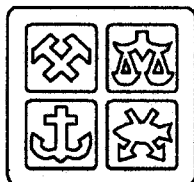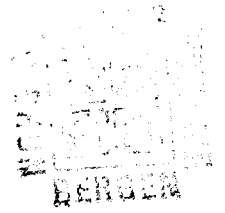# FIVE ESSAYS ON THE DYNAMICS OF FISCAL POLICY

by

Gaute Torsvik

A dissertation submitted for the degree of dr. oecon.

Norwegian School of Economics
and Business Administration
Helleveien 30, N-5035 Bergen, Norway

To my parents

# Acknowledgements

# Contents

# Outline of the thesis

This thesis contains five essays about economic policy in a dynamic setting. The first four chapters are - in one way or another - about time inconsistency and the value of making commitment to economic policy plans. The last chapter considers the effect of a corporate income tax when the firms' equilibrium behaviour is determined in a dynamic game.

Chapter 1 gives a survey of the recent literature in "the time inconsistency field" of economic theory. I start by asking why future governments want to deviate from the policy plan announced by the current government. If we rule out the possibility that new information becomes available as time passes, there are in principle two reasons why future governments want to carry out another policy than the one that is announced; i) the future government can have different objectives or preferences than the current government. ii) the constraints that the government faces are different when the policy is about to be implemented than they were when the plan was announced.

If future governments want to deviate from past plans we say that the government's planning is time - or dynamically - inconsistent. The current government will be aware of the possibility of time-inconsistency and will therefore try to commit future governments' policy actions. A key question is if, and how, a pre-commitment to a policy plan can be made, for a formal precommitment to a policy plan violates the basic principle of democracy. In democracy future governments cannot be reduced to be the

"implementers" of a policy announced by governments in the past. This does not, however, mean that today's government is totally constrained from

influencing future policy. Many variables that are controlled by the current political leaders influence what policy future political leaders will chose when they are in power. Much of the discussion in chapter 1 is concerned with how the current government can "commit" future governments through manipulation of state variables, and what impact this possibility has on both public and private economic behaviour.

In the last section in chapter 1 I allow future governments to receive new information as time passes. If there is uncertainty about "the state of the world" in the future, there seems to be a trade-off between the value of being flexible to choose the right policy given the realized state of the world, and the cost of flexibility caused by time inconsistency. The question of optimal policy rules in presence of uncertainty is discussed more thoroughly in chapter 3.


In the second essay - chapter 2 - I discuss a specific policy issue where credibility problems seem important. I ask what policy a government should use to reduce unemployment problems in a low productivity backward area. The conventional wisdom is that the best, or most effective, way to reduce regional unemployment problems is to give labour cost subsidies to firms that locate their production facilities there. Still, the mainstay in most governments' regional policy is capital grants. I show that the contradiction between the advice given by economists and the policy that is actually chosen by politicians, can be explained if the dynamics of the regional policy problem is modelled more carefully. That is, I show that capital grants can be the optimal regional incentive policy if we recognize that regional policy must be implemented sequentially and that investments in the rural area are - to some degree - irreversible.

Making a commitment to a specific economic policy will be desirable since it reduces the time inconsistency problem. But tying one's hands also has its obvious disadvantages by making it impossible to adjust policy if something *unexpected* should happen. It is therefore costly to commit to a specific policy - e.g., a specific tax rate - if there is uncertainty about the "state of the world" in the future and the government wants to respond differently in each state. This policy dilemma is studied in chapter 3 (which is co-authored by Trond Olsen). We consider a situation where the government is uncertain about its future revenue requirement. It can raise money by taxing income from two different projects (there are two different tax bases). One of the two projects is irreversible, the other is fully reversible. We characterise optimal taxes from an ex ante point of view. We find that the optimal tax rate policy is both state contingent and imply differentiated tax rates. So in our model it is not optimal to level-the-playing-field; it is not optimal to tax capital income at a uniform tax rate. We also show that the optimal differentiation of taxes has important implications for the optimal distribution of the tax burden over time.

In the last section of this chapter we acknowledge that the tax rate policy we find to be optimal is very complicated and that it therefore can be difficult to commit to such a policy. So in this section we compare the welfare effects of two simpler tax rules. One where income from the two projects is taxed uniformly (no tax rate differentiation) and one where the government announces a constant tax rate on income from the durable project. Contrary to what one should expect, based on the insight from the Ramsey principle of tax smoothing, we find that in terms of welfare, the latter rule may dominate the first one.

Chapter four is about voluntary contributions to a public good. The problem is classic, but instead of studying a case where individuals contribute to the good I consider the contribution game when there are groups that are the decision makers. Individuals in each group differ in their valuation of the public good. The amount that will be contributed from a group therefore depends on whom it is that makes the contribution decision in each group. I consider two different collective decision making procedures, i) direct voting and, ii) a system with representation. Representative democracy provides a mechanism for the median voter in each group to commit herself in the contribution game. Since there is strategic interaction between groups I show that the median voter will choose a representative with lower taste for the good than she herself has. Representative democracy reinforces therefore the tragedy of the commons, and a better solution would be achieved if each group had to make their contribution decision by direct voting. So in this policy game the possibility for a decision maker - a group - to make a binding commitment to a policy plan lowers welfare.

The last chapter is also about the dynamics of fiscal policy, but here I am not concerned with the issue of time inconsistency. The assumption we make about the equilibrium behaviour in the private sector is of course important for the results we derive about the welfare effects of different tax policies. A static model with perfect competition is by far the most common framework used for studying normative and positive taxation problems. In chapter five I consider the effects of a corporate income tax in a model with imperfect competition and dynamic optimizing behaviour. I show that the "well known" neutrality result of a profit tax does not generalize to a model where industry output and prices are determined in a dynamic game with tacit collusion between the firms.

# Chapter One:

# SEQUENTIAL IMPLEMENTATION OF FISCAL POLICY*

## Abstract

If future governments want to deviate from past plans we say that its planning is time - or dynamic - inconsistent. I this chapter I start by asking why future governments may want to deviate from the policy plan announced by the current government. Another important question is if and how a pre-commitment to a plan can be made. A formal pre-commitment violates the basic principle of democracy. Future governments cannot be reduced to be the "implementers" of the policy announced by governments in the past. This does, however, not mean that today's government is constrained from influencing future policy. Many variables that are controlled by the leaders today influence what policy future governments will chose when they are in power. Much of my discussion is concerned with how the current government can "commit" future governments through manipulation of state variables, and what impact this possibility has on current public and private economic behaviour.

# 1 Introduction

*Rationality*

A rational agent on the standard economic definition, is one who has consistent and complete preferences at any given point of time. This agent will make choices that can be predicted both by himself and by others, e.g. by economists.

What about rationality in a temporal setting?

If time is introduced into an optimization problem it is usually dealt with by reducing the dynamic choice to a static choice of an optimal plan or strategy. The economic agent looks ahead to all decisions that he will be called upon to make and

---

considers his choice today as a part of an overall strategy of choices. In this setting rationality means complete and consistent preferences over feasible *plans*. This "static" approach is often applied when economic policy is analysed; consequently a lot of interesting dynamic aspects connected to the choice of - and implementation of - economic policy are overlooked.

For usually one does not ask *whether* the optimal plan will be carried out. As time passes the government may get both opportunities and incentives to change its future choices from those that were planned. The incentives to deviate from the announced plan - and the private households' adjustment given the governments' opportunities and incentives - should be taken into consideration when *current* decisions are made. If this is done one will see that the government may try to make commitments in order to foreclose certain future opportunities that otherwise would be available. But the opposite may also happen, that is, the government will make more "flexible choices" today in order to face a uncertain future with as many opportunities as possible.

If we go fifteen or twenty years back in time the vast majority of the literature on public sector dynamics took the static approach of reducing the dynamic problem to a static problem, a problem of choosing the optimal policy plan from a set of feasible plans. The credibility of different plans was not discussed. In the last 10 years this has changed. Sustainability of policy plans - credibility of policy plans - time consistency or inconsistency of policy plans - the value of making a commitment to policy plans - are all terms that have appeared invariably in the journal literature the last decade.

*The dynamics of economic policy in a mixed economy - a neoclassical framework*

Within the Keynesian paradigm, where economic resources are (potentially) unemployed and individuals are myopic and face liquidity constrains, fiscal and monetary policy can be used for macroeconomic stabilization. An alternative approach is to investigate the effects economic policy has on the resource allocation in an economy where the price mechanism ensures that all economic resources are utilized. One could say that this second approach concerns the long run of the economy while the Keynsian paradigm concerns the short run. The argument is that over time the economy will be drawn towards the full employment equilibrium but, due to macroeconomic shocks along the way, it will never quite reach it.

In this chapter I focus on dynamic aspects of economic policy within the long run and full-employment-of-all-economic-resources perspective. Here the households' economic choices are the outcome from an intertemporal optimization problem. Markets are cleared in every period and if the government wants to raise revenue to produce public goods or, if it wants to regulate the "private sector", it can manipulate some policy instruments but it cannot achieve complete command over resource allocation.

More precisely, policy makers choose a sequence of policy instruments - possibly contingent on the history - so as to maximize the utility of the individuals they represents. The policy must be chosen subject to an intertemporal budget constraint and subject to the constraint that the system must be in a competitive equilibrium. In solving this program the government faces a two stage optimization problem: It chooses some policy plan to which private individuals or households make their best responses and then, when the government search for the best policy, it must take these best responses into consideration - this is the constraint that the system must be in competitive equilibrium.

More formally:

Consider a situation where a government in period 1 announces a policy plan for the next $m$ periods. Denote such a plan $x_1^m = (x_1, x_2, \dots, x_m)$. Let $X_1^m$ be the set of feasible plans in period 1. Denote policy planned in periods from $j$ to $m$, announced in period $i$ by ${}_i x_j^m$, where $i \leq j$. The government has preferences over economic outcomes, $o_i^m = (o_i, o_{i+1}, \dots, o_m)$ that is generated by a policy plan.

The feasible set of outcomes, $O_i^m$, is constrained by the policy instruments that are available, by the best response private households have to different policy plans, and of course by some resource and technology constraints.

Assume for a start that there are many identical households in the economy, and let the equilibrium behaviour of the representative one be given by the best response mapping; $g_i(x_i^m) = y_i^m = (y_i, y_{i+1}, \dots, y_m)$, where $y_i$ is the value of the choice variable ( possibly a vector) controlled by the representative household.

Given the technological and resource constraints and households best reaction to different policy plans, the feasible set of policy outcomes, as seen from period 1, is given by $O_1^m = \{x_1^m, g_1(x_1^m)\}$.

Let the government's preferences at time $i$ be represented by a function from $O_i^m \rightarrow R$; $w_i(o_i^m)$.

The static approach to the problem of finding the optimal policy plan in period 1, would be to search for the plan $x_1^m \in X_1^m$ that generates the outcome, $o_1^m$, that is evaluated as being best according to the utility function at that time. This policy plan is often called the Ramsey equilibrium and it is given by

$$ {}_1 x_1^{m*} = argmax\ w(x_1^m, g_1(x_1^m)) $$
$$ x_1^m \in X_1^m $$

I call this a "static" way to solve a dynamic problem, but of course this approach also involves dynamics in the sense that some of the technology constraints are dynamic stock - flow relationships governed by difference or

differential equations. The approach is static in the important sense that it builds on an once-and-for-all-optimization in period 1. The question of whether the optimal policy plan in period 1 actually will be followed up by future governments, is ignored. One does not ask: "If plans can be reconsidered as time passes, will the ex-ante optimal plan be followed in the future" ? Using the notation introduced above the question - usually not asked - is whether $_i x_j^{m*}$ $=_j x_j^{m*}$ for all $i < j < m$?

There is no problem with using the once-and-for-all-optimization approach to study the dynamics of economic policy if the government never changes its mind - and thus its plans - or if it never change it in some systematic way as time passes. But, as is pointed out in recent research, this may not be the case. On the contrary, one will often find that the government in period $j$, has incentives to deviate from the announced policy. That is, one will often find that $_1 x_j^{m*} \neq_j x_j^{m*}$ for $j > 1$. An important question is; why?

In principle there are two different reasons why optimal plans are dynamically inconsistent:

I) *Time-inconsistent preferences.*

The preferences, represented by the welfare function, evaluating different sub-plans (plans for period $j$ to $m$) - are different in period $j$ than they were in period 1, for $j > 1$.

II) *Time-inconsistent constraints.*

The set of possible outcomes from period $j$ to $m$ can be different when period $j$ arrives: Something that wasn't feasible when the problem was viewed from distance, that is, in earlier periods, has now become feasible .

*The similarity between inconsistent preferences and constraints*

Both dynamically inconsistent preferences and constraints have in some sense the same implication; past decisions about behaviour in what now has become the present will not be followed up. The initial policy plan is *time inconsistent*.

Donald McCloskey, in his "Economical writing", McCloskey [1986], uses "the time inconsistency problem" as an example of economic jargon that hide a five-cent thought in a five-dollar word. He claims that *"The time inconsistency problem simply is the economics of changing one's mind"*.

I think the idea is worth more. But McCloskey is certainly right about one thing: When a policy plan is time inconsistent it obviously means that the government has changed its mind about what is the best policy to pursue. But such a unified treatment of the phenomenon doesn't give any information about the *reasons* why a government has changed its mind. And the *reasons* are of interest if one is trying to find solutions to time inconsistency problems.

*The difference*

A change in the government's preferences between period 1 and $j$, where $1 < j$ leads to a time-inconsistent policy plan in the following way: In period $i$ the government *prefers another policy outcome* - another element in $O_j^m$ - than it did in period 1. In order to generate the policy outcome which now (in period $j$) is ranked as the best, it must choose a different policy from the one it announced in period 1. In this way a change in preferences over policy outcomes generates a change in the preferred economic policy.

The most straightforward way to think about time inconsistent policy caused by a change in constraints is as an invention of new policy instruments, like if a new "taxation technology" is discovered and this gives the government incentives to change tax policy. More interesting is it that the same policy instruments (the same taxation technology) may yield different responses from households if they are implemented *surprisingly* in period $j$ - than they would if they were announced in period 1. If this is the case we can say that the private individuals'

best response function to different policies has changed between period 1 and *j* and this may give the government incentives to change policy.

In both cases the government would like to take action in earlier periods (before period *j*) in order to decrease the probability of a deviation from the announced plan. If such strategic actions are made we can say the government is precommiting itself to a specific strategy or policy plan. One of the main conclusions that are made in this literature is that a precommitment to the ex ante optimal plan often will be desirable. Economic policy should be govern by rules not discretion. But, on the other hand, tying one's hands also has its obvious disadvantages, by making it impossible to adjust behaviour - policy - if something *unexpected* should happen.

These are the topics I will address in this chapter. I will study in more detail the different reasons why governments want to deviate from the announced policy, and possible strategies that can be used in order to eliminate - or decrease the incentive for - policy deviation.

I focus on fiscal policy. This is not a binding restriction. In a neo-classical framework the only impact of monetary policy is as a tax on cash holding .

## 2 Time-inconsistent preferences

With time inconsistent preferences a plan made in period 1 is not followed up in period *j*, the *reason* being that preferences over feasible outcomes, from period *j* and onwards, are different in period 1 and *j*, with $1 < j$. The government has followed the optimal plan in periods preceding period *j*, and the set of feasible outcomes is the same as it was when viewed from period 1, *but preferences have changed.*

Preferences over different feasible policy outcomes may change, as time passes, for different reasons.

• The most important one - given that the issue I study is the choice of economic policy over time - is that a new government with preferences diverging from the one in power in period 1 is elected in period $j$.

• Inconsistent time-preferences is another reason why governments may announce time inconsistent policy: The relative importance - the discounting of utility - of periods $j$ and $j+1$ is then no longer the same when period $j$ arrives as it was when viewed from period 1 ( This problem was first pointed out by Strotz [1955-56])

• Dynamically inconsistent plans can also be due to endogenously changing preferences. The decision-maker goes then through a character modification that is contingent on his past decisions. At the individual level habit forming through drugs use is the standard example (von Weiszäcker [1971] is the central reference here, see also Hammond [1976]).

Once again one could claim that these phenomena could be treated in one unified framework since the common denominator is announcement of plans that will not be followed up because *preferences have changed.* But - to repeat - I think this is an unwise approach because it suppress the *reasons why* preferences change. Preferences change for different reasons in different contexts. And in order to say something practical about the consequences of a deviation from planned policy, and in order to be able to point out possible ways to reduce the incentives to deviate, we have to know the source of the change.

## 2.1 Election of a new government

Economic policy is often analysed in simple models where the private sector is represented by one household and the government is modelled as a "social planner". In these models the planners preferences, whether he is a benevolent

one trying to maximize the utility of the representative household, or a malevolent one trying to maximize something else, are stationary and exogenously given. This is clearly a very unrealistic description of the relationship between a government and its constituency. One could defend the approach as being "harmless" in the sense that nothing of relevance is lost by modeling economic policy in this way. Recent research has, however, shown that this is not true, many interesting aspects of economic policy are indeed lost by abstracting from the political institutions and mechanisms one can find in a democracy.

Here I am interested in how the political system in a democracy can generate changes in the preferences of the fiscal authorities as time passes, and how this issue can be modelled in order to analyse its implications.

The principle of democracy implies that the majority of citizens are - at every point of time - free to choose the policy that should be implemented. Thus, if economic policy was determined through direct democracy - e.g., referendum - a change in the preferences over policy outcomes would have to be explained by a change in the majority's preferences. A change in the majority's preferences could come about in two ways: Either as a change in preferences on the individual level, that is, as a change in the taste of the decisive voter or - if we assume that each individual's preferences are stationary - as a change in the identity of the decisive voter, e.g., by the entrance of new voters and the departure of old one.

Direct democracy is, however, a time consuming way to make collective decisions. Direct democracy would also make the policy unstable and unpredictable. This problem is stressed by Elster [1979, page 88] :

> " A direct democracy - either in the sense that all citizens vote on all issues instead of electing representatives, or in the sense that representatives can be recalled at any time - will tend towards zig - zag policies and constant reevaluation of past plans; it will be incontinent, vacillating and inconsistent"

In modern societies the majority at one time protects itself against a constant re-evaluation of past decisions by electing representatives that make decisions on behalf of them. If governments were elected to be in power, and could not be recalled, for a very long period of time, this would indeed increase the predictability of future policy. But this is not democracy. A compromise is to have periodic elections of governments[1]. So, if we are trying to find out how the political system generate changes in the government's preferences and thus leads to time inconsistent policy plans, the election process seems to be the right place to start looking.

### Elections and a change in the government's objectives

In order to say anything about why and how elections can influence the choice of economic policy one has to specify what objectives politicians have, and what information voters have about these objectives and about the abilities of the politicians.

Two extreme answers are given to the question why politicians choose to be politicians. One is that they enjoy being in power and that remaining in office as long as possible is their only objective. Politicians are then said to be purely "office motivated". The other extreme is to say that being in office gives the politicians no kick in itself. In this case it is the policy that matters, and politicians are purely ideologically motivated.

### Office motivated politicians

For convenience let there be only two different political parties in the economy[2]. Both parties will follow the same policy if they are governed by purely office motivated politicians. With no taste for the policy itself both choose policy

---

[1] See Rogoff [1990] for an interesting discussion of how often elections should be held.
[2] It is not very satisfying to take the number of parties as exogenously given, but this is what is done in the literature.

goals that maximizes the probability of being re-elected, in general this is the policy preferred by the median voter (the "median voter theorem").

If voters are non-rational and backward looking when they form their expectations about the future, they can be fooled. Policy makers will take advantage of this and choose their policy in order to influence voters to re-elect them. Nordhaus, in his important work on political business cycles uses this model to conclude that periodic elections has the negative effect of giving politicians incentives to give priority to short run measures at the cost of lower welfare in the longer run. In addition to the assumption that policy makers have opportunistic objectives, that their only goal is to "stay in power as long as possible", the Nordhaus type of political business cycles relies on nominal rigidities and voter myopia.

It is, however, shown, e.g. by Rogoff and Sibert [1988] and Rogoff [1990,b] and Persson and Tabellini [1990], that one can get cycles with opportunistic policy makers even if voters form their expectations in a rational way. The driving force behind these cycles is imperfect information about the politicians *ability* to solve economic problems. Politicians will try to signal - through the choice of economic policy - high ability when they are up for elections.

Note that with purely "office motivated" politicians elections *cannot* generate a change in the governments preferences. Preferences are stable but the policy may be distorted by the election process because economic policy is used by the incumbent politicians as means for winning the next election, or as means for signalling abilities if these are not perfectly known by the public.

*The partisan model*

With ideologically motivated politicians the two parties have different preferences over *policy outcomes*. The groups that make up the constituency for political parties have different preferences over policy outcomes. Parties are modelled with different objective functions. In this situation a change in

governments - election of a new government - generates a change in the policy makers' preferences and thus to a change in economic policy.

The partisan model was first developed by Hibbs [1977]. He pointed out that there will be political business cycles when there is a change in governments and these governments have different ideologies. Hibbs was not very specific about individuals' expectation formation. Alesina [1987], [1988], show that there will be cycles even if voters have rational expectations. He does this in the context of monetary policy and the inflation/ unemployment trade off that exists when the nominal wage is fixed through long term contracts.

In fiscal policy - which I focus on - this two party system, or some version of it, has been studied by Persson and Svensson [1989], Alesina and Tabellini [1989], [1990] and Glazer [1989].

In the paper by Torsten Persson and Lars Svensson there are two "political parties". The parties differ with respect to the preferred level of public spending. There is a left wing party ($l$) that prefers high government consumption and there is a right wing party ($r$) that prefers less government spending. I will use a simplified version of their model to illustrate how *time inconsistent preferences* generates *time inconsistent policy*.

There are two periods (1 and 2). Per period utility of a government - or the representative individual in the group that makes up the constituency for the government - is given by $W(c_i, g_i; \gamma)$, where $i = 1,2$ and $\gamma$ is a taste parameter that can take two values $\gamma^l > \gamma^r$. For a given consumption vector, $(c_i, g_i)$, the left wing government has a higher marginal rate of substitution between public and private consumption than the conservative has, that is[3] ;

$$\frac{W_g(c_i, g_i, \gamma^l)}{W_c(c_i, g_i, \gamma^l)} > \frac{W_g(c_i, g_i, \gamma^r)}{W_c(c_i, g_i, \gamma^r)} .$$

Let $T_i$ be the amount of taxes that is collected in period $i$. For simplicity let the interest rate be zero, that is, there is assumed to be no discounting.

---

[3]When one of the arguments in the function is used as a subscribt this means the derivative of the function with respect to this argument.

Assume first that there exists a constitutional amendment requiring the public budget to be balanced in every period, that is, $g_i = T_i$, $i = 1,2$.

There is an excess burden - a deadweight loss- from taxation. I model this in the simplest possible way by assuming that if $E$ is the amount of resources available in the economy in each period (GDP) and $T_i$ are the taxes that are collected in period $i$, then only $E - \mu(T_i)$ remains available for private consumption $(c_i)$, where $\mu(T_i) > T_i$, $\mu_T(T_i) > 1$ and $\mu_{TT}(T_i) > 0$. The tax-cost function captures increasing marginal excess burden of taxation[4].

Given the assumptions above the only problem left, that is, the only problem not taken care of by the assumptions, is the choice of public consumption in period 1 and 2. Public expenditures preferred by government $j$, $g^{j*}$, is given by the f.o.c;

$$\frac{W_g(c_i \, g_i \gamma^j)}{W_c(c_i \, g_i \gamma^j)} = \mu_T(\, g^{j*} \,), \text{ where } j = (r,l),$$

which implies that $g^{l*} > g^{r*}$.

Now consider a situation where the right wing government is in power in period 1. The optimal sequence of public expenditures for this government is to choose $g = g^{r*}$ in both periods. But if there is a positive probability that a left wing government preferring, $g^{l*} > g^{r*}$ is elected in the second period then there is a positive probability that *the initial plan is time inconsistent*. That is, time inconsistent preferences leads to a time inconsistent plan for optimal policy.

Alesina and Tabellini [1987] and [1990] consider a situation with a positive probability that a government, preferring different *kinds* of public consumption than the incumbent one, is elected in the future. Thus there is a possibility for time inconsistent policy.

---

[4]This is modelled more carefully in the paper by Persson and Svensson. There excess burden is a convex function of the tax amount collected because taxes are levied on income from endogenously determined labour supply.

In Alesina and Tabellini [1989] there are two groups - workers and capitalists - who disagree, in a very natural way, about how the tax burden should be distributed between the two groups. In their model the parties representing these groups alternate randomly in being in office. This creates time inconsistent policy.

In Glazer [1989] the future median voter may value a government project differently from the current one. This possibility of dynamic inconsistent preferences may give rise to dynamic inconsistent policy plans, e.g., if the future governments can close the project.

All these possibilities of time inconsistent policy preferences give the incumbent government - knowing that it can be replaced by another government preferring other policy outcomes - incentives to behave differently than it would if it knew with certainty that it would be in power also in the future. Indeed, the main question asked and answered in the papers by Persson and Svensson [1989] and Alesina and Tabellini [1987],[1990] is how time inconsistent preferences influence the choice of a government's debt policy.

Instead of imposing the constraint that the public budget must balance in every period Persson and Svensson assume that the public sector faces only an overall intertemporal budget constraint. Then there is a possibility for the first period government to finance some of its expenditures by issuing debt. The question of how time inconsistent preferences influence the preferred level of debt - and fiscal policy in more general terms - will be taken up later in section 3 where I discuss how a government can reduce or perhaps even eliminate time inconsistency through sophisticated planning.

## 2.2 Inconsistent time-preferences

In a democratic political system the replacement of one government by another in periodic elections is perhaps the most natural and interesting source to how a change in preferences generates dynamic inconsistent policy plans.

Another reason why economic agents may not obey past decisions was pointed out by Robert H. Strotz almost four decades ago, Strotz [1955-56]. The central message in his famous article was that individuals often have an inconsistent attitude towards time itself. Strotz showed that dynamic inconsistency will arise if the relative weight an agent puts on the utility in different periods - the discounting of utility - changes as time passes. Plans are consistent over time if, and only if, the marginal rate of substitution between "consumption" in two following periods is *independent* of how far away - the distance is measured in time - these periods are. Whether a planner actually deviates from the initial plan depends of course on the degree of commitment he has made to it. The point is that *if* he is free to reconsider his plan at later dates, he will disobey it unless utility is discounted so that the marginal rate of substitution between two periods is independent of the distance to these periods.

An illustration of the problem can be made within a simple model of an economy that span three "generations" and where the public sector - the government - has to pay a fixed amount $g^*$ at the end of the third period[5].

Assume that aggregated production is exogenously given, normalized to 1 in each period. The public sector collects taxes $T_i$ in period $i = 1,2,3$, for simplicity I assume a zero interest rate on public assets. The intertemporal budget constraint is therefore given by $\sum_{i=1}^{3} T_i = g^*$.

In other words, it is impossible for the government to smooth its expenditures over time, but the financing of the public project - the tax burden - can freely be

---

[5]This example corresponds to a more general situation where there is a probability of a sharp increase in governments outlay in some future period. My example is closely related to one in Elster [1979].

distributed between the three periods. For simplicity I assume that the public outlay in period three, $g^*$, generates no utility.

In period $i$ consumption of the private good is $c_i = 1 - T_i$. I will restrict the analyses to time separable utility function, per period utility is represented by $V_i(c_i)$ where $V_{ic}(c_i) > 0$ and $V_{icc}(c_i) < 0$. The government in period 1 chooses a plan, a sequence of taxes, in order to,

maximise; $$W(c_1, c_2, c_3) = \sum_{i=1}^{3} V_i(c_i)$$

subject to, $$\sum_{i=1}^{3} T_i = g^*.$$

Each generation cares for the following generations, but utility is discounted, that is, each generation has preferences for the near future over the more distant future. Time-preferences imply that $V_{1c}(c) > V_{2c}(c) > V_{3c}(c)$, for all $c \in [0,1]$.

The utility function in the second period is assumed to be given by; $W(c_2, c_3) = V_1(c_2) + V_2(c_3)$ and utility in the third period is $W(c_3) = V_1(c_3)$.

As Elster points out this formulation makes it clear in what sense the;

*"utility function is constant, and hence allows a discussion of the consistency and inconsistency of time preferences that is independent of the problem of preferences that change over time"* (Elster [1979] page 69)

In accordance with the notation used in the introduction let $_1T_1^{3*} = (T_1^*, T_2^*, T_3^*)$ be the optimal plan that solves the first order conditions;

$$V_{1c}(c_1^*) = V_{2c}(c_2^*) = V_{3c}(c_3^*),$$

where $c_i^* = 1 - T_i^*$.

Dividing with $V_{3c}(c_3^*)$ this first order condition reads;

$$\frac{V_{1c}(c_1^*)}{V_{2c}(c_3^*)} = \frac{V_{2c}(c_2^*)}{V_{3c}(c_3^*)} = 1.$$

The optimal continuation plan for the government when the second arrives, given that $T_1*$ was implemented, must solve this equation;

$$\frac{V_{1c}(c_2)}{V_{2c}(c_3)} = 1.$$

Thus, the optimal plan in period one, $_1T_1{}^{3*}$, is time consistent, that is, $_1T_2{}^{3*} = {}_2T_2{}^{3*}$, if and only if;

$$\frac{V_{1c}(c)}{V_{2c}(c)} = \frac{V_{2c}(c)}{V_{3c}(c)}$$

If we take the special case with equivalent per period utility functions, say $V_i(c_i) = b_i V(c_i)$, $i = 1,2,3$ - or, in order to get a numerical example, let us be even more specific: Let $V(c_i) = \ln c_i$. This gives the following welfare function, $W(c_1,c_2,c_3) = b_1 \ln c_1 + b_2 \ln c_2 + b_3 \ln c_3$, where $1 \geq b_1 > b_2 > b_3$.

The ex-ante optimal plan, $_1T_1{}^{3*}$, is given by the vector $(T_1, T_2, T_3)$ that solves;

$$\frac{b_1}{1 - T_1} = \frac{b_2}{1 - T_2} = \frac{b_3}{1 - T_3},$$

subject to the constraint that $\sum_{i=1}^{3} T_i = g*$.

When period two arrives consumption in period two and three is chosen in order to solve:

$$\frac{b_1}{1 - T_2} = \frac{b_2}{1 - T_3}$$

Hence, the optimal plan in period one is time consistent - will be followed up - if, and only if, $\frac{b_1}{b_2} = \frac{b_2}{b_3}$. This requirement is fulfilled if, and only if, utility is discounted exponentially[6].

---

[6]This is what Koopmans [1960] call "stationary time preferences".

## Numerical example of inconsistent time preferences

Let $g^* = 1$ and let $b_1 = 1$, $b_2 = 0.9$ and $b_3 = 0.85$. Given these parameters the optimal plan in the first period is $_1T_1^{3*} = (0.273, 0.345, 0.382)$. Maximization of $W(c_2, c_3) = b_1 lnc_2 + b_2 lnc_3$, given $T_1^* = 0.27$ gives $_2T_2^{3*} = (0.330, 0.397)$ which is <u>not</u> the continuation of the initial plan.

In the numerical example utility is discounted non-exponentially, and consequentially time preferences are inconsistent and planning is time inconsistent. An important question is whether exponential discounting of utility *realistic,* or in more general terms if it is realistic to assume that the marginal rate of substitution between "consumption" in two following periods is *independent* of how far away - in time - these periods are? In my opinion there are good reasons to belive that the answer to this question is negative. Pollak and Phelps [1968] argue that there is a general and realistic mechanism that generates inconsistent time preferences: If agents give an absolute priority to the present over future periods, which then are given gradually lower weights - as is the case in the numerical example above - they will discount future utility non-exponentially. For, the future as seen from today is nothing but the present when "tomorrow" arrives, and with a coherent attitude towards time - the absolute priority of the present - more weight will be given to consumption "tomorrow" when it actually arrives than consumption "tomorrow" was given as viewed from distance. The psychology behind this inconsistent way of discounting utility is very nicely captured by Elster [1979], page 71:.

> "..[I]n the notion of time preferences we bundle two problems that can and should be separated: the absolute priority of the present, and the shading-off of the future. The absolute priority of the present is somewhat like my absolute priority over all other persons: I am I - while they are all "out there". The shading-off is a perspectival phenomenon that admits of degrees of "out-thereness" the far future is like a distant relative, while the near future is like a close one"

## 2.3 Endogenous change in preferences

Endogenous change in preferences over policy outcomes is different from both inconsistent time-preferences and exogenous change in preferences (election of a new government). But the consequences are the same; plans, that are optimal from todays point of view, will not be followed up in the future ( See von Weiszäker [1971] and Hammond [1976]). To be sure, there must be a span of time in order to make it meaningful to talk about a change in planned behaviour at all, but time itself - and the agents attitude towards time - is not the important issue when we talk about preferences that change endogenously. Preferences that change endogenously is to be explained as a character modification of the decision-maker, where the change of character is based on past decisions. At the individual level habit forming through drugs use is the standard example.

The following example is based on Hammond [1976]. Consider an individual who at time 1, $t_1$, has to decide whether to take drugs ($d$) or to refrain ($r$). If he chooses $d$ at 1 he has to make a new decision of whether to continue with drugs or refrain from it (see figure 1).
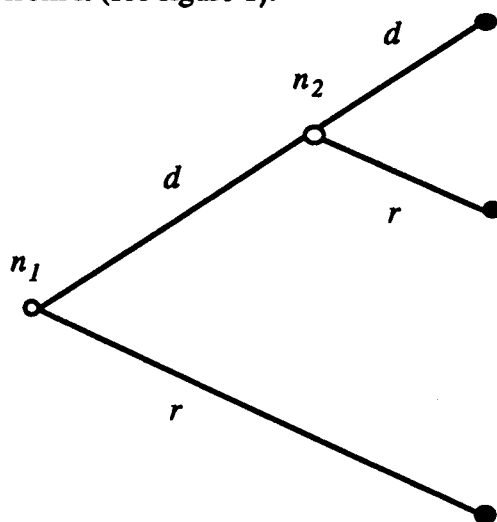


*figure 1*

The optimal path as seen from time 1, $n_1$, is ($d,r$). By choosing this path he gets the initial thrill of drugs but gives it up before it seriously impairs his health.

But in node $n_2$ he has become an addict, and by definition this means that he prefers $d$ to $r$ at this stage. The optimal plan initially, $(d,r)$, is not time consistent because *preferences change along the path.*

Economic policy decisions can, just like drug using, be addictive. Consider for example the decision about the size of the public sector. Assume that the current majority for some reason wants temporarly to increase the public sector. To be more concrete assume that the current government wants to increase the fraction of individuals that work in the public sector from $h_0$ to $h_1 > h_0$ for one period before one returns to the old fraction, $h_0$. This policy plan will be time consistent if, and only if, the preferred size of the public sector does not depend on how many "insiders" there are in the public sector. The initial policy plan will therefore be dynamic inconsistent if - as is reasonable to assume - the preferred size of the public sector is an increasing function of the number of individuals that work in the sector. If that is the case the preferences over the size of the public sector are endogeneous.

I have discussed three different reasons why policy preferences may change over time and how this can lead to dynamic inconsistent planning.

First preferences can change *exogenously.* There is an exogeneous change in preferences if new politicians, with different objectives than those currently in power, are elected or if the governments' time preferences are inconsistent. Second, a government's policy preferences can change endogenously over time.

A totally different reason why economic policy plans are dynamic inconsistent is that the policy constraints a government face change over time. But before I turn to study that issue I will discuss what a government can and will do if it faces a planning problem and knows that its policy preferences may change over time. How does a government tackle the problem of time-inconsistent preferences?

# 3 Solving the problem of inconsistent preferences through sophisticated planning

## 3.1 Consistent planning

How will - or should - the incumbent government act if it knows that the preferences may change endogenously or exogenously over time. The structure of the problem faced by the current government is the same in each of the inconsistency situations: It is playing a game against future governments that have different policy *preferences* than itself. This can be thought of as a dynamic principal agent problem where the principal is the present government and the agents are the future governments. Note that there need not be a physical change in governments, future governments can be just another version - a later and different version - of the current government.

In the literature these kinds of inconsistency or "weakness of will" have been discussed mainly in relation to some individual intertemporal consumer problem ( See Strotz [1955-56], Pollak [1968], Peleg and Yaari [1973] , Hammond [1976] and Elster [1979]). Strotz pointed out that an individual - being weak and being aware of it - had two strategies available to make plans dynamically consistent : He could either precommit himself to the optimal plan or he could choose the best plan from the subset of plans that are time consistent.

It is misleading to say that consistent planning and precommitment are to different *strategies* that can be used to make plans dynamically consistent. Consistent planning means sophisticated planning and sophisticated planning means that the planner recognises and takes into account the possibility for an exogenous or endogeneous change in preferences as time passes. A sophisticated planner will choose a plan - the one evaluated to be the best - from the subset of plans that actually will be followed up in the future. How large the subset of consistent plans is, is basically a question of technology and not a question of

choice or strategy. And if the ex-ante optimal plan is contained in the subset of consistent plans then, in Strotz' terminology, it is possible for the decision maker to precommit himself. Precommitment is therefore nothing but consistent planning in a spesific institusional framework where it is possible for a planner to bind himself to the initial optimal plan. Ulysses had access to such a precommitment technology. Ulysses had a rope in his boat and could therefore (literally) bind himself:

> *"but you must bind me hard and fast, so I cannot stir from the spot where you will stand me ... and if I beg you to release me, you must tighten and add to my bonds"*
> - The Odyssey.

Here is another example that illustrates that precommitment is consistent planning in a spesific context. Consider the person tempted to try drugs. First, imagine that the person is stranded on an isolated island, and that he arrives at the island with a large amount of drugs but there is no drugs on the island except what he brings with him. In this situation it would be relatively easy to commit to the first best solution; using drugs for a while but stop before it seriously impairs his health; $(d,r)$. This can be realised by throwing away part of the drugs such that using what he has left behind would give him the thrill but not a health damage. In this context the ex ante optimal plan, $(d,r)$, is a member of the subset of plans that are dynamic consistent. In a game theoretic formulation and related to the game tree in figure 1, precommitment amounts to removing the branch $d$ from $n_2$, or more precise; *the option of making a binding commitment is the same as having the option to play a game with a game tree representation where the d branch from node $n_2$ is removed.*

Precommitment is not always possible. If the potential drugs user lived in a big city, it would be almost impossible to foreclose the option of "getting more drugs". Hence in this situation the player is playing a game where all the branches in the game three are intact. If precommitment is not possible the potential drugs user, if he is sophisticated and recognizes how drugs use will change his preferences, chooses to refrain from drugs taking altogether in node $n_1$. The

subset of time consistent plans, that is, the set of plans that will actually be followd up in this game is; $(d,d)$ and $(r)$, and among these $(r)$ is the best; "refrain" in node 1 is better than addiction.

This illustrates that the question of whether it is possible to make binding commitments to a plan is a question technology and not a question of strategy. It is a question of whether or not the planner has access to the right tools, to the right "means of commitment".

Related to economic policy the key question is: Can the present government bind future governments to follow - or continue - a specific policy-plan? If it could - it would. The government in the paper by Persson and Svensson would of course prefer to bind the future government to implement what $it$ views as the optimal supply of the public good; $g^{r*}$ and a government with inconsistent time preferences would like to commit itself to implement $_1T_1{}^3*$. In this last case a government that cannot commit itself to a policy plan - but is sophisticated - will reason as follows: "In the next period an absolute priority to the present will be given, just as it is in this period, hence "I" must take this into consideration when "I" choose tax policy in this period". The government will then see that for every $T_1$ it chooses there follows an allocation of $T_2$ and $T_3$. This function from $T_1 \rightarrow$ $(T_2, T_3)$, call it $H(T_1)$, is given by;

maximization of $V_1(c_2) + V_2(c_3)$ subject to $c_i = 1 - T_i$ and $T_1 + T_2 + T_3 = g^*$, for $i = 2,3$.

