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Political Uncertainty: Valuation and Decision Making with a Focus on Oil Investments

by

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A dissertation submitted for the degree of dr. oecon

To my parents

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Chapter 1

Introduction

Abstract

In this chapter I present the objective of the dissertation and give an overview of the contents and the contributions of the chapters to follow. I discuss alternative definitions of the term political uncertainty and how political uncertainty may be analyzed.

1 Objective

My objective with this dissertation is to examine how political uncertainty[1], and especially uncertainty regarding expropriation and taxation, influences the value of real investments and investors' optimal decision making when managing these investments. Even though the results presented are applicable to real investments in general, I have as a rule focused on natural resource investments and investments in oil fields in particular. Besides being an important sector by itself, the natural resource sector has the advantage that the finished products often are traded on international commodity exchanges. This facilitates the use of the contingent claims methodology when evaluating the investments, and in particular when pricing the future sales revenue from the investment. Hopefully, the analyses presented in this dissertation will capture the essence of the problem, and give insights into how political uncertainty affects the value of assets and optimal decision making.

2 Political Uncertainty

The uncertainties studied in this dissertation belong mainly to the class of political uncertainty. At a more general level, one might ask what political uncertainty is, and what it is not. Jodice (1985) delineated political risk from other types of risk by stating:

> "Political risk is distinguished from the customary economic risks of business (marketing competition, availability of inputs) including macroeconomic trends that affect business performance; and risk arising out of social changes (labor, unionism, feminism, race relations) that are not an output of the political system. Of course, at the margin, these putative economic and social factors may be political products (i.e. laws governing collective bargaining) and at that point the distinctiveness of political risk disappears. The interrelationship of these factors has inclined practitioners to speak of country risk. Either way, one has to look at the political process in order to shape judgements about the likelihood of nationalization or expropriation or changing administrative behaviour."

[1] In Knight (1921) a distinction is made between risk and uncertainty. Risk refers to situations where probabilities can be calculated, and uncertainty refers to situations where probabilities cannot be calculated. I will not differ between these terms. As a rule I will use the term uncertainty. I use the term risk when it is natural in the context, e.g., when established terms, like "country risk", are used.

The almost all-encompassing meaning of the term political was also noted by Lax (1983),

> "The adjective *political* carries a host of meanings. In its most narrow usage, it denotes the organizational and decision-making process of governments. At its broadest, the term can be used to encompass virtually all the interactions between the units in a system (for example, people in a country or states in the international community). To avoid the pitfalls of being either encyclopedic or myopic in scope, we shall treat the term *political* as referring to the class of decisions and events that concern the authoritative allocation of values and resources or that otherwise involve issues of legitimacy, authority, or the use of force."

The quotations from Jodice (1985) and Lax (1983) are in the tradition of political science, and not specifically of finance theory. In the political science tradition I also cite Jodice (1985)'s definition of political risk, which concerns foreign investments.

> "Changes in the operating conditions of foreign enterprises that arise out of political process, either directly through war, insurrection, or political violence, or through changes in government policies that affect the ownership and behaviour of the firm. Political risk can be conceptualized as events, or a series of events, in the national and international environment that can affect the physical assets, personnel, and operation of foreign firms."

A point worth commenting on is the distinction between political stability and stability in policy. A country may have an unstable political climate with frequent changes of government, but still have a stable regulatory environment for investments. On the other hand, a country may be politically stable, but change regulations affecting investments frequently. In this paper I focus on situations where the policy regulating the investment may change, i.e. instability in policy.

Political uncertainty may be grouped into three categories, which are:

1. *Uncertainty in regulatory framework*, such as taxes, legal protection of property rights, safety regulations, and other regulations based on one or several nations' official authority.
2. *Uncertainty related to behavior from the state, or politically controlled companies, in the market place*. An example of this is uncertainty regarding the volume of oil

produced by OPEC or by Saudi Arabia.

3. *Uncertainty caused by political conflict.* This category includes external or internal war, or other types of major upheavals affecting investments. Examples of such upheavals are social unrest and the fall of communism,

Political uncertainty increases the complexity when analyzing investments. Factors, which in more stable environments usually are treated as parameters, are turned into variables. Even if one abstracts from the complexity and concentrates on one variable, which represents political uncertainty, the question is the same: "Which regulatory regime for the investment, or political conditions affecting the investment, will be in place ?" It is the qualifying term "political" which makes political uncertainty different from other types of uncertainty. In this dissertation, and in most formal analyses in finance theory, the situations studied are simplified so there is little doubt concerning what the political uncertainty is. The focus of the analysis determines how political uncertainty is included in the formal analysis. The political uncertainty belonging to the three categories are created by decisions made by governments, state companies, opposition groups, or other "political" decision makers. One can say that political uncertainty is created by uncertain political decision making. One way of categorizing analyses involving political uncertainty is according to the level of detail in the modeling of the political decision making process, and to the extent, measured in number

Number of regime variables		*Low*	*High*
	Many	*The focus is on the effect of interaction be-tween regime variables*	*Analyses with high relevance, but often too complex for analytical clarity*
	Few	*Analyses focusing on the effect of uncertainty*	*The focus is on a realistic description of political decision making*

Level of specification of political decision making is -

Figure 1 Focus of analyses including political uncertainty.

of regime variables, political uncertainty is included in the analyses. See Figure 1. The need for clarity usually necessitates that one can expand the analysis in one of the dimensions only.

In this dissertation I study mainly the effects of political uncertainty on real investments. The three categories cover most of what might be termed political uncertainty related to investments in real assets. With a different focus, the term political uncertainty may have a somewhat different, but related, meaning. If the focus is, e.g., to study political uncertainty related to valuation of mainly financial assets, such as stocks and bonds, the term political uncertainty would probably in most cases be used in connection with the possiblility of shocks in the financial markets caused by some kind of "political event", e.g., a war or a revolution[2].

Tax rates, indicator variables for the event of expropriation, and other regime variables are determined by governments. In this dissertation I use different approaches when modeling the dynamics of the regime variables. In chapter three and four, the regime variables are exogenous, whereas in chapter five the government's decision making is determined as a part of the solution. These approaches complement each other when trying to understand the effect of political uncertainty on optimal decision making and the value of investments.

3 Overview of Chapters

In addition to this introductory chapter, the dissertation consists of four chapters. I have

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[2] When studying such shocks in financial markets, an important question is whether a risk premium is required for assets influenced by political uncertainty. While political uncertainty related to one or more nations vital to the world economy may be considered as systematic, political uncertainty in a given country not vital to the world economy is probably not. To an internationally well diversified investor holding a large portfolio of stocks from many countries, this specific uncertainty may be considered to be diversifiable. In this respect, political uncertainty would be comparable to other types of non-systematic event uncertainties, like. e.g., the probability of a technical break-down or the probability of fire in a factory.

Political uncertainy may, however, be different from these types of uncertainties. In many situations the probability of a given event, or shock, may vary considerably over time. The level and the dynamic behavior of the political uncertainty is especially important in relation to the timing of investments. This is especially true when the investment is *irreversible*. As an example, related to oil investments, by including the value of optimal decision making related to when to invest, when to temporarily close down production, or when to abandon the oil field, the value of the investment opportunity may be considerably increased as compared to value if no such decision making were taken into account.

aimed at making each chapter self contained, and there is therefore some overlap in contents and discussion of issues. The aim has been to use consistent notation in the dissertation, but because the chapters' contents and methodological approach vary this has not been completely obtained. The use of symbols and notation therefore vary between the chapters. I have provided lists of the most frequently used symbols as appendices to chapters three, four, and five.

I start in chapter two by reviewing selected literature relevant to investors' optimal decision making in the presence of political uncertainty. My search for literature revealed that there is no homogenous body of literature related to valuation and decision making under political uncertainty. It seems that at the end of the sixties and in the seventies the focus was on analyzing and predicting events like expropriation and wars. The majority of analyses were primarily not in the main stream of finance or financial economics, but more often in the political science tradition. The review is primarily limited to literature explicitly dealing with the problem of asset valuation under political uncertainty, and investors' decision making implied from the solution to such valuation problems. In the introduction to the review, I discuss general principles for analyzing political uncertainty in a formal way, and the meaning of frequently used terms like country risk. I summarize the reviewed articles, and suggest future research. Political uncertainty can broadly be analyzed in two ways, by explicitly or implicitly including political uncertainty in the analysis. The simplest way is to look at irreversible regime shifts. Some situations, like expropriation or default, are suited for models with binary, irreversible regime shifts. In one-period models there is no distinction between reversible and irreversible regime shifts. In an implicit modeling of political uncertainty, it is assumed that total uncertainty includes political uncertainty. In such approaches, there is a lack of specification when the effect of increased political uncertainty is analyzed. The review chapter serves as a background for the following chapters, but I also hope it may serve as a reference or starting point for other financial economists interested in the topic.

In chapter three I address analytical and empirical issues related to the use of suitable risk indices in the evaluation of investments affected by political uncertainty. I suggest a method whereby an unobservable state variable, governing the type of policy regime, can be deduced from the risk indices. I show how this approach can be combined with the contingent claims approach to price assets influenced by events where the probabilities of the events are functions of risk indices. I derive a set of closed-form valuation formulas which may, e.g., be used to evaluate political risk insurance contracts and the value of investments under expropriation risk. For a set of risk indices I also show how relevant parameters in the indices' evolutionary equations may be estimated. To my knowledge, this is the first attempt to include risk indices directly in the valuation of investments by using the contingent claims methodology.

Whereas I in chapter three explain how risk indices can be used when evaluating investments, chapter four may be regarded as an example of how this approach can be used when analyzing specific problems. Occasionally situations arise where the operating conditions or the regulations applying to an investment will largely depend on the outcome of events taking place at a fixed future date. Examples of such "watershed events" are the first all-racial election in South Africa and the hand-over of rule of Hong Kong from Great Britain to China. In chapter four I study the investor's incentive to wait until the date when the uncertainty is resolved when there is a possibility of deferring the investment decision today until this future date. I consider specifically the situation where either the numerical value of a royalty rate, or an expropriation, will be determined at a future date. For a set of examples I show that the incentive to wait in case of political uncertainty may be lower than the case with no political uncertainty if the correlation between the risk index and the cash flow from the investment is negative. It is therefore not necessarily so that increased political uncertainty will increase the incentive to wait. This fact has been noted by other authors, but I am able to model this in a new way due to the results developed in chapter three.

A government's lack of credibility when promising future taxation and regulation of foreign direct investments, is often regarded as an obstacle to foreign investment. As shown in chapter five, the total lack of inter-period credibility does not necessarily prevent investment from taking place. If the government in the host country is not able to undertake the investment activity itself, both the government and the investor can benefit from negotiating a series of agreements where the investor gets a share of the revenue generated from previous investments against making new investments. This assumes that intra-period agreements are respected by the parties. Based on an example, the conclusion is somewhat different than one might expect. The investor's utility from the investment, or net present value, when considering to invest in a country with intra-period credibility only is never lower than the utility from a similar investment opportunity in a country with inter-period credibility. I also consider the effect of the investor's possibility to defer production, or investment, on the investor's utility from the investment. Based on an example, I show that increased flexibility to defer decisions does not necessarily increase the value of the investment project.

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Chapter 2

Asset Valuation and Investors' Optimal Decision Making in the Presence of Political Uncertainty: A Review of Selected Literature

Abstract

In this chapter I review selected literature relevant to investors' optimal decision making in the presence of political uncertainty. The review is limited to literature explicitly dealing with the problem of asset valuation under political uncertainty, and with investors' decision making implied from the solution to such valuation problems. Political uncertainty can broadly be categorized in three groups: uncertainty in the regulatory framework for investments, uncertainty related to behavior from state or governmental market participants, and uncertainty caused by political conflict. In the introduction to the review, I discuss general principles for analyzing political uncertainty in a formal way, and the meaning of frequently used terms like country risk. I summarize the reviewed articles, and suggest future research.

1 Introduction

In this chapter I review selected literature analyzing effects of political uncertainty[1] on asset values and on investors' optimal decision making.

I will distinguish between three categories of political uncertainty:

1. *Uncertainty in regulatory framework*, such as taxes, legal protection of property rights, safety regulations, and other regulations based on one or several nations' official authority.
2. *Uncertainty related to behavior from the state, or politically controlled companies, in the market place.* An example of this is uncertainty regarding the volume of oil produced by OPEC or Saudi Arabia.
3. *Uncertainty caused by political conflict.* This category includes external or internal war, or other types of conflicts affecting investments.

When considering political uncertainty in general, some comments are appropriate regarding how this type of uncertainty can be included in formal analyses. Perhaps the simplest approach is not to specify the political uncertainty *per se*, but to assume that the political uncertainty is included in the total uncertainty of an investment. As an example of this, consider the uncertainty in the oil price. In the real options literature, the oil price, S, is assumed to develop according to a pre-specified process, such as a geometric Brownian motion with constant parameters of the form

$$dS_t = S_t \alpha dt + S_t \sigma dB_t \ , \qquad (1)$$

where α and σ are constants, and where dB_t is the increment of a standard Brownian motion.

[1] In Knight (1921) a distinction is made between risk and uncertainty. Risk refers to situations where probabilities can be calculated, and uncertainty refers to situations where probabilities cannot be calculated. I will not differ between these terms. As a rule I will use the term uncertainty. If the term risk is used in the referred literature, I will do the same. Risk is also used when treating established terms, like country risk.

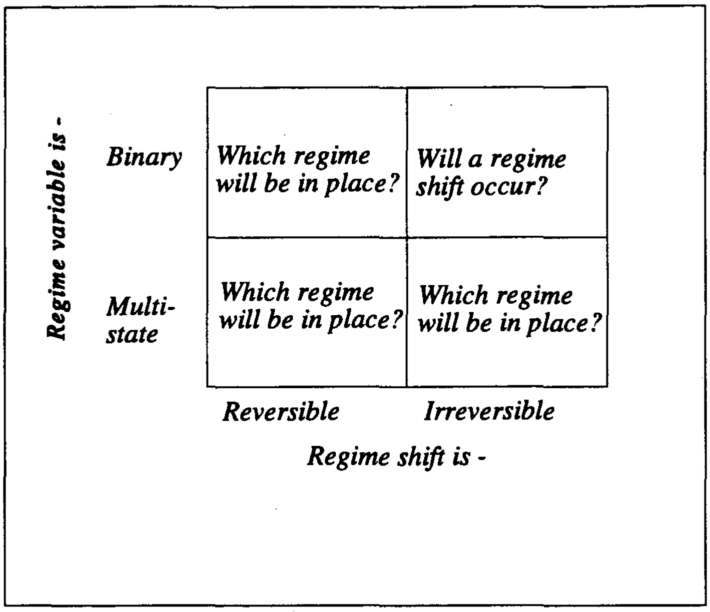

Figure 1.1 Main question capturing the political uncertainty

The uncertainty in the oil price captured by such a process reflects total uncertainty, including political uncertainty. For the oil price, all the three types of political uncertainty is clearly relevant as explanatory variables. The role of OPEC and the effect of political conflicts in the Arab Gulf has clear implications for the oil price. Type 1 uncertainty, like the possibility of introduction of a tax on fuel in the USA, does also influence on the oil price. Increased political uncertainty can then be included in the analysis by increasing the uncertainty in the stochastic process for the oil price, which is achieved by increasing the volatility, i.e., the numerical value of σ.

When the political uncertainty is included explicitly in a formal analysis, it must be done in such a way that it captures the essence of the situation being analyzed. The specific inclusion of political risk is often done in the form of *regime shifts*. The simplest regime shift models are the "either-or" models, of which the irreversible shift models are the most simple. As an example, consider a single variable x which is determined by political decision making, and is thus assumed to capture the political uncertainty. If x describes an "either-or" situation, x will be a binary variable, with possible numerical values x_0 and x_1. If the regime shift is irreversible, and $p_{t,s}$ is the time t probability that x_1 will be in place at a future date $s>t$, the

Number of regime variables	Low	High
Many	The focus is on the effect of interaction between regime variables	Analyses with high relevance, but often too complex for analytical clarity
Few	Analyses focusing on the effect of uncertainty	The focus is on a realistic description of political decision making

Level of specification of political decision making is -

Figure 1.2 Focus of analyses including political uncertainty.

political uncertainty regarding the future value of x is then characterized by $p_{t,s}$, x_0, and $\Delta x = x_1 - x_0$. The uncertainty is highest when $p_{t,s} = 0.5$, and the dispersion is increasing with increasing $|\Delta x|$. Many real-world situations can be analyzed within such a model. The best example is perhaps expropriation of an investment, or the non-payment of a loan. In these situations the question capturing the uncertainty is "Will a regime shift occur ?". In more complex models, the question is which type of regime will be in place at a certain date, and how the regimes will vary during a time period. In such models, it is not obvious what is meant by the term "increased political uncertainty"[2].

The focus of the analysis also determines how political uncertainty is included in the analysis. We see that the three types of political uncertainty are created by decisions made by governments, state companies, opposition groups, or other "political" decision makers. One can say that political uncertainty is created by uncertain political decision making. One way of categorizing analyses involving political uncertainty is according to the level of detail in the modeling of the political decision making process, and to the extent, measured in number

[2] See page 30 for a discussion of increased uncertainty when a Poisson process governs the regime shifts.

of regime variables, political uncertainty is included in the analyses. See Figure 1.2. The need for clarity usually necessitates that one can expand the analysis in one of the dimensions only.

At a general level, it is also worth pausing to consider the meaning of uncertainty in a formal model. The absence of political uncertainty, i.e., political certainty, does not imply that the regime variable will not change. Assume that the cash flow from an investment at a given time *t*, π_t, is modeled as a function of a set of state variables at time *t*, x_t, a set of decisions the investor can make, g_t, and a set of constants *K*,

$$\pi_t(x_t, g_t, K) \ . \qquad (2)$$

Going from the certain to the uncertain case involves moving the tax rate from *K* to x_t. This means increasing the number of state variables, or the dimension of uncertainty. The total dispersion in π_t is a result of all three factors, but uncertainty in π_t is usually linked to exogenously specified uncertainty in the set of state variables x_t. Take as an example uncertainty in a tax-rate. At a given time the investor is not certain which tax rule will apply at a future date. If the tax rate is a deterministic function of x_t, time, or the investor's decisions, the numerical value of the tax rate will change over time. In this case the tax rate does not however increase the *dimension of uncertainty*. In this paper I will mainly study literature where the uncertainty about political decisions increases the total dimension of uncertainty. This mean that I do not include literature about valuation and decision making under politically determined *constraints*.

One way to measure risk is by using ratings, or indices[3]. A rating, or index, is generally a rule, or function, ψ, which to a set of characteristics in a set *H* assigns an element in an ordered set Ψ. In case of a risk rating, the risk is assumed to increase, or decrease, with the number in the order of the elements in Ψ. Country risk indices measure the risk for foreign investors when investing in a given country. The term country risk is primarily used in cross-border lending. When the borrower is a government, the credit risk is known as sovereign risk, or sovereign credit risk. Credit risk is the risk that the borrower will not completely fulfill the obligations in the loan agreement such that the credit provider, or lender, suffers

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[3] I will not distinguish between the use of the terms rating or index.

losses. In the literature about cross-border lending, the term country risk can be given a precise economic content. Consider the value of a one period discount bond issued by a government with principal I. If the loan is fully repaid, the holder of the bond will receive I. If the country will not pay in full, the bond holder will only receive a fraction k. With a default probability of p, the probability of payment in full is (1-p), the risk free interest rate is r, and assuming that no risk compensation is required (the probability of default is non-systematic), the present value of the bond is given by

$$X_0 = \frac{I}{1+r}(1-p) + \frac{Ik}{1+r}p \ , \tag{3}$$

or

$$r = [\frac{I}{X_0} - 1](1-p) + [\frac{Ik}{X_0} - 1]p \ , \tag{4}$$

where the expressions in brackets are equal to the ex post rate of return in case of full or fractional payment, respectively. If $k=0$, and the ex post return in case of no default is $z \equiv [I/X_0 - 1]$, then the spread, i.e., the default risk premium, on the bond, s, is

$$s = z - r = \frac{p}{1-p}(1+r) \ . \tag{5}$$

With the assumptions made, the spread is directly related to the probability of default. The spread should then increase with an index measuring the probability of default. Such a clear economic interpretation for country risk indices is not always the case. I have in Figure 1.3 shown how the term country risk is, and can be, used for three types of foreign investment, lending, equity investment and foreign direct investment (FDI). When the term country risk is used, it is often meant to measure the possibility of loss only. The borrowers are categorized into two groups, the government and government guaranteed borrowing, and borrowing from private companies without public guarantee. Calverley (1990) distinguishes between country risk for sovereign risk and what he calls generalized (non-sovereign) country risk. He defines generalized country risk "...the risk of country-wide factors, whether

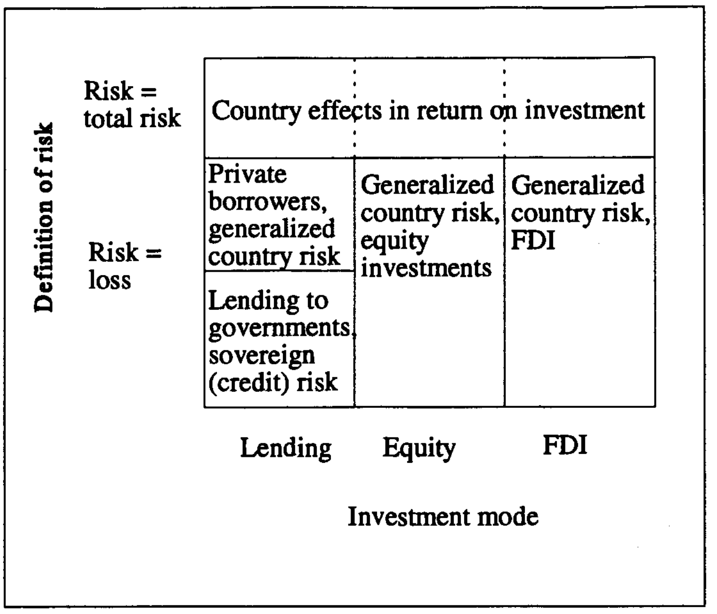

Figure 1.3 Use of the term country risk

economic or political, affecting the credit-worthiness of private sector borrowers" (p. 189). I have used Calverley's term, and extended the definition of generalized country risk to cover equity investment and FDI.

Calverley continues to propose a way of assessing the generalized country risk by considering three characteristics of the country, namely, 1) General health of the economy, 2) Stability of policy, and 3) Political stability. General health of the economy includes such factors as the country's debt burden, liquidity position, and macro economic management. Stability of policy means the stability in policy towards economic management and regulation of business activities in the country. Political instability means major discontinuities such as revolution, civil war, or war with other countries. The use of sub-criterions, or sub indices, are a typical way of constructing a country risk index. As an example, The International Country Risk Guide (ICRG) rating system is shown in Table A.2 in the Appendix, and the rating criterions for the Institutional Investor Country Credit Rating are given in Table A.1 of the Appendix. The ICRG index consists of three sub indices: Economic Risk, Financial Risk, and Political Risk, which again consists of sub indices. Notice that the Political Risk index cannot be related to specific risks for investments. The investment specific risk is found in the Financial Risk index. Relating this to Calverley (1990), the Political Risk index measures political stability, whereas the Financial Risk index measures policy stability.

There are many ways to structure a review article on political uncertainty. One could focus on type of uncertainty, review literature where the primary concern is policy making, or focus on the effect on private investors. I am concerned with decision making. In rational decision making, the decision solves a specified problem. Persson and Tabellini (1994) grouped political decisions into two groups: those solving an explicit choice problem, and those maximizing an arbitrary popularity function. For investors, rational decision making is often assumed to aim at maximizing the market value of an investment. By investors' decision making I mean such decisions as whether to invest or not, to abandon investments, close down operations temporarily, etc. In this review I will focus primarily on literature where optimal decisions are implied from the solution of a valuation problem. The valuation problem is typically to determine the market value of the investment, conditioned on the investor's decision making. I will, however, also include literature where valuation only is considered.

Decision making as such is the concern of many methodological frameworks. In game theory the behavior of rational players is analyzed in situations where the players interact. The interaction between the players are important because one player's behavior affects the payoff to the other players. The concern of game theory is often to describe, or predict, the players' decision, but not to determine the market value of the investment or decisions. I have therefore chosen as a general rule not to include game theory in this review. However, in stochastic games, the real options approach has been used to value investments where the payoff is determined by the outcome of the game. As these games involve the solution to a valuation problem, they could be included. I am not aware of any literature dealing with such games involving political uncertainty. The literature covering political risk analysis (PRA) is mainly rooted in the political science tradition. Subramanian, Motwani, and Ishak (1993) categorized research in the PRA tradition into four research streams. The first category is the definition of political risk. The second one covers normative issues such as articles advocating the importance of the political risk analysis function. The third one contains conceptual models for risk assessment. Category four covers current practices in PRA. The

PRA literature is a valuable source of information when trying to assess political uncertainty, but because I am focusing on asset valuation and valuation-induced decision making, I do not include this tradition either.

In the introduction to his book, Merton (1990) discusses the issues covered by modern finance theory. According to Merton, the theory covers the area of financial management of firms, financial management of households, intermediation, capital market, micro investment theory, and most of economics of uncertainty. The literature I have selected is in the finance tradition. I have chosen not to include more macro-oriented literature covering uncertainty in fiscal and monetary policy.

The reviewed articles are listed in Tables 1.1 and 1.2. The articles are listed in chronological order.

ARTICLE	UNCERTAINTY	DECISION/ VALUATION**
Ekern (1971)	Tax rate	Portfolio composition
Shapiro (1978)	Expropriation	Valuation
Brennan and Schwartz (1982a)	Regulation of regulated companies	Invest
Brennan and Schwartz (1982b)	Regulation of regulated companies	Invest
Brennan and Schwartz (1985) *	Expropriation	Invest, Open, Close, Abandon
Johnson and Stulz (1987)*	Default	Valuation
Mahajan (1990)	Expropriation	Invest, Structure the investment
Teisberg (1993)	Regulation of regulated companies	Invest, Wait, Abandon
Hassett and Metcalf (1993)	Tax credit	Invest, Wait, Choose scale of investment
Pindyck (1993)	Regulation	Invest, Wait, Abandon
Dixit and Pindyck (1994), chapter 9.2 B	Tax credit	Invest, Wait
Teisberg (1994)	Regulation of regulated companies	Invest, Wait, Abandon
Claessens and Penacchi (1996)	Default	Valuation
Lessard (1996)	Country risk including political risk	Valuation
Cherian and Perotti (1997)	Taxation	Invest, Valuation

* The asterisk means that the article is not primarily dealing with political uncertainty, but political uncertainty is included in the analysis, e.g., as an example.
** For literature mainly concerned with valuation, I have used the term"Valuation".

Table 1.1 Overview of reviewed literature, mainly theoretical

ARTICLE	CONTENT/ MAIN ISSUE
Kobrin (1978)	Relationship between political stability and flow of foreign direct investments.
Pindyck and Solimano (1993)*	Relationship between political stability and variance in the value of output from a country.
Howell and Chaddick (1994)	Test of the predictive power of three risk indices.
Erb, Harvey, and Viskanta (1994)	The economic content of Institutional Investor's country credit rating; application to fixed income papers.
Erb, Harvey, and Viskanta (1995)	The economic content of Institutional Investor's country credit rating; application to equity investments.
Erb, Harvey, and Viskanta (1996a)	The economic content of five risk measures; application to fixed income papers.
Erb, Harvey, and Viskanta (1996b)	The economic content of five risk measures; application to equity investments.
Diamonte, Liew, and Steven (1996)	Testing trading strategy for equity investment when using the ICRG political risk index.
Melvin and Tan (1996)	The relationship between the ICRG political risk index (and its sub-indices) and the bid-ask spread of foreign currencies.
Brunetti and Weder (1997)	Testing the relationship between measures of "institutional uncertainty" and investment rates.

* The asterisk means that the article is not primarily dealing with political uncertainty, but political uncertainty is included in the analysis, e.g., as an example.

Table 1.2 Overview of reviewed literature, mainly empirical

2 Review of Selected Literature

2.1 Regulated Companies - Rate of Return Regulation

Regulated companies are often utilities, like water works, gas providers, or electric power plants. The task for the regulator is to set output prices such that the regulated company earns an appropriate rate of return for the shareholders. According to Brennan and Schwartz (1982a) two criteria are used in USA to regulate the output prices for these companies. The criteria are the comparable earnings standard, and the capital attraction standard. The comparable earnings standard means that the output prices should be set so that the earnings for the regulated company are similar to the earnings of a comparable, unregulated, firm. The capital attraction standard means that the return should be such that the company finds it attractive to make new investments. According to popular beliefs, both standards require that the allowed rate of return should be set equal to the cost of capital, which is defined as the rate of return an investor should expect to earn on investment in other firms of equivalent risk. The implicit justification of this view is that this approach will cause the market value of the regulated company to be equal to the value of the rate base on which the return is allowed. The point of Brennan and Schwartz is that this approach does not take into consideration the regulatory uncertainty. They define (on page 509) a *consistent regulatory policy* as "..a procedure for determining the holding of a rate hearing and setting the allowed rate of return at the hearing such that, when properly anticipated by investors, the procedure causes the market value of the regulated firm to be equal to the value of the rate base at the time the hearing is held."

In the article, they studied the effect of rate of return regulation of the return x on a firm's rate base B. Note that B is generally not the market value of the rate base, but reflects the level, or size, of the rate base. The return x follows an Ito process of the form

$$dx = \mu(x)dt + \sigma(x)dz \ , \tag{6}$$

where dz is the increment of a Brownian motion. The instantaneous earning rate is xB. With a net payout rate to the owners of $p(x)$, the increase in the rate base B is given by

$$dB = (x - p(x))Bdt \ . \tag{7}$$

The regulatory policy is defined as a rule for holding a regulatory hearing, represented by an instantaneous probability that the hearing will be held during the next increment of time, $\pi(x)$, and a rule for determining the outcome of the hearing, $x^*(x)$. $x^*(x)$ is the allowed rate of return on the rate base. The market value of the firm, $F(x,B)$, is determined in a general equilibrium model like in Cox, Ingersoll and Ross (1985), but where jumps governed by a Poisson process are added. In the appendix, they state the assumptions, which are:

1. The investors have time-additive von Neuman-Morgenstern utility functions. The utility functions are logarithmic, and defined over the rate of consumption of a single consumption good.
2. There are no taxes or transaction costs in the economy, trading takes place continuously, and the market is always in equilibrium.
3. The state of the economy is completely described by aggregate wealth and an s-dimensional vector of state variables whose behavior is governed by a system of stochastic differential equations, which are a combination of a standard Gauss-Wiener process and a Poisson process.

In this model all financial assets must satisfy a fundamental partial differential equation. For the regulated company, this partial differential equation is

$$\frac{1}{2}\sigma^2(x)F_{xx} + \mu(x)F_x + (x - p(x))BF_B + p(x)B + \pi(x)[F(x^*(x),B) - F(x,B)] = rF + \lambda\sigma(x)F_x \,. \quad (8)$$

The left hand side of (8) is equal to the expected return on the market value of the firm. The first three terms reflect the expected return due to the changes in x and B, the fourth term is the net dividend to the owners, and the fifth term reflects the effect of regulation. The right hand side of (8) is the required return in market equilibrium, where r is the constant risk free interest rate and $\lambda\sigma(x)$ is the covariance between changes in x and the rate of return on aggregate wealth. An increase in λ means that the systematic risk increases[4].

