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International Redistribution;

Normative Foundations and Issues of Implementation

By

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Chapter 1

INTRODUCTION*

1. International Redistribution: The Main Issues

Most of us are well aware of the fact that many people in this world live in poverty. On a regular basis reports from television and newspapers tell us about people who starve to death, who are deprived of adequate shelter and who suffer from painful and serious diseases. Most of us who live in the rich part of the world are moreover aware of the fact that we, at a relatively minor cost, are able to help these poor people out of starvation, homelessness, and illness. For very few do this awareness lead to notable and adequate actions.

This observation raises some basic questions: What should we do about world poverty? What are our obligations towards poor people in the world? For many people, including myself, the large issue of global inequality and poverty constitutes a moral paradox. A major motivation for this thesis has been to increase my own, and hopefully also other person's, understanding of this important and difficult problem. Broadly speaking, there are two lines of analysis, which are helpful when discussing the desirability of international redistribution. First, it is

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obvious that the fundamental question above is of a normative nature. Hence, it is useful with a thorough understanding of theories of distributive justice and how these may be applied to questions of global justice. However, a knowledge of the alternatives from which we can choose is necessary even if one were to succeed in finding such a normative foundation for decision making. Positive analysis can clarify the consequences of different policies of international redistribution, and this is obviously an important basis for making sensible decisions. Thus, the second relevant line of analysis for the discussion of international redistribution is of a positive nature.

This focus seems to presume that rich countries in the world are willing to act according to a normative doctrine. It is pertinent to ask whether an understanding of positive and normative theory can change real world policies of international redistribution. International redistribution is for a large part determined by donations of foreign aid from rich societies. In most of these countries governments are elected by its citizens. If these citizens vote according to their own interests, then the governments of rich countries may make decisions that favour the well-being of the country's citizens. On this background one might argue that the decisions which affect international distribution are not affected by normative considerations. According to this perspective, international redistribution of wealth can come about only to the extent it makes people in donor countries better off. If this view is correct, it is not possible to change international distribution through increased awareness of normative theory.

Power and self-interest are, in my view, important determinants of real world distributive policies. However, if self-interest is the only factor which affects behaviour, then the possibility of improving the world seems rather grim. The actions of people and government can not, according to such a viewpoint, be affected by moral considerations. Hence, it is futile to investigate what different normative theories recommend regarding international redistribution. I will not try to argue that such a view of the world is incorrect. The issue touches upon philosophical questions about human nature that I make no attempt to answer. However, my choice of perspective is a different one. I assume that our decisions can be changed by awareness of our moral obligations. My point of view is that increased understanding of normative theory can influence our efforts to improve living conditions in poor countries, and hence, that a study of this subject is relevant for the question of international redistribution.

The specific analytical discussions in the chapters of this thesis are confined to fairly narrow topics in normative and positive theory. Through this introduction I hope to show how these chapters are relevant for the broader issue of international redistribution. First, however, I will present some rough estimates of the magnitude of world poverty.

2. The World Poverty Problem

According to estimates by the World Bank, there were 1110 million "poor" people, 630 million "extremely poor" people in the world in 1985 (World Bank (1990)). Poor people are defined as people with less available resources than the most widely used poverty line of $ 370 annually. The extremely poor are those with less than $ 275 available annually.[1] Almost 25% of the world's population are consequently defined as poor. From a policy perspective it is of considerable interest to analyse what it would cost to abolish world poverty. There are obviously substantial problems concerning calculations of such costs. A proper answer requires knowledge of incentive effects of aid, general equilibrium effects of redistribution, administrative costs and so forth. Some of these problems are discussed later in this thesis. However, we get an idea of the magnitude of these costs by assuming that lump sum transfers are possible, and ignoring all equilibrium effects etc. Hence, I calculate the aggregate difference between per capita GDP in poor countries and the respective poverty lines. These differences are multiplied by the population, to get the cost of abolishing poverty in the country in question. Based on this method I find that increasing the average purchasing power of all poor countries op to $ 370 per capita would cost approximately $ 150 billion annually. The less ambitious goal of raising purchasing power up to $ 275 per capita in all poor countries would cost approximately $ 38 billion annually.[2]

[1] The poverty lines are measured at purchasing power parity (PPP). There are several problems, both conceptually and practically, in defining poverty lines. See Ravallion et.al (1991), Ravallion (1992) and Tungodden (1994), (1996) for elaborations on these issues.

[2] I have used data for 1993 (World Bank (1995)). The numbers are found by the following method: For each country with GDP below the poverty line, I have calculated the difference between GDP/capita and the poverty

What sacrifices are needed from the rich countries in order to achieve abolishment of world poverty? As a starting point it can be noted that the amount disbursed as official development assistance in 1993 was \$ 68.5 billion (OECD (1994)). According to the above figures, raising the purchasing power of poor countries up to \$ 370 per capita requires an additional \$ 150 billion, or a total of about \$ 230 billion annually. In other words one would have to give more than three times the current amount of aid in order to reach this goal. This would be achieved if approximately 1.2% of GDP in the high-income countries were given as foreign aid. The funds required to raise purchasing power of all countries up to \$ 275 per capita requires that approximately 0.6% of GDP in high income countries are given as foreign aid.[3]

The above numbers focus on poverty, and the cost of alleviating this problem. It is far from obvious that this should be the goal we should focus on. Some might claim that a fair redistribution of wealth requires equalisation of resources between different people. The realism of ever implementing such a suggestion is of course open to question. Again assuming that there are no efficiency losses caused by redistributive policies, equalisation of purchasing power between nations implies that each person would get approximately \$ 5120 available annually.[4]

To me the calculated costs of alleviating poverty are surprisingly low. However, I will point at four different reasons why these numbers may be unrealistic: First, I have calculated the cost of raising the average purchasing power within a country up to the poverty line. An implicit assumption is thus that one can obtain complete equalisation within a country without any loss of aggregate production. A somewhat more realistic approach would calculate the cost of raising the purchasing power of all poor individuals within a country without reducing the income of the relatively rich people. I have not conducted such a calculation, but it is clear that the cost of such a policy would be higher than the estimates above. Taking into account the problem of targeting policies toward the poor would increase these costs even further (see Datt and Ravallion (1991) and Besley and Coate (1992)). Second, I assume that there are no distortive effects of reallocating a dollar from persons in the industrialised world towards poor

line. This difference is multiplied with each country's population. Subsequently these numbers for all poor countries are added together.

[3] The high-income countries are defined as countries with more than 10 000 \$ GDP per capita annually. The figures are based on data from 1993 in World Bank (1995).

people. As is well known from theory and practice, redistributive policies will almost always produce distortive incentives for both recipients and donors. The magnitude of these effects are hard to estimate, but we must generally assume that they are greater than zero. In chapter 4 and 5 of this thesis I analyse possible distortive effects of foreign aid in more detail. Third, the policies I hint at are likely to have substantial general equilibrium effects. Specifically, raising the income of a starving population will increase the demand for food, and this is likely to cause a price increase for food products. Substantial price increases will increase the cost of alleviating world poverty. Finally, there are reasons to believe that the administrative costs of international redistribution are considerable. I have suggested that aid budgets should be increased to more than three times their current level in order to abolish poverty. It is likely that this would necessitate substantial and costly improvements in aid administration. We can conclude that there are several reasons to expect that the real costs of abolishing poverty are well above the figures calculated above.

In the next to sections I will give a brief overview over normative and descriptive issues which are particularly relevant to the question of international redistribution: In section 3 I will discuss a selection of theories of justice which attempt to address distributive problems at a global level. In section 4 I turn to descriptive problems, with special emphasis on foreign aid. The main purpose is to illuminate the issue of international redistribution by interpreting selected parts of some relevant theories. I do not intend to give a survey of all relevant perspectives on this issue, nor do I attempt to present all aspects of any single theory.

3. The Normative Foundation for International Redistribution

The problem of global justice concerns our moral obligations towards people in different parts of the world. A first question that comes to mind is whether the principles of justice that should apply in a global context are different than those that should apply for example within a country or within a family. Are there any morally relevant differences between people who live in other parts of the world and people within a country? Is there an asymmetry between

[4] The figures are from UNDP (1994). Measures of real GDP are measured using purchasing power parity.

national and international distributive justice? I will start by discussing what the so-called universal normative theories say about these questions.[5]

3.1 Universal Theories of Justice

A universal theory of justice is based on the idea that there are certain basic moral principles which hold for all people irrespective of time, place and culture. Specific normative rules are deduced from these. From a universal vantage point it is possible that the normative desirability of particular actions are contingent on the circumstances. These contingencies must however follow from the general normative principles. For example, one may be able to justify killing Adolf Hitler, but not Mahatma Gandhi. According to a utilitarian doctrine (to be discussed below) the first act could generate an increase in aggregate well-being, whereas the second act could more reasonably be expected to have the opposite effect.

Utilitarianism is the most prominent theory among universal theories of justice. Its basic doctrine tells us that the morally desirable actions are those which maximise the sum of utilities for all individuals.[6] John Stuart Mill's "Utilitarianism" (1863) is an important early formulation of this view. More recently John Harsanyi (1953) (1955), (1976) has defended the same view by using an impartial spectator argument: The desirable norms for a society are found by asking what a rational individual would choose if he did not know which person he would become in the world. The basic idea is that morality requires us to divest of our personal interest in this world. Behind a so-called "veil of ignorance" individuals are able to do this. Harsanyi claims that a rational and impartial individual would attach an equal probability to attaining any position in society. Based on an "equal probability approach" Harsanyi arrives at utilitarianism as the morally desirable social choice rule.

[5] The discussion in this section draws on Cappelen (1994).

[6] There are many versions of utilitarianism. One distinction is between act and rule utilitarianism (see J. Harsanyi (1979)). The example in the text is based on an act utilitarian view, in which the desirability of every alternative action is evaluated by their «sum utility ranking». Rule utilitarianism, on the other hand, claims that one should find different sets of directly applicable normative rules and choose the set which attains the highest sum of utilities. Within a rule utilitarian approach it is conceivable that the rule «you shall not kill» attains a higher sum of utilities than alternative general rules. However, it is unclear why it is impossible, within a rule utilitarian framework to make detailed rules which are highly dependent on specific circumstances. If such contingencies could be specified in sufficient detail, rule utilitarianism would coincide with act utilitarianism.

Welfarism can be thought of as a generalisation of utilitarianism.[7] According to a welfarist approach the desirability of social states must be judged solely on the basis of vectors of utilities, but not necessarily on the sum of utilities in each state. A strict egalitarian view (in utility space) would maximise utility for the least well off individual. Such a rule is still welfarist, because it takes into account only information about utilities (namely the utility of the worst off individual). Different "welfarist" positions can be represented by different Bergson-Samuelson welfare functions, which in various ways take into account a society's moral attitudes towards equality.[8] Within social choice theory the anonymity axiom is widely used and accepted. Expressed in a "welfarist" context it says that two social states are equally good if the only difference between the states is the "ownership" of the utility numbers. In more general terms anonymity can be thought of as a rule which demands that "different individuals should be treated equally". A reasonable interpretation of anonymity would be that people in different countries should be treated as equals. Thus the anonymity axiom may be said to be in conflict with theories of asymmetric justice.

It seems appropriate to include John Rawls among universal theorists. A fundamental part of his reasoning is that justice is defined by the normative principles one would choose behind "a veil of ignorance". Note that this argument for justice is similar to Harsanyi's defence for utilitarianism. However, Rawls argues that such an impartial individual would choose different principles than those Harsanyi arrives upon. He rejects Harsanyi's approach where each individual, behind a veil of ignorance, assigns an equal probability of becoming any individual in society. Rawls claims that rational individuals would choose institutions according to the following two principles of justice: The first principle requires that everyone shall have as extensive liberties as possible, as long as they are equal for everyone (this principle is modified in Rawls (1993)). The second principle is relevant for questions of distributive justice, and states that a society is just if (i) it gives priority to the interests of the

[7] «Welfarism» has been proposed by Sen (1977) as a term for normative theories which depend only on utility information.

[8] Arrow's possibility theorem (Arrow (1951)) shows that if no interpersonal comparisons of utility are possible, and if certain other reasonable axioms are accepted, any social choice rule must dictatorial. In order to avoid this problem, utilitarianism must adopt an assumption of utility being cardinally comparable between people. In practice cardinal comparability can be thought of as interpersonal comparability of marginal utilities of income. Other welfarist approaches do also require some sort of interpersonal comparability of utility. The seminal article on the relation between interpersonal comparability of utility and social choice rules is Deschamps and Gevers (1978). See also Sen (1986) for an overview of this literature.

least well off individuals (the difference principle), and (ii) it has fair equality of opportunity. The first principle has priority over the second.

The difference principle (part (i) of the second principle of justice) is often interpreted as a "maximin utility rule": One should try to achieve as high as possible utility for the least well off person in a society. This is a misinterpretation. Rawls is explicit when stating that the difference principle is obtained if one maximises an *index of primary goods* for the least well of persons. These primary goods are (1) liberties, (2) opportunities to attain powers and prerogatives of offices and positions of responsibility, and (3) income and wealth. Rawls is not very precise when discussing how the primary goods shall be defined and measured, nor does he explain how the different primary goods shall be weighted in an index. However, he states explicitly that the index shall not be a measure of individual utility. Part (ii) of Rawls' second principle can be interpreted as some version of opportunity egalitarianism. He is even less explicit when describing how this principle shall be operationalised. It is clear, however, that Rawls justifies "fair equality of opportunity" by claiming that individuals to some extent must take responsibility for their own choices and preferences. A society in which the opportunities to achieve success are distributed fairly, is consequently not unjust even though individual achievements (such as realised utility) are unequal. Thus, Rawls' principles of distributive justice constitute an explicit departure from a welfarist approach. I focus on this aspect of Rawls' theory in chapter 2 and 3 of this thesis.

Various universal theories may, as we see, be very different in content. A common feature, however, seems to be that they can not justify any asymmetric treatment of "insiders" and "outsiders" of a nation. From a universal viewpoint, it does not seem defensible to give special attention even to members of one's family. The moral obligations that are implied by universal theories are likely to be very demanding. This is revealed if we think about the implications of for example a utilitarian view. A normal assumption would be that the marginal utility of income is higher for poorer people than for richer people. If this assumption is adopted, utilitarianism implies that any person should redistribute income to poorer people until he becomes equally poor! For the question of global redistribution, a utilitarian doctrine seems to demand substantial increases in resource transfers to developing countries. For any single person, a strict interpretation of utilitarianism implies that he should give as charity any material wealth above the minimum level in the world. Utilitarianism may therefore lead to "an overload of obligations". Moreover, most of us feel that we have special obligations towards our closest family. Thus, it seems fair to claim that many universal theories, and in particular utilitarianism, have implications that run contrary to our moral intuitions.

This observation has given rise to a so-called "assigned responsibility model" which attempts to reconcile a utilitarian way of thinking with our moral intuition (see e.g. R. Goodin (1988)). According to this view a major problem with classical utilitarianism is the fact that it seems impossible to follow its demands. It may therefore be desirable to give every individual special duties towards a group of people, for example his family or fellow citizens. These special duties are however derived from the basic goal of maximising total utility. The construction of assigned responsibility areas must be judged as a convenient way of delegating responsibilities so that the utilitarian goal is more effectively achieved.

It is important to note that the assigned responsibility approach can give rise to asymmetric treatment of fellow citizens and people in developing countries. This asymmetry is however not a basic part of the utilitarian principles, but must be regarded as a possible device which makes the basic goals easier to implement. Moreover, it is far from obvious that the current global division in responsibility areas is a desirable way to organise the global society. It does not seem likely that the goal of maximising total utility can be attained within a system in which the poor take care of the poor and the rich take care of the rich. Even though the assigned responsibility approach may legitimise that we have different obligations towards different people, the current division of responsibilities seems hard to reconcile with this line of reasoning.

3.2 Particularistic Theories of Justice

The term "particularistic theory" is here used as a common term for philosophical views that within their fundamental structure legitimise differences in our moral obligations towards people from different societies. According to such a view morality arises from particular relationships between persons within a group. A particularistic theory can be thought of almost as an antonym to a "universal" theory. Whereas a universal theory holds that principles of justice apply universally to all persons and within all societies, a particularistic theory claims that normative rules arise from specific relationships between people within a group.

The communitarian views are presented for a large part as responses to utilitarianism. Utilitarianism presumes that some fundamental moral truth exists as an independent entity, which human beings are able to reveal through rational reasoning. The communitarian approach opposes this view in fundamental ways.[9] Michael Waltzer denies that morality can be thought of as a truth that exists independently of human beings (Waltzer (1983)). Rather he claims that; "Justice is a human construction, and it is doubtful that it can be made in only one way" (ibid. pg. 5). Moreover; "Every substantive account of distributive justice is a local account... We are (all of us) culture-producing creatures; we make and inhabit meaningful worlds... Justice is rooted in the distinct understandings of places, honours, jobs, things of all sorts, that constitute a shared way of life. To override those understandings is (always) to act unjustly" (Ibid. pg. 314). According to a communitarian view the moral obligations of an individual must be understood in a historical, social and cultural context. The impartial point of departure, which is common in universal thinking, is considered an abstract construct, which reveals a lack of understanding of what a person is. An individual can not be parted from his/her social, cultural and historical inheritance. Rather all these characteristics define for a large part an individual as a member of a community. A proper understanding of morality must take into account the fact that human beings are social creatures, and that normative rules have been constructed in a social context. Thus there can be many different acceptable normative rules, and, more importantly, these are defined within a membership group that shares a way of life.

An important ingredient in communitarian views is the conception of justice as a human or social construct. This does in my view also pose a problem. Is a communitarian view bound to be relativistic? Is it impossible from this vantage point to make substantial moral judgements? The immediate answer seems to be yes. A likely response from a communitarian would be: if the process of creating normative rules respects the opinions and values among the members of the community, the resulting institutions are just. It is also possible to make certain

[9] D. Miller (1976), M. Sandel (1982) and M. Waltzer (1983) are considered proponents of communitarian views.

inferences about which normative considerations are "true" within a society, based on knowledge of this community's basic values. But if alternative social institutions can come about through acceptable processes and if they furthermore are consistent with its society's values, how can we evaluate these different alternatives? In my view it is difficult, within a communitarian framework, to come up with an acceptable response to this question. To a certain extent communitarianism is bound to be relativistic. It seems like there must be normative questions that can not be answered within the communitarian approach.

Another normative theory within the particularistic framework can be called the mutual benefit approach. Gauthier (1986) is the author who most recently has expressed a view that fits this label, but Hobbes and Hume can be viewed as predecessors of the mutual benefit approach. The basic idea within this line of reasoning is that the moral obligations between individuals arise from a common participation in mutually beneficial co-operation. Gauthier perceives principles of justice as a manifestation of a hypothetical but rational bargaining process, in which individuals decide upon the division of the gains from co-operation. The "threat point" for bargaining is defined as what each individual would obtain if no co-operative actions were conducted and if no coercive powers were used between individuals. A rational way to construct social institutions would be to arrange activities so that a "social surplus" in some way is maximised. This surplus arises from the benefits that can be gained through joining forces. However, there is a conflict between individuals with respect to the division of this surplus. According to Gauthier rational individuals will reconcile this conflict as if they participated in a non co-operative bargaining process. The principles of justice can therefore be considered as rules which both co-ordinate individuals' actions so that the social benefits are "maximised" and reconcile potential conflicts with regards to the division of this surplus.

As explained above, Rawls is most often interpreted as adopting a universal approach to normative questions. However, Rawls can also be understood as defending a "mutual advantage" view of justice. Describing some basic features of society he writes; "There is an identity of interests since social co-operation makes possible a better life for all than any would have if each were to try to live solely by his own efforts. There is a conflict of interests since men are not indifferent as to how the greater benefits produced by their collaboration are distributed..." (Rawls (1971) pg. 126). In this context it is important to note that the two principles of justice apply, according to Rawls, primarily within "well-ordered societies" (Rawls (1971) pg. 453-462). It seems clear that Rawls has western democracies in mind when he describes such societies. Thus, it is possible, within Rawls' general framework, to defend asymmetric treatment of fellow citizens and foreigners.

This interpretation of Rawls' theory has given rise to some debate. Pogge (1989) argues that Rawls' approach must be interpreted in a global context. There is no moral reason to limit the domain of the principles of justice to specific geographic areas. The central part of Rawls' theory is "ideal": Justice is regarded as the principles one would arrive upon if one were to choose institutions behind a veil of ignorance, not knowing the position one would obtain in the real world. This framework for justifying normative rules does not leave room for asymmetric considerations. How can it be possible, within Rawls' general framework, to defend that our moral obligations are more extensive for fellow citizens than for foreigners? Rawls' own argument for limiting the applicability of the principles of justice is of a pragmatic nature: There is a high likelihood of finding "overlapping consensus" within well ordered societies.[10] It seems incoherent, however, to claim that such a pragmatic argument can defend that we, fundamentally speaking, have different moral obligations for people inside and outside our society. In a response to Pogge, Rawls opens up for the possibility of moral obligations extending beyond the borders of well-ordered societies. (Rawls (1993)). He does argue, however, that there to some extent is an asymmetry of obligations for citizens within these societies and the "foreigners". It is thus somewhat unclear whether Rawls appropriately can be interpreted as a "mutual advantage" theorist.

The idea that morality originates from agreements among individuals with mutually beneficial interests clearly legitimises the view that principles of justice can have limited geographical scope. The mutual benefits of co-operation may very well be greatest within a certain area such as a nation or a state. It follows directly that our moral obligations are stronger towards people within such a co-operative group than they are towards people outside this group. Thus both communitarian theories of justice and the mutual advantage theories have embedded in their basic logic that moral obligations towards people who belongs to different groups may be different. In this way these theories may justify an asymmetric treatment of people in our own country and poor people in developing countries.

Even though particularistic theories may legitimise asymmetric justice, it should be emphasised that the current division of areas of obligations may not be in accordance with the prescriptions of these theories. The states are the most important areas of joint moral obligations in the world today. But these states do often not coincide with areas of joint cultural and historical inheritance. Thus a communitarian might be willing to extend moral obligations to people outside current state borders, or confine them to people in smaller areas within a state. From a mutual benefit point of view one might argue that the area of joint interests are much larger than reflected in current state borders. One may for example claim that the gains from global division of labour are substantial, and that the benefits from world trade should be divided in a different way than today.

4. The Effects of Global Redistributive Policies

In the previous section I briefly discussed a very selective part of political philosophy, with a particular emphasis on its relevance for questions regarding international redistribution. Normative considerations provide a necessary, but not sufficient foundation for making the "right" decisions. The other important basis for choice is a solid knowledge of the alternatives from which we can choose. In this section I discuss some main issues which affects the possibility of successful redistribution between countries. In particular I focus on the possible effects of increased foreign aid. As a starting point one might ask what the effects of a "status quo" policy are. Is the difference between rich and poor countries likely to be permanent or transitory? The section starts with a discussion of the convergence hypothesis, which addresses this question. Our beliefs regarding the possibility of achieving growth in developing countries provide an important basis for discussing possible effects of foreign aid. Next, I briefly discuss the population problem in the context of foreign aid. I conclude this section by arguing that the commitment problem must be considered an important distinguishing feature of international redistribution.

[10] The term «overlapping consensus» is used by Rawls in the meaning that there within a society is a common

4.1 The Convergence Hypothesis

When studying the effects of different policies towards developing countries, it is pertinent to ask what the effects of current policies are. What are the effects of a status quo policy? This question opens up the vast problem area of finding the causes of development and underdevelopment. A very relevant question is whether the poor countries in the world today eventually will start to prosper and reach the welfare level of the western societies. Or is the inferior "productivity" in poor countries of a more permanent nature? The need for, and design of, redistributive policies clearly depends on the answers to these questions.

This fundamental question is discussed in the literature on the "convergence hypothesis". The basic Solow-Swan model (see Solow (1956)) predicts that different countries with equal rates of savings, equal rates of population growth, equal rates of capital depreciation, and equal constant returns to scale production functions, will converge to identical steady states in which production per capita and consumption per capita are constant across countries. This theory predicts that even if two countries experience very different initial wealth levels, they will still end up in a situation where consumption and production per capita is quite even. In the Ramsey model saving is treated as a variable which agents choose optimally, in contrast with the Solow-Swan model where the rate of saving is an exogenous parameter. The results, however, are quite similar. In the Ramsey model different countries with equal "discount factors" will see their level of wealth convergence to the same steady state, irrespective of differences in initial levels of wealth. It should be noted that these strong results are based on very restrictive assumptions of equal production functions, equal population growth and equal rates of depreciation. If these fail, which they seem likely to do, the result no longer holds. Inferior production technology in developing countries may for example account for differences in welfare between poor and rich countries. Differences in exogenously determined technological improvements might also explain inequalities between countries.

In the past decade growth theory has undergone substantial developments. The recent literature on endogenous growth tries to explain the growth process by focusing on spillover effects between individual capital accumulation and aggregate productivity (seminal articles

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understanding of fundamental normative values.

are Romer (1986) and Lucas (1988)). The basic idea is that an individual chooses his level of capital accumulation according to the individual payoff from his decisions. An increase in one individual's level of capital will however increase productivity for other individuals as well. Hence, there is a positive externality associated with individual capital accumulation. A relevant aspect in this context is the accumulation of human capital or education. It may for example not be profitable to become an engineer in a poor developing country, because profitable use of such an education demands other people with similar knowledge. If there already existed a substantial number of engineers in the country, it could be individually profitable to pursue this career. Sensible use of a kind of knowledge might require a network of other individuals who possess the same kind of understanding of certain problems.[11] A common assumption in this literature is that the production function for an individual has constant returns to scale, whereas for the whole economy there is increasing returns to scale. This might give rise to a situation in which differences in initial wealth between two countries implies that they converge to two different steady states, even though the economies are identical in other respects. A country may be trapped in a steady state with low capital accumulation, in which individuals have no incentives to increase savings and investments. However, for levels of capital above some critical level, the individual incentives to invest improve, and the economy converges to a better steady state. Within the framework of endogenous growth theory, it is thus possible with permanent differences in production per capita only because two countries differ in their initial capital level.

These differences in theoretical predictions have lead economists to study the empirical validity of these opposing hypotheses. There is an extensive and non-conclusive literature that attempts to answer the question of convergence (see for example Mankiw et.al. (1992), Barro et.al. (1995), and Levine and Renelt (1992)). The question I focus on is the global distribution of wealth. I am consequently primarily interested in whether production in the poor countries of the world converges to the steady states of the developed countries. Sala-i-Martin (1996) finds that there is a tendency for convergence within the developed world, but there is no evidence of convergence between rich and poor countries on the world. The following figure displays this finding:

[11] This problem is similar to the topic of «network externalities» studied in industrial economics (see Farrell and

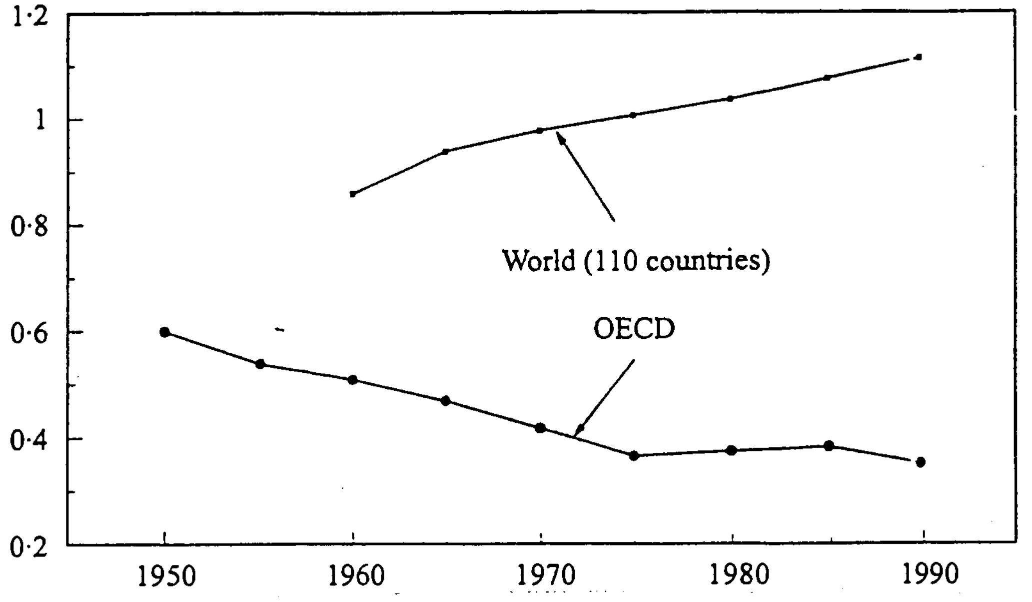

Figure 1: Divergence of GDP per capita within the world and the OECD. (Source: Sala-i-Martin (1996)).

The vertical axis measures the dispersion of real per capita GDP within a group of countries.[12] A high measure of dispersion means that there are relatively big differences in real per capita GDP within this group of countries. The figure shows that between 1960 and 1990 the OECD countries have become more equal, whereas for the world as a whole the differences in production have become larger. This study does consequently suggest that differences in per capita production may be permanent and even increasing. It is certainly possible to make a case for the hypothesis that a status quo policy will lead to permanent differences in production between rich and poor countries in the world.[13]

The finding presented in figure 1 shows that a cross-country interpretation of the convergence hypothesis in the Solow-Swan model is unjustified: Different countries do not converge to the

Saloner (1986) and Katz and Shapiro (1985)).

[12] The analysis is conducted using data from 110 countries in the time period from 1960 to 1990. The figure displays whether there is σ -convergence. A group of countries experiences σ -convergence if the variation of their real per capita GDP tends to decrease over time. The variation at a specific point in time is measured by the standard deviation of the natural logarithm of the individual countries' real per capita GDP.

[13] The finding should be interpreted with caution; it does not provide a final answer to the question of convergence. For policy purposes we are primarily interested in the future pattern of growth, and this may be different from the past.

same steady state.[14] It is possible to interpret the lack of convergence as a defence for endogenous growth theory. But of course there are many other possible causes for this phenomenon. For the purpose of this chapter it is still interesting to look into the possible policy implications of endogenous growth theory. An important part of this theory is that the initial level of capital may be decisive for the steady state a country ends up in. There may be a critical level of capital, where countries that have more capital than this level end up in a "good" steady state and poorer countries end up in an inferior steady state. Taken at face value, this theory seems to suggest that aid policy should follow a "big push" approach. If aid can increase capital above some critical level, the country will converge to a superior steady state.

Endogenous growth theory can be interpreted as having fairly optimistic implications regarding the possible effects of foreign aid. Large investments in poor countries may improve their economic conditions substantially. It is, however, pertinent to point out that endogenous growth theory is not the only possible explanation for inequality in the world. Differences in economic conditions may be caused by differences in production technology, differences in capital depreciation, differences in population growth and differences in the rates of savings (caused for example by differences in "patience" in different countries). If either of these factors differ, the steady states will generally be different between countries as well. Increasing the level of capital does not necessarily lead to convergence to a new and better steady state. The literature on "conditional convergence" debates whether each country converges to a steady state. However, this literature accepts that different countries may converge to different steady states due to differences in production functions, population growth and so forth. The empirical literature has however focused on conditional convergence in within certain regions in the world. One has not analysed this question in a global perspective. However, it is quite possible that there exist fundamental differences between countries that will not be altered if capital accumulation is increased. In this context it is

[14] It is not clear that the Solow-Swan model predicts equal steady states in different countries. A more reasonable interpretation might be that it predicts one country to converge to a steady state, but not that all countries will converge to the same steady state. A cross country interpretation of the convergence hypothesis requires that different countries have equal population growth, equal production functions, and equal depreciation of capital. These assumptions are very strict and not central in the original formulation of neo-classical growth models.

interesting to note that the empirical literature on the effects of foreign aid has not, generally speaking, been able to find significant positive effects of aid on economic growth.[15]

What are the policy implications of these alternative explanations for the divergence of per capita production in the world? In my view the two opposing theories give quite different implications regarding the desirability of alternative aid policies. Modern endogenous growth theory suggests that a substantial increase in a country's capital stock might enable it to move to a superior steady state. This could imply that foreign aid should be directed to large and capital intensive projects such as power plant construction, the educational system, and general infrastructure projects. In the short run such policies may not be beneficial for the individuals. But in the long run it may lead to economic growth and prosperity. It seems like a large portion of real world aid programs is based on such a view. An explicit goal in Norwegian aid policy is to help developing countries to prosper so that they eventually will not need aid. If, on the other hand, the reason for inequalities is fundamental differences technology, preferences, population growth or capital depreciation, one should use aid resources for quite different purposes. If long-term economic growth is an unrealistic option, it seems more sensible to alleviate immediate problems for poor people. In that case aid resources might more sensibly be used for poverty alleviation, hospitals, housing projects etc. We may conclude that the optimal design of aid policy to a large extent depends on our views regarding the fundamental reason for diverging living standards in rich and poor countries.

4.2 The Population Problem

A popular argument against increased foreign aid is that it will lead to population growth, and not to improvements in living standards in developing countries. The Mechanism which is assumed to be at work are similar to the ones originally described by Malthus ("An Essay on the Principle of Population", (1798)). Malthus argued that it is an inescapable fact that large parts of the population are bound to live at the level of subsistence. The reasoning should be well known: The human population has the capacity to grow at a geometric rate. If there were no countervailing effects, the population would soon become almost infinitely large. Food production, however, can increase only at an arithmetic rate. These opposing facts imply,

[15] For a survey of the empirical literature of the effects of aid see White (1992). These issues are also discussed

according to Malthus, that it is impossible to maintain a standard of living substantially above the subsistence level. Harsh as this conclusion may seem, the reasoning has a strong and intuitive appeal. Indeed the same kind of mechanism is at work in modern growth theory in which fertility is made an endogenous variable. In such a framework it is entirely possible to get "tangled up" in an inferior steady state. Increased GDP per capita may increase the population growth, which again puts a limit on GDP per capita in a steady state (see e.g. Barro and Becker (1989) and Galor and Weil (1996)). Other authors have claimed that the choice of the number of children within a family is influenced by several factors in intricate ways (see e.g. Becker (1960) and Lee et.al. (1991)). Giving birth may for example be an insurance device in order to secure the living standard when parents get old. In that case increased living standards may reduce the need for children, and hence also fertility. It should be clear that these matters are very relevant for the desirability of foreign aid.

I will not at all discuss the theoretical plausibility of these different mechanisms. Whether these effects are important in the real world is an empirical question. The history of the industrial revolution tells us that increased population growth has been a temporary feature in the western world. For the industrialised countries, one experienced a period with lower mortality rates and unchanged fertility rates. After a limited period, however, fertility rates were reduced as well. Malthus' prophecy was consequently not valid for the western part of the world. But it may still be possible that increased foreign aid leads to increased population growth in developing countries. One hypothesis is as follows: Increased aid leads to an immediate increase in living standards, which again leads to population growth through reduced mortality or increased fertility. Eventually the population increases to a level where per capita GDP is at its original level. An important question is whether this hypothesis receives support in empirical studies. In particular it seems important to know how increased living standards affects fertility. Birdsall (1988) addresses this question in an investigation of the relation between per capita income and fertility rates in the developing countries. The results can be summarised in figure 2:

in chapter 4 and 5 of this thesis.

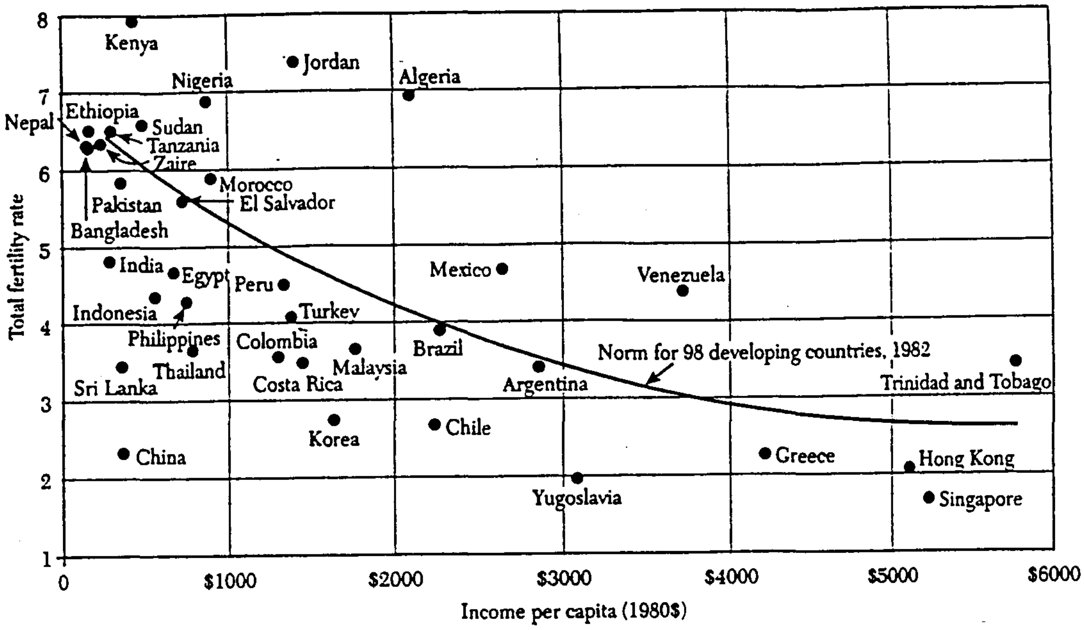

Figure 2: Fertility in relation to income in developing countries, 1982.[16]
Source: Birdsall (1988).

The interesting fact is that the fertility rate within the developing world decreases with per capita income. This finding weakens the position that foreign aid is less desirable because it leads to higher populations. However, it should be noted that the growth in population is determined both by fertility rates and mortality rates. It is obviously possible that aid could reduce mortality rates substantially, and thereby increase the population. There are obviously also other limitations to the possible interpretations of the above finding.[17] However, the finding is an indication that foreign aid may only to a limited extent lead to population growth.

[16] Fertility is measured as the number of live births a woman would expect to give if she were to live through all her childbearing years and to bear children at each age in accordance with the prevailing age-specific fertility rates.

[17] One weakness is the fact that cultural factors vary between developing countries, and these may be very important for fertility choices. As we can observe from figure 2 the African countries have generally higher fertility rates than other developing countries. Furthermore we see that these countries are also generally speaking very poor. Dividing the countries in groups according to similarity of cultures would seem to weaken the above finding. Using time series analysis would also illuminate possible interesting patterns regarding the effect of material well-being on fertility.

4.3 What is Special about International Redistribution?

In the discussion of the convergence hypothesis I tried to argue that foreign aid should not necessarily be expected to increase growth and lead to a superior steady state. Trying to fulfil some immediate needs may in many cases seem more realistic. In such a context foreign aid can be considered as a global social security system. An important task is to analyse the possible functional and dysfunctional effects of such a policy. The literature on taxation and public finance has provided important insights regarding the effects of social security and redistribution within a country (e.g. Mirrlees (1971), Atkinson (1973) and Phelps (1973)). An important question is whether there is a need for a specific theoretical study of international redistribution. Are there any conceptual differences between redistribution within a country and redistribution between countries? I will argue that such differences exist; in particular I will focus on the institutional framework in which international redistribution takes place. My main claim is that redistribution within a western society is governed by a set of (credible) rules. Policies of international redistribution, however, are to a larger extent discretionary. Thus, I consider the commitment problem to be an important and distinguishing feature of international redistribution. In the following paragraphs I will defend this view.

In most western societies there are policies which in some way attempt to limit inequality between its different citizens. The tax system and social security system are the most important instruments for that purpose. It should be emphasised that such redistributive policies are decided upon and implemented through fairly long lasting and complicated processes: The citizens elect politicians based for example on their platform on taxes and social security. The state bureaucracy subsequently implements this set of rules. The process of changing rules and practices of redistribution may take many years. It seems reasonable that an individual's choice of effort to raise income is made more often, maybe on a continuous basis. Thus, it takes longer time for the government to change its policies than it takes the other agents to change their actions.[18] One might therefore claim that governments in democratic societies face limited commitment problems when deciding on a domestic

[18] This claim may need some further discussion. The choices which determine an individual's income level may be made a long time before the payoff from these choices are revealed. A particularly striking example is the choice of education. The payoff from education will usually appear 10-40 years after the educational choices are made. Within this time the government may change its tax system. It may therefore not be entirely clear cut whether a government faces a commitment problem regarding domestic redistributive policies.

redistributive policy. This discussion is by no means conclusive. A substantial number of economists have started to focus on the commitment problems that a government faces for policies within a country. For questions of capital taxation and monetary policy, economists have argued that a government's lack of ability to commit must be regarded as a serious problem.[19] However, my argument is that it seems possible for a government to commit on redistributive policies. It seems like the decision processes within western democracies constitute favourable institutions for avoiding commitment problems regarding redistribution within a country. In my opinion there are reasons to believe that lack of ability to commit constitutes a more serious problem for redistributive policies between countries, compared to redistribution within a country.

Decision processes regarding international redistribution are quite different from those within a country. In the case of foreign aid the most relevant agents are the governments in donor and recipient countries.[20] Furthermore, a very important criterion for disbursing aid is the severity of the problems in a recipient country. Countries with more pressing needs will generally receive larger amount of aid. Note that it takes relatively short time for a donor to change its level of foreign aid. Aid budgets are in most cases decided upon on an annual basis. In contrast to this it generally takes very long time for a recipient of aid to implement policies which leads to increased economic growth. Growth enhancing policies will (assuming that such policies are available) most realistically have an impact after 20 or 30 years. Consequently, it takes much longer time for a recipient country to increase its level of per capita GDP than it does for the donor to change its level of aid. This makes a perfect case for commitment problems for the donor. The donor must take the level of wealth in recipient countries as a given, and subsequently choose an aid budget.

Another important feature of international redistribution is the fact there are relatively few agents. Each donor has a limited number of countries which receives aid from them. This opens up the possibility of strategic interaction between donors and recipients, and between

[19] Recently there has emerged a substantial literature on the commitment problem for policy makers within a country (see Kydland and Prescott (1977) and Fischer (1980)). This literature has to a large extent focused on commitment problems regarding inflationary monetary policies. The problem of capital taxation has also been discussed.

[20] Obviously large international organisations such as the World Bank and UN are important donors of aid. It seems likely that the commitment problem is equally big for these organisations compared to states.

different recipients. In chapter 4 and 5 of this thesis I analyse the impact of aid in a setting of such strategic interaction. The commitment problem for the donor is the problem which I focus on.

5. Outline of the Thesis

In chapter 2 of this thesis I analyse the so called indexation approach, as proposed initially by John Rawls (Rawls (1971), (1993)), and later by Amartya Sen (Sen (1980), (1992)). These lines of thought have influenced the United Nations, among others, to use the so-called human development index (HDI) as a measure for the level of development in a country. Rawls view regarding distributive justice is represented by the difference principle, which roughly states that social institutions should be arranged so that they are to the greatest benefit to the least advantaged group in society. It has been common within normative economics to formalise this idea by a "maximin utility rule" or a "leximin utility rule". This is an invalid interpretation. Rawls explicitly states that the interests of the least advantaged group should be measured by an index of primary goods. The primary goods are necessary prerequisites that are common for all people in order to achieve success in life. The primary goods are thus prerequisites for success, and not achieved utility. In my interpretation of Rawls the index of primary goods can be used for a complete ranking of the positions of different individuals.

Sen has elaborated on the indexation approach, but objects to the view that primary goods should measure the interests of individuals. He claims that functionings are more fundamentally important for people. Functionings are "states of being", and can be such things as "being in good health", "being adequately nourished" or "being happy". An important feature of Sen's approach is the possibility for the index of functionings to give an incomplete ranking of different individual's position. Sen suggests that one position can be ranked as better than another only if everyone agrees about this ranking.

Chapter 2 of the thesis, "The Indexation Approach is Incompatible with the Pareto Principle", is concerned with the possibility of using the indexation approach as a basis for social choice. The chapter starts out with an analysis of Rawls' difference principle. It is shown that a necessary condition for this version of the indexation approach to be compatible with the

Pareto principle is that people have identical preferences. If this assumption is rejected, the indexation approach and the Pareto principle are incompatible. Next, I analyse Sen's approach to ranking of positions, with particular emphasis in the possibility of incomplete ranking. I show that this does not solve the problem. A necessary condition for compatibility with the Pareto principle is still that different individuals have equal preferences. If one accepts that individuals have different preferences, the Pareto principle and the incomplete indexation approach are incompatible. Variety of preferences seems like a very reasonable assumption about individuals. Furthermore, the Pareto principle has a strong and intuitive appeal as a normative criterion. Thus, the results of chapter 2 can be considered as a criticism of the basis for the indexation approach.

Chapter 3 of the thesis, "Is Opportunity Egalitarianism a Sound Criterion for Social Choice?" analyses whether opportunity egalitarianism may constitute a foundation for making normative decisions. The point of departure is the suggestion from a variety of writers that some version of opportunity egalitarianism may be a sensible rule for distributive purposes. Rawls has suggested that "fair equality of opportunity" should be adopted as a principle of distributive justice. Other authors have similar proposals. An important distinguishing feature of these suggestions is the fact that opportunities, and not only end states, are regarded as objects of normative importance. In the chapter I argue that a reasonable operationalisation of opportunity egalitarianism must pay attention to opportunity sets and not only to the chosen bundles of goods. I study the possibility of such criteria in different economic environments, assuming that lump sum transfers are possible. I propose the "minimal opportunity egalitarian criterion" as a reasonable requirement for opportunity egalitarian views. This principle states that a social state is better than another if the worst opportunity set in the latter social state is completely contained in the worst opportunity set in the former social state. This implies that an improvement in the worst individual's opportunity set must be considered a social improvement. However, it is demonstrated that there does not generally exist a social choice rule which satisfies the minimal opportunity egalitarian criterion and the Pareto principle. Next, I show that even when the domain of the minimal opportunity egalitarian criterion is restricted to competitive equilibria, the incompatibility result persists. Finally, I study situations where different individuals have different wage earning capacities. In this context, I argue that the "extended opportunity egalitarian criterion" is a reasonable requirement for an opportunity egalitarian view. It is shown that this criterion can not be used as a basis for social choice. The conclusion seems to be that it is hard to find reasonable operationalisations of opportunity egalitarianism which provide a sound foundation for normative decisions.

In chapter 4 of the thesis, "Competing for Aid", I turn to a positive analysis of the effects of foreign aid. The chapter is based on the fundamental view that the donor of aid faces a commitment problem when allocating resources between different recipients. I construct a dynamic game in which two recipients simultaneously choose their level of investment, and a donor subsequently allocates aid between the recipients. The donor is fundamentally concerned with the well-being of the recipients, and does consequently give more aid to poorer countries. Knowing this, the recipients will overallocate resources to current consumption at the expense of investments. I elaborate on the effects of aid in different settings where these fundamental mechanisms are at work. I show that if recipients have access to an international credit marked, investments will be efficient. However, the recipients will choose en excessive level of indebtedness. Next I develop a game in which each recipient chooses between long term and short-term investments. I show that when the recipients compete for aid, they will overallocate resources to long term investments. Finally, I focus on the recipients' choice of risk exposure. I show that competition for aid does not distort the choice of risk-taking by recipients.

In the final chapter of the thesis, "The Commitment Problem in an Infinite Horizon Game of Aid Donations", I analyse the commitment problem when a donor and a recipient interact in an infinitely lasting game. I construct a neo-classical growth model, in which investments are determined by the sum of savings by the recipient and aid donations by the donor. I restrict the analysis to games in which the recipient and the donor plays stationary Markovian strategies. This means the only factor which can influence savings and aid donations in any period is the current level of production in the country. I show that any Markov-perfect equilibrium of the game must converge to a steady state. Furthermore, if the players use twice continuously differentiable strategies, the outcome of the game can never be Pareto optimal.

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Chapter 2

THE INDEXATION APPROACH IS INCOMPATIBLE WITH THE PARETO PRINCIPLE*

Abstract

This chapter analyses Rawls' and Sen's proposals of using an index of primary goods or functionings as a basis for social choice. First, I formalise Rawls' suggestion of judging an individual's degree of success by an index of primary goods. Based on an indexing function which provides a complete ranking of all individuals' vectors of primary goods, I define two axioms based on the indexation approach; "the minimal indexing rule" and "the leximin indexing rule". I show that a social ranking of the available social states which satisfies the Pareto principle and either of these two indexing rules exists if and only if all individuals have identical preferences and the indexing function represents these preferences. Next, I analyse Sen's proposal on how to compare individual positions: He suggests that individual states can be ranked only when everyone agrees upon such a ranking. A weak version of egalitarianism, based on this idea, is formalised. The "unanimous egalitarian criterion" states that a unanimous improvement of the worst position in a social state must be regarded as a social improvement. The main result of this chapter is that, given a set of reasonable conditions on the profile of preferences, a quasi-ordering which ranks social states according to the unanimous egalitarian criterion and the Pareto principle exists if and only if all individuals have identical preferences.