Which is the same as maximization of: $V_1(1 - T_1) + V_2(1 - g^* + T_1 + T_2)$. And from the first order condition of maximizing this function with respect to $T_2$ we find $T_2$ implicitly defined as a function of $T_1$:

$$T_2(T_1) = \frac{V_{1c}(1-T_2{}^*)}{V_{2c}(1 - g^* + T_1 + T_2)} - 1 = 0$$

$$T_3(T_1) = g^* - T_1 - T_2(T_1).$$

A government choosing the plan "evaluated to be the best among those plans that actually will be followed up in the future" will choose $T_1$ in order to maximize $W(c_1, c_2, c_3)$ given $H(T_1)$.

In the numerical example this function from $T_1$ to $T_2$ and $T_3$ is given by:

$$T_2 = \frac{0.9 - T_1}{1.9},$$

$$T_3 = 1 - T_1 - \frac{0.9 - T_1}{1.9}.$$

One consistent plan is the one where the present government starts out by implementing the plan that is optimal given the initial (but inconsistent) time preferences but the government in period two maximizes according to the "new" time preferences, $T_1 = T_1^* = 0,273, T_2 = 0,33$ and $T_3 = 0,397$. This is however not the optimal plan. And a sophisticated government would not choose this plan it would take $H(T_1)$ into consideration when maximizing and if this is done the government will choose $T_1 = 0,283$, $T_2 = 0,325$ and $T_3 = 0,392$.

So when we study intertemporal fiscal policy a important question is whether the government has access to a commitment technology.

Some authors e.g. Blanchard and Fischer [1989, page 600] argue that the government can commit itself to implement a specific policy plan. In their view making policy through contracts - or by law, constitutional or less fundamental - is one way for a government to solve the time inconsistency problem. They argue that policy making by law will make it impossible or difficult for the government to deviate from the announced policy.

I am more sceptical and the main reason for my scepticism is that the basic principle of democracy is violated if the current government could bind future governments through making policy by the law, or through signing contracts. In a democracy the majority should - at every point of time - be free to choose its policy. It would therefore not be in the spirit of democracy if the present

government could dictate future fiscal policy simply by implementing the policy plan by a law. In a democracy where legislators can pass a law, they can repeal it as well. A traditional law needs only a simple majority vote and it is hard to see why policy making by such a law is a more credible commitment to the initial plan than sweet promises to stick to it.

The point that *democracy forbids the existence of a formal commitment technology in fiscal policy* - a point based on the argument that in a democracy future majorities should not be reduced to slaves of the present majority - is important. It's important first of all because it leads our attention to other channels or mechanisms through which governments must influence future policy. We then discover that the present government can influence future policy *indirectly* by manipulating variables that - in some way - link the present and the future. That is, the government can in many cases make actions today that change the set of consistent plans. That is what I will discuss now.

## 3.2 Strategic behaviour - manipulation of state variables in order to change the set of consistent plans

Strategic behaviour by manipulation of state variables, describes a situation where the present government strategically chooses the level of some variables that link the present and the future. Strategic refers to the motivation behind the manipulation; it is done to change the payoff different policy outcomes give the "new government" in the future.

The idea is easiest to understand if it is compared to a situation where the decision maker can precommit himself to the optimal plan. The best way to illustrate precommitment is by using the game-tree representation of a game on extensive form. A planner who is able to precommit to the optimal plan is a planner who can choose to play a game where all branches - except those along the

optimal path - are struck off. To give an example look at a two period planning problem where the government that is in power in each period can choose policy from a set $A_i = \{a,b,c\}$, $i = 1,2$. Assume for simplicity that the government's preferences are time separable. The government holding office in the first period prefers $a$ to $b$, and $c$ is viewed as the worst alternative. The optimal plan is therefore to implement $a$ in both periods, that is, $\{a,a\}$. When the second period arrives preferences have changed - a new government is elected, time preferences are inconsistent or preferences are different because the planner has gone through some character modification. Whatever the reason is, at this stage the government prefers $c$ to $b$, and $a$ is now regarded as the worst alternative.

If it is possible for the present government to precommit future governments - if it has access to a commitment technology - it is playing *a different game* than if such a technology is not available. This is illustrated in figure 2. If it is possible to precommit to a plan the government can - in node $n_0$ - decide to bind future governments. This means that it is playing the game at left in figure 2.

Commitment possible:

Commitment *not* possible:
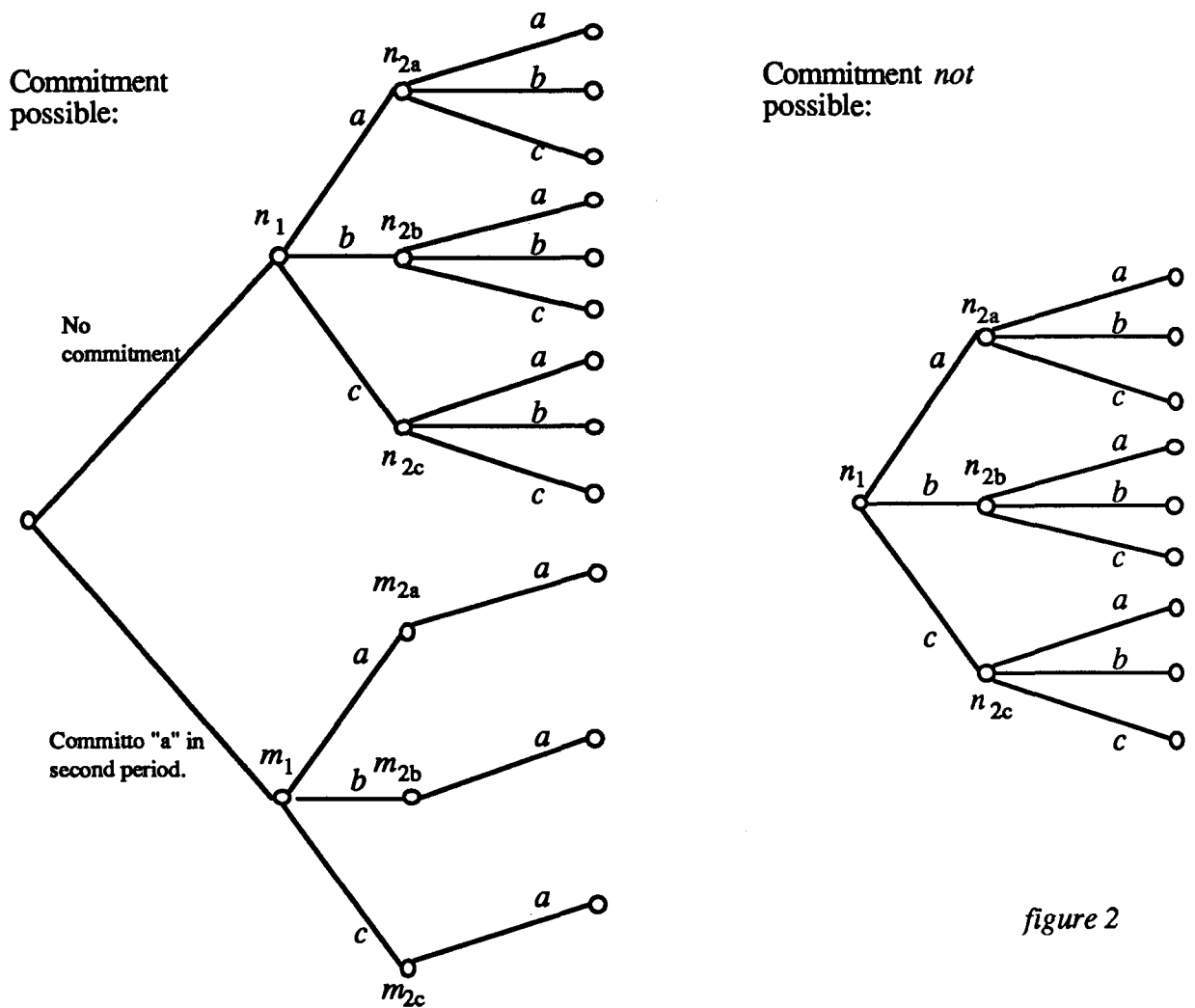
*figure 2*

But as I have already indicated a few times, the basic rules of democracy precludes this kind of precommitment to policy plans. The first period government cannot force the one in power in the second period to implement policy choice, $a$. Future policy cannot be fixed by the present government, for in a democratic political system one government cannot limit the set of policy instruments that are available for future governments. Hence the first period government must take $A_2 = \{a,b,c\}$ as given.

This does, however, not mean that the government - passively - will sit down and make consistent planning. That is, the government will not passively accept that policy $c$ will be chosen in the second period and announce $(a, c)$ as the optimal

consistent plan. A *sophisticated* government will say to itself: " We cannot limit the set of future policy instruments, but is there anything else we can do to influence future governments so that they will to make policy choices that are more in line with our own preferences"? A government - *a sophisticated government* - will see that there exists a general mechanism for influencing future policy: The present government can make decisions today that alter the way future governments evaluate the consequences of different policy choices when they are in office. The present government can change future policy by manipulating variables that change the payoff different policy choices generate for future governments. Such variables are called *state-variables*.

So even though a government cannot limit the set of policy options faced by future governments - even though it cannot choose "commit to $a$" in node $n_0$ - it can in general make decisions that alter the payoff from choosing $a,b$ or $c$. In the example above the first period government may do something to make it very costly for the second period government to choose $c$. In this way the current government can perhaps induce the future government to choose $b$, and that choice is - as seen from the present governments point of view - a better alternative than $c$. If this is true we can say that the first period government - by strategically choosing the level of some state variables - makes $\{a,b\}$ an element in the set of consistent plans.

Individuals often use Indirect strategies to change future choices when preferences are time inconsistent, see Elster [1979] for lots of examples. Here is one: A person can make it more costly to change the decision of cease smoking - a decision made in a moment of strong will - through making it an official statement rather than a secret one. For a person who likes to be viewed as a decisive individual with a firm character - an individual who does not succumb to the temptation of the moment - telling friends and family about the decision of giving up smoking may be an effective way to prevent backsliding.

In my view it is in this 'manipulation-of-state-variables' perspective one should view policy making by law. It is already noted that in a democratic society laws

can be changed; this means that even though economic policy is made by law it does not preclude future governments from changing policy since they have the option to change the law. One may, however, argue that policy making by law imposes deviation costs similar to those the smoker - who announces his decision to cease smoking officially - faces; the embarrassment of reversal and the simple fact that it requires a distinctive action may make it more costly to change policy if it is made by law. Even more costly will a change in policy be if it is made by a constitutional law. Policy making through constitutional laws is therefore a signal of a particularly strong commitment since constitutional laws cannot be changed without going through a lengthy and difficult process. Matters fixed by constitutional laws are therefore more fundamental and enduring than matters governed by mere laws.

Whether the incumbent government can choose state variables in order to distort choices made by future governments is of course also a question of having access to the right technology: Are there any state variables to manipulate? This is, however, a more interesting question because here we cannot simply state that some principle - the principle of democracy - gives us the answer.

*The costs of using indirect strategies in influencing future policy*
*- the costs of manipulating state variables*

It is difficult to say anything absolute about the desirability of making a commitment to a policy plan in the case where time inconsistency of optimal plans is caused by a change in the government's preferences ( e.g. election of a new government). If the present government can commit future governments, with different objectives, to pursue the policy the present one finds optimal, this is definitely favourable for the one in power initially, but is not so nice for future median voters. But even though it is difficult to characterise such a situation as either being good or bad, it is possible to say something about how efficiently the manipulation of policy is made. That is, the following question can be answered: Given that certain future policy outcomes can be induced by the present

government through manipulation of state variables, is it done in the most efficient way, or can it be done more efficiently?

One key observation in this dynamic principal-agent game, is that the incumbent government is willing to incur a loss in efficiency - it is willing to pay a price - in order to induce the new government to make choices that are more in accordance with those preferred by itself. In other words the non-cooperative solution leaves room for inefficiency. It is possible that both governments could get higher payoff if they could cooperate, that is, if they could write a binding contract between themselves. To be more concrete, look back at the game illustrated in figure 2 (on page 31). Assume that the first period government can manipulate a state variable such that $b$ is implemented in period 2. The first period government is willing to do this even if the manipulation in itself generate inefficiency. It is willing to do this as long as the distortions that are imposed are smaller than the difference in payoff between policy $c$ and $b$.

It is therefore not surprising that manipulation of state variables in general distorts the economy and that both governments could do better if they wrote a contract simply stating that "the policy the first period government can induce by manipulation of state variables - policy choice $b$, in the example - is to be implemented in the second period".

I now turn to how strategic choice of state variables has been discussed in the literature.

*Debt as a state variable*

In section 2.2, I gave a sketch of the problem analysed in Persson and Svensson [1989]. In their model a right wing government with a preferred level of public consumption, $g^{r*}$, is in power in the first period. This government knows that it will be replaced by a left wing government preferring a higher level of

public consumption. Given this situation Persson and Svensson study what debt and tax policy the right wing government will choose.

If we adhere to the assumptions made in section 2.1, that there is a requirement of a balanced budget in every period, there is no problem to solve: In the first period public consumption is $g^{r*}$ and private consumption is $E - \mu(g^{r*})$ in the second period respectively, $g^{l*}$ and $E - \mu(g^{l*})$.

It is more interesting to consider a situation where the first period government is free to run a deficit or a surplus. This is interesting because the debt handed over to the second period government is a state variable - the deficit influences the valuation of public consumption in the second period. To see this note that taxation gives rise to a distortion ($\mu_T > 0$) and the marginal efficiency loss is an increasing function ($\mu_{TT} > 0$) of the tax-rate which means that the second best optimal tax-structure - given an arbitrary level of government consumption in the second period - is to smooth taxation between periods (see Barro [1979] for more about tax smoothing and government debt). The tax smoothing principle tells the first period right wing government to run a surplus to minimize the intertemporal deadweight loss of taxation. Put differently, the right wing government would run a surplus if public expenditures in the the second period were exogenously given and equal to, $g^{l*}$. But, in this model, public consumption in the second period is not exogenously given, it's chosen by the left wing government and the more taxes that are collected in the first period the higher will public consumption be in the second period.

The budget constraint faced by the government in the second period is given by, $T_2 = g^l + D$, where $D$ is the deficit from the first period that must be paid back in the second period $D = g^{r*} - T_1$. Substituting this into the first order condition for optimal public consumption in the second period gives:

$$\frac{W_g(c_i, g_i; \gamma)}{W_c(c_i, g_i; \gamma)} = \mu_T(g^{l*} + D).$$

This condition shows that optimal supply of public goods in the second period is a function of the deficit in the first period. Hence the first period government can influence the level of public consumption in the second period through its debt policy, and it is easy to check that a higher deficit in the first period means lower public consumption in the second period. Persson and Svensson show that if the government in power in period one knows that it will be replaced by a new government with preferences for higher public expenditures, it will strategically collect too little taxes in the first period. By collecting little taxes the conservative government increases the marginal cost of public consumption in the second period and so it induces the left wing government to make choices more in accordance with its own preference structure. But the strategic debt policy is not without costs. The total level of government consumption is financed in an inefficient way since the tax burden is not perfectly smoothed between periods. But as is shown by Persson and Svensson the incumbent government is willing to pay this price in terms of not minimizing the efficiency loss for a given government expenditure in order to get less public consumption in period 2.

A numerical example will be illustrative. Let $g_i$, $c_i$ and $T_i$ be public consumption, private consumption and taxes collected in period $i = 1,2$. The right wing government's preferences is represented by:

$$W^r(g_1, c_1, g_2, c_2) = \sum_{i=1}^{2} \gamma^r \ln g_i + c_i.$$

The left wing government has preferences represented with:

$$W^l(g_1, c_1, g_2, c_2) = \sum_{i=1}^{2} \gamma^l \ln g_i + c_i.$$

The government's budget constraint in period 1 is given by: $T_1 = g_1 - D$. In the second period the government must pay back the deficit in first period, $D$: $T_2 = g_2 + D$. Normalize total production in each period to be one, $E = 1$. The government collects a fraction of the production through taxation, $T_i \in (0,1)$.

In order to find optimal policy in the first period the right wing government must solve the left wing government's maximization problem, that is, it must find how $g_2$ depends on the deficit, $D$ (it must optimize backwards). Assume $\gamma^l = 0{,}6$ and $\gamma^r = 0{,}2$ and let the tax cost function be given by, $\mu(T_i) = (1 + T_i)T_i$.

In the second period the left wing government solves:

*Maximize*            $0{,}6\ln g_2 + c_2$,

subject to the constraint:    $c_2 = 1 - (1 + g_2 + D)(g_2 + D)$

From the first order condition it can find $g_2$ as a function of $D$, $g_2(D)$, this gives the optimal level of public consumption in the second period as a function of the deficit in the first period:

$$g_2(D) = \frac{-(2D + 1) + \sqrt{(2D + 1)^2 + 4.8}}{4}$$

$$\Rightarrow g_{2D}(D) = -0{,}5 + 0{,}5[(2D + 1)^2 + 4{,}8(2D + 1)]^{-0{,}5} < 0 \quad \text{for all } D.$$

The right wing government takes this best response function into its maximization problem, which then is given by:

*Maximize*: $0{,}2\ln g_1 + 2 - (1 + g_1 - D)(g_1 - D) + 0{,}2\ln g_2(D) - (1 + g_2(D) + D)(g_2(D) + D)$

Solving this problem gives (* for optimal values) $g_1{}^* = 0{,}14$, $D^* = -0{,}02$ and $g_2{}^* = 0{,}37$. This implies that $T_1{}^* = 0{,}16$ and $T_2{}^* = 0{,}35$ which means that taxes are far from efficiently distributed (smoothed) between periods. Given $g_1{}^*$ and $g_2{}^*$ the excess burden is not minimized. If the government could write a binding contract specifying that $g_1{}^*$ and $g_2{}^*$ should be implemented, the first period government would choose $D = -0{,}115$ ($< -0{,}02$, i.e. a higher surplus) such that $T_1 = T_2 = 0{,}255$. Such a contract would generate higher utility for both governments.

So far it is shown that a right wing government - for strategic reasons - will accumulate too little assets. Another question Persson and Svensson ask is if it could happen that a right wing government finds it optimal to run a budget deficit $D > 0$, in the first period? In other words, is it possible that the conservative government runs a deficit that is higher than the deficit it would choose if it was certain to be in power in both periods (if it where in power in both periods it would choose $D = 0$). Such a government is called a stubborn conservative by Persson and Svensson, and it is not difficult to construct preferences so that a conservative government finds it optimal to run a deficit (is stubborn). Consider for example a situation where the right wing government has an extreme aversion against public expenditures getting above a certain level. To be more specific let $W^r_g(c,g) = - \infty$ for $g > 0,3$. For $0 \leq g \leq 0,3$, preferences are assumed to by represented by the utility function above.

Given these preferences the solution to the right wing government's maximization problem is clearly to choose a deficit that induces the left wing government to implement public expenditures equal to 0,3, that is, the conservative will choose $D$ such that $g_2(D) = 0,3$. If we solve this problem we find that $D = 0,2$. This government is a stubborn conservative one, it runs a deficit. Note also that in this situation the right wing government will increase its own supply of the public good, with $D = 0,2$ it is optimal to choose $g_1 = 0,2$. The reason for the increase in $g_1$ is of course that the marginal cost of public consumption in the first period is very low when the deficit is high.

Alesina and Tabellini [1989] and [1990] consider a situation with a positive probability that a government, preferring different *kinds* of public consumption from the incumbent one, is elected in the future. Here as in Persson and Svensson - where the conflict of interests were between the *level* of public consumption - the result can be that the incumbent government leaves too much debt (hand over too little wealth) for public consumption in the second period. The economic intuition

is straightforward: Consider an economy where individuals lives in two periods. The median voter in this period - period one - decides two things: The level of resources that should be used for public consumption in the first period (how much debt should be handed over to the second period) and how public consumption is divided between goods $g$ and $h$. Assume that the present median voter prefers much public consumption of good $g$ and little of good $h$. A positive probability that another person - preferring good $h$ to good $g$ - is the median voter in the second period will induce the median voter in period one to use "too much" resources. Why? The median voter is assumed to have convex preferences, he would like to smooth (public) consumption between periods. But of course if there is a positive probability that the bundle of public consumption in the second period is worth little to him (consists of much $h$ and little $g$ ), he may "over-consume" in this period when he is the one deciding the composition of public consumption.

### Empirical evidence

There is empirical evidence that politics matters; left and right wing parties seem to choose different policies, see Paldam [1989] and Alesina and Roubini [1990]. The point that it is possible to reduce future government spending by running a deficit now, is an old one. But whether governments really look forward and make decisions today with *strategic motives* is another question entirely, a question that is important but hard to verify.

### Constitution as a state-variable

There is a clear parallel between the argument made by Persson and Svensson and that made by Brennan and Buchanan [1977]. Brennan and Buchanan considered a situation where the majority (in period 1) knows that the state will be ruled by a Leviathan - a budget or revenue maximizing politician or bureaucrat - in the future. The majority in the first period can influence the choice made by the Leviathan by determining the basic rules or principles of taxation in the constitution (here the constitution is the state variable). In order to find the optimal tax

constitution the majority must trade off two different distortions: If the cost of taxing is relatively low - if distortions are small - one will get too much government consumption in the future. But, on the other hand, one would also like to see that a given level of government consumption is financed in a way that minimizes distortions.

This is exactly the same trade-off that the incumbent government in Persson and Svensson faces. The conclusion is the same: The government is willing to pay the price of greater- than-necessery tax distortions in order to get less public expenditures. In Brennan and Buchanan this is achieved through specifying a tax constitution with a narrow, rather than a broad tax base, and with a progressive rather then a proportional tax-rate. These results are in sharp contrast to the conclusions following from the normative theory of optimal taxation.

*Durability of projects as state-variable - policy uncertainty and a bias towards durable projects*

Amihai Glazer [1989] applies the idea that state variables can be used to influence future policy to study a more specific question. He shows that the present government - or median voter - can strategically precommit future governments through "building" durable projects. This creates a bias towards durable public projects even in situations where non-durable projects have cost advantages.

It is easy to show the logic behind his results. Imagine an economy that lasts for two periods. For simplicity there is no discounting between periods. The incumbent government chooses between two projects: One durable, modelled as giving gross benefits that the incumbent government values to be equal to $V$ in both periods (total gross benefit, for the incumbent government is $2V$). This project costs $C_d$. Another project is nondurable, it gives the same gross benefits $V$, but only for one period. A nondurable project costs $C_s$.

Glazer shows that a long-lived project will be preferred to a short-lived one in two different situations where such a project would not have been chosen if there were no policy uncertainty, that is, if the incumbent government knew it would be in power in both periods.

The first case he considers is when $2V < C_d < 2C_s$. Clearly neither of the projects would be implemented if the incumbent government knew it was to be in power in both periods. But this is not necessarily true if there is a positive probability that the present government will be replaced by a government that values the benefits of a short run project to be greater than its costs. In the presence of this possibility it can be optimal to build a long lived project in period one. The reason being that the incumbent government;

> *"fears that if a durable project is not built in period 1, then in period 2 the electorate will nevertheless build a short-lived project. If durable projects are efficient in the sense that $C_d < 2C_s$, the voter may find that the welfare is higher when a durable project is built"* [ Glazer page 1210]

Next Glazer shows that a durable project may be preferred in situations where the incumbent government views both projects as profitable, but the short lived project as more profitable than the durable one, that is if $2V > C_d > 2C_s$. This can happen if there is a positive possibility $p > 0$, that a government that will not build a short-lived project in period 2 is elected (a government with low valuation of the project's benefits is elected with probability $p > 0$). Let $C_d = 2C_s + e$, where $e$ is a positive number indicating that the short-lived project has a cost-advantage. The net benefit from building a durable project is $2V - C_d$, while the expected net benefit from building a short-lived project is $V - C_s + (1-p)(V - C_s)$ ( in period two there is a probability $p > 0$ that a short lived project is not built). The short-lived project is preferred to the durable one if: $2V - C_d < V - C_s + (1-p)(V - C_s)$..

Substitute $2C_s + e$ for $C_d$ and simplify the expression to get that the durable project is preferred to the short-lived if, and only if: $p > \dfrac{e}{V - C_s}$ .

This inequality will be fulfilled for $p$ sufficiently large and $e$ sufficiently small, that is, a durable project can be chosen even if the short-lived projects has a cost advantage.

# 4 Time-inconsistent Constraints

So far I have discussed policy inconsistency as a result of dynamic inconsistent preferences. Economic policy can, however, be time inconsistent even if the government's preferences over policy outcomes are perfectly stable over time. In fact most of what is written about time inconsistency problems in economic policy assume that preferences are stable. What is changing, as time passes, is the constraints imposed by the equilibrium behaviour in the private sector. The point is that if households' make decisions that to some extent are irreversible then their current actions depend on future policy and irreversibility means that households' reactions to a change in future policy depends on whether this change was announced in advance. This difference in policy responsiveness makes it optimal for the government to deviate from its announced policy. The government has changed its mind about what economic policy it should pursue, but the reason is no longer a change in its preferences over the set of feasible policy outcomes, the reason is that there has been a change in the feasible set of policy outcomes.

I think some examples will make the idea of time inconsistency caused by inconsistent constraints, clearer.

## The "capital levy problem"

The "capital levy problem" is the classical example of dynamic inconsistency in fiscal policy. The government's preferences over policy outcomes are stable: In the most stylized version the objective is to maximize the typical individuals utility, given some intertemporal revenue constraint ( that is, to minimize the "deadweight loss" from taxation). The government's budget constraint must be financed through taxation of two different tax-bases; income from capital and income from labour supply. Capital income is generated from savings done in the past. It is precisely this fact, that capital income in period $j$ is comes from a decision made in some earlier period $j^\cdot < j$, which gives raise to time inconsistent tax policy: Savings respond negatively - ignoring income effects - to increased taxation of capital income in period $j$ if the policy is announced before period $j'$, (before the saving decision is made in the private sector). In period $j$ the saving decision - made in period $j'$ - is irrevocably done. Higher taxation of the income that comes from this capital stock does therefore not create any excess distortions. On the other hand such a reform raises revenue and leaves therefore room for a reduction in labour income taxes, and a reduction in labour taxes will reduce the total efficiency loss from taxation since the magnitude of this tax-base is *not predetermined* by decisions made in the past[6]. Hence, the optimal thing to do for the government, when period $j$ arrives, is to increase the tax rate on income originating from savings done in previous periods, and lower the tax rate on labour-income. The ex-ante optimal tax plan - the plan announced before individuals in the private sector have made a commitment through their saving decision - is time inconsistent.

---

[6] Investment in human capital implies that this statement should be modified. But still it is true that at least one dimension of the labour supply - namely how many hours one is willing to work in period $j$ - is a decision workers are free to choose in period $j$.

In order to get more insight into what is going on - and to have a basic model to refer to - I will present the argument in a simple deterministic two-period model (this is the model used by Stanly Fisher [1980]).

A lot of identical individuals, represented by one of them, live for two periods (1 and 2). They consume in the end of both periods ( $c_1$, $c_2$), but work ($l$) only in the second. Borrowing money is not possible. To get any consumption at all in the first period each individual are given an endowment $e$ in the beginning of the first period. For simplicity I assume that the investment technology is linear. The resources that are available in the end of period one; $Re$, where $R > 1$ can either be consumed or saved ($s$) for consumption in the second period.

The government has an exogenously given income requirement; $G$, in period two. It does not levy any taxes in the first period, but in order to pay for its spending in the second period taxes are levied on capital and labour income, $\tau_s$ and $\tau_l$ , in this period, let $\tau_i \in [0,1]$, $i = s,l$. The timing of important events is shown in figure 3 .



| $t_0$ | Period 1 | $t_1$ | Period 2 | $t_2$ |

The government announces policy for period 2

Individuals invest their endowment e.

Individuals decide how much to consum and how much to save.

The government implements period two policy.
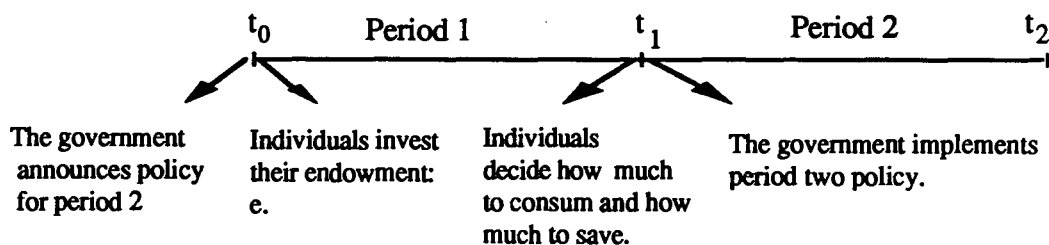
*figure 3*

The representative individual's behaviour is summarized in the following programming problem:

Maximize: $\qquad\qquad\qquad\qquad U(c_1, c_2, l),$

subject to $\qquad\qquad\qquad\qquad c_1 + s = Re$

$$c_2 = wl(1 - \tau_l) + Rs - (R - 1)s\tau_s$$

The utility function is assumed to have the usual convexity properties. From the first order conditions we find labour supply and saving as functions of the tax parameters; $s = s(\tau)$ and $l = l(\tau)$, where $\tau = (\tau_s, \tau_l)$. Substituting these expressions into the utility function gives us the indirect utility function, $V(\tau)$. The government's problem is then given by:

Maximize: $\Lambda(\tau, \lambda) = V(\tau) + \lambda(g(\tau) - G)$
$\tau$

Where $g(\tau) = \tau_s (R - 1)s(\tau) + \tau_l l(\tau)$ and $\lambda$ is the Lagrange multiplier.

Define $\tau^*_s, \tau^*_l$ to be the tax parameters that solve the first order condition. That is,

$$\frac{V_{\tau_s}(\tau^*_s, \tau^*_l)}{G_{\tau_s}(\tau^*_s, \tau^*_l)} = \frac{V_{\tau_l}(\tau^*_s, \tau^*_l)}{G_{\tau_l}(\tau^*_s, \tau^*_l)} \qquad\qquad (*)$$

This equation is an instance of the well-known Ramsey rule; the marginal distortion of the last unit of revenue should be equated across different tax bases.

The tax policy that is optimal from an ex-ante point of view, $(\tau^*_s, \tau^*_l)$, is in general not time consistent: Equation (*) does not hold when period two arrives (ex-post). It is easy to see why: If increased taxation of capital income is announced in the first period it will generate a substitution effect in favour of

consumption in the first period. This creates an efficiency loss. But when period two arrives then the capital is a stock variable. And taxation of income from an exogenous given tax base is of course a lump sum tax. It is therefore optimal to increase the tax rate on capital income and reduce taxes on labour income when period two arrives. In this example the best thing for the government to do ex post is to raise the tax rate on capital income to one; $\tau^*_s = 1$. The tax rate on labour income must be given by the tax rate that solves the budget constraint when $\tau^* = 1$,

$$G - (R - 1)s(\tau^*) - \tau_l l (\tau^*_s = 1, \tau_l) = 0.$$

In appendix 1 I have gone through these arguments more carefully in a quasi linear model.

*Why is time inconsistency a problem?*

Why do we talk about the incentive to implement a tax reform in period 2 - a lump sum tax on existing capital ( capital accumulated in the past ) - as a *problem*. After all the government is benevolent and the policy reform is implemented to increase the welfare of the representative individual. Higher tax rates on income generated from tax bases that have become more inelastically supplied is consistent with - or follows directly from - the Ramsey principle of taxation. Why then, do we speak of the *time inconsistency problem*?

Time inconsistent policy - caused by time inconsistent constraints - is a problem because intelligent individuals in the private economy will see through the structure of the government's problem. They will realize that income from savings done in period 1 will be taxed at a high tax rate when push comes to shove - that is, when the second period arrives - and they will base their saving /consumption decision in the first period on this. So if it is possible for the government to change its tax policy in the second period people will predict high tax rates on capital income and low tax rates on labour

income and little will be saved. In the two period model above it was optimal to implement 100% taxation of capital income in period two. Zero saving is the best response households have to this policy. The equilibrium in this game is therefore characterized by a low (no) capital stock and poor economic performance. This outcome is clearly sub optimal, proved simply by the fact that a high tax rate on capital income, and low tax rate on labour income, was a feasible tax policy to announce ex-ante, but it was dominated by the ex-ante optimal policy ( See in the appendix for a numerical example).

### Is time inconsistent constraints a more general phenomenon ?

The desirability of a one-time tax on existing wealth - a capital levy - has long been recognized in economic literature (see Eichengreen [1989] for more about the history of "the capital levy"). The credibility problem that follows for capital taxation was first discussed in Kydland and Prescott [1977], [1980] and in Fischer [1980][8].

Real or physical capital is just one type of wealth. The same credibility problem is, however, present for all types of wealth accumulation since the decision of how much wealth to hold at any time must - logically - have been made in the past, and increased taxation of income from tax bases that are determined in the past will do not give any excess distortions.

### Inflation and debt repudiation.

In the literature it has for example been pointed out that time inconsistency of policy constraints will make it tempting for governments to inflate the economy. There are two reasons for this.

First, unexpected inflation is equivalent to a lump sum tax on the holders of money. In a two period model with money one will typically announce some inflation in the second period in order to smooth taxation between

---

[8]But - although not often referred to by those writing about time inconsistency in capital taxation - Kelvin Lancaster had pointed out the same credibility problem in a somewhat different context, in his article in The Journal of Political Economy in 1973.

different tax-bases (and inflation is a tax on money). Ex-ante inflation creates an efficiency loss through the negative substitution effect on holding money. To find optimal inflation the government must trade-off the inflation distortion against distortions created by taxation of other bases, see Phelps, E.S [1973]. But whatever the ex ante optimal inflation rate is the government has an incentive to increase inflation when the second period arrives and individuals show up with a certain amount of money, for then the tax base - the quantity of money - is given, see Calvo [1978].

Second, inflation is a way for the government to devalue the real value of its nominal debt to the private sector.[9] A government need, however, not inflate the economy to get rid of the burden of public debt. It can simply refuse to pay it back. This fear of repudiation of public debt may make it difficult for the government to sell any debt all - even if it is indexed, see Calvo [1987] - this, in turn, can make it difficult for the government to smooth taxation between periods.

### Dynamic inconsistent tax policy in an economy without wealth taxation

Wealth , whether it is held in real capital, money or in bonds or securities issued by the government, is logically determined by decision made in the past. Thus savings and wealth will in general give rise to time inconsistent constraints since the tax base is elastic ex ante - before assets are accumulated - but completely inelastic ex post - after assets are accumulated.

The other major tax base in an economy is labour income. Labour supply in one period, say *j*, is of course not completely fixed by decisions made in the past[10]. Labour supply will therefore not become totally inelastic when period *j* arrives. But dynamic inconsistency arises in fiscal policy whenever

---

[9]There is still another reason why the government would like to increase the quantity of money ex-post. If unemployment for some reason is to high and the (short-run) Phillips curve is not vertical (because of (nominal) wage contracts) surprise inflation can be used to increase employment, see Kydland and Prescott [1977]. This literature is surveyed in Persson[1988], Persson and Tabellini [1990] and Rogoff [1986].

[10]See ftn. 9.

there is a *change* in the elasticity of a tax - base. The relevant question is therefore whether the elasticity of labour supply in period $j$ - with respect to after tax wage in period $j$ - is the same in period $j'$ and $j$, where $j' < j$? If not, policy constraints are time inconsistent and the government will most likely announce time inconsistent policy. And in general the elasticity of the labour income tax-base also will change over time. This means that fiscal policy will be dynamic inconsistent even if there is no "capital levy problem" present.

What is going on can be illustrated by informal reasoning in a two period model. A benevolent government produces public goods, $G$, only in the second period, it can tax labour income in both periods but is constrained from taxing capital income ( otherwise we would run into the "capital levy problem"). The two step maximization problem faced by the government in the beginning of the first period is solved by first finding the best response functions in the private sector, that is the labour supply as function of tax-parameters; $l_i(\tau_1,\tau_2)$ where $i =1,2$ , then the government decides how much it should produce of public goods given the budget constraint, $\tau_1 l_1(\tau_1,\tau_2) +$ $\tau_2 l_2(\tau_1,\tau_2) = G$ (pre tax wage is assumed to be fixed equal to one and interest rate to zero). Let $L(\tau_2; \tau_1{}^*)$ be the labour supply as a function of tax rate in period two when the first period, and the labour supply in that period, is history. The response in $L(\tau_2; \tau_1{}^*)$ for a small change in $\tau_2$ will never be greater than the response in $l_2(\tau_1, \tau_2)$, or in other words the elasticity of labour supply will in general decrease as time passes. It is easy to understand why; households are prevented from making any intertemporal adjustments if there is an increase in the second period taxation and this increase is announced in the end of the first period. This means that individuals have more flexibility to adjust to a change in $\tau_2$ in the beginning of period 1 than they have in the end of period 1. It follows from the Le Chatleier - Samuelson principle that the response in $L(\tau_2{}^*; \tau_1{}^*)$ will always be equal or smaller than the response in $l_2(\tau_1{}^*,\tau_2{}^*)$ for a given increase in $\tau_2{}^*$. But we cannot conclude from this that the government always has incentives to change taxation ex-post, one

must also take into consideration the effect announced changes in second period taxation has on first period labour supply. But the argument above makes it clear that there exists situations where the government has incentives to increase taxation and produce more public goods in the second period than was announced in period 1. That is, even in economies where labour supply is the only tax base can policy be dynamic inconsistent (See Persson and Tabellini [1990] for more about this ).

The dynamic inconsistency that occurs in the taxation of labour income is, in my view, primarily of academic interest and has less practical relevance than the "capital levy problem". First and foremost because it is unrealistic to assume that one can implement a tax system where workers are treated differently depending on how many "periods" they have left to work before retirement. Such a tax structure would require a lot of information about each worker and it would give substantial possibilities for arbitrage between workers at different stages in their career[11]. On the other hand, the only information needed to impose a capital levy is the distinction between old and new capital.

But even for capital taxation and the debt policy predictions from theory are counterfactual: In most economies one does not observe confiscation - or sharp increases in taxation - of old capital. This must mean that lump sum taxation - in terms of a surprise tax - must generate welfare costs that are not captured by the standard model. This is an important observation since it is

---

[11]If the government has this much information about each individual tax payer other more serious credibility problems will arise. It is now widely accepted that limited information about the tax payers abilities provides the reason for distortionary taxes. But if tax authorities should keep track of the tax history of every individual tax payer they would collect a lot of information about his ability as time passes. One should therefore assume that less distortionary taxes could be implemented after a while. Rational tax payers will see trough this - they will recognize that the information revealed by past actions can be used to implement future taxes which leaves them less rents - and, hence, they will be reluctant reveal their type. Hence, unless the government can commit not to use the information it accumulates, one will end up with severe "pooling problems" and highly distortionary behaviour in earlier periods - this is called the ratchet effect in the theory of regulation.

difficult to make consistent calculations of the total welfare impact of a tax reform with lump sum elements, if we are not able to recognize the costs that follows from lump sum taxation.

# 5 Solutions to the Time Inconsistency problem.
## - Policy constraints are in general time inconsistent - but there is no general time inconsistency problem, why?

The standard optimal tax model predicts that governments will be tempted to behave dynamic inconsistently with respect to e.g., capital taxation. In real life we don't observe the kind of economic behavior one would predict - not in a large scale anyway - if dynamic inconsistency was a serious problem. Wealth is taxed moderately in most economies, and households do accumulate capital. So, there must be some mechanisms out there, mechanisms that are not included in the standard model, that restrain the use of capital levies. A positive theory of fiscal policy should be able to point out these mechanisms, and from a normative point of view it becomes important to find new institutions that can make it credible that the announced fiscal policy will be implemented. The design of such "commitment-institutions" may give greater welfare gains than the search for the optimal policy that are based on an assumption of once-and-for-all-optimization.

*What do we have to explain?*

Confiscation of wealth is, from an ex - post point of view, an attractive type of taxation (lump - sum), the problem is that in a perfect foresight equilibrium the attractiveness will be perceived by households and it will lead to low capital accumulation and poor economic performance. This leads to the time inconsistency

problem that can be summarized as a situation where *"The temptation to push the economy towards the 1st best, drives the equilibrium from the 2nd to the 3rd best"*, Torsten Persson [1988].