[4] In equilibrium the excess expected return on asset i is equal to the covariance between the rate of return on asset i and the rate of return on aggregate wealth, i.e., $\alpha_i - r = \sigma_{i,w}$. The required rate of return for asset i is then: $\alpha_i = r + \lambda\sigma_i$, where λ is the standard deviation of the rate of return on aggregate wealth multiplied by the correlation coefficient between the rate of return on asset i and the rate of return on aggregate wealth.

After having established (8), Brennan and Schwartz define a new variable, $y(x) \equiv F(x,B)/B$, which they name the *normalized value of the firm*. We see that the normalized value of the firm is equal to the market value of the firm, given the current rate of earning and the level of the rate base, divided by the level of the rate base. Equation (8) is then reformulated by inserting $y(x)$,

$$\frac{1}{2}\sigma^2(x)y_{xx} + y_x(\mu(x) - \lambda\sigma(x)) + (x - r - r(x))y + p(s) + \pi(x)[y(x^*(x)) - y(x)] = 0 \ . \qquad (9)$$

We see again that the influence of the regulatory policy on the value is captured in the last term on the left hand side. Brennan and Schwartz note that if $\partial x^*/\partial x = 0$ and $\partial\pi/\partial x = 0$, then as $\pi \to \infty$, $y(x) \to y(x^*)$. This represents a situation with a "policy of continuous" regulation under which the firm always earns the allowed rate of return. In case of no regulation, $\pi(x)=0$.

Brennan and Schwartz state that for a consistent regulatory policy $y(x^*)=1$, or $F(x^*,B)=B$. This means that if a hearing is held, and the allowed rate of return is x^*, the market value of the firm at the time of announcement of x^* is equal to the current value (or level) of the rate base.

In an explicit model, they make three assumptions. The rate of return process (6) has constant parameters μ and σ, the output capacity of the firm is proportional to the rate base, and the firm is required to maintain capacity equal to potential demand which is growing at the constant rate g. From (7), this means that the net payout rate is $(x-g)B$. With this specific model, they value the firm in the case of no regulation and with two models for holding rate hearings. The case with a constant probability of a hearing, $\pi(x)=\pi$, is named *stochastic regulatory hearings*. The second model for rate hearings is a model where hearings are held when the rate of return x reaches pre-specified upper or lower bounds. This is named *deterministic regulatory hearings*.

The article contains numerical examples for the case when $\sigma = 0.005$, $\mu = 0.0$, $\lambda = 0.14$, $r = 0.08$, and $g = 0.06$. I show the firm value for three cases in Figure 2.1. Under stochastic regulatory hearings π=0.1 and x^*=0.086. For deterministic regulatory hearings, the upper trigger point, x_u, is 0.18, and the lower trigger point, x_l, is 0.03. In this case x^*=0.092.

For the unregulated case x^*=0.099. In the case with stochastic regulatory hearings, the value of the firm will rotate clockwise with increasing π. When π becomes large, the normalized value of the firm will be parallel to the x-axis and will pass through 1.0. In case of deterministic regulatory hearings, the normalized value of the firm will get closer to 1.0 as the rate of return x gets closer to the upper and lower trigger levels.

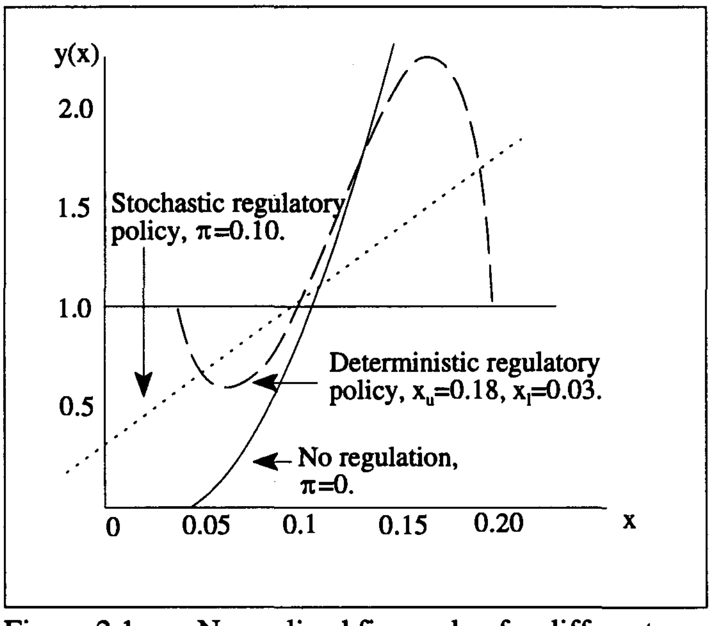

Figure 2.1 Normalized firm value for different regulatory policies.
Source: Figure 1 in Brennan and Schwartz (1982a)

The investment incentives for the regulated firm are evaluated by studying an investment of size I, which will generate an instantaneous earnings rate ρI, where ρ is assumed to be described by the same evolutionary equation as the existing return x_0, i.e., (6). The effect of the investment on the rate of return is

$$x(I) = \frac{x_0 B + \rho I}{B + I} , \qquad \textbf{(10)}$$

and

$$\frac{dF}{dI} = F_x \frac{dx}{dI} + y(x) \ , \tag{11}$$

where $dB/dI = 1$. By differentiating (11) with respect to I, and setting $I = 0$, the gross present value of a marginal investment is

$$\left.\frac{dF}{dI}\right|_{I=0} = y(x) + y_x(x)(\rho - x) \ . \tag{12}$$

An investment will be undertaken if (12) exceeds unity. In Figure 2.2, the gross present value of a marginal investment is shown for the three cases of regulatory policy when a low return investment (ρ=0.08) is considered. The corresponding values for a high return project (ρ=0.2) are shown in Figure 2.3. The effect of regulation on investment incentives are measured as the difference in the present value of the same investment project of a regulated and an unregulated firm.

The low return project will not be undertaken in case of no regulation or when hearings are stochastic. In case of deterministic hearings, the incentive to undertake the investment increases strongly as the return reaches the upper trigger point for the regulatory hearing. The intuition is that the low return project is undertaken in order to reduce the probability of a regulatory hearing which will reduce the return to x^*. The high return project will always be undertaken in the case of no regulation and with stochastic regulatory hearings. With deterministic regulatory hearings, there is a strong disincentive to undertake the project as the return x gets closer to the upper and lower trigger points.

Whereas the investment policy in the examples of Brennan and Schwartz (1982a) is exogenous (g is a constant), the investment policy in Brennan and Schwartz (1982b) is endogenous. Here the investors are allowed to determine the investment rate within upper and lower bounds. The investment rate is treated as a policy control. The return on new investments are, for illustration, supposed to be of the form

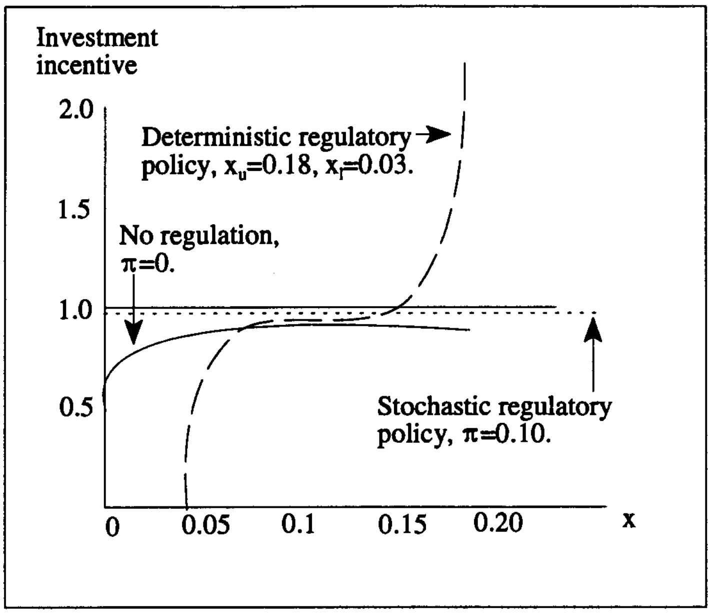

Figure 2.2 Investment incentives for investment in a project with relatively low profitability.
Source: Figur 2 (a) in Brennan and Schwartz (1982a)

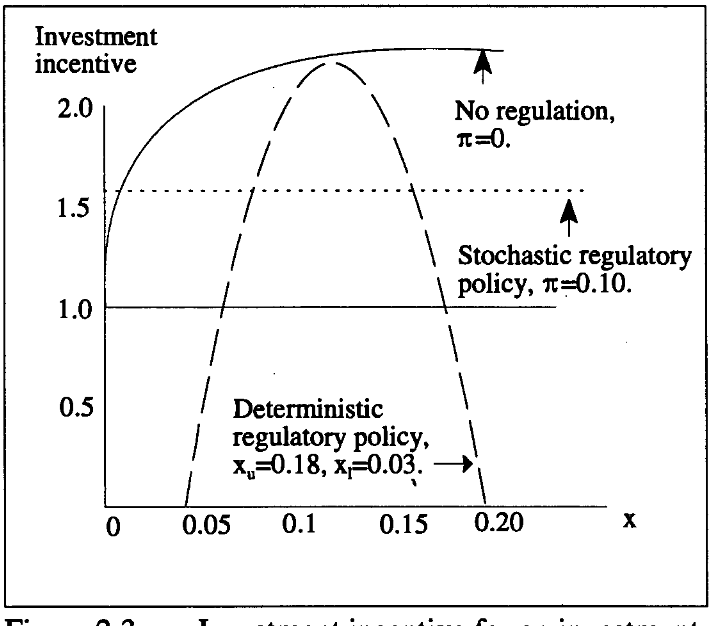

Figure 2.3 Investment incentive for an investment with relatively high profitability.
Source: Figure 2(b) in Brennan and Schwartz (1982a)

$$\rho = a + bgcx \ , \qquad (13)$$

where $b<0$ and $c>0$. The return on new investment is thus increasing with the rate of return on the existing rate base and declining with the amount invested. In an example, Brennan and Schwartz make the following assumptions: ρ=0.05-g+x, $g\in(-0.1,0.1)$, $\lambda\sigma_w = 0.14$ and r=0.08. The regulatory policy is consistent with $y(x^*)=1$, and $\pi(x)=0.1$ or $\pi(x)=2|x-0.1|$.

The value of the firm under the two alternatives for regulatory hearings are shown in Figure 2.4. We see that if the probability of a hearing increases when the return moves away from the allowed rate of return, x^*=0.1, the normalized market value will not diverge far away from one. In Figure 2.5 we see the optimal investment rate, $\hat{g}$, under the different rules for holding a hearing.

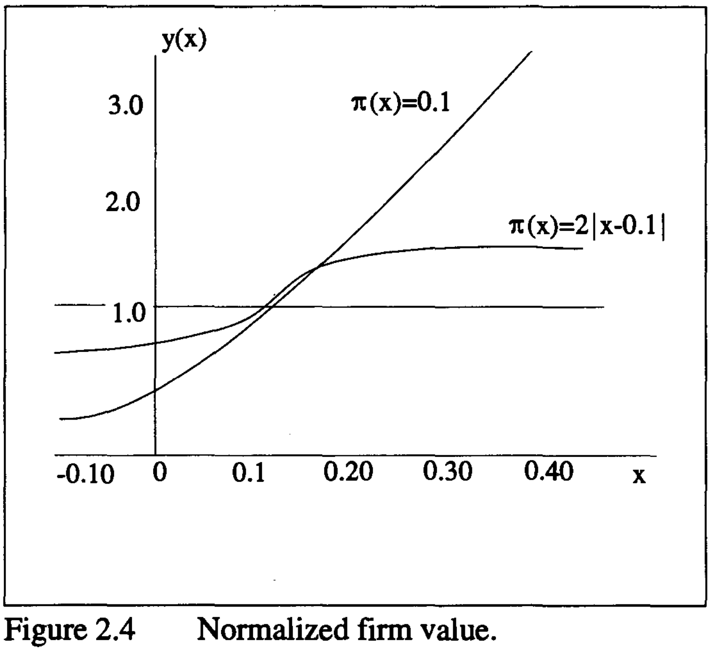

Figure 2.4 Normalized firm value.
Source: Figure 2 A) in Brennan and Schwartz (1982b)

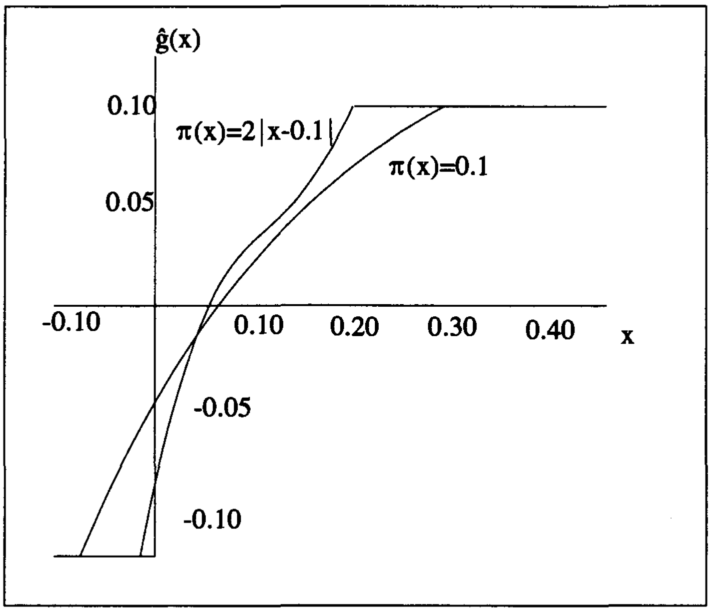

Figure 2.5 Investment policy.
Source: Figure 2B) in Brennan and Schwartz (1982b)

In Teisberg (1993) and (1994) the effect of three policies: cost allowance policy, financing cost policy and abandonment policy, are evaluated with regard to the policies' effect on a regulated company's decision to invest, wait, or abandon an investment. The equilibrium market value of an investment project, $F(V,K)$, is a function of the current market value of a completed project, V, and the costs of completing construction, K. The firm's decision variable is the rate of investment, which is bounded upwards by k. Above an investment threshold V^*, it is optimal to invest at rate k. Below V^*, the investment rate is zero. Below a second threshold, V^0, it is optimal to abandon the investment project.

The market value of a completed regulated investment project evolves according to the following stochastic differential equation (equation (1) in the articles)

$$dV_t = (\mu - \delta(V_t))V_t dt + \sigma V_t dZ_t \ , \qquad \textbf{(14)}$$

where μ is the expected market return of a non-regulated company, $\delta(V_t)$ is the "rate of foregone earnings", "rate of return shortfall", or "net convenience yield", and dZ_t is the increment of a Brownian motion. The cost allowance policy is described by V_t. Equation (14) is assumed to reflect the uncertainty stemming from a review of the completed projects' investment, together with rate of return regulation. The value of the completed project is therefore uncertain. Note that this uncertainty is exogenously given by (14). The financing cost policy is characterized by $\delta(V_t)$. There are three ways a regulator can treat the firm's costs related to the investment: as expenses, as construction work in progress (CWIP), or as an allowance for funds used during construction (AFUDC). She states that regulators can use combinations of these three policies. If the firm expects partial AFUDC disallowance, then $0<\delta(V_t)<\mu$. A higher fraction of disallowance corresponds to higher numerical value of $\delta(V_t)$. It is assumed that $d[\delta(V_t)]/dV_t \geq 0$, since "..regulators are less likely to allow further increases in the value of a completed project as the value gets higher...". The abandonment policy, or salvage value (SAL) in case of abandonment, is characterized by the allowed fraction z of previous used expenditures for the project which the firm recovers. This means that if accumulated expenditures are K, then SAL(K)=zK.

We see that with deterministic $\delta(V_t)$ and z, the only uncertainty is linked to the diffusion part of equation (14). An explanation is not given for the difference between equation (14) and an evolutionary equation for a similar non-regulated investment project, except that $\delta(V_t)$ would not have been included for a non-regulated project. In fact, Teisberg refers to a case with constant δ as a case with no profit restrictions. Would equation (14) reflect the value of a completed non-regulated project if $\delta(V_t)$ were not included, or if the δ was a constant ? If yes, then the diffusion part of (14) is identical for a regulated and an unregulated company. In numerical examples, the value of the project and the optimal decision are derived. This is done for different assumptions about regulation. The article thus complements the original article of Majd and Pindyck (1987). In the introduction to Teisberg (1994), she notes that the analysis can apply for unregulated companies where the value of a completed project is influenced by taxation or the possibility of nationalization.

Pindyck (1993) studies the implications of cost uncertainty for irreversible investment decisions, and uses as an example investment in a nuclear power plant. He specifies two types of uncertainty relevant for costs, technical uncertainty and input cost uncertainty. In the latter group he includes "unpredictable changes in government regulation" as a source for cost uncertainty. Technical uncertainty can only be resolved by undertaking the project, while input cost uncertainty is external to what the firm does. Pindyck assumes that the value of a finished project, V, is certain, but that the cost of completing the project, K, is uncertain. The payoff from completing the project is then comparable to the payoff of a put option, $\max[0, V-K]$. The costs to completion follows a controlled diffusion process, where the investor decides whether to invest at a given rate, or not invest. The effect of uncertain regulation on the cost is not specified specifically. The technical uncertainty is treated as independent of the overall economy, whereas this may not be the case for input cost uncertainty. The effect of the two types of uncertainty is that technical uncertainty makes investment more attractive, whereas input cost uncertainty makes investment less attractive.

2.2 Taxation

Ekern (1971) studied the effect of uncertain taxation of asset return in a one period model with two assets, one with a risk free return and one with a stochastic return. A change in political risk is defined " ... in terms of a corresponding change in a dispersion shift parameter which indicates the stretching or compression of the probability distribution around its expected value." He studied three problems. The effect of uncertainty in taxes on investors choice between the risk free and the risky asset, if tax uncertainty disturb the market equilibrium, and the effect of a change in political uncertainty on tax revenue and social welfare in a country. Ekern assumes that the tax is stochastically independent of asset return. For the general case, a clear relationship between increasing political uncertainty and portfolio composition cannot be established. For a special case with quadratic utility function, he finds that an increase in political uncertainty reduces the portion of wealth invested in the risky asset.

Hassett and Metcalf (1994) analyze the effect of an uncertain tax credit π on the investment threshold. With a pre-tax investment amount I, the after tax investment amount is $I\pi$, i.e., π is the portion of the investment expenditure the investor has to pay. Because the tax credit is uncertain, the investor does not know for certain the size of the tax credit at future points in time. The investment threshold, or hurdle, is the value which the investment project must exceed in order for an investment to take place. In the model the firm chooses when to undertake an investment project, and the amount of capital, *K*, employed in the project. The number of units of output are given by the production function F(*K*), where F' > 0, and F''< 0. The price of output, p, is a geometric Brownian motion with constant parameters. The cost of capital, p_k, is also a geometric Brownian motion with fixed parameters. In principle, these processes can be correlated. The cost of capital is the net present value of the costs. Uncertainty regarding the tax credit is assumed to be captured by the diffusion part in the stochastic differential equation describing p_k. The firm's problem is to find the ratio $(p/p_k)^*$, which describes the optimal time to invest. *K* will increase with this ratio. In this model, the investmen threshold is increasing with increasing variance in output prices and/or cost of capital. This effect is known from the literature.

With these results as a starting point, Hassett and Metcalf consider another model, in which two tax credits, or policy regimes, are possible, π_0 and π_1. Since there is no uncertainty related to the size of the tax credit per se, the uncertainty is related to which of the policy regimes will be in place at a given time. The shift between the two states is modeled as Poisson processes, where $\lambda_1 dt$ is the probability that the tax credit π_1 will be introduced at the next increment of time given that the tax credit today is π_0. Similarly, $\lambda_0 dt$ is the probability that the tax credit π_0 will be introduced during the next increment of time if the tax credit today is π_1. In this model the cost of capital, p_k, is constant. The after tax cost of capital is $(1-\pi)p_k$. The output price is given by a geometric Brownian motion.

Hassett and Metcalf provide a discussion of what increased uncertainty means in this context. They would prefer to have a mean preserving spread. This can in principle be done in two ways. The first method is to let the values of λ's be given and adjust π_0 and π_1 such that the expected tax credit, E(π), is unchanged. The effect of the difference between π_0 and π_1 is called "the spread between rate" effect. The second method is to vary one of the λ's and adjust π_0 and π_1 such that the mean is preserved. The effect of adjusting one or both of the λ's , is called the "frequency effect". The uncertainty is however not necessarily increasing with increasing λ's. Let $\lambda_0 = \lambda_1 = \lambda$. Then, with a high λ, the instantaneous probability of a switch between 0 and 1 is close to 1. In such a situation the variation will be very high, but there will almost be no uncertainty. Hassett and Metcalf state that in a continuous time setting, there is highest uncertainty when there is a probability of transition from the current state over the next year, equal to 0.5, which corresponds to a λ of 0.69. This number is found by solving the equation

$$1 - e^{-\lambda * 1} = 0.5 \quad . \tag{15}$$

The left hand side of this equation is the cumulative distribution function for the exponential distribution with argument one, i.e., the probability that at least one "jump" or "transition" takes place in one year.

They first study (by running simulations) the effect of changes in frequency on the investment threshold. They set $\lambda_0 = \lambda_1 = \lambda$, let λ range from 0 to 1, and consider the case where π_0 = 0.0[5] and $\pi_1 = 0.15$. See Figure 2.6. In the case that no tax credit is in place, the trigger price increases with increasing probability that an investment credit will be introduced. If an investment credit is in place, the trigger price will first decrease when λ increases, but when λ increases above 0.20 the trigger price starts increasing again. They then let $\lambda_0 = 0.33$, and vary λ_1 between 0 and 1. In this case the trigger price is increasing with increasing λ_1 whether or not a tax credit is in place at time 0. The implication is that there is an increasing incentive to wait when the probability that a tax credit is in place increases. They then let λ_1 = 0.33, and vary λ_0 between 0 and 1. In this case the trigger price is decreasing with increasing λ_1, whether a tax credit is in place or not at time 0. The implication is that there is less incentive to wait when the probability that a tax credit is not in place increases. Changing both λ_0 and λ_1 have thus offsetting effects. They continue to study how mean preserving spreads in the level of the tax credit affect investment. In the example, the increase in spread reduces the trigger price. Note that when the spread is changed, the probabilities for type of regime in place will also change. Several effects are therefore considered at the same time. Hassett and Metcalf conclude that whereas the effect of increased uncertainty on the investment threshold in the initial model with geometric

[5] Metcalf and Hassett write that $\pi_0 = 0.05$ (page 22 in the article), but this must be a misprint judged from the following discussion.

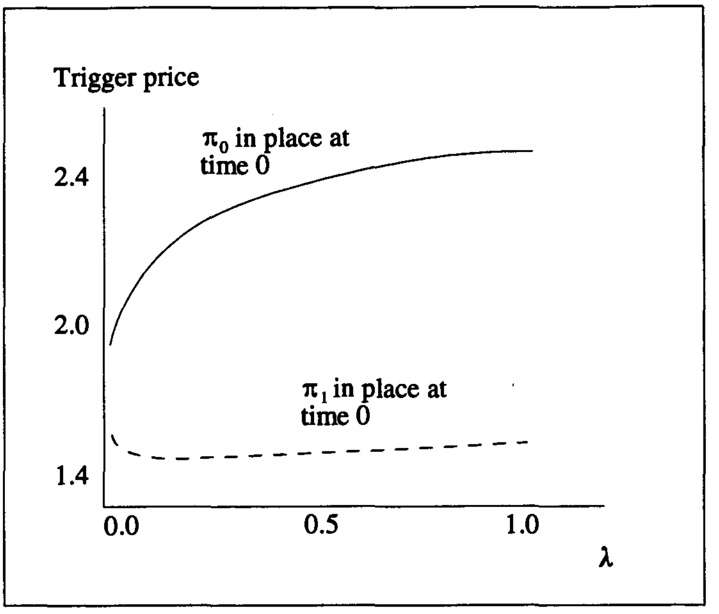

Figure 2.6 Price which triggers investment. Source: Figure 1 in Hassett and Metcalf (1994)

Brownians motions is increasing with increased uncertainty, the effect of uncertainty in tax regime when using Poisson processes is not that simple. It all depends on the form of policy uncertainty.

Dixit and Pindyck (1994), chapter 9.2 B, use the same model as Hassett and Metcalf (1994), where the change in tax credit policy is governed by Poisson processes. The amount of capital is however fixed. The analysis on the effect on uncertainty on the investment threshold have the same conclusions as Hassett and Metcalf.

Cherian and Perotti (1997) start by modeling an economy where the government is either of a type imposing a tax on foreign investments or not. The model is game-theoretic, and the investors base their expectations about the type of government by observing whether a tax is introduced or not. They then state the theoretical model's implications for investment flows and volatility of asset prices. The implications of the model on asset prices are then tested by examining a time series of prices of options written on Hang Seng stocks, i.e., stocks listed on Hong Kong's stock exchange.

In the theoretical model, Cherian and Perotti assume a multi-period framework. The government first announces, at time 0, a favorable fiscal policy promoting foreign investment. The government is either contrary (committed) or favorable (uncommited) to a future tax on foreign capital. The government and the foreign investors are assumed to be risk neutral. The committed government is averse to taxation, while the uncommited government maximizes the expected stream of tax revenue, discounted at a rate δ per period. The government is unable to credibly reveal its true type and the investors learn about the true type of government only by observing its actual policy. At time zero the investor expects the government to be committed with probability p_0 and uncommitted with probability $(1-p_0)$. In general, p_t is referred to as the government's credibility or reputation for commitment at time t. The game is played over an infinite time horizon, and the government's strategy for each period is either not to impose a tax, or to impose a tax of size τ.

Capital investment is fully reversible, meaning that the investment can be costlessly scaled down within one period. Or alternatively, the capital stock fully depreciates in one period. The pre-tax cost of capital is a constant r. The production function, $R(K)$, is twice differentiable with positive, declining marginal productivity, i.e., $R'(K)>0$, $R'(0)=\infty$, and, $R''(K)<0$. The only source of uncertainty in the model is the government's tax policy.

Cherian and Perotti then find the Perfect Bayesian Equilibrium, described for each period $t>0$ by:

a. The governments reputation p_t, which is computed from the prior p_{t-1}.

b. The optimal strategy for both types of governments. The governments are playing mixed strategies, meaning that they randomize over the actions $\{0, \tau\}$. The committed and uncommitted governments choose to tax with probability λ_t and μ_t, respectively, at time t. By assumption, λ_t is set equal to zero for all points in time.

c. An investment rule K_t for the investors, which is a function of the history of the game and the investor's belief about the future tax policy.

In each period the investing firm chooses the optimal investment programme, K_t, such that

$$\max_{\{K_t\}}\{\mathrm{E}_t[\mathrm{R}(K_t)-(r+\tau I_t)K_t]\} \quad , \tag{16}$$

where I_t is an indicator function equaling one if taxation is imposed at time t, and zero if not. The investor maximizes the expected return in excess of the opportunity cost of capital, r, and expected tax payment, τI_t. The expected numerical value of I_t at time t is Θ_t, where

$$\mathrm{E}_t[I_t]=\Theta_t=\mu_t(1-p_t)+\lambda_t p_t=\mu_t(1-p_t)\ . \tag{17}$$

The first order condition determining the amount of capital invested is:

$$\mathrm{R}'(K_t)=r+\tau\mu_t(1-p_t)=r+\tau\Theta_t\ . \tag{18}$$

Cherian and Perotti analyze the solution to the game, i.e., how the game will be played. In proposition one they state that an opportunistic government will choose to tax in all following sub-periods after the first time the government introduces a tax. They state in proposition two that a pure strategy of immediate taxation is not optimal and that the opportunistic government will randomize between taxing now and waiting for at least one period before introducing the tax. They establish the time T when an uncommited government will impose a tax with probability one. An *equilibrium path* is developed where the government is indifferent between imposing the tax at time t or at t+1. In proposition four they state that "capital accumulation will increase while the hazard rate will decrease over time as long as no taxation is observed". When the investors do not observe an introduction of the tax, they increase their expectation of the government being of the committed type. Because of the production function, this will increase the amount of capital invested.

Given the prior belief p_t, the government's reputation at time t+1 is given by

$$p_{t+1}=\frac{\text{Pr(no tax at time t | government is committed)Pr(committed)}}{\text{Pr(no tax at time t)}}=\frac{p_t}{1-\Theta_t}. \tag{19}$$

After having analyzed the solution to the dynamic game, they then examine how financial prices and conditional volatility evolves in an economy of the type modeled.

The ex-post realized profit at time t is $\pi(K_t,I_t) \equiv R(K_t) - (r+\tau I_t)K_t$, from (16). The price at time t of a claim to the expected profit at time $t+i$ is

$$P_t \equiv \frac{\Theta_t q_t(i)\pi(K_t,1) + (1-\Theta_t)q_t(i)\pi(K_t,0) + (1-q_t(i))\pi(K(1),0)}{(1+r)^i} \quad , \tag{20}$$

where $q_t(i)$ is the probability at time t that the uncommited government does not tax untill time t+1. The value of an equity claim is equal to "..the discounted sum of the perpetual stream of P_t.

The conditional variance of posterior beliefs p_t is given by

$$\sigma_t^2 = (1-\Theta_t)(p_t - p_{t-1})^2 + \Theta_t(0-p_{t-1})^2 = (p_{t-1})^2\Theta_t/(1-\Theta_t) \ . \tag{21}$$

The first term in (21), $(p_{t-1})^2$, is increasing in t, while the last term, $\Theta_t/(1-\Theta_t)$, will decrease with t and with the limit equal to zero. Cherian and Perotti state that the volatility of equity prices will in principle correspond to σ_t^2, see Figure 2.7. At first the conditional volatility will increase as the government's credibility increases, but then decrease as the credibility converges to one. The empirical implication of the model is that "...implied volatility, which in an efficient market is the market's conditional expectation of future volatility, would reflect a policy risk component which tends to decline over time".

The implied volatility, derived by using the dividend yield adjusted Black-Scholes option pricing formula, are examined for *covered warrants*, which are (despite of the name) standard options with maturity of approximately two years. The sample includes thirteen warrants written on eleven stocks listed on the Hong Kong Stock Exchange. The sample period is 1992-1994. Weekly price data were used.

