac{1}{1+r\Delta t} \left[1 - \sigma^2 \frac{\Delta t}{(\Delta x)^2} \right]$$

$$D = \frac{1}{1+r\Delta t} \left[\left(\kappa(\alpha - x_j) \frac{\Delta t}{2\Delta x} \right) + \frac{1}{2} \sigma^2 \frac{\Delta t}{(\Delta x)^2} \right]$$

B , C and D may be regarded as the discounted probabilities of a decrease, no change, and an increase in the value of ownership, respectively.

In the geometric mean reversion case we transform the variables in order to derive an underlying variable of $V_{t,x}$ with a constant instantaneous standard deviation. Let ϕ be a variable given by

$$\phi_t = \ln X_t$$

where X_t is given by (2). Then by Ito's lemma it follows that

$$d\phi_t = \left(\kappa(\alpha - \ln X_t) - \frac{1}{2} \sigma^2 \right) dt + \sigma dZ_t$$

Hence, we have an alternative partial differential equation to (19) for the value of ownership in the geometric mean reversion case, given by

$$-rV_{t,x} + \frac{dV}{dt} + \left(\kappa(\alpha - \ln X_t) - \frac{1}{2} \sigma^2 \right) \frac{dV}{d\phi} + \frac{1}{2} \sigma^2 \frac{d^2V}{d\phi^2} + C_t = 0$$

We then approximate the value, by the discrete counterpart $V_{i,j}$, given by

$$V_{i-1,j} = BV_{i,j-1} + CV_{i,j} + DV_{i,j+1} + F$$

where

$$B = \frac{1}{1+r\Delta t} \left[- \left(\left(\kappa(\alpha - \ln x_j) - \frac{1}{2} \sigma^2 \right) \frac{\Delta t}{2\Delta\phi} \right) + \frac{1}{2} \sigma^2 \frac{\Delta t}{(\Delta\phi)^2} \right]$$

$$C = \frac{1}{1+r\Delta t} \left[1 - \sigma^2 \frac{\Delta t}{(\Delta\phi)^2} \right]$$

$$D = \frac{1}{1+r\Delta t} \left[\left(\left(\kappa(\alpha - \ln x_j) - \frac{1}{2} \sigma^2 \right) \frac{\Delta t}{2\Delta\phi} \right) + \frac{1}{2} \sigma^2 \frac{\Delta t}{(\Delta\phi)^2} \right]$$

By recursively solving for $V_{i-1,j}$ we get the value of ownership depending on the present freight rate level.

We must choose the size of Δt , Δx and $\Delta \phi$ so that the probabilities are positive and less than unity. Nonetheless, at some level of x and ϕ one of the discounted probabilities B , C or D inevitably will be negative. These levels give us the maximum and minimum values of the grid. In these cases the probabilities have to be adjusted to secure convergence to the true $V_{t,x}$ ².

Value of flexibility

Below we carry out some simulations in order to illustrate the importance to the value of a VLCC of the options to lay up and to scrap. We use the same assumptions with regard to parameter values as previously, except for the level α which has in the Ornstein Uhlenbeck case been adjusted upward to the estimated level of the geometric mean reversion process. Due to this, the values of the vessel are almost identical, irrespective of the process chosen, in the case of no flexibility. Then we have a proper starting point for comparing the value of flexibility given on the underlying process. We apply the following estimates of the parameters;

Table 3; Base case parameter values

Parameter	Ornstein Uhlenbeck	Geo. mean reversion
α	17 486	10.54
σ	8 870	0.97
κ	0.22	0.3
r	1.5%	1.5%

We suppose that the VLCC in question may live for 30 years from now, i.e. about the feasible technical lifetime of a new vessel. If we assume full flexibility the shipowner may operate the vessel at the prevailing freight rates, he may lay up the vessel or scrap it. The daily operation costs, w , are set to USD 8,000 per day. The lay up costs, m , are assumed to be USD 2,700 per day. At any time the vessel may be sold for scrap at USD 5 mill, i.e. P_t is fixed.

² For details see Hull & White (1990).

The table below shows the value of a vessel given a present time charter equivalent freight rate level of USD 15,000 per day, the assumed freight rate process, and the degree of flexibility. Note that we have increased the mean reversion level of the Ornstein-Uhlenbeck process to that of the geometric mean reversion process in order to make the results more easy to compare.

Table 4; Value of a vessel that may be operated for 30 years.

Degree of flexibility	Ornstein Uhlenbeck		Geo. mean reversion	
Only operation	47 503 293	100.0%	47 502 115	100.0%
Operation or scrapping	47 668 984	100.3%	47 619 883	100.2%
Operation and lay ups	53 767 989	112.3%	52 316 357	110.1%
Full flexibility	53 788 927	112.4%	52 363 337	110.2%

If continuous operation is the only choice for the next thirty years, and then the vessel is sold for demolition at USD 5 mill, there will be almost no differences in values regardless the process chosen. The Ornstein Uhlenbeck process may give negative values and thus, the option to lay up the vessel will be particularly valuable in the Ornstein Uhlenbeck case. Therefore, as we take all flexibility into consideration the difference in value between the Ornstein Uhlenbeck case and the geometric mean reversion case is about USD 1.5 mill.

We also see from the table above that the option to scrap is of nearly no value to a new vessel. For comparison we look at a vessel that has at most ten more years to live. All other assumptions are as above.

Table 5; Value of a vessel that may be operated for 10 years.

Degree of flexibility	Ornstein Uhlenbeck		Geo. mean reversion	
Only operation	27 701 043	100.0%	28 675 673	100.0%
Operation or scrapping	28 254 896	102.0%	29 068 835	101.4%
Operation and lay ups	31 015 753	112.0%	31 182 355	108.7%
Full flexibility	31 085 720	112.2%	31 339 193	109.3%

A shorter remaining lifetime reduces the possibility of very low freight rates, and therefore the importance of the option to lay up the vessel is reduced. As we are approaching the maximum age, the present value of the expected future cash flow falls and the probability of an early exercise of the option to

scrap increases. Thus, we have that the value of the option to lay up a vessel decreases and the value of the option to scrap increases as a vessel gets older.

Summary and conclusions

We believe that both economic intuition and our empirical findings, indicate that the geometric mean reversion specification is more appropriate than the Ornstein-Uhlenbeck process as regards describing the stochastic nature of the time charter equivalent spot freight rates in the VLCC market.

A correctly specified freight rate process can be very useful for estimating the value of shipping assets, such as the value of a vessel. If the Ornstein-Uhlenbeck process is chosen, flexibility is more highly valued than in the geometric mean reversion case. The reason is mainly that the process is not downward restricted. Therefore, if we look at operation only, the value in the Ornstein-Uhlenbeck case is almost identical to the value in the geometric mean reversion case. However, as flexibility increases the value in the Ornstein-Uhlenbeck case increases more than the value in the geometric mean reversion case.

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Ch. 2;

Spot versus Time Charter Markets - The Case of VLCCs

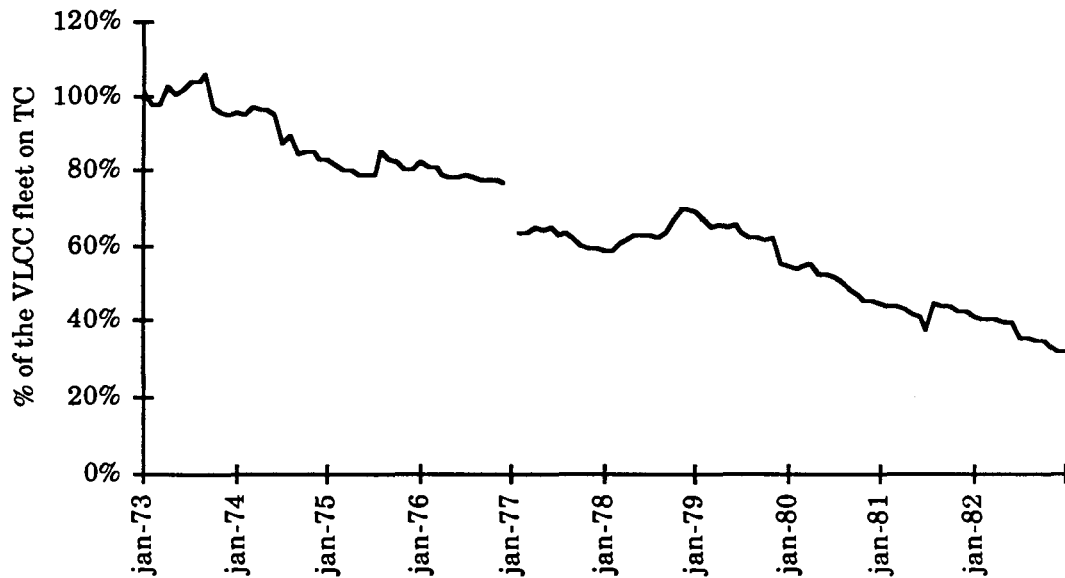
Abstract

This paper suggests a model for describing the equilibrium of the spot charter market and the TC market and the distribution of vessels between them. We represent the agents of the market by a representative shipowner and a representative charterer. Both are assumed to be risk averse. First we study time charter equilibrium given demand uncertainty. In this case the shipowners prefer a fixed income in the time charter market to an uncertain income in the spot market, given equal expected values. However, we find that the charterers prefer to hire vessels in the spot market at an uncertain freight rate to fixing the freight rate in advance in the time charter market, given equal expectations. The reason for this is that the spot market exposure reduces the total gain uncertainty of the representative charterer. Consequently, the equilibrium time charter freight rate will always be below the expected time charter equivalent spot freight rate in the case of only demand uncertainty. Thereafter we study the effect of supply uncertainty on the time charter equilibrium. We represent this by uncertainty in the capital stock and in the price of fuel. As long as demand for oil transport is inelastic, which seems to be the case in the VLCC market, the equilibrium time charter rate is below the expected time charter equivalent spot rate. However, for elastic demand this may not hold.

Introduction

The importance of the time charter market compared to the voyage charter market has varied, but mainly declined, during the history of the Very Large Crude Carriers (VLCCs). From the first VLCCs were constructed in the late 1960's until 1975, spot freight rates were exceptionally volatile and on average very high. Most VLCC owners made huge profits. During this period volatility both in demand and supply was substantial. Scarcity of tonnage as well as yard capacity prevailed in the wake of a sharp unexpected rise in demand. In addition, such large vessels had never before been constructed, and this implied technical uncertainty. In these early years almost all VLCCs either were hired on time charters or owned directly by the major oil companies. Figure 6 below shows the percentage of the independent fleet of crude oil carriers above 175,000 dwt. hired on time charters from 1973 to 1982. The figures also include time charter vessels relet on other new time charters. Hence, the percentage may exceed 100%. The broken line is due to changes in the reporting procedures applied by the source. However, the trend is clear, during the 1970's and 1980's time chartering was substantially reduced.

Figure 6; Percentage of the independent VLCC fleet on TC, 1973 to 1982

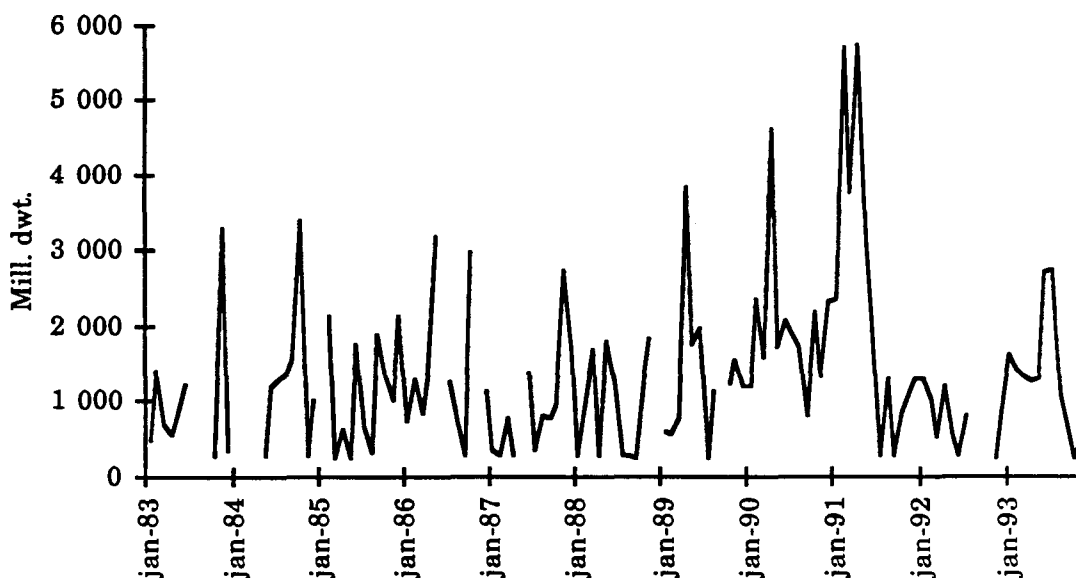


Source: Drewry

The fall in time chartering came together with a stabilisation of the freight rate at a low level. Demand fell from the historically high levels of the early 1970's and in addition, the market suffered from too much capacity, both of vessels and yards. Technical uncertainty was almost removed by the fact that construction of new vessels almost seized and the existing fleet proved to be a technological success. The price of oil rose and became unstable. This implied that fuel cost also became volatile, and in unfavourable circumstances spot freight rates hardly covered the cost of bunkers. Consequently, a number of vessels were mothballed.

The risk attitude of the shipowners and the charterers influences the equilibrium in the spot and time charter markets. During the late 1970's and the 1980's the risk attitude of the shipowners may have changed. According to a panel study by Lorange & Norman (1973) shipowners seem to be risk prone. However, according to Eckbo (1977) the shipping crisis may have made shipowners more risk averse.

During the 1980's time chartering was kept at a low level. Figure 7 shows VLCCs fixed on time charters from 1983 to 1993 measured in dwt. per month. Note, that for some months no figures are reported by the source.

Figure 7; Time charter fixtures per months 1983 to 1993

Source: Drewry

Spot freight rates rose to a higher level for the period 1989 to 1992. Simultaneously, time chartering peaked. During these years a number of new vessels were ordered. These contracts were signed in anticipation of substantial scrapping due to technical attrition of the early 1970's vessels. Hence, there was apparently a comprehension of high technical uncertainty among the agents.

The literature, as early as Koopmans (1939), has been aware of the relation between high freight rates and a large number of time charters. As already indicated, technical uncertainty seems to be related to extensive time chartering. However, high freight rates and uncertainty are often related. As capacity becomes scarce due to an unexpected large increase in demand, freight rates rise. This triggers construction of new vessels. As yard capacity becomes more restricted, delivery dates in the future become less predictable. Jumps in freight rates are often also related to supply shocks. Historically, we have seen these effects due to wars, the closure or reopening of the Suez channel or long range crude oil pipe lines as well as congestion in harbours. Even adverse weather conditions may have a positive impact on freight rates.

Most studies of the time charter market share the approach applied by Eriksen & Norman (1976). They assume that time charter operation generally reduces risk for both shipowners and charterers. The risk averse shipowners prefer a time charter contract to spot operation if the expected values are equal. Therefore, the time charter rate will lie below the expected time charter equivalent spot rate. This is in accordance with the model presented below in the case of demand uncertainty. However, they assume that the charterers minimise transportation costs. If so, the risk averse charterer should accept to pay a higher time charter rate than the expected time charter equivalent spot rate. Therefore, the inverse demand curve for time charters should be above the time charter equivalent spot rate. Hence, we believe it is inconsistent to conclude that the equilibrium time charter rate will always be below the equivalent expected spot rate if the charterers are cost minimisers.

Eriksen & Norman argue that to keep the number of time chartered vessels close to the expected total demand implies a risk of having more vessels on time charter than own requirements. However, the risk of being oversupplied is restricted, as Koopmans points out, by the fact that the charterer may relet any surplus capacity in the spot market at the prevailing freight rate.

Koopmans focuses on reletting, or rather lack of reletting, by the major oil companies as a major factor for understanding both the time charter and the voyage charter markets. In some periods the oil companies compete for time charter tonnage in fear of being without transportation in the future. Assume that a major part of the total independent fleet is on time charter to a restricted number of oil companies. These companies are free to retain own surplus time charter capacity from the spot market. If they choose to do so, other oil companies, being without transportation, must restrict sales. Consequently, the oil companies with excess transport capacity enjoy high oil prices and large sales due to restricted competition. As Koopmans argues, if some major oil companies use the tanker market for restricting the access of others to the oil markets, an oligopoly model is probably needed to describe the time charter and spot markets. However, Koopmans is studying the oil market in the mid-1920's, and the structure of today's market is quite different.

Generally, the crude oil tanker market is often thought of as one of the most perfectly competitive markets of the world. See e.g. standard text books like Porter (1983) or Kreps (1990). This assumption is justified by a large number of shipowners and cargo owners, almost free or none-exclusive access to extensive market intelligence, easy entry and exit and a truly international market. Throughout, we follow this traditional assumption of the perfect competitiveness of the tanker market.

From the emergence of the international oil tanker business in the 1880's until the late 1960's the world saw a tremendous technological improvement and a huge increase in the average and maximum size of the tanker vessels. However, the steady growth in economies of scale during almost a century came to a halt in the early 1970's. The VLCCs of today are hardly more efficient than the mid-1970 vessels. Slightly more sophisticated machinery and high tensile steel are probably pros of today's vessels, whereas a double hull requirement reduces efficiency. All in all, there is at present no strong indications that the near future will bring about vessels of substantially higher efficiency than those of today. Therefore, the present VLCCs are very homogenous.

In this paper we focus on equilibrium in the time charter and the spot markets and on the distribution of tankers between the two markets. The spot freight rate is the price paid for a single voyage. It includes all transportation costs at sea due for the charterer. A time charter contract specifies the period in which the vessel is at the disposal of the charterer. Normally, it is for a longer period than the duration of a spot voyage. The time charter rate does not cover voyage related costs. That is, in addition to the time charter rate, the charterer must pay for fuel, channel charges and harbour fees that will incur during the time charter period. The spot rate less voyage related costs, is known as the time charter equivalent spot rate.

The time charter rate is equivalent to the sum of a succession of forwards on spot rates less voyage related costs (see the appendix). In this paper we simplify by assuming that the duration of the time charter is equal to the length of a representative voyage in the spot market, i.e., we mainly focus on the relation between the spot and the forward markets for shipping services.

Some degree of default risk is involved in all time charter agreements. Nonetheless, in our simple model we do not take into account any heterogeneity of this kind in the demand and supply for time charters, and hence, we assume that all time charters are risk free.

Equilibrium in the spot and time charter markets

We assume that transportation is produced by a constant return to scale technology. This seems to be a reasonable approximation and in line with the literature. A number of characteristics support this assumption. There seems to be no scarcity of fuel or potential seamen, i.e., the main production factors that are variable in the short-run. In a medium-term perspective, the present marginal vessel can easily be duplicated by close to identically efficient units. Moreover, high quality vessels can be constructed at a number of yards all over the world.

Among possible homogenous representations of the technology, we choose the following Cobb-Douglas aggregated production function,

$$Q_t = b_t^\gamma k_t^{1-\gamma} \quad (1)$$

where Q_t is the total production of transport services, measured in tonnes or tonnemiles, k_t is the capital stock, i.e., the total VLCC fleet, and b_t is a bundle of short-term inputs, all at time t . The exponent γ is assumed to be positive and less than unity.

The supply and demand for spot versus time charter vessels are strongly related to the uncertainty incumbent in the market. Let the stochastic nature of the economy be defined by a probability space (Ω, \mathcal{F}, P) with the usual properties. Let a two dimensional standard Brownian motion, $Z_t = \{Z_t^1, Z_t^2\}$, be defined on this probability space. We assume that Z_t^1 and Z_t^2 are uncorrelated. Further, let the filtration of Z_t be given by $F = \{\mathcal{F}_t; t \geq 0\}$ where \mathcal{F}_t is a sub-sigma algebra of \mathcal{F} , generated by $\{Z_s, 0 \leq s \leq t\}$.

The charterers may cover their demand for transport by hiring vessels in the spot voyage market at the prevailing freight rate, X_t , or they may use vessels they have rented on a Time Charter (TC) basis. At time t , the total fleet, k_t , is either in the spot market, k_t^s , or on time charters, k_t^{TC} , that is

$$k_t = k_t^s + k_t^{TC} \quad (2)$$

The gain of the charterers

The charterers pay a predetermined rate, R_t , per unit of time charter vessels hired, and consequently, the rate is independent of the actual market conditions at time t , only on expectations, the risk attitudes of the agents and the market at the time the contracts are fixed, i.e. at time s . Hence, we assume that the time charter equilibrium, i.e. the price and volume, is \mathcal{F}_s measurable, $\mathcal{F}_t \subset \mathcal{F}_s$, for some $s < t$. That is, the number of vessels on time charters and the time charter rate are determined at time s .

If the total supply of the TC fleet rented by the charterers at time s , proves to be insufficient at time t , the charterers must hire more capacity in the spot market. On the other hand, a charterer may find that his time charter fleet is larger than his own requirements. Fortunately, he may relet any surplus capacity in the spot market. On an aggregated basis, total supply from time charter and spot vessels must be equal to total demand. That is,

$$Q_t = Q_t^s + Q_t^{TC} \quad (3)$$

The time charter rate does not cover voyage related costs, and these are therefore payable for the charterer. Consequently, the utilisation of the time charter fleet is dependent on the choice of the charterer. The total cost for the charterer of renting vessels at time t , is

$$C_t = X_t Q_t^s + C_t^{TC} \quad (4)$$

The first part is the cost of renting spot vessels and the second part is the cost of renting time charter vessels. We assume that

$$C_t^{TC} = w b_t^{TC} + R_t k_t^{TC} \quad (5)$$

where w is a cost per unit of the short-term input bundle, b_t . The total supply from the time charter vessels is then given from (1) by

$$Q_t^{TC} = (b_t^{TC})^\gamma (k_t^{TC})^{1-\gamma} \quad (6)$$

Inserting (3), (5) and (6) in (4), we have the total cost of the charterers given by

$$C_t = X_t \left(Q_t - (b_t^{TC})^\gamma (k_t^{TC})^{1-\gamma} \right) + w b_t^{TC} + R_t k_t^{TC} \quad (7)$$

The charterer chooses the utilisation policy of the TC fleet that minimises the total transportation cost at any time t . From the first order condition for optimal utilisation with respect to b_t^{TC} , it follows that the optimal short-term input bundle is

$$b_t^{TC} = \left(\frac{w}{\gamma X_t} \right)^{\frac{1}{\gamma-1}} k_t^{TC} \quad (8)$$

Directly from (6) and (8) we then have that the supply from the time charter fleet is given by

$$Q_t^{TC} = \left(\frac{w}{\gamma X_t} \right)^{\frac{\gamma}{\gamma-1}} k_t^{TC} \quad (9)$$

We assume that the charterers are profit maximisers in the crude oil market, but they do not assume that their export policy influences market prices. Let the charterer be an oil company extracting oil in one area and refining in another. By the assumption of perfect competition in the shipping market, the charterer takes the spot freight rate as given. In addition to costs of sea transport, expenses related to extraction, refining and distribution on-shore incur before the oil reaches the end consumer. For simplicity we assume that all these costs are linear and given by $c_t Q_t$, where c_t is a constant unit cost. In order to maximise profits the charterers adjust sales in order to make marginal revenues equal to total marginal costs. Approximately, there is a one-to-one relation between Q_t and the total delivery to the refineries. Therefore, we can write the total gain of the charterers at time t

$$S_t = \mathcal{P}_t (Q_t + q_t) - C_t \quad (10)$$

where $\mathcal{P}_t = p_t - c_t$, p_t is the price of oil and q_t is oil that is already available at the refinery. It follows that Q and q are assumed to be measured in the

same units, e.g. tonnes. An optimal chartering policy implies that (10) is maximised with respect to total sales, where total sales are equal to the total amount of oil that is available at the refinery. The first order condition for optimal Q_t gives that

$$p_t = X_t \quad (11)$$

Now it follows by simple manipulation that the maximum gain function, dependent on the number of vessels hired on time charter, is given by

$$S_t^* = S_t^*(k_t^{TC}; R_t, X_t, k_t) = \left(\frac{w}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} X_t^{\frac{1}{1-\gamma}} (1-\gamma)k_t^{TC} - R_t k_t^{TC} + X_t q_t \quad (12)$$

The profit of the shipowners

The shipowners maximise profit at any time t . The total aggregated profit is given by total income from spot operation less voyage costs related to the spot operation, plus the already predetermined income from TC operation, that is

$$\Pi_t = X_t Q_t^s - w b_t^s + R_t k_t^{TC} \quad (13)$$

First order condition for optimally chosen b_t^s entails that

$$b_t^s = \left(\frac{w}{\gamma X_t}\right)^{\frac{1}{\gamma-1}} k_t^s \quad (14)$$

Then it follows that the total supply of spot vessels at time t , is given by

$$Q_t^s = \left(\frac{w}{\gamma X_t}\right)^{\frac{\gamma}{\gamma-1}} k_t^s \quad (15)$$

Simple manipulation gives the maximum profit function of the shipowners depending on the number of vessels on time charter

$$\Pi_t^* = \Pi_t^*(k_t^{TC}; R_t, X_t, k_t) = \left(\frac{w}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} X_t^{\frac{1}{1-\gamma}} (1-\gamma)(k_t - k_t^{TC}) + R_t k_t^{TC} \quad (16)$$

Total supply

From (9) and (15) we see that total short-term supply of transport services is not influenced by the number of time charter contracts to the number of spot contracts.¹ This is obvious from the constant return to scale assumption and the fact that both the charterers and the shipowners face the same constant unit cost of the short-term inputs. We then have from relation (3), (9) and (15) that total supply can be written

$$Q_t = \left(\frac{w}{\gamma X_t} \right)^{\frac{\gamma}{\gamma-1}} (k_t^s + k_t^{TC}) \quad (17)$$

The total costs for the charterers less the profits of the shipowners are equal to the total minimised short-run operation costs, that is,

$$C_t^* - \Pi_t^* = \gamma \left(\frac{w}{\gamma} \right)^{\frac{\gamma}{\gamma-1}} X_t^{\frac{1}{1-\gamma}} k_t = w b_t \quad (18)$$

We have that the total gains and profits of the market are given by

$$S_t^* + \Pi_t^* = \left(\frac{w}{\gamma} \right)^{\frac{\gamma}{\gamma-1}} X_t^{\frac{1}{1-\gamma}} \{1-\gamma\} k_t + X_t q_t \quad (19)$$

The capital stock is increased by construction of new vessels and reduced by demolition. The shipowners order new vessels, aiming at maximised profits in time, given some time preferences. Nevertheless, we can leave aside the problem of optimal control of the fleet size, k , for all t , since we consider the choice between time charters versus spot charters in a short-term perspective. That is, we circumvent the control problem by assuming that the size of the fleet at time t cannot be influenced by the agents when the size of k_t^{TC} has to be decided upon, i.e., at time $s < t$.

¹ In a richer model one should note that the supply function may actually differ depending on whether the vessel is on time charter or operating spot. This follows from the fact the freight rate level that triggers lay-up will be different for the charterer of a time charter vessel than for a shipowner operation spot. A charterer that relets a time charter vessel in the spot market pays for all variable and fixed costs except for voyage related costs, through the freight rate. Laying the vessel up only removes the voyage related costs. For a shipowner operating spot, laying up means that all variable costs, both voyage and non-voyage related costs, are removed. Consequently, the charterer that relets time charter vessels will be more hesitant to lay up and hence, accepts to sail for a lower freight rate level.

Demand uncertainty and the time charter equilibrium

We assume that the spot freight rate X_t follows a geometric Brownian motion given by

$$dX_t = \mu X_t dt + \sigma X_t dZ_t^1 \quad (20)$$

where μ is the instantaneous expected growth rate of the process and σ is the standard deviation of the incremental relative change in the spot freight rate. The increment of the standard Brownian motion, dZ_t^1 , is as defined above. Be aware that the freight rate only follows this process for a given development of the fleet size. An increase in the capacity will reduce the freight rate, whereas demolition will make it increase. (See e.g. Tvedt 1995a) We assume that the fleet size k_t is known at time s , i.e., there is no supply uncertainty.

Further, let both the shipowners and the charterers be risk averse. Assume that the preferences of the shipowners for profits in time are represented by an additive separable utility functional of the form

$$\Phi(\Pi) = \int_0^T e^{-\rho t} \phi(\Pi_t^*) dt \quad (21)$$

Equivalently, we assume that the preferences of the charterers are given by the additive separable utility functional defined by

$$\Psi(S) = \int_0^T e^{-\rho t} \psi(S_t^*) dt \quad (22)$$

where $\phi(\cdot)$ and $\psi(\cdot)$ are both increasing and concave Bernoulli utility functions and ρ is a common rate of time preference. $\Phi(\Pi)$ and $\Psi(S)$ are assumed to satisfy technical conditions for Fubini's theorem.

Let the rate of income less voyage costs, of the spot fleet at time t , \tilde{X}_t , be defined by

$$\tilde{X}_t = \frac{\Pi_t^* - R_t k_t^{TC}}{k_t^*} = \left(\frac{w}{\gamma} \right)^{\frac{\gamma}{\gamma-1}} X_t^{\frac{1}{1-\gamma}} (1-\gamma) \quad (23)$$

Practitioners usually refer to \tilde{X}_t per day as the time charter equivalent spot rate.

From the structure of the problem it follows that the time charter market for delivery at time t will be cleared at time $s < t$, and only at that time. According to the model, activity before and after this point in time does not influence the equilibrium. In this respect, the model reduces to a two period problem, at time s the time charter equilibrium is settled and at time t the spot market clears. Further, we have that at time t , the predetermined time charter equilibrium does not influence the spot market. The problem for the shipowner at time s , of choosing the optimal number of time charter vessels at time t , will then be to maximise expected utility with respect to the size of k_i^{TC} given the rate R_t . This maximum can be taken at any time s for the corresponding delivery date t , independent of the markets at any other points of time. The optimal k_i^{TC} chosen by the shipowner at time s can then be found by solving

$$\max_{k_i^{TC}} E \left[e^{-\rho t} \phi(\Pi_t^*) \mid \mathcal{F}_s \right] \quad (24)$$

The first order condition for the optimal choice of k_i^{TC} is

$$\hat{\Phi}' = E \left[\frac{d\phi(\Pi_t^*)}{d\Pi} [-\tilde{X}_t + R_t] \mid \mathcal{F}_s \right] = 0 \quad (25)$$

Equivalently, we have the first order condition for optimal k_i^{TC} for the charterers given by

$$\hat{\Psi}' = E \left[\frac{d\psi(S_t^*)}{dS} [\tilde{X}_t - R_t] \mid \mathcal{F}_s \right] = 0 \quad (26)$$

From equations (25) and (26) we derive the equilibrium time charter freight rate, R_t , and the part of the fleet on time charter, k_i^{TC} . The equilibrium is determined by the size of the total fleet, k_i , and the probability distribution of X_t , which depends on the spot freight level at time s , x_s .

Assume that R_t is zero. Then we have that $\hat{\Phi}' < 0$ for all k_i^{TC} , since by assumption $\frac{d\phi(\Pi_i^*)}{d\Pi} > 0$ and $\tilde{X}_t > 0$ for $x_t > 0$, because zero is an absorbing level to the freight rate process. Hence, the first order condition for optimally chosen k_i^{TC} is never satisfied. Expected utility will increase as long as k_i^{TC} is reduced. Consequently, the shipowners will prefer to set $k_i^* = k_t$, and there will be no vessels on time charter.

If the time charter rate is zero, the charterers will prefer to rent as many vessels as possible. Formally, we have that $\hat{\Psi}' > 0$ for all k_i^{TC} , since we assume $\frac{d\psi(x_t)}{dx_t} > 0$ and $\tilde{X}_t > 0$ for $x_t > 0$. Thus, the charterers prefer to set $k_i^{TC} = k_t$, since the first order condition for optimal choice of k_i^{TC} is never satisfied.

For simplicity, we assume for a moment that $q_t = 0$. Then, from definition (23) we can write relation (12) and (16) as

$$S_t^* = \tilde{X}_t k_i^{TC} - R_t k_i^{TC} \quad (27)$$

and

$$\Pi_t^* = \tilde{X}_t (k_t - k_i^{TC}) + R_t k_i^{TC} \quad (28)$$

As long as the expected value of the uncertain \tilde{X}_t is below or equal to the certain time charter freight rate R_t , the risk averse shipowners prefer to fix all their vessels in the time charter market. This will obviously maximise $E[e^{-\rho t} \phi(\Pi_t^*) | \mathcal{F}_s]$.

However, this will not be true for the charterers. From (27) we see that the total gain at time t is less volatile in the case that k_i^{TC} is zero. Then the total gain is also zero. That is, $\max_{k_i^{TC}} E[\psi(S_t^*) | \mathcal{F}_s] = \psi(0)$ a. s. if $E[\tilde{X}_t | \mathcal{F}_s] \leq R_t$. The main reason for this is that in equilibrium high demand for transport is accompanied by high freight rates whereas low demand is accompanied by low freight rates. Thus, correlation between demand and supply subdues the volatility in the gain of the charterers. In our model, a fixed time charter rate does not influence the correlation between demand and supply. Hence, both high and low demand are accompanied by the same fixed time charter rate and thus, the total gain becomes more erratic. We thereby have

that the risk averse charterers prefer to pay the uncertain spot rate rather than the certain time charter rate, if the expected values are equal. Consequently, in equilibrium, we must have for all k_i^{TC} that $0 < R_t < E[\tilde{X}_t | \mathcal{F}_s]$.

In equilibrium, at time s , the shipowners and the charterers must be indifferent to whether their vessels are fixed in the time charter market or operated in the spot market, at time t . Thus, under a certainty equivalent probability measure, Q , we must have for any positive k_i^{TC} , that

$$R_t = E^Q[\tilde{X}_t | \mathcal{F}_s] = (1 - \gamma) \left(\frac{w}{\gamma} \right)^{\frac{\gamma}{\gamma-1}} E^Q \left[X_t^{\frac{1}{1-\gamma}} | \mathcal{F}_s \right] \quad (29)$$

To be more precise about the actual spot versus time charter equilibrium and the corresponding certainty equivalent probability measure, we need more knowledge about the utility functions². Below we give one example in which we assume a utility function for the representative shipowner with constant relative risk aversion. Since the gain function of the charterers can take negative values we cannot use such a utility representation for the representative charterer. Hence, we assume that the risk attitude of the representative charterer is given by a constant absolute risk aversion. Given the utility functions, we derive the equilibrium time charter rate and the number of time charter fixtures.

Utility functions

Assume that the utility function of the shipowner is given by $\phi(\Pi_t^*) = \frac{\Pi_t^{*\vartheta}}{\vartheta}$, where ϑ is a constant relative risk coefficient, $0 < \vartheta < 1$. Since the gain of the charterers can be negative we have chosen a different form for the utility function in this case. Let the utility function of the charterer be given by $\psi(S_t^*) = -\frac{1}{\eta} e^{-\eta S_t^*} + \frac{1}{\eta}$, where $\eta > 0$. From these utility functions we have the equilibrium conditions

$$\hat{\Phi}' = E \left[\Pi_t^{*\vartheta-1} [R_t - \tilde{X}_t] | \mathcal{F}_s \right] = 0 \quad (30)$$

² Often, problems of this kind can be solved using arbitrage arguments. However, to use this approach in our case seems challenging since most constructions will entail prohibitive transaction costs.

and

$$\hat{\Psi}' = E \left[e^{-\eta s_i} [\tilde{X}_i - R_i] | \mathcal{F}_s \right] = 0 \quad (31)$$

From (28) we see that relation (30) can be written

$$\hat{\Phi}' = E \left[\left(\tilde{X}_i (k_i - k_i^{TC}) + R_i k_i^{TC} \right)^{\vartheta-1} [R_i - \tilde{X}_i] | \mathcal{F}_s \right] = 0 \quad (32)$$

and, it follows from (27) that (31) is equal to

$$\hat{\Psi}' = E \left[e^{-\eta [\tilde{X}_i - R_i] k_i^{TC}} [\tilde{X}_i - R_i] | \mathcal{F}_s \right] = 0 \quad (33)$$

Numerical example in the case of demand uncertainty

From the above, we know that the time charter rate will always be below the expected time charter equivalent spot rate, as long as both the charterers and the shipowners are risk averse. We continue numerically in order to get a better understanding of the effects due to shifts in time charter demand and supply. For the base case time charter equilibrium we use the parameter and variable values listed in table 1 below.

For simplicity, assume a freight rate development without a trend. Hence, we set μ equal to zero. As regards the choice of σ , γ and ω , they are mainly in accordance with the values used in Tvedt (1995b).

Table 1; base case values for the time charter equilibrium

	Parameter/variable	Value
Geo. Brownian motion	μ	0
	σ	0.025
Vessel supply	γ	0.24
	ω	0,000000000000244
	k	500
	x_0	0.0003
Risk attitude	ϑ	0.2
	η	0.00000003

We assume that there are 500 vessels in the market, which is approximately equal to the present number of tankers in the VLCC segment.

Let the time charter contracts be fixed forward one quarter of a year. A longer time horizon will imply higher uncertainty. The effect of increased volatility will be studied below.

Given the above parameter values and number of vessels, the market clears, at time s , at an output of 4.3 billion tonne miles per day and a freight rate per tonnemile of *USD 0.0003*, i.e., $x_0 = 0.0003$. In the model this freight rate level is equivalent to a time charter equivalent spot rate per day of *USD 19,761.-* .

For the representative shipowner we have set the constant relative risk coefficient $\nu = 0.2$. The risk aversion of the charterer is given by the constant absolute risk aversion coefficient $\eta = 0.00000003$.

In the case of $k_i^{TC} = 0$ we see from relation (33) that the conditional expected time charter equivalent spot rate is equal to the time charter rate, i.e. $E[\tilde{X}_t | \mathcal{F}_s] = R_t$. Further, it follows from the exponential form of the first factor in (33) that for the representative charterer, an increase in k_i^{TC} must be compensated by a reduction in R_t . Hence, the demand curve lies below the conditional expected time charter equivalent spot rate.

For $k_i^{TC} = 0$ we see that relation (32) reduces to

$$\hat{\Phi}' = k_i^{\vartheta-1} E[R_t \tilde{X}_t^{\vartheta-1} - \tilde{X}_t^{\vartheta} | \mathcal{F}_s] = 0 \quad (34)$$

We have that $R_t < E[\tilde{X}_t | \mathcal{F}_s]$ in order for this to hold. For any two variables a and b we have that $\text{cov}(a, b) = E[ab] - E[a]E[b]$. Hence, it follows that $E[X_t | \mathcal{F}_s] E[X_t^{\vartheta-1} | \mathcal{F}_s] = E[X_t^{\vartheta} | \mathcal{F}_s] - \text{cov}(X_t, X_t^{\vartheta-1} | \mathcal{F}_s)$. Because $\vartheta < 1$, we must have that $\text{cov}(X_t, X_t^{\vartheta-1} | \mathcal{F}_s) < 0$. Therefore, it follows that

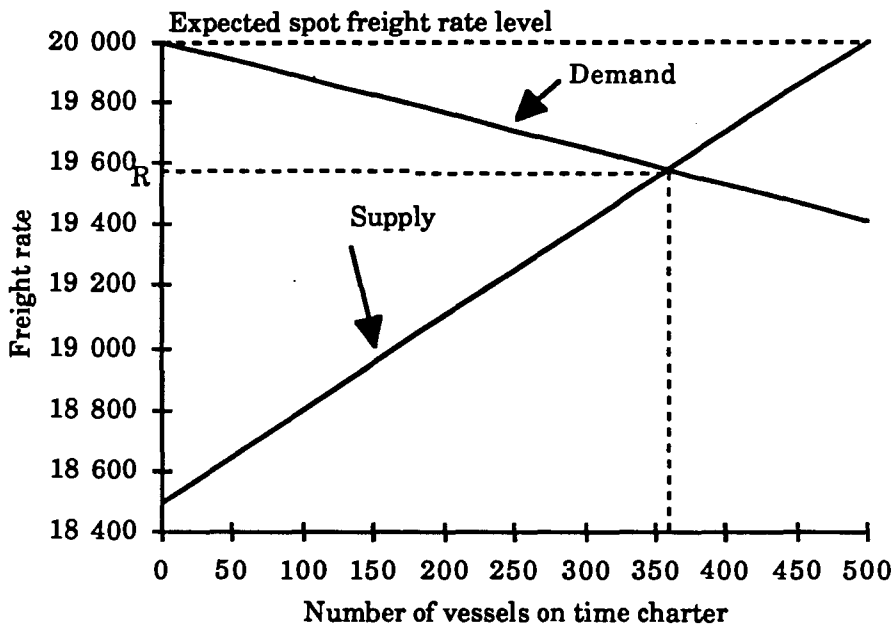
$$E[X_t | \mathcal{F}_s] > \frac{E[X_t^{\vartheta} | \mathcal{F}_s]}{E[X_t^{\vartheta-1} | \mathcal{F}_s]} = E^Q[X_t | \mathcal{F}_s] = R_t \quad (35)$$

Further, for $k_t^{TC} = k_t$, that is, the whole fleet is on time charter, there is no uncertainty for the shipowner and relation (32) reduces to

$$\hat{\Phi}' = (R_t k_t)^{\sigma-1} E[R_t - \bar{X}_t | \mathcal{F}_s] = 0 \quad (36)$$

The time charter rate will then simply be the conditional expectation of the time charter equivalent spot rate, that is, $R_t = E[\bar{X}_t | \mathcal{F}_s]$.

Figure 1; base case time charter equilibrium



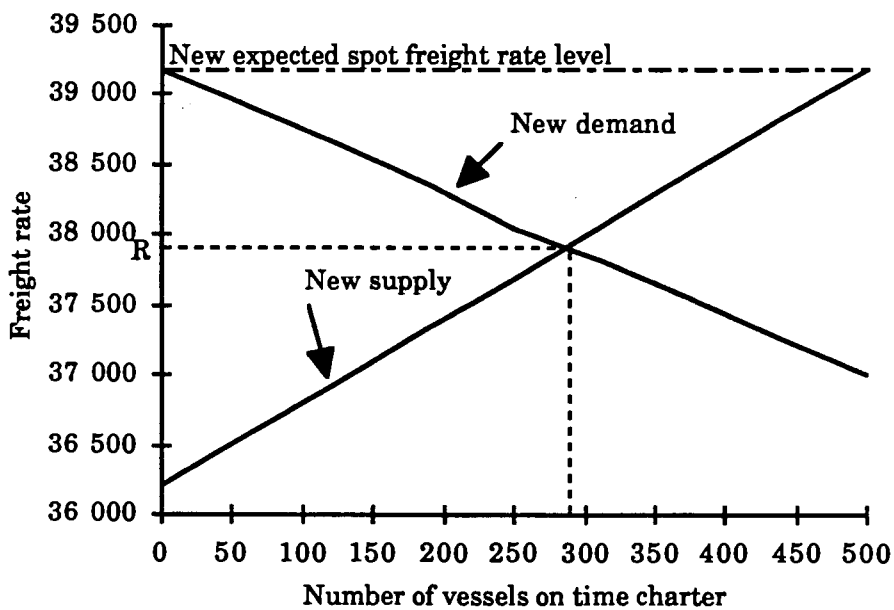
In the base case the expected time charter equivalent spot rate is $E[\bar{X}_t | \mathcal{F}_s] = 20,000$.-. The corresponding equilibrium time charter rate is USD 19,580, i.e. $R_t = 19,580$. We see that approximately 360 vessels, or 70% of the fleet, are hired in the time charter market.