The puzzle is that in real life we do not observe a 3rd best situation with no (low) capital accumulation.

### What explanations can there be?

The problem that inefficient choices are made ex-ante because there is a fear of opportunism ex-post, occurs invariably in economic relationships. It arises when buyers or sellers in a market invest in relationship specific assets, e.g. when a worker invests in job-specific human capital or when a firm learn to use a technology supplied only by one upstream vendor.

The general problem is that when $A$ gets locked into a relationship with $B$ through some relationship specific investments, his bargaining strength is considerably weakened ex-post, that is, after the investment is undertaken. Ex-post $A$ cannot make a credible threat that he will switch to another partner $C$ if he does not get "his share" of the return of the investment. $B$ has substantial bargaining power if the investment made by $A$ has no value outside their relationship. And in the destructive world of economics he will take advantage of his bargaining power unless he somehow has commited himself not to do so. For this reason it is difficult to bring about the efficient amount of relationship specific investments: $A$ will not make any investment, or - in more general terms - make too little investment in a relationship with $B$ if $B$ is essential for the return to be generated. A better outcome would be achieved if $B$ could commit himself not to take advantage of the change in $A$'s responsiveness ex-ante and ex-post. Since credibility problems in fiscal policy is just a specific example of a more general opportunism problem, two questions should be asked:

The first is: What is - or could be - done to solve the *general* problem where "aggressive" ex post incentives generate large distortions in choices made ex ante?

The second is: Could these solutions be to any help in order to solve the specific problem where a government faces inconsistent policy constraints and thus a credibility problem?

An obvious remedy for opportunism problems in economics would be for the parties to write a long term contract. True, the problem of ex ante inefficiency due to opportunism would be solved if *complete* long term contracts specifying all relevant variables, could be written.[12] But it is unrealistic to assume that such contracts can be written. It will often be impossible to specify all contingencies in the contract in advance, and if not impossible, prohibitive costly to do so. Another complication, pointed out by Hart and Moore [1988], is that the relevant contingencies must be specified in such a way that if there is a violation of the contract then it must be enforceable in court. It is therefore realistic to assume that only incomplete contracts, leaving much scope for opportunism, can be written. This means that even if one legally could treat the government as an ordinary contract partner, it would be close to impossible to make irrevocable commitments - through contracts - not to take advantage of the lock in effects that follows from some of the decisions made in the private sector.

Additional problems arise when the government is one of the contracting parties. A government cannot be treated like an ordinary contractor. First and foremost political democracy prohibits one government to determine the fiscal policy in the future: In a democracy it is impossible for a government to sign binding contracts on how fiscal issues should be solved in the future. This point was stressed in section 3. Second, even if a contract is not violating the spirit of democracy - or if there is no democracy - one still has to deal with the fact that the government is the body in control of laws and rules that should guarantee enforceability of contracts. This would not be a problem if the government could commit itself not to change the law. But - again - in a democracy it cannot. The

---

[12]Not only would it solve the "opportunism" problem of diverging ex-ante and ex-post incentives it would also make about half of what is written about economics the last ten years superfluous.

point is that the present government cannot limit the set of future policy instruments. Thus, signing contracts or policy-making by law cannot be the solutions to credibility problems caused by inconsistent policy constraints.

We must therefore direct our attention towards institutions and mechanisms that increase the costs of implementing a policy reform - institutions that make it costly to deviate from announced policy. This leads us back to section 3 where I claimed that the current government must resort to indirect means in its manipulation of future policy. The only way to assure that the future policy makers will adhere to the initial plan is by raising the marginal cost (or reduce the marginal gain) from implementing a surprise policy. It is from this perspective one can claim that policy making by law has an impact on future policy. These ideas were discussed in section 3.

The point of departure, the reason why a policy plan is time inconsistent , is not the same here as in section 3 where time inconsistent policy was caused by inconsistent preferences. This difference has e.g. an important bearing on what we can say about the *desirability* of a commitment. In section 3 I argued that it is difficult to say anything definite about whether the possibility of making a precommitment to a specific policy plan is a vice or a virtue. When time inconsistency is caused by a change in the government's preferences different governments view precommitment differently. There is no such ambiguity when time inconsistency is caused by a change in the policy constraints imposed by a change in individuals response to economic policy. The government's preferences are assumed to be stable in this situation and hence we can say, without making any reservations, that the possibility of making a credible commitment to adhere to the ex-ante optimal policy, will increase welfare. To be sure, if uncertainty is introduced there arises a trade off between the flexibility to meet exogenous shocks with the optimal policy and the value of commitment in order to avoid dynamic inconsistency problems. I will return to this tradeoff later, for the present I assume that there is no exogenous uncertainty.

It is not only the way to view commitment that is different here and in section 3, the kind of commitments we are looking for, are also different.

In section 3 manipulation of state variables were discussed from a general perspective. Here the aim is to find out what is wrong with a theory that claims there are credibility problems whenever constraints imposed on fiscal policy are dynamically inconsistent. Something must be wrong - or missing - in the standard model since we do not observe serious credibility - or time inconsistency - problems in real life. What I will do in this section is to discuss what institutions and mechanisms (state variables) it is that can make it costly for the government to make a specific type of policy deviation. I will look for mechanisms that make it costly to implement a policy reform which takes advantage of the fact that certain choices made by individuals reduce their flexibility to adjust to a change in policy.

I have chosen to split the potential cost-sources of changing policy into two groups; short run costs and long run costs.

## 5.1 Short run costs

### 5.1.1 Distributional effects

In the two period model outlined above all individuals are equal. In reality they are not. In a society individuals differ both with respect to their preferences and endowments.

A capital levy - or a lump sum tax on old capital - will always be worthwhile for efficiency reasons, since it generates only income effects. But with heterogeneous individuals the income effects are important because they will be unevenly distributed across the population. In this case the tax structure does not only determine efficiency, that is, the excess burden of taxation, but also the distribution of income and welfare. This gives us the

familiar efficiency-equity tradeoff, a tradeoff between efficient taxation and the preferred distribution of the tax burden.

A surprise tax on existing capital will of course injure people in proportion to how much capital they have accumulated. In order to show how distributional preferences can reduce or even eliminate the incentive to increase taxation ex-post, imagine a society with only two groups, capitalists and workers. Capitalists live for two periods and accumulate all the capital in the economy. Workers make only one decision and that is the supply of work in the second period (I simplify and assume that the labour supply in the first period is exogenously given and that workers cannot save).

The government maximizes a social welfare function, $\alpha U(c_1,c_2) + V(C,l)$, where $c_i$ is consumption done by the capitalist in period $i$ =1,2 and $C$ is consumption by the worker in the second period , $l$ is labour supply and $\alpha$ is the relative weight given to the utility of capitalists. This welfare function is maximized under the constraint that some exogenous given government expenditures, $G$, must be financed through taxation of capital and labour income.

The basic structure of this problem is not different from the "representative household" model: Fix the welfare weight $\alpha$ and we have the "representative household" model. This means that for a given welfare weight the government will run into the familiar time inconsistency problem. The story is the same as before: Ex-ante the government announces a tax structure that minimizes the excess burden of taxation, but now the policy is *adjusted* for distributional considerations. But ex-post - when capitalists have made their saving decisions - an increase in taxation of capital income is non-distortionary from an efficiency point of view. The only way to restore optimality is through an increase in capital taxation.

There is, however, one difference between this model and the "representative household" model: As long as the welfare function is nice and concave it is not optimal to confiscate *all* capital income. The negative

distribution effect following from such a policy will outweigh the efficiency gain.

The key observation in this setup is that the ex-post optimal tax rate of capital is a decreasing function of $\alpha$, the welfare weight given to the capitalist's utility[13]. The more the social planner cares for the capitalists the more lenient will capital income be taxed ex-post. This implies that the *problem* of dynamic inconsistent capital taxation, that is, high tax rates on capital and low capital accumulation in the steady state equilibrium, is less serious the more weight utility of capitalists are given in the welfare function.

This example illustrates how distributional considerations can reduce the time inconsistency problem, a point discussed extensively in a more general framework by Rogers [1986]. The example also hints that from a normative point of view it is possible for a government - or a social planner - to make the ex-ante optimal tax structure credible by choosing the "right" welfare weight. To achieve this the social planner should distort his preferences in favour of the capitalists. He should choose a welfare function that gives more weight to capitalists than his "real preferences" does.

*Economic equilibrium and political institutions*

What does it mean that a social planner chooses a welfare function? This question can be turned into a more interesting one of how a society should choose a social planner or a government. The example above suggests that features of the economic equilibrium may have feedback effects on the political equilibrium in a society.

I will illustrate this point in the simplest possible setting. Consider a two-period model of an economy with $N$ individuals distinguished by how much capital, $e$, they are endowed with when they entered the economy. Labour

---

[13]The same is of course true for the ex-ante optimal tax-rate on capital income.

endowments and preferences are the same for every individual but capital is distributed over some closed interval. Assume that the government has only a redistributive function: It collects a fixed amount, $G$, of revenue from taxing capital and labour income and distributes it back to the private sector through equal lump sum transfers (each individual receives $\frac{G}{N}$ ). The only issue determined by the government is how the tax burden should be divided between labour and capital income. Of course individuals have different opinions about how the distribution problem should be solved. Those that are in possession of much capital prefer relative low taxes on capital income while those making a living mainly out of labour supply would prefer low taxes on labour income and higher taxes on capital income.

From this general set-up one could construct a simple model generating single peaked preferences, where the median voter - the median in the distribution over endowments - determines the tax-policy. From this model one can derive the result that the median voter would elect a social planner - a government - with different endowment , and therefore different preferences over tax structures, than he himself has. This follows directly from the observation that time inconsistency implies that the decisive voter - the one with the median endowment - deviates ex-post from the policy he finds optimal ex-ante. In a rational expectation equilibrium this yields little capital accumulation and poor economic performance. The median voter would therefore try to commit himself in order to make the ex-ante optimal tax structure credible. Electing a social planner endowed with much capital is one way for the median voter to make such a commitment. The median voter induces - through strategic election of a social planner - the policy he prefers ex ante to be implemented ex post[14]. This idea is developed in Persson and Tabellini [1989].

---

[14]The elected social planner with high endowments of capital will announce a very low taxation of capital - lower than the median voter prefers - but he will increase taxation ex-post to the level preferred ex-ante by the median voter.

*Policy conclusions*

It is far from realistic to assume that the objectives of the fiscal authorities are uni -dimensional and that preferences are governed solely by how much wealth it is in position of. Indeed, if it was like this, why bother with elections at all, one could achieve the same simply by transfering (or extracting) the "right" amount of wealth to (from) the social planner. In real life things are more complicated. In order to draw any policy conclusions at all one needs a more general model for government behaviour.

Unfortunately the results above turn out not to be particularly robust. Small complications of the model alter the conclusion that a more capitalist friendly government should be elected. Torsten Persson and Guido Tabellini [1990,b] construct for example a model where it is optimal for the median to elect a *less* capitalist friendly social planner. Here capital taxation is studied in an open economy with mobile capital. Again voters are distinguished by different capital endowments and the distribution of endowments generates a distribution of preferred level of taxation within each country. Their main result is that although integration of the European market in 1992 reduces capital mobility costs, and therefore generates harsher fiscal competition between countries, it will not necessarily result in inefficient low tax rates on income from the mobile capital bases (which is the conventional wisdom in this tax competition literature). Given the new economic environment in 1992, they show that the median voter in every country would vote in favour of a government with preferences to the "left" of his own preferences. The elected government has a low level of initial wealth and will therefore prefer higher redistributive taxes. Increased capital mobility will therefore not necessarily reduce the taxation of returns on mobile capital since the political effect; the choice of a more left winged government, tend to offset the economic effect[15].

---

[15]Note that the strategic effects points in opposite directions; transfer wealth to the elected representative in order to avoid the time inconsistency problem; extract wealth from the one elected in order to avoid to lenient taxation when capital becomes more mobile. One should

In both examples above representative democracy generates a better outcome than referendum: Strategic voting reduces the inefficiency generated by an anticipated capital levy on accumulated wealth and it reduces the problem of inefficient lenient taxation of capital income in situations with high jurisdictional mobility.

To summarize: Preferences over the distribution of income and welfare between heterogeneous households can reduce the dynamic inconsistency problem in taxation of capital income. More specifically it is shown that dynamic inconsistency is less of a problem the more the social planner cares about the welfare of the capitalists. I have also discussed how the economic equilibrium can change the political equilibrium.

### 5.1.2. Inertia in the political decision process

The point that the *way* policy plans are announced is a state variable, viz., policy making by law makes the *process* of changing policy somewhat more difficult or costly, hints in direction of a more a general argument: Every institution or mechanism that slows down the decision making process and gives a bias towards the status quo reduces the time inconsistency problem.

In economics we often speak about the government as a social planner. In reality governments are made up of a multitude of agencies, departments and so on. This organisation structure imposes institutional constraints on the decision process and leads to bureaucratic inertia and a certain amount of sluggishness. This is often viewed as an inefficiency in the organisation of the state and of course it has obvious disadvantages. But it is interesting to note that a lengthy and bureaucratic decision process in the public sector reduces the probability of ex-post opportunism and can therefore reduce the time inconsistency problem.

---

therefore expect increased capital mobility to reduce the time inconsistency problem in taxation of capital income.

### 5.1.3. Political costs of changing policy

Economic policy is constrained by political feasibility. Put differently, changing policy imposes "political costs" and these costs may prevent the government from implementing a policy reform that otherwise seems perfectly reasonable.

In his fascinating book, "Exit, voice, and loyalty", Albert Hirschman put forward the hypothesis that members in a group who cannot easily exit from it if an adverse policy reform is proposed, will use their voice to prevent the policy from being implemented. Translated into the terminology of public economics this reads: Those who supply tax bases that are more or less fixed, that is, tax bases with low supply elasticity of with respect to after tax return, will use their "voices" if taxes are increased. A surprise tax on existing wealth is of course motivated exactly by the inelasticity of this tax base; tax payers cannot "escape" because their tax base is inelastically supplied. But then one can argue that when they cannot escape they will use their "voice" in order to prevent the tax reform from being implemented. When "voices" are raised costs are imposed on the elected government. "Voice" can therefore discipline the government not to implement a surprise policy that takes advantage of the reduced responsiveness in the private sector.

An extreme example of such political costs is reported in The Economist (18-2-89):

> *"In December 1986, 15 people died in riots after the government announced it would stop subsidising"breakfast meal", a staple of the Zambian diet. The subsidy stayed."* [16]

A less extreme example of a situation where "voices" made the implementation of a policy reform very costly, is the introduction of a poll tax in the U.K.

---

[16]This citation is drawn from Kanbur and Myles [1990]. In their paper political constraints are imposed in a mechanic and static way, simply by restricting the feasible "policy space".

I have argued that an increase in taxation of a tax base imposes costs coming from two different sources;

*a*: High degree of exit - high elasticity - which means high efficiency loss,

*b*: Adverse distribution effects.

Here I argue that in addition to these two types of costs there is a third:

*c*: Tax-payers use of voice.

An increase in taxation generates low (no) costs from source *a* if the tax base is inelastically supplied. This is precisely what motivates the government to reconsider its optimal plan for capital taxation and therefore creates the credibility problem - ex-post this tax base inelastically supplied. I have already discussed how adverse distribution effects, costs from source *b*, can outweigh the efficiency gain and how this can reduce or even eliminate credibility problems. In my view costs from source *c*, "voice", is an equally important cost factor. Following Hirschman we can argue that the lower the degree of elasticity - the fewer exit possibilities there are - the more "voice" will there be if taxes are increased. With no possibility for exit, "voice" is the only alternative. Hence, if a tax base has low elasticity with respect to after tax return more resources will be used in order to increase the political costs of implementing a policy reform that increases the tax rate on income from this tax base. The political costs caused by "voice" depends on two factors - first the loudness of each "voice" and second how many voices there are[17]. This opens up for multiple equilibria. It seems that one can get a stable situation where few invests in assets that give little flexibility ex -post because there is a fear of increased taxation ex-post, and since there are few investors and the government has an incentive to increase taxation ( the "voice" cost is small). Another equilibrium is characterized by a situation where many invests in the irreversible asset because they think there is no fear of an increase in taxation ex -post, and their expectations are fulfilled since the "voice cost" of such a policy is very high (a lot of "voices").

---

[17]The loudness of a voice could be interpreted as the bargain power to those raising their "voice" - how influential they are.

The key observation with respect to the time inconsistency problem is that if politicians are sensitive to tax payer's use of voice they will favour tax reforms that increases taxes on bases that are *elastically supplied*. Such tax reforms are in sharp contrast to those advocated by economists, and explains therefore very well why there is a contradiction between the predictions from conventional *normative* theory in economics and real life policy.

This is not the right place to elaborate this idea any further, but in my opinion one can gain insight into real life phenomena by thinking along these lines. It is for example hard to find any other explanations than lack of political feasibility (high political costs - a lot of "voices" will be raised) as to why capital invested in owner-occupied housing still is so lenient taxed. Increased taxation of the imputed income produced by owner-occupied housing would both have a favourable lump sum element - increased taxation of old capital - at the same time it would "level the playing field" between the taxation of income from "productive and non-productive" assets.

## 5.1.4. Tax evasion

If people are prevented from adjusting their tax bases and taxes are increased, they may use their voices in order to increase the political costs of such a policy reform. A more drastic option is to refrain from reporting the income to the tax authorities. If a surprise tax on wealth increases the degree of tax evasion this would certainly make the policy less attractive. Data show some evidence that there is a positive correlation between the tax rate and tax evasion of capital income, see Poterba [1987]. I think that the surprise element of this tax reform - implementing another policy than the one announced - would reinforce this correlation, since a surprise policy would be regarded as unfair and thus in a way justify evasion of taxation.

## 5.2. LONG-RUN COSTS

### 5.2.1. Dynamic games and reputation

Time inconsistency problems in economic policy is just another example of the general point stressed in game theory, namely that it can be optimal for a player to limit his own strategy space - to commit himself not to take certain actions - if he is playing against rational opponents. This can be a wise strategy because by commiting oneself one can change the optimal behaviour of other players. The best known example of a problem where individual incentives brings about an inefficient outcome in a game situation is probably the prisoners dilemma: Given the dominance of playing "confess" , the prisoners would be better off if they could eliminate the possibility to confess ex-ante.

We know from the theory of games that there exists mechanisms that are able to solve the "puzzle" that the outcome in a game is Pareto dominated by other feasible outcomes. Repetition of the game is one such mechanism. If a "prisoners dilemma" game is repeated over and over, both the theory of supergames and experiments suggest that the outcome of the game can be improved. The reason is simple, repeated play allow players to respond to each other's actions, each player must therefore consider the reactions of his opponents when making his present decision. If the prisoners dilemma game is repeated, cooperation in the present round has a potential benefits it does not have if the game is played only once since it can extract cooperative behaviour from the other player in the *future*.

The analogy between the prisoners' dilemma and the dynamic inconsistency problem suggests that repetitions of this policy game may reduce the incentive the government has to deviate from the announced policy. If the government pushes up the tax rates on existing wealth this may induce people not to accumulate capital

in the future. Given such reactions it may be rational for the government to stick to the announced behaviour (announced taxes).

It is easy to construct policy games of the supergame type - games with (almost) perfect information and no state variables - where the ex-ante optimal policy - the Ramsey policy - is a sustainable plan, see Chari and Kehoe [1990]. In general what is needed for the "reputation" mechanism to work is an infinite number of repetitions , or uncertainty about when the final round of the game will be played. Furthermore since the cost of a surprise tax - or opportunism in general - comes in the future the social planner must not be too impatient. If the government is concerned only - or mainly - about what happens today, losses, showing up in the future, would not count for much. Impatience would therefore abolish the reputation effect and therefore the credibility of the ex-ante optimal policy.

One could claim that these are simple ideas and therefore need no equations for their explication. I do not agree, and I will sketch the formalization done by Chari and Kehoe [1990]. In their specification each period - say a calender year - is partitioned into two time intervals, or sub periods. In each repetition of the game - in each period - a two stage game is played. Thus, each round of the game corresponds exactly to the basic two period model I have presented above and I will use the notation used there and in the specific example in the appendix.

Period 1                              Period 2

Stage one                Stage 2        Stage 1              stage 2

Individuals   Individuals decide how   The government    Individuals   Individuals decide how   The government
invest their  much to consum and       implements policy invest their  much to consum and       implements policy
endowment.    howmuch to save for      in 1. period.     endowment.    howmuch to save for      in 2. period.
              the 2.stage.                                             the 2.stage.
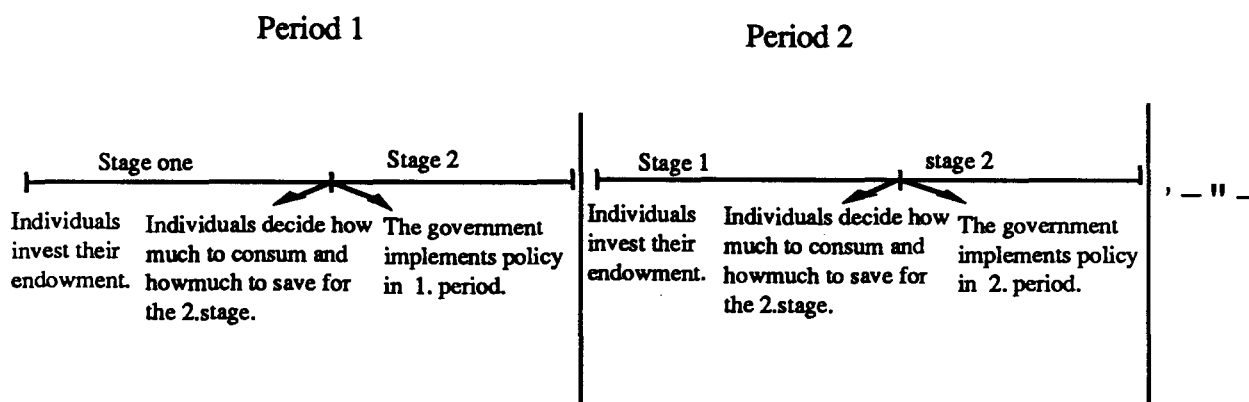
In the beginning of each round households receive resources that either can be consumed at the first stage or at the second stage. It is assumed by Chari and Kehoe that capital can be stored only between stages within each period and not between periods. To me this seems to be a bad way to model taxation and capital accumulation, and below I will argue in favour of an alternative, more dynamic approach. There are infinitely many periods in the game and the government is controlling a sequence of tax rates, starting in period 0; $T = ((\tau_{s0}, \tau_{l0}), (\tau_{s1}, \tau_{l1}), ...)$. Households have to determine two things, labour supply and how much they should save of the resources received at stage one in each period $i$.

Payoffs:

$V^*$ is defined as the per period utility for the representative household if ex-ante optimal - Ramsey optimal - taxes are implemented in the second stage of a round in the game, that is if $(\tau_s^*, \tau_l^*)$ is implemented, (See appendix 1).

Time inconsistency arises because the government can improve welfare by increasing the tax rate on capital income ex post (in the second stage). Using the notation in the appendix $V^{**}$ is the value of the per period utility if the representative household chooses saving and labour supply based on the announced policy - the ex-ante optimal policy - but the government cheats and deviates optimaly from the announced policy. Optimal deviation imply higher (100 %) taxation of income from the given capital stock and lower taxation on labour income. We know that this increases utility of the representative household: $V^{**} > V^*$. If households anticipate high taxation of capital income on the second stage and adjust optimaly - save nothing - welfare will be $V^0$ which is lower than $V^*$. Hence we have the following ranking of utility levels: $V^{**} > V^* > V^0$ ( see the appendix for proof). In a static game - only one repetition - we know that there is a unique equilibrium with payoff $V^0$.

The interesting question is whether or not the sequence of ex-ante optimal tax rates, $T^* = ((\tau_{s0}^*, \tau_{l0}^*), (\tau_{s1}^*, \tau_{l1}^*), ...)$ is a sustainable plan in a game stretching over infinitely many periods.

Strategies:

The sequence of tax rates up to period $j$ and the saving allocation done by households in these periods is called the "history" of the game in period $j$, and is denoted $h_{j-1}$.

A strategy in the game is a mapping from histories into allocation choices, saving, labour supply and taxes.

Assume that households announce that they will play according to a strategy, $\sigma$, saying that if the government cheats and deviates from the announced policy in one period each household chooses zero saving in every future round of the game.

Equilibrium:

Given the households' strategy and given that the government, if it deviates from announced policy, deviates as best as it can. then the gain from cheating is : $V^{**} - V^*$. The loss in each future period is $(V^* - V^o)$, which means that the discounted value of the loss is: $\frac{\delta}{1-\delta} (V^* - V^o)$, where $\delta \in [0,1)$ is the discounting factor. Hence given strategy $\sigma$ the net gain $(ng)$ from implementing a tax reform (optimal cheating) in one period is:

$$ng = V^{**} - V^* - \frac{\delta}{1-\delta} (V^* - V^o).$$

It follows immediately that the net gain of a capital levy decreases as $\delta$ - the degree of patience - increases. And since $V^{**} - V^*$ is finite there exists a critical value of $\delta$, call it $\delta^* < 1$ for which $ng = 0$. Thus for every $\delta > \delta^*$ the net gain from cheating is negative and the second best policy - the Ramsey policy - $T^*$ is an equilibrium policy if households play according to $\sigma$.

So far it is shown that the government will adhere to $T^*$ as long as households play according to strategy $\sigma$ and $\delta > \delta *$. Whether $T^*$ is a sustainable plan or not in a more general sense, depends on the credibility of the strategy $\sigma$.[18] It follows, however, from our one period analysis that households choosing $s_i = 0$ and the government choosing $\tau_{si} = 1$, is a Nash Equilibrium for every period $i \geq j$ whatever the history $h_{j-1}$ looks like; $\sigma$ is therefore a credible strategy (ignoring the problem of re-negotiations which perhaps is not a very serious problem in this set-up with a continuum of savers).

Conclusion:

A surprise tax - a capital levy - gives a short run gain in terms of an immediate increase in welfare but the policy generates future losses, and these losses can make it optimal for the government to follow the announced policy. Whether this mechanism works or not depends on the weight a government assigns to the future - it depends on the government's time preferences. Formally I have shown that $T^*$ is a sustainable plan provided that the government is not too impatient ( if $\delta > \delta*$).

The requirement that the government should not be "too impatient" may be too restrictive. Impatience, that is, the incentive to give priority to short run measures at the cost of lower welfare in the longer run, is often emphasised as one of the consequences of the political system. Politicians are in power for a short period of time only and will therefore be tempted to give priority to the present since there are less incentives to take present losses in order to build a reputation that mainly will gain their successors. But of course how impatient governments are - how

---

[18] This is where one usually starts to talk about sub-game perfection in these repeated games. It turns out that sub game perfection is not a very usefull equilibrium consept in a game like this where there is one big player and a continum of atomistic households, see the paper by Chari and Kehoe for more about this.

heavily they discount future gains and losses - is a question that can be answered only through empirical studies.

*Another more realistic and dynamic formulation of the policy game*

In the supergame formulation there are no state variables. The amount and the distribution of wealth - the capital stock - would be an obvious candidate for a state variable in this game. Charie and Kehoe exclude this possibility when they assume that it is not possible to transfer wealth from one period to another. Although such a modeling strategy simplifies the analyses a lot, it has the serious drawback of making it impossible to study aspects that intuitively seem important for the time inconsistency problem. One important state variable would be the debt position of the government. One would expect time inconsistency to be a more serious problem in countries where the public sector is heavily indebted, examples are Belgium and Italy where the debt/ GNP ratio is larger than one. It is hard to see how the public sector can solve a severe debt problem without implementing some kind of capital levy. Another important factor would be the wealth position and the portfolio that is held by private households. It is also easy to imagine that the temptation to impose a capital levy depends on *who* it is that can claim the income generated by the assets. It is for example reasonable to assume that the temptation to implement an ex-post increase in taxation is higher if assets are owned mainly by foreigners .

## 5.2.2   Social contract

Kotlikoff, Persson and Svensson, (KPS) [1988] study the time inconsistency problem in an overlapping generations model. Households live for two periods and they face exactly the same allocation problem as in the basic two period model above. The difference is that there are two generations present in every period. KPS assume that the young and the old generation have separate governments

called counsels. Only the elderly consume the public good, and their consumption is financed by taxation of the "old's" capital and labour income. The counsel for the old has, therefore, an incentive to impose higher taxation of capital income and lower taxation of labour income, than was announced when they were young (in the first period). Households perceive this and consume more of their initial wealth when they are young. How can the time inconsistency problem be solved in this model? Clearly the reputation mechanism does not work since households have finite lives, they live for two periods only, so there is no *future* punishment to worry about. The key observation is, however, that in this framework punishment can come immediately: The young generation can punish the old generation if their council deviates from the ex-ant optimal policy, which means that time inconsistency would not arise if a mechanism could be designed to make a punishment across generations credible. Kotlikoff, Persson and Svensson claim that a social contract could be the solution. They introduce a social contract that is owned by the old generation. The contract explicitly states that a capital levy shall not be implemented. The contract can possibly be sold to the young generation.

Assume that the young generation is willing to buy this contract if, and only if, the clause that rules out a surprise tax on capital income is observed. Clearly, the council of the old generation sticks to the announced policy if they can get a good price for the contract (the price must be higher than the value of the efficiency gain one would get by imposing a capital levy). The price is bounded from above by what it costs a young generation to "design" their own contract - KPS call this transaction costs. The crucial question is, however, whether the out-of-equilibrium-threat, the threat that the contract will not be bought if a capital levy is imposed, is credible. If it is not credible the old generation would both implement a capital levy and sell the contract and then we are in the middle of the time inconsistency problem again. KPS show, however, that the threat can be made credible if every generation plays according to a strategy saying that a broken contract will never be bought. Such a strategy profile implies that a broken contract has no value because the young generation cannot resell it when they are

old. Given these strategies no generation will unilaterally deviate from the out-of-equilibrium-threath.

Hence, if every generation plays according to this strategy, and if transaction costs are not to low, the time inconsistency problem will be solved. That is, with a social contract the ex-ante optimal policy of no confiscation of capital will be credible. This is proved formally by KPS.

Does this "social contract-solution" give any descriptive or normative insight ?

From a descriptive point of view it gives little insight. There exists no asset or social contract like this in real life, consequently this cannot be the explanation to why we do not observe an overall time inconsistency problem in societies. Research need, however, not be descriptive. But even if the scope of the analysis is normative , I find the social contract solution unsatisfying. First the solution with no capital levy in equilibrium is but one of many sub game perfect equilibria. Furthermore it is sustained by boothstrapping strategies: A broken contract has no value simply because every generation *thinks* it has no value[19]. The punishment is symmetric - it hurts both the old and the young generation - which implies that the solution is not renegotiation proof. A young generation would buy the contract, even if it was broken by the old, if they knew they could sell it to the next generation (for example if they could hide the information that it was broken). Hence, both the young and the old generation have incentives to renegotiate their strategies if a contract is broken. The threat that a young generation will punish the old generation by not buying the contract if a capital levy is implemented, is therefore not credible if we allow for joint deviations from the out-of-equilibrium-path , that is, a social contract does not solve the time inconsistency problem if we require strategies to be renegotiation proof.

---

[19]A contract's condition, whether it is broken or not, is not a state variable and does therefore not influence the set of actions that can be made in the future, or the pay-off these actions give.

# 6 Uncertainty and the value of flexibility

A government with inconsistent preferences, or a government facing inconsistent policy constraints, will try to precommit its future behaviour. A government, facing a time inconsistency problem would like to implement *a policy rule* in order to remove future policy discretion. But, to repeat, in a democracy future policy cannot be fixed by the present government. The government currently in power must use other and more indirect means to reduce future governments' flexibility and policy discretion. As long as we operate in a deterministic world, and the inconsistency is caused by inconsistent policy constraints, there is no question about the advantage of being precommited to implement specific policy actions in the future.

Precommitment has, however, its obvious disadvantages if there is uncertainty about "the state of the world" in the future. Then there seems to be a tradeoff between the *value of being flexible* in order to choose the right policy *given* the realized state of the world, and the *cost of flexibility* caused by time inconsistency. This trade off has been formalised in the literature on time inconsistent monetary policy, see e.g Canzoneri [1985], Rogoff [1985] and Torben Andersen [1989]. In Rogoff [1985] it is proved that a government will not precommit to a fixed money supply if the economy randomly is hit by supply shocks and stabilisation policies are worth while, that is, if wages and prices are not perfectly flexible. It seems to be a safe guess that the same is true for fiscal policy. One would for example not expect it to be optimal for a government to bind itself to implement a specific tax rate on capital income in the future if e.g. there is uncertainty about the government's revenue requirement in the future. In order to be more precise about this let us extend the basic two period model by including uncertainty about the revenue requirement in the second period. This is shown in figure 4.

| $t_0$ | Period 1 | $t_1$ | Period 2 | $t_2$ |

The government announces policy for period 2. At this stage the revenue requirement is uncertain.

Individuals invest their endowment: e.

Individuals decide how much to consum and how much to save.

The government implements period two policy, after the state of the world - the revenue requirement - is realised.
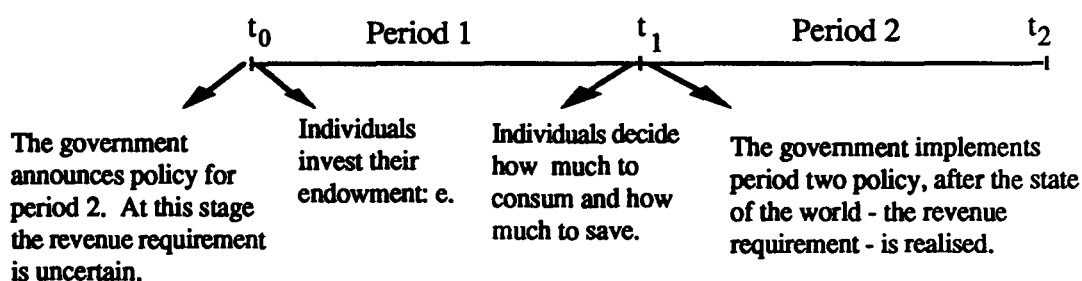
*figure 4.*

As an illustration imagine a very simple state space with only three possible revenue realisations, $G^l < G^m < G^h$; each is realised with a probability of one third. We know from the theory of optimal taxation that it is optimal to smooth taxes across different economic activities. The Ramsey principle of tax smoothing follows from the fact that the efficiency loss generated by taxing an economic activity is a convex function (approximately quadratic) of the tax rate. A commitment to tax capital income at a specific tax-rate is therefore inefficient if future revenue requirements are uncertain. Assume for example that the government precommits itself to implement $\tau^*_s$ in period two, where $\tau^*_s$ is defined to be the optimal tax rate on returns to savings *if* revenue requirement $G^m$ is realised. Now then if $G^h$ is realised all revenue above $G^m$ - the unexpected revenue requirement - must be financed solely by higher taxes on labour income. This is inefficient. But on the other side, complete discretion about second period policy leads to time inconsistency and inefficiency in terms low capital accumulation.

Hence if preommitment is viewed as a commitment to implement a specific policy action - a specific tax rate - it has its obvious costs and these costs must be compared with the value of commitment in terms of eliminating the time inconsistency problem. But in principle this dilemma, or tradeoff, can be avoided; the government can precommit to a *state contingent* policy rule. State contingent rules make it possible to smooth tax rates in every state of the world without creating a time inconsistency problem. In our concrete example the government

commits to a more complicated rule, a state contingent rule of capital income taxation that specify higher-than- normal tax rates on saving if the revenue requirement is high and vice versa if it is low. With this rule the government is precommited not to take advantage of the change in the elasticities of saving ex-ante and ex-post, but it is not precommited from smoothing taxes between all tax bases given the realised revenue requirement. It is therefore misleading when Alan Auerbach argues that one of the costs of using rules rather than discretion is that:

"..[T]he rules may constrain the government from responding to realizations in a stochastic economic environment, i.e., instituting a capital levy in times of severe fiscal need, or making the transition from income to consumption tax after it has been discovered on the basis of definite economic research that the latter is superior"

(Auerbach 1990, page 31).

*Optimal commitment rules and uncertainty*

To solve the time inconsistency problem the government need a policy rule that restricts future policy choices. In this policy rule it is only necessary to specify future policy along dimensions where private choices that are made today predetermine - more or less - the households' future behaviour. In the two period model a rule is needed only for capital income taxation, taxation of labour income does not need to be specified in period one since - at least in the model - labour supply in period two is independent of choices made in the first period.

With this in mind it makes sense to distinguish taxation rules with respect to their complexity. A natural classification is to say that tax rules are simpler the fewer tax rates it is that are contingent on the realised state of the world and the fewer states they are contingent on. Tax rates on all tax bases cannot be constant over all states; fulfilment of the revenue requirement prevent this. It is, however, possible to let all the unexpected revenue requirement be financed by taxes on bases that are fully reversible such that the tax rates specified in the rule are the same over all states: The government can fix the tax rate on capital income and let the unexpected high (low) revenue requirement be financed by high (low) tax rates on labour income.

Above I pointed out that the advantage of tax smoothing makes such simple rules inefficient. I will now argue that there exist arguments in favour of simple rules, e.g. a constant, non-contingent tax rate on capital income deriving from irreversible capital investment.

## Arguments in favour of simple rules

In general a policy rule is defined as a mapping from the governments information set to its policy actions. In the example of state contingent revenue requirement it is a mapping from the set of realised revenue requirements to tax rates on capital and labour income. One obvious constraint on the complexity of rules is asymmetric information. Taxes can only be contingent on states that are publicly known. Another point is that if the state space is very complicated and the rule specifies taxes that are contingent on every possible state, it may be difficult - or close to impossible - for households to verify whether the government actually behaves according to the rule or if it behaves opportunistically. Hence, in real life the government must announce simple rules e.g. rules where capital is taxed at the same rate across many different states of the world[20]. The argument in favour of a simple rule in a complex world, that is, rules that are simple relative to the state space, is fairly transparent and need no formal analyses.

There is, however, another and more interesting argument in favour of fixed or non contingent - "open-loop" - taxation of irreversible capital investment. This argument is based on the insight that in presence of uncertainty being flexible so that one can reconsider choices that were made in the past, has a value in itself - it

---

[20] See Barry Eichengreen (1989) for more about the difficulties with state contingent rules. In the literature on time inconsistent monetary policy this trade-off between flexibility to meet supply shocks with optimal monetary policy and the value of commiting to a specific monetary policy to avoid high inflation in equilibrium, is discussed by Kenneth Rogoff (1985), see also Persson and Tabellini (1990). Rogoff disregard contingent rules as a possible practical solution. Instead he argues that it might be possible for the society to choose between different "inflation-costs functions". The set of such functions is given by the set of possible central bankers. On the one side one would choose a "conservative" central banker, one who view inflation as very costly, in order to get low inflation in equilibrium. On the other hand it is not optimal to choose an ultra conservative banker since this one will never meet exogenous supply shocks with the right policy. It is, however, hard to think of a similar institutional arrangement in the taxation of capital.

gives the decision maker an option value. This problem cannot be analysed in the standard two period model illustrated in figure 4 since there is only one investment project available in that model; there is no portfolio choice present in the beginning of period one. The assumption of one type of capital is, however, not the important one, the crucial assumption is that investments are completely reversible. Capital invested at $t_0$ can, without costs, be converted to consumption at time $t_1$. If this is true one can separate the portfolio decision made at time $t_0$, and the consumption/saving decision made at time $t_1$: Individuals allocate their endowment at time $t_0$ in order to maximize the value of the portfolio at time $t_1$ they need not look further ahead because their current portfolio choice does not put any constraints on allocation choices that can be made in the future. This kind of separation is not possible if some of the projects or capital assets are irreversible - modelled for example as giving a payoff only in the end of the second period. A state contingent tax rule generates uncertainty about future tax rates and investments in liquid assets give an option to increase consumption in period one if the bad state - high revenue requirement - is realised in period two. This option is not present if capital is invested in an irreversible project. Hence, state contingent tax rates distort the portfolio choice in favour of liquid assets. This distortion is avoided if the government binds itself to implement a fixed tax rate on income generated from irreversible investments.