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1. Introduction

In the past 25 years, many economists and political philosophers have argued against traditional welfare economics or "welfarism" as an appropriate approach to normative questions in economics. Welfarism has been defined by Amartya Sen (1977) as a common term for normative theories which view utility or preference information as the only relevant information when evaluating the desirability of a social state. In alternative approaches one make use of additional information when comparing different social situations. The indexation approach was pioneered by John Rawls (1971, 1993). He argued that one should equalise an *index of primary goods*. Amartya Sen (1980, 1985, 1992) argued that *functionings* are the objects of importance for individuals, and that equality should be achieved in this space. Hence, in the indexation approach, the interest of an individual is measured by an index of the objects which are considered valuable. This index value is not intended as a measure of utility, and the approach does consequently suggest a non-welfarist social choice rule.[1]

The strategy of this chapter is to formalise some of Rawls' and Sen's main ideas, and to discuss whether these can serve as a sound foundation for social choice. Specifically, I propose the Pareto principle as a basic criterion for social choice. A main problem is whether one can construct a sensible social choice rule which satisfies the Pareto principle and makes use of index values. It should be noted that the indexation approach is proposed as a practicable method for making interpersonal comparisons. A well-known problem with welfarism is that it relies on utility being interpersonally comparable at some level. Arrow (1963) showed that if utility was only <u>intra</u>personally comparable and a set of other reasonable axioms were satisfied, no ordering of social states could exist.[2] In a seminal article Deschamps and Gevers (1978) showed that there is a close link between different social choice rules and the possibility of making interpersonal comparisons: If utility is ordinally comparable between persons, the leximin social choice rule must be adopted; if utility is cardinally comparable between persons, the utilitarian principle must be adopted.[3] However, these results may not be very constructive without an established basis for making such interpersonal comparisons. In this chapter I focus on the suggestions by Rawls and Sen on

[1]In this text the terms "preference" and "utility" will have a standard behavioural content. The term "well-being" will be understood as the quality of an individual's life. According to some theories the well-being of an individual is determined by fulfilment of preferences or increased utility. In other theories an individual's well-being can be determined partly independently of this person's tastes. For further elaborations on these issues see Parfit (1984) Appendix I and Scanlon (1993).

[2]Arrow's axioms are (1) unrestricted domain, (2) independence of irrelevant alternatives, (3) the Pareto principle and (4) non-dictatorship. Moreover Arrow demands that a social ranking must be an ordering; a binary relation which is reflexive, transitive and complete.

[3]See e.g. Sen (1986) and Blackorby et. al. (1984) for elaborations on the interrelationships between interpersonal comparability of well-being and different forms of welfarism.

how to rank individual positions without relying on comparisons of utility between individuals.[4] They suggest that one should base rankings of individual positions on "objects" which to some extent are observable; namely primary goods or functionings. As I will come back to later, it seems reasonable to interpret Rawls as proposing an index which gives a <u>complete</u> ranking of positions, whereas Sen suggests an <u>incomplete</u> ranking. This chapter can be understood as an analysis of whether these versions of the indexation approach can reasonably be used as a basis for social choice.

The indexation approach is not only a theoretical construct, but has also been influential as a basis for decision making in practical redistributive policies. The United Nations have followed this approach by using the "Human Development Index" (HDI) instead of per capita GNP as the primary measure of welfare.[5] The HDI is suggested by the Norwegian government as a decisive factor when distributing aid to developing countries. It is consequently of practical importance to analyse whether the indexation approach has a sound conceptual foundation.

Before proceeding to the analytic sections of the chapter, I will discuss some aspects of the philosophical underpinnings of the indexation approach.

1.1 The Indexation Approach

Maybe the most influential non-welfarist approach in the literature of political philosophy is developed by John Rawls in "A Theory of Justice" (1971), with further elaborations in "Political Liberalism" (1993). His theories include a principle of equal liberty which has strict priority over issues regarding the distribution of material goods. Rawls' second principle concerns questions of distributive justice. This principle states roughly that a social state is just if it is organised according to (a) "the difference principle" and (b) the principle of "equality of fair opportunity". In this chapter I will discuss the difference principle, which roughly states that society should be organised according to the interests of the least well off individuals in the society. An important part of this principle, and the entire theory of Rawls, is that the interests of the least advantaged individuals are measured by an *index of primary goods*. Normative economists, who tend to operationalise Rawls as a "utility egalitarian",

[4]Both Sen and Rawls discuss <u>ordinal</u> comparisons, or rankings, of individual positions.
[5]The human development index is a weighted measure of a country's (1) average literacy, (2) life expectancy at birth and (3) per capita GNP. For further details see e.g. UNDP (1994).

often neglect this part of his theory.[6] Rawls' focus on primary goods is however essential for the non-welfarist content of his theory.

Rawls is concerned with the situation for the worst off people in a society. The interests of this group are, in his approach, determined by an index of primary goods. These are "rights and liberties, opportunities and powers, income and wealth" (1971, pg. 92). Rawls seems to have two main reasons for his focus on primary goods. First, Rawls perceives primary goods as objects which indirectly are valuable for all individuals. Different individuals will however differ in their specific ultimate goals, and in how they use primary goods to achieve these. Primary goods are the objects which everyone can agree upon as valuable. Rawls does not develop a specific or detailed theory of "the good" which can be regarded valid for all individuals. Such a theory presumes that the specific well-being experienced by each individual is interpersonally comparable. This is not, according to Rawls, a reasonable assumption. His ideas are rather based on the concept of "a thin theory of the good". Rawls accepts that different individuals have different goals, and primary goods are thought of as basic prerequisites for the pursuit of any valuable end in life. The index value can therefore not be understood as a precise measure of well-being.[7] Rather it measures an individual's so called "expectations" for success.

There is also a more practical reasoning behind Rawls' focus on primary goods. As noted in the introductory paragraphs, an important problem for welfarism is that it is based on interpersonal comparability of utility or well-being. Such comparisons are generally not thought to be directly accessible. A fundamental problem for welfarism is consequently that its theoretical insights can not, in a fairly non-controversial manner, be interpreted into practical redistributive policies. An important motivation for Rawls' introduction of primary goods is to develop an implementable basis for redistribution. This is stated explicitly by Rawls: "The thought behind the introduction of primary goods is to find a practicable public basis of interpersonal comparisons based on objective features of citizens' social circumstances open to view..." (Rawls (1993) pg 181). The focus on an index of primary goods, instead of well-being, as the basis for normative comparisons in Rawls' approach must be understood also as an attempt to give his theories an operational content.

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[6]Rawls accepts differences between individuals if these benefit the worst off segment in a society. It is more precise to say that Rawls argues in favour of a "maximin" or "leximin" rule, and not in favour of strict equality. I will use the terms "equality" and "egalitarianism" in a general sense; they cover also maximin and leximin social choice rules.

[7]This point has been stated explicitly by Rawls: "...fair shares of primary goods are clearly not intended as a measure of citizens' expected overall psychological well-being, or of their utility..." (Rawls 1993, pg 187/188).

In order to obtain a specific index value for each individual it is necessary to put weights on each of the primary goods. The problem of weighting primary goods has not been addressed or discussed thoroughly by Rawls. He argues that people who have relatively small amounts of one primary good also tend to have little of other primary goods. It is consequently satisfactory, in Rawls' view, to rank individuals according to one primary good; income. In the remainder of this chapter I will assume that there are important situations in which the vector of primary goods for one individual is not dominated by the vector of primary goods for all other individuals. In these situations the problem of weighting different primary goods is fundamental. In section 3 I interpret Rawls' proposal for an index as providing a complete ranking of the least advantaged positions in a society. This means that for every two bundles of primary goods, the index will be able to say which one is better (or that they are equally good). The index can thus never be undetermined regarding the ranking of two individual states. It should be noted that Rawls is not explicitly demanding completeness of the index. However, he does not discuss the possibility of incompleteness, and thus it seems like he imagines that the index of primary goods is complete. Regardless of whether this interpretation is correct, an analysis of such a complete index is of considerable interest. This is the focus of the analysis in section 3.

Amartya Sen's normative theories share many important characteristics with the views of Rawls (Sen (1980, 1985, 1992)). Sen objects to Rawls' approach, however, by noting that primary goods are not of intrinsic value to people. He argues that it is important to focus more directly on the objects of fundamental value, which he has given the term "functionings". Functionings are states of being, and can be such things as being in good health, being adequately nourished, being happy and so forth. People will generally differ in their ability to transform primary goods into functionings, and the distinction between these concepts is therefore important. Equalisation of an index of primary goods does not imply an equal index of functionings. According to Sen functionings, not primary goods, are the relevant objects of value.

The question of weighting different functionings to obtain rankings of different individuals' well-being is acknowledged by Sen as an important topic. He suggests that a ranking of well-being associated with bundles of functionings should be sensitive to differences in preferences. It seems rather uncontroversial to rank one individual state as worse than another if everyone prefers the latter situation to the former. A problem occurs, however, if different people have different views as to what constitutes the worst situation. Sen suggests a solution to this problem, in which a ranking may be incomplete. If everyone agrees that one bundle of functionings is better than another bundle, then it seems uncontroversial to state that the well-being associated with the preferred bundle is higher than the well-being associated with the "inferior" bundle. In situations where people do not agree it may not be reasonable to rank the relative well-being in two alternative individual states. Thus, Sen suggests that a ranking of positions in society may be incomplete. This approach will be analysed explicitly in section 4 of this chapter.

1.2 Discussion and Analysis of the Indexation Approach

Arneson (1990) discusses the philosophical basis for Rawls' use of an index of primary goods. He argues that this approach can be defended on two grounds: The index can represent an objective measure of what constitutes "the good" for people. This measure must be correct regardless of the individuals' subjective perception of welfare. According to this defence there is one objective measure of well-being of individuals which is identical for all individuals; the index of primary goods. However, this interpretation conflicts with Rawls' pluralistic view; different individuals are assumed to have different goals in life. The second possible defence of an index is that it represents subjective welfare for all persons. If this defence is adopted, one must however accept that the indexing function differs between people when preferences vary. This conflicts with Rawls' description of an index of primary goods.

Roemer (1996) argues, in line with Arneson's second defence, that the indexing approach must be extended by individual indices which vary between people. Roemer argues that this does not necessarily make the approach welfarist. It is possible to let each index be a representation of each individual's well-being and still let it contain additional morally relevant information which is not contained in utility measures. These points will be discussed more carefully in section 3.

The first formal analysis of the indexation approach is in Plott (1978). He adopts assumptions about a social choice rule (including the Pareto criterion) which are quite similar to the general axioms used by Arrow (1963). In addition he requires, in accordance with Rawls' theory, that the best social state must be defined as one which maximizes the minimum index value. Plott's basic result is that there under these assumptions must be one unique individual who determines the weights of the indexing function. This individual can be perceived as a dictator. Plott's analyses and results have a very close resemblance to Arrow's well known "possibility theorem": Given a set of axioms, the ranking of social states must coincide with the preferences of one individual; a dictator.

Gibbard (1979) uses a different set of assumptions to reach a result of inconsistency between the Pareto criterion and a "minimal difference principle". This result is interesting. However, the minimal difference principle is not necessarily an acceptable way to operationalise the indexation approach. The minimal difference principle states that if the minimum income in a society is improved, and all prices remain the same, this must be considered a social improvement. An alternative interpretation of Rawls' theory seems to be that an index should take into account all the primary goods, and not only income. If this approach is adopted it is entirely possible to construct indexing functions which rank individual states differently from the ranking obtained by Gibbards minimal requirement.

Blair (1988) shows that, given a set of assumptions, one primary good must have lexicographic importance in the construction of an index. This means that the ranking of positions must be done solely according to the ranking of one primary good. A critical feature of his analysis, however, is that primary goods are assumed to be only ordinally comparable. This basis for Blair's analysis is criticised by Sen (1991). The assumption of only ordinal comparability of any primary good is too restrictive. Sen argues that individuals will have preferences over primary goods (or functionings), and these can be used as a basis for the construction of an index which allows for trade-offs between different primary goods. Sen explicitly proposes a procedure for making incomplete rankings.

A common feature of the above papers is that they are not explicit on how an index should be constructed (as in Gibbard (1979)). Rather, they suggest some conditions for an indexing function. It is not at all clear that Rawls or Sen would agree that an index would have to meet these requirements. In the analysis of this chapter I explicitly discuss the complete and incomplete indexing approach, as suggested by Rawls and Sen. Moreover, Blair (1988) and Plott (1978) require that a social choice rule must be an ordering; a reflexive, transitive and complete binary relation. It is well known from social choice theory that these are rather heavy demands to put on a welfare function. In contrast I do not require a ranking to be complete. This makes the inconsistency results of this chapter stronger than if I required the ranking to be an ordering. The rest of the chapter is structured as follows: In section 2 the formal framework for the analysis is presented. In section 3 I show properties of social choice rules based on complete indexing functions. In section 4 Sen's approach of incomplete ranking is formalised and analysed. In the concluding section 5 I discuss the main findings of the chapter.

2. Formal Framework

Let $N := \{1,2,\dots,n\}$ be a set of individuals. Let $x_i = (x_{i1},\dots,x_{im})$ be an individual state for individual i, and let X_i be the set of all possible individual states for i. This set is defined by $X_i := E^m, \forall i \in N$, where E^m is the m-dimensional Euclidean space. I assume that $m \geq 2$. I adopt the following standard definitions; $x_i \geq y_i \Leftrightarrow \forall k \in \{1,\dots,m\}, x_{ik} \geq y_{ik}$. If, additionally, $\exists l \in \{1,\dots,m\}: x_{il} > y_{il}$ then $x_i > y_i$. A social state x is an individual state for each individual; $x = (x_1,\dots,x_n)$. The set of all possible social states X is defined by $X := \times_{i\in N} X_i = E^{m\times n}$.

An individual preference relation, R_i, is an ordering on $E^m = X_i = X_j$. An ordering is a reflexive, transitive and complete binary relation. R_i can be interpreted as weak preference. R_i defines strong preference, P_i, and indifference, I_i, in the following standard manner: $x_i P_i y_i \Leftrightarrow (x_i R_i y_i \wedge not(y_i R_i x_i))$ and $x_i I_i y_i \Leftrightarrow (x_i R_i y_i \wedge y_i R_i x_i)$. Let $\overline{R} = (R_1,\dots,R_n)$ be a profile of orderings, and let $\overline{\boldsymbol{R}}$ be the set of all possible profiles of orderings.

In section 3 of this chapter I discuss the possibility of using an index of "goods" (primary goods or functionings) as a relevant piece of information when constructing a ranking of the social states. An individual indexing function is a function which assigns a value to any individual state; $f_i : X_i \to E^1$. A social indexing function is a vector of such individual indexing functions; $F = (f_1,\dots,f_n)$, $F: X \to E^n$. Let $\boldsymbol{F}$ be the set of all possible indexing functions. I adopt the following standard definitions; $[\forall i \in N, f_i(x_i) \geq f_i(y_i)]$ $\Leftrightarrow F(x) \geq F(y)$. If, additionally $\exists j \in N : f_j(x_j) > f_j(y_j)$ then $F(x) > F(y)$.

Based on relevant pieces of information I will attempt to construct a ranking of the social states. I make an important methodological choice by <u>not</u> putting much structure on the social ranking. Throughout the chapter I allow the ranking to be <u>incomplete</u>; there may exist social states which will not be ranked by the constructed social choice rule. Specifically, I will study rankings of the social states which can be represented by a quasi-ordering, R of X. A quasi-ordering is defined by a binary relation that is <u>transitive</u> and <u>reflexive</u>.[8] A binary relation is <u>transitive</u> if, for all $x,y,z \in X, [xRy \wedge yRz] \Rightarrow xRz$. A binary relation is <u>reflexive</u> if, for all $x \in X, xRx$. Let $\boldsymbol{R_{QO}}$ be the set of all possible quasi-orderings of X. Furthermore, a quasi-ordering, R, defines a "strict partial ordering", P, in the following standard manner: $xPy \Leftrightarrow [xRy \wedge not(yRx)]$. This strict ranking is a strict partial ordering; a <u>transitive</u> and <u>asymmetric</u> binary relation. Transitivity of P is defined as above. A binary relation, P, is

[8] The definitions of quasi-orderings and strict partial orderings are adopted from Sen (1970).

asymmetric if, for all $x, y \in X, [xPy] \Rightarrow not[yPx]$. The transitivity of P follows from Lemma 1*a in Sen (1970). The asymmetric property of P follows from its definition: $xPy \Leftrightarrow [xRy \wedge not(yRx)] \Rightarrow not(yPx)$.

The general strategy is to construct a "social evaluation functional" G, which assigns a "social ranking" $R \in \boldsymbol{R}_{QO}$ to each element $(\overline{R}, F) \in \boldsymbol{R}^* \times \boldsymbol{F}^*$, where the asterixes denote admissible domains. I will choose some suitable restrictions on the domain, so that $\boldsymbol{R}^* \subseteq \overline{\boldsymbol{R}}$ and $\boldsymbol{F}^* \subseteq \boldsymbol{F}$. Hence, $G: \boldsymbol{R}^* \times \boldsymbol{F}^* \rightarrow \boldsymbol{R}_{QO}$. I will furthermore adopt axioms on G.

Before describing possible domain restrictions, it is worthwhile to explain the effect of such restrictions. This chapter is primarily concerned with "non-existence" results; I will show that there exist admissible profiles of preferences for which it is impossible to construct a "reasonable" quasi-ordering. Such a result will of course also hold for a larger domain. The effect of restricting the domain is consequently to make non-existence results "more powerful". Conversely, if a quasi-ordering can be constructed only if some implausible domain condition is met, the usefulness of this quasi-ordering may be questioned.

The social indexing function will be required to consist of n identical individual indexing functions. This restriction is based on Rawls' description of the indexing function. According to Rawls, two individuals who experience the same bundle of primary goods must be ranked as equally well off by the "index of primary goods". Hence, the individual indexing function is required to be identical for all individuals.

<u>Condition II; Identical Indexingfunctions:</u>
The social indexingfunction $F = (f_1, \ldots, f_n)$ satisfies condition "identical indexingfunctions" if; $f_i(a) = f_j(a), \forall i, j \in N, \forall a \in E^m$.

The set of admissible profiles of preferences can be restricted in various ways:

<u>Condition SRP; Self Regarding Preferences:</u>
The profile of preferences $\overline{R}$ satisfies condition "self regarding preferences" if, for all $i \in N$ and all $x, y, z, w \in X$ such that $x_i = z_i$ and $y_i = w_i$, we have; $x_i R_i y_i \Leftrightarrow z_i R_i w_i$.

<u>Condition CP; Continuous Preferences:</u>
The profile of preferences $\overline{R}$ satisfies condition "continuous preferences" if for all $i \in N$ and all $a \in E^m$, the sets $A_i^{0+} := \{\forall x \in E^m : xR_i a\}$ and $A_i^{0-} := \{\forall x \in E^m : aR_i x\}$ are closed.

Condition M; Monotonicity:
The profile of preferences $\overline{R}$ satisfies condition "monotonicity " if for all $i \in N$ and all $x_i, y_i \in X_i, [\forall k \in \{1,2,...,m\}: x_{ik} > y_{ik}] \Rightarrow x_i P_i y_i$.

Condition IP; Identical Preferences:
The profile of preferences $\overline{R}$ satisfies condition "identical preferences" if for all $i \in N$ and all $a,b \in E^m$ we have; $aR_i b \Leftrightarrow aR_j b$.

The condition of self-regarding preferences (SRP) is adopted throughout the chapter, mainly for expository purposes. It should be uncontroversial to claim that a reasonable "social ranking" should work also when individuals are "selfish". Condition CP (continuous preferences) is essential in the analysis of section 4. As shown in Debreu (1959) continuity of preferences is sufficient to ensure that preferences can be represented by a utility function. This assumption is standard in mainstream economics. Condition M (monotonicity) is in accordance with the general approach of this chapter: The focus is on distribution of goods for which it seems reasonable to assume that all individuals prefer more to less. Again, adopting such an assumption is standard in economics. Assuming identical preferences (condition IP) is controversial. It seems quite unlikely that this condition will hold in the real world. The theoretical foundation for a "social ranking" is in my view seriously weakened if its existence hinges on a condition of identical preferences. This point will be discussed more carefully in sections 3 and 4.

Lastly, I define a domain restriction regarding the relation between the indexing function and the profile of preferences. The condition demands that the social indexing function actually is a representation of the profile of preferences.

Condition (IRP); The Social Indexing Function Represents the Profile of Preferences:
A social indexing function and a profile of preferences $(\overline{R}, F)$ satisfy "condition IRP" if for all $a,b \in E^m$ and all $i \in N$ we have; $aR_i b \Leftrightarrow f_i(a) \geq f_i(b)$.

There is a close relationship between condition II, condition IP and condition IRP: As previously argued, one can reasonably interpret Rawls as defending identical individual indexingfunctions (condition II). If we assume that condition II holds, then condition IRP implies condition IP. The reason is as follows: Condition IRP demands that the individual indexing function must represent all individual's preferences. In order for one function to represent the preferences of all individuals, all individuals must have identical preferences.

Hence condition IRP implies condition IP, when condition II holds. Condition IRP will be used in section 3 of this chapter, where I analyse properties of a social choice rule based on a particular indexing function. Condition IP will be used in section 4, where such a function no longer is used.

The social evaluation functional G may be demanded to satisfy a variety of requirements. The most widely accepted axiom for social choice is the Pareto criterion. I adopt the strong Pareto criterion:

Axiom P: For all $x, y \in X$, if $\left[\forall i \in N, x_i R_i y_i\right]$ then xRy.
If, additionally, $\left[\exists j \in N : x_j P_j y_j\right]$ then xPy.

In the analysis of this chapter I don't use any axioms which make use of so called interprofile information, such as independence of irrelevant alternatives (e.g. Arrow (1963)). In this chapter I study the possibility of constructing a social ranking based on information about a single profile of preferences (and other pieces of information). The domain restrictions are conditions that are put on any single element in the domain. Thus, I conduct what has become customary to label a *single profile analysis* (see e.g. Blackorby et.al. (1990)).

3. The Indexing Problem with Complete Rankings of Well-being

As a starting point, I will discuss briefly whether my formulation is a reasonable interpretation of the views of Rawls. It is important to note that preferences and the indexing function are defined on bundles of goods, such as primary goods or functionings. It is quite clear that Rawls wants the indexing function to be defined over primary goods. It is, however, less obvious that preferences should be defined over the same set of alternatives. According to Rawls a vector of primary goods can be regarded as prerequisites for a valuable life, but it does not directly determine a high level of well-being. I claim that it is in line with this thinking to define individual preferences over a set of primary goods. It seems reasonable that individuals have (direct) preferences over the objects of intrinsic value. According to Rawls, primary goods are used to obtain objects which are of intrinsic value to individuals. Hence, it seems natural that individuals have indirect preferences over primary goods, which are derived from their preferences over objects of intrinsic value.

In Rawls' theory, social states are ranked according to the lowest index value in every social state. Thus, Rawls incorporates an element of anonymity in the social choice rule; the desirability of a social state should not depend on the name of the individual who is in the worst state. I will start out with an analysis of a version of the indexation approach which does not adopt anonymity as a basic requirement for a social choice rule. The minimal requirement I put on an indexing rule is that it must be increasing in its arguments: If at least one individual experiences a larger index value, and no one gets any less, this must be considered a social improvement. This condition can be expressed in the following "Minimal Indexing Rule":

Axiom MIR: For all $x,y \in X$; $(F(\mathrm{x}) \geq F(\mathrm{y})) \Rightarrow \mathrm{x}Ry$, $(F(\mathrm{x}) > F(\mathrm{y})) \Rightarrow \mathrm{x}Py$.

A first result regarding Rawls' proposal of a complete indexing rule is as follows:

Proposition 1

A social evaluation functional, G, which satisfies Axiom P and Axiom MIR exists if and only if the social indexing function represents the profile of preferences (Condition IRP).

Proof:

Only if:

Suppose condition IRP fails. Then there exists an individual $i \in N$ and bundles $a,b \in E^m$ such that one of the following two must hold; (1) $aP_ib \wedge f_i(b) \geq f_i(a)$, or (2) $aR_ib \wedge f_i(b) > f_i(a)$. Consider two social states x,y which differ only in the individual state of i such that $x_i = a$ and $y_i = b$. Case (1): Axiom P implies xPy and Axiom MIR implies yRx, which is a contradiction. Case (2): Axiom P implies xRy and Axiom MIR implies yPx, which leads to a contradiction. Consequently, a quasi-ordering on X does not exist.

If:

When all individuals' preferences are represented by the indexingfunction F, it is easily verified that Axiom P and Axiom MIR are equivalent. Axiom P defines a quasi-ordering, and is thus a permissible social ranking.

Q.E.D.

Proposition 1 applies for general indexing rules. The literature discussed in this chapter is however concerned with egalitarian ethical theories, which appropriately can be formalised by a leximin indexing social choice rule. A leximin social choice rule ranks social states according to the interest of the worst off individual. When comparing two social states which are equally good for the worst off person, social states are ranked according to the interest of the second worst off person, and so forth. A leximin rule is anonymous; social states are ranked according to the worst individual position regardless of the name of the person who is in this situation. In a leximin *indexing* social choice rule, the interest of the worst off individual is determined by the value assigned to his consumption bundle by the indexing function. In order to formalise the leximin indexing rule, I define a permutation function for each social state which reorder the individuals according to their ranking by the function f in the relevant social state. A permutation function $\sigma_x : N \to N$ is formally defined by; $f_i(x_i) < f_j(x_j) \Rightarrow \sigma_x(i) < \sigma_x(j)$ and $f_i(x_i) = f_j(x_j) \Rightarrow \sigma_x(i) < \sigma_x(j) \vee \sigma_x(i) > \sigma_x(j)$ (if two or more individuals have the same index value they are ranked in any strict order). There exists (at least) one permutation function for each possible social state, each of which is defined in a similar way as above. The inverse of the permutation function, $\sigma_x^{-1}(i)$, is denoted $\rho_x(i) = \sigma_x^{-1}(i)$. The formal statement; " $f_{\rho_x(i)}(x_{\rho_x(i)}) < f_{\rho_y(i)}(y_{\rho_y(i)})$" means; "the i th worst off individual in social state x has a lower index value than the i th worst off individual in social state y". An egalitarian axiom based on the indexing approach, the Leximin Indexing Rule (LIR), is proposed:

Axiom LIR: $\left[\exists j \in N : \forall i < j \left[f_{\rho_y(i)}(y_{\rho_y(i)}) = f_{\rho_x(i)}(x_{\rho_x(i)}) \wedge f_{\rho_y(j)}(y_{\rho_y(j)}) > f_{\rho_x(j)}(x_{\rho_x(j)}) \right]\right] \Rightarrow yPx.$

The following result shows that an egalitarian version of the indexation approach does not solve the problem expressed in Proposition 1:

Corollary (to Proposition 1)

A social evaluation functional, G, which satisfies Axiom P and Axiom LIR exists if and only if the social indexing function represents the profile of preferences (Condition IRP).

Proof:

Only if:

It is easily verified that any two alternatives, which are ranked by Axiom MIR, are ranked identically by Axiom LIR. Thus, the proof of the "only if" part of Proposition 1 applies.

If:

When all individuals have identical preferences which are represented by the indexing function, it is easily verified that Axiom LIR implies Axiom P. Thus, the quasi-ordering defined by Axiom LIR is a permissible social ranking.

Q.E.D.

Proposition 1 and its corollary show that a social indexing function F must represent the profile of preferences if such a function shall be used in the construction of a social choice rule. As noted previously, this has strong implications for the set of permissible profiles of preferences. A reasonable interpretation of Rawls is to claim that condition II holds (identical individual indexingfunctions). If a social indexing function with identical individual indexingfunctions represents the profile of preferences, then all individuals must have identical preferences. Hence, if this interpretation of Rawls is accepted, proposition 1 and its corollary show us that the "indexation approach" may be meaningful only to the extent that individuals have identical preferences over the set of primary goods or functionings.

These results are easy to understand intuitively, and are almost self evident when the formal basis for the analysis is established. The minimal indexing rule (Axiom MIR) is very similar to a minimal requirement on a social welfare function which has individual utilities as its arguments. A reasonable requirement on such a welfare function is that higher utility values imply higher social welfare (the welfare function is strictly increasing). However, the minimal indexing rule uses index values and not utility values as arguments. The arguments coincide only if the social indexing function represents the profile of preferences. If the indexing function does not represent the profile of preferences, it is possible that an individual's index value increases whereas his utility does not. This is the reason for contradictory rankings of social states by the Pareto principle and the minimal indexing rule. It is easily seen that the conflict persists when the minimal indexing rule is replaced by the leximin indexing rule.

Proposition 1 and its corollary contains a positive result as well: the indexation approach is not in conflict with the Pareto principle if the social indexing function represents the profile of preferences (condition IRP). If we require that the individual indexing functions are identical, individuals must have identical preferences in order for condition IRP to hold. In my view, however, this condition is unrealistic. An assumption of identical preferences seems like an unreasonable restriction to put on the set of possible profiles of preferences. Moreover, such an assumption seems incompatible with some of Rawls' main ideas. Primary goods are something everyone needs in order to live valuable lives. However, Rawls admits different people to have different ultimate goals in their lives. If people have different preferences over ultimate ends then one must admit indirect preferences over primary goods to differ as well.

Arneson (1990) and Roemer (1996) have realised some of these problems. They suggest that a way out of this problem is to abandon the condition of identical individual indexingfunctions, and use n different functions, f_i, each of which represents the preferences of the individual in question. Both Arneson and Roemer are of course aware that their suggestion is not an adequate description of Rawls' views. It is clear that Rawls describes only <u>one</u> function f, which shall be the standard for comparisons between individuals. The proposal must be regarded as a possible extension of the indexation approach, which is meant to be compatible with the general framework of Rawls' theory. Roemer, in particular, asserts that it is possible to develop a non-welfarist theory based on individual indices rather than one index. To me it seems questionable that such an approach is compatible with Rawls' main views. In the introduction to this chapter I quoted Rawls where he argued that a main reason for the use of an index is that it constitutes a; "...practicable public basis of interpersonal comparisons based on objective features of citizens' social circumstances open to view..." (Rawls (1993) p 181). An important justification for the use of an index is to make his theory implementable in the sense that normative policies are dependent only on observable factors (thereby avoiding the problems associated with interpersonal comparisons of well-being). A "multiple indices approach" can consequently be compatible with Rawls' main ideas only to the extent that differences between indices is determined solely by information which is objectively observable. But how can it be possible, based only on objectively observable differences between people, to discover individual preferences? Maybe one can make some inferences about preferences based for instance on observed choices: Nevertheless, preferences are generically non-observable. These multiple indices can not both represent the preferences of all individuals and be compatible with Rawls' requirement that social choice rules must depend only on objective and observable features.

I now turn to an analysis of Sen's incomplete ranking of well-being. He regards functionings and not primary goods as the relevant objects of value. He does nevertheless suggest that a version of the indexation approach can be used to evaluate the relative well-being experienced by different individuals.

4. Incomplete Rankings of Well-being and The Unanimous Egalitarian Criterion

Sen suggests that differences in individual rankings over functionings should be reflected in the valuation of different vectors of functionings (Sen 1985, 1991, 1992). If different individuals rank two bundles differently, then it may not be sensible to judge one of these as worse or better than the other. Sen does consequently propose that rankings of well-being should be incomplete when individuals disagree. Sen suggests an approach where it is possible with a ranking of different individuals' well-being only if everyone agrees about these rankings. The analysis in section 3 used an indexing function which gave a complete ranking of individual positions. The non-existence results were obtained exactly because individuals did not agree with the rankings given by the indexing function. The arguments in section 3 are consequently not valid as a critique against a ranking of individual positions as proposed by Sen. In this section of the chapter I will analyse a social choice rule that is based on an incomplete ranking of individual positions. I will assume, in accordance with Sen, that individual positions can be ranked only when individuals unanimously agree with this ranking.

Sen does not propose any specific ways to incorporate his unanimous ranking into a social choice rule. It is, however, clear that he is favourable to egalitarian approaches. I will focus on a criterion where the ranking of social states is determined by the ranking of the worst positions in the relevant situations.[9] I propose the "unanimous egalitarian criterion": If in one social state it is possible to determine a worst position (by unanimous ranking of individual states), and if all positions in another social state are unanimously better than this worst position, then the latter social state must be considered superior to the former.

I will formalise the criterion described above as an axiom on a social evaluation functional, G'. However, the domain of this mapping must be redefined. The indexing function as described in section 2 and 3 have no use in the present context. Hence, the social evaluation functional is a mapping with the set of all admissible profiles of preferences as the domain and the set of all possible quasi-orderings as the range; $G' : \boldsymbol{R}^{*} \rightarrow \boldsymbol{R}_{QO}$. In this section I will make use of the following kind of judgements; "According to individual k, consuming the bundle of person i in social state x is (weakly) preferred to consuming the bundle of person j in social state y" $\left(x_i R_k y_j\right)$. This kind of judgement may be used as a way of determining a worst position in a social state, and to compare different individual states in different social states. The "Unanimous Egalitarian Criterion" (UEC) is based on such comparisons. Its basic idea is that if everyone agrees that all individual states in one social state is better than the worst individual state in another social state, then the former social state is preferable. The formal definition of the criterion is as follows:

<u>Axiom UEC:</u>
For all $x, y \in X$, if $\left[\exists i \in N : \forall j, k \in N, \left[x_j R_k x_i \wedge y_j P_k x_i\right]\right]$ then yPx.

The idea behind this criterion may need some further discussion. First, the criterion demands that one can identify a worst position in a social state: There must be an individual state x_i in social state x which everyone agrees is "at least as bad" as any other position in this social state; $\left[\exists i \in N : \forall j, k \in N, \left[x_j R_k x_i\right]\right]$. Next, it demands that every individual state in an alternative social state y is considered better than this "worst" individual state x_i by all

[9]As will become clear, the arguments below hold equally well if the "unanimous ranking approach" were applied to the i th worst position. The focus on an egalitarian rule is thus not essential for the result.

individuals; $\left[\forall j,k \in N,\left[y_j P_k x_i\right]\right]$. If these two requirements are met, then social state y must be regarded as better than social state x; yPx.

The unanimous egalitarian criterion is illustrated in figure 1. In the figure I assume that there are only two individuals, and that the individual states are 2-dimensional.

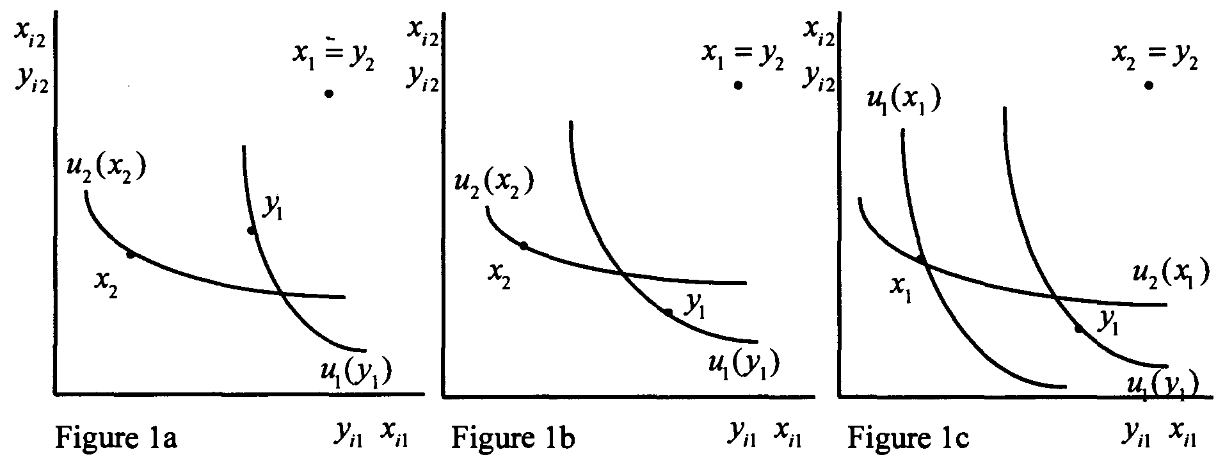

Figure 1a Figure 1b Figure 1c

In figure 1a both individuals prefer any of the other positions to that of being individual 2 in social state *x*. Specifically, the worst individual state in social state y (y_1) is strictly preferred by both individual 1 and 2 to the worst individual state in social state x (x_2). The unanimous egalitarian criterion does consequently demand that social state y must be ranked as strictly better than social state x. In figure 1b the worst off individual is uniquely defined in both *x* and *y*. The two individuals do however not agree about which position is worse; being individual 1 in social state *y* or being individual 2 in social state *x*. These two social states are consequently not ranked by the unanimous egalitarian criterion.

Figure 1c illustrates a situation where the unanimous egalitarian criterion does not apply, but where there nevertheless are good reasons to rank the social states. The individuals disagree whether the worst individual state in social state *x* is worse or better than the worst individual state in social state *y*. The unanimous egalitarian criterion does consequently not rank these two social states. The difference from figure 1b, however, is that individual 1 is defined as being worst off in both social states. Moreover, individual 1 regards *y* as better than social state *x* (and individual 2 is indifferent). Social state *y* does consequently Pareto dominate social state *x*. The Pareto criterion is not implied by the unanimous egalitarian criterion, and must be added as an additional requirement on a social choice rule (if it is considered desirable). The unanimous egalitarian criterion is generally not applicable for situations in which there are more than one possible candidate for being worst off. This illustrates that the unanimous egalitarian criterion can not be regarded as a complete description of a social choice rule. The unanimous egalitarian criterion must consequently be understood as a *minimal requirement* for a social choice rule.

One can interpret the unanimous egalitarian criterion as a method for making interpersonal comparisons of the level of well-being by any two individuals. The method is simply to say that one individual position is worse than another if everyone prefers the latter to the former. In order to make such comparisons one needs only information about individual preferences over bundles experienced by different individuals.

The following proposition shows that Axiom UEC may not be very useful for the construction of a "social ranking":

Proposition 2

Suppose the following conditions hold; Continuous Preferences (CP), Monotonicity (M), and Self-Regarding Preferences (SRP). A social evaluation functional, G', which satisfies Axiom P and Axiom UEC exists if and only if all individuals have identical preferences (condition IP).

Proof:

Lemma 1

If conditions CP, M and SRP hold, and condition IP does not hold, then there exist two individuals $i,j \in N$, and four bundles $a,b,c,d \in E^m$ such that;

$$dP_jb \wedge bP_jc \wedge cP_ja \text{ and}$$
$$cP_ia \wedge aP_id \wedge dP_ib \text{ and}$$
$$\forall k \in N:\left[cP_ka \wedge dP_kb\right].$$

Proof of Lemma 1:

Define the sets; $A_i^{0+}:=\left\{\forall x \in E^m: xR_ia\right\}$, $A_i^{0-}:=\left\{\forall x \in E^m: aR_ix\right\}$, $A_i^{+}:=\left\{\forall x \in E^m: xP_ia\right\}$, $A_i^{-}:=\left\{\forall x \in E^m: aP_ix\right\}$. Moreover, define an open neighbourhood of a as $N_\varepsilon(a):=\left\{\forall x \in E^m: d(x,a)<\varepsilon\right\}$, $\varepsilon>0$.

When condition IP does not hold, there exist bundles $a,b' \in E^m$ and individuals $i,j \in N$ such that; $aR_ib' \wedge b'P_ja$. By condition CP the set A_j^{0-} is closed, and hence the set $A_j^{+}=E^m - A_j^{0-}$ is open. Notice that $b' \in A_j^{+}$. Because A_j^{+} is open there exists an open neighbourhood of b' which is contained in $A_j^+; N_\varepsilon(b') \subset A_j^+$. Consider the vector $b = \left(\left(b_1' - \frac{\varepsilon}{m}\right), \ldots, \left(b_m' - \frac{\varepsilon}{m}\right)\right)$, $\varepsilon > 0$. It is immediate that $b \in N_\varepsilon(b') \subset A_j^+$, and hence; bP_ja. Moreover, by condition M and by transitivity of preferences; aP_ib.

By condition CP the set $B_j^- := \{\forall x \in E^m : bP_jx\}$ is open. Note that bP_ja, so that $a \in B_j^-$. Thus, there exists an open neighbourhood of a which is contained in B_j^-; $N_{\bar{\varepsilon}}(a) \subset B_j^-$. Consider the vector $c = \left(\left(a_1 + \frac{\bar{\varepsilon}}{m}\right), \ldots, \left(a_m + \frac{\bar{\varepsilon}}{m}\right)\right)$. It is immediate that bP_jc. By condition M we have $cP_ia \wedge cP_ja$.

Finally, consider the set $A_i^- = E^m - A_i^{0+}$ which by condition CP is open. We have; aP_ib, and consequently; $b \in A_i^-$. There exist an open neighbourhood of b which is contained in $A_i^-; N_{\bar{\bar{\varepsilon}}}(b) \subset A_i^-$. Choose a vector $d = \left(\left(b_1 + \frac{\bar{\bar{\varepsilon}}}{m}\right), \ldots, \left(b_m + \frac{\bar{\bar{\varepsilon}}}{m}\right)\right)$. We have; $d \in N_{\bar{\bar{\varepsilon}}}(b)$, and hence; aP_id. By condition M we have $dP_ib \wedge dP_jb$.

By construction of *c* and *d*, and condition M, we have: $\forall k \in N: [cP_ka \wedge dP_kb]$.

Lemma 1 is hence proven.

<u>Proof of the "only if" part of Proposition 2:</u>
Define the bundle $e := ((max\{a_1, b_1, c_1, d_1\} + \varepsilon), \ldots, (max\{a_m, b_m, c_m, d_m\} + \varepsilon)), \varepsilon > 0$. Consider the social states $x, y, z, w \in X$, which are defined by; $x := (x_i = e, x_j = c)$, $y := (y_i = a, y_j = e)$, $z := (z_i = e, z_j = b)$, $w := (w_i = d, w_j = e)$, and for all $k \in N \setminus \{i, j\}$ let $x_k = y_k = z_k = w_k = e$. By Lemma 1 and Axiom UEP we have; $xPy \wedge wPz$. By Lemma 1 and Axiom P we have; $yPw \wedge zPx$. By transitivity of P we have; $xPw \wedge wPx$. This contradicts the asymmetric property of P.

Hence, if conditions CP, M and SRP hold, and condition IP does not hold, there can not exist a quasi-ordering of X which satisfies Axiom P and Axiom UEC.

<u>If:</u>
I will show that when individuals have identical preferences, a version of the leximin rule conforms to Axiom P and Axiom UEC:

Let R_k be the ordering for a representative individual. For all social states, $x \in X$, define a permutation function, $\sigma_x : N \to N$, as follows; $x_i P_k x_j \Rightarrow \sigma_x(i) > \sigma_x(j)$ and $x_i I_k x_j \Rightarrow \sigma_x(i) > \sigma_x(j) \vee \sigma_x(i) < \sigma_x(j)$. Define the inverse of the permutation function by; $\rho_x(i) = \sigma_x^{-1}(i)$. Hence, $\rho_x(1)$ is the individual who is ranked as worst off in social state x. The leximin rule can now be defined by;

The leximin rule: $\left[\exists j \in N : \forall i < j \left[y_{\rho_y(i)} I_k x_{\rho_x(i)} \wedge y_{\rho_y(j)} P_k x_{\rho_x(j)}\right]\right] \Rightarrow yPx.$

It is straightforward to show that the leximin rule above is a quasi-ordering which satisfies both the Pareto principle and the unanimous egalitarian criterion, provided that preferences are identical. Q.E.D.

Proposition 2 is proven by showing that, when individuals have different preferences, there must exist social states for which there does not exist a transitive and reflexive binary relation which satisfies Axiom P and Axiom UEC. The argument is illustrated in figure 2. By Lemma 1 there exist bundles $a,b,c,d,e \in E^m$ for which the preferences are as illustrated in figure 2. The relevant social states are then defined in an appropriate manner, as in the proof of Proposition 2.

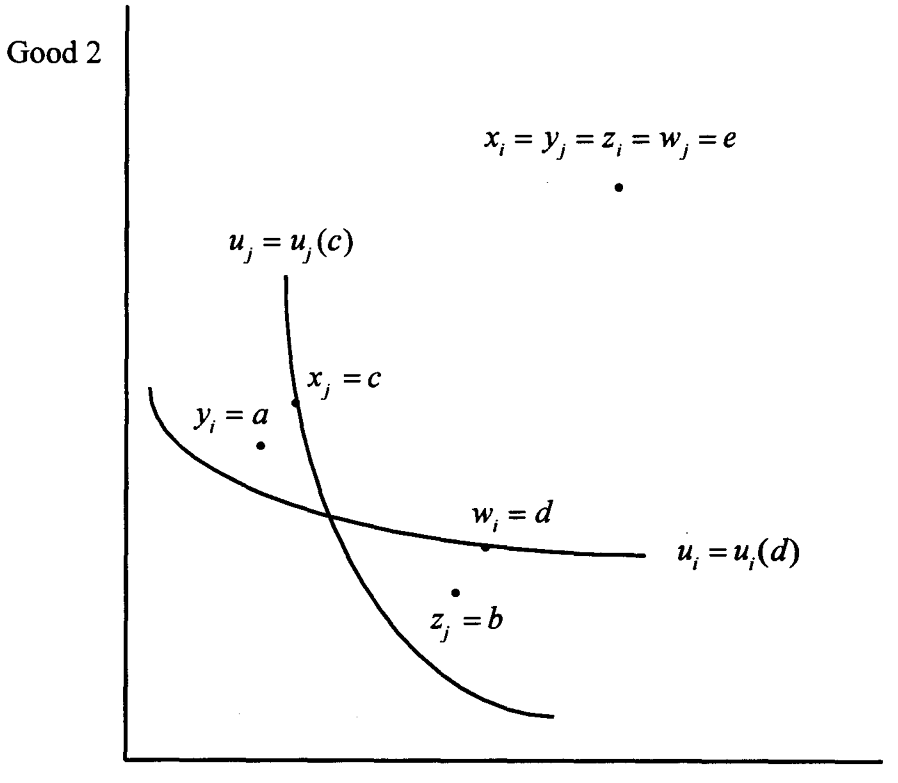

Figure 2

Axiom P demands that the following social rankings must hold: $yPw \wedge zPx$. By Axiom UEP we have; $xPy \wedge wPz$. Transitivity gives the following rankings; $xPw \wedge wPx$, which contradicts the asymmetric property of P.

Proposition 2 shows that it is impossible to construct a reasonable social choice rule which satisfies both the Pareto principle and the unanimous egalitarian criterion.

As hinted at in footnote 9, I claim that Proposition 2 does not depend upon the fact that the unanimous egalitarian criterion is concerned with the worst position in a society. A more general criterion would demand that a social state is improved if <u>some</u> position unanimously is ranked better than originally. I will define such a criterion. First define the relations; $x_j R_{i,j} x_i \Leftrightarrow \left[x_j R_i x_i \wedge x_j R_j x_i\right]$ and $x_j P_{i,j} x_i \Leftrightarrow \left[x_j P_i x_i \wedge x_j P_j x_i\right]$. Next, define a general unanimous criterion: $\left[\left[\exists i,j \in N{:}\, x_j R_{i,j} x_i \wedge y_j P_{i,j} x_i \wedge y_i P_{i,j} x_i\right] \wedge \left[\forall k \in N \setminus \{i,j\}{:}\, y_k R_k x_k\right]\right] \Rightarrow yPx$. This criterion is similar to Axiom UEC. If the set of individuals consisted of only i and j the two criteria would in fact be equivalent. The general unanimous criterion may however rank social states even if there are other positions in social state x which are unanimously worse than x_i and x_j. If all individuals other than i and j are at least as well off in y as they are in x, and Axiom UEC applies for i and j, then by the general unanimous criterion y must be better than x. This new criterion may be thought of as the unanimous egalitarian criterion applied to two individuals, and the Pareto criterion applied for all the other individuals. The non-existence result would persist if this criterion replaced the egalitarian principle analysed in Proposition 2. This is realised by changing the alternatives in the proof of Proposition 2 so that there are $k \in N \setminus \{i,j\}$ who unanimously are ranked worse than individual i and j, and who are indifferent between social states x,y,z and w. The general unanimous criterion would rank these alternatives identically as the unanimous egalitarian criterion. The non-existence result would consequently hold for this more general criterion as well.

Throughout this chapter I have implicitly assumed that individual preferences are fully observable. Implementation of Axiom UEC, for example, demands that such information is available for policy makers. Preferences are however not directly observable. Implementation of the unanimous egalitarian criterion seems to rely on truthful revelation of preferences by the individuals in a society. This may seem unrealistic. If revelation of preferences is not in an individual's best interest it seems more reasonable to assume that he will conceal this information. There are many situations where application of the unanimous egalitarian criterion would lead to reduced well-being for some individuals. These individuals may thereby have an incentive to misrepresent their preferences. Hence, Axiom UEC can be criticised for not being possible to implement in real world situations. More realistic criteria for social choice must take into account whether individuals have incentives to reveal their true preferences.

Such questions of implementation of social choice rules and public policy is discussed thoroughly both in the literature of social choice and public finance.[10] I regard these topics as highly relevant. However, for two reasons I do not find the analysis in this chapter to be fully subject to this criticism. I will limit my arguments to proposition 2. First, the main insight in proposition 2 is that Axiom UEC does not work as a criterion even in an ideal world where preferences are observable. It seems unlikely that the additional problem of implementability makes Axiom UEC a more viable criterion. Second, it is possible to rephrase the unanimous egalitarian criterion in a way that makes it implementable. Consider the following "dominance version" of Axiom UEC: "If all individuals in social state x have more of all goods than the worst off individual in social state y, then social state x must be better than y". This version of Axiom UEC depends only on comparisons of bundles, which in principle are publicly observable. Hence the "dominance version" of Axiom UEC can be implemented without information about individual preferences. Moreover, if we are willing to assume that all individuals prefer more to less, the two criteria are similar: If we apply only the dominance version of Axiom UEC we know that all alternatives would be ranked identically by the original Axiom UEC. The obvious question is whether this implementable version of Axiom UEC suffers from the same problems as the original unanimous egalitarian criterion. The answer is yes. This is realised by noting that whenever Axiom UEC is applied in the proof of proposition 2, one bundle actually dominates another. Proposition 2 will consequently still hold if a dominance version of Axiom UEC is adopted. Consequently, the problem of inconsistency with the Pareto criterion persists, even if we consider a version of Axiom UEC that is based on observable factors.