According to the model, the expected time charter equivalent spot rate always lies above the time charter rate. However, since spot rates tend to stay at very low levels for long periods and only occasionally make upward jumps which are soon reverted, the expected spot rate will usually lie above the actual spot rate. Consequently, even the time charter rate can lie above the actual spot rate which materialises, and still be below the conditional expected spot rate. This seems to have been the prevailing situation during most of the 1980's. See e.g. Stray (1992).

Effects of high freight rates

Empirically, high freight rates seem to entail high volatility. Figure 2 shows a new equilibrium given by a higher initial freight rate level. We assume that the present spot level is $x_0 = 0.0005$ giving a present time charter equivalent spot rate of *USD 38,700*. Further, the expected time charter equivalent spot rate at the time of delivery is *USD 39,200*. Since the freight rate follows a geometric Brownian motion, we know that volatility increases geometrically with the freight rate. Hence, the relative standard deviation rate of X_t remains unchanged by this new higher freight rate level.

Figure 2; Equilibrium given a higher initial freight rate level



The main effect of a higher initial freight rate level is that both the level of R_t and the level of $E[\tilde{X}_t | \mathcal{F}_t]$ have increased. Observe that both the slope and the relative level of the supply curve are unchanged. They are unaffected due to the combination of an unchanged relative standard deviation and the fact that the representative shipowner is assumed to have a constant relative risk aversion.

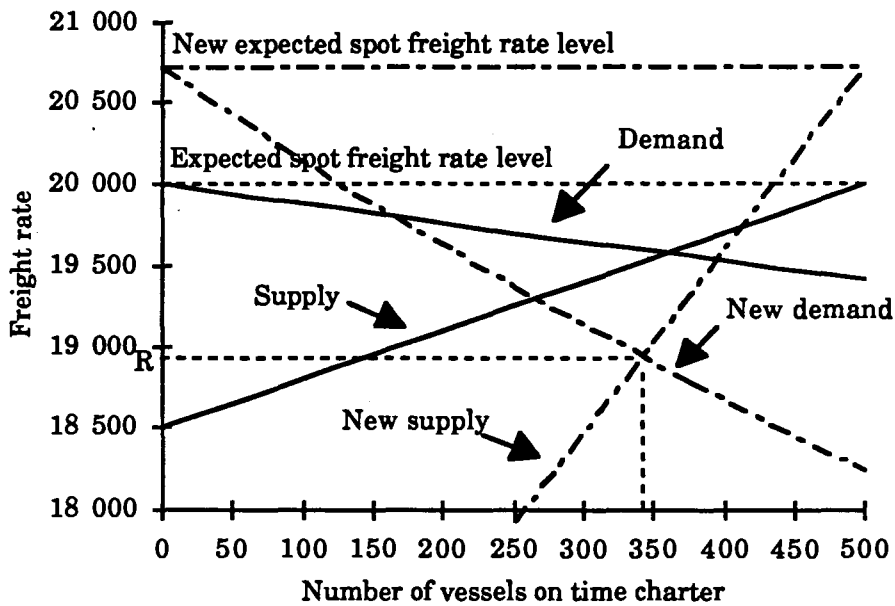
However, since the representative charterer has a constant absolute risk aversion, the demand function is influenced by the change in the freight rate level and the slope of the demand curve becomes steeper. In this case the higher present spot rate level reduces the number of time charter contracts, and the time charter rate in percentage of the expected time charter equivalent spot rate falls. Evidently, other utility representations

may give totally different results. A good description of risk attitude is vital in order to determine the volume effect.

Effects of higher volatility

Figure 3 below shows the change in the time charter equilibrium given an increase in volatility for an unchanged initial freight rate level. We double the value of σ to 0.05.

Figur 3; Effects of higher volatility on the time charter equilibrium



Higher volatility increases the slope of the supply curve. This is due to the risk aversion of the shipowners. Observe also that the expected time charter equivalent spot rate increases with the volatility coefficient σ . At $k_i^{TC} = k_i$ we have that $R_t = E[\tilde{X}_t | \mathcal{F}_t]$, which, in our case, has increased due to higher volatility. At $k_i^{TC} = 0$ we have from relation (33) that R_t , relative to $E[\tilde{X}_t | \mathcal{F}_t]$, is reduced because of the increase in volatility. All in all, the supply curve becomes steeper.

The demand curve also becomes steeper. Because of higher volatility, the charterer must be compensated for the extra risk of hiring time charter vessels.

As long as the market initially clears at a time charter level below the crossing of the old and the new supply curves or the crossing of the old and

the new demand curves, as in our figure, the effect of an increase in volatility is a lower time charter rate. The effect on the number of time charter fixtures is not obvious. The downward shift in demand has a negative impact on the number of fixtures, and the shift in supply has a positive impact on the number of fixtures. In our case the total effect is negative, but it may also be positive. Again, the elasticity of demand and supply, given by the risk attitude of the representative agents, determines the effect on the number of fixtures of time charter vessels.

If the market initially clears at a level above the crossing of the supply functions, the shift in demand has a negative impact, and the shift in the supply curve has a positive impact on the time charter freight rate. In that case the number of time charter fixtures will be reduced. The reverse will be the case if the market initially clears at a level above the crossing of the demand functions.

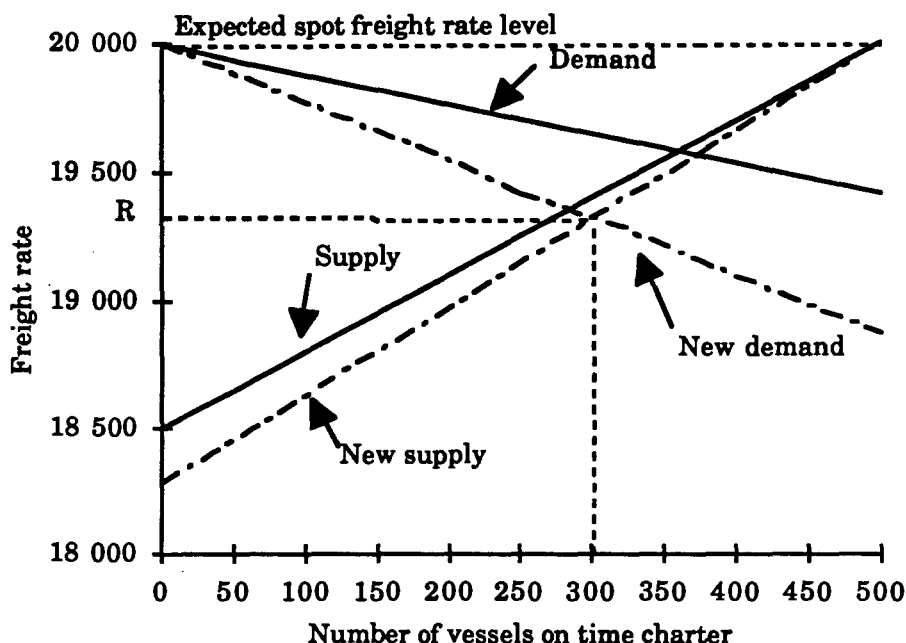
If we ignore the effect on the expected value of an increase in σ , the pure effect of increased volatility is that the slope of the demand and the supply relations get steeper. The equilibrium time charter rate falls, but the effect on the number of fixtures will depend on the risk attitude of the agents.

Effects of increased risk aversion

The risk aversion of the two parties is a key element in order to determine the effect of the above changes on the number of fixtures and the freight rate level. In figure 4 we show the effect on the time charter equilibrium of an increase in the risk aversion. The relative risk aversion of the representative shipowner is defined by $-\phi''\Pi_i^*/\phi' = (1-\vartheta)$ and consequently, risk aversion falls as ϑ increases. The absolute risk aversion of the representative charterer is given by the risk coefficient $-\phi''/\phi' = \eta$.

Figure 4 below shows the effect of more risk averse shipowners and charterers. We have set $\vartheta = 0.1$ and $\eta = 0,00000006$

Figure 4; Effects of higher risk aversion on the time charter equilibrium



A higher risk aversion gives a steeper demand function and a lower risk aversion a more gentle slope. These effects can be seen from relation (33) where $\lim_{\eta \downarrow 0} \hat{\Psi}' = 0$ for $R_t = E[X_t | \mathcal{F}_t]$, i.e. in the risk neutral case. Higher risk aversion means that the representative charterer demands a lower fixed freight rate to take on the extra risk of hiring vessels in the time charter market. Thus, increased risk aversion among the charterers reduces both the equilibrium time charter rate and the number of time charter fixtures.

The slope of the supply function gets steeper as the risk aversion of the shipowners increases, and vice versa. This follows from similar arguments as above. In the risk free case, where $k_t^{TC} = k_t$, then $R_t = E[\tilde{X}_t | \mathcal{F}_t]$, which is unaffected by the change in risk aversion. However, higher risk aversion means that for all $k_t^{TC} < k_t$ the shipowner will accept a lower time charter rate than before.

The total effect on the time charter rate is negative if both parties become more risk averse, and the effect is positive if both parties become less risk averse. Observe that the effect on the number of time charter fixtures is only uniquely determined if the changes in risk aversion of the two parties go in opposite directions.

Supply uncertainty and the time charter equilibrium

In the basic model above we only take demand uncertainty into consideration, represented by a stochastic freight rate. This simplification can be justified since demand is much more uncertain than supply, especially in our short-run perspective. However, for completeness we focus in this part on supply uncertainty and its effect on the time charter equilibrium and the relation to the spot market. As will be evident from the discussion below, we develop to a large extent this model of supply uncertainty by drawing on results from above. We focus on two sources of supply uncertainty, firstly, the size of the fleet and secondly, the price of the short term inputs.

Although the fleet size is usually almost deterministic, there have been short periods of extensive uncertainty related to the capacity of the fleet. We model this by letting k_t be a stochastic variable. Assume that the increment of k_t is given by the geometric Brownian motion

$$dk_t = \alpha k_t dt + \hat{\beta} k_t dZ_t^2 \quad (37)$$

where α is the instantaneous drift and β is the standard deviation of the incremental relative change in the capital stock. The standard Brownian motion, Z_t^2 , is as defined above. Evidently, k_t will never be negative, but occasionally $k_t < k_t^{TC}$. This happens in the case that the shipowner is unable to supply the agreed number of vessels. We assume that a shipowner who fails to deliver a time charter vessel must compensate the charterer by the difference between the prevailing equivalent spot rate and the time charter rate. Let this also be the case at an aggregated level. Consequently, the gain and profit functions, (12) and (16), remain unchanged.

Changes in total capital stock will influence the spot market equilibrium. Consequently, we need to describe the oil market in more detail than above in order to study market equilibrium under both demand and supply uncertainty. However, in the extended model developed below, all the above findings follow from the special case of no supply uncertainty.

Assume that the inverse demand function for crude oil is given by a constant price elasticity function of the form

$$P_t = \left(\frac{Q_t}{Y_t} \right)^{\frac{1}{\varepsilon}} \quad (38)$$

where $-\varepsilon$ is the price elasticity of demand and Y_t is a stochastic demand scalar. Let the increment of Y_t be given by a geometric Brownian motion

$$dY_t = \hat{\mu}Y_t dt + \hat{\sigma}Y_t dZ_t^1 \quad (39)$$

The instantaneous drift term, $\hat{\mu}$, and the standard deviation, $\hat{\sigma}$, of the relative change in Y_t , are both constants. The standard Brownian motion, Z_t^1 , is the same as the one in equation (20) and is independent of Z_t^2 .

For the representative charterer, the marginal revenue of transport to the refineries must be equal to marginal costs, which is simply assumed to be the freight rate. From (11) and (38) it then follows that

$$X_t = \left(\frac{Q_t}{Y_t} \right)^{\frac{1}{\varepsilon}} \quad (40)$$

Substituting the optimal total supply from relation (17) for Q_t , we get the equilibrium freight rate at time t , given by

$$X_t = \zeta \left(\frac{Y_t}{k_t} \right)^\gamma \quad (41)$$

where

$$\zeta = \left(\frac{w}{\gamma} \right)^{\frac{-\gamma}{(\gamma-1)\varepsilon-\gamma}} \quad (42)$$

and

$$\varsigma = \frac{(\gamma-1)}{(\gamma-1)\varepsilon-\gamma} \quad (43)$$

Relation (40), together with the increments of the capacity and demand scalar from (37) and (39), imply that the increment of the freight rate at time t , is given by Ito's lemma as follows

$$dX_t = \left(\hat{\mu} - \alpha + (\varsigma - 1)\hat{\sigma}^2 + (\varsigma + 1)\hat{\beta}^2 \right) \varsigma X_t dt + \varsigma \hat{\sigma} X_t dZ_t^1 - \varsigma \hat{\beta} X_t dZ_t^2 \quad (44)$$

Assume that $\mu = (\hat{\mu} - \alpha + (\zeta - 1)\hat{\sigma}^2 + (\zeta + 1)\hat{\beta}^2)\zeta$ and $\sigma = \zeta\hat{\sigma}$ and let $\beta = \zeta\hat{\beta}$, then relation (44) simplifies to

$$dX_t = \mu X_t dt + \alpha X_t dZ_t^1 - \beta X_t dZ_t^2 \quad (45)$$

Observe that (45) is identical to the geometric Brownian motion in (20) if $\hat{\beta} = 0$, that is, in the case of a deterministic supply development. Hence, the structure of the freight rate is unchanged by the introduction of supply uncertainty, only the trend and degree of volatility are altered. Further, the freight rate and the level of the capital stock will be correlated due to Z_t^2 , which wholly or in part generates both processes. Consequently, the profit function of the shipowner, relation (28), will have two correlated sources of uncertainty, the freight rate, X_t , and the capital stock, k_t .

The gain function of the charterer has only one source of uncertainty, i.e. X_t . This follows from (12), since the gain function in this case is independent of k_t . In the case of $q_t = 0$ the gain of the charterer will be zero almost surely if no vessels are rented on time charter. But, the introduction of supply uncertainty will increase freight rate uncertainty. Hence, the effect of increasing supply uncertainty on time charter demand in the case of $q_t = 0$, will be equivalent to the effect of increasing demand uncertainty.

Generally, we have that the maximal gain function (12) can be written

$$S_t^* = \tilde{X}_t k_t^{\tau c} - R_t k_t^{\tau c} + X_t q_t \quad (46)$$

It follows that the time charter equivalent spot rate \tilde{X}_t , in terms of k_t and Y_t , is given by

$$\tilde{X}_t = \Gamma Y_t^\nu k_t^{-\nu} \quad (47)$$

where $\nu = \frac{-1}{(\gamma - 1)\varepsilon - \gamma}$ and $\Gamma = (1 - \gamma) \left(\frac{w}{\gamma} \right)^{\frac{-\gamma}{1-\gamma}(1-\nu)}$

Hence, the optimal gain function, in terms of \tilde{X}_t , k_t and Y_t , is

$$S_i^* = (\tilde{X}_i - R_i)k_i^{TC} + \zeta Y_i^\zeta k_i^{-\zeta} q_i \quad (48)$$

We have that $\zeta \geq 0$ and $\nu \geq 0$. From (47) we see that an increase in the capital stock, k_i , reduces the time charter equivalent spot rate, \tilde{X}_i . Hence, from (48) it follows that the supply uncertainty cannot be reduced by hiring time charter vessels since the value of the last term is positively correlated with \tilde{X}_i . If the total capital stock, k_i , increases, the time charter equivalent spot rate, \tilde{X}_i , falls and the charterer loses on the vessels hired at a higher time charter rate. In addition, total deliveries of oil increase and the price of oil declines. Hence, the value of the oil already available at the refineries, given by the last term of (48), is reduced.

A reduction in k_i means that the charterer makes a gain on the time charter contracts, oil prices go up and profit increases.

Therefore, the charterers cannot reduce the supply risk exposure by renting vessels in the time charter market. On the contrary, a high k_i^{TC} only means that the gain fluctuates even more. Hence, the charterers will never accept to pay a higher time charter rate than the expected time charter equivalent spot rate.

From the demand relation above, the profits of the representative shipowner are given by

$$\Pi_i^* = \Gamma Y_i^\nu (k_i^{1-\nu} - k_i^{-\nu} k_i^{TC}) + R_i k_i^{TC} \quad (49)$$

If $0 < \nu < 1$, i.e., demand is elastic, we see that an increase in total capital stock, k_i , increases the value of the first term. The relative reduction in freight rate is less than the relative increase in supply. Hence, total profit from spot operation increases. A lower freight rate also increases the gain from having fixed vessels in advance on time charters. All in all, an increase in k_i has a purely positive impact on profits, irrespective of the number of time charter vessels. Analytically, we have that

$$\frac{\partial \Pi_i^*}{\partial k_i} = \Gamma Y_i^\nu ((1-\nu)k_i^{-\nu} + \nu k_i^{-\nu-1} k_i^{TC}) > 0 \quad \text{for } 0 < \nu < 1 \quad (50)$$

Equivalently, a reduction in k_t will have a negative effect on profits, given elastic demand. If demand for oil is elastic, it is thus not possible to hedge against risk due to supply uncertainty, by fixing vessels in advance in the time charter market.

If demand for oil is inelastic, it will be possible for the shipowner to reduce exposure to supply uncertainty by using the time charter market. An increase in k_t reduces profits, but as the spot freight rate falls, the shipowner gains from having fixed vessels in the time charter market.

Bunker price uncertainty

We now turn to the case of an uncertain price of the short term input bundle. It is natural to consider w_t as the price of bunkers, and it should therefore be closely related to the price of oil, p_t . The price of oil is to some degree influenced by the cost of transport, and changes in w_t may partly be due to changes in X_t . Hence, an increase in the demand for sea transport may rise the price of bunkers. However, since the cost of sea transport is a minor part of the price of oil, the price of bunkers is probably only to a small degree influenced by the freight rate. Previously, in the case of demand uncertainty, we ignored this relation.

Assume that volatility in w_t is the only source of uncertainty. For $\varepsilon > 1$ we have that $0 < \nu < 1$, and for $\varepsilon < 1$, it follows that $\nu > 1$. In the case of elastic demand, i.e., $\varepsilon > 1$, the spot freight rate, X_t , increases as the price of bunkers increases whereas the time charter equivalent spot freight rate, \tilde{X}_t , decreases. This follows from relations (17), (40) and (47). From (46) we see that, for a positive q_t , the charterers can hedge against bunker price risk by hiring more time charter vessels. However, this is only true as long as demand is elastic. If $\varepsilon < 1$, then both the spot rate and the time charter equivalent spot rate increase, as the bunker price increases. Hence, more time charter vessels will only expand the exposure of the charterers.

The shipowner can hedge against bunker price risk by using the time charter market, regardless of the elasticity of demand. This follows from equations (47) and (49). If all vessels are on time charters, then there will be no bunker layout payable for the shipowner, and hence, bunker price risk is eliminated.

Summary and main conclusions

We have suggested a model for describing the time charter market and the distribution of vessels between the time charter market and the spot charter market for VLCCs. Our model may be useful for pointing out some factors that influence the time charter market.

In the introduction we review some stylistic facts about the historical development of the time charter market. Then we derive aggregated supply in the VLCC market by assuming a constant return to scale Cobb-Douglas production function. We let aggregated demand in the crude oil market be given by a function with constant price elasticity. We introduce a representative shipowner and a representative charterer, who are both risk averse. The shipowner maximises profits by providing transport services and the charterer maximises total gain by selling crude oil less the cost of transport.

According to our model, the time charter rate always lies below the expected time charter equivalent spot rate in the case of demand uncertainty and in the case of supply uncertainty and inelastic demand. In these cases the shipowners can hedge using the time charter market, but the charterers cannot. We find that higher volatility or increased risk aversion reduce the time charter rate relative to the expected equivalent spot rate.

Demand in the VLCC market is probably inelastic, but for completeness we derive that for elastic demand and uncertain capital stock neither the shipowner nor the charterer can hedge by using the time charter market. In the case of elastic demand and uncertain bunker price both parties can hedge.

Since we are not able to say anything about volume effects without knowledge of the elasticity of the supply and demand functions, further research is needed in order to relate our model to observations. It is easier to be conclusive on the effect on the time charter rate relative to the expected time charter equivalent spot rate. Empirically however, we easily run into problems. Often it is difficult to find observations of time charter rates for a number of durations. In addition, it is not possible to observe the conditional expected time charter equivalent spot rate.

This paper only serves as a preliminary discussion of the relation between the spot and the time charter markets and a major motivation is to point out fields for empirical research. However, theoretically there exist numerous natural extensions. A first step may be to study a more generalised model. Our approach also call for a refinement of the production function. See e.g. Evans (1988 & 1994). We also think it may be useful to study the effect of deviations from the perfect competition assumption in the oil market or the time charter market.

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Appendix

Time charters of different durations

In this paper we have regarded a time charter as a forward on a time charter equivalent spot charter. A normal spot charter last on average about 60 days, whereas time charters may have a duration of up to the life time of a vessel. Thus, the present value of a time charter is the certainty equivalent value of a succession of spot rates discounted by a risk free interest rate. The value of a time charter with duration equivalent to n voyages is given by

$$V_t^{TC} = \sum_{k=1}^n E^Q \left[e^{-\int_t^{t_k} r_u du} \tilde{X}_{t_k} | \mathcal{F}_t \right] \quad (51)$$

where r_u is the instantaneous risk free interest rate at time u , \tilde{X}_{t_k} is the time charter equivalent spot rate at time t_k , and Q is a certainty equivalent probability measure.

It seems reasonable to assume that the spot freight rate follows a mean reversion process. See e.g. Bjerksund & Ekern (1995) or Tvedt (1995a). Naturally, at the time of exertion, the value of the forward rate process will be equal to the spot rate process given by relation (20) and (45). In order to determine the value of (51) we therefore need to specify a spot process for each n voyage charters. A number of authors have argued that for very long-term charters the average time charter rate approaches a long-term mean freight rate level that depends on the cost structure in the industry. If the spot rate is above this long-term level, the time charter rate falls with the length of the time charter period. Conversely, if the spot rate is below this long-term level the time charter rate increases with the duration of the contract. Zannetos (1966) discusses this in detail. The trends of the spot processes should be specified to give such a mean reversion property.

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Ch. 3;

A model of the short run freight rate formation in the VLCC market

Abstract

This paper focuses on the spot charter market for Very Large Crude Carriers (VLCCs) in a short term perspective. We develop a model in which the number of vessels is assumed fixed. Further, we assume that the intervals between each available cargo are irregular. By nature there must be a one-to-one matching of vessels and cargoes. Each vessel is assumed to be identical to all the other vessels in the market except for its distance to the loading area. The charterers are characterised by the point of time at which they prefer to dispatch their cargoes. The set of stable matches is restricted upward by a maximum freight rate level given in the case the shipowner sets the rate and restricted downwards by a minimum freight rate level given in the case the charterer sets the rate. We assume price competition among the agents in the market. In order to assign a unique freight rate to every match we introduce a weight function that may depend on the "psychology" of the market. We compare results from a default run of the model to market observations. The model fails in one major respect. It is unable to explain the quite striking fact that the activity level of the previous week influences the freight rate development during the following week.

A short introduction to the VLCC market

Every year some 640 million tonnes of crude oil are exported from the Persian Gulf region (1991 figures including export from the Persian Gulf and the Red Sea). About 49% per cent of the world's shipment of crude oil originates from this area. Pipe lines take care of some of the transport requirement, but the major part of the oil is moved by tankers. The Very Large Crude Carriers (VLCCs) and the Ultra Large Crude Carriers (ULCCs) are mainly trading on the Persian Gulf. 79% of all crude oil exported by VLCC/ULCCs comes from the Persian Gulf or the Red Sea. Furthermore, both the Near East and Africa have each a 7% stake in the over all export carried by these vessels. Smaller vessels are not competitive as regards moving large volumes of oil over long distances. Therefore, the VLCCs are sheltered from competition under normal circumstances. The major oil consumption areas are North America, Western Europe and the Far East. 23% of all oil carried by VLCCs are delivered in Europe and the Mediterranean, 19% in North America and 24% in Japan.

Normally, there is no return cargo that is suitable for being transported in a VLCC from the consumption areas back to the Gulf. Thus, the VLCCs return in ballast immediately after discharging the crude oil. Therefore, the

shipowner must include the back haul when calculating the freight rate he needs for accepting a cargo. When a shipowner selects a destination, he must take into consideration the total profit of the round trip as well as his expectations of the freight market at the time of return to the Gulf.

Our impression is that the shipowners only to a moderate extent take the return date into consideration when choosing among different cargoes. However, the voyage length of the main round trips do differ to some degree. The Persian Gulf to Japan and back takes about 50 days, the Persian Gulf to USA and back takes about 82 days and the Persian Gulf to Rotterdam and back takes about 65 days.

The shipowner may to a certain extent influence his time of arrival either by slow steaming or speeding up. It seems as if slow steaming is very unusual, especially in times of low and moderate bunker prices. An average working speed at 14 knots is a reasonable estimate at bunker oil prices slightly below USD 100 per tonne.

The first VLCCs were built in the late 60's. Since then there has been no major technical break-through to make vessels significantly different as regards short term marginal costs. Occasionally, when the price of bunker is high, tankers equipped with diesel engines have a cost advantage to turbine tankers. However, the majority of the fleet is turbine tankers, at present about 55%. Generally, it seems as if the turbine tankers, constituting the old generation of tankers, have both a higher speed flexibility and capability.

Table 1; The world fleet of VLCCs, ULCCs and large Ore / Oil carriers 1991

Owner type	# of owners	# of vessels	Average #	% of fleet
Large indep.	68	281	4.1	55%
Small indep.	65	93	1.4	18%
Oil major	9	66	7.3	13%
Oil producers	9	52	5.8	10%
Other Oil	8	22	2.8	4%
Total	159	514	3.2	100%

Source: Clarkson Research Studies Limited, Autumn 1991

The supply side is characterised by a large number of shipowners. The tanker owners include the oil majors, independent shipowners and to an increasing degree oil producers. Table 1 above shows the structure of ownership.

There is some concentration of ownership. The 22 largest owners (i.e. 14%) control 53% of the total fleet. 32 owners control four or more vessels. However, there are as many as 82 owners that control one single tanker.

Evidently, the different fractions of the group of shipowners may have divergent interests. Actually, the oil majors and the oil producers will probably profit from keeping the freight rates very low even though they are shipowners.

The proportion of the fleet owned by the independents has fluctuated to some degree. In 1991 73% was owned by independents. Back in 1973 it was at a low 66%. Oil producers operating as major owners of VLCCs is a reoccurring phenomenon. Historically, the "seven sisters" were both major owners of crude oil tankers and dominant producers. Today their importance as producers has declined. Today's major producers, the national Arabic corporations, seem to be increasing their involvement in tanker ownership.

In a short term perspective the oil companies control a larger part of the VLCC fleet than their direct investments in vessels. A major part of their transport requirement is covered by independently owned vessels hired on time charter. The part of the total fleet that at any time has been on time charter has varied. In 1973 the long term charter business reached a peak with 52% of the total fleet on time charter. In 1991 it was only 16%. Occasionally, oil companies control more vessels than they require. Any surplus capacity will be free for hire in the single voyage market - the spot market. This reletting of vessels by the oil companies may in the short term change the number of vessels that are competing in the spot market.

The brokers play a major part in the spot market. They are mediators between shipowners and charterers. In addition, they provide the two parties with market intelligence. Due to the brokers, all parties will, with only minor effort, be aware of the position of any vessel in the market, will

be informed about most free cargoes and will know the prevailing freight rate level as well as historical freight rates.

When a charterer needs a vessel for transportation of oil from the Gulf he approaches his exclusive broker or a number of brokers. The charterer's broker announces to his network of shipowner's brokers that there is a free cargo. Immediately after receiving the request the shipowner's broker starts searching for appropriate vessels. He finds a vessel in a suitable position, gets a first offer from the shipowner, and passes it on to the charterer's broker. The offer from the shipowner will be valid for a specified limited time. The charterer is now in possession of at least one offer, probably more. He will normally not accept these first offers, but makes counteroffers. The shipowner is now free to accept the offer, to make a counter offer, or to cut further bargaining and to work another cargo. During the bargaining between the charterer and the shipowner both parties risk that the other party prefers to make a deal with somebody else. Therefore, in order to offer the shipowners and the charterers sound advice it is vital for the brokers to know the competition from other vessels as well as alternative cargoes available. The position and cost efficiency of other vessels, the time preference of shipowners and charterers as well as alternative cargoes, are all key aspects in the bargaining process.

Clearly, a shipowner may compete for more than one cargo at a time. However, he cannot make an offer for a cargo as long as he is tied to an offer for another cargo. Therefore, it is reasonable to assume that a shipowner will always focus on giving offers for the cargoes of which he is in the best position to get.

In the same way, the charterer usually receives offers from a number of shipowners. However, he will only bother to give counter offers to the shipowners which are in the best positions to lift his cargo at the right time at a fair freight rate.

Imagine that an owner has an uncharted vessel waiting in the Gulf. The shipowner prefers to get the vessel hired as soon as possible, however, at a reasonable freight rate. Thus, he would like to get the earliest cargo available. Other vessels will be interested in this cargo as well. However, later on there will be more cargoes. The shipowner knows that in order to get this first cargo he must offer a rate that makes the charterer prefer his

vessels to those of the competitors. However, this does not hold if there are no other vessels interested in the cargo at this freight rate level. If that was the case, the shipowner could charge a freight rate only marginally below the level that would make other shipowners interested, and still get the cargo.

Above, we have implicitly assumed that the freight rate is set by the shipowner and that the charterers take the freight rate as given. However, the fixing of the freight rate is a matter of offers and counter offers. It may be argued that in some instances the freight rate is just as much dictated by the charterer. If the charterer sets the freight rate for a given match of a vessel and a cargo he must see to it that he pays a freight rate to the shipowner that makes the shipowner prefer his cargo to the competitors. Yet, if no other charterer is interested in this vessel at the given freight rate levels then the charterer may lower his offer. He may lower his offer until the freight rate makes other charterers only marginally uninterested.

Part one, An assignment model

We assume that new cargoes become available irregularly. We denote a cargo by a number, j , in accordance with its appearance. The charterer's most preferred point of time for dispatch is given by t_j . We assume that he is not able to lift his cargo before this date. However, if there is no tanker available at his most preferred point of time, dispatch will be postponed. For every incremental delay there will be a cost of b .

Vessels are differentiated by their present location. The distance from the Gulf will decide when a vessel will be able to load, and this will be the only unique characteristic of a vessel. We denote the call of a vessel at the Gulf by i and the time of arrival by t^i . We assume that all vessels are sailing at the same constant speed. That is, there is no strategic slow-steaming. Further, we simplify by considering only one representative trade. This is equivalent to assuming that all possible round-trips take exactly the same time. Thereby, we may describe the differences between the vessels by their position along a line, the time axes, showing the distance in time to arrival in the Gulf. Thus, vessels that have the same position may be regarded as equal. Vessels that are waiting for cargo have position zero. Vessels that are just about to leave the Gulf will obviously be the last to be ready for loading and will consequently have the most remote position on the time axis. In between, we have the position of all the other vessels.

Since vessels in the model are not allowed to slow-steam or to speed up, they may find themselves situated in the Gulf without any cargo immediately available. We assume that the incremental cost of waiting will be given by a .

In the model, the demand for shipping services is totally inelastic to freight rates. This is probably an acceptable approximation in this short term perspective and as long as the freight rate does not go sky high. The cost of sea transport as a proportion of the price of oil to the end consumers, is low. In addition, it takes time to change consumption from petroleum derivatives to other energy sources.

Matching

We have that the waiting cost for the shipowner of vessel i from accepting cargo j is given by

$$\alpha_{ij} = \max\{0, a(t_j - t^i)\}$$

That is, if the vessel arrives before the preferred time of dispatch then costs of waiting will incur.

The delay costs for cargo owner j from using vessel i is given by

$$\beta_{ij} = \max\{0, b(t^i - t_j)\}$$

That is, if the vessel arrives after the preferred time of dispatch the cargo owner will suffer costs of delay.

It follows from the definitions of α_{ij} and β_{ij} above that the cost of matching vessel i to cargo j is given by

$$g_{ij} = \alpha_{ij} + \beta_{ij} = \max\{\alpha_{ij}, \beta_{ij}\}$$

Our model is a short term model with a finite horizon. We close the model when the total number of calls and the number of cargoes are both equal to n , where $\max\{t_n, t^n\}$ gives our time horizon. Let the set of all possible matches, Ω , be given by

$$\Omega = \{\langle i, j \rangle \mid i \in \{1, \dots, n\}, j \in \{1, \dots, n\}\}$$

The total cost of matching the n cargoes with the n calls is

$$v(\omega) = \sum_1^n g_{\omega}$$

where $\omega \subset \Omega$, is a one-to-one matching of n cargoes and n calls, $\omega = (\omega_1, \omega_2, \dots, \omega_n)$, where $\omega_k = \langle i, j \rangle$. The total number of subsets ω of Ω is $n!$. Define \mathcal{F} as the set of all $n!$ possible one-to-one matches of n cargoes and n calls. Let the match among the $n!$ possible matches that minimise the total cost $v(\omega)$ be given by ω^* . The minimum cost function is then

$$\Phi = \min_{\omega \in \mathcal{F}} v(\omega) = v(\omega^*)$$

Proposition 1: Matching the first free cargo to the first free vessel, i.e. $\omega = \hat{\omega} = (\langle 1, 1 \rangle, \langle 2, 2 \rangle, \dots, \langle n, n \rangle)$, makes $v(\hat{\omega}) = \Phi$.

Proof: We have $v(\hat{\omega}) = g_{11} + g_{12} + \dots + g_{ii} + \dots + g_{kk} + \dots + g_{nn}$ where $t^i < t^k$ and $t_i < t_k$. Assume another match $\tilde{\omega}$ where cargo i and k have changed vessels, i.e. $v(\tilde{\omega}) = v(\hat{\omega}) - g_{ii} - g_{kk} + g_{ik} + g_{ki}$. Then it follows directly from linearity of costs that for any of the six possible permutations of t^i , t_i , t^k and t_k will $v(\tilde{\omega}) \geq v(\hat{\omega})$. Generalising to any rematching of $\hat{\omega}$ we have that for all $\omega \in \mathcal{F}$ are $v(\omega) \geq v(\hat{\omega})$. \square

The match $\hat{\omega} = (\langle 1, 1 \rangle, \langle 2, 2 \rangle, \dots, \langle n, n \rangle)$ is optimal but need not be a unique cost minimising match. In the case that some $t^i = t^k$ or $t_j = t_i$ then there will be some $\omega \in \mathcal{F}$ different from $\hat{\omega}$ that makes $v(\omega) = v(\hat{\omega})$.

Competition and market clearance

Each match is accompanied by a freight rate. The freight rate will be a result of the bargaining between the shipowner and the charterer. Let the core of the bargain be the set of freight rates restricted by an upper freight rate limit in the case when the freight rate is set solely by the shipowner and a lower limit when the freight rate is set by the charterer.

To find the upper limit we assume that the shipowners make offers for potential cargoes. In this formulation the charterers are not strategic players, and are supposed to just accept the best offer available. We assume price competition among the shipowners.

To find the lower limit we assume that the charterers make offers for the available vessels. The shipowners accept the best offer they receive. In this case price competition among the charterers is assumed.

We start out by letting the freight rate be dictated by the shipowner. In total there will be n^2 possible single matches of cargoes and calls. We assign a unique freight rate to every single match. All possible freight rates are then given by

$$x = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ x_{n1} & & \dots & x_{nn} \end{bmatrix}$$

where x_{ij} is the freight rate received by vessel i from carrying cargo j .

Shipowner i prefers to lift cargo j to cargo l if

$$x_{ij} \geq x_{il} - \alpha_{il} + \alpha_{ij} \quad (1)$$

That is, the shipowner prefers the cargo which gives the highest freight rate after adjusting for waiting costs. Remember that our model assume a representative voyage and therefore, we circumvent the problem the shipowners in practice may not ignore that different cargoes can imply different return dates.

In order for i to get the job of lifting cargo j , at least one of the two relations (2) and (3) below must be satisfied. That is, vessel i is preferred by j to all other vessels or no other vessel is interested in lifting cargo j .

The cargo owner j will accept the most favourable offer he receives. He will prefer the offer from i to the offer from any vessel k if

$$x_{ij} \leq x_{kj} + \beta_{kj} - \beta_{ij} \quad \forall k = 1, \dots, n \quad (2)$$

All vessels k different from i prefer other cargoes than j if for all vessels k there exists at least one cargo l such that

$$x_{kl} \geq x_{kj} - \alpha_{kj} + \alpha_{kl} \quad (3)$$

If relation (2) holds with inequality, shipowner i may increase the freight rate for lifting cargo j without losing the job. Thus, the best choice will be to set x_{ij} such that (2) holds with equality for at least one k and with inequality for the rest.

However, as long as (3) holds with inequality it is optimal for i to increase x_{ij} . Thus, there is at least one match (k, l) , when the freight rate for the match (i, j) , x_{ij} , is optimally chosen, such that

$$x_{kl} = x_{kj} - \alpha_{kj} + \alpha_{kl} \quad (4)$$

For this vessel k relation (2) must hold. Otherwise, k will be preferred by j to i , whereas k is indifferent between the cargoes j and l . That is, for an optimally chosen x_{ij}

$$x_{ij} = x_{kl} + \beta_{kj} - \beta_{il} \quad (5)$$

From (4) and (5) we have that i sets the freight rate x_{ij} such that there is at least one match (k, l) such that

$$x_{ij} = x_{kl} + \beta_{kj} - \beta_{il} + \alpha_{kj} - \alpha_{kl} \quad (6)$$

and (2) or (3) hold for all other matches.

In stead of paying x_{kl} for vessel k , l may pay x_{il} for vessel i . He prefers vessel k as long as

$$x_{il} \geq x_{kl} + \beta_{kl} - \beta_{il} \quad (7)$$

From (1) and (7) we then have that

$$x_{ij} \geq x_{kl} + \beta_{kj} - \beta_{il} - \alpha_{il} + \alpha_{ij} \quad (8)$$

In order for both the expressions (6) and (8) to hold we must have that

$$g_{kj} + g_{il} \geq g_{ij} + g_{kl} \quad (9)$$

Proposition 2: The upper limit freight rate vector \bar{x} , generated by equation (6) and satisfying expression (9), entails that the first free cargo is matched to the first free vessel. The freight rate vector is given by the trace of x , $\bar{x} = tr(x)$.

Proof: Assume that $t^i < t^k$ and $t_j < t_l$ or $t^i > t^k$ and $t_j > t_l$. Then (9) holds. Then assume that $t^i > t^k$ and $t_j < t_l$ or $t^i < t^k$ and $t_j > t_l$ and (9) does not

hold. Thus, we have that $\langle i, j \rangle$ and $\langle k, l \rangle$ are unique stable matches only if $t^i < t^k$ and $t_j < t_l$ or $t^i > t^k$ and $t_j > t_l$ for all i, k, j and l . Since there are n calls and n cargoes it follows that $i = j$ and $k = l$ for the matches to be stable. Then it also follows that the freight rate vector \bar{x} will be the trace of x . \square

Let the market equilibrium matches, $\bar{\omega}$, be the one-to-one matching of n cargoes and n calls which materialises in the market.

Lemma 1: Given price competition among the shipowners and no strategic interaction by the cargo owners, the vector of market equilibrium matches, $\bar{\omega}$, is among the cost minimising vectors, i.e. $v(\bar{\omega}) = \Phi$.

Proof: Follows from proposition 1 and 2. \square

We now turn to the lower limit freight rate vector \underline{x} . We assume price competition among the cargo owners and that the shipowners are free to lift the cargo that is most favourable to them. However, the cargo owners set the freight rate.

A cargo owner j prefers vessel i to vessel k as long as

$$x_{ij} \leq x_{kj} + \beta_{kj} - \beta_{ij} \quad (10)$$

However, in order for cargo owner j to get vessel i either relation (11) or (12) must hold. That is, vessel i must either prefer the offer from cargo owner j to all other offers i receives,

$$x_{ij} \geq x_{il} - \alpha_{il} + \alpha_{ij} \quad \forall l = 1, \dots, n \quad (11)$$

or all the other cargo owners must prefer other vessels than i . That is, for all cargo owners l different from j , there must be at least one vessel k such that

$$x_{kl} \leq x_{il} - \beta_{kl} + \beta_{il} \quad (12)$$

If (11) holds with inequality, it will be optimal for cargo owner j to reduce the freight rate x_{ij} until it holds with equality for at least one l and with inequality for the rest. Moreover, if (12) holds with inequality it will be

optimal to reduce the freight rate x_{ij} further. Thus, there will be at least one match $\langle k, l \rangle$ such that for an optimally chosen x_{ij}

$$x_{kl} = x_{il} - \beta_{kl} + \beta_{il} \quad (13)$$

For this cargo l relation (11) must hold with equality. If not, l will be preferred by i to j , whereas l is indifferent between vessel i and k .

$$x_{ij} = x_{il} - \alpha_{il} + \alpha_{ij} \quad (14)$$

From (13) and (14) we have that there must be at least one match $\langle k, l \rangle$ such that

$$x_{ij} = x_{kl} + \beta_{kl} - \beta_{il} - \alpha_{il} + \alpha_{ij} \quad (15)$$

Shipowner k prefers cargo l as long as

$$x_{kj} \leq x_{kl} - \alpha_{kl} + \alpha_{kj} \quad (16)$$

From (10) and (16) it follows that the freight rate must satisfy

$$x_{ij} \leq x_{kl} + \beta_{kj} - \beta_{ij} - \alpha_{kl} + \alpha_{kj} \quad (17)$$

It follows that relation (15) and (17) hold if

$$g_{kj} + g_{il} \geq g_{ij} + g_{kl} \quad (18)$$

Proposition 3: The lower limit freight rate vector \underline{x} , generated by equation (15), entails that the first free cargo is matched to the first free vessel. The freight rate vector is given by the trace of x , $\underline{x} = \text{tr}(x)$.

Proof: Observe that the right hand side of relation (15) is identical to x_{ij} in equation (8) and that the right hand side of relation (17) is equal to x_{ij} as defined in equation (6). In order for both (15) and (17) to be true, we must have $t^i < t^k$ and $t_j < t_l$, or $t^i > t^k$ and $t_j > t_l$. This is evident by the same reasoning as in the proof of proposition 2. Thus, given n calls and n cargoes then we must have $i = j$ and $k = l$ for the matches to be stable. Consequently, the freight rate vector \underline{x} will be the trace of x . \square

Lemma 2: Given price competition among the cargo owners and no strategic interaction by the shipowners, the vector of market equilibrium matches, $\underline{\omega}$, is among the cost minimising vectors, i.e. $v(\underline{\omega}) = \Phi$.

Proof: Follows from proposition 1 and 3. \square

Since the right hand side of relation (17) is identical to x_{ij} in equation (6) then it follows that $\bar{x} \geq \underline{x}$.