The problem of taxation when revenue requirement is state contingent is discussed in more detail in chapter 4.

# 7 Conclusions and further remarks

The ideas that are presented and surveyed in this paper represents first of all a *new* way to think about economic policy. The standard model is replaced by a more complicated framework where economic policy is determined as the outcome of a

*dynamic game.* Game theoretic models require a precise description of each player's payoff function and strategy set. Assumptions that are tacitly made in the standard approach are therefore uncovered when fiscal policy over time is studied as a game. Most important is the exposition of the implicit assumption that the government sticks to the plan that is optimal at the beginning of the planning period. For a long time this assumption was approved, it was not considered as ad hoc or as a short cut. I think one reason for this was that the formal framework one used to solve intertemporal fiscal problems, was the *static* Ramsey model.

Frank Ramsey [1927] addressed the following question: A government needs a fixed revenue. It collects taxes through the levy of flat rate excise taxes, $\tau$, on consumption goods. The government's problem is to find the vector of tax rates, $(\tau_1, \tau_2, ..)$, which minimizes the excess burden of taxation. Pigou [1947] was perhaps the first to argue that the Ramsey formulation could be applied to study fiscal policy over time. One could simply interpret the vector indexation to refer to different time periods instead of different goods as in the Ramsey model. One could apply the static model to find the optimal time path of taxes given an intertemporal budget constraint. To solve the problem, where one optimizes over a set of functions and not over a set of points, one must use optimal control theory or dynamic programming. Hence, new mathematical techniques were required. But in a more fundamental sense both problems, the static and the dynamic, were treated in the same manner since they were both viewed and solved as once-and-for-all-optimization problems. This is of course the natural way to think about the static Ramsey problem where taxes on all goods must be chosen simultaneously. But when the decision problem is an intertemporal one, this is different. In a dynamic setting decisions can - at least logically - be made sequentially: It is possible that the decision maker is free to reoptimize and reconsider past decisions about future policy as time passes.

One of the central messages in this paper is that the principle of democracy makes it impossible for a government to choose economic policy once-and-for-all. Hence, when we study the dynamics of economic policy it is not only "in

principle" that it is possible to think of this as a sequential problem, indeed, the rules of democracy imply that implementation of economic policy over time must be made sequentially. A government cannot precommit to a policy plan. The commitment assumption made in the traditional approach does therefore not describe very well how policy is implemented in real life. A relevant question is if this matters; "does it matter whether policy is chosen sequentially or once-and-for-all "? As Finn Kydland and Edward Prescott showed in their very important 1977 paper, it does.

## The applicability of the Ramsey framework

From a normative perspective economists give advise to a society to follow specific policy plans. With rational individuals plans are operational only if individuals belive that the announced policy will be carried out in the future, that is, rational individuals will only base their present behaviour on announced policy if they can see that it is optimal to implement the announced policy when the future arrives. This observation puts the applicability of the Ramsey framework seriously into question because a government that chooses its fiscal policy as prescribed by the Ramsey principle will not, in general, implement the policy initially found to be optimal. In other words the optimal Ramsey policy is, in general, time inconsistent.

As a defense for the Ramsey framework one could claim that changing policy is so costly that it prevents future governments from deviating from the announced policy. The argument is that even though we know that economic policy *has to* be implemented sequentially *it can be modelled* as if the present government faces an once-and-for-all-optimization-problem. It is tempting to argue that the absence of large scale credibility problems in the implementation of economic policy indicates that the problem of time inconsistent policy constraint can be ignored, and that we can solve dynamic policy problems by using the once-and-for-all-optimization-

framework. I think this view is fundamentally wrong. If it is deviation costs that discipline the government to implement the announced policy, then these costs should be modelled explicitly. Taking a short cut here will prevent us from getting insight into many interesting dynamic aspects of fiscal policy.

# Appendix

*An example of time inconsistency in capital taxation in a two period model with quasi linear preferences and linear technology.*

There are two periods. Households make only a saving decision in the first period: How much it should save (s) of the resources (e) it receives in period 1? In the second period it must determine labour supply (l) and consumption (c). Technology is assumed to be linear and preferences quasi linear. The government has a fixed revenue requirement (G) that must be financed by taxation of capital and labour income in the second period.

With these assumptions the household's problem can be written as:

*Maximize:* $\qquad U(c_{-1}, c, l) = Dc_{-1} + c + \ln(1 - l),$ $\qquad$ (h.1)

where $(D - 1) > 0$ is the rate of time preferences and $l \in [0,1)$

subject to the constraint: $\qquad c_{-1} = e - s,$

$$c = s[R - \tau_s(R - 1)] + w(1 - \tau_l)l. \qquad (h.2)$$

The government's objective is to:

*Maximize:* $\qquad U(c_{-1}, c, l),$ $\qquad$ (g.1)

subject to the budget constraint: $\quad G = \tau_s(R - 1)s(\tau) + \tau_l wl(\tau),$ $\qquad$ (g.2)

$s(\tau)$ and $l(\tau)$ are households' best response functions - best response in terms of optimal saving- and labour supply for different tax structures.

Let $V(\tau_s, \tau_l)$ be the indirect utility function; $V(\tau_s, \tau_l) = \max \{U(c_{-1}, c, l), \text{ given } (h.2)\}$

Assume $D > R$ and let $\tau_s^* \in (0,1)$ be defined $[R - \tau_s^*(R - 1)] = D$, $\tau_s^*$ is the tax rate that make the households indifferent between consumption in the first and the second period. By using an "epsilon argument" one can assume that $s = e$ if $\tau_s = \tau_s^*$. Labour supply is then given by $l(\tau) = 1 - \dfrac{1}{w(1 - \tau_l)}$. I also assume that $G > \tau_l(R - 1)e$ when $\tau_s = 1$, that is, the government cannot finance its expenditures solely by capital income taxation.

*First result*

Ex ante it is optimal to announce tax rates: $\tau_s^*$ and $\tau_l^*$, where $\tau_s^*$ is defined above and $\tau_l^*$ is given by: $\tau_l^*(1 - \dfrac{1}{w(1 - \tau_l^*)}) = G - (R - D)e$. This equation have two roots, the government should choose the one that is "on the right side of the laffer - curve", that is, the one that gives a positive derivative of the left side with respect to $\tau_l^*$. Given $\tau_s^*$ and $\tau_l^*$, denote the utility level of the representative individual $V^*$.

*Proof:*

Assume that $\tau_s > \tau_s^*$, this implies $s = 0$. Higher taxation of labour income is therefore required in order to fulfil the governments budget constraint. This means higher excess burden of labour taxation. In addition this policy imposes a

excess burden on the choice of intertemporal consumption equal to $(R - D)e$. Such a policy is therefore inferior to $\tau_s = \tau_s^*$.

Assume that $\tau_s < \tau_s^*$, this implies $s = e$ which means that there is no excess burden of capital taxation. This is the case for all $\tau_s \leq \tau_s^*$, but with $\tau_s < \tau_s^*$ the government collects less revenue from capital taxation than if $\tau_s = \tau_s^*$. Thus taxation of labour income must be increased in order to get enough revenue. Higher tax rate on labour income leads to higher total excess burden of taxation. Choosing $\tau_s < \tau_s^*$ is therefore inferior to choosing $\tau_s = \tau_s^*$.

## Second result

The ex ante optimal tax structure $(\tau_s^*, \tau_l^*)$, is not time consistent: It is not optimal for the government to implement this policy when the second period arrives. Formally: Define $(\tau_s^{**}, \tau_l^{**}) = \text{argmax } V(\tau_s, \tau_l)$ *given* that $s = e$. Dynamic inconsistency imply that $(\tau_s^*, \tau_l^*) \neq (\tau_s^{**}, \tau_l^{**})$. Let $V^{**}$ denote the value of the utility function if $(\tau_s^*, \tau_l^*)$ is announced and household base their choices on this policy but $(\tau_s^{**}, \tau_l^{**})$ is implemented.

## Proof.

When period two arrives households have made their consumption decision in period 1. If households base their decision on the announced tax rates this means that savings $s$ is set equal to $e$. But at this time - in period two - an increase in capital income taxation such that $\tau_s > \tau_s^*$ does not generate any excess burden. Higher tax rate on income from a given tax base generates, however, more government revenue, hence, increased capital taxation ex-post leaves room for lower taxation of labour income. Such a tax reform is welfare improving since there is an excess burden of labour taxation and this excess burden is reduced with a lower tax rate.

This argument can be repeated for every $\tau_s < 1$. This implies that $\tau_s^{**} = 1$ and $\tau_l^{**}$ is given by the tax rate that solves:

$$\tau_l^{**} (1 - \frac{1}{w(1 - \tau_l^{**})}) = G - (R - 1)e.$$

## Third Result

Given the 1. and 2. result households will choose $s = 0$ regardless what policy the government announces ex ante. Labour taxes alone must therefore finance government expenditures this leads both to high excess burden both in terms of a high marginal tax on labour and a large distortion in intertemporal consumption. Denote the utility level where $s = 0$, $V^o$.

## Proof:

We know that $\tau_s^{**} = 1$. Since $D > 1$ this imply that $s(\tau) = 0$.

## Forth Result

We have the following ranking of utilities; $V^{**} > V^* > V^o$.

## Proof:

$V^{**} > V^*$ follows from the fact that $(\tau_s^*, \tau_l^*) \neq (\tau_s^{**}, \tau_l^{**})$. If $V^{**} \leq V^*$ then $\tau_s^* = \tau_s^{**}$ but the 2. result shows that this cannot be true. Furthermore $V^* < V^o$

cannot be true by definition of $V^*$. If $V^* = V^o$ then $\tau_s{}^* = \tau_s{}^{**}$ but the 2. result contradicts this.

## A Numerical example

Parameter specification: $w = 2,5$ $e = 2, D = 1,05$ and $R = 1,1$ and $G = 0,25$.

*Optimal policy ex - ante.*

With these parameter values we get $\tau_s{}^* = 0,5 \Rightarrow s = e = 2$ and the tax income from capital taxation is 0,1. Labour supply is given by $l(\tau_l) = 1 - \dfrac{1}{2.5(1 - \tau_l)}$. The government revenue requirement defines $\tau_l{}^*$ by $[1 - \dfrac{1}{2.5(1 - \tau_l{}^*)}]2,5\tau^* = 0,15$

which gives $\tau_l{}^* = 0,11$ and labour supply $l(\tau_l{}^*) = 0,55$.

Maximized utility of the representative individual is then $V^* = 2,53$.

*Optimal policy ex - post.*

Ex -post, that is, when the second period arrives, it is optimal to increase the tax rate on capital income to 1. Capital taxation collects then a revenue of 0,2, leaving only 0,05 to be financed by taxation of labour income. Optimal labour taxes are $\tau_l{}^{**} = 0,03$ and labour supply is $l(\tau_l{}^{**}) = 0,59$.

Maximized utility of the representative individual is $V^{**} = 2,77$.

*Equilibrium.*

Private households will see through the government's incentives to raise taxes on capital ex-post, and they will choose ex ante not to make themselves vulnerable, that is , they will not not accumulate capital, $s = 0$. Government expenditures then

have to be financed solely by labour taxes. This means that the tax rate on labour income must be 0,2 and labour supply is 0,5. The welfare level is $V^0 = 2,40$.

# References

Alesina, A. [1987]: " Macroeconomic Policy in a Two-Party System as a Repeated Game" *Quarterly Journal of Economics*, 102, 651-78.

Alesina, A [1988]: "Credibility and Policy Convergence in a Two-Party System with Rational Voters" *American Economic Review*, 78, 796-805.

Alesina, A and N. Roubini [1990]: "Political Cycles: Evidence form OECD Countries" *Paper presented at the CEPR/IIES conference June* 14 - 16, 1990.

Alesina, A. and Tabellini, G. [1989]: "External Debt, Capital Flights and Political Risk" *Journal of International Economics*, 27, 199 - 220.

Alesina, A. and Tabellini, G. [1990]: " Voting on the Budget Deficit" *American Economic Review*, 80, 36 -49.

Andersen, T.M. [1989]:"Credibility of Policy Announcements" *European Economic Review*, 33,13-30.

Auerbach, A. [1990]: " Public Sector Dynamics" *NBER working paper #* 3508

Barro, R.J. [1979]: "On The Determination of The Public Debt" *Journal of Political Economy*, 87, 940 - 971.

Blanchard, O.J and S Fischer [1989]: "Lectures on Macroeconomics" *The MIT Press*.

Brennan, G. and J. M. Buchanan [1977]: "Towards a Tax Constitution for Leviathan" *Journal of Public Economics*, 8, 255 - 274.

Calvo, G [1978]: " On the Time Consistency of Optimal Policy in a Monetary Economy" *Econometrica*, 46, 1411-28.

Calvo, G [1988]: " Servicing the Public Debt" *American Economic Review*, 78, 647-661.

Canzoneri, M. [1985]: "Monetary Policy Games and the Role of Private Information" *American Economic Review*, 75, 1056-1070.

Chari, V.V. and Kehoe, P.J [1990]: " Sustainable Plans" *Journal of Political Economy*, 98, 783 - 802.

Eichengreen, B. [1989]: " The Capital levy in Theory and Practice" *NBER working paper* # 3096.

Elster, J. [1979]:" Ulysses and the Sirens; Studies in Rationality and Irrationality" *Cambridge University Press*.

Fischer, S. [1980]: "Dynamic Inconsistency, Cooperation and the Benevolent Dissembling Government" *Journal of Dynamics and Control*, 2, 93-107.

Glazer, A. [1989]: "Politics and The Choice of Durability", *American Economic Review*, 79, 1207 - 1213.

Hart, O. and J. Moore [1988]: " incomplete Contracts and Renegotiation" *Econometrica* 56, 755 - 785.

Hammond, P. [1976]:"Changing Tastes and Coherent Dynamic Choice" *Review of Economic Studies*, 43, 159-173.

Hibbs, D. [1977]: "Political Parties and Macroeconomic Policy" *American Political Sience Review*, 71, 1467 - 87.

Koopmans, T. [1960]: "Stationary Ordinal Utility and Impatience" *Econometrica*, 28, 289-309.

Kotlikoff, L. and T. Persson and L.G. Svensson [1988]: "Social Contracts as Assets: A Possible Solution to the Time-Consistency Problem" *American Economic Review*, 78, 662-677.

Kydland, F. and E. Prescott [1977]: "Rules Rather than Discretion; The Inconsistency of Optimal Plans" *Journal of Political Economy*, 85, 473-490.

Kydland, F. and E. Prescott [1980]: "Dynamic Optimal Taxation, Rational

Expectations and Optimal Control" *Journal of Dynamics and Control*,

2, 79-91.

Lancaster, K. [1973]: " The Dynamic Inefficiency of Capitalism" *Journal of

Political Economy*, 81, 1092 - 1109.

McCloskey, Donald [1986]: "Economical writing" *Economic Inquiry* , *188 - 221*..


Nordhaus, W. [1975]:" The Political Business Cycle" *Review of Economic

Studies*, 42, 169-90.

Paldam, M. [1989]: "Politics matter after all: Testing Alesina's Theory of Partisan

Cycles" *Aarhus University working paper*.

Peleg, B. and M. Yaari [1973]: " On the Existence of a Consistent Course of

Action when Tastes are Changing" *Review of Economic Studies*, 40,

391-401.

Persson, T [1988]: " Credibility of Macroeconomic Policy: A Broad Survey"

*European Economic Review*, 32, 519-32.

Persson, T. and L. Svensson [1989]: " Why a Stubborn Conservative would

run a Deficit; Policy with Time Inconsistent Preferences " *Quarterly Journal

of Economics*, 104, 325-345.

Persson, T. and G. Tabellini [1990,a]: " Macroeconomic Policy, Credibility and

Politics" *Harvard Academic Publishers*.

Persson, T. and G. Tabellini [1989]: "Representative Democracy and Capital

Taxation." manuscript.

Persson, T. and G. Tabellini [1990,b]: " The Politics of 1992; Fiscal Policy and

European Integration" *NBER working paper* # 3460.

Phelps, E.S [1973]:" Inflation on the Theory of Public Finance" *Swedish Journal

of Economics* 75, 67-89.

Phelps, E.S and R.A. Pollak [1968]: "On Second-Best Savings and Game

Theoretic Equilibrium Growth" *Review of Economic Studies*, 35, 185-199.

Pigou, A.C [1947]: " A Study in Public Finance" *Macmillan, London*.

Poterba, J. [1987]:" Tax Evasion and Capital Gains Taxation" *American Economic Review*, 76, 234-239.

Ramsey, F.P. [1927]: " A Contribution to the Theory of Taxation" *Economic Journal*.37, 47-61.

Rogers, C.A [1986]: "The Effects of Distributive Goals on the Time Inconsistency of Optimal Taxes" *Journal of Monetary Economics*,17, 251-269.

Rogoff, K. [1985]: "The Optimal Degree of Commitment to an Intermediate Monetary Target" *Quarterly Journal of Economics*, 100, 1169-1190.

Rogoff, K. [1990,a]: "Equilibrium Budget Cycles" *American Economic Review*, 80, 21-35.

Rogoff, K. [1990,b]: "How often should Elections be held" *Paper presented at the CEPR/IIES conference June* 14 - 16, 1990.

Rogoff, K. and A. Sibert [1988]: "Elections and Macroeconomic Policy Cycles" *Review of Economic Studies*, 55, 1-16.

Strotz, R. [1955-56]: " Myopia and Inconsistency in Dynamic Utility Maximization" *Review of Economic Studies, 23, 165-180.*

Weiszäcker, C.C. von [1971]: "Notes on Endogenous Change of Taste" *Journal of Economic Theory* 3, 345-372.

# Chapter two:

## How should a government induce production and employment in rural areas?
## The problem of time-inconsistent plans*

### Abstract

Capital grants are the mainstay of most countries' regional-incentive packages. From an economic point of view it is hard to understand why governments choose to give subsidies to reduce the cost of capital. For if the objective is to reduce the unemployment problem in a specific district, it seems reasonable to encourage labour-intencive firms to establish their production in the area. In this article I discuss this discrepancy between the policy recommended by economists and the policy carried out by politicians. I show that giving capital grants can be the best regional policy if important dynamic aspects of the policy problem are carefully considered.

# 1 Introduction

Normative questions concerning economic policy over time are usually studied in a framework where a social planner - the government - chooses a policy plan that maximizes a specific welfare function. The government takes the equilibrium behaviour in the private sector and the budget constraints into consideration when it chooses a policy plan. This model has been used by e.g., Atkinson and Sandmo [1980] to study optimal taxation over time. It has been used also to give advice for optimal production - and regulation - in public enterprises, and the

---

same framework has been applied to answer the more specific question of how one should choose policy instruments in order to reach desired production and employment targets in a backward area.

The framework that is used to study these dynamic policy problem is static in the sense that it assumes once-and-for-all-optimization, an assumption that corresponds to a situation where the current government determines the policy that future governments must carry out. This assumption is unrealistic. For in a democracy the government should be free to choose its own policy, it should not be dictated by decisions made by past governments. Assuming once-and-for-all-optimization violates therefore the basic principle of a democratic political system. The traditional model, where the current government announces a policy plan under the assumption that it is not possible to deviate from the plan in the future, does therefore not give an accurate description of how economic policy is carried out in real life.

The problem of making the once-and-for-all-optimization assumption was first pointed out in the late seventies in some important papers by Finn Kydland and Edvard Prescott and Guillermo Calvo. These papers spurred a new game theoretic modelling of economic policy. In the new approach the choice and implementation of economic policy is studied as a sequential problem where the future governments' incentives and possibilities to deviate from the announced policy, and the consequences this have for economic behaviour both in the private and the public sector, are modelled explicitly. One important lesson we have learned from the new way to model the dynamics of economic policy, is that the possibility of a change in policy midstream can introduce distortions and efficiency losses of a magnitude that is far greater than the losses generated by credible policy plans, even if these plans are sub-optimal from an ex-ante point of view. Put differently, often there is a greater potential for welfare gains in designing

institutions for pre-commitment than from designing optimal policy plans that are based on the assumption that the current government optimizes once-and-for-all. In this paper I study one example of this general principle.

### Regional problems and regional policy

I am concerned with an economy confronted with regional problems. The seriousness of this issue is indicated by the fact that all European Community countries struggle with some kind of economic regional problems (Allen et al [1981]). The concrete problems differ of course between countries but the basic ingredients are the same: There are backward regions where productivity is low and the economic situation is depressed and insufficient job alternatives - unemployment - is the main problem. The government (the majority) wants therefore new factories to locate their production facilities in the depressed area. In the first section of this paper I will take this objective as given. I will use the new game theoretic framework to discuss what policy instruments the government should use to generate production and employment in a rural area. The question is; how should the government stimulate new investments in the rural area?

The reason why I find the question of optimal regional policy interesting is the observation that politicians tend to choose policy instruments that are exactly the opposite of those usually recommended by economists. Politicians choose to give capital cost subsidies to attract new investment in backward areas with low productivity. Economists, argue that this is the wrong policy and that labour cost reductions is a better regional policy.

Before I turn to this contradiction between theory and reality I want to say something about the more fundamental question of why subsidies are given to a low productivity area at all.

At first sight the "work to workers" seems like a bad policy. The market signals that workers are more productive in other regions and hence that they should move to more centralized areas where their productivity is higher. There may, however, be substantial costs related to such a migration process. Large out-migration from rural areas to central areas will for example easily create crowding problems in central districts. In addition most of the capital in rural areas - houses, plants and other constructions - cannot easily be moved from the area and this type of capital provides no benefits if people leave the region. A migration process will probably also be very selective, leaving the most unable and unproductive behind. A segregation like this can be costly. More specific arguments, such as military considerations are also used as an argument in favour of bringing "work to workers". The argument is that a migration process is costly because it depopulates sensitive border areas.

The arguments above suggest that a migration flow from outlying regions can be costly, and should therefore perhaps be avoided. This does, however, not necessarily imply intervention from the government. Economic logic tells us that if efficiency is all what we are concerned about intervention is justified only if there are real costs or gains that are *external* to the decision makers in the market. To be sure, many of the migration costs, and the gains to the society from having people living in these areas (military considerations etc.), are truly external to each household in the region. Still, this does not explain why a government should choose a specific regional policy, e.g., a policy giving incentives to new investments in the low productivity region.

Following the same economic logic, justification for the "work to workers" policy must be based on a view that location of new investments in the depressed region has positive effects that are external to investors. The most important argument in this direction is undoubtedly that new investments permit utilization of

resources - in particular human capital - that otherwise would remain unused. The argument makes sense only if there are imperfections in the price mechanism. The imperfection usually pointed out is a rigid wage level: The wage level does not adjust downwards in situations of excess supply of labour. The reason can be that unions bargain for a nation-wide (minimum) wage. If the wage is set higher than the competitive wage, given by the productivity of the marginal worker in a region, there will be unemployment in regions where productivity is low. If this is the case the shadow price of labour is lower than the market price and this provides an argument for government intervention. The government should give incentives in order to stimulate firms to locate their plants in the rural area.

In principle incentives could be given through lump sum transfers to firms that locate their production facilities in the rural area. In practice i.e., in most countries, regional incentives are given through subsidies that reduce the production costs. If we restrict the regional policy programs to provide cost reducing subsidies, the government has two policy alternatives: It can either reduce the cost of capital or it can reduce the cost of labour.

Economists argue in favour of using labour cost subsidies to attract new production to low productivity regions (see Lind and Serck-Hansen [1972]). The "economist-argument" is basically the one hinted at above: An imperfect labour market with sticky wages and unemployment gives a shadow price of labour in the rural area that is lower than the market price (the wage). Investors should therefore be given incentives to establish labour intensive projects in the rural area, and labour intensive production is encouraged by a regional policy that reduces the cost of labour - and not by a policy that lowers the price of capital. The price of capital, which is assumed to be mobile between sectors, ought to be the same in every region.

In a less technical language;

*"For example, if the objective is to reduce unemployment it would seem reasonable to use a labour cost subsidy to encourage the hiring of more workers or the location of more labor-intensive firm.....[T]here is a certain irony in the use of capital investment inducement by so many governments as an instrument in depressed-region policy. They will tend to favour capital-intencive firms who may have few workers when the stated policy goal is the reduction of unemployment, and such inducement could in fact lead to some substitution of capital for labour"*
Smith [1971], page 251.

From the table below we can see that in spite of the advice given by economists, capital subsidies - given through capital grants, is the regional policy preferred by the governments in most countries. In Allen et.al [1980] different major regional incentives are characterized either as: Capital grants (investment credits) (CG), interest related subsidies (IRS), tax concession (TC), generous deprecation allowances (DA) or labour subsidies (LS).

| COUNTRY | INCENTIVE TYPES | | | | |
|---|---|---|---|---|---|
| | CG | IRS | TC | DA | LS |
| Belgium | | x | | x | |
| Denmark | x | x | | | |
| France | x | | x | | |
| Germany | x | x | | x | |
| Ireland | x | | x | | |
| Italy | x | x | x | | x |
| Luxemburg | x | | x | | |
| Netherlands | x | | | | |
| United Kingdom | x | x | | | |

Table 1 (From Allen page 228)

Out of the ten European countries surveyed, all countries except Belgium had *capital grants* as the prime component of the their regional-incentives packages. Only one country, Italy, used labour cost subsidies as an incentive to generate new investment in regional areas[1].

---

[1] In Norway there has been a differentiated pay-roll tax between central and rural areas since 1975.

What I want to do in this paper is to shed some light upon the lack of consistency between the policy that economists recommend, and the policy that is actually carried out by the governments. To do this I develop a simple model that captures the problem of low productivity and unemployment in the rural area. In a static once-and-for-all-optimization framework the best policy is to give incentives through labour cost reductions. But I show that this conclusion may change if important dynamic aspects of the policy problem are considered more carefully. I show that in a dynamic model where policy is implemented sequentially it can be optimal for the current government to give investment grants. That is so even if the main problem in the rural area is lack of job alternatives. The reason is that investments made in the rural area are partly irreversible; some of the capital costs are sunk when physical capital is moved to one sector of the economy. This makes it tempting for the government to withdraw the promised subsidy after investments are made. Giving up-front investment grants is one way to avoid the credibility problem.

I also discuss the problem of time inconsistent preferences. Here I show that even if it is known with certainty that the current government will stick to the announced labour subsidy program, it may still be optimal to give investment grants. This happens if there is a positive probability that a new government, one giving low priority to regional unemployment problems, is elected in the future.

In the last section I discuss, in a broader perspective than the model I develop provides, whether lack of credibility is an important issue in regional policy.

## 2  A simple static model

Consider an economy where one single product can be produced in two different geographical districts; either in a central $(C)$ or in a rural $(R)$ area. In this economy there is an investor/entrepreneur denoted $E$ that has a production idea. In developing this idea she makes choices along two dimensions: She decides *where* to locate the production plant and she decides what *technology* to use in the production of the good. With respect to technology the important feature, the one that generates the results I want to show, is that it is possible to develop the production idea with technologies that differ with respect to how capital intensive they are. Here the technology choice is made as simple as possible for expositional reasons. I assume that the entrepreneur can choose between two different technologies; a capital intensive technology, $a$, which requires capital and labour in amounts $k^a$ and $l^a$ to be operative and a labour intensive technology, $b$. Let $k^a > k^b$ and $l^a < l^b$.

Assume that the firm produces for a certain period of time and that the market prices for the inputs and the output are the same in every period. The price a firm can sell its products for, and the price it can buy its capital equipment for, is, for simplicity, taken as unity. It costs $w$ to hire a worker for one period. Prices are assumed to be the same in both the rural and the central region[2]. The economy is assumed to be small and open to international capital flows so the firm and the government can lend and borrow money at a constant interest rate, $r > 0$.

I assume that for a given rental price on capital and labour the two projects, $a$ and $b$, have the same costs and both projects produce the same amount of output, $q^C$, if production is established in the central area.

---

[2] Recall the assumption of a labour union that bargains over one nationwide wage-level. I need not make the assumption of one wage level, what is needed is imperfection in the labour market in the rural area such that the wage is higher than the productivity of the marginal worker.

If we assume no capital deprecation both projects yield the same profit per period as long as it is operative:

$$\pi^C = q^C - rk^a - wl^a = q^C - rk^b - wl^b$$

If the production idea is developed in the rural area technology $a$ and $b$ produces an output $q^R$. Productivity is low in this region which means that $q^R < q^C$. It follows that less profits are earned if the firm is located in the rural area $\pi^C > \pi^R$. $E$ will therefore locate production in the central area if there is no intervention from the government.

### The government's objective.

The government is concerned about the unemployment problems in the rural region. To reduce unemployment it can introduce a regional policy reform to make it more attractive to invest in the rural area. I consider two alternative programs; a labour subsidy and an investment credit (capital grants). The first corresponds to a case where the government pays a fraction $s \geq 0$ of the wage that the workers receive. In the second program the government covers a relative part $g \geq 0$ of the capital equipment expenditures. The entrepreneur will obviously choose a labour intensive technology if labour costs subsidies are given and a capital intensive project if capital cost subsidies are offered. The profits earned per period will then be, $\pi^R(s, 0) = q^R - w(1 - s)l^b - rk^b$ in the first case, and $\pi^r(0,g) = q^R - wl^a - r(1 - g)k^a$ in the latter.

It is, however, realistic to assume that giving incentives through regional subsidies imposes costs to the society. That is, the government faces a shadow cost of public funds, $\lambda > 0$. The idea is that one krone spent by the government (through subsidies) must be raised through distortionary taxes (taxes are levied on tax bases that are endogenously given). It costs the society $1 + \lambda$ krone to give one krone in subsidies. The government will therefore try to minimize the transfers per new working place. More formally the government's objective is to

choose a regional policy program $(s,g)$ to minimize $[wsl + rgk]/l$, given the constraint that $\pi^R(s,g) \geq \pi^C$ (This constraint will be binding since it is costly to transfer money to firms in the low productivity region).

The solution to this problem is clearly for the government to choose labour cost subsidies: Both programs require the same amount of subsidies but if the labour cost subsidy program is chosen more unemployed workers are given new job alternatives. Total subsidies each period are equal to $ws^*l^b(1 + \lambda)$, where $s^*$ is the subsidy that gives $\pi^R(s^*, 0) = \pi^C$. Of course, if the cost is higher than the value of getting $l^b$ workers employed in the area the government should not undertake any rural policy program at all. To get a interesting case I assume that this is not so.

This model, or some version of this model, is used by economists when they argue in favour of labour subsidies. The reasoning above overlooks, however, important dynamic aspects of the regional problem.

# 3 Dynamics and the time inconsistency of policy constraints

*Irreversible capital investment*

One aspect that is missing in the model above is the irreversibility of capital investments. Capital equipment once allocated to the rural area will be - more or less - locked into the area. Much of the capital cannot physically be moved to central areas (without substantial costs) if production is located in the outlying region. Investors must for example leave buildings and plants behind if they decide to reallocate the production to the central area.

*The sequential structure of policy making.*

In the model above it was tacitly assumed that the current government determines future policy as well as today's policy. But, as pointed out in the introduction, in real life economic policy is not chosen once-and-for-all by the current government. If the current government could precommitment itself to a policy plan this would violate the basic principle of a democratic political system. The choice of regional policy must therefore be studied as a sequential policy problem. And when economic policy is made sequentially there is a new dimension to distinguish different policy reforms along; whether or not it is possible to deviate from the announced policy - or, in a more continuous world - how costly it is to deviate from the announced reform.

In the standard once-and-for-all-optimization framework the only difference between the two kinds of subsidy programs I consider is the production factor the subsidy is given to. If a capital grant of magnitude $g$ is given, the yearly rental price for a unit of capital equipment is equal to $r(1 - g)$, if, on the other hand, a labour subsidy of magnitude $s$ is given the yearly labour rental price is given by $w(1 - s)$. If we model the dynamics with more care another crucial difference between labour subsidies and capital grants appears: Capital grants are given up-front while labour subsidies are announced for the future. So while there is nothing in the model that commits the government to follow up announced labour cost reductions future governments cannot reconsider capital grants that are given in the past. Since policy is implemented sequentially the credibility of announced labour subsidies depends on whether the future governments find it optimal to follow up the announced labour subsidies *after* the firm is located in the rural area. Of course *if* labour cost subsidies are credible, we already know that this program is preferable to capital grants. If not, we may have an argument in favour of capital cost subsidies, since giving capital grants is one way for the government to make a

precommitment to a regional policy. These issues are analysed in a simple model below.

## A model

The important dynamic aspects of the regional policy problem, that is, the irreversibility of investments and the fact that economic policy must be chosen sequentially, can be studied in a two period model. In the first period the government announces a regional policy and the owners of the production choose where to locate the firm. In the beginning of the second period - which can be thought of as the time period for which the project is operative - the government decides whether the announced plan will be followed up or rejected and the owners of the firm decide where to continue their production.

Related to the time horizon the stages are as follows:

The entrepreneur decides:

*Where the plant should be located and what technology they should use in production*

*Where to continue production.*

— time

The government decides:

*What regional policy plan should be announced*

*Whether to adhere to or deviate from the announced policy*

The irreversibility of capital investments is captured by letting a fraction $\mu \in$ [0, 1] of the capital invested be sunk in the rural area if investment is made there. Only a fraction $(1 - \mu)$ of the initial capital equipment can be used in the production if the owners of the firm decides to move from the rural to the central area.[3]

In this section I assume that the government's preferences are the same in every period. Its aim is to get new employment in the rural area, but since it is costly to transfer money, it wants to minimize the transfers that are needed per new job alternative.

The owners of the production idea make a location and technology choice so as to maximize the expected value of the firm's profit flow.

Let labour subsidies of a magnitude $s^*$ per period be announced and assume that the firm locates its plant in the rural area.

The government's ex post incentives depends of course on what the firm will do if subsidies are withdrawn. If labour subsidies are repudiated but the firm carries on its production in the rural area the value of the firm is $\delta \pi^R$, where $\delta$ is a parameter that captures both the length and the discounting of the second period.

Another option is to move the production to the central area where productivity is higher, in this case the value of the firm is $\delta[\pi^C - \mu r k^b]$.[4]

The firm will not be moved to the central area if the degree of irreversibility is large, or if the difference between profits in the central and the rural area is small, so that $\pi^C - \mu r k^b \leq \pi^R$. Let $\hat{\mu}$ be the degree of irreversibility that makes this equation hold with equality, $\hat{\mu} = (\pi^C - \pi^R)/rk$.

[3] In some cases it can be more realistic to let the costs of moving from the rural to the central area be independent of the capital invested. One could for example assume that the firm's owner had to pay a fixed cost to move production. Non of the conclusions in this paper depend on the way "moving costs" are modelled.

[4] I assume that resources are productive in the central area immediately after they are moved from the rural area.

Let us first consider the case where, $\mu > \hat{\mu}$. The degree of irreversibility is then so large that the firm will not move from the rural area if the labour cost subsidies are withdrawn. In this case the number of working places in the rural area are the same whether the government sticks to the announced plan or withdraws the labour subsidies. But by withdrawing the labour subsidies the government saves the transfer costs. So ex post - after the firm is established in the rural - it is optimal for the government to deviate from its announced plan: It is optimal to reject the subsidies. The firm recognises this ex ante and will therefore not establish its production facilities in the rural area and announcement of labour subsidies will not induce new investment in the rural area.

If this model captures the dynamics of the regional policy giving capital cost subsidies is the only way to get new employment in the low productivity area. Put differently, there are two sub game perfect equilibria in this policy game. One where the government announces labour cost subsidies and the firm doesn't establish production in the central area and no subsidies are given in the last stage by the government. The other sub-game perfect equilibrium is the one where the government announces capital subsidies, the firm establish production in the rural area and later there is no decision to make for the government.

Suppose the value of each new working place in the low productivity region is high, then, given the two sub game perfect equilibria, the best regional policy is to give capital grants. The time inconsistency - the lack of credibility - of announced labour subsidies can therefore explain why capital grants is the mainstay in most countries regional policy programs. The reason why economist traditionally have reached another conclusion is that the regional policy problem has not been modelled as a sequential problem but as a problem where the current government optimizes once-and-for-all.

So far I have considered the case where the degree of irreversibility of investments in the rural area is high and the firm will therefore not move from the area if subsidies are withdrawn. It is, however, not necessary to assume large lock in effects of investments in the rural area to reach the conclusion that labour subsidies are non-credible. The government will of course not reject the whole subsidy if the degree of irreversibility is small. If $\mu < \hat{\mu}$ the government will reduce the subsidy only so much that the firm, from an ex post point of view, is indifferent between producing in the rural or in the central area. The entrepreneur then decides to locate the firm in the central area since this, from an ex ante point of view, gives higher profit. Only if the project is "on wheel", that is, if capital is completely flexible, $\mu = 0$, will the government stick to its promised labour subsidies.

We can therefore conclude that this model predicts that the governments will use capital grants and not labour subsidies as their regional policy if new employment opportunities in the low productivity region are given high priority and investments are - to some degree - irreversible.

# 4 Exogenous policy uncertainty (time inconsistent policy preferences)

In this section I will show that a capital subsidy can be the optimal regional policy even if the current government does not face the credibility problem studied in the preceding section. There I focused on the time inconsistency of policy constraints. The firm's reaction to different policies changed over time; the irreversibility of investments in the rural area imply that the location decision ex post is less "elastic" than it was ex ante and this gave the government incentives to reconsider the promised labour cost subsidy. The model I used was highly

stylized. In particular it overlooked that it can be costly to deviate from the promised labour subsidies (the announced policy). It is for example well known that repeated play and reputation effects can solve general opportunism problems that occur in economic relationships which stretches over time. The reputation argument could be applied to the regional policy problem. It would then be possible to show that it is optimal for the government to follow the announced plan. That is, if the government must handle regional problems also in the future, cheating on the promised policy today imposes costs by making it impossible to use labour subsidies as regional incentives in the future, and given these future costs it may be optimal to follow the announced policy. Here I assume that the current policy makers are farsighted, they are concerned about future as well as current regional unemployment problems. The current government will therefore stick to the announced labour subsidies unless the state of the world changes. That is, there is a possibility that the state of the world changes in the future and that the optimal policy response to this shock is to withdraw the labour subsidy. The probability of a policy reversal is then exogenous. An example could be a sudden exogenous disappearance of a tax base. For a given expenditure level tax rates on other economic activities must be raised and it will then be extra tempting to eliminate the labour subsidies since the shadow price of government revenue is high in this case (efficiency loss from taxation is high).

Another interpretation, which I prefer, is to think of the shock as a change in the government's preferences. Consider for example a case where there are two possible types of governments:   A left wing government and a right wing government. The left wing government is farsighted and concerned about the unemployment problems and the depressed economic situation in low productivity regions, and - due to the adverse reputation effects and long run losses a rejection of announced subsidies will have - it will stick to the announced plan ex post. The

right wing government is assumed to be less concerned about regional unemployment problems and does therefore not care about its "reputation" and consequentially this government will stop the flow of labour subsidies if it is elected.

Let us consider this situation more carefully. Assume that the left wing government is in power initially and that it implements a regional policy reform. Investors know the government's preferences and can see that the left wing government will not deviate from the plan. But investors also assign a positive probability to the event that a right wing government will be elected later (but before the capital is exhausted), and they know that if this happens the subsidies will be withdrawn.

The points I want to make can be illustrated in the same simple two period framework as used above. We know that the total amount of subsidies that is needed to induce new investment is the same whether labour or capital cost subsidies are chosen. If there is no policy uncertainty a labour cost subsidy of magnitude $s^*$ creates $l^b$ new job alternatives in the rural area. The subsidy per new working place is therefore $s^*w$. If capital cost subsidies are chosen fewer unemployed workers will get a new job alternative and the subsidy per worker, $y$, is therefore higher, $s^*w < y$ (labour subsidies are more efficient). The total cost of getting one more worker employed is then $s^*w(1 + \lambda)$ if labour subsidies are given and $(1 + \lambda)y$ if capital grants are given.

Politicians on the left wing is much concerned about regional unemployment problems, so it will pay the costs it take to get new investments and job alternatives in the area. They value each new working place to be worth $z$, where $z > (1 + \lambda)y > (1 + \lambda)s^*w$.