The concept of unanimous rankings of well-being can be thought of as an attempt to incorporate anonymity into a non-welfarist social choice rule. In welfarist theories anonymity does normally apply to ownership of utilities. The unanimous egalitarian criterion however is

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[10] The general topic has been addressed in the social choice literature (Gibbard (1973), Satthertwaite (1975) are classic references). The so called Gibbard-Satthertwaite theorem roughly states that one can not construct a voting scheme (a social choice rule) where the voters will always tell the truth. The literature of asymmetric information does furthermore analyse questions of implementation when preferences are not fully observable. In this tradition Mirrlees (1971) provided a classic result about optimal taxation when individual's have private information about their own productivity or wage earning capacity.

in some sense anonymous with respect to ownership of bundles. If everyone agrees that the relevant bundle is improved then this must be regarded as a social improvement, regardless of who owns this bundle. It is straightforward that complete anonymity with respect to ownership of bundles would result in conflicts with the Pareto principle: This kind of anonymity would demand that a social choice rule were indifferent between situations where the ownership of bundles has been changed. Such changes can however easily constitute Pareto improvements when people have different preferences. The unanimous egalitarian criterion is however anonymous in a more limited sense; it is only when all individuals agree that the worst bundle is <u>improved</u> that ownership is unimportant. This kind of anonymity would not apply to situations where Pareto improvements are obtained through changed ownership of bundles. As Proposition 2 shows, however, this limited form of anonymity is not sufficient to remove the problems of inconsistency with the Pareto criterion.

This insight casts doubt on the relevance of an approach where relative well-being values are determined only over individual states where people rank alternatives identically. Such a concept of well-being ranking can not, as Proposition 2 shows, serve as a basis for reasonable social choice.

5. Concluding Remarks

The starting point for the discussion in this chapter has been the non-welfarist views presented in the literature of political philosophy and normative economics. In conventional normative economic analysis, preference- or utility information is the only object of intrinsic value. The indexation approach uses an index of inputs to determine the interests of individuals, thereby avoiding interpersonal comparisons of utility. A general conclusion to this chapter is that a social choice rule can not be compatible both with the indexation approach and the Pareto criterion. In section 3 I studied complete indexing functions, as proposed by John Rawls. Minimal indexing rules and the leximin indexing rule were found to be in direct conflict with the Pareto criterion. In section 4 I discussed social choice rules based on incomplete ranking of well-being, an approach which is associated with Amartya Sen. It was found that the unanimous well-being ranking of bundles does not constitute a solid foundation for social choice, if the Pareto criterion is accepted.

One may think of three possible resolutions to the inconsistencies which are discussed in this chapter: (1) The Pareto criterion is abandoned as a criterion for social choice. (2) The indexation approach is abandoned, or (3) alternative operationalisations of the indexation approach are developed.[11]

Sacrificing the Pareto criterion is to me a drastic suggestion, but may be consistent with some versions of input based normative theories. Several authors claim that fulfilment of preference is not of intrinsic value to society. Sen (1992) and Elster (1983) use examples of endogenous preferences to illustrate that preferences can not be considered as an independent or constant entity. Elster argues that a slave may become accustomed to a life without freedom. Therefore, he might not prefer emancipation. Nevertheless, Elster argues, it is not irrational from a moral point of view to argue against slavery. Dworkin (1981 (a) and (b)) assumes that individuals to some extent are able to choose their preferences, and on this background he finds that people should be held morally responsible for their preferences. It may therefore be argued that preferences can not be considered as a constant or fundamental entity; they are products of cultural influences and free choices. Preferences can not, on this background, be considered as ultimate ends for society. If one accepts that preference fulfilment is not an intrinsic goal for society, then it is not necessarily troublesome, from a moral point of view, to prefer social states which are Pareto dominated.

This criticism presumes that there are situations where preference fulfilment does not imply increased well-being: In other words, individuals do not necessarily choose the alternatives which give them highest well-being. This view may of course in some instances be correct. In my opinion, this is a difficult standpoint which needs be more thoroughly explained and justified. I will not discuss this point at length. Note, however, that the argument is valid only against the so-called preference interpretation of the Pareto principle. John Broome (1991) makes a distinction between two possible interpretations of the efficiency criterion: (1) Situation x is better than situation y if everyone <u>prefers</u> the former to the latter (the preference interpretation). (2) Situation x is better than situation y if everyone has higher <u>well-being</u> in the former situation (the betterness interpretation). It should be straightforward to adopt the betterness interpretation of the Pareto criterion throughout this chapter. The criticism mentioned above does consequently not apply against this version of the Pareto principle. An argument against the betterness version of the Pareto criterion must be based on a general criticism of well-being as a relevant concept for normative analysis. This is a much more difficult and far reaching task than criticizing the view that only preference information can form the basis for questions of social choice.

[11]One might also argue that transitivity is not a reasonable assumption to use for some social choice rules. This fourth possibility will not be discussed in this chapter.

Proponents of the indexation approach should acknowledge that indexing rules are in conflict with the Pareto principle (in terms of preference or well-being). If the approach is maintained, the results in this chapter necessitate a thorough discussion and criticism of the Pareto criterion; the two are not compatible.

The second possible way out of the inconsistency problems is to abandon the indexation approach altogether. Based on its fundamental problems, this is a reasonable conclusion. The third possibility is however to formulate the indexation approach differently than in this chapter. Remember that one of the most important reasons for the use of an index was the need for an <u>implementable</u> normative theory. Since preference fulfilment or utility information is not directly observable, an important argument against welfarism is the lack of operational and practical content in this approach. The indexation approach was partly thought of as a way to solve this problem.

One has to acknowledge the informational problems in welfare economics which Rawls and others have pointed out. The indexation approach, as described by Rawls, is however not a satisfactory solution to these problems. The incompatibility with the Pareto criterion is an indication of this. A different line of analysis would be to address the informational problems explicitly; one could maximise some welfare function with constraints concerning the availability of utility information. It is possible that some version of the indexation approach is the outcome of such a procedure. A fully implementable social choice rule must be contingent on tangible objects; possibly such objects as primary goods or functionings. The indexation approach could therefore be operationalised based on such a general framework. Such an analysis would, however, be fully compatible with a general welfarist approach.

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Chapter 3

IS OPPORTUNITY EGALITARIANISM A SOUND CRITERION FOR SOCIAL CHOICE?*

Abstract

This chapter analyses how opportunity egalitarian criteria can be used in pure distribution problems. In different economic environments I analyse whether such criteria can be used as guidance for lump sum transfers of income. I define the "Minimal Opportunity Egalitarian Criterion" (MOEC). This principle states that, if one individual's opportunity set is completely contained in another individual's opportunity set, then a lump sum transfer to the former individual from the latter individual constitutes a social improvement. First, I analyse this criterion for an economic environment in which all individuals face equal prices, but where prices and social endowments of goods may vary between social states. For this set of social states it is demonstrated that a social choice rule which satisfies MOEC and the Pareto principle does not generally exist. Second, I assume that the set of social states is restricted to (Pareto efficient) competitive equilibria for which the social endowment of goods is kept fixed. It is shown that a social choice rule which satisfies MOEC and the Pareto indifference principle does not generally exist, for this restricted set of social states. Finally, I analyse social states for which both initial endowments of goods and individual price vectors are kept fixed for all possible social states. Prices differ between individuals due to different wage earning capacities. I argue that the "Extended Opportunity Egalitarian Criterion" (EOEC) is a reasonable operationalisation of opportunity egalitarianism in this context. However, it is demonstrated that a social choice rule which satisfies EOEC does not generally exist in this environment.

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1. Introduction

Several authors have discussed principles of fairness which appropriately can be labelled "opportunity egalitarianism". A common feature for such views seems to be that individuals are given some responsibility for their own actions. One may accept differences in outcomes or achievements, as long as these are caused by an individual's choices and not by differences in opportunities. There are, however, few precise definitions of opportunity egalitarianism. Moreover, there are different opinions regarding what opportunities one should equalise. From a libertarian vantage point one might argue that equal opportunities are achieved if all individuals have the same <u>formal</u> rights. Thus, if individuals are not discriminated regarding their opportunity to get education and jobs, to elect a government and so forth, opportunities are distributed in a fair manner. However, other authors want a far-reaching interpretation of opportunity egalitarianism, where differences in opportunities justify redistributive policies. Some may argue that for example differences in talent may cause differences in opportunities to attain power and wealth. Thus, one might argue that redistributive policies should seek to equalise a more extended concept of opportunities. Some of these fundamental topics will be discussed in this introductory section.

My main focus, however, is on problems which are common to all opportunity egalitarian views. Thus I adopt a fairly general understanding of the concept. There are, as I see it, two main parts of opportunity egalitarianism; the focus on opportunities rather than outcomes, and the preference for an equal distribution of opportunities. It should be noted that this constitutes a departure from a traditional welfarist approach in which only utility information is relevant for the ranking of social states.[1] In this chapter I make an attempt to formalise the discussion of opportunity egalitarianism by proposing several explicit normative rules. I will argue that these proposals convey some of the main ideas and features of opportunity egalitarianism. Based on these explicit proposals I discuss whether opportunity egalitarianism can constitute a sound foundation for social choice. First, I will try to clarify some of the fundamental ideas which have lead authors to propose opportunity egalitarian principles.

1.1 The Philosophical Basis for Opportunity Egalitarianism

In the introductory paragraph of this chapter, I suggested that libertarianism might be considered an opportunity egalitarian view. This is not a conventional interpretation. My main justification for this categorisation is that libertarian views are explicitly focusing on the process by which an outcome has come about, and not on the outcome as such. Robert Nozick

[1]For further elaborations on welfarism see Sen (1977) or Blackorby et.al. (1984).

(1974) is a prominent proponent of such a libertarian view. A central idea in his work is that judgements of the fairness of a social state should be based on how the actual distributions came about. Nozick proposes two basic principles of justice. (1) "Justice in acquisition". According to this principle everyone is entitled to his own natural talents and abilities, as well as an appropriate part of the natural world as long as nobody is left worse off. (2) "Justice in transfer". The basic content of this principle is that all exchanges from an initial just acquisition must come about voluntarily. A simplified (possibly oversimplified) version of Nozick's view can be described as "self-ownership with voluntary exchanges". It is clear that an important part of Nozick's theory is that justice is not determined by characteristics about end states, but about the process which lead to the specific outcomes. This is the part of his theory which may be called opportunity egalitarian.

John Rawls (1971, 1993) is among the most influential authors who have put forward opportunity egalitarian views. His second principle of justice (regarding distributive questions) consists of the difference principle <u>and</u> the principle; "equality of fair opportunity".[2] This principle means more than making the opportunities of different individuals <u>formally</u> equal. Rawls argues against a so called "meritocratic society". Such a society is likely to arise from a situation in which all individuals have the same formal opportunities, and assignments to all positions are based on merit. In this society differences in "talents" (in a general meaning of the word) are likely to lead to differences in achievements. Rawls is opposed to such an organisation of society. Differences in both genetic and cultural inheritance are, according to Rawls, morally arbitrary. As Rawls puts it; "no one deserves his greater natural capacity nor merits a more favourable starting place in society" (Rawls (1971) pg. 102). Unfair distributions of both natural talents and social contingencies are, according to Rawls, relevant reasons for compensation.

The opportunity egalitarian content of Rawls' theory becomes clearer when discussing his focal normative goal for questions of material distribution, namely maximising the "expectations" of the least advantaged group in society. The proposed mechanism for this is to equalise an <u>index of primary goods</u>. As I showed in chapter 2 of this thesis, this approach has substantial limitations. For the purpose of this chapter it is important to note that the primary goods index is not intended as a measure of achieved individual well-being. Rawls argues that an equal index of primary goods only can achieve equal "expectations" with regards to well-being. According to Rawls, different individuals may have very different goals for their lives. Primary goods are defined as necessary means to achieve any "rational" goal; they are prerequisites to achieve well-being. Rawls is consequently not primarily interested in

[2]Fulfilment of these two principles describes what Rawls calls «a state of democratic equality».

the actual outcomes, but is rather concerned with a fair distribution of the objects which may enable an individual to achieve his goals.

The works of Sen (1980, 1985, 1992) are central among those who regard freedom as a separate object of value. Sen argues that the objects that are relevant for the well-being of individuals are "functionings", which can be such things as "being in good health", "avoiding premature mortality", "having self-respect" and so on. Capabilities denote the set of possible vectors of functionings. Capabilities constitute an individual's opportunity set in the space of functionings, and do therefore represent the extent of freedom which an individual experiences. Sen argues that freedom has intrinsic value; a value that goes beyond the fact that increased freedom often enable people to choose a better outcome. He illustrates this by an example where in one situation an individual faces an opportunity set from which he prefers and chooses one specific alternative. In another situation the exact same alternative is assigned to this individual; the individual is given no freedom to choose among the alternatives. Sen claims that the former situation can be better than the latter, and consequently that freedom has intrinsic value.

Sen does not propose how these ideas can be made precise and formal. He does, however, suggest that freedom may have intrinsic importance in two different ways. (1) It is possible that a person's well-being is determined both by the actual outcome, and the extent of freedom an individual experiences before this outcome comes about. (2) The extent of freedom may not influence the well-being of an individual directly. Nevertheless, the overall good for society might be positively affected by individual freedom. As I will discuss later, I choose the second interpretation as the basis for the analysis of this chapter.

Ronald Dworkin (1981a, 1981b) has argued in favour of resource egalitarianism as opposed to welfare egalitarianism. One of his central arguments for resource egalitarianism is that people must take responsibility for their own tastes or preferences. Dworkin uses the examples of "expensive tastes" and "tastes for champagne" to illustrate that welfare egalitarianism might imply a subsidy of luxury goods. Such a conclusion runs contrary to common moral intuitions. This problem is avoided if one equalises resources instead of welfare.[3] Dworkin's argument for focusing on resources is not that welfare does not express ultimate ends for the respective individuals, but that the concept is irrelevant for moral questions. He clearly argues against the view that only end states are relevant for normative considerations. Resources determine a set of opportunities for each individual, and equality should be achieved in this

[3]Even though Dworkin calls his normative views "resource egalitarianism", it seems fairly clear that the general reasoning behind his view, and the mechanism he proposes, fits well into the general interpretation of opportunity egalitarianism presented in this paper: End states are not the objects of intrinsic importance in Dworkin's approach.

space. An important part of Dworkin's work is that it suggests that individual responsibility must have a separate and important part in moral questions. Dworkin bases his argument on the assumption that individuals are able to choose their preferences. These choices should, according to Dworkin, be within the sphere of individual responsibility.[4]

There are some attempts to formally incorporate freedom or opportunities as an important piece of information for individual well-being and social choice. Fleurbaey (1995) has analysed the necessary conditions for achieving equal opportunity sets among individuals.[5] In his analysis individual talents, external resources and the "intensity of will" determine an outcome which an individual exerts. He concludes that conditions which ensure equal opportunity sets are fairly restrictive. The argument in Gibbard (1979) was originally directed against the possibility of constructing a reasonable index of primary goods, as proposed by Rawls. Gibbard proposed a "minimal difference principle", which states that social states should be ranked according to its minimum income, provided that prices of all goods are equal in the social states to be ranked and between different individuals.[6] He shows that a social choice rule which satisfies this principle and the Pareto criterion must be intransitive. There are some recent attempts to solve the task of finding a well-being ranking based on information about an outcome and an opportunity set (the first way in which Sen believes freedom can be given intrinsic value). Bossert, Pattanaik and Xu (1994) characterise different rules for ranking the well-being associated with an opportunity set, where freedom is given an intrinsic importance. Based on different sets of axioms, they are able to characterise lexicographic ranking rules: either preference fulfilment (utility) is more important than "better" opportunity sets, or the converse is true. Thus, the ranking rules which Bossert et.al. discuss do not allow trade-offs between utility and freedom in the determination of well-being. The result presented in Gravel (1994) is less optimistic with regards to the possibility of finding a well-being ranking of pairs consisting of actual outcomes and opportunity sets. He proposes a set of axioms and shows that there does not exist ranking rules which generally conform to the proposed axioms.[7] The axioms appear to be reasonable, and this conclusion

[4]Dworkin proposes a mechanism to achieve equalisation of resources: Through redistributions of income one should try to imitate the wealth distributions which would have been obtained if everyone were uncertain with regards to which talent they would be endowed with at birth, and they were able to insure themselves against this risk. Roemer (1985, 1986) has forcefully criticised this specific mechanism for not satisfying an axiom which reasonably can be demanded in a resource egalitarian approach. It seems reasonable to say that the important contribution of Dworkin is his main ideas, not the specific mechanism which he proposes.

[5]The necessary condition for the possibility of equal opportunity is that there must exist some compensation in the external resources for unfortunate internal talents, and this necessary compensation must be independent of the individual's execution of will.

[6]An example in which different people experience different price vectors over relevant goods is the case of different people having different talent and consequently different wages. When people have different wage rates the price of leisure will differ between individuals.

[7]An important methodological difference between Gravel and Bossert et.al is that Gravel requires the well-being ranking to be an ordering, whereas Bossert et.al. assume that the ranking only needs to be a quasi-ordering. An

may therefore be a serious criticism of the approach suggested by Sen. Gravel's own suggestion is that freedom can not be given intrinsic importance, but should be regarded valuable only to the extent it increases the utility of individuals.

In contrast with the analyses of Bossert et. al. (1995) and Gravel (1994), my main interest is not in how freedom can influence individual well-being, but in how well-being and freedom both can be utilised in a social choice rule. Hence, I focus on whether it is possible to construct a reasonable "social ranking" of available social states based on information both about fulfilment of preferences and the opportunity sets.[8] As previously mentioned, Sen has discussed two different ways in which freedom can influence social choice.[9] First, being free to choose can be regarded as an important constituent of living, and may therefore be a functioning in itself. Well-being is determined by a vector of functionings, and freedom can be directly important in determining well-being because it is regarded as a functioning. This interpretation hints at an approach where individual well-being is determined by (a) the actual outcome and (b) freedom. This is the approach which Bossert et. al. (1995) and Gravel (1994) follow. Based on the vector of well-being associated with different social states one can construct a ranking of the relevant alternatives. The second way in which freedom can be important for social choice is, according to Sen, that it is regarded as a good for society even though it might not be directly important for any single individual. Freedom can therefore be good for society even though it does not affect the level of well-being experienced by the individuals in society. The "overall good" can in this perspective be thought of as a function of both the well-being and the freedom of every individual. According to this second view rankings of different social states must be based on information about well-being as well as freedom. The approach in this chapter follows this second line of reasoning. I discuss how information about both opportunity sets and preferences (which I assume represent well-being) can be utilised in a "social ranking" of the available alternatives.

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ordering is a binary relation which is reflexive, transitive and complete. A quasi-ordering differs from an ordering by being incomplete in the sense that two alternatives need not be ranked. It may appear to be a conflict between the conclusions in the two papers. This methodological difference is however an important reason for the differences in conclusion: Gravel realises that there exist quasi-orderings of well-being which satisfy his axioms.

[8]In this chapter I assume that an individual chooses the alternative which maximises well-being. Thus, there are no important distinctions in my use of the words preference fulfilment, utility and well-being.

[9]These interpretations of Sen's views are based on Sen (1992) p 40-42.

1.2 Formalising Opportunity Egalitarianism

A foundation for the analysis in this chapter is that a ranking of social states according to an opportunity egalitarian view make use of information about opportunity sets. This seems like a reasonable interpretation. The starting point for many proponents of opportunity egalitarianism is a rejection of utilitarianism or welfarism. Thus, opportunity egalitarian views must take into account information beyond fulfilment of preferences. When opportunity egalitarianism is regarded as a non-welfarist approach, then the distribution of opportunity sets seems obvious as a piece of relevant non-utility information. However, an important question is how one should define the opportunities to be equalised. One has to decide upon the space of valuable objects. Obviously one may want to achieve equality in many different dimensions: One may want an equal opportunity of gaining knowledge, of participating in public decisions, of achieving central positions in a society, of enjoying a reasonable level of material well-being and so on. Rawls, Sen and Dworkin propose different spaces which they argue are relevant for equalisation; primary goods, functionings and resources respectively. Other possibilities can also be imagined. The definition of valuable objects is important for the normative recommendations given by an opportunity egalitarian criterion: If one achieves completely identical opportunity sets in primary goods, one might still find that opportunities for functionings are different among individuals.

I do not attempt to resolve any of these questions. I have an agnostic view regarding what objects one should consider valuable. However, the basis for my analysis is that opportunity sets, measured in any space of valuable objects, must be regarded as important when ranking social states. The opportunity set of an individual is the set of all bundles of valuable objects it is possible for that person to acquire, given his endowments and abilities. Thus, I analyse some properties of opportunity egalitarian views which hold regardless of what one considers valuable goods.

The term "opportunity egalitarianism" suggests that one should redistribute resources until the opportunity sets of different individuals are equal. The suggestion presumes that this is actually possible, which may or may not be true. Let me illustrate this. Equalisation of wealth ensures equal opportunity sets only if we consider tradable goods as the space of valuable objects, and if all individuals are faced with the same prices for all these goods. In that case different individuals' opportunity sets (which are equal to their budget sets) are identical. The problem becomes more complicated if leisure is added as a relevant object of value. Due to different wage rates, people will differ in the time they have to work in order to earn a certain income. Thus, if we equalise income, people may enjoy different levels of leisure. Moreover, we can easily see that when peoples' productivity and wage rates differ, we can never make opportunity sets equal through lump sum transfers. This is because different people have different slopes on their opportunity constraint: The price of leisure is higher for high productivity individuals than for people with low productivity. Another relevant example can be a policy which, through lump sum transfers, attempts to equalise the opportunity of achieving knowledge and the opportunity of enjoying a reasonable level of material well-being. The problem is that any person has the possibility to use his material wealth to achieve knowledge (buy books, pay for good teachers and so on). Moreover, a given amount of money may to various degrees result in increased knowledge for different individuals, due to differences in talents or social background. In this case the slope of the opportunity constraint is different due to different abilities to acquire knowledge. Thus, it may not seem reasonable to assume that one can equalise opportunity sets through lump sum transfers. The general problem is that people have different natural abilities to transform one valuable object into another. This is equivalent to saying that two individuals may experience different relative prices on the valuable goods. In section 4 of this chapter, I analyse how one can operationalise opportunity egalitarianism when individual price vectors vary. In section 3 I discuss the same topic when individuals face equal price vectors.

This chapter studies the desirability of opportunity egalitarian criteria applied to some specific economic environments. When defining the set of available social states I explicitly try to capture some main features of the real world. I construct some very simple models which determine the set of available social states. The economic environments differ with regards to the endowments of goods and prices which are available in alternative social states. In parts of the analysis I demand that the economic environments are competitive equilibria. Furthermore, I study pure distribution problems, or situations in which normative rules give advice regarding redistribution of income between individuals. Thus, throughout the chapter I assume that it is possible with lump sum redistribution of income or wealth. The chapter attempts to analyse whether opportunity egalitarian criteria constitute a sound foundation for redistributive policies in alternative economic environments.

The remainder of the chapter is structured as follows. In section 2 I define the formal framework. Section 3 analyses opportunity egalitarian criteria based on a presumption that all individuals face equal price vectors in any single social state. Prices may however change as the social state change. In section 4 I am concerned with the problem which occurs when different individuals face different price vectors. In this section I assume that each individual's price vector is fixed. In this framework I discuss the possibility of constructing an opportunity egalitarian criterion for social choice. The main topics are discussed more generally in the concluding section 5.

2. Definitions and Formal Framework

Let $N := \{1,...,n\}$ be a set of individuals. An individual outcome, x_i, is a vector of goods; $x_i = (x_{i1},...,x_{im}) \in E^m_{0+}$, where E^m_{0+} is the non-negative orthant of the m-dimensional Euclidean space. An individual outcome is the bundle of goods which that person consumes in the relevant social state. I assume that $m \geq 2$. I adopt the following standard notation; $[x_{ir} \geq y_{ir}, \forall r \in \{1,...,m\}] \Leftrightarrow x_i \geq y_i$. If, additionally, $[\exists s: x_{is} > y_{is}]$ then $x_i > y_i$. A social outcome, x, is an n-dimensional vector of individual outcomes: $x = (x_1,...,x_n) \in E^{mn}_{0+}$. An individual price vector, p_i, is an m-dimensional vector of prices; $p_i = (p_{i1},...,p_{im}) \in E^m_+$, where E^m_+ is the strictly positive orthant of the m-dimensional Euclidean space. I allow the price vector to differ between individuals. A social price vector is an n-dimensional vector of individual price vectors; $p = (p_1,...,p_n) \in E^{mn}_+$. An individual endowment, $e_i = (e_{i1},...,e_{im}) \in E^m_{0+}$, is an m-dimensional vector of initial belongings of the goods of value, each of which can be traded for the prices given by the individual prices. A social endowment, $e = (e_1,...,e_n) \in E^{mn}_{0+}$, is an n-dimensional vector of individual endowments. The individual transfer for individual i, T_i, is the net amount which an individual receives from "society" in terms of a numeraire good. The social transfer, $T = (T_1,...,T_n) \in E^n$, is an n-dimensional vector of individual transfers. An individual state, $\bar{x}_i$, is a quadruple consisting of an individual outcome, an individual price vector, an individual endowment, and an individual transfer; $\bar{x}_i = (x_i, p_i, e_i, T_i) \in E^{3m+1}$. A social state is defined by a social outcome, a social price vector, a social endowment, and a social transfer; $\bar{x} = (x, p, e, T) \in E^{(3m+1)n}$.

The set of possible social states varies with the context and will be described in each section of the chapter. I will study three different "economic environments" which differ in the set of possible social states ($\bar{X}^k \subset E^{(3m+1)n}, k = \{1,2,3\}$). Each individual has preferences defined over his possible individual outcomes. Hence, an individual's preference relation, R_i, is an ordering on the m-dimensional Euclidean space; E^m_{0+}.[10] An ordering is a reflexive, transitive and complete binary relation. Let $\bar{R} = (R_1,...,R_n)$ be a profile of orderings, and let $\bar{\boldsymbol{R}}$ be the set of all possible profiles of preferences. The other important part of information for social rankings is the different individuals' opportunity sets. Let individual i's opportunity set be defined by; $OS_i(p_i, e_i, T_i) := \{a \in E^m_{0+} : p_i \cdot a \leq p_i \cdot e_i + T_i\}$. For notational convenience I will sometimes write $OS_i(p_i, e_i, T_i) = OS_i(\bar{x})$ where the triple (p_i, e_i, T_i) is the individual price vector, individual endowment, and individual transfer associated with social state $\bar{x}$. Define

[10] The binary relation R_i defines strong preference, P_i, and indifference, I_i, in the following standard manner: $x_i P_i y_i \Leftrightarrow (x_i R_i y_i \wedge not(y_i R_i x_i))$ and $x_i I_i y_i \Leftrightarrow (x_i R_i y_i \wedge y_i R_i x_i)$.

$OS(p,e,T) = (OS_1(p_1,e_1,T_1),...,OS_n(p_n,e_n,T_n))$ as a profile of opportunity sets. For each social state there is one profile of opportunity sets. Let $\boldsymbol{OS^k}$ be the set of all profiles of opportunity sets for the set of possible social states $\overline{X}^k$.[11]

Throughout this chapter I try to construct a "social ranking" of the set of social states; R of $\overline{X}^k$, based on information about individual preferences and profiles of opportunity sets. I will assume that the social ranking, R, is a quasi-ordering; a <u>transitive</u> and <u>reflexive</u> binary relation. A binary relation is transitive if for all $\bar{x},\bar{y},\bar{z} \in \overline{X}^k$, $[\bar{x}R\bar{y} \wedge \bar{y}R\bar{z} \Rightarrow \bar{x}R\bar{z}]$. A binary relation is reflexive if for all $\bar{x} \in \overline{X}^k$,$[\bar{x}R\bar{x}]$. Let $\boldsymbol{R^k}$ be the set of all possible quasi-orderings on $\overline{X}^k$. The general strategy of this chapter is to analyse whether there exists a "social evaluation functional", G^k, which for each admissible profile of preferences $\boldsymbol{R^*} \subseteq \overline{\boldsymbol{R}}$ and the profile of opportunity sets $\boldsymbol{OS^k}$, defines a quasi-ordering of the set of social states: $G^k : \boldsymbol{R^*} \times \boldsymbol{OS^k} \rightarrow \boldsymbol{R^k}$.

The analyses of this chapter will show the difficulty of finding a social choice rule based on an opportunity egalitarian criterion. For the set of social states and set of profiles of preferences which will be permitted, I will show that a social evaluation functional does not exist, given the demands which are put on a social ranking. In the following I will make several assumptions about the set of possible social states and the set of attainable profiles of preferences. The purpose of these assumptions is to avoid non-existence results which hinges on some unrealistic social states or profiles or preferences. A non-existence result is established if we can find <u>one</u> element in the domain for which it is impossible to define a quasi-ordering on the set of social states. Consequently, if a non-existence result can be established for a narrowly defined domain, it will also hold for all domains which includes the original one. The same logic applies for restrictions on the set of possible social states, $\overline{X}^k$. Narrowly defined domains and possible social states make a non-existence result stronger. Moreover, non-existence results are more easily established if we demand much "structure" on a social ranking. It is quite common within social choice theory to demand that the social ranking must be an <u>ordering</u>: a transitive, reflexive and complete binary relation. In contrast, I demand that the social ranking is a <u>quasi-ordering</u>: a transitive and reflexive, but not necessarily complete, binary relation. All orderings are consequently also quasi-orderings. If

[11]My definition of an opportunity set is similar to a traditional budget set as described in standard consumer theory. However, I allow for differences in different individuals' price vectors, and this makes it possible to discuss situations in which different individuals have different slopes on their "opportunity constraints». This feature of an individual's opportunity set is more likely to appear if the relevant space of goods is the space of functionings, primary goods, or both leisure and aggregate consumption. In such cases it is reasonable that different individuals face different price vectors on the space of valuable goods. This is due to the fact that different individuals have different abilities to convert for example leisure into consumption or money into health. My formulation is thereby sufficiently general to describe situations in which the slope of different individuals opportunity constraint varies.

we find that it is impossible to construct a quasi-ordering, we can conclude that it is also not possible to construct an ordering. By allowing incompleteness I do not require that a social ranking, R, is always able to rank two social states. This is an important methodological choice. The non-existence results in this chapter are derived without requiring very much structure on the binary relation which represents social preferences.

I will adopt standard assumptions regarding the set of admissible profiles of preferences. First, it is important to note that individuals have preferences which are defined on their individual outcome. This implies that they do not care about the outcome for other individuals in the society. Moreover, I will assume that everyone prefers more to less. Lastly, I assume that preferences are strictly quasi-concave (which implies strictly convex indifference curves). Strict quasi-concavity ensures that all individuals have a unique optimal choice of individual outcome, given a linear constraint in their opportunity sets. These assumptions about preferences are formally expressed as follows:

Condition SRP; Self Regarding Preferences:
The profile of preferences $\overline{R}$ satisfies condition "self regarding preferences" if, for all $i \in N$ and all $\overline{x}, \overline{y}, \overline{z}, \overline{w} \in \overline{X}^k$ such that $x_i = z_i$ and $y_i = w_i$, we have; $x_i R_i y_i \Leftrightarrow z_i R_i w_i$.

Condition M; Monotonicity:
The profile of preferences $\overline{R}$ satisfies condition "monotonicity" if for all $i \in N$ and all $x_i, y_i \in E_{0+}^m$, we have $\left[\forall k \in \{1,2,\ldots,m\} : x_{ik} > y_{ik}\right] \Rightarrow x_i P_i y_i$.

Condition SQC: Strict Quasi-Concavity:[12]
The profile of preferences $\overline{R}$ satisfies condition "strict quasi-concavity" if for all $i \in N$ and all $x_i, y_i \in E_{0+}^m$, $x_i \neq y_i$, such that $x_i R_i y_i$, we have $\left(\alpha x_i + (1-\alpha) y_i\right) P_i \left(x_i\right)$ for all $\alpha \in \langle 0,1 \rangle$.

Next, I will make some general assumptions on the set of possible social states. First, I will assume that all individuals actually choose their optimal individual outcome given their opportunity sets. It is hard to defend any kind of opportunity egalitarian criterion if individuals do not choose the best element in their opportunity set. Moreover, for social choice rules based on opportunity egalitarian criteria, non-existence results are easily established if suboptimal individual choices are possible. Second, I will assume that lump sum transfers are possible. Thus, I focus on pure distribution problems in which the government has the opportunity to levy non-distortionary taxes. This might not be a realistic assumption.

[12] Strict quasi-concavity is usually maintained as an assumption about utility functions, and not about preferences directly. If there exists a utility function which can represent an individual's preferences, however, the adopted assumption is equivalent to a standard assumption of strictly quasi-concave utility functions (see Kreps (1990)).

However, it is interesting to investigate the possibility of creating a social choice rule based on an opportunity egalitarian criterion, given "favourable" conditions for redistribution. The kind of decision problem which I have in mind is that the "government" initially chooses a profile of lump sum transfers based on criteria of efficiency or equity. Subsequently, individuals choose the optimal outcome given the opportunity set they face. To formalise my assumptions about available social states, it is convenient to define an optimal individual outcome and an optimal social outcome associated with a social state. The optimal individual outcome associated with social state $\bar{x} = (x', p', e', T')$ is defined as: $x_i^*(p_i', e_i', T_i') := \{x_i \in OS_i(p_i', e_i', T_i') : x_i R_i a, \forall a \in OS_i(p_i', e_i', T_i')\}$. For convenience I denote the optimal individual outcome by; $x_i^*(\bar{x})$. The associated optimal social outcome is defined as the profile of optimal individual outcomes: $x^*(\bar{x}) := (x_1^*(\bar{x}), \ldots, x_n^*(\bar{x}))$. The assumptions of attainable social states can be formalised as follows:

<u>Condition IR: Individual Rationality:</u>
For all social states $\bar{x} = (x', p', e', T') \in \bar{X}^k$ we have; $x' = x^*(\bar{x})$.

<u>Condition LST: Lump Sum Transfers:</u>
Suppose $\bar{x} = (x^*(\bar{x}), p, e, T) \in \bar{X}^k$, where $\sum_{i=1}^n T_i = 0$. Then, for all T' such that $\sum_{i=1}^n T_i' = 0$, $\bar{x}' = (x^*(\bar{x}'), p, e, T') \in \bar{X}^k$.

These assumptions will be maintained throughout the chapter. However, I will study three different economic environments, or sets of possible social states, which differ in other respects. In each of these economic environments I make different assumptions regarding the possible social price vectors and social endowments. This will be explained more carefully in the sections to come.

Throughout this chapter I will study the possibility of establishing a social evaluation functional, G^k, which for each element in the domain, $\boldsymbol{R}^* \times \boldsymbol{OS}^k$, defines some element in the range, $\boldsymbol{R}^k$. I have discussed possible restrictions on the domain and in the set of social states. Moreover, I have made precise what demands I put on a social ranking. I will now turn to a description of axioms which can be put on the social evaluation functional. The most widely accepted axiom for social choice is the Pareto criterion, defined as:

<u>Axiom P:</u> For all $\bar{x}, \bar{y} \in \bar{X}^k$, if $[\forall i \in N, x_i R_i y_i]$ then $\bar{x} R \bar{y}$.
If, additionally $[\exists j \in N : x_j P_j y_j]$ then $\bar{x} P \bar{y}$.

This Axiom is normally called the strong Pareto criterion. Note that the quasi-ordering defined by R above, also defines the Pareto indifference principle: If both $\bar{x}R\bar{y}$ and $\bar{y}R\bar{x}$ according to Axiom P, then $\bar{x}I\bar{y}$. The Pareto indifference principle demands that if all individuals are indifferent between their individual outcomes in two social states, then the social choice rule must rank the two social states as equally good. The Pareto indifference principle is used in the proof of proposition 2. For later reference I define the concept of Pareto efficiency:

Definition; Pareto efficiency:
A social state $\bar{x}$ is strongly Pareto superior to social state $\bar{y}$ if and only if $\left[\left[\forall i \in N, x_i R_i y_i\right] \wedge \left[\exists j \in N : x_j P_j y_j\right]\right]$. Social state $\bar{x}$ is Pareto efficient if and only if there does not exist any other social state $\bar{z} \in \bar{X}^k$ which is strongly Pareto superior to $\bar{x}$.

In the analyses to come it is convenient to use some results about the "strict preference", P, and "indifference", I, defined by the "weak social preference", R. P and I are defined as follows; $\bar{x}P\bar{y} \Leftrightarrow \left(\bar{x}R\bar{y} \wedge not\left(\bar{y}R\bar{x}\right)\right)$ and $\bar{x}I\bar{y} \Leftrightarrow \left(\bar{x}R\bar{y} \wedge \bar{y}R\bar{x}\right)$. The strict preference relation, P, and indifference relation, I, defined by a quasi-ordering exhibit the following properties:

Lemma 1
$\forall \bar{x},\bar{y},\bar{z} \in \bar{X}^k$, (1) $\bar{x}P\bar{y} \wedge \bar{y}I\bar{z} \Rightarrow \bar{x}P\bar{z}$, (2) $\bar{x}I\bar{y} \wedge \bar{y}P\bar{z} \Rightarrow \bar{x}P\bar{z}$, (3) $\bar{x}I\bar{y} \wedge \bar{y}I\bar{z} \Rightarrow \bar{x}I\bar{z}$, (4) $\bar{x}P\bar{y} \wedge \bar{y}P\bar{z} \Rightarrow \bar{x}P\bar{z}$. Moreover; (5) $\bar{x}P\bar{y} \Rightarrow not\left(\bar{y}P\bar{x}\right)$.

Proof: The four first properties of transitivity are contained in Lemma 1*a in Sen (1970). Property (5) (asymmetry) is immediate: $\bar{x}P\bar{y} \Leftrightarrow \left[\bar{x}R\bar{y} \wedge not\left(\bar{y}R\bar{x}\right)\right] \Rightarrow not\left(\bar{y}P\bar{x}\right)$.
Q.E.D.

Thus, the binary relation P is a "strict partial ordering"; a transitive and asymmetric, but not necessarily complete, binary relation.

In the following sections I will analyse the possibility of establishing a social choice rule based on the Pareto criterion and different opportunity egalitarian criteria. This will be done in three different economic environments. In section 3 I assume that all individuals face identical price vectors in each social state. The price vector and social endowment may however differ between different social states. Later in this section I study whether an opportunity egalitarian criterion can be useful if the set of possible social states is restricted to competitive equilibria in which endowments are fixed. In section 4 I will analyse situations in which the social endowment and social price vector is fixed. The individual price vectors may however differ between individuals.

3. Opportunity Egalitarianism when Individuals Face Equal Prices

3.1 Social Choice Rules Based on "the Minimal Opportunity Egalitarian Criterion"

I will formalise an opportunity egalitarian principle which is concerned with the opportunity set of the worst off individual. It seems reasonable to operationalise an opportunity egalitarian view by a criterion which claims that a redistribution between two individuals is justified if this implies an improvement for the individual who has the "worst" opportunity set among the two. However, this kind of rule necessitates criteria which enable us to rank the opportunity sets of different individuals. Consider a situation in which one opportunity set is a strict subset of another opportunity set. In such a situation the individual who faces the larger opportunity set can choose to consume all the bundles of valuable goods which are available for the other individual. Additionally, he can choose some bundles which the other individual does not have the opportunity to choose. It seems reasonable to say that the larger opportunity set should be ranked as strictly better than the smaller set. This ranking of opportunity sets can be used as a minimal requirement for a social choice rule. Axiom MOEC states that if there are two social states, $\bar{x}$ and $\bar{y}$, which are identical for all individuals except for individuals *i* and *j* in such a way that individual *i*'s opportunity set in social state $\bar{y}$ is a strict subset of individual *j*'s opportunity set in social state $\bar{y}$, and individual *i*'s opportunity set in social state $\bar{y}$ is a strict subset of individual *j*'s opportunity set in social state $\bar{x}$, and individual *i*'s opportunity set in social state $\bar{y}$ is a strict subset of individual *i*'s opportunity set in social state $\bar{x}$, then social state $\bar{x}$ must be ranked as strictly better than $\bar{y}$:

<u>Axiom MOEC (the Minimal Opportunity Egalitarian Criterion):</u>

Consider two social states $\bar{x},\bar{y} \in \bar{X}^k$ for which the individual states differ only for individuals $i, j \in N$. If $OS_i(\bar{y}) \subset OS_j(\bar{y})$ and $OS_i(\bar{y}) \subset OS_j(\bar{x})$ and $OS_i(\bar{y}) \subset OS_i(\bar{x})$ then $\bar{x}P\bar{y}$.

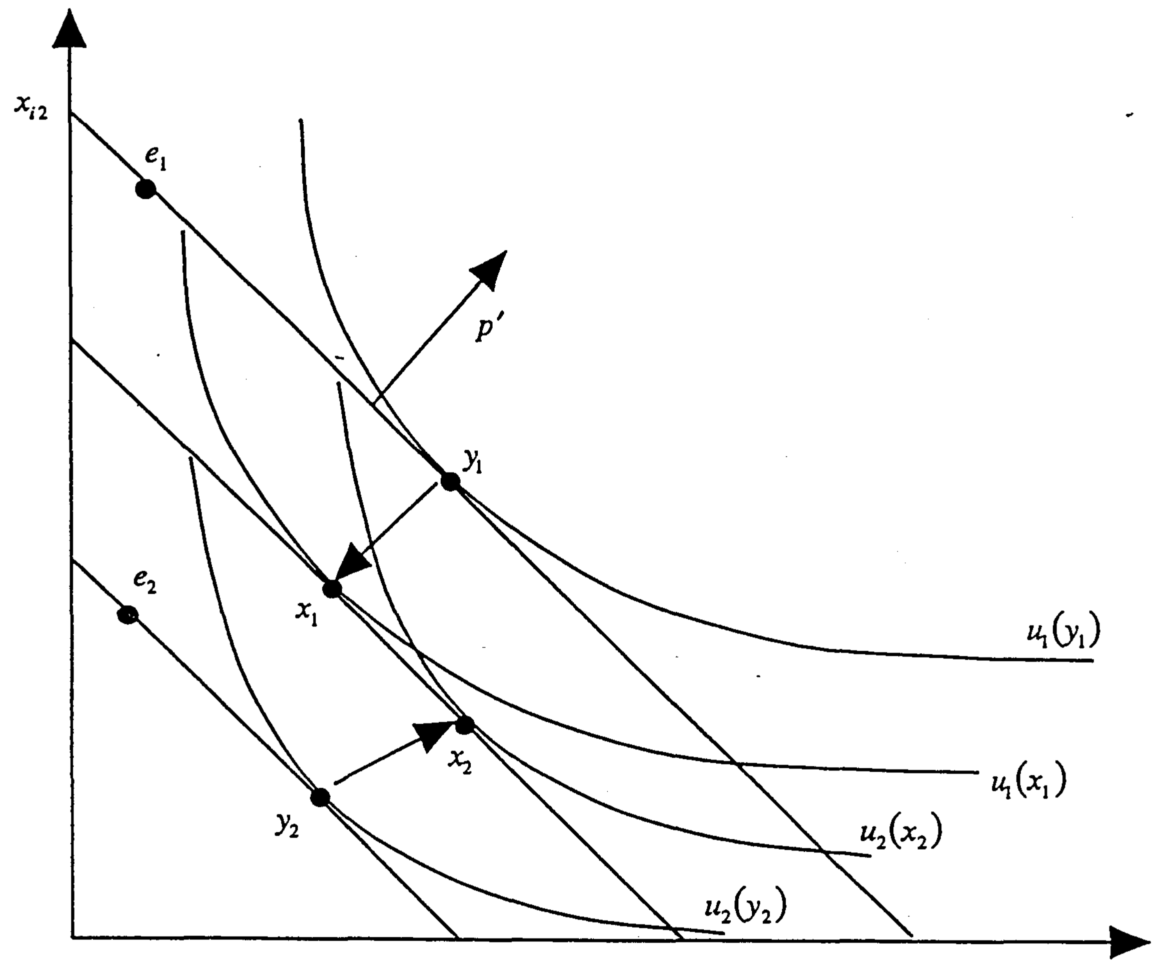

Figure 1

Axiom MOEC is illustrated in figure 1. There are two individuals, two valuable goods, one social endowment, and one social price vector for which individual price vectors are equal. The two social states $\bar{x}$, $\bar{y}$ differ in the vector of lump sum transfers. In social state $\bar{y}$ there are no lump sum transfers, and individual 2's opportunity set is completely contained in individual 1's opportunity set. In social state $\bar{x}$ there has been redistribution of income from individual 1 to individual 2, such that the two individuals' opportunity sets have become identical. Moreover, both individuals have a strictly better opportunity set in social state $\bar{x}$ than individual 2 has in social state $\bar{y}$. Hence, Axiom MOEC ranks social state $\bar{x}$ as better than $\bar{y}$.

I will let $\bar{X}^1$ denote the set of possible social states in the analysis to come. It is necessary to define this set. I assume that all individuals face the same price vector for any single social state. Hence; $\forall i,j \in N, p_i(\bar{x}) = p_j(\bar{x}), \forall \bar{x} \in \bar{X}^1$. However, I put no restrictions on how the common individual price vector may vary with changes in social states. Hence, I define the set of possible social price vectors as all strictly positive prices on each of the goods, where the price of a good is equal for all individuals in any social state. This set is equal to the strictly positive m-dimensional Euclidean space; E_{+}^{m}. Moreover, I put no restrictions on the set of possible social endowments. This set is equal to; $E_{0+}^{m\times n}$. Thus, the set of admissible pairs of social prices and endowments is defined by: $E_{+}^{m} \times E_{0+}^{m\times n}$. Finally, I maintain the assumptions that lump sum transfers are possible (Condition LST) and that individuals' choose the best element in their opportunity set (Condition IR).

Definition:

The set of social states $\underset{\sim}{X}^{1}$ is defined as all quadruples (x,p,e,T) such that, (1) for each social state all individuals face identical individual price vectors, (2) social endowments are unrestricted; $e \in E^{mn}$, (3) condition LST is satisfied, and (4) condition IR is satisfied.

The following result shows that a social evaluation functional, $G^{1}: \boldsymbol{R}^{\bullet} \times \boldsymbol{OS}^{1} \to \boldsymbol{R}^{1}$, does not exist for this set of possible social states:[13]

Proposition 1

Suppose the set of profiles of preferences $\boldsymbol{R}^{\bullet}$ *satisfies conditions "Self-regarding Preferences", "Monotonicity", and "Strict Quasi Concavity". There does not exist a social evaluation functional,* G^{1}*, which satisfies Axiom P and Axiom MOEC.*

Proof:

It suffices with an example of an admissible profile of preferences for which there are no quasi-orderings on the set of social states which satisfies Axiom P and Axiom MOEC. The proof is conducted by use of an example in which there are two valuable goods and two individuals. The argument is easily generalised to situations with more valuable goods and individuals. The four social states, $\bar{x},\bar{y},\bar{z},\bar{w}$, are illustrated in figure 2. All individual outcomes are 2-dimensional bundles, and are denoted x_{1} and so forth. The price vector for social states $\bar{x}$ and $\bar{y}$ is p^{1} (for both individuals), for social states $\bar{z}$ and $\bar{w}$ the price vector is p^{2}. In social states $\bar{x}$ and $\bar{y}$ the social endowment is, (e_{1},e_{2}), in social states $\bar{z}$ and $\bar{w}$ the social endowment is (e_{1}',e_{2}'). In both social states $\bar{y}$ and $\bar{w}$ the social transfer is $(0,0)$, in social state $\bar{x}$ the social transfer is $(T_{1}>0,T_{2}<0)$, in social state $\bar{z}$ the social transfer is $(T_{2}>0,T_{1}<0)$. Social states $\bar{x}$ and $\bar{z}$ are reached by redistributive policies from $\bar{y}$ and $\bar{w}$, respectively. The social state $\bar{y}$ is defined by $\bar{y}=\left((y_{1},y_{2}),(p^{1},p^{1}),(e_{1},e_{2}),(0,0)\right)$, and

[13]This result has similarities with a result in Gibbard (1979). Gibbard proposed the so called "minimal difference principle". The principle states that, if all prices in two social states are equal, and the minimum income in one social state is higher than the minimum income in another, then the social choice rule must regard the former as better than the latter. Gibbard shows that a social choice rule which satisfies this criterion and the Pareto principle is intransitive. My analysis differs from Gibbard's in that the minimal opportunity egalitarian criterion is more general than the minimal difference principle. The important insights behind the two alternative formulations are, however, quite similar.

similarly for the other social states. It is easily verified that these social states are elements of the set of available social states; $\bar{x},\bar{y},\bar{z},\bar{w} \in \bar{X}^1$. The individuals' preferences are described by the indifference curves in figure 2, and are admissible.