From proposition 2 and 3 we have that the first free cargo is matched to the first free vessel. Consequently, from (6) the freight rates \bar{x} will be generated by

$$\bar{x}_{ii} = \bar{x}_{i+1,i+1} + \beta_{i+1,i} - \beta_{i,i} + \alpha_{i+1,i} - \alpha_{i+1,i+1} \quad (19)$$

and from equation (15) \underline{x} must satisfy

$$\underline{x}_{ii} = \underline{x}_{i+1,i+1} + \beta_{i+1,i+1} - \beta_{i,i+1} - \alpha_{i,i+1} + \alpha_{i,i} \quad (20)$$

Proposition 4: Given $t^i < t^{i+1} < t_i < t_{i+1}$ or $t_i < t_{i+1} < t^i < t^{i+1}$ it follows that $\bar{x}_{ii} = \underline{x}_{ii}$ given the freight rate for the match $(i+1, i+1)$. Otherwise, \bar{x}_{ii} will always be larger than \underline{x}_{ii} .

Proof: There will be six permutations of t^i , t_i , t^{i+1} and t_{i+1} given stable matching. Further, using the notation $[z]^+ = \max[0, z]$, we have that

$$\begin{aligned} \beta_{i+1,i} - \beta_{i,i} &= b \left(\min \left[[t^{i+1} - t_i]^+, [t^{i+1} - t^i]^+ \right] \right) \\ \beta_{i+1,i+1} - \beta_{i,i+1} &= b \left(\min \left[[t^{i+1} - t_{i+1}]^+, [t^{i+1} - t^i]^+ \right] \right) \\ \alpha_{i+1,i+1} - \alpha_{i,i+1} &= -a \left(\min \left[[t_{i+1} - t^{i+1}]^+, [t_{i+1} - t_i]^+ \right] \right) \\ \alpha_{i,i} - \alpha_{i,i+1} &= -a \left(\min \left[[t_{i+1} - t^i]^+, [t_{i+1} - t_i]^+ \right] \right) \end{aligned}$$

Then it follows from (19) and (20) that for any permutation $\bar{x}_{ii} \geq \underline{x}_{ii}$ and in the special cases that $t^i < t^{i+1} < t_i < t_{i+1}$ or $t_i < t_{i+1} < t^i < t^{i+1}$ we have $\bar{x}_{ii} = \underline{x}_{ii}$. \square

The freight rate

According to our model the freight rate x will be in the range \underline{x} to \bar{x} , $x \in [\underline{x}, \bar{x}]$. In order to say anything more about the freight rate outcome of the bargaining, we must specify the bargaining power of the two parties in more detail. We suggest to let the freight rate vector be given by a weighted sum of \underline{x} and \bar{x} . That is,

$$x_{ii} = \eta_{ii} \underline{x}_{ii} + (1 - \eta_{ii}) \bar{x}_{ii} \quad (21)$$

where $\eta_{ii} \in [0, 1]$ gives the bargaining power of the charterer and $(1 - \eta_{ii})$ gives the bargaining power of the shipowner for the match $\langle i, i \rangle$.

Proposition 5: The freight rate vector x generated by relation (21) satisfies the same stability conditions as \bar{x} and \underline{x} .

Proof: For a given $x_{i+1, i+1}$ it follows from relation (2) and (3) that $x_{ii} \leq x_{i+1, i+1} + \beta_{i+1, i} - \beta_{i, i} + \alpha_{i+1, i} - \alpha_{i+1, i+1}$. This is the upper freight rate stability condition. From (19) it follows that the right hand side of the above inequality is equal to \bar{x}_{ii} . From (8) we have that $x_{ij} \geq x_{i+1, i+1} + \beta_{i+1, i+1} - \beta_{i, i+1} - \alpha_{i, i+1} + \alpha_{i, i}$. As already shown above, \bar{x}_{ii} satisfies this condition. However, this inequality can also be derived from relation (11) and (12) and it follows from equation (20) that the right hand side of this inequality is equal to \underline{x}_{ii} . Thus, the inequality is the lower freight rate stability condition. Further, we have that \underline{x}_{ii} must satisfy (17). However, this inequality is equal to the upper freight rate stability condition. Therefore, it follows that both \bar{x}_{ii} and \underline{x}_{ii} satisfy the same stability conditions. Hence, any linear combination of \bar{x}_{ii} and \underline{x}_{ii} will also satisfy the same two stability conditions. \square

Now we are able to relate the vector of matches which is consistent with the freight rate vector x , to the cost minimising vector.

Theorem: Given price competition both among the shipowners and the cargo owners and bargaining between the shipowners and cargo owners, the vector of market equilibrium matches, ω , is among the cost minimising vectors, i.e. $v(\omega) = \Phi$.

Proof: Follows from lemma 1 and 2 and proposition 5. \square

From the theorem and proposition 5 it follows that there is a close connection between the range \underline{x} to \bar{x} and the payoff of the core of a traditional assignment game. See e.g. the "elongated core" of Shapley & Shubik (1972) and Roth & Sotomayor (1992) for further references. However, we have a slightly different formulation due to the need for a tailor made model for our short term perspective. The most special feature is that the preferences, represented by the waiting costs and the costs of delay, depend on the history of the game. The time of arrival of a vessel will be equal to the date of departure of this vessel's preceding match plus the days used for one round trip. If a vessel must wait for a cargo the next arrival date will be postponed accordingly, and consequently, waiting reduces supply in the future. Thus, our assignment game has a dynamic nature.

The form of the bargaining weight function η is not related to the competition among shipowners and among cargo owners. Thus, we must introduce something more than just the waiting costs of the shipowners and the delay costs of the cargo owners in order to derive a unique freight rate. These forces may be what practitioners tend to call the "psychology" of the market. They may just as well depend on historical as well as future characteristics of the market. One such factor may be the activity level during recent weeks. Market reports from brokers put much emphasis on this in order to predict future freight rate development.

The cost functions

Above, we have assumed constant cost of waiting and delay, per unit of time. This is probably very unrealistic as well as unnecessarily restrictive. The same results will hold if we let the total costs be convex in time. That is, the cost of waiting may be specified as $c_i^\alpha = c_i^\alpha \left([t_i - t^s]^+ \right)$ and the cost of delay as $c_i^\beta = c_i^\beta \left([t^s - t_i]^+ \right)$, where $s \in (i, i + 1)$ in both cases and

$$\frac{d^2 c^\alpha}{dt_i^2} \geq 0, \quad \frac{d^2 c^\beta}{dt_i^2} \geq 0.$$

Further, the cost of waiting is closely linked to the freight rate level. Generally, the cost of waiting is mainly the alternative cost of not sailing

and thus, generating profits. However, if the freight rate goes below the voyage related costs, then the shipowner may prefer to lay up his vessel to continue trading and consequently, the alternative cost of waiting in the Gulf to sailing may be zero or negative. However, to let c_t^a depend on x may change the above stability conditions for some specifications of the function.

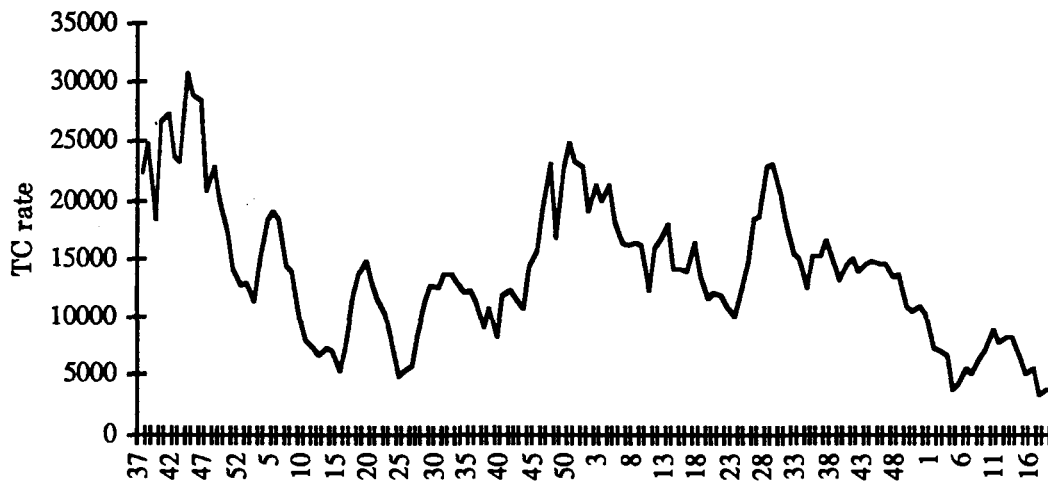
The cost of delay c_t^b is most probably not very strongly influenced by the freight rate. The cost of not dispatching the cargo at the most preferred point of time, will be associated with the expected development of the oil price. Normally, delayed shipment means reduction of storage in the consumption region. The cost of delay will thus be related to the convenience yield of being in possession of oil close to the consumers, compared to storage in the production region. Thus, it reflects the charterer's risk of stock-out. The difference between the value of optimal storage and the reduced storage from delay of shipment gives the cost of delay. Therefore, to let c_t^b be over-proportional in the time lag is probably reasonable, but it should not depend to any large degree on the freight rate level.

Part Two, Simulations and empirical findings

Observations from the VLCC market

Figure 1 shows the actual time charter equivalent spot freight rate development from August 1991 to May 1994. The time charter equivalent spot rates are derived from the operation of a crude oil tanker of 280,000 dwt. built in the mid 1970's and equipped with a steam engine.

Figure 1; Time charter equivalent VLCC spot rates, weekly average
August 1991 to May 1994



Source: R. S. Platou

These years represent a fairly good period for the tanker industry. During the late 1970's and the 1980's the freight rates were generally very low. This depressed market followed a period during the early 1970's of sky high rates. Therefore, the first years of the 1990's represent something in between the extremes of the 1970's and the 1980's. For this period we have weekly observations of some of the factors that are assumed by the model to determine the freight rate. Hence, we use observations from the early 1990's to test out some hypotheses related to the model. However, the available observations are not tailor made for testing the model. We will return to this below.

The following observations, in addition to the average freight rates, are available; the number of fixtures during the week, the number of vessels waiting in the Gulf at the end of the week, the number of vessels that are assumed to arrive in the Gulf in two weeks' time and the number of vessels

that are assumed to arrive in four weeks' time. Appendix A presents graphs of these observations.

Totally, we have 139 observations of the freight rate and the number of fixtures. Concerning the number of vessels waiting, four observations are missing, and in the data sets of vessels arriving two and four weeks later, eleven observations are missing. For the graphs and estimations below we have substituted these missing observations with linear combinations of the observations immediately before and after.

Simulations

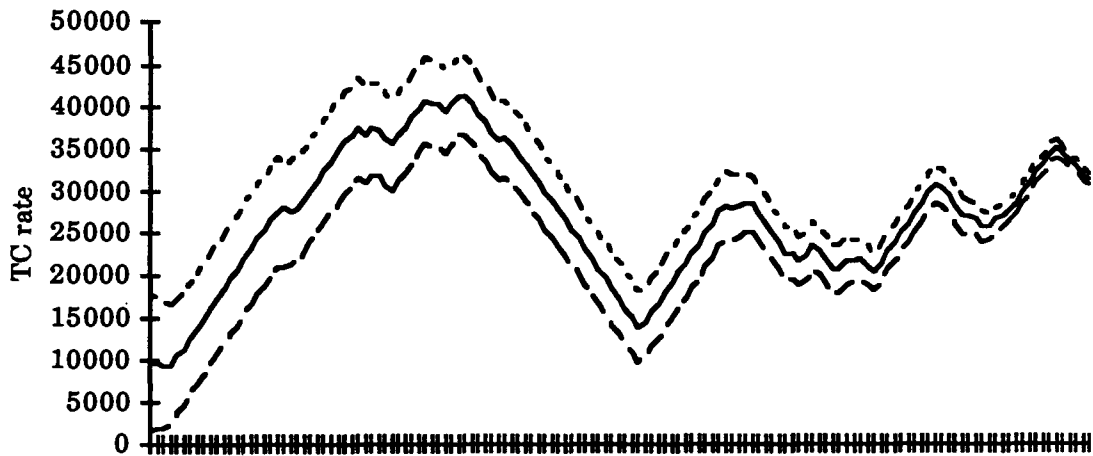
In the model we have a fixed number of 180 spot vessels. This is approximately the number of vessels that operated in the spot market in late December 1993 including all crude oil tankers and combis over 200,000 dwt. We assume that each vessel uses 60 days for one round-trip. At present this should be close to the average distance. Consequently, if there is no waiting there will be on average three vessels arriving in the Gulf each day. This average number should be higher than the demand intensity to secure that the freight rate on average does not go sky high.

We let the most preferred time of dispatch of the cargoes be distributed randomly with equal probability for equally large time intervals. All parties will know these preferred points of time before the freight rate is negotiated. The average number of cargoes per day is set slightly below three.

In appendix A below each of the plots of the available observations, there is a graph showing the equivalent data derived from a default run of the model. For this simulation, we have used linear cost of delay and cost of waiting both of USD 150 per day. We fix the freight rate for the last match at USD 30,000.- per day and solve the model backwards .

Figure 2 below shows the graph of the freight rate derived from the default run of the model. The stable outcomes are restricted by the upper and lower graphs. The freight rates, which we have chosen to compare with the observations, are generated from the average of the upper and lower bounds. That is, we have fixed η at 0.5.

Figure 2; simulated freight rates



Hypotheses

In order to indicate the goodness of the model we compare the available market observations to equivalent data derived from the default run. The most fundamental characteristic of the model is that the freight rates are determined by the structure of the market at present and not by any factors of the past. In line with this, we expect that a given weekly average freight rate, as reported above, is influenced by the number of vessels that are unhired and the number of vessels that are arriving in two and four weeks' time. These observations are obviously present characteristics of the market.

The number of fixtures during the week is of a more mixed character as regards explaining the average freight rate. There may be a correlation between the number of fixtures during the week and the number of vessels waiting at the end of the week. Present competition will be influenced. To be conclusive, from a model perspective, we need knowledge of the exact timing of arrivals and preferred points of dispatch. Nonetheless, according to our model, the number of fixtures the preceding week does not add anything to the understanding of the present freight rate formation.

Main empirical findings

We use ordinary least square estimation with the dependent variable being the change in the average freight rate from the Thursday of registration to the next, and the independent variables being the registered number of fixtures the preceding week, the number of waiting vessels on the given

Thursday, the number of vessels arriving in two weeks and the number of vessels arriving in four weeks. The estimation gives the following results;

Table 2; Change in the freight rate versus specified variables

<i>Variable</i>	<i>Estimated coefficient</i>	<i>Standard deviation</i>	<i>t-value</i>
No. of fixtures	149.58	42.18	3.55
No. waiting	110.55	57.27	1.93
No. in two weeks	-67.37	33.79	-1.99
No. in four weeks	-17.71	14.86	-1.19
Constant	-166.58	1373.3	-0.12

The regression yields a R^2 of 17.1% and a R^2 - adjusted of 14.5%. The Durbin-Watson statistic of 2.17 indicates no positive first order auto correlation in the residuals.

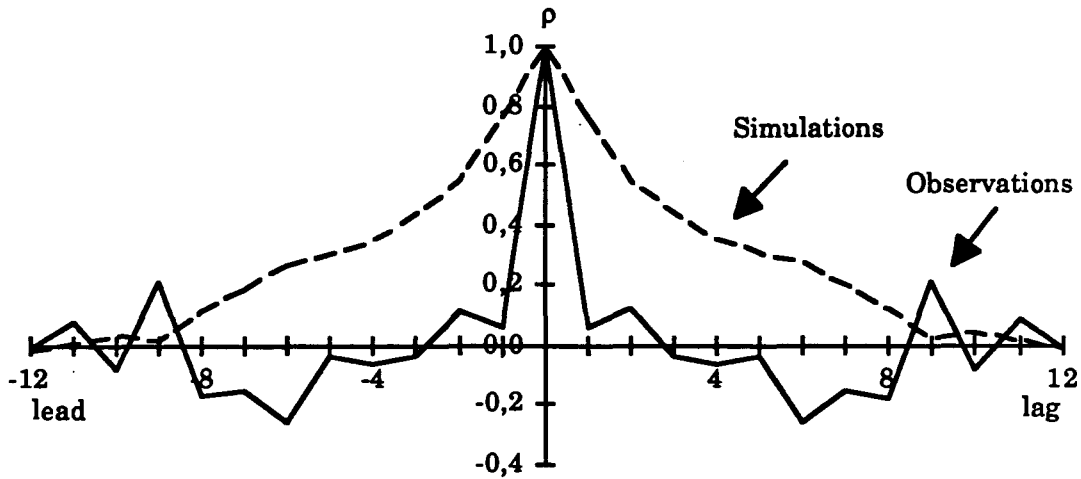
Using a 5% level of significance we can rule out that the estimated coefficients for the number of fixtures, the number of waiting vessels and the number of vessels arriving in two weeks are equal to zero. The coefficients for the number of vessels arriving in four weeks and for the constant, are not significantly different from zero.

As expected, the number of vessels arriving in the future has a negative impact on the development of the average freight rate. However, contrary to our expectations, the number of fixtures in the preceding week has a very strong positive impact. Further, it may seem strange that a high number of vessels waiting entails that the freight rate may be expected to rise during the next week, and vice versa. However, the direction of this result is in accordance with the assumptions of our model. We will return to this below.

Main results from the default run

We have carried out the same estimations on the data set derived from the default run of the model. We find that there is first order auto correlation in the residuals as we regress the change in the simulated freight rate against the four independent variables. Thus, we can already conclude that the freight rate generated by the model has another structure than the observed freight rate. The difference is easily seen in figure 3 below. It shows the correlation function of the change in the freight rate both for the observed freight rate and the simulated freight rate.

Figure 3. Correlation function for the freight rate, observed and simulated



To remove the first order auto correlation we use the Hildreth-Lu procedure. We take the general difference of the change in the freight rate and the general difference of the number of fixtures, the number of vessels waiting and the number of vessels arriving in two and four weeks' time. This gives a first order auto correlation coefficient of 0.75. Then, we get the following estimated parameters;

Table 3; Regression of the general difference of the change in the freight rate against the general difference of the specified variables. Data from a default run of the model.

Variable	Estimated coefficient	Standard deviation	t-value
No. of fixtures	4.75	12.38	0.38
No. waiting	47.70	12.84	3.71
No. in two weeks	-3.66	10.94	-0.33
No. in four weeks	2.58	12.49	0.21
Constant	-170.25	331.87	0.51

The Durbin-Watson statistic is 2.00. We have an R^2 of 11.8% and the R^2 - adjusted of 9.1%.

Contrary to our findings in the market observations, the number of fixtures in the preceding period has no significance in explaining the freight rate development in the week to come. Further, the importance of the number of vessels waiting as the week begins, seems to be higher as regards

explaining the freight rate development. The number of vessels arriving in the future is not significant at a 5% level.

Comparison of simulations and observations

Both in the observations and in the model, a high number of vessels waiting to be hired entails that the freight rate level will increase during the next week, keeping everything else unchanged. From a model perspective, this is obvious. Assume that the number of vessels that are waiting is above normal, that is, an above normal number of vessels failed to get hired last week. Consequently, the competition last week was probably tougher than usual, and the freight rate was depressed. Hence, next week the market will most probably return to more normal conditions and competition will be relaxed. Thus, we may expect improved freight rates.

In the observations a high number of vessel arriving in two weeks' time gives lower freight rates in the close future. For the model, however, we do not find any significance. The number of vessels arriving in four weeks' time has no significance neither in the observations nor in the model.

In our model, past characteristics like the number of fixtures last week have no importance in determining the development of the freight rate in the week to come. Evidently, this is not the case in the VLCC market where the past really does matter. Figure 4 below may clarify this differences between the model and the real world as regards the importance of the number of fixtures during the preceding week. The figure shows the partial correlation between the change in the freight rate during the week until the Thursday of registration, and the leaded and lagged number of weekly fixtures.

In the default run the number of fixtures for any lead or lag is hardly correlated with the change in the freight rate. Contrary to this, in the observations we see a strong correlation between the number of fixtures in the preceding week and the change in the freight rate.

Figure 4; Correlation between change in freight rate and leaded and lagged number of fixtures

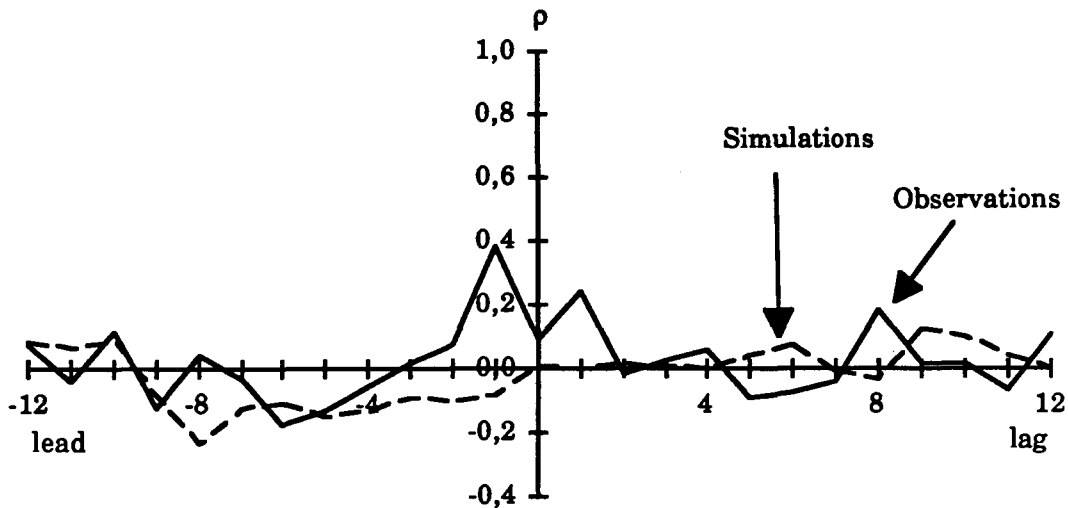


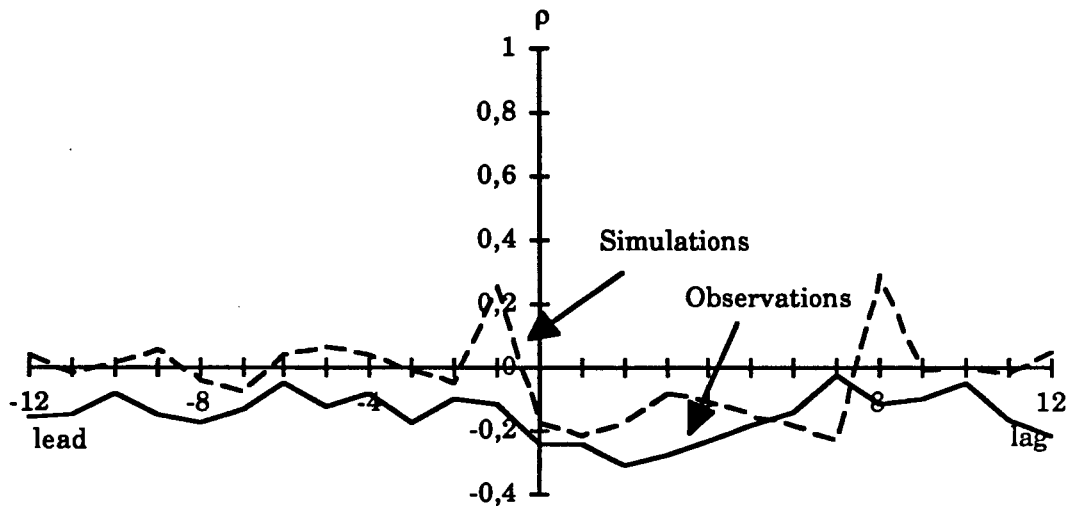
Figure 5 below shows the correlation between the number of fixtures and leads and lags of the number of vessels waiting at the end of the week. Not surprisingly, a high number of fixtures entails that the number of vessels waiting at the end of the week is reduced. In this respect the results of our model are more or less identical to observations. Fewer vessels waiting for cargo means leaner competition. In our model, this does not lead to rising freight rates. Due to perfect foresight, the leaner competition has already been taken into account the preceding week. Anticipation of less competition makes it more favourable to wait, and the freight rate level will be high also the preceding week. Thus, there will not be any major change in the freight rate level.

However, in the observations, a high number of fixtures the preceding week really does make the freight rate rise. See figure 4. Foremost, the perfect foresight assumption should be relaxed. The positions of the vessels are well known but the number of cargoes in the future is more uncertain. However, it seems as if the number of fixtures is generated by white noise¹. Thus, a high activity level one week does not indicate a high activity level

¹ Using the Bartlett and the Box-Pierce tests, the autocorrelation coefficients of the number of fixtures using 15 lags were not significantly different from zero neither individually nor jointly, using critical 5% and 10% values respectively. Further, lags up to one year do not indicate jointly significant coefficients. We are not able to disclose any seasonal pattern. A larger sample is probably needed.

the following week. Therefore, the number of fixtures disclosed at the end of the week does not provide us with more information about future market characteristics than the number of vessels waiting, which is revealed simultaneously.

Figure 5; Correlation between the number of fixtures and leaded and lagged number of vessels waiting at the end of the week



Nevertheless, it is relevant to raise the question why the freight rate does not rise immediately as the high number of fixtures are disclosed, but apparently rises during the following week. One reason may be found in the way the observations are reported. Taking the difference between two average freight rates blurs some of the dynamics. Being an average rate, the reported freight rate at the beginning of the week includes possible low freight rates from the start of the preceding week, at a time when the number of fixtures and thus the number of vessels waiting, was unknown. Hence, the difference between the two average freight rates may not be zero, even though the freight rate at the beginning of the week is equal to that at the end of the week.

We are aware that the above mentioned points do not give exhaustive explanations for the failure of our model to replicate the relation between the number of fixtures and the freight rate development. After all, this relation is well known by practitioners. Leading brokers often refer to the recent activity level when predicting future freight rate development. Our findings very much support such a reasoning. However, this leads to a very simple chartering rule for the shipowners to outperform the market: If the

number of fixtures the last week has been below average, then fix the tanker as soon as possible. If the number of fixtures is above average then be patient. However, risk aversion may explain part of this phenomenon.

Some additional remarks should be made as regards differences between properties of the observations and the simulated values in figure 5. The one week lead of the number of vessels waiting in the default run is due to the fact that high capacity ensures that no cargo owners must wait for dispatch, and accordingly the number of fixtures will be high. Observe also, that a high number of fixtures entails that the number of vessels waiting in eight weeks will also be high. Our model has only one representative voyage that takes 60 days, i.e. more than eight weeks, so this is obvious.

Conclusions

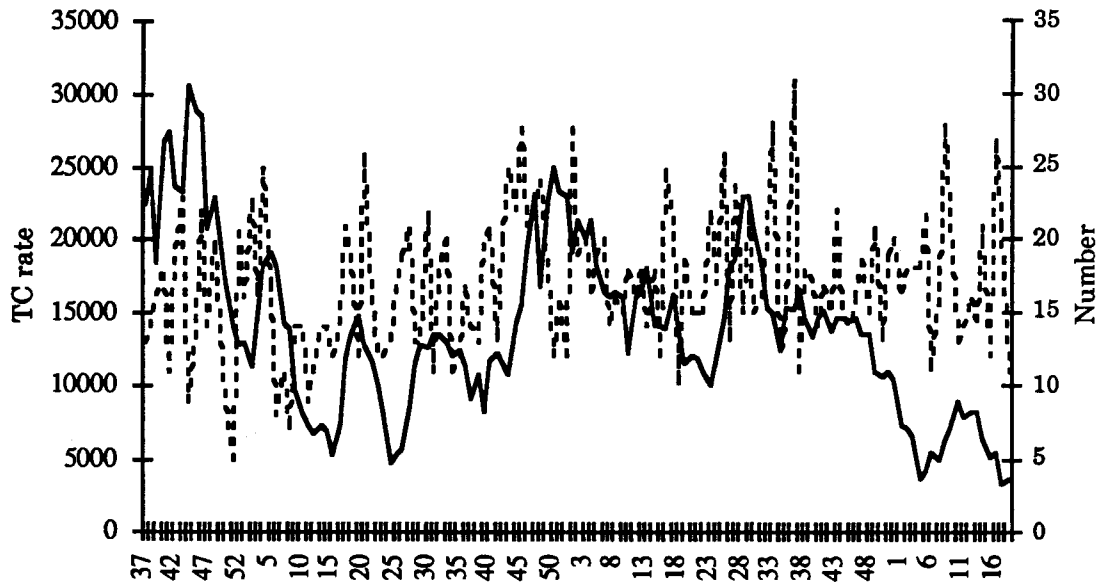
Our theoretical approach seems to take account of the effect on the freight rate development of present and future supply in an acceptable way. However, the model totally fails to explain the apparently very strong relation between past activity level and the freight rate development. We have suggested some explanations. Nevertheless, it seems as if the question of why past characteristics of the market influence the development of the freight rate, is still open .

Acknowledgements

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Appendix

Figure 6; time charter equivalent VLCC spot rates and the number of fixtures per week



Source: R. S. Platou

Figure 7; simulated freight rates and the number of fixtures per week

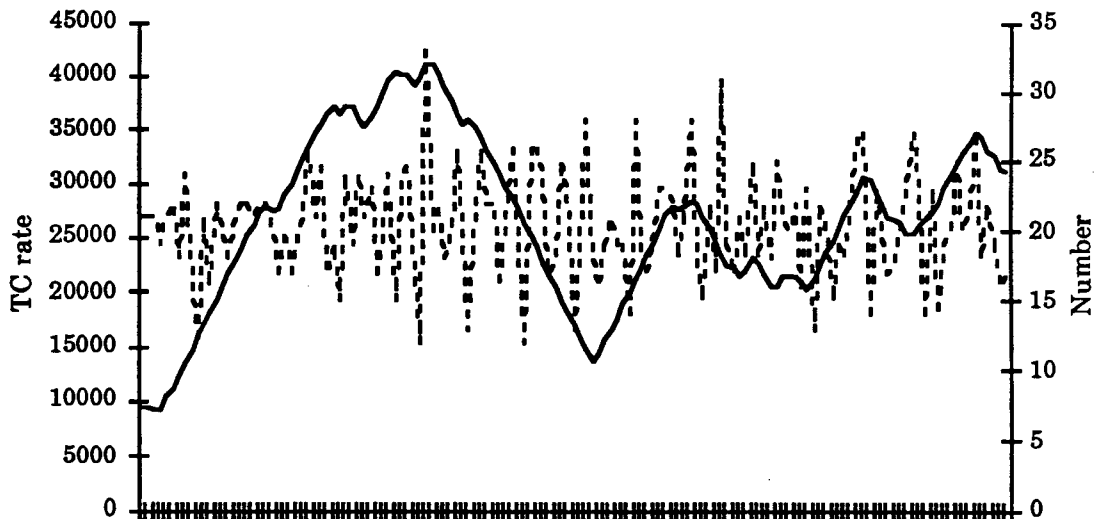
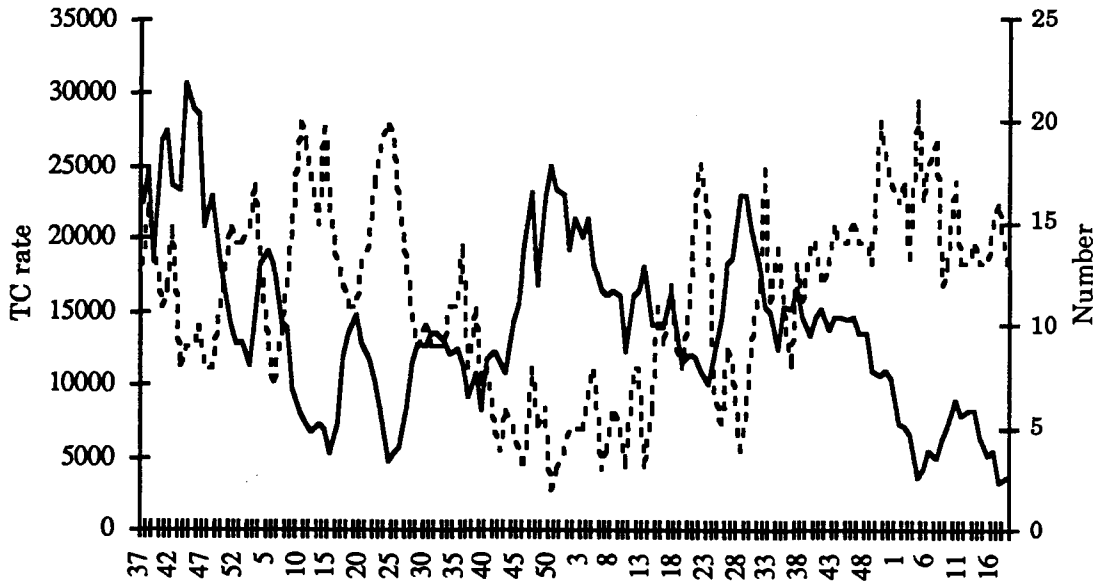


Figure 8; time charter equivalent VLCC spot rates and the number of vessels waiting at the end of the week



Source: R. S. Platou

Figure 9; simulated time charter equivalent VLCC spot rates and the number of vessels waiting at the end of the week

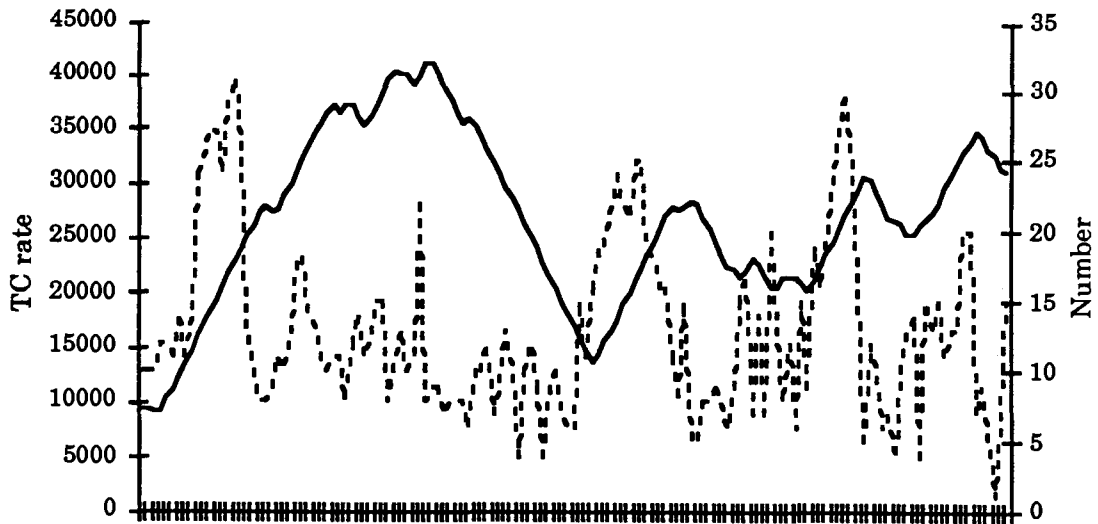
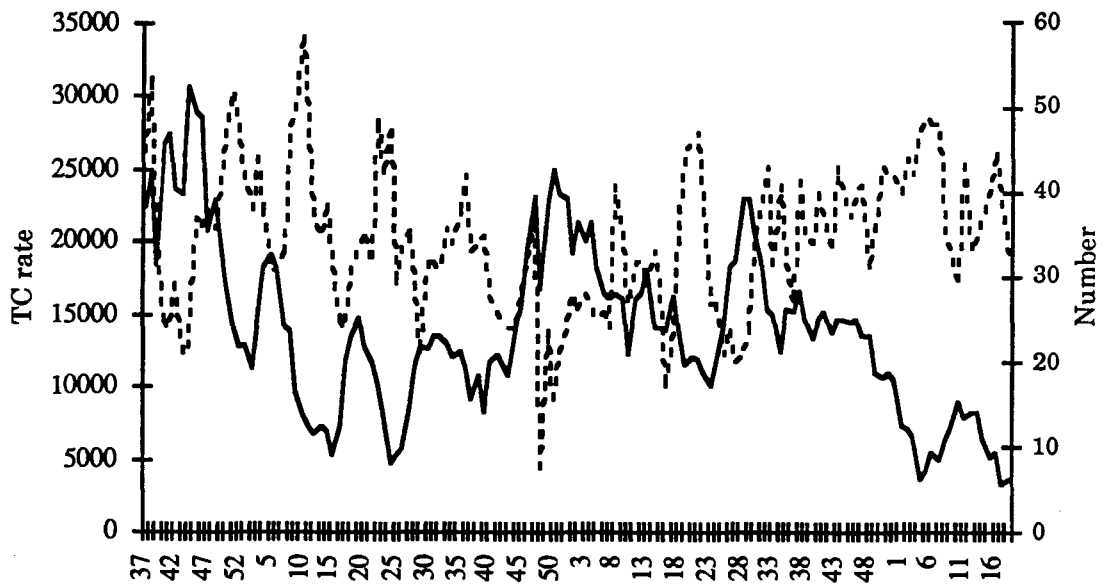


Figure 10; time charter equivalent VLCC spot rates and the number of vessels arriving during the next two weeks



Source: R. S. Platou

Figure 11; simulated time charter equivalent VLCC spot rates and the number of vessels arriving during the next two weeks

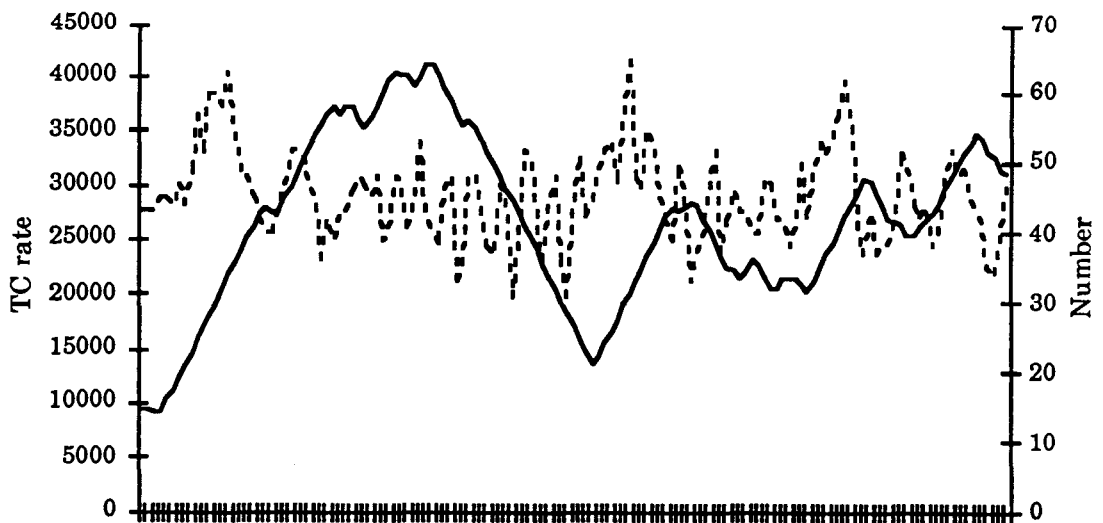
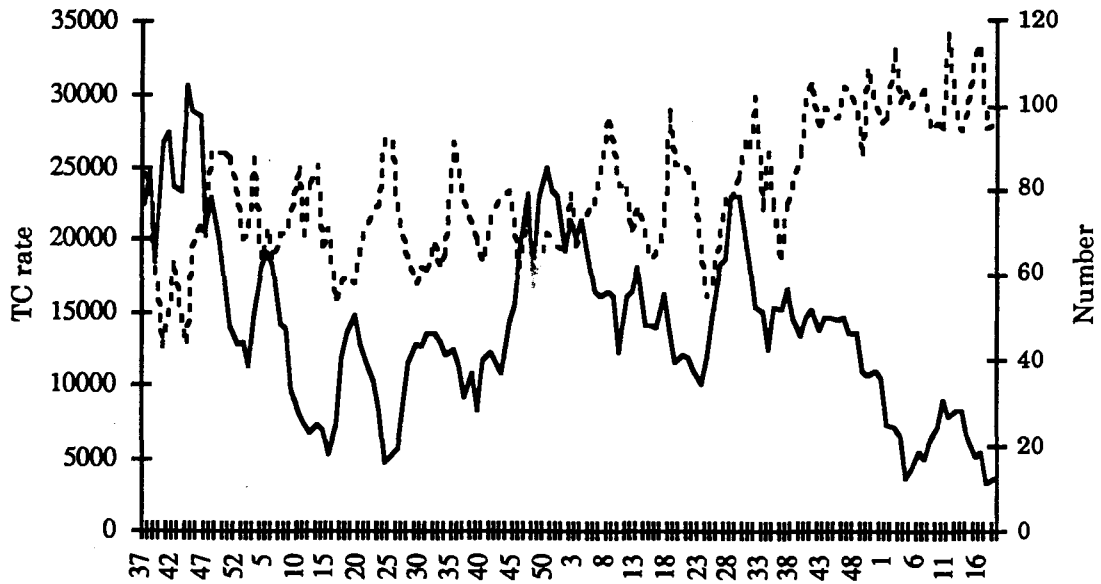
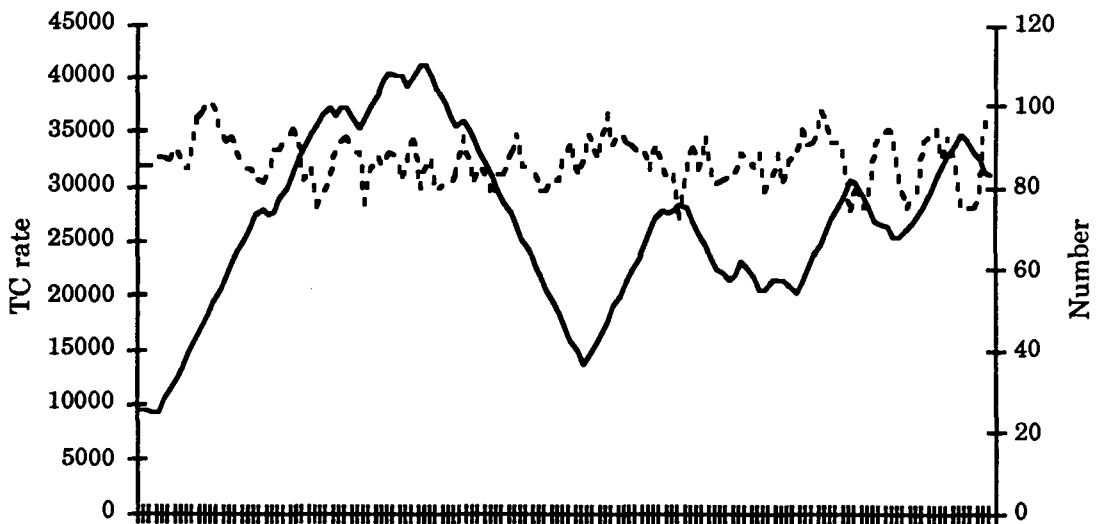


Figure 12; time charter equivalent VLCC spot rates and the number of vessels arriving during the next four weeks



Source: R. S. Platou

Figure 13; simulated time charter equivalent VLCC spot rates and the number of vessels arriving during the next four weeks



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Ch. 4;

The BFI and the BIFFEX - Stochastic Properties and Valuation

Abstract

In this article we address the question of valuation in the BIFFEX market. First, we try to disclose the stochastic structure of the BFI. We suggest that a mean reversion process is a good description of the index. Then we derive a futures price formula and indicate how to price a European option on a futures. We then compare the forecasting performance of the stochastic process with the strategy of using the futures as forecasts of the settlement value. We find that using the futures perform better only for very short forecasting horizons. This indicates that the futures prices are adjusted for risk.

Introduction

Ten years have passed since the Baltic International Freight Futures Exchange (BIFFEX) was initiated. Contrary to numerous prophecies, the BIFFEX is still in operation, and is the sole alternative for organised trading of freight rate risk.