The system of equations estimated was

$$Sigma_{i,t} = b_0 + b_1 \mathrm{x} T_{i,t} + b_2 \mathrm{x} PV_{i,t} + b_3 \mathrm{x} HisVol_{i,t} + b_4 \mathrm{x} Vlm_{i,t} + b_5 \mathrm{x} \beta_{i,t} \ . \tag{22}$$

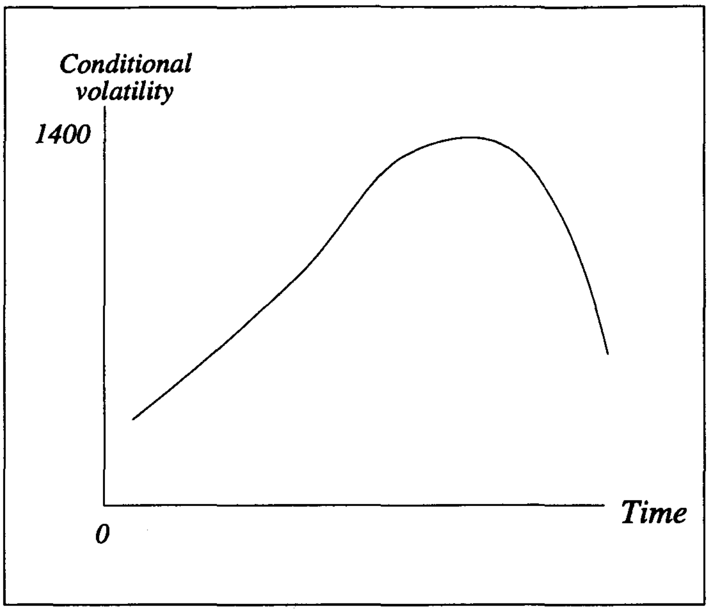

Figure 2.7 Conditional stock price volatility under a production function of the type $R=K^{0.5}$
Source: Cherian and Perotti (1997), Figure 4

Sigma is the implied volatility and the trend variable is calendar time, *T*. The other variables are the absolute difference between the value of the stock price and the value of the strike, *PV*, historical volatility, *HisVol*, the warrant trading volume, *Vlm*, and the beta of the stock measured with respect to the Hang Seng Index.

The hypothesis is that the coefficient b_1 should be negative, meaning that the implication of the theoretical model cannot be rejected. The regression resulted in a negative coefficient for calendar time, significant at the five per cent level.

2.3 Expropriation

In Shapiro (1978) a traditional discounted cash flow approach is used to value foreign direct investments. The investment project can be expropriated. It is assumed that the risk of expropriation does not influence the rate of return requirement for the investment. The argument is that international investments are to a large degree independent of national investments. The result of this diversification is thus that the risk in the return on international investments is unsystematic.

In the seminal article of Brennan and Schwartz (1985), a mine is valued under optimal decision making regarding when to open, close or abandon the mine. The cash flow of the mine when open is modeled as a stochastic differential equation. Included in the cash flow are two tax rates on the value of the mine, one when the mine is open and another when the mine is closed. They state that these tax rates can be interpreted as the intensities of Poisson processes governing the event of expropriation. With this interpretation they assume that there is no risk premium associated with the possibility of expropriation. Note that this implies that the probability of expropriation when the mine is closed can be different than the probability of expropriation when the mine is opened. In Figure 2.8 I have shown their Figure 1, the value of a mine when resources are infinite. *V* is the value of an open mine and W is the value of a closed mine. S is the spot price of the output from the mine. S_1^* is the spot price at which the mine is closed, and S_2^* is the spot price at which the mine is opened. The cost of closing and opening the mine are k_1 and k_2, respectively. The value of the mine in Figure 2.8 does not include expropriation risk. If expropriation risk is introduced for the mine when it is open only, this would shift the line *V* downwards, and therefore increase both S_1^* and S_2^*. If expropriation is possible when the mine is open, the incentive to keep the mine closed is increased. If there is expropriation risk only when the mine is closed, this would shift *W* down, and cause that both S_1^* and S_2^* are lowered. In this case, the incentive is

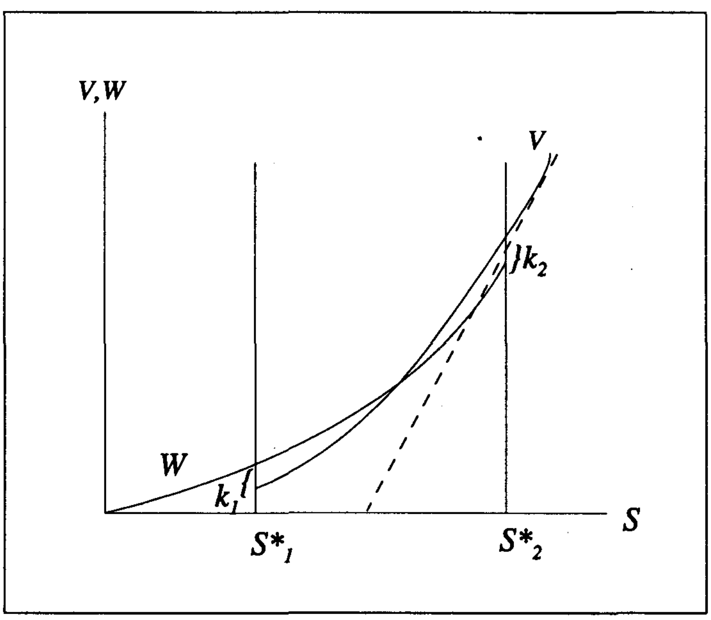

Figure 2.8 The value of a mine.
Source: Brennan and Schwartz (1985)

to keep the mine open.

In Mahajan (1990) the net present value of an investment project under expropriation risk is

$$\text{NPV}^* = \text{NPV} - C \ , \tag{23}$$

where NPV is the value without expropriation uncertainty, and where C is the present value of an option to expropriate the investment. The government's opportunity to expropriate a given investment with market value S, is viewed as a call option on S with a stochastic exercise price X. X is the cost to the government when expropriating the investment. The total costs to the host government are consisting of three parts. The first part is direct and indirect compensation paid to the investor. The second part is the difference between the value of the investment before and after expropriation, i.e., the value to the government, and not necessarily market value. The third part of the costs are reduced benefits from reduced future inflow of direct investments, and direct penalties from the investor's home government.

Mahajan assumes that the market value of the equity develops according to a geometric Brownian motion,

$$\frac{dS}{S} = \alpha_s dt + \sigma_s dz_s \ , \tag{24}$$

and that the costs with expropriation also evolves according to a geometric Brownian motion,

$$\frac{dX}{X} = \alpha_x dt + \sigma_x dz_x \ . \tag{25}$$

The instantaneous coefficient of correlation between S and X is ρ_{SX}. Mahajan simplifies the analysis by assuming that the investment has a fixed time horizon, T. A fixed horizon is typically the case for joint ventures and for projects involving licensing and other contractual agreements. He further assumes that no dividends will be paid by the investment in the time leading up to T. This assumption was necessary in order to obtain a closed form solution to the valuation of the option to expropriate. With these assumption, it can be shown that it does not pay to exercise the call option before time T, as is the case for a European stock option when the stock is not paying dividends.

The "hedge security", or "twin asset", which is perfectly correlated with X, is assumed to develop according to equation (25), but with the exception that α_x is replaced by r_h, the expected rate of return on the hedge portfolio. A political risk insurance contract which compensates the investor fully for the market value of the project in case of expropriation, will provide a perfect hedge. If only partial insurance is available, Mahajan claims that a hedge can still be created by buying partial insurance and issuing bonds to the host country. If the country expropriates, the investor will then default on its bonds. The closed form valuation formula for the call option is (Mahajan's equation (7))

$$C = SN\left\{\frac{\ln(S/X) + [r_h - \alpha_x + (\sigma^2/2)]T}{\sigma\sqrt{T}}\right\} - Xe^{-(r_h-\alpha_x)T}N\left\{\frac{\ln(S/X) + [r_h - \alpha_x - (\sigma^2/2)]T}{\sigma\sqrt{T}}\right\}, \quad \textbf{(26)}$$

where $\sigma^2 = \sigma_S^2 - 2\rho_{SX}\sigma_S\sigma_X + \sigma_S^2$. We recognise this problem as the problem considered by Margrabe (1978), i.e., finding the pricing formula for the value of exchanging one asset for another[6]. The expropriation risk is a function of the relationship S/X today, the volatilities and covariance of the processes, and the time horizon T.

An important implication of (23) is that the investor should structure the investment such that NPV* is maximized. The value of expropriation risk can be reduced, e.g., by reducing the value of the investment to the government if expropriation takes place.

2.4 Default

Johnson and Stulz (1987) used option pricing techniques to value assets under default risk. They model the repayment capacity for the writer of an option as a stochastic variable. In section III A, they give an example of a bond insurance, where it is assumed that a change in government will cause default. The probability that the current government will be in place at

[6] Equation (26) corresponds to the closed-form valuation formula, given in Hull (1993) p. 423, for the value of the right to exchange one asset for another. In Hull's formula, "yields" or "dividends" are included. Compared with equation (26), the "yield" on asset one (q_1) is $r_h - \alpha_x$ and the yield on asset two (q_2) is zero.

a future date T, is modeled by the exponential distribution, $e^{-\lambda T}$. It is implicitly assumed independence between change of government and asset value (repayment capacity), and value of the bond. The value of the bond insurance can be seen as the value of a put option, V(P). Considering the effect of uncertainty regarding the type of government, the value of the put option is $V(P)(1-e^{-\lambda T})$. They find that an increase in time to maturity has an ambiguous effect. The present value of the promised payment falls, while the probability of a regime change increases with time to maturity.

Claessens and Penacchi (1996) used the observed market prices of Brady bonds to estimate the likelihood of Mexican default. There exist two types of Brady bonds. Both bonds have an original maturity of thirty years, with the principals fully collateralized by thirty year US Treasury zero coupon bonds. Both bonds have oil recapture clauses, which gives the creditors a share in Mexico's oil export revenue if oil prices increase by a specified percentage in the years 1997 and beyond. The bonds have also a rolling guarantee covering up to eighteen months of interest payments. This rolling guarantee is collateralized by an escrow account at the Federal Reserve Bank of New York. The bonds differ in rate of return and size of the principal. One is a discount bond with a principal equal to sixty five per cent of the original face value, and with a floating interest rate of LIBOR +13/16. The other is a par bond with the principal equal to the original face value, and a fixed interest rate of 6.25 per cent.

The event of default on the interest payments is assumed to be governed by an unobservable state variable z_t. It is assumed that z_t follows the arithmetic Brownian motion process

$$dz_t = \mu\, dt + \sigma\, dq \ , \qquad (27)$$

where dq is a standard Wiener process. Claessens and Penacchi make the assumption that the country's default risk is diversifiable. This implies that z_t contains no systematic risk and is uncorrelated with the risk free interest rate. The rolling guarantee is covering τ periods of interest payments. It is assumed that when non payment by Mexico occurs for the first time, the guarantee is called upon in full. The time interval t_2 is defined as the interval $[t_2, t_2 + \tau]$, and the time interval t_1 is defined as the interval $[(t_2 - \tau), t_2 = t_1 + \tau]$. The interest rate guarantee is paid during time interval t_2 if the non payment is announced, or made clear, during the time interval t_1. Let ρ be the stopping time $\rho = \{\min \rho : z_\rho \leq 0, 0 \leq \rho \leq T\}$, the first time that the unobservable variable z hits zero. Assume that an interest payment is due at time t', which belongs to the time interval t_2. The time 0 probability that the bond holder will not receive this interest payment is

$$(1-\psi_0(t_2)) = P(\min z_t < 0, t \in (t_1,t_2)) - P(\rho \in (t_1,t_2)) \ , \tag{28}$$

where the first term of the element on the right hand side of (28) is the probability that z_t is negative during the time interval (t_1,t_2), and the last element is the probability that z_t hits the 0-barrier for the first time during this interval. $\psi_0(t_2)$ is thus the time 0 probability that an interest payment due during interval t_2 will be received by the bond holder.

The valuation formulas for the time 0 value of interest payment due at time t, I_t, for the par bond is given by

$$v_0^{par}(I_t) = p_0(t) I_t \psi_0(t_2) \ , \tag{29}$$

where $p_0(t)$ is the time 0 price of a default free zero coupon bond paying USD 1 at time t, and $\psi_0(t_2)$ is the time 0 probability that the bond holder receives the interest payment over the interval t_2. If D is the level of principal (e.g., 65% of original face value) and s is the spread over the yield on a default free six month bond, issued six months prior to the interest payment date (remember that the Mexican discount bond has a spread of 13/16 per cent over LIBOR), the value of the discount bond's interest payment[7] is

[7] This footnote contains the explanation of equation (30), and corresponds to footnote 7 in Claessens and Pennacchi. At time 0 the investor can borrow $p_0(t)$ which will be repaid with the amount 1 at time t. At time 0 the investor invests $\exp(1/2s)p_0(t-1/2)$ in a bond maturing at time $t-1/2$. The net expenditure at time 0 is then $\exp(1/2s)p_0(t-1/2) - p_0(t)$. At time $t-1/2$ the cash flow from the second investment at time 0 is $\exp(1/2s)$. This cash flow can be reinvested in a bond at time $t-1/2$ which matures at time t. Making the investment at time $t-1/2$ produces a cash flow at time t equal to $\exp(1/2s)\exp(1/2R(t-1/2,1/2)) = \exp(1/2R(t-1/2,1/2)+s)$, where $R(t-1/2,1/2)$ is the continously compounded yield on a six-month default-free bond issued at time $t-1/2$. The net cash flow at time t, after repayment of the borrowed amount, is $\exp(1/2R(t-1/2,1/2)+s) - 1$ which is equal to the floating rate bond's semiannual coupon payment at time t. The cost at time 0 of producing this cash flow is, as we have seen, $\exp(1/2s)p_0(t-1/2) - p_0(t)$. In order to exclude an arbitrage opportunity, this must also be the time 0 value of the floating rate bond's semiannual coupon payment at time t.

$$\nu_0^{\text{discount}}(D,t) = [e^{(1/2)s}\mathrm{p}_0(t-1/2) - \mathrm{p}_0(t)]\,D\,\psi_0(t_2) \ . \tag{30}$$

Since (29) and (30) are nonlinear function of z_t/σ and μ/σ only, Claessens and Penacchi simplify by setting $\sigma = 1$. In order to find μ and z_t, they use the generalized Kalman filter. The measurement equation is

$$V_t = \nu_t(z_t;\mu) + \epsilon_t \ , \tag{31}$$

where V_t is a vector of observed secondary market prices of debt at time t, and ν_t is the vector of "true" debt prices. ϵ_t is a vector of measurement errors, which is assumed to be serially uncorrelated and distributed N(0,R), where R is the covariance matrix. Equation (27) is rewritten in discrete form as

$$z_t = z_{t-\Delta t} + \mu\,\Delta t + \omega_t \ , \tag{32}$$

where $\omega_t \sim \mathrm{N}(0,\Delta t)$. (32) is the transition equation in the Kalman algorithm.

The data consisted of time series for the period 1990-1995 of prices for the two Brady bonds, and estimates of the prices of zero-discount bonds from a one-factor Vasicek (1977) bond pricing model. Maximum likelihood estimates of μ and R are first developed. Then "smoothed" estimates of the zs are computed. This "smoothed" time series of z_t is used to evaluate other bonds not used directly in the estimation. Theoretical and actual prices for Aztec bonds are provided. The comparison made is "visual", and the fit seems reasonably close.

The contribution of this article is that it shows a procedure for evaluating the rolling interest guarantee, and a procedure for estimating the unobservable state variable z_t which then can be used to price other assets influenced by this variable.

2.5 Risk Indices and Other Risk Measures

Erb, Harvey, and Viskanta (1996b) investigated the economic content of five risk measures of country risk. The five risk measures were the Institutional Investor's country credit rating (IICR), the International Country Risk Guide composite index (IICRC), as well as the sub indices for political (ICRGP), financial (ICRGF), and economic (ICRGE) risk. For a specification of these indices, see Table A.2 in the Appendix. The date consisted of time series for 117 countries for the period 1984 to 1995. Developed countries had higher index values (lower risk) than the emerging countries. During this period there was a tendency that those countries with a high (low) risk index at the start of the period had a lower (higher) risk index at the end of the period. Erb, Harvey, and Viskanta call this effect the "mean reversion" in the risk levels. The mean reversion is especially evident for the ICRG financial and composite indices for countries with equity markets. Least evidence for mean reversion was found for the credit risk.

In Table 2.1 I refer the correlation between the five risk measures. This table corresponds to Table 5 in the article. In the upper triangle, the correlation is between changes, and in the lower triangle the correlation is between the levels of the risk measures. The highest correlation is between the ICRG composite index and the three sub-indices. A high correlation between the composite index and the sub-indices was to be expected because the composite index is a weighted average of the sub-indices. Note that the correlation betwen the level of the ICRG financial index and the credit rating is only 0.26, and based on changes, 0.03 only. This is approximately the same figures as for the correlation between the ICRG political risk index and the financial risk index.

Source	IICR	ICRGC	ICRGP	ICRGF	ICRGE
IICR		-0.03	0.01	**0.03**	-0.09
ICRGC	0.35		**0.79**	**0.54**	**0.43**
ICRGP	0.30	**0.83**		0.25	0.06
ICRGF	**0.26**	**0.60**	0.35		0.05
ICRGE	0.10	**0.52**	0.24	0.25	

Table 2.1 Correlation of risk measures, levels (upper triangle) and changes (lower triangle), semi-annual observations, January 1984-July 1995.
Source: Table 7 in Erb, Harvey, and Viskanta (1996b)

In Table 7 in the article, they provide a correlation analysis between the risk measures and the mean return, volatility of the return, and the beta against the world market portfolio. This table is reproduced here as Table 2.2. The betas are against the Morgan Stanley Capital International (MSCI) World Index and the return data are from MSCI and International Financial Council (IFC). For all countries as a whole, the correlations between the risk measures and beta are positive. This is contrary to what one should expect. It means that higher index values (lower risk) correspond to higher betas. The relationship is a result of the fact that emerging markets have lower betas with respect to the world market portfolio than developed countries, see Harvey (1995). We see from the table that for the emerging countries, the correlation between the risk measures and the betas are consistently positive. Concentrating on the emerging countries, increased risk indices (lower risk) is negatively correlated with geometric return and level of volatility. The only exception is for the ICRG political risk measure, which is positively correlated with volatility.

Erb, Harvey, and Viskanta (1996b) also tested portfolio strategies based on upgrades and downgrades of countries. The portfolios were rebalanced every six months, and if the index did not change, the country was kept in the portfolio. The upgrade portfolios had higher average returns than the downgrade portfolios. The ICRG political risk measure was never the most important one. Financial, political, and credit risk were unable to distinguish between high and low returns in the portfolio strategy. They also investigate the cross-sectional relationship between the equity return and the risk measures, and the relationship between the risk measures and fundamental variables such as book-to-price, dividend-to-

Country Sample	IICR	ICRGC	ICRGP	ICRGF	ICRGE
All countries					
Geometric return	-0.23	-0.15	-0.13	-0.16	-0.16
Volatility	-0.52	-0.45	-0.31	-0.49	-0.59
Beta-MSCI World	0.24	0.43	0.44	0.40	0.30
Developed countries					
Geometric return	0.18	-0.15	-0.28	-0.08	0.21
Volatility	-0.46	-0.41	-0.38	-0.47	-0.15
Beta-MSCI World	0.09	-0.15	-0.24	-0.04	0.06
Emerging countries					
Geometric return	-0.26	-0.06	-0.02	-0.08	-0.12
Volatility	-0.16	-0.08	**0.20**	-0.16	-0.45
Beta-MSCI World	0.03	0.42	0.46	0.35	0.20

Table 2.2 Sample period correlation between average risk measures and price moments.
Source: Erb, Harvey, and Viskanta (1996b)

price, and price-to-cash ratios.

Erb, Harvey, and Viskanta (1995) contain similar analyses as the (1996b) article, but here they use only the Institutional Investor country credit measure.

Diamonte, Liew and Steven (1996) used the ICRG political risk index to test trading strategies in equity (represented by stock indices) in developed and emerging markets. The data covered 21 developed countries and 24 emerging countries. The time period is from 1985 to 1989. There is a overlap between this article and Erb, Harvey and Viskanta (1996b). While Diamonte et al. test ex-post strategies, i.e. strategies based on information not available at the point of trading, Erb et al. also tested ex-ante strategies.

Whereas Erb, Harvey, and Viskanta (1996b) dealt with the relationship between five country risk measures and return on equity, Erb, Harvey and Viskanta (1996a) study the relationship between the same five risk measures and fixed income return. They compare the rank correlation between the four ICRG risk measures, the Institutional Investor's country credit rating and the country credit rating of Standard & Poor and Moody's Investor service. The results are shown in Table 2.3 (Exhibit 6 in the article). We see that the Institutional Investor's credit rating has a higher rank correlation with Standard & Poor's and Moody's ratings than the ICRG indices, but that the ICRG financial risk index has a relatively high correlation with the same ratings.

The data consist of return on fixed income from 20 developed countries, with data from the Salomon Brothers World Government Bond Index. The time period is 1985-1995. When

Index/rating	S&P	Mo	ICRGC	ICRGP	ICRGF	ICRGE	IICR
S&P		0.84	0.31	-0.03	**0.68**	0.26	**0.92**
Mo			0.45	0.21	**0.78**	0.21	**0.85**
ICRGC				0.83	0.77	0.62	0.38
ICRGP					0.46	0.20	0.01
ICRGF						0.38	0.71
ICRGE							0.35

Table 2.3 Rank correlation between country ratings/indices, December 1995.
Source: Erb, Harvey, and Viskanta (1996a).

describing the data, they find that higher returns are generally related to higher risk (lower index values). They use the risk measures in trading strategies for these fixed-income securities. Portfolios based on risk levels show that the spread in raw returns is positive in the unhedged case, and that this holds also for beta adjusted returns. For hedged portfolios, the result is mixed. Portfolio strategies based on ex-post changes in risk, show that upgrade portfolios uniformly perform better than downgrade portfolios. On an ex-ante basis, this still holds.

The article also contains cross sectional analyses of returns where the risk measures are explanatory variables.

In Erb, Harvey, and Viskanta (1994) similar analyses are done as in the (1994a) article, but only for the Institutional Investor's country credit measure.

Howell and Chaddick (1994) tested the predictive power of three methods for political risk evaluation. The Economist's approach was to categorize countries according to such criterions as if they have "bad neighbors", "generals in power", etc. The BERI approach is described in Coplin and O'Leary (1994), and the same is the Coplin O'Leary system of Political Risk Services (PRS). A loss index reflecting losses to investors due to political events was estimated for 36 countries. The estimation was based on reports from The Overseas Private Investment Corporation (OPIC), but the authors adjusted the index to properly reflect losses. The loss index ranged from 0 to 10, where 10 indicate high losses. The time period covered was 1987-1992. The loss index for this period was then compared to the risk indices made in 1986 for the countries, and according to the three approaches. The coefficients of correlation are shown in Table 2.4, together with the levels of significance. We see that the BERI and PRS approaches have the highest coefficients of correlation. For both the PRI and PRS higher index values corresponds to less risk. For The Economist, the opposite is true. The authors' do not comment upon whether this has been adjusted for when estimating the coefficients of correlation.

	Coefficient of correlation	Level of significance
The Economist	0.33	0.053
BERI	0.51	0.006
PRS	0.57	0.001

Table 2.4 Correlation between country risk measures and actual Losses. Source: Howell and Chaddick (1994)

Melvin and Tąn (1996) used the ICRG political risk index, see Table A.2 in the Appendix, as an explanatory variable when modeling foreign exchange market bid-ask spreads. The sample data covered thirty-six countries and currencies for the time period March 1987 to August 1990. The thirty-six countries included both industrialized and emerging markets. Monthly observations were used. The observations of percentage bid-ask spreads were the average of daily bid-ask spreads for the month. Melvin and Tan ran first a cross-sectional regression for each month. They state that the estimated coefficients of the risk indices were larger for more recent months. Based on this, they assumed that a structural change happened in June 1989, as a result of the unstability caused by the Tiananmen Square conflict in China, and included a dummy variable for the period following June 1989. They do not report these results beyond stating that the dummy variable was significant. They then report the results from running the regression equation, a random effects model,

$$\nu_{it} = \alpha_0 + \alpha_1 \sigma_{it} + \alpha_2 CR_{it} + \epsilon_{it} + u_i \quad , \qquad (33)$$

where ν_{it} is the bid-ask spread of country i in month t, σ_{it} is the standard deviation of changes in the daily bid-ask spread in country i in month t, CR_{it} is the value of the political risk measure (either the ICRG political risk index or one of its sub-indices) for country i in month t, ϵ_{it} is the observation-specific disturbance, and u_i is a country-specific disturbance "...which could be viewed as the collection of factors not in the regression that are specific to that country." The regression equation is reported separately for the period March 1987-May 1989 and the period June 1989-August 1990. For both the time periods, the estimated coefficients reflecting the political risk measure (either the index itself or its sub-indices) were all negative, implying that a reduction in the risk measure, i.e., increased political risk, increases the bid-ask spread. Many of the estimated coefficients were significant at a one per cent level of significance. For the first time period R^2 ranged from 0.10 to 0.26, and from 0.05 to 0.15 for the last time period. When all the sub-indices and the political risk index were included in the same regressions, the signs of the estimated coefficients were both negative and positive. This may be caused by colinearity between the different risk measures.

2.6 Political Stability and Country Risk

The aim of Pindyck and Solimano (1993) is to explore the empirical relevance of irreversibility and uncertainty for aggregate investment behavior. In one of their analyses, they investigate the relationship between political instability variables and the volatility in the value of output of a country. The instability variables are probability of government change, the average numer of assasinations, government crisis, riots, revolutions, and constitutional changes per year. The time period was 1950-1985. The relationship betweeen these variables and the volatility of output is weak. They conclude on page 286 that this analysis "... suggests that strikes, riots, revolutions and other forms of political turmoil ... may have little to do with uncertainty over the return on capital, and, hence with investment."

The Pindyck and Solimano article can be compared to Kobrin (1978). Kobrin used indices for political conflict, i.e., for turmoil, internal war and conspiracy, to study the relationship between foreign direct investment and these indices. The article suggests that the only significant relationship is a negative relationship between focused, generally covert, anti-regime violence and foreign direct investment. The time period covered is 1964-1967.

Brunetti and Weder (1997) examined the effect of institutional uncertainty, represented by indicators for *government instability*, *political violence*, *policy uncertainty*, and *enforcement uncertainty* on yearly investment rates for the period 1974-89. The data covers sixty countries, balanced across regions and across levels of development. They do not consider private investment per se, but use data for total investment. They note that private and total investments tends to be highly correlated. As the endogenous variable they use average rate of total investment per unit GDP.

The effect of the presented variables were all negative, meaning that the effect is to "reduce the investment rate". The variables that were negative at ten per cent level of significance in all model specifications are listed in Table 2.5. In the table is also listed the effect on the investment rate of an increase in the uncertainty measure. As an example, an increase in the number of changes in institution by one standard deviation would, ceteris paribus, reduce the investment rate by 1.8 per cent.

Variable name	**Effect of one standard deviation rise in variable value on investment rate in percentage points**
* *Government instability indicators*	
Number of revolutions	-1.8
Number of coups	-1.1
* Political violence indicators	
Number of political executions	-1.5
Number of war casualties	-1.5
Violent Social Change	-1.9
Terrorism	-1.3
* *Policy uncertainty indicators*	
Number of changes in institution	-1.8
Volatility of the real exchange rate distortion	-2.1
Volatility of the black market premium on foreign exchange	-1.6
* *Indicators of uncertainty in enforcement*	
Corruption-ICRG	-2.7
Low rule of law	-2.8

Table 2.5 The effect of indicators showing a ten per cent level of significance in all specifications. Source: Box 1 in Brunetti and Weder (1997)

Lessard (1996) study how country risk can be incorporated in analyses of offshore projects. By offshore projects Lessard primarily means foreign direct investments (FDI) and to a large extent he focuses on FDI in emerging countries. Lessard's article is dealing with two issues. The first is that a FDI should be structured such that the parties who participate in the FDI should allocate specific risks of the project among themselves, such that each party bears the risk where he has a comparative advantage in bearing the particular risk. According to Lessard, comparative advantage in risk-bearing may be because 1) information is not equally available to all investors, 2) investors may have different degrees of influence over outcomes, and 3) investors may differ in their ability to diversify risks. Lessard notes that these three reasons constitutes violation of the underlying assumptions of the CAPM and other "equilibrium-based" valuation approaches.

For a specific investment example, an investment in an Argentine independent power plant by a Chilean investor, Lessard illustrates the specific risk types of the project and the type of participants or investors in the project, see Table 2.6. The risk types are related to construction, operations, demand, institutional, currency, country, and world market. By institutional risk, Lessard means risk that "...involves all of the uncertainties about how the rules of the game are likely to change", meaning the rules set by regulators and other official authorities. As possible participants/investors in the project, Lessard uses the following categories: operator/ strategic investor, local strategic investor, local portfolio investor, local public authority, international portfolio investor (i.e., the "market"), and international policy lender (e.g., the World Bank).

The second issue Lessard is dealing with is the question of general principles regarding risk and valuation of FDI. He specifies two general types of risk. Two-sided or "symmetric" risk factors are factors with similar upside and downside. Examples of two-sided risks are fluctuations in exchange rates or interest rates. Downside or "asymmetric" risk are risks whose potential downside impacts are greater than their potential upside impacts. Examples of downside risks are expropriation and war damages.

Whereas an increase in downside risk reduces the expected cash flows, this is not necessarily true for an increase in two-sided risks. In a discounted cash flow (DCF) valuation approach with a use of a risk adjusted discount rate, it is the *unconditional* expected cash flow that should be discounted according to the finance theory. An unconditional expected cash flow

Investor/participant in project	**Should take on risk related to**
Operator/strategic investor	Construction, operations, institutional
Local strategic investor	Construction, operations, demand, institutional
Local portfolio investor	Institutional
Local public authority	Demand
International portfolio investor	Demand, curency, country, world market
International policy lender	Demand, country

Table 2.6 Types of investors/ participants in the project, and the types of risk they have comparative advantage in bearing.
Based on Table 1 in Lessard (1996)

in this context means "...cash flows expected under each future scenario weighted by the probability of that scenario". Lessard notes that the cash flow estimates used in practice are based on the *most-likely* future scenario. When there is a substantial downside risk, the unconditional expected cash flow is lower than the expected cash flow conditioned on the most likely scenario. To illustrate this, consider an investment paying USD 1.- a year from now. If there is a probability of ten per cent of the payment being expropriated, the unconditional expected cash flow is USD 0.90. The expected cash flow based on the most likely scenario, i.e., no expropriation, is however USD 1.-.

Lessard notes that if the structure of downside risk is simple and the impact is expected to grow at a compund rate over time, the weighted average discount rate can be adjusted according to the following formula:

$$r_{\text{adjusted}} = r_{\text{normal}} + \text{adjustment for downside risk} \qquad \textbf{(34)}$$

Lessard refers to Appendix 15.1 in Levi (1990)[8] for the specification of equation (34).

In order to find the cost of equity for offshore invetments, Lessard shows how a project beta can be used. The market premium for systematic risk is assumed to be the same as in the investor's home country. He simplifies and assumes that the offshore project has the same risk as the local economy when compared to a project in the home country, i.e.,

$$\text{offshore project beta} = \text{beta of comparable home country project x country beta.} \qquad \textbf{(35)}$$

Having found the cost of capital, the value of the project can then be found by the DCF approach. Lessard assumes that it is the expected cash flow conditioned on the most likely scenario that is used in the DCF approach, but that expected cash flow is adjusted downwards to take into account downside risk. This downside risk adjustment may be based on, e.g., bond risk premiums, political risk insurance premiums, and political risk ratings.