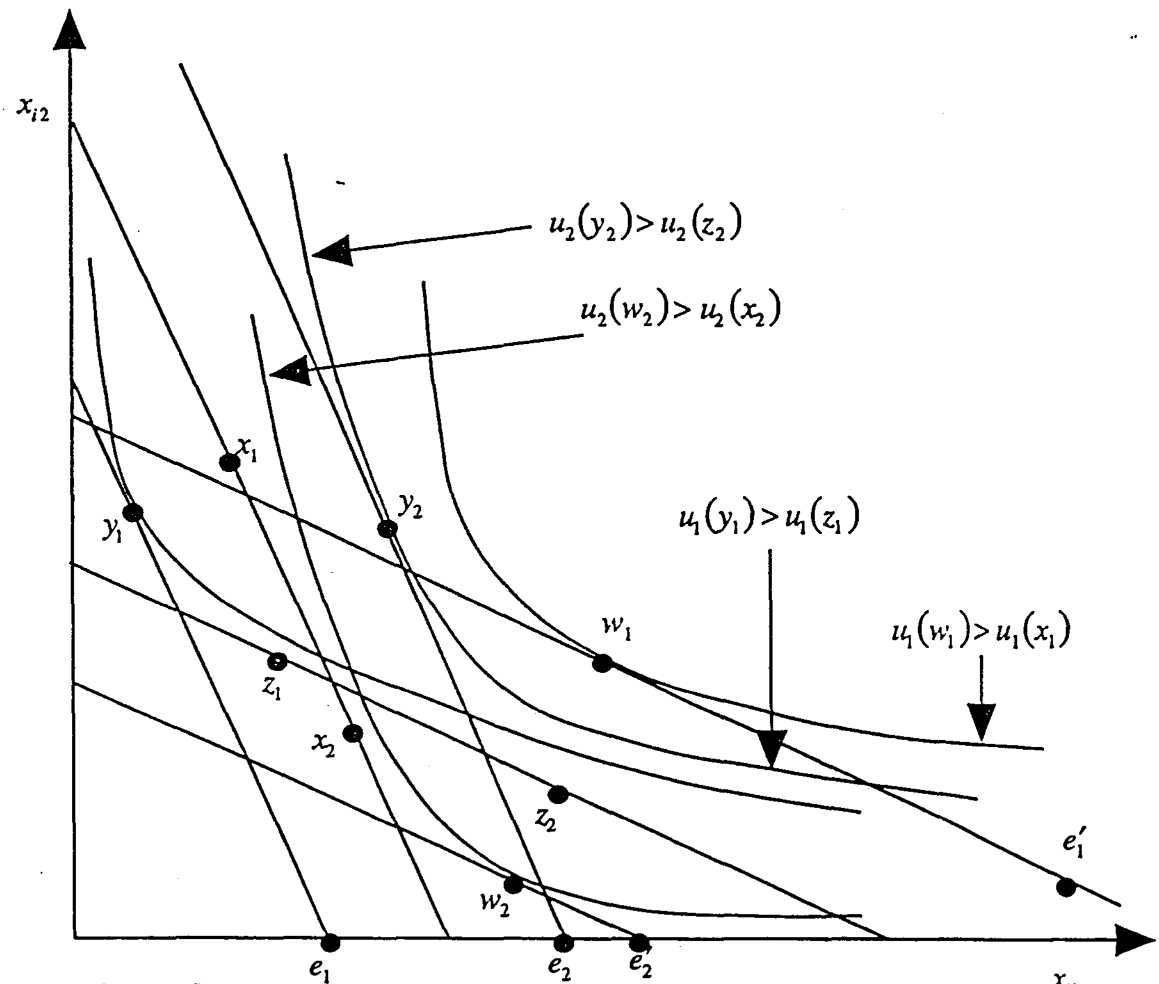

Figure 2

Observe that $OS_2(\bar{w}) \subset OS_2(\bar{z}) \wedge OS_2(\bar{w}) \subset OS_1(\bar{z})$. By Axiom MOEC this implies; $\bar{z}P\bar{w}$. Furthermore, observe that $OS_1(\bar{y}) \subset OS_1(\bar{x}) \wedge OS_1(\bar{y}) \subset OS_2(\bar{x})$, which by Axiom MOEC implies; $\bar{x}P\bar{y}$. For the preferences drawn in figure 2, observe that Axiom P implies: $\bar{y}P\bar{z}$ and $\bar{w}P\bar{x}$. By transitivity of P we have; $\bar{z}P\bar{w} \wedge \bar{w}P\bar{x} \Rightarrow \bar{z}P\bar{x}$ and $\bar{x}P\bar{y} \wedge \bar{y}P\bar{z} \Rightarrow \bar{x}P\bar{z}$. These rankings of $\bar{x}$ and $\bar{z}$ contradict the asymmetric property of P (Lemma 1).
Q.E.D.

Proposition 1 can be understood better by realising that in the above example only social states $\bar{y}$ and $\bar{w}$ are Pareto efficient. But these two social states are dominated according to Axiom MOEC by social states $\bar{x}$ and $\bar{z}$, respectively. Note that the minimal opportunity egalitarian criterion ranks two alternatives whether they are Pareto efficient or not. There is nothing that precludes Axiom MOEC from judging a Pareto inefficient social state as better than a Pareto efficient social state. At the same time, the Pareto criterion does not explicitly take into account the opportunity sets in the different social states. These facts imply that it is possible to create "circular rankings" such that $\bar{y}$ is better than $\bar{w}$ is better than $\bar{x}$ is better than $\bar{z}$ is better than $\bar{y}$. Such a ranking does not satisfy the definition of a quasi-ordering.

Proposition 1 shows that one can not construct a reasonable social choice rule based on the Pareto criterion and the minimal opportunity egalitarian criterion in the way described above. This result is noteworthy for several reasons. First, both the Pareto criterion and the minimal opportunity egalitarian criterion are stated as <u>minimal requirements</u> for a social choice rule. The suggested social ranking does not demand completeness; two alternatives which are not ranked by either of the two principles need not be ranked by the social choice rule. The formulation of the social choice rule demands only that <u>if</u> either the Pareto or the minimal opportunity egalitarian criterion (or both) rank two alternatives, then the social choice rule must rank the alternatives accordingly. Second, the minimal opportunity egalitarian criterion seems like an uncontroversial operationalisation of opportunity egalitarianism; it implies that redistribution is a social improvement if the opportunity set of one individual is contained in the opportunity set of another individual (and redistribution goes to the worst off individual). If opportunity egalitarianism shall have any operational content, then it seems very reasonable that the minimal opportunity egalitarian criterion should be included. Third, the Axiom MOEC does not contradict the Pareto criterion directly; any two alternatives which can be ranked by this axiom can not be ranked by the Pareto principle. However, as proposition 1 shows, it is impossible to combine these two criteria in a transitive and asymmetric social choice rule.

3.2 The Minimal Opportunity Egalitarian Criterion when Endowments are Fixed and Prices are Determined in Competitive Equilibria

The above insight should motivate further investigations into alternative formulations of opportunity egalitarianism. It is pertinent to point out a weakness about the analysis which led to Proposition 1: The set of possible social states were chosen without a very solid justification. Specifically, the proof of Proposition 1 hinges on the availability of completely different social endowments. Moreover, prices can be chosen in any possible way. It is not easy to see that these social states can come about in any realistic economic environment. By choosing the set of possible social states in that way, one implicitly assumes that the resources and production technology in an economy can be chosen freely. This is obviously not realistic.

At first sight this criticism might seem serious. In my view, however, this may not be the case. It seems perfectly legitimate to claim that a social choice rule should apply also for hypothetical situations. A normative rule should convey basic value judgements and should consequently not depend on the specific circumstance in which it is applied. If a normative proposal does not work in a set of (hypothetical) situations, then it is reasonable to claim that the explicit proposal can not reflect our basic value judgements. Another problem with restrictions on the set of social states for which a normative rule applies is the following: Whether a social state is considered better than another, in such cases, may depend on the availability of other social states. One might find that social state $\bar{x}$ is preferable to social state $\bar{y}$ if these are the only two alternatives, whereas the ranking might be reversed if other options are available. It seems like our moral intuition demand that the desirability of two social states must be judged solely on the basis of characteristics about these two alternatives.

On the other hand one might argue that the main purpose of normative rules is to provide guidance when we are faced with moral problems of the real world. It may be superfluous to demand that a social choice rule should be "well-behaved" in all hypothetical situations, regardless of whether these alternatives are realistic. If criteria of justice are developed for real world problems only, then maybe it should not be disturbing that one can develop paradoxical results when the rule is applied to unrealistic situations.

I make no attempt to resolve this discussion. However, it is in line with the general focus of this chapter to analyse the possibility of reasonable normative rules which are applicable to specific economic environments. Hence, the lack of theoretical justification for the set of available social states which are assumed to exist above, is regarded as a weakness for the purpose of this analysis. Thus, the set of social states will be described and justified quite differently in the following paragraphs. I will assume that the economic environment is one of a <u>pure exchange economy</u> in which prices are consistent with a <u>competitive equilibrium</u>. I will maintain the assumptions Condition LST and Condition IR.

The set of social endowments can be any element in $E_{0+}^{m\times n}$. I consider a pure exchange economy in which individual endowments can be exchanged one-for-one between individuals. The set of possible social price vectors will be defined as those which can occur in competitive equilibria. This implies that in any single social states each individual faces the same price vector. A competitive equilibrium is defined in the standard way as: (1) a social outcome $x \in E^{m\times n}$ and a price vector $p = (p_1,...,p_m) \in E_{+}^{m}$, for which (2) all individuals choose their most preferred individual outcome given their opportunity set; $x = x^*(\bar{x})$, and (3) supply equals demand for all goods; $\sum_{\forall i \in N}(x_{ir} - e_{ir}) = 0, \forall r \in \{1,...,m\}$. Again the assumptions of individual rationality (Condition IR) and of available lump sum transfers (Condition LST) are maintained. This leads to the following definition of $\overline{X}^2$:

Definition:

The set of social states $\overline{X}^2$ is defined as all quadruples (x,p,e,T) such that, (1) for each social state all individuals face an identical individual price vector which is consistent with a competitive equilibrium, (2) social endowments are unrestricted; $e \in E^{mn}$, (3) condition LST is satisfied, and (4) condition IR is satisfied.

Proposition 2 tells that there does not exist a social evaluation functional, $G^2 : \boldsymbol{R}^{*} \times \boldsymbol{OS}^{2} \rightarrow \boldsymbol{R}^{2}$, which satisfies Axiom P and Axiom MOEC.

Proposition 2

Suppose the set of profiles of preferences $\boldsymbol{R}^{*}$ satisfies conditions "Self-regarding Preferences", "Monotonicity", and "Strict Quasi Concavity". There does not exist a social evaluation functioning, G^2, which satisfies Axiom P and Axiom MOEC.

Proof:

An example will suffice. All arguments are illustrated in figure 3. There are two individuals with preferences which are represented by the indifference curves in figure 3. The indifference curves are "dented" in a way that ensures that the expansion path for individual i is given by the function: $x_{i2}^{*}(p_i,e_i,T_i) = \alpha x_{i1}^{*}(p_i,e_i,T_i) + a_i, a_1 > 0, a_2 < 0$, for all $(p_{i1},p_{i2}) : (p_{i1}/p_{i2}) \in [(p_1'/p_2'),(p_1/p_2)]$. This is an admissible profile of preferences. This expansion path is affine with equal slope but different intercepts for the two individuals, for the price vectors in the example. I construct the four social states; $\overline{x},\overline{y},\overline{z},\overline{w}$. In all social states the social endowment is kept fixed, $e' = (e_1,e_2)$. In social state $\overline{x}$ prices are p' and transfers are zero. The social outcomes are individually rational; $x = (x_1,x_2) = x^{*}(\overline{x})$. Thus; $\overline{x} = ((x_1,x_2),(p',p'),(e_1,e_2),(0,0))$. Social state $\overline{w}$ has come about from $\overline{x}$ through redistribution from individual 2 to individual 1. The social outcome is changed to $w = (w_1,w_2)$, but prices are constant at p'. Hence; $\overline{w} = ((w_1,w_2),(p',p'),(e_1,e_2),(T_1(\overline{w}) > 0, T_2(\overline{w}) < 0))$. The social outcome y in social state $\overline{y}$ is identical to the social outcome w in social state $\overline{w}$; $y = (y_1 = w_1, y_2 = w_2)$. The price vector in $\overline{y}$ is however changed to p. Moreover, in social state $\overline{y}$ there are substantial transfers from individual 2 to individual 1: $\overline{y} = ((y_1 = w_1, y_2 = w_2),(p,p),(e_1,e_2),(T_1(\overline{y}) > 0, T_2(\overline{y}) < 0))$. Social state $\overline{z}$ has come about through a reduction in the transfers from individual 2 to individual 1, compared to social state

$\bar{y}$. Prices are kept at p, but the social outcome z becomes identical to the social outcome x. Hence; $\bar{z} = \left(\left(z_1 = x_1, z_2 = z_2\right), \left(p, p\right), \left(e_1, e_2\right), \left(T_1(\bar{y}) > T_1(\bar{z}) > 0, T_2(\bar{y}) < T_2(\bar{z}) < 0\right)\right)$. These four social states are competitive equilibria, as we can observe from the facts that; (1) both individuals choose bundles which maximise utility given their budget constraints and (2) total demand is equal to the fixed total supply in each equilibrium.

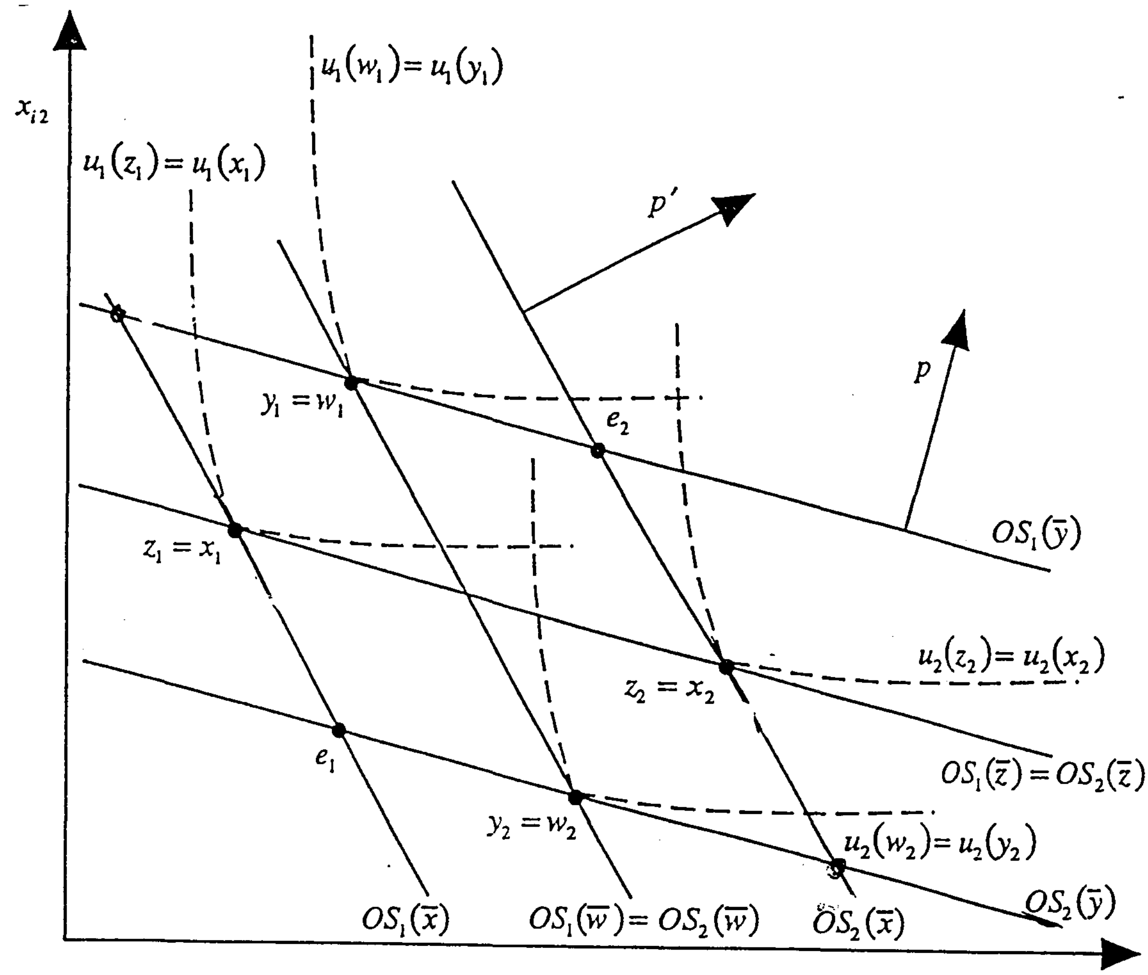

Figure 3

By Axiom MOEC we have: $\bar{w}P\bar{x} \wedge \bar{z}P\bar{y}$. By Axiom P (Pareto indifference) we have $\bar{y}I\bar{w} \wedge \bar{x}I\bar{z}$. By transitivity of the binary relations, P, I, (Lemma 1) we have; $\bar{y}I\bar{w} \wedge \bar{w}P\bar{x} \Rightarrow \bar{y}P\bar{x}$ and $\bar{x}I\bar{z} \wedge \bar{z}P\bar{y} \Rightarrow \bar{x}P\bar{y}$. This contradicts the asymmetric property of P (property 5, Lemma 1).
Q.E.D.

The minimal opportunity egalitarian criterion is basically a criterion which ranks social states according to the opportunity set of the worst off individual. In a pure exchange economy this principle is similar to a requirement of equalised income or expenditure. The problem with this approach is that expenditure is a value assigned to a bundle of goods, where this value is determined by a price vector. It is entirely possible that one bundle of goods is given a higher value than some alternative bundle of goods for one price vector, whereas for another price vector this ranking is reversed (this is the case in the proof of Proposition 2). Prices are consequently very important in the ranking of different social states according to the minimal opportunity egalitarian criterion. Proposition 2 is most easily understood by realising that the minimal opportunity egalitarian criterion can rank one specific social outcome (or physical allocation) as both better and worse than another allocation, depending on the prevailing prices. In figure 3 the social outcome x is considered better than social outcome y when prices are p, according to Axiom MOEC. When prices are p', however, the ranking is reversed. Preferences on the other hand depend only on the physical allocation, and are not affected by price changes. By the Pareto principle the social choice rule is indifferent between different equilibria in which the physical allocations are unchanged. Prices are therefore only of indirect importance for rankings according to the Pareto principle. This is the underlying tension between the minimal opportunity egalitarian criterion and the Pareto principle.

4. Opportunity Egalitarian Criteria when Endowments and Prices are Fixed

Throughout section 3 I assumed that the individual price vector in any single social state was identical for all individuals. The price vector could however vary for different social states. In section 3.1 I gave no specific explanation for how different social price vectors or social endowments could come about. In section 3.2 I demanded that the endowments were fixed, and that prices were consistent with a competitive equilibrium. A crucial part in the proof of Proposition 2 was the property that one specific social outcome (or physical allocation) could be supported as a competitive equilibrium for different price vectors (in fact infinitely many). This property relies on the specific L- shaped indifference curves in the example.[14] Thus, in the analysis of section 3.2, relative prices did not reflect the marginal rates of substitution for the consumers in the economy. The relative prices could not be interpreted as the social "cost" of using one more unit of a good, in terms of another (numeraire) good. To the contrary, prices were chosen more or less at random.

In this section I describe production possibilities more explicitly. A price vector reflects an individual's "cost of producing" one unit of a good in terms of another good. Throughout the analysis I will define leisure and aggregate consumption as the only valuable goods. An individual's price vector shows how many units of consumption he must sacrifice in order to

[14]This property allowed me to change the ranking of two alternative social outcomes as the prevailing prices changed. However, my choice of preferences were not general. One needs «kinked» indifference curves in order to get undetermined prices in a competitive equilibrium. If preferences are «smooth» (continuously differentiable indifference curves) and quasi-concave (indifference curves are convex), then a competitive equilibrium assigns essentially one price vector to each physical allocation (social outcome). Thus, for a more «normal» profile of preferences the argument underlying Proposition 2 does not hold.

increase leisure with one unit. The price of consumption may therefore be interpreted as an individual's productivity or wage earning capacity. This productivity may differ between individuals. I will, however, assume that this price is constant for each individual independent of the social state in question. Moreover, I will assume that the social endowment is fixed. This endowment will essentially be the amount of leisure which is available for an individual when he chooses not to consume. Thus, in this section I assume that the social endowment and the social price vector is constant across all possible social states. The set of possible social states is defined by all the social outcomes which can come about through lump sum redistribution, given the fixed social endowment and the fixed social price vector.

Definition:
The set of social states $\overline{X}^3$ is a quadruple (x, p, e, T), for which the pair consisting of a social price vector and a social endowment is any one element in $E_{+}^{m\times n} \times E_{0+}^{m\times n}$, and for which the social outcome, x, and the vector of transfers, T, satisfies Condition LST and Condition IR.

Throughout this section I analyse situations with two goods; leisure and consumption. Thus, it may seem like the opportunity egalitarian criteria I propose are analysed for a very limited set of situations. As argued in the introduction of this chapter, however, the same basic problems are likely to occur in for a wide range of situations. The basic feature of the problems I study is that different individuals have different abilities to convert income into other valuable goods. Different individuals may differ in their ability to use income to gain knowledge, political power, good health and so forth. These situations share the feature that the "implicit" prices of a good differ between individuals. Thus, an analysis of situations in which there is a fixed social endowment and a fixed social price vector (which may differ between individuals) is relevant for more situations than the specific models may indicate.

4.1 Equalising Opportunity Sets Violates the Pareto Principle

As I have already argued there are many relevant cases for which it is impossible to equalise opportunity sets through lump sum transfers of income or wealth. A straightforward interpretation of opportunity egalitarianism would demand different individual's opportunity sets to be identical also in these cases. If this suggestion is to be taken seriously, one has to analyse the desirability of redistributive policies which are distortionary. In the following paragraphs I will argue that such policies will violate the Pareto principle. The argument is conducted assuming that there are two individuals, $N = \{1,2\}$, that leisure *(l)* and aggregate consumption *(c)* are the only valuable goods, that each individual have the same endowment of time, $\overline{L}$, and that productivity (the price of leisure) differs between individuals; $w_1 > w_2$. The set of possible social states are all those which satisfy the aggregate production constraint: $\sum_{i \in N} c_i \leq \sum_{i \in N} w_i(\bar{L} - l_i)$. I will assume that individuals may have different preferences over bundles of leisure and consumption; $u_i = u_i(c_i, l_i)$.

First of all, it is important to note that the set of all Pareto efficient social outcomes can be achieved. The question is whether it is possible to achieve a Pareto efficient outcome if all individuals face identical opportunity sets. Figure 4 illustrates why all policies which make opportunity sets equal, will violate the Pareto principle, for the configuration of preferences in this example.

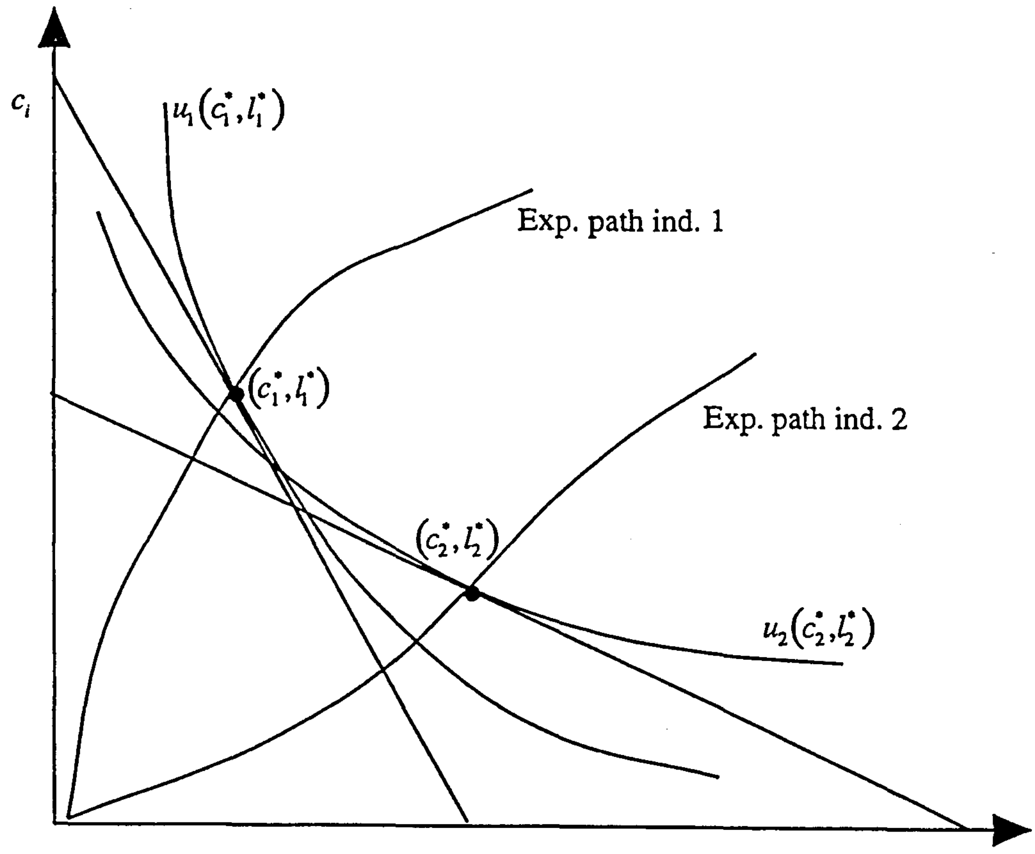

Figure 4

We observe that the productivity of individual 1 is higher than the productivity of individual 2. In figure 4 the social outcome $((c_1^*, l_1^*), (c_2^*, l_2^*))$ is achieved through a lump sum redistribution of income from individual 1 to individual 2. It is immediate that this allocation is Pareto efficient. Moreover, the set of Pareto efficient social outcomes equals the set of optimally chosen bundles for all possible profiles of lump sum transfers. All Pareto efficient outcomes must consequently be located on each of the two individuals' expansion paths, for the prices defined by the individual productivity. Equality of opportunity implies that the two individuals must be faced with the same menu of bundles to choose from. Our task is consequently to find two identical opportunity sets, where each individual finds it optimal to choose a feasible bundle on his expansion path. We know that these equal opportunity sets must at least contain the two bundles which constitute some feasible Pareto efficient social state.

Let us now consider a policy where the equal opportunity sets consist only of the two optimal bundles (c_1^*, l_1^*) and (c_2^*, l_2^*) showed in figure 4. If, when faced with these opportunities, individual 1 would prefer (c_1^*, l_1^*) and individual 2 would prefer (c_2^*, l_2^*), the resulting social state would be Pareto efficient among the feasible alternatives. The problem, however, is that each of the two individuals in figure 1 actually prefers the other bundle in the opportunity set. This alternative allocation of bundles is not feasible given the aggregate budget constraint. We can consequently not offer both individuals a free choice between these two bundles, and end up with a feasible and Pareto efficient social state.

The next question is whether there exists any opportunity set which could be offered to both individuals, where the choices by the individuals would result in a feasible and Pareto efficient social state? For the preferences drawn in figure 4, the answer is no. The reason is that any other Pareto efficient social state would involve one individual consuming a bundle further to the Northeast on his expansion path, and the other individual consuming a bundle on his expansion path to the Southwest of the original bundle. By inspecting the preferences in figure 1, one realises that the individual who experiences a decrease in "purchasing power" would always prefer the other individual's bundle. If this individual were free to choose between the two bundles he would choose the other one. But this would still not be possible within the aggregate budget constraint. The conclusion is that it is not possible, given the preferences drawn in figure 4, to let the individuals have equal opportunity sets and to achieve a Pareto optimal social state.[15]

4.2 Non-Existence of Social Choice Rules Based on the "Extended Opportunity Egalitarian Criterion" (EOEC)

The discussion above illustrates that a criterion of equal opportunity sets is not desirable when individuals face different price vectors. Thus, the opportunity egalitarian criterion must be adapted to the context of this section. I will argue that Axiom MOEC must be extended in

[15]The problem of finding a Pareto efficient social state resulting from a choice situation where the individuals have equal opportunity sets, is similar to the problem of finding envy free and Pareto efficient ("fair") social states (see Varian (1974) and Thompson and Varian (1985)). A social state is defined as envy free if no individual prefers a bundle consumed by any other individual. An envy free and Pareto efficient social state is defined as fair. Varian (1974) shows that there does not generally exist fair competitive equilibria. The problem in my example is that in every Pareto efficient social state there is at least one individual who prefers (or envies) the bundle of the other individual. One can consequently not offer these opportunity sets to both individuals. Thus demanding equality of opportunity sets violates the Pareto principle. This insight is similar to Varian's non-existence result of fair competitive equilibria.

order to be a relevant criterion for the problems of interest in this section. A basic question in this section is to what extent redistribution is desirable when individuals have different wage earning capacities. Axiom MOEC does not call for lump sum transfers in such cases. The reason is as follows: A redistribution of income from the high productivity individual to the low productivity individual makes it impossible for the highly productive individual to spend his entire endowment of time for leisure (and consume nothing). The low productivity individual will however have the possibility of not working, and still consume a positive amount. Thus, any redistribution in favour of the low productivity individual would give him the opportunity to consume some bundles which would not be available to the highly productive individual. However, it seems reasonable to claim that the high productivity individual faces a more favourable situation than the low productivity individual, and that lump sum transfers are warranted. In particular one might argue that one must look at the part of the opportunity sets which are relevant for the individuals in question. It does not seem like a strong argument against redistribution that a high productivity individual will no longer have the opportunity to not work and not consume. This opportunity is not relevant for his actual choices. Hence, it seems reasonable to extend the applicability of an opportunity egalitarian criterion in a direction that allows for redistribution also in situations where one opportunity set is not completely contained in another.

I will adopt the following criterion for justifiable redistribution: If individual 1 has the opportunity to consume the optimal bundle of individual 2, but individual 2 does not have the opportunity to consume individual 1's optimal bundle, then I will say that lump sum transfers from individual 1 to 2 are legitimate. This notion of opportunity egalitarianism is based on whether an individual has the opportunity to consume the bundle actually chosen by another individual. It is furthermore based only on pairwise comparisons of the opportunity sets of any two individuals. This extension of the Minimal Opportunity Egalitarian Criterion is called the "Extended Opportunity Egalitarian Criterion". Its formal definition is as follows:[16]

<u>Axiom EOEC (the Extended Opportunity Egalitarian Criterion):</u>
Consider two social states $\bar{x}, \bar{y} \in \bar{X}^3$ *for which the individual states differ only for individuals* $i, j \in N$. *If* $\left[y_i \in OS_j(\bar{y}) \wedge y_j \notin OS_i(\bar{y})\right]$ *and* $\left[OS_i(\bar{y}) \subset OS_i(\bar{x})\right]$ *and* $\left[x_j \notin OS_i(\bar{x})\right]$ *then* $\bar{x} P \bar{y}$.

As noted previously, the set of social states for which Axiom EOEC applies differ only in the social transfers and the associated optimal social outcome. Hence, Axiom EOEC applies for

[16]This result is similar to the result in Varian (1974). He has shown that fair allocations exist in economies without production (in pure exchange economies), but the existence result does not extend to economies with production.

pure distribution problems. Axiom EOEC is illustrated in figure 5. In figure 5 (a) individual i has the opportunity to choose the bundle of valuable goods chosen by individual j. Individual j on the other hand does not have the opportunity to consume the bundle of valuable goods chosen by individual i. In this situation Axiom EOEC defends redistribution in favour of individual j. The Axiom is formulated such that it does not justify "excessive redistribution", or a redistribution which leads to a new situation where one can defend a redistribution in the opposite direction from individual j to i. Figure 5 (b) and 5 (c) shows two situations in which Axiom EOEC does not defend redistribution. In figure 5 (b) both individuals have the opportunity to choose the bundle chosen by the other individual. In this case it seems reasonable to claim that neither of the individuals have good reason to claim larger transfers. In figure 5 (c) on the other hand neither of the individuals have the opportunity to choose the other individual's bundle. Both individuals could argue that his opportunities are worse than those of the other individual. This situation is however symmetric; based on comparisons of opportunity sets one can not easily judge one individual as more unfortunate than another. In such a situation Axiom EOEC does not defend redistribution between the two individuals.

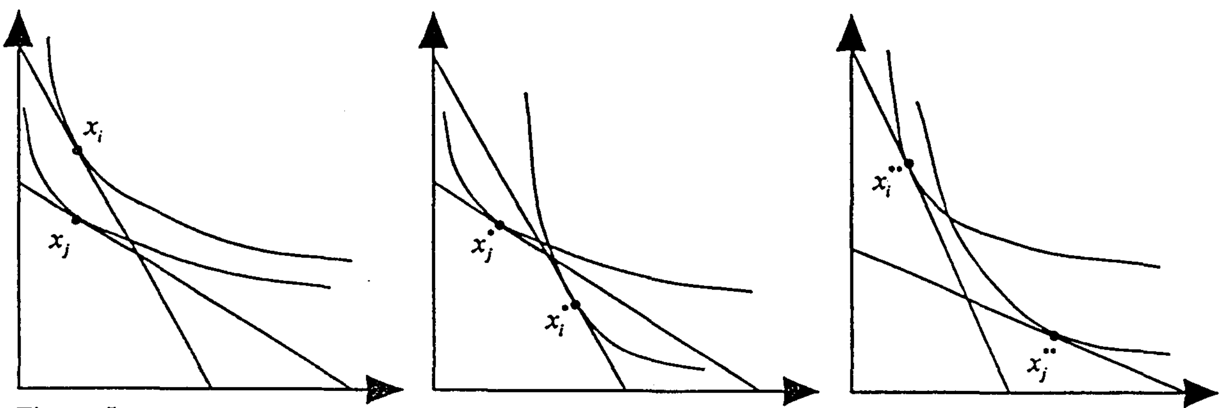

Figure 5a Figure 5b Figure 5c

A central question is whether Axiom EOEC is a sensible criterion for social choice. In my view it is reasonable to extend the applicability of opportunity egalitarianism beyond the minimal opportunity egalitarian criterion, for the cases discussed in this section. Axiom EOEC seems like a reasonable approach. There might be raised objections to Axiom EOEC based on normative arguments. The main objective in this chapter, however, is to analyse whether the criterion can be used as a basis for social choice. Therefore, I study whether there exists a social evaluation functional; $G^3 : \boldsymbol{R}^* \times \boldsymbol{OS}^3 \to \boldsymbol{R}^3$. Proposition 3 gives a negative answer to this question:

Proposition 3

Suppose the set of possible profiles of preferences, R^*, satisfies Condition SRP, Condition M and Condition SQC. There does not exist a social evaluation functional, G^3, which satisfies Axiom EOEC.

Proof:

An example will suffice. All arguments are illustrated in figure 6.

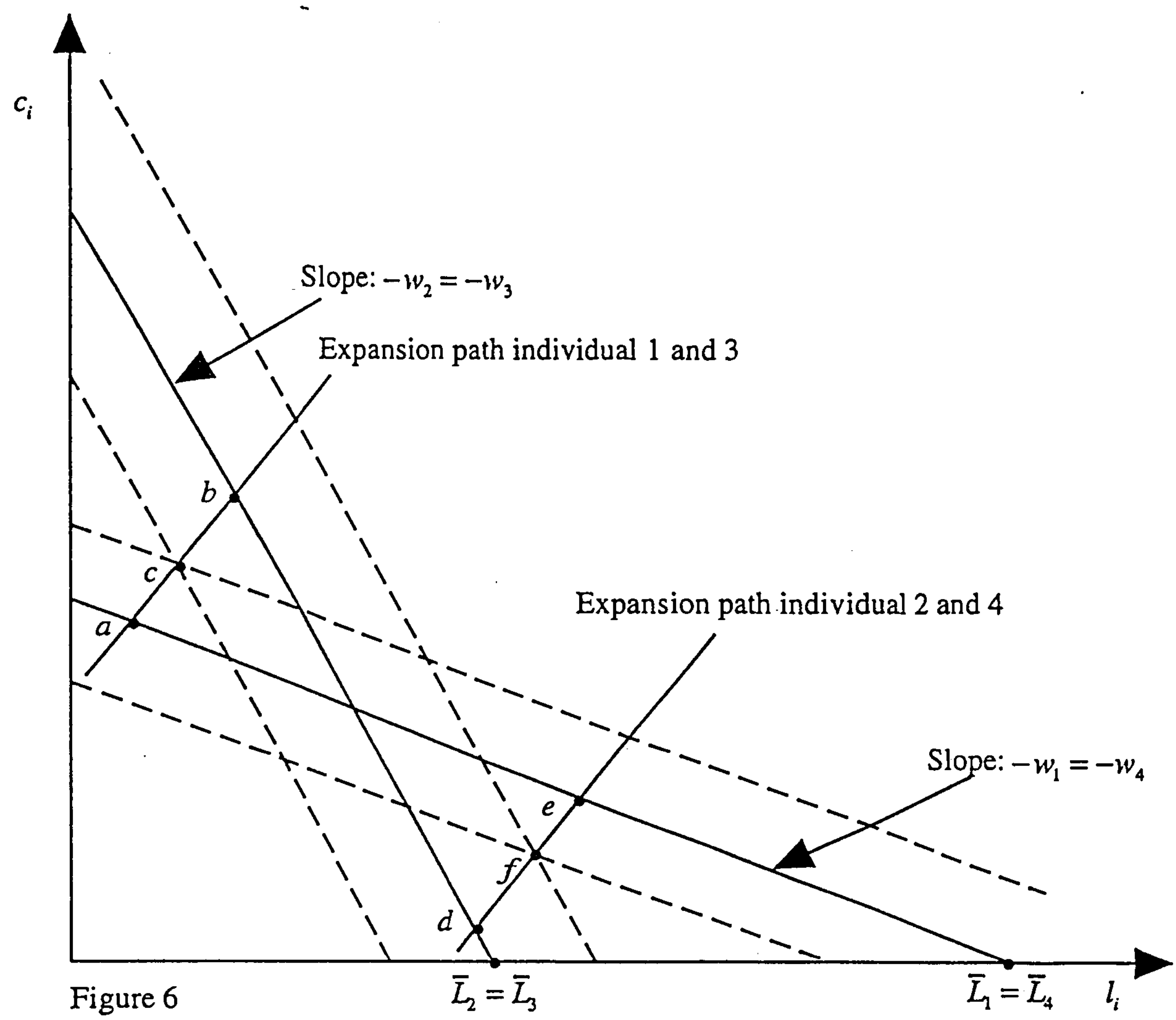

Figure 6

There are four individuals, $N = \{1,2,3,4\}$. $\bar{L}_i$ is individual i's endowment of leisure where $\bar{L}_1 = \bar{L}_4 > \bar{L}_2 = \bar{L}_3$. Individual productivity differs in the following way: $w_1 = w_4 = \underline{w} < w_2 = w_3 = \bar{w}$. These endowments and prices are kept constant for all possible social states. I assume that preferences are such that the expansion path for individual 1 at price w_1 is equal to the expansion path for individual 3 at price w_3. Similarly, I assume that the expansion paths for individuals 2 and 4 are equal to each other at prices w_2 and w_4, respectively. Furthermore, I assume that all individual's expansion paths are linear with equal slopes, but that individual 2 and 4 always prefer relatively more leisure than consumption, compared with individuals 1 and 3. The expansion paths are illustrated in figure 6. I now turn to an analysis of the different social outcome which can come about through lump sum redistribution, given the prices, endowments and preferences. Consider the bundles a,b,c,d,e,f, illustrated in figure 6. Consider the following social outcomes:

$x=(x_1=a,x_2=d,x_3=b,x_4=e)$, $y=(y_1=c,y_2=d,y_3=c,y_4=e)$,

$z=(z_1=c,z_2=f,z_3=c,z_4=f)$, $v=(v_1=a,v_2=f,v_3=c,v_4=e)$.

It is easily verified that social outcome y has come about through a redistribution from 1 to 3 compared to social outcome x. Social outcome z has come about through redistribution from individual 4 to 2 compared to social outcome y. Social outcome v has come about through redistribution from individual 1 to 4 compared to social outcome z. And finally social outcome x has come about through redistribution from individual 2 to 3 compared to social outcome v. Axiom EOEC implies the following strict rankings; $\bar{x}P\bar{v} \wedge \bar{v}P\bar{z} \wedge \bar{z}P\bar{y} \wedge \bar{y}P\bar{x}$. By transitivity of P we have $\bar{x}P\bar{x}$. This contradicts the asymmetric property of P.

Q.E.D.

The underlying logic behind Proposition 3 is more easily understood by closely inspecting figure 6. In social state $\bar{x}$ individual 2 and 3 have identical opportunity sets, and individual 1 and 4 have identical have identical opportunity sets. However, in this social state individual 1's chosen bundle of goods is contained in individual 3's opportunity set, and individual 2's chosen bundle of goods is contained in individual 4's opportunity set. Consequently, in social state $\bar{x}$ it is legitimate to redistribute resources from individual 3 to 1 and from individual 4 to 2. This is exactly what happens when moving from social state $\bar{x}$ to social state $\bar{z}$ (in two steps). The problem, however, is that in this new social state $\bar{z}$ individual 4's opportunity set is completely contained in individual 1's opportunity set, and individual 3's opportunity set is completely contained in individual 2's opportunity set. The redistribution has therefore created a new social state where new redistributions are legitimate, according to Axiom EOEC. This new redistribution does, however, take us back to social state $\bar{x}$. Hence, Axiom EOEC can defend social rankings of the type; "social state $\bar{x}$ is better than social state $\bar{v}$ is better than....is better than social state $\bar{x}$". This is not legitimate when the ranking is strict. One can also think of the result in a more intuitive way: Axiom EOEC may seem like a fairly good criterion for redistributions of income between any two individuals. When there are more than two individuals in the economy, however, the criterion seems less applicable. A legitimate redistribution between two individuals (according to Axiom EOEC) may create a situation which demands new transfers. If these transfers are implemented, it might be legitimate with even new redistributions. Proposition 3 can be interpreted as a result which says that a chain of legitimate lump sum transfers might not converge to a situation in which no more transfers are defensible by Axiom EOEC. An even simpler interpretation is that comparison of opportunity sets is not a reasonable basis for determination of redistributive policies.

Proposition 3 holds for different definitions of the space of valuable goods. As long as it is impossible to equalise opportunity sets through lump sum transfers, and when preferences differ between individuals, a similar argument can be carried through. The fact that lump sum transfers can not equalise opportunity sets is likely to be true for a large class of economic environments. The example in which functionings are considered the valuable goods illustrates this point. An important reason for differences in the level of functionings between different individuals is that individuals may differ in their ability to convert tradable goods into functionings, and also in their ability to exchange one functioning into another. This implies that the slope of the "opportunity constraint" in the functioning space in general will differ, and lump sum transfers will not be sufficient to ensure equal opportunity sets. In these cases it seems hard to establish a foundation for redistributional policies based on comparisons of opportunity sets.

5. Concluding Remarks

The analyses in this chapter have been concerned with properties of normative rules which formalise opportunity egalitarianism. A fundamental characteristic of these rules is that one is not only concerned with outcomes, but base evaluations of justice on comparisons of opportunity sets as well. I have tried to argue that the suggested and analysed criteria are reasonable interpretations of opportunity egalitarianism. The general conclusions of the analyses are however negative. It seems to be very difficult to include both an opportunity egalitarian criterion and the Pareto principle in a social choice rule. In my view this constitute a serious problem for, and a challenge to, proponents of opportunity egalitarian views.

The normative approaches in this chapter explicitly takes into account additional information beyond what is possible to represent by utility- or preference information. Opportunity egalitarianism is thereby an example of a non-welfarist normative view. The Pareto criterion is one of three axioms which is normally used to characterise welfarism, and obviously any non-welfarist view must conflict with at least one of these axioms.[17] The inconsistency between opportunity egalitarianism and the Pareto principle may on this background seem less surprising.[18] The Pareto criterion is however quite uncontroversial, at least when it is interpreted in terms of well-being (the "betterness" interpretation). According to this

[17]Welfarism is implied by the axioms; (1) Pareto indifference, (2) unrestricted domain and (3) binary independence of irrelevant alternatives. For a proof see Blackorby et.al (1984).

[18]Elster (1983) and Sen (1992) criticises the sole use of preference information in the evaluation of social states.

interpretation a social state is improved if at least one individual increases his well-being, without anyone experiencing a lower well-being (strong Pareto).[19] In my view the Pareto criterion is very reasonable as a minimal requirement for a social choice rule when the betterness interpretation of the principle is adopted.

Provided that one finds the Pareto criterion reasonable, should one abandon opportunity egalitarianism based on the results in this chapter? I believe such a conclusion is premature. It is possible that my interpretations of opportunity egalitarianism are unable to capture some important aspects of such a fairness norm, and that other operationalisations are able to deal with the problems which I have pointed at in this chapter.

I have defined individual preferences and well-being over vectors of valuable objects or inputs. As previously noted this formulation excludes the possibility of individuals valuing freedom beyond the fact that freedom might make better outcomes possible. My approach is fairly standard, but it might not capture a possibly important justification for opportunity egalitarianism: Freedom might be intrinsically important for people. If this is true preferences and well-being should be defined not over outcomes only, but over pairs consisting of an outcome and an opportunity set. This line of reasoning is exactly the one which was hinted at by Sen (1985, 1992), and which has been analysed formally by Gravel (1994) and Bossert et.al (1994). The insights gained from this research are important, but inconclusive with regards to whether it makes sense to rank well-being based on both opportunity sets and outcomes. The approach adopted in this chapter does, however, not allow people to assign intrinsic importance to freedom in this way. It is possible that this alternative approach will prove more promising in formalising opportunity egalitarianism.

I believe that the concept of individual responsibility constitute an even more important normative justification for opportunity egalitarianism. An essential idea in opportunity egalitarianism is that one accepts that actual outcomes may differ, if "free choices" and not unfair opportunities cause these differences. It seems like this idea of free choice and individual responsibility in some sense is incompatible with standard economic decision theory. The predominant decision theoretic model in economics can be described as one in which individuals have a predetermined and fixed ranking of alternatives, and where changes in behaviour is explained by changes in opportunity sets. For a given profile of preferences, the outcomes are completely determined by a profile of opportunity sets. In my view the notion of individual responsibility is meaningless if preferences are exogeneously given, beyond the control of the individual in question. The concept of individual responsibility can only make sense if the object of responsibility is within the control of the agent who is

[19]Elaborations on the betterness interpretation of the Pareto criterion are found in Broome (1991).

considered responsible. Why should an individual be held responsible for outcomes and not for opportunities, when opportunities fully determine the outcomes? In a standard model of rational choice people have no autonomous control over outcomes. Outcomes are determined by exogenously given preferences and the opportunity sets which individuals face. Individual responsibility does therefore not seem to be a reasonable notion if such a standard behavioural model of rationality is applied. Hence, it seems like a proper defence of opportunity egalitarianism must be based on an alternative theory of individual decisions.

Implicitly it seems like opportunity egalitarian views assume alternative models of individual behaviour. Individuals are given responsibility for their own actions. This suggests that individuals to some extent have "free will", and that irrational actions are possible. Normative theories which use the concept of individual responsibility often claim that individuals should be held responsible for outcomes which are caused by "bad choices".[20] Sometimes it is assumed that an individual's ability to choose a good outcome is determined by the amount of "will" which is exerted.[21] This approach conflicts with a standard model of rational behaviour where "will" has no explanatory power. In standard models people simply choose the best element in an opportunity set. Individual responsibility for the consequences of ones choices can only make sense if individuals in some sense are "free to choose". Individual responsibility does therefore seem to be linked to the difficult philosophical concept; "free will". These concepts are obviously very difficult to incorporate into decision theory and normative theory. It seems reasonable to demand that proponents of opportunity egalitarianism clarify the behavioural assumptions on which their theories are based. A general conclusion of this chapter seems to be that opportunity egalitarianism is not a sensible foundation for social choice when it is based on standard approaches concerning preferences and choice.

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[20]Arneson (1989) is quite explicit on this; he claims that the sensible notion of opportunity egalitarianism is one where people have equal opportunity for welfare. This doctrine demands not only that welfare should be equal if people choose the best element in their opportunity set, but also that the welfare from choosing the second and third best option should be equalised between individuals. Obviously the welfare associated with suboptimal choices is irrelevant if people behave rationally.

[21]As in Fleurbaey (1995).

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Chapter 4

COMPETING FOR AID*

The Effects of Aid from a Non Committing Donor on Investment and Lending Behaviour

Abstract

This chapter studies the distortive effects of foreign aid on recipient countries' investment and lending behaviour. The primary focus is on the donor's commitment problem when allocating aid between recipients: A utilitarian donor will always allocate more aid to the poorer country. This creates possibly adverse incentives regarding the recipient's intertemporal decisions. Four different dynamic games are developed, in which two recipients first choose investment policy, and the donor subsequently allocates aid resources between the two recipient countries. The chapter contains proofs of existence of a subgame-perfect equilibrium (SPE) in all but one of these games. (1) In a basic model it is shown that the recipients in a SPE of the game choose too low investments. (2) Next, it is shown that if the recipients have access to an international credit market, investments will be efficient, but the level of debt will be excessively high. (3) In an extended dynamic game I show that in a SPE a too high fraction of investments is allocated to long-term projects relative to short-term projects. (4) Finally, in another extension of the game, I demonstrate that foreign aid does not lead to distortions regarding the recipients' choice of risk-taking.

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1. Introduction

Few people would deny that poverty is one of the most important economic problems in the world today. In 1985 there were 1110 million "poor" people and 630 million "extremely poor" people in the world.[1] Many of these are deprived of adequate access to food, shelter and medical attention. At least in the short run, it is unlikely that poverty can be alleviated through simple policy reforms which improves productivity in poor countries. In order to solve this problem it seems necessary with transfers of resources from rich to poor countries. In fact, substantial amounts are allocated as foreign aid in order to improve living conditions in poor countries: In 1993 approximately 68.5 billion US$ was granted as official developing funds to developing countries (OECD (1994)).