BIFFEX quotes futures on the Baltic Freight Index (BFI). BFI is an arithmetic weighted dry bulk freight rate index. The index is compiled from actual observed freight rates on certain prespecified representative routes (See appendix *d*). These rates are provided daily by eight to twelve major London shipbrokers, the panel, or in lack of observations, the rates are substituted by the panellists' views of the fair rate level on the individual trades. Eight contracts are traded at any time. There is always a contract for the current month, the "spot contract", and for the month to come, the "prompt contract". Further, the January, April, July and October contracts are traded up to eighteen months ahead. Settlement of the contracts is the first trading day of the following month, and the settlement value is the average of the last five days' BFI values.

Gullinane (1991) addresses the question of who are actually using the BIFFEX. From a survey study of the shipowners' attitude towards and the use of the BIFFEX, the majority of the responders, if using the market at all, were using it for speculative purposes, not for hedging. This may seem a bit surprising. The main part of the shipowners' income is derived from the freight market, and thus, they are heavily exposed to the large freight rate fluctuations. Therefore, the shipowners are the most obvious hedgers on the BIFFEX. After all, the profits of the charterers are only partly related to the

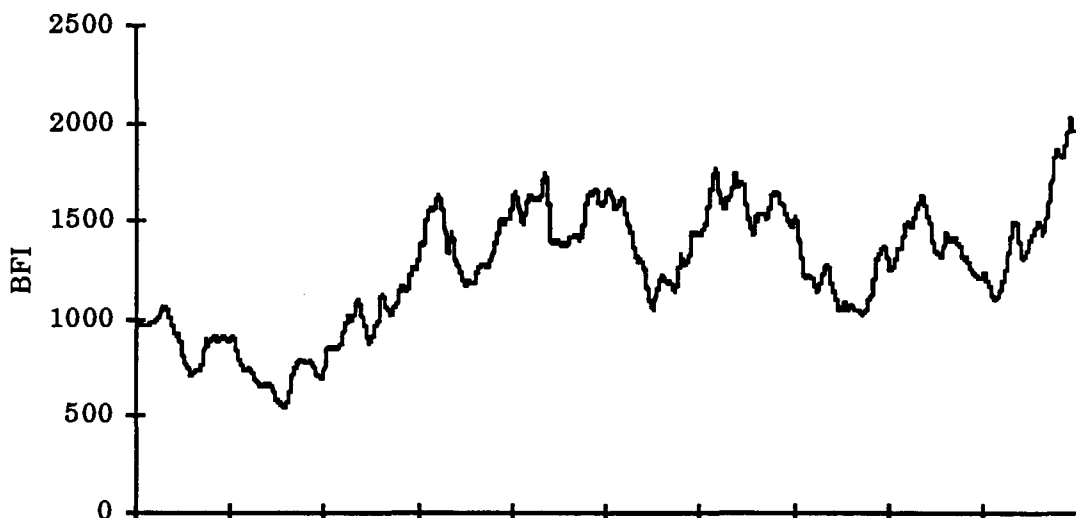
cost of sea-transport. (Some investigation of the attitude of the charterers should be called for).

The first part of this article focuses on the choice of a stochastic process for describing the path followed by the BFI. The motivation for this search is mainly to derive tools for better valuation of contracts based on the BFI. Part two addresses this problem, primarily by discussing the theoretical price of futures, and secondarily, by sketching how to improve practical valuation of options on the BFI. In section three we test the strategy of using the futures prices versus using the stochastic process for forecasting the settlement value.

Part one, The dynamic structure of the BFI

Using an ARIMA approach, Gullinane (1992) finds that an AR(3)R representation is suitable for very short term predictions of the BFI. Hence, speculation on the spot contract three to five days before closing day seems to be promising. In the case of longer lead time the AR(3)R model was outperformed by Holt's method. Simple moving average models did not perform competitively.

Figure 1; The BFI March 1985 to January 1995



Source: BIFFEX and Lloyd's List

The initial value of the BFI, the notation of the first of March 1985, was set to 1000. Since then the index has fluctuated between 553.5 in 1986 to 2067 in January 1995. According to Gray (1990, ch. 3.3), the low 1986 notation

represents the bottom income level that makes the shipowner prefer trading to laying the vessels up. That is, an index value just above 500 represents a lower bound to the BFI. Historically, also according to Gray, it seems as if 1650 is an upper resistance level to the BFI. Obviously, as the freight rate rises, the demand for bulk carriers is reduced. At the same time, more vessels are attracted by the dry bulk trades, i.e. the combined carriers leave oil for dry bulk. Further, the shipowners try to increase the efficiency of the dry bulk fleet if freight rates rise. In a long term perspective, new capacity will be available by construction of new vessels. All this imply that the freight rate will be reverted. Consequently, the BFI is restricted downwards and the extreme high levels are rare. Taking this into consideration, a mean reversion representation of the freight rate may be appropriate. We test the goodness of the fit to observations of the Cox-Ingersol-Ross (CIR) term structure process, a mean reversion process with arbitrary absorbing level (The MRA process), and an Ornstein Uhlenbeck process.

The primitives

We define a standard Brownian motion, Z_t , restricted to a given time interval $[0, T]$. Z_t is defined on a complete probability space (Ω, \mathcal{F}, P) where Ω is the set of states of the world with generic elements ω , \mathcal{F} is a sigma field, i.e. a set of events and P is a probability measure; $P: \mathcal{F} \rightarrow [0, 1]$, $P(\emptyset) = 0$ and $P(\Omega) = 1$. We also specify a filtration F of sub-sigma fields of \mathcal{F} , $F = \{\mathcal{F}_t: t \in [0, T]\}$. The filtration gives how information is disclosed as time passes. We have that $\mathcal{F}_t \subset \mathcal{F}_s$ for all $t \leq s$.

The CIR term structure process; basic properties:

If we postulate that the freight rate index follows the CIR term structure process, we have that the increment of the index is given by

$$dX_t = \kappa(\alpha - X_t)dt + \sigma\sqrt{X_t}dZ_t \quad (1)$$

where X_t is the index value at time t , dZ_t is the increment of a standard Brownian motion as defined above and κ , α and σ are constants. According to this formulation the index value is chi-square distributed.

The process has mean reversion properties since the drift is positive as the index is low, that is, below α , and negative as the index is high, i.e., above α . Further, the volatility of the index increases as the freight rate rises. Zero is a low bound to the process, though it is not an absorbing level.

The conditional expectation of X_τ at time t , is

$$E[X_\tau | \mathcal{F}_t] = e^{-\kappa(\tau-t)} x_t + \alpha (1 - e^{-\kappa(\tau-t)})$$

and the conditional variance is

$$\text{Var}[X_\tau | \mathcal{F}_t] = x_t \frac{\sigma^2}{\kappa} (e^{-\kappa(\tau-t)} - e^{-2\kappa(\tau-t)}) + \alpha \frac{\sigma^2}{2\kappa} (1 - e^{-\kappa(\tau-t)})^2$$

For more details see appendix c.

The mean reversion with arbitrary absorbing level (The MRA process); basic properties:

If the index follows the MRA process, the increment of the index is postulated to be given by

$$dX_t = \kappa(\alpha - \ln(X_t - \lambda))(X_t - \lambda)dt + \sigma(X_t - \lambda)dZ_t \quad (2)$$

As above, X_t is the index value at time t , dZ_t is the increment of a standard Brownian motion and κ , α and σ are constants. Let λ be an arbitrary absorbing level for the process. See Tvedt (1995) for details in the case of λ equal to zero, and appendix a for the general case. For this specification it follows that the index less the absorbing level is lognormally distributed.

Like the CIR process, the MRA process has mean reversion properties. However, in this case, the mean reversion is stronger for high than for low index values for the same absolute deviation of $\ln(X_t - \lambda)$ from α . Also, the MRA process exhibits increasing volatility as the index level rises, though the structure of this relation is different from that of the CIR representation.

The logarithm of $(X_t - \lambda)$ is Gaussian with conditional expectation of $\ln(X_\tau - \lambda)$ at time t , given by

$$E[\ln(X_\tau - \lambda) | \mathcal{F}_t] = e^{-\kappa(\tau-t)} \ln(x_t - \lambda) + \left(\alpha - \frac{1}{2} \frac{\sigma^2}{\kappa} \right) (1 - e^{-\kappa(\tau-t)})$$

Further, the conditional variance of the process is

$$\text{Var}[\ln(X_\tau - \lambda)|\mathcal{F}_t] = \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa(\tau-t)})$$

For more details, see appendix *a*.

The Ornstein-Uhlenbeck process, basic properties:

If we assume that the BFI follows an Ornstein-Uhlenbeck process the increment of the index is given by

$$dX_t = \kappa(\alpha - X_t)dt + \sigma dZ_t \quad (3)$$

As before, X_t is the index at time t , dZ_t is the increment of a standard Brownian motion and κ , α and σ are constants. See Bjerk Sund and Ekern (1995) for details and some applications to the pricing of shipping assets. By this assumption, it follows that the index is normally distributed.

The Ornstein-Uhlenbeck process has the same drift as the CIR process. However, the diffusion is independent of the index level. The process has no absorbing level or boundaries. That is, the process may take negative values.

Solving (3) for a given initial index level x_t , we have that

$$X_\tau = e^{-\kappa(\tau-t)}x_t + \alpha(1 - e^{-\kappa(\tau-t)}) + e^{-\kappa(\tau-t)}\sigma \int_t^\tau e^{\kappa s} dZ_s,$$

which is Gaussian. It follows from stochastic calculus that the conditional expectation of X_τ at time t , is

$$E[X_\tau|\mathcal{F}_t] = e^{-\kappa(\tau-t)}x_t + \alpha(1 - e^{-\kappa(\tau-t)})$$

and the conditional variance is given by

$$\text{Var}[X_\tau|\mathcal{F}_t] = \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa(\tau-t)})$$

Estimating the parameters of the CIR term structure process.

The square root of the CIR process is Gaussian. Thus, we have the following discrete version of the process

$$\sqrt{X_t} = \beta_0 + \beta_1 \sqrt{X_{t-1}} + \varepsilon_t \quad (4)$$

where the error term is supposed to be normally distributed. Consequently, the parameters are easily estimated by ordinary least square.

Estimating the parameters of the mean reversion process with arbitrary absorbing level

We try out alternative specifications of the MRA process, one with absorbing level at zero and some with absorbing levels above 550. As already mentioned, Gray (1990) argues that a general freight rate level below the low 1986 index level of 553.5 will make the shipowners prefer lay-up to continuing operation. Since we cannot take logs of negative values, this lowest observed index value makes an upper bound to the absorbing level.

The following discrete version is used for estimating the parameters,

$$\ln(X_t - \lambda) = \beta_0 + \beta_1 \ln(X_{t-1} - \lambda) + \varepsilon_t \quad (5)$$

where the parameters are given by $\beta_0 = \left(\alpha - \frac{1}{2} \frac{\sigma^2}{\kappa} \right) (1 - e^{-\kappa})$ and $\beta_1 = e^{-\kappa}$.

We have that relation (5) is Gaussian with $\varepsilon_t \sim N \left[0, \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa}) \right]$. Thus, it follows that

$$\text{Var}[\ln X_t | \ln X_{t-1}] = \text{Var}[\varepsilon_t] = \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa}) = \frac{\tilde{\varepsilon}^T \tilde{\varepsilon}}{N-1}$$

Now the parameters are readily estimated by using ordinary least square.

Estimating the parameters of the Ornstein-Uhlenbeck process

The Ornstein-Uhlenbeck process is Gaussian. Therefore, we may use ordinary least square directly on the discrete counterpart of the process in

order to estimate the parameters of the process. We have the discrete version of the process given by

$$X_t = \beta_0 + \beta_1 X_{t-1} + \varepsilon_t \quad (6)$$

where $\beta_0 = \alpha(1 - e^{-\kappa})$ and $\beta_1 = e^{-\kappa}$.

The error term is normally distributed, that is, $\varepsilon_t \sim N\left[0, \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa})\right]$

It then follows that

$$\text{Var}[X_t | X_{t-1}] = \text{Var}[\varepsilon_t] = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa}) \approx \frac{\tilde{\varepsilon}^T \tilde{\varepsilon}}{N-1}$$

We now turn to the results of the estimation of the parameters for the suggested processes.

Empirical findings

Daily observations, excluded week-ends and holidays, from March 1985 to end December 1993 are used for estimating the parameters. This gives a sample of 2,269 observations. The BFI composition has been changed a number of times. Originally, a major part of the index was made up by Handy size routes. However, the last of these routes was removed in November 1993. During the first years, the BFI was solely a spot index. Today also four time charter contracts have been included. We have ignored any effects on the BFI values these new formations may have caused. In the table below the results of the estimations are presented together with some sample statistics.

Table 1; Estimated parameters and sample statistics; CIR, MRA and O-U

Model	Coefficients		Durbin-h	R ² -adj.
	β_0	β_1		
CIR *)	0.017767 (0.90568)	0.99953 (1791.0)	30.162	0.9993
MRA**) $\lambda = 0$	0.0030753 (0.80436)	0.99958 (1853.5)	30.335	0.9993
MRA***) $\lambda = 550$	0.008453 (1.2390)	0.99870 (936.28)	18.069	0.9974
MRA***) $\lambda = 552.585$	0.019027 (1.8419)	0.99703 (616.75)	0.000078	0.9941
MRA***) $\lambda = 553$	0.02755 (2.2122)	0.99568 (510.79)	-6.9188	0.9914
O-U****)	0.74688 (0.99758)	0.99948 (1699.6)	30.083	0.9992

Numbers in brackets are t - values

*) The Cox-Ingersol-Ross process.

**) Mean reversion with absorbing level equal to zero.

***) Mean reversion with absorbing level as specified.

****) The Ornstein-Uhlenbeck process.

For all the specifications, except the MRA with an absorbing level of 552.585 (MRA*), the Durbin-h statistic suggests that the autocorrelation of the residuals is too high for the specifications to be correct. The Durbin-h statistic is asymptotically standard normal. For this fairly large sample the critical value is about 1.96, and thus, we may reject normality for any of the other specifications. The MRA* has significant parameters at a 10% level of significance and β_1 is significant at a 5% level as well. However, β_0 is not significantly different from zero at a 5% level. Despite this shortcoming, it seems as if the MRA* is by far the most promising specification.

In the MRA* case the variance of the residual is reported to be 0.0035897. Together with the values of β_0 and β_1 , and from the above discrete counterpart of the model, it is straight forward to estimate the parameters of the process. They are as follows;

$$\hat{\kappa} = 0,00297$$

$$\hat{\sigma} = 0,06000$$

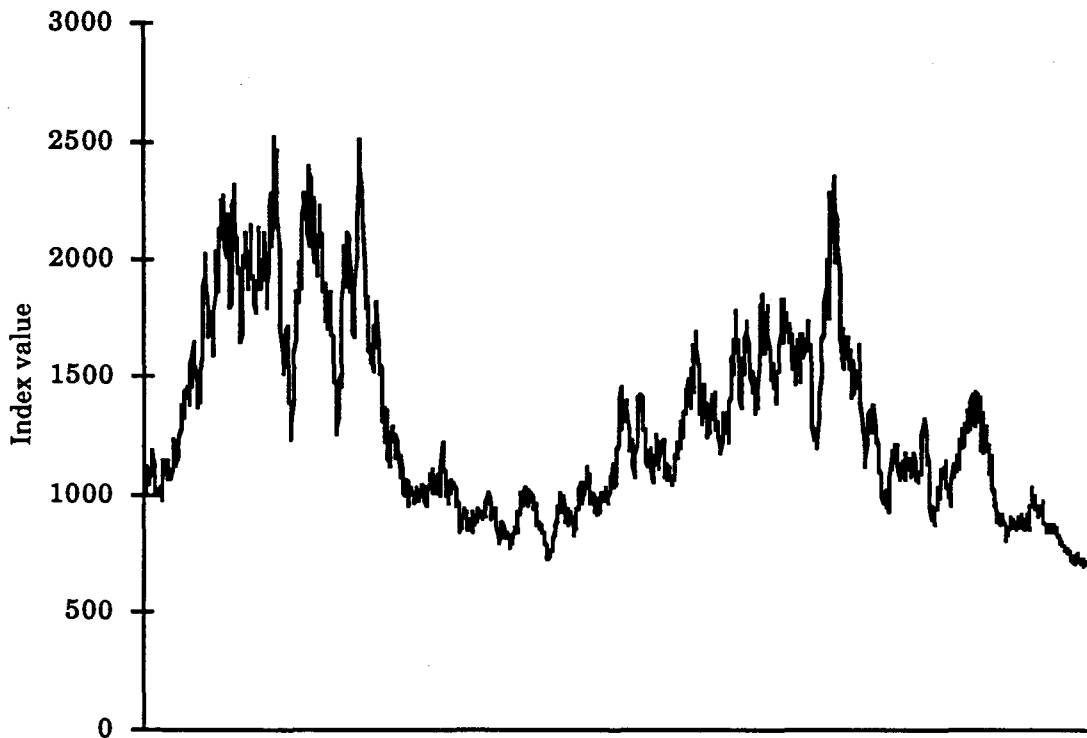
$$\hat{\alpha} = 7.01162$$

Inserting these estimates in the above MRA process, we have that the increment of X_t in the MRA* case is given by

$$dX_t = 0.00297(7.01162 - \ln(X_t - 552.585))(X_t - 552.585)dt + 0,06(X_t - 552.585)dZ_t \quad (7)$$

The graph below shows a path derived from the MRA* process. The time horizon is equivalent to a realisation lasting 2,000 days.

Figure 2; Simulated BFI values using MRA*



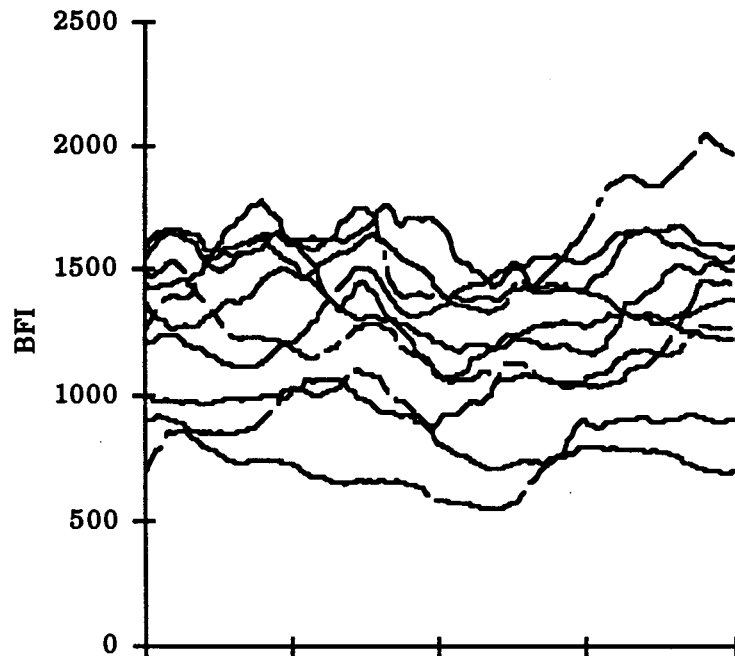
In this default run the index starts out at 1,000, reaches a maximum at 2,522, a minimum at 640, and an average at 1,255. The above path is, of course, only one of an infinite number of possible developments of the index. The index will occasionally take values above those shown by the graph. However, the index will never go below 552.585.

Seasonal variation

It is a well known fact that the dry bulk markets are fairly dull during the summer months. This is quite apparent in the graphs below. Figure 3 shows

the annual development of the BFI from 1985 to 1994. Observe the general lower level of the BFI between the second and third quarters each year.

Figure 3, Annual development of the BFI, 1985 to 1994.

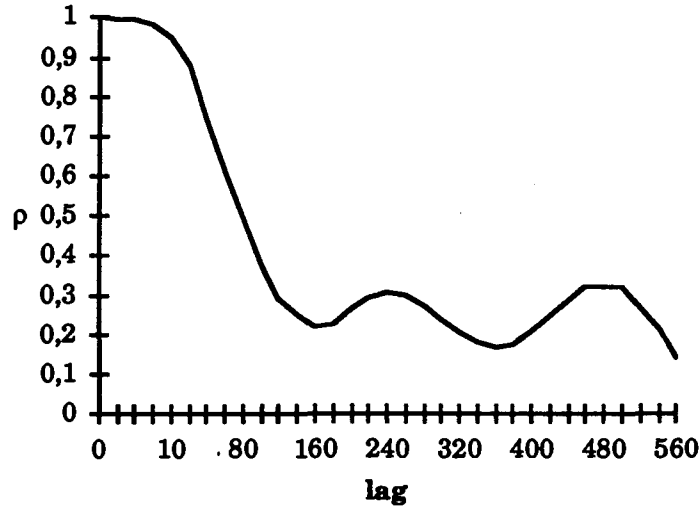


Source: BIFFEX

There are a number of reasons for the cyclical behaviour of the BFI. The index is at present compiled from 10 different routes. For the precise composition see appendix *d*. However, as much as 30% of the weight is on spot grain trades. Therefore, seasonal fluctuations of the deep-sea transport of grain may be an obvious first explanation. The by far largest exporter of grain is North America. The EC and Australia are also significant contributors. The main import areas the last decade have been the Far East including Japan, the former Soviet Union, the Middle East, Africa, China and Central America (Importance in descending order). From 1985 to 1990 the Soviet Union and China experienced the most rapid growth and highest volatility in demand for grain. On the other extreme, Japan has had a very stable volume of import.

The seasonal pattern is most easily seen in the autocorrelation function for the BFI below.

Figure 4; A sample autocorrelation function for the BFI with 560 lags.



The function is derived from the available BFI observations until December 1993 and by applying 560 lags. There are about 250 observations each year. Therefore, note the peaks at 250 and 500 which clearly indicate an annual cycle in the BFI.

The fact that there are seasonal fluctuations in the BFI calls for a modification of the specification of the stochastic process. We suggest changing the drift term of the MRA process to include a sine term, i.e. we introduce a mean reversion process with arbitrary absorbing level and seasonal variation (MRAS). We suggest a process with the following incremental change

$$dX_t = \kappa(\alpha + \varphi \sin(\gamma t + \theta) - \ln(X_t - \lambda))(X_t - \lambda)dt + \sigma(X_t - \lambda)dZ_t \quad (8)$$

The added sine term makes the degree of convergence depend on time. The new parameters introduced are φ , γ and θ .

By the use of stochastic calculus it then follows that the index value at time τ , X_τ , given the index value at time t , X_t , is

$$X_\tau = e^{\hat{\Gamma}_\tau + \hat{\Psi}_\tau + \hat{\Lambda}_\tau} + a \quad (9)$$

where

$$\hat{\Gamma}_\tau = e^{-\kappa(\tau-t)} \ln(X_t - \lambda) + \left(\alpha - \frac{1}{2} \frac{\sigma^2}{\kappa} \right) (1 - e^{-\kappa(\tau-t)})$$

$$\hat{\Psi}_t = -\frac{\varphi\gamma\kappa}{\kappa^2 + \gamma^2} \left((\cos(\gamma\tau + \theta) - e^{-\kappa(\tau-t)} \cos(\gamma t + \theta)) \right) \\ - \varphi \left(\frac{\gamma^2}{\kappa^2 + \gamma^2} - 1 \right) (\sin(\gamma\tau + \theta) - e^{-\kappa(\tau-t)} \sin(\gamma t + \theta))$$

and

$$\hat{\Lambda}_\tau = e^{-\kappa\tau} \sigma \int_t^\tau e^{\kappa z} dZ_t$$

Observe the fact that a given pair of cosine and sine values uniquely establish a point on the cycle, i.e. telling us whether the seasonal effect has a negative or positive impact on the freight rate development.

From appendix *b* we have that the conditional expected value at time τ , of the log of the process less the absorbing level, is given by

$$E[\ln(X_\tau - \lambda) | \mathcal{F}_t] = e^{-\kappa(\tau-t)} \ln(x_t - \lambda) + \left(\alpha - \frac{1}{2} \frac{\sigma^2}{\kappa} \right) (1 - e^{-\kappa(\tau-t)}) \\ - \frac{\varphi\gamma\kappa}{\kappa^2 + \gamma^2} \left((\cos(\gamma\tau + \theta) - e^{-\kappa(\tau-t)} \cos(\gamma t + \theta)) \right) \\ - \varphi \left(\frac{\gamma^2}{\kappa^2 + \gamma^2} - 1 \right) (\sin(\gamma\tau + \theta) - e^{-\kappa(\tau-t)} \sin(\gamma t + \theta)) \quad (10)$$

and the conditional variance is

$$\text{Var}[\ln(X_\tau - \lambda) | \mathcal{F}_t] = \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa(\tau-t)}) \quad (11)$$

Naturally, the variance is not influenced by the introduction of the deterministic seasonal sine waves.

Estimating the MRAS

The autocorrelation function indicates seasonal variations in the index value. Given n observations each year, γ must be set to $2\pi/n$ in order to estimate annual fluctuations. Let $\tau = t + 1$. We then have a function that is linear in the log of the preceding period's value of the process, less the absorbing level, and in the sine and cosine functions of time.

Let the parameters κ , σ , α , φ , and θ , give the process which has the highest probability of having generated our sample. We estimate these parameters by maximising the likelihood function

$$f = \prod_{k=1}^m \rho_k$$

where ρ_k is the conditional density of $Y_k = \ln(X_k - \lambda)$, and is given by

$$\rho_k = \left[\pi \frac{\sigma^2(1-e^{-2\kappa})}{\kappa} \right]^{-1/2} e^{\left[\frac{-\kappa(Y_k - Y_{k-1}e^{-\kappa} - \Xi)}{\sigma^2(1-e^{-2\kappa})} \right]}$$

where

$$\Xi = \frac{\varphi\kappa}{\kappa^2 + \gamma^2} [\sin(\gamma\kappa)(A\gamma + B\kappa) - \cos(\gamma\kappa)(B\gamma + A\kappa)],$$

$$A = \sin(\gamma + \theta) - e^{-\kappa} \sin(\theta)$$

and

$$B = \cos(\gamma + \theta) - e^{-\kappa} \cos(\theta).$$

The simulation method used is quite vulnerable to initial values, and we are not able to rule out that the results presented below represent local maxima only.

From the estimation of the MRA above, we have that an absorbing level equal to 552.585 gives the lowest possible first order autocorrelation. We choose the same absorbing level for estimating the MRAS. This level does not give the highest value of the log-likelihood function. However, since the log-likelihood function seems to reach a maximum for infinitely low absorbing levels and to reach a minimum for values close to the upper limit of 553.5, the log-likelihood function is not a useful criterion for selecting the most appropriate absorbing level.

The results from the estimation are presented below

Table 2; The MRAS model - results from ML estimation.

Model	Coefficients				
	κ	σ	α	φ	θ
MRAS	0.00315	0.05983	6.9701	-1.6433	5.7443
$\lambda = 552.585$	(2.060)	(67.190)	(13.991)	(-1.711)	(14.531)

The value of the log-likelihood function is 3167. Asymptotically, all the parameters except φ , are different from zero at a significance level of 5%. At a 10% level also φ is significant. However, note that φ and θ are very closely related. θ has no meaning if φ is zero. Also be aware that θ may well be equal to zero. This simply indicates that the seasonal wave has reached an extreme value at a given point of time during the year. Further, the θ reported above is one of an infinite number of parameter values that are equivalent. The reported θ is the lowest positive value. The equivalent values are obtained by adding 2π . This is, of course, due to the sine specification, and the choice of θ among the equivalent efficient values has no implication or economic interpretation. For the same reason, substituting $-\varphi$ and $\theta + \pi$ for φ and θ gives an equivalent set of parameter values to the one reported above.

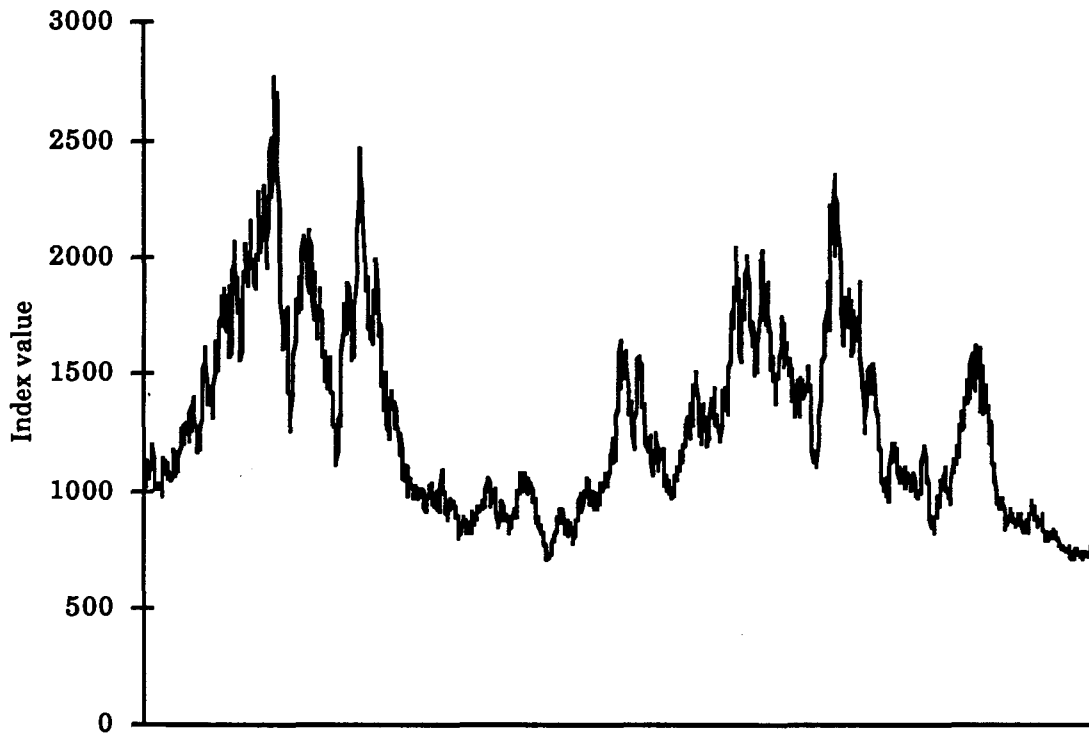
The parameters above give us the following increment of the freight rate process

$$dX_t = 0.0032(6.97 - 1.64 \sin(0.017t + 5.74) - \ln(X_t - 552.59))(X_t - 552.59)dt + 0.0598(X_t - 552.59)dZ_t$$

We see that the added sine term is maximal for $t \approx -60$. That is, 60 days before the new year, or primo November, the seasonal fluctuation reaches a maximum. The minimum is reached at $t \approx -243$ or primo May.

We have used this relation to simulate the BFI values. A graph equivalent to 2000 observations are shown below. The path of Z_t is the same as the one used to generate the MRA graph above.

Figure 5; Simulated BFI values using MRAS 552.585



This graph also starts out at the basis value of 1,000, reaches a maximum at 2,769, a minimum at 627 and has an average of 1,237. There are certainly some differences between this graph and the graph of the MRA above. The goodness of the seasonal representation is indicated in the correlation functions below.

We have derived correlation functions for the MRA and the MRAS using simulated index values, a total of 5,000 data points.

Figure 6; A correlation function for a MRA

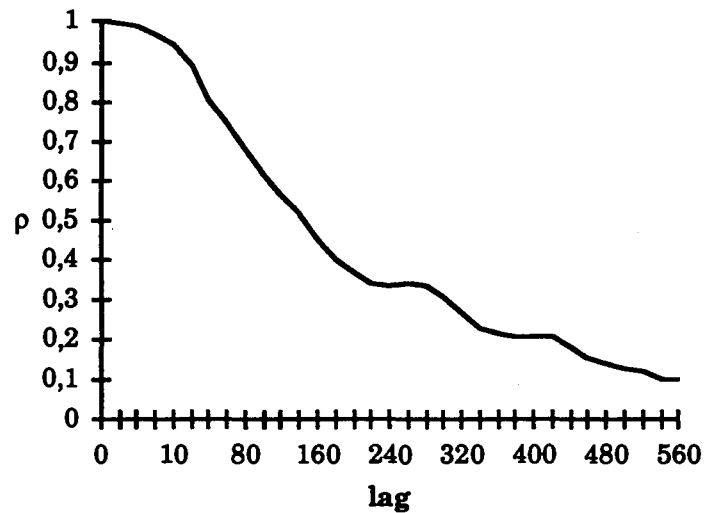
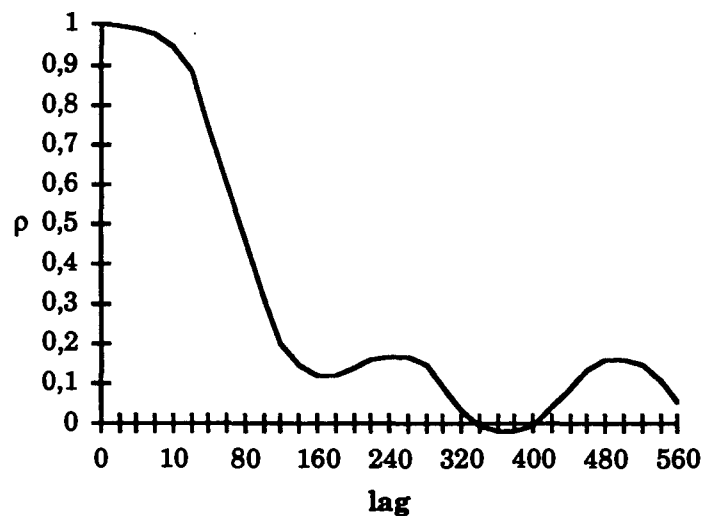


Figure 7; A correlation function for a MRAS



By using the MRAS representation we obtain a similar structure of the correlation function as the sample correlation function derived from the observed BFI values. Peaks at 250 and 500 interrupt the stationarity of the process. The MRA representation is by definition stationary, and due to an insufficiently simulated sample the irregularities of the graph in figure 6 has not been totally smoothed out.

Part two,

A futures contract and a European call option on a futures

A Futures Contract - by "text book" arbitrage arguments

In this section we leave the discussion of the actual structure of the BFI for a moment, and assume that the index can be described by a one dimensional Ito process X_t , with increment;

$$dX_t = \mu_t dt + \sigma_t dZ_t \quad (12)$$

Take a futures contract on the BFI index with settlement at time T . Let the futures price process, or the settlement price process at time t , be given by Φ_t . The actual spot price process of the futures contract is zero. In the BIFFEX market, like in most other equivalent markets, the change in the futures price process, the variation margin, is credited the holder of the futures contract once a day, and can be considered as an accumulated dividend process associated with the futures contract. This is known as a daily resettlement procedure. In its simplest form the procedure is as follows: if the price falls, the holder of the contract has to pay to the exchange an amount equal to the change in his open position since the last resettlement. If the price goes in his favour, the holder is paid equivalently. The holder may take the opposite position at any time, and hence, there will be no further resettlements.

The settlement or delivery value of the futures contract will be X_T , which is obviously an \mathcal{F}_T measurable variable. By the structure of the contract, we have that the futures price process at the time of settlement must be equal to the delivery value, $\Phi_T = X_T$.

By that time, the net gain or loss of a futures position from time t , will be given by the total net resettlement gain. For simplicity, we assume continuous resettlement. Further, let θ_t be the futures position process, that is, giving the number of futures contracts held at time t by the given holder. The total net resettlement gain can then be written

$$\int_t^T \theta_s d\Phi_s \quad (13)$$

As already stated, the true spot price process of a futures contract is zero. Thus, under an equivalent martingale measure, Q , the deflated gain process

$$\Phi^Y = \int Y d\Phi_t \quad (14)$$

is a martingale, where $Y_t = \exp\left(\int_0^t -r_s ds\right)$ and r is a deterministic bounded short rate process. Since Y_t is bounded and can never be zero or negative, then also Φ_t must be a martingale under the measure Q . Consequently, it follows that the futures price process at time τ , is given by

$$\Phi_\tau = E^Q[X_T | \mathcal{F}_\tau] \quad (15)$$

where $\tau \in [0, T]$.

To derive the equivalent martingale measure, Q , we here use an arbitrage argument. In practice it may be a bit complicated to construct such an arbitrage by the way the index is compiled. We could imagine a ship operator who positions his large number of own and hired vessels in such a way that, at the time of settlement of the futures contract, his dry bulk market exposure replicates the BFI.

Although it may be hard to find, we assume that there exists some self financing strategy apart from the futures that, at the time of settlement, T , has the value

$$\Theta_T = X_T \exp\left(\int_\tau^T r_s ds\right) \quad (16)$$

Our aim is to construct a portfolio of futures contracts and borrowing and lending at the short rate, in such a way that the cash flow from this portfolio is identical to the cash flow from a self financing strategy which has the value Θ_T at the settlement date.

Two identical cash flows should have the same value, and hence, if $\Theta_\tau \neq \Phi_\tau$, there is an arbitrage.

Our strategy is to keep θ_t futures contracts at time t . Let this futures position process, θ_t , be $\theta_t = 0$ for $t \notin [\tau, T]$, and for $t \in [\tau, T]$, let it be equal to

$$\theta_t = \exp\left(\int_{\tau}^t r_s ds\right) \quad (17)$$

The amount invested at the short rate at time t , is given by V_t . For $t \notin [\tau, T]$ let $V_t = 0$. At time τ the price of a futures contract is invested at the short rate, i.e. $\Phi_{\tau} = V_{\tau}$. Any resettlement gain is also immediately invested at the short rate and consequently, the incremental change in the amount invested at the short rate at any time $t \in [\tau, T]$, will be given by

$$dV_t = r_t V_t dt + \theta_t d\Phi_t \quad (18)$$

Since the true spot price of a futures contract is zero, the market value at time t of this self financing strategy is equal to the amount invested at the short rate, V_t .

We use Ito's lemma to derive the market value of the strategy. Expanding the function $g(V, t) = V_t \exp\left\{-\int_{\tau}^t r_s ds\right\}$, it follows that

$$dg_t = (-r_t V_t dt + dV_t) \exp\left\{-\int_{\tau}^t r_s ds\right\} = (\theta_t d\Phi_t) \exp\left\{-\int_{\tau}^t r_s ds\right\} \quad (19)$$

Integrating from τ to T , we have that

$$g_T - g_{\tau} = \int_{\tau}^T \exp\left(-\int_{\tau}^t r_s ds\right) \theta_t d\Phi_t dt \quad (20)$$

Substituting the function for g , it follows that

$$V_T \exp\left(-\int_{\tau}^T r_t dt\right) = \int_{\tau}^T \exp\left(-\int_{\tau}^t r_s ds\right) \theta_t d\Phi_t dt + V_{\tau} \quad (21)$$

From above we have that $\Phi_{\tau} = V_{\tau}$, and therefore

$$V_T \exp\left(-\int_{\tau}^T r_t dt\right) = \int_{\tau}^T \exp\left(-\int_{\tau}^t r_s ds\right) \theta_t d\Phi_t + \Phi_{\tau} = \Phi_T \quad (22)$$

By the definition of the futures contract, $\Phi_T = X_T$. Consequently, it follows that

$$V_T = \Phi_T \exp\left(\int_{\tau}^T r_s ds\right) = X_T \exp\left(\int_{\tau}^T r_s ds\right) \quad (23)$$

But we know that

$$\Theta_T = X_T \exp\left(\int_{\tau}^T r_s ds\right) \quad (24)$$

and thus, the value of the portfolio at time T , consisting of futures contracts and borrowing and lending at the short rate is equal to Θ_T , i.e. $V_T = \Theta_T$. In order to rule out any arbitrage, then must also $V_{\tau} = \Theta_{\tau}$, since both assets are constructed by self financing strategies. According to our strategy, $V_{\tau} = \Phi_{\tau}$, and thereby we get a relation from which we can derive the equivalent martingale measure Q ,

$$\Phi_{\tau} = \Theta_{\tau} = E_{\tau}^Q[X_T | \mathcal{F}_{\tau}] \quad (25)$$

A futures contract - the MRAS case

If the futures contract is not redundant in that a self financing strategy, except for the futures, with value Θ_t at time $t \in [\tau, T]$, is not available, then we cannot use the arbitrage arguments above to derive the measure Q and the futures price process, Φ_t , for $t \in [\tau, T)$. Here we suggest one approach in the case that the spot process is given by the MRAS specification.

First it is appropriate to relate some properties of the BFI to those usually assumed in dynamic asset pricing theory. Often the underlying process of a derivative describes the dynamics of an asset price. The market value of an asset is equal to the market agents' present valuation of the future cash flow generated by this asset. E.g. the value at time t of an asset that pays no dividend between time t and T , is equal to the expected value, under a certainty equivalent measure, Q , of the asset price at time T , discounted by the risk free interest rate. From this it follows that Q is an equivalent

martingale measure to the discounted asset price process. Hence, the asset price itself is partly a consequence of the measure Q . This property does not apply to the BFI index. There are a number of reasons why the BFI, or a discounted index value for that case, is not a martingale under a certainty equivalent probability measure. Most fundamentally; the BFI is an index of prices of shipping services and not of asset values. Services are by nature none-durable and therefore cannot be stored. Thus, the present price is only to a very limited degree dependent on expectations of future prices. This is easily demonstrated by the fact that there are seasonal fluctuations. The high winter freight rate levels are evidently not equal to the expected value under a certainty equivalent martingale measure, of the low summer freight rates. The freight rate is a part of the dividend process of a vessel, and therefore, in conventional asset pricing theory, an asset corresponds to a vessel or a company stock in the shipping markets and not to a freight rate or a freight rate index. Consequently, in deriving the futures price process we cannot rely on any martingale properties of the BFI under Q .

Let Q be a certainty equivalent measure to our original measure P . Generally, we have that the futures price at time t is the expected value at time t , of the spot price at the time of settlement $T \geq t$, under the certainty equivalent measure Q , i.e., $\Phi_t = E_t^Q[X_T | \mathcal{F}_t]$. As shown above, by the principle of convergence, Q is also a martingale measure to the futures price process, Φ_t . That is, since $\Phi_T = X_T$, almost surely, we have that $\Phi_t = E_t^Q[\Phi_T | \mathcal{F}_t] \forall t \leq T$, and Φ_t is a (Q, \mathcal{F}_t) martingale.

Define a process, ζ_t , by

$$\zeta_t = \frac{\kappa(\alpha + \varphi \sin(\gamma + \theta) - \ln(X_t - \lambda))(X_t - \lambda) - v_t}{\sigma(X_t - \lambda)}, \quad (26)$$

assuming that v_t is so that ζ_t satisfies Novikov's condition;

$$E \left[\exp \left(\frac{1}{2} \int_0^T \zeta_s^2 ds \right) \right] < \infty \quad (27)$$

Let a stochastic process ξ_t be given by

$$\xi_t = \exp\left(-\int_0^t \zeta_s dZ_s - \frac{1}{2} \int_0^t \zeta_s^2 ds\right) \quad \forall t \leq T \quad (28)$$

Then we define the measure Q by $dQ \stackrel{\text{def}}{=} \xi_T dP$, so that ξ_T is a Radon Nikodym derivative. Then it follows from Girsanov's theorem, that ξ_t is a martingale under the measure P , and the increment of the BFI is equal to

$$dX_t = v_t dt + \sigma(X_t - \lambda) d\tilde{Z}_t \quad (29)$$

where \tilde{Z}_t is a standard Brownian motion under the probability measure Q .