The right wing government, which has a probability $p$ of being elected in the second period, gives low priority to regional unemployment problems and values each new job alternative to be worth $h$, where $h < (1 + \lambda)s^*w < (1 + \lambda)y$.

What regional policy will a left wing government choose in this situation; will it give a subsidy to reduce labour costs or will it give capital grants to reduce capital costs?

With no policy uncertainty, that is, if the left wing government knows that it will be in power also in the following period, it will obviously announce labour subsidies[5].

The point I will make is that labour subsidies are not necessarily the best policy if there is a probability that a right wing government is elected in the future. There are two reasons why the left wing government may prefer capital subsidies to labour subsidies when policy preferences are -potentially - time inconsistent.

First since there is uncertainty about whether labour subsidies will be given in the second period and since investments are irreversible investors will require a higher than $s^*$ subsidy per worker in order to be willing to establish project $b$ in the rural area. This can make it better for the government to use capital subsidies.

Second if the degree of irreversibility is so low that the firm moves its production facilities to the central area if subsidies are withdrawn in the second period, it can be optimal for the left wing government to give capital subsidies to commit the right wing government from increasing the unemployment in the rural area if it should be elected in the second period.

Formally the arguments go as follows.

The value of a firm that locates in the rural area when the left wing government announces a labour subsidy plan, with a subsidy $s$ per worker, is given by,

---

[5] Remember that labour subsidies are assumed to be credible since the cost of deviating from the policy is so large for the left wing government that it finds it optimal to follow the announced plan, even if the firm - because of irreversible investments - would not move from the area if subsidies where taken away in the second period

$$v(s) = \delta\{(1 - p)\pi^R(s) + p\max[\pi^R, \pi^C - \mu rk^b]\}.^6$$

Let the degree of irreversibility be high so that $\pi^R > \pi^C - \mu rk^b$. In this case $v(s) = \delta\{\pi^R(s) + p[\pi^R - \pi^R(s)]\}$. A risk neutral investor will invest in the rural area if, and only if,

$$\delta\{\pi^R(s) + p[\pi^R - \pi^R(s)]\} \geq \delta\pi^C, \text{ which yields } \pi^R(s) - \pi^C \geq p[\pi^R(s) - \pi^R].$$

The term on the right hand is the premium investors require if labour subsidies are announced in the first period and there is policy uncertainty. Clearly if $p > 0$ the labour subsidy required to make this inequality hold is larger than $s^*$. So even if we know that labour subsidies are more efficient when there is no policy uncertainty, $s^*w < y$, in presence of policy uncertainty it may be optimal to give capital cost subsidies since investors demand a compensation for the possibility that subsidies may be rejected in the future (in the second period).

Consider next a case where the degree of irreversibility is below the critical value; $\mu < \hat{\mu}$. The firm then moves its production facilities from the rural area if labour subsidies are taken away. Investors will, however, demand compensation in terms of higher labour subsidies per worker if there is a possibility that a right wing government will be chosen in the second period and it is costly to move production to the central area. By using the same argument as above we can therefore conclude that it can be less expensive for the left wing government to give incentives through capital subsidies.

When the degree of irreversibility is so low that the firm leaves the low productivity region if the right wing government is elected, there is yet another

---

[6] $\pi^R$ is the profits earned if no subsidies are given $\pi^R(0,0) \equiv \pi^R$.

reason why the left wing government may choose to give capital grants. By giving capital grants in the first period the left wing government can assure that there will be no increase in the number of unemployed people in the rural area in the second period. This is so even if the right wing government should be elected. If the left wing government values each working place in the districts highly it may be optimal to choose capital subsidies.

To see this more formally consider the extreme case where investments in the rural area exhibits no hysterisis effects at all; $\mu = 0$. If capital grants is chosen the left wing government gets a payoff $[z- (1+\lambda)y]l^a$. If labour subsidies are given there is a probability $p$ that the firm moves from the rural area, and the expected payoff for the left wing government is thus, $(1-p)[z - (1+\lambda)s^*w]l^b$

Capital subsidies are chosen if,

$$[z- (1+\lambda)y]l^a \geq (1-p)[z - (1+\lambda)s^*w]l^b .$$

This inequality will hold for high values of $z$ and $p$. That is, it is optimal for the left wing government to choose capital subsidies, even if the investments' degree of irreversibility is low (zero), if the current government gives high priority to solving the regional unemployment problems and the probability that a right wing government is elected in the future is high.

# 4 Is lack of credibility an issue in regional policy?

Dynamic inconsistency of optimal policy plans is a general problem that recently has received much attention in the economics literature. Regarding tax and subsidy policy, the issue of dynamic inconsistency arises when allocation of

resources responds differently to changes in economic policy ex ante, when the plan is announced, and ex post, when the policy is to be implemented. All existing wealth in the private sector gives the government a standing invitation to collect revenue through a surprise tax. This revenue collected from higher taxation on wealth accumulated in the past is "distortion free" and such a lump sum tax should be exploited by a government that aims at minimizing the deadweight loss from taxation. The problem is that in a perfect foresight equilibrium individuals will see that the government has incentives to impose a capital levy and consequentially they will not save. But this is not what happens in real life. In real life households save and governments don't confiscate the income from past savings. Why is the theory's predictions counterfactual? There can be two reasons; first, perhaps the government does not aim at minimizing the deadweight loss from taxation and second deviating from the announced policy can impose costs that are overlooked in the simple Ramsey model.

Some economists (for example Blanchard and Fischer) take the fact that individuals accumulate wealth, and that there is no expropriation of the wealth, as an evidence that the society has found ways to deal with potential time inconsistency problems. Does this mean that there is some general mechanism out there that solves all inconsistency problems and that we gain no new insight by having this possibility in mind when we study the dynamics of economic - e.g., regional - policy?

I don't think so, I think it is important to examine the credibility problems related to economic policy in a less aggregated way than one usually does. It is correct, I think, that individuals who live in a stable democracy do not fear - or expect - total expropriation of their wealth in the future. But for some assets there may be a positive probability of a "surprise tax" in the future. Such a surprise tax can be implemented either because the government behaves

opportunistically and takes advantage of the irreversibility of private allocation decisions or because there is a change in the government's preferences. So the central question is: Is the temptation to cheat on announced regional subsidies large? Some indications suggest that the answer is yes.

I have already mentioned that the most important negative effect - or cost - of a surprise tax is the adverse reputation effect that follows from a "deviation policy". If the government behaves opportunistically today individuals start to expect that it will cheat on its policy also in the future. But as a matter of logic, the loss of reputation is less of a problem the longer the time span is between every repetition of the game. And assume now that as a response on announced labour cost subsidies there will be massive investments in the low productivity region. If these investments are irreversible, the government will not face the negative reputation effect of an "opportunistic" policy in a long time. The reputation argument is therefore less compelling in the "regional policy game" than it is for other assets like savings in general. Here the negative reputation effect of cheating is more serious since there is no time lag between each repetition of the game.

In addition I have shown that even if the current government is disciplined by the value of their reputation, it can be optimal to give capital grants if there is policy uncertainty, e.g. if there is a possibility that a government which is less concerned with regional policy problems is elected in the future.

And of course the empirical finding that most countries use capital grants to make investments in low productivity regions more attractive is an indication that a government face credibility problems when it announces regional labour cost subsidies. Within a model of rational choice it is hard to find any other explanation than credibility problems, to why governments prefer to give capital subsidies as long as unemployment is the major problem.

The model I have used predicts that if a labour subsidy is announced for a specific district it should have only small effects on investment and employment in that area. In an empirical evaluation of the effects of a reduced pay-roll tax in the Norbotten area in Sweden Bohm and Lind [1989] found that the policy did not give a significant reduction in the unemployment problems. It is interesting to note that the authors argue (on page 21) that one reason to why there was no response to the announced labour cost reduction could be that," *the firms did not believe that the reduction in the pay-roll tax would last"* .

# References

Allen, K. [1980]: " Regional Policy in the European Community" *St. Martins Press New-York.*

Atkinson, A. and A. Sandmo [1980]: " Welfare Implications of the Taxation of Saving" *The Economic Journal,* 90, 529 - 549.

Bohm, L. and T. Lind [1989]: "Policy -evaluation Quality: A Quasi-Experimental Study of Regional Employment Subsidies in Sweden" mimeo. Dept. of Economics, University of Stockholm.

Blanchard, O.J and S Fischer [1989] "Lectures on Macroeconomics" *The MIT Press.*

Calvo, G [1978]: " On the Time Consistensy of Optimal Policy in a Monetary Economy" *Econometrica,* 46, 1411-28.

Kydland, F. and E. Prescott [1977]: "Rules Rather than Discretion; The Inconsistency of Optimal Plans" *Journal of Political Economy,* 85, 473-490.

Lind, T. and J. Seck-Hansen [1972]: "Regional Subsidies on Capital and Labour"

*Swedish Journal of Economics* 68 - 84.

# Chapter three:

## (Co-author: Trond E. Olsen)

## Irreversible Investments and Optimal Tax and Debt Policy*

### Abstract

Capital types differ, among other things , with respect to their degree of irreversibility. The main goal for the tax reforms that have been enacted in several countries in the past decade has been to reduce the diversity of tax rates on income from different capital types. In this paper we show that if there is uncertainty about the government's future revenue requirement it is in general not optimal to tax income from investment projects of different durability at the same tax rate. The conventional wisdom concerning the optimal distribution of the tax burden over time is also challenged. It is not optimal to smooth the tax burden over time if taxes can be levied on income from capital types which leave investors with different degrees of flexibility. We find that even if the expected revenue requirement in the future is much higher than the current revenue requirement it is optimal for the government to run a deficit in the first period.

## 1 Introduction.

The main goal for the tax reforms that have been enacted in several countries in the past decade has been to reduce the diversity of tax rates on income from different capital types. The theoretical argument in favour of a "levelled-playing-field" is that it provides a neutral tax system which then assures efficient production. The desirability of efficiency on the production side of the economy was pointed out by Diamond and Mirrlees [1971]. They showed that in the

---

absence of profits - i.e., under constant returns to scale - or if profits can be taxed away by the government, then production efficiency should be maintained.

It is well known that there are situations where it can be optimal to deviate from the "level-the-playing-field" principle of taxation . Auerbach [1979], Christiansen and Kvinge [1989], Feldstein [1990] and Richter [1988] have shown that if there are other distortions on the supply side in the economy, for instance if there is one untaxed asset or if there is an intertemporal distortion caused for example by a general income tax, then it is in general optimal to differentiate taxation of capital income. In second best situations like this the government should e.g take differences in investment responses to changes in after tax return, into account when tax rates are chosen. Optimal differentiated tax rates will be contingent on differences in the slope of marginal productivity curves across capital types. Responsive investments should be taxed more leniently than less responsive investments (the inverse elasticity rule).

In this paper we present a *new argument in favour of heterogeneous taxation* of capital income. We argue that it can be optimal for the government to tax income from different capital types at different tax rates even in cases where all capital types have the same slope of the marginal productivity curve. What distinguishes capital types in our model is their degrees of irreversibility. That is, we consider a situation where individuals can put their resources into projects which give them different degrees of flexibility in the future. It is well known from the theory of real and financial "options" that flexibility - the possibility to reallocate resources in the future - is valuable if there is uncertainty about the future returns of current choices. This means that the aggregate level of irreversible investments will be very sensitive to uncertainty about future returns and, as is pointed out by R. Pindyck, it therefore becomes;

> "..*important to understand how investment depend on factors that are at least partly under government control*.." [Pindyck 1991; 1141].

In this paper we study the problem of optimal policy design in presence of uncertainty in a model where individuals make irreversible allocation decisions.

Based on the option argument it is easy to understand that if irreversible and reversible capital types are exposed to the same risk about future returns there will be a bias towards reversible projects and little investment in irreversible capital. If there is uncertainty about future government spending this is exactly the effect of taxing according to the "level-the-playing-field" principle. Investors then know that all capital types will be taxed at the same rate in the future, but they are uncertain about the *level* of these tax rates. Big spending requires large tax revenues and hence high tax rates on all types of capital, while the opposite is true if spending is low. Homogeneous taxation will, because of the option argument, discriminate against irreversible investments. Uncertainty about government spending seems therefore to provide an argument in favour of differentiated taxation. This conjecture is confirmed in our analysis. The government should differentiate taxation and the differentiation should be made on the basis of capital types' degrees of irreversibility.

How, then, should optimal taxes be differentiated? What is the optimal "slope" of the "playing field"? The argument in favour of different taxes on capital types with different degrees of irreversibility is based on the effect uncertainty about future taxes has on current investment. Uncertainty about future tax rates has no effect on current investments in reversible capital but it clearly has a negative incentive effect on current investments in irreversible capital. It seems therefore natural to conjecture that irreversible capital types should be made less exposed to tax uncertainty by letting more of the uncertainty about future taxes be allocated to reversible projects. In the light of this argument the tax policy we actually find to be optimal is surprising. For in section three we demonstrate that this conjecture is false; it is not optimal to levy more of the tax rate risk on income from reversible capital types. On the contrary, relative to a "level-the-playing-field" policy, the government should *increase* the differences in tax rates between high and low states of government spending on irreversible investment, and *decrease* the variability in taxes on reversible capital types. This policy in itself will, compared with a levelled playing field, reinforce the distortion of

investments away from irreversible capital types. There is, however, a remedy, and that is to reduce *current* taxation on irreversible projects. In this way the government makes investment in such assets more attractive.

The way taxes are differentiated between capital types with different degrees of irreversibility has also important implications for *optimal debt policy*. The point usually made with respect to optimal intertemporal distribution of taxes, is that a government should *smooth* the tax burden over time if its revenue requirement is expected to change in the future (See for example the articles by Robert Barro [1979], Robert Lucas and Nancy Stockey [1983] and Lucas [1986]). The neoclassical model views debt accumulation as a way to spread over time the costs of distortionary taxation. For example, if there is a probability of a sharp increase in the the government's need of revenue in the future, the advice usually given by economists is that the government should run a surplus in periods before the increase is expected. Or put differently, in the words of Robert Lucas;

> " *One implication of the Ramsey principle is that tax rates ought to be smoothed over periods of differing government demands, relative to endowment. In practice this principle requires deficit spending during wars and depressions, balanced (in present value sense) by surplus in times of relative peace and prosperity. This is a common sense principle which has long been recognised"*

Lucas [1986], page 130.

Our results on optimal debt policy contradicts this common sense principle of tax smoothing. In a two period model we find that it can be optimal for the government to run a *deficit* in the first period even in cases where the expected revenue requirement in the second period is considerably higher than what it is in the first period.

Why do we get this result? Usually when the tax smoothing principle is discussed there is only one tax base (see Barro [1979] and Bohn [1990]). In our model the government collects its revenue from two tax bases, it is taxing income from both irreversible and reversible projects. Our result follows from the link

between present and future capital supply that is induced by investment opportunities that are irreversible.

We think our normative results have theoretical interest as well as practical relevance. Time inconsistency is, however, a problem. The tax policy we find to be optimal from an ex ante perspective is not time consistent. Announced taxes distort investment in irreversible projects, but when the future arrives irreversible investment decisions are irrevocably made. At this stage increased taxation has only income effects. A government trying to minimize the excess burden of taxation will take advantage of the possibility to get distortion free tax-revenue by increasing taxation of irreversible projects.

If this is how capital taxation and investments are determined in the real world, then there is no room for normative analyses. The search for an optimal tax or debt policy from an ex-ante point of view is spoilt effort since investors will recognise that the optimal plan is not credible. But, luckily, there must be a flaw in the reasoning since its predictions are counterfactual: People do put their savings into irreversible investment projects, and the income they earn is not confiscated by the government.

The reason why the time inconsistency problem is not as serious as theory predicts must be that although opportunistic policy gives an efficiency gain in the short run, it imposes other kinds of costs. If we discussed our problem in a model with many "periods" and repeated play, both theory and common sense would predict a confiscatory policy to have negative incentive effects on future investments.

The commitment assumption we make in the two first sections of this paper can be interpreted as an assumption to the effect that deviations from an announced policy generate long run costs that are large enough to make the announced policy credible.
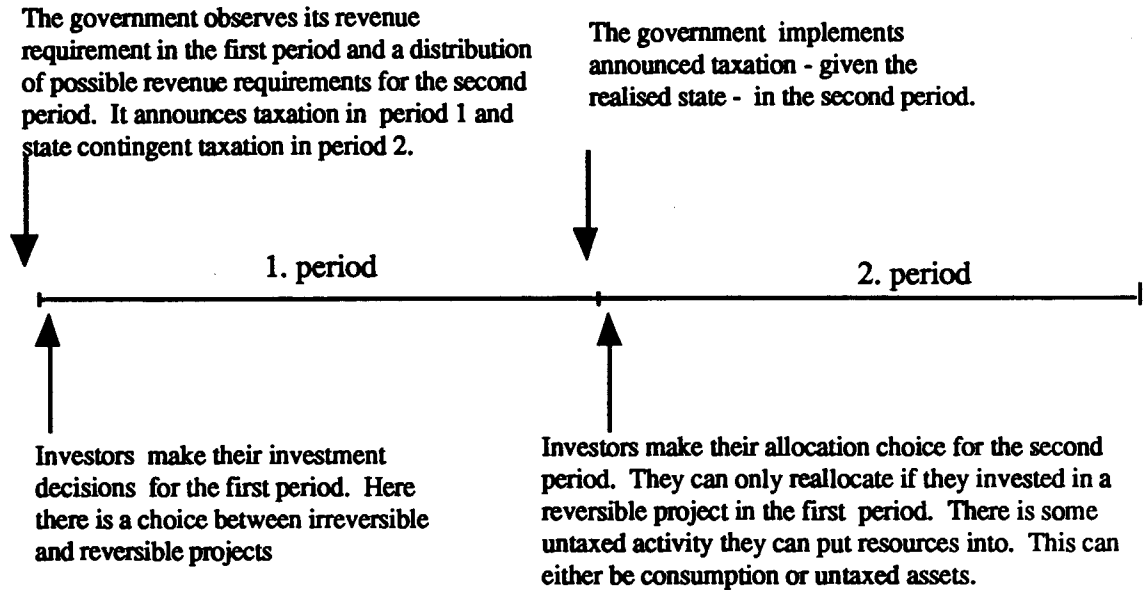
Another point worth discussing is that very complicated policy announcements may not function well as a restriction on future policy. If the announced policy is complicated it might be difficult to see through whether the government is behaving

opportunistically or if the realization of the state of the world in fact justifies high taxes. It can therefore be natural to make the restriction that a government can only commit to "simple rules" or principles of taxation. The optimal tax plan in our model is far from simple; it is both state contingent and specifies heterogeneous taxation of capital income. It could be made less complicated in two ways; less differentiation, in the limit this would imply a level-the-playing-field policy; or it could be made contingent on fewer states, in the limit this implies a fixed tax rate in every state. In the third section we compare the welfare effects of both principles, and contrary to what one should belive based on "the Ramsey principle" of tax smoothing, we find that the second principle may give better results than "level-the-playing-field". This result is not in line with the conclusion Bizer and Judd [1989] draw when they study capital taxation and uncertainty. They find that randomization of the tax rate policy will in general increase welfare. But these authors do not consider a case where capital investment is irreversible and thus they does not capture the option argument in favour of a stable tax rate policy.

# 2 The setting

The model includes two periods. Investors can choose to invest in assets with different durability in the beginning of the first period. We consider a situation where there is uncertainty about the future tax burden, modelled here as there being uncertainty about the tax burden in the second period. This uncertainty could be induced by uncertainty about the government's expenditures in the future. We study a second best situation where investors can allocate resources into some untaxed "activity" between the first and the second period. The "escape asset" can either be interpreted as consumption or some untaxed investment possibility, e.g. investments abroad ( the asset need not be untaxed, we only require that it is taxed at a fixed rate whatever state is realised in the second period.)

The basic structure of the situation we study is given below.

The government observes its revenue requirement in the first period and a distribution of possible revenue requirements for the second period. It announces taxation in period 1 and state contingent taxation in period 2.

The government implements announced taxation - given the realised state - in the second period.

1. period                2. period

Investors make their investment decisions for the first period. Here there is a choice between irreversible and reversible projects

Investors make their allocation choice for the second period. They can only reallocate if they invested in a reversible project in the first period. There is some untaxed activity they can put resources into. This can either be consumption or untaxed assets.

## A model

In the beginning of period $i$ there are fixed endowments $e_i$ that can be allocated to three different projects $a$, $b$ and $c$. The economy produces an aggregate output $Y$ using these three different types of capital. Let $k^a$, $k^b$ and $k^c$ be the amounts of capital invested in each project. Capital is transformed into output according to the following simple aggregate production function,

$$Y_i = f(k_i^a) + f(k_i^b) + k_i^c r \qquad i = 1,2. \tag{1}$$

We assume that income from capital type, $c$, cannot be taxed. If we let $\phi^{j,i}$ $\in (0, 1)$ be the tax rate levied on income from project $j = a,b$ in period $i = 1,2$ we can write the government's budget constraint as,

$$g_i = \phi_i^a f'(k_i^a)k_i^a + \phi_i^b f'(k_i^b)k_i^b. \tag{2}$$

The problem we study is quite general, but for expositional reasons we have chosen this simple form of the production function and the government's budget constraint. In order to have a well behaved optimization problem, we assume in addition that the elasticity of the marginal product of capital, $\varepsilon(k) = -(f''/f')k$, is non-decreasing, $\varepsilon'(k) > 0$, and ssatisfies $\varepsilon(k) \leq 1$. Our results would not change if there was decreasing returns on the untaxed "activity" but it simplifies a lot to assume constant returns since all (uninteresting) income effects of taxation are eliminated by this assumption. One interpretation of the model is that $k^c$ is the amount of capital allocated to the world capital market where the return on capital is given by $r$, and this capital income cannot be taxed by the government.

None of our results are dependent on the assumption that both taxable projects have an output function of the same functional form. We could have used a more general functional form where different capital types had different slopes of their marginal productivity curves, $(f^{a''}(k^a) \neq f^{b''}(k^b)$, for $k^a = k^b)$, or where the cross derivatives of the marginal productivities of capital types were either positive or negative (not zero as in our additive production function (1)). With a more general technology we would have to consider arguments in favour of differentiated taxation based on differences in ex ante supply ( or cross) elasticities of different capital types. These arguments are, however, well understood and need not be analysed here, see for example Feldstein [1990]. The point is that if the output function of projects $a$ and $b$ had the same functional form it would be optimal to level-the-playing-field if the capital types were of the same durability (i.e., if investment in $a$ and $b$ imposes the same degree of irreversibility). So if we find that it is optimal to differentiate taxation of capital income in our model this must be based on differences in projects' degree of irreversibility.

In a model with decreasing returns to scale there are profits. We have assumed that profits are left untaxed. We make this assumption in order not to get too many tax instruments to handle. Alternatively we could let the government tax

away all pure rents. This would change the government's budget constraints but not our main conclusions. Another possibility would be to include labour as a production factor and to let projects $a$ and $b$ have constant returns to scale with respect to labour and capital. What is profits in our specification, $f(k) - f'(k)k$, would then be the wages received by the workers. In this case we could model labour income as just another reversible tax base (that is not done here).

We consider a situation where the government's future revenue requirements are uncertain, that is, $g_2$ is a random variable which can take values both higher and lower than the revenue requirement in the first period. For simplicity we assume a two point distribution where the revenue requirement takes the value $\bar{g}$ with probability $p$ and $\underline{g}$ with probability $(1-p)$, where $\bar{g} > g_1 > \underline{g}$.

The level of government consumption is not a choice variable in our model. In principle we could let government spending be determined in the model. Uncertainty about future government consumption could e.g. correspond to a situation with uncertainty about future governments' preferences; "will a left wing government with preferences for high government consumption be elected in the future"? Modelled like this we would have to tackle the problem of strategic choice of economic policy, where the government making policy today knows that it might be replaced by another government in the future and thus has incentives to strategically manipulate state variables - such as debt - in order to influence future policy choices. (See e.g. Persson and Svensson [1989] and Alesina and Tabellini [1990]). Such strategic aspects of economic policy would make our discussion less clear-cut. For that reason we consider a case where government consumption is exogeneously given.

The way we model uncertainty about the future level of public spending corresponds for example to a situation where there is a probability of a "war" at some date in the future, (Lucas and Stokey [1983]). It could also correspond to a case where government *consumption* is constant over time but consumption can be financed by taxation of capital and labour income (not modelled here) *and* by lump sum revenues of uncertain magnitude. The tax burden levied on capital

income will then be contingent on the realisation of these lump sum revenues. This is a good description of the situation in countries like Norway and Britain where the uncertain lump sum element of revenue is income from oil reserves in the North - Sea.

### The capitalists' allocation problem

Capitalists are risk neutral and allocate the fixed capital stock in order to maximize the net return of capital. In order to capture irreversibility in a stark form we assume that it is impossible to reduce the level of capital invested in project $a$, hence $k^{a,1} \leq k^{a,2}$. Each capitalist is assumed to be small and she takes the marginal productivity of capital in project $a$ and $b$ as given when she makes an allocation choice. Their objective is to choose period one investments, $k_1^j, j = a,b,c$, and state contingent period two investment policies $k_2^a(k_1^a, \phi_2^a, \phi_2^b)$ so as to maximize;

$$v = f'(k_1^a)(1 - \phi_1^a)k_1^a + f'(k_1^b)(1 - \phi_1^b)k_1^b + rk_1^c$$
$$+ \delta E[f'(k_2^a)(1 - \phi_2^a)k_2^a + f'(k_2^b)(1 - \phi_2^b)k_2^b + rk_2^c],$$

where E denotes expectation with respect to the probability distribution for the second period tax rates. Total resource constraints are given by $e_i = k_i^a + k_i^b + k_i^c$, and the irreversibility constraint is given by $k_1^a \leq k_2^a$. $\delta$ is the capitalists' and the government's discount factor which is equal to $[1/(1 + r)]$.

Without the irreversibility constraint, $k_1^a \leq k_2^a$ it would be trivial to find optimal taxes. If all projects are fully reversible there is no need to announce taxes for the second period. When capital can freely be re-allocated between periods the choices made today have no implications for what choices can be made tomorrow and the decisions made today will therefore not be contingent on the policy announced for the future. Given our specification of production functions it is, in the case with no irreverrsibility constraint, optimal to "level-the-playing-field"

between capital types within both periods ( and within both states in the second period). If $\overline{\phi}_2^a$ and $\underline{\phi}_2^b$ denote the tax-rates that are implemented in the high revenue requirement state, and $\underline{\phi}_2^a, \underline{\phi}_2^b$ are the tax-rates imposed if the revenue requirement is low in the second period, the optimal tax policy would imply:

$$\overline{\phi}_2^a = \underline{\phi}_2^b > \phi_1^a = \phi_1^b > \underline{\phi}_2^a = \underline{\phi}_2^b.$$

It is considerably more interesting to study the problem of finding optimal taxes if making an investment in one of the taxable capital types, here $a$, is an irreversible decision. That is what we will do in the next section.

First we consider a situation where there is some constitutional amendment requiring the public budget to be balanced in every period (and for every realised revenue-requirement in the second period) .

# 3 Balanced budgets and optimal tax policy

Given the structure of the revenue requirements where $\overline{g} > g_1 \geq \underline{g}$ it seems natural first to consider tax policies where $\overline{\phi}_2^a > \phi_1^a > \underline{\phi}_2^a$. In particular, "level-the-playing-field" is, as we have seen, a policy in this class. Note, however, that such policies expose investors to risks and therefore generate option values associated with holding reversible capital. If an irreversible investment decision is made, there is no possibility to re-allocate resources to consumption or untaxed assets ($c$) if the high tax state ($\overline{\phi}_2^a$) is realised. Uncertainty about future taxes will therefore have a negative impact on $current$ investments in irreversible capital.

The uncertainty about future tax rates is under the control of the government. The government can reduce, or even eliminate, the uncertainty about tax rates on irreversible capital income , e.g. by announcing a fixed - state-independent - tax rate on irreversible investments ($a$). In this case all unexpected revenue must be financed by taxes on income from project $b$. This would increase future uncertainty

about tax rates for project $b$ but since this is a fully reversible project, increased uncertainty about future returns has no effect on current investment in asset $b$. Given the asymmetric impact uncertainty about future tax rates has on current investments in $a$ and $b$, it is not quite clear whether the optimal tax policy has a structure where $\bar{\phi}_2^a > \phi_1^a > \underline{\phi}_2^a$. We shall therefore first prove that such a structure is indeed optimal.

If the government announces a tax plan where $\bar{\phi}_2^a > \phi_1^a > \underline{\phi}_2^a$ the supply of capital to different projects is given implicitly by the first order conditions to the capitalists' maximization problem:

$$f^{\,\backprime}(k_1^a) = f^{\,\backprime}(\bar{k}_2^a) = \frac{r(1 + \delta p)}{(1 - \phi_1^a) + \delta p(1 - \bar{\phi}_2^a)} \tag{3}$$

$$f^{\,\backprime}(\underline{k}_2^a) = \left(\frac{r}{1 - \underline{\phi}_2^a}\right) \tag{4}$$

$$f^{\,\backprime}(k_i^b) = \left(\frac{r}{1 - \phi_i^b}\right) \text{ for } i = 1,2 \tag{5}$$

Relation (5) says that investments in each period are carried on to the point where marginal returns on taxed and untaxed capital are equalised. This is the standard condition for optimal investment if capital is fully reversible. From (3) we can see how the irreversibility of type $a$ capital distorts the investment made in this project in the first period. For this tax structure, the investment decision is based on taxes in the first period and on the tax rate announced to be implemented in the bad state in the second period. In this case the investors do not take account of the tax rate announced for the good state. The reason is of course that by investing in $c$ they have an option to increase investment in $a$ if the good state with low taxes is realised in the second period. But, since $a$ is an irreversible investment project, there is no offsetting possibility for a reduction in the capital supplied to

project $a$ if the bad state is realised. This asymmetry gives rise to a portfolio distortion and an efficiency loss in the first period.

In its design of the tax policy the government can eliminate the asymmetry created by the possibility to increase but not reduce investment in $a$. The government can do so by announcing taxes for the second period such that investors *never* find it optimal to increase investment in project $a$ in the second period. Announcement of a fixed tax-rate in the second period - the same in every state - which is higher than the tax-rate in the first period would be an example of such a tax policy.

More generally, in order to make the first period investment decision bind in all states in the second period, the government must announce taxes that give,

$$k_1^a = \bar{k}_2^a = f^{-1}\left(\frac{r(1 + \delta p)}{(1 - \phi_1^a) + \delta p(1 - \overline{\phi_2^a})}\right) \geq f^{-1}\left(\frac{r}{1 - \underline{\phi_2^a}}\right) = \underline{k}_2^a$$

In terms of tax rates this implies $\underline{\phi_2^a} \geq \phi^* = \dfrac{\phi_1^a + \delta p \,\overline{\phi}_2^a}{1 + \delta p}$. Our first result, stated as Lemma 1, shows that such a tax structure will never be optimal if the revenue requirement satisfy $\bar{g} > g_1 \geq \underline{g}$ .

## Lemma 1

*If* $\bar{g} > g_1 \geq \underline{g}$ *it is never optimal to announce taxes such that the first period investment is binding in the good state. That is, it is never optimal to announce*

$$\underline{\phi_2^a} \geq \phi^* = \frac{\phi_1^a + \delta p \,\overline{\phi}_2^a}{1 + \delta p} \quad .$$

This lemma is proved formally in an appendix.

It follows from Lemma 1 that optimal taxes are set such that investors will increase investment in $a$ if the good state is realised. It is relatively easy to

understand why such a policy is optimal. For consider a situation where the opposite is true, that is, where $\underline{\phi}_2^a > \phi^*$, and the investment decision made in period 1 thus is strictly binding whatever state is realised in the future. In this case a reduction in $\underline{\phi}_2^a$ will have a positive incentive effect on investment in project $a$ in the first period and hence in both states in the second period. The government can therefore reduce taxation of income from project $b$ both in the first period and in the high revenue requirement state in the second period. On the other hand, taxation of capital income of type $b$ must be increased if the good state is realised in the second period. But since this tax rate is low initially ($\underline{\phi}_2^a$ is high) the cost of such an increase, in terms of higher excess burden, is low. A reduction in $\underline{\phi}_2^a$ therefore improves efficiency, and this argument can be repeated for every $\underline{\phi}_2^a >$ $\phi^*$.

Setting $\underline{\phi}_2^a = \phi^*$ cannot be optimal either. In this case a reduction in $\underline{\phi}_2^a$ has no effect on the first period investment in project $a$. And if the good state is realised in period two it is optimal to have the same amount of capital in projects $a$ and $b$. It is then optimal to set $\phi_2^a = \underline{\phi}_2^b$ which is lower than $\phi^*$.

According to Lemma 1, the government should set taxes such that the supply of capital of type $a$ will be based on considerations where future tax rates enter asymmetrically. Only the tax rate in the bad state in the second period is taken into account. One should expect this asymmetry to have implications for optimal taxes. In particular one should expect that it is not optimal to level the playing field, as we know would be optimal if both projects $a$ and $b$ were perfectly reversible.

In order to confirm this conjecture we must solve the government's maximization problem. The government's aim is to maximize expected aggregate output subject to the state contingent budget constraints, and subject to the constraints that capital will be allocated by investors in order to equalize expected net returns. The government's objective function is thus,

$$Y = f(k_1^a) + f(k_1^b) + (e - k_1^a - k_1^b)r + \delta\{p[f(\bar{k}_2^a) + f(\bar{k}_2^b) + (e_2 - \bar{k}_2^a - \bar{k}_2^b)r] +$$

$$(1-p)[f(\underline{k}_2^a) + f(\underline{k}_2^b) + (e_2 - \underline{k}_2^a - \underline{k}_2^b)r]\}, \tag{6}$$

where $e_2$ is equal to $(e - k_1^a)(1 + r) + f'(k_1^a)(1 - \phi_1^a)$. The budget constraints are:

$$g_1 = \phi_1^a f'(k_1^a)k_1^a + \phi_1^b f'(k_1^b)k_1^b \tag{7}$$

$$\bar{g} = \bar{\phi}_2^a f'(\bar{k}_2^a)\bar{k}_2^a + \bar{\phi}_2^b f'(\bar{k}_2^b)\bar{k}_2^b \tag{8}$$

$$\underline{g} = \underline{\phi}_2^a f'(\underline{k}_2^a)\underline{k}_2^a + \underline{\phi}_2^b f'(\underline{k}_2^b)\underline{k}_2^b \tag{9}$$

The constraints that capital owners allocate their capital in order to maximize their after tax income are given by equations (3), (4) and (5).


*Derivation of optimal tax rates*

If the good state is realised in the second period we know, from Lemma 1, that the optimal tax structure implies that investors will increase their holdings of type $a$ capital. In this state it is "as if" both projects are fully reversible. Thus, it is optimal to tax income from both projects at the same tax-rate; $\underline{\phi}_2^a = \underline{\phi}_2^b$.

The derivation of optimal taxes for the first period and for the bad state in the second period is more involved and is relegated to an appendix. Here we give an intuitive argument. We define $\mu_1$ to be the multiplier associated with the government's budget constraint in the first period. Let $\bar{\mu}_2$ and $\underline{\mu}_2$ be the multipliers associated with the high and the low revenue requirement constraints, respectively, in the second period. Define $\bar{\lambda}_2$ to be the current value of the multiplier associated with the high revenue requirement in the second period, i.e. the current shadow price of government revenue if this state is realised,

$$\bar{\lambda}_2 = \frac{\bar{\mu}_2}{\delta p}$$

Given the tax structure implied by Lemma 1 the stocks of irreversible capital ($a$) in the first period and in the high revenue requirement state in the second period are equal. The effect on investment in $a$ of increased taxation in the bad state in the second period can therefore always be neutralised by an appropriate reduction in the tax rate in the first period. As shown below it follows from this that the shadow price of government revenue in the first period, $\mu_1$, and the current value of the shadow price in the bad state in the second period, $\bar{\lambda}_2$, must be equal in optimum. It will become clear that the equality between these two shadow prices has some quite interesting implications.

To see that $\mu_1 = \bar{\lambda}_2$, suppose one is at an optimum and suppose the government conducts the following experiment: It increases the tax rate in the first period, $\phi_1^a$, and decreases the tax rate in the bad state in the second period such that the first period supply of capital of type $a$ is left unchanged. If there is a one unit increase in the tax rate in the first period, the offsetting reduction in the second period (bad state) tax rate must be equal to $-1/\delta p$ in order to leave the capital stock unchanged (see equation (3)). This experiment induces an increase in the government's revenue of one dollar in the first period and a loss in revenue equal to $-1/\delta p$ in the second period (in the bad state). There is no effect on the excess burden of taxation of the policy reform since the supply of capital is unchanged. The marginal value of increased revenue in the first period is equal to $\mu_1$, while the value of a loss in revenue of $-1/\delta p$ dollar in the high revenue state in the second period is equal to $-1/\delta p\,\bar{\mu}_2$, which is precisely the shadow price of government revenue measured in current value terms, $\bar{\lambda}_2 = \bar{\mu}_2/\delta p$. The government is free to make an experiment where tax rates are changed in the opposite direction, that is, increase $\bar{\phi}_2^a$ and decrease $\phi^{a,1}$ such that the supply of capital is constant. It cannot be an optimum if the government can gain something by making an "experiment" in either of the two directions. Hence in optimum $\mu_1$

must be equal to $\bar{\lambda}_2$ .[1] We can now turn to the derivation of the optimal tax rates. The tax rates for the good state are trivial since these are the same as for the fully flexible case. There are then four tax rates ($\phi_1^a$, $\phi_1^b$, $\bar{\phi}_2^a$, $\bar{\phi}_2^b$) and three capital stocks ($k_1^a$, $k_1^b$, $\bar{k}_2^b$) left to be determined. These capital stocks affect expected production via the following terms

$$(1 + \delta p)[f(k_1^a) - rk_1^a] + f(k_1^b) - rk_1^b + \delta p[f(\bar{k}_2^b) - r\bar{k}_2^b] \tag{10}$$

The two budget constraints that are of interest is those corresponding to $g_1$ and $\bar{g}$ . In optimum we know that the value of a dollar of tax income collected in the second period is $\delta p$ times the value of a dollar collected in the first period ( the shadow prices satisfy $\mu_1 = \bar{\mu}_2/\delta p$ ). The two budget constraints can therefore be combined to a single constraint of the following form

$$g_1 + \delta p\,\bar{g} = \phi^*\,(1 + \delta p)f'(k_1^a)k_1^a + \phi_1^b f'(k_1^b)k_1^b + \bar{\phi}_2^b \delta p f'(\bar{k}_2^b)\bar{k}_2^b \tag{11}$$

where $\phi^* = (\phi_1^a + \delta p\,\bar{\phi}_2^a)/(1 + \delta p)$

The capitalists' investment rule for the irreversible project depends on the tax rates $\phi_1^a$ and $\bar{\phi}_2^a$ only via their average $\phi^*$ (see equation (3)). But then we see that the optimal taxation problem reduces to a problem of finding the vector of tax rates ($\phi^*$, $\phi^{b,1}$, $\bar{\phi}_2^b$) that maximizes the production function (10) subject to fullfilling the budget constraint (11). Since $\alpha f(\cdot)$ has the same elasticity as $f(\cdot)$ for any positive constant $\alpha$, it follows directly from the elaciticety rule for taxation that it is optimal to set the tax rates $\phi^*$, $\phi_1^b$ and $\bar{\phi}_2^b$ all equal, thus

$$\phi_1^b = \bar{\phi}_2^b = (\phi_1^a + \delta p\,\bar{\phi}_2^a)/(1 + \delta p).$$

This means that investments in $a$ and $b$ should be taxed at different rates both in the first period and in the bad state (high revenue requirement) state in the second period.

---

[1] A formal proof is given in appendix 2.