Given the large amounts of foreign aid, and the importance of the poverty problem, it is pertinent to ask how efficient foreign aid is in promoting growth, poverty alleviation and other goals in poor countries. The answer provided by available empirical studies is at best ambiguous. In a major evaluation of aid projects Cassen (1986) draws the general picture that the available microevaluations of specific aid projects are positive, but at the macro level it is harder to find positive effects of aid.[2] A possible explanation for this so called micro-macro paradox is that foreign aid is fungible (Singer (1965), Pack and Pack (1993)): A donor's disbursement of aid may be directed to a successful development project. However, it is quite possible that the recipient regardless of aid donations would have carried out this project. If this is the case increased aid will effectively be used for other purposes than the specific project it was intended for.

If fungibility of aid is a widespread phenomenon, then obviously macroeconomic evaluations are more relevant than microevaluations of the effect of aid. White (1992) provides a survey of much of the empirical literature on the macroeconomic effects of aid. The available studies do not seem to have found evidence of a significant positive effect of foreign aid on economic growth in recipient countries (see Papanek (1973), Voivodas (1973), Dowling and Hiemenz (1982), El Shibly (1984), Mosley et.al (1987), Brewster and Yeboah (1994)). It has been argued that an important reason for this finding is that aid partially crowds out savings. Thus, foreign aid may only to a limited extent lead to higher aggregate investments. Capital accumulation is generally considered as a main determinant of growth. A crowding out effect of aid on savings is therefore a possible explanation for the lack of correlation between aid and

[1] All numbers are from the World Bank (1990). The poor people are defined as those with an annual income less than $ 370, and the extremely poor are those with less than $ 275. The numbers are measured at purchasing power parity (PPP).

[2] A substantial sample of World Bank projects are reported to have an average internal rate of return (IRR) of approximately 16% (Cassen (1986)).

growth. To some extent the empirical studies seem to support such a claim: Aid seems to have a negative impact on domestic savings (see Boone (1996), Griffin (1970), White (1992)). A tentative conclusion from these empirical studies is that foreign aid may have limited macroeconomic effects, and that this may be due to a crowding out effect of aid on savings.

The above observations warrant thorough theoretical examinations of how aid disbursements influence various decisions by a recipient country. Intertemporal decisions by recipients of aid seem particularly relevant in this respect. The link between foreign aid and intertemporal decisions may be studied in a variety of ways. I will highlight and discuss some fundamental modelling assumptions which are adopted in this chapter. Specifically I assume that: (1) Governments in recipient countries are benevolent, (2) donors and recipients of aid have symmetric information, and (3) donors are unable to commit to optimal aid policies. These assumptions need further justification and discussion.

(1) Benevolent recipients

Throughout this chapter I assume that the donor and recipients of aid agree on what constitutes a welfare improvement for the population in a recipient country. Thus the donor and recipient agree on the country's welfare function. Decision-makers in a recipient country (governments) are furthermore benevolent in the sense that they will try to maximise welfare in their country. In this paper the differences in interest between donor and recipients stem solely from the fact that the donor cares about the welfare in all (two) recipient countries, whereas each recipient cares only about his own well-being. Thus, I do not analyse problems which arise when recipients and the donor have different opinions about fundamental values and goals.

This does not mean that such problems are irrelevant. "Casual empiricism" certainly supports the view that it is inadequate in many cases to treat the governments in developing countries as benevolent. Governments may want to direct additional resources to the political elite, to excessive rearmament or to some special interest group or tribe. Furthermore, the decision processes in developing countries may be dysfunctional, for example due to rent seeking (see e.g. Boone (1996) and Bjorvatn (1996)).

The reason for not focusing on these problems is twofold: First, I believe that there are many important cases in which governments in recipient countries legitimately can be regarded as "benevolent". Many governments in developing countries are democratically elected and implement policies according to the interests of their population. Secondly, differences in value judgements between donors and recipients raise some fundamental questions regarding choice of aid policy and research methodology. An attempt to influence the political priorities by a recipient may conflict with the principle of respecting a sovereign country's value judgements. It is obviously very hard to choose the "correct" welfare function when there is disagreement between donors and recipients. It is not obvious that one should choose the donor's judgements as the correct one. By assuming symmetry in donor and recipient preferences, I simply abstract from these difficult issues. As a "benchmark case" it is relevant to study the effects of aid when donors and recipients share the basic value judgements. If donors and recipients have different opinions regarding developmental goals, the incentive problems are likely to be enhanced. However, I show that foreign aid will have distortive effects even when donors and recipients have identical judgements of welfare in a recipient country.

(2) Symmetric information

In the model to be presented below, the donor has complete knowledge of the preferences (utility functions) of the recipient countries. Furthermore, at the time he makes his allocations of aid the donor knows all the relevant previous actions by the recipients. In other words, I assume that there is symmetric information between donor and recipients. Problems of moral hazard or adverse selection do not apply in the model of this chapter. There are a few papers which analyse foreign aid under the assumption of asymmetric information (see Murshed and Sen (1995), Pietrobelli and Scarpa (1992) and Pedersen (1995b)). Models with asymmetric information have, in my view, limited relevance in the context of foreign aid. Many of the major determinants of economic performance in developing countries are observable both for donors and decision-makers in a recipient country. Important determinants of growth, such as capital accumulation, the level of education, and technological progress, are probably observable for the donors of aid. It is for example entirely possible for a donor to make aid disbursements conditional on investments in education, infrastructure and so forth. Lack of observability is not likely to be the serious constraint for the donors when designing an aid policy.

(3) Donors are unable to commit

There are reasons why aid policies may be inefficient even if governments in recipient countries are benevolent, and donors and recipients have symmetric information. I will focus on the incentive effects of foreign aid when donors are unable to commit to an efficient aid policy. This problem resembles what Buchanan (1975) has called "the Samaritan's dilemma", which also applies to the case of foreign aid. The general problem is that an altruistic donor will increase disbursements of aid to a recipient which is in greater need. A recipient will react by not allocating sufficient effort or resources to activities which improve his economic performance. The result is a Pareto inferior allocation. If the donor were able to commit to a certain aid policy, he could improve the well-being for himself and for the recipient.

I will argue that the commitment problem is likely to be a serious problem in the case of foreign aid. First, it seems likely that donors are genuinely concerned about the well-being of the poor people in the world. It is hard to defend an assumption of purely selfish motives when explaining the substantial amounts which are given as aid. Thus, altruism can be regarded as an important motivation for disbursements of foreign aid. Furthermore, I model situations in which the donor, when making his choices, regards the level of production in a developing country as predetermined (the donor is a "Stackelberg follower"). I claim that this modelling assumption captures an important structural relationship between donors and recipients. It may last 20 to 40 years before growth-enhancing policies by a recipient country takes effect. At any given time the recipient government has limited opportunities to improve the living condition for its population. The donor, however, usually decides upon an aid budget on an annual basis. Thus, at the time the donor decides about his aid budget, the level of production in a recipient country can be regarded as fixed. When the recipient make decisions about growth policies, it is a reasonable presumption that increased production will lead to reductions in aid. In my view, the commitment problem for the donor constitutes a fundamental limitation on the opportunity to effectively transfer wealth from rich to poor countries.

In the analysis below I focus on a situation where two recipient countries compete for a fixed amount of aid supplied by one donor. The donor always allocates the largest amount of aid to the country with inferior economic performance. If a country invests less and thereby reduces its future GDP, this will lead to an increase in future receipts of aid. This will be at the expense of the other country, since there is a fixed amount of aid available. To a large extent this seems to be realistic description of the situation for recipients of aid. Development aid from the United Nations and from the different national aid agencies constitutes a major proportion of total aid disbursements. These aid agencies have a fixed annual budget. Furthermore it has been established that poorer countries receive larger amounts of aid per capita (see Trumbull and Wall (1994)). Thus, if one country becomes poorer it receives a larger amount of aid. This must imply that other countries receive smaller amounts. Similarly, it has become an explicit part of official Norwegian aid policy that different countries (within a specified region) must compete for a fixed amount of aid resources. This chapter tries to analyse the effects of this competition on investment and lending behaviour by recipient countries.

The analysis in this chapter has many linkages to other parts of the economics literature. The basic problem in the "Samaritan's dilemma" (Buchanan (1975)) is that a potential recipient of aid may behave in a socially inefficient manner in order to acquire more extensive funds from an altruistic donor. This idea has been used to analyse effects of social security systems within a country (see Bruce and Waldman (1991) and Coate (1995)). These two papers focus on "spill-over effects" between donors, which occur because donations from one individual increases the well-being for other altruistic persons as well. Because such an externality exists, charity tends to be inefficiently low. These papers focus on the effect of altruism on the behaviour of donors. In contrast this chapter focus on how altruistic behaviour by one donor affects the behaviour of recipients who compete for aid. The analysis in my chapter has links also to the topic of "partnerships" (see Farrell and Scotchmer (1988), and Gaynor and Gertler (1995)). This literature studies different kinds of institutions in which the partners receive an equal share of the profit from a partnership. The model in this chapter has this property as well: The total resources available are shared equally between the recipient countries. In such settings the agents tend to behave in a socially inefficient manner, because each individual pays the full cost of an increased effort whereas the gains are shared among the "partners".

Finally there is a literature which specifically analyses incentive effects of foreign aid. Pedersen (1995a) shows that an aid agency which seeks to alleviate poverty in a poor country will reduce the government's own efforts for this purpose. Pedersen considers a situation with only one recipient of aid. Furthermore the focus is on the specific problem of poverty alleviation. Pedersen also analyses how an aid agency might set up optimal contracts when he has incomplete information about the recipient country (Pedersen (1995b)). A basic assumption is that the donor of aid and the recipient country's government have different goals regarding the use of resources. Furthermore, the donor has limited information about the donor (hidden actions/hidden type). In this setting Pedersen analyses how a donor should design an optimal contract in order to induce a recipient to undertake measures against poverty. Bjorvatn (1996), Svensson (1995), Lahiri and Raimondos (1995) and Kemp and Kojima (1985) elaborate further on different aspects of the incentive effects of aid.

The analysis in this chapter explicitly considers a situation where two recipient countries compete for a fixed amount of aid. This formulation is different from the models in Bruce et.al. 1991) and Coate (1995) in which there are many donors and one recipient. The basic model in section 2 is however quite similar to standard models of "partnerships". The standard result that charitable donations lead to adverse incentive effects persists in this setting. Moreover, the result that discretionary donations of aid may lead to adverse incentive effects is well established in the literature. The major novelties in this chapter are, in my view, the extensions of the basic model in sections 3, 4 and 5. In these sections I analyse how competition for aid affects the recipients' choice of indebtedness and investments, long-term versus short-term investments, and risk-taking. It is of considerable interest to get a more detailed knowledge regarding the effect of aid on different aspects of a recipient country's intertemporal decisions. The choices of indebtedness, of time perspective on investments, and risk-taking are all very important determinants of the growth and economic structure in a developing country. To my knowledge, these issues have not previously been analysed in the literature about foreign aid.

The rest of the chapter is organised as follows. In section 2 I construct a basic two-stage model and demonstrate that competition for aid leads to underinvestments. I extend the basic model in section 3, by giving the recipients access to a perfect international credit market. I show that investments will be efficient in this context, whereas the recipients will choose an inefficiently high level of indebtedness. In section 4 of the chapter I focus on how competition for aid affects the choice of long-term versus short-term investments. In the strategic situation which is described, recipients will tend to invest too much in long-term projects relative to short-term projects. Finally, in section 5 I analyse risk-taking by countries which receive aid from a non-committing donor. I demonstrate that the recipients will choose an efficient level of risk-taking.

2. A Two-Period Model of Aid-Donations

I construct a single good, two-period model, where the intertemporal decision by the recipient countries is in focus. Initially I will analyse the outcome of a game in which the recipient countries do not have access to an international capital market; the only way they can reallocate wealth between periods is through real investments. Subsequently, I will study a situation where a recipient can borrow or lend money at a constant rate of interest.

In the beginning of the first period ($t = 0$) each of the recipient countries, $i = 1,2$, is endowed with a level of wealth (w_{0i}). Both countries then simultaneously choose a level of investments in the first period I_i. Their investments are related to consumption in the first period (c_{0i}) in the following way: $c_{0i} = w_{0i} - I_i$. Investments determine the level of wealth in the next period; $w_{1i} = f(I_i)$. In the second period the donor of aid first observes the recipient countries' level of wealth, and then decides on how to divide a fixed total amount of aid between the two recipients: $a_1 + a_2 \leq \bar{a}$. After the donor has chosen an aid policy, the recipient countries consume the sum of their wealth and aid; $c_{1i} = w_{1i} + a_i$. The timing of the choices is illustrated below:

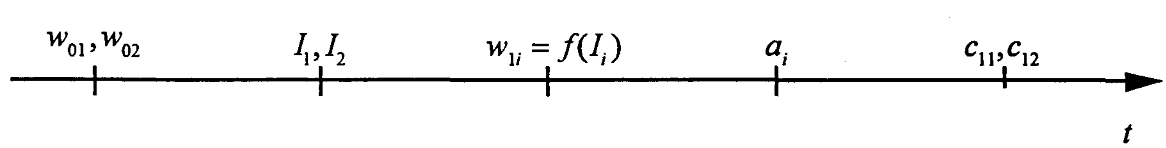

Figure 1

The payoff to the recipients is given by a standard additively separable utility function; $U_i = u(c_{0i}) + \delta u(c_{1i})$. I assume that the recipients have identical utility functions and discount rates. The budget constraint for each country is: $c_{1i} \le f(I_i) + a_i$. The donor has a utilitarian payoff; he prefers allocations which increase the sum of the two recipient countries' utilities. The donor's payoff is thus: $U_D = U_1 + U_2$.

The structure of the game is depicted in the figure below:

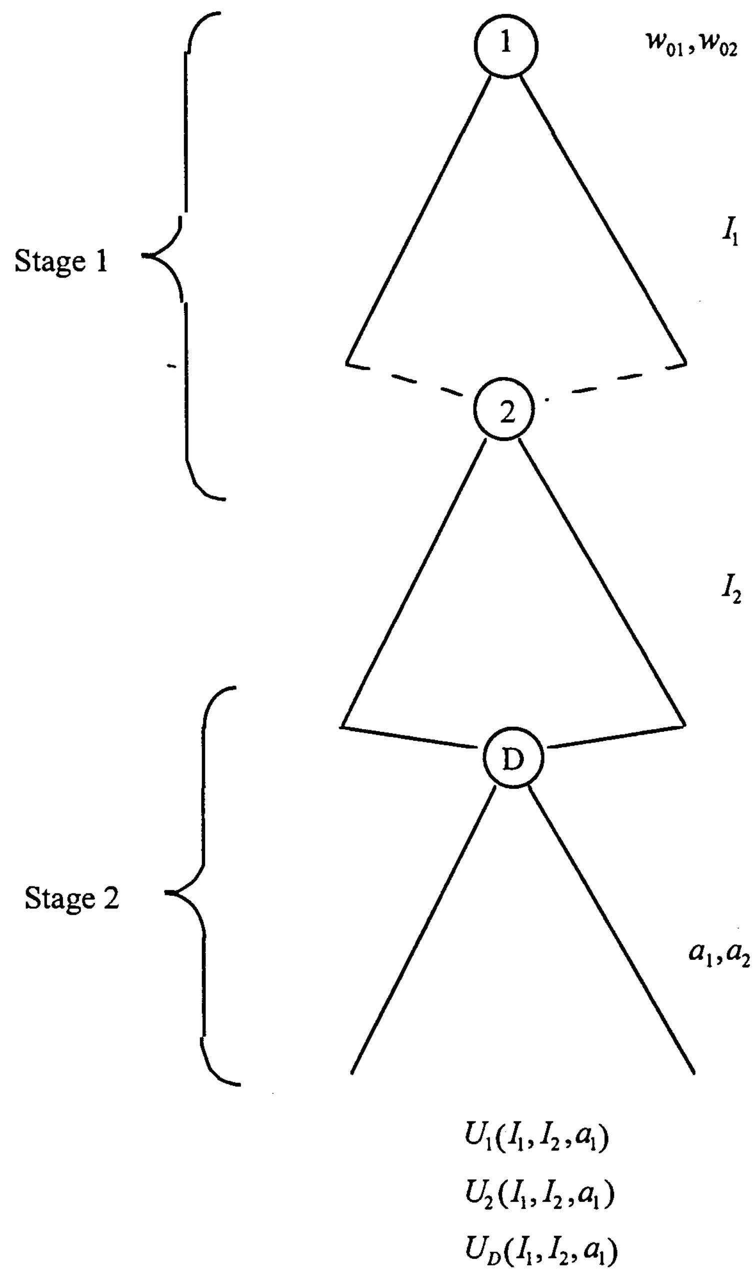

Figure 2

I adopt the following structural assumptions on the problem:

Assumption 2.1: *f, u are twice continuously differentiable* (C^2),

$$u' > 0, u'' < 0, u'(0) = \infty,\ f' > 0, f'' < 0, f'(0) = \infty.$$

Assumption 2.2: $\left\{\max_{i \in 1,2}\left[f(w_{0i}) - f(0)\right]\right\} < \bar{a}$.

Assumption 2.1 is standard. Assumption 2.2 states that the donor will always have sufficient resources to ensure equal consumption by both recipients. This assumption is adopted in order to ensure interior solutions, in which the donor allocates a positive amount of aid to both recipients for all possible strategies by the recipient countries. This assumption is useful for establishing a general theorem of existence of a subgame-perfect equilibrium in the game.[3]

2.1 Pareto Efficient Allocations

Before analysing the game it will be useful to analyse Pareto efficient allocations in this framework. A Pareto optimal allocation is found by solving the following problem:

2.1
$$\max_{I_1,I_2,a_1}\left\{\begin{array}{l}\lambda_1\left[u(w_{01}-I_1)+\delta u(f(I_1)+a_1)\right] \\ \lambda_2\left[u(w_{02}-I_2)+\delta u(f(I_2)+\bar{a}-a_1)\right]\end{array}\right\},$$
$$\text{subject to}\quad a_1 \in [0,\bar{a}],\quad \mathrm{I}_i \in [0,w_{0i}].$$

Here $\lambda_i > 0$.

By assumption 2.1 and 2.2 a solution (I_1^*, I_2^*, a_1^*) to problem 2.1 must satisfy its first order conditions:

2.2
$$\lambda_1 u'(f(I_1^*)+a_1^*) = \lambda_2 u'(f(I_2^*)+\bar{a}-a_1^*),$$

2.3
$$u'(w_{0i}-I_i^*) = \delta f'(I_i^*)u'(f(I_i^*)+a_i^*),\quad i=1,2.$$

Equation 2.2 determines how aid should be allocated between the two countries. Equation 2.3 gives us a rule for Pareto efficient intertemporal allocation of the good. Equation 2.3 will be useful for comparisons with the outcomes of the game, which I will analyse next.

[3]If assumption 2.2 does not hold then each recipients' best response function in stage 1 (subgame 1) will be discontinuous in the other recipient's strategy (when the donor plays an optimal strategy in stage 2 (subgame 2)). This is due to the fact that the optimal aid disbursements will not be concave in the recipient's strategy when assumption 2.2 fails. Therefore, the objective function in problem 2.7 will not be concave. As a consequence the recipient's payoff function will not be quasiconcave in its strategy, and one recipient's best response function becomes discontinuous in the other recipient's strategy. The general assumptions adopted by Dasgupta and Maskin (1986), which ensure existence of equilibria, do thereby not hold. It can furthermore be shown that when assumption 2.2 does not hold, there are utility functions satisfying assumption 2.1 for which an equilibrium in subgame 1 does not exist.

2.2 *Subgame-perfect Equilibria in the Two-Stage Aid-Game*

In this section I derive subgame-perfect equilibria (SPE) in the aid game described above. Before doing so, I want to emphasise the underlying assumptions which legitimise the use of this equilibrium concept. I model a situation in which the donor has no ability to commit to an efficient aid policy. The donor has utilitarian goals, and he will choose according to these regardless of the history of the game. In the simple model above, a country can be poor in the second period either because its initial wealth is low, or because it chooses to consume a large proportion of its wealth in the initial period. The donor, however, cannot make his policy conditional on the causes for a country's low level of wealth. The only relevant information at the time when the donor is to make his choice, is the countries' level of wealth in that period. The extensive form of the game, with the donor choosing his strategy after the two recipients have made their choice of strategy, captures this idea of a donor's inability to commit. The use of the subgame-perfect equilibrium concept guarantees that the strategies must be sequentially rational.

I will study four different extensive form games in this chapter. The method for finding subgame-perfect equilibria will in principle be the same throughout. A SPE is defined as a Nash equilibrium of the game, with the additional condition that the equilibrium strategies constitute a Nash equilibrium in each proper subgame as well. The following notation is used: A_i is the action space for player i at the relevant stage of play, h^s is one particular history (sequence of previous actions) at stage s, and H^s is the set of all possible histories of the game at stage s. I will use the method of "backward induction" when finding the subgame-perfect equilibrium in each of the games to be analysed: First I solve for the Nash equilibria (NE) at the last stage of the game. Nash equilibria at this stage will simply consist in the optimal action by the donor evaluated at all possible nodes at this last stage. At the second to last stage the two recipient countries choose their levels of investment. I construct a "reduced" strategic form game in which their payoff functions are determined by the donor's equilibrium strategy at the last stage. For any history of the game, a Nash equilibrium in this reduced form game is defined by a pair of strategies for which neither of the countries can increase their payoff by changing their strategy.[4] The optimal strategy for each recipient will in general depend on the other recipient's strategy and on the history of the game at that stage. Each recipient's best response function can consequently be expressed as a function of the history and of the other country's strategy: $I_i^{br}\left(h^s, I_j\right): H^s \times A_j \rightarrow A_i$. The history of the game is treated as a fixed

[4] In sections 2, 3 and 5 of this chapter, the games consist only of two stages, and the second to last stage is consequently also the first stage. In section 4, however, I analyse a four stage game in which the history of the game is important at the second to last stage.

parameter. A NE of this reduced form game consists of a pair of actions (scalars), $(\hat{I}_1, \hat{I}_2) \in A_1 \times A_2$, which constitute a "mutual best response" for the specific history of the game: $\hat{I}_1 = I_1^{br}(h^s, \hat{I}_2) \wedge \hat{I}_2 = I_2^{br}(h^s, \hat{I}_1)$. This method of backward induction is used until the initial stage of the game is reached. When this process is completed we have found strategies which constitute a NE of all the proper subgames. Consequently the strategies constitute a subgame-perfect equilibrium for the entire game. I now use this method to analyse the SPE of the initial two-stage game:

STAGE 2: At stage 2 of the game the donor faces the following problem:

2.4 $$\max_{a_1 \in [0, \bar{a}]} \left[u(f(I_1) + a_1) + u(f(I_2) + \bar{a} - a_1) \right], \forall I_1, I_2.$$

The donor's optimal strategy $\hat{a}_1 : [0, w_{01}] \times [0, w_{02}] \to [0, \bar{a}]$ must solve the first order condition for problem 2.4:

2.5 $$\begin{aligned} & u'(f(I_1) + \hat{a}_1) = u'(f(I_2) + \bar{a} - \hat{a}_1) \\ & \Leftrightarrow \hat{a}_1 = \frac{f(I_2) - f(I_1) + \bar{a}}{2}, \forall I_1, I_2 \in [0, w_{01}] \times [0, w_{02}]. \end{aligned}$$

The donor's equilibrium strategy is a function from the set of all possible histories of the game to the set of possible actions for the donor: $\hat{a}_1 : [0, w_{01}] \times [0, w_{02}] \to [0, \bar{a}]$.

Let $\hat{c}_{2i}$ be country i's consumption in period two in an outcome of the game. The first order condition implies that the donor will equalise consumption by the two countries in the second period, such that each recipient receives half of the available resources. Given the Nash equilibrium in the last stage, consumption by both countries in any outcome of the game is determined by:

2.6 $$\hat{c}_{1i} = \frac{f(I_i) + f(I_j) + \bar{a}}{2}.$$

STAGE 1: In the "reduced" simultaneous move game at stage 1 each recipient country anticipates the equilibrium behaviour by the donor at the second stage. In this game, each of the two countries solves the following problem:

2.7 $$\max_{I_i \in [0, w_{0i}]} \left[u(w_{0i} - I_i) + \delta u\left(\frac{f(I_i) + f(I_j) + \bar{a}}{2} \right) \right].$$

Each recipient's optimal strategy in stage one is a function of the strategy by the other recipient at stage 1; a best response function; $I_i^{br}(I_j)$, $I_i^{br}:[0, w_{0j}] \to [0, w_{0i}]$ (in this reduced form game there is no history at the second to last stage). A recipient's best response function must solve the following first order condition to problem 2.7:

2.8 $$u'(w_{0i} - I_i^{br}) = \frac{1}{2}\delta f'(I_i^{br}) u'\left(\frac{f(I_i^{br}) + f(I_j) + \bar{a}}{2} \right), \forall I_j \in [0, w_{0j}].$$

A Nash equilibrium in the reduced form stage one game is a pair $\hat{I}_1, \hat{I}_2$ (scalars) such that $\hat{I}_1 = I_1^{br}(\hat{I}_2), \hat{I}_2 = I_2^{br}(\hat{I}_1)$. A subgame-perfect equilibrium of the game consists of a triple of strategies; $(\hat{I}_1, \hat{I}_2, \hat{a}_1(\cdot))$ such that: (i) the donor's strategy, $\hat{a}_1(\cdot)$ (a function), is given by $\hat{a}_1(\cdot)$ as defined by equation 2.5, and (ii) the two recipients' strategies, $\hat{I}_1, \hat{I}_2$, satisfy the condition; $\hat{I}_1 = I_1^{br}(\hat{I}_2), \hat{I}_2 = I_2^{br}(\hat{I}_1)$, where $I_1^{br}(\cdot)$ and $I_2^{br}(\cdot)$ are defined by equation 2.8. An <u>outcome</u> of a subgame-perfect equilibrium is a triple of scalars $(\hat{I}_1, \hat{I}_2, \hat{a}_1)$ consisting of the two recipients' equilibrium strategies, and the donor's equilibrium strategy evaluated at the two recipients' equilibrium strategies; $\hat{a}_1 = \hat{a}_1(\hat{I}_1, \hat{I}_2)$.

I can now present the result regarding existence of a subgame-perfect equilibrium of the above game:

<u>Proposition 2.1</u>
Under assumptions 2.1 and 2.2 there exists a unique subgame-perfect equilibrium $(\hat{I}_1, \hat{I}_2, \hat{a}_1(\cdot))$ *of the 2-stage aid game. The equilibrium is in pure strategies and is given by the solutions to the first order conditions 2.5 and 2.8.*

Proof: See Appendix 1.

The following proposition provides a characterisation of the SPE:

Proposition 2.2

The subgame-perfect equilibrium of the 2-stage aid game entails too low investments for both recipients. This means that there is a feasible allocation with higher investments for both recipients which is:

(i) Pareto superior to the SPE, and

(ii) Pareto efficient.

Proof:

Consider an outcome of the game, $(\hat{I}_1,\hat{I}_2,\hat{a}_1)$, in which investments are determined by equation 2.8. Next, fix the aid allocation at $\hat{a}_1$, and let each country choose an optimal level of investment for this allocation of aid. These new levels of investments are determined by equation 2.3, for $a_1^* = \hat{a}_1$. Comparing equations 2.3 and 2.8 establishes that these new levels of investments are higher than previously for both recipients; $\hat{I}_i < I_i^*$. It is immediate that this new allocation is Pareto superior to the original allocation in the game. Moreover, it is possible to choose (λ_1, λ_2) in equation 2.2 such that a Pareto optimal allocation of aid equals the allocation of aid in the game; $a_1^* = \hat{a}_1$. The new allocation is consequently Pareto efficient.
Q.E.D.

The intuition for the above result is quite straightforward. When a recipient decides on a level of investment, he makes a trade-off between consumption today and consumption tomorrow generated by investments today. In a subgame-perfect equilibrium a recipient will pay the full cost of investments in terms of lower current consumption. He will, however, keep only half of the increase in future production generated by investments today. This is because the donor in equilibrium will equalise consumption between the two recipients in the last period. As a consequence, an increase in one country's wealth in the last period will be shared equally between the two recipient countries, through reallocations of the donor's aid budget. Due to this sharing of future resources, the recipients are faced with a disincentive to invest, compared to a situation in which the countries received a fixed amount of aid. Pareto improvements may therefore be gained by increasing the investments by both recipients. The investment choices by a recipient country is compared with a Pareto optimal allocation in figure 3 below:

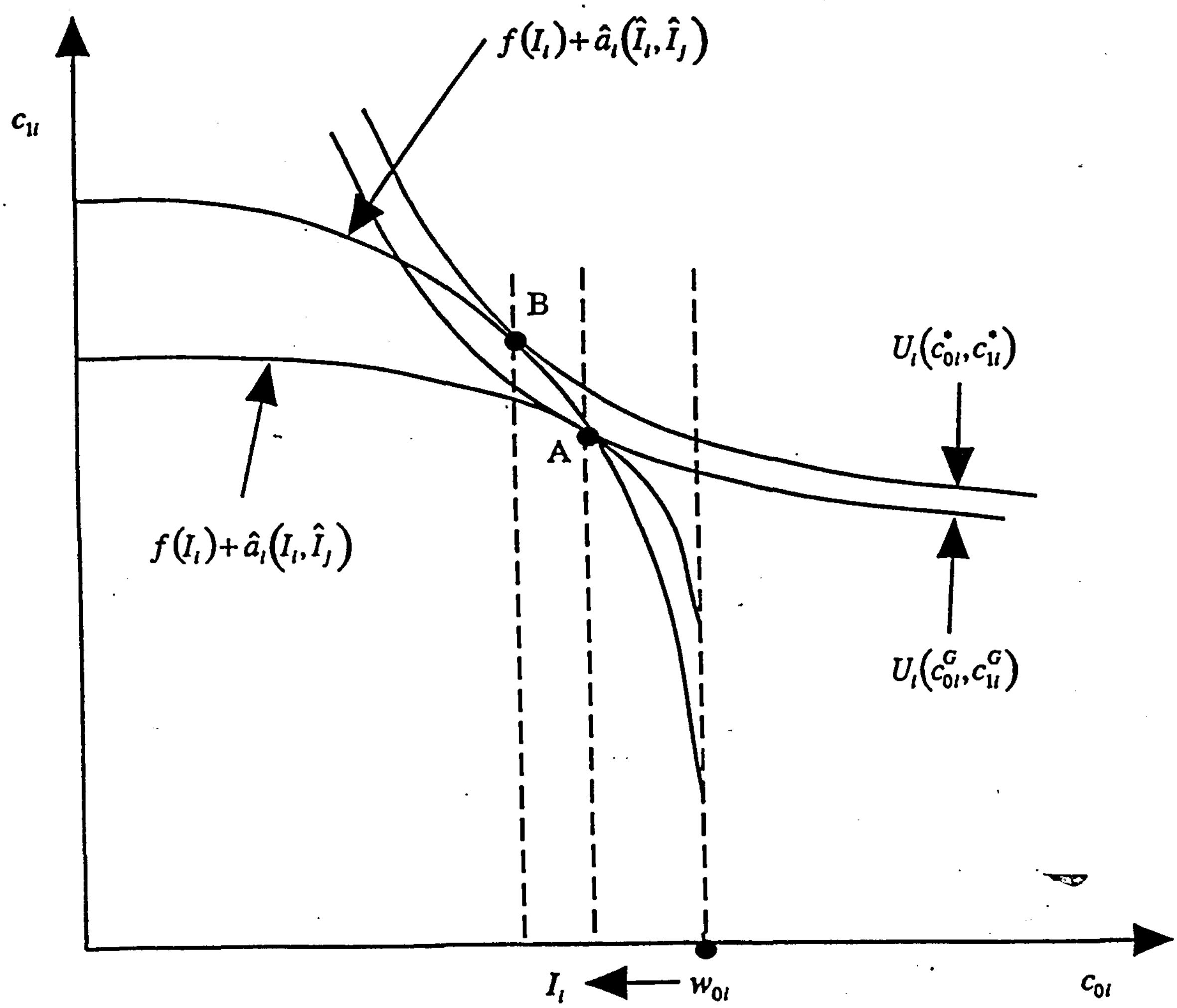

Figure 3

Point A shows the allocation in a SPE of the game, with consumption vector $\left(c_{0i}^{G}, c_{1i}^{G}\right)$. In a SPE the indifference curve is tangent to the consumption possibility curve in the game $\left(f\left(I_i\right)+\hat{a}_i\left(I_i,\hat{I}_j\right)\right)$. The investment in this case is the horizontal distance between w_{0i} and the point A. In the outcome of a SPE the recipient receives the amount $\hat{a}_i=\hat{a}_i\left(\hat{I}_1,\hat{I}_2\right)$ as aid. If the recipient country received this amount of aid regardless of investments, its consumption possibility curve would be equal to $f\left(I_i\right)+\hat{a}_i$. The slope of this curve is twice as steep (in absolute value) as the consumption possibility curve in the game for all levels of investment. A Pareto efficient allocation is located at point B, in which the recipient country's indifference curve is tangent to the consumption possibility curve when a fixed amount of aid is received. Here the consumption vector is $\left(c_{0i}^{*}, c_{1i}^{*}\right)$. In this efficient case the investments are increased to the horizontal distance between w_{0i} and the point B.

3. Effects of Aid when Recipients have Access to a Perfect International Credit Market

In section 2 I analysed a game in which the recipient countries did not have the possibility of using an international credit market to finance investments. This may be a reasonable description for many developing countries. After the international debt crisis in the 1980's, credit from commercial banks have been difficult to obtain for many developing countries. A large part of those who have received international credit faces an upper limit on total indebtedness.[5] For these countries the only relevant way to increase investment is through reduced consumption. However, the international credit market has improved since the 1980's. Developing countries can to an increasing extent use international loans to finance investments. It is of considerable interest to analyse how access to a credit market affects the recipients' investment behaviour.

In section 2 the utilitarian donor creates a distortion on the recipient's intertemporal allocations through its investment decisions. In that context the only way a recipient country could ensure itself a larger portion of the donor's aid budget, was by reducing its investments. By reducing investments it makes sure that its wealth in period two is reduced, which again causes an increase in the amount of aid received. If, however, a recipient country has access to an international credit market it may reduce its wealth in the last period by two different kinds of decisions: It can reduce investments or it can increase borrowing. I now turn to an analysis of the decisions of a recipient country in such an environment.

The fundamental structure of the game is as described in section 2. In stage 1 of the game, however, the two countries choose both a level of investment and a level of debt (D_i). I assume that the level of debt can be chosen at any level in a closed interval; $D_i \in \left[\overline{D}, \overline{\overline{D}}\right]$.[6] Each country can borrow at a fixed interest rate. The debt payments in the second period become: $D_i(1+r)$. The investments are: $I_i = w_{0i} - c_{0i} + D_i$. The intertemporal budget constraint becomes: $c_{1i} \leq f\left(w_{0i} - c_{0i} + D_i\right) - (1+r)D_i + a_i$.

In order to get interior solutions I need to alter the structural assumptions in the analysis in this section. In this section I keep assumption 2.1, whereas assumption 3.2 below replaces

[5]A reason why creditors set limits on the total indebtedness by a developing country may be that an excessively indebted country may choose not to service its debt. There is an extensive literature on sovereign countries' debt, which analyse such situations (see e.g. Eaton and Gersovitz (1981)). Many models of sovereign countries' debt show that credit will be rationed.

assumption 2.2:

Assumption 3.2:

- $\left(\overline{D},\overline{\overline{D}}\right)$ are chosen such that there exist $D',D'' \in\left[\overline{D},\overline{\overline{D}}\right]$ for which $f'\left(w_{0i}+D_i'\right)-(1+r)>0$, and $f'\left(w_{0i}-c_{0i}+D_i''\right)-(1+r)<0, i=1,2$.

- $\max_{i\in 1,2}\left[\max_{D_i,c_{0i}}\left[f(w_{0i}-c_{0i}+D_i)-D_i(1+r)\right]-\min_{D_j,c_{0j}}\left[f\left(w_{0j}-c_{0j}+D_j\right)-D_i(1+r)\right]\right]<\overline{a}$.

Assumption 3.2 is similar to assumption 2.2 in the previous section. It ensures that the donor has sufficient resources to equalise consumption between the two recipient countries, regardless of their choices of investments and indebtedness. In order to obtain this I constrain the possible level of indebtedness. Furthermore, given assumption 3.2 neither recipient will choose indebtedness according to the maximum or minimum possible level (no corner solutions). This is necessary for optimal solutions to be given by the first order conditions to the problems below.

I will first study the Pareto efficient allocations in this framework.

3.1 Pareto Efficient Allocations

A Pareto optimal allocation, when countries have access to a capital market with a fixed interest rate, is found by solving problem 3.1 below:

$$\max_{\{I_1,I_2,a_1,D_1,D_2\}}\left\{\begin{array}{l}\lambda_1\left[u\left(w_{01}-I_1+D_1\right)+\delta u\left(f\left(I_1\right)-(1+r)D_1+a_1\right)\right]+\\ \lambda_2\left[u\left(w_{02}-I_2+D_2\right)+\delta u\left(f\left(I_2\right)-(1+r)D_2+\overline{a}-a_1\right)\right]\end{array}\right\}$$

3.1 subject to

$$a_1\in\left[0,\overline{a}\right],$$
$$I_i\in\left[0,w_{0i}+D_i\right],$$
$$D_i\in\left[\overline{D},\overline{\overline{D}}\right].$$

Where $\lambda_i>0$.

The first order conditions to problem 3.1 are:

[6]The reason I restrict the possible levels of debt to such a closed interval is solely for ease of expositon. The assumption makes it possible with a straightforward application of Brouwer's fixed point theorem when proving proposition 3.1.

3.2 $$\lambda_1 u'\left(f\left(I_1^*\right)-(1+r)D_1^*+a_1^*\right)=\lambda_2 u'\left(f\left(I_2^*\right)-(1+r)D_2^*+\bar{a}-a_1^*\right),$$

3.3 $$u'\left(w_{0i}-I_i^*+D_i^*\right)=\delta(1+r)u'\left(f\left(I_i^*\right)-(1+r)D_i^*+a_i^*\right),$$

3.4 $$(1+r)=f'\left(I_i^*\right),\ i=1,2.$$

Equations 3.2 and 3.3 are similar to the results in section 2. Equation 3.4 highlights the most important change in conditions for Pareto optimality when countries can borrow money in an international credit market. The optimal level of investment can be determined independently of initial wealth and intertemporal preferences. Equation 3.4 resembles the Fischer separability result (Fischer (1930)). When an investor faces a fixed interest rate, optimal investment decisions can be made independently of intertemporal consumption decisions. The investment decisions which maximise the net present value of investments will coincide with the decisions which maximise utility. This is exactly what equation 3.4 tells us.

3.2 Subgame-perfect Equilibria

STAGE 2: Define the wealth for recipient i in the last period as: $w_{1i}=f\left(I_i\right)-(1+r)D_i$. The donor must solve the following problem:

3.5 $$\max_{a_1\in[0,\bar{a}]}\left[u\left(w_{11}+a_1\right)+u\left(w_{12}+\bar{a}-a_1\right)\right].$$

The donor's equilibrium strategy, $\hat{a}_1(\cdot)$, must solve the first order condition for problem 3.5:

3.6 $$\begin{aligned}&u'\left(w_{11}+\hat{a}_1\right)=u'\left(w_{12}+\bar{a}-\hat{a}_1\right)\\ \Leftrightarrow\ &\hat{a}_1(\cdot)=\frac{f(I_2)-(1+r)D_2-\left(f(I_1)-(1+r)D_1\right)+\bar{a}}{2},\ \forall\, I_1,D_1,I_2,D_2.\end{aligned}$$

The donor's equilibrium strategy is a mapping from all possible histories of the game to the set of possible actions at stage 2: $\hat{a}_1:\left[0,w_{01}+D_1\right]\times\left[\overline{D},\overline{\overline{D}}\right]\times\left[0,w_{02}+D_2\right]\times\left[\overline{D},\overline{\overline{D}}\right]\rightarrow[0,\bar{a}]$.

As before the donor will seek to equalise consumption by the two countries in the second period. Let recipient i's consumption in any equilibrium of stage two of the game be denoted by; $\hat{c}_{1i}$. We can easily see that in stage two of the game, each recipient's equilibrium consumption equals half of the available resources:

$$3.7 \qquad \hat{c}_{1i} = \frac{f(I_2)-(1+r)D_2+\left(f(I_1)-(1+r)D_1\right)+\bar{a}}{2}, \text{ for all } I_1, D_1, I_2, D_2.$$

STAGE 1: In stage 1 of the game each of the two countries solves the following problem:

$$3.8 \qquad \max_{\substack{I_i \in [0, w_{0i}+D_i], \\ D_i \in [\underline{D}, \overline{D}]}} \left[u(w_{0i} - I_i + D_i) + u\left(\frac{f(I_i)-(1+r)D_i + f(I_j)-(1+r)D_j + \bar{a}}{2} \right) \right].$$

Each recipient's best response function, $\left(I_i^{br}(I_j, D_j), D_i^{br}(I_j, D_j)\right)$, in this reduced strategic form game is defined by the solution to the following pair of first order conditions:

3.9

$$u'\left(w_{0i} - I_i^{br} + D_i^{br}\right) = \frac{1}{2}\delta f'\left(I_i^{br}\right)u'\left(\frac{f(I_i^{br})-(1+r)D_i^{br} + f(I_j)-(1+r)D_j + \bar{a}}{2} \right), \forall I_j, D_j.$$

3.10

$$u'\left(w_{0i} - I_i^{br} + D_i^{br}\right) = \frac{1}{2}\delta(1+r)u'\left(\frac{f(I_i^{br})-(1+r)D_i^{br} + f(I_j)-(1+r)D_j + \bar{a}}{2} \right), \forall I_j, D_j.$$

Equations 3.9 and 3.10 have the following implication:

$$3.11 \qquad (1+r) = f'\left(I_i^{br}\right), \ i = 1,2.$$

By equation 3.11 the recipient's optimal choice of investments is independent of all other strategic decisions; $I_i^{br} = I_i^{br}(r)$. This implies that each recipient's optimal choice of indebtedness can be expressed as a function only of the other recipient's choice of indebtedness: $D_i^{br} = D_i^{br}(D_j)$. A SPE in this game is a triple of strategies $\left((\hat{I}_1, \hat{D}_1), (\hat{I}_2, \hat{D}_2), \hat{a}_1(\cdot)\right)$ such that the following holds: (i) $\hat{a}_1(\cdot)$ solves equation 3.6. (ii) $\hat{I}_1, \hat{I}_2$ solves equation 3.11, and $\hat{D}_1 = D_1^{br}(\hat{D}_2), \hat{D}_2 = D_2^{br}(\hat{D}_1)$ where $D_i^{br}(\cdot)$ is defined by equations 3.9 and 3.10.

Existence and uniqueness of a SPE in this game is established by the following proposition:

Proposition 3.1

Under assumptions 2.1 and 3.2 there exists a unique subgame-perfect equilibrium $\left((\hat{I}_1, \hat{D}_1), (\hat{I}_2, \hat{D}_2), \hat{a}_1(\cdot)\right)$ *of the 2-stage aid game with an international credit market. The equilibrium is in pure strategies, and is given by the solutions to the first order conditions 3.6, 3.9 and 3.10.*

Proof: See appendix 2.

A comparison of a Pareto optimal allocation and the outcome of the SPE gives the following result:

Proposition 3.2

Consider the subgame-perfect equilibrium of the 2-stage aid game, where recipient countries have access to an international credit market. This SPE entails;

> *(i) efficient investment for both recipients* $\left(\hat{I}_i = I_i^*\right)$, *and*
>
> *(ii) too high indebtedness* $\left(\hat{D}_i > D_i^*\right)$ *for both recipients. By this last statement is meant that there exists a feasible allocation, in which borrowing for both recipients is lower than in the SPE, which is both Pareto efficient and Pareto superior to the SPE of the game.*

Proof:

Consider an outcome of the game, $\left((\hat{I}_1, \hat{D}_1), (\hat{I}_2, \hat{D}_2), \hat{a}_1\right)$, in which investments and borrowing are determined by equations 3.11 and 3.10, respectively. A comparison of equation 3.11 and equation 3.4 establishes that investments are efficient. Next, fix the aid allocation at $\hat{a}_1$, and let each country choose an optimal level of indebtedness for this allocation of aid. These new levels of indebtedness are determined by equation 3.3, for $a_1^* = \hat{a}_1$ and $I_i^* = \hat{I}_i$. Comparing equations 3.3 and 3.10 establishes that these new levels of indebtedness are lower than previously for both recipients; $\hat{D}_i > D^*$. It is immediate that this new allocation is Pareto superior to the original allocation in the game. Moreover, it is possible to choose (λ_1, λ_2) in equation 3.2 such that a Pareto optimal allocation of aid equals the allocation of aid in the

game; $a_1^* = \hat{a}_1$. The new allocation is consequently both Pareto efficient and Pareto superior to the allocation in the game.
Q.E.D.

Part (i) of proposition 2 is somewhat surprising; there are no distortions on investments in the aid game, when the recipients have access to an international credit market. The intuition for this result is as follows. Suppose a recipient has decided on an optimal level of consumption in the first period. The country can choose many different combinations of investment and debt which keep consumption in the first period constant. These decisions will only affect the country's level of wealth in the last period. In the last period, however, both countries know that they will consume half of the total resources which are available (by total resources I mean ($w_{11} + w_{12} + \bar{a}$)). But this means that each country, for any given level of consumption in the first period, will try to maximise the total resources available tomorrow. Maximisation of the total resources available tomorrow is only achieved when the net present value of the investments is maximised. Hence, the aid recipients invest efficiently in the context described above.

Part (ii) of the proposition shows that in this game there are still distortions to a country's intertemporal decisions: The countries tend to borrow too much money in the international credit market, compared with an efficient allocation. Consequently they consume too much in the initial period and too little in the last period. The fundamental reason for this effect is that increased indebtedness for one recipient will be paid partly by the other recipient, through redistributions of the aid budget. Consequently, each recipient chooses an excessively high level of indebtedness. This result is more or less the same as when recipient countries did not have access to the international credit market, as analysed in section 2. The difference is that the intertemporal distortion in consumption will happen solely because the recipients choose too much borrowing, not because they choose too low investments. Figure 4 below illustrates a recipient country's choice in a SPE compared to a Pareto optimal allocation, when it has the possibility of borrowing money at a fixed interest rate.

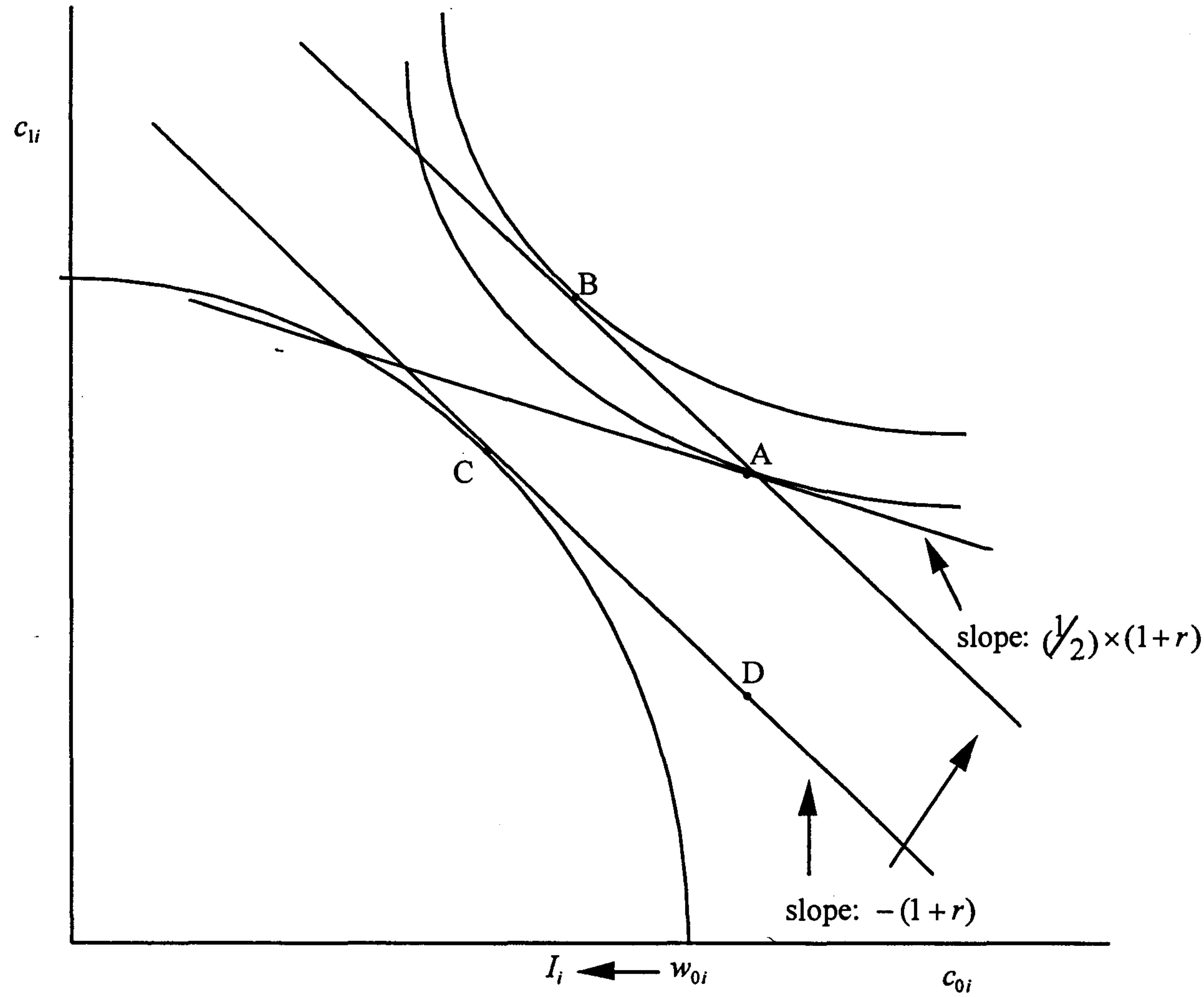

Figure 4

In the figure the country invests the horizontal distance between w_{0i} and the point C. This is the efficient level of investment. In a SPE the country chooses consumption vector A. At this point the country's indifference curve is tangent to a line with slope $-\frac{1}{2}(1+r)$. In equilibrium he receives the vertical distance between points A and D as aid. This allocation can be compared with a Pareto efficient allocation in which the countries receive the same disbursements of aid as in the SPE. The investments are still the horizontal distance between w_{0i} and the point C. The Pareto efficient consumption vector, on the other hand, is at point B. At this point the country receives the same amount of aid as in the SPE. However, it is keeping the total benefit from investments. In the efficient allocation the indifference curve is tangent to the line going through point A with slope $-(1+r)$. In this efficient solution it consumes less in the initial period and more in the last period, compared with the outcome of the game.