From (25), and since Q is an equivalent probability measure to P , it follows that the futures price process can be written

$$\Phi_t = E_t^Q[X_T | \mathcal{F}_t] = \frac{E_t[\xi_T X_T | \mathcal{F}_t]}{\xi_t} \quad (30)$$

Relation (30) gives the futures price process under our original probability measure P . In order to derive an analytical representation of the futures price process we need, however, to know the process v_t . Apart from the arbitrage argument of the above section, a precise representation of v_t is not possible to derive without further specifications and assumptions. Finally, the goodness of a choice of an approximation to the true v_t , among possible candidates, ends up as an empirical question. Due to its simplicity and attractiveness for empirical testing, we assume that

$$v_t = \kappa(\zeta + \varphi \sin(\gamma + \theta) - \ln(X_t - \lambda))(X_t - \lambda) \quad (31)$$

where all parameters are as before, except for ζ , which is an unknown constant.

The process ζ_t has the interpretation of the price of diffusion risk, and in our special case $\zeta_t = \bar{\zeta} = \frac{\kappa(\alpha - \zeta)}{\sigma}$. Observe that for $\zeta = \alpha$, we have that $\zeta_t = 0$, which implies that the probability measures are equal, $Q = P$, and the market agents are risk neutral.

Now we can calculate the futures price process. We have from (30) that

$$\Phi_t = E \left[\exp \left(-\tilde{\zeta} Z_T - \frac{1}{2} \tilde{\zeta}^2 (T-t) + \hat{\Gamma}_T + \hat{\Psi}_T + e^{-\kappa T} \sigma \int_t^T e^{\kappa s} dZ_s \right) + \xi_T \lambda | \mathcal{F}_t \right] \quad (32)$$

By the martingale property $E[\xi_T | \mathcal{F}_t] = \xi_t = 1$, and by stochastic calculus we have that

$$\begin{aligned} & E_t \left[\exp \left(e^{-\kappa T} \sigma \int_t^T e^{\kappa s} dZ_s - \tilde{\zeta} Z_T \right) \right] \\ &= \exp \left(\frac{\sigma^2}{4\kappa} (1 - e^{-2\kappa(T-t)}) - \frac{\tilde{\zeta}^2}{2} (T-t) - \frac{\sigma \tilde{\zeta}}{\kappa} (1 - e^{-\kappa(T-t)}) \right) \end{aligned} \quad (33)$$

Then it follows that the futures price process can be written

$$\Phi_t = \exp \left(e^{-\kappa(T-t)} \ln(x_t - \lambda) + \left(\zeta - \frac{\sigma^2}{2\kappa} \right) (1 - e^{-\kappa(T-t)}) + \hat{\Psi}_T + \frac{\sigma^2}{4\kappa} (1 - e^{-2\kappa(T-t)}) \right) + \lambda \quad (34)$$

Estimating a measure for the risk attitude

From (34) it is straight forward to calculate the adjusted level ζ as a function of the already estimated parameters κ , σ , and λ and the observable index value, X_t , the futures value, Φ_t , and the corresponding time horizon $(T-t)$. We have that

$$\zeta = \frac{\ln(\Phi_t - \lambda) - e^{-\kappa(T-t)} \ln(X_t - \lambda) + \frac{\sigma^2}{2\kappa} (1 - e^{-\kappa(T-t)}) - \hat{\Psi}_T - \frac{\sigma^2}{4\kappa} (1 - e^{-2\kappa(T-t)})}{(1 - e^{-\kappa(T-t)})} \quad (35)$$

We use observations of the spot index and the futures during the period January 1991 to December 1993, to estimate ζ . If ζ is significantly different from α we can rule out that the market agents are risk neutral.

We have a total sample of 8371 observations of ζ . This sample does not include ζ for contracts of less than ten days to settlement. We have removed these observations since the settlement value of the futures is equal to the average of the last five trading days' BFI value, and not as we have assumed, the value at the last day of trading. Hence, our model for the futures price process is strongly biased for very short time horizons. Therefore, we have excluded the short contracts from the sample.

We derive an average of $\hat{\zeta}$ of 6.75. If we use the MRAS specification, we receive a standard deviation of 0.82, and the standard deviation in the MRA case is 0.53. The estimated value of α is 6.97 and 7.01 for the MRAS and the MRA specifications, respectively. As we would expect, if shipowners were risk averse, $\hat{\alpha} > \hat{\zeta}$, but due to the high variance of the estimates, we cannot reject that ζ is different from $\hat{\alpha}$ at a 5% level of significance. Hence, from this test we cannot rule out that the market agents are risk neutral.

We have postulated that ζ is constant for all levels of the index and for all time horizons. We estimate the parameters of the following relation

$$\zeta_i = \beta_0 + \beta_1 X_t + \beta_2 (T-t)_i \quad (36)$$

where ζ_i is the calculated level of ζ for futures contract i at time t with time horizon $(T-t)$ and with observed level of the index X_t . If the explanatory power of the model is high and β_1 or β_2 is significantly different from zero, then this will indicate that our v_t is a bad choice.

The estimation of the parameters gives the following results in the MRAS and MRA cases

Table 3; Estimated parameter values of equation (36)

Model	Coefficients			R ² -adj
	β_0	β_1	β_2	
MRA	6.50 (150.1)	0.0001 (3.7)	0.0004 (12.2)	2.1%
MRAS	6.38 (90.1)	0.0002 (7.1)	0.0003 (4.1)	0.8%

Numbers in brackets are t - values

For both specifications, all parameters are significantly different from zero at a 5% level of significance. This goes in favour of rejecting that our choice of v_t is a good one. However, note the very low R² - adjusted statistics. But β_0 is clearly most significant, which is in agreement with suggesting that ζ is constant.

The volumes are generally very low for contracts with duration above half a year. Therefore, we have estimated the parameters of relation (36) for ζ_i derived from contracts with 10 to 180 days to settlement. Our sample is then reduced to 3394 observations.

Table 4; Estimated parameter values of equation (36) - restricted sample

Model	Coefficients			R ² -adj
	β_0	β_1	β_2	
MRA	6.85 (70.3)	-0.00009 (-1.3)	-0.0008 (-2.8)	0.2%
MRAS	6.63 (38.9)	0.00009 (0.8)	-0.001 (-2.6)	0.2%

Numbers in brackets are t - values

For these contracts, for which the market is less thin, β_1 is not different from zero at a 5% level of significance. That is, the variation in the estimated ζ cannot be explained by variation in the level of the BFI index. The time horizon still has a significant effect, but the signs of the parameter values have changed. Also observe that the R² - adjusted is very close to zero. Therefore, it seems as if the simplified version of v_t replicates the market fairly precisely for the shorter contracts.

The price of a European call option on a futures

At present, there is an unformalised market for options on BIFFEX futures. Some practitioners use the Black-Scholes formula to price these options. As the BFI most probably does not follow a geometric Brownian motion, not even over short time intervals, we think practitioners should consider other pricing formulas than Black-Scholes'.

If we accept the MRAS process as a better description of the BFI, the next step is to model the risk attitude of the market participants. As a starting point, we apply the certainty equivalent measure Q as specified above.

A European call option on a BIFFEX futures is a right, but not an obligation, to buy the futures at a given price at a given date. Let this given date be the settlement date of the futures. Thus, by the convergence principle, the option will only be exercised if the BFI is above the agreed

level at the settlement date of the option. The profit of the holder of the option is given by

$$C_T = [(\Phi_T - \psi)\chi_A] = [(X_T - \psi)\chi_A] \quad (37)$$

where χ_A is the indicator function of the event A , where $A \in \{\omega: X_T(\omega) - \psi \geq 0\}$ and ψ is the exercise value. The present value of the option at time t will then be given by the expectation of the deflated C_T under the certainty equivalent probability measure, where the deflator is given by the deterministic short rate process Y ,

$$C_t = e^{-r(T-t)} E^Q[(X_T - \psi)\chi_A | \mathcal{F}_t] \quad (38)$$

From (29) we have that $X_t = v_t dt + \sigma(X_t - \lambda) d\bar{Z}_t$, where \bar{Z}_t is a standard Brownian motion under Q . Then we can write (38) as

$$C_t = e^{-r(T-t)} \int_{-\infty}^{\infty} \left(\exp(\bar{\Gamma}_T + \hat{\Psi}_T + e^{-\kappa T} \sigma y) + \lambda - \psi \right) \chi_A f(y) dy \quad (39)$$

where $\bar{\Gamma}_T = e^{-\kappa(T-t)} \ln(x_t - \lambda) + \left(\varsigma - \frac{\sigma^2}{2\kappa} \right) (1 - e^{-\kappa(T-t)})$ and $f(\cdot)$ is the density function of the stochastic variable $\int_t^T e^{-\kappa Z_s} dZ_s$. We have that

$$\int_t^T e^{-\kappa Z_s} dZ_s \sim N \left[0, \frac{e^{2\kappa T} - e^{2\kappa t}}{2\kappa} \right] \quad (40)$$

The option is exercised if $\exp\left(\bar{\Gamma}_T + \hat{\Psi}_T + e^{-\kappa T} \sigma \int_t^T e^{-\kappa Z_s} dZ_s\right) + \lambda \geq \psi$ and therefore, it follows that $\chi_A = 1$ as long as

$$\int_t^T e^{-\kappa Z_s} dZ_s \geq \frac{\ln(\psi - \lambda) - \bar{\Gamma}_T - \hat{\Psi}_T}{e^{-\kappa T} \sigma} = y^* \quad (41)$$

Then we can write (39) as

$$C_t = e^{-r(T-t)} \int_{y^*}^{\bar{y}} \left(\exp(\tilde{\Gamma}_T + \hat{\Psi}_T + e^{-\kappa T} \sigma y) - (\psi - \lambda) \right) \frac{1}{\sqrt{\frac{2\pi(e^{2\kappa T} - e^{2\kappa t})}{2\kappa}}} \exp\left(\frac{-y^2}{\frac{e^{2\kappa T} - e^{2\kappa t}}{\kappa}}\right) dy \quad (42)$$

Let the last part of expression (42) be given by V_t^1 and define the variables

$$\hat{y} = \frac{y}{\sqrt{\frac{e^{2\kappa T} - e^{2\kappa t}}{2\kappa}}} \text{ and the constant } \hat{y}^* = \frac{y^*}{\sqrt{\frac{e^{2\kappa T} - e^{2\kappa t}}{2\kappa}}}. \text{ Then we have that}$$

$$\begin{aligned} V_t^1 &= e^{-r(T-t)} \int_{\hat{y}^*}^{\bar{\hat{y}}} (\psi - \lambda) \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-\hat{y}^2}{2}\right) d\hat{y} \\ &= e^{-r(T-t)} (\psi - \lambda) N(-\hat{y}^*) \end{aligned} \quad (43)$$

where $N(\cdot)$ is the standardised normal distribution function.

Let the first part of (42) be given by V_t^2 , and by defining the variable

$$\tilde{y} = \frac{y}{\sqrt{\frac{e^{2\kappa T} - e^{2\kappa t}}{2\kappa}}} - \sigma \sqrt{\frac{1 - e^{-2\kappa(T-t)}}{2\kappa}} \text{ and } \tilde{y}^* = \frac{y^*}{\sqrt{\frac{e^{2\kappa T} - e^{2\kappa t}}{2\kappa}}} - \sigma \sqrt{\frac{1 - e^{-2\kappa(T-t)}}{2\kappa}} \text{ we}$$

derive, after some computation, that

$$\begin{aligned} V_t^2 &= e^{-r(T-t)} \int_{\tilde{y}^*}^{\bar{\tilde{y}}} \exp\left(\tilde{\Gamma}_T + \hat{\Psi}_T + \frac{\sigma^2}{4\kappa} (1 - e^{-2\kappa(T-t)})\right) \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-\tilde{y}^2}{2}\right) d\tilde{y} \\ &= e^{-r(T-t)} \exp\left(\tilde{\Gamma}_T + \hat{\Psi}_T + \frac{\sigma^2}{4\kappa} (1 - e^{-2\kappa(T-t)})\right) N(-\tilde{y}^*) \end{aligned} \quad (44)$$

But from (30) and (34) we know that

$$\exp\left(\tilde{\Gamma}_T + \hat{\Psi}_T + \frac{\sigma^2}{4\kappa} (1 - e^{-2\kappa(T-t)})\right) = E^Q[\Phi_T | \mathcal{F}_t] - \lambda = \Phi_t - \lambda \quad (45)$$

and hence, relation (44) reduces to

$$V_t^2 = e^{-r(T-t)} (\Phi_t - \lambda) N(-\tilde{y}^*) \quad (46)$$

Then, by adding together V_t^1 and V_t^2 , we get the following formula for the price of a European call option on a futures contract

$$C_t = e^{-r(T-t)} \left\{ (\Phi_t - \lambda)N(z) - (\psi - \lambda)N\left(z + \sigma\sqrt{\frac{1 - e^{-2\kappa(T-t)}}{2\kappa}}\right) \right\} \quad (47)$$

where

$$z = \frac{\ln\left(\frac{\Phi_t - \lambda}{\psi - \lambda}\right) + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa(T-t)})}{\sigma\sqrt{\frac{1 - e^{-2\kappa(T-t)}}{2\kappa}}} \quad (48)$$

This option pricing formula does not require that we know the values of the parameters α , φ , γ and θ in order to derive the value of the option. This follows from the martingale properties of Φ_t under the equivalent probability measure Q . The effects of the level, α , the risk adjusted level, ζ , and the seasonal patterns, are already incorporated in Φ_t , i.e., knowledge of the trend is not necessary for pricing the option. Therefore, we only need to know the parameters which are embodied in the variance of the log of freight rate, κ , σ , and λ , together with the observable futures price, the corresponding time horizon, and the strike value.

Part three, Forecasting the BFI, futures prices vs. the MRAS

Gullinane (1992) suggests that it is possible to forecast the settlement value three to five days ahead better than using the futures price process' as a prediction of the settlement value. As discussed above, the futures price is a market price and should therefore not be viewed as a prediction of the settlement value. However, in the case that the market agents are risk neutral the futures price process is given by $E [X_T | \mathcal{F}_t]$ and the futures price will be the market's best prediction of the settlement value, under the quadratic loss function as a penaliser.

In this part we mainly focus on a longer horizon than Gullinane and investigate the predictive power of the MRAS specification. Our main purpose is to indicate the goodness of the specification. However, we also compare the predictions of the MRAS with the accuracy of the futures prices as forecasts of the settlement value. It is reasonable to believe that the market possesses at least as much information about market characteristics as the estimated MRAS process. If our processes systematically outperforms the futures price process in predicting the settlement value, then the futures price process is probably adjusted for risk and the market agents are risk averse.

The MRAS process incorporates three dynamic aspects; lognormally distributed error terms of the process less the absorbing level, mean reversion properties and seasonal fluctuations. The model is somewhat restrictive since it is Markov. Thus, the present BFI level, the present date and the settlement date are the only observations needed for deriving the expected value of the index at the time of settlement. For the estimations above we use data from 1985 until end 1993. To test the forecasting ability of the MRAS versus the futures prices, we use daily observations of all futures contracts traded from the first of January 1994 to the first of February 1995, and with settlement during this period. In addition, we have included the very rude forecasting procedure of assuming that the settlement value will simply be equal to the BFI value on the forecasting date, i.e. that the BFI follows a random walk.

The dynamic properties of the MRAS, especially the mean reversion assumption, are mainly long term. Therefore, we would expect that the MRAS fares better compared to the risk adjusted expectations of the market, inherent in the futures values, for long term predictions than for

forecasting the spot and prompt contracts' settlement values. Relative to using the futures as predictions, the farther away from settlement the better the predictions of the MRAS.

Given risk neutral market agent, we expect that the predictive power of the futures prices would be better than the predictive power of the MRAS. Firstly, the past BFI values are available to all market participants, free of charge. Thus, the MRAS only includes information that may be taken into account by the agents of the market. Secondly, we do not update the parameters by incorporating the previous day's BFI observation to the data set. Thus, except for the present level of the BFI, the MRAS does not gain more information during the year whereas the agents learn from the market development. Thirdly, the market will also have some knowledge of future characteristics like changed trends in demand and total supply.

Further, we should expect that the futures prices, in the case of risk neutral market agents, and the MRAS give at least as good forecasts as a random walk prediction.

The measure used for comparing the forecasting ability of the alternative forecasting procedures is Mean Squared Errors (MSE) defined by

$$MSE = \left[\sum_1^n (P - A)^2 / n \right]$$

where P is the predicted value, A is the actual value at settlement and n is the number of forecasts.

For the spot and prompt contracts we estimate the MSE for each day of trading. Thus, the MSEs shown in table 5 and 6 below are estimated from forecasts one day to one month ahead in the spot contract case, and one month to two months ahead in the prompt case. The figures reported are the MSE for each contract and the total MSE for the forecasts given by using the futures values, the MRAS, and the random walk hypothesis. The results of the forecasting power of the MRA specification are also included. In order to facilitate comparison, the parameter values used by the MRA are equal to those estimated for the MRAS model. In addition, the MSE in percentage of the MSE of the futures is reported. The lowest MSEs are printed in bold and the second best MSEs are printed in italic.

Table 5; Mean Squared Errors for different forecasts of the spot contracts

Contract	Futures	MRAS	%	MRA	%	R. W.	%
January	937	2 475	264%	1 391	148%	596	64%
February	997	2 239	225%	3 825	384%	2 034	204%
March	1 999	9 649	483%	2 327	116%	5 275	264%
April	6 058	21 713	358%	9 127	151%	11 963	197%
May	618	5 267	852%	935	151%	934	151%
June	1 580	656	41%	2 322	147%	1 808	114%
July	4 033	3 053	76%	1 633	40%	2 591	64%
August	1 326	5 081	383%	2 030	153%	1 651	125%
September	8 132	2 511	31%	13 924	171%	15 416	190%
October	2 619	1 589	61%	9 335	356%	8 017	306%
November	7 326	2 039	28%	6 935	95%	4 715	64%
December	305	1 291	424%	1 075	353%	677	222%
January	4 146	1 224	30%	810	20%	1 502	36%
Total	3 083	4 522	147%	4 282	139%	4 398	143%

Looking at the bottom line, using the futures as forecasts, outperform all the other models. Thus, we can not be conclusive on the risk attitude of the market agents. Our theoretical MRAS model does worst of all the specifications. Even the random walk assumption proves to be better. Hence, to assume mean reversion and seasonal fluctuation when predicting the spot contract does not seem to be fruitful. Nevertheless, the pure mean reversion model performs slightly better.

The MSEs of the individual months show clear patterns. Using the futures performed best early in 1994 and the theoretical models performed best during the autumn of 1994. During April the BFI increased whereas the MRAS predicted a seasonal slump. The market could foresee this change in the freight rate pattern, whereas the MRAS only reproduced the old pattern. This is a good illustration of the shortcomings of the MRAS model.

Table 6; Mean Squared Errors for different forecasts of the prompt contracts

Contract	Futures	MRAS	%	MRA	%	R. W.	%
February	17 545	18 899	108%	20 800	119%	11 559	66%
March	2 154	8 118	377%	1 019	47%	3 310	154%
April	27 806	120 012	432%	48 029	173%	73 273	264%
May	16 002	69 173	432%	17 929	112%	27 233	170%
June	1 326	1 849	139%	27 080	2042%	23 105	1742%
July	47 799	21 040	44%	6 549	14%	11 508	24%
August	10 204	1 404	14%	1 329	13%	3 292	32%
September	49 040	961	2%	23 305	48%	27 490	56%
October	78 926	14 495	18%	110 383	140%	115 705	147%
November	14 476	20 308	140%	24 130	167%	16 671	115%
December	42 537	2 034	5%	33 445	79%	22 480	53%
January	2 901	7 162	247%	3 267	113%	996	34%
Total	23 901	21 958	92%	24 405	102%	25 894	108%

Comparing the prompt contract with the spot contract, the one month increase in the forecasting horizon implies that the total MSE of using the futures as predictors, increase eight times. Evidently, the other models perform approximately as well as using the futures for predictions. Actually, the MRAS model outperforms the futures strategy by 8%. Also for the prompt contract, using the futures as forecasts made the best result in the beginning of the year and the MRAS during the last part of the year.

Only four contracts with a longer forecasting horizon than two months, were traded during 1994, i.e. the April, July, October and January contracts. We have calculated the MSE of these four contracts for forecasts given each trading day during the third and fourth month before settlement. The results are shown in table 7 and 8.

Table 7; Mean Squared Errors for different forecasts three months ahead

Contract	Futures	MRAS	%	MRA	%	R. W.	%
April	28 815	94 193	327%	26 882	93%	52 049	181%
July	12 454	14 918	120%	2 812	23%	1 814	15%
October	191 735	10 299	5%	140 317	73%	151 927	79%
January	65 403	8 986	14%	41 019	63%	16 061	25%
Total	22 954	9 877	43%	16 233	71%	17 065	74%

Table 8; Mean Squared Errors for different forecasts four months ahead

Contract	Futures	MRAS	%	MRA	%	R. W.	%
April	11 310	62 889	556%	8 242	73%	26 099	231%
July	21 219	79 834	376%	7 945	37%	21 673	102%
October	215 214	31 966	15%	178 591	83%	210 221	98%
January	77 338	33 711	44%	71 790	93%	35 525	46%
Total	25 006	16 031	64%	20 505	82%	22 578	90%

The strategy of using the futures as forecasts are beaten by all the other strategies for both three and four months' lead time. In total, the MRAS gives the lowest MSE, but as for the spot and prompt contracts, the MRAS specification failed in forecasting the settlement values during the first part of 1994. The forecasts of the MRA give the second lowest MSE. Further, note that the MRA beats the strategy of using the futures, for all contracts. Hence, recognising that the BFI has a mean reverting structure gives systematically better forecasts of the settlement value than using the futures as predictions. Even the random walk outperforms the futures. Hence, there is a strong indication that the market adjusts the futures price process for risk.

We do not report in detail the MSE for longer forecast horizons since only the July, October and January contracts are available. However, the results support the above findings. Using the futures as predictions were outperformed by at least one of the other forecasts for all lead times other than those reported above, except for the July contract five months ahead. For this single forecast the futures were slightly better than the MRA. Consequently, it seems as the futures prices in the BIFFEX market for contracts other than the spot and prompt contracts systematically deviate from the conditional expected value. As already noted, this may be due to risk adjustments. A testing period of only one year is very short and we

believe the question of discrepancy from expected values ought to be a field for some further research. There is no reason for such a study to be restricted to Markov processes.

Summary and conclusions

Among the models tested, it seems as if the most promising approach is that of describing the movement of the BFI index using a mean reversion with an absorbing level above zero. However, we also have to take seasonal variations into consideration. This is taken care of, on a fairly ad hoc basis, by adding a sine term.

In part two we sketch how a futures on the BFI could be priced in the absence of arbitrage. However, as we have already pointed out, it seems unlikely that it is possible to construct a risk free portfolio by replicating the BFI in the real shipping markets. Therefore, in order to take risk attitude into account, we suggest a certainty equivalent probability measure. Further, we derive a pricing formula for a European call option on a futures.

Part three discusses the strategy of using the futures versus the MRAS process as regards predicting the settlement value of the BFI. Only for short forecast horizons do the futures outperform our process. As expected, the MRA and the MRAS models give the best description of the BFI if the time perspective is fairly long. However, the most striking finding is that the MRA outperforms the futures for all but the spot and prompt contracts. Hence, it seems that the futures price process is not the market's prediction of the settlement value, but is subject to risk adjustment by the market participants.

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Appendix a. The future value of a mean reversion process with arbitrary absorbing level

We have that the incremental change in the MRA process is given by

$$dX_t = \kappa(\alpha - \ln(X_t - \lambda))(X_t - \lambda)dt + \sigma(X_t - \lambda)dZ_t, \quad (2)$$

Where dZ_t is the increment of a standard Brownian motion Z_t , i.e., $dZ_t \sim N(0, dt)$, as defined in the text, and λ is an arbitrary absorbing level for the process.

From dividing by $(X_t - \lambda)$ and multiplying by the integrating factor $e^{\kappa t}$, we have that

$$e^{\kappa t} \frac{dX_t}{(X_t - \lambda)} = e^{\kappa t} \kappa(\alpha - \ln(X_t - \lambda))dt + e^{\kappa t} \sigma dZ_t, \quad (1a)$$

Define the function $g(x_t, t) = -e^{\kappa t}(\alpha - \ln(x_t - \lambda))$ and apply Ito's lemma. Then it follows that

$$\begin{aligned} dg(X_t, t) &= \frac{dg}{dt} dt + \frac{dg}{dx} dX + \frac{1}{2} \frac{d^2g}{dx^2} dX^2 = d\{-e^{\kappa t}(\alpha - \ln(X_t - \lambda))\} \\ &= -e^{\kappa t} \kappa(\alpha - \ln(X_t - \lambda))dt + e^{\kappa t} \frac{dX_t}{(X_t - \lambda)} - \frac{1}{2} e^{\kappa t} \frac{dX_t^2}{(X_t - \lambda)^2} \end{aligned}$$

Rearranging, we get

$$d\{-e^{\kappa t}(\alpha - \ln(X_t - \lambda))\} + \frac{1}{2} e^{\kappa t} \frac{dX_t^2}{(X_t - \lambda)^2} = -e^{\kappa t} \kappa(\alpha - \ln(X_t - \lambda))dt + e^{\kappa t} \frac{dX_t}{(X_t - \lambda)}$$

From (2) and (1a) we see that the above relation can be written

$$d\{-e^{\kappa t}(\alpha - \ln(X_t - \lambda))\} + \frac{1}{2} e^{\kappa t} \sigma^2 dt = e^{\kappa t} \sigma dZ_t,$$

Rearranging again, and integrating from time zero to time τ , we have that

$$-e^{\kappa \tau}(\alpha - \ln(X_\tau - \lambda)) + e^{\kappa 0}(\alpha - \ln(x_0 - \lambda)) = \sigma \int_0^\tau e^{\kappa t} dZ_t - \frac{1}{2} \sigma^2 \left(\frac{e^{\kappa \tau} - 1}{\kappa} \right)$$

Further manipulations give

$$\ln(X_\tau - \lambda) = e^{-\kappa\tau} \ln(x_0 - \lambda) + \left(\alpha - \frac{1}{2} \frac{\sigma^2}{\kappa} \right) (1 - e^{-\kappa\tau}) + e^{-\kappa\tau} \sigma \int_0^\tau e^{\kappa s} dZ_s \quad (2a)$$

Define

$$\Gamma_\tau = e^{-\kappa\tau} \ln(x_0 - \lambda) + \left(\alpha - \frac{1}{2} \frac{\sigma^2}{\kappa} \right) (1 - e^{-\kappa\tau})$$

and

$$\Lambda_\tau = e^{-\kappa\tau} \sigma \int_0^\tau e^{\kappa s} dZ_s$$

It follows then that the value of the process at time τ , given the level at time zero, is

$$X_\tau = e^{\Gamma_\tau + \Lambda_\tau} + \lambda \quad (3a)$$

We have that

$$\Lambda_\tau \sim N \left[0, e^{-2\kappa\tau} \sigma^2 \int_0^\tau e^{2\kappa t} dt \right] = N \left[0, \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa\tau}) \right] = N[\tilde{\mu}, \tilde{\sigma}^2]$$

Then it follows by the moment generating function $\phi(\tau)$ that

$$\phi(\tau) = E[e^{\Lambda_\tau} | \mathcal{F}_0] = e^{\tilde{\mu} + \frac{\tilde{\sigma}^2 \tau^2}{2}} = e^{\frac{\sigma^2}{4\kappa} (1 - e^{-2\kappa\tau})}$$

Then we have that the conditional expectation of X_τ

$$E[X_\tau | \mathcal{F}_0] = e^{\Gamma_\tau + \frac{\sigma^2}{4\kappa} [1 - e^{-2\kappa\tau}]} + \lambda \quad (4a)$$

We have that (2a) is Gaussian, and thus, it follows that the conditional expectation of the log of the process less the absorbing level is

$$E[\ln(X_\tau - \lambda)|\mathcal{F}_0] = e^{-\kappa\tau} \ln(x_0 - \lambda) + \left(\alpha - \frac{1}{2} \frac{\sigma^2}{\kappa} \right) (1 - e^{-\kappa\tau})$$

and the variance is

$$\text{Var}[\ln(X_\tau - \lambda)|\mathcal{F}_0] = \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa\tau})$$

b. The future value of a mean reversion process with arbitrary absorbing level and sinus fluctuations

The procedure used here is analogue to that of appendix a. We start out with the increment of the MRAS process

$$dX_t = \kappa(\alpha + \varphi \sin(\gamma t + \theta) - \ln(X_t - \lambda))(X_t - \lambda)dt + \sigma(X_t - \lambda)dZ_t \quad (8)$$

Where, as before, dZ_t is the increment of a standard Brownian motion Z_t , i.e., $dZ_t \sim N(0, dt)$, λ is the arbitrary absorbing level. The term $\varphi \sin(\gamma t + \theta)$ gives the direct dependence on time of the drift term.

As above, we divide the increment of the process by $(X_t - \lambda)$ and multiply by the integrating factor $e^{\kappa x}$. Thereafter, define the function $g(x, t) = -e^{\kappa x}(\alpha + \varphi \sin(\gamma t + \theta) - \ln(x - \lambda))$ and apply Ito's lemma. Hence, it follows that

$$\begin{aligned} dg(X_t, t) &= \frac{dg}{dt} dt + \frac{dg}{dx} dX + \frac{1}{2} \frac{d^2g}{dx^2} dX^2 = d\{-e^{\kappa x}(\alpha + \varphi \sin(\gamma t + \theta) - \ln(X_t - \lambda))\} \\ &= -e^{\kappa x} \kappa(\alpha + \varphi \sin(\gamma t + \theta) - \ln(X_t - \lambda))dt - e^{\kappa x} \varphi \gamma \cos(\gamma t + \theta)dt \\ &\quad + e^{\kappa x} \frac{dX_t}{(X_t - \lambda)} - \frac{1}{2} e^{\kappa x} \frac{dX_t^2}{(X_t - \lambda)^2} \end{aligned}$$

Then rearrange to get

$$\begin{aligned} d\{-e^{\kappa x}(\alpha + \varphi \sin(\gamma t + \theta) - \ln(X_t - \lambda))\} + e^{-\kappa x} \varphi \gamma \cos(\gamma t + \theta)dt + \frac{1}{2} e^{\kappa x} \frac{dX_t^2}{(X_t - \lambda)^2} \\ = -e^{\kappa x} \kappa(\alpha + \varphi \sin(\gamma t + \theta) - \ln(X_t - \lambda))dt + e^{\kappa x} \frac{dX_t}{(X_t - \lambda)} \\ = e^{\kappa x} \sigma dZ_t \end{aligned}$$

Inserting for dX_t and integrating from time t to time τ , gives

$$\begin{aligned}
 & -e^{\kappa\tau}(\alpha + \varphi \sin(\gamma\tau + \theta) - \ln(X_\tau - \lambda)) + e^{\kappa t}(\alpha + \varphi \sin(\gamma t + \theta) - \ln(X_t - \lambda)) \\
 & = \sigma \int_t^\tau e^{\kappa s} dZ_s - \frac{1}{2} \sigma^2 \left(\frac{e^{\kappa\tau} - e^{\kappa t}}{\kappa} \right) \\
 & - \frac{\varphi\gamma}{\kappa^2 + \gamma^2} (e^{\kappa\tau} (\kappa \cos(\gamma\tau + \theta) + \gamma \sin(\gamma\tau + \theta)) - e^{\kappa t} (\kappa \cos(\gamma t + \theta) + \gamma \sin(\gamma t + \theta)))
 \end{aligned}$$

Then we have that

$$\begin{aligned}
 \ln(X_\tau - \lambda) & = -e^{-\kappa(\tau-t)} \ln(X_t - \lambda) + \left(\alpha - \frac{1}{2} \frac{\sigma^2}{\kappa} \right) (1 - e^{-\kappa(\tau-t)}) \\
 & - \frac{\varphi\gamma}{\kappa^2 + \gamma^2} (\kappa \cos(\gamma\tau + \theta) + \gamma \sin(\gamma\tau + \theta) - e^{-\kappa(\tau-t)} (\kappa \cos(\gamma t + \theta) + \gamma \sin(\gamma t + \theta))) \quad (1b) \\
 & + \varphi (\sin(\gamma\tau + \theta) - e^{-\kappa(\tau-t)} \sin(\gamma t + \theta)) + e^{-\kappa\tau} \sigma \int_t^\tau e^{\kappa s} dZ_s
 \end{aligned}$$

By defining

$$\begin{aligned}
 \hat{\Gamma}_\tau & = e^{-\kappa(\tau-t)} \ln(X_t - \lambda) + \left(\alpha - \frac{1}{2} \frac{\sigma^2}{\kappa} \right) (1 - e^{-\kappa(\tau-t)}) \\
 \hat{\Lambda}_\tau & = e^{-\kappa\tau} \sigma \int_t^\tau e^{\kappa s} dZ_s
 \end{aligned}$$

and

$$\begin{aligned}
 \hat{\Psi}_\tau & = -\frac{\varphi\gamma}{\kappa^2 + \gamma^2} (\kappa \cos(\gamma\tau + \theta) + \gamma \sin(\gamma\tau + \theta) - e^{-\kappa(\tau-t)} (\kappa \cos(\gamma t + \theta) + \gamma \sin(\gamma t + \theta))) \\
 & + \varphi (\sin(\gamma\tau + \theta) - e^{-\kappa(\tau-t)} \sin(\gamma t + \theta)) \\
 & = -\frac{\varphi\gamma\kappa}{\kappa^2 + \gamma^2} (\cos(\gamma\tau + \theta) - e^{-\kappa(\tau-t)} \cos(\gamma t + \theta)) \\
 & - \varphi \left(\frac{\gamma^2}{\kappa^2 + \gamma^2} - 1 \right) (\sin(\gamma\tau + \theta) - e^{-\kappa(\tau-t)} \sin(\gamma t + \theta))
 \end{aligned}$$

we can write the value of the process at time τ , given the level at time t , as

$$X_\tau = e^{\hat{\Gamma}_\tau + \hat{\Psi}_\tau + \hat{\Lambda}_\tau} + \lambda \quad (9)$$

The conditional expectation of X_τ given the filtration up to time t , is then

$$E[X_\tau | \mathcal{F}_t] = e^{\hat{r}_t + \hat{\psi}_t + \frac{1}{2}\sigma^2 \left[\frac{1 - e^{-2\kappa(\tau-t)}}{2\kappa} \right]} + \lambda \quad (2b)$$

The log of the index value less the absorbing level is Gaussian. Consequently, we may write the expected value of the log of the index less the absorbing level at time τ , given the filtration up to time t , as

$$\begin{aligned} E[\ln(X_\tau - \lambda) | \mathcal{F}_t] &= e^{-\kappa(\tau-t)} \ln(X_t - \lambda) + \left(\alpha - \frac{1}{2} \frac{\sigma^2}{\kappa} \right) (1 - e^{-\kappa(\tau-t)}) \\ &\quad - \frac{\phi\gamma\kappa}{\kappa^2 + \gamma^2} \left((\cos(\gamma\tau + \theta) - e^{-\kappa(\tau-t)} \cos(\gamma t + \theta)) \right) \\ &\quad - \phi \left(\frac{\gamma^2}{\kappa^2 + \gamma^2} - 1 \right) \left((\sin(\gamma\tau + \theta) - e^{-\kappa(\tau-t)} \sin(\gamma t + \theta)) \right) \end{aligned}$$

The variance will be

$$\text{Var}[\ln(X_\tau - \lambda) | \mathcal{F}_t] = \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa(\tau-t)})$$

However, since we have added a sine term the process will not be stationary.

Appendix c. The future value of the CIR process

The incremental change in the CIR process is given by

$$dX_t = \kappa(\alpha - X_t)dt + \sigma\sqrt{X_t}dZ_t \quad (1)$$

Where dZ_t is the increment of a standard Brownian motion Z_t , i.e., $dZ_t \sim N(0, dt)$.

We multiply each side by the integrating factor $e^{\kappa t}$ to get

$$e^{\kappa t}dX_t = e^{\kappa t}\kappa(\alpha - X_t)dt + e^{\kappa t}\sigma\sqrt{X_t}dZ_t \quad (1c)$$

Like in the above appendixes, we define a function $g(x_t, t) = -e^{\kappa t}(\alpha - x_t)$ and use Ito's lemma to get

$$\begin{aligned} dg(X_t, t) &= \frac{dg}{dt}dt + \frac{dg}{dx}dX + \frac{1}{2}\frac{d^2g}{dx^2}dX^2 = d\{-e^{\kappa t}(\alpha - X_t)\} \\ &= -e^{\kappa t}\kappa(\alpha - X_t)dt + e^{\kappa t}dX_t \end{aligned}$$

And by rearranging (1c) we have that

$$d\{-e^{\kappa t}(\alpha - X_t)\} = e^{\kappa t}\sigma\sqrt{X_t}dZ_t$$

By integrating from time zero to time τ , it follows that

$$-e^{\kappa\tau}(\alpha - X_\tau) + (\alpha - x_0) = \sigma \int_0^\tau e^{\kappa t}\sqrt{X_t}dZ_t$$

Define

$$v_\tau = \sigma \int_0^\tau e^{\kappa t}\sqrt{X_t}dZ_t$$

and we have the value of the process at time τ given by

$$X_\tau = v_\tau e^{-\kappa\tau} + \alpha - (\alpha - x_0)e^{-\kappa\tau} \quad (2c)$$

The expectation of the stochastic part, given the filtration up to time zero, is $E[\vartheta_\tau|\mathcal{F}_0] = 0$. Thus, it follows that the conditional expectation of the CIR process at time τ , is given by

$$E[X_\tau|\mathcal{F}_0] = x_0 e^{-\kappa\tau} + \alpha(1 - e^{-\kappa\tau}) \quad (3c)$$

The variance of the process is by definition

$$\text{Var}[X_\tau|\mathcal{F}_0] = E[X_\tau^2|\mathcal{F}_0] - (E[X_\tau|\mathcal{F}_0])^2$$

From (2c) above we have that

$$X_\tau^2 = \vartheta_\tau^2 e^{-2\kappa\tau} + \alpha^2 - (\alpha - x_0)^2 e^{-2\kappa\tau} + 2(\vartheta_\tau e^{-\kappa\tau} \alpha - \vartheta_\tau e^{-2\kappa\tau} (\alpha - x_0) - \alpha(\alpha - x_0) e^{-\kappa\tau})$$

Further, from the properties of the stochastic intergral and from Fubini's theorem we have that

$$\begin{aligned} E[\vartheta_\tau^2|\mathcal{F}_0] &= E\left[\sigma^2 \int_0^\tau e^{2\kappa s} X_s ds | \mathcal{F}_0\right] = \sigma^2 \int_0^\tau e^{2\kappa s} E[X_s|\mathcal{F}_0] ds \\ &= \sigma^2 \int_0^\tau e^{2\kappa s} E[(x_0 - \alpha)e^{-\kappa s} + \alpha | \mathcal{F}_0] ds \\ &= \frac{\sigma^2}{\kappa} (x_0 - \alpha)(e^{\kappa\tau} - 1) + \frac{\sigma^2}{2\kappa} \alpha (e^{2\kappa\tau} - 1) \end{aligned}$$

From (3c) we see that

$$(E[X_\tau|\mathcal{F}_0])^2 = (x_0 - \alpha)^2 e^{-2\kappa\tau} + \alpha^2 + 2\alpha(x_0 - \alpha)e^{-\kappa\tau}$$

Then it follows by simple calculation that the conditional variance at time τ given the filtration up to time zero, is

$$\text{Var}[X_\tau|\mathcal{F}_0] = x_0 \frac{\sigma^2}{\kappa} (e^{-\kappa\tau} - e^{-2\kappa\tau}) + \alpha \frac{\sigma^2}{2\kappa} (1 - e^{-\kappa\tau})^2$$

Appendix d

The Baltic Freight Index, including changes of 3 November 1993

	Route	Cargo size	Commodity	Weight
1	US Gulf/North Continent	55,000	Light grain	10%
1a.	64,000 DWAT, Hitachi Type, Trans Atlantic Round-Voyage (TC)			10%
2	US Gulf to 1 Combo Port South Japan	52,000	Heavy grain	10%
2a.	64,000 DWAT, Hitachi Type, Skaw Passero Range/Taiwan Japan Range (TC)			10%
3	US North Pacific to 1 Combo Port South Japan	52,000	Heavy grain	10%
3a.	64,000 DWAT, Hitachi Type, Trans Pacific Round-Voyage (TC)			10%
6	Hampton Roads/Richard Bay to South Japan	120,000	Coal	7.5%
7	Hampton Roads to Rotterdam	110,000	Coal	7.5%
8	Queensland to Rotterdam	130,000	Coal	7.5%
9	64,000 DWAT, Hitachi Type, Fare East to Europe (TC)			10%
10	Tubarao to Rotterdam	130,000	Iron Ore	7.5%
				100%

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Ch. 5;

The Structure of the Freight Rate, A Stochastic Partial Equilibrium Model for the VLCC Market

Abstract

In this paper we present a stochastic partial equilibrium model for the tanker market. Our aim is to relate the time charter equivalent freight rate derived from this model to processes previously suggested in the literature. We assume that demand for transport in crude carriers exhibits constant freight rate elasticity. The dynamics of demand is given by a stochastic development generated by a geometric Brownian motion. Further, we assume constant return to scale in the aggregated supply of transport services, depending mainly on bunker consumption and total tonnage available. The shipbuilding industry supplies new tonnage, but since there are costs related to changing the output from the yards the production of new vessels are not instantaneously adjusted to changes in demand. We find that the freight rates generated by the model has mean reversion properties and is lognormally distributed for given intervals. There is higher volatility when freight rates are high and lower volatility as freight rates are low.

Introduction

We have argued that knowledge of the stochastic nature of the freight rate is vital in order to value assets in the shipping industry. The most common approach in the literature has been to suggest alternative stochastic processes that may represent the freight rate in an appropriate way, e.g. Mossin (1968), Bjerksund & Ekern (1993) and Tvedt (1995 b). However, we may get a better understanding of the characteristics of the freight rate movements if we model the underlying demand and supply relations in some detail. This is the main aim of this paper.

Our approach is to construct a partial equilibrium model. Our model includes the demand and supply for crude oil transport and the shipbuilding markets.

Demand for crude oil tankers fluctuates to a significant degree. The number of shipments varies with the seasons. Cold winters in Europe and the USA lead to reduced stocks in the consumption areas and increased demand for tankers for refill.

Demand for crude oil tankers is usually measured in tonne-miles, i.e. the number of tonnes of crude oil transported during a given period (e.g. one year) times the total number of miles sailed. High volatility in the oil price

increases activity among the oil traders. Cargoes may be loaded without the final destination having been decided upon. The ship master will under way be instructed where to go and the vessel may be re-routed a number of times. Thus, oil trading may increase demand for tankers without any change in the overall consumption of oil, since increased trading may entail longer sailing distances and thereby higher tonne-mile demand.