We summarize our observations in Proposition 1 (the formal proof is given in Appendix 2),

## Proposition 1

*Suppose the revenue requirements satisfy $\bar{g} > g_1 \geq \underline{g}$ . Then optimal tax rates satisfy;* $\phi_1^b = \bar{\phi}_2^b = (\phi_1^a + \delta p\, \bar{\phi}_2^a) / (1 + \delta p)$ *and* $\bar{\phi}_2^a > \phi^{a,1} > \bar{\phi}_2^b = \phi_1^b > \underline{\phi}_2^a = \underline{\phi}_2^b$.

*Given these tax rates equal amounts of capital are allocated to projects a and b in the first period as well as in in both states in the second period:*

$$k_1^a = k_1^b = \bar{k}_2^a = \bar{k}_2^b < \underline{k}_2^a = \underline{k}_2^b .$$

As we can see the tax system is *both state contingent and differentiated*. It is not optimal to level the playing field. In fact, relative to a tax policy where all capital income is taxed at the same tax rate (a "level-the-playing-field" tax policy), the optimal tax policy increases the difference between the tax rates in the low and in the high revenue state for the irreversible project $a$ . It is thus optimal for the government to make investments in irreversible capital relatively more risky than investments in reversible capital. Ceteris paribus this makes the irreversible project even less attractive than under the levelled playing field regime. The remedy is, however, to attract investment in capital type $a$ by a reduction in the tax rate for project $a$ in the current period.

The picture of a "level-the-playing-field" tax structure and the optimal tax structure is as follows:

*"level-the-playing-field"*

$$\phi^b_2 \qquad \phi^b_1 \qquad \overline{\phi}^b_2$$
$$\underline{\phi}^a_2 \qquad \phi^a_1 \qquad \overline{\phi}^a_2$$

*Optimal tax structure.*

$$\phi^b_2 \qquad \overline{\phi}^b_2 = \phi^b_1$$
$$\underline{\phi}^a_2 \qquad \phi^a_1 \qquad \overline{\phi}^a_2$$
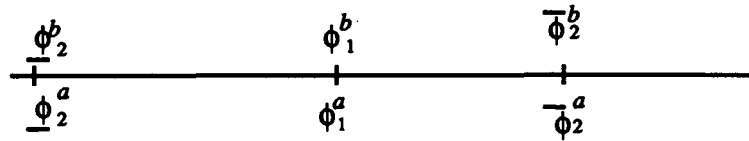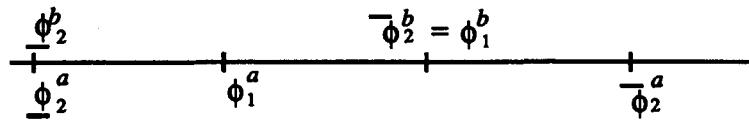
To understand why taxes should be differentiated as indicated in Proposition 1 we start out with a tax policy that levels the playing field. That is, let income from both projects, $a$ and $b$, be taxed at the same tax rate in both states in the second period. This yields a tax structure, where

$$\overline{\phi}^a_2 = \overline{\phi}^b_2 > \phi^a_1 = \phi^b_1 > \underline{\phi}^a_2 = \underline{\phi}^b_2.$$

With homogeneous taxation of capital income, capital supply will be given by (see equations (3) - (5)),

$$k^a_1 < k^b_1, \quad \overline{k}^a_2 > \overline{k}^b_2 \text{ and } \underline{k}^a_2 = \underline{k}^b_2.$$

Assume now that the government undertakes a tax reform: The tax rate on project $a$ in the bad state in the second period is increased while the first period tax rate on this project is reduced by such an amount that the first period investment in $a$ is left unchanged. In the picture - with reference to a levelled playing field - this amounts to moving $\overline{\phi}^a_2$ to the right and $\phi^a_1$ to the left. The government now collects more revenue in the bad state in the second period and less revenue in the first period. To fulfil its revenue requirement it must reduce taxation of income from $b$ in the second period (bad state) and increase taxation in the first period (moving $\overline{\phi}^b_2$ to the left and $\phi^b_1$ to the right). The tax reform's effect on welfare comes therefore solely from the impact it has on the supply of capital of type $b$. And as we can see, the change in tax rates will bring about a decrease in first period supply of $b$, and an increase in the supply in the bad state in the second

period. If we take into consideration both the discounting and the concavety of the production function we find that the increase in the second period capital of type $b$ gives a welfare gain that dominates the first period loss. The concavety of the production function is important since the supply of capital $b$ is higher in the first period than in the bad state in the second period under a levelled playing field.

This experiment indicates in what direction taxes should be differentiated. As we can see from Proposition 1, and from the picture of the optimal tax rates, $\overline{\phi}_2^a$ should be increased and $\phi_1^a$ reduced all the way until the first period and the second period - bad state - tax rate on $b$ are equalised. Taxes are then chosen such that equal amounts of capital are allocated to projects $a$ and $b$ in the first period as well as in in both states in the second period.

The exact tax rates can now be found as follows. Let $k(\phi, r)$ denote the solution to $f'(k(\phi, r)) = r/(1 - \phi)$ and define $H(\phi, r) \equiv \phi f'(k(\phi, r))k(\phi, r)$. The common tax rate $\underline{\phi}$ for projects $a$ and $b$ in the low revenue state is then found by solving $g = 2H(\underline{\phi}, r)$.

If $\phi^*$ denotes the common (effective) tax rate given in Proposition 1, then all capital stocks entering the budget constraints for the first period and for the high revenue state in the second state are equal, and given by $k(\phi^*,r)$. From these two budget constraints - (7) and (8) - we then obtain, $(g_1 + \delta p \overline{g})/(1 + \delta p) = 2H(\phi^*, r)$.

This equation determines $\phi^*$, which now obviously exceeds $\underline{\phi}$. The tax rates for project $a$ in the first period and in the high state in the second period can be found similarly by using the budget constraint and the relation $\phi^*$

$$ = (\phi_1^a + \delta p \overline{\phi}_2^a)/(1 + \delta p). $$

Proposition 1 shows that optimal capital taxation changes dramatically when one of the capital types is irreversible. An interesting question is what happens to the aggregated production level if we compare a situation where all capital types are

fully flexible to a situation where one of the capital types are irreversible. The answer is given in Proposition 2.

## Proposition 2

*Given the government's current revenue requirement, and the distribution of its state contingent future requirements, then expected production will always be higher if one of the capital types is irreversible than if all capital types were perfectly flexible.*

Proof:

From Proposition 1 we know that $\phi_1^b = \overline{\phi}_2^b = (\phi_1^a + \delta p \, \overline{\phi}_2^a) / (1 + \delta p)$ and $k_1^a$ $= k_1^b = \overline{k}_2^a = \overline{k}_2^b = k^*$. The revenue requirements in the first period and in the bad state in the second period then yield

$$\hat{g} \equiv (g_1 + \delta p \, \overline{g}) / (1 + \delta p) = 2\phi^* f^{\backprime}(k^*) k^* = 2[f^{\backprime}(k^*) - r]. \tag{12}$$

The last equality in (12) follows from the capitalists' allocation rule (3). This term is concave in $k$. This imply that $k^* > \hat{k}$ where $\hat{k} = k_1^f \, 1/(1 + \delta p) + k_2^f \delta p/(1 + \delta p)$ and $k_i^f$ ($f$ for flexibility) are the capital stocks that solve $(1 - \phi_i^f) f^{\backprime}(k_i^f) = r$ and $g_i = 2\phi_i^f f^{\backprime}(k_i^f) k_i^f$ for $i = 1;2$. The production function is concave in $k$, which imply that $f(k^\wedge)(1 + \delta p) > f(k_1^f) + \delta p \, f(k_2^f)$, and since $k^* > \hat{k}$ it follows that expected production is higher when one capital type is irreversible.

# 4 Optimal Debt policy

Assume now that the government is dispensed from the balanced budget requirement and that it can lend or borrow money in the first period. Obviously welfare is never lower in this case. It might be optimal to have a balanced budget in which case there is no change in welfare, but in general it will be optimal to run a deficit or a surplus, and then welfare will be higher. The issue we investigate here is *when* the government should borrow and *when* it should lend money.

Our results are surprising as they are in sharp contrast to the tax smoothing principle advocated by Barro [1979] and several others. Consider for example the following situation. Let the expected revenue requirement in the second period be much higher than the revenue needed in the first period. Moreover let the revenue requirements satisfy $\bar{g} > g_1 = \underline{g}$ such that in no states in the second period are the required tax revenues lower than in the first period.

This case is illustrated by the figure below;

$$\text{——} \quad \bar{g} \qquad \text{happens with probability } p > 0.$$

$$\text{——} \; g_1 \qquad\qquad \text{——} \; \underline{g} \qquad \text{happens with probability } (1 - p)$$

| 1. period | 2. period |
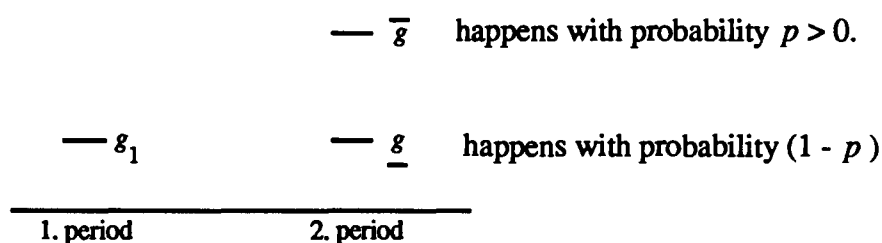
In this situation, should the government run a deficit or a surplus in the first period? One would expect an economist to answer " a surplus". The excess burden of taxation is a convex function of the amount of taxes collected. Based on this it should be optimal to smooth the tax level over time. This would be the right conclusion if all tax bases where fully reversible. In our model where one capital type is irreversible " a surplus" is, however, the wrong answer. In this case it is optimal for the government to run a deficit.

The reason why the common principle of tax smoothing does not apply in our model is the irreversibility of project $a$, and the way optimal taxes are be differentiated between this project and the reversible capital type ($b$). In the following we will show this formally. We also discuss what implications it has for optimal tax rates.

Let $s$ be the public surplus accumulated in the first period. The government can lend and borrow freely at the world market interest rate $r$. The government's optimization problem is as before except for the budget constraints, which are now given by:

$$g_1 + s = \phi_1^a f'(k_1^a)k_1^a + \phi_1^b f'(k_1^b)k_1^b \tag{13}$$

$$\bar{g} - s\delta^{-1} = \bar{\phi}_2^a f'(\bar{k}_2^a)\bar{k}_2^a + \bar{\phi}_2^b f'(\bar{k}_2^b)\bar{k}_2^b \tag{14}$$

$$\underline{g} - s\delta^{-1} = \underline{\phi}_2^a f'(\underline{k}_2^a)\underline{k}_2^a + \underline{\phi}_2^b f'(\underline{k}_2^b)\underline{k}_2^b \tag{15}$$

Consider the Lagrangian for this problem. As before, let $\mu_1$, $\bar{\mu}_2$ and $\underline{\mu}_2$ be the multipliers associated with the (new) budget constraints (13), (14) and (15) respectively. The partial derivative of the Lagrangian with respect to the surplus $s$ is then given by:

$$L_s = -\mu_1 + \delta^{-1}(\bar{\mu}_2 + \underline{\mu}_2)$$

$$= -\mu_1 + p\,\bar{\lambda}_2 + (1-p)\,\underline{\lambda}_2 \quad \text{, where } \bar{\lambda}_2 \equiv \frac{\bar{\mu}_2}{\delta p} \text{ and } \underline{\lambda}_2 \equiv \frac{\underline{\mu}_2}{\delta(1-p)}$$

All other derivatives of the Lagrangian are the same as in the balanced budget case. From the previous section we therefore know that when $s = 0$ and taxes are set optimally, then the multipliers $\mu_1$ and $\bar{\lambda}_2$ are equal (see the discussion

preceding Proposition 1). This means that we can write the derivative of the Lagrangian with respect to $s$ - evaluated at $s = 0$ - as,

$$L_s|_{s=0} = \mu_1[-1 + p] + (1 - p)\lambda_2 \qquad (16)$$

The envelope theorem implies that equation (16) gives an expression for the welfare effects of a marginal change in $s$, given optimal choice of tax rates. Moreover, it is easy to see that for the tax structure given in Proposition 1, the state contingent current value shadow price, $\lambda_2$, for the good state is lower than the shadow price for government revenue in period 1, $\mu_1{}^2$. The expression in (16) - which captures the marginal value of a surplus evaluated at $s = 0$ - is clearly strictly *negative* as long as $p < 1$. That is, it pays the government to run a (marginal) deficit in the first period.

The next two issues that are natural to discuss are: How large should the government deficit be and - second - how are optimal tax rates on income from capital $a$ and $b$ characterized when the government freely can distribute the tax burden over time?

The reasoning that lead to the conclusion that it is optimal to run a deficit if revenue requirements satisfy $\bar{g} > g_1 \geq \underline{g}$ suggests that it is optimal to run a deficit as long as the optimal tax structure is characterised by Proposition 1. That is, it suggests that the government should run a deficit as long as the effective tax revenues satisfy $\underline{g} - \delta^{-1}s \leq g_1 + s \leq \bar{g} - \delta^{-1}s$. Recall that Proposition 1 was based on Lemma 1. A closer inspection of the proof of this lemma shows that its conclusions are valid if the assumption $\bar{g} > g_1 \geq \underline{g}$ is replaced by the weaker assumptions $\bar{g} > g_1$ and $\underline{g} < (g_1 + \delta p \bar{g})/(1 + \delta p)$. (See the last line of the proof in Appendix 1.) Hence Proposition 1 is also valid under this weaker set of

---

[2] The multipliers satisfy $(1 + \mu_1)\phi_1^b = \mu_1\varepsilon(k_1^b)$ and $(1 + \lambda_2)\phi_2^b = \lambda_2\varepsilon(k_2^b)$. Given $\varepsilon'(k) \geq 0$ the inequality $\phi_1^b > \phi_2^b$ then implies $\mu_1 > \lambda_2$.

assumptions. In the present context this means that it is optimal to run a deficit as long as the effective tax revenues satisfy,

$$\bar{g} - \delta^{-1}s > g_1 + s$$

and                                         (17)

$$\underline{g} - \delta^{-1}s < [g_1 + s + \delta p(\bar{g} - \delta^{-1}s)]/(1 + \delta p)$$

For a sufficiently large deficit ($s^* < 0$) the inequality

$$\underline{g} - \delta^{-1}s < [g_1 + s + \delta p(\bar{g} - \delta^{-1}s)]/(1 + \delta p),$$

will be replaced by an equality.

One can show that further increases of the deficit will not increase welfare (see Appendix 3). Hence we have,

## Proposition 3

• *If* $\bar{g} > g_1$, $\underline{g} < (g_1 + \delta p\,\bar{g})/(1 + \delta p)$ *the optimal tax and debt policy entails running a deficit of magnitude* ($s^*$), *where* $s^* < 0$ *is the solution to*

$$\underline{g} - \delta^{-1}s = [g_1 + s + \delta p(\bar{g} - \delta^{-1}s)]/(1 + \delta p).$$
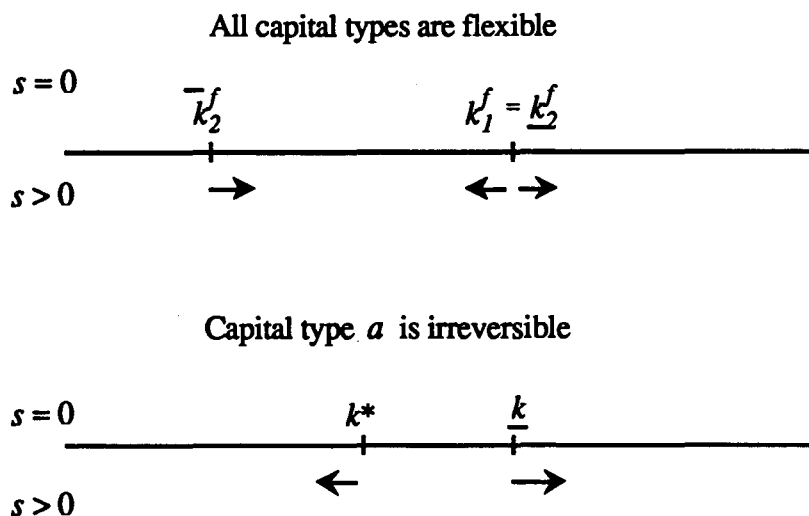
• *Optimal taxes satisfy* $\phi_1^a < \bar{\phi}_2^b = \phi_1^b = \underline{\phi}_2^a = \underline{\phi}_2^b$

$= (\phi_1^a + \delta p\,\bar{\phi}_2)/(1 + \delta p)$, *implying that capital stocks are equalised across all states and periods:* $k_1^a = k_1^b = \bar{k}_2^a = \bar{k}_2^b = \underline{k}_2^a = \underline{k}_2^b$.

A proof of Proposition 3 is given in Appendix 3.

*Conclusions*

The question of whether the government should run a deficit or a surplus in the first period does not depend on the expected magnitude of the future revenue requirement relative to the current revenue requirement. *It can be optimal for the government to run a deficit even if the expected revenue requirement in period two is ten times the revenue needed in the first period.* We have shown that this statement is true in cases where there is a positive probability that the revenue requirement in the future will be higher than in the present period, and the government can collect taxes from both reversible and irreversible tax bases. This result is in sharp contrast to the common sense principle of tax smoothing, which says that since the excess burden of taxation is a convex function of the tax rate, the government should run a surplus in the first period if the expected revenue requirement in the future is higher than it is at present. It is, however, important to recognize that it is not the smoothing of tax rates in itself which gives higher welfare. In the situation we consider, all income effects of taxation are eliminated and the important thing to smooth between periods and states are tax bases - which are capital stocks - and not tax rates. In the traditional analysis there is only one fully reversible tax base. A higher tax rate levied on this tax base means less of it, and the conclusion of tax rate smoothing then follows. But it is really the smoothing of the tax base that drives the argument.

We think the following argument explains why we do not get the standard tax smoothing result. Considered a case where $\bar{g} > g_1 = \underline{g}$. Let us first assume that the government's budget must balance; $s = 0$. Given optimal tax rates in the fully flexible case, which means homogeneous taxation, and in the case where $a$ is an irreversible capital type the allocation of capital is as shown in the figure

All capital types are flexible

$s = 0$

$\overline{k_2^f}$  $k_1^f = \underline{k_2^f}$

$s > 0$  $\rightarrow$  $\leftarrow \rightarrow$

Capital type $a$ is irreversible

$s = 0$

$k^*$  $\underline{k}$

$s > 0$  $\leftarrow$  $\rightarrow$

Assume now that the government runs a marginal surplus in the first period which obviously is the right direction to go if all capital types are the fully flexible. (Expected government outlay is higher in the future than in the current period thus it is optimal for the government to run a surplus now so that the tax burden is smoothed over time). Given a concave production function capital stocks moves in the right direction when $s > 0$. The opposite is true in if capital type $a$ is irreversible. If the government runs a surplus in this case it drives the capitalstocks in the wrong directions. The optimal policy is therefore to run a deficit (s < 0). As we can see from Proposition 3, the optimal tax and debt policy generates perfect tax base smoothing; the capital stocks are equal across all periods and states, but the tax rates or the tax burden are far from smoothed between periods.

We study the problem in a more realistic setting where there are more than one tax base, and where tax bases differ with respect to degrees of irreversiblity. In this case tax rate smoothing does not necessarily imply tax base smoothing. And as we can see, the optimal tax and debt policy generates perfect tax base smoothing; the capital stocks are equal across all periods and states, but the tax rates or the tax burden are far from smoothed between periods.

# 5 Time inconsistency and policy announcement as a commitment.

We have assumed that the government is commited to the policy it announces. It is easy to see that this assumption is binding. If policy can be changed costlessly between period 1 and 2 it is optimal to set $\bar{\phi}_2^a = 1$ and $\underline{\phi}_2^a = \underline{\phi}_2^b$ when period two arrives. The ex-ante announcement of optimal tax rates in the high revenue requirement state is therefore not credible. The government runs into the well known time-inconsistency problem in taxation of capital income from irreversible investment decisions. But there are solutions to these kind of problems. There must be, since we do not observe overall credibility problems as predicted by the theory in real economies.

The solution we find most appealing is the argument that opportunistic policy where the government changes policy in order to take advantage of irreversible investment decisions made in the private sector, imposes long run costs which make the policy less attractive. When the future arrives and the government is tempted to impose a levy on "old capital", there are new investment projects under planning. A confiscatory policy today indicates that the same policy will be followed in the future, and hence it has an adverse effect on investments done today (on new capital).

This idea is formalized by Chari and Kehoe [1990]. They show that the announced policy - the ex-ante optimal policy - can be made credible if only the government is patient enough. Announced taxes are of relevance in their model, that is, private individuals base their expectations about future tax rates on the announced policy. They expect the announced policy to be implemented since a deviation from the announced policy has adverse implications for future investments in irreversible projects. In Chari and Kehoe a two period model is used as the "stage game" which is repeated over and over. Their stage game is a two period model which is a simpler version of the one we have studied. In their

model there is only one asset and there is no uncertainty about future government spending. We could also embody our two period model into a more dynamic framework with repeated investment and taxation decisions. In this way the ex ante optimal tax policy could be made credible, unless, of course, the government is too impatient.

The complexity of the situation we analyse, where there are many assets and uncertainty about the government's revenue requirement in the second period, raises some (additional) problems with the "commitment through reputation" approach. A policy announcement works as a restriction on future policy actions since a deviation from the announced policy induces "punishment" and low capital accumulation in the future. If the state space is very complicated and the optimal rule specifies taxes that are contingent on every possible state, it may be difficult - or close to impossible - for private investors to verify whether the government actually behaves according to the rule or if it behaves opportunistically. Hence, one could argue that in real life the government cannot announce too complicated rules as long as these rules should function as a commitment - that is, as a restriction on future policy.

In our model the ex-ante optimal tax plan is far from simple. It is both state-contingent, as income from project $a$ is taxed differently in different states, and it is differentiated,as income from project $a$ is taxed differently than income from project $b$. To be sure optimal tax policy is not "inaccessible" as long as we have restricted the analysis to only two assets and two possible states of the world. But with many capital types that differ with respect to the degree of irreversibility, and with a more complicated state space the optimal tax rule would soon become too complicated to have a value as a commitment.

A tax policy which levels the playing field - taxes all capital income at the same tax rate - is definitely a much simpler principle of taxation. And, recognising the credibility problem and the need for commitment, a policy announcement's simplicity is of relevance. Thus, even if it is not the optimal tax policy, one could

say that a level-the-playing-field principle of taxation has an advantage of being simple. But there are other simple rules of taxation , and in the light of the time inconsistency problem it seems worthwhile to compare the welfare effects of different simple rules.

### Simple rules of taxation.

In principle a policy rule could be made less complicated in two ways. One alternative is to make it less differentiated, in the limit this would imply a level-the-playing-field policy. Another alternative is to let the tax policy be contingent on fewer states, which - in the limit - implies announcement of a fixed tax rate applicable in every state.

We will compare the welfare effects of these two simple taxation rules; (i) level-the-playing-field and (ii) a fixed tax rate on income from irreversible capital types. It is impossible to do this analytically so we have to resort to numerical examples.

Note first that announcing a fixed tax rate on capital income from project $a$ implies a tax rate structure that goes in the opposite direction from the optimal tax structure. Relative to a level-the-playing-field policy we know that the optimal tax structure increases the difference between high and low state tax rates on income from project $a$ . Furthermore, traditional insights about optimal taxation weigh against implementing a fixed-tax-rate-policy. That is, if both capital types were irreversible it would definitely be unwise to let *all unexpected* revenue realisations be taken account for through tax rate adjustment on one taxbase only. If for example government spending is unexpectedly high, it seems to be inefficient to let all the unexpected part of the revenue requirement be financed by increased taxation on project $b$ alone (or more generally, on reversible tax bases). The Ramsey smoothing principle of taxation indicates therefore that a fixed-tax-rate-policy is a particularly inefficient way to tax capital income. We have already challenged

many of the traditional insights concerning optimal taxation so the possibility that a fixed-tax-rate-policy could give higher welfare than a tax smoothing policy with harmonized taxation of capital income, should not be precluded. This is particularly a possibility when the revenue requirement in the bad state in the second period is considerably higher than the revenue requirements for both the first period and for the good state in the second period. The reason is of course that this revenue structure requires that high taxes must be announced for the bad state in the second period. Hence there will be little investment in irreversible capital types in the current period. This portfolio distortion is also clearly reinforced if the probability that the bad state should happen, increases.

This indicates what parameter values we should choose in order to find interesting cases where the fixed-tax-rate-policy dominates the alternative simple rule, viz. to level the playing field. In Proposition 4 the main results are reported.

## Proposition 4

*If the revenue requirement in the bad state in the second period is considerably higher than the revenue requirement in the first period - and there is a high probability that the bad state will happen - then a fixed-tax-rate-policy can give higher welfare than a level-the playing-field-tax-policy.*

The proposition is proved by numerical examples which can be found in Appendix 4.

In these examples we have used a production function $f(k) = 2k^{0,5}$ such that $\varepsilon(k) = 0,5$.

First we let $\underline{g} = g_1 = 4$ and $\bar{g} = 8$. We then find that a fixed tax rate policy dominates level-the-playing-field if the probability that the bad state happens is larger than 0,8, ($p > 0,8$). The increase in welfare under a fixed tax rate regime relative to the welfare when taxation is harmonized increases as $p$ increases( see Appendix 4).

If there is an increase in the revenue requirement in the bad state in the second period ($\bar{g}$ = 12) we find that the critical probability where a fixed tax rate policy starts to dominate a "level-the-playing-field" policy is lower. In this case it happens for some $p$ between 0,6 and 0,7. Note also that gross domestic production (welfare) is considerably higher under fixed tax rate policy if it is very likely that the high revenue requirement will be realised. If $p = 0,9$ production is 4,4% higher under a fixed tax rate policy.

# 6 Concluding comments.

In this paper we have studied the problem of characterizing the optimal capital income tax when capital types differ with respect to their degrees of irreversibility, and there is uncertainty about the government's future revenue requirement. The distinction between irreversible and reversible capital types is important because it changes many of the conventional results about optimal taxation.

First, we show that in this model it is not optimal to level-the-playing-field. That is, income from different capital types should not be taxed at the same tax rate. The argument we present in favour of heterogeneous taxation has been ignored in the debate concerning the design of the tax reforms that recently have been undertaken in so many countries. The main goal of these reforms has been to harmonize taxation of income from different capital types, a policy that we show to be sub-optimal.

Next, we find that the presence of capital types that leave investors with different degrees of flexibility in the future, has important and surprising implications for the optimal distribution of the tax burden over time. The well known tax smoothing argument, which says that one should run a surplus (deficit) in the first period if the required revenue in the future is much higher (lower) than in the current period, does not generalize to a model with tax bases of different

durability. It is optimal to run a deficit in the present period even if the expected revenue future requirement is much higher - say ten times - than the current revenue requirement. This surprising result is driven by the way tax rates are optimally differentiated between irreversible and reversible capital types.

One could argue that the optimal tax plan is too complicated as tax rates are both state contingent and differentiated, and that the simplicity of a levelled-playing-field makes this an attractive policy rule from a practical point of view. In the last section we compare the welfare effects of an alternative simple rule where irreversible investments are taxed at a the same tax rate irrespective of the realised revenue requirement in the future. We show that this latter fixed-tax-rate-rule can give a better solution than a levelled-playing-field.

## Appendix 1

Proof of Lemma 1.

Note first that $\underline{\phi}_2^a \geq \phi^*$ is equivalent to $\underline{\phi}_2^a \geq (\phi_1^a + \delta E \phi_2^a)/(1 + \delta)$. Since the irreversibility constraint is binding in both states for $\underline{\phi}_2^a \geq \phi^*$, the capital allocated to project $a$ will be given by

$$k_1^a = \overline{k}_2^a = f^{-1}(\frac{r(1 + \delta)}{(1 - \phi_1^a) + \delta(1 - E\phi_2^a)})  \qquad (3')$$

Suppose now that is optimal to let $\underline{\phi}_2^a > \phi^*$. Consider the maximization of expected production (6) subject to the revenue requirements (7) - (9) and the incentive constraint (3'), (4) and (5). Let $\mu_1$, $\overline{\mu}_2$ and $\underline{\mu}_2$ be the multipliers associated with the revenue constraints (7), (8) and (9). Moreover define

$\overline{\lambda}_2 \equiv \frac{\overline{\mu}_2}{\delta \overline{p}}$ and $\underline{\lambda}_2 \equiv \frac{\underline{\mu}_2}{\delta \underline{p}}$ where $\overline{p} = \Pr[g_2 = \overline{g}]$ and $\underline{p} = (1 - \overline{p})$.

We then obtain the following first order conditions:

$$(1 + \mu_1)\phi_1^b = \mu_1 \varepsilon(k_1^b) \tag{a,1}$$

$$(1 + \overline{\lambda}_2)\overline{\phi}_2^b = \overline{\lambda}_2 \varepsilon(k_2^b) \tag{a,2}$$

$$(1 + \underline{\lambda}_2)\underline{\phi}_2^b = \underline{\lambda}_2 \varepsilon(k_2^b) \tag{a,3}$$

$$(1 + \mu_1)\phi_1^a + \delta E\phi_2^a + \overline{\lambda}_2\delta\overline{p}\ \overline{\phi}_2^a + \underline{\lambda}_2\delta\underline{p}\ \underline{\phi}_2^a = \tag{a,4}$$

$$\{\mu_1[1 + \delta(1 - E\phi_2^a)] + \overline{\lambda}_2\delta\overline{p}\ \overline{\phi}_2^a + \underline{\lambda}_2\delta\underline{p}\ \underline{\phi}_2^a\}\varepsilon(k_1^a)$$

$$(1 + \mu_1)\phi_1^a + \delta E\phi_2^a + \overline{\lambda}_2\delta\overline{p}\ \overline{\phi}_2^a + \underline{\lambda}_2\delta\underline{p}\ \underline{\phi}_2^a = \tag{a,5}$$

$$\{\mu_1\phi_1^a + \overline{\lambda}_2[1+\delta- \phi_1^a - \delta\underline{p}\ \underline{\phi}_2^a] + \underline{\lambda}_2\delta\underline{p}\ \underline{\phi}_2^a\}\varepsilon(k_1^a)$$

$$(1 + \mu_1)\phi_1^a + \delta E\phi_2^a + \overline{\lambda}_2\delta\overline{p}\ \overline{\phi}_2^a + \underline{\lambda}_2\delta\underline{p}\ \underline{\phi}_2^a = \tag{a,6}$$

$$\{\mu_1\phi_1^a + \underline{\lambda}_2[1+\delta- \phi_1^a - \delta\overline{p}\ \overline{\phi}_2^a] + \overline{\lambda}_2\delta\overline{p}\ \overline{\phi}_2^a\}\varepsilon(k_1^a)$$

From the last two equations it follows that the current value of the multipliers for the two states in period 2 must be equal, $\overline{\lambda}_2 = \underline{\lambda}_2$ . (Equating the expressions inside the curly brackets and rearranging yields $(\overline{\lambda}_2 - \underline{\lambda}_2)(1 - \phi_1^a +\delta(1 - E\phi_2^a)) = 0$ , hence the claim follows.) From (a.4) and (a.5) it then follows that this common value for the period-2 multipliers must be equal to the multiplier for the period 1 revenue requirement:

$$\mu_1 = \overline{\lambda}_2 = \underline{\lambda}_2$$

Equations (a,1) - (a,4) then yield,

$$\phi_1^b = \overline{\phi}_2^b = \underline{\phi}_2^b = (\phi_1^a + \delta E\phi_2^a)/(1 + \delta), \qquad (a,7)$$

and equal capital stocks across all states and periods $(k_1^a = \overline{k}_2^a = \underline{k}_2^a = k_1^b)$.

Let $k$ denote this common stock, let $h(k) \equiv f'(k)k$, and let $\hat{\phi}$ denote the common tax rate in (a.7). From the budget constraints we then have,

$$g_1 = (\phi_1^a + \hat{\phi})h(k)$$

$$\overline{g} = (\overline{\phi}_2^a + \hat{\phi})h(k)$$

$$\underline{g} = (\underline{\phi}_2^a + \hat{\phi})h(k)$$

By assumption $\underline{\phi}_2^a \geq \phi^* \equiv (\phi_1^a + \delta p\,\overline{\phi}_2^a)/(1 + \delta p)$. The budget constraints then imply $\underline{g} \geq (g_1 + \delta p\,\overline{g})/(1 + \delta p)$, which is a contradiction since $\underline{g} \leq g_1 < \overline{g}$. This completes the proof.

## Appendix 2
Derivation of Proposition 1

The government maximizes expected production (6) subject to the constraints given by equations (3) - (9). Let $\mu_1$ be the multiplier associated with the budget constraint in the first period. Let $\overline{\mu}_2$ and $\underline{\mu}_2$ be the multipliers associated with the high and low revenue requirement in the second period. Let $p$ be the probability that $g_2 = \overline{g}$.

After some algebra the first order conditions for the tax rates for capital type $b$, $\phi_1^b$, $\overline{\phi}_2^b$, $\underline{\phi}_2^b$ and $a$, $\phi_1^a$, $\overline{\phi}_2^a$, $\underline{\phi}_2^a$ can be written in the following way:

$$(1 + \mu_1)\phi_1^b = \mu_1 \varepsilon(k_1^b) \tag{a2,1}$$

$$(\delta p + \bar{\mu}_2)\bar{\phi}_2^b = \bar{\mu}_2 \varepsilon(k_2^b) \tag{a2,2}$$

$$(\delta(1-p) + \mu_1)\underline{\phi}_2^b = \mu_1 \varepsilon(\underline{k}_2^b) \tag{a2,3}$$

$$(1 + \mu_1)\phi_1^a + (\delta p + \bar{\mu}_2)\bar{\phi}_2^a = \{\mu_1(1 + \delta p) + \bar{\phi}_2^a(\bar{\mu}_2 - \delta p \mu_1)\}\varepsilon(k_1^a) \tag{a2,4}$$

$$(1 + \mu_1)\bar{\phi}_2^a + (\delta p + \bar{\mu}_2)\bar{\phi}_2^a = \{\bar{\mu}_2(1 + \frac{1}{\delta p}) + \phi_1^a(\mu_1 - \frac{\bar{\mu}_2}{\delta p})\}\varepsilon(k_1^a) \tag{a2,5}$$

$$(\delta(1-p) + \underline{\mu}_2)\underline{\phi}_2^a = \underline{\mu}_2 \varepsilon(\underline{k}_2^a) \tag{a2,6}$$

By using the definition $\bar{\lambda}_2 \equiv \bar{\mu}_2/\delta p$ we can rewrite equations (a2,4) and (a2,5) to obtain,

$$(1 + \mu_1)\phi_1^a + \delta p(1 + \bar{\lambda}_2)\bar{\phi}_2^a = \{\bar{\lambda}_2(\delta p + 1) + \phi_1^a(\mu_1 - \bar{\lambda}_2)\}\varepsilon(k_a^1) \tag{a2,7}$$

$$(1 + \mu_1)\phi_1^a + \delta p(1 + \bar{\lambda}_2)\bar{\phi}_2^a = \{\mu_1(1 + \delta p) + \bar{\phi}_a^2 \delta p(\bar{\lambda}_2 - \mu_1)\}\varepsilon(k_1^a) \tag{a2,8}$$

The left side is the same in both equations. The expression in the parentheses on the right side of the equation must therefore also be equal. Using this information we get the following equation,

$$(\mu_1 - \bar{\lambda}_2)[1 - \phi_1^a + \delta p(1 - \bar{\phi}_2^a)] = 0.$$

Optimal tax rates are obviously lower than 100%. The first order conditions for optimal taxation therefore requires $\mu_1 = \bar{\lambda}_2$.

We can then write the first order conditions (a.2,1), (a.2,2) and (a.2,3) as follows,

$$(1 + \mu_1)\phi_1^b = \mu_1 \varepsilon(k_1^b) \qquad (a2,9)$$

$$(1 + \mu_1)\bar{\phi}_2^b = \mu_1 \varepsilon(k_2^b) \qquad (a2,10)$$

$$(1 + \mu_1)(\phi_1^a + \delta p\, \bar{\phi}_1^a) = \mu_1(1 + \delta p)\, \varepsilon(k_1^a) \qquad (a2,11)$$

Optimal tax rates in the first period and in the high revenue requirement state in the second period must therefore satisfy:

$$\frac{\phi_1^b}{\varepsilon(k_1^b)} = \frac{\bar{\phi}_2^b}{\varepsilon(\bar{k}_2^b)} = \frac{\phi_1^a + \delta p\bar{\phi}_2^a}{1 + \delta p} \Big/ \varepsilon(k_1^a) \qquad (a2,12)$$

For $\varepsilon'(k) \geq 0$ it is easy to see that, for any tax rates and capital stocks which satisfy (a2,12) and the relevant incentive constraints, the enumerators in (a2,12) must all be equal. Similarly, from (a.2,3) and (a.2,6) we obtain

$$\frac{\phi_2^a}{\varepsilon(\underline{k}_2^a)} = \frac{\phi_2^b}{\varepsilon(\underline{k}_2^b)} = \frac{\lambda_2}{1 + \lambda_2} \cdot$$

When $\varepsilon'(k) \geq 0$ this yields $\underline{\phi}_2^a = \underline{\phi}_2^b$. This completes the proof.

**Appendix 3**

Proof of Proposition 2.

The analysis in the text preceding Proposition 3 shows that welfare increases (strictly) with higher debt as long as $s > s^*$ ( where $s^* < 0$) So, consider $s < s^*$ . The effective revenue requirements $g_1(s) \equiv g_1 - s$, $\underline{g}(s) \equiv \underline{g} + \delta^{-1}s$ and $\overline{g}(s)$ $\equiv \overline{g} + \delta^{-1}s$ then satisfy

$$\underline{g}(s) > [g_1(s) + \delta\overline{g}(s)]/(1 + \delta p) \qquad (*)$$

From the proof of Proposition 1 in Appendix 2 it follows that in this case it is *not* optimal to implement a tax structure which induces the irreversibility constraint to bind only in the bad state. ( If we solved the government's maximization problem under the revenue constraint (*) we would find that the first order conditions require $\underline{\phi}_2^a > \phi^*$ which imply that optimal second period tax rates must be binding in both states).

The proof of Lemma 1 in Appendix 1 shows that for the optimal such tax structure, where $\underline{\phi}_2^a > \phi^*$, the current value shadow prices of government revenue are all equal, $\mu_1 = \overline{\lambda}_2 = \underline{\lambda}_2$ . In this case the welfare effect of a marginal increase in the surplus $s$ is given by

$$L_s = -\mu_1 + \delta p \, \overline{\lambda}_2\delta^{-1} + \delta(1-p) \, \underline{\lambda}_2\delta^{-1}$$
$$= 0$$

Hence $L_s < 0$ for all $s > s^*$ and $L_s = 0$ for all $s < s^*$ This means that setting $s = s^*$ maximizes welfare. The optimum is, however, not unique. Every $s < s^*$ gives the same welfare level. But for any such $s$ the tax rates and the capital stocks will satisfy the conditions stated in Proposition 3.

## Appendix 4

Numerical example.

In this numerical example the production function is given by $f(k) = 2k^{0,5}$ such that $\varepsilon(k) = 0,5$.

In the first example we have revenue requirements $\underline{g} = g_1 = 4$ and $\bar{g} = 8$. $\delta$ is equal to 1. $W^l$ and $W^f$ denote welfare under level the playing field and under the optimal fixed tax rate policy[3].

Welfare is measured as the value added $(f(k))$ from real investments under the two tax systems. The column indexed by % shows how many per cent higher welfare under a fixed-tax-rate-policy is.