[8] Lessard does not refer to a specific edition of Levi's book. The comparable appendix of Levi (1990) is appendix 15.2. In this appendix Levi considers the cash flow from an investment that may be completely confiscated. The probability that a confiscation occurs in any year is a constant λ. The probability of receiving a cash flow for year t is then $(1-\lambda)^t$. When the cash flow from the investment is a constant $\overline{CF}$, and when the life time of the cash flow is infinite, the value of the investment is $\overline{CF}(1-\lambda)/(DR_e+\lambda)$, where DR_e is the discount rate in case of no risk of confiscation.

The contribution of Lessard's article is that he analyzes the different types of risk for FDI, including political risk, in a coherent way by using a standard CAPM-approach. He also presents how practitioners analyze such investments.

3 Summary and Discussion

Political uncertainty can broadly be analyzed in two ways, by explicitly or implicitly including political uncertainty in the analysis. For the multi-period models, I have summarized the two approaches in Tables 3.1 and 3.2. In order to obtain analytical clarity, the models must be simple. As seen from Table 3.1, many of the authors use a binary regime variable. The simplest way is to look at irreversible regime shifts. Some situations, like expropriation or default, are suited for models with binary, irreversible regime shifts. In one period models there is no distinction between reversible and irreversible regime shifts. The most elaborate model in the review with respect to the modeling of the political uncertainty, is Brennan and Schwartz (1982a and b). In implicit modeling of political uncertainty, it is assumed that total uncertainty includes political uncertainty. In such approaches, there is a lack of specification when the effect of increased political uncertainty is analyzed.

Empirical research is hampered by the lack of data. Historical data for events like expropriation and default, may also be considered obsolete for prediction purposes due to changes in the political climate. This is especially true after the fall of the Berlin wall and communism. The use of political risk indices seems promising. The challenge here is to link these general indices to specific events which have a clear effect on the cash flow for investments. Since these indices reflects judgement about political conditions which may be hard to model explicitly for a financial economist, they may serve as a useful input to the stringent mathematical models used in the finance literature.

I am not able to see any clear-cut and simple relationship between optimal decision making and political uncertainty. In many cases, it is not obvious what is meant by terms like increased political uncertainty, as the Hassett and Metcalf (1994)'s discussion shows. This means that optimal decision rules must be determined from case to case, depending on the type of asset and the type of political uncertainty.

Article	Future Regime Variable is	Probability of Future Regime is Modeled as	Is probability of regime independent of underlying economic variable(s) ?	Is change of regime variable ("size of jump") independent of underlying economic variable(s) ?
Hassett and Metcalf (1994)	Binary, reversible	Poisson process, constant intensity	Yes	Yes
Mahajan (1990)	Binary, irreversible	Relation between two geometric Brownian motions at a given time	No	Yes
Brennan and Schwartz (1982a)	Multi state, reversible	Poisson process, stochastic intensity	No	No
Brennan and Schwartz (1982a)	Multi state, reversible	Poisson process, stochastic intensity	No	No
Johnson and Stulz (1987)	Binary, irreversible	Poisson process, constant intensity	Yes	Yes
Brennan and Schwartz (1985)	Binary, irreversible	Poisson process, constant intensity	Yes	Yes
Claessens and Penacchi (1996)	Binary, irreversible	Stopping time for arithmetic Brownian motion	Yes	Yes
Cherian and Perotti (1997)	Binary, reversible	Binomial variable with Bayesian update of probabilities	Yes, from investors' perspective No, from the government's perspective	Yes

Table 3.1 Explicit dynamic modeling of political uncertainty

Article	Variable influenced by political uncertainty	Modeling of uncertainty
Hassett and Metcalf (1994)	Net present value of costs of investment	Geometric Brownian motion
Pindyck (1993)	Net present value of costs of investment	Geometric Brownian motion
Teisberg (1993) and (1994)	Value of completed project	Geometric Brownian motion

Table 3.2 Implicit dynamic modeling of political uncertainty

Future research involving political uncertainty could focus on the effect of governmental incentives to promote private investment in areas with high political uncertainty. Examples of such incentives are guarantees and investment subsidies. Many governments also provide political risk insurance. The effect of political risk insurance, public and private, could be analyzed with respect to the incentive they create to invest. For an overview of political risk insurance providers, see Hashmi (1995). If possible, the use of risk indices or other procedures to evaluate the actual risk, should be used in such analyses. In the same spirit, one could investigate the risk that state-owned companies take on in areas with high political uncertainty, and compare governments' decision making with the optimal decision making of private investors. In order to focus on a better modeling of political decision making, the use of stochastic game theory could provide useful insights into the problem of valuation and optimal decision making from both investors' and policy makers' point of view.

Appendix Risk Measures

Factor	OECD 1979	OECD 1994	Emerging 1979	Emerging 1994	Rest of World 1979	Rest of World 1994
Economic outlook	1	1	2	3	3	4
Debt service	5	2	1	1	1	1
Financial reserves/ current account	2	3	4	4	4	3
Fiscal Policy	9	4	9	7	6	6
Political outlook	6	6	7	9	8	9
Access to capital markets	6	6	7	9	8	9
Trade balance	4	7	5	5	5	5
Inflow of portfolio investment	7	8	8	8	7	8
Foreign direct investment	8	9	6	6	9	7

Table A.1 Ranking of critical risk factors in Institutional Investor's country credit ratings by rankings, 1979 and 1994. Source: Erb, Harvey, and Viskanta (1996b).

Political Risk (PR)	Max Points	Financial Risk (FR)	Max Points	Economic Risk (ER)	Max Points
PR1 Economic expectation vs. reality	12	FR1 Loan default or unfavorable loan restructuring	10	ER1 Inflation	10
PR2 Economic planning failures	12	FR2 Delayed payment of suppliers' credit	10	ER2 Debt service as a percent of export of goods and services	10
PR3 Political leadership	12	FR3 Repudiation of contracts by governments	10	ER3 International liquidity ratios	5
PR4 External conflict	10	FR4 Losses from exchange controls	10	ER4 Foreign trade collection experience	5
PR5 Corruption in government	6	FR5 Expropriation of private investments	10	ER5 Current account balance as a percentage of goods and services	15
PR6 Military in politics	6			ER6 Parallel foreign exchange rate	5
PR7 Organized religion in Politics	6				
PR8 Law and order tradition	6				
PR9 Racial and nationality tensions	6				
PR10 Political terrorism	6				
PR11 Civil war	6				
PR12 Political party development	6				
PR13 Quality of bureaucracy	6				
Maximum Possible Rating	**100**		**50**		**50**

Composite Risk Rating (CRR)=(PR+FR+ER)/2.

General Principle: The higher the rating, the lower risk.

Table A.2 The ICRG rating system. Source: Coplin and O'Leary (1994).

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Chapter 3

Modeling Political Uncertainty by the Use of Risk Indices: A Contingent Claims Approach with a Focus on Oil Investments

Abstract

This chapter addresses analytical and empirical issues related to the use of suitable risk indices in the evaluation of investments affected by political uncertainty. I suggest a method whereby an unobservable state variable, governing the type of policy regime, can be deduced from the risk indices. I estimate parameters of the stochastic process characterizing the deduced state variable for a set of risk indices. The deduced variable can be used directly when evaluating investments. I show how this approach can be combined with the contingent claims approach to price assets influenced by political uncertainty.

◊ I thank Campbell R. Harvey, Duke University and NBER, for letting me access his data on the risk indices of International Country Risk Guide and the country credit ratings of Institutional Investor. I also thank Delphi Economics, Oslo, for letting me access their time series covering the Morgan Stanley Capital International World Index, Eurodollar interest rates and the Brent Blend oil price. I also thank Statoil, Stavanger, for their data support.

1 Introduction

When investing internationally, rather than domestically, the presence of political uncertainty may create additional evaluation difficulties for the investor when analyzing the investment. Political uncertainty in a foreign country can of course affect the profitability of investments domestically as well, but the consequences of political uncertainty in the host country for the investment has a more direct effect on the profitability of investments located in that country. Political uncertainty increases the complexity when analyzing investments. Factors, which in more stable environments usually are treated as parameters, are turned into variables. I will use the term regime variables when describing variables representing political uncertainty. A "regime" is a collection of on or more regime variables. Regime variables can, e.g., be a tax rate, an allowed ownership share, or an indicator variable indicating whether repatriation restrictions are in place for a country or not. Even if one abstracts from the complexity and concentrates on one variable representing political uncertainty, the question is often the same: "Which regime will be in place ?" The natural way to answer this question is to specify the possible types of regimes, and the probability of each regime. When the additional question about valuation of a cash flow partially determined by the type of regime is raised, further analysis is needed. This analysis may involve finding how the different regimes in a country covary with, say, observable prices of tradet assets, or the world stock return. Country risk indices, or sub indices of these indices, may assist the investor in both these tasks, that of estimating which regime will be in place, and of determining a value for the cash flow.

In this chapter I want in particular to study how risk indices can be included in an evaluation using a contingent claims approach. When using the arbitrage free valuation methodology, specific requirements must be imposed on the stochastic processes in order to obtain a solution to the valuation problem. It is therefore important to examine whether the stochastic properties of the indices, or some function[1] of the indices, is of a form consistent with this methodology. Such an examination requires an empirical analysis. Ideally, in such an empirical analysis two relationships should be investigated. The first relationship is between the indices and the regime variable, and the second is the relationship between theoretical

[1] The term "function", or transformation, will be made clear in section two.

values and actual values of assets. The problem, or the challenge, with the first relationship is to find data for regime variables. An analysis of the second relationship should include a risk index as one of several explanatory variables determining the theoretical value of an asset. The problems are to find values of foreign direct investments, to specify the regime variables, and to properly describe the effect of the regime variable on the profitability of the investment.

The empirical analysis I conduct in this paper is an investigation of the stochastic properties of underlying, not directly observable, processes generating the indices themselves. This may serve as a first step towards a more comprehensive empirical analysis. This paper will hopefully give some answers to whether, and how, risk indices may be useful in the evaluation of foreign investments.

In the next section I suggest a method for modeling the relationship between a risk index and a regime variable. In section three I deal with the question of valuation of assets and in particular questions related to risk indices and valuation. I have in Appendix 1 included a summary of the main results from the theory of arbitrage free pricing, which is used in the examples. Section four contains a study of the stochastic properties of a selection of risk indices. I then show how the obtained results can be used, by presenting numerical examples, in section five. In the final section I summarize and comment upon the main results.

2 The Relationship between Risk Indices and Regime Variables

I start by describing two approaches, termed the direct and the indirect approach, which may be used to establish a relationship between a risk index and a regime variable. In sub-section 2.3 I then comment generally on transformation of indices. I start by describing the direct approach.

2.1 The Direct Approach

Assume that only two policy regimes are possible, termed "G" (Good) and "B" (Bad)[2]. The reason for choosing a binary variable is that I then can represent the government's problem of selecting regime as a binary choice problem. One way to analyze binary choice problems empirically, has been to use index function models, or, random utility models[3]. Index function models can, e.g., be used to investigate consumers' decisions.

An Index[4] Function Model

As an example[5], consider a consumer contemplating to buy a certain good. Let the indicator variable for whether a good is bought at time t be $y_t \in \{0,1\}$. The indicator variable equals 1 if a purchase is made, and 0 if it is not. Before the consumer is deciding whether to buy the good, she makes a cost benefit analysis of the purchase. The marginal net benefit from the purchase, i.e., marginal benefits less marginal costs, is y_t^*. A purchase is made if the net benefit is positive, which implies that

$$y_t = \begin{cases} 1 & \text{if } y_t^* > 0 \\ 0 & \text{if } y_t^* \leq 0 \ . \end{cases} \tag{1}$$

Assume that an estimate, $\hat{y}_t$, of y_t^* is made. This estimate may, e.g., be the output of a regression model. The relationship between $\hat{y}_t$ and y_t^* is given by

$$y_t^* = \hat{y}_t + \epsilon_t \ , \tag{2}$$

where ϵ_t is noise at time t. Assume that $\hat{y}_t$ is an unbiased estimate, i.e. $E(\epsilon_t) = 0$, and that ϵ_t is normally distributed with variance σ_t^2. The time t probability that a purchase will be made at that date, or that $y_t^* > 0$, is then

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[2] The regimes are named "Good" and "Bad" from the perspective of the investor. From the government's perspective, the ranking may be opposite.

[3] For an introduction to such models, see, e.g., Greene (1993) page 642 and 643.

[4] The term "index" in this context does not refer to a risk index, it refers to a model of the type presented here.

[5] This example is based on Greene (1993) page 642.

$$p_t = \mathrm{P}(y_t = "1")$$

$$= \mathrm{P}(y_t^* > 0) = \mathrm{P}(\epsilon > -\hat{y}_t).$$

Due to the symmetry of the normal distribution, we have that

$$p_t = \mathrm{P}(\epsilon_t < \hat{y}_t) \ , \tag{3}$$

or,

$$p_t = \int_{-\infty}^{b_t} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = \mathrm{N}(b_t) \ , \tag{4}$$

where $\mathrm{N}(b_t)$ is the cumulative distribution function for the standard normally distributed variable with argument b_t, and where

$$b_t = \frac{\hat{y}_t}{\sigma_\epsilon} \ . \tag{5}$$

This example captures the essence of a binary choice situation, and the model can be interpreted to characterize situations with other decision makers than consumers. Consider a government deciding whether regime "G" or "B" shall apply at time t. The indicator variable equals 0 if policy regime "B" is applying, and 1 if policy regime "G" is applying. The government's net benefit from selecting regime "B"[6] is here y_t^*. For practical use we do not have an estimate of $\hat{y}_t$ readily available from a regression model. However, $\hat{y}_t$ may be obtained in a different way. It seems reasonable to assume that, in some cases, the probability of type or regime can be found by conditioning on the level of a risk index for a country. The numerical value of a country's risk index expresses the degree of risk in that country. How

[6] In order to make the presentation simple, I assume that it is a given government, or central planner, that makes the decision. In a more realistic example, one could condition the decision making on which type of government is in place, e.g., a "left wing" or a "right wing" government. Alternatively, one could consider the citizens of a country as decision makers. They will elect the government which then implements the policy. The interpretation of the net benefits of making a decision, y_t^*, will depend on the assumption about the decision maker. For the case with a central planner, y_t^* may represent the welfare level of the citizens if "G" is chosen, less the welfare level if "B" is chosen.

clearly the type of risk and the specification of risk is stated, varies between different risk indices. The usefullness of a risk index in this paper is determined by the informational content of the index, i.e., the degree of certainty regarding the probability of regime "G" that can be obtained by conditioning on the level of the index. At one extreme no information regarding the probability p_t is obtained by conditioning on the level of the risk index. At the other extreme p_t is completely determined by the risk index. I will make the critical assumption that p_t is a deterministic function, g(·), of the numerical value of an observable risk index, or rating at time t, ψ_t. The probability of regime "G" is then

$$p_t = g(\psi_t) \ . \tag{6}$$

We then have

$$g(\psi_t) = p_t = N(b_t)$$

and

$$b_t = N^{-1}(p_t) \ . \tag{7}$$

By making the standardizing assumption that σ_ϵ is one[7], we have from (5) that

$$\hat{y}_t = b_t \ . \tag{8}$$

Dynamic Policy Making

The policy making may be termed reversible[8] if the government, either continuously or at given intervals, decides which regime variable will apply. At a given time t, before the regime variable is announced, the investor will consider the regime variable as the outcome

[7] The assumption that $\sigma_\epsilon = 1$ is not critical. It is only the relationship between $\hat{y}$ and σ_ϵ that is important.

[8] An alternative is to model irreversible policy making. This may be analyzed in a slightly different way than the case with reversible policy making. Assume that the present policy, at time t, is "G". In the time period $[t,T]$, the policy may change, if at all, to "B". The time the change occurs is τ, where $\tau \equiv \{\inf s: y_s^* > 0,\ t \le s \le T\}$. A risk index might then be used to find $P(\tau \in [t,T])$, i.e., $P(\tau \in [t,T]) = f(\psi_t)$, for some function f(·). The analysis then proceeds as in the case of reversible policy making, but where y_t^* is used instead of $\hat{y}_t$. This approach assumes that there is no noise, ϵ_t.

of a lottery, where the probability of regime "G" is p_t. The "success" probability is not dependent on the type of regime at time $t-1$, only on the current level of the risk index, ψ_t, or alternatively, the country's deduced marginal benefit from selecting "G", $\hat{y}_t$. If the process of $\hat{y}_t$ is known, the investor may estimate the 'success' probability in future lotteries.
A candidate for a process describing the evolution of $\hat{y}_t$, is the arithmetic[9] Brownian motion

$$d\hat{y}_t = \mu_{\hat{y}} dt + \sigma_{\hat{y}} dB_t \ , \tag{9}$$

where dB_t is the increment of a Wiener process and where $\mu_{\hat{y}}$ and $\sigma_{\hat{y}}$ are constants. From a time series of numerical values of an index, $\psi = \{\psi_1, \psi_2, ..., \psi_n\}$, a time series of the deduced, unobservable, country's net benefit from choosing "G", $\hat{y} = \{\hat{y}_1, \hat{y}_2, ..., \hat{y}_n\}$, can be obtained and the parameters $\alpha_{\hat{y}}$ and $\sigma_{\hat{y}}$ may be estimated. Whether or not the parameters of a process for $\hat{y}_t$ may be estimated depends on the function g(·). As an example, consider the case where the probability of regime "G" is a constant p_1 if the index is below a critical index level $\overline{\psi}$ and a constant p_2 if not. If the index for a time period has been fluctuating, but never crossed the critical index level $\overline{\psi}$, the deduced observations $\hat{y}$ are identical for all observations. In order to get observations of $\hat{y}_t$s that facilitate estimation, restrictions must be put on the function g(·). It should preferably be continuous and monotonic in order to get a one-to-one relationship between index observations and induced observations of $\hat{y}_t$.

Because it may be perfectly reasonable to assume a non-continous function g(·), I suggest an extra step in the analysis. This extra step involves finding first an underlying process generating the risk index itself. The function g(·) can then be applied in the second step, but now restrictions may not be put on g(·) for empirical estimation concerns. I term the procedure involving the extra step as the indirect approach.

2.2 The Indirect Approach

A risk index is typically bounded between a maximum and minimum numerical value. Define a variable, q_t, by

[9] By using an arithmetic Brownian motion, $\hat{y}_t$ may be negative. This is not the case for, e.g., a geometric Brownian motion.

$$q_t \equiv f(\psi_t) \equiv \frac{\psi_t - \psi^{MIN}}{\psi^{MAX} - \psi^{MIN}} \ , \tag{10}$$

where ψ_t is the numerical value of the index at time t, and where ψ^{MAX} and ψ^{MIN} are the respective maximum and minimum values of the index. Because the variable q_t will be between on and zero, q_t may be interpreted as a probability. This probability is such that

$$\psi_t = \psi^{MAX} q_t + \psi^{MIN}(1-q_t) \ , \tag{11}$$

i.e., the observation of the index at time t, ψ_t, is the expected numerical value of a (hypothetical) lottery which pays ψ^{MAX} with probability q_t and ψ^{MIN} with probability $(1-q_t)$. This is a "shadow lottery" of the index, defined such that the expected payoff of the lottery at a given time is always equal to the numerical value of the index. The announcement of the level of the risk index is therefore tantamount to the announcement of the 'success probability' q_t. One way to interpret the probability q_t is as the probability that the government of the country is of "ψ^{MAX}-type". A risk index is usually constructed such that the highest value of the index refers to the situation with no risk and the lowest level of the risk refers to a situation with highest possible risk. The producer of the risk index may then consider the government of a country to be one of two types, a highest possible risk government, i.e., a "ψ^{MIN}-government", or a no risk government, i.e., a "ψ^{MAX}-government". The analyst's "willingnes" to categorize it as a "ψ^{MAX}-government" at time t if the analyst has perfect information is captured by the variable x_t^*. The indicator variable x_t equals one if the country's government is of "ψ^{MAX}-type" and zero if it is of "ψ^{MIN}-type", i.e.,

$$x_t = \begin{cases} 1 & \text{if } x_t^* > 0 \\ 0 & \text{if } x_t^* \leq 0 \ . \end{cases} \tag{12}$$

Due to lack of transparency and the government's possible lack of credibility, it is likely that the analyst cannot determine for certain the type of government. The analyst has probably positive information about the government, i.e., information indicating that it is of a "ψ^{MAX}-type", and negative information indicating that the government is of a "ψ^{MIN}-type". The variable $\hat{x}_t$ may be regarded as the analyst's subjectively weighted stock of information at time t. The weighting of the information is determined by the analyst's assesment of the information's importance and relevance. The relationship between x_t^* and $\hat{x}_t$ is given by

$$x_t^* = \hat{x}_t + \nu \ , \tag{13}$$

where $\hat{x}_t$ is the analyst's estimate, and, ν is the noise, a normally distributed random variable with zero mean and variance σ_ν^2. The time t probability that x_t is one is then

$$q_t = \mathrm{P}(x_t = "1")$$

$$= \mathrm{P}(x_t^* > 0) = \mathrm{P}(\nu > -\hat{x}_t) = \mathrm{P}(\nu < \hat{x}_t) = \mathrm{N}(k_t) ,$$

and $k_t = \hat{x}_t / \sigma_\nu$. If the minimum numerical value of the index is zero, which is often the case, we note that

$$\psi_t = \mathrm{h}(\hat{x}_t) = \psi^{MAX} \mathrm{N}(\hat{x}_t / \sigma_\nu) \ . \tag{14}$$

With a time series of an index, ψ, the function from (11) and assumptions about σ_ν, a time series $\hat{x}$ can be obtained, as for $\hat{y}$ in the direct approach. I will use the indirect approach in section four, when deducing observations of $\hat{x}_t$. I assume that the evolutionary equation for $\hat{x}_t$ is

$$d\hat{x}_t = \mu_{\hat{x}} dt + \sigma_{\hat{x}} dB_t \ , \tag{15}$$

where dB_t is the increment of a Wiener process and where $\mu_{\hat{x}}$ and $\sigma_{\hat{x}}$ are constants. Equation (15) has an interesting interpretation. It represents the arrival of new relevant information which is either positive or negative ($d\hat{x}_t$ is positive or negative). If $\mu_{\hat{x}} / \sigma_{\hat{x}}$ for one country is higher than for another, the future information the analyst receives is more predictable for the first country.

Having applied the indirect approach first, the function g(·) may then be applied to find the probability, p_t, of a specific policy regime.

$$p_t = g(\psi_t) = g(f^{-1}(q_t)) = g(f^{-1}(N(\hat{x}_t/\sigma_v))) \qquad \textbf{(16)}$$

By inserting from (16), we get

$$N(\hat{y}_t/\sigma_\epsilon) = g(f^{-1}(N(\hat{x}_t/\sigma_v))) \qquad \textbf{(17)}$$

An alternative to using the function g(·) is to relate $\hat{y}_t$ and $\hat{x}_t$ directly, e.g., by an affine transformation[10]

$$\hat{y}_t = \beta_0 + \hat{x}_t\beta_1 \ , \qquad \textbf{(18)}$$

for constants β_0 and β_1. According to equation (18), the estimate of the country's net benefit of introducing regime "G" is a linear function of the index producer's willingnes to categorize the government as a "ψ^{MAX}-government", or the estimated stock of weighted information indicating the type of government. By inserting (18) in (2) we get

$$y_t^* = \beta_0 + \hat{x}_t\beta_1 + \epsilon_t \ , \qquad \textbf{(19)}$$

where we recall that ϵ_t is noise at time t with zero mean. I present in Figures 2.1 and 2.2 examples of the relationship between the probability p_t and an index $\psi_t \in [0,100]$ where I have used (18) and (19). If ψ^{MAX} represents the no-risk situation, it may be reasonable to assume that the probability of regime "G" increases with increasing values of the index. Note that there is a continuous relationship between p_t and ψ_t. In Figure 2.1 $\beta_0 = 0$ and $\beta_1 = 1$, and the value of σ_ϵ is varied. When the standard deviation is one, the relationship between p_t and ψ_t is linear, as implied from (10). In some cases it may be reasonable to assume that the probability of regime "G" is high for index values just above 50, or low for index values not far below 50. This is the case when the standard deviation is less than one.

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[10] A model of the type given by (18) and (19) may be preferable because, in some instances, closed form valuations formulas can be found for claims where the payoff is a function of p_t, see sub-section five.

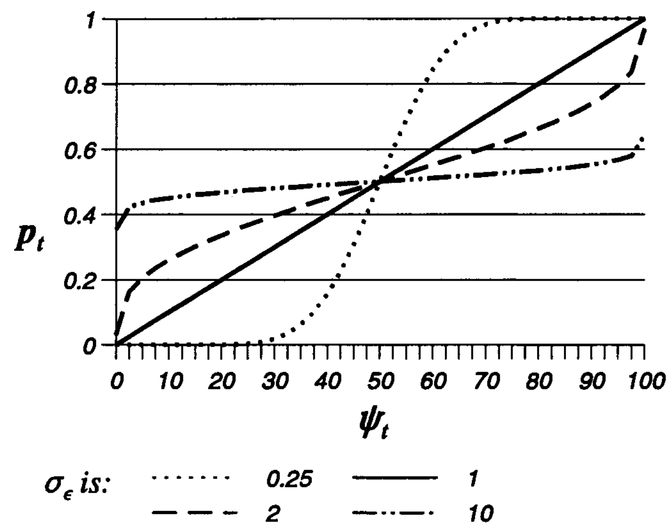

Figure 2.1 The relationship between p_t and ψ_t when $\hat{y}_t$ is an affine transformation of $\hat{x}_t$ and where $\beta_0 = 0$ and $\beta_1 = 1$

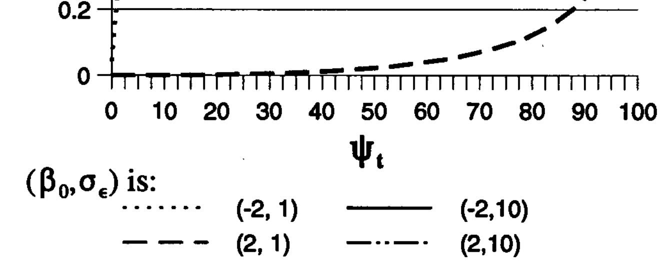

Figure 2.2 The relationship between p_t and ψ_t when $\hat{y}_t$ is an affine transformation of $\hat{x}_t$ and where $\beta_1 = 1$

As the standard deviation goes to zero, the curve relating p_t to ψ_t will be zero for $\psi_t \in [0,50]$ and equal to one for $\psi_t \in \langle 50,100]$. When the standard deviation of the noise is larger than one, the probability of regime "G" is relatively high only for high levels of the index, or low probability for low levels of the index. When the standard deviation approaches infinity, the curve will be flat, indicating a fifty-fifty chance of regime "G" being chosen irrespective of the index level. In Figure 2.2 I show the relationship between p_t and ψ_t when $\beta_1 = 1$ and where I vary β_0 and σ_ϵ. When the parameters are $\beta_0 = -2$ and $\sigma_\epsilon = 1$ and , the probability of regime "B" is large for relatively low index values. When the noise increases, the schedule goes toward a flat line at $p_t = 0.5$. When the parameters $\beta_0 = 2$ and $\sigma_\epsilon = 1$ are used, p_t is relatively high only for high index values. And as for the previous case, the curve will be flat at $p_t = 0.5$ when the standard deviation approaches infinity. Figures 2.1 and 2.2 illustrate the point that, by selecting appropriate parameters of the noise in (19), a wide range of possible relationships between the probability of policy regime and the level of index can be modeled

2.3 Comments on Index Transformations

The reason why a transformation of an index should be considered, is that the transformed index may better facilitate a solution to the problem of establishing the value of claims contingent on the index. When the aim is, as in this chapter, to use the arbitrage free pricing methodology when pricing such claims, the most intuitive approach is to let the transformed index represent either a (ex-dividend) price process or the accumulated return from capital appreciation from holding a hypothetical asset. If the aim is to solely rely on the absence of arbitrage, the transformed index should be continuous. In the presence of jumps in the process, the pricing must be based on equilibrium arguments.

For the general problem of transforming the index into a new variable, the question of a government's binary choice problem may not enter into the consideration at all. I am however of the opinion that the intuitive explanation of the transformation when cast as a binary choice problem, and the use of a probit model as in sub-sections 2.2 and 2.3, may be preferred to an arbitrary transformation where the new (deduced) variable does not have a logical or intuitive interpretation.

3 Risk Indices and Valuation

3.1 Assumptions about Tradeable Assets

In order to use risk indices when finding the value of claims with payoff conditioned upon the level of the index I first define an asset where the price at time t of the asset, $Z_t^{(\hat{x})}$, is given by

$$Z_t^{(\hat{x})} = \mathrm{e}^{\hat{x}_t} \ . \tag{20}$$

The price of this (hypothetical) asset at time t is equal to the exponential of the numerical value of the deduced variable at time t, $\hat{x}_t$. The variable $\hat{x}_t$ is here interpreted as the accumulated continuously compounding interest rate from price changes of asset $Z_t^{(\hat{x})}$. The prices of the asset at time T, $t \le T$, is

$$Z_T^{(\hat{x})} = Z_t^{(\hat{x})} e^{\hat{x}_T - \hat{x}_t} \ . \tag{21}$$

The capital appreciation/depreciation of the asset is determined by the change in the deduced variable, or implicitly by the change in the numerical value of the risk index, i.e.,

$$\ln(Z_T^{(\hat{x})} / Z_t^{(\hat{x})}) = \hat{x}_T - \hat{x}_t = \mathrm{h}^{-1}(\psi_T) - \mathrm{h}^{-1}(\psi_t) \tag{22}$$

where $\mathrm{h}^{-1}(\cdot)$ is given by (14). The assumed evolutionary equation for $\hat{x}_t$ is given by (15). By applying Ito's lemma on (20) we get

$$dZ_t^{(\hat{x})} = Z_t^{(\hat{x})}(\mu_{\hat{x}} + \frac{1}{2}\sigma_{\hat{x}}^2)\, dt + Z_t^{(\hat{x})} \sigma_{\hat{x}} dB_t^{(1)} \ , \tag{23}$$

a geometric Brownian motion with constant parameters and where $dB_t^{(1)}$ is the increment of a standard Brownian motion. The return of the hypothetical asset $Z^{(\hat{x})}$ is perfectly correlated with the deduced variable $\hat{x}$ and thereby also with the risk index ψ_t. Instead of using equation (23) directly, I define $\mu_{\hat{x}} \equiv (\alpha_{\hat{x}} - 0.5\,\sigma_{\hat{x}}^2)$, insert this new variable into (23) and get the more traditional equation

$$dZ_t^{(\hat{x})} = Z_t^{(\hat{x})}\alpha_{\hat{x}}dt + Z_t^{(\hat{x})}\sigma_{\hat{x}}dB_t^{(1)} \ . \tag{24}$$

I will in section five demonstrate how the contingent claims pricing methodology can be used to value assets where the payoffs are functions of a risk index. In particular I will study how investments in oil assets can be priced. I assume that the evolutionary equation for the spot price of crude oil, S_t, is given by

$$dS_t = S_t\alpha_S dt + S_t\sigma_S(\rho_{S,\hat{x}}dB_t^{(1)} + \sqrt{1-\rho_{S,\hat{x}}^2}dB_t^{(2)}) \ . \tag{25}$$

where α_S, σ_S, and $\rho_{S,\hat{x}}$ are constants. The standard Brownian motions $B^{(1)}$ and $B^{(2)}$ are uncorrelated. The parameter $\rho_{S,\hat{x}}$ may be interpreted as the coefficient of correlation[11] between the stochastic components of $Z_t^{(\hat{x})}$ and S_t, i.e., $\rho_{S,\hat{x}} \in [-1,1]$. Observe that if $r = B^{(1)}$ and $v = \rho_{S,\hat{x}}B^{(1)} + \sqrt{1-\rho_{S,\hat{x}}^2}B^{(2)}$, then

$$(r,v) \sim \mathrm{N}\left(\begin{pmatrix}0\\0\end{pmatrix}\begin{pmatrix} t & t\rho_{S,\hat{x}} \\ t\rho_{S,\hat{x}} & t\end{pmatrix}\right).$$

The random variables r and v are normally distributed random variables where each variable has a variance of t and where their coefficient of correlation is $\rho_{S,\hat{x}}$.