The long term dynamics of the tonne-mile demand will mainly be governed by changes in total world consumption of crude oil and in the geographical demand and supply pattern. Obviously, demand for oil will mainly depend on the price of oil in relation to alternative sources of energy and on the growth of the world economy. Today, the extraction policy of the most efficient producers, the majority of whom is organised in the OPEC cartel, decides the price of oil. Throughout the history of OPEC the oil price has been unstable. In periods of tight OPEC quotas and thereby high oil prices, producers outside OPEC have acquired a larger market share. This includes North Sea fields and reopening of marginal wells in the USA. High oil prices reduce demand for oil, as consumers substitute other energy sources for oil. The main import regions for crude oil are the USA, West Europe, Japan and the NIC countries. That is, as OPEC restrict own production in order to raise oil prices, market shares are taken by producers closer to the main consumption areas. Therefore, high oil prices are often related to low overall consumption of oil and shorter transport distances. Both effects reduce the need for tanker capacity.

The total supply of VLCCs and ULCCs are about 117 million dead weight tonnes (dwt.), i.e. 450 vessels (Clarkson, May 1991). The total tonne-mile supply is to a certain extent flexible in the short run. The two major factors that influence the degree of utilisation of the fleet are lay-up and speed. If the freight rate plus cost of lay-up do not cover variable costs, the ship owner will be better off by laying up the vessel. If only voyage related costs are covered, the shipowner will be just as well off by waiting for an improved market than accepting available cargoes. Therefore, the freight rate will very seldom fall below the voyage related costs. The main voyage related costs are fuel, harbour charges and channel fees.

Besides reduced lay-up, increased speed is the most effective means of providing additional supply. Speed may be adjusted instantaneously, and will be a trade-off between additional generated income and increased

bunker consumption, off-hire and maintenance costs. Costs increase exponentially with increased speed. The relation between speed and voyage costs differs between vessels, but there are two main categories of VLCCs - the turbine tanker and the motor tanker. The majority of the world fleet are turbine tankers. They are less fuel efficient and thus vulnerable to markets with high oil and bunker prices and low freight rates.

In a bad market the shipowner may prefer to sell his vessel for demolition. Therefore, capacity may readily be adjusted permanently downwards. Apparently there is no shortage of scrapping capacity.

At full technical utilisation of the fleet only newbuilding will increase the supply. To increase the capital stock takes time. The most efficient Japanese yards construct a VLCC in a few months. However, in some instances there may be a lack of available yards and the minimum period from ordering to delivery may be substantially prolonged.

However, yard capacity may be expanded to take care of future increases in demand for tankers. The construction capacity may gradually be enlarged by increased productivity. However, there may be a need for investments in new docks and infrastructure to increase capacity sufficiently. In addition, unqualified workers have to be trained. Large changes in construction capacity therefore entail some initial investments.

Short term equilibrium in the tanker market

In deriving the short term equilibrium below we mainly follow Dixit (1991).

Demand

We assume that demand for shipping services is given by a constant freight rate elasticity function. That is, the demand for VLCCs at time t , Q_t , measured in tonne-miles per year, is given by

$$Q_t = Y_t X_t^{-\epsilon} \quad (1)$$

where X_t is the time charter equivalent spot rate. We assume the elasticity ϵ to be above one. This is done to simplify computations and to clarify the presentation of the main points. This assumption may easily be generalised

to allow ε to be above zero by using a slightly more complicated model. See Tvedt (1995 a) for this extension and for applications of the model.

The scalar Y_t gives the dynamic of demand. We postulate that Y_t follows a geometric Brownian motion. Thus, Y_t makes stochastic movements in the demand curve. The incremental change in Y_t is given by

$$dY_t = \mu Y_t dt + \sigma Y_t dZ_t \quad (2)$$

where μ is the instantaneous expected growth rate, σ is the standard deviation of the incremental relative change in Y_t and dZ_t is the increment of a standard Brownian motion, i.e. $dZ_t \sim N[0, dt]$.

Supply

Tonne-mile demand and supply must be equal at any time. In the model the tonne-mile production at a given point of time, t , depends on the stock of vessels, k_t , and a utilisation indicator, b_t . In most respects, b_t may approximately be regarded as the total bunker consumption of the fleet. We assume that we have constant return to scale in aggregated supply. This is probably a reasonable approximation due to the fact that the fleet may be replicated by new and more or less technologically identical tonnage. In the model the incremental tonne-mile supply is then given by

$$Q_t = b_t^\gamma k_t^{1-\gamma} \quad (3)$$

where γ is positive and below one. An appropriate γ will probably be closer to zero than one.

Given the present fleet, improved utilization should for low tonne-mile production, represent reentering of less efficient vessels, for medium high tonne-mile production, improved utilization should represent increase in speed from a moderate to high level. To raise utilization further by increasing speed to a maximum, will demand an extremely large additional bunker consumption.

The short term variable costs are given by

$$VC_t = wb_t = w \left(\frac{Q_t}{k_t^{1-\gamma}} \right)^{\frac{1}{\gamma}} = wQ_t^{\nu} k_t^{-(\nu-1)} \quad (4)$$

where $\nu = \frac{1}{\gamma}$ and w are constants.

From this we have the short term marginal costs

$$\frac{dVC_t}{dQ_t} = w\nu Q_t^{\nu-1} k_t^{-(\nu-1)}$$

Short term equilibrium

From (1) it follows that the freight rate at time t is given by

$$X_t = \left(\frac{Q_t}{Y_t} \right)^{-\frac{1}{\varepsilon}}$$

We assume perfect competition in the tanker market. Short term market clearance with price equal short term marginal cost gives

$$w\nu \left(\frac{Q_t}{k_t} \right)^{\nu-1} = \left(\frac{Q_t}{Y_t} \right)^{-\varepsilon}$$

where $\varepsilon = \frac{1}{\gamma}$. With some further manipulations it follows that the total tonne-miles produced and the freight rate at time t are given by (see appendix I.a.)

$$Q_t = k_t \left(\frac{Y_t}{k_t} \right)^{\frac{\varepsilon}{\varepsilon+\nu-1}} \left(\frac{w}{\gamma} \right)^{-\frac{1}{\varepsilon+\nu-1}} \quad (5)$$

and

$$X_t = \left(\frac{Y_t}{k_t} \right)^{\frac{\varepsilon(\nu-1)}{\varepsilon+\nu-1}} \left(\frac{w}{\gamma} \right)^{\frac{\varepsilon}{\varepsilon+\nu-1}} \quad (6)$$

Observe that by this representation the freight rate X_t only depends on the ratio of the demand scalar over the capital stock, and not on the absolute magnitude of the variables.

Dynamic equilibrium in the tanker market

Following Lucas and Prescott (1971) the competitive equilibrium assumption entails that the present value of the instantaneous total market surpluses less the investment costs are maximised. This is equivalent to maximising total welfare. Thus, for our purpose, we can focus on the optimal investment policy on an aggregated social level. Thereby, we circumvent the problem of the different parties in the market having divergent interests. A shipowner maximises his own profit from investing in tonnage and running his vessels. The customers maximise their utility of applying the vessels and the yards maximise their profit from building the ships. In a note we focus on the individual behaviour of the different agents in the shipping markets. See Tvedt (1995 a).

Total freight market surplus

The total instantaneous surplus in the freight market, i.e. consumer plus producer surplus, is given by the integral under the inverse demand curve less variable costs from zero to the market clearing tonne-mile level. That is, for an optimally chosen supply, Q_t^* , total market surplus is given by

$$S_t^* = \int_0^{Q_t^*} \left(\frac{s}{Y_t} \right)^{-\epsilon} ds - VC(Q_t^*)$$

Inserting (5) for Q_t^* , we get after some manipulations (see appendix I.b.)

$$S_t^* = ck_t^{1-\phi} Y_t^\phi \quad (7)$$

where

$$\phi = \frac{1}{\gamma + \epsilon - \gamma\epsilon}$$

and

$$c = \left(\frac{e}{\phi(1-e)} \right) \left(\frac{w}{\gamma} \right)^{\frac{e-1}{e+\nu-1}}$$

The dynamics of investment

Steel and man hours are the main components for building a VLCC. Steel is indeed a variable production factor. However, this is not so for the work hours. To increase the work force gradually is not very costly in terms of hiring costs. However, large increases will often mean that untrained labour must be hired. Costs of loss of efficiency and training costs may be substantial.

To lay off workers in a large scale may not be very expensive for the ship yard, except for the loss of skill and experience that may be costly to retain if the yard later on decides to increase its capacity. However, in many countries the shipbuilding industry has been or is a major employer. Large reductions in the shipbuilding activity may cause severe regional unemployment problems.

Historically, the number of workers and real capital needed for constructing a vessel has been in steady decline. Key concepts for explaining this is increased automatisisation, better organisation and more skilled workers. However, there are large differences between the work hours needed by the most efficient Japanese yards and young VLCC building nations like China.

We have applied a very simple representation of the shipbuilding industry. We must admit that our representation is mainly chosen by its mathematical tractability. However, we think it is still useful for describing some of the fundamentals of the market.

We assume that it is costless to keep the ratio of yard capacity over the present fleet, α , constant. Further, we assume that to alter the yard-fleet ratio entails costs of change.

At any time we assume that the given capacity of the ship yards is fully employed. That is, deliveries are equal to capacities. This is of course only an approximation, especially as far as real capital invested in dock facilities and equipment are concerned. Then, we have that the incremental change in the fleet is given by the total level of the fleet. That is,

$$dk_t = \alpha_t k_t dt$$

where a_t is the marginal relative change in the fleet at time t .

The cost of increasing the yard capacity by increasing the yard-fleet ratio is given by q_i and the cost of reduction is given by q_r .

Then, at the time of a control and for a given k_t , it follows that

$$q_i k_t d(a_t) = q_r k_t \xi_t$$

where ξ_t is the chosen shift in the yard-fleet ratio at time t . Substituting q_r for q_i gives the equivalent relation for a reduction of the ratio. Due to the cost of control the shifts will not be infinitesimal.

The production cost of one ship unit is assumed constant and given by p_i . Then the incremental investment outlay is given by $p_i a_t k_t dt$.

Potential freight capacity is adjusted downwards by demolition. We represent the decay of the capital stock by a constant rate, δ , of depreciation. We are aware that this is not an accurate description of the actual scrapping pattern of large crude oil tankers. Scrapping is to a large extent a strategic decision, that is, the timing when a vessel is sold for demolition is related to the expectation of profits generated in the future. However, as the vessel gets older, maintenance and running costs normally increase. Further, old vessels are gradually removed from important trades due to international and local legislation.

The decay of the capital stock is given by

$$dk_t = -\delta k_t dt$$

The vessels are sold as scrap at a fixed price per unit, p_s . Then, the incremental income to the shipowners from demolition is given by $p_s \delta k_t dt$.

The net incremental increase in the total fleet is then given by

$$dk_t = (a_t - \delta) k_t dt. \quad (8)$$

An optimal control problem, the general case

Let the present value at time zero of the total freight market surplus less the net cost of increasing the fleet at time t is given by

$$F(\bar{X}_t) = e^{-\rho t} c k_t^{1-\theta} Y_t^\theta - e^{-\rho t} p_t a_t k_t + e^{-\rho t} p_t \delta k_t$$

where $e^{-\rho t}$ is a discount factor where ρ is constant and \bar{X}_t is the state of the system as defined below.

Let m be a fixed cost of increasing construction capacity and n a fixed cost of decreasing capacity.

Then it follows from the discussion above that the cost of adjustment is

$$\bar{K}(\bar{X}_{\theta_j}, \xi_j) = e^{-\rho t} \begin{cases} m + q_i \xi_j k_t & ; \xi_j > 0 \\ n + q_r \xi_j k_t & ; \xi_j < 0 \\ 0 & ; \xi_j = 0 \end{cases}$$

where ξ_j is the jump in a_t at time θ_j

The maximum of the present value of the total consumer and producer surpluses less the costs of keeping an optimal investment path is given by the value function $\Phi(\bar{x})$.

$$\Phi(\bar{x}) = \sup_{\omega} E^{\bar{x}} \left[\int_0^{\infty} F(\bar{X}_t) dt - \sum_{j=1}^N \bar{K}(\bar{X}_{\theta_j}, \xi_j) \right] \quad (9)$$

where the controls are given by ω , where

$$\omega = (\theta_1, \theta_2, \dots, \theta_N; \xi_1, \xi_2, \dots, \xi_N), \quad N < \infty,$$

and θ_l is the time of the first control and ξ_l is the size of the first jump in a_t , and so forth.

We have that the value function is dependent on the state of the system, \bar{X}_t , where

$$\bar{X}_t = \begin{bmatrix} s+t \\ k_t \\ Y_t \\ a_t \end{bmatrix}$$

The incremental change in \bar{X}_t between each change in a , is given by

$$d\bar{X}_t = \begin{bmatrix} 1 \\ (a_t - \delta)k_t \\ \mu Y_t \\ 0 \end{bmatrix} dt + \begin{bmatrix} 0 \\ 0 \\ \sigma Y_t \\ 0 \end{bmatrix} dZ_t$$

It follows from the assumptions above that the process $\{\bar{X}_t; t \geq 0\}$ is a Markov process, and hence has an infinitesimal generator \mathcal{A} ,

$$\mathcal{A} = \frac{\partial}{\partial t} + (a_t - \delta)k_t \frac{\partial}{\partial k} + \mu Y_t \frac{\partial}{\partial y} + \frac{1}{2} \sigma^2 Y_t^2 \frac{\partial^2}{\partial y^2}$$

Our optimal control problem may be handled by applying the fairly new approach of formulating the quasi-variational inequalities

$$\mathcal{A}\Phi + F \leq 0 \quad (10)$$

$$\Phi(\bar{x}) \geq \mathcal{M}\Phi(\bar{x}) \quad (11)$$

$$\{\mathcal{A}\Phi + F(\bar{x})\} \{\Phi(\bar{x}) - \mathcal{M}\Phi(\bar{x})\} = 0 \quad (12)$$

where \mathcal{M} is the shift operator, defined by

$$\mathcal{M}H(\bar{x}) = \text{Sup}_{\xi} \{H(\bar{x}, \xi) - \bar{K}(\bar{x}, \xi)\} \quad (13)$$

In our case we have

$$H(\bar{x}, \xi) = \Phi(s, k, y, a + \xi)$$

and

$$\bar{K}(\bar{x}, \xi) = e^{-\rho t} (m\chi_{\xi>0} + n\chi_{\xi<0} + q_i \xi \chi_{\xi>0} + q_r \xi \chi_{\xi<0})$$

where χ_A is the indicator function of the event A .

We try a solution of the form $\Phi(\bar{x}) = e^{-\rho t} \Psi(\bar{x})$ for the value function, where $\Psi(\bar{x})$ is a time homogenous function. Consequently, we may write the relations (10) and (11) as

$$-\rho\Psi + (a_i - \delta)k_i \frac{\partial\Psi}{\partial k} + \mu y_i \frac{\partial\Psi}{\partial y} + \frac{1}{2} \sigma^2 y_i^2 \frac{\partial^2\Psi}{\partial y^2} + ck_i^{1-\phi} y_i^\phi - p_i a_i k_i + p_s \delta k_i \geq 0 \quad (14)$$

$$\psi(\bar{x}) \leq \sup_{\xi} \left\{ \Psi(k, y, a + \xi) - (m\chi_{\xi>0} + n\chi_{\xi<0} + q_i \xi \chi_{\xi>0} + q_r \xi \chi_{\xi<0}) \right\} \quad (15)$$

Between each point of time of adjustment, relation (10) must hold with equality. We try the form $\Psi(\bar{x}) = kv(g)$ of the value function where $g(y, k) = \frac{y}{k}$. Then it follows that relation (14) may be written

$$(a_i - \delta - \rho)kv(g) + (a_i - \delta)k_i^2 \frac{\partial v}{\partial g} \frac{\partial g}{\partial k} + \mu y_i k_i \frac{\partial v}{\partial g} \frac{\partial g}{\partial y} + \frac{1}{2} \sigma^2 y_i^2 k_i \frac{\partial^2 v}{\partial g^2} \left(\frac{\partial g}{\partial y} \right)^2 + ck_i g_i^\phi - (p_i a_i - p_s \delta)k_i = 0$$

Since $\frac{\partial g}{\partial y} = \frac{1}{k}$ and $\frac{\partial g}{\partial k} = -\frac{y}{k^2}$, (14) reduces to an ordinary differential equation, that is

$$(a_i - \delta - \rho)v(g) + (\mu - a_i + \delta)g_i \frac{dv}{dg} + \frac{1}{2} \sigma^2 g_i^2 \frac{d^2 v}{dg^2} + cg_i^\phi - (p_i a_i - p_s \delta) = 0$$

To solve for the homogenous part we try the form $v(g) = g^\gamma$. This gives us the following relation

$$(a_i - \delta - \rho)g^{\gamma_i} + (\mu - a_i + \delta)\gamma_i g g^{\gamma_i-1} + \frac{1}{2} \gamma_i (\gamma_i - 1) \sigma^2 g^2 g^{\gamma_i-2} = 0$$

Dividing by g^γ and solving for γ we get two values

$$\gamma_i = \frac{a_i - \delta + \frac{1}{2} \sigma^2 - \mu \pm \sqrt{(a_i - \delta + \frac{1}{2} \sigma^2 - \mu)^2 + 2\sigma^2(\rho - a_i + \delta)}}{\sigma^2}$$

Now it is straight forward find a particular solution of the inhomogenous equation (See appendix I. c.). A general solution to the ordinary differential equation is then constructed from this particular solution and the solution above, as follows

$$v(g) = \frac{2cg^{\phi}}{\sigma^2} \left(\frac{1}{(\phi - \gamma_1)(\gamma_2 - \phi)} \right) + \frac{p_1 a - p_2 \delta}{a - \delta - \rho} + C_1 g^{\gamma_1} + C_2 g^{\gamma_2}. \quad (16)$$

Thus, the value function can be written

$$\Phi(\bar{x}) = e^{-\rho t} k_t v(g_t). \quad (17)$$

The optimal construction capacity level, a^* , will then be a function, though in most cases not continuous, of g . That is $a^* = a^*(g)$.

The optimal control problem, a special case

We make the following simplifications: We assume that the costs of changing construction capacity are given by

$$\bar{K}(\bar{X}_{\theta_j}, \xi_j) = e^{-\rho t} \begin{cases} q_i \xi_j k_t & ; \xi_j > 0 \\ q_r \xi_j k_t & ; \xi_j < 0 \\ 0 & ; \xi_j = 0 \end{cases}$$

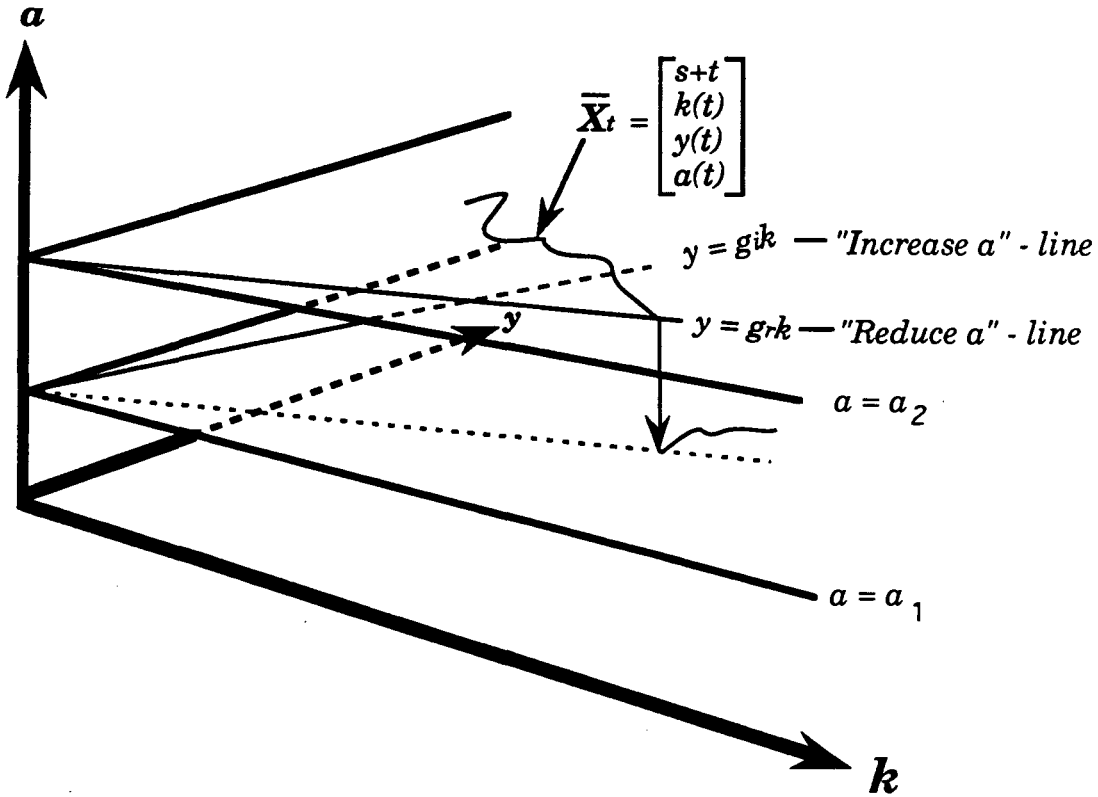
that is, m and n are set to zero.

Let $\gamma_{i1} > 0$ and $\gamma_{i2} < 0$. A sufficient condition for this to be true is that $\rho \geq a_i - \delta$. This assumption makes it easier to derive the optimal controls.

Further, we assume that the construction capacity can only take two values, a_1 and a_2 , where $a_1 < a_2$. Then we have that also ξ can take only two values, if $a = a_1$ then $\xi = a_2 - a_1$ and if $a = a_2$ then $\xi = a_1 - a_2$.

We presuppose that there is a fixed trigger level, g_i , that will initiate increase in the construction capacity. Equivalently, there is a fixed level, g_r that triggers reduction in the capacity. These trigger levels form linear relations between the demand scalar y and the capital stock k , so whenever $a = a_1$ and $y = g_i k$ the construction capacity is increased to a_2 and whenever $a = a_2$ and $y = g_r k$ the construction capacity is reduced to a_1 .

Figure I, Optimal control in the special case



Our problem is now reduced to solving

$$\mathcal{A}\Phi + F = 0 \quad \begin{array}{l} \text{if } a = a_1 \text{ and } y < g_i k \\ \text{or } a = a_2 \text{ and } y > g_r k \end{array} \quad (18)$$

$$\Phi(\bar{x}) = \mathcal{M}\Phi(\bar{x}) \quad \begin{array}{l} \text{if } a = a_1 \text{ and } y > g_i k \\ \text{or } a = a_2 \text{ and } y < g_r k \end{array} \quad (19)$$

and as above we have that

$$\mathcal{A}\Phi + F \leq 0 \quad \text{everywhere and} \quad (20)$$

$$\Phi(\bar{x}) \geq \mathcal{M}\Phi(\bar{x}) \quad \text{everywhere.} \quad (21)$$

The shift operator is now simply

$$\mathcal{M}\Phi(\bar{x}) = \begin{cases} \mathcal{M}\Phi(t, k, y, a_1) = \Phi(t, k, y, a_2) - e^{-\rho t} q_i (a_2 - a_1) k \\ \mathcal{M}\Phi(t, k, y, a_2) = \Phi(t, k, y, a_1) - e^{-\rho t} q_r (a_2 - a_1) k \end{cases} \quad (22)$$

Assume that $a = a_1$ and $y < g_i k$. That is, construction of new vessels is at the lowest level. Then we have that

$$v(g) = \frac{2cg^{\rho}}{\sigma^2} \left(\frac{1}{(\phi - \gamma_1^1)(\gamma_2^1 - \phi)} \right) + \frac{p_i a_1 - p_s \delta}{a_1 - \delta - \rho} + C_1 g^{\gamma_1^1} + C_2 g^{\gamma_2^1} \quad (23)$$

Where $\gamma_j(a_1) = \gamma_j^1$ for $j = 1, 2$.

If there is a total collapse in demand, the value function will only be the present value of vessels sold for demolition less the present value of the cost of deliveries. That is

$$\lim_{g \rightarrow 0} v(g) = \frac{p_i a_1 - p_s \delta}{a_1 - \delta - \rho}$$

Thus, we must have that $C_2 = 0$.

and

$$v(g) = \frac{2cg^{\rho}}{\sigma^2} \left(\frac{1}{(\phi - \gamma_1^1)(\gamma_2^1 - \phi)} \right) + \frac{p_i a_1 - p_s \delta}{a_1 - \delta - \rho} + C_1 g^{\gamma_1^1} \quad (24)$$

whenever $a = a_1$ and $y < g_i k$.

Next, assume that $a = a_2$ and $y > g_i k$. Further, let g be infinitely large. The probability of exercising the option to reduce the capacity is then zero. The value function will then be the present value of future shipping services less the cost of future deliveries and plus the income from scrapped vessels.

$$\Phi(\bar{x}) = E^{\bar{x}} \left[\int_0^{\bar{y}} F(\bar{X}_t) dt \right] = E^{\bar{x}} \left[\int_0^{\bar{y}} (e^{-\rho t} c k_t^{1-\rho} Y_t^{\rho} - e^{-\rho t} p_i a_2 k_t + e^{-\rho t} p_s \delta k_t) dt \right]$$

Assume that $a = a_2$ until $y = \bar{y}$. Thereafter all activity ceases. Let the first time $y = \bar{y}$ be τ . Then we have the value function

$$\Phi(\bar{x}) = E^{\bar{x}} \left[\int_0^{\tau} F(\bar{X}_t) dt \right]$$

with the solution

$$\Phi(\bar{x}) = e^{-\rho t} k v(g)$$

and the following border conditions:

Take a $g = \hat{g}$ such that $\hat{g} \in \{g; y \rightarrow 0\}$. We have that

$$\lim_{g \rightarrow 0} v(g) = \frac{p_i a_2 - p_s \delta}{a_2 - \delta - \rho} + C_2 g^{\gamma_1^2}$$

where $\gamma_j(a_2) = \gamma_j^2$ for $j = 1, 2$.

But for $g = \hat{g}$ the terminal date is very far into the future. The value function will therefore simply be the present value of future delivery costs and income from demolition. That is $C_2 = 0$ and

$$\lim_{\substack{g \rightarrow 0 \\ \tau \rightarrow \infty}} v(g) = \frac{p_i a_2 - p_s \delta}{a_2 - \delta - \rho}$$

Then take $g = \hat{g}$ such that $\hat{g} \in \{g; y \rightarrow \bar{y}\}$. Then $\tau \rightarrow 0$, almost surely and the value function will be zero. Then we must have that

$$C_1 = -\frac{2cg^{\phi - \gamma_1^2}}{\sigma^2} \left(\frac{1}{(\phi - \gamma_1^2)(\gamma_2^2 - \phi)} \right) + \frac{p_i a_2 - p_s \delta}{a_2 - \delta - \rho} g^{-\gamma_1^2}$$

If we let $\bar{y} \rightarrow \infty$ then also $\hat{g} \rightarrow \infty$. We assume that $\gamma_1^2 > \phi$ and thus we have that $C_1 \rightarrow 0$. Therefore, the present value of the market surplus with production of new capital at the maximum level will be

$$\Phi(g) = \left(\frac{2cg^{\phi}}{\sigma^2} \left(\frac{1}{(\phi - \gamma_1^2)(\gamma_1^2 - \phi)} \right) + \frac{p_i a_2 - p_s \delta}{a_2 - \delta - \rho} \right) e^{-\rho t} k$$

We return to the original problem with $a = a_2$ and $y > g, k$. Now it follows from above that

$$\lim_{g \rightarrow -} v(g) = \frac{2cg^*}{\sigma^2} \left(\frac{1}{(\phi - \gamma_1^2)(\gamma_2^2 - \phi)} \right) + \frac{p_1 a_2 - p_s \delta}{a_2 - \delta - \rho}$$

and therefore we must have that $C_1 = 0$ and

$$v(g) = \frac{2cg^*}{\sigma^2} \left(\frac{1}{(\phi - \gamma_1^2)(\gamma_2^2 - \phi)} \right) + \frac{p_1 a_2 - p_s \delta}{a_2 - \delta - \rho} + C_2 g^{\gamma_2^2} \quad (25)$$

whenever $a = a_2$ and $y > g_r k$.

Let $v(g) = v_1(g)$ whenever $a = a_1$ and $y < g_i k$ and let $v(g) = v_2(g)$ whenever $a = a_2$ and $y > g_r k$

By a "value matching" argument we have that the value function at the trigger level before the control must be equal to the value function after the control less the cost of the control applied, in order for the control to be optimal. At the trigger level for an increase in capacity we have that

$$v_1(g_i) = v_2(g_i) - q_i(a_2 - a_1) \quad (26)$$

and at the trigger level for a reduction we have

$$v_1(g_r) = v_2(g_r) + q_r(a_2 - a_1). \quad (27)$$

By (26) and (27) then relation (19), which includes the shift operator, is satisfied.

By the "high contact" principle we have, in the case of an increase

$$\frac{dv_1(g_i)}{dg} = \frac{dv_2(g_i)}{dg} \quad (28)$$

and in the case of a reduction

$$\frac{dv_1(g_r)}{dg} = \frac{dv_2(g_r)}{dg} \quad (29)$$

where

$$\frac{dv_1(g)}{dg} = \frac{2c\phi g^{\theta-1}}{\sigma^2} \left(\frac{1}{(\phi - \gamma_1^1)(\gamma_2^1 - \phi)} \right) + C_1 \gamma_1^1 g^{\gamma_1^1 - 1}$$

and

$$\frac{dv_2(g)}{dg} = \frac{2c\phi g^{\theta-1}}{\sigma^2} \left(\frac{1}{(\phi - \gamma_1^2)(\gamma_2^2 - \phi)} \right) + C_2 \gamma_2^2 g^{\gamma_2^2 - 1}$$

Then we have four equations, (26) to (29), for deciding the values of g , g_r , C_1 and C_2 . Exact values of these four variables are now readily derived by simulation for given parameter values.

From the quasi variational inequality formulation the optimal solution must, in addition, satisfy the following conditions: Whenever $a = a_1$ and $y = g, k$ or $a = a_2$ and $y = g, k$ then $\mathcal{M}\Phi + F > 0$. Further, whenever $a = a_1$ and $y < g, k$ or $a = a_2$ and $y > g, k$ then $\Phi(\bar{x}) > \mathcal{M}\Phi(\bar{x})$.

The structure of the freight rate

From relation (6) we have that the freight rate, X_t , is given by the demand scalar, Y_t , over the fleet size, k_t , defined by $G_t(Y_t, k_t) = Y_t/k_t$. The dynamics of the fleet size is subject to control given by the optimal construction strategy of the agents in the shipping markets. Thus, the path followed by the freight rate is changed due to changes in construction capacity.

From Ito's lemma we have that the change in the demand scalar over the capital stock, dG , is given by

$$dG = \frac{\partial g}{\partial k} dK + \frac{\partial g}{\partial y} dY + \frac{1}{2} \frac{\partial^2 g}{\partial y^2} dY^2 = (\mu - a_t + \delta)Gdt + \sigma GdZ_t \quad (30)$$

Further, let

$$\varsigma = \frac{e^{(v-1)}}{(e+v-1)}$$

and

$$\zeta = \left(\frac{w}{\gamma} \right)^{\frac{1}{e+v-1}}$$

Then the freight rate relation (6) can be written

$$X_t = \zeta(G_t)^\zeta$$

The dynamic of the freight rate is then given by Ito's lemma as

$$dX_t = \frac{dx}{dg} dG_t + \frac{1}{2} \frac{d^2x}{dg^2} dG_t^2 \quad (31)$$

By simple manipulation we have that

$$dX_t = \zeta \left(\mu + \delta + \frac{1}{2} \sigma^2 (\zeta - 1) - a_t \right) \zeta G_t^\zeta dt + \zeta \sigma \zeta G_t^\zeta dZ_t$$

But by substituting X_t for $\zeta(G_t)^\zeta$ we have the structure of the freight rate given by

$$dX_t = \zeta \left(\mu + \delta + \frac{1}{2} \sigma^2 (\zeta - 1) - a_t \right) X_t dt + \zeta \sigma X_t dZ_t$$

The optimal construction level, a^* , depends on G_t . But since the freight rate, X_t , is uniquely determined by G_t , then we may write

$$a^* = a^*(G_t) = a^*(X_t)$$

Then we have the structure of the freight rate, given optimally chosen construction capacity given by

$$dX_t = \zeta \left(\mu + \delta + \frac{1}{2} \sigma^2 (\zeta - 1) - a^*(X_t) \right) X_t dt + \zeta \sigma X_t dZ_t \quad (32)$$

From the diffusion term we observe that the volatility is proportional to the freight rate. Further, the drift term depends on the level of the freight rate. If $a^*(X_t)$ is above $\mu + \delta + \frac{1}{2} \sigma^2 (\zeta - 1)$ then the drift is negative, and vice versa. The exact form of $a^*(X_t)$ depends on the optimal control. However, it is reasonable to believe that the marginal change of $a^*(X_t)$ from a change in the freight rate is non-negative, since the probability of very high freight rates in the future increases as the freight rate rises.

In our special case where $a^*(X_t)$ can take two levels, a_1 and a_2 , the structure of the freight rate will at any time follow one of two geometric Brownian motions. We have that

$$dX_t = \zeta \left(\mu + \delta + \frac{1}{2} \sigma^2 (\zeta - 1) - a_1 \right) X_t dt + \zeta \sigma X_t dZ_t, \quad (33)$$

if $a = a_1$ and $X_t < X_i = X(G_i)$

$$dX_t = \zeta \left(\mu + \delta + \frac{1}{2} \sigma^2 (\zeta - 1) - a_2 \right) X_t dt + \zeta \sigma X_t dZ_t, \quad (34)$$

if $a = a_2$ and $X_t > X_r = X(G_r)$.

In the general case the freight rate will also follow a geometric Brownian motion between each time of control. However, because of the controls the overall structure will be of a mean reverting nature if $a_i < \mu + \delta + \frac{1}{2} \sigma^2 (\zeta - 1) < a_j$ for some levels a_i and a_j . In the case of no cost of changing the construction capacity, there will be a bang-bang solution to the impulse control problem, i.e. the trigger levels for increase and reduction in construction capacity will be equal.

Previously we have suggested that the freight rate follows a geometric mean reversion process (Tvedt 1995 b). The change in the freight rate is then assumed to be of the form

$$dX_t = \kappa (\alpha - \ln X_t) X_t dt + \hat{\sigma} X_t dZ_t. \quad (35)$$

If we let $\kappa = \zeta$, $\alpha = \mu + \delta + \frac{1}{2} \sigma^2 (\zeta - 1)$, $\ln(X_t) = a^*(X_t)$ and $\hat{\sigma} = \zeta \sigma$ then we have that the processes are identical. However, we will hardly find that $a^*(X_t) = \ln(X_t)$ for any versions of the optimal control problem. Yet, for both $a^*(X_t)$ and $\ln(X_t)$ the marginal change from a change in the freight rate level is non-negative. Thus, for lack of ability to solve the general case of the impulse control problem, relation (35) seems to be a reasonable approximation to the structure of the freight rate in our model.

Summary and conclusions

Our aim has been to investigate the structure of the freight rate by bridging the gap between representing the freight rate by a stochastic process and the classical partial equilibrium arguments. Rigidities, represented by the

cost of changing the ship yard output, make the freight rate follow a mean reversion process. Without any costs of change the capacity will instantaneously be adjusted to meet any change in demand.

We have approached the problem using a continuous stochastic setting. Demand is given by a constant freight rate elasticity function and supply depends on a Cobb-Douglas production function. New vessels are added to the fleet by deliveries from the yards. The total incremental yard output is given by a fraction of the existing fleet, but this fraction is subject to control. This optimal control problem is solved using the quasi-variational inequalities approach.

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This article has benefited greatly from comments and suggestions from Knut K. Aase, Victor D. Norman and Bernt Øksendal.

Appendix

a. Deriving equilibrium supply and freight rate in the short run

Price equal short term marginal cost gives

$$\left(\frac{Q_t}{Y_t}\right)^{-\epsilon} = wv\left(\frac{Q_t}{k_t}\right)^{v-1}$$

Inserting (3) for Q_t gives

$$\frac{w}{\gamma} b_t^\gamma k_t^{\gamma-1} - \left(\frac{b_t^\gamma k_t^{1-\gamma}}{Y_t}\right)^{-\epsilon} = 0$$

Rearranging this expression we get

$$k_t \left(\frac{Y_t}{k_t}\right)^{\frac{\epsilon}{\epsilon+v-1}} \left(\frac{w}{\gamma}\right)^{-\frac{1}{\epsilon+v-1}} = b_t^\gamma k_t^{1-\gamma}$$

Thus, we have (5), the tonne-mile supply in the short run

$$k_t \left(\frac{Y_t}{k_t}\right)^{\frac{\epsilon}{\epsilon+v-1}} \left(\frac{w}{\gamma}\right)^{-\frac{1}{\epsilon+v-1}} = Q_t$$

If we substitute (1) for Q_t we have

$$k_t \left(\frac{Y_t}{k_t}\right)^{\frac{\epsilon}{\epsilon+v-1}} \left(\frac{w}{\gamma}\right)^{-\frac{1}{\epsilon+v-1}} = Y_t X_t^{-\epsilon}$$

We then have (6), the freight rate in the short run, given by

$$\left(\frac{Y_t}{k_t}\right)^{\frac{\epsilon(v-1)}{\epsilon+v-1}} \left(\frac{w}{\gamma}\right)^{\frac{\epsilon}{\epsilon+v-1}} = X_t$$

b. Deriving the incremental total market surplus

Total market surplus is given by

$$S_t^* = \int_0^{Q_t^*} \left(\frac{s}{Y_t} \right)^{-e} ds - VC(Q_t^*)$$

Given that $e > 1$ then it follows from (4) that

$$S_t^* = \frac{Y}{1-e} \left(\frac{Q_t^*}{Y_t} \right)^{1-e} - w Q_t^{*v} k_t^{-(v-1)}$$

Inserting (6) for Q_t we get

$$S_t^* = \frac{Y}{1-e} \left(\frac{k_t \left(\frac{Y_t}{k_t} \right)^{\frac{e}{e+v-1}} \left(\frac{w}{\gamma} \right)^{-\frac{1}{e+v-1}}}{Y_t} \right)^{1-e} - w \left(k_t \left(\frac{Y_t}{k_t} \right)^{\frac{e}{e+v-1}} \left(\frac{w}{\gamma} \right)^{-\frac{1}{e+v-1}} \right)^v k_t^{-(v-1)}$$

it follows that

$$S_t^* = \frac{1}{1-e} k^{\frac{(e-v-1-ev)}{e+v-1}} Y^{\frac{ev}{e+v-1}} \left(\frac{w}{\gamma} \right)^{\frac{(e-1)}{e+v-1}} - w k^{\frac{(e-v-1-ev)}{e+v-1}} Y^{\frac{ev}{e+v-1}} \left(\frac{w}{\gamma} \right)^{\frac{v}{e+v-1}}$$

$$\text{let } \phi = \frac{1}{\gamma + \varepsilon - \gamma \varepsilon} = \frac{ev}{(e+v-1)} \text{ and } c = \left(\frac{e}{\phi(1-e)} \right) \left(\frac{w}{\gamma} \right)^{\frac{e-1}{e+v-1}}$$

then we have the total surplus given by

$$S_t^* = c k_t^{1-\phi} Y_t^\phi \quad (7)$$

c. Deriving a solution to the inhomogenous equation

To derive a solution to the particular equation we try a solution of the form

$$\hat{C}_1(g)g^{\gamma_1} + \hat{C}_2(g)g^{\gamma_2} + C_3 = v(g)$$

where we assume that

$$\frac{d\hat{C}_1}{dg}g^{\gamma_1} + \frac{d\hat{C}_2}{dg}g^{\gamma_2} = 0$$

Then we have that

$$\frac{dv}{dg} = \gamma_1\hat{C}_1(g)g^{\gamma_1-1} + \gamma_2\hat{C}_2(g)g^{\gamma_2-1}$$

and

$$\frac{d^2v}{dg^2} = \gamma_1(\gamma_1-1)\hat{C}_1(g)g^{\gamma_1-2} + \gamma_1\frac{d\hat{C}_1}{dg}g^{\gamma_1-1} + \gamma_2(\gamma_2-1)\hat{C}_2(g)g^{\gamma_2-2} + \gamma_2\frac{d\hat{C}_2}{dg}g^{\gamma_2-1}$$

Inserted in the differential equation we have

$$\begin{aligned} p_1a_1 - p_2\delta - cg^\phi &= (-\rho + a - \delta)\{\hat{C}_1(g)g^{\gamma_1} + \hat{C}_2(g)g^{\gamma_2} + C_3\} \\ &\quad + (\mu - a + \delta)g\{\gamma_1\hat{C}_1(g)g^{\gamma_1-1} + \gamma_2\hat{C}_2(g)g^{\gamma_2-1}\} \\ &\quad + \frac{1}{2}\sigma^2g^2\left\{\gamma_1(\gamma_1-1)\hat{C}_1g^{\gamma_1-2} + \gamma_1\frac{d\hat{C}_1}{dg}g^{\gamma_1-1} + \gamma_2(\gamma_2-1)\hat{C}_2g^{\gamma_2-2} + \gamma_2\frac{d\hat{C}_2}{dg}g^{\gamma_2-1}\right\} \end{aligned}$$

The homogenous part of the equation is by definition zero so

$$p_1a - p_2\delta - cg^\phi = \frac{1}{2}\sigma^2\gamma_1\frac{d\hat{C}_1}{dg}g^{\gamma_1+1} + \frac{1}{2}\sigma^2\gamma_2\frac{d\hat{C}_2}{dg}g^{\gamma_2+1} + (a - \rho - \delta)C_3$$

Let $C_3 = \frac{p_1a - p_2\delta}{a - \rho - \delta}$. Further we have that

$$\frac{d\hat{C}_1}{dg} = -\frac{d\hat{C}_2}{dg}g^{\gamma_2-\gamma_1}$$

Inserted in the above relation it follows that

$$\frac{-2cg^{\phi}}{\sigma^2} = -\gamma_1 \frac{d\hat{C}_2}{dg} g^{\gamma_2+1} + \gamma_2 \frac{d\hat{C}_2}{dg} g^{\gamma_2+1}$$

That is

$$\frac{d\hat{C}_2}{dg} = \frac{2cg^{\phi-\gamma_2-1}}{\sigma^2(\gamma_1-\gamma_2)}$$

and

$$\hat{C}_2 = \frac{2cg^{\phi-\gamma_2}}{\sigma^2(\gamma_1-\gamma_2)(\phi-\gamma_2)} + C_2$$

It follows directly that

$$\frac{d\hat{C}_1}{dg} = \frac{-2cg^{\phi-\gamma_1-1}}{\sigma^2(\gamma_1-\gamma_2)}$$

and

$$\hat{C}_1 = \frac{-2cg^{\phi-\gamma_1}}{\sigma^2(\gamma_1-\gamma_2)(\phi-\gamma_1)} + C_1$$

Then we have the particular solution to the differential equation given by

$$v(g) = \frac{2cg^{\phi}}{\sigma^2} \left(\frac{1}{(\phi-\gamma_1)(\gamma_2-\phi)} \right) + \frac{p_i a - p_s \delta}{a - \delta - \rho} + C_1 g^{\gamma_1} + C_2 g^{\gamma_2}$$

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Ch. 6;

The Stochastic Partial Equilibrium Model for the VLCC Market - Extensions and Applications.