Table 1,

| | "Level the playing field" | | | | Fixed tax rate policy: | | | | | | |
| $p$ | $\phi_1$ | $\bar{\phi}$ | $\underline{\phi}$ | $W^l$ | $\phi_1^a$ | $\phi_1^b$ | $\bar{\phi}_2^b$ | $\underline{\phi}_2^b$ | $\phi_2^a$ | $W^f$ | % |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0,5 | 0,116 | 0,252 | 0,113 | 136,1 | 0,074 | 0,165 | 0,24 | -0,013 | 0,255 | 135,5 | -0,44 |
| 0,6 | 0,117 | 0,254 | 0,113 | 134,9 | 0,071 | 0,169 | 0,233 | -0,02 | 0,266 | 134,4 | -0,37 |
| 0,7 | 0,117 | 0,255 | 0,113 | 133,7 | 0,068 | 0,173 | 0,218 | -0,03 | 0,277 | 133,5 | -0,15 |
| 0,8 | 0,118 | 0,256 | 0,113 | 132,4 | 0,066 | 0,176 | 0,208 | -0,033 | 0,285 | 132,4 | 0 |
| 0,9 | 0,118 | 0,257 | 0,113 | 131,1 | 0,064 | 0,18 | 0,195 | -0,4 | 0,296 | 131,5 | 0,3 |

---

[3] The optimal fixed rate tax policy is derived by solving the same program that leads to Proposition 1 except that an additional constraint viz. that the tax rate in the second period is constant $\underline{\phi}_2^a = \bar{\phi}_2^a = \phi^{a,2}$, is imposed.

In the next example everything is as in table 1 except that the revenue requirement in the bad state is increased to 12, $\bar{g}$ = 12.

Table 2,

| | | "Level the playing field" | | | | | Fixed tax rate policy: | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $p$ | $\phi_1$ | $\bar{\phi}$ | $\underline{\phi}$ | $w^l$ | $\phi_1^a$ | $\phi_1^b$ | $\bar{\phi}_2^b$ | $\underline{\phi}_2^b$ | $\phi_2^a$ | $w^f$ | $\%$ |
| 0,5 | 0,121 | 0,459 | 0,113 | 127,5 | -0,013 | 0,298 | 0,22 | -0,19 | 0,609 | 123,6 | --0,7 |
| 0,6 | 0,123 | 0,47 | 0,113 | 123,9 | 0,01 | 0,261 | 0,33 | -0,154 | 0,51 | 123,2 | -0,56 |
| 0,7 | 0,125 | 0,481 | 0,113 | 120,4 | 0,01 | 0,262 | 0,326 | -0,155 | 0,52 | 121,3 | 0,74 |
| 0,8 | 0,126 | 0,493 | 0,113 | 116,6 | 0,006 | 0,266 | 0,310 | -0,160 | 0,53 | 119,4 | 2,4 |
| 0,9 | 0,127 | 0,504 | 0,113 | 112,6 | 0,003 | 0,271 | 0,293 | -0,165 | 0,54 | 117,6 | 4,4 |

# References

Alesina, A. and Tabellini, G. [1990]: " Voting on the Budget Deficit" *American Economic Review*, 80, 36 -49.

Auerbach [1979]: " The Optimal Taxation of Heterogeneous Capital" *Quarterly journal of Economics*, 93, 589 - 612.

Barro, R.J. [1979] "On The Determination of The Public Debt" *Journal of Political Economy*, 87, 940 - 971.

Bizer, D. and K. Judd [1989]" Taxation and Uncertaninty" *America Economic Review Papers and Proceedings*, 79, 331 - 336.

Bohn, H. [1990]: " Why do We have Nominal Government Debt" *Journal of Monetary Economics*, 21, 127 - 140.

Chari, V.V. and Kehoe, P.J [1990]: " Sustainable Plans" *Journal of Political Economy*, 98, 783 - 802.

Christiansen V. and T. Kvinge [1988]:" Optimal differensiering av
kapitalbeskatning" *Noras rapport nr.* 3.

Diamond, P. and J.A. Mirrlees [1971]: "Optimal Taxation and Public Production I:
Production Efficiency" *American Economic Review* 61, 8 -27.

Feldstein, M. [1990]: " The Second Best Theory of Differential Capital Taxation"
*Oxford Economic papers* , 256 - 267.

Lucas, R. [1986]: "Principles of Monetary and Fiscal Policy" *Journal of Monetary
Economics* 17, 117 - 134.

Lucas, R and N. Stokey [1983]: " Optimal Fiscal and Monetary Policy in an
Economy without Capital" *Journal of Monetary Economics"* 8, 3, 319 -329

Persson, T. and L. Svensson [1989]:" Why a Stubborn Conservative would Run a
Deficit; Policy with Time Inconsistent Preferences " *Quarterly Journal of
Economics*, 104, 325-345.

Pindyck, R. [1991]: "Irreversibility, Uncertainty, and Investment " *Journal of
Economic Literature*, XXIX, 110-1148.

Richter, W.F. [1988]: " The Optimal taxation of Risky Capital Income; An
Elasticity Rule" *Paper prepared for The conference on "tax reform for tax
neutrality"* Bielefeld May 30 - June 2, 1988.

Chapter four:

# When Groups Contribute To a Public Good: The Importance of Institutional Framework For making Collective Decisions*.

**Abstract**

In this paper I discuss how the institutional framework for making collective decisions influences the outcome of a game where groups contribute to a public good. Representative democracy invites each group to act strategically in the election of representatives. I show that this strategic effect reinforces "the tragedy of the commons", The society - all groups taken together - has therefore incentives to restrict groups from making collective decisions through a system of representation.

# 1 Introduction

Consider a society that is partitioned into different groups. These groups could be distinguished by geographical borders, such as; governments, counties, municipalities and so on. The basic insight from the model can, however, be extended to a wide variety of partitions into different jurisdictions, e.g. if the "society" is a firm then we can think of the sub units as different divisions within the firm. Housing estates in a local community could be another example. I study a situation where each group in the society is invited to contribute to a public

---

good, that is, a good that is available to all members of the society once it is supplied?

This situation is of interest for two reasons. First, private provision of public goods is the standard example of "the tragedy of the commons". Each consumer cares only about his own consumption of private goods, but for the total supply of the public good. With this externality the public good will be under-supplied in a non-cooperative contribution equilibrium. Second, since each contributor to the public good is a group of individuals, one must take into consideration the possibility that individuals,within each group, have different taste for the good.

Each group must therefore make two decisions: The fundamental one is of course to decide how much their group should contribute towards the public good. But before this decision can be made the individuals in a group must figure out what procedure they should use for making the collective decision. With respect to the second question I assume that the contribution decision can be made in two different political or institutional frameworks

i) Members in each group can vote directly over the size of the contribution; **referendum.**

ii) They can elect a decision maker who subsequently decides the actual contribution; **representative democracy.**

The timing of events in the decision problem is important: At time $t$ different groups simultaneously choose how much their group should contribute to the public good. The institutional framework for making the collective contribution decision must of course have been decided earlier, that is, at $t' < t$. The time structure is shown in figure 1.
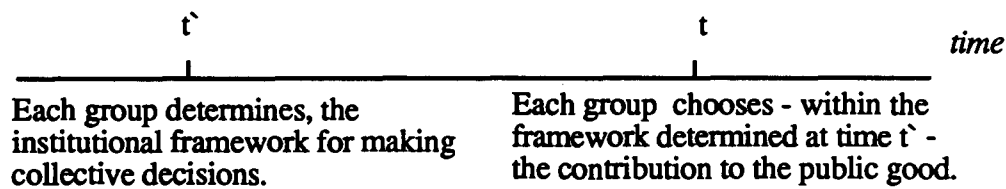
```
       t`                                    t
       |                                     |                    time
───────┴─────────────────────────────────────┴──────────
   Each group determines, the          Each group chooses - within the
   institutional framework for making   framework determined at time t` -
   collective decisions.                the contribution to the public good.
```

*figure 1*

The institutional framework turns out to be important for the equilibrium supply of the public good. This suggests that the society - the union of the groups - has incentives to determine, or restrict, the procedure for making collective decisions within groups.

The decision problem described here is best analysed as a two stage game.

## Two stage games

In economics there exist a large number of two stage games with strategic interaction between periods, (see Shapiro [1990] for references of two stage games in the literature on Industrial Organization). The driving force in these models is a general point stressed in game theory, namely that it can be optimal for a player to limit his own strategy space if he is playing against rational opponents. If he, on the other hand, is playing a game against nature (ordinary decision problem), eliminating some options will never be a wise strategy. The reason for this difference is that in a game with rational opponents, commiting oneself from making certain actions in the future can alter the optimal behaviour of other players. A strategic action at stage one must fulfil two requirements in order to function as a credible commitment: It must be irreversible and it must be visible to the other players.

Thomas Schelling [1960], in his essays on bargaining, was perhaps the first to apply the idea of strategic commitments to bargain and game like situations. He pointed out that in a strategic environment a stupid negotiator - stupid in the sense of not being aware of the damage of a possible disagreement and a conflict -

could achieve better results than a perfectly rational one. Observing this Schelling argued that a principal could bind himself through the use of a bargaining agent having an incentive structure that is different from the principal's. The idea of commitment through delegation has later been applied by several authors writing in the area of industrial organisation. It is pointed out that the owners of a firm can find it in their interest to delegate decision power to a manager and give him incentives that diverges from the owner's own objective that is to maximize profits. For example, if owners, for strategic reasons, gain by making a commitment to produce more than the quantity implied by profit maximization at the second stage, they could do so by arranging a contract where managers are given a wage which is an increasing function of revenue less only a fraction of true costs. Managers will then, to some extent, be revenue maximisers. The firm will be more aggressive in the second period and rival firms will reduce their output. This will shift profits from rival firms to the aggressive firm ( see Vickers [1985], Sklivas [1987] and Fersthman and Judd [1987] for more about these ideas).

### Commitments and political institutions.

The value of commitments in a political or in a collective decision process to avoid a zig zag policy, in terms of constant reevaluation of past decisions, is an old idea (see Elster [1979], page 88). Representative democracy can be used as a way to get more stable and predictable collective decisions. Instead of letting all individuals in a group vote directly over issues, decision power is delegated to representatives who thereafter make decisions on behalf of the group.

Traditionally it has been pointed out that the main problem with representation - without a constant recall option - is the problem of finding representatives with preferences that agrees with the majority's. (See Rogoff [1990]). But, as should be evident from the discussion above, if a group faces a strategic environment the problem may be the opposite; the majority or the median voter

will search for representatives with preferences diverging - in some optimal way - from the median voters' own preferences.

The idea that representative democracy is a device for the median voter to strategically commit in a policy game, is applied in Persson and Tabellini [1989]. They discuss how the well known problem of time inconsistent capital taxation can be reduced in a framework of representative democracy. The median voter can reduce - or even eliminate - the incentive to increase taxation on existing assets, by voting in favour of a policy makers that care more about the welfare of asset owners (the capitalists) than the median voter does. A better solution is thereby achieved by representation.

Another interesting idea, following the same line of reasoning, is discussed in Persson and Tabellini [1990,b]. They show that changes in the economic environment, which disturbs an initial non-cooperative equilibrium, may trigger reactions in the underlaying political equilibrium. Their key point is that changes in the economic environment will bring about changes in the political equilibrium, changes that will tend to offset the effects of the "new economic environment". In particular they show that although integration of the European market in 1992 reduces capital mobility costs, and therefore generates harsher fiscal competition between countries, it will not necessarily lead to inefficient low tax rates on income from mobile tax bases (capital). In their model voters have different endowments and the distribution of endowments generate a distribution of preferred levels of redistributive taxation within each country. Given the new economic environment in 1992, they show that the median voter in every country will react by electing representatives further to the "left" of himself. These representatives have a low level of initial wealth and prefer higher redistributive taxes. The political effect, the choice of a more left wing agent, therefore tends to offset the economic effect of reduced capital mobility costs.

In both examples above representative democracy generates a better outcome than referendum: Strategic election of representatives reduces the inefficiency

generated by an anticipated capital levy and it reduces the problem of inefficient lenient taxation of capital income in situations with high jurisdictional mobility[1] .

In the game I study - a situation where groups contribute to a public good - the result is the opposite. Here a referendum generates a better outcome than representative democracy. In fact representative democracy substantially reinforces "the tragedy of the commons" .

Leif Johansen [1977] argued that representative democracy could reduce - or even eliminate - the revelation problem one faces in the supply of public goods. His point was that individuals have less incentives to hide their true preferences for the public good in a democratic system with representation. In this paper I ignore the revelation problem. I focus on the externality that exists, and the underprovison of the good that follows, when a public good is supplied through voluntary contributions. I show that representative democracy adds to this general "underprovision problem" since it invites median voters in a group to choose their representatives strategically. In equilibrium each group will elect representatives with a low taste for the public good. They do this to force other groups to supply more of the good.

Referendum would give a better outcome, but I show that referendum is not a sustainable political institution. In a non cooperative equilibrium every group will make their contribution decision in a representative democracy.

---

[1] Note that the strategic effects point in the opposite directions; transfer wealth to the elected representative to avoid the time inconsistency problem; extract wealth from the ione elected in order to avoid too leneient taxation when capital becomes more mobile. One should therefore expect increased capital mobility to reduce the time inconsistency problem in taxation of capital income.

## 2 The model

Individual $i$ in group $j$ generates utility by consuming a private good $(c)$ and by consuming the total supply of the public good $(G)$. I assume that utility can be represented by:

$$U_i^j = \ln(c_i^j) + \gamma_i^j \ln(G), \quad \text{where } G = \sum_{j=1}^{J} G^j \tag{1}$$

$\gamma_i^j$ is an individual taste parameter. It is distributed with a bounded support, $S^j$, in group $j$. Define $\underline{\gamma}^j$ to be the infinum to the support.

Utility is non decreasing in both arguments for all individuals, which implies $\underline{\gamma}^j \geq 0$ for $j = 1,...,J$. Let $\gamma_m^j$ be the taste parameter for the median member in group $j$. Assume a non-degenerate distribution of taste parameters within each group, that is $\underline{\gamma}^j < \gamma_m^j$ for $j = 1,...,J$.

Each individual receives an endowment of one unit that can either be consumed as a private good, or it can be contributed to a public good that is subsequently consumed. For simplicity I assume a linear technology and that it takes one unit of the private good to produce one unit of the public good. Hence the budget constraint for individual $i$ in group $j$, is given by:

$$c_i^j + g_i^j = 1 \tag{2}$$

In order to emphasize that it is groups that make the contribution decision, I assume that there is no problem in revealing the true preferences about the public good within each group. This is a strong assumption, much of the literature about public goods is concerned precisely about the incentive individuals have to hide their true preferences for the good. One could argue - as is done by Leif Johansen [1977] - that the focus on the problem of correct revelation of preferences is

somewhat misplaced. If the supply of a public good is determined by voluntary contributions from groups one still faces the problem of underprovision of the good, even when there is no preference revelation problem within each group. The reason is of course the externality that exists since the contributors pays the full cost but receives only part of the benefits of a contribution to the public good. This is the aspect of public goods I focus on. I assume that there exists a convention saying that the total supply of the public good from one group, is shared alike within the group. This means that individual $i$ in group $j$ must supply an amount $\frac{G^j}{N^j}$ to the public good if group $j$'s total contribution to the good is $G^j$ and $N^j$ are the number of members in the group. In order to focus attention on the essential strategic effect I assume there are only two groups, $j=1,2$, in the society and that both groups have $N$ members[2].

Given these assumptions the individual maximization problem can be written as:

$$Maximize: \quad \ln(c_i^j) + \gamma_i \ln(G) \tag{3}$$

$$subject\ to: \quad c_i^j + \frac{G^j}{N} = 1$$

The aspect that makes this to an interesting problem is that it need not be the same individual who determines the contribution decision in a referendum as in a representative democracy. Voters must therefore optimize backwards. Before they make a decision at the first stage - the procedure-choosing stage of the game - they must reason out how the equilibrium contributions at the second stage depends on $who$ it is that make the contribution decision in each group.

---

[2] Assume that N is an uneven number.

## 3   The Second Stage

Each member in a group is identified by their taste parameter for the public[21] good. Assume that member $d$, with taste-parameter $\gamma_d^j$, makes the contribution decision in group $j = 1,2$. Focus on group 1 and consider a non cooperative situation where individual $d$ takes the contribution from group 2 as given when she chooses group 1's contribution in order to maximize (3). The first order condition to this problem implicitly defines group 1's optimal contribution as a function of the contribution made in group 2. The best response function is given by:

$$G_d^1 \equiv R^1(G_d^2) = \frac{\gamma_d^1 N - G_d^2}{1 + \gamma_d^1} \tag{4}$$

The equilibrium locus ( best reaction functions) has a negative slope. The situation is illustrated in figure 2:

Figure 2
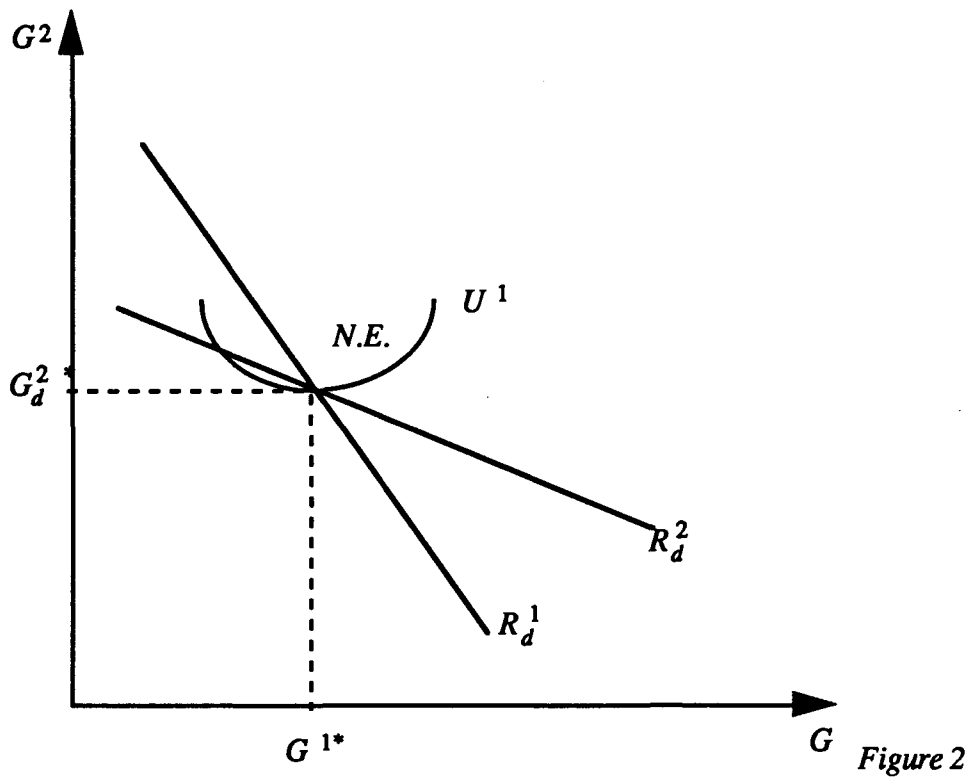
It is easy to check from (4) that these best reactions curves intersect only once and that they intersect in the "right way" to assure that the non-cooperative Nash - equilibrium is "stable"[4] .

By substituting group two's best response function into (4) we can express the equilibrium contribution from group 1 as a function of the decisive members taste-parameters:

$$G_d^1 = \frac{(\gamma_d^1 + \gamma_d^1 \gamma_d^2 - \gamma_d^2 )N}{\gamma_d^1 + \gamma_d^1 \gamma_d^2 + \gamma_d^2} \tag{5}$$

Total supply of the public good is given as the sum of the contributions made in each group:

$$G_d^* = \frac{\gamma_d^1 \gamma_d^2 2N}{\gamma_d^1 + \gamma_d^1 \gamma_d^2 + \gamma_d^2} \tag{6}$$

Given the decisive voters in each group the optimal supply of the public good is found by maximizing the sum of the decisive voters' utility. This is the solution one would get if the decisive voter in each group internalized the externality and cooperatively decided how much of the good they should supply:

$$G_d^{**} = \frac{(\gamma_d^1 + \gamma_d^2)2N}{\gamma_d^1 + 2 + \gamma_d^2} \tag{7}$$

It is straightforward to check that $G_d^* < G_d^{**}$ for all $d$ (the tragedy of the commons).

There is one final thing to note about the second stage equilibrium. If a member with a higher taste parameter for the public good determines the contribution from group 1, this has two effects on the total supply of the public good. First, the contribution made from group 1 will increase: Group 1's reaction curve shifts outwards. Second, it follows, as a consequence of downward sloping reaction curves, that for a *given* decisive voter in group 2, the contribution from this group is reduced.

Analytically:

$$\frac{\partial G^{1*}}{\partial \gamma_d^1} = \frac{2N\gamma_d^1(1 + \gamma_d^2)}{(\gamma_d^1 + \gamma_d^1 \gamma_d^2 + \gamma_d^2)^2} > 0 \tag{6}$$

$$\frac{\partial G^{2*}}{\partial \gamma_d^1} = \frac{-2N\gamma_d^2}{(\gamma_d^1 + \gamma_d^1 \gamma_d^2 + \gamma_d^2)^2} < 0$$

# 4 The procedure-choosing stage of the game

The utility function given by (1) is clearly quasi concave. Preferences over $(c,G)$ are therefore single peaked and Black's median voters theorem can therefore be applied. In other words, only the policy - the contribution decision - preferred by the member with the median taste parameter, $\gamma_m$, can sustain a majority decision rule within each group. Thus, the median voter is the decisive voter if the contribution decision is made by simple majority voting (referendum).

In a representative democracy the contribution decision is made by the person elected to represent the group. The interesting question is therefore who it is that determines - or, who is the decisive voter - in the election of representatives? The answer is stated formally below.

## Lemma 1

*In representative democracy the median voter is determining who will be elected to make the contribution decision on behalf of the group.*

### Proof:

Assume that member $d$, with taste parameter $\gamma_d$, elects a member that makes the contribution decision in group 1. The elected member, $e$, has a taste parameter $\gamma_e$. The median voter theorem can be applied to the choice of representatives if $\gamma_d$ is a strictly increasing function of $\gamma_e$, that is, the claim above is proved if $\partial \gamma_e / \partial \gamma_d > 0$ for all $\gamma > 0$.

In order to find member $d$ will elect to represent the group I substitute the contribution decision made by an individual with taste parameter $\gamma_e$, given by equation (5) into $d$'s utility function. In addition I use the budget constraint to express private consumption as a function of the contribution made to the public

good. After these operations are done one gets the following maximum value function for member $d$ in group 1:

$$V_d^1 = \ln \left( \frac{2\gamma_e^2}{\gamma_e^1 + \gamma_e^1 \gamma_e^2 + \gamma_e^2} \right) + \gamma_d^1 \ln \left( \frac{2N\gamma_e^2 \gamma_e^1}{\gamma_e^1 + \gamma_e^1 \gamma_e^2 + \gamma_e^2} \right) \qquad (7)$$

Taking the partial derivative of (7) with respect to $\gamma_d^1$ gives:

$$\frac{\partial V_d^1}{\partial \gamma_d^1} = \frac{\gamma_d^1 \gamma_e^2 - \gamma_e^1 (1 + \gamma_e^2)}{(\gamma_e^1 + \gamma_e^1 \gamma_e^2 + \gamma_e^2) \gamma_e^1} \qquad (8)$$

Let the interior solution to the problem; $\max_\gamma V_d$ be denoted, $\gamma_e^{1*}$ . The solution is found by setting (8) equal to zero and solve for $\gamma_e^1$,

$$\gamma_e^{1*} = \frac{\gamma_d^1 \gamma_e^2}{1 + \gamma_e^2} \qquad (9)$$

As we can see from equation (9), $\gamma_e^{1*}$ is a monotonically increasing function of $\gamma_d^1$,

$\partial \gamma_e^{1*} / \partial \gamma_d^1 = \gamma_e^2 / (1 + \gamma_e^2) > 0$. The median voter theorem can therefore be applied in the election of representatives.

In the case of representative democracy, we know from lemma 1, that the median voter is the one determining who is to be elected to make the contribution decision for the group. It follows that representation will be a strictly preferred procedure - or institutional framework - for making the contribution decision, if and only if the median voter elects a member with a taste-parameter strictly different from $\gamma_m$. If he vote in favour of one with taste parameter $\gamma_m$ this means that he is indifferent between representation and referendum.

This observation makes it easy to establish the main result of this paper, which is summarized in the following proposition.

## Proposition 1

*The median voters in both groups prefer representative democracy to direct voting. An agent with lower taste parameter for the public good - than the median voter has himself - is elected to make the group's contribution decision. So if groups vote strategically when they choose their representatives the supply of public goods is lower under a system with representation than it would be with direct voting.*

*Proof.*

This proposition can be proved simply by looking at figure 2. Let $d = m$ for group 1 (Lemma 1). By inspection of the figure it should be clear that the median voter in group 1 will not choose to make the contribution decision himself, that is, he would not choose representatives with a taste-parameter $\gamma_e^1 = \gamma_m^1$ in the first stage of the game. The median voter's indifference curve is flat at the point

$(G_d^{1*}, G_d^{2*})$ ( $d = m.$), a small shift inwards in 1's reaction curve, that is, choosing a member with a lower taste parameter than $\gamma_m^1$, has no first order effect on the median voters utility *given* the supply of public good from group 2. But as we can see 2's reaction curve is downward sloping. This means that choosing $\gamma_e^1 < \gamma_m^1$, brings the median voter in group 1 to an indifference curve above the one he reaches when $\gamma_e^1 = \gamma_m^1$ . Utility is increasing in this direction. It is therefore not an equilibrium where both groups make their contribution decision through referendum.

In fact there does not exists a sub-game perfect equilibrium where any group chooses to make their contribution to the public good in a referendum. I can show this formally by using the expression for optimal choice of representatives given by equation (9). The median voter chooses a representative with taste

parameter, $\gamma_e^{1*} = \gamma_m^1 \gamma_e^2/(1 + \gamma_e^2)$. If we use the corresponding equation for group 2 we can write,

$$\gamma_e^{1*} = \frac{\gamma_m^1 \gamma_m^2 - 1}{1 + \gamma_m^2} \tag{10}$$

We can see from (10) that if there are members in each group that generate no utility from the public good, if $\gamma^j = 0$ for $j = 1,2$, then the product of the median voters taste parameters must be larger than one in order to get a positive amount of public good supplied. To be guarantied a positive supply of the public good in the case with representation I assume $\gamma_m^j > 1$ for $j = 1,2$.

Above an interior solution is taken for granted. That is, it is assumed that the median voter in each group will not choose an agent with lower taste parameter than the one who values the public good the least within the group. But it is of course no guarantee that this constraint is non binding in equilibrium. It may be that $\frac{\partial V_m^j}{\partial \gamma_e^j} > 0$ evaluated at $\gamma_e^j = \gamma^j$, in this case is $\gamma^j$ the solution to the

median voters maximization problem. The member chosen to represent group $j$ will therefore have a taste parameter given by: $\max (\gamma_e^{j*}, \gamma^j)$.

It is easy to see from (9) that $\gamma_e^{j*} < \gamma_m^j$, and $\gamma^j < \gamma_m^j$ by the assumption that there are individuals with different taste for the public good within each group. It is therefore possible to conclude that both groups elect members with lower taste for the public good than the median voter. A system of representation gives therefore a lower supply of the public good since $\frac{\partial G^{j*}}{\partial \gamma^j} > 0$ for all $\gamma^j \in S^j$.

*Remarks*

With representation there are two representatives who decide on the participation of their groups. One could then argue that it seems more natural to

consider some kind of a bargaining mechanism. That is, it seems more reasonable to let the two representatives bargain on the amount that each group will contribute towards the public good. Clearly, since the outcome of the bargain depends on the type of the players the strategic effects in election of representatives still holds.

It is also worth emphasizing a crucial difference between this model and the model of delegation in industrial organization which I referred to in the introduction ( see page 4). The delegation literature has been criticised since the equilibrium contracts are not renegotation proof. That is, there is an incentive for owners and leaders of the firm to secretly renegotiate the initial contract when stage two arrives. Writing such a contract has therefore no commitment value. The model I use is not subject to this criticism. Here decision power is delegated to representatives with *different preferences* than the median voter. These representatives are not willing to reconsider their contribution decision since, given their preferences, it is their best choice.


Although it is not directly related to the problem of private contributions to a public good it is interesting to consider the introduction of a pre-strike ballot in Britain in 1984 (The employment act of 1984) in the light of this paper. Under the new legislation a trade union lacks legal immunity for industrial action unless it first obtains a majority in favour of a strike in a secret ballot of the workers concerned. This law restricts the way a group (unions) can make collective decisions. Given the requirement of a majority vote in favour of a strike it is not possible for the union - or the median voter in the union - to elect their leaders strategically, that is, they cannot gain by electing leaders who view the cost of an open conflict as lower than does the median union member.

Another interesting problem where the political dimension seems to be important for the economic outcome is for the opposite problem of global financing of local public goods. This will be the case if a program delivers benefits to specific groups but is financed by a global tax. Assume that representatives bargain over the provision of local public goods. In this case

seems to be optimal for a group to elect a member that has a higher taste for the good than the median voter has.[3]

Proposition 1 shows that the institutional framework for making collective decisions turns out to be important for the outcome in this game. *How* important, can be illustrated with an example:

### Numerical illustration

Assume that the median voter in both groups have a taste parameter equal to 1,2. The efficient supply of the public good (the optimal supply from the majority's point of view) would be $1,08N$ (we get this number from equation (7)). Total supply in an equilibrium with direct voting would be $0,75N$ (follows from equation (6). In a non-cooperative equilibrium with representative democracy the median voter would elect representatives with taste parameters equal to 0,2 (follows from equation (10)). This would give a total supply of the public good equal to $0,18N$. The supply of the public good - with direct voting - is in this case more than *four times higher* than if the contribution is determined through representative democracy.

The importance of the strategic effect suggests that the society - the union of groups - has incentives to restrict the freedom that each group has in choosing their procedure for making collective decisions. Of course, if all decisions could be made at the society level, and specifically if the supply of the public good could be made cooperatively, all externalities would be internalized and the tragedy of the commons would be eliminated. In many situations it is unrealistic to assume that it is possible to establish this degree of cooperation across groups. One may, however, argue that it is possible to implement some basic rules at the

---

[3] Chari and Cole discuss the politics of global financing of local public goods but they do not consider a case where the members of each grop has different taste for the good.

society level. One could, for example, argue that it is possible to determine <u>how</u> collective decisions in groups should be made.

In proposition two it is pointed out under what circumstances the median voter in both groups would vote in favour of a proposal saying that the contribution decisions should be made in a referendum.

## Proposition 2

*Direct voting will give a better outcome for both median voters if a representation game is characterized by an interior solution - that is, if* $\gamma_e^j = \gamma_e^j *$.

This proposition states that $V_m^j(\gamma_m^1, \gamma_m^2) > V_m^j(\gamma_e^1*\gamma_e^2*)$ for $j = 1, 2$. It is straightforward to understand the result for $\gamma_m^1 = \gamma_m^2$. In this case we have a symmetric situation and both groups will contribute the same amount to the public good. We know that the public good is under-supplied in the equilibrium with referendum, $G_m^* < G_m^{**}$, and since the supply under representation is even lower, $G_e^* < G_m^*$, it follows that referendum will be preferred to representation by the median voter in both groups.

It is a bit more demanding to understand why this would be true if the median voters differ in their valuation of the public good. If for example $\gamma_m^1 > \gamma_m^2$ - and particularly if the difference is large - one may argue that the median voter in group $j$ would prefer representation since this gives a small amount of the good in equilibrium. This is not true. To see this note that it follows from (9) that for an interior solution in a representative democracy $\gamma_e^j$ is strictly increasing in $\gamma_m^j$ which in turn means that the contribution made from group $j$ is strictly increasing in the median voters taste parameter. Hence if the difference between the median voter in group 1 and 2 is large group 1 supply most of the public good. The important thing is that both groups supply less than the median voter would do if he determined the contribution directly.

Proposition 2 requires an interior solution in the representation game. There is less scope for acting strategically in the election of representatives if the group is homogeneous in the sense that all the members have the same - or almost the same - taste for the public good. Then, if a homogeneous group is playing the contribution game against a more heterogeneous group, one may find that the second group prefer representative democracy to referendum. This result follows immediately from the observation that in a representative democracy it is possible to have a sub-game perfect equilibrium where the group that values the public good highest, contributes least to it.

That is, even if, $\gamma_m^1 > \gamma_m^2$ one can get $\gamma_e^1 < \gamma_e^2$ which imply $G^{1*} < G^{2*}$.

This result occurs precisely when the taste parameters in one group, say 1, is distributed over a small interval close to the median voter's taste parameter so that the median voter is constrained from choosing the most preferred representatives (not an interior optimum). The following numerical example illustrates this:

Let $\gamma_m^1 = 1,2$ and $\gamma_m^2 = 1,3$, and assume that $\chi^1 = 1,1$ and $\chi^2 = 0$. With these parameters the equilibrium in a representative democracy is given by $\gamma_e^2 = \gamma_e^{2*} = 0,68$ and $\gamma_e^1 = \chi^1 = 1,1$. Contribution from each group is then: $G^{1*}=0,46$ and $G^{2*}=0,13$.

Given these parameters the median voter in group 2 reaches a higher utility level under representation than with referendum. An interesting implication of this result is that groups with a wide difference in opinions (preferences) will come off best in a situation where groups contribute to a public good through representative democracy.

# 4 Conclusions and further remarks

I have shown how a representative democracy could strategically be used in a situation where groups contribute towards a public good. For this reason a system with representation reenforces the tragedy of the commons. A better result could be achieved if the collective decision within each group where made through direct voting or referendum.

There are other important differences between representative democracy and referendum. For example will the equilibrium locus or best response functions in the space of taste parameters be increasing in a system of representation: If the median voter in one group elects an agent with higher taste parameter, then the median voter in group $h$ will also choose an agent who prefers more of the public good. This is seen by differentiating $(9)$ with respect to $\gamma_e^2$, which gives

$$\frac{\partial \gamma_e^{1*}}{\partial \gamma_e^2} = \frac{\gamma_d^1}{1 + \gamma_e^2} > 0.$$

That is, the marginal value of behaving strategically is reduced if the other group act less strategically. The choice of taste parameters - or agents - is therefore strategic complements.

The choice of taste parameters - or agents - are therefore strategic compliments.

With direct voting we know that contribution from one group is a strategic substitute to contributions from other groups (reaction curves are falling( see Bulow, Geanakoplos, Klemperer [1985]). This difference between referendum and representative democracy have some important implications.

First, if there was a change in the rules of the game so that groups move sequentially, not simultaneously, this would have different consequences in a referendum than in a representative democracy. One difference is that sequential moves in referendum imply a decrease in the total supply of public goods compared with a situation of simultaneous moves. The opposite is true under representation, here, the total supply of public goods is higher if groups move

sequentially (this is a standard result based on the difference between strategic substitutes and complements) .

Another observation, also based on the difference between strategic substitutes and complements, which I think is important , is that in a situation with direct voting where the contribution decision is strategic complements, there is a first mover advantage, while in a representative democracy moving first will be a disadvantage. This difference plays an important role in a more dynamic setting. To see this let there be two groups contributing to a public good. Both groups have private information about their exact preferences for the public good. A game with a first mover disadvantage leads, in this situation, to a "war of attrition". Each group finds it optimal to wait the other out. Contribution to a public good will not occur before one group concedes. This happens when a group finds the loss of not being able to consume the public good, equal to the gain of waiting for the other group's concession. The temporization of the supply of the public good will obviously be inefficient. There may, therefore, be a substantial gain from changing the rules in order to get a game with a first mover advantage. This suggests that there is another reason - in addition to the efficiency loss through the strategic election effect - for choosing referendum as the institutional framework when groups contribute to a public good ( See Bliss and Nalebuff (1984)) for more about this).

# References:

Bliss C. and B. Nalebuff [1984]: " Dragon-Slaying and Ballroom Dancing: The
   Private Supply of Public Goods" *Journal of Public Economics* 15, 1 - 12.

Bulow, J. , J. Geanakoplos and P. Klemperer [1985]: "Multimarket Oligopoly:
   Strategic Substitutes and Compliments" *Journal of Political Economy* 93,
   455 - 511.

Elster, J [1979]: "Ulysses and the sirens". *Cambridge University Press*

Fershtman, C, K. Judd [1987]: "Equilibrium Incentives in Oligopoly"
*American Economic Review* ,77, 927-940.

Johansen. L [1977]: " The Theory of Public Goods: Misplaced Emphasis?"
*Journal of Public economics*, 7, 147-152.

Persson, T. and G. Tabellini [1989]: "Representative Democracy and Capital
Taxation." manuscript.

Persson, T. and Tabellini, G [1990,b]: " The Politics of 1992: Fiscal policy and
European Integration" NBER working paper nr 3460.

Rogoff, K. [1990]: "How Often Should Elections be Held." Paper presented on
CEPR/IIES conference June 14 - 16, 1990

Schelling, T. [1960]: " The Strategy of Conflict" Cambridge Mass: *Harvard
University Press*.

Shapiro, C. [1990]: " Theories of Oligopoly Behaviour" *Handbook of Industrial
Organisation*.

Sklivas, S.D. [1987]: " The Strategic Choice of Managerial Incentives"
*Rand Journal of Economics*, 18, 452-458.

Vickers [1984]: " Delegation and The Theory of The Firm"
*Economic Journal supplement*, 95, 138 - 147.

# Chapter five:

# Tacit collusion and the impact of a corporate profit tax*

## ABSTRACT

Public finance questions are usually studied in models with perfect competition. This is so although imperfect competition and the theory of industrial organisation has been a - or should I say *the* - central research field in economics the last decade. In this paper I study the impact of a corporate profit tax in an industry with imperfect competition. The conventional wisdom is that tax levied on pure profit is neutral. I show that this result does not generalize to models with imperfect competition. I show that a tax levied on pure corporate profit can have a real impact on resource allocation if prices and output are determined in a repeated game between firms.

# 1 Introduction

In recent years the analysis of imperfect competition has come a long way. The application of game theory has considerably improved our understanding of strategic interaction, and particularly the dynamic interaction, between firms. In spite of this, corporate taxes are usually studied in models with perfect competition between firms. To date there appears to have been few applications of the new theories of industrial organisation to questions concerning the impact of corporate taxes. This lack of interest is hard to understand because it is far from obvious that the answers that are usually given to public finance questions - based on the conventional model with perfect competition - are the correct answers when competition is imperfect and there is strategic interaction between firms. On the

---

contrary, the few studies that exist, concerning the impact of corporate taxation in presence of imperfect competition, indicate that the conventional knowledge about public finance issues does not generalize to models with imperfect competition.

In a static context Katz and Rosen [1985] show that a tax that increases the cost of production may lead to *higher after-tax profit* for firms in an oligopolistic industry. The reason is that higher production costs will cause a change in the best response functions to every firm in the industry. Each firm's optimal output given other firms' production decisions is lower the higher the variable costs. The effect of a capital tax that leads to higher production costs, is a reduction in each firm's output. The interesting result is that, in terms of profit, the positive effect of a lower industry output can dominate the negative direct effect of a cost increase. The reason is of course that in oligopolistic industries production is higher than the output that maximizes total profit.

Myles [1989] studies an optimal taxation problem in a general equilibrium model where there is imperfect competition in one sector of the economy. He takes the "strategic effect" - the output contraction - of taxes into consideration in the characterisation of optimal indirect taxes. He shows that imperfect competition in one sector alters the structure of optimal taxes.

Taxes or a subsidies, which increases or reduces production costs, have an impact on the firms' best response functions, and therefore on the strategic interaction between firms. This provides an argument for government intervention that is not present in an industry with perfect competition. In the literature on optimal trade policy it has been argued that a government could use a tax or a subsidy strategically in order to shift profit from foreign to domestic firms, see Brander and Spencer [1985].

Besely and Suzumura [1989] use a more general two-stage-competition model to show that the effects of a tax/subsidy are different from the impact the same policy instruments would have in an industry with perfect competition.

*The corporate profit tax*

One of the oldest and best established truths about the effect of a corporate profit tax, dates at least back to Henry George and says that a corporate profit tax is perfectly efficient, in the sense that it does not distort economic behaviour, *if* the tax base is the pure profit earned by a firm. That is, the conventional wisdom is that a profit tax - a tax on pure non-competitive rents - has no impact on prices and output in the market as long as the tax rate is less than 100%. Such a tax has therefore no impact on the allocation of economic resources.