4. Recipient Countries' Choice of Long-Term vs Short-Term Investments

In sections 2 and 3 I analysed how the presence of competition for aid affected the level of investments. The quantity which is used for investments is only one important aspect of an investment policy. A developing country must also decide on the duration of its investments. In other words it must allocate investment resources between projects which yield a payoff in the near future, and projects which will increase production a long time from the time of investment. In this section I analyse how competition for aid affects a recipient country's choice of long-term versus short-term-investments. For this purpose I extend the two-stage game in sections 2 and 3 to a four-stage game. In the first stage the two countries choose the level of investment in projects which gives a payoff after one period, and in projects which gives a payoff after two periods. The short-term investment in period zero is denoted I_{0i}, and gives a payoff of $f(I_{0i})$ one period later. The long-term investment is denoted I_i^L, and gives a payoff of $g(I_i^L)$ two periods later. The short-term production function is the same as in section 2 and 3. The long-term production function satisfies similar assumptions: $g' > 0, g'' < 0, g'(0) = \infty$. At stage two, after having observed the payoff from the short-term investments, the donor chooses its allocation of aid between the two countries in that period (a_{11}, a_{12}). He has a fixed amount of resources to allocate to the recipients in this period; $a_{11} + a_{12} \leq \bar{a}_1$. At stage 3 the two countries again choose the level of investments in a short-term project (I_{1i}) which gives a payoff one period later $(f(I_{1i}))$. The production function for short-term investments is constant over time. At stage 4 of the game the donor first observes the payoff from both the long-term investments which were made two periods ago ($g(I_i^L)$); and the payoff from short-term investments made in the previous period $(f(I_{1i}))$. Then he chooses how to allocate a fixed sum of aid between the recipients at this last stage; $a_{21} + a_{22} \leq \bar{a}_2$. At this last stage the recipients of aid consume the sum of their payoffs from long and short-term investments, in addition to foreign aid received in that period.

The utility of each recipient is given by: $U_i = u(c_{0i}) + \delta u(c_{1i}) + \delta^2 u(c_{2i})$. The consumption for a recipient in the three periods is given by; $c_{0i} = w_{0i} - I_{0i} - I_i^L$, $c_{1i} = f(I_{0i}) - I_{1i} + a_{1i}$, $c_{2i} = f(I_{1i}) + g(I_i^L) + a_{2i}$. As before the donor's utility equals the sum of the recipient's utilities: $U_D = U_1 + U_2$.

The timing of events are shown in figure 5.

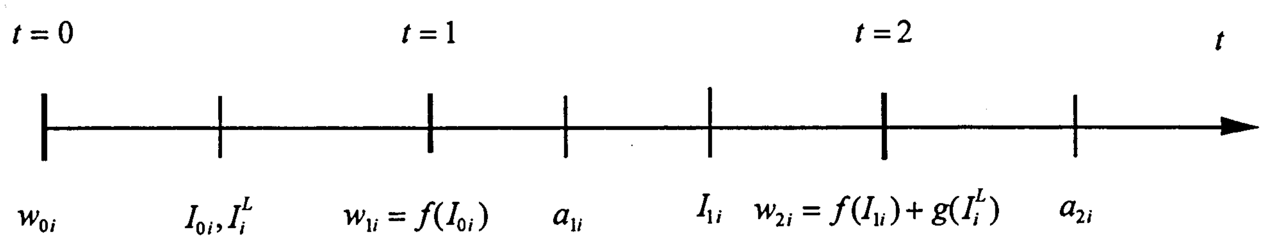

Figure 5

The structure of this four-stage and three-period game is shown in figure 6.

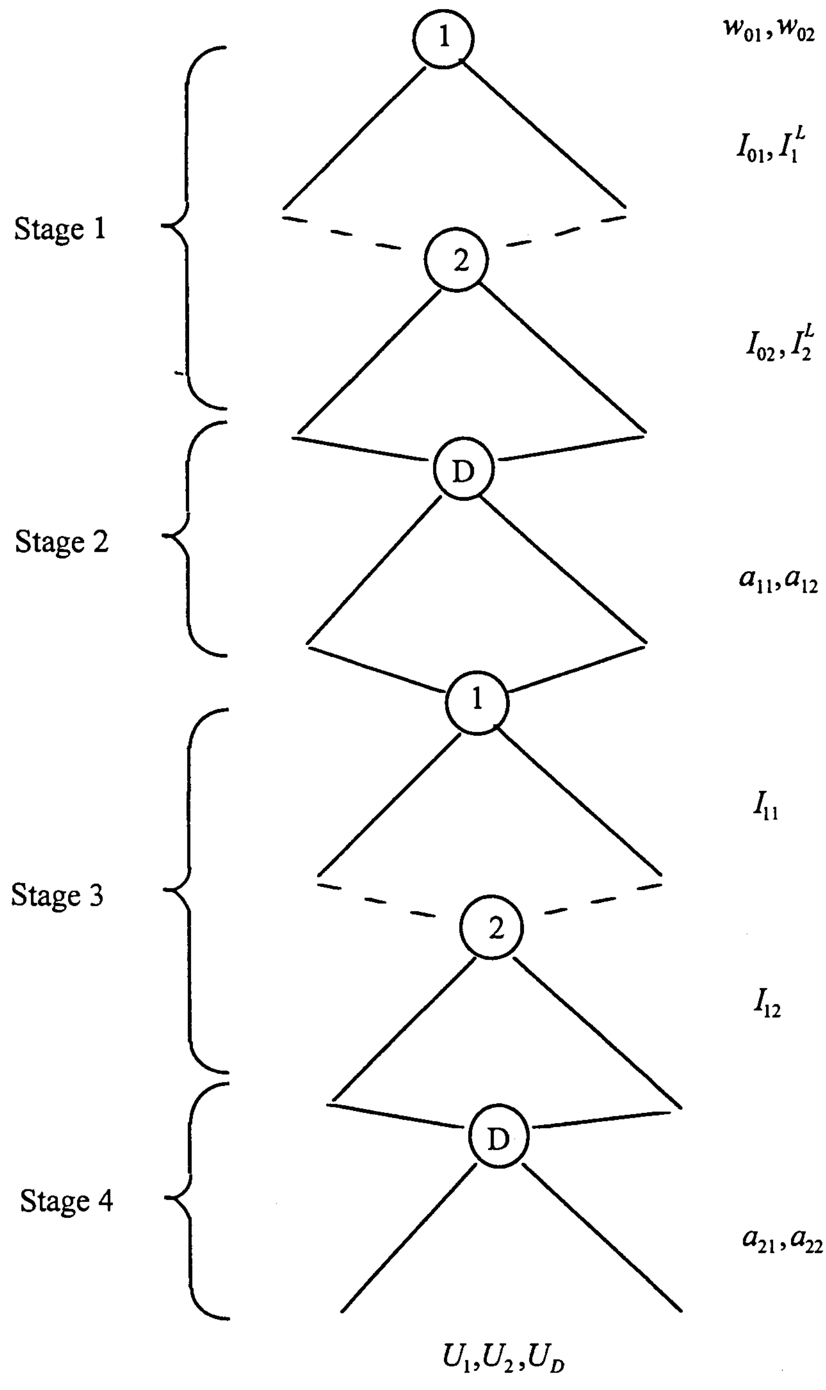

Figure 6

The primary focus of the analysis in this section is whether competition for aid leads to distortions in the allocations of long-term versus short-term investments.

The basic assumptions in the analysis are:

Assumption 4.1: f, g, u *are three times continuously differentiable* (C^3),

$u' > 0, u'' < 0, u'(0) = \infty$, $f' > 0, f'' < 0, f'(0) = \infty$,

$g' > 0, g'' < 0, g'(0) = \infty$.

Assumption 4.2:

- $\left[f(w_{0i}) - f(0)\right] < \bar{a}_1$.
- $\left[g(w_{0i}) + f\left(f(w_{0i}) + \bar{a}_1\right)\right] - \left[f(0) + g(0)\right] < \bar{a}_2$.

Assumption 4.1 replaces assumption 2.1. Note that I assume that production and utility functions are three times continuously differentiable. This is necessary to ensure that the maximand for the donor in stage 2 of the game (the maximand in equation 4.13) is twice continuously differentiable. For further elaborations see appendix 3.1. Assumption 4.2 states that the donor's endowment of aid is sufficient to ensure equalisation of the two recipients' resources (after aid has been received) in both period 1 and 2.

In the following paragraphs I calculate conditions for efficiency, with particular emphasis on a condition for efficient allocation of resources between short-term and long-term investments. I also define what is meant by overallocation of resources to long-term investments.

4.1 *Efficient Allocation of Resources between Long-Term and Short-Term Investments.*

A Pareto optimal allocation is found by solving the following problem:

$$\max_{\left\{\substack{I_{01}, I_{02}, I_1^L, I_2^L, I_{11}, I_{12}, \\ a_{11}, a_{21},}\right\}} \left\{ \begin{array}{l} \lambda_1 \left[\begin{array}{l} u\left(w_{01} - I_{01} - I_1^L\right) + \delta u\left(f\left(I_{01}\right) + a_{11} - I_{11}\right) \\ + \delta^2 u\left(f\left(I_{11}\right) + g\left(I_1^L\right) + a_{21}\right) \end{array} \right] + \\ \lambda_2 \left[\begin{array}{l} u\left(w_{02} - I_{02} - I_2^L\right) + \delta u\left(f\left(I_{02}\right) + \bar{a}_1 - a_{11} - I_{1i}\right) \\ + \delta^2 u\left(f\left(I_{12}\right) + g\left(I_2^L\right) + \bar{a}_2 - a_{21}\right) \end{array} \right] \end{array} \right\}$$

4.1 subject to

(i) $a_{11} \in \left[0, \bar{a}_1\right]$ (ii) $a_{21} \in \left[0, \bar{a}_2\right]$,

(iii) $I_{0i} \in \left[0, w_{0i} - I_i^L\right]$, (iv) $I_i^L \in \left[0, w_{0i} - I_{0i}\right]$

(v) $I_{1i} \in \left[0, f\left(I_{0i}\right) + a_{1i}\right]$.

By assumptions 4.1 and 4.2 the first order conditions are necessary for a Pareto optimal allocation:

$$4.2(a_{21}) \qquad \lambda_1 u'\left(f\left(I_{11}^*\right)+g\left(I_1^{L*}\right)+a_{21}^*\right)=\lambda_2 u'\left(f\left(I_{12}^*\right)+g\left(I_2^{L*}\right)+\bar{a}_2-a_{21}^*\right),$$

$$4.3(a_{11}) \qquad \lambda_1 u'\left(f\left(I_{01}^*\right)-I_{11}^*+a_{11}^*\right)=\lambda_2 u'\left(f\left(I_{02}^*\right)-I_{12}^*+\bar{a}_1-a_{11}^*\right),$$

$$4.4(I_{0i}) \qquad u'\left(w_{0i}-I_{0i}^*-I_i^{L*}\right)=\delta f'\left(I_{0i}^*\right)u'\left(f\left(I_{0i}^*\right)-I_{1i}^*+a_{1i}^*\right),\ i=1,2,$$

$$4.5(I_{1i}) \qquad u'\left(f\left(I_{0i}^*\right)-I_{1i}^*+a_{1i}^*\right)=\delta f'\left(I_{1i}^*\right)u'\left(f\left(I_{1i}^*\right)+g\left(I_i^{L*}\right)+a_{2i}^*\right),\ i=1,2,$$

$$4.6(I_i^L) \qquad u'\left(w_{0i}-I_{0i}^*-I_i^{L*}\right)=\delta^2 g'\left(I_i^{L*}\right)u'\left(f\left(I_{1i}^*\right)+g\left(I_i^{L*}\right)+a_{2i}^*\right),\ i=1,2.$$

Equation 4.4-4.6 yields:

$$4.7 \qquad \frac{f'\left(I_{0i}^*\right)\times f'\left(I_{1i}^*\right)}{g'\left(I_i^{L*}\right)}=1,\ i=1,2.$$

Equation 4.7 is a rule for Pareto efficient allocation of resources between a long-term investment strategy and a short-term investment strategy. The long-term investment strategy consists in marginally increasing long-term investments in period 0, keeping consumption in period 1 and aid allocations in period 1 and 2 constant. The short-term investment strategy consists in a marginal increase in short-term investments in period 0, and reinvesting the increased production from this investment in a new short-term project in period 1 $\left(dI_{1i}/dI_{0i}=f'\left(I_{0i}\right)\right)$. The payoff from the short-term investment strategy becomes; $-u'\left(c_{0i}\right)+\delta^2 f'\left(I_{0i}^*\right)f'\left(I_{1i}^*\right)u'\left(c_{2i}\right),\ i=1,2$. The payoff from the long-term investment strategy is; $-u'\left(c_{0i}\right)+\delta^2 g\left(I_1^{L*}\right)u'\left(c_{2i}\right),\ i=1,2$. In an optimal allocation the payoff from these two alternative investment strategies must be equal, which implies equation 4.7.

An equilibrium of the game yields overinvestments in long-term projects if the long-term investment strategy yields a lower marginal payoff than the short-term strategy. If the triple $\left(\hat{I}_{0i},\hat{I}_{1i},\hat{I}_i^L\right)$ are outcomes in an equilibrium of the game, overaccumulation in long-term investments are equivalent to:

$$4.8 \qquad \frac{f'\left(\hat{I}_{0i}\right)\times f'\left(\hat{I}_{1i}\right)}{g'\left(\hat{I}_i^L\right)}>1,\ i=1,2.$$

The numerator in the quotient in equation 4.7 and 4.8 is the increased production in period 2 by the short-term investment strategy. The denominator is the increased production in period 2 by increasing the long-term investment strategy. The criterion in equation 4.8 simply states that overallocation of resources to long-term investments occurs if the long-term investment strategy yields a <u>lower</u> production increase in period 2 than the the short-term investment strategy.

I now turn to an analysis of subgame-perfect equilibria of the game.

4.2 Subgame-perfect Equilibria

<u>STAGE 4:</u> The donor's strategy in the last subgame is in general a function of the history of the game. Let h^4 be one possible history at the stage 4 of the game and let H^4 be all such possible histories. The donor's equilibrium strategy consists in allocating aid according to the following rule:

4.9 $$\hat{a}_{21}(\cdot)=\frac{f(I_{12})+g(I_2^L)-(f(I_{11})+g(I_1^L))+\bar{a}_2}{2},\forall h^4 \in H^4.$$

This strategy implies that both recipients consume equally much in the last stage of the game:

4.10 $$\hat{c}_{2i}=\frac{f(I_{12})+g(I_2^L)+f(I_{11})+g(I_1^L)+\bar{a}_2}{2}.$$

<u>STAGE 3:</u> At stage three of the game each of the recipients solves the following problem:

4.11 $$\max_{I_{1i}}\left\{u(f(I_{0i})+a_{1i}-I_{1i})+\delta u\left(\frac{f(I_{1i})+g(I_i^L)+f(I_{1j})+g(I_j^L)+\bar{a}_2}{2}\right)\right\}$$

subject to $\qquad I_{1i}\leq f(I_{0i})+a_{1i}.$

Each recipient's best response function for one possible history of the game at stage 3 (h^3) is defined by the solution to the first order condition:

4.12 $$u'(f(I_{0i})+a_{1i}-I_{1i}^{br})=\frac{1}{2}\delta f'(I_{1i}^{br})u'\left(\frac{f(I_{1i}^{br})+g(I_i^L)+f(I_{1j})+g(I_j^L)+\bar{a}_2}{2}\right).$$

In the reduced strategic form game of stage three, each recipient's optimal choice of investments can be expressed as a function of the history of the game and the other recipient's choice of investments: $I_{1i}^{br} = I_{1i}^{br}\left(h^3, I_{1j}\right)$. A Nash equilibrium of this game is defined by a pair of strategies $\hat{I}_{11}, \hat{I}_{12}$ such that; $\hat{I}_{11} = I_{11}^{br}\left(h^3, \hat{I}_{12}\right) \wedge \hat{I}_{12} = I_{12}^{br}\left(h^3, \hat{I}_{11}\right)$, where $I_{1i}^{br}(\cdot)$ is defined by equation 4.12. In the proof of proposition 4.1 (appendix 3.2) I show that the reduced strategic form game has a unique Nash equilibrium for all possible histories of the game. In a SPE of this game all strategies must be specified as mappings from the set of all possible histories to the set of possible actions by the recipients at stage 3. By inspection of equation 4.12 we see that the optimal strategy by each player at stage 3 depends only on a subset of the previous history. Hence, player i's equilibrium strategy in stage 3 can be expressed as; $\hat{I}_{1i}\left(I_{01}, I_1^L, I_{02}, I_2^L, a_{11}\right)$, where the functions; $\hat{I}_{11}(\cdot), \hat{I}_{12}(\cdot)$, solve equation 4.12 simultaneously. In the analysis of SPE of the subgame starting in stage 2 of the game, it is of particular interest to study how the donor's choice of aid allocation affects the equilibrium outcome of stage 3 in the game. Thus, I suppress the other arguments, and express the recipient's equilibrium strategies in stage 3 as follows: $\hat{I}_{1i} = \hat{I}_{1i}\left(a_{11}\right)$.

For notational convenience I define the total resources available in period two as: $\overline{w}_2 = f\left(I_{11}\right) + g\left(I_1^L\right) + f\left(I_{12}\right) + g\left(I_2^L\right) + \overline{a}_2$.

<u>STAGE 2:</u> At stage two of the game, the donor solves problem 4.13:

$$4.13 \quad \max_{\{a_{11}\}} \left\{ \begin{aligned} & u\left(f\left(I_{01}\right) + a_{11} - \hat{I}_{11}\left(a_{11}\right)\right) + \delta u\left(\frac{f\left(\hat{I}_{11}\left(a_{11}\right)\right) + g\left(I_1^L\right) + f\left(\hat{I}_{12}\left(a_{11}\right)\right) + g\left(I_2^L\right) + \overline{a}_2}{2}\right) + \\ & u\left(f\left(I_{02}\right) + \overline{a}_1 - a_{11} - \hat{I}_{12}\left(a_{11}\right)\right) + \delta u\left(\frac{f\left(\hat{I}_{11}\left(a_{11}\right)\right) + g\left(I_1^L\right) + f\left(\hat{I}_{12}\left(a_{11}\right)\right) + g\left(I_2^L\right) + \overline{a}_2}{2}\right) \end{aligned} \right\}$$

$$\text{subject to;} \qquad a_{11} \in \left[0, \overline{a}_1\right].$$

Assuming interior solutions, an optimal strategy $\left(\hat{a}_{11}(\cdot)\right)$ for the donor must solve the first order condition for all possible information sets (histories) $h^2 \in H^2$:

$$u'\left(f(I_{01})+\hat{a}_{11}-\hat{I}_{11}\right)\left[1-\frac{\partial\hat{I}_{11}}{\partial a_{11}}\right]+\frac{1}{2}\delta u'\left(\frac{\overline{w}_2}{2}\right)\left[\frac{\partial\hat{I}_{11}}{\partial a_{11}}f'(\hat{I}_{11})+\frac{\partial\hat{I}_{12}}{\partial a_{11}}f'(\hat{I}_{12})\right]+$$

4.14

$$u'\left(f(I_{02})+\overline{a}_1-\hat{a}_{11}-\hat{I}_{12}\right)\left[-1-\frac{\partial\hat{I}_{12}}{\partial a_{11}}\right]+\frac{1}{2}\delta u'\left(\frac{\overline{w}_2}{2}\right)\left[\frac{\partial\hat{I}_{11}}{\partial a_{11}}f'(\hat{I}_{11})+\frac{\partial\hat{I}_{12}}{\partial a_{11}}f'(\hat{I}_{12})\right]=0.$$

The first order condition for the recipients (equation 4.12) enables us to manipulate the above first order condition into:

4.15 $$u'(c_{11})\left[1+\frac{\partial\hat{I}_{11}}{\partial a_{11}}\right]=u'(c_{12})\left[1-\frac{\partial\hat{I}_{12}}{\partial a_{11}}\right].$$

Both symmetric and asymmetric solutions to the donor's problem are, in theory, conceivable. By a symmetric solution I mean a situation in which the donor chooses to allocate his resources so that the amount which is available for consumption and investments in period 1 is equalised between the recipient countries. This concept is central for the discussion in this section, and will therefore be defined formally:

Definition

A *symmetric equilibrium* in the subgame starting in stage 2 is an equilibrium outcome in which the donor allocates its resources according to the following rule: $f(I_{01})+\hat{a}_{11}=f(I_{02})+(\overline{a}_1-\hat{a}_{11})$.

The focus of this chapter is on symmetric equilibria. However, it is worthwhile to discuss the possibility of asymmetric equilibria in the model of this chapter. The donor allocates resources so that the sum of the recipients' discounted utilities is maximised. The sum of the two recipients' utility in the *current* period is maximised when their consumption is equal. Thus, maximisation of the sum of the two recipient's utility in the current period is an argument for choosing a symmetric solution. However, if the donor chooses to allocate more resources to one of the recipients, this will influence both consumption and investments in period 1. It is conceivable that an asymmetric solution will increase the discounted sum of the two recipients' *future* utility, because total investments in an asymmetric solution may be higher than in the symmetric solution. When one moves from a symmetric solution towards an asymmetric solution, the richer country increases its investments and the poorer country reduces its investments. It is conceivable that the increase in investments by the richer country exceeds the reduction in the poorer country's investment. We know that the donor prefers increased total investments, because there is an underallocation of resources to investments in any SPE of the subgame starting in stage 3. Thus, this possibility of achieving increased total investments by allocating resources asymmetrically, may lead the donor to choose an asymmetric solution as his optimal strategy at stage 2.

As already pointed out, I find it more relevant to study symmetric equilibria. There are intuitive, empirical and theoretical reasons for this. Intuitively it seems unreasonable that donors of aid should contribute to inequality between recipients. Moreover, empirical findings show a strong tendency to allocate larger amounts to poor countries than to relatively rich countries (see Tumbull and Wall (1994)). From a theoretical perspective it is important to note that a symmetric solution is always a local maximum for the donor in subgames starting in stage 2. This result is presented formally:

<u>Lemma 4.1</u>

Consider any subgame-perfect equilibrium of the subgame starting at stage 2. If assumptions 4.1 and 4.2 hold, the symmetric solution is a local maximum for the donor.

Proof: See appendix 3.1.

Throughout the analysis, I will only study symmetric equilibria. I have not been able to find sufficient conditions for the symmetric solution to be the global maximum. As already argued, however, there are good reasons to focus on this case. In the symmetric equilibrium the donor will allocate its aid resources according to the rule:

4.16 $$\hat{a}_{11} = \frac{f(I_{02}) - f(I_{01}) + \bar{a}_1}{2}, \forall I_{01}, I_{02}.$$

Hence, the donor's equilibrium strategy at stage 2 of the game is $\hat{a}_{11}(\cdot)$ as defined by equation 4.16. Consumption in period one by each recipient in the symmetric equilibrium is thus:

4.17 $$\hat{c}_{1i} = \frac{f(I_{01}) + f(I_{02}) + \bar{a}_1}{2} - \hat{I}_{1i}.$$

In a symmetric solution it is clear that $\hat{c}_{11} = \hat{c}_{12}$ and $\partial \hat{I}_{11} / \partial a_{11} = - \partial \hat{I}_{12} / \partial a_{11}$.

STAGE 1: Finally, I will find Nash equilibria at stage 1 of the game, when this game is analysed as a strategic form game in which the outcome is determined by the equilibrium outcome at stage 2, 3, and 4, for all possible actions at stage 1. I will assume that the symmetric equilibrium is played from stage 2 and on. The recipients will solve the following problem:

$$\max_{\{I_{0i},I_i^L\}}\left\{\begin{aligned}&u\left(w_{0i}-I_{0i}-I_i^L\right)+\delta u\left(\frac{f(I_{0i})+f(I_{0j})+\bar{a}_1}{2}-\hat{I}_{1i}(\cdot)\right)\\&+\delta^2 u\left(\frac{f(\hat{I}_{1i}(\cdot))+g(I_i^L)+f(\hat{I}_{1j}(\cdot))+g(I_i^L)+\bar{a}_2}{2}\right)\end{aligned}\right\} \tag{4.18}$$

$$\text{subject to} \qquad \left(I_{0i},I_i^L\right)\in\left\{R_+^2 : I_i^L+I_{0i}\le w_{0i}\right\}.$$

In appendix 3.2 I show that the functions $\hat{I}_{1i}(\cdot)$, $\hat{I}_{1j}(\cdot)$ are twice continuously differentiable in I_{0i}, I_i^L. This property, and an assumption of an interior solution to problem 4.18, ensures that the optimal solution $\left(I_{0i}^{br}, I_i^{L^{br}}\right)$ for each recipient must solve the first order conditions to problem 4.18. By the envelope theorem these can be expressed as:

$$\begin{aligned}&u'\left(w_{0i}-I_{0i}^{br}-I_i^{L^{br}}\right)=\\&\frac{1}{2}\delta f'\left(I_{0i}^{br}\right)u'\left(\frac{f\left(I_{0i}^{br}\right)+f\left(I_{0j}\right)+\bar{a}_1}{2}-\hat{I}_{1i}(\cdot)\right)+\frac{1}{2}\delta^2 f'\left(\hat{I}_{1j}(\cdot)\right)u'\left(\frac{\overline{w}_2}{2}\right)\frac{\partial\hat{I}_{1j}}{\partial I_{0i}},\forall I_{0j},I_j^L,\end{aligned} \tag{4.19}$$

$$\begin{aligned}&u'\left(w_{0i}-I_{0i}^{br}-I_i^{L^{br}}\right)=\\&\frac{1}{2}\delta^2 u'\left(\frac{f(\hat{I}_{1i}(\cdot))+f(\hat{I}_{1j}(\cdot))+g\left(I_i^{L^{br}}\right)+g\left(I_j^L\right)+\bar{a}_2}{2}\right)\left[g'\left(I_i^{L^{br}}\right)+f'\left(\hat{I}_{1j}\right)\frac{\partial\hat{I}_{1j}}{\partial I_i^L}\right],\forall I_{0j},I_j^L.\end{aligned} \tag{4.20}$$

An equilibrium of the reduced strategic form game of stage 1 is a pair of strategies; $\left(\left(\hat{I}_{01},\hat{I}_1^L\right),\left(\hat{I}_{02},\hat{I}_2^L\right)\right)$, such that $\left(\hat{I}_{0i},\hat{I}_i^L\right)=\left(I_{0i}^{br}\left(\hat{I}_{0j},\hat{I}_j^L\right),I_i^{L^{br}}\left(\hat{I}_{0j},\hat{I}_j^L\right)\right)$ hold simultaneously for both countries, and where the best response function $\left(I_{0i}^{br}(\cdot),I_i^{L^{br}}(\cdot)\right)$ is defined by equations 4.19 and 4.20. I will assume that there is a subgame-perfect equilibrium of the game, in which strategies are determined by the first order conditions. Let $\left(\hat{I}_{01},\hat{I}_1^L,\hat{I}_{11},\hat{I}_{02},\hat{I}_2^L,\hat{I}_{12}\right)$ be the levels of investment which occur in an equilibrium. From the recipient's first order condition in the subgame starting in stage 3 (equation 4.12), we can manipulate equations 4.19 and 4.20 into:[7]

$$4.21 \quad g'(\hat{I}_1^L) - f'(\hat{I}_{01})f'(\hat{I}_{11}) = -\frac{1}{2} f'(\hat{I}_{01})f'(\hat{I}_{11}) + f'(\hat{I}_{12})\left[\frac{\partial \hat{I}_{12}}{\partial I_{01}} - \frac{\partial \hat{I}_{12}}{\partial I_1^L}\right].$$

This condition determines how investments are allocated between long-term and short-term projects in equilibria of the game. Simple manipulation of equation 4.8 shows that there is overinvestment in long-term projects relative to short-term projects if:

$$4.22 \quad g'(\hat{I}_1^L) - f'(\hat{I}_{01})f'(\hat{I}_{11}) < 0.$$

If the left-hand side of 4.22 is equal to zero, the allocation of investments between long- and short-term investments is Pareto efficient.

A symmetric subgame-perfect equilibrium of the 4-stage aid game consists of a 3-tuple of strategies; $\left((\hat{I}_{01}, \hat{I}_1^L, \hat{I}_{11}(\cdot)), (\hat{I}_{02}, \hat{I}_2^L, \hat{I}_{12}(\cdot)), (\hat{a}_{11}(\cdot), \hat{a}_{12}(\cdot))\right)$ for which the following holds: The pair, $\left((\hat{I}_{01}, \hat{I}_1^L), (\hat{I}_{02}, \hat{I}_2^L)\right)$, is a (Nash) equilibrium in the reduced stage 1 game; $\left(\hat{I}_{0i}, \hat{I}_i^L\right) = \left(I_{0i}^{br}\left(\hat{I}_{0j}, \hat{I}_j^L\right), I_i^{Lbr}\left(\hat{I}_{0j}, \hat{I}_j^L\right)\right)$. The donor's strategy, $\hat{a}_{11}(\cdot)$ solves equation 4.14 for all possible histories at stage 2 of the game. The pair, $\left(\hat{I}_{11}(\cdot), \hat{I}_{12}(\cdot)\right)$, constitute a (Nash) equilibrium in stage 3; $\hat{I}_{11} = I_{11}^{br}\left(\hat{I}_{12}\right) \wedge \hat{I}_{12} = I_{12}^{br}\left(\hat{I}_{11}\right)$, for all possible histories at stage 3. And, finally, at stage 4 the donor's equilibrium strategy $\hat{a}_{21}(\cdot)$ is defined by equation 4.9 for all possible histories at stage 4 of the game.

I have not been able to establish existence of a subgame-perfect equilibrium for this 4-stage game, based solely on standard assumptions on utility- and production functions. Formally, the game in this section is an infinite-action, multistage game with imperfect information. To my knowledge there are only existence results on such games when the players have <u>perfect</u> information. Harris (1985), and Hellwig and Leininger (1987) are examples of research which establish existence of subgame-perfect equilibria in infinite-action, multistage games, but

[7]Equation 4.21 is obtained by first using equation 4.12 to eliminate $u'(c_{11})$ from the right hand side of equation 4.19. Setting the right hand sides of equation 4.19 and 4.20 equal to each other yields the result.

where the players move sequentially (perfect information). The results in these papers are therefore not applicable for the game in this section. Hence, I consider another obvious approach to establish existence of an equilibrium. By backward induction I try to find (Nash) equilibria in each subgame. Each subgame can be analysed as a reduced (strategic form) game, in which payoffs are defined by subsequent equilibrium strategies. Hence, the problem consists in finding equilibrium strategies for the last stage of the game, and then showing that there exists a Nash equilibrium at the second to last stage, given the equilibrium strategies in the last stage. This method is successful for all stages but the first. At the first stage, however, I am unable to establish existence of a Nash equilibrium in the reduced stage 1 game. The problem is the following: At stage 2 I am not able to establish continuity of the donor's equilibrium strategy, $\hat{a}_{11}(\cdot)$, with respect to $I_{01}, I_1^L, I_{02}, I_2^L$. I can not exclude the possibility of asymmetric solutions, in which the donor's optimal strategy at stage 2 is to allocate aid so that the two recipient countries have very different amounts at their disposal. Moreover, it can be optimal for the donor to "switch" from a symmetric to an asymmetric equilibrium for small changes in previous actions: $I_{01}, I_1^L, I_{02}, I_2^L$. This again implies that one can not guarantee continuity of the best response functions at stage 1 of the game. Hence, Brouwer's fixed-point theorem does not apply.

A third strategy for establishing existence of equilibria in this game is to look for general results regarding existence of Nash equilibria in strategic form games with infinite action spaces. Dasgupta and Maskin (1986) establishes sufficient conditions for existence of a pure-strategy Nash equilibrium for such games. One of the conditions is that each player's payoff function has a "continuous maximum". Intuitively speaking, this condition is fulfilled if one player's best (optimal) payoff is a continuous function of the other player's actions. This condition is however not fulfilled in the reduced strategic form game at stage 1. The problem is the same as described above; a small change in the investment level for one recipient of aid might lead the donor to implement substantial reallocations of aid at stage 2. Hence, one player's payoff might be altered dramatically when the other player changes his actions marginally. The characterisation of equilibria in this section will consequently be done based on the assumption that an equilibrium actually exists. Furthermore, I assume that any such solution satisfies the first order conditions for optima.

My main interest in this section is whether competition for aid leads to a distortion in the recipients' allocation of investments between long-term and short-term projects. This question is answered by the following proposition:

Proposition 4.1

In any interior, symmetric subgame-perfect equilibrium of the 4-stage aid game, the recipients overallocate resources to long-term projects relative to short-term projects.

Proof:

See appendix 3.2.

Comparing equations 4.21 and 4.22 shows that there are two elements on the right hand side of equation 4.21 which distorts the recipient's choice of term structure for investments projects in equilibria of the game. The first element, $-(1/2)\times f'(\hat{I}_{01})f'(\hat{I}_{11})$, reflects an effect which can be called "double punishment" of short-term investments. The second element, $f'(\hat{I}_{12})\left[\frac{\partial \hat{I}_{12}}{\partial I_{01}}-\frac{\partial \hat{I}_{12}}{\partial I_1^L}\right]$, is a strategic effect. When a recipient changes his long-term and short-term investments in the initial period, the other recipient's choice of investements in the intermediate period will be affected. Let me explain these effects in more detail.

The so-called "double punishment" effect has a fairly straightforward explanation: A long-term project yields increased production in only the last period. If, on the other hand, the recipient chooses to allocate his resources to a series of short-term projects, his production will be increased on both period 1 and period 2. The donor's allocation rule for aid implies that the recipient keeps only half of any increase in production. From a recipient's point of view, the donor's aid policy is equivalent to a 50% tax rate on any increased production caused by investments. The effective "tax rate" on a long-term project is 50%. The effective "tax rate" on a series of short-term projects, however, is 75%. One may say that the recipient will be "punished" twice if he uses a short-term investment strategy, whereas he will be "punished" only once if he uses a long-term investment strategy. This effect clearly distorts the recipients' investment decisions towards overallocating resources to long-term investment projects.

There are, however, some strategic effects which partially offsets this distortion. When a recipient changes the investment policy, it will affect the other player's equilibrium investment in the intermediate period (period 1). As a first observation, note that any increase in wealth in the last period will cause a reduction in period 1 investments by both recipients. Similarly, an increase in wealth in period 1 will cause an increase in period 1 investments by both recipients. When one recipient changes his short- and long-term investment strategy, this affects the total wealth in both period 1 and 2. Thus, as a consequence investments by the other recipient in period 1 will change. Let me explain this more carefully.

If a recipient increases his long-term investments, the total wealth and consumption by both recipients increases in the last period. An increase in consumption in the last period causes a reduction in investments in the intermediate period (period 1). Consequently, the other recipient reduces his period 1 investments when one recipient increases his long-term investment. This affects the recipient negatively; when the other player reduces his investments the recipient will have less to consume in period 2. Thus, this strategic effect discourages the recipient from increasing his long-term investments. Furthermore, if the recipient increases his short-term investments, the total resources in period 1 increase. This makes investments in period 1 more desirable for both recipients. Consequently, the other recipient will increase his investments in period 1. This increase is favourable for the recipient, because total wealth and consumption in period 2 increases. Thus, this strategic effect makes it more desirable with short-term investments. There are consequently two strategic effects which makes it more desirable with short-term investments as opposed to long-term investments. Proposition 4.1 shows that the direct "double punishment" effect, which distorts investments towards too much long-term investments, is stronger than the strategic effects which make long-term projects less desirable. The conclusion is that, in a symmetric equilibrium, there is overallocation of resources to long- term investments relative to short-term investments.

In this section I have analysed the effect of competition for aid on the allocation of resources into investment projects of various length. One might ask how this result would be affected if the recipients had access to an international credit market. We recall from section 3 that in such a context for the two-stage game, the recipients invested efficiently. A similar result can easily be established in this four-stage game: If the recipient countries could choose a series of short-term investments and loans, with the same termination date, they would choose efficient short-term investments. Similarly, if the country could choose a long-term investment and a long-term loan (with the same termination date), long-term investments would be efficient as well. There would consequently not be a distortion in the allocation of resources between short-term and long-term projects. The underlying reason and intuition for this result is exactly the same as described in section 3, and I will not go through a detailed analysis of this topic in this chapter.

5. The Effect of Aid on a Recipient Country's Risk-Taking

In the past sections I have treated the payoff from investments as fully predictable. However, the profitability of investments for a majority of projects depends on the outcome of random events which will be revealed after the investment decision has been made: The payoff from investing in a water drainage system may for example depend on future rainfall, the profitability of investments in oil-drilling or mining will depend on uncertain market prices and so forth. Thus, riskiness of projects is present for a wide range of development projects. Furthermore, it is obvious that the presence of aid changes the risk exposure which a recipient country faces from an investment. In general aid will depend negatively on the level of GDP: If a very good outcome occurs, aid disbursements are likely to be reduced. Aid will most likely increase if the recipient country experiences a bad outcome. It does therefore seem likely that the presence of aid will affect a recipient country's preferences towards risky projects. Taking this fact into account seems to be relevant when analysing the effects of aid. The question which will be analysed in this section is whether aid leads to inefficient risk-taking by the recipient country.

The topic will be analysed by constructing a simple portfolio model in which the agents have the opportunity to invest in a safe and a risky asset. The analysis of this section is similar to the approach adopted in the literature which explores the effect of taxation on risk-taking (see Mossin (1968) and Stiglitz (1969)).

For this purpose I develop the following three-stage game: In the beginning of the game each of the recipient countries is endowed with a fixed amount of resources which shall be used for investments$\left(I_i\right)$.[8] The recipients allocate these resources between two assets; a safe and a risky one. At stage one the recipients simultaneously decide on the proportion $\left(\alpha_i\right)$ of its investment resources which shall be allocated to a risky project with payoff $\left(1+\tilde{x}_i\right)$ per unit invested. The remaining resources will be invested in a safe asset with payoff $(1+r)$. I will assume that the countries can choose the risk exposure $\left(\alpha_i\right)$ from some closed interval: $\alpha_i \in\left[\overline{\alpha},\overline{\overline{\alpha}}\right]$. I do not preclude that the recipients can "go short" on the risky project $(\overline{\alpha}<0)$, or that they can borrow money in order to invest more than their total resources in the risky project$\left(\overline{\overline{\alpha}}>1\right)$. In stage two of the game "Nature" chooses the outcome of the stochastic variables $\left(\tilde{x}_1,\tilde{x}_2\right)\in X_1\times X_2\equiv X$. I will assume that the support of the stochastic variable belongs to a closed interval: $\tilde{x}_i \in\left[\overline{x},\overline{\overline{x}}\right]\equiv X_i$. Furthermore I assume that $\tilde{x}_1,\tilde{x}_2$ are identically distributed with distribution function $\Pi: X_i \to [0,1]$, and a corresponding density function π. The wealth of the recipient after the outcome of the risky project $\left(x_i'\right)$ is revealed is:

[8]With this game structure I do not analyse how aid might affect the magnitude of total investments when projects are risky. I assume that the recipients already have decided on the amount to be allocated for investments. The analysis regards how aid affects the recipients' choice of risk exposure, assuming that the total amount which will be used for investments is constant for both countries.

$w_i = (1-\alpha_i)I_i(1+r) + \alpha_i I_i(1+x_i') = I_i[(1+r)+\alpha_i(x_i'-r)]$. In stage three the donor allocates a fixed amount of resources between the two recipient countries: $a_1 + a_2 \leq \bar{a}$. The consumption of each country in stage three equals the sum of realised wealth and disbursements of aid: $c_i = w_i + a_i$. The recipient countries' payoff is defined by its expected utility; $U_i = \underset{\tilde{x}_1,\tilde{x}_2}{E}\left[u(c_i(\tilde{x}_1,\tilde{x}_2))\right]$. As in the previous sections, the donor is a utilitarian who seeks to maximise the sum-of the recipient countries' utilities; $U_D = U_1 + U_2$. The structure of this dynamic game is depicted in figure 7.

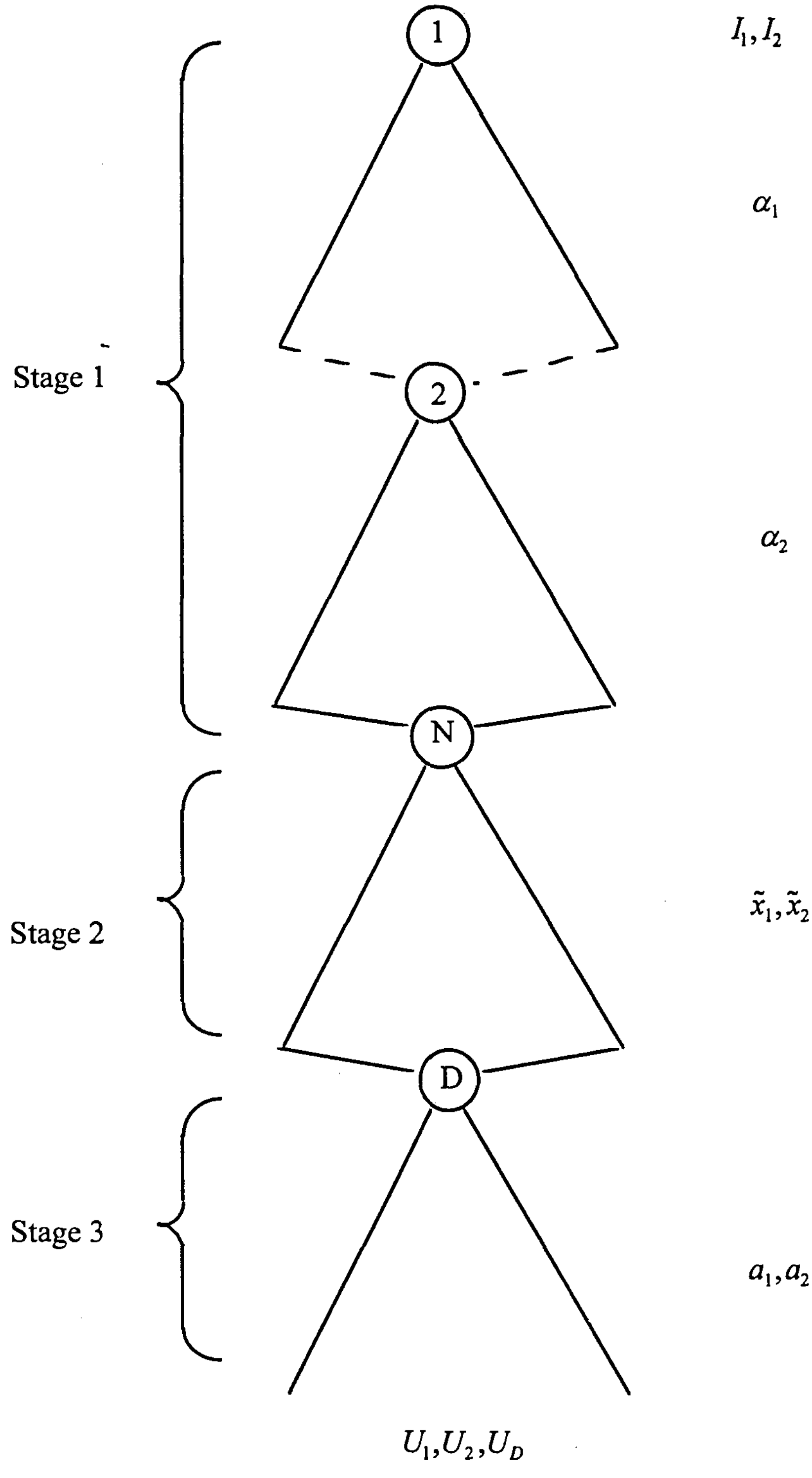

Figure 7

The assumptions which will be maintained throughout this section are:

Assumption 5.1: π *is continuous on the domain* $[\underline{x},\overline{\overline{x}}]$, $\underline{x} < r$, $\overline{\overline{x}} > r$. u *is twice continuously differentiable* (C^2), $u' > 0, u'' < 0$.

Assumption 5.2: *The correlation coefficient between the two random variables is less than one;* $\rho(\tilde{x}_1, \tilde{x}_2) < 1$.

Assumption 5.3: $\max_{i,j \in \{1,2\}, i \neq j} \left| I_i[(1+r) + \overline{\overline{\alpha}}(\overline{\overline{x}} - r)] - I_j[(1+r) + \overline{\overline{\alpha}}(\underline{x} - r)] \right| < \overline{a}$

In the analysis below I will confine myself to interior solutions in the utilitarian problem and in subgame-perfect equilibria. Assumption 5.3 is sufficient to ensure that there will never be corner solutions regarding allocations of aid.

For later comparisons I will start out by analysing the solution to a utilitarian program, with special emphasis on the risk-taking. Obviously a utilitarian allocation is also Pareto optimal. The reason for comparing a SPE with a utilitarian allocation (instead of the set of Pareto optimal allocations) will become apparent.

5.1 Risk-Taking in a Utilitarian Program

A utilitarian allocation is given by a solution to the following problem:

5.1
$$\max_{\{\alpha_1,\alpha_2,a_1\}} \left\{ \underset{\tilde{x}_1,\tilde{x}_2}{E} \left[\begin{array}{l} u\left(I_1\left[(1+r)+\alpha_1(\tilde{x}_1 - r)\right] + a_1(\tilde{x}_1,\tilde{x}_2)\right) + \\ u\left(I_2\left[(1+r)+\alpha_2(\tilde{x}_2 - r)\right] + \overline{a} - a_1(\tilde{x}_1,\tilde{x}_2)\right) \end{array} \right] \right\}.$$

$$\text{subject to} \quad \begin{array}{l} a_1(\tilde{x}_1,\tilde{x}_2) \in [0,\overline{a}], \quad \forall (\tilde{x}_1,\tilde{x}_2) \in X, \\ \alpha_i \in [\underline{\alpha}, \overline{\overline{\alpha}}]. \end{array}$$

In order to ensure a unique and interior solution in both a utilitarian program and in a SPE of the game, I adopt the following ad-hoc assumption:

Assumption 5.4: *The maximand in problem 5.1 is strictly concave globally.*

By assumptions 5.1, 5.2, 5.3 and 5.4 the first order conditions are sufficient for an optimum. In a utilitarian program the optimal disbursement of aid $a_1^U(\tilde{x}_1, \tilde{x}_2)$ must satisfy:

5.2
$$u'\left(c_1\left(x_1',x_2'\right)\right)=u'\left(c_2\left(x_1',x_2'\right)\right),\ \forall\left(x_1',x_2'\right)\in X$$
$$\Leftrightarrow I_1\left[(1+r)+\alpha_1\left(x_1'-r\right)\right]+a_1^U\left(x_1',x'\right)=$$
$$I_2\left[(1+r)+\alpha_2\left(x_2'-r\right)\right]+\bar{a}-a_1^U\left(x_1',x'\right),\ \forall\left(x_1',x_2'\right)\in X$$

When aid resources are allocated according to a utilitarian rule, each recipient country will consume:

5.3 $$c_i^U\left(x_1',x_2'\right)=\tfrac{1}{2}\times\left[I_1\left[(1+r)+\alpha_1\left(x_1'-r\right)\right]+I_2\left[(1+r)+\alpha_2\left(x_2'-r\right)\right]+\bar{a}\right],\ \forall\left(x_1',x_2'\right)\in X.$$

Moreover optimal risk-taking in a utilitarian program (α_1^U,α_2^U) must satisfy the following condition:

5.4
$$\underset{\tilde{x}_1,\tilde{x}_2}{E}\left[u'\left(c_i^U\left(\tilde{x}_1,\tilde{x}_2\right)\right)I_i\left[\tilde{x}_i-r\right]\right]=0,\ \ i=1,2.$$
$$\Updownarrow$$
$$\underset{\tilde{x}_1,\tilde{x}_2}{E}\left[u'\left(\frac{\left[I_1\left[(1+r)+\alpha_1^U\left(x_1'-r\right)\right]+I_2\left[(1+r)+\alpha_2^U\left(x_2'-r\right)\right]+\bar{a}\right]}{2}\right)\left[\tilde{x}_i-r\right]\right]=0.$$

Optimal allocation of aid $\left(a_1^U\left(\tilde{x}_1,\tilde{x}_2\right)\right)$ and optimal risk-taking (α_1^U,α_2^U) in a utilitarian program must consequently satisfy equations 5.3 and 5.4 respectively.