Abstract

In this paper we review some of the literature of empirical studies of the bulk shipping markets. We focus on estimation of demand and supply in the freight markets and relate previous specifications to the short term equilibrium of the SPE model for the VLCC market (Tvedt 1995 a). Then, we elaborate some on the assumptions of the dynamics of the model and compare these to the literature and to observations. Having specified a base case and made some modifications to the model, we study the dynamics of the freight rate. Thereafter, we do some sensitivity studies of the effect of changes in parameter values on the trigger levels and on the structure of the freight rate.

Introduction

The stochastic partial equilibrium (SPE) model for the VLCC market suggests the following structure to the increment of the freight rate X_t

$$dX_t = \kappa(\alpha - a^*(X_t))X_t dt + \hat{\sigma}X_t dZ_t. \quad (1)$$

Whence, $\kappa = \zeta$, $\alpha = \mu + \delta + \frac{1}{2}\sigma^2(\zeta - 1)$ and $\hat{\sigma} = \zeta\sigma$, all of which are constants. The process $a^*(X_t)$ gives the percentage of new vessels delivered at time t , to the total fleet. For details see Tvedt (1995 a).

The freight rate is assumed to follow a geometric Brownian motion between every point of time of any major, i.e. non-continuous, change in the production capacity of new vessels. However, each shift means that the trend of the geometric Brownian motion is altered, and this is the source of the mean reverting nature of the freight rate.

The time charter equivalent freight rate has to some extent been studied in Tvedt (1995 b), and it seems as if a special version of the specification above is a good description of the structure of the freight rate. The structure of the freight rate generated by the SPE model will critically depend on the values of the parameters. In this paper we will mainly focus on the simple version of the model with only two levels of construction capacity. But before we turn to the question of specifying the dynamics of the model we review some of the literature of supply and demand in a static setting.

Short term equilibrium

Demand

A common approach when building models for the tanker market is to assume that the demand is totally inelastic to freight rates. Classical as well as recent works like Tinbergen (1934), Koopmans (1939), Norman & Wergeland (1981), Hawdon (1978), Charemaza & Gronicki (1981), Vergottis (1988), Beenstock & Vergottis (1989 a and b) and Lensberg & Rasmussen (1992) are all assuming totally inelastic demand. See Vergottis (1988) for a review of the early literature. However, Norman & Wergeland (1981) indicate that the elasticity of oil import to the price of oil would be about 0.5. The cost of oil transport relative to the price of oil varies substantially, from a low two percent to a high fifty percent. Normally, the ratio is about ten percent. Thus, the freight rate elasticity of demand would in most cases be about 0.05. Strandenes & Wergeland (1982) focus on the fact that the elasticity of demand for tankers does not only stem from the consumer price elasticity of import of oil. As pointed out in most of the works cited above, tonne miles is the proper measure of demand for tankers. The sailing pattern of the fleet may also be influenced by the freight rate. Thus, the freight rate elasticity of the tonne mile demand depends both on the oil price elasticity of consumption and the relation between the sailing pattern and the freight and oil prices. Strandenes & Wergeland calculate the deviation of total tonne miles actually sailed to an estimated distance minimising the sailing pattern. Changes in the deviation from changes in the freight rate are used to estimate the freight rate elasticity of the demand for tankers. Contrary to the dry bulk markets, the elasticity of the tanker market is found to be very low. The reported estimate of the elasticity is 0.005, which almost justifies the assumption of totally inelastic demand.

An early model not assuming totally inelastic demand is that of Wergeland (1981). However, this is a model of the dry bulk market. The demand relation used is

$$Q_t = aT_t^c X_t^{-\varepsilon}$$

where Q_t is the demand for tonne miles, T_t is the volume of world trade in tonnes, X_t is the freight rate and a , c , ε are parameters. Compared to our specification

$$Q_t = Y_t X_t^{-\epsilon}, \quad (2)$$

where Y_t is a demand scalar given by a geometric Brownian motion, the similarities are quite apparent. However, to specify Y_t to resemble aT_t^c would probably not be the best choice for the tanker market. The demand in Norman & Wergeland depends on the total oil consumption, the import propensity of oil consumption and the average transport distance for shipment in large tankers. Along these lines, we assume that demand is determined by overall oil import and the average transport distances.

It is difficult to be conclusive about the actual source of the freight rate elasticity of tonne mile demand. However, we follow Strandenes and Wergeland and assume that the freight rate elasticity stems from changes in the trading pattern. Hence, by this hypothesis we may estimate the trend and volatility of Y_t from observations of crude oil imports measured in tonnes, without being concerned about elasticities in demand and supply. The level of Y_t is set to make the magnitude of Q_t represent the tonne mile demand per time unit. If X_t is the time charter equivalent spot rate then Q_t must represent tonne mile demand per day per vessel.

Supply

The supply of tonne miles is extensively studied in the literature. These include the works of Tinbergen, Koopmans, Norman & Wergeland, Vergottis, Evans (1988) and Beenstock & Vergottis.

Tinbergen assumes that the tonne mile demand is determined by the total tanker tonnage, k_t , the freight rate level, X_t , and the price of bunker, w_t . According to his assumptions, aggregated supply is given by totally inelastic demand and the following relation clears the market

$$X_t = \nu k_t + \vartheta Q_t + \eta w_t$$

where Greek letters are all parameters. Fleet size and freight rate are postulated to have a positive impact on supply whereas the bunker price is supposed to have a negative impact. Using annual observations from 1870 to 1913 he receives the following estimates of the parameters

$$\nu = -1.6 \quad \vartheta = 1.7 \quad \eta = 0.4$$

Rearranging the above equation, we get

$$Q_t = 0.94k_t + 0.59X_t - 0.24w_t$$

The signs are as expected. Further, we see that there is close to a one-to-one relation between tonne mile supply and the total fleet.

In line with Tinbergen, Koopmans assumes that demand is not influenced by the freight rate. He suggests the following relation between tonne mile supply and the size of the fleet, k_t , the freight rate, X_t , and an operation cost index, w_t . The index is mainly determined by the cost of bunker.

$$Q_t = k_t \left(\frac{X_t}{w_t} \right)^\beta \quad (3)$$

We see that there is a linear relation between tonne mile supply and the size of the fleet. Further, the freight rate is supposed to be positively related to the ratio of the freight rate to the operation costs. Using observations from 1920 to the early 1930's he obtains an estimate of β of 0.15.

The relation of Koopmans has been the dominant representation of supply in later studies of the tanker market. In Vergottis, the following special version of (3) in log linear form is estimated

$$\ln X_t = \nu + \vartheta(\ln Q_t - \ln k_t) + \ln w_t + \eta \ln d_t$$

In this case k_t represents the operating fleet, adjusted for lay-ups, tankers used as storage and combies in oil. Further, he adds the average distance, d_t , to the equation. The above relation is estimated as a part of a system of equations using 3SLS. Observations from 1962 to 1985 gave the following estimates of the parameters (t - values in brackets).

$$\begin{array}{lll} \nu = 0.47 & \vartheta = 3.25 & \eta = -2.28 \\ (1.06) & (17.57) & (-8.52) \end{array}$$

Rearranging to get the equation on the form of Koopmans, we have that

$$Q_t = k_t \left(0.63 \frac{X_t}{w_t} \right)^{0.31} d_t^{7.41}$$

Ignoring the differences in specifications between Koopmans and Vergottis, Vergottis' estimate of β is double that of Koopmans.

In Beenstock and Vergottis the average sailing distance, d_t , is left out. They use data from 1962 to 1986. Written in the form of Koopmans, they get the following result

$$Q_t = k_t \left(19.65 \frac{X_t}{w_t} \right)^{0.32}$$

Evidently, the value of β is not much influenced by dropping the average sailing distance from the equation.

From our representation using a Cobb-Douglas production function, supply is given by equating marginal costs to the freight rate,

$$\frac{dVC_t}{dQ_t} = w \nu Q_t^{\nu-1} k_t^{-(\nu-1)} = X_t$$

where $\nu = \frac{1}{\gamma}$ and w are constants and VC_t is total variable costs in the short run at time t . Then we have that

$$Q_t = k_t \left(\gamma \frac{X_t}{w_t} \right)^{\frac{\gamma}{1-\gamma}}$$

Observe that our representation is a restricted form of the equation of Koopmans or Beenstock & Vergottis where $\beta = \frac{\gamma}{(1-\gamma)}$. Applying the results of Beenstock & Vergottis and assuming that our Cobb-Douglas specification is correct, we should expect that the constant term γ is related to β in the following manner, $\gamma = \frac{\beta}{(1+\beta)}$. The reported β of Beenstock & Vergottis of 0.32 should imply a constant term γ of 0.24. However, the reported level of 19.69 must be viewed in relation to the cost index w relative to the freight rate index. However, we see that the estimated

constant of Vergottis of 0.63, when the average sailing distance is included, is closer to the derived γ . Since Vergottis assumes totally inelastic demand, the negative relation between freight rate and the average sailing distance must be due to more tonne mile capacity as the average distance increases, because each vessel calls on fewer ports and less time is spent on loading and discharging. However, as noted above, the trading pattern may to some degree be influenced by the freight rate. Increased freight rates favour shorter hauls and thus, demand and the average trading distance are reduced. Hence, the significance of the average trading distance may be due to both demand and supply effects.

Base case assumptions

For the short term equilibrium we use as a base case the following parameter and state variable values.

Table 1; Base case values for the short term equilibrium

	Parameters	Values
Demand	ϵ	0.005
	Y_0	4,916,000,000
Supply	γ	0.24
	k_0	124,600,000

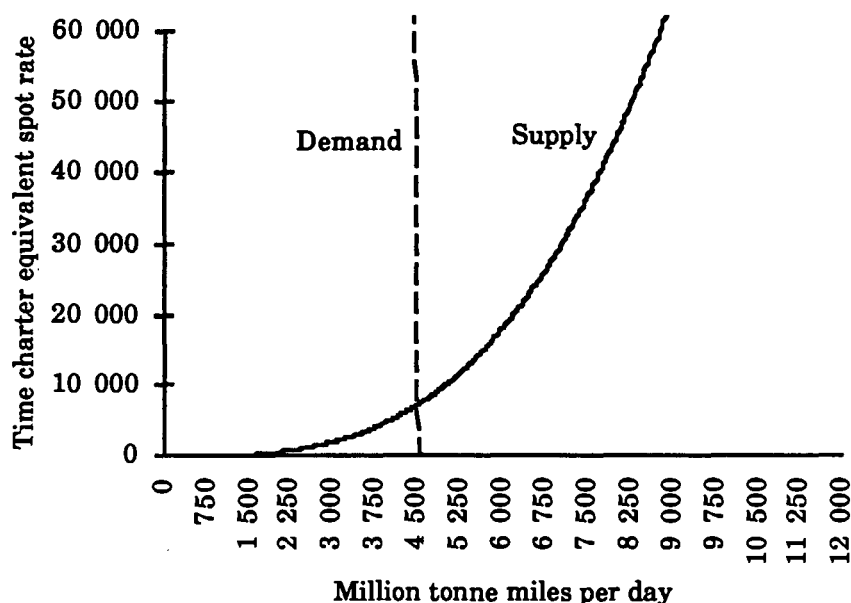
The chosen demand elasticity is that of Strandenes & Wergeland. The index for oil consumption, Y_0 , is set to equate demand with supply at the freight rate level end 1985. The average time charter equivalent spot rate in 1985 for a 280.000 dwt. turbine tanker was about USD 8,000 per day. The exponent of the Cobb-Douglas supply function is set in accordance with the findings of Vergottis and Beenstock & Vergottis. The total tonnage of crude oil tankers above 200.000 dwt was end 1985 about 124,4 mill. dwt¹. The operation cost index must be set such that it makes the total tonne mile supply for crude oil carriers above 200,000 dwt equal to the average for 1985. The total supply was 1,722 billion tonne miles per year², that is, an average of 4.7 billion tonne miles per day.

Figure 1 below shows static supply and demand, given the base case parameter and state variable values

¹ Source: Drewry Shipping Consultants

² Including tankers and combies, Source: World Bulk Trades 1985

Figure 1; Supply and demand in the VLCC market



What may appear to be the most striking fact from observing this graph is that supply is fairly elastic to freight rate even in the short run, whereas demand is very inelastic.

An extension of the SPE model

In the SPE model the freight rate elasticity of demand, ϵ , must be above one. However, all empirical studies indicate that the freight rate elasticity is positive but close to zero. In that case, the consumer plus producer surplus will be infinite in the SPE model. To circumvent this, we follow Dixit (1991) in deriving the short term equilibrium, though our interpretation of the model is slightly different.

We assume that at some freight rate level, \hat{x} , VLCCs are defeated by substitutes. This level may very well be extremely high, but at some freight rate other vessels and pipe lines can economically be substituted for large tankers. Further, at freight rate levels far above historical levels, the price of oil including the cost of transport may be so high that other sources of energy take over for oil. Thus, the need for VLCCs is restricted. Hence, it is probably realistic to let the demand for large tankers be more elastic to freight rates when the freight rate is very high. We model this by assuming that the level \hat{x} is a ceiling to the freight rate. Then, we have that the total market surplus in the VLCC market is finite for all positive ϵ .

For a given \hat{x} there will be a unique demand over capacity ratio \hat{g} . From Tvedt (1995) we have that $G_t = Y_t/k_t$. The exponent of the Cobb-Douglas production function is given by $\gamma \in (0, 1)$, the elasticity of demand parameter $e = 1/\varepsilon$, and w is the price of the short term inputs. Then it is straight forward to compute the total market surplus to be

$$S_t^* = k_t c (G_t^\phi - \phi \hat{g}^{\phi-1} G_t) \quad \text{if} \quad G_t < \hat{g}$$

and

$$S_t^* = k_t \hat{c} \hat{g}^\phi \quad \text{if} \quad G_t > \hat{g}$$

where

$$\phi = \frac{1}{\gamma + \varepsilon - \gamma \varepsilon}$$

$$c = \left(\frac{e}{\phi(1-e)} \right) \left(\frac{w}{\gamma} \right)^{\frac{e-1}{e+\nu-1}}$$

and

$$\hat{c} = (1-\gamma) \left(\frac{w}{\gamma} \right)^{\frac{e-1}{e+\nu-1}}.$$

As for the basic SPE model, the problem is to regulate the capital stock in such a manner that the present value of the total market surplus as specified above, less the cost of producing new vessels and regulating the construction capacity, is maximised.

Our new value function is then

$$\Phi(\bar{x}) = \sup_{\omega} E^{\bar{x}} \left[\int_0^{\infty} F(\bar{X}_t) dt - \sum_{j=1}^N \bar{K}(\bar{X}_{\theta_j}, \xi_j) \right] \quad (4)$$

where

$$F(\bar{X}_t) = e^{-\rho t} k_t c (G_t^\phi - \phi \hat{g}^{\phi-1} G_t) - e^{-\rho t} p_t a_t k_t + e^{-\rho t} p_t \delta k_t \quad \text{if} \quad G_t < \hat{g}$$

and

$$F(\bar{X}_t) = e^{-\rho t} k_t \hat{c} \hat{g}^\phi - e^{-\rho t} p_i a_i k_t + e^{-\rho t} p_i \delta k_t \quad \text{if } G_t > \hat{g}$$

Also this problem may readily be handled by applying the quasi variational inequalities setting. However, in this case we find the value function to be

$$\Phi(\bar{x}) = e^{-\rho t} k_t v(g_t)$$

where

$$v(g) = \frac{-\hat{c} \hat{g}^\phi}{a - \delta - \rho} + \frac{p_i a - p_i \delta}{a - \delta - \rho} + \hat{C}_1 g^{\gamma_1} + \hat{C}_2 g^{\gamma_2} \quad \text{if } G_t > \hat{g}$$

and

$$v(g) = c \left(\frac{\phi \hat{g}^{\phi-1}}{\rho - \mu} g - \frac{2}{\sigma^2 (\phi - \gamma_1) (\gamma_2 - \phi)} g^\phi \right) + \frac{p_i a - p_i \delta}{a - \delta - \rho} + C_1 g^{\gamma_1} + C_2 g^{\gamma_2} \quad \text{if } G_t < \hat{g}$$

As above, γ_1 and γ_2 depend on a .

In the special case where a can only take two values a_1 and a_2 , the value function is given by

$$\Phi(\bar{x}) = e^{-\rho t} k_t \begin{cases} \hat{v}(g_t) & \text{if } g > \hat{g} \\ v_2(g_t) & \text{if } g < \hat{g}, a = a_2 \\ v_1(g_t) & \text{if } g < \hat{g}, a = a_1 \end{cases}$$

since we assume that it will always be optimal to keep the production of new vessels at the maximum level if $g > \hat{g}$. That is, we assume that the trigger level for increasing the capacity is below \hat{g} . For very high \hat{g} this will obviously be true.

To assist us in deriving the actual value functions, we have the following border conditions:

If there is no demand g is absorbed at zero. The value function will then be equal to the present value of the price paid for future deliveries less scrapping. Thus, we have that

$$\lim_{g \rightarrow 0} v_1(g) = \frac{p_1 a_1 - p_s \delta}{a_1 - \delta - \rho}$$

and consequently $C_2^1 = 0$.

Further, if demand is very high compared to the fleet, the freight rate will be restricted by \hat{x} . Thus, the total market surplus even for infinitely high demand will be restricted. Then we have that

$$\lim_{g \rightarrow \infty} \hat{v}(g) = \frac{-\hat{c}\hat{g}^\phi}{a_2 - \delta - \rho} + \frac{p_1 a_2 - p_s \delta}{a_2 - \delta - \rho}$$

and thus we must have that $\hat{C}_1 = 0$.

Therefore, the parts of the value function dependent on g are given by

$$\hat{v}(g) = \frac{-\hat{c}\hat{g}^\phi}{a_2 - \delta - \rho} + \frac{p_1 a_2 - p_s \delta}{a_2 - \delta - \rho} + \hat{C}g^{\gamma_1^2} \quad \text{if } g > \hat{g}$$

$$v_2(g) = c \left(\frac{\phi \hat{g}^{\phi-1}}{\rho - \mu} g - \frac{2}{\sigma^2 (\phi - \gamma_1^2) (\gamma_2^2 - \phi)} g^\phi \right) + \frac{p_1 a_2 - p_s \delta}{a_2 - \delta - \rho} + C_1^2 g^{\gamma_1^2} + C_2^2 g^{\gamma_2^2}$$

if $g < \hat{g}$, and $a = a_2$, and

$$v_1(g) = c \left(\frac{\phi \hat{g}^{\phi-1}}{\rho - \mu} g - \frac{2}{\sigma^2 (\phi - \gamma_1^1) (\gamma_2^1 - \phi)} g^\phi \right) + \frac{p_1 a_1 - p_s \delta}{a_1 - \delta - \rho} + C_1^1 g^{\gamma_1^1} \quad \text{if } a = a_1$$

As for the basic model we have the following "value matching" conditions:

It will be optimal to increase the construction capacity if

$$v_1(g_i) = v_2(g_i) - q_i(a_2 - a_1) \quad (5)$$

and reduce capacity if

$$v_1(g_r) = v_2(g_r) + q_r(a_2 - a_1). \quad (6)$$

where g_i and g_r are the trigger levels for an increase and a reduction respectively. In addition, at the level that caps the freight rate we must have that

$$\hat{v}(\hat{g}) = v_2(\hat{g}) \quad (7)$$

Further, we have the "high contact" conditions

$$\frac{dv_1(g_i)}{dg} = \frac{dv_2(g_i)}{dg} \quad (8)$$

$$\frac{dv_1(g_r)}{dg} = \frac{dv_2(g_r)}{dg} \quad (9)$$

$$\frac{d\hat{v}(\hat{g})}{dg} = \frac{dv_2(\hat{g})}{dg} \quad (10)$$

for an increase and a reduction in capacities and at the price ceiling.

The six equations (5) to (10) determine $C_1^1, C_1^2, C_2^2, \hat{C}, g_i$ and g_r for given parameter values and \hat{g} .

The dynamics of the VLCC market

Demand

As indicated above, it is reasonable to let the growth and fluctuation of Y_t be represented by growth and fluctuations in the demand for import of oil. A number of statistics are available for estimating trend and volatility. We use monthly observations from October 1982 to December 1993 of crude oil transported out of the Arabian Gulf and the Red Sea in million metric tonnes³. Other equivalent data series did not prove to give very different results.

In order to test whether the above specified observations may have been generated by a geometric Brownian motion, we use OLS to estimate parameters of the following relation;

$$\frac{Y_t - Y_{t-1}}{Y_{t-1}} = \mu + \beta_1 \left(\frac{Y_{t-1} - Y_{t-2}}{Y_{t-2}} \right) + \varepsilon_t$$

According to our assumptions β_1 should be equal to zero and the error term normally distributed. The estimation gave the following parameter values

Table 3; Estimated parameters using monthly observations

Parameters	Values	t - value
μ	0.111	2.91
β_1	-0.362	-4.42

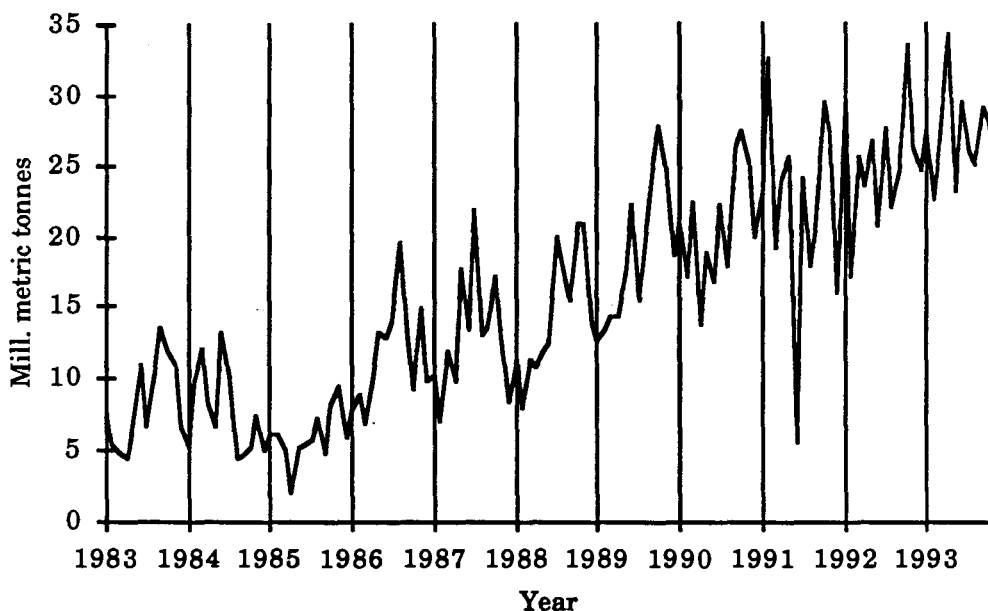
The regression gives a R^2 - adjusted of 12.4%, a Durbin-Watson statistic of 2.11 and a Durbin- h of -1.97. If the observations were generated by a geometric Brownian motion we would expect that the R^2 - adjusted should be close to zero, since any deviation of the relative change in Y_t from the constant level should be pure white noise. However, this is obviously not the case, since the parameter β_1 is significantly different from zero. Thus, we can conclude that the demand indicator Y_t could not have been generated by a geometric Brownian motion.

In the graph below we have plotted the observations. A monthly increase of 11% as the estimated μ above indicates is obviously incorrect. One obvious reason why the demand does not follow a geometric Brownian motion is

³ Source: Fearnleys: World Bulk Trades 1982 to 1993

seasonal variation. These systematic fluctuations are easily observed, especially in the late 1980's. However, using standard ad-hoc measures to remove seasonal fluctuations⁴, the demand still does not follow a geometric Brownian motion.

Figure 2; Monthly crude oil export from AG and the Red Sea, million metric tonnes



Source: Fearnley's

The assumption that Y_t follows a geometric Brownian motion is fundamental for the SPE model. We are not able to incorporate the seasonal fluctuations in the model. To carry on, we ignore the seasonal fluctuation and estimate the trend and volatility in demand by using annual observations. However, we should keep in mind that the trigger levels will change with the seasons and that capacity will be set as a trade-off between meeting the seasonal peaks and depths. We use total tonne mile shipments of crude oil per year by vessels above 50.000/60.000 dwt. from 1972 to 1992⁵, though this is slightly in conflict with our assumptions for the demand relation. We estimate the parameters for an equivalent equation as above, which gives us the following parameter values

⁴ See e.g. Pindyck & Rubinfeld (1991)

⁵ Source: Fearnley's

Table 4; Estimated parameters using annual observations

Parameters	Values	t - value
μ	0.017	0.50
β_1	0.350	1.52

The standard deviation of the residual, σ , is 15%. The regression gave an R^2 - adjusted of 6.5% and a Durbin-Watson statistic of 1.92. The Durbin-h statistic is not available due to high volatility. Apparently, neither of the coefficients are significantly different from zero at a 95% level of confidence. Thus, we are not able to reject that the tonne mile demand follows a geometric Brownian motion. Therefore, we use the above trend of 1.7% per year and annual volatility of 15% as our base case assumptions for the dynamics of demand.

The newbuilding market

Few models of the shipping market specify the construction of vessels in any detail. One exception is Lensberg & Rasmussen (1992) in which the marginal cost of constructing a vessel increases with the number of vessels ordered in the period, and decreases with the construction capacity. Construction capacity is assumed to be directly related to the size of the fleet.

In the SPE model the construction of new vessels is assumed to be proportional to the size of the fleet. That is, the construction of new vessels at time t is given by $a_t k_t$, where a_t is constant between each regulation and k_t is the size of the fleet. Occasionally, it will be optimal to regulate the size of a_t . However, such changes cannot be made without costs. Hence, the construction capacity of vessels follows costlessly the change in the total fleet. To break this relation entails costs of change.

It is reasonable to believe that the number of shifts in a_t is reduced and the size of the shifts becomes larger as the costs of change rise. If this is true, then we will expect that our reduced version of the SPE model, the two tier version, will be an appropriate approximation in the case of very high costs of change.

Therefore, our first concern is to get an idea of the costs of changing a_t . If these costs prove to be substantial, then we should expect to find a two tier structure in the construction of new vessels.

The construction capacity⁶

In a few years' time, a very large part of the present tanker fleet must be replaced. Whether there is sufficient ship yard capacity to build the necessary number of new vessels or not, is the concern of a number of recent studies. There are a number of ways of extending the shipbuilding capacity. Over the last few decades increased automatisation has considerably enlarged the annual output per dock. However, to increase the capacity of building VLCCs, shipyards often expand their smaller docks to be able to handle these huge vessels. However, in some cases entirely new yards may be constructed. Such "Green field" development takes time, normally three to five years. Today, reopening of previously closed sites is an option. However, this is not done without substantial costs. Technologically, a yard deteriorates fast. Thus, green field development is often preferred to reopening of old yards.

The major variable cost components in constructing a VLCC are labour, steel, equipment and the main engine. The relative importance of labour input is much larger for building VLCCs than for any other type of vessel in international bulk trade. According to Hellesjø and Mohn (1994) the cost of constructing a VLCC can be divided into 43% labour costs, 27% cost of steel, 19% for equipment and 11% for the main engine. These figures are for a single hull configuration. The new double hull vessels will demand a higher proportion of man hours and steel. The corresponding figures for a product tanker, a smaller and slightly more advanced vessel, is 36% labour, 38% steel, 13% equipment and 13% for the main engine.

Automatisation and improved skills have significantly decreased the number of man hours needed for constructing a VLCC. In the 1970's about one million man hours were needed. Today the most efficient and technological advanced Japanese yards use about 400.000 hours. However, major shipbuilding countries with low wage levels like China and South Korea, are less labour efficient than Japan when constructing VLCCs.

⁶ This part is mainly based on JAMRI (1985 -1994)

To increase capacity obviously entails substantial costs that will be sunk after investment. These costs also include training of the work force. This is a major concern of many shipbuilding nations. In Japan hardly anyone entered the shipbuilding industry during the 1980's and the average age of the work force is getting alarmingly high, about 42 years. In Japan, the industry is not attractive to young people. Salaries are relatively poor and the status is low. To rebuild the Japanese shipbuilding industry to historical high levels in case of a new boom will be expensive. From the beginning of the VLCC era in the early 1970's to the mid 1980's, almost 50% of the world production of vessels was constructed in Japan. In 1993 only 20% of the world's order book was on Japanese hands. Large cuts in employment, partly through large scale early retirement schemes in 1979 and in 1987, have reduced the number of employees in the Japanese shipbuilding industry from the peak level of 273,000 in 1974 to 84,600 in 1988. The capacity measured in docks has also to a large extent been reduced. The "equipment capacity" in 1988 was only 47% of that of the peak years.

As the major VLCC supplier, Japan was hard hit by the late recessions in the tanker market. However, European shipbuilding suffered at least as much. In 1987 total employment in the sector was only 95,400, a decline to 32% of the peak level of 1975. Especially Sweden and Norway implemented heavy cuts in the employment with residual ratios of only 8% and 16%, respectively. These changes were not made without reluctance and hesitation. In Sweden, direct subsidies to the yards increased from zero in 1975 to SEK 4.5 billion, about USD 650 million, in 1979. In Norway, the direct subsidies did not reach the same levels, but in 1980 government guarantees and loans reached NOK 14 billion, i.e. about USD 2 billion. Seen in relation to the total employment in 1975 of 25.000 and 22.000 in Sweden and Norway, respectively, the government support to the industry was amazingly high.

Also today the shipbuilding industry is heavily subsidised. The subsidies are given in a number of different forms, from direct transfers to guarantees and subsidised financing. Subsidies are especially prominent in Europe. European countries with present VLCC capacities are (estimated national subsidy level in brackets) Denmark (8%-13%), Germany (15%-23%), U.K. (9%-12%), and France (15%-50%).

Contrary to the European countries, Japan does not directly subsidise the shipbuilding industry. However, the enterprise structure of Japan sees to it that the yards to a large extent are kept in operation during recessions. Usually, Japanese yards are part of large corporations that have the strength to bear losses for long periods. Further, there are strong ties between the Japanese steel industry and the yards. In addition, Japanese shipowners almost always build their vessels at Japanese yards. Evidently, orders are sometimes made in order to keep up the activity level at the yards. This was especially prominent during the depression in the mid 1980's. During this period independent owners did hardly order any new vessels at all. The main part of the VLCC order book of Japanese yards was on a domestic account, sponsored by the government.

Today, the main VLCC builder besides Japan is South Korea. The major South Korean yards are very well suited for constructing VLCCs. They entered the market as a major shipbuilder in 1973 with the completion of the Hyundai yard - the world's largest. In spite of notoriously extending the capacity just in head of difficult times for the shipping industry, namely in 1973, 1979 and 1981, the Koreans have succeeded in acquiring a dominant position in the construction of VLCCs. However, the large industrial groups owning the yards have been forced to accept huge losses, and in 1988 the South Korean government had to provide a large refinancing scheme for the industry. In the early 1980's the shipbuilding industry in South Korea experienced severe labour conflicts.

During 1993 South Korea passed Japan for the first time in the number of orders received. South Korea and Japan had 38% and 32%, respectively, of the total new ordering of the world in 1993. At present, South Korea has a cost advantage over Japan of about 20% percent. Japan is, however, still attractive as a shipbuilder due to better quality and reliability. In anticipation of the near future renewal of the world fleet, South Korean yards have announced extensive plans for increased shipbuilding capacity. Competitors in Europe, and especially in Japan, express anxiety for future over capacity in the shipbuilding industry. There is some concern that after a short boom the shipbuilding industry and consequently also the rest of the shipping industry, will experience the same depressed markets as in the late 1970's.

Cost of a new VLCC

Today, Japan is the marginal producer of VLCCs and has the largest potential capacity. However, as noted, South Korea is the largest producer at the moment. Other countries have only limited capacity, the most efficient being China, Brazil, Taiwan, Denmark and the U.K. Estimations of the supply curve for the shipbuilding industry indicate that the curve is rather flat until the total of the Japanese technical building potential is exploited (See Hellesjø, Mohn & Wergeland).

The majority of the VLCCs are single hull vessels. However, new environmental standards demand a double hull configuration. Since these vessels are more labour and steel intensive, the price is consequently higher. The price of a new double hull 280.000 dwt. crude carrier is today about USD 100 million, whereas an equally large single hull vessel would probably cost 20% less.

According to Hellesjø, Mohn & Wergeland, it is probably practically possible to produce from 44 to 68 vessels a year with today's yard facilities. The high case implies that all docks that are large enough are solely employed with VLCC construction. In the low case, yards that are able to construct VLCCs have an optimal portfolio of different vessels on their order books.

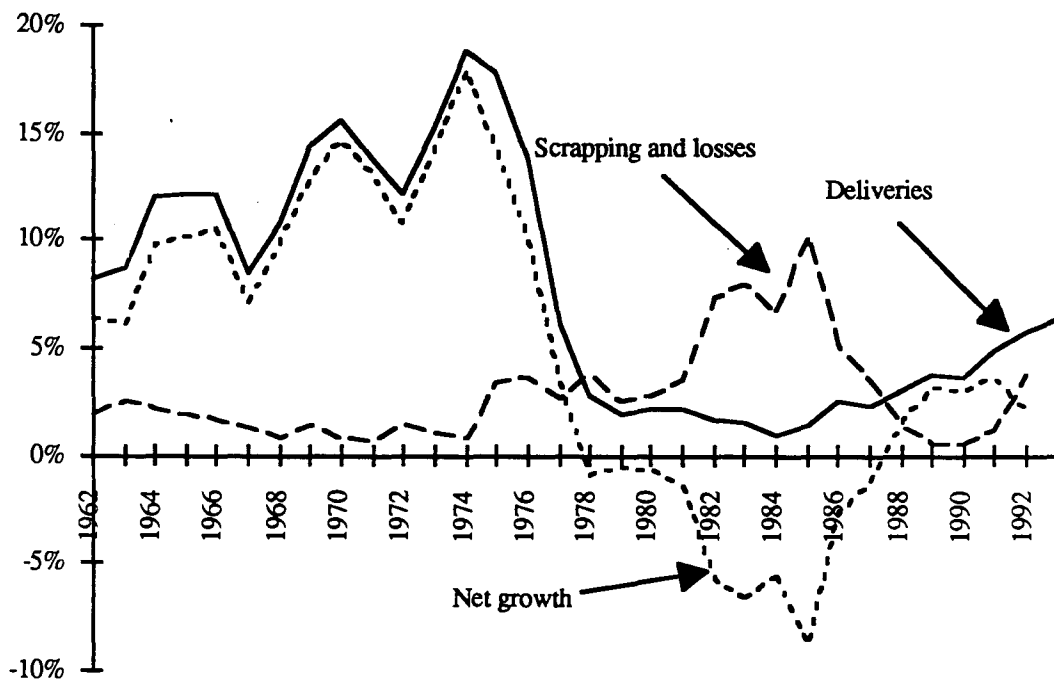
Recent developments in orders⁷

Our model postulates that there are rigidities in the adjustment of the construction of new vessels. Even in periods of extensive economical scrapping, new vessels are ordered (See figure 3 below). Our model does not differentiate between different kinds of investors in the VLCC market. The model maximises the total welfare of the market, including the shipbuilding sector. Thus, the investment behaviour does not represent the behaviour of an independent shipowner. However, a large part of the fleet is owned by international oil companies like Shell, Exxon and BP. Other major owners are the shipping companies of large oil producing nations, like Vela of Saudi Arabia. Consequently, many of those who demand shipping services are also major shipowners. These owners are obviously interested in maximising their consumption surpluses as well as the producer surpluses from their VLCC ownership.

⁷ Arne Osmundsvaag, NHH, made us aware of this development.

Few countries, if any, will suffer more from high freight rates than Japan. Their are also the number one shipbuilding nation of the world. To maintain shipbuilding at a high level secure moderate freight rates as well as jobs.

Figure 3; Annual deliveries, scrapping and net growth of tankers as a percentage of total tanker fleet



Data source: Fearnley's

To recapitulate, some of the investors in the shipbuilding industry are concerned only with the producer surplus, some take consumer and producer surplus into account, whereas others also consider the effect on the shipbuilding industry when ordering new vessels. The figures below indicate that different types of shipowners invest at different points of time in the shipping cycle.

Table 5; Historical orders by type of shipowner

Year of order	Number of contracts	Ordered by				
		Japanese interests	Korean interests	Oil companies	Oil producers	Independent
1984	6	66.7%	0%	33.3%	0%	0%
1985	10	100%	0%	0%	0%	0%
1986	19	47.4%	15.8%	10.5%	0%	23.3%
1987	17	17.6%	0%	0%	0%	82.4%
1988	8	87.5%	0%	0%	0%	12.5%
1989	24	41.7%	0%	4.2%	0%	54.2%
1990	52	30.8%	0%	3.8%	15.4%	50%
1991	30	33.3%	0%	3.3%	20%	43.3%
1992	17	23.5%	0%	5.9%	58.8%	11.8%
1993	9	33.3%	33.3%	11.1%	0%	22.2%

Source: Arne Osmundsvaag

Apparently, in times of large orders the major part is made by independent shipowners, and when orders are few the majority are made by Japanese and Korean interests. It goes without saying that Japanese interests order their vessels at Japanese yards, and correspondingly, Korean interests order at Korean yards. It also seems as if the oil companies are providing a "guarantee" for over capacity by orders late in the building boom.

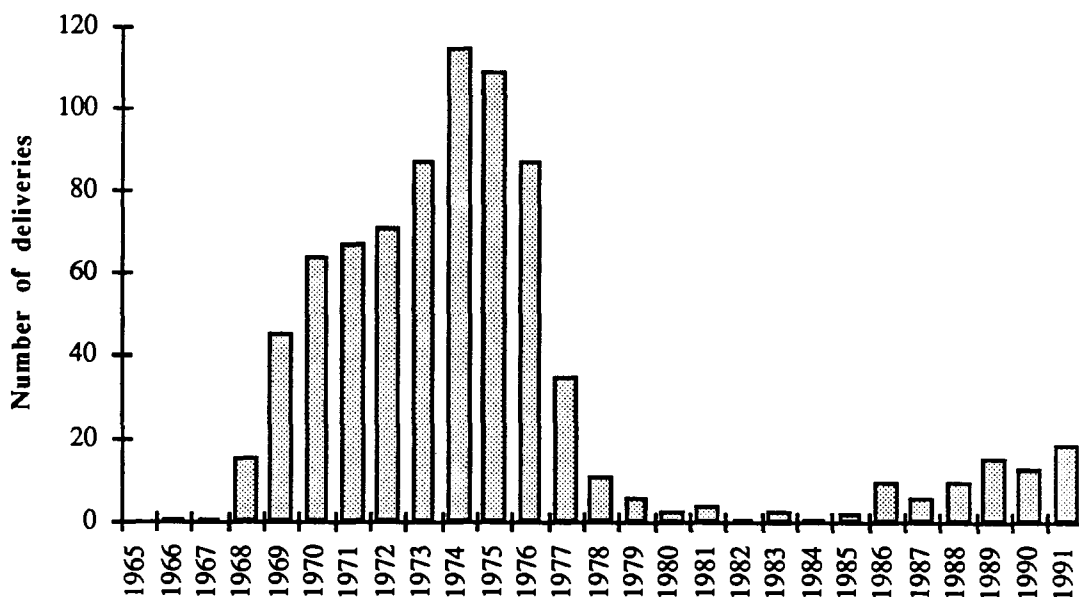
Summing up the cost of change

The cost to society of large changes in the production capacity of the shipbuilding industry seems to be substantial. To give accurate estimates of the actual level of these costs is beyond our ambitions. The level of the costs of change that we indicate below as base case assumptions, is not very well founded and is only meant as a basis for illustrating our main points.

The to tier version of the SPE model - simulations

From figure 3 it seems as if production of new tankers measured as deliveries in percent of the total fleet, has been either at a high or at a low level. In the early 1970's the percentage was nearly 20. After the shipping crisis and until the beginning of the 1990's production never reached 5%. This is the general picture for tankers. However, the construction of VLCCs has followed an even more erratic pattern. This is to a major extent due to the short history of this market. The first VLCCs were constructed in the late 1960's, and even today a major part of the fleet consists of tankers built during the first decade of the history of this market. An equivalent figure to figure 3, but for the VLCC market only, is therefor rather meaningless. However, the figure below showing the number of VLCCs delivered from the first vessel in 1966 until 1991, indicates the large jumps in production level during this period.

Figure 4; Number of deliveries of VLCCs, ULCCs and OO 1966 to 1991



Source: Clarkson (1991)

Presumably, the future will not be as shifting as the early years of the market. Thus, for our base case we use production levels more in accordance with the level of the tanker market in total, or slightly above, with a high production level of about 24% and a low production level of 5% of the current fleet. Further, we assume an annual scrapping of vessels marginally below 6%. Thus, we assume approximately zero net growth in the fleet in the case of low level of deliveries.

Summary of base case assumptions for simulation

We use a time unit of one week for our simulations of the freight rate path. Below, parameter values used in the base case, all on a weekly basis, are listed.

Table 6; Base Case assumptions

Parameter	Value
μ	0.0003
σ	0.021
ρ	0.005
ε	0.005
γ	0.24
w	0.000000000000244
a_1	0.001
a_2	0.004
δ	0.0011
p	75,000,000.-
q_i	5,000,000,000.-
q_r	3,000,000,000.-
p_s	7,500,000.-

The value of μ and σ for the dynamics of demand is equivalent to the above estimate of 1,7% growth and 15% standard deviation on an annual basis. The discount factor, ρ , is equal to a rate of 30% per year, which is sufficient to secure that $\rho \geq a_i - \delta$ always. The value of ε for the demand function and the value of γ for the Cobb Douglas supply function are as referred above, in accordance with the current literature. In the base case the cost index, w , is adjusted somewhat compared to the value derived if calibrating the level solely from the 1985 observations reported above. We will revert to this below. The values of a_1 and a_2 are equivalent to construction levels of 5% and 24%, respectively, and depreciation, δ , is of about 6% annually. The price of a new vessel is set at USD 75 million. This is below the price paid for a VLCC, but the actual price probably contains some of the cost of changing the construction capacity. These costs are specified explicitly at USD 5 billion for increasing the new building capacity to the high level and USD 3 billion for decreasing the capacity to the low level. The price of a vessel sold for demolition is set at USD 7.5 million.

The cost index, w , if estimated by using the 1985 observations for total demand, dwt capacity and freight rate, is higher than the value reported above. If we use this unadjusted value we derive unrealistically high trigger levels for changing the production capacity. Probably, we are over estimating the tonne mile capacity of the 1985 reported stock of vessels. Thus, in calibrating the model we get a too high cost index level in order to clear the market at a freight rate of USD 8.000 per day. The trigger level for an increase is then as high as USD 136,000 per day and the trigger level for a reduction is as high as USD 28,000 per day. The index value used is equivalent to a freight rate in the model of USD 500 per day given the 1985 observations of supply and demand.

In the base case we have set the level of \hat{g} as high as 5 million. Consequently, the upper ceiling to the freight rate is of no effect to the trigger values. Then, for freight rate elasticity above one, the SPE model and the modified SPE model applied here are equal for most applications.

Results of the base case

In the base case it is optimal to increase the construction capacity when the freight rate reaches USD 70.000 per day from below and to reduce capacity when the freight rate hits USD 14,500 per day from above. This is equivalent to a demand over capital ratio of 1226 and 741, respectively. In 1985 the actual ratio was 263, i.e. tonne mile demand per week over total tonnage.