The aim with this paper is to discuss whether the neutrality result of a profit tax holds good in an industry with imperfect competition. First I show that the no-impact result of a profit tax is valid in a static model. More interesting is it that the neutrality result does not generalize to models with imperfect competition and dynamic interaction between firms. I show that a non-linear tax levied on pure corporate profit may change producer behaviour if prices and quantities are determined in a dynamic game with tacit collusion between the firms.

It is easy to understand why a corporate profit tax can have a real impact if market behaviour is decided in a dynamic game between firms. If oligopolists interact repeatedly this provides a setting for collusive behaviour between firms. In this setting it is possible for firms to restrict industry output and get high prices in equilibrium, prices that are not enforceable if firms make an once-and-for-all production decision. The success of the oligopolists in tacitly supporting a collusion scheme depends upon their ability in a credible manner to punish defectors from the scheme. To find the optimal strategy a producer compares the instantaneous gain in profit from defecting, with future losses. Oligopolists will tacitly collude if the instantaneous gain is less than the discounted sum of future losses. A nonlinear tax on a firm's profit can *change the difference between the instantaneous gain and the future loss*. That is, a nonlinear profit tax can for example make it more difficult for oligopolists to collude by making *defection*

*more profitable* or *punishment less severe* than what would be the case without a profit tax. This in turn will give lower prices (higher output) in equilibrium.[1]

After I have made some general points about the impact of a nonlinear profit tax I discuss the implications of more specific non-linearities in the corporate profit tax. I show that the "tacit collusion-model" provides a theoretical foundation for the empirical results reported in Krzyzaniak and Musgrave [1963]. Their results indicate that corporations shifted more than 100 % of the corporate income tax, that is, firms earned higher profit, net of taxes, after an increase in the tax rate on corporate income. This result cannot be explained in a model of perfect competition or in a static model of imperfect competition. In these models the corporate income tax, if it is viewed as a tax on pure profit, is neutral and therefore should not be shifted at all. The Krzyzaniak/Musgrave result can, however, be explained in a model where industry output is determined in a dynamic game between oligopolists. And if the punishment of breaking out of the cartel is severe, so that profit is driven below zero during the punishment phase of the game, it is even possible to show that an increase in the tax rate of proportional profit tax levied on all positive profit levels can increase the firms net profit.

In the next section I introduce a static model of a symmetric oligopoly. I discuss the impact of a corporate profit tax in this model. After that I study the effects of a profit tax in a dynamic model. In the last section I briefly discuss what insights this study provides for normative issues concerning the corporate income tax.

---

[1] Repeated play and implicit collusion will in general change policy conclusions. show for example that, contrary to what is true in a static model, import and export quotas can promote competition in a model with repeated play and collusion.

## 2 A Static model

Consider a non competitive industry with gross industry-wide profit given by

$$\Pi(Q) = [p(Q) - c]Q$$

$Q$ is total production, $c$ is the constant unit cost, and the inverse demand curve is given by $p(Q)$ where $p'(Q) < 0$ and $p''(Q) \leq 0$.

If taxes, $T$, are deducted from gross profit we get net industry profit:

$$\Pi(Q) - T.$$

Define $Q^m$ to be: $\text{argmax}_Q \colon \Pi(Q)$.

Define $Q^o$ by: $\Pi(Q^o) = 0 \Rightarrow p(Q^o) = c$.

*No Taxes.*

Assume first that there are no taxes, $T = 0$ for all $Q$, and that there is only one firm in the industry. This firm will then produce $Q^m$ and earn monopoly profit $\Pi(Q^m) = \Pi^m$. I am interested in a situation where there are more than one, but few, producers in the industry. For expositional convenience let there be $n$ identical firms and let the distribution of output quotas, $q_i$ for firm $i$, be symmetric.

We know that $Q^m$ is not enforceable as the industry output if firms do not cooperate and make their production decision (price or quantity) only once. It is also well known that the output - and the corresponding price - that is enforceable in a single decision non-cooperative equilibrium, depends on whether quantity or price that is the firms' choice variable. Prices are driven down to the constant unit cost if prices are the firms' choice variable, while a price above unit cost is sustainable if quantities are the firms' choice variables. In order to make some

preliminary observations about the impact of taxes in an industry with imperfect competition, I first consider a quantity setting game.

## Cournot competition

Let $q_{-i}$ denote the quantity produced by each producer except firm $i$, i.e. $q_{-i}=$ $(q_1, ...q_{i-1}, q_{i+1}, ...q_n)$ and let $Q_{-i}$ be industry output when $q_i = 0$.

$\pi^i(q_i, q_{-i})$ is firm $i$'s profit if every other firm produces $q_{-i}$ and firm $i$ produces $q_i$.

It follows from the assumptions of constant unit costs and a concave demand function that $\pi^i(q_i, q_{-i})$ has a single peak (it is first increasing and then decreasing) for every $Q_{-i} \in [0, Q^o)$. In other words each firm has a unique best response in terms of output given the output produced by other firms. Firm $i's$ best response is denoted $r^i(q_{-i})$ and is implicitly given by the first order condition;

$$\pi^i_{q_i}(q_i; q_{-i}) = 0.$$

We have $n$ such first order conditions for $n$ profit maximizing firms. These can be used to solve for the $n$ unknown production quantities. The solution yields a symmetric pure strategy Nash-equilibrium. Denote the solution $q^n$. That is $q^n$ is the best response for each firm given that the $n-1$ other firms produce $q^n$, $\pi^i_{q_i}(q_i^n,$ $q_{-i}^n) = 0$, denote this function $r(q_{-i})$. $Q^n$ is the industry output given by the Nash-equilibrium. $Q^n$ must be strictly higher than monopoly output, $Q^m$ since $\pi^i_{q_i}(q_i^m; q_{-i}^m$ $) > 0$. $Q^o$, the output level that gives zero profit, is an upper bound of the quantity produced in the non cooperative equilibrium, i.e., $Q^m < Q^n \leq Q^o$.

## Introduction of a corporate profit tax

The impact of taxes in an industry with imperfect competition depends here, as in an industry with perfect competition, on how a firms' tax base is defined. The new and interesting aspect of taxation in industries with imperfect competition, is

the impact taxes have on the *strategic interaction* between firms. The papers referred to in the introduction show that this strategic effect is important for the impact of taxes that directly alter production costs, such as taxes levied on inputs or output. Is the same true if the tax base is non-competitive rents or pure profit? We know that a profit tax has no impact on output or prices in markets with perfect competition - or monopoly - as long as the marginal tax-rate is lower than 100%. It is easy to show that this is the case also in a one stage game between oligopolists. Since I will use this result later I state it formally in Proposition 1.

## Proposition 1

*A profit tax with a marginal tax rate strictly lower than 100% - for all profit levels - has no impact on the best response function for each individual firm, this tax has therefore no impact on output and prices since the Nash Equilibrium is not changed.*

### Proof.

Let $\psi(\pi^i(q_i, q_{-i})) \leq \pi^i(q_i, q_{-i})$ be after-tax profit for firm $i$. Proposition 1 states that $r^i(q_{-i})$ is the solution to $\max_{q_i} \psi(\pi^i(q_i, q_{-i}))$ as long as $\psi()$ is a strictly increasing function (marginal tax rate less than 100%). This statement is obviously true since $\psi()$ is a positive monotone transformation of the profit function. Hence, for a given choice of quantity by other firms, $q_{-i}$, it is the same argument that maximizes pre- and after-tax profit. That is, if the tax base is pure profit, the tax has no impact on best response functions.

### Remark.

The only restriction imposed on the tax function is that the marginal tax rate should be lower than 100%. Note that as long as the tax-rate is below this upper bound it can be highly non-linear without having any impact on real variables.[2] It can be progressive (regressive) in the traditional sense viz. that the ratio of after-tax profit to pretax profit is an increasing (decreasing) function of pretax profit, i.e.,

$d[\psi(\pi)/\pi]/d(\pi) < (>) 0$. Or it can be progressive or regressive in the more strict sense of increasing or decreasing marginal tax-rate, i.e., $\psi''(\pi) < 0$ or $\psi''(\pi) > 0$.

# 3 Taxation in a Dynamic model of imperfect competition

## Collusive behaviour

An important feature of reality is that firms are operative over a long time and make a sequence of decisions, rather than an once-and-for-all production decision. With many periods and repeated play the strategic interaction between firms gets a new dynamic dimension. If one firm considers to change its price or output today it must consider other firms *reactions*, that is, it must ask; " how will other firms react if we change our prices/quanteties today"? A price reduction by one firm can for example induce a price war in the future. Hence repeated play allows firms to respond to each other's actions and, as is well known, this can bring about collusive behaviour and give lower output and higher prices in equilibrium.

There are many different theories that can explain collusive behaviour between firms. The underlying mechanism is, however, the same in every model: There is a constant temptation to deviate from the production plan since the collusive output is set lower than the non cooperative equilibrium output. On the other hand, such a deviation may lead to lower profit in the future, and this can make it in the firm's interest to resist the temptation and stick to the production plan. More formally, let $\pi^c$ be the profit a firm earns in each period if all firms abide to the plan. In every period - if there has not already been a deviation from the production plan - a firm must calculate the value of two different strategies: i) Abide to the plan and earn $\pi^c$ in both the present period and in future periods: ii) Deviate from the plan and earn high profit, $\pi^{ch}$, in the current period but lower profit, $\pi^n$,

in future periods.[3] All the different theories that can explain tacit collusion has as a common structure the following ranking of profit; $\pi^{ch} > \pi^c > \pi^n$.[4]

I want to study the impact a corporate profit tax has on equilibrium prices and output in a model with tacit collusion. Given the vast number of theories and models that can explain tacit collusion, a natural question is: What model should I use? Theories that predict tacit collusion between firm's in equilibrium differ in their way to motivate the link between one firms deviation and future punishments. In other words, models differ in their way to specify the reasons for retaliation. There are at least three different arguments that can be used to explain why one gets a price war after one firm has cheated on the "cartel agreement". First we have the model of repeated games - a model where the single decision game is repeated over and over. Here firms condition their present behaviour on past outcomes, the history, only because other firms do so. In these models a price war is driven by "bootstrap" behaviour. It is rational for firms to increase their output or charge a lower price if one firm breaks the cartel "agreement" since every firm expects other firms to follow the same strategy. Another approach is to model state variables that give an explicit connection between current and past behaviour. If there are short run price rigidities (menu costs) prices can be taken as state variables, and then one can explain the reactions to a price cut - a price war - as the outcome when other firms try to regain their market shares that has been eroded when one firm cheats on the production plan (Maskin and Tirole [1988]). A third explanation to why a deviation by one firm brings about a price war is imperfect information. A deviation will then reveal information about the firm's type and given this information it can be optimal for other firms to play non-collusive from now on.

The results I derive about the impact of a corporate profit tax are based on - or driven by - the structure that all models of tacit collusion have common, viz. the ranking of profit, $\pi^{ch} > \pi^c > \pi^n$. Hence these results could in principle be derived

---

[3] It is possible that the profits earned in the punishment phase of the game is nonstationary (as in the stick-and-carrot punishment I will consider later). In this case $\pi^n$ should be interpreted as the per period equivalent to the profits earned after a deviation is detected.

[4] Formally $\pi^{ch}$ is defined as $\pi^{ch} = \pi(r(q_{-i}^c), q_{-i}^c)$.

within any of the theories I referred to above. I choose to work with the repeated game specification since this is the simplest and most familiar model. But it is worth to emphasize that the results are not model specific. This should be emphasized since a serious critique against the new theory of industrial organisation is that it has given us a vast number of different models and that close to everything of observed market behaviour can be explained if only the "right" model is selected. A central question is then what model is best suited to explain behaviour in a particular industry? How one should go about testing this is very much an open question. It is therefore an important observation that the results I derive about the impact of a tax on corporate profit are robust in the sense that they do not depend on the specific model that is used.

### A repeated game analysis

I consider a game ($G^\infty$) where the static game introduced in the first section is repeated infinitely many times and where every firm produces its share, $q^c$, of the cartel quantity, $Q^c$, unless one firm deviates from this plan. If one firm cheats in period $t$ the cartel dissolves and each firm increases its production. First I assume that collusion is enforced by "grim strategies". That is, firms revert permanently to the competitive behaviour, the static Nash Equilibrium, $q^n$, after one firm deviates from the production plan.

It is well known that there is not a unique sub-game perfect equilibrium in the repeated game, $G^\infty$. On the contrary, for a given discounting factor there is a large set of equilibria. This poses an obvious problem when one wants to study how a change in tax parameters changes equilibrium prices and output. In order to do comparative statics one must select one equilibrium out of the set of equilibria, one must - in other words - assume that firms coordinate on a "focal equilibrium". In a symmetric game the "focal equilibrium" is often assumed to be the symmetric equilibrium on the pareto frontier. I will make the same assumption here. I study the impact of a profit tax on the best symmetric equilibrium that can be enforced by "grim trigger strategies". This assumption raises, however, another

question. Abreu [1986] has shown that revision to the static solution if one firm deviates - the "grim trigger strategies" - is in general not the optimal punishment. Specifically, in a symmetric quantity setting game, where the static cournot profit is strictly positive ($\pi^n > 0$), it is possible to punish deviations harsher by introducing a symmetric stick-and-carrot punishment (severe punishment in one period before one returns to the collusive production plan). With a punishment like this it is possible to sustain "more collusion" than with "grim trigger strategies". One could therefore argue that it is restrictive to assume that deviations are punished by "grim trigger strategies": If one selects the equilibrium that maximizes the industry profit, subject to the constraint that no firm breaks the cartel agreement, as the "focal equilibrium", it seems inconsistent to assume that the industry does not use maximal punishments in order to push the pareto frontier as far out as possible. What kind of punishment there will be in an oligopolistic industry if a firm deviates from the production plan, is of course an empirical question. One cannot a-priori wave aside grim strategies. Davidson and Deneckere [1990] argue for example that optimal punishments often are very complicated (especially in games where players (firms) are in some sense different (asymmetric games) ) and that,

> " it is highly unlikely that such strategies [ Abreu's optimal punishment] would ever be implemented in the real world. On the other hand, the standard trigger strategies require only simple calculations and are easily understood by industry participants"

Davidson and Deneckere [1990], page 527.

There are two reasons to why I choose to study the impact of a profit tax in a model with "grim trigger strategies". One reason is that it is particularly easy to characterize the impact of a profit tax when profit in different stages of the game are stationary - as is the case when firms use grim strategies. Another point is that if we consider a game with Bertrand competition then trigger strategies are indeed optimal (in the sense that there are no punishments that can enforce more collusive

behavior). Secondly, I want to compare my discussion with the analysis done by Davidson and Martin [1985] and they consider the impact of a *capital income tax* in a quantity setting game where collusion is enforced by "grim trigger strategies".[5]

Given the "grim trigger strategies" the net gain from cheating, $g$, is:

$$g = ( \pi^{ch} - \pi^c) - \frac{\delta}{1-\delta} (\pi^c - \pi^n) \tag{1}$$

$\delta = e^{-r\Delta}$, where $r$ is the discount rate used by producers and $\Delta$ is the length of each period. The term in the first parenthesis is the extra profit a firm earns if it deviates optimally from the production plan in the current period and the second term is the discounted loss in profit due to the permanent reversion to a higher output in the future. Collusion is enforceable as long as $g \leq 0$. I assume that the firms, when they operate as a cartel, try to reduce output as much as possible given the constraint that $g \leq 0$. It is in the collective interest of firms to reduce the industry output under collusion, $Q^c$, to get it as close as possible to the monopoly output, $Q^m$. But they must pay attention to each firm's temptation to violate the implicit agreement and this temptation increases when output is reduced. That is easy to show. Equation (1), the enforcebility condition can be rewritten as,

$$\delta \geq \frac{\pi^{ch} - \pi^c}{\pi^{ch} - \pi^n}. \tag{2}$$

Profits earned in the punishment phase of the game, $\pi^n$, are independent of the output decision made in the cartel - $q^n$ is independent of $Q^c$. $\pi^c$ and $\pi^{ch}$ are on the other hand both unequally determined by $Q^c$, and both $\pi^c$ and $\pi^{ch}$ are strictly

---

[5] Including Abreu's maximal punishment in a quantity setting repeated game will not change any of the results derived under the assumption of grim strategies ( the ranking of profits is still $\pi^{ch} > \pi^c > \pi^n$). In an example I show that stick-and-carrot punishment are of particular interest when it comes to the practical relevance of the general results.

decreasing in $Q^c$. Hence we can conclude that the right hand side of equation (2) is a strictly decreasing function of $Q^c$.

Formally the cartel's aim is to:

Maximize $\pi^c$ over $Q^c$, subject to the constraint that $g \le 0$.

Let $Q^{c*}$, be the solution to this problem. I assume that collusion is efficiently constrained so that $Q^{c*} < Q^m$.

The maximum enforceable profit $\pi^{c*}$ depends critically on the discount rate. For a given discount rate optimal cartel behaviour can be written as,

$$\delta = \frac{\pi^{ch} - \pi^{c*}}{\pi^{ch} - \pi^n} \tag{3}$$

This equation makes it clear how strategic interaction between firms over time and tacit collusion opens up for new effects of taxation. We can for example see from equation (3) how a tax, which alters the discount factor, also will change the enforceable industry output, $Q^{c*}$. A tax that increases the discount factor ($\delta$), e.g., a tax that decreases the interest rate after taxes will make it possible for the cartel to reduce output ( an increase in $\delta$ allows for an increase in the right hand side of equation (3), that is, for a reduction in $Q^{c*}$). The impact a tax on capital income has on the interest rate, and consequentially on the output in an industry with tacit collusion, is studied by Davidson and Martin [1985]. Their point is precisely that a tax on capital income has a dynamic effect in addition to the cost-increase-effect studied by e.g., Katz and Rosen [1985]. The net return of capital - the after-tax interest rate - is the discount rate used by a profit maximizing firm. The lower the interest rate is the higher is the discounted value of the loss from breaking out of the cartel. A tax that lowers after-tax return on capital makes it therefore easier for the cartel to sustain collusive behaviour. This explains their

surprising result that a general tax on an exogenously given tax base - capital - is shifted. The standard prediction is that for a fixed capital stock the after-tax rate of return will fall to offset the tax and therefore that the pretax return is left unchanged. Such a tax should therefore have no real effects. But, as the argument above show, a lower after-tax rate will change industry output and prices if the output is determined in a dynamic game between oligopolists.

Davidson and Martin [1985] do not consider a tax similar to the corporate profit tax. Related to equation 2 they investigate the effects of a capital tax that changes the left-hand side of the equation while I investigate the impact of a tax on corporate profit that may change the right hand side of equation (3). I assume that the discount factor is exogenously given. This is a natural assumption since a tax levied on pure profit should not have a direct impact on the interest rate.[6]

Clearly a profit tax that changes the right hand side of the equation, that is, a tax that changes the nominator relative to the denominator, will also change the self enforcing industry output, e.g., a tax that leads to a fall in the nominator relative to the denominator makes it possible for the cartel to reduce equilibrium output level, $Q^{c*}$.

These effects will be studied more carefully below.

*Tacit collusion and the effects of a tax on corporate profit*

As a first observation, note that it follows from Proposition 1 that pre-tax profit in the deviation stage, $\pi^{ch}$, and in the punishment phase of the game, $\pi^n$, are not changed by a profit tax since the marginal tax-rate is assumed to be less than 100% ( the best response functions for firms are not changed by a profit tax (Prop. 1)). The corresponding output is therefore not changed by a profit tax. It follows that if after-tax profit is given by $\psi(\pi) \leq \pi$, we can write equation 3 as

---

[6] Put differently, their results are driven by the timing of gains and losses if a firm cheats; firms get the gain from cheating immediately while losses come in the future. A tax which changes the importance of the future relative to the present, that is, a tax that changes the interest rate will then clearly have an impact on the equilibrium. My results are driven by the difference in profit earned in the different phases of the game ( Collusion, deviation and punishment).

$$\delta = \frac{\psi(\pi^{ch}) - \psi(\pi^{c})}{\psi(\pi^{ch}) - \psi(\pi^{n})} \tag{4}$$

From this equation it immediately follows that a proportional tax on profit has no impact on prices and output. If all profit levels in the interval $[\pi^{n}, \pi^{ch}]$ are taxed at the same tax rate then the usual no impact result of a profit tax applies. With a proportional corporate income tax both the gain and the penalty (the nominator and the denominator, in equation 2) is scaled down by the same rate and the equilibrium will not be changed.

More interesting is it that we also can see directly from equation (4) the result stated in the following proposition:

## Proposition 2

*A non-linear tax on pure non-competitive profit can have real effects, that is, a non-linear tax on profit can induce a change in equilibrium prices and output.*

Let $Q^{c**}$ be the lowest enforceable output after a corporate or profit tax $[\pi - \psi(\pi)]$ is imposed. Proposition two states that for a given $\delta$, $Q^{c**}$ need not be equal to $Q^{c*}$. The reason is of course that a non-linear tax on profit can change the gain - relative to the loss - from a deviation. If this is the case there also will be a change in output and in the corresponding profit earned in collusion. With non-lineareties in the tax scheme it is for example possible that the reduction in the extra profit earned by cheating is scaled down more than the future losses (the nominator is scaled down at a higher rate than the denominator). A nonlinear tax of this type leads to a lower industry output and higher prices in equilibrium ( for a given $\delta$ equation (4) will not hold for $\pi^{c} = \pi^{c*}$, and the only way to reassure equality

between the left and right hand side is by reducing the industry output that imply an increase in $\pi^c$).

Formally, a tax scheme leading to a change in equation 2 such that $(\psi(\pi^{ch}) - \psi(\pi^c))/(\psi(\pi^{ch}) - \psi(\pi^n)) \neq (\pi^{ch} - \pi^{c*})/(\pi^{ch} - \pi^n)$ will also changes the equilibrium output in the industry, $Q^{c**} \neq Q^{c*}$. A tax scheme, which change equation (4) such that $(\psi(\pi^{ch}) - \psi(\pi^c))/(\psi(\pi^{ch}) - \psi(\pi^n)) < (\pi^{ch} - \pi^{c*})/(\pi^{ch} - \pi^n)$, would lead to a fall in output, $Q^{c**} < Q^{c*}$.

Proposition 2 says that if output and prices in an industry are determined in a dynamic game and there is tacit collusion between firms, then a tax levied on pure profit can have a real impact. This observation is interesting since it contradicts the standard neutrality result: A profit tax - even if it is non-linear - will not have any real effects in a perfectly competitive industry or in an industry with static imperfect competition (Proposition 1) but, as is shown here, it can have real effects in a model with tacit collusion.

*Progressive - regressive tax.*

It is not possible to make any general conclusions about the impact of a regressive relative to a progressive profit tax without making more specific assumptions about the tax scheme or about the profit earned in different phases of the game ($\pi^{ch}$, $\pi^c$, $\pi^n$). Relative to a proportional tax a general regressive tax will increase both the gain from cheating, which draw in the direction of a lower enforceable output than $Q^c$, and the future losses, which draw in the opposite direction. It is, however, possible to make more precise conclusions if one makes the assumption that the punishment is harsh so that each firm earns zero profit in the punishment phase of the game, $\pi^n = 0$.

**Proposition 3**

*A progressive corporate income tax leads to lower output and higher prices if there is constrained tacit collusion between firms and $\pi^n = 0$. The opposite is true for a regressive tax.*

This is straightforward to understand and formally it can be shown that if $\pi^n$ = 0, and therefore $\psi(\pi^n) = 0$, it follows that the condition that must be fulfilled to get a lower collusion output,

$$(\psi(\pi^{ch}) - \psi(\pi^c))/(\psi(\pi^{ch}) - \psi(\pi^n)) < (\pi^{ch} - \pi^{c*})/(\pi^{ch} - \pi^n) \quad \text{reduces to}$$

$$\psi(\pi^{ch})/\pi^{ch} < \psi(\pi^c)/\pi^c,$$

which is the definition of a progressive tax (the ratio of after-tax profit to pretax profit is a decreasing function of pretax profit).

Proposition 3 is interesting because we know that the "grim strategies" of eternal reversion to the one stage equilibrium give zero profit in the punishment phase if the one stage game is specified as a price setting game instead of a quantity setting game. We can therefore conclude that a progressive profit tax always makes it easier for firms to enforce collusion in a dynamic game with Bertrand competition.

*Strictly progressive and regressive taxes.*

It is possible to get more general results if we consider a profit tax that is regressive or progressive in the more strict interpretation of increasing or decreasing marginal tax-rates.

## Proposition 4

*A profit tax will lead to higher prices and lower output - relative to a situation without profit taxes, or a situation with a proportional tax rate -if the marginal tax-rate is strictly increasing, $\psi''(\pi) < 0$. The opposite is true if the marginal tax rate is strictly decreasing.*

Since we have, $\pi^{ch} > \pi^c > \pi^n$, it follows that a profit tax with a strictly increasing marginal tax rate will scale down the numerator more than the denominator in equation 4. An output $Q^{c**} < Q^{c*}$ is therefore enforceable. In

other words, industry output can be lowered without causing the cartel to break up if a progressive (in the strict interpretation) profit tax is implemented.

# 4 Applications

A puzzling empirical finding about the impact of the corporate tax is the result reported in Krzyzaniak and Musgrave [1963]. Their study shows that corporations shifted more than 100% of the tax (an increase in the tax rate increases the after-tax profit). As far as I know there is no model that can explain this result. True, Davidson and Martin [1985] explain how a *capital income tax* can be shifted, but, as Alan Auerbach writes,

> "*this paper* (Davidson and Martin [1985] ) *does not focus specifically on the effects of a tax similar to the corporate income tax.[]. Thus, we are still without a model to justify the Krzyzaniak-Musgrave result.*
>
> (Alan Auerbach [1990] page 24).

The corporate income tax has been viewed both as a tax on capital used in the corporate sector, and as a tax on entrepreneurship and pure non-competetive rents in the corporate sector. High international mobility of capital implies that the supply of capital is highly elastic in a "local" perspective - a tax on capital in one country will drive it elsewhere. This, and the fact that firms can substitute debt for equity capital, and in most countries interest on debt can be deducted from gross corporate income, imply that a tax on corporate income should be viewed as a tax that is mainly falling on non-competitive rents earned within the corporate sector, (see for example Stiglitz [1976]). The empirical results reported in Krzyzaniak and Musgrave [1963] can therefore not be explained in a model of perfect competition or in a static model of imperfect competition. In these models the corporate income tax, if it is viewed as a tax on pure profit, is neutral and should therefore not be shifted at all.

The main insight from the discussion above is that a corporate tax levied on pure non-competetive rents can have real effects if output and prices are determined in a dynamic game between oligopolists. This suggests that a model with imperfect competition and tacit collusion can explain the Krzyzaniak /Musgrave result. For this to be the case there must be non-lineareties in the corporate income tax. If e.g., a corporate income tax with a strictly increasing marginal tax rate was introduced it follows directly from Proposition 4 that one would get higher prices in equilibrium and the tax would therefore be borne - fully or partly - by consumers. It is, however, not realistic to let the corporate tax be specified with an increasing marginal tax rate. If we look at the corporate tax in different countries we will typically find that profit are taxed at a fixed tax rate. But a fixed tax rate does of course not exclude non-lineareties. Besides the fixed tax rate one also will typically find that the corporate tax system allows for extensive tax deductions. Firms can reduce their taxable income through allocations to untaxed reserves and funds or through capital investments that allow for generous deprecation allowances. Tax deductions with a constant tax rate will give a non linear corporate profit tax.

I will now show that if we allow profit below a certain level, say $\pi^o > 0$, to be exempted from taxation, and if we let profit above $\pi^o$ be taxed at a fixed tax rate $\tau$, then the increased tax burden that follows from an increase in the tax rate may very well be shifted over to consumers (the Krzyzaniak/Musgrave result).

For this to happen let $\pi^n > \pi^o > \pi^c$. The enforceability condition (equation 3) can now be written as:

$$\delta \geq \frac{(\pi^{ch} - \pi^c)(1 - \tau)}{\pi^{ch}(1 - \tau) - \pi^n + \tau\pi^o} \qquad (5)$$

Assume that $\pi^{c*}$ is the maximal enforceable profit if the tax rate is equal to $\tau^*$. That is, if $\tau = \tau^*$ then $\pi^{c*}$ makes equation (5) holds with equality. An illustration of the situation is given in figure 1.
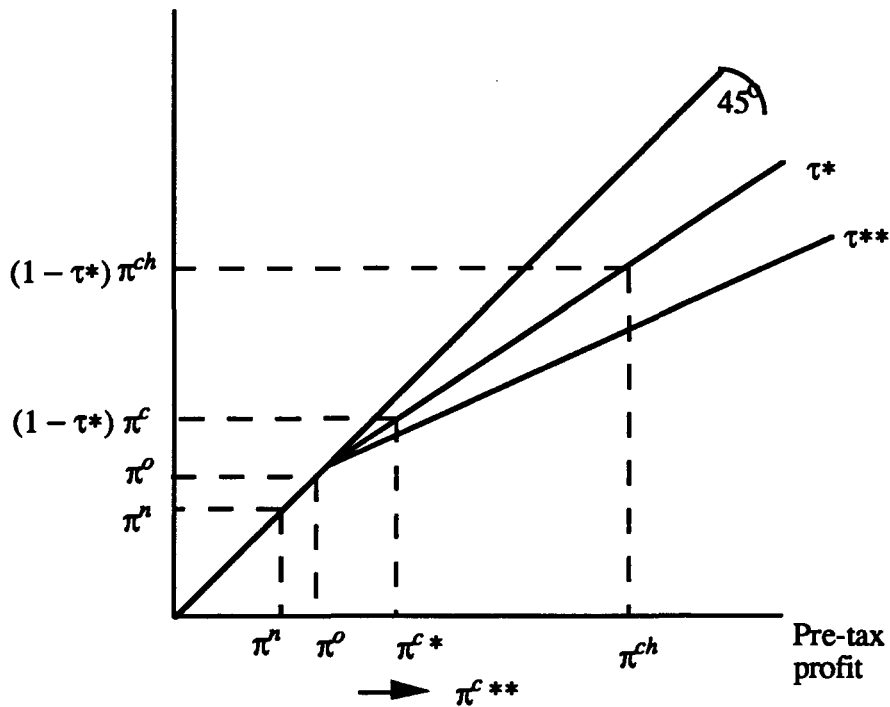
$(1 - \tau^*) \pi^{ch}$

$(1 - \tau^*) \pi^c$

$\pi^o$

$\pi^n$

$45$

$\tau^*$

$\tau^{**}$

$\pi^n$   $\pi^o$   $\pi^c *$   $\pi^{ch}$

$\pi^c **$

Pre-tax profit

figure 1.

What happens if the corporate tax rate increases from $\tau^*$ to $\tau^{**}$? What is the new maximal self enforced profit, $\pi^{ch**}$, after the tax increase? From the figure it is easy to see what will happen. The gain from cheating on the cartel agreement is scaled down relatively more than the loss in profit (because $\pi^o > \pi^n$). For a given discounting factor there will be a reduction in the industry output and each firm will earn higher profit before taxes. The increased tax burden that follows when corporate profit is taxed at a higher tax rate is therefore (partly) shifted over to the consumers (the Krzyzaniak/Musgrave result). This result is of course in sharp contrast to the standard neutrality result of a profit tax. It is easy to show formally that an increase in the tax rate will reduce industry output. The derivative of the right hand side of equation 5 with respect to the tax-rate is given by, $[ - (\pi^{ch} - \pi^c)( \pi^o(1 + \tau) - \pi^n)]/d^2$ , where $d$ is the denominator. This expression is negative

as long as $\tau > 0$ and $\pi^o > \pi^n$, which means that the right hand side of the equation decreases when the tax rate increases as long as the profit that is exempted from taxation is higher than the profit firms earn during the "punishment". The left hand side is assumed not to be changed by a change in the tax rate (the discount rate is assumed to be fixed). To restore equality between the left and the right hand side of equation (5), after there has been an increase in the tax rate, the industry output must be reduced. Firms will therefore earn higher profit before taxes if there is an increase in the profit tax rate.

If we consider changes in the level of profit that are tax exempted we find that maximal enforceable profit $\pi^{c*}$ is increasing in $\pi^o$ as long as $\pi^o > \pi^n$. This is easy to understand, and formally it can be shown by taking the derivative of the right hand side of equation (5) with respect to $\pi^o$. This yields, $[-(\pi^{ch} - \pi^{c*})(1 - \tau)\tau] / d^2$, which clearly is negative.


The "shifting result" discussed above, requires that some profit earned by the firm is exempted from taxation. This is not unrealistic. More restrictive is it, perhaps, that in order to find that an increase in the corporate income tax rate will be shifted, it must be assumed that the profit that is exempted from taxation is larger than the profit earned in the punishment phase of the game ($\pi^o > \pi^n$). This assumption might seem somewhat artificial here where the punishment profit is given by reversion to the static Cournot solution. This assumption is, less restrictive or artificial if the punishment is more severe such that profit is driven down to zero or perhaps even below zero in this phase of the game . I will therefore consider two cases where profit is lower (than the static Cornout solution) during the price war after one firm breaks the production agreement. First I consider a game with price competition - a Bertrand game. After that I will say something about Abreu's optimal punishment.


*Bertrand competition*

If we consider a price setting game (Bertrand competition) we know that $\pi^n = 0$. This means that an increase in the tax-rate levied on corporate profit would lead to higher prices in equilibrium - the profit tax would be shifted - as long as some of the profits is tax exempted, that is, as long as $\pi^o > 0$.

In this case it is also easy to show that more than 100% of the tax can be shifted, so that firms earn higher after tax profit if there is an increase in the profit tax. If there are $n$ producers in the industry and they are playing a price setting game, then $\pi^{ch} = n \, \pi^c$ (or to be correct, a bit lower than $n \, \pi^c$, since a deviating firm has to set the price slightly lower than the collusive price). Hence if we substitute $\pi^{ch} = n \, \pi^c$ into equation (3) we can see that with no corporate tax the condition for self enforced collusion is given by $\delta \geq (n - 1)/n$, that is, it is independent of the profit earned in collusion, $\pi^c$. It is therefore equally difficult to collude on a price that is lower than the monopoly price as it is to collude on setting the monopoly price. I stick to the assumption of a "focal equilibrium" on the pareto frontier which means that if there is any collusion between firms at all, they will set the monopoly price and the monopoly profit, $\Pi^m$, is divided between the firms.

Consider a case where $n = 2$ and there are no corporate profit taxes. Collusion is possible if and only if $\delta \geq 0,5$. If a corporate tax with $\pi^o > 0$ and $\tau > 0$ is imposed, we can see directly from equation (5) that if $\pi^n = 0$ then collusion will be possible if and only if $\delta \geq \alpha$ where $\alpha$, is strictly less than 0,5. This clearly imply that if $\delta \in [\alpha, 0,5]$ and a tax like the one considered here, is imposed then one would go from a situation with marginal cost pricing to a situation with monopoly pricing and more than 100 % of the tax burden will clearly be borne by the consumers. Here is a numerical example that illustrates this.

### Numerical example

Let there be two firms in the industry, $n = 2$. Demand is assumed to be linear and given by $Q = 10 - p$. Unit costs are equal to 1. Industry profit is maximized if firms set a price equal to 5,5 ( $\Pi^m = 20,25$). Let $\delta = 0,45$. This means that without a corporate tax the discount factor is too low to enforce collusion and in

equilibrium prices are set equal to the unit cost, 1. Suppose a corporate tax with a tax rate $\tau = 0,5$ and $\pi^o = 2,5$ is imposed. Collusion can now be enforced if (substituting the numbers into (5);

$$\delta \geq \frac{10.125 \cdot 0.5}{20.25 \cdot 0.5 + 0.5 \cdot 2.5} = 0,445.$$

Collusion can therefore be enforced after the corporate tax is imposed. After the tax is imposed, firms charge the monopoly price (5,5) from consumers and each firm earn profits, net of taxes, equal to 6,31 compared with 0 before the tax was imposed. Clearly, far more than 100% of the corporate tax is borne by the consumers.

### Stick and carrot punishment

As I mentioned in the introduction, punishment by "grim strategies" with eternal reversion to the static Cournot solution are not optimal in quantity-setting games. There exists sub-game perfect equilibria in which more collusive output can be enforced by harsher punishment. This is well known from the work of Dilip Abreu. In a symmetric Cournot game optimal punishments are sometimes symmetric stick-and-carrot punishment (this is true for discount factors close to 1, Abreu [1986]). These are characterised by a short period with low prices (the stick) before one returns to the collusive outcome (the carrot). Symmetric stick-and-carrot punishments are only optimal if following this path - getting both the stick and the carrot - assures the lowest possible average per period profit. Clearly if there are no "exit costs" the minmax profit cannot be lower than zero since a firm can always choose to withdraw from the market. This means that the profit earned during the one period with a price war must be negative (the argument is that since the firms earn a positive profit after the first period with a price war and the annual per period profit is zero the stick must yield negative profit, see Abrue (1986) page 204 - 205). This mean that the shifting result I have discussed

above can be derived under the weak assumption that the tax system treats positive and negative profit asymmetrically ( a different tax-rate is levied on positive and negative profit).

## 5 Normative implications

This study can also shed some light on normative issues. For example, it becomes clear that viewing high profit as windfall gains that can be taxed away without having negative incentive effects, might be wrong. If output, prices and profit are determined in a dynamic game then high profit in one period can correspond to a situation where one firm cheats on the cartel agreement. Clearly, in this case a progressive profit tax - high tax on high profit - will have adverse incentive effects: It gives producers no incentive to break out of the cartel since most of the extra profit they will earn by cheating, will be taxed away. Thus, highly collusive behaviour with high prices and low welfare is enforceable if such a tax scheme is introduced.

## 6 Conclusions

Public finance questions are usually studied in models with perfect competition. This is so although imperfect competition and the theory of industrial organisation has been a - or should I say the - central research field in economics the last decade. Anthony Atkinson writes that it is "*striking, for example, how little reference there is to the impact of taxation in Tirole's The theory of Industrial Organization*"(Atkinson [1990] page 48). Yes, it is striking, because if we accept that these models describe real market behaviour then investigating the impact of economic policy - e.g., taxes - should be given high priority. I think of one reason why this research field still is described to "offer considerable scope for *future*

research" is the problem that the new game theoretic approach to study imperfect competition offers several different models and that close to everything of observed market behaviour can be explained if only the "right" model is selected. One should therefore not expect to find any robust policy conclusions. And investigating how a tax alters the equilibrium in one specific model seems somewhat less challenging. It is therefore interesting that the results I have presented in this paper are robust and not model specific. A nonlinear profit tax will in general have an impact on output and prices if these are determined in a dynamic game between oligopolists.

## REFERENCES:

Atkinson, A. [1990]; "The Distribution of The Tax Burden: 30 Years After "The Theory of Public Finance"" Discussion paper WSP/51

Besley T. and K. Suzumura [1989]: " Taxation and Welfare in an Oligopoly with Strategic Commitment" *International Economic Review*, 33, 1413-1433.

Davidson, C. and L. Martin [1985]: " General Equilibrium Tax Incidence Under Imperfect Competition; A Quantity Setting Supergame Analysis" *Journal of Political Economy*, 93, 1212 - 1223.

Davidson C. and R Deneckre [1990]: "Excess Capacity and Collusion" *International Economic Review*, 31, 521-541.

Katz, M. L. and H. S. Rosen [1985]: " Tax Analysis in an Oligopolistic Model" *Public Finance Quarterly*, 13, 9 - 20.

Maskin and Tirole [1988]: "A Theory of Dynamic Oligopoly, II: Price competition, Kinked Demand Curves, and Edgeworth Cycles" *Econometrica*, 56, 249-269.

Myles, G. D. [1989]: " Ramsey Tax Rules for Economies with Imperfect Competition" *Journal of Public economics*, 38, 95 - 116

Krzyzaniak and Musgrave [1963]: " The Shifting of the Corporation Income Tax" *Johns Hopkins Press, Baltimore.*

Stiglitz, J. [1986]: " The Corporation Tax " *Journal of Public economics*, 5, 303 - 311.