The solution to the stochastic differential equation (25) is

$$S_t = S_0\exp(\alpha_S - \frac{1}{2}\sigma_S^2)t + \rho_{S,\hat{x}}\sigma_S B_t^{(1)} + \sigma_S\sqrt{1-\rho_{S,\hat{x}}^2}dB_t^{(2)}) \ . \tag{26}$$

The processes used here are well known from the contingent claims literature.

3.2 Rate of Return Adjustment

If I want to use the processes (24) and (25) to describe the price processes of traded assets, it

[11] This means that the type of regime variable and the oil price may be correlated.

is important to be aware of the fact that these processes, or "securities", are not actually traded. The expected gain from holding such "home made" securities must be adjusted by including dividends. I will refer to the traded asset with price equal to the spot price of oil as $Z_t^{(S)}$. Assume that there is a constant proportional dividend yield, δ_i, on $Z_t^{(i)}$, $i \in \{S, \hat{x}\}$. The received dividend for holding the asset over the next increment of time is deterministic and is given by

$$dD_t^{(i)} = \delta_i Z_t^{(i)} dt \ . \tag{27}$$

When investors buy asset $Z_t^{(i)}$, the expected gain by holding the asset over the next increment of time is

$$\mu_t^{(i)} = \frac{E_t[dZ_t^{(i)} + dD_t^{(i)}]}{Z_t^{(i)}} \ , \tag{28}$$

i.e., the expected appreciation of the asset and a dividend payment. If $\mu_t^{(i)}$ can be determined, e.g., from an equilibrium pricing model like CAPM, then $\mu_t^{(i)}$ and $dZ_t^{(i)}$ can be used in (28) to determine the dividend process given by $dD_t^{(i)}$. The term $dD_t^{(i)}$ serves as the drift, or, rate of return adjustment.

It is important to be aware of the fact that if the contingent claims pricing methodology is used for a state variable which is not the price of an asset traded in financial markets, or the price of a commodity for which there exists a futures market, an equilibrium model such as CAPM is needed to determine the required drift of the state variable's stochastic process[12]. If CAPM is used, and it is assumed that the correlation between the increments of the state variable and the return on the market portfolio is zero, the required drift is equal to the instantaneous risk free interest rate[13].

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[12] For an instructive discussion of this point, see, e.g., Schwartz (1994), particularly pages 1926 to 1928.

[13] Compare footnote two on page five.

In order to present the required expected incremental return in a CAPM-setting for the general case where the ex-dividend price process of the asset is given by a geometric Brownian motion, I proceed with a simple example. Assume that the dynamics on the return on the market portfolio, M, is given by [14]

$$\frac{dM_t}{M_t} = \alpha_M dt + \sigma_M dB_t^{(1)} \ , \tag{29}$$

The evolutionary equation for an asset Z is

$$\frac{dZ_t}{Z_t} = \alpha_Z dt + \sigma_Z(\rho_{Z,M} dB_t^{(1)} + \sqrt{1-\rho_{Z,M}^2}\, dB_t^{(2)}) \ . \tag{30}$$

We see that of the return in Z_t, only $\sigma_Z \rho_{Z,M}$ reflects the systematic risk, for which the holder of the asset should be compensated. The risk $\sigma_Z\sqrt{1-\rho_{Z,M}^2}$ is unsystematic. The same level of systematic risk as Z_t possesses can be obtained by holding a portfolio, P, with a portion, w of P in the market portfolio and a portion (1-w) of P in the riskless asset. By holding the riskless asset the investor will receive a risk free return of r. The incremental return on this portfolio will be

$$\frac{dP_t}{P_t} = (1-w) r\, dt + w(\alpha_M dt + \sigma_M dB_t^{(1)}) \ , \tag{31}$$

and by choosing the weight such that $w = \rho_{Z,M}\sigma_Z/\sigma_M$, we get

$$\frac{dP_t}{P_t} = (r + \frac{\alpha_M - r}{\sigma_M}\rho_{Z,M}\sigma_Z)dt + \sigma_Z\rho_{Z,M} dB_t^{(1)}.$$

The expected required drift of this portfolio gives the required expected drift of asset Z_t, α_Z^*, such that the investors will hold the asset. In other words,

$$\alpha_Z^* = r + \lambda\rho_{Z,M}\sigma_Z \ , \tag{32}$$

[14] The standard, independent, Brownian motions $B^{(1)}$ and $B^{(2)}$ in (29) and (30) are not the same as in equations (24) and (25).

where $\lambda = (\alpha_M - r)/\sigma_M$. The required incremental return on the asset is equal to the risk free interest rate added the price of market risk, λ, multiplied by the volume of market risk that asset Z_t possesses, $\rho_{Z,M}\sigma_Z$. The difference between the required and actual "expected incremental return" is named the drift adjustment, δ_Z, where $\delta_Z = \alpha_Z^* - \alpha_Z$. If the drift adjustment is positive, δ_Z is known as the rate of return shortfall, or convenience yield. A positive δ_Z expresses how much the investor must be compensated for the next increment of time in addition to the expected capital appreciation in order to hold the asset during that period. A negative δ_Z expresses what the investor is willing to pay in addition to the price of the asset for receiving the expected capital appreciation.

We see that for an asset where the incremental return is given by (31), the corresponding continuously compounded return over the time interval $(T-t)$ is given by

$$\ln(P_T/P_t) = [(1-w)r + w(\alpha_M - \frac{1}{2}\sigma_M^2)](T-t) + w\sigma_M(B_T^{(1)} - B_t^{(1)})$$

$$= \left[r + \frac{\alpha_M - \frac{1}{2}\sigma_M^2 - r}{\sigma_M} \rho_{Z,M}\sigma_Z \right](T-t) + \sigma_Z\rho_{Z,M}(B_T^{(1)} - B_t^{(1)}),$$

or,

$$\ln(P_T/P_t) = r(T-t) + [(\alpha_M - \frac{1}{2}\sigma_M^2(T-t) - r(T-t))]\beta_Z + \sigma_Z\rho_{Z,M}(B_T^{(1)} - B_t^{(1)}) \ , \tag{33}$$

where

$$\beta_Z = \frac{\mathrm{Cov}[(\ln(Z_T/Z_t), \ln(M_T/M_t)]}{\mathrm{Var}(\ln(M_T/M_t)} = \frac{\rho_{Z,M}\sigma_Z\sigma_M(T-t)}{\sigma_M^2(T-t)} = \frac{\rho_{Z,M}\sigma_Z}{\sigma_M} \ . \tag{34}$$

We see that the required expected continuously compounded return is equal to the risk free return with the addition of a risk adjustment. This risk adjustment equals the market premium multiplied by the beta for the asset. The beta is equal to the ordinary least squares regressor between the continuously compounded return on the market portfolio and the hypothetical asset Z_t. Note that (32) can be rewritten as

$$\alpha_Z^* = r + (\alpha_M - r)\,\beta_Z\ , \tag{35}$$

where the beta is the same as in (34). This offers a recipe for finding α_Z^* and δ_Z: Estimate the parameters of the process Z_t, then find beta by a regression analysis and use (35) to find α_Z^* and from this deduce δ_Z.

4 Examining Selected Risk Indices for Oil Producing Countries

4.1 Introductory Remarks

I examine the indices of International Country Risk Guide and the Institutional Investor's country credit ratings. Some, or all, of these indices have been used in analyses by Erb, Harvey, and Viskanta (1994, 1995, 1996a and b), by Diamonte, Liew, and Stevens (1996), and by Melvin and Tan (1996). In addition to describing and presenting the indices, I want to a) apply the approach presented in sections two and three on an empirical data set, and b) estimate the parameters of the assumed processes. As a starting point, I summarize the analysis' assumptions:

A1 A country's risk index is a transformation of a, not directly observable, state variable. By applying a "reverse transformation", the state variable can be obtained from observations of the risk index.

A2 The "reverse transformation" in A1 is given by "the indirect approach", described in sub-section 2.2. I have chosen to use the "indirect approach", even though the "direct approach" with different functions g(·) could have been used for different countries. In my opinion the indirect approach seems reasonable with an intuitive interpretation, and it is useful when comparing results across countries.

A3 The dynamic behavior of the state variable governing a given risk index is captured by an arithmetic Brownian motion with constant parameters.

A4 The spot price of crude oil is a geometric Brownian motion with constant parameters. The state variable governing the risk index and the log of relative crude oil prices have a constant correlation coefficient. In short, the evolutionary equations for a given risk index and the spot price of crude oil are as described in section three.

A5 Assets are priced in accordance with the CAPM, as described in section three.

In sub-section 4.3 I use assumptions A1 and A2 when finding the deduced variable $\hat{x}_t$ for a set of risk indices.

Two testable implications of assumption A3 are that the increments of the deduced variable are normally distributed and independent. These implications are tested in sub-section 4.4. If the testable implications are rejected, the reason may be that any one of assumptions A1, A2, or A3 are incorrect.

As mentioned in the introduction, I consider the research in this section as a first step towards a more comprehensive analysis, e.g., where the deduced variable $\hat{x}_t$ can be used as one of several explanatory variables when explaining investment flows between countries or the level of stock indices in different countries. I have therefore included in this section a rather comprehensive presentation of results for individual countries. I have chosen not to search for alternative stochastic processes for the deduced variable or the oil price, which may have fitted the data better. The possibilities to reach a good fit for an individual country are many when the direct approach and the function $g(\cdot)$ are combined with alternative stochastic processes for the evolutionary development of $\hat{y}_t$. I feel that such an approach may be relevant when considering a select few number of countries, but not in a more standard analysis of a large number of countries, as presented here. In principle, each country could have a specific function $g(\cdot)$ and a distinct stochastic process for $\hat{y}_t$.

In sub-section 4.5 I estimate the parameters for the assumed stochastic processes given by equations (23), (25), and (27), i.e., α_i, σ_i, $\rho_{S,\hat{x}}$, and δ_i, $i \in \{S, \hat{x}\}$. The degree of systematic risk in $\hat{x}_t$ is found by estimating betas for different countries, as explained in sub-section three. This has implications for the investor when calculating required risk premiums in politically unstable countries when the investor applies a CAPM and the risk is measured by the examined indices. Ex ante, I would expect that the betas are not significantly different from zero for most countries, especially since I use a world market portfolio[15]. If a high level of $\hat{x}_t$ corresponds to a situation with low political risk, a positive coefficient of correlation between the deduced variable and the oil price, $\rho_{S,\hat{x}}$, indicates that an "oil-investor" is facing low political risk when oil prices are high. If the coefficient of correlation is negative, the opposite is true. The coefficient of correlation has important implications for the valuation of oil investments, as we will see in sub-section five and in chapter four of the dissertation.

4.2 The Data

The forty-four countries specified in the BP (British Petroleum) Statistical Review 1997 are listed in Table 4.1. The first columns contain the countries' oil production in 1995 and remaining reserves at the end of 1995. The five largest oil producers in 1995 were Saudi Arabia, USA, the Russian Federation, Iran, and China. The countries having the largest proven reserves were Saudi Arabia, Iraq, The United Arab Emirates, Kuwait, and Iran. The March 1996 levels of the International Country Risk Guide's (ICRG) risk indices for political risk (PR), financial risk (FR), economic risk (ER), and composite risk (CR) are then reported. For a specification of the ICRG risk indices, see Table 4.2. The ICRG indices are weighted sums of economic indicators and/or ratings of a set of characteristics of the country. The ratings are made by ICRG-experts. For a detailed description of how the indices are made, I refer to Coplin and O'Leary (1994). ICRG has not specified what constitutes "high" or "low" risk, but a general classification in risk categories for the composite risk index has been offered, see Table 4.3. High/low levels of the ICRG risk indices corresponds to low/high levels of risk. An index level for the ICRG CR below fifty is considered as "very high risk." The average of the countries' ICRG composite risk indices was 68.7. The countries with highest composite risk, i.e., lowest CR, were Iraq, Angola, Algeria, Cameroon, and Congo.

[15] See footnote two on page five.

The countries with the lowest risk according to ICRG CR were Brunei, Denmark, Norway, and USA. The important factors determining the Institutional Investor's country credit rating (IICCR) are reported in Table 4.4. The assessment of the countries are made by people working in business. The factors considered to be important for a country's credit rating change over time, and differ between types of countries. Note especially the importance of debt service. For the OECD countries debt service was ranked only as number five in 1979, while it was ranked as number two in 1994, making debt service an important factor for good credit ratings for all types of countries. As for the ICRG indices, high/low IICCR-values corresponds to situations with low/high risk. The average of the IICCR was 41.3. The lowest rated countries were Iraq, Angola, Congo, and Uzbekistan. The highest rated countries were USA, United Kingdom, Norway, and Denmark.

The purchase of political risk insurance for investments in a country is an indication of the fact that investor regard political risk is a concern. The penultimate column in Table 4.1 indicates whether the country is a member of the Multilateral Investment Guarantee Agency (MIGA) and the ultimate column shows how much of MIGA's outstanding obligations at the end of June 1996 were in the country. Of the forty-four countries listed in Table 4.1, six of the countries were not members of MIGA. These were Australia, Brunei, Iran, Iraq, Mexico, and Syria. Political risk insurance from MIGA had been purchased for investments in sixteen of the countries, and these insurances constituted 52.4 per cent of MIGA's total outstanding liabilities. The standard premiums for MIGA's political risk insurance contracts are reported in Table 4.5. The actual premiums paid may differ from the standard rates due to specific risk-characteristics of the insured project. Note that the standard premiums are highest for the oil and gas sector. The standard premium of 1.25% for insurance against expropriation implies that if USD 100 is insured, the premium (per year) is USD 1.25.

For the spot price of oil I use prices of the Brent Blend crude oil. As the risk free interest rate I use the six month Eurodollar rate. I use the Morgan Stanley Capital International World Index (MSCIWI), measured in US dollars, to represent the market portfolio. MSCIWI is a value weighted index reflecting reinvestment of dividends. The oil prices, the return on the market portfolio, and the Eurodollar interest rate are all end of the month observations

Country	Oil Production[1] 1995 Bls/day[2]	%	Reserves end '95[1] Thousand mill. Barrels	%	ICRG, March 1996[3] PR	FR	ER	CR	March '96, IICCR[4]	Member[5] of MIGA?	% of liabilities[6]
Algeria	1,325	2.0	9.2	0.9	48	36	28.0	56.0	21.5	Y	
Angola	630	0.9	5.4	0.5	50	21	38.5	55.0	12.5	Y	
Argentina	750	1.1	2.2	0.2	76	35	34.0	72.5	38.4	Y	5.4
Australia	575	0.9	1.6	0.2	80	44	36.5	80.5	71.0	N	
Azerbaijan	185	0.3	1.2	0.1	NA	NA	NA	NA	NA	Y	
Brazil	715	1.1	4.2	0.4	64	34	33.0	65.5	35.8	Y	7.2
Brunei	175	0.3	1.4	0.1	82	47	48.0	88.5	NA	N	
Cameroon	105	0.2	0.4	0.0	52	29	31.5	56.5	18.5	Y	
Canada	2,390	3.5	7.2	0.7	81	46	38.5	83.0	79.9	Y	
China	2,990	4.4	24.0	2.4	68	38	38.0	72.0	56.4	Y	4.9
Colombia	590	0.9	3.5	0.3	58	39	35.0	66.0	46.7	Y	
Congo	185	0.3	1.5	0.1	56	29	28.5	57.0	14.2	Y	
Denmark	190	0.3	1.0	0.1	85	48	42.0	87.5	80.3	Y	
Egypt	920	1.4	3.9	0.4	56	31	31.5	59.5	25.7	Y	
Equador	395	0.6	2.1	0.2	60	40	38.0	69.0	34.0	Y	1.9
Gabon	355	0.5	1.3	0.1	59	34	35.0	64.0	25.1	Y	
India	785	1.2	5.8	0.6	62	36	36.0	67.0	45.8	Y	
Indonesia	1,575	2.3	5.2	0.5	65	39	37.0	70.5	51.8	Y	4.5
Iran	3,705	5.5	88.2	8.7	65	35	33.0	66.5	23.6	N	
Iraq	545	0.8	100.0	9.8	37	19	12.5	34.5	8.4	N	
Kazakhstan	440	0.7	5.3	0.5	NA	NA	NA	NA	19.2	Y	0.8
Kuwait	2,105	3.1	96.5	9.5	71	43	43.0	78.5	54.1	Y	2.2
Libya	1,415	2.1	29.5	2.9	59	34	34.0	63.5	29.9	Y	
Malaysia	735	1.1	4.3	0.4	75	43	41.0	79.5	68.4	Y	
Mexico	3,065	4.5	49.8	4.9	66	40	33.0	69.5	41.2	N	
Nigeria	1,890	2.8	20.8	2.0	54	23	24.0	50.5	14.8	Y	
Norway	2,995	4.4	8.4	0.8	84	46	45.0	87.5	82.0	Y	
Oman	870	1.3	5.1	0.5	70	42	40.0	76.0	52.5	Y	
Papua New Guinea	100	0.1	0.4	0.0	63	35	38.0	68.0	33.0	Y	3.4
Peru	125	0.2	0.8	0.1	59	34	34.5	64.0	27.2	Y	6.9
Quatar	460	0.7	3.7	0.4	66	39	33.5	69.5	53.8	Y	
Romania	140	0.2	1.6	0.2	72	36	30.0	69.0	30.9	Y	
Russian Federation	6,200	9.2	49.0	4.8	58	29	42.0	64.5	19.9	Y	4.8
Saudi Arabia	8,885	13.2	261.2	25.7	65	43	38.0	73.0	55.1	Y	0.4
Syria	610	0.9	2.5	0.2	69	33	32.0	67.0	24.6	N	
Trinidad & Tobago	145	0.2	0.5	0.0	63	37	37.5	69.0	36.4	Y	2.2
Tunisia	90	0.1	0.4	0.0	70	36	36.0	71.0	44.8	Y	2.9
United Arab Emirates	2,485	3.7	98.1	9.6	67	41	39.0	73.5	60.8	Y	
United Kingdom	2,755	4.1	4.3	0.4	80	46	35.0	80.5	88.2	Y	
USA	8,290	12.3	29.6	2.9	82	46	37.5	83.0	90.9	Y	
Uzbekistan	175	0.3	0.3	0.0	NA	NA	NA	NA	14.9	Y	2.2
Venezuela	2,840	4.2	64.5	6.3	65	33	31.0	64.5	30.1	Y	2.6
Vietnam	150	0.2	0.5	0.0	69	26	26.0	60.5	30.3	Y	0.1
Yemen	335	0.5	4.0	0.4	67	35	27.0	64.5	NA	Y	
Sum/Avg	66,385	98.3	1010.4	99.4	65.8	36.6	34.9	68.7	41.3	38Y,6N	52.4

[1] Source: BP Statistical Review of World Energy 1996. [2] Thousand barrels per day. [3] International Country Risk Guide: PR= political risk, FR=financial risk, ER=economic risk, and CR=composite risk. [4] Institutional Investor's country credit rating. [5] Source: MIGA Annual Report 1996. [6] Percentage of MIGA's total outstanding liabilities of USD 2.3 billion as per June 30 1996, according to MIGA Annual Report 1996.

Table 4.1 Country characteristics

Political Risk (PR)	Max Points	Financial Risk (FR)	Max Points	Economic Risk (ER)	Max Points
Economic expectation vs. reality	12	Loan default or unfavorable		Inflation	
10					
Economic planning failures	12	loan restructuring	10	Debt service as a percent of export	
Political leadership	12	Delayed payment of suppliers'		of goods and services	10
External conflict	10	credit	10	International liquidity ratios	5
Corruption in government	6	Repudiation of contracts by		Foreign trade collection experience	5
Military in politics	6	governments	10	Current account balance as a	
Organized religion in Politics	6	Losses from exchange		percentage of goods and services	15
Law and order tradition	6	controls	10	Parallel foreign exchange rate	5
Racial and nationality tensions	6	Expropriation of			
Political terrorism	6	private investments	10		
Civil war	6				
Political party development	6				
Quality of bureaucracy	6				
Maximum Possible Rating	**100**		**50**		**50**

Composite Risk Rating (CR)=(PR+FR+ER)/2.

General Principle: The higher the rating the lower risk.

Source: Coplin and O'Leary (1994)

Table 4.2 The ICRG indices

Composite Risk	Risk Category
00.0 - 49.5	Very high risk
50.0 - 59.5	High risk
60.0 - 69.5	Moderate risk
70.0 - 84.5	Low risk
85.0 -100.0	Very low risk

Source: Coplin and O'Leary (1994), p. 249.

Table 4.3 Risk categories for the ICRG composite risk index

	OECD		Emerging		Rest of World	
Factor	1979	1994	1979	1994	1979	1994
Economic outlook	1	1	2	3	3	4
Debt service	5	2	1	1	1	1
Financial reserves/ current account	2	3	4	4	4	3
Fiscal Policy	9	4	9	7	6	6
Political outlook	6	6	7	9	8	9
Access to capital markets	6	6	7	9	8	9
Trade balance	4	7	5	5	5	5
Inflow of portfolio investment	7	8	8	8	7	8
Foreign direct investment	8	9	6	6	9	7

Source: Erb, Harvey, and Viskanta (1996b)

Table 4.4 Ranking of critical risk factors in Institutional Investor's country credit ratings, 1979 and 1994

	MANUFACTURING/SERVICES		NATURAL RESOURCES		OIL AND GAS	
TYPE OF RISK	CURRENT[a]	STANDBY[b]	CURRENT	STANDBY	CURRENT	STANDBY
CURRENCY TRANSFER	0.50 %	0.25 %	0.50 %	0.25 %	0.50 %	0.25 %
EXPRO-PRIATION	0.60 %	0.30 %	0.90 %	0.45 %	1.25 %	0.50 %
WAR/CIVIL DISTURBANCE	0.55 %	0.25 %	0.55 %	0.25 %	0.70 %	0.30 %

Source: MIGA's "Investment Guarantee Guide". a. Contract is running b. Contract is on hold and not active

Table 4.5 MIGA premium rates. Annual rates in per cent of insured amount measured in nominal units.

The estimation of the process parameters are based on the time period covering nine years, from 1988 to 1996. One of the main events in the oil market during this period was the Gulf War. Iraq invaded Kuwait on August 2 1990 and operation Desert Storm withdrew from Kuwait on February 27 1991. Events affecting the political risk during this period was, e.g., the fall of the Berlin-wall and the opening-up in China with the establishment of free economic zones.

4.3 Finding the Deduced Variable

In order to deduce the observations of $\hat{x}_t$, I first use the equation [16]

$$\psi_t = \psi^{MAX} q_t = \psi^{MAX} \mathrm{N}(k_t)) \tag{36}$$

to find k_t. By assuming a constant σ_v, I then deduce $\hat{x}_t$ by computing

$$\hat{x}_t = \sigma_v k_t \ . \tag{37}$$

The assumption about σ_v has implications for the observed time series of $\hat{x}_t$. When $\hat{x}_t$ develops according to an arithmetic geometric Brownian motion, the increment is given by

$$d\hat{x}_t = \sigma_v (\alpha_{\hat{x}} - \frac{1}{2}\sigma_{\hat{x}}^2)dt + \sigma_v \sigma_{\hat{x}} dB_t^{(1)} \ , \tag{38}$$

where $\alpha_{\hat{x}}$ and $\sigma_{\hat{x}}$ are the parameters when the standard deviation of the "noise", σ_v, is one. The effect of increasing the value of σ_v is that the absolute increment in the deduced variable, $d\hat{x}_t$, is increased.

The numerical values of the ICRG indices are integers. This means that the data is "censored". The effect is that if the drift and the variance of the true process is small, we will not expect to observe any changes in the risk index during a short time interval because the expected change is not sufficiently large to make the index change from one integer to

[16] For the ICRG indices and Institutional Investor's country credit rating , ψ^{MIN} is zero.

another. With longer time intervals between observations, we would expect the censoring to play a lesser role. In Figure 4.1 I show the schedule for a simulated time series of $\hat{x}_t$, assuming $\sigma_v = 1$, the corresponding time series of the index ψ_t of the form given by (36) where $\psi^{MAX} = 100$, and the time series of $\hat{x}_t$ deduced from the index observations. Only integer values were allowed for the risk index. The match between the simulated and deduced time series of $\hat{x}_t$ seems rather good in this case. In Figure 4.2 I show the schedules for the variable deduced from the index in Figure 4.1 for different assumptions about the standard deviation of the noise, σ_v. A small, compared to a large, value of σ_v "smooths" the observed time series of $\hat{x}_t$. When σ_v is small, only a small change in $\hat{x}_t$ is needed to produce a given change in the index. The effect of different numerical values of σ_v on the deduced variable for Norway is shown in Figures 4.3 and 4.4. The indices for Norway during the period was rather stable, except for the ICRG political risk index. We see the same effect of increasing the numerical value of σ_v as in Figure 4.2, large numerical values of σ_v "magnifies" $\hat{x}_t$ (which is obvious from equation (37)) and increases the absolute changes in $\hat{x}_t$ if these are non-zero. I will for the remaining of this chapter assume that $\sigma_v = 1$ when deducing the time series of $\hat{x}_t$.

4.4 Properties of the Stochastic Processes

With the assumed process, the increments of $\hat{x}_t$, $z_j^{(\hat{x})} = \hat{x}_j - \hat{x}_{j-1}$, are normally distribution with mean $\sigma_v(\alpha_{\hat{x}} - 0.5\sigma_{\hat{x}}^2)\Delta t$ and variance $\sigma_v^2(\sigma_{\hat{x}}^2 \Delta t)$, where Δt is the time interval between the observations measured in years. With data consisting of $n+1$ observations of $\hat{x}_t$, the estimator for the mean of the increments is

$$\text{Est.}(\sigma_v(\alpha_{\hat{x}} - 0.5\sigma_{\hat{x}}^2)\Delta t) = \bar{z}^{(\hat{x})} = \frac{1}{n}\sum_{j=1}^{n} z_j^{(\hat{x})} , \tag{39}$$

and the estimator[17] for the variance of the increments is

$$\text{Est.}(\sigma_v^2 \sigma_{\hat{x}}^2 \Delta t) = s_{\hat{x}}^2 = \frac{1}{n-1}\sum_{j=1}^{n} (z_j^{(\hat{x})} - \bar{z}^{(\hat{x})})^2 . \tag{40}$$

[17] For a description of estimation of volatility and drift of a standard process of the type presented here, see, e.g., Campbell, Lo, and MacKinlay (1997), pp. 361-366.

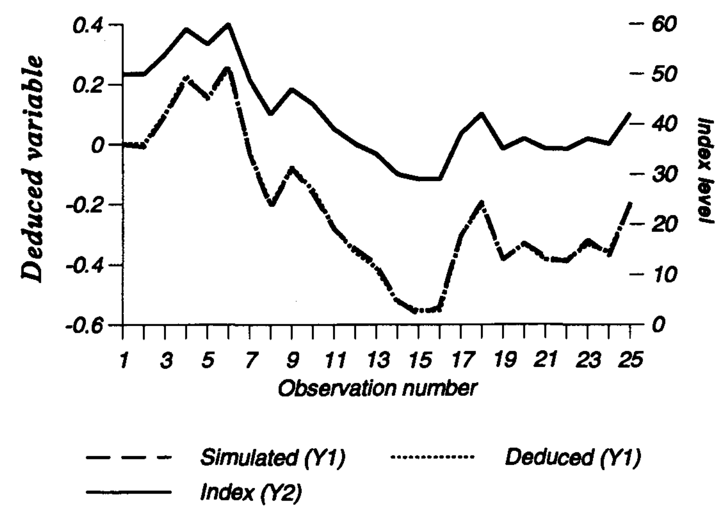

Figure 4.1 Simulated risk index

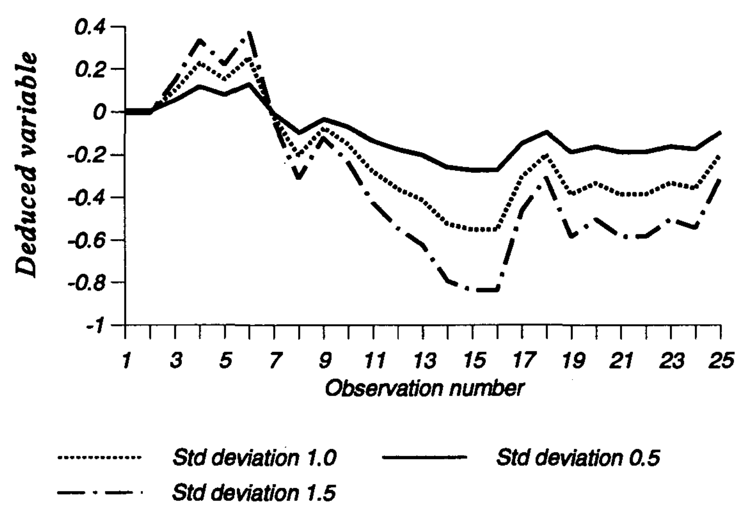

Figure 4.2 Deduced observations of $\hat{x}_t$ for different assumptions about σ_v

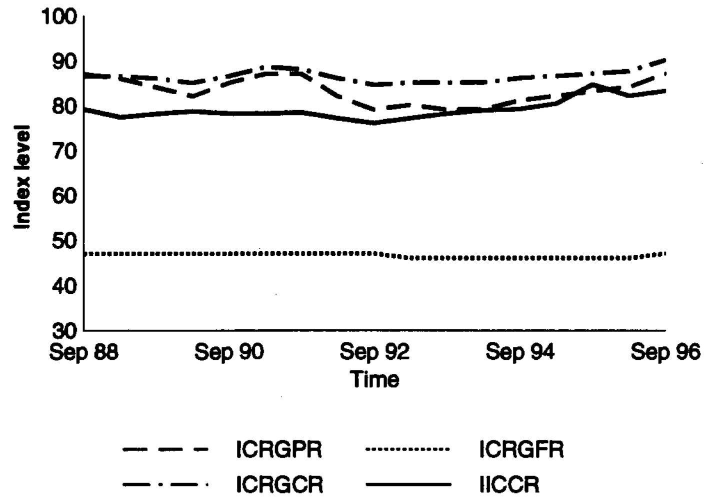

Figure 4.3 Risk indices for Norway

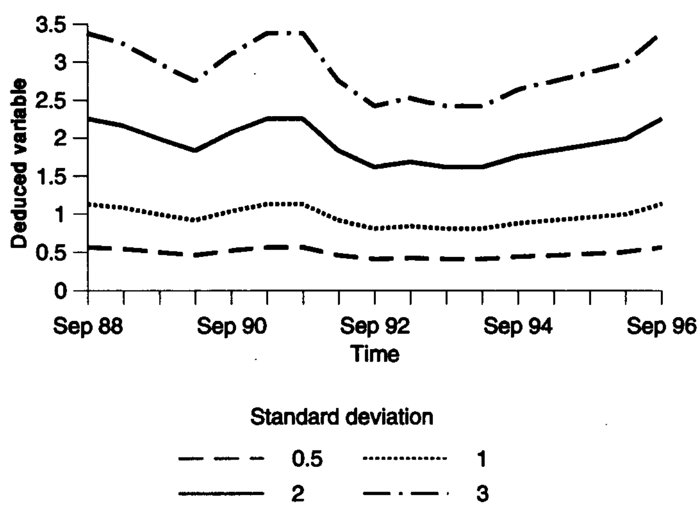

Figure 4.4 Deduced $\hat{x}_t$ observations for Norway's ICRG PR index for different assumptions about σ_v

A negative mean of the increments of $\hat{x}_t$, i.e., $\bar{z}^{(\hat{x})} < 0$, implies that the risk index at the end of the sample period is lower than at the start of the period, i.e., the risk increased during the sample period. A positive mean of the increments implies that the risk index increased over the sample period. A mean significantly different from zero means that the hypothesis of "no trend" in the deduced variable (and therefore also in the risk index) can be rejected. When the change in the deduced variable during the sample period, i.e., $\hat{x}_{96} - \hat{x}_{88}$, is interpreted as the accumulated continuously compounded rate of capital appreciation from holding the (hypothetical) asset $Z_t^{(\hat{x})}$, $\bar{z}^{(\hat{x})}$ can be thought of as the average percentage capital appreciation over the time interval Δt.