For the given base case assumptions the parameters of the increment of the freight rate process of the model are

$$\kappa = \zeta = 3.12$$

$$\alpha = \mu + \delta + \frac{1}{2} \sigma^2 (\zeta - 1) = 0.0018$$

and

$$\hat{\sigma} = \zeta \sigma = 0.065$$

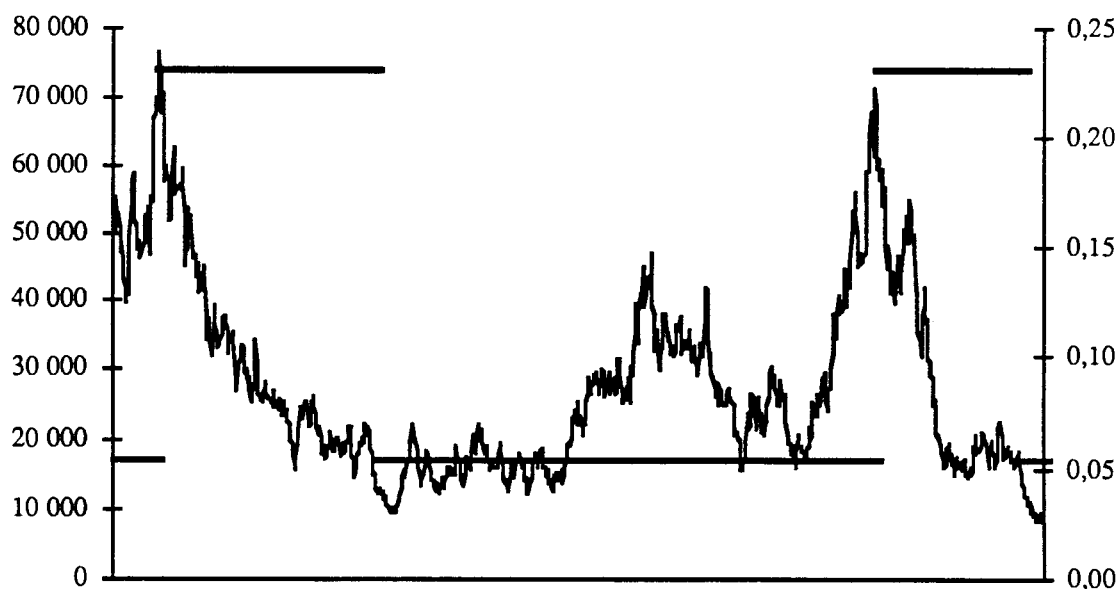
It follows that the increment of the process is given by

$$dX_i = 3.12(0.0018 - a_i)X_i dt + 0.065X_i dZ_i ; i \in (1,2). \quad (11)$$

where $a_1 = 0.001$ and $a_2 = 0.004$ and the prevailing level is a consequence of optimal control, given costs of change. Observe that if the current level of deliveries is equal to a_1 then the process has a positive trend. If the level is a_2 then the trend will be negative. Further, the downward trend is stronger than the upward trend, both due to the fact that a_2 deviate stronger from α than a_1 , and because the process itself is geometric.

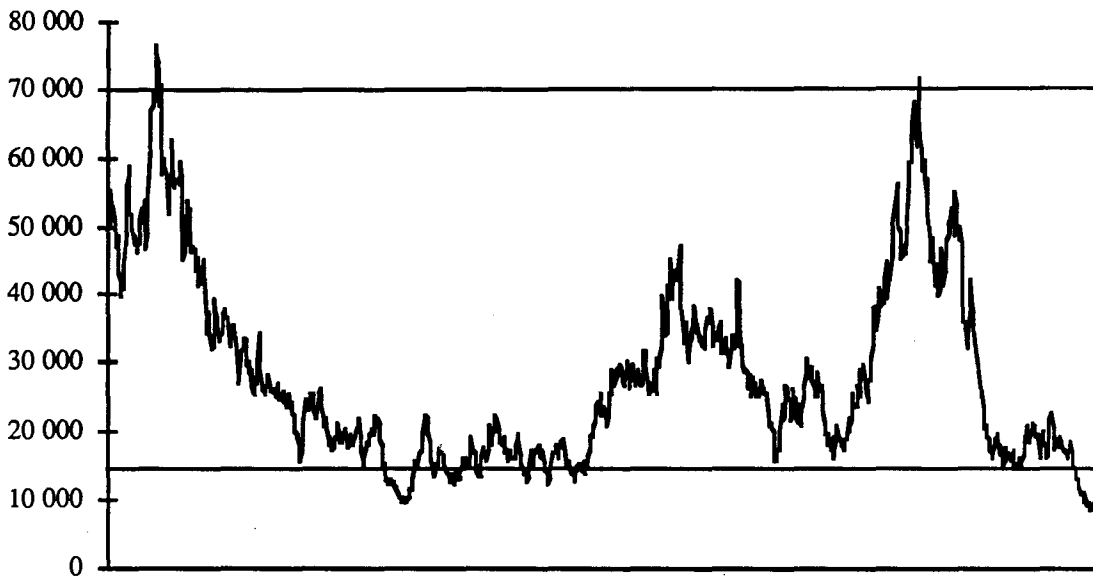
Figure 5 below show a sample trace of the freight rate simulation generated by the SDE above together with the level of deliveries of new vessels. Figure 6 shows the same trace together with the upper and lower trigger levels. The trace replicates weekly observations for a period of about 20 years.

Figure 5; Simulated TC equivalent freight rates per day and the level of deliveries from the SPE model



Observe that as soon as the freight rate reaches the trigger level for increasing deliveries, the trend of the process is reverted downwards. After a few years freight rates are back at a low level and the shipbuilding industry is contracted. After a long and slow growth phase, the shipping market once again experiences high freight rates and high deliveries for a restricted period.

Figure 6; Simulated TC equivalent freight rates per day and the trigger levels from the SPE model



Effects on trigger levels and the freight rate of changes in parameter values

The effect on trigger levels has been extensively studied in the literature for quite similar models (see e.g. Dixit 1989, 1991 and Dixit & Pindyck 1994). For completeness, we study the effect on the SPE model of a one percent increase in each of the parameter values from the base case assumption in table 6. The resulting change in the trigger freight rate levels and the parameter values of the incremental change of the freight rate are presented below.

As the trend of the relative change in the demand scalar, μ , increases, so does of course also the trend factor of the relative change in the freight rate, given the construction level of new vessels. However, the positive effect on the freight rate is dampened by the fact that the high construction level of vessels is initiated at a lower trigger freight rate. The mean reversion effect is further strengthened by a lower trigger freight rate for the low construction level. Hence, higher future demand is met by higher construction of new vessels. The reduction for both the trigger levels is 0.1%, and thus, the absolute gap between the upper and lower trigger levels is evidently reduced.

Table 7; The effect of a one percent increase in the parameter value from B.C.

Parameter	Effect on x_i	Effect on x_r	Effect on α	Effect on $\hat{\sigma}$	Effect on κ
μ	-0.1%	-0.1%	0.2%	-	-
σ	-0.2%	-0.5%	0.5%	1.0%	-
ρ	0.5%	0.6%	-	-	-
ε	-0.01%	0.01%	-0.005%	-0.02%	-0.02%
γ	-7.3%	-5.9%	-0.5%	-1.3%	-1.3%
w	0.2%	0.2%	-	-	-
a_1	0.2%	0.3%	-	-	-
a_2	0.4%	0.7%	-	-	-
δ	-0.3%	-0.4%	0.6%	-	-
p	0.5%	1.0%	-	-	-
q_i	0.2%	-0.1%	-	-	-
q_r	0.1%	-0.1%	-	-	-
p_s	-0.01%	-0.01%	-	-	-

From (1) we have that a higher standard deviation for the relative change in the demand scalar, σ , gives an increased trend factor of the relative change of the freight rate. Further, the standard deviation of the relative change in the freight rate increases proportionally with the increase in σ . As above, a higher trend factor for the relative change in the freight rate decreases the upper and lower trigger levels. However, another effect comes into force. Higher volatility increases the option value of changing the construction capacity. A larger alternative cost of exercising the option makes the gap between the increase and decrease trigger freight rate levels wider. Here, this expanding effect of the increased volatility is stronger than the contracting effect of the increased trend. However, the upper trigger level is reduced due to the trend effect and in spite of the volatility effect.

A higher discount factor solely influences the trigger levels. More emphasis is placed on present costs and less on future low supply of vessels. Thus, the trigger freight rates are increased and the freight rate will in general be at a higher level.

Less elastic demand reduces the trend of the relative change in the freight rate. However, the trigger levels are hardly influenced. Thus, the recovery

of the freight rate after a shipbuilding boom will be slower as the elasticity is reduced.

An increased γ changes the technology of the fleet by reducing the productivity of capital in favour of inputs that can be adjusted in the short run. To keep up supply, the overall level of the capital stock must be increased and thus the level of the trigger freight rates are reduced. Further, the optimal level of the freight rate is now more easily obtained by adjusting the short term inputs. Thus, both the trend and the volatility of the relative change in the freight rate are reduced.

An increased cost index reduces the advantage of a large fleet and the trigger values rise.

If the low construction level is increased, then deliveries at the low level will be sufficiently high for higher freight rates and both the trigger levels are increased. The effect of a higher upper construction level also increases both trigger levels, since this higher level makes the market go into over capacity more rapidly. Therefore, the reluctance to increase capacity and the willingness to decrease capacity are higher. For both the increased low and high construction level the trend of the relative change in the freight rate is reduced. Thus, reversion to low freight rates is faster but recovery to high rates is slower.

Higher depreciation increases the need for replacements. Therefore, the trigger levels are reduced for this small increase in scrapping. The total effect on the trend of the relative change in the freight rate is, between each control, equivalent to a reduction in the construction levels of new vessels.

An increased construction cost of new vessels makes the optimal fleet smaller, and the trigger levels are higher. The general freight rate level will be higher to defend investments in the more expensive vessels.

Higher costs of change naturally reduce the willingness to make adjustments in the construction capacity. Thereby, the gap between the high and low trigger freight rate is increased.

A high price of scrap increases the value of a vessel and optimal fleet size becomes higher. Therefore, the trigger levels are reduced.

Concluding remarks

When we use classical empirical studies to determine the parameters of the SPE model together with some simple estimations in those cases of which we are not aware of any published results, we find that even the two-tier version may give some insight into the dynamics of the freight market. However, a number of important features are missing in this first attempt. First of all, the model does not take into consideration time to build, both the vessel itself and the construction of new yards. As far as yard capacity is concerned, an extension will probably not change the main mean reversion structure of the model. The decision to extend a yard will be taken in anticipation of the freight rate reaching a given level by the time the yard is completed. In addition, at a certain cost, the completion of the yard may be speeded up, postponed or abandoned during the construction period. Compared to our model, time to build may effect the freight rate both ways. If demand proves to be higher than expected when initiating the development of new docks, then the freight rate in the booming market may go sky high. On the contrary, if demand fails to meet expectations the short freight rate peaks may disappear all together. Thus, time to build will most probably give larger deviations in the magnitude of the cyclical peaks.

In this paper we have for the two tier version of the model, fixed the size of a_1 and a_2 . A natural extension is to regulate the size optimally. However, more advanced simulation techniques must then be applied.

Between each change in the construction capacity the freight rate follows a geometric Brownian motion. If this should prove to be close to real market conditions, then pricing of assets depending on a short cash flow horizon could be made on the basis of a geometric Brownian motion assumption for the freight rate. For example, in the BIFFEX market major agents price OTC options using the Black-Scholes formula. Our model gives arguments for such a practice between each change in the production level of vessels, if the freight rate is fairly far from a trigger level. However, the goodness of the use of the Black-Scholes formula in shipping markets should be an empirical question.

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Ch. 7;

Note one;

The Stochastic Partial Equilibrium Model for the VLCC Market - Characteristics of the Shipbuilding Market

Abstract

In this extension of Tvedt (1994 a. and b.) we suggest different paths to follow in order to improve the characterisation of the shipbuilding market in the SPE model. We apply a static industry cost curve in accordance with the literature and apply simple dynamics. A multi-tier version of the SPE model is then derived. The original SPE model does not take into consideration time to build. Here we suggest one way of taking account of this without abandoning the Markov properties. We introduce a state variable specifying the order book. The activated yard capacity adds to the order book, whereas deliveries are a fixed proportion of the order book.

Introduction

In Tvedt (1994 a.) we derive from a partial equilibrium model, a stochastic process describing the freight rate of the VLCC market. The mean reverting property of the process depends on $\alpha^*(X_t)$, which is the construction of new vessels at time t as a percentage of the fleet. Between any change in $\alpha^*(X_t)$ the freight rate follows a geometric Brownian motion given by

$$dX_t = \kappa(\alpha - \alpha^*(X_t))X_t dt + \hat{\sigma}X_t dZ_t. \quad (1)$$

where, κ , α and $\hat{\sigma}$ are constants. Tvedt (1994 b.) gives estimates of these parameters and extends the basic model. Below we elaborate this extended model further by introducing a somewhat richer characterization of the shipbuilding market and derive a multi-tier version of the SPE model.

Supply in the shipbuilding market

Hellesjø, Mohn and Wergeland (1994) present a detailed study of capacities and the cost structures of the world shipbuilding industry. They develop a cost index showing the average production cost per compensated gross register tonne of the different shipbuilding nations relative to the level of Germany.

Constructing VLCCs and ULCCs demands very large docks. Therefore, only a limited share of the world's yards are capable of handling these vessels.

The table below shows the present annual VLCC building capacity of the world and its distribution on yards and nations.

Table 1, The World's VLCC producers

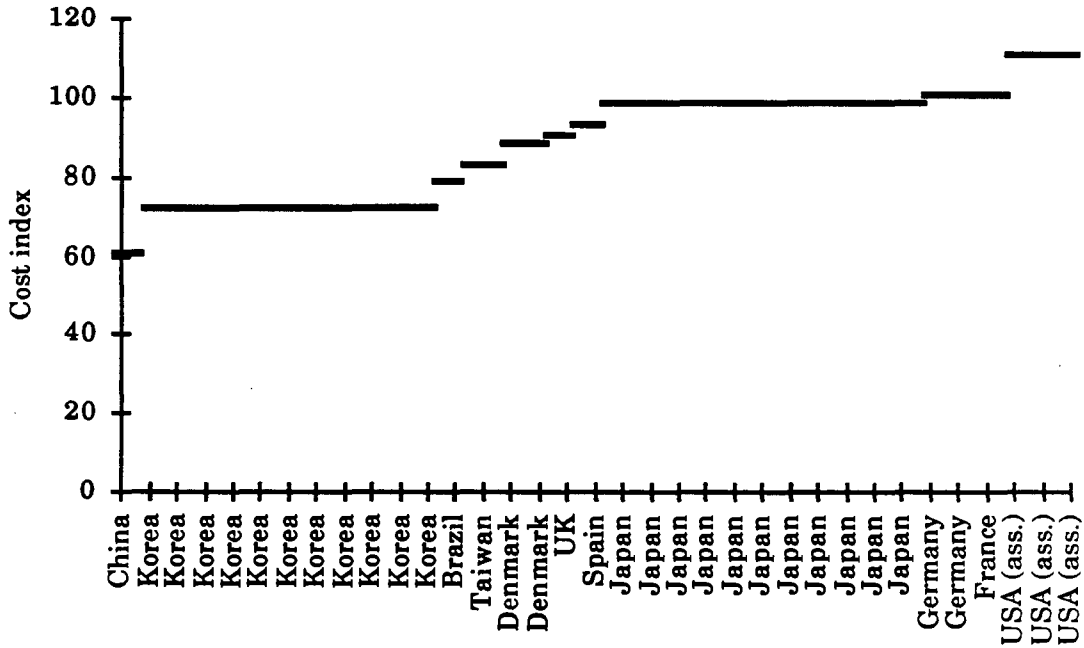
Nationality	Yard	Annual capacity
Japan	Mitsubishi	4
	IHI	3
	Hitachi	5
	Mitsui	2
	Kawasaki	2
	NKK	2
	Sumitomo	2
	Sasebo	2
Total Japan		22
Korea	Daewoo	12
	Hvundai	6
	Samsung	3
Total Korea		21
USA	Newport News	3
	Bethlehem	2
Total USA		5
Germany	HDW	2
	Bremer	1
Total Germany		3
Taiwan	CSBC	3
Denmark	Odense	3
France	Chantiers	3
China	Dalien	2
Brazil	Ishiras	2
UK	H&W	2
Spain	AESA	2
World total		68

Source: Hellesjø & Mohn (1994)

Evidently, Japan and South Korea are the dominant players in the market for VLCC construction. At present South Korea has a cost advantage over Japan. However, at today's production levels and cost structures Japan is

the marginal producer. If we relate the cost index of Hellesjø, Mohn and Wergeland to the construction capacities of table 1 we have the following relation between marginal average construction costs and capacities.

Figure 1, Capacities and cost levels of the world's shipbuilding nations



Data source: Hellesjø, Mohn and Wergeland

Given that each yard has a constant average cost and we assume price equal marginal cost in the shipbuilding market, then figure 1 can be viewed as a short term static supply function for the construction of VLCCs.

A multi-tier version of the SPE model

As shown above, it is reasonable to model the supply function of the yards as a step function. Assuming price equal marginal costs, we model the price of a new vessel as a function of the level of construction, that is,

$$p = p(a_i) \tag{2}$$

where $\frac{dp}{da} \geq 0$.

Further, we assume that the jumps in a_i are exogenously determined. Hence, we may use actual observations of the world shipbuilding facilities to estimate the steps in a_i . As in the pervious versions of the SPE model the

total production of new vessels depends on the total stock, and is given by $a_t k_t$. Therefore, as the total stock of vessels increases, the yard facilities are assumed to follow suit. It may be argued that this dynamic specification of the shipbuilding market is unrealistically simple.

As before, to let a_t jump entails costs of change. In addition to letting the cost be linear in the size of the change, we may also let the cost depend on the individual new yard facility that is triggered. That is, we let the cost of adjustment be given by

$$\bar{K}(\bar{X}_{\theta_j}, \xi_j) = \begin{cases} q_i(a_t) \xi_j k_t & ; \xi_j > 0 \\ q_r(a_t) \xi_j k_t & ; \xi_j < 0 \\ 0 & ; \xi_j = 0 \end{cases} \quad (3)$$

If we incorporate the inverse supply function for the shipbuilding market given by relation (2) and the new cost of change function (3), our value function will be given by

$$\Phi(\bar{x}) = \sup_{\omega} E^{\bar{x}} \left[\int_0^{\infty} F(\bar{X}_t) dt - \sum_{j=1}^N \bar{K}(\bar{X}_{\theta_j}, \xi_j) \right] \quad (4)$$

where

$$F(\bar{X}_t) = e^{-\rho t} k_t c (G_t^* - \phi \hat{g}^{*1} G_t) - e^{-\rho t} p_i(a_t) a_t k_t + e^{-\rho t} p_r \delta k_t \quad \text{if } G_t < \hat{g}$$

and

$$F(\bar{X}_t) = e^{-\rho t} k_t \hat{c} \hat{g}^* - e^{-\rho t} p_i(a_t) a_t k_t + e^{-\rho t} p_r \delta k_t \quad \text{if } G_t > \hat{g}$$

A solution to this stochastic control problem may be reached by the same procedures as the earlier versions of the model. Then we have the value function

$$\Phi(\bar{x}) = e^{-\rho t} k_t v(g_t)$$

where

$$v(g) = \frac{-\hat{c} \hat{g}^*}{\alpha - \delta - \rho} + \frac{p_i(\alpha) \alpha - p_r \delta}{\alpha - \delta - \rho} + \hat{C}_1 g^{\gamma_1} + \hat{C}_2 g^{\gamma_2}$$

if $G_t > \hat{g}$, and

$$v(g) = c \left(\frac{\phi \hat{g}^{\rho-1}}{\rho - \mu} g - \frac{2}{\sigma^2 (\phi - \gamma_1) (\gamma_2 - \phi)} g^\rho \right) + \frac{p_i(a)a - p_s \delta}{a - \delta - \rho} + C_1 g^{\gamma_1} + C_2 g^{\gamma_2}$$

if $G_t < \hat{g}$.

We assume that we have n possible levels of a_t . Then it follows that the value function is

$$\Phi(\bar{x}) = e^{-\rho t} k_t \begin{cases} \hat{v}(g_t) & \text{if } g > \hat{g} \\ v_n(g_t) & \text{if } g < \hat{g}, a = a_n \\ \cdot \\ v_j(g_t) & \text{if } g < \hat{g}, a = a_j \\ \cdot \\ v_2(g_t) & \text{if } g < \hat{g}, a = a_2 \\ v_1(g_t) & \text{if } g < \hat{g}, a = a_1 \end{cases}$$

Following the same line of reasoning as in Tvedt (1995b) and given that $\gamma_2^1 < 0$ and $\gamma_1^n > 0$, then we have that $\hat{C}_1 = 0$ and $C_2^1 = 0$. Further, we have the following "value matching" conditions

$$v_j(g_i^j) = v_{j+1}(g_i^j) - q_i(a_j)(a_{j+1} - a_j) \quad \forall j \in \{1, \dots, n-1\} \quad (5)$$

and

$$v_{j-1}(g_r^j) = v_j(g_r^j) + q_r(a_j)(a_j - a_{j-1}) \quad \forall j \in \{2, \dots, n\} \quad (6)$$

and the corresponding "high contact" conditions

$$\frac{dv_j(g_i^j)}{dg} = \frac{dv_{j+1}(g_i^j)}{dg} \quad \forall j \in \{1, \dots, n-1\} \quad (7)$$

and

$$\frac{dv_{j-1}(g_r^j)}{dg} = \frac{dv_j(g_r^j)}{dg} \quad \forall j \in \{2, \dots, n\} \quad (8)$$

Moreover, at the absorbing level the solution must satisfy the "value matching" condition

$$\hat{v}(\hat{g}) = v_n(\hat{g}) \quad (9)$$

and the "high contact" condition

$$\frac{d\hat{v}(\hat{g})}{dg} = \frac{dv_n(\hat{g})}{dg} \quad (10)$$

From (7), (8), (9) and (10) we have $4(n-1)+2$ equations to solve for the $4(n-1)+2$ variables $C_1 = \{C_1^1, C_1^2, \dots, C_1^n\}$, $C_2 = \{C_2^2, C_2^3, \dots, C_2^n\}$, \hat{C} , $g_i = \{g_i^1, g_i^2, \dots, g_i^{n-1}\}$ and $g_r = \{g_r^2, g_r^3, \dots, g_r^n\}$ for given parameter values and \hat{g} .

The solution entails that the drift of the freight rate process changes every time G_t hits g_i from below or g_r from above, that is, $a^*(X_t)$ increases as X_t hits x_i from below and decreases as X_t hits x_r from above, $x_i = \{x_i^1, x_i^2, \dots, x_i^{n-1}\}$ and $x_r = \{x_r^2, x_r^3, \dots, x_r^n\}$. As already noted, the number of steps in the supply function, n , and the size of the steps $\xi_i = \{\xi_i^1, \xi_i^2, \dots, \xi_i^{n-1}\}$ and $\xi_r = \{\xi_r^2, \xi_r^3, \dots, \xi_r^n\}$ are predetermined. From our specification it follows that $\xi_i = -\xi_r$.

The model is easily extendible to take account of demolition motivated by anticipation of low future freight rates. We allow for negative values of a_t and define $p_i(a | a < 0) = p_i$.

The SPE model with order book

One major weakness with the original SPE model is that it does not take into consideration the time lag between ordering and delivery of tankers. In order to take account of this "time to build" we introduce a fifth state variable, the order book at time t , ω_t . We assume that the dynamics of the order book is given by

$$d\bar{\omega}_t = a_t k_t dt - \eta \bar{\omega}_t dt \quad (11)$$

where $a_t k_t dt$ is the initiation of new shipbuilding projects at time t and $\eta \bar{\omega}_t dt$ is deliveries from the yards at time t . As before, a_t is subject to control, whereas η is a constant rate, $0 \leq \eta \leq 1$. Thus, η represents the degree of "time to build" in the industry.

The net increase in the capital stock is then given by

$$dk_t = \eta \bar{\omega}_t dt - \delta k_t dt \quad (12)$$

where, as before, $\delta k_t dt$ is the physical depreciation at time t .

The state variables of the system is now given by the vector

$$\bar{X}_t = \begin{bmatrix} s+t \\ \bar{\omega}_t \\ k_t \\ Y_t \\ a_t \end{bmatrix}$$

Between any change in a_t we have the incremental change in the state of the system given by

$$d\bar{X}_t = \begin{bmatrix} 1 \\ a_t k_t - \eta \bar{\omega}_t \\ \eta \bar{\omega}_t - \delta k_t \\ \mu Y_t \\ 0 \end{bmatrix} dt + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \sigma Y_t \\ 0 \end{bmatrix} dZ_t$$

Evidently, we keep the Markov properties of the process $\{\bar{X}_t; t \geq 0\}$ even though we have introduced an order book. It follows that the process has a generator, \mathcal{A} , given by

$$\mathcal{A} = \frac{\partial}{\partial t} + (a_t k_t - \eta \bar{\omega}_t) \frac{\partial}{\partial \bar{\omega}} + (\eta \bar{\omega}_t - \delta k_t) \frac{\partial}{\partial k} + \mu Y_t \frac{\partial}{\partial y} + \frac{1}{2} \sigma^2 Y_t^2 \frac{\partial^2}{\partial y^2}$$

Our optimal control problem is similar to those in Tvedt (1995a) and (1995b), however, the specification of the state of the system has changed.

Further, the total market surplus less the cost of adjusting the fleet is now given by

$$F(\bar{X}_t) = e^{-\rho t} c k_t^{1-\phi} Y_t^\phi - e^{-\rho t} p_i(\alpha) \eta \bar{\omega}_t + e^{-\rho t} p_s \delta k_t$$

That is, we assume payment of the vessels at delivery. Using the quasi-variational inequalities approach we have that between any change in α_t the value function must satisfy the following partial differential equation

$$\begin{aligned} -\rho \Psi + (\alpha_t k_t - \eta \bar{\omega}_t) \frac{\partial \Psi}{\partial \bar{\omega}} + (\eta \bar{\omega}_t - \delta k_t) \frac{\partial \Psi}{\partial k} + \mu y_t \frac{\partial \Psi}{\partial y} + \frac{1}{2} \sigma^2 y_t^2 \frac{\partial^2 \Psi}{\partial y^2} \\ + c k_t^{1-\phi} y_t^\phi - p_i(\alpha_t) \eta \bar{\omega}_t + p_s \delta k_t = 0 \end{aligned}$$

where $\Phi(\bar{x}) = e^{-\rho t} \Psi(\bar{x})$. Let $\Psi(\bar{x}) = kv(g, h)$ where $g = \frac{y}{k}$ and $h = \frac{\bar{\omega}}{k}$. The interpretation of g is as before and h is the order book in percent of the total fleet. Then the value function must satisfy

$$\begin{aligned} (-\rho + \eta h - \delta)v(g, h) + (\alpha - \eta h - \eta h^2 + \delta h) \frac{\partial v}{\partial h} + (\mu - \eta h + \delta)g \frac{\partial v}{\partial g} \\ + \frac{1}{2} \sigma^2 g^2 \frac{\partial^2 v}{\partial g^2} + c g_t^\phi - p_i(\alpha) \eta h - p_s \delta = 0 \end{aligned}$$

It follows that $\alpha^* = \alpha^*(g, h)$, that is, optimal new ordering will depend on the order book in percent of the fleet and the demand scalar relative to the fleet size.

We have that $X_t = \zeta(G_t)^\xi$, where ζ and ξ are constants. Thus, the freight rate is solely determined by G_t . The increment of the freight rate can then be written

$$dX_t = \kappa(\alpha - \alpha^*(X_t, h))X_t dt + \hat{\alpha} X_t dZ_t. \quad (11)$$

Hence, we have that the expected change in the freight rate will depend on own level and on the order book relative to the total fleet.

The trigger freight rate levels for changes in a^* will then be a function of the percentage of the fleet on order, that is, $x_i = \{x_i^1(h), x_i^2(h), \dots, x_i^{n-1}(h)\}$ and $x_r = \{x_r^2(h), x_r^3(h), \dots, x_r^n(h)\}$.

Summary and concluding remarks

The aim of this note has been to indicate paths to follow in order to make the SPE model more realistic by focusing on the shipbuilding market. First we establish that the two-tier version of the model is easily extended to a multi-tier version. Hence, we can incorporate a more realistic supply curve for the shipbuilding industry.

Then we indicate how to develop a model that takes account of the effect of an order book. The lag from a vessel is ordered to delivery influences the freight rate dynamics. We suggest that deliveries at any time are given by a fixed fraction of the order book. The order book is Markov, and hence, all the state variables of the system remain Markov even though "time to build" is taken into consideration.

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Note two;

**Stochastic Continuous Time Markov Models with "Time to Build"
- Formulation and a sketch of a possible solution**

Abstract

In this note we discuss the possibility of deriving a solution to the control problem introduced in Tvedt (1995). Although we are not able to find a closed form solution, some characteristics of the optimal controls are presented.

Introduction

In a model with instantaneous and costless adjustments any demand or supply shocks are immediately met by changes in input mix and volumes. For example, a constant return to scale economy with fixed input prices will not experience any price changes from a demand shock if production can be adjusted instantaneously. Two major characteristics applying to most real world investment settings, and which influence the degree of adaptability of the production technology, are cost of change and time to build. Here we focus on the problem of modelling a production technology with a lag between the decision to change input mix and the actual change. We model this by assuming an order book of production facilities, hereafter referred to as an order book of capital. We assume that at any time a given percentage of the order book adds to the capital stock.

The basic problem

We consider some continuous and bounded running reward function $F(\bar{X}_t)$ with state variables, \bar{X}_t , given by

$$\bar{X}_t = \begin{bmatrix} s+t \\ \varpi_t \\ k_t \\ Y_t \end{bmatrix}$$

Where ϖ_t is the order book, k_t the capital stock, and Y_t is a stochastic process, all at time t .

The capital stock is assumed to increase by a fixed fraction of the current order book. Let this fraction be given by η . Naturally, the same portion will be deducted from the order book. Further, assume geometric decay of the capital stock at a constant rate δ . The order book may, at any time, be

replenished. The new orders at time t are given by ξ_t . Hence, we have the dynamics of the order book and the capital stock respectively, given by

$$d\varpi_t = \xi_t - \eta\varpi_t dt$$

$$dk_t = \eta\varpi_t dt - \delta k_t dt.$$

The stochastic process Y_t is the source of the stochastic nature of the running reward function. Here we let Y_t be given by a geometric Brownian motion with incremental change at time t given by

$$dY_t = \mu Y_t dt + \sigma Y_t dZ_t \quad (1)$$

where the constants μ and σ are the instantaneous expected growth rate of Y_t and the standard deviation of the instantaneous relative change in Y_t , respectively, and Z_t is a standard Brownian motion, i.e., $dZ_t \sim N[0, dt]$. However, other choices may be more appropriate in applications.

Now we have that the incremental change in \bar{X}_t between each new order, is given by

$$d\bar{X}_t = \begin{bmatrix} 1 \\ -\eta\varpi_t \\ \eta\varpi_t - \delta k_t \\ \mu Y_t \end{bmatrix} dt + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \sigma Y_t \end{bmatrix} dZ_t$$

The process $\{\bar{X}_t; t \geq 0\}$ is Markov, and thus it follows that the process between any new order has an infinitesimal generator, \mathcal{A} , given by

$$\mathcal{A} = \frac{\partial}{\partial t} - \eta\varpi_t \frac{\partial}{\partial \varpi} + (\eta\varpi_t - \delta k_t) \frac{\partial}{\partial k} + \mu Y_t \frac{\partial}{\partial y} + \frac{1}{2} \sigma^2 Y_t^2 \frac{\partial^2}{\partial y^2}$$

Each new order entails costs. Here we suggest a simple representation of these cost. We assume a fixed cost per unit of capital ordered and a cost per new order that is linear in the size of the order book. That is; we let the present value of the cost of new order number j , at time θ_j , be given by

$$\bar{K}(\bar{X}_{\theta_j}, \xi_j) = e^{-\rho t} (m\varpi_{\theta_j} + q\xi_j)$$

Optimal regulation of the order book follows from the value function or performance criterion $\Phi(\bar{x})$

$$\Phi(\bar{x}) = \sup_{\omega} E^{\bar{x}} \left[\int_0^{\infty} e^{-\rho t} F(\bar{X}_t) dt - \sum_{j=1}^N \bar{K}(\bar{X}_{\theta_j}, \xi_j) \right] \quad (2)$$

where $\omega = (\theta_1, \theta_2, \dots, \theta_N; \xi_1, \xi_2, \dots, \xi_N)$, $N < \infty$.

To search for a solution to this optimal control problem we specify the quasi-variational inequalities

$$\mathcal{A}\Phi + F \leq 0 \quad (3)$$

$$\Phi(\bar{x}) \geq \mathcal{M}\Phi(\bar{x}) \quad (4)$$

$$\{\mathcal{A}\Phi + F(\bar{x})\} \{\Phi(\bar{x}) - \mathcal{M}\Phi(\bar{x})\} = 0 \quad (5)$$

where \mathcal{M} is the shift operator, defined by

$$\mathcal{M}\Phi(\bar{x}) = \text{Sup}_{\xi} \{ \Phi(\bar{x}, \xi) - \bar{K}(\bar{x}, \xi) \} \quad (6)$$

where \bar{x} is the state variable after control of ω_i .

Between any change in the order book we have that (3) must hold with equality. We try a solution of the form $\Phi(\bar{x}) = e^{-\rho t} \Psi(\bar{x})$ for the value function, where $\Psi(\bar{x})$ is a time homogenous function. Thus, it follows that between any change in the order book (3) can be written

$$-\rho\Psi - \eta\omega_i \frac{\partial\Psi}{\partial\omega} + (\eta\omega_i - \delta k_i) \frac{\partial\Psi}{\partial k} + \mu y_i \frac{\partial\Psi}{\partial y} + \frac{1}{2} \sigma^2 y_i^2 \frac{\partial^2\Psi}{\partial y^2} + F(\bar{x}) = 0 \quad (7)$$

For the homogenous part of the equation, we try a solution of the form

$$\Psi(\bar{X}) = k g^{\gamma} h^{1-\gamma},$$

where $g = \frac{y}{k}$ and $h = \frac{\varpi}{k}$. It then follows from the homogenous part of equation (7) that

$$\frac{1}{2}\sigma^2\gamma^2\Psi + (\eta + \mu - \frac{1}{2}\sigma^2)\gamma\Psi + (-\rho - \eta)\Psi = 0$$

We then have that the values of γ are given by

$$\gamma = \frac{\frac{1}{2}\sigma^2 - \mu - \eta \pm \sqrt{(\frac{1}{2}\sigma^2 - \mu - \eta)^2 + 2\sigma^2(\rho + \eta)}}{\sigma^2}$$

Observe that one root is larger and one smaller, than zero. Define $\gamma_1 > 0$ and $\gamma_2 < 0$. Now we have a solution to the homogenous part of the partial differential equation given by (3) between any change in the order book, as follows

$$\Phi(\bar{x}) = e^{-\rho k_i} (C_1 g_i^{\gamma_1} h_i^{1-\gamma_1} + C_2 g_i^{\gamma_2} h_i^{1-\gamma_2})$$

The solution to the inhomogenous equation will depend on the form of the running reward function $F(\bar{X}_i)$. In most cases this function will be independent of the order book. However, except for some trivial problems, probably most solutions to the inhomogenous equation will be dependent on the order book. Nonetheless, we do for a moment assume that the inhomogenous part of (7) has a solution independent of the order book and is given by $V(y, k)$. From the quasi-variational inequality formulation we then have that when the order book is replenished then $\Phi(\bar{x}) = \mathcal{M}\Phi(\bar{x})$. Substituting the above solutions for the general $\Phi(\bar{x})$ and $\mathcal{M}\Phi(\bar{x})$ we have that

$$\Phi(\bar{x}) = V(y_i, k_i) + e^{-\rho k_i} (C_1 g_i^{\gamma_1} h_i^{1-\gamma_1} + C_2 g_i^{\gamma_2} h_i^{1-\gamma_2})$$

and

$$\mathcal{M}\Phi(\bar{x}) = V(y_i, k_i) + e^{-\rho k_i} (C_1 g_i^{\gamma_1} \tilde{h}_i^{1-\gamma_1} + C_2 g_i^{\gamma_2} \tilde{h}_i^{1-\gamma_2}) + e^{\rho k_i} (m\varpi_i + q\xi_i)$$

where $\tilde{h} = \frac{\varpi + \xi}{k}$, and ξ is chosen optimally.

When relation (4) holds with equality we have that

$$C_1 g_t^{\gamma_1} h_t^{1-\gamma_1} + C_2 g_t^{\gamma_2} h_t^{1-\gamma_2} = C_1 g_t^{\gamma_1} \tilde{h}_t^{1-\gamma_1} + C_2 g_t^{\gamma_2} \tilde{h}_t^{1-\gamma_2} + m h_t + \frac{q \xi_t}{k_t}$$

This is often referred to as the "value matching" condition. For optimal regulation we have that the "high contact" condition must hold, that is,

$$\frac{d\Phi(\bar{x})}{d\varpi} = \frac{d\mathcal{M}\Phi(\bar{x})}{d\varpi}$$

Then it follows that

$$(1-\gamma_1)C_1 g_t^{\gamma_1} h_t^{-\gamma_1} + (1-\gamma_2)C_2 g_t^{\gamma_2} h_t^{-\gamma_2} = (1-\gamma_1)C_1 g_t^{\gamma_1} \tilde{h}_t^{-\gamma_1} + (1-\gamma_2)C_2 g_t^{\gamma_2} \tilde{h}_t^{-\gamma_2} + m + q \frac{d\xi_t}{d\varpi}$$

at the time of a change in the order book.

The geometric Brownian motion, Y_t , generated by (1) has an absorbing level in zero. If the running reward function, $F(\bar{X}_t)$, is zero in the case that Y_t is absorbed in zero, a feature that is true for a number of interesting applications, then

$$\lim_{y \rightarrow 0} \Phi(\bar{x}) = 0$$

In order for this to hold, $C_2 = 0$. Consequently, skipping subscripts, we hereafter write the "value matching" and "high contact" conditions as follows

$$C g_t^{\gamma} h_t^{1-\gamma} = C g_t^{\gamma} \tilde{h}_t^{1-\gamma} + m h_t + \frac{q \xi_t}{k_t}$$

$$(1-\gamma)C g_t^{\gamma} h_t^{-\gamma} = (1-\gamma)C g_t^{\gamma} \tilde{h}_t^{-\gamma} \left(1 + \frac{d\xi_t}{d\varpi} \right) + m + q \frac{d\xi_t}{d\varpi}$$

One candidate for an optimal increase relation ξ is $\xi_t = (a-1)\varpi$. If this relation should prove to be optimal, the constant a less one gives the percentage increase in the order book. Then it follows that

$$\tilde{h} = \frac{\varpi + \xi}{k} = a \frac{\varpi}{k} = a h$$

The "value matching" and the "high contact" conditions can then be written

$$C \left(\frac{g_t}{h_t} \right)^\gamma (1 - a^{1-\gamma}) = m + q(a - 1) \tag{8}$$

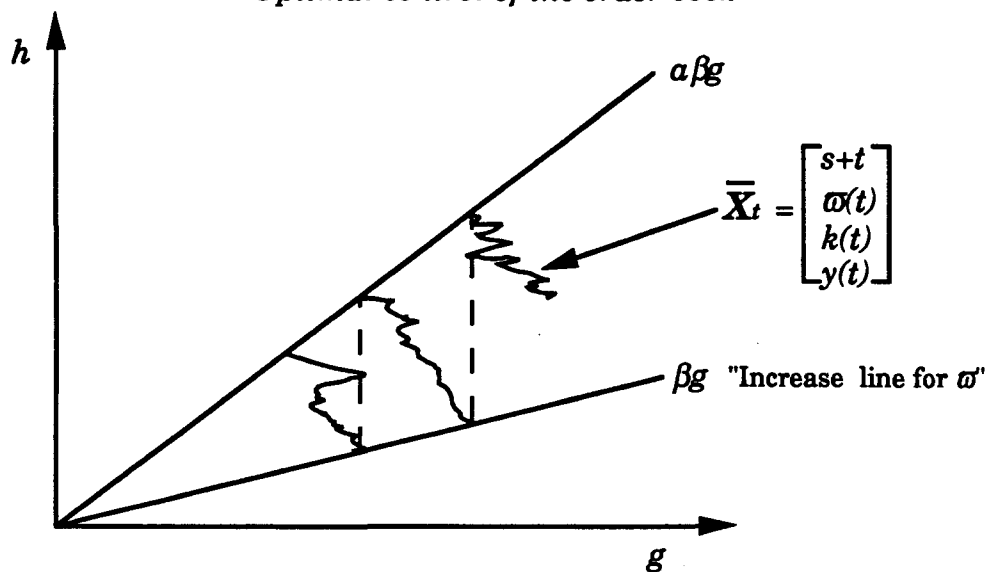
$$(\gamma - 1)C \left(\frac{g_t}{h_t} \right)^\gamma (1 - a^{1-\gamma}) = m + q(a - 1) \tag{9}$$

From (8) it follows that the trigger level of h will be linearly related to the trigger level of g , that is,

$$h_t = \beta g_t = \left[\frac{m + q(1 - a)}{C(1 - a^{1-\gamma})} \right]^{-\frac{1}{\gamma}} g_t$$

From the assumptions above we also have that $\tilde{h}_t = a\beta g_t$. The picture below indicates the continuity area of the process. When Y_t over the order book reaches a given level, $\frac{y}{\omega} = \beta$, the order book is replenished by a fixed percentage of the order book, a . As time passes, the order book decreases, and the value of h_t falls as long as $\eta(1 + h_t) > \delta$. The direction of g_t depends on the path followed by Y_t , the rate of decay of the capital, and deliveries from the order book.

Optimal control of the order book



Combining the "value matching" condition (8) and the "high contact" condition (9), we get that the value of a is independent on the state values and C , and is given by

$$a = \frac{m}{q} + 1 \quad (10)$$

Let the cost of initiating a change in the order book, m , be prohibitively high. However, if change is preferred, the optimal increase in the order book will be very large in order to reduce the expected number of changes in the future. To see this, let m go to infinity. Then the optimal a also goes to infinity.

Assume that m is very low. Then it will be optimal to adjust the order book frequently to match any changes in demand for new capital. Our problem is not defined for m equal to zero, but from (10) we see that on the margin, no costs of initiating new orders lead to infinitesimal increases in the order book. That is, $a - 1$ goes to zero. Then it follows that the number of changes will be infinite.

If we let the cost of a unit of new capital, q , be infinite, we see from (10) that $a - 1$ goes to zero. Naturally, there will be no new orders in this case. On the contrary, if capital is almost free, any new orders will be very large. Let q go to zero and a goes to infinity. However, our problem is not defined for q equal to zero.

Concluding remarks

In this note we discuss the possibility of solving an optimal control problem with a capital stock that is controlled through an order book. A fixed fraction of the order book adds to the capital stock. The main remaining problem is to find a particular solution to a partial differential equation. The solution will depend on ϖ , but the reward function, $F(\bar{X}_t)$, is not dependent on ϖ . In economic applications one interesting reward function would be $F(\bar{X}_t) = ck^{1-\phi}Y^\phi$, where c and $\phi > 1$ are positive constants.

In our opinion this paper gives a useful formulation for modelling a number of phenomena in economics as well as other fields, where impulse controls to a system have delayed effects on state variables. Further, this

formulation should be suitable for deriving solutions by applying simulation techniques.

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