The central question in this section is whether we get optimal risk-taking as the outcome in a subgame-perfect equilibrium of the game. I now turn to an analysis of this problem.

5.2 Risk-Taking in Subgame-perfect Equilibria of the Aid-Game

STAGE 3

In stage 3 of the game the donor will solve the following problem:

$$\max_{\{a_1\}}\left\{\underset{\tilde{x}_1,\tilde{x}_2}{E}\begin{bmatrix} u\left(I_1\left[(1+r)+\alpha_1(\tilde{x}_1-r)\right]+a_1(\tilde{x}_1,\tilde{x}_2)\right)+ \\ u\left(I_2\left[(1+r)+\alpha_2(\tilde{x}_2-r)\right]+\bar{a}-a_1(\tilde{x}_1,\tilde{x}_2)\right)\end{bmatrix}\right\}$$

5.5

$$\text{subject to} \quad a_1(\tilde{x}_1,\tilde{x}_2)\in[0,\bar{a}], \quad \forall(\tilde{x}_1,\tilde{x}_2)\in X.$$

Some manipulation of the first order condition to problem 5.5 yields:

5.6 $$\hat{a}_1(\cdot)=\frac{-I_1\left[(1+r)+\alpha_1(x_1'-r)\right]+I_2\left[(1+r)+\alpha_2(x_2'-r)\right]+\bar{a}}{2}, \ \forall(\alpha_1,\alpha_2).$$

5.7 $$\hat{c}_i(x_1',x_2')=\tfrac{1}{2}\times\left[I_1\left[(1+r)+\alpha_1(x_1'-r)\right]+I_2\left[(1+r)+\alpha_2(x_2'-r)\right]+\bar{a}\right].$$

Here $\hat{c}_i$ denotes the consumption for recipient i in any SPE of the game. Notice that the donor's allocation of aid resources coincides exactly with the aid allocation in a utilitarian program.

STAGE 1

The question is how the two recipients will choose their risk exposure, knowing that the donor will insure some of their risk through aid allocations as in equation 5.6. The countries' choice of risk exposure will solve the following problem:

5.8

$$\max_{\{\alpha_i\}}\left\{\underset{\tilde{x}_1,\tilde{x}_2}{E}\left[u\left(\hat{c}_i(\tilde{x}_1,\tilde{x}_2)\right)\right]\right\}$$
$$\text{subject to} \quad \alpha_i\in\left[\bar{\alpha},\bar{\bar{\alpha}}\right].$$

Consumption in stage 3 of the game is given by equation 5.7. Risk-taking in a SPE of the game must consequently solve the following first order condition to problem 5.8:

$$\underset{\tilde{x}_1,\tilde{x}_2}{E}\left[u'\left(\hat{c}_i(\tilde{x}_1,\tilde{x}_2)\right)I_i\left[\frac{\tilde{x}_i-r}{2}\right]\right]=0$$

5.9 $$\Updownarrow$$

$$\underset{\tilde{x}_1,\tilde{x}_2}{E}\left[u'\left(\frac{\left[I_i\left[(1+r)+\alpha_i^{br}(x_i'-r)\right]+I_j\left[(1+r)+\alpha_j(x_j'-r)\right]+\bar{a}\right]}{2}\right)\left[\tilde{x}_i-r\right]\right]=0,\forall\alpha_j.$$

A subgame-perfect equilibrium of the game consists of strategies $(\hat{a}_1(\cdot), \hat{\alpha}_1, \hat{\alpha}_2)$, where $\hat{a}_1(\cdot)$ is defined by equation 5.6, and gives an optimal strategy for all possible choices of risk-taking at stage 1, and $\hat{\alpha}_1 = \alpha_1^{br}(\hat{\alpha}_2) \wedge \hat{\alpha}_2 = \alpha_2^{br}(\hat{\alpha}_1)$ where $\alpha_i^{br}(\cdot)$ is defined by equation 5.9.

<u>Proposition 5.1:</u>
Under assumptions 5.1, 5.2, 5.3 and 5.4 there exists a unique subgame-perfect equilibrium of the 3-stage aid game. The equilibrium is in pure strategies and satisfies the first order conditions 5.6 and 5.9.

Proof: See Appendix 4.

A comparison of risk-taking in a SPE of the game and risk-taking in a utilitarian program yields the following result:

<u>Proposition 5.2:</u>
Risk-taking in a SPE of the 3-stage aid game coincides with risk-taking in the utilitarian program, and is thus Pareto optimal.

Proof:
By assumptions 5.1, 5.2, 5.3 and 5.4 the first order conditions (equations 5.4 and 5.9) are sufficient for risk-taking in a utilitarian program and in a SPE. Any risk-taking in a SPE $(\hat{\alpha}_1, \hat{\alpha}_2)$ must satisfy equation 5.9. These same values of risk-taking solve equation 5.4 as well. Risk-taking in a SPE does consequently coincide with utilitarian risk-taking.
Q.E.D.

Proposition 5.2 tells us that competition for aid does NOT lead to distortions with regard to risk-taking by a recipient country, compared to a utilitarian solution. Obviously a utilitarian solution is Pareto efficient, and there is consequently no efficiency loss caused by the recipients' choice of risk exposure when they compete for aid.

The intuition for this result should be clarified. First, note that in the equilibrium of the game both risk-sharing and risk-taking is efficient. Risk is shared efficiently when the marginal rate of substitution for consumption in any two states of the world is equal for the two recipient countries. The aid policy used by the utilitarian donor actually implies that the <u>marginal utility</u> of consumption for the two countries is equal in any state of the world. When this condition is fulfilled, the marginal rate of substitution for two recipients must also be equal for any two states of the world. Hence, the utilitarian donor ensures that the two recipient countries share a given "amount" of risk efficiently.

However, proposition 5.2 also shows that the two recipient countries choose an efficient amount of risk, when they face a utilitarian donor. The recipients of aid choose to invest an efficient fraction of their wealth in a risky project. This result is a consequence of the fact that risk-sharing is efficient in the equilibrium. When the two countries share risk efficiently, they face "correct" incentives when choosing their level of risk exposure. Hence, the amount spent on risky investments coincides with risk exposure in a Pareto efficient outcome.

Notice, however, that the presence of aid does affect the recipient's choice of risk exposure, compared with an autarkic situation. In an autarkic situation, the country would be exposed to all the risk caused by variations in the payoff of the risky project. Both in the game and in the utilitarian solution, the country is exposed to only half the risk which is caused by variations in the payoff in its own project. Optimal risk-taking in an autarkic situation will therefore generally be different than optimal risk-taking in the game. It should also be noted that I have assumed that the two countries have a fixed amount which shall be allocated between risky and safe investments. I do not analyse whether this fixed amount is chosen in an efficient manner, when investors face risky as well as risk-free projects. It seems quite clear that the amount that is spent on investments will generally be too low, even if projects are risky. The analysis of this section has however focused on how competition for aid may affect the allocation of aid between risky and risk-free projects. The presence of a utilitarian donor does not lead the recipient countries to inefficient risk-taking.

6. Concluding Remarks

The focus of this chapter has been how the presence of a utilitarian donor of aid affects investment and borrowing decisions by recipient countries. Specifically, I have constructed different versions of a game in which there are two recipient countries which compete for a fixed amount of aid. If a country is relatively bad off it receives a greater amount of aid from the donor. If it improves its economic performance it will experience a reduction in aid disbursements. The structural reason for this is that a utilitarian donor which is unable to commit will equalise the marginal utility of consumption in different countries at any point in time. Thus, the donor will supply aid so that the available resources for poor and less poor countries are equalised, regardless of the reason for a recipient's bad economic performance. This creates an intertemporal distortion on the choices by the recipients: A country which is able to improve its own economic performance through investment or lending decisions will keep only a fraction of this improvement. In this general framework I have studied various central aspects of intertemporal decisions when recipients compete for aid.

The main conclusions are as follows. First, I develop a simple two-period, two country model, in which the recipients of aid in the first period allocate wealth for consumption or for investment which generates production in the last period. The donor will equalise consumption in the last period, through an egalitarian aid policy. The recipients will therefore underallocate resources for investments, and rather use an excessively high proportion of their resources for present consumption. In section 3 I extend the model such that the recipient countries have the opportunity to use the international credit market to finance investments and consumption. In this setting I show that recipient countries will choose an efficient level of investment. However, the presence of an international credit market will create a distortion in the intertemporal allocation of consumption through borrowing decisions. By being heavily indebted the recipients will make their future economic situation worse, which again leads to higher future disbursements of aid. Recipient countries will therefore choose a too high level of indebtedness. In section 4 I focus on how competition for aid may affect recipient countries' choice of long-term versus short-term projects. I show that a recipient country will allocate a too large proportion of total investments to long-term projects. Finally, in section 5 I study whether the presence of aid leads to distortions regarding a recipient country's risk-taking. It is shown that when countries compete for aid, they will choose an exposure to risk which equals risk exposure in a utilitarian program. Risk-taking in a subgame-perfect equilibrium of the game is consequently efficient.

As I have highlighted several places, the structural reason for the distortive effects of aid in this chapter, is the donor's inability to commit. Apart from the commitment problem, the models in this chapter contain no features which would make it difficult for the donor to design an optimal aid policy. It seems pertinent to discuss whether an altruistic donor can overcome this problem. One way to achieve this would be to give "in kind" donations: A donor could give aid as a subsidy on the actual investment by a recipient country and thereby avoid the distortive effects of aid. It is of course possible for an aid agency to give disbursements only to investment projects (in fact such policies are to a large extent applied). The question, however, is whether such a policy would lead to higher aggregate investments. An argument against the effectiveness of such "in kind" donations is the so-called fungibility of foreign aid. The problem is that a country most likely would have financed substantial investments in the absence of aid. Funding of investment projects by a donor will quite possibly lead the recipient country to reduce its own funding of such projects. The excess money can then be used by the government in a recipient country for whatever seems desirable. Thus, in the presence of a fungibility problem it will be quite difficult to assure that additional funds are used entirely according to a donor's wishes. Giving aid as subsidies on investment might therefore not lead to increased investment in the aggregate.

On this background it seems more realistic to treat all aid donations as lump sum transfers to recipient countries. In this setting it seems reasonable to assume that the commitment problem will be a major problem when trying to implement optimal and non-distortive aid policies. In practice such policies would imply that recipient countries were made fully responsible for the consequences of their own decisions, even though these decisions were made be another government 30 years ago. It is hard to imagine an aid agency which credibly can commit to punishing a country for such past choices. In my view it is appropriate to regard donors' inability to commit as an important cause for distortive effects of aid.

The results in this chapter have been obtained in extensive form games of two, three and four stages. The qualitative results of the chapter are likely to remain valid in longer lasting games, provided that the duration of the games is finite and certain. It is not obvious that it is appropriate to analyse aid donations as a game with finite horizon. It is well known from game theory that it is possible to support a wide range of outcomes as subgame-perfect equilibria in games with an infinite horizon (the folk theorem). Thus, the results of this chapter are likely to hinge upon whether the horizon is assumed to be finite or infinite. An important question is whether or not it is appropriate to analyse donor-recipient relationships as finitely lasting. In my view there is not a clear-cut answer to this question, and there are arguments in favour of both approaches. One could argue that many donor-recipient relationships last for very long periods of time, possibly infinitely. However, even though aid may be disbursed for long time periods, the agents which represent the donor and recipients may have finite horizons. Heads of aid agencies are employed for a limited time. The governments in recipient countries will not stay in power forever. It is therefore not unlikely that the central decision-makers in aid agencies and recipient countries apply strategies which are contingent only on events in their time of governance. Another argument which supports the use of finite horizon models is that an important goal for aid agencies and developing countries is to generate growth such that recipient countries are able to support themselves. If this goal is achieved it is likely that aid disbursements are curtailed. In such settings finite horizon models are appropriate. However, even though the individual players might change over time each player might care about the well-being of the next generation of players. In such settings it is quite possible that each player behaves as if he were living infinitely. Moreover one might argue that it is a substantial uncertainty with regards to the termination date of a finitely lasting game. It is well known that an infinite horizon model is appropriate if there is always a possibility that the game last for more periods. This discussion of whether finite or infinite horizon models are more appropriate for the analysis of foreign aid relationships is not conclusive: I believe both approaches are able to shed light on some of the problems which can occur when donor's are unable to commit. The commitment problem in an infinite horizon setting is analysed in the final chapter of this thesis.

Appendices

Before proving various propositions in the chapter, I will briefly discuss the general methodology when proving existence and uniqueness of a SPE.

Existence: Consider a function $T: X \rightarrow X$. Brouwer's fixed point theorem states that if (1) the set X is a compact, convex and non-empty subset of a Euclidean space, and if (2) the function T is continuous, then there exists a fixed point: $\exists x' \in X: T(x') = x'$.

Uniqueness: Suppose $X \subset R$. There exists at most one fixed point of the function T if, at any potential fixed point $x' \in X$, the slope of T is less than one: $\partial T(x')/\partial x < 1$.

Note furthermore that I analyse SPE of the extensive form games by a backward induction technique: I start out by finding a Nash equilibrium in the last stage of the game. In the second to last stage I analyse only Nash equilibria in a "reduced" game in which all players anticipate that the equilibrium strategy will be played in the last stage. Thus, I find only the subgame-perfect Nash equilibria in the subgame which starts at the second to last stage of a game. This technique is repeated until a Nash equilibrium is found in the "reduced" subgame starting at the first stage of the game. It is standard that the strategies which are found by this technique equals the SPE of the game.

Appendix 1: Proof of proposition 2.1.

There are three parts of the proposition; (1) existence of a SPE, (2) uniqueness of the SPE and (3) the SPE is in pure strategies.

(1) Existence.

In stage 2 of the game an equilibrium strategy, $\hat{a}_1(\cdot)$, exists if there exists a solution to problem 2.4. Such a solution exists by assumption 2.1 and 2.2. The solution must satisfy the first order condition (equation 2.5).

Assumptions 2.1 and 2.2 ensure that there for each country, *i*, exists a unique best response function, $I_i^{br}(\cdot)$, defined by equation 2.8. Since f', u' are continuous (f, u are C^2), it follows that $I_i^{br}(\cdot)$ is continuous. Moreover, this best response function has a compact, convex and non-empty range and domain: $I_i^{br}: [0, \overline{w}] \rightarrow [0, \overline{w}]$, where $\overline{w} \equiv max\{w_{01}, w_{02}\}$. Define the function; $T_1(\cdot) \equiv \left(I_{01}^{br}(\cdot), I_{02}^{br}(\cdot)\right)$, $T_1 : [0,\overline{w}] \times [0,\overline{w}] \rightarrow [0,\overline{w}] \times [0,\overline{w}]$. An equilibrium in stage 1 is equivalent to a fixed point of the mapping T_1. T_1 is a continuous function, the domain (and range) is non-empty, convex and compact. Hence, there exist an equilibrium of the game.

(2) Uniqueness.

An equilibrium in stage 1 of the game is equivalent to: $\hat{I}_1 = I_1^{br}\left(I_2^{br}(\hat{I}_1)\right) \wedge \hat{I}_2 = I_2^{br}(\hat{I}_1)$. I will show that in any equilibrium, the function, $I_1^{br}\left(I_2^{br}(\cdot)\right)$, has a slope strictly between zero and one; $\partial I_1^{br}\left(I_2^{br}(\hat{I}_1)\right)/\partial I_1 \in \langle 0,1 \rangle$. Hence, there is at most one equilibrium of the game. Notice that; $\partial I_1^{br}\left(I_2^{br}(\hat{I}_1)\right)/\partial I_1 = \left[\partial I_1^{br}(\hat{I}_2)/\partial I_2\right] \times \left[\partial I_2^{br}(\hat{I}_1)/\partial I_1\right]$. Total differentiation of equation 2.8 and performing some algebraic manipulations establishes:

$$\frac{\partial I_1^{br}\left(I_2^{br}(\hat{I}_1)\right)}{\partial I_1} = \frac{\partial I_1^{br}(\hat{I}_2)}{\partial I_2}\frac{\partial I_2^{br}(\hat{I}_1)}{\partial I_1} = \frac{A_1}{A_1 + B_1 + C_1} \times \frac{A_2}{A_2 + B_2 + C_2}, \text{ where}$$

$$A_i = 1/4\,\delta\left(f'(\hat{I}_i)\right)^2 u''(c_{1i}) < 0 \wedge B_i = u''(c_{0i}) < 0 \wedge C_i = 1/2 \times \delta f''(\hat{I}_i) u'(c_{1i}) < 0.$$

Thus, $\partial I_1^{br}\left(I_2^{br}(\hat{I}_1)\right)/\partial I_1 \in \langle 0,1 \rangle$, and uniqueness is established.

(3) All the best responses are functions, and the SPE which has been proven to exist is in pure strategies.
Q.E.D.

Appendix 2: Proof of proposition 3.1

(1) Existence.

In stage 2 of the game, an equilibrium strategy, $\hat{a}_1(\cdot)$, exists by assumptions 2.1 and 3.2, and is given by the solution to the first order condition (equation 3.6).

Assumptions 2.1 and 3.2 ensure that there for each country, i, exists a best response function, $\left(I_i^{br}, D_i^{br}\right)$, which is defined by the first order conditions (eq. 3.9, 3.10 and 3.11). By equation 3.11 the optimal investment strategy is a constant; $I_i^{br} = I_i^{br}(r)$. The maximand in 3.8 is strictly concave in D_i, and the optimal debt strategy, D_i^{br}, is therefore unique. Thus, the pair of best responses for each recipient, I_i^{br}, D_i^{br}, are actually *functions*. Moreover, since u' is continuous (by assumption 2.1), the best response functions are continuous. A Nash equilibrium in stage 1 of the game is defined by a quadruple $\left(\hat{I}_1, \hat{D}_1, \hat{I}_2, \hat{D}_2\right)$ satisfying the following conditions: $\hat{I}_1 = I_1^{br}(r) \wedge \hat{I}_2 = I_2^{br}(r) \wedge \hat{D}_1 = D_1^{br}\left(\hat{D}_2\right) \wedge \hat{D}_2 = D_2^{br}\left(\hat{D}_1\right)$. Define the function $T_2 = \left(I_1^{br}, I_2^{br}, D_1^{br}, D_2^{br}\right)$,

$T_2 : [0, \overline{w}] \times [0, \overline{w}] \times \left[\overline{D}, \overline{\overline{D}}\right] \times \left[\overline{D}, \overline{\overline{D}}\right] \rightarrow [0, \overline{w}] \times [0, \overline{w}] \times \left[\overline{D}, \overline{\overline{D}}\right] \times \left[\overline{D}, \overline{\overline{D}}\right]$. It is easily verified that all the conditions of Brouwer's fixed-point theorem are satisfied, and consequently that there is a fixed point of the mapping T_2. Hence, an equilibrium of the game exists.

(2) Uniqueness.

The optimal investment strategy for each country is a constant, $I_i^{br} = I_i^{br}(r)$, and hence unique. I will show that the equilibrium is unique in debt strategies as well. In any equilibrium the following must hold: $\hat{D}_1 = D_1^{br}\left(D_2^{br}\left(\hat{D}_1\right)\right) \wedge \hat{D}_2 = D_2^{br}\left(\hat{D}_1\right)$. Hence, uniqueness of a SPE is established if I can verify that, at any fixed point of the mapping, $D_1^{br}\left(D_2^{br}(\cdot)\right)$, its slope is strictly between one and zero. Notice that:

$$\frac{\partial D_1^{br}\left(D_2^{br}\left(\hat{D}_1\right)\right)}{\partial D_1} = \frac{\partial D_1^{br}\left(\hat{D}_2\right)}{\partial D_2} \frac{\partial D_2^{br}\left(\hat{D}_1\right)}{\partial D_1}.$$

Total differentiation of the first order condition (equation 3.10) yields:

$$\frac{\partial D_1^{br}\left(\hat{D}_2\right)}{\partial D_2} = -\frac{1/4 \times \delta(1+r)^2 u''\left(c_{11}\right)}{u''\left(c_{01}\right) + 1/4 \times \delta(1+r)^2 u''\left(c_{11}\right)} \in \langle -1, 0 \rangle,$$
$$\frac{\partial D_2^{br}\left(\hat{D}_1\right)}{\partial D_1} = -\frac{1/4 \times \delta(1+r)^2 u''\left(c_{12}\right)}{u''\left(c_{02}\right) + 1/4 \times \delta(1+r)^2 u''\left(c_{12}\right)} \in \langle -1, 0 \rangle.$$

Thus, $\partial D_1^{br}\left(D_2^{br}\left(\hat{D}_1\right)\right) / \partial D_1 \in \langle 0, 1 \rangle$, and an equilibrium is unique.

(3) Pure strategy equilibrium

All best responses are functions, and the equilibrium is thus in pure strategies.

Q.E.D.

Appendix 3.1: Proof of Lemma 4.1.

First, demonstrate that the maximand in problem 4.13 is twice continuously differentiable (C^2).

A.1. The maximand in problem 4.13 is C^2

By inspection we see that the maximand in problem 4.13 is C^2 if the period 1 investments in an outcome of a SPE, $\hat{I}_{11}(a_{11}), \hat{I}_{12}(a_{11})$, are C^2. In appendix 3.2 I show that there is a unique SPE of the subgame starting in stage 3. This equilibrium is defined by: $\hat{I}_{11} = I_{11}^{br}(\hat{I}_{12}, a_{11}) \wedge \hat{I}_{12} = I_{12}^{br}(\hat{I}_{11}, a_{11})$. This is equivalent to the following system of equations: $\hat{I}_{11} = I_{11}^{br}(I_{12}^{br}(\hat{I}_{11}, a_{11}), a_{11}) \wedge \hat{I}_{12} = I_{12}^{br}(\hat{I}_{11}, a_{11})$. Simple calculation establishes that investments in the equilibrium outcome in subgame 3 vary with aid according to the following pair of equations:

$$\text{A3.1} \quad \frac{\partial \hat{I}_{11}}{\partial a_{11}} = \frac{\dfrac{\partial I_{11}^{br}}{\partial a_{11}} + \dfrac{\partial I_{11}^{br}}{\partial I_{12}} \dfrac{\partial I_{12}^{br}}{\partial a_{11}}}{1 - \dfrac{\partial I_{11}^{br}}{\partial I_{12}} \dfrac{\partial I_{12}^{br}}{\partial I_{11}}} \text{ and } \frac{\partial \hat{I}_{12}}{\partial a_{11}} = \frac{\partial I_{12}^{br}}{\partial I_{11}} \frac{\partial \hat{I}_{11}}{\partial a_{11}} + \frac{\partial I_{12}^{br}}{\partial a_{11}} = \frac{\dfrac{\partial I_{12}^{br}}{\partial a_{11}} + \dfrac{\partial I_{12}^{br}}{\partial I_{11}} \dfrac{\partial I_{11}^{br}}{\partial a_{11}}}{1 - \dfrac{\partial I_{11}^{br}}{\partial I_{12}} \dfrac{\partial I_{12}^{br}}{\partial I_{11}}}.$$

The best response function for recipient 1 in stage 3 of the game must satisfy the first order condition (equation 4.12), and similarly for recipient 2:

$$u'\left(f(I_{01}) + a_{11} - I_{11}^{br}\right) = \frac{1}{2}\delta f'\left(I_{11}^{br}\right) u'\left(\frac{f\left(I_{11}^{br}\right) + g\left(I_1^L\right) + f\left(I_{12}\right) + g\left(I_2^L\right) + \bar{a}_2}{2}\right)$$

Total differentiation of this equation with respect to a_{11} and I_{12} yields:

$$\frac{\partial I_{11}^{br}}{\partial a_{11}} = \frac{u''(c_{11})}{u''(c_{11}) + 1/2\,\delta f''\left(I_{11}^{br}\right) u'\left(\frac{W_2}{2}\right) + 1/4\,\delta\left(f'\left(I_{11}^{br}\right)\right)^2 u''\left(\frac{W_2}{2}\right)},$$

$$\frac{\partial I_{11}^{br}}{\partial I_{12}} = -\frac{1/4\,\delta f'(I_{11}^{br}) f'(I_{12}) u''(\frac{\bar{w}_2}{2})}{u''(c_{11}) + 1/2\,\delta f''(I_{11}^{br}) u'(\frac{\bar{w}_2}{2}) + 1/4\,\delta (f'(I_{11}^{br}))^2 u''(\frac{\bar{w}_2}{2})}.$$

From the first order condition we have; $1/2\,\delta u'(\frac{\bar{w}_2}{2}) = \frac{u'(c_{11})}{f'(I_{11}^{br})}$ and $1/2\,\delta f'(I_{11}^{br}) = \frac{u'(c_{11})}{u'(\frac{\bar{w}_2}{2})}$.

Using these facts in the above two equations yields:

A3.2
$$\frac{\partial I_{11}^{br}}{\partial a_{11}} = \frac{\frac{u''(c_{11})}{u'(c_{11})}}{\frac{u''(c_{11})}{u'(c_{11})} + \frac{f''(\hat{I}_{11})}{f'(\hat{I}_{11})} + \frac{f'(\hat{I}_{11})}{2}\frac{u''(c_{21})}{u'(c_{21})}} \in \langle 0,1 \rangle.$$

A3.3
$$\frac{\partial I_{11}^{br}}{\partial I_{12}} = -\frac{\frac{f'(\hat{I}_{11})}{2}\frac{u''(c_{21})}{u'(c_{21})}}{\frac{u''(c_{11})}{u'(c_{11})} + \frac{f''(\hat{I}_{11})}{f'(\hat{I}_{11})} + \frac{f'(\hat{I}_{11})}{2}\frac{u''(c_{21})}{u'(c_{21})}} \in \langle -1,0 \rangle.$$

Similar calculations establish that the best response function of recipient 2 satisfies the following conditions: $\partial I_{12}^{br}/\partial a_{11} \in \langle -1,0 \rangle \wedge \partial I_{12}^{br}/\partial I_{11} \in \langle -1,0 \rangle$. By inspection of equations A3.2 and A3.3 (and the equivalent equations for recipient 2) we find that the best response function for country i is C^2 with respect to a_{1i} and I_{1j} if the functions f,u are C^3. This holds by assumption 4.1. Inspection of equation A3.1 shows that when the best response functions are C^2, the equilibrium outcomes $\hat{I}_{11},\hat{I}_{12}$ are C^2 in a_{11}. We conclude that the maximand in equation 4.13 is C^2 in a_{11}. Any allocation is a local maximum to problem 4.13 if and only if it satisfies the first and second order conditions.

A.2. The first and second order conditions hold in a symmetric equilibrium

The following conditions hold in a symmetric equilibrium: $\hat{I}_{11} = \hat{I}_{12}, c_{11} = c_{12}$. This implies that the best response functions of country 1 and 2 are "mirror images" of each other in the following sense: $\partial I_{11}^{br}/\partial I_{12} = \partial I_{12}^{br}/\partial I_{11}$, $\partial I_{11}^{br}/\partial a_{11} = -\partial I_{12}^{br}/\partial a_{11}, \partial^2 I_{11}^{br}/\partial a_{11}^2 = -\partial^2 I_{12}^{br}/\partial a_{11}^2$. This again implies that the outcome of the subgame starting at stage 3 has the following properties: $\partial \hat{I}_{11}/\partial a_{11} = -\partial \hat{I}_{12}/\partial a_{11}, \partial^2 \hat{I}_{11}/\partial a_{11}^2 = -\partial^2 \hat{I}_{12}/\partial a_{11}^2$. The donor's first order condition (equation 4.15) is consequently satisfied in a symmetric solution.

The second order condition for problem 4.13 is:

$$\begin{aligned}
&u''(c_{11})\left[1-\frac{\partial \hat{I}_{11}}{\partial a_{11}}\right]^2 - u'(c_{11})\frac{\partial^2 \hat{I}_{11}}{\partial a_{11}^2} + u''(c_{12})\left[1+\frac{\partial \hat{I}_{12}}{\partial a_{11}}\right]^2 - u'(c_{12})\frac{\partial^2 \hat{I}_{12}}{\partial a_{11}^2} \\
\text{A3.4}\quad &+\frac{1}{2}\delta u''\left(\frac{\overline{w}_2}{2}\right)\left[f'(\hat{I}_{11})\frac{\partial \hat{I}_{11}}{\partial a_{11}} + f'(\hat{I}_{12})\frac{\partial \hat{I}_{12}}{\partial a_{11}}\right] + \\
&\delta u'\left(\frac{\overline{w}_2}{2}\right)\left[f''(\hat{I}_{11})\left(\frac{\partial \hat{I}_{11}}{\partial a_{11}}\right)^2 + f'(\hat{I}_{11})\frac{\partial^2 \hat{I}_{11}}{\partial a_{11}^2} + f''(\hat{I}_{12})\left(\frac{\partial \hat{I}_{12}}{\partial a_{11}}\right)^2 + f'(\hat{I}_{12})\frac{\partial^2 \hat{I}_{12}}{\partial a_{11}^2}\right] < 0
\end{aligned}$$

One can not in general determine whether the second order condition holds, without making assumptions regarding the sign of the third derivatives of the functions u and f (one can not establish global concavity of the maximand in problem 4.13). In a symmetric solution, however, the following properties hold; $\hat{I}_{11} = \hat{I}_{12}, c_{11} = c_{12}$, $\partial \hat{I}_{11}/\partial a_{11} = -\partial \hat{I}_{12}/\partial a_{11}, \partial^2 \hat{I}_{11}/\partial a_{11}^2 = -\partial^2 \hat{I}_{12}/\partial a_{11}^2$. Using these properties in equation A3.4 yields;

$$2u''(c_{11})\left[1-\frac{\partial \hat{I}_{11}}{\partial a_{11}}\right]^2 + 2\delta u'\left(\frac{\overline{w}_2}{2}\right)f''(\hat{I}_{11})\left(\frac{\partial \hat{I}_{11}}{\partial a_{11}}\right)^2 < 0.$$

Thus, the second order condition holds in a symmetric equilibrium.

We conclude that the symmetric solution is always a local maximum for the donor at stage 2 of the game.
Q.E.D.

Appendix 3.2: Proof of proposition 4.1.

Notice that an equilibrium outcome $(\hat{I}_{11}, \hat{I}_{12})$ in a symmetric equilibrium can be expressed as:

$\hat{I}_{11} = I_{11}^{br}\left(I_{01}, I_1^L, I_{02}, I_2^L, \hat{I}_{12}\right)$, $\hat{I}_{12} = I_{12}^{br}\left(I_{01}, I_1^L, I_{02}, I_2^L, I_{11}^{br}\left(I_{01}, I_1^L, I_{02}, I_2^L, \hat{I}_{12}\right)\right)$. Moreover, inspection of equation 4.12 reveals that the following conditions hold in a symmetric equilibrium: $\hat{I}_{11} = \hat{I}_{12}, c_{11} = c_{12}, \partial I_{11}^{br}/\partial a_{11} = -\partial I_{12}^{br}/\partial a_{11}$, $\partial I_{11}^{br}/\partial I_{12} = \partial I_{12}^{br}/\partial I_{11}$, $\partial I_{11}^{br}/\partial I_{01} = \partial I_{12}^{br}/\partial I_{01}$, $\partial I_{11}^{br}/\partial I_1^L = \partial I_{12}^{br}/\partial I_1^L$. It follows from this that:

A3.5
$$\frac{\partial\hat{I}_{12}}{\partial I_{01}} = \frac{\dfrac{\partial I_{12}^{br}}{\partial I_{01}} + \dfrac{\partial I_{12}^{br}}{\partial I_{11}}\dfrac{\partial I_{11}^{br}}{\partial I_{01}}}{1 - \dfrac{\partial I_{11}^{br}}{\partial I_{12}}\dfrac{\partial I_{12}^{br}}{\partial I_{11}}} = \frac{\dfrac{\partial I_{12}^{br}}{\partial I_{01}}}{1 - \dfrac{\partial I_{12}^{br}}{\partial I_{11}}}, \quad \frac{\partial\hat{I}_{12}}{\partial I_1^L} = \frac{\dfrac{\partial I_{12}^{br}}{\partial I_1^L} + \dfrac{\partial I_{12}^{br}}{\partial I_{11}}\dfrac{\partial I_{11}^{br}}{\partial I_1^L}}{1 - \dfrac{\partial I_{11}^{br}}{\partial I_{12}}\dfrac{\partial I_{12}^{br}}{\partial I_{11}}} = \frac{\dfrac{\partial I_{12}^{br}}{\partial I_1^L}}{1 - \dfrac{\partial I_{12}^{br}}{\partial I_{11}}}.$$

Equal conditions hold for recipient 1. The first order condition for recipient 2 in stage 3 of the game (equation 4.12), evaluated in a symmetric equilibrium, is:

$$u'\left(\frac{f(I_{01}) + f(I_{02}) + \bar{a}_1}{2} - I_{12}^{br}\right) = \frac{1}{2}\delta f'\left(I_{12}^{br}\right)u'\left(\frac{f\left(I_{12}^{br}\right) + g\left(I_2^L\right) + f\left(I_{11}\right) + g\left(I_1^L\right) + \bar{a}_2}{2}\right).$$

Total differentiation of this equation with respect to I_{11}, I_{01} and I_1^L yields:

A3.6
$$\frac{\partial I_{12}^{br}}{\partial I_{11}} = -\frac{1/4\,\delta f'\left(\hat{I}_{11}\right)f'\left(\hat{I}_{12}\right)u''\left(c_{22}\right)}{u''\left(c_{12}\right) + 1/2\,\delta f''\left(\hat{I}_{12}\right)u'\left(c_{22}\right) + 1/4\,\delta\left(f'\left(\hat{I}_{12}\right)\right)^2 u''\left(c_{22}\right)},$$

A3.7
$$\frac{\partial I_{12}^{br}}{\partial I_{01}} = \frac{f'\left(\hat{I}_{01}\right)}{2} \times \frac{u''\left(c_{12}\right)}{u''\left(c_{12}\right) + 1/2\,f''\left(\hat{I}_{12}\right)u'\left(c_{22}\right) + 1/4\,\delta\left(f'\left(\hat{I}_{12}\right)\right)u''\left(c_{22}\right)},$$

A3.8
$$\frac{\partial I_{12}^{br}}{\partial I_1^L} = -\frac{1}{2}\frac{g'\left(I_1^L\right)}{f'\left(I_{12}\right)} \times \frac{1/2\,\delta\left(f'\left(\hat{I}_{12}\right)\right)^2 u''\left(c_{22}\right)}{u''\left(c_{12}\right) + 1/2\,f''\left(\hat{I}_{12}\right)u'\left(c_{22}\right) + 1/4\,\delta\left(f'\left(\hat{I}_{12}\right)\right)u''\left(c_{22}\right)}.$$

Substituting this into equation A3.5, and evaluating these in a SPE, yields:

A3.9
$$\frac{\partial \hat{I}_{12}}{\partial I_{01}} = \frac{f'(\hat{I}_{01})}{2} \times \frac{u''(c_{12})}{u''(c_{12}) + 1/2\,\delta f''(\hat{I}_{12})u'(c_{22}) + 1/2\,\delta\left(f'(\hat{I}_{12})\right)^2 u''(c_{22})}$$
$$= \frac{f'(\hat{I}_{01})}{2} \times k,\ k \in \langle 0,1 \rangle.$$

A3.9
$$\frac{\partial \hat{I}_{12}}{\partial I_1^L} = -\frac{1}{2}\frac{g'(\hat{I}_1^L)}{f'(\hat{I}_{12})} \times \frac{1/2\,\delta\left(f'(\hat{I}_{12})\right)^2 u''(c_{22})}{u''(c_{12}) + 1/2\,\delta f''(\hat{I}_{12})u'(c_{22}) + 1/2\,\delta\left(f'(\hat{I}_{12})\right)^2 u''(c_{22})}$$
$$= -\frac{1}{2}\frac{g'(\hat{I}_1^L)}{f'(\hat{I}_{12})} \times l,\ l \in \langle 0,1 \rangle.$$

Using these values in equation 4.21 enable us to determine the allocation of resources between long-term and short-term investments:

$$g'(\hat{I}_1^L)\left[1 - \frac{1}{2}l\right] = f'(\hat{I}_{01})f'(\hat{I}_{11})\left[\frac{1}{2} + \frac{1}{2}k\right]$$
$$\Updownarrow$$
$$\frac{f'(\hat{I}_{01})f'(\hat{I}_{11})}{g'(\hat{I}_1^L)} = \frac{1 - \frac{1}{2}l}{\frac{1}{2} + \frac{1}{2}k} = 1 + \frac{1 - l - k}{1 + k}.$$

Evaluating $(-l-k)$ gives:

$$-l-k = -\frac{u''(c_{12}) + 1/2\,\delta\left(f'(\hat{I}_{12})\right)^2 u''(c_{22})}{u''(c_{12}) + 1/2\,\delta f''(\hat{I}_{12})u'(c_{22}) + 1/2\,\delta\left(f'(\hat{I}_{12})\right)^2 u''(c_{22})} \in \langle -1,0 \rangle.$$

This implies that: $\frac{f'(\hat{I}_{01})f'(\hat{I}_{11})}{g'(\hat{I}_1^L)} > 1$. Hence, there is overaccumulation of resources into long-term investments compared to short-term investments in any symmetric SPE of the game. Q.E.D.

Appendix 4: Proof of proposition 5.1.

(1) Existence:

In stage 2 of the game, an equilibrium strategy, $\hat{a}_1(\cdot)$, exists by assumptions 5.1 and 5.3. Its solution is given by the unique solution to the first order condition (equation 5.9).

Assumptions 5.1, 5.2, 5.3 and 5.4 ensure that there for each country, i, exists a unique best response function, $\alpha_i^{br}(\cdot)$, defined by equation 5.9. Since the density function, π, is continuous, it follows that $\alpha_i^{br}(\cdot)$ is continuous. Moreover, these best response functions have a compact, convex and non-empty range and domain: $\alpha_i^{br}:[\overline{\alpha},\overline{\overline{\alpha}}]\to[\overline{\alpha},\overline{\overline{\alpha}}]$. Define the function; $T_4(\cdot)\equiv(\alpha_1^{br}(\cdot),\alpha_2^{br}(\cdot))$, $T_4:[\overline{\alpha},\overline{\overline{\alpha}}]\times[\overline{\alpha},\overline{\overline{\alpha}}]\to[\overline{\alpha},\overline{\overline{\alpha}}]\times[\overline{\alpha},\overline{\overline{\alpha}}]$. An equilibrium in stage 1 is equivalent to a fixed point of the mapping T_4. T_4 is a continuous function, the domain (and range) is non-empty, convex and compact. Hence, there exist an equilibrium of the game.

(2) Uniqueness:

Notice that an equilibrium of the game is equivalent to; $\hat{\alpha}_1 = \alpha_1^{br}(\alpha_2^{br}(\hat{\alpha}_1)) \wedge \hat{\alpha}_2 = \alpha_2^{br}(\hat{\alpha}_1)$. I will demonstrate that the slope of the mapping $(\alpha_1^{br}(\alpha_2^{br}(\hat{\alpha}_1)))$ must be less than 1 in any potential equilibrium, and thus that there is at most 1 SPE of the game. We have that:

A4.1
$$\frac{\partial(\alpha_1^{br}(\alpha_2^{br}(\hat{\alpha}_1)))}{\partial\alpha_1}=\frac{\partial\alpha_1^{br}(\hat{\alpha}_2)}{\partial\alpha_2}\frac{\partial\alpha_2^{br}(\hat{\alpha}_1)}{\partial\alpha_1}.$$

Differentiation of equation 5.9 establishes that;

A4.2
$$\frac{\partial\alpha_1^{br}}{\partial\alpha_2}=-\frac{I_2}{I_1}\frac{\underset{x_1,x_2}{E}\left[u''(\hat{c}_1(x_1,x_2))\times(x_1-r)\times(x_2-r)\right]}{\underset{x_1,x_2}{E}\left[u''(\hat{c}_1(x_1,x_2))\times(x_1-r)^2\right]}.$$

A similar expression holds for $\partial\alpha_2^{br}/\partial\alpha_1$. In the following I will denote the function in equation A4.1 as follows: $\alpha_1^{br}(\alpha_2^{br}(\alpha_1))\equiv(\alpha_1)^2(\alpha_1)$. We get that:

A4.3

$$\frac{\partial(\alpha_1)^2(\hat{\alpha}_1)}{\partial\alpha_1}=\frac{\underset{x_1,x_2}{E}\left[u''(\hat{c}_1(x_1,x_2))\times(x_1-r)\times(x_2-r)\right]}{\underset{x_1,x_2}{E}\left[u''(\hat{c}_1(x_1,x_2))\times(x_1-r)^2\right]}\frac{\underset{x_1,x_2}{E}\left[u''(\hat{c}_1(x_1,x_2))\times(x_1-r)\times(x_2-r)\right]}{\underset{x_1,x_2}{E}\left[u''(\hat{c}_1(x_1,x_2))\times(x_2-r)^2\right]}.$$

Now redefine parameters so that $x=\left(-u''(\hat{c}_1(x_1,x_2))\right)^{\frac{1}{2}}(x_1-r)$ and $y=\left(-u''(\hat{c}_1(x_1,x_2))\right)^{\frac{1}{2}}(x_2-r)$. Substituting these into the above equation yields:

A4.4 $$\frac{\partial\left(\alpha^{br}\right)^2}{\partial\alpha_1}=\frac{\underset{x,y}{E}[xy]}{\underset{x,y}{E}[x^2]}\frac{\underset{x,y}{E}[xy]}{\underset{x,y}{E}[y^2]}.$$

By the Cauchy-Schwartz' inequality; $\left(\underset{x,y}{E}[xy]\right)^2\leq\underset{x,y}{E}[x^2]\underset{x,y}{E}[y^2]$. The inequality is strict if the variables x,y are not proportional to each other: $x\neq by$, where b is a constant. Notice that $x\neq by\Leftrightarrow x_1\neq bx_2$. Moreover, it is only when the correlation coefficient between two random variables equals 1 that one of the variables can be expressed as a linear function of the other. Thus the Cauchy-Schwartz' inequality is strict when we assume that the correlation coefficient between the two random variables is less than one: $\rho(x_1,x_2)<1$. Under this assumption we get:

A4.5 $$\frac{\partial\left(\alpha_1^{br}\right)^2(\hat{\alpha}_1)}{\partial\alpha_1}\in[0,1\rangle.$$

Thus, under the premises in the proposition there will exist at most one fixed point of the mapping $\left(\alpha_1^{br}\right)^2$. We have already established existence of a Nash equilibrium. This equilibrium must thus be unique.

Q.E.D.

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Chapter 5

THE COMMITMENT PROBLEM IN AN INFINITE-HORIZON GAME OF AID DONATIONS*

Abstract

The basic question of this chapter is whether a donor's inability to commit to an optimal aid policy leads to Pareto inferior capital accumulation in an infinite-horizon game in which the players use Markovian strategies. The donor cares both about the welfare of the recipient and the cost of supplying aid, and is thus liable to increase aid disbursements when production in a recipient country is reduced. I develop an infinitely lasting game in which the recipient and the donor in each period simultaneously choose savings and aid donations. Production in the next period is determined by the sum of domestic savings and aid disbursements. The concept of Markov-perfect equilibrium (MPE) in pure stationary strategies in this game is defined. It is shown that the level of capital in any such equilibrium converges to a steady state. The main result of the chapter is that when the players use twice continuously differentiable strategies, capital accumulation in a MPE of the game can never be Pareto optimal.

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1. Introduction

Substantial amounts are each year used for foreign aid (68.5 billion US$ in 1993 (OECD (1995)). The empirical studies of the macroeconomic effects of aid are at best inconclusive: Papanek (1973), Mosley (1987) and White (1992) are not able to detect significant positive effects of aid on economic growth in recipient countries. A possible explanation for this is that aid has an unfavourable effect on intertemporal decisions by recipient countries. Boone (1996), Griffin (1970) and White (1992) find indications that aid reduces domestic savings and thus partially crowds out investments. These findings highlight the importance of a thorough theoretical understanding of how foreign aid affects decisions in the recipient country. In particular it seems relevant to study the effects of aid on intertemporal choices by recipients.

Analyses of the effects of aid must be based on some understanding of the fundamental source of inefficiency. As I will elaborate on later, the focus of this chapter is on the commitment problem. However, there are other possible conceptual perspectives, which I will discuss briefly: One possible explanation for the lack of aid effectiveness can be that decision-makers in recipient countries do not seek to fulfil developmental goals. Aid resources may be used for increased military expenses, favours for interest groups, or for other "non-productive" purposes. In such cases it may not be reasonable to assume that the choices of governments in developing countries are aimed at fulfilling the needs of the country's population. Even though there are examples where such a description is pertinent, there are also cases in which it seems more sensible to assume that the governments in recipient countries are concerned with the well-being of its population. In this chapter I choose to focus on situations in which the decision-makers in recipient countries are benevolent. There is consequently no disagreement as to what constitutes a welfare improvement in the recipient country. Another possible perspective is that there are important asymmetries in information between recipients and donors. One could argue that donors have less information than recipients about important determinants of development in the recipient country. When such informational asymmetries are important it is relevant to use models of adverse selection and moral hazard. There are examples of such studies (Murshed and Sen (1995), Pietrobelli and Scarpa (1992) and Pedersen (1995b)). However, it seems to me that many important determinants of development are as observable for donors as they are for recipients. Investment policies and poverty alleviation programs are important examples of development policies which are possible to observe for donors. Thus there are important cases in which recipients and donors have symmetric information.

The commitment problem, however, seems to be fundamental for donors of aid, even when other problems are disregarded. The commitment problem is most easily understood in the context of a two-stage game. In the second stage of a game the donor chooses how much aid resources to direct to a development country. The donor of aid is fundamentally concerned with the well-being of the population in the recipient country in addition to the costs associated with aid disbursements. Such altruistic motives imply that a poorer recipient receives larger amounts of aid. If a donor is unable to commit, he can not curtail aid to a poor country because its economic policy has been unwise. In the first stage of the game, the recipient will predict this behaviour from the donor. If investments are reduced in this initial period, production in the next period will be reduced as well. But this negative effect will partially be offset by increased aid disbursements from the altruistic and non-committing donor. In this simple two stage setting it is quite obvious that aid from a non-committing donor distorts the recipient's intertemporal decisions. Consequently, the commitment problem will lead to Pareto inefficient allocations.

There are several reasons for modelling the relationship between recipients and donors of aid based on such assumptions. First, it seems quite reasonable to regard altruistic motives as a fundamental reason for the existence of foreign aid. It is hard to imagine that the substantial amounts which are given as aid would exist in such magnitudes if the donor's choices were governed by self-interest only. Assuming that donors are unable to commit seems realistic as well. A committing donor would be able to curtail aid if unwanted policies were implemented, regardless of the effects such aid curtailment would have on the recipient country's population. Such a policy would give "correct" incentives for the decision-makers (governments). But in practice an optimal aid policy would also punish the population in a recipient country for decisions made by the country's government (or maybe even decisions made by a previous government). Such policies seem unrealistic. There is also empirical support for the claim that increased production leads to reduced disbursements of aid (Trumbull and Wall (1994)).

There are some studies of effects of aid when the donor is unable to commit (see for example Svensson (1995) and Bjorvatn (1996)). These articles all assume that the duration of the game between donors and recipients is finite and certain. In this chapter I study the same general topic in an infinite horizon context. Such analyses are not available in the literature at the present. It is disputable whether finite or infinite horizon models are most relevant in the context of foreign aid. Typically aid relationships are long lasting. Even though the duration might be finite, it seems reasonable to assume that the exact date of termination is uncertain.

As is well known from game theory, the set of equilibrium outcomes expands if we assume that the game lasts infinitely, or has an uncertain termination date (the folk theorem). Almost any outcome can be supported as a subgame-perfect equilibrium (SPE) in such games, and indeed Pareto optimal allocations may be the outcome. In this chapter I restrict attention to so called Markov-perfect equilibria (MPE). In such equilibria, strategies can depend only upon the payoff relevant history of the game. In the setting of the aid game of this chapter, this amounts to letting the savings and aid disbursements depend only upon the country's wealth level in the current period.

The restriction of my analysis to Markov-perfect equilibria is a central, and possibly controversial, modelling assumption. As already noted this assumption leaves out a wide range of possible SPE outcomes. First, it is important to note that restricting the agents to play Markovian strategies, does not imply that we end up with equilibria which are not subgame perfect.[1] Any Markov-perfect equilibrium is also a subgame-perfect equilibrium. One may think of a MPE as a subgame-perfect equilibrium of an alternative game in which the players have access only to information about the payoff relevant history (the wealth level of a recipient country). In such a setting it is impossible to maintain equilibria which are supported by strategies which depend upon histories which do not affect the wealth level in a recipient country. Thus, one way to defend a restriction of SPE to MPE may be that players have limited capacity to process information. Another argument in favour of MPE is that such equilibria may be the outcome of an evolutionary process. Recently Maskin and Tirole (1995) have found that Markov-perfect equilibria may be the only outcomes when the "survival" of strategies is determined evolutionary. At the end, however, the legitimacy of the modelling strategy of this chapter is a question of whether the concept of a subgame-perfect equilibrium is relevant for infinitely repeated games. Obviously this question is a fundamental topic in game theory, which I make no attempts to resolve in this chapter. The reason for my special interest in Markov-perfect equilibria is of more pragmatic nature. I believe that aid policies in practice are dependent on factors which are relevant for the future opportunities of donors and recipients. It seems less likely that outcomes are sustained by sophisticated mutual punishment strategies (bootstrapping). I think the restriction to Markov-perfect equilibria actually is a more realistic description of the strategic relationship between donors and recipients. The basic question of this chapter is thus whether the inability to commit constitutes a problem in an infinitely repeated game where the players in equilibrium are restricted to play Markovian strategies.