In order to determine whether to use monthly, quarterly, or bi-annual observations, I first performed an analysis for the period 1984-1996, see Appendix 3 (especially Table 6 in the appendix). Monthly observations are available for the ICRG indices, while the IICCR are published twice per year. The hypotheses that the increments of the deduced variable are normally distributed or that the incrments are zero-correlated can be rejected for almost all countries based on monthly and quarterly observations. This may be caused by the "censoring" due to integer index values. Based on bi-annual observations the hypotheses could not be rejected for guite a large number of countries. For the rest of the analysis I therefore use bi-annual observations of the risk indices. I further limit the analysis by excluding the ICRG economic risk index. This index is based on economic and financial measures, and as such does not concentrate on political uncertainty. These economic measures are, however, included in the ICRG composite risk index.

I report in Tables 4.6.A-D summary statistics for the sample of increments of $\hat{x}_t$ deduced from the ICRG PR, FR, CR, and, the IICCR for the period 1988-1996. Eight countries had an estimated negative mean of the changes in the deduced variable for the ICRG PR. These were Algeria, Brazil, Cameroon, Canada, Gabon, Mexico, United Kingdom, and Venezuela. Six countries, Indonesia, Iran, Malaysia, Romania, Syria, and Vietnam, had a mean significantly different from zero at a significance level of five per cent. Only Iran had a mean different from zero at one per cent significance level. For the ICRG FR, the sample mean was negative for two countries, Nigeria and USA. For Iran, Libya, Malaysia, and Romania the mean was significantly different from zero at a significance level of five per cent. Iran, Libya, and Malaysia had a mean different from zero at one per cent significance level. United Kindom had during the period an index value of fifty, i.e., no risk. The deduced variable is then infinity, and the statistics for United Kindom are therefore not reported. For the increments of $\hat{x}_t$ deduced from the ICRG CR, only Algeria and United Kingdom had a negative sample mean. Six countries, Argentina, Indonesia, Iran, Malaysia, Peru, and Syria had a mean significantly different from zero at a significance level of one per cent. While the means of the increments of the variables deduced from the ICRG-indices were predominantly positive, a large number of estimated sample means of the variables deduced from the IICCR were negative. Eighteen countries had a negative sample mean. These countries, i.e., those with a negative mean, were also the only ones with a mean significantly different from zero, all at a significance level of five per cent.

A measure of the stability in $\hat{x}_t$ during the sample period is obtained by dividing the mean of the changes by the standard deviation of the changes. A high positive/negative value of this measure indicates that the "trend" of the index during the period has been stable. A low positive/negative value of this measure indicates either that the index changed little over the period, i.e., the mean is close to zero, or that there was an "unstable trend". This measure is shown for the increments of $\hat{x}_t$ deduced from the four indices in Figures 4.5-4.7. Those countries with largest positive or negative mean, $\bar{z}^{(\hat{x})}$, are those countries with the most "stable trends". The standard deviation of the increments seems to be approximately the same for all countries, because the visual impression is that there is an almost linear relationship between $\bar{z}^{(\hat{x})}$ and $\bar{z}^{(\hat{x})}/s_x$. This seems especially to be the case for the IICCR.

Most of the countries had an increase in the risk indices during the sample period. This may be an indication of positive correlation between the indices. By weighting the deduced variables of the countries with equal weights and calculating coefficients of correlation, I find that the ICRG indices were significantly positively correlated both when levels and changes are considered, see Table 4.7. This is also true when the coefficients of correlation are calculated directly from the risk indices, see Table 4.8. However, when comparing the deduced variable from Institutional Investor's country credit ratings with the variables deduced from the ICRG indices, or the indices themselves, the correlation is negative based on levels. The coefficient of correlation for the levels of ICRG FR and IICCR is significantly

Country	Mean	t-value(1)	Variance	Coeff. of Skewness	Excess Kurtosis	B-J(2), p-value	Studentized range(3)	ρ_1(1),(4)	ρ_2(1),(4)
Algeria	-0.0227	-1.07	0.0077	-0.846	-0.298	0.35	3.42	0.157	-0.285
Angola	0.0163	0.64	0.0111	-0.296	1.069	0.59	4.36 h*	0.226	0.016
Argentina	0.0308	1.73	0.0054	1.262	1.171	0.06	3.54	-0.265	-0.099
Australia	0.0161	0.68	0.0097	0.183	0.666	0.82	4.27	0.507 *	0.106
Azerbaijan	NA								
Brazil	-0.0016	-0.08	0.0062	-0.937	0.071	0.29	3.05 l*	-0.505 *	-0.256
Brunei	0.0142	1.31	0.0020	1.296	3.656	0.00 **	4.72 h**	0.013	-0.023
Cameroon	-0.0030	-0.21	0.0034	-1.995	4.399	0.00 **	3.96	-0.139	-0.077
Canada	-0.0022	-0.14	0.0044	-0.717	1.158	0.30	3.77	0.448	0.028
China	0.0095	0.42	0.0088	-0.018	1.781	0.33	4.58 h*	0.277	-0.239
Colombia	0.0044	0.34	0.0029	-0.018	-0.953	0.72	3.36	0.341	0.150
Congo	0.0045	0.28	0.0042	-1.823	6.248	0.00 **	4.63 h**	0.280	0.121
Denmark	0.0053	0.31	0.0048	0.778	0.635	0.37	3.75	-0.339	0.147
Egypt	0.0121	0.62	0.0064	0.155	-0.01	0.97	3.87	-0.318	-0.040
Equador	0.0238	0.95	0.0106	0.354	-0.537	0.76	3.51	-0.151	-0.690
Gabon	-0.0030	-0.23	0.0030	0.971	3.702	0.00 **	4.53 h*	-0.179	-0.375
India	0.0255	0.81	0.0168	0.964	1.707	0.10	4.02	0.104	-0.029
Indonesia	0.0408	2.61 *	0.0042	0.051	0.546	0.90	3.87	0.051	-0.110
Iran	0.0600	2.98 **	0.0069	0.813	-0.218	0.39	3.37	0.002	0.198
Iraq	0.0080	0.20	0.0279	-0.373	0.308	0.79	3.83	-0.039	-0.102
Kazakhstan	NA								
Kuwait	0.0447	0.67	0.0749	-1.65	7.659	0.00 **	5.26 h**	0.130	-0.137
Libya	0.0314	1.18	0.0120	0.462	-0.622	0.64	3.38	-0.064	-0.099
Malaysia	0.0305	2.23 *	0.0032	0.625	0.79	0.46	3.90	0.401	-0.053
Mexico	-0.0048	-0.23	0.0073	-1.628	4.232	0.00 **	4.34 h*	-0.426	0.233
Nigeria	0.0148	1.18	0.0027	-0.093	-1.156	0.62	3.47	-0.245	-0.090
Norway	0.0000	0.00	0.0078	-0.679	0.622	0.45	3.86	0.357	-0.239
Oman	0.0235	1.42	0.0047	0.219	0.311	0.90	3.95	-0.128	0.157
Papua New Guinea	0.0094	0.61	0.0040	-0.645	2.046	0.13	4.42 h*	0.104	-0.139
Peru	0.0298	1.39	0.0078	-1.065	2.074	0.04 *	4.31 h*	-0.338	-0.055
Quatar	0.0288	1.32	0.0081	0.782	-0.346	0.40	3.34	-0.098	-0.084
Romania	0.0370	2.43 *	0.0039	0.802	-0.104	0.40	3.67	0.105	-0.145
Russian Federation	0.0221	0.84	0.0055	-1.079	1.427	0.33	3.10	-0.340	-0.326
Saudi Arabia	0.0227	0.88	0.0113	0.138	0.404	0.92	4.05	0.013	-0.063
Syria	0.0426	2.51 *	0.0049	1.032	0.699	0.19	3.73	0.081	0.263
Trinidad & Tobago	0.0170	1.12	0.0039	-0.902	1.018	0.22	3.83	-0.378	-0.073
Tunisia	0.0294	1.29	0.0088	1.179	1.136	0.09	3.74	-0.080	-0.041
United Arab Emirates	0.0395	1.64	0.0099	2.424	6.93	0.00 **	4.11	-0.317	0.030
United Kingdom	-0.0071	-0.47	0.0039	-0.92	0.731	0.25	3.67	0.084	0.115
USA	0.0066	0.34	0.0066	0.358	0.187	0.82	3.94	0.010	0.013
Uzbekistan	NA								
Venezuela	-0.0048	-0.17	0.0138	-1.603	3.627	0.00 **	4.18	-0.224	0.023
Vietnam	0.0408	2.20 *	0.0058	2.443	6.599	0.00 **	4.19	-0.232	0.323
Yemen	0.0485	1.26	0.0104	1.099	-0.913	0.44	2.35	-0.251	-0.472

Table 4.6.A Statistics for sample of increments of $\hat{x}_t$ deduced from the ICRG political risk index. Time period: 1988-1996. Bi-annual observations

Country	Mean	t-value[1]	Variance	Coeff. of Skewness	Excess Kurtosis	B-J[2], p-value	Studentized range[3]	ρ_1[1][4]	ρ_2[1][4]
Algeria	0.0372	1.90	0.0065	0.591	-1.146	0.38	2.73 l**	-0.208	0.202
Angola	0.0180	0.98	0.0057	1.133	1.585	0.07	4.10	-0.109	0.399
Argentina	0.0586	1.78	0.0184	0.156	0.848	0.75	4.05	0.265	0.302
Australia	0.0237	1.25	0.0061	0.741	1.020	0.32	3.97	0.398	0.027
Azerbaijan	NA								
Brazil	0.0126	0.39	0.0174	-0.019	0.016	1.00	3.80	-0.136	0.033
Brunei	0.0088	0.46	0.0061	0.112	4.563	0.00 **	5.12 h**	0.286	-0.013
Cameroon	0.0030	0.23	0.0030	1.192	3.424	0.00 **	4.57 h*	0.050	0.357
Canada	0.0000	0.00	0.0113	1.802	8.000	0.00 **	5.18 h**	0.144	0.391
China	0.0266	0.76	0.0210	1.165	1.679	0.05	4.07	0.531 *	0.119
Colombia	0.0168	0.53	0.0167	1.051	1.808	0.07	4.02	0.428	0.123
Congo	0.0391	1.59	0.0103	1.981	5.477	0.00 **	4.54 h*	0.009	-0.157
Denmark	0.0491	1.59	0.0162	3.018	9.605	0.00 **	3.85	0.111	-0.028
Egypt	0.0299	1.08	0.0130	0.984	1.920	0.07	4.30	-0.124	0.024
Equador	0.0614	1.83	0.0191	2.485	7.621	0.00 **	4.49 h*	0.182	0.209
Gabon	0.0132	1.46	0.0014	1.258	2.836	0.01 **	4.81 h**	-0.090	-0.090
India	0.0319	1.11	0.0140	0.301	0.284	0.85	3.98	-0.045	0.281
Indonesia	0.0614	1.36	0.0345	1.040	0.567	0.19	3.66	0.613 **	0.320
Iran	0.0757	2.19 *	0.0204	1.168	2.128	0.03 *	4.20	0.166	0.284
Iraq	0.0199	0.25	0.1080	0.780	4.071	0.00 **	4.90 h**	0.126	-0.048
Kazakhstan	NA								
Kuwait	0.0466	0.34	0.3230	-2.172	8.705	0.00 **	5.09 h**	-0.155	0.015
Libya	0.0550	2.45 *	0.0086	1.042	0.194	0.21	3.46	0.266	0.125
Malaysia	0.0662	2.18 *	0.0157	0.143	0.141	0.96	3.91	0.669 **	0.471
Mexico	0.0420	1.14	0.0231	-0.926	0.973	0.21	3.95	-0.006	-0.001
Nigeria	0.0000	-0.00	0.0051	-0.136	0.604	0.86	4.20	-0.071	0.124
Norway	0.0000	0.00	0.0028	0.000	8.000	0.00 **	5.67 h**	0.000	0.000
Oman	0.0405	1.34	0.0156	2.389	7.124	0.00 **	4.41 h*	0.034	0.416
Papua New Guinea	0.0033	0.14	0.0094	-0.831	1.807	0.12	3.82	0.461	0.039
Peru	0.0687	1.83	0.0241	-0.972	2.897	0.01 *	4.64 h**	-0.017	-0.509 *
Quatar	0.0425	1.45	0.0146	2.049	6.643	0.00 **	4.73 h**	0.071	0.367
Romania	0.0651	3.99 **	0.0045	0.809	-0.472	0.37	3.12	-0.040	0.250
Russian Federation	0.0064	0.29	0.0040	-0.306	0.154	0.94	3.16	0.402	0.005
Saudi Arabia	0.0665	1.76	0.0242	2.103	4.298	0.00 **	4.05	0.114	-0.117
Syria	0.0518	1.84	0.0134	3.018	10.403	0.00 **	4.49 h*	0.149	0.015
Trinidad & Tobago	0.0260	1.64	0.0043	1.183	3.807	0.00 **	4.73 h**	0.030	-0.114
Tunisia	0.0504	1.41	0.0218	3.037	10.233	0.00 **	4.19	-0.108	-0.101
United Arab Emirates	0.0657	2.07	0.0171	2.572	7.868	0.00 **	4.20	0.196	0.147
United Kingdom	NR								
USA	-0.0178	-0.52	0.0203	-0.253	4.187	0.00 **	4.91 h**	-0.532	-0.032
Uzbekistan	NA								
Venezuela	0.0095	0.27	0.0211	0.518	1.257	0.39	4.27	-0.104	-0.190
Vietnam	0.0485	1.68	0.0142	1.831	3.011	0.00 **	3.94	-0.189	0.430
Yemen	0.0237	1.02	0.0038	0.755	-0.748	0.66	2.59	0.333	-0.437

Table 4.6.B Statistics for sample of increments of $\hat{x}_t$ deduced from the ICRG financial risk index. Time period: 1988-1996. Bi-annual observations

Country	Mean	t-value(1)	Variance	Coeff. of Skewness	Excess Kurtosis	B-J(2), p-value	Studentized range(3)	ρ_1(1),(4)	ρ_2(1),(4)
Algeria	-0.0038	-0.25	0.0040	-0.578	-0.391	0.59	3.47	0.122	-0.390
Angola	0.0059	0.35	0.0047	0.111	-0.045	0.98	3.63	0.480	-0.181
Argentina	0.0442	2.18 *	0.0070	0.907	0.970	0.22	3.94	-0.117	-0.079
Australia	0.0138	0.98	0.0034	-0.420	-0.280	0.76	3.60	-0.579 *	0.101
Azerbaijan	NA								
Brazil	0.0108	0.66	0.0045	-0.010	-0.737	0.82	3.12	-0.380	-0.213
Brunei	0.0194	2.02	0.0016	1.236	1.597	0.05 *	3.79	-0.153	-0.060
Cameroon	0.0007	0.06	0.0029	0.982	3.384	0.00 **	4.47 h*	-0.225	0.148
Canada	0.0000	0.00	0.0018	-0.172	0.696	0.81	4.28	0.612 **	0.157
China	0.0142	0.57	0.0103	-0.503	1.055	0.47	4.13	0.282	-0.090
Colombia	0.0091	0.59	0.0040	0.429	-0.261	0.75	3.62	0.229	0.338
Congo	0.0149	1.10	0.0031	-1.999	6.339	0.00 **	4.49 h*	0.258	-0.160
Denmark	0.0168	1.97	0.0012	-0.026	-0.757	0.82	3.42	-0.261	0.164
Egypt	0.0203	1.50	0.0031	0.253	-0.469	0.84	3.77	-0.354	0.085
Equador	0.0356	1.53	0.0092	1.059	0.692	0.17	3.65	0.043	-0.066
Gabon	0.0047	0.43	0.0021	-0.044	-1.321	0.54	3.08	-0.017	-0.327
India	0.0262	1.12	0.0093	0.400	0.463	0.74	3.84	0.120	0.119
Indonesia	0.0387	2.44 *	0.0043	0.867	0.513	0.31	3.82	0.460	0.121
Iran	0.0602	2.86 *	0.0076	0.785	-0.530	0.38	3.11	-0.037	0.189
Iraq	0.0041	0.13	0.0173	-0.462	-0.764	0.60	3.19	0.079	-0.234
Kazakhstan	NA								
Kuwait	0.0348	0.43	0.1130	-2.673	10.450	0.00 **	5.09 h**	-0.072	-0.008
Libya	0.0368	1.66	0.0083	0.407	-0.739	0.65	3.39	-0.047	0.042
Malaysia	0.0343	2.51 *	0.0032	0.638	1.013	0.39	4.26	0.268	-0.156
Mexico	0.0135	0.69	0.0065	-2.173	6.611	0.00 **	4.47 h*	-0.064	-0.323
Nigeria	0.0037	0.24	0.0040	0.303	0.654	0.75	4.27	-0.020	0.208
Norway	0.0105	0.78	0.0031	0.325	0.588	0.76	3.95	0.333	-0.308
Oman	0.0278	1.66	0.0048	1.123	0.502	0.15	3.33	-0.159	0.146
Papua New Guinea	0.0090	0.48	0.0060	-1.892	3.986	0.00 **	3.88	0.305	0.053
Peru	0.0422	2.23 *	0.0061	-1.693	3.854	0.00 **	4.10	-0.167	-0.099
Quatar	0.0262	1.33	0.0066	0.186	1.081	0.63	4.32 h*	-0.024	0.228
Romania	0.0272	1.72	0.0043	-0.566	-0.221	0.62	3.67	0.009	-0.319
Russian Federation	0.0208	0.83	0.0050	-0.833	-0.518	0.60	2.68 l**	-0.111	-0.236
Saudi Arabia	0.0301	1.37	0.0082	2.108	7.000	0.00 **	4.53 h*	0.177	-0.127
Syria	0.0415	2.74 *	0.0039	0.431	-0.589	0.68	3.36	0.162	0.385
Trinidad & Tobago	0.0190	1.46	0.0029	0.616	4.262	0.00 **	5.03 h**	-0.328	-0.042
Tunisia	0.0346	1.78	0.0064	1.359	2.335	0.01 *	4.13	-0.166	-0.105
United Arab Emirates	0.0354	1.61	0.0082	2.565	8.710	0.00 **	4.42 h*	-0.123	-0.027
United Kingdom	-0.0132	-1.24	0.0019	-0.116	-0.314	0.95	3.64	-0.045	0.038
USA	0.0037	0.28	0.0030	0.067	-0.810	0.79	3.28	-0.309	0.181
Uzbekistan	NA								
Venezuela	0.0047	0.19	0.0105	-0.738	1.612	0.18	4.20	-0.143	0.065
Vietnam	0.0443	2.42 *	0.0057	2.064	4.752	0.00 **	3.98	-0.325	0.492 *
Yemen	0.0358	1.04	0.0083	-1.060	0.890	0.46	2.85 h**	-0.231	-0.604

Table 4.6.C Statistics for sample of increments of $\hat{x}_t$ deduced from the ICRG composite risk index. Time period: 1988-1996. Bi-annual observations.

Country	Mean	t-value(1)	Variance	Coeff. of Skewness	Excess Kurtosis	B-J(2), p-value	Studentized range(3)	ρ_1(1)(4)	ρ_2(1)(4)
Algeria	-0.0329	-3.33 **	0.0017	-0.495	0.622	0.62	3.93	-0.208	0.358
Angola	0.0021	0.18	0.0022	-0.144	0.134	0.96	4.04	0.060	0.132
Argentina	0.0235	1.58	0.0038	-0.355	0.336	0.80	4.07	0.654 **	0.659 **
Australia	0.0017	0.25	0.0008	-0.596	-0.153	0.60	3.55	0.456	0.288
Azerbaijan	NA								
Brazil	0.0144	1.72	0.0012	0.325	-0.226	0.85	3.77	0.487 *	0.548 *
Brunei	NA								
Cameroon	-0.0310	-4.57 **	0.0008	-0.318	-0.752	0.71	3.22	0.356	0.250
Canada	-0.0150	-2.14 *	0.0008	-0.499	0.217	0.69	3.80	0.274	-0.123
China	-0.0117	-0.95	0.0026	-1.773	3.759	0.00 **	3.94	0.571 *	0.156
Colombia	0.0114	0.97	0.0024	-1.991	4.965	0.00 **	3.91	0.096	0.157
Congo	0.0021	0.31	0.0008	-0.342	0.370	0.81	3.93	-0.189	0.257
Denmark	0.0150	2.56 *	0.0006	-0.380	0.529	0.74	4.15	0.159	0.437
Egypt	0.0060	0.64	0.0015	-0.909	0.502	0.28	3.90	0.727 **	0.534 *
Equador	0.0198	2.81 *	0.0008	-0.069	-0.785	0.80	3.44	0.396	0.264
Gabon	-0.0136	-2.09	0.0007	0.373	0.538	0.74	4.09	0.319	-0.040
India	-0.0053	-0.44	0.0025	-1.541	4.079	0.00 **	4.19	0.470	0.331
Indonesia	0.0133	2.62 *	0.0004	1.221	1.128	0.08	3.82	0.168	0.360
Iran	0.0129	1.03	0.0027	0.115	-0.903	0.74	3.67	0.513 *	0.514 *
Iraq	-0.0168	-0.82	0.0072	-1.687	3.562	0.00 **	4.25	0.149	0.106
Kazakhstan	0.0041	0.22	0.0029	-1.524	3.773	0.02 *	3.33	-0.795 *	-0.133
Kuwait	-0.0057	-0.16	0.0213	-3.720	14.826	0.00 **	4.66h**	-0.081	-0.205
Libya	0.0086	0.81	0.0019	-0.629	0.300	0.55	3.87	-0.308	-0.201
Malaysia	0.0205	4.39 **	0.0004	-0.857	0.619	0.31	3.63	0.459	0.033
Mexico	0.0218	1.85	0.0024	-1.896	5.398	0.00 **	4.33h*	0.291	0.373
Nigeria	-0.0118	-1.31	0.0014	0.493	-0.536	0.64	3.50	-0.001	0.358
Norway	0.0062	0.44	0.0034	0.840	3.049	0.01 *	4.62h**	-0.171	0.083
Oman	0.0037	0.49	0.0010	-0.314	0.772	0.70	4.15	0.106	0.025
Papua New Guinea	-0.0071	-1.01	0.0008	-0.437	1.006	0.53	4.13	-0.074	0.120
Peru	0.0327	2.16 *	0.0039	-0.408	-0.972	0.57	3.20	0.494 *	0.493 *
Quatar	-0.0018	-0.18	0.0016	-2.328	8.236	0.00 **	4.74h**	0.098	-0.185
Romania	-0.0038	-0.39	0.0016	-0.747	0.920	0.34	3.96	0.549 *	0.366
Russian Federation	-0.0092	-0.48	0.0029	-1.034	1.084	0.40	3.14	-0.475	0.545
Saudi Arabia	-0.0078	-1.00	0.0010	-0.889	2.371	0.04 *	4.33h*	0.176	-0.152
Syria	0.0132	1.91	0.0008	0.957	2.194	0.05 *	4.19	-0.137	-0.235
Trinidad & Tobago	-0.0006	-0.05	0.0022	-0.870	1.066	0.23	3.83	0.472	0.465
Tunisia	0.0189	2.55 *	0.0009	-0.961	1.267	0.15	3.92	-0.126	0.126
United Arab Emirates	0.0074	0.69	0.0019	-2.364	8.309	0.00 **	4.77h**	-0.103	-0.197
United Kingdom	0.0040	0.45	0.0013	-1.652	3.935	0.00 **	4.41h*	0.042	-0.550 *
USA	-0.0011	-0.08	0.0028	-0.853	0.211	0.35	3.62	0.258	0.017
Uzbekistan	-0.0025	-0.17	0.0017	-1.070	2.384	0.18	3.36	-0.019	0.468
Venezuela	-0.0061	-0.51	0.0024	0.672	0.240	0.52	3.65	0.379	0.207
Vietnam	0.0574	5.20 **	0.0011	-0.491	-1.150	0.65	3.02l*	0.503	-0.307
Yemen	NA								

The number of observations are 17 for all countries, except for the Russian Federation (8), Yemen (7), Kazakhstan (8), Uzbekistan (8), and Vietnam for the IICCR index (8). (1) * and ** indicates whether the estimate is significantly different from zero, using a two sided test and a significance level of 0.05 and 0.01, respectively. (2) The p-value of the Bera-Jarque test of normality, based on the statistic $J = n[(\text{coeff. of skewness})^2/6 + (\text{excess kurtosis})^2/24]$. In case of normality, J is χ^2-distributed with two degrees of freedom. The reported p-value is the probability of observing a J statistic equal to or lower than the sample statistic J. (3) h* and h** indicates that in a normal distribution with n observations, the probability of the observed studentized range being this high is less than 0.05 and 0.01, respectively. Similarly, l* and l** means that in a normal distribution with n observations, the probability of the observed studentized range being this low is less than 0.05 and 0.01. (4) Coefficient of correlation between observations, where one observation is lagged one or two periods.

Table 4.6.D Statistics for sample of increments of $\hat{x}_t$ deduced from the Institutional Investor's country credit ratings. Time period: 1988-1996. Bi-annual observations

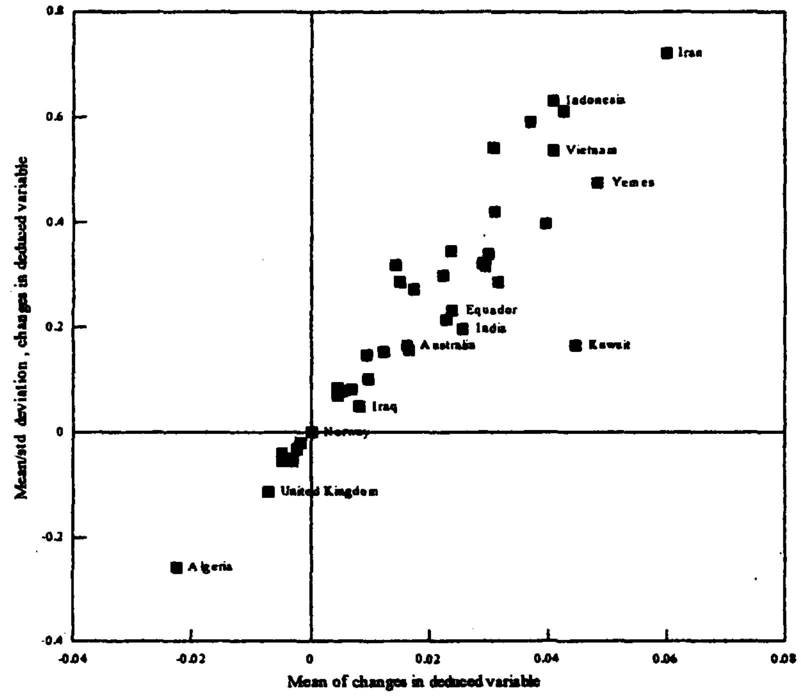

Figure 4.5 ICRG political risk index. Time period: 1988-1996. Bi-annual observations

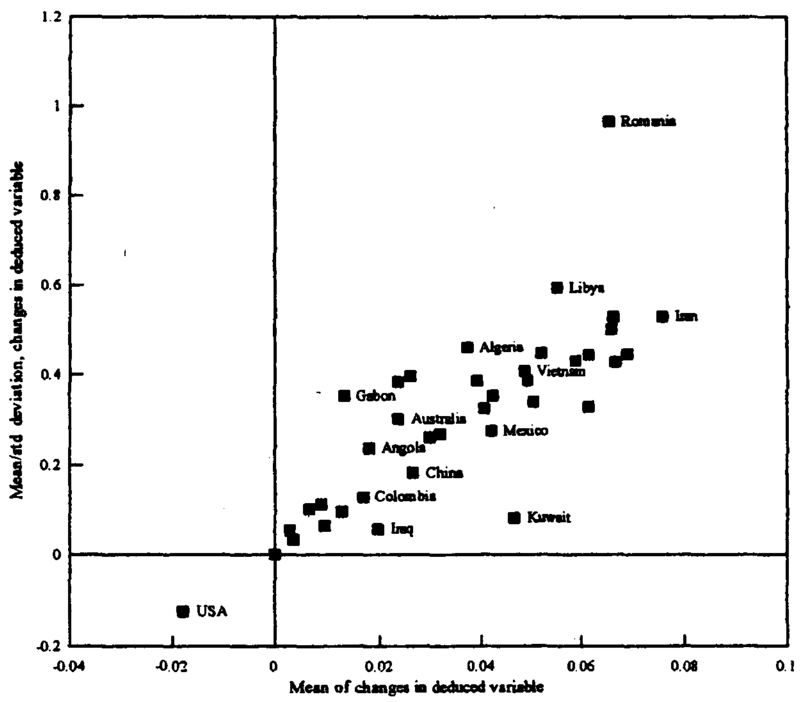

Figure 4.6 ICRG financial risk index. Time period: 1988-1996. Bi-annual observations

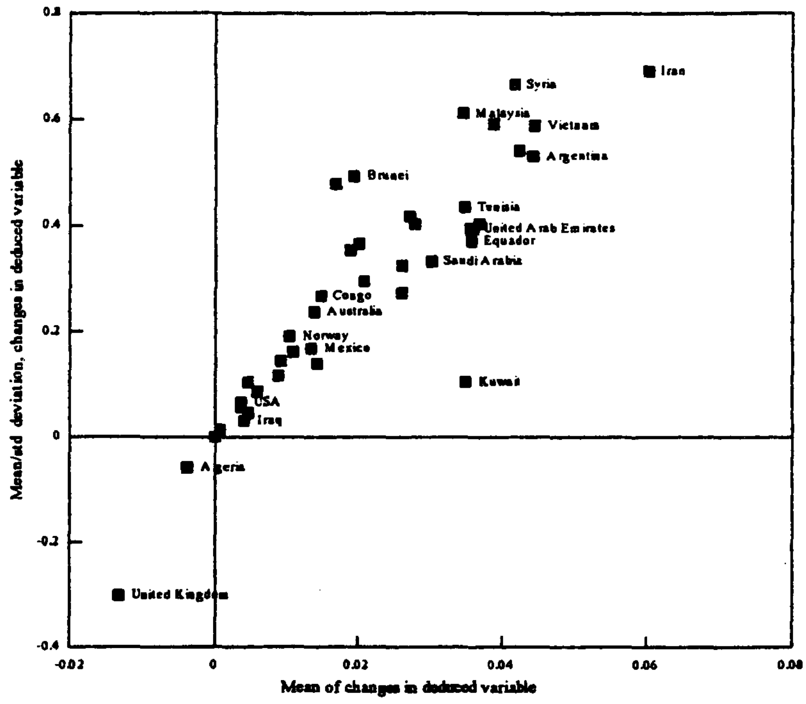

Figure 4.7 ICRG composite risk index. Time period 1988-1996. Bi-annual observations

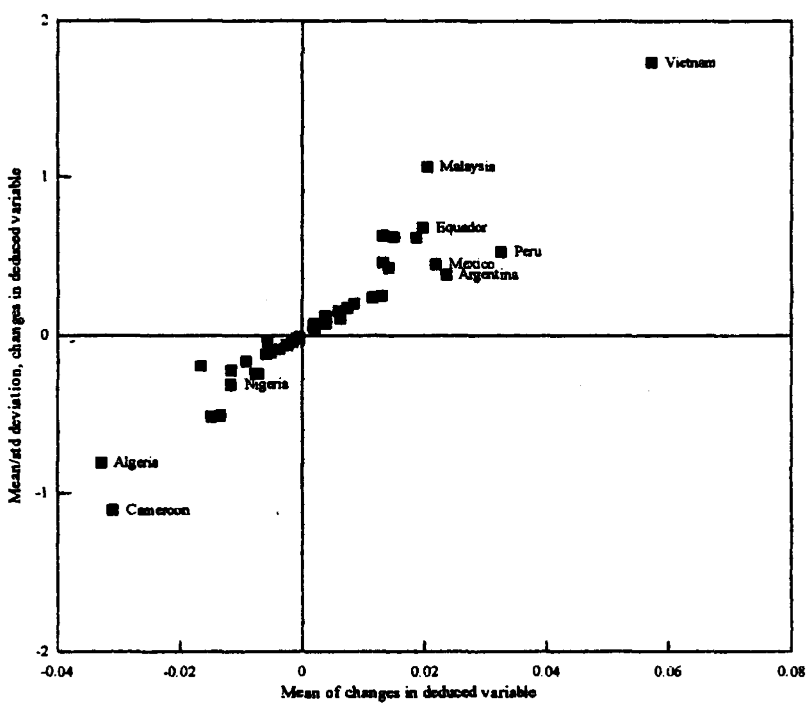

Figure 4.8 Institutional Investor's country credit ratings. Time period: 1988-1996. Bi-annual observations

	ICRG PR	ICRG FR	ICRG CR	IICCR
ICRG PR		0.934**	0.981**	-0.292
ICRG FR	0.539*		0.982**	-0.579*
ICRG CR	0.750**	0.783**		-0.434
IICCR	0.234	-0.358	0.047	

* Significantly different from zero at significance level 0.05. ** Significantly different from zero at significance level 0.01

Table 4.7 Correlation between average values (equally weighted) of deduced observations, $\hat{x}_t$, level (upper right triangle) and changes (lower left triangle). Time period: 1988-1996. Bi-annual observations

	ICRG PR	ICRG FR	ICRG CR	IICCR
ICRG PR		0.952**	0.984**	-0.318
ICRG FR	0.596*		0.988**	-0.562*
ICRG CR	0.765**	0.783**		-0.446
IICCR	0.173	-0.331	0.020	

* Significantly different from zero at significance level 0.05. ** Significantly different from zero at significance level 0.01

Table 4.8 Correlation between average index values (equally weighted), level (upper right triangle) and changes (lower left triangle). Time period: 1988-1996. Bi-annual observations

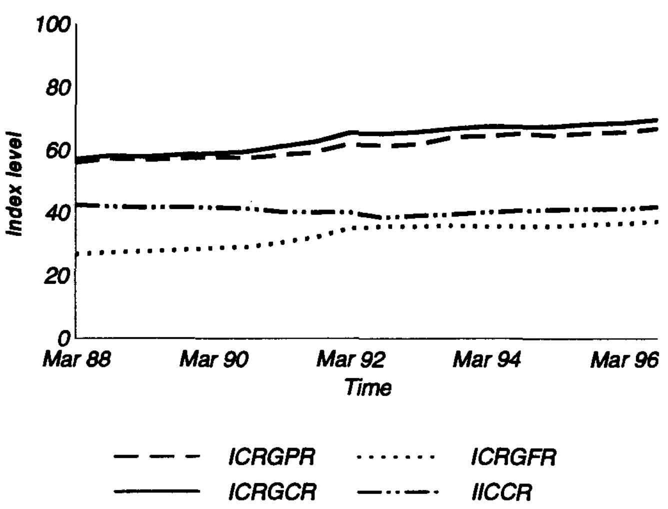

Figure 4.9 Average of risk indices for the sample period, equal weighting of countries

Country	ICRG PR	ICRG FR	ICRG CR	IICCR
Algeria		✓		
Angola	✓			
Argentina				✓
Australia	✓		✓	
Azerbaijan	NA	NA	NA	NA
Brazil	✓			✓
Brunei	✓	✓	✓	NA
Cameroon	✓	✓	✓	
Canada		✓	✓	
China	✓	✓		✓
Colombia				✓
Congo	✓	✓	✓	
Denmark		✓		
Egypt				✓
Equador		✓		
Gabon	✓	✓		
India				✓
Indonesia		✓		
Iran		✓		✓
Iraq		✓		✓
Kazakhstan	NA	NA	NA	✓
Kuwait	✓	✓	✓	✓
Libya				
Malaysia		✓		
Mexico	✓		✓	✓
Nigeria				
Norway		✓		✓
Oman		✓		
Papua New Guinea	✓		✓	
Peru	✓	✓	✓	✓
Quatar		✓	✓	✓
Romania				✓
Russian Federation			✓	
Saudi Arabia		✓	✓	✓
Syria		✓		✓
Trinidad & Tobago		✓	✓	
Tunisia		✓	✓	
United Arab Emirates	✓	✓	✓	✓
United Kingdom		NR		✓
USA		✓		
Uzbekistan	NA	NA	NA	
Venezuela	✓			
Vietnam	✓	✓	✓	✓
Yemen			✓	NA
Sum ✓[1]	15/26	26/14	17/24	20/21

A mark "✓" is inserted if the hypothesis of the increments of $\hat{x}_t$ being normally distributed and/or the coefficient of correlation between lagged increments is zero can be rejected based on the tests reported in Table 4.6A-D using a significance level of five per cent.
[1] Number of countries with ✓-marks/ number of countries, for which data are available or reported, with no ✓-marks.

Table 4.9 Summary of results for the tests of whether the increments of the deduced variable $\hat{x}_t$ are normally distributed and uncorrelated. Time period: 1988-1996. Bi-annual observations.

different from zero. The estimated coefficient of correlation between the changes in ICRG FR and IICCR, or between the variables deduced from these indices, is negative but not significantly different from zero. Figure 4.9 shows that from 1988 to 1992 the IICCR was steadily increasing, while the ICRG FR was decreasing.

The coefficients of skewness, the excess kurtosis, and the studentized range are reported in Tables 4.6A-D in order to indicate if the changes in $\hat{x}_t$ are normally distributed. The coefficient of skewness measures the asymmetry of the distribution and the excess kurtosis indicates the thickness of the tail of the distribution. For the normal distribution, the expected value of both statistics is zero. I also report the *p*-value of a Bera-Jarque test of normality based on the coefficient of skewness and excess kurtosis. The studentized range is defined as the difference of the largest and smallest observation divided by the sample standard deviation. Finally I report the coefficient of correlation between lagged increments, where ρ_k is the coefficient of correlation between $z_j^{(\hat{x})}$ and $z_{j+k}^{(\hat{x})}$. According to the process assumptions, the coefficient of correlation between lagged increments should be zero.

Table 4.9 contains an overview of those countries where the reported tests resulted in a rejection of the hypothesis of normality and/or the hypothesis of zero correlation betweeen lagged increments. Those countries where the tests showed significant rejection of the hypotheses, at a five per cent significance level, are marked with a ✓-symbol. A ✓-mark indicates that the hypothesis of an arithmetic Brownian motion with constant parameters can be rejected, and that further analysis is needed, e.g., by selecting a different stochastic process. Table 4.9 shows that there were more ✓-marks for the ICRG FR and the IICCR than for the ICRG PR and CR. Based on this, the arithmetic Brownian motion process is less likely to describe the dynamic behavior of a variable deduced from the ICRG FR or IICCR than of a variable deduced from the ICRG PR or CR. Note that for quite a large number of countries the hypothesis about the underlying stochastic process cannot be rejected, even though the rather simple model presented in section two, the "indirect approach", is used to deduce the variables.

For the change in the state variable governing the Brent Blend oil prices, I use the observation $z_j^{(S)} = \ln(S_j / S_{j-1})$, i.e., the logarithm of relative prices where the time period between price observations is Δt. The estimators for the mean and variance of $z_j^{(S)}$ are given by (39) and (40), but where $z_j^{(S)}$ replaces $z_j^{(\mathcal{E})}$. The statistics for the sample period are reported in Table 4.10. For the whole period, the coefficient of correlation, either lagged one or two periods, is significantly different from zero at one per cent significance-level, and the test based on the studentized range statistic indicates that the hypothesis of normally distributed increments can be rejected. By excluding the period for the Gulf War, only the coefficient of correlation between the lagged increments for quarterly data are significantly different from zero. Statistics are also reported for the period before and after the Gulf War.

Period	Observations	N	Mean	t-value[1]	Variance	Coeff of Skewness	Excess Kurtosis	B-J[2], p-value	Studentized range[3]	ρ_1[1],[4]	ρ_2[1],[4]
Whole period	Bi-annual	17	0.0239	0.29	0.1170	0.252	2.340	0.13	4.65 h**	-0.546 *	0.090
	Quarterly	35	0.0079	0.21	0.0520	1.848	7.331	0.00 **	5.83 l**	-0.190	-0.417 *
	Monthly	105	0.0026	0.28	0.0095	0.645	3.988	0.00 **	7.37 h**	0.237 *	-0.017
- e xcl. Gulf War	Bi-annual	15	0.0225	0.42	0.0427	0.967	1.635	0.13	4.02	-0.485	0.029
	Quarterly	32	0.0032	0.13	0.0194	0.339	-0.591	0.58	3.95	-0.091	-0.401 *
	Monthly	98	0.0029	0.39	0.0054	-0.102	-0.224	0.83	5.01	0.018	-0.115
Pre Gulf War	Montlhly	31	0.0022	0.14	0.0083	0.089	-0.442	0.86	4.06	0.090	-0.212
Post Gulf War	Monthly	67	0.0032	0.41	0.0042	-0.314	-0.449	0.44	4.15	-0.043	-0.041

Whole period: 1988-1996. Gulf War: August 1990-February 1991. (1) * and ** indicates whether the estimate is significantly different from zero, using a two sided test and a significance level of five and one per cent, respectively. (2) The p-value of the Bera-Jarque test of normality, based on the statistic $J = n[(\text{coeff. of skewness})^2/6 + (\text{excess kurtosis})^2/24]$. In case of normality, J is χ^2-distributed with two degrees of freedom. The reported p-value is the probability of observing a J statistic equal to or lower than the sample statistic J. (3) h* and h** indicates that in a normal distribution with n observations, the probability of the observed studentized range being this high is less than 0.05 and 0.01, respectively. Similarly, l* and l** means that the probability of the observed studentized range being this low is less than 0.05 and 0.01. (4) Coefficient of correlation between observations, where one observation is lagged one or two periods.

Table 4.10 Statistics for sample of the logarithm of relative Brent Blend oil prices

4.5 Estimates of Process Parameters

In order to estimate the drift adjustment, $\delta_{\mathcal{E}}$, I first perform a regression analysis to estimate a beta according to a traditional CAPM. For the return on the world market portfolio, M, I use the observation $z_j^{(M)} = \ln(M_j/M_{j-1})$ where the time period between observations, Δt, is a half year. As mentioned in section 4.2, I use the Morgan Stanley Capital International World Index, measured in US dollars, as the market portfolio. For the risk free interest rate I use the observation of the six month Eurodollar interest rate, $r_j^{f,obs}$, and convert it to a continuously compounded rate for a half year, i.e., $r_j = \ln(1 + r_j^{f,obs}/2)$. I find the ordinary least squares (OLS) estimate for beta, $\hat{\beta}_{\mathcal{E}}$, by running the regression equation

$$z_j^{(\hat{x})} - r_{j-1} = a + (z_j^{(M)} - r_{j-1})\,\beta_{\hat{x}} + u_j \ . \qquad \textbf{(41)}$$

where u_j is the error term. The left hand side of equation (41) is equal to the change in the deduced variable over the time interval Δt, i.e., the rate of capital appreciation from holding asset $Z_t^{(\hat{x})}$, in excess of the risk free interest rate.

A positive beta means that high excess return on the market portfolio corresponds to high expected excess return from holding the asset $Z_t^{(\hat{x})}$. If the interest rate was constant over the sample period, a positive beta would mean that high excess return on the market portfolio would correspond to an increase in the risk index, i.e. lower risk, and a negative beta would imply the opposite: a high market return would correspond to an increase in risk as measured by the index. For most countries we would expect a beta close to zero. The important fact to be aware of is that the variables deduced from the indices are not related in any clear way to prices of actually traded assets. A priori, it does not seem clear to me that the estimated betas should be different from zero, unless perhaps for the big countries like USA. For large countries influencing the world economy, we would probably expect that decreasing levels of risk corresponds to high levels of market return i.e., a negative beta.

I report in the first part of Table 4.11.A-D the results from running the regression-equation (41) for the ICRG PR, FR, CR, and the IICCR. Very few of the estimated betas are significantly different from zero. For the ICRG PR, only the beta for Papua New Guinea (positive) is significant, at one or five per cent level of significance. For the ICRG FR the estimated beta for Australia was negative and significant at a one per cent level. The estimated betas for Papua New Guinea and Trinidad and Tobago were positive and significant at a one and five per cent level, respectively. For the ICRG CR the only significant negative beta was for Argentina (one per cent level). Papua New Guinea and Trinidad and Tobago had significant positive estimated betas at, respectively, one and five per cent level of significance. For the IICCR, China and Papua New Guinea had positive betas, both significant at the five per cent level. For all indices the reported R^2 is low. For most of the countries, the hypothesis that beta is zero cannot be rejected. According to the CAPM, a beta equal to zero implies that the required expected rate of return from holding the asset $Z_t^{(\hat{x})}$ is equal to the risk free interest rate. The implication is that for most of the countries, the risk measured by the indices may be considered as non-systematic.

The results of the regressions reported in Table 4.11.A-D are, however, based on the assumption that the standard deviation of the noise, σ_v, is equal to one. The OLS-estimator of beta is

$$\hat{\beta}_x = \text{Cov}(\sigma_v z^{(\hat{x}:\sigma_v=1)} - r, z^{(M)} - r) / s_M^2 \quad , \tag{42}$$

where $z^{(\hat{x}:\sigma_v=1)}$ is the increment in the deduced variable when σ_v is equal to one. If the risk free interest rate r is a constant, an increase in σ_v would increase the beta estimate due to scaling effects. Because the risk free interest rate is not a constant, the effect on the beta from selecting different values of σ_v is not obvious. I report in Table 4.12 estimates of betas for different assumptions about σ_v for five countries: Iraq, Nigeria, Norway, United Kingdom, and USA. For all of the countries except Nigeria, the absolute value of the estimated beta increases with increasing values of σ_v, but none of the estimated betas became significantly different from zero. The explained variance, as measured by R^2, showed only minor changes. It therefore seems, based on these results, that selecting values of σ_v different from one will not improve the fit, as measured by R^2, or the level of significance of the estimated beta.

Because σ_v and Δt are constants, the estimator for $\alpha_{\hat{x}}$ is

$$\hat{\alpha}_{\hat{x}} = \frac{\bar{z}^{(\hat{x})}}{\sigma_v \Delta t} + \frac{1}{2}\sigma_v \hat{\sigma}_{\hat{y}}^2 \tag{43}$$

and the estimator for the parameter $\sigma_{\hat{x}}$ is

$$\hat{\sigma}_{\hat{x}} = \sqrt{\frac{s_{\hat{x}}^2}{\sigma_v^2 \Delta t}} \ . \tag{44}$$

Equation (43) is derived by solving equation (39) with respect to $\alpha_{\hat{x}}$ and equation (44) is equal to equation (40) solved with respect to $\sigma_{\hat{x}}$.

Based on the estimates from the regression, an estimate of the rate of return adjustment, given assumptions about r and α_M, may be found by using the equation

$$\hat{\delta}_{(i)} = r + (\alpha_M - r)\hat{\beta}_{(i)} - \hat{\alpha}_{(i)} \ , \ i \in \{\hat{x}, S\} \ . \tag{45}$$

The logarithm of the relative oil prices, $z_j^{(S)}$, is

$$\ln(S_j/S_{j-1}) = (\alpha_S - \frac{1}{2}\sigma_S^2)\,\Delta t + \sigma_S(m_j - m_{j-1}) \ . \tag{46}$$

where $m_j = \rho_{S,\hat{x}}(B_j^{(1)} - B_{j-1}^{(1)}) + \sqrt{1-\rho_{S,\hat{x}}^2}(B_j^{(2)} - B_{j-1}^{(2)})$. The estimator of the covariance between the state variable governing the oil price and the variable deduced from the risk index is

$$\hat{\text{Cov}}_{S,\hat{x}} = \frac{1}{n-1}\sum_1^n (z_j^{(\hat{x})} - \bar{z}^{(\hat{x})})(z_j^{(S)} - \bar{z}^{(S)}) \ , \tag{47}$$

and the sample coefficient of correlation is

$$r_{S,\hat{x}} = \frac{\hat{\text{Cov}}_{S,\hat{x}}}{s_S s_{\hat{x}}} \ . \tag{48}$$

According to the process assumptions, $z_j^{(\hat{x})}$ and $z_j^{(S)}$ are distributed according to a bivariate normal distribution, i.e.,

$$(z_j^{(\hat{x})}, z_j^{(S)}) \sim \text{N}\left(\begin{pmatrix} \sigma_v(\alpha_{\hat{x}} - \frac{1}{2}\sigma_{\hat{x}}^2)\Delta t \\ (\alpha_S - \frac{1}{2}\sigma_S^2)\Delta t \end{pmatrix} \begin{pmatrix} \sigma_v^2\sigma_{\hat{x}}^2\Delta t & \text{Cov}_{S,\hat{x}} \\ \text{Cov}_{S,\hat{x}} & \sigma_S^2\Delta t \end{pmatrix}\right),$$

where $\text{Cov}_{S,\hat{x}} = \rho_{S,\hat{x}}\sigma_v\sigma_{\hat{x}}\sigma_S\Delta t$. Note that the coefficient of correlation between the deduced variable $\hat{x}_t$ and the logarithm of relative oil prices is not affected by the choice of σ_v. Also note that if $c_j = B_j^{(1)} - B_{j-1}^{(1)}$, and $m_j = \rho_{S,\hat{x}}(B_j^{(1)} - B_{j-1}^{(1)}) + \sqrt{1-\rho_{S,\hat{x}}^2}(B_j^{(2)} - B_{j-1}^{(2)})$, then

$$(c_j, m_j) \sim \mathrm{N}\left(\begin{pmatrix}0\\0\end{pmatrix}\begin{pmatrix}\Delta t & \rho_{S,\pounds}\Delta t\\ \rho_{S,\pounds}\Delta t & \Delta t\end{pmatrix}\right).$$

If $c_j' = a_\pounds + b_\pounds c_j$ and $m_j' = a_S + b_S m_j$ for constants $a_\pounds$, $b_\pounds$, a_S, and b_S, then

$$(c_j', m_j') \sim \mathrm{N}\left(\begin{pmatrix}a_\pounds\\a_S\end{pmatrix}\begin{pmatrix}b_\pounds^2\Delta t & b_\pounds b_S\rho_{S,\pounds}\Delta t\\ b_\pounds b_S\rho_{S,\pounds}\Delta t & b_S^2\Delta t\end{pmatrix}\right). \tag{49}$$

We get that $c_j' = z_j^{(\pounds)}$ and $m_j' = z_j^{(S)}$ by letting $a_\pounds = \sigma_v(\alpha_\pounds - \frac{1}{2}\sigma_\pounds^2)\Delta t$, $b_\pounds = \sigma_v\sigma_\pounds$, $a_S = (\alpha_S - \frac{1}{2}\sigma_S^2)\Delta t$, and, $b_S = \sigma_S$. The estimator for $\rho_{S,\pounds}$ is therefore given by

$$\hat{\rho}_{S,\pounds} = r_{S,\pounds}\ . \tag{50}$$

A positive coefficient of correlation between the deduced variable and the state variable governing the oil prices means that the risk, as measured by the index, is reduced when the oil price increases. When the coefficient of correlation is negative, an increase in the oil price is likely to occur together with an increase in risk. There are some intuitive explanations for why the correlation should be positive or negative. If the country is mainly dependent on the production and sale of oil for its revenue, a reduction in the oil price may lead to political turmoil, i.e., increased risk (positive correlation). A large drop in the oil revenue combined with a lack of willingness to cut back on public spending may reduce the country's credit rating. If the country is a major oil producer, a political uncertain situation in the country may lead the participants in the oil market to believe that there is a chance for a reduction in the supply of oil. This can cause the oil prices to rise. In this instance the risk indices and the oil price are negatively correlated. A negative coefficient of correlation may also be expected if the country is a large net importer of oil. An increase in the oil price will increase the cost of an important input factor and may cause the economy to slow down. This may again lead to political instability due to, e.g., unemployment. The credit rating of

Country	Regression a	t-value(1)	β	t-value(1)	R^2	DW(2)	5%	1%	Estimated process parameters $\hat{\rho}_{S,£}$ (1)	$\hat{\alpha}_£$	$\hat{\sigma}_£$	$\hat{\delta}_£$ (3)
Algeria	-0.0563	-2.51 *	0.1940	0.71	0.032	1.606			-0.21	-0.0376	0.1240	0.1063
Angola	-0.0140	-0.53	-0.0652	-0.20	0.003	1.467			-0.07	0.0437	0.1491	0.0165
Argentina	-0.0012	0.06	-0.1190	-0.52	0.018	2.336			0.38	0.0670	0.1040	-0.0086
Australia	-0.0133	-0.51	-0.1370	-0.43	0.012	2.274			-0.06	0.0419	0.1391	0.0160
Azerbaijan	NA											
Brazil	-0.0279	-1.50	-0.3730	-1.64	0.153	2.720	I		0.39	0.0030	0.1112	0.0471
Brunei	-0.0176	-1.62	0.0473	0.36	0.008	2.040			-0.26	0.0303	0.0629	0.0335
Cameroon	-0.0354	-2.45 *	0.1030	0.58	0.022	1.970			-0.12	-0.0025	0.0822	0.0682
Canada	-0.0340	-2.02	0.0506	0.25	0.004	1.039	+	I	0.16	0.0000	0.0937	0.0640
China	-0.0201	-0.82	-0.1160	-0.39	0.010	1.371	I		-0.15	0.0279	0.1328	0.0306
Colombia	-0.0257	-1.99	-0.0770	-0.49	0.016	1.423			0.06	0.0118	0.0758	0.0480
Congo	-0.0295	-1.80	0.2150	1.08	0.071	1.447			0.14	0.0131	0.0917	0.0562
Denmark	-0.0259	-1.46	0.0012	0.01	0.000	2.567			0.34	0.0154	0.0980	0.0470
Egypt	-0.0033	-0.13	-0.3200	-1.03	0.066	2.095			-0.09	0.0307	0.1132	0.0212
Equador	-0.0184	-0.90	-0.0500	-0.20	0.003	2.516			-0.33	0.0581	0.1453	0.0026
Gabon	-0.0360	-2.49 *	0.1400	0.79	0.040	2.276			-0.25	-0.0031	0.0780	0.0699
India	-0.0024	-0.07	-0.2550	-0.62	0.025	1.504			0.22	0.0678	0.1832	-0.0138
Indonesia	0.0099	0.62	-0.0201	-0.10	0.001	2.070			0.26	0.0857	0.0913	-0.0241
Iran	0.0304	1.50	-0.1200	-0.49	0.015	1.975			-0.35	0.1269	0.1176	-0.0685
Iraq	-0.0257	-0.59	0.1970	0.37	0.009	2.027			-0.17	0.0439	0.2363	0.0249
Kazakhstan	NA											
Kuwait	-0.0020	-0.03	1.2040	1.51	0.132	1.701			-0.54 *	0.1642	0.3870	-0.0625
Libya	0.0046	0.17	-0.3450	-1.05	0.068	2.164			-0.27	0.0747	0.1547	-0.0236
Malaysia	0.0024	0.19	-0.2400	-1.49	0.129	1.286	I		0.42	0.0642	0.0797	-0.0097
Mexico	-0.0370	-1.72	0.0748	0.28	0.005	2.551			0.19	-0.0024	0.1204	0.0672
Nigeria	-0.0165	-1.27	0.0133	0.08	0.000	2.465			-0.34	0.0323	0.0734	0.0305
Norway	-0.0340	-1.53	0.2200	0.81	0.042	1.249	I		0.17	0.0078	0.1247	0.0617
Oman	-0.0050	-0.30	-0.2070	-1.01	0.064	2.447			-0.12	0.0516	0.0966	0.0040
Papua New Guinea	-0.2920	-2.34 *	0.5760	3.77 **	0.486	1.862			-0.34	0.0227	0.0896	0.0584
Peru	-0.0002	-0.01	-0.0898	-0.32	0.007	2.581			0.08	0.0674	0.1246	-0.0080
Quatar	-0.0005	-0.02	-0.1450	-0.52	0.017	2.196			-0.14	0.0657	0.1272	-0.0082
Romania	0.0099	0.66	-0.3150	-1.72	0.165	1.578			0.35	0.0779	0.0887	-0.0259
Russian Federation	-0.0546	-1.25	1.2090	1.47	0.264	2.341	NR	NR	0.05	0.0497	0.1050	0.0521
Saudi Arabia	-0.0108	-0.41	0.1790	0.55	0.020	1.945			-0.30	0.0566	0.1501	0.0116
Syria	0.0113	0.61	0.0207	0.09	0.001	1.707			-0.37	0.0900	0.0987	-0.0270
Trinidad & Tobago	-0.0169	-1.09	0.2130	1.12	0.078	2.537			-0.60 *	0.0379	0.0886	0.0314
Tunisia	0.0001	0.00	-0.1430	-0.49	0.016	1.844			-0.42	0.0675	0.1324	-0.0099
United A. Emirates	0.0103	0.41	-0.1510	-0.49	0.016	2.532			-0.43	0.0890	0.1409	-0.0316
United Kingdom	-0.0420	-2.57 *	0.2870	1.44	0.121	1.652			-0.55 *	-0.0103	0.0886	0.0820
USA	-0.0261	-1.22	0.1180	0.45	0.013	1.951			0.48	0.0198	0.1150	0.0463
Uzbekistan	NA											
Venezuela	-0.0410	-1.46	0.3940	1.15	0.080	2.420			0.11	0.0042	0.1658	0.0710
Vietnam	0.0063	0.32	0.2620	1.08	0.072	2.170			-0.01	0.0874	0.1081	-0.0165
Yemen	-0.0727	-1.53	2.3590	2.53	0.561	2.820	NR	NR	-0.42	0.1074	0.1444	0.0320

Table 4.11.A Results for the regression to estimate beta when the variable $\hat{x}_t$ is deduced from the ICRG political risk index, and estimates of parameters in the evolutionary process for $\hat{x}_t$. Time period: 1988-1996. Bi-annual observations

Country	Regression a	t-value(1)	β	t-value(1)	R^2	DW(2)	5%	1%	Estimated process parameters $\hat{\rho}_{S,\hat{x}}$(1)	$\hat{\alpha}_{\hat{x}}$	$\hat{\sigma}_{\hat{x}}$	$\hat{\delta}_{\hat{x}}$(3)
Algeria	0.0091	0.47	-0.2350	-0.98	0.061	2.624	I		-0.35	0.0810	0.1141	-0.0263
Angola	-0.0146	-0.76	0.1120	0.48	0.015	1.856			0.04	0.0416	0.1068	0.0243
Argentina	0.0344	1.06	-0.5450	-1.38	0.112	1.477			0.22	0.1355	0.1920	-0.0910
Australia	0.0004	0.04	-0.6130	-3.96 **	0.510	1.500			0.43	0.0535	0.1105	-0.0112
Azerbaijan	NA											
Brazil	-0.0105	-0.34	-0.6210	-1.65	0.154	2.179			0.49 *	0.0426	0.1863	-0.0005
Brunei	-0.0229	-1.23	0.0428	0.19	0.002	1.514			0.02	0.0237	0.1106	0.0400
Cameroon	-0.0280	-1.99	-0.0147	-0.09	0.000	1.795			0.24	0.0090	0.0774	0.0529
Canada	-0.0320	-1.25	0.0690	0.22	0.003	1.822			0.37	0.0113	0.1501	0.0533
China	-0.0048	-0.13	0.0226	0.05	0.000	0.902	+	I	-0.09	0.0743	0.2050	-0.0112
Colombia	-0.0141	-0.44	-0.0203	-0.05	0.000	0.976	+	I	-0.23	0.0502	0.1828	0.0114
Congo	0.0043	0.17	0.2820	0.89	0.050	1.862			0.14	0.0884	0.1432	-0.0168
Denmark	0.0204	0.62	-0.1900	-0.47	0.015	1.640			-0.15	0.1145	0.1801	-0.0584
Egypt	0.0338	0.98	-0.2800	-0.67	0.029	1.682			-0.29	0.0727	0.1612	-0.0195
Equador	0.0001	0.00	-0.1060	-0.31	0.006	2.384			-0.19	0.1419	0.1954	-0.0830
Gabon	-0.0174	-1.74	-0.0428	-0.35	0.008	1.438			-0.07	0.0278	0.0529	0.0331
India	0.0043	0.14	-0.2740	-0.74	0.035	1.823			0.24	0.0778	0.1671	-0.0245
Indonesia	0.0400	0.95	-0.7550	-1.47	0.126	1.106	+		0.30	0.1573	0.2627	-0.1196
Iran	0.0490	1.41	-0.3480	-0.82	0.043	1.809			-0.34	0.1717	0.2017	-0.1208
Iraq	-0.0138	-0.17	0.1980	0.19	0.002	1.749			0.01	0.1477	0.4648	-0.0789
Kazakhstan	NA											
Kuwait	-0.0199	-0.15	2.7450	1.70	0.161	2.264			-0.68 **	0.4162	0.8037	-0.2642
Libya	0.0283	1.26	-0.3450	-1.26	0.095	1.331	I		-0.01	0.1186	0.1309	-0.0676
Malaysia	0.0403	1.43	-0.4090	-1.19	0.086	0.890	+	I	0.29	0.1480	0.1772	-0.0991
Mexico	0.0124	0.33	-0.1190	-0.26	0.004	2.012			0.29	0.1071	0.2150	-0.0486
Nigeria	-0.0315	-1.87	0.0275	0.13	0.001	1.523			-0.11	0.0051	0.1009	0.0581
Norway	-0.0307	-2.33 *	-0.0326	-0.20	0.003	1.498			0.12	0.0028	0.0748	0.0585
Oman	0.0087	0.27	0.0521	0.13	0.001	1.877			-0.27	0.0966	0.1764	-0.0326
Papua New Guinea	-0.0393	-2.18 *	0.8890	4.04 **	0.521	1.290	I		-0.33	0.0161	0.1371	0.0753
Peru	0.0384	0.98	-0.0697	-0.15	0.001	2.127			-0.03	0.1614	0.2194	-0.1014
Quatar	0.0183	0.64	-0.5400	-1.56	0.139	1.953			-0.14	0.0995	0.1706	-0.0548
Romania	0.0352	2.22 *	-0.0958	-0.49	0.016	2.305			0.34	0.1348	0.0953	-0.0756
Russian Federation	-0.0277	-0.68	0.2470	0.32	0.017	0.954	NR	NR	0.43	0.0169	0.0894	0.0535
Saudi Arabia	0.0439	1.18	-0.6670	-1.47	0.126	1.275	I		-0.05	0.1572	0.2199	-0.1166
Syria	0.0265	0.95	-0.4540	-1.34	0.107	1.630			-0.15	0.1170	0.1638	-0.0695
Trinidad & Tobago	-0.0111	-0.82	0.4560	2.75 *	0.335	1.541			-0.69 **	0.0562	0.0926	0.0210
Tunisia	0.0262	0.72	-0.5400	-1.22	0.090	1.942			-0.19	0.1227	0.2090	-0.0780
United A. Emirates	0.0429	1.43	-0.6450	-1.76	0.171	1.684			-0.26	0.1486	0.1850	-0.1073
United Kingdom	NR											
USA	-0.0470	-1.31	-0.1510	-0.34	0.008	2.398			-0.06	-0.0153	0.2016	0.0727
Uzbekistan	NR											
Venezuela	-0.0278	-0.80	0.4770	1.13	0.078	1.607			0.01	0.0401	0.2052	0.0378
Vietnam	0.0143	0.47	0.2360	0.64	0.026	1.881			0.04	0.1113	0.1685	-0.0412
Yemen	-0.0364	-1.02	0.8730	1.25	0.238	1.551	NR	NR	-0.01	0.0512	0.0873	0.0396