[1] I will discuss this point in a more formal context in section 4.1 of this chapter.

The subject of this chapter may be interpreted as a question with a general game theoretic relevance. One can think of the general question as: In an infinite horizon game, in which the players have partially conflicting interests, is it possible to maintain a Pareto optimal allocation as the outcome of a Markov-perfect equilibrium? In the literature there are some lines of analysis which seek to answer this question in different contexts, but using models which are formally similar to the one presented below. First, in the context of a common property fishery several authors have studied the "tragedy of the commons" when the strategies can only depend upon the stock of fish each period (Levhari and Mirman (1980), Sundaram (1989), Dutta and Sundaram (1992, 1993a, 1993b)). In this literature there are no established results which shows generally that extraction of the resource will lead to a Pareto inferior outcome. In the literature of economic growth one has analysed capital accumulation in an economy where the agents derive utility from their own consumption and from the well-being of one or more succeeding generations (altruism). One result from this literature is that Markov-perfect equilibria in such games may yield lower consumption for each generation than other feasible programs (see Ray (1987) and Kohlberg (1976)). However, Pareto efficient outcomes are also possible in this context. Third, in the literature of industrial organisation Fudenberg and Tirole (1983) study the equilibrium accumulation of capital among producers competing in an oligopolistic market.[2] In this setting they show that co-operation can be supported by Markovian strategies.

The chapter is organised as follows: In section 2 I outline the basic model. I find necessary conditions for a Pareto optimal allocation in section 3. In section 4 I define the concept of Markov-perfect equilibrium in the present context, and discuss some problems associated with establishing existence of a MPE. MPE of the game are characterised in section 5. The central finding is that one can not obtain a Pareto optimal allocation in a Markov-perfect equilibrium. Section 6 contains concluding remarks.

2. The Model

In the beginning of each period the recipient country has a level of wealth (w_t) available for consumption and saving. The recipient consumes (c_t) and receives foreign aid (a_t) from a donor, and this determines the investments (ψ_t): $\psi_t = w_t - c_t + a_t$. Production in the succeeding period is a function of the investments in the current period: $w_{t+1} = f(\psi_t)$. The game which will be analysed later can formally be described by a 6-tuple; $G^6 = \langle N, A, g, f, W, \delta \rangle$. The game has discrete time and last infinitely; $t \in T = \{0,1,2,...\}$. (1)

[2]Co-operation between different producers is equivalent to a Pareto optimal allocation among the relevant producers in the market.

The set of players N consists of the donor and the recipient; $N := \{D, R\}$. (2) A is the Cartesian product of the donor's action space (A^D) and the recipient's action space (A^R); $A := A^D \times A^R$. In each period the donor can choose any level of aid between zero and some maximum level $\overline{\overline{a}}$. The donor's action space is thus; $a_t \in A^D := [0, \overline{\overline{a}}]$. The recipient can in any period choose a "planned level of consumption" ($\widetilde{c}_t$) as any non-negative amount up to the maximum possible level of resources. The recipient's action space is; $\widetilde{c}_t \in A^R := [0, \overline{\overline{a}} + \overline{\overline{w}}]$, where $\overline{\overline{w}}$ is the maximum possible level of initial wealth. In order to rule out "non-feasible" solutions (in which $c_t > w_t + a_t$), the recipient's "actual consumption" (c_t) will be defined as; $c_t := \min\{(w_t + a_t), \widetilde{c}_t\}$.[3] (3) The set of per period payoff functions is denoted $g := \{u, u - \omega\}$. The recipient's payoff is defined by a per period utility function $u(c_t)$ (a function of actual consumption), which satisfies the following standard conditions: $u' > 0, u'' < 0, u'(0) = \infty$. The donor's per period payoff equals the payoff to the recipient minus a cost of supplying aid; $u(c_t) - \omega(a_t)$. The cost function, $\omega : A^D \to R$, is assumed to be increasing, convex and with properties which excludes corner solutions; $\omega'(a) > 0, \omega''(a) > 0, \omega'(0) = 0, \omega'(\overline{\overline{a}}) = \infty$.[4] I will assume that both the recipient's and the donor's per period payoff functions are bounded (see assumptions 2 and 3 below). (4) The production function ($f : I \to W$) determines tomorrow's level of wealth as a function of investments today; $w_{t+1} = f(w_t - c_t + a_t)$. The set of possible investments is defined by; $I := [0, (\overline{\overline{w}} + \overline{\overline{a}})]$. The production function satisfies the following conditions; $f' > 0, f'' < 0, f(0) = 0, f'(0) = \infty, f'(\overline{\overline{w}} + \overline{\overline{a}})$ is finite. (5) The set of decision nodes in this game is the set of all possible histories. I will however restrict my analysis to Markov-perfect equilibria, or subgame-perfect equilibria in which the players use "Markovian" strategies. The payoff relevant information in this game is the wealth level. Consequently, in the equilibria I study each player's strategy will be a mapping from the set of possible wealth levels to the relevant set of possible actions. The set of possible wealth levels is defined by; $W := [0, \overline{\overline{w}}]$, where $\overline{\overline{w}} := \lim_{b \to \infty} f(b)$. I will assume that the initial wealth level lies in this interval; $w_0 \in W$. This ensures that the wealth level in any period is in W. (6) Finally, the common discount factor $\delta \in (0,1)$ determines in the obvious manner the sum of discounted payoffs for both the donor and the recipient. The recipient and the donor will seek to maximise the discounted sum of their per period payoff:

[3] I have chosen the players' action spaces to be time-independent. This is convenient when defining stationary (time-independent) Markovian strategies later in the chapter. When action spaces are not dependent on time there is a possibility of non-feasible levels of consumption ($c_t > w_t + a_t$). In order to rule out such outcomes I make the distinction between actual and planned consumption.

[4] Note that with this cost function, the donor will always give a positive amount of aid, even if the recipient is richer than the donor. This is obviously an unrealistic assumption which is adopted mainly for expository purposes. Furthermore, in the cases I study the recipients are considerably poorer than the donors.

(1)
$$U^D\left(\{a_t\}_{t=0}^{\infty},\{c_t\}_{t=0}^{\infty},w_0\right)=\sum_{t=0}^{\infty}\delta^t\left[u(c_t)-\omega(a_t)\right],$$
$$U^R\left(\{a_t\}_{t=0}^{\infty},\{c_t\}_{t=0}^{\infty},w_0\right)=\sum_{t=0}^{\infty}\delta^t\left[u(c_t)\right].$$

The following functional assumptions are adopted:

Assumption 1:
The production function f *is twice continuously differentiable* (C^2).
$f'>0, f''<0, f(0)=0, f'(0)=\infty, f'(\overline{\overline{w}}+\overline{a})$ is finite. $\exists\ \overline{\overline{w}}\in R_+:\ \lim_{b\to\infty} f(b)=\overline{\overline{w}}$.

Assumption 2:
The utility function u *is twice continuously differentiable* (C^2), *and bounded;*
$\exists B\in R:\ |u(c)|<B,\ \forall c\in[0,\overline{a}+\overline{\overline{w}}]$. *Moreover;* $u'>0, u''<0, u'(0)=\infty$.

Assumption 3:
The cost function ω *is twice continuously differentiable* (C^2), *and bounded;*
$\exists C\in R:\ |\omega(a)|<C,\ \forall a\in[0,\overline{a}]$. *Moreover;* $\omega'(a)>0, \omega''(a)>0, \omega'(0)=0, \omega'(\overline{a})=\infty$.

I will start out by showing necessary conditions for a Pareto optimal allocation in this context.

3. Pareto Optimal Allocations

A Pareto optimal solution is found by maximisation of a weighted sum of the two players' payoffs, subject to the given constraints. In problem (2) below $\alpha\in[0,1]$ is the weight which is put on the donor's utility and $(1-\alpha)$ is the weight put on the recipient's utility. Let the set of possible consumption- and aid levels be defined as follows: $\Gamma_P(w_t):=\left\{(a,c):a_t\in[0,\overline{a}]\wedge c_t\in[0,w_t+a_t]\right\}$. A Pareto optimal allocation is given by the solution to problem (2) below:

$$
(2) \qquad \begin{aligned} &\max_{(\{c_t\},\{a_t\})} \sum_{t=0}^{\infty} \delta^t \left[u(c_t) - \alpha\omega(a_t) \right] \\ &s.t. \qquad w_0 \text{ fixed}, \\ &\qquad\qquad w_{t+1} = f(w_t - c_t + a_t), \\ &\qquad\qquad a_t, c_t \in \Gamma_P(w_t). \end{aligned}
$$

I will find necessary (and sufficient) conditions for an optimum to this problem. First I will find a condition for optimal allocation of consumption and aid in any single period. This is done by maximising the objective function (2) with respect to c_t and a_t, holding all other variables constant. This operation yields:

$$
(3) \qquad u'(c_t^*) - \alpha\omega'(a_t^*) = 0.
$$

The asterixes denote optimal allocations of consumption and aid in any single period. Next, I will use the so-called "variational approach" in order to find a necessary (and sufficient) condition for an optimal sequence of consumption and aid. The idea behind this method is that, along an optimal path, it must be impossible to increase discounted utility by a changing the allocation of consumption (or aid) between two consecutive periods and leaving the path unchanged thereafter. In the following I will make such a change in consumption at time t and $t+1$ without altering any other relevant decisions. Note that $w_{t+2} = f(f(w_t - c_t + a_t) - c_{t+1} + a_{t+1})$. Total differentiation of this equation, keeping w_{t+2} constant, yields; $\partial w_{t+2}/\partial c_t = 0 = f'(\psi_{t+1})\left[-f'(\psi_t) - \partial c_{t+1}/\partial c_t\right]$. This again implies that; $\partial c_{t+1}/\partial c_t = -f'(\psi_t)$. Maximising the objective function in (2) with respect to c_t and c_{t+1}, holding all other variables fixed, yields:

$$
(4) \qquad \delta^t u'(c_t^*) - \delta^{t+1} f'(\psi_t^*) u'(c_{t+1}^*) = 0,
$$

where the asterixes denote levels of consumption, investment (and so on) along an optimal path. This equation is known as the Euler equation, and is a necessary condition for optimal allocation of consumption in any two consecutive periods. Theorem 4.15 in Stokey and Lucas (1989) proves that when assumptions 1, 2 and 3 hold, equation (3), (4) and the following transversality condition are necessary and sufficient conditions for a solution to the dynamic optimisation problem in 2:

$$
(5) \qquad \lim_{t\to\infty} \delta^t \left[u'(c_t^*) - \alpha\omega'(a_t^*) \right] w_t^* = 0.
$$

A standard result in neo-classical growth theory is that an optimal path defines a sequence of wealth, consumption and aid which converges to a steady state.[5] This result holds also for my slightly modified model with aid donations. An optimal steady state is defined by: $\lim_{t\to\infty} w_t^* = w^*, \lim_{t\to\infty} a_t^* = a^*, \lim_{t\to\infty} c_t^* = c^*$. I define savings in an optimal steady state as: $\psi^* := w^* - c^* + a^*$. In a steady state the level of consumption and aid must reproduce the steady state level of wealth. This, together with equation (3) and (4) evaluated in a steady state, yields:

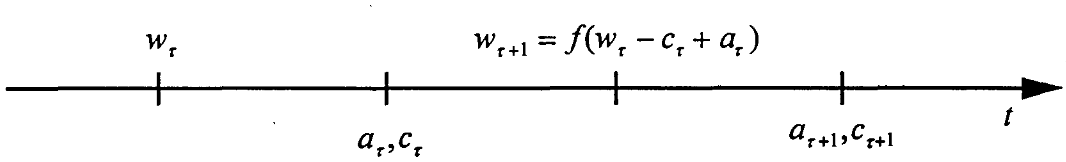
$$(6) \qquad (i)\ 1 = \delta f'(\psi^*),\ (ii)\ u'(c^*) = \alpha\omega'(a^*),\ (iii)\ \ w^* = f(w^* - c^* + a^*).$$

These conditions for a Pareto optimal steady state will be compared with the outcome in a game situation. Equation 6 (i) will prove illuminating for such comparisons. This equation is the necessary condition for Pareto optimal *wealth accumulation*, and says that the discounted value of an increased production caused by a marginal increase in investment must equal the consumption which is foregone by the investment.

4. Markov-perfect Equilibrium in the Aid Game

4.1 Definition of a Markov-perfect Equilibrium

The game which will be at the centre of attention in the following is the infinite-horizon game $G^6 = \langle N, A, g, f, W, \delta \rangle$ described in the previous section. It has the following structure: A country's wealth level is revealed at any point in time. After having observed this, the donor and the recipient simultaneously choose their level of aid and consumption. The timing of the game is depicted in the figure below.



I am interested in subgame-perfect equilibria in which the players use pure stationary Markovian strategies, or Markov-perfect equilibria. If the agents play <u>Markovian</u> strategies their actions in any period can depend only on the wealth level in that period, but actions in

[5] This result is proved in Appendix A to chapter 2 in Blanchard and Fischer (1989).

two *different* periods may depend on calendar time. If, furthermore the strategies are stationary then the actions must be identical in two different periods in which the wealth level is the same. This leads to the following definition:

Definition:
A pure stationary Markovian strategy (psMs) for the donor is a time-independent function; $a{:}W \rightarrow A^D$*, and a psMs for the recipient is a time-independent function;* $c{:}W \rightarrow A^R$.

If the donor uses a psMs the recipient's problem becomes:

$$
(7) \qquad \begin{aligned}
&\max_{\{\tilde{c}_t\}} && \sum_{t=0}^{\infty} \delta^t u(c_t) \\
&s.t. && c_t = \min\{\tilde{c}_t, (w_t + a(w_t))\} \\
& && w_0 = \text{fixed} \\
& && w_{t+1} = f(w_t - c_t + a(w_t)) \\
& && \tilde{c}_t \in \Gamma_R = [0, \overline{\overline{w}} + \overline{\overline{a}}].
\end{aligned}
$$

In this problem $\{\tilde{c}_t\}$ denotes the time sequence of planned consumption from date zero to infinity. If the recipient uses a psMs the donor's problem becomes:

$$
(8) \qquad \begin{aligned}
&\max_{\{a_t\}} && \sum_{t=0}^{\infty} \delta^t [u(c(w_t)) - \omega(a_t)] \\
&s.t. && w_0 = \textit{fixed} \\
& && w_{t+1} = f(w_t - c(w_t) + a_t) \\
& && a_t \in \Gamma_D = [0, \overline{\overline{a}}].
\end{aligned}
$$

I will define a Markov-perfect equilibrium in the standard way as a subgame-perfect equilibrium, with the additional condition that the strategies used are Markovian (psMs). In order to define a Markov-perfect equilibrium I will introduce some notation: A history h^k at stage k is defined by all previous actions at that stage; $h^k \equiv \{c_t, a_t\}_{t=0}^{k-1}$. Moreover, define H^k as the set of all possible histories up to stage k. Each possible history defines a proper subgame, which is denoted; $G(h^k)$. Pure stationary Markovian strategies are functions from the set of possible wealth levels to the action spaces for each of the players; $a{:}W \rightarrow A^D$, $c{:}W \rightarrow A^R$. Let the set of all feasible pure stationary Markovian strategies for the recipient and the donor be denoted; Θ_R, Θ_D, respectively. Next, let the sets of "unconstrained" strategies for the recipient and the donor be denoted; S_R, S_D, respectively, where s_R, s_D are typical elements in these sets. The sets will not be defined formally, but each of their elements must be "feasible" in the sense that they must fulfil the intertemporal budget constraints. Moreover, the sets are unconstrained in the sense that each of their elements can be made conditional on all available information in any subgame, $G(h^k)$, for each possible history $h^k \in H^k$, for all stages k. Lastly, let the continuation payoff for the donor and the recipient for the history h^k when the players use strategies s_R, s_D, be denoted: $U^D(s_R, s_D, h^k), U^R(s_R, s_D, h^k)$. Continuation payoffs are defined in the obvious way. We are now ready to define a Markov-perfect equilibrium:

Definition:

A Markov-perfect equilibrium is a pair of pure stationary Markovian strategies, $\hat{c} \in \Theta_R, \hat{a} \in \Theta_D$ such that, for all possible proper subgames $G(h^k)$ of G^6 defined by all possible histories $h^k \in H^k$, and for all alternative pairs of strategies; $s_R \in S_R, s_D \in S_D$, the following condition holds: $U^R(\hat{c}, \hat{a}, h^k) \geq U^R(s_R, \hat{a}, h^k) \wedge U^D(\hat{c}, \hat{a}, h^k) \geq U^D(\hat{c}, s_D, h^k)$.

It is important to note that the above formulation requires that neither player can gain from a deviation to any feasible strategy in any possible subgame (which includes history-dependent strategies). Hence the definition ensures that a Markov-perfect equilibrium is also a subgame-perfect equilibrium. In section 5 of this chapter I will characterise equilibria in which both players use stationary Markovian strategies. The legitimacy of such an analysis depends on whether it is optimal for one player to use a stationary Markovian strategy if the other player applies such a strategy. Note that the only relevant information for an agent facing problem (7) or (8) is the functional forms and the level of wealth. Looking at these problems we see that the other player's (Markovian) strategy can be treated as a part of the production function, where the production function becomes a composite function. We can conclude that one player faces a standard single agent optimisation problem when the other player uses a Markovian strategy. Moreover, it is well known from the literature of dynamic programming that a Markovian strategy will be optimal in such single agent problems.[6] Consequently, if either player uses a Markovian strategy, it will be optimal for the other player to use a Markovian strategy as well. This result has an important implication: A strategy that is optimal among all Markovian strategies is optimal among the unconstrained set of strategies.

[6] The setting in which optimal single player solutions to dynamic Markovian problems can be expressed by time-independent functions is described in more detail in chapter 4 of Stokey and Lucas (1989) (see Theorem 4.8).

4.2 *Existence of Markov-perfect Equilibria in the Aid Game*

This chapter does not contain a proof of existence of a Markov-perfect equilibrium in the aid game. However, it is useful with a closer look at this issue. First, I will define best response functions for the two players. Let the sequence $\{c_t^*, w_t^*\}_{t=0}^{\infty}$ be a solution to the recipient's problem 7, when the donor uses the strategy $\hat{a}(\cdot)$. I can define a pure stationary Markovian strategy $\hat{c}(\cdot)$ which is equivalent to this optimal sequence in the following sense: $c_t^* = \hat{c}(w_t^*), \forall t$. Obviously, this optimal Markovian strategy depends on which Markovian strategy is played by the donor. Hence, one can define a best response mapping, $c^{br}(\cdot)$ which assigns an optimal Markovian strategy for the recipient to each feasible Markovian strategy by the donor. The best response mapping can be described as a function with the set of possible Markovian strategies by the donor as the domain and the set of possible Markovian strategies by the recipient as the range: $c^{br}(\cdot):\Theta_D \to \Theta_R$. A best response function for the donor can be defined in a similar way: $a^{br}(\cdot):\Theta_R \to \Theta_D$. An alternative definition of Markov-perfect equilibrium is therefore a pair of Markovian strategies, $\hat{a}(\cdot) \in \Theta_D, \hat{c}(\cdot) \in \Theta_R$, for which the following condition is satisfied: $\hat{c}(\cdot) = c^{br}(\hat{a}(\cdot)) \wedge \hat{a}(\cdot) = a^{br}(\hat{c}(\cdot))$.[7] Next, define the mapping $T := (c^{br}, a^{br})$, which has the following range and domain: $T:\Theta_R \times \Theta_D \to \Theta_R \times \Theta_D$. An equilibrium of the game is equivalent to a fixed point of the mapping T. Note that the domain and range of this mapping is a function space, which is infinite dimensional. The Schauder-Tychonoff infinite-dimensional fixed-point theorem is applicable for such cases (see Smart (1974) pg. 15). This theorem states that any compact, convex non-empty subset of a locally convex space has the fixed-point property (i.e. every continuous mapping from such a space into itself must have a fixed point). A proof of existence must start out with a specification of the domain which ensures that T maps into a subset of this domain. A problem occurs because it can be demonstrated that the best response to a *k*- times continuously differentiable strategy is at most a *k-1*- times continuously differentiable function, provided that $k > 1$ (in the interior of W). This suggests an approach in which the domain is chosen to be infinitely many times continuously differentiable. A best response to such a function will also be infinitely many times continuously differentiable (given some further assumptions on the domain). However, this suggestion raises at least one substantial problem. The Shauder-Tychonoff theorem demands that the best response functions must be continuous according to some legitimate norm on the domain. It is,

[7] Note that this way to define a MPE differs from the definition used in that the equilibrium strategies must be best responses to the other player's Markovian strategies, when each player can choose only among Markovian strategies. The definition used in this chapter allows each player to choose among the unconstrained set of strategies. However, as the discussion in section 4.1 should have made clear, this restriction in the strategy set is not important: It is always optimal for a player to use a Markovian strategy if all other players use Markovian strategies. Hence, the sets of Markov-perfect equilibria which satisfy the two alternative definitions are identical.

however, not easy to work with legitimate norms which allow their elements to be infinitely many times continuously differentiable (the sup norm for example is not a norm in such a space). Moreover, establishing compactness of the space remains a problem. I continue with a characterisation of Markov-perfect equilibria of the game, despite the lack of a proof of existence.

5. Characterisation of Markov-perfect Equilibria

5.1 Convergence of Wealth Levels in MPE to Steady States

An important motivation for this study is to compare the outcomes of MPE to a Pareto optimal outcome. A proper characterisation of a Markov-perfect equilibrium is therefore an essential task, which will be carried out in this section. In section 3 I claimed that a Pareto optimal allocation rule would give a state path which converge to some steady state. Fortunately this holds also for state paths generated by MPE. Comparisons of outcomes of the game to Pareto optimal outcomes can therefore be done based on comparisons of steady states. Let us first establish that any state path generated by MPE converges to a steady state.

Proposition 1

Any sequence of wealth levels generated by a MPE in pure stationary strategies is weakly monotone and converges to a steady state; $\lim_{t\to\infty} w_t = \overline{w}$. *Let* $(\hat{c},\hat{a})$ *be the pair of equilibrium strategies. The equilibrium levels of consumption and aid converge;* $\lim_{t\to\infty} \hat{c}(w_t) = \overline{c},\ \lim_{t\to\infty} \hat{a}(w_t) = \overline{a}$.

Proof:
See Appendix 1

5.2 Suboptimal Outcomes when Strategies are Twice Continuously Differentiable (C^2).

In the following I will assume that the equilibrium strategies $(\hat{c},\hat{a})$ are twice continuously differentiable (C^2):

Assumption 4:
The pure stationary Markovian strategies by the recipient and the donor, $(\hat{c},\hat{a})$, *are assumed to be twice continuously differentiable* (C^2).

Note that I adopt an assumption regarding the strategies which the players use in an equilibrium. Ideally, one would like to derive the players' optimal strategies, and not make assumptions regarding their properties. The characterisation of MPE in proposition 2 is constrained to equilibria in which the players use twice continuously differentiable (Markovian) strategies. It is in principle possible that there are equilibria in which strategies are not C^2. Such equilibria may have different characteristics than the ones I describe in proposition 2. From a practical perspective, however, it seems unlikely that donors or recipients use strategies which imply very large changes in their decisions for rather small changes in the level of wealth in the recipient country. Nevertheless, the assumption of twice continuously differentiable strategies is a limitation to my analysis. This assumption is used when describing MPE in this section.

Let us first look at the donor's optimal strategy when the recipient uses pure stationary Markovian strategy which is C^2. As when calculating necessary and sufficient conditions for a Pareto optimal solution, I will use the "variational approach" to find a solution to problem 8. Along an optimal path it must be impossible to increase the utility for the donor by reallocating aid between any two subsequent periods, and letting the path remain unchanged thereafter. Note that the wealth level in period $t+2$ is defined by; $w_{t+2} = f\big(f(w_t + a_t - c(w_t)) - c(f(w_t + a_t - c(w_t))) + a_{t+1}\big)$. When w_{t+2} is kept constant, the aid level in period $t+1$ can be expressed as a function of the aid level in period t; $a_{t+1} = a_{t+1}(a_t)$. Total differentiation of the equation above with respect to a_t yields: $\partial w_{t+2}/\partial a_t = 0 \Rightarrow \quad f'(\psi_{t+1})\big[f'(\psi_t)(1 - c'(f(\psi_t))) + \partial a_{t+1}/\partial a_t\big] = 0$. This implies; $\partial a_{t+1}/\partial a_t = -f'(\psi_t)(1 - c'(f(\psi_t)))$. The donor's maximisation problem over two periods becomes:

$$(9) \qquad \max_{a_t \in [0,\bar{a}]} \left\{ \begin{array}{l} u(c(w_t)) - \omega(a_t) + \\ \delta\big[u(c(f(w_t - c(w_t) + a_t))) - \omega(a_{t+1}(a_t))\big] \end{array} \right\}.$$

Assumptions 1, 2, 3 and 4 ensure that the first order condition to problem (9) must hold in an optimum. Hence, we arrive at the following Euler equation for optimal allocation of aid.

$$(10) \qquad -\omega'(\hat{a}(w_t)) + \delta \left[\begin{array}{l} u'(c(f(\psi_t))c'(f(\psi_t))f'(\psi_t) + \\ \omega'(\hat{a}(w_{t+1}))f'(\psi_t)[1 - c'(f(\psi_t))] \end{array} \right] = 0.$$

A transversality condition must be added to the Euler equation above.

$$(11) \qquad \lim_{t\to\infty} \delta^t w_t^* \left[u'(\hat{c}(w_t^*))\hat{c}'(w_t^*) + [1-\hat{c}'(w_t^*)]\omega'(\hat{a}(w_t^*))\right] = 0.$$

From Theorem 4.15 in Stokey and Lucas (1989) we know that, when assumptions 1, 2, 3, and 4 are adopted, equation (10) is a necessary condition for a maximum. If, additionally, the transversality condition (11) and a second order condition to the problem are fulfilled, we know that the strategies must be optimal (the second order condition is stated in appendix 2).

We know that $\lim_{t\to\infty} (w_t, c_t, a_t, \psi_t) = (\overline{w}, \overline{c}, \overline{a}, \overline{\psi})$. Furthermore, all the functions in the first order condition are continuous (including c'). Evaluating the first order condition (10) in a steady state yields (after some algebraic manipulations and with suppressed arguments):

$$(12) \qquad 1 = \delta f'\left[1 - c'\left(1 - \frac{u'}{\omega'}\right)\right].$$

This condition determines the donor's optimal aid strategy at a steady state, $\hat{a}(\overline{w})$, when the recipient plays; $c(\cdot)$.

We will now investigate properties of the recipient's optimal strategy; $\hat{c}(\cdot) = c^{br}(a(\cdot))$. With a similar reasoning as when studying the donor's choices, we analyse the recipient's optimal allocation of consumption between two periods when the wealth levels in the following periods are kept constant. The wealth level two periods from the current period is defined by; $w_{t+2} = f\left(f\left(w_t - c_t + a(w_t)\right) + a\left(f\left(w_t - c_t + a(w_t)\right)\right) - c_{t+1}\right)$. When w_{t+2} is kept constant, the level of consumption in period $t+1$ can be expressed as a function of the level of consumption in period t; $c_{t+1} = c_{t+1}(c_t)$. We know that $\partial w_{t+2}/\partial c_t = 0 \Rightarrow f'(\psi_{t+1})\left[f'(\psi_t)\left(1 + a'(f(\psi_t))\right) + \partial c_{t+1}/\partial c_t\right] = 0$. This implies $\partial c_{t+1}/\partial c_t = -f'(\psi_t)\left(1 + a'(f(\psi_t))\right)$. The recipient's maximisation problem over two periods becomes:

$$\max_{c_t \in [0, w+a(w)]} \left\{u(c_t) + \delta\left[u(c_{t+1}(c_t))\right]\right\}.$$

It is clear that assumptions 1, 2, and 3 ensure that we can disregard corner solutions; $c_t \neq 0 \wedge c_t \neq w + a(w)$. The recipient's optimal strategy $\hat{c}(\cdot) = c^{br}(a(\cdot))$ must consequently solve the following Euler condition:

(13) $$u'(\hat{c}(w_t)) - \delta u'(\hat{c}(w_{t+1}))f'(\psi_t)[1 + a'(w_{t+1})] = 0, \forall w_t \in W.$$

The transversality condition is:

(14) $$\lim_{t\to\infty} \delta^t w_t^* [u'(\hat{c}(w_t^*))[1 + \hat{a}'(w_t^*)]] = 0.$$

Again, if assumptions 1, 2, 3, and 4 hold, equation (13) and (14) and a second order condition are sufficient for an optimum (the second order condition to the recipient's problem is stated in appendix 2).

Taking limits as $t \to \infty$ the first order condition becomes:

(15) $$1 = \delta f'(\overline{\psi})[1 + a'(\overline{w})].$$

Equation (10) and (13) are necessary conditions for the donor's and recipient's optimal strategies respectively. In an equilibrium each equation must hold when the opponent plays his equilibrium strategy. The following proposition characterises the outcome of Markov-perfect equilibria in the aid game:

<u>Proposition 2</u>
Consider an outcome of a MPE when the agents play twice continuously differentiable strategies. No such outcome can be Pareto efficient.

Proof:
See Appendix 2.

Proposition 2 tells us that capital accumulation in a steady state of the game can never be Pareto optimal. Equation 6 (i) $(1 = \delta f'(\psi(w^*)))$ is the rule for Pareto optimal capital accumulation in a steady state. It says that a marginal increase in investments must yield increased production the next period which, when discounted, is equal to the consumption which is foregone by the investment. In a steady state of the game, the recipient chooses capital accumulation according to equation (15) $(1 = \delta f'(\overline{\psi})[1 + \hat{a}'(\overline{w})])$. We can easily see that the recipient will choose an optimal level of capital accumulation only if the level of aid is unaffected by changes in the level of wealth in the steady state $\left(\hat{a}'(\overline{w})=0\right)$. It is only when this condition is fulfilled that the recipient country will get the full benefits from increased investments. Proposition 2 tells us that this condition $\left(\hat{a}'(\overline{w})=0\right)$ can never occur in a MPE of the game. The reason for this can be thought of as follows: If the recipient gets a fixed amount of aid it will increase consumption if the level of wealth increases $\left(\hat{c}'(\overline{w})>0\right)$. If a country experiences an increase in wealth it will consequently also reduce its marginal utility of wealth. The donor will consequently want to reduce aid disbursements to a country when its wealth increases $\left(\hat{a}'(\overline{w})<0\right)$. The condition $\left(\hat{a}'(\overline{w})=0\right)$ can consequently never hold in a Markov-perfect equilibrium. Thus, there can never be Pareto optimal capital accumulation in such an equilibrium of the game.

6. Concluding Remarks

The question which this chapter has tried to answer is whether the commitment problem for a donor of aid is avoided in the setting of an infinite horizon model. In a finite horizon context the donor is faced with the so-called "Samaritan's dilemma" (Buchanan (1975)). An altruistic donor wants to alleviate poverty in poor countries. This fact implies that poorer countries receive larger disbursements of aid. Recipients will understand this, and will underallocate resources to investments and rather choose to increase current consumption. Thus the altruistic donor is in a sense a victim of his good intentions: If he had applied an aid policy which was insensitive to changes in the economic conditions in recipient countries, both he and the recipient would have experienced a higher utility. The main result of this chapter is that this dilemma persists in an infinite horizon setting, assuming that the donor and recipient play Markovian strategies. Confining the analysis to Markov-perfect equilibria is essential for this result. If the players could make history dependent strategies the result would be unlikely to hold.

There are three interesting problems in the context of the constructed game which this chapter leaves unanswered. First, proving existence of a Markov-perfect equilbrium of the infinite horizon aid game is an interesting future task. Secondly, the chapter shows that capital accumulation is inefficient, but does not establish whether an outcome of the game leads to over accumulation or under accumulation of capital. Intuitively one would expect outcomes of the game to yield too low capital accumulation. Thirdly, the main result of the chapter concludes that a MPE of the game gives Pareto inferior solutions when the players use twice continuously differentiable strategies. However, it has not been established that a similar result holds when players use general Markovian strategies. All these questions are interesting areas for further research.

Appendix 1: Proof of proposition 1.

Let the pair of strategies $(\hat{a},\hat{c})$ be a MPE in pure stationary strategies. Then these functions define an equilibrium investment function in the following way: $\hat{\psi}(w):=w+\hat{a}(w)-\hat{c}(w),\forall w\in W$. The equilibrium strategy by the donor defines a function $\hat{R}:W\rightarrow R$ as follows: $\hat{R}(w):=w+\hat{a}(w)$, $\forall w\in W$. Note that $\hat{c}(w)=\hat{R}(w)-\hat{\psi}(w),\forall w\in W$. Furthermore let $\{(w'_{-1}),(w'),(w'_{+1})\}$ denote a selection in a sequence of wealth levels generated by a MPE which "goes through" w' with w'_{-1} as the immediate predecessor and w'_{+1} as the immediate successor. Moreover, define $V^R(w)$ as the maximum continuation payoff for the recipient when the wealth level is w. $V^D(w)$ is defined in a similar way.

The proof of proposition 1 follows from three lemmas.

Lemma 1

Consider a MPE in pure stationary strategies of the aid game, and let $\hat{a}$ be the donor's equilibrium strategy. In any such equilibrium the following condition holds: $w'>w\Rightarrow w'+\hat{a}(w')>w+\hat{a}(w)$.

Proof:

Suppose the Lemma were not true. Then there would exist $w,w'\in W,w'>w$ for which $w'+\hat{a}(w')\leq w+\hat{a}(w)$. I will show that, if the donor's strategy satisfies such a condition, the recipient's best response $\hat{c}$ will not generate the sequence $\{\ldots,w'_{-1},w',w'_{+1},\ldots\}$. Consider the alternative strategy $\breve{c}$. It is defined such that $f\left(w'_{-1}+\hat{a}(w'_{-1})-\breve{c}(w'_{-1})\right)=w$ and $f\left(w+\hat{a}(w)-\breve{c}(w)\right)=w'_{+1}$. Since $w'>w$ we know that $\breve{c}(w'_{-1})>\hat{c}(w'_{-1})$, and since $w+\hat{a}(w)\geq w'+\hat{a}(w')$ we know that $\breve{c}(w)\geq\hat{c}(w')$. Consider now the payoff to the recipient from choosing strategy $\hat{c}$ instead of strategy $\breve{c}$ starting at wealth level w'_{-1}. A necessary condition for $\hat{c}$ to be a best response to the strategy $\hat{a}$ at w'_{-1} is: $u\left(\hat{c}(w'_{-1})\right)+\delta u\left(\hat{c}(w')\right)+\delta^2V^R\left(w'_{+1}\right)\geq u\left(\breve{c}(w'_{-1})\right)+\delta u\left(\breve{c}(w)\right)+\delta^2V^R\left(w'_{+1}\right)$. This implies: $\left[u\left(\hat{c}(w'_{-1})\right)-u\left(\breve{c}(w'_{-1})\right)\right]+\delta\left[u\left(\hat{c}(w')\right)-u\left(\breve{c}(w)\right)\right]\geq 0$. But this contradicts the facts that $\breve{c}(w'_{-1})>\hat{c}(w'_{-1})$ and $\breve{c}(w)\geq\hat{c}(w')$. The strategy $\hat{c}$ can therefore not be a best response to the strategy $\hat{a}$ at w'_{-1}, and the sequence $\{\ldots,w'_{-1},w',w'_{+1},\ldots\}$ can not be generated by a MPE in pure stationary strategies.

Q.E.D.

Lemma 2

An equilibrium investment function is weakly increasing: $w' > w \Rightarrow \hat{\psi}(w') \geq \hat{\psi}(w), \forall w, w' \in W$.

Proof:

Suppose the lemma were not true. Then there would exist $w, w' \in W, w' > w$ for which $\hat{\psi}(w') < \hat{\psi}(w)$. We will consider whether the strategies which imply such an investment function can be mutual best responses (a MPE). Define an alternative strategy $\breve{c}$ by: $\breve{c}(w) = \hat{R}(w) - \hat{\psi}(w')$ and $\breve{c}(w') = \hat{R}(w') - \hat{\psi}(w)$. This alternative strategy is feasible since $\hat{\psi}(w') < \hat{\psi}(w)$ (by hypothesis) and since $w' + \hat{a}(w') > w + \hat{a}(w)$ (Lemma 1). In a MPE the following conditions must hold:

$$u\left(\hat{R}(w') - \hat{\psi}(w')\right) + \delta V^R\left(f(\hat{\psi}(w'))\right) \geq u\left(\hat{R}(w') - \hat{\psi}(w)\right) + \delta V^R\left(f(\hat{\psi}(w))\right)$$

$$u\left(\hat{R}(w) - \hat{\psi}(w)\right) + \delta V^R\left(f(\hat{\psi}(w))\right) \geq u\left(\hat{R}(w) - \hat{\psi}(w')\right) + \delta V^R\left(f(\hat{\psi}(w'))\right).$$

Adding up the LHS and the RHS of these two inequalities, and rearranging yields:

$$u\left(\hat{R}(w') - \hat{\psi}(w')\right) - u\left(\hat{R}(w) - \hat{\psi}(w')\right) \geq u\left(\hat{R}(w') - \hat{\psi}(w)\right) - u\left(\hat{R}(w) - \hat{\psi}(w)\right)$$

$$\Rightarrow \int_{\hat{\psi}(w)}^{\hat{\psi}(w')} \int_{\hat{R}(w)}^{\hat{R}(w')} u''(x-y)dxdy \leq 0.$$

By Lemma 1 we know that $\hat{R}(w') > \hat{R}(w)$. But the above inequality can not hold for $\hat{\psi}(w') < \hat{\psi}(w)$, since $u'' < 0$ (strict risk aversion). We conclude that $\hat{\psi}$ is a weakly increasing correspondence.

Q.E.D.

Lemma 3

Any sequence $\{w_t\}_0^\infty$ generated by a MPE in pure stationary strategies must be weakly monotone: $w_{t'+1} \geq w_{t'} \Rightarrow \left(w_t \geq w_{t'}, \forall t > t'\right)$.

Proof:

Consider a sequence $\{w_t\}_0^\infty$ generated by a MPE. Suppose, without loss of generality, that $t' = 0$. The fact $f' > 0$ and that $\hat{\psi}$ is weakly increasing gives, for $w_{t'+1} \geq w_{t'} : f\left(\hat{\psi}\left(w_{t'+1}\right)\right) \geq f\left(\hat{\psi}\left(w_{t'}\right)\right) \Rightarrow w_{t'+2} \geq w_{t'+1} \Rightarrow w_{t'+2} \geq w_{t'+1} \geq w_{t'}$. By induction we get the result. Q.E.D.

Proof of Proposition 1:

We know that any sequence $\{w'_t\}_0^\infty$ is bounded above and below. Moreover by Lemma 3 the sequence is weakly monotone. From basic mathematical analysis (e.g. Simon and Blume (1994) Theorem 29.2) we know that every bounded weakly monotone sequence converges. We have thus established that the sequence of wealth level $\{w'_t\}_0^\infty$ generated by a MPE is weakly monotonous, and converges to a steady state; $\lim_{t\to\infty} w_t = f\left(\hat{\psi}\left(w_{t-1}\right)\right) = \overline{w} \in W$.

Next, consider the convergence of $\{a_t\} = \{\hat{a}(w_t)\}$, $\{c_t\} = \{\hat{c}(w_t)\}$. We know that all entries of these sequences are contained in closed and bounded sets. By the Bolzano-Weierstrass Theorem we know that all bounded and closed sequences have convergent sub-sequences (see e.g. Simon and Blume (1994), theorem 29.5). Consequently the question of convergence of the sequences $\{a_t\}$ and $\{c_t\}$ reduces to examining whether each of these sequences may have two (or more) sub-sequences converging to different limit points. This will be proven to be impossible by contradiction.

Throughout the proof we will assume that $\{w_t\}$ is non-decreasing (by Lemma 2 the sequence is monotone). The proof for non-increasing wealth levels follows identical arguments.

Consider first $\{a_t\}$, and suppose that there exist sub-sequences, $\{w_{t_l}\},\{w_{t_k}\}$, such that $\lim_{l\to\infty}\{\hat{a}(w_{t_l})\} = \tilde{a} > \overline{a} = \lim_{k\to\infty}\{\hat{a}(w_{t_k})\}$. Choose a sub-sequence $t_{k(l)}$ of t_k such that $t_{k(l)} > t_l$ for all l, which implies that $w_{t_{k(l)}} \geq w_{t_l}$ (for non-decreasing sequences). From Lemma 1 we know that $w_{t_{k(l)}} + \hat{a}\left(w_{t_{k(l)}}\right) \geq w_{t_l} + \hat{a}\left(w_{t_l}\right)$. Taking limits as $l \to \infty$ we get; $\overline{w} + \overline{a} \geq \overline{w} + \tilde{a}$, which is a contradiction of the original assertion.

Next consider $\{c_t\}$, and suppose there exist sub-sequences $\{w_{t_l}\},\{w_{t_k}\}$ for which $\lim_{l\to\infty}\{\hat{c}(w_{t_l})\} = \overline{\overline{c}} > \overline{c} = \lim_{k\to\infty}\{\hat{c}(w_{t_k})\}$. Chose now a sub-sequent $t_{k(l)}$ of t_k such that $t_{k(l)} > t_l$ for all l. From Lemma 2 we have: $w_{t_{k(l)}} + \hat{a}\left(w_{t_{k(l)}}\right) - \hat{c}\left(w_{t_{k(l)}}\right) \leq w_{t_l} + \hat{a}\left(w_{t_l}\right) - \hat{c}\left(w_{t_l}\right)$. Taking limits as $l \to \infty$ we get; $\overline{w} + \overline{a} - \overline{c} \leq \overline{w} + \overline{a} - \overline{\overline{c}}$. This contradicts the original assertion.

Q.E.D.

Appendix 2: Proof of Proposition 2.

By inspecting the recipient's first order condition (15) it is clear that Pareto optimal capital accumulation can be obtained in a steady state only if $\hat{a}'(\overline{w}) = 0$. The strategy in this proof is simply to show that there can never exist an equilibrium where the derivative of the donor's optimal strategy with respect to wealth is zero in a steady state. To this aim I will first calculate the second order condition to the donor's maximisation problem (for notational simplicity I will denote this expression $SOC_D(w_t)$ in the following):

$$
\begin{aligned}
&-\omega''(\hat{a}(w_t)) + \delta f''(\psi_t)\left[u'(c_{t+1})c'(w_{t+1}) + \omega'(\hat{a}(w_{t+1}))(1-c'(w_{t+1}))\right] \\
&+ \delta u''(c_{t+1})\left(c'(w_{t+1})f'(\psi_t)\right)^2 + \delta\omega''(\hat{a}(w_{t+1}))\hat{a}'(w_{t+1})(f'(\psi_t))^2(1-c'(w_{t+1})) \\
&+ \delta c''(w_{t+1})(f'(\psi_t))^2\left[u'(c_{t+1}) - \omega'(\hat{a}(w_{t+1}))\right] = SOC_D(w_t) < 0
\end{aligned}
$$

Evaluating the second order condition as the game converges to a steady state yields (suppressing arguments):[8]

$$
\begin{aligned}
&-\omega'' + \delta f''\left[u'c' + \omega'(1-c')\right] \\
&+ \delta u''(c'f')^2 + \delta\omega''\hat{a}'(f')^2(1-c') \\
&+ \delta c''(f')^2\left[u' - \omega'\right] = SOC_D(\overline{w}) < 0
\end{aligned}
$$

Algebraic manipulation of the donor's first order condition (equation (12)) yields: (i) $\omega'/\delta f' = u'c' + \omega'(1-c')$ and (ii) $(u'-\omega') = \omega'(1-\delta f')/\delta f'c'$. Substituting these expressions into the second order condition yields:

$$
\begin{aligned}
&SOC_D(\overline{w}) = -\omega'' + \delta\frac{f''}{f'}\omega' + \delta u''(c'f')^2 + \\
&\delta\omega''\hat{a}'(f')^2(1-c') + \delta\frac{c''}{c'}f'\omega'\left[1-\delta f'\right] < 0.
\end{aligned}
$$

Total differentiation of the donor's first order condition (10), with respect to w_t, yields:

$$
-\omega''(\hat{a}(w_t))\hat{a}'(w_t) + \left[1 + \hat{a}'(w_t) - c'(w_t)\right] \times \left[SOC_D(w_t) + \omega''(\hat{a}'(w_t))\right] = 0
$$

8From Proposition 1 we know that that $\lim_{t\to\infty}(w_t, c_t, a_t, \psi_t) = (\overline{w}, \overline{c}, \overline{a}, \overline{\psi})$. Moreover all the functions in the above expression are by assumption continuous (including c''). The limit value of the second order condition as $t \to \infty$ is consequently equal to the second order condition evaluated in a steady state.

This implies;

$$\hat{a}'(w_t) = -\left[1 - c'(w_t)\right] \times \left[1 - \frac{-\omega''(\hat{a}(w_t))}{SOC_D(w_t)}\right]$$

Evaluated in a steady state, and suppressing arguments, we get:

$$\hat{a}'(\overline{w}) = -[1-c'] \times \left[1 - \frac{-\omega''}{SOC_D(\overline{w})}\right] =$$

$$-[1-c'] \times$$

$$\left[1 - \frac{-\omega''}{-\omega'' + \delta f''[u'c' + \omega'(1-c')] + \delta u''(c'f')^2 + \delta\omega''\hat{a}'(f')^2(1-c') + \delta c''(f')^2[u' - \omega']}\right].$$

The above equation can generally be written:

$$\hat{a}'(\overline{w}) = -[1-c'] \times r, \ \ r < 1.$$

We know (from equation 15) that a MPE generates optimal capital accumulation in a steady state ($1 = \delta f'$) if and only if the donor's strategy satisfies the condition; $\hat{a}'(\overline{w}) = 0$. Note that $r = \left(1 - \frac{-\omega''(\hat{a}'(\overline{w}))}{SOC_D(\overline{w})}\right)$, and define r^* as the value this quotient attains at a Pareto efficient steady state $(\overline{w} = w^*)$. Inspection of the second order condition evaluated in a Pareto efficient steady state reveals that; $r^* \in \langle 0,1 \rangle$. We consequently have that in an efficient steady state the donor's optimal strategy must satisfy:

$$\hat{a}(w^*) = -[1-c'] \times r^*, \ \ r^* \in \langle 0,1 \rangle.$$

I now turn to describing the recipient's optimal strategy in a steady state; $\hat{c}(\cdot) = c^{br}(a(\cdot))$. The second order condition to the recipient's maximisation problem is:

$$u''(c_t) + \delta u''(c_{t+1})(f'(\psi_t))^2[1 + a'(w_{t+1})]\hat{c}(w_{t+1})$$
$$+ \delta u'(c_{t+1}) f''(\psi_t)[1 + a'(w_{t+1})] + \delta u'(c_{t+1})(f'(\psi_t))^2 a''(w_{t+1}) = SOC_R(w_t) < 0.$$

Taking limits as $t \to \infty$, suppressing arguments and rearranging, the second order condition becomes:

$$u'' + \delta u''(f')^2[1+a']\hat{c}' + \delta u' f''[1+a'] + \delta u'(f')^2 a'' = SOC_R(\overline{w}) < 0.$$

I will calculate the derivative of the recipient's best response function with respect to wealth. By totally differentiating the recipient's first order condition, with respect to w_t, I get:

$$u''(c(w_t))\hat{c}'(w_t) + [1 + a'(w_t) - \hat{c}'(w_t)] \times [SOC_R(w_t) - u''(\hat{c}(w_t))] = 0.$$

Taking limits, suppressing arguments, and rearranging yields:

$$\text{(A3.2)} \quad \hat{c}'(\overline{w}) = (1+a')\left[1 - \frac{u''}{SOC_R(\overline{w})}\right].$$

We can consequently write the above equation as:

$$\hat{c}'(\overline{w}) = (1+a') \times s, \; s < 1.$$

Let us look at an equilibrium in which each player's strategy will have to be a best response to the other player's strategy. In equilibrium the following conditions must consequently hold simultaneously:

$$\text{(A3.3)} \qquad \hat{c}' = (1+\hat{a}') \times s \;\; \wedge \;\; \hat{a}' = -[1-\hat{c}'] \times r. \, .$$

where $r < 1, s < 1$.

I will show that a MPE in twice continuously differentiable strategies can never converge to an optimal steady state (where optimal capital accumulation is defined by; $1 = \delta f'$). From equation (15) we see that a MPE converges to an optimal steady state if and only if (i) $\hat{a}' = 0$ in equilibrium, and if (ii) the following pair of equations holds.

$$\hat{c}' = (1+\hat{a}') \times s \;\; \wedge \;\; \hat{a}' = -[1-\hat{c}'] \times r^*, \;\; s < 1 \wedge r^* \in \langle 0,1 \rangle.$$

Solving this system of equations with respect to $\hat{a}'$ yields:

$$\hat{a}' = -r^*\left(\frac{1-s}{1-r^*s}\right).$$

We see that the condition $\hat{a}' = 0$ can only hold if $r^* = 0$. But we have already established that $r^* \in \langle 0,1 \rangle$. We have thereby shown that it is not possible to have a MPE in twice continuously differentiable strategies which converges to a Pareto optimal steady state.

Q.E.D.

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