Table 4.11.B Results for the regression to estimate beta when the variable $\hat{x}_t$ is deduced from the ICRG financial risk index, and estimates of parameters in the evolutionary process for $\hat{x}_t$. Time period: 1988-1996. Bi-annual observations

Country	Regression a	t-value[1]	β	t-value[1]	R^2	DW[2]	5%	1%	Estimated process parameters $\hat{\rho}_{S,\hat{x}}$[1]	$\hat{\alpha}_{\hat{x}}$	$\hat{\sigma}_{\hat{x}}$	$\hat{\delta}_{\hat{x}}$[3]
Algeria	-0.0353	-2.17 *	0.0030	0.15	0.002	1.684			-0.13	-0.0035	0.0896	0.0659
Angola	-0.0249	-1.48	-0.0289	-0.14	0.001	0.987	+	I	-0.04	0.0165	0.0974	0.0448
Argentina	0.0199	1.07	-0.5310	-2.35 *	0.269	1.913			0.44	0.0954	0.1183	-0.0504
Australia	-0.0156	-1.03	-0.1300	-0.70	0.032	2.565			-0.01	0.0311	0.0825	0.0270
Azerbaijan	NA											
Brazil	-0.0153	-0.99	-0.3940	-2.09	0.225	2.480			0.48	0.0261	0.0951	0.0233
Brunei	-0.0128	-1.27	0.0789	0.64	0.027	2.187			-0.15	0.0404	0.0560	0.0245
Cameroon	-0.0304	-2.27 *	0.0023	0.01	0.000	2.470			0.03	0.0044	0.0760	0.0580
Canada	-0.0323	-3.02 **	0.0918	0.70	0.032	0.884	+	I	0.14	0.0018	0.0595	0.0636
China	-0.0141	-0.55	-0.2210	-0.70	0.031	1.302	I		-0.06	0.0387	0.1438	0.0164
Colombia	-0.0213	-1.39	-0.0576	-0.31	0.006	1.592			-0.31	0.0223	0.0898	0.0381
Congo	-0.0201	-1.52	0.2920	1.81	0.179	1.340	I		0.02	0.0328	0.0788	0.0390
Denmark	-0.0144	-1.43	0.0043	0.03	0.000	1.944			0.22	0.0348	0.0496	0.0277
Egypt	0.0078	0.32	-0.2650	-0.90	0.051	1.689			-0.11	0.0436	0.0788	0.0100
Equador	-0.0110	-0.78	0.0109	0.06	0.000	2.511			-0.45	0.0803	0.1356	-0.0177
Gabon	-0.0264	-2.30 *	-0.0035	-0.03	0.000	1.958			-0.13	0.0116	0.0643	0.0507
India	-0.0031	-0.12	-0.1420	-0.45	0.013	1.467			0.21	0.0617	0.1363	-0.0040
Indonesia	0.0091	0.61	-0.1180	-0.65	0.027	1.337	I		0.29	0.0817	0.0926	-0.0232
Iran	0.0032	1.49	-0.2150	-0.83	0.043	2.102			-0.28	0.1279	0.1229	-0.0726
Iraq	-0.0291	-0.86	0.1560	0.38	0.009	1.786			-0.24	0.0255	0.1862	0.0419
Kazakhstan	NA											
Kuwait	-0.0202	-0.26	1.8480	1.97	0.206	2.154			-0.68 **	0.1826	0.4754	-0.0599
Libya	0.0092	0.41	-0.2780	-1.02	0.065	2.130			-0.14	0.0819	0.1292	-0.0287
Malaysia	0.0065	0.52	-0.2560	-1.68	0.159	1.615			0.46	0.0718	0.0797	-0.0179
Mexico	-0.0171	-0.85	-0.0408	-1.66	0.002	2.126			0.22	0.0335	0.1139	0.0275
Nigeria	-0.0270	-1.79	-0.0392	-0.21	0.003	2.307			-0.34	0.0114	0.0895	0.0497
Norway	-0.0236	-1.73	0.2250	1.35	0.109	1.111	+		0.06	0.0241	0.0787	0.0456
Oman	-0.0021	-0.12	-0.1010	-0.48	0.015	2.380			-0.13	0.0604	0.0977	-0.0014
Papua New Guinea	-0.0324	-2.52 *	0.7920	5.05 **	0.629	1.217	I		-0.54 *	0.0239	0.1093	0.0643
Peru	0.0085	0.43	0.1970	0.82	0.043	2.322			-0.33	0.0905	0.1103	-0.0217
Quatar	-0.0030	-0.15	-0.1530	-0.61	0.024	2.079			-0.24	0.0589	0.1147	-0.0016
Romania	-0.0012	-0.08	-0.2110	-1.06	0.069	1.612			0.31	0.0587	0.0924	-0.0033
Russian Federation	-0.0557	-1.31	1.2040	1.50	0.272	2.099	NR	NR	-0.03	0.0466	0.1002	0.0551
Saudi Arabia	-0.0006	-0.03	-0.0372	-0.13	0.001	1.467			-0.22	0.0683	0.1279	-0.0072
Syria	0.0117	0.75	-0.0983	-0.51	0.017	1.675			-0.27	0.0870	0.0884	-0.0279
Trinidad & Tobago	-0.0169	-1.51	0.3670	2.67 *	0.322	2.428			-0.74 **	0.0408	0.0759	0.0335
Tunisia	0.0066	0.33	-0.2440	-1.00	0.063	2.066			-0.33	0.0755	0.1131	-0.0212
United A. Emirates	0.0085	0.38	-0.3260	-1.21	0.089	2.129			-0.31	0.0791	0.1280	-0.0274
United Kingdom	-0.0457	-3.85 **	0.0993	0.68	0.030	1.932			-0.34	-0.0246	0.0621	0.0901
USA	-0.0279	-1.92	0.0305	0.17	0.002	2.325			0.45	0.0103	0.0776	0.0530
Uzbekistan	NA											
Venezuela	-0.0282	-1.13	0.1350	0.44	0.013	2.180			0.13	0.0199	0.1446	0.0469
Vietnam	0.0104	0.53	0.2140	0.90	0.051	1.800			0.01	0.0943	0.1066	-0.0250
Yemen	-0.0069	-0.11	0.4510	0.36	0.026	2.509	NR	NR	-0.70	0.0800	0.1289	-0.0029

Table 4.11.C Results for the regression to estimate beta when the variable $\hat{x}_t$ is deduced from the ICRG composite risk index, and estimates of parameters in the evolutionary process for $\hat{x}_t$. Time period: 1988-1996. Bi-annual observations

Country	Regression a	t-value[1]	β	t-value[1]	R^2	DW[2]	5%	1%	Estimated process parameters $\hat{\rho}_{\xi\xi}$[1]	$\hat{\alpha}_\xi$	$\hat{\sigma}_\xi$	$\hat{\delta}_\xi$[3]
Algeria	-0.0650	-0.66 **	0.0749	0.62	0.025	2.133			0.08	-0.0641	0.0576	0.1289
Angola	-0.0287	-2.70 *	-0.0316	-0.24	0.004	2.153			0.17	0.0063	0.0665	0.0550
Argentina	-0.0091	-0.52	0.1110	0.52	0.017	0.654	+	+	-0.20	0.0507	0.0868	0.0153
Australia	-0.0304	-3.08 **	0.0733	0.73	0.034	1.016	+	I	0.06	0.0042	0.0398	0.0605
Azerbaijan	NA											
Brazil	-0.0192	-2.00	0.1840	1.57	0.141	1.010	+	I	-0.03	0.0299	0.0488	0.0384
Brunei	NA											
Cameroon	-0.0639	-7.60 **	0.1360	1.33	0.105	1.018	+	I	0.37	-0.0612	0.0395	0.1280
Canada	-0.0474	-6.18 **	0.0938	1.00	0.063	1.348	I		-0.15	-0.0292	0.0410	0.0946
China	-0.0480	-3.92 **	0.4000	2.68 *	0.323	0.923	+	I	-0.21	-0.0208	0.0718	0.0962
Colombia	-0.0229	-1.79	0.2470	1.58	0.143	1.728			-0.03	0.0252	0.0688	0.0452
Congo	-0.0282	-3.45 **	-0.0645	-0.65	0.027	1.331	I		-0.09	0.0050	0.0396	0.0553
Denmark	-0.0172	-2.21 *	0.0786	0.83	0.043	1.000	+	I	0.04	0.0305	0.0340	0.0344
Egypt	-0.0131	-1.50	0.1380	1.28	0.099	1.021	+	I	-0.04	0.0134	0.0543	0.0534
Equador	-0.0267	-2.33 *	0.1140	0.81	0.042	0.484	+	+	-0.11	0.0404	0.0411	0.0256
Gabon	-0.0459	-5.34 **	0.0862	0.82	0.043	1.135	I		-0.23	-0.0266	0.0380	0.0917
India	-0.0379	-2.77 *	0.1140	0.68	0.030	0.903	+	I	-0.03	-0.0081	0.0709	0.0742
Indonesia	-0.0183	-4.30 **	0.0374	0.72	0.033	2.274			-0.34	0.0271	0.0296	0.0365
Iran	-0.0162	-1.44	-0.1530	-1.11	0.075	1.397			-0.08	0.0286	0.0732	0.0288
Iraq	-0.0452	-2.13 *	-0.2120	-0.82	0.043	1.474			0.50 *	-0.0264	0.1199	0.0818
Kazakhstan	-0.0142	-0.39	-0.1090	-0.16	0.004	2.273	NR	NR	0.05	0.0112	0.0765	0.0476
Kuwait	-0.0275	-0.79	-0.7250	-1.71	0.164	1.497			0.49 *	0.0099	0.2064	0.0287
Libya	-0.0207	-1.88	-0.1450	-1.08	0.072	2.293			-0.27	0.0192	0.0621	0.0384
Malaysia	-0.0101	-1.97	-0.0457	-0.73	0.035	0.672	+	+	-0.22	0.0414	0.0272	0.0194
Mexico	-0.0076	-0.64	-0.1380	-0.95	0.057	1.607			-0.12	0.0460	0.0686	0.0118
Nigeria	-0.0423	-4.40 **	-0.0501	-0.43	0.012	1.916			-0.07	-0.0222	0.0525	0.0829
Norway	-0.0261	-1.70	0.0870	0.46	0.014	2.215			-0.23	0.0159	0.0826	0.0493
Oman	-0.0276	-3.09 **	0.0067	0.06	0.000	1.433			0.07	0.0084	0.0444	0.0542
Papua New Guinea	-0.0413	-0.59 **	0.2310	2.71 *	0.329	1.999			-0.53 *	-0.0134	0.0411	0.0833
Peru	0.0012	0.07	0.0298	0.14	0.001	0.796	+	+	-0.13	0.0693	0.0884	-0.0060
Quatar	-0.0313	-2.98 *	-0.1260	-0.95	0.057	1.202	I		0.28	-0.0019	0.0567	0.0602
Romania	-0.0353	-3.12 **	0.0281	0.20	0.003	0.660	+	+	0.25	-0.0059	0.0571	0.0692
Russian Federation	0.0026	0.09	-0.7890	-1.41	0.248	1.317	NR	NR	0.65	-0.0154	0.0766	0.0520
Saudi Arabia	-0.0382	-0.42 **	-0.0636	-0.57	0.021	1.168	I		0.16	-0.0146	0.0458	0.0748
Syria	-0.0180	-2.06	0.0087	0.08	0.000	1.798			-0.36	0.0273	0.0405	0.0353
Trinidad & Tobago	-0.0350	-2.39 *	0.0593	0.36	0.008	0.924	+	I	0.03	0.0010	0.0665	0.0633
Tunisia	-0.0132	-1.61	0.0732	0.73	0.035	2.184			-0.00	0.0387	0.0432	0.0260
United A. Emirates	-0.0210	-1.90	-0.2170	-1.61	0.147	1.398			0.31	0.0168	0.0623	0.0385
United Kingdom	-0.0277	-2.62 *	0.0392	0.30	0.006	1.585			0.45	0.0093	0.0513	0.0543
USA	-0.0352	-2.54 *	0.2290	1.35	0.108	1.589			-0.03	0.0006	0.0742	0.0692
Uzbekistan	-0.0023	-0.10	-0.5280	-1.18	0.189	2.000	NR	NR	0.20	-0.0033	0.0590	0.0484
Venezuela	-0.0371	-2.90 *	-0.0170	-0.07	0.000	1.206	I		-0.34	-0.0098	0.0698	0.0716
Vietnam	0.0434	2.11	-0.2250	-0.55	0.042	0.928	+	I	-0.16	0.1159	0.0468	-0.0610
Yemen	NA											

The number of observations are 17 for all countries, except for the Russian Federation (8), Yemen (7), Kazakhstan (8), Uzbekistan (8), and Vietnam for the IICCR index (8). [1] * and ** indicates whether the estimate is significantly different from zero, using a two sided test, at a significance level of five and one per cent, respectively. [2] Conclusions for the Durbin Watson statistic are presented, where the levels of significance are five and one per cent. "+" and "-" indicates that the hypothesis of no first order serial correlation can be rejected for the alternative hypothesis of, respectively, positive and negative serial correlation. "I" means that the test is inconclusive and "NR" means not reported. [3] Computed based on the assumption that, $\alpha_M = 0.095$, $r = 0.06232$, and the estimated beta.

Table 4.11.D Results for the regression to estimate beta when the variable $\hat{x}_t$ is deduced from the Institutional Investor's country credit ratings, and estimates of parameters in the evolutionary process for $\hat{x}_t$. Time period: 1988-1996. Bi-annual observations

Country	a	t-value(1)	β	t-value(1)	R^2	DW(2)	5%	1%
Iraq								
Std. deviation "noise" = 0.5	-0.0286	-1.28	0.1150	0.42	0.012	1.960		
Std. deviation "noise" = 1.0	-0.0257	-0.59	0.1970	0.37	0.009	2.027		
Std. deviation "noise" = 2.0	-0.0198	-0.23	0.3590	0.34	0.008	2.059		
Std. deviation "noise" = 10.0	0.0274	0.07	1.6610	0.32	0.007	2.084		
Nigeria								
Std. deviation "noise" = 0.5	-0.0241	-3.61 **	0.0237	0.29	0.006	2.356		
Std. deviation "noise" = 1.0	-0.0165	-1.27	0.0133	0.08	0.000	2.456		
Std. deviation "noise" = 2.0	-0.0015	-0.06	-0.0074	-0.02	0.000	2.459		
Std. deviation "noise" = 10.0	0.1190	0.91	-0.1730	-0.11	0.001	2.425		
Norway								
Std. deviation "noise" = 0.5	-0.0328	-2.87 *	0.1270	0.91	0.052	1.242	I	
Std. deviation "noise" = 1.0	-0.0340	-1.53	0.2200	0.81	0.042	1.249	I	
Std. deviation "noise" = 2.0	-0.0364	-0.82	0.4060	0.75	0.036	1.244	I	
Std. deviation "noise" = 10.0	-0.0555	-0.25	1.8920	0.70	0.032	1.236	I	
UK								
Std. deviation "noise" = 0.5	-0.0368	-4.09 **	0.1610	1.46	0.125	1.423		
Std. deviation "noise" = 1.0	-0.0420	-2.57 *	0.2870	1.44	0.121	1.652		
Std. deviation "noise" = 2.0	-0.0524	-1.68	0.5410	1.42	0.118	1.778		
Std. deviation "noise" = 10.0	-0.1350	-0.90	2.5670	1.39	0.114	1.882		
USA								
Std. deviation "noise" = 0.5	-0.0288	-2.57 *	0.0758	0.55	0.020	1.844		
Std. deviation "noise" = 1.0	-0.0261	-1.22	0.1180	0.45	0.013	1.951		
Std. deviation "noise" = 2.0	-0.0205	-0.49	0.2010	0.39	0.010	1.995		
Std. deviation "noise" = 10.0	0.0238	0.12	0.8690	0.35	0.008	2.023		

(1) * and ** indicates whether the estimate is significantly different from zero, using a two sided test, at a significance level of five and one per cent, respectively. (2) Conclusions for the Durbin Watson statistic are presented, where the levels of significance are five and one per cent. "+" and "-" indicates that the hypothesis of no first order serial correlation can be rejected for the alternative hypothesis of, respectively, positive and negative serial correlation. "I" means that the test is inconclusive and "NR" means not reported.

Table 4.12 Results for the regression to estimate beta when the variable $\hat{x}_t$ is deduced from the ICRG political risk index for different assumptions about the standard deviation of "noise", σ_v. Time period: 1988-1996. Bi-annual observations

the country may also drop. For the sample, we would expect that the coefficient of correlation, $\rho_{S,\hat{x}}$, is negative for both Iraq and Kuwait due to the Gulf War.

The second part of Table 4.11.A-D contains the estimated coefficients of correlation between the log of the relative oil prices and the increments of the deduced variable, estimates of the parameters $\alpha_{\hat{x}}$, $\sigma_{\hat{x}}$, and $\delta_{\hat{x}}$ for $\hat{x}_t$ deduced from the ICRG PR, FR, CR and IICCR.

For Kuwait, the coefficient of correlation is significantly different from zero at five per cent level of significance, and negative, for all cases except for the IICCR. For Iraq, $\rho_{S,\hat{x}}$ is negative for the ICRG PR and CR, but positive for the ICRG FR and the IICCR. For Norway the estimated coefficient of correlation is positive for the ICRG-indices, but negative for the IICCR. The estimated coefficient of correlation for USA is positive for the ICRG PR and CR, but negative for the ICRG FR and IICCR. For all the ICRG indices, the variable deduced for Trinidad and Tobago is significantly different from zero at five per cent significance level.

The reported rate of return adjustments, the $\delta_{\hat{x}}$s, are computed by using the estimated betas and by assuming an instantaneous return on the market portfolio equal to 9.5 % and a risk free interest rate of 6.23%. The return on the market portfolio is estimated based on sample data and the risk free interest rate is the average, annualized, six month Eurodollar interest rate for the sample period. If the estimated expected increase in the price of the asset $Z_t^{(\hat{x})}$ is zero[18], i.e., $\hat{\alpha}_{\hat{x}} = 0$, the drift adjustment is equal to the risk free interest rate. Note that all estimated parameters of the processes are in nominal terms, i.e., they include inflation.

In Table 4.13 I report the results of the regressions to estimate the beta for the Brent Blend oil price process. The estimated beta for the log of relative oil prices is negative when data for the whole sample period is used. When monthly observations are used, the estimated beta is significant at the one per cent level. On of the regressions includes an indicator variable equaling one during the Gulf War. When running the regression on data for the time before or after the Gulf War, the estimated betas are positive, but not significantly different from zero.

In Table 4.14 I report the estimated market return and the parameters of the oil price process. The estimated drift adjustment, δ_S, is negative for all cases, except when monthly data is used for the period from 1988 until the start of the Gulf War. The drift adjustment is computed by using $\hat{\alpha}_M$, the annualized average risk free interest rate, and the estimated beta for each sample. Note that the drift adjustment is therefore not a direct estimate of the parameter δ_S for the sample period. It is an estimate of the *required* drift adjustment given assumptions about α_M and r. In the real options literature when, e.g., analyzing the value of waiting, it is usually assumed that the convenience yield is positive. A negative convenience

[18] Standing at time t, the expected value of the asset at time T, $t \leq T$, is $Z_t^{(\hat{x})}\exp(\alpha_{\hat{x}}(T-t))$ because $Z_T^{(\hat{x})}$ is log-normally distributed.

yield would, in a standard analysis, never make the alternative to invest more valuable than the value of deferring the investment decision. The drift adjustment, or convenience yield, is usually estimated by using observations of spot oil prices and prices of futures contract on the same oil price. By using a general equilibrium model like CAPM the estimate may be different from the estimate based on futures prices. The price of a futures contract on one barrel of oil at time t maturing at time T, $t \leq T$, has a theoretical price of $S_t \exp(-\delta_S(T-t))$ when δ_S is a constant. A situation where the convenience yield is positive corresponds to a situation with backwardation in the futures market, i.e., the price of the futures contract is lower than the price of the spot price of oil. When the convenience yield is negative, there is a situation with contango in the futures market.

Brennan (1991) estimated the convenience yield for No. 2 heating oil traded at the New York Mercantile Exchange (NYMEX). Based on data for the period September 1980 to December 1994 he used spot and futures prices and derived a maximum likelihood estimate of convenience yield equal to 0.06588 (6.588%) per year . Gibson and Schwartz (1991) also used futures prices when estimating convenience yield for the West Texas Intermediate (WTI) oil price. They used a term structure of convenience yield. Note that when the convenience yield, δ_S, is assumed constant, the implied term structure of convenience yield is flat. They used weekly data from November 1986 to November 1988. For this period the mean of the one month forward convenience yield, annualized, ranged from 11.55% for two months ahead to 7.45 % for eight months ahead. Gibson and Schwartz (1991) presented an estimate of the term structure at August 1988, in their Table 2. At this date the forward convenience yield is increasing with number of periods, but the forward convenience yields one and two months ahead are negative, as most of the estimates in Table 4.14.

The estimates of the volatility parameter for the Brent Blend oil price, $\hat{\sigma}_S$, range from 0.23 to 0.48, and the highest estimate is for the whole period with bi-annual observations. With quarterly and monthly observations, the effect of the Gulf War on the estimates of the volatility is reduced. Gibson and Schwartz (1991) also estimated the implied volatility of spot prices of the WTI based on put and calls traded at NYMEX. The average implied volatility was 0.33.

Based on these articles, the estimated convenience yield presented in Table 4.14 seems to have wrong signs according to what we would expect, but that the estimates of the volatility seems to be more in line with previous findings.

The estimated annualized market premium range from 3.2 % for the whole sample period based on bi-annual observations, to 4.6 % based on monthly data. This seems low. The highest estimated market premium, 5.6 %, is found for the period after the Gulf War when using monthly data.

Period	Observations	N	a	t-value(1)		β	t-value(1)		Indicator	R^2	DW(2)	5%	1%
Whole Period	Bi-annual	17	0.0129	0.161		-1.564	-1.60			0.146	2.97	-	I
	Monthly	105	-0.0004	-0.042		-0.643	-2.64	**		0.063	1.562	+	
Whole Period,	Bi annual	17	0.0175	0.198		-1.578	-1.56		-0.0375	0.147	2.984	-	I
Indicator	Monthly	105	0.0004	0.041		-0.647	-2.64	**	-0.0116	0.064	1.567	+	I
Pre Gulf War	Monthly	31	-0.0053	-0.316		0.125	0.30			0.003	1.700		
Post Gulf War	Monthly	67	-0.0017	-0.205		0.231	0.87			0.012	2.074		

(1) * and ** indicates whether the estimate is significantly different from zero, using a two sided test, at a significance level of five and one per cent, respectively. (2) Conclusions for the Durbin Watson statistic are presented, where the levels of significance are five and one per cent. "+" and "-" indicates that the hypothesis of no first order serial correlation can be rejected for the alternative hypothesis of, respectively, positive and negative serial correlation. "I" means that the test is inconclusive and "NR" means not reported.

Table 4.13 Results of the regressions to estimate beta for the oil price process.

Period	Observations	N	$\hat{\alpha}_M$	$\hat{\sigma}_M$	Average annualizid interest rate	Market premium	$\hat{\alpha}_S$	$\hat{\sigma}_S$	α_S^*	$\hat{\hat{\delta}}_S$ (1)
Whole Period	Bi-annual	17	0.0947	0.1148	0.0623	0.0323	0.1648	0.4837	0.0117	-0.1531
	Quarterly	35	0.0802	0.1140	NR	NR	0.1019	0.3951	NR	NR
	Monthly	105	0.1070	0.1321	0.0606	0.0464	0.0891	0.3385	0.0308	-0.0583
Excl Gulf War	Bi-annual	15	0.0949	0.0903	0.0594	0.0354	0.0876	0.2921	0.0040	-0.0836
	Quarterly	32	0.0858	0.0917	NR	NR	0.0388	0.2411	NR	NR
	Monthly	98	0.1110	0.1172	0.0592	0.0517	0.0677	0.2556	0.0260	-0.0417
Pre Gulf War	Monthly	31	0.1294	0.1422	0.0859	0.0435	0.0767	0.3154	0.0913	0.0146
Post Gulf War	Monthly	67	0.1025	0.1048	0.0469	0.0557	0.0641	0.2254	0.0598	-0.0044

(1) The drift adjustment is calculated based on the estimated α_M, the annualized average risk free interest rate, and the estimated beta for each sample.

Table 4.14 Estimated parameters for market return and the oil price process

5 Asset Valuation - Examples

This section contains examples of how future regulatory regimes can be modeled by using risk indices and how investments, where the cash flow is influenced by future regulatory regimes, can be priced. I have limited the examples to those where I am able to present closed-form valuation formulas.

5.1 State Prices

The most general result, which facilitates a wide range of applications, is the derivation of state prices, where the "states of the world" is determined by combinations of levels of an oil price and a risk index. The first contingent claim I consider is a claim with a payoff at time T, $Z_T^{(C1)}$, equal to

$$Z_T^{(C1)} = \begin{cases} K & \text{if } \psi_T \geq \overline{\psi} \text{ and } S_T \geq \overline{S} \\ 0 & \text{otherwise} \end{cases} , \tag{51}$$

where K is a constant. The payoff is conditioned on the risk index and the oil price both being equal to, or above, critical levels $\overline{\psi}$ and $\overline{S}$. The second contingent claim will at time T have a payoff, $Z_T^{(C2)}$, equal to the oil price, but conditioned on critical levels of the oil price and the risk index, i.e.,

$$Z_T^{(C2)} = \begin{cases} S_T & \text{if } \psi_T \geq \overline{\psi} \text{ and } S_T \geq \overline{S} \\ 0 & \text{otherwise} \end{cases} . \tag{52}$$

If the risk free interest rate is constant, the processes of the hypothetical asset, $Z_T^{(\mathfrak{X})}$, and the oil price are given by equations (24) and (25), and the index is given by

$$\psi_t = \psi^{MAX} \mathrm{N}(\ln(Z_t^{(\mathfrak{X})}) / \sigma_v) , \tag{53}$$

closed-form solutions can be found for the value of these contingent claims. These formulas are derived in Appendix A2. The formulas are derived by finding the expected future payoff under an equivalent martingale measure, as explained in Appendix 1. The value of the first contingent claim at time t , $t \leq T$, is

$$Z_t^{(C1)} = Ke^{-r(T-t)} N(a_t, a'_t, \rho_{S,\pounds}) \ , \tag{54}$$

where

$$a_t \equiv \frac{\ln(Z_t^{(\pounds)}/\overline{Z}) + (r - \delta_\pounds - \frac{1}{2}\sigma_\pounds^2)(T-t)}{\sigma_\pounds\sqrt{(T-t)}}$$

and

$$a'_t \equiv \frac{\ln(S_t/\overline{S}) + (r - \delta_S - \frac{1}{2}\sigma_S^2)(T-t)}{\sigma_S\sqrt{(T-t)}} \quad .$$

The parameter $\overline{Z}$ is defined by the (unique) value of $Z_T^{(\pounds)}$ which makes the equation

$$\overline{\psi} = \psi^{MAX} N(\ln(Z_T^{(\pounds)}) \ , \tag{55}$$

hold, and $N(\cdot,\cdot,\rho_{S,\pounds})$ is the bivariate normal distribution with coefficient of correlation $\rho_{S,\pounds}$. The value of the claim is dependent on the risk index, or equivalently, $Z_t^{(\pounds)}$, and the spot price of oil, both at time t. Note that the value of the claim is also dependent on the coefficient of correlation between the deduced variable and the oil price, $\rho_{S,\pounds}$. If we let $\overline{Z}$ be equal to zero, the value of the claim at time t is equal to the second term in the Black and Scholes' option pricing formula, when the underlying asset pays constant proportional dividends. By letting $\overline{S}$ be equal to zero, the claim is only dependent on $Z_t^{(\pounds)}$, or the risk index today, in a similar way.

The value of the second contingent claim is

$$Z_t^{(C2)} = S_t e^{-\delta_S(T-t)} N(b_t, b'_t; \rho_{S,\pounds}) \ , \tag{56}$$

where

$$b_t \equiv \frac{\ln(Z_t^{(\hat{x})}/\overline{Z}) + (r - \delta_{\hat{x}} + \rho_{S,\hat{x}}\sigma_S\sigma_{\hat{x}} - \frac{1}{2}\sigma_{\hat{x}}^2)(T-t)}{\sigma_{\hat{x}}\sqrt{(T-t)}}$$

and

$$b'_t \equiv \frac{\ln(S_t/\overline{S}) + (r - \delta_S + \frac{1}{2}\sigma_S^2)(T-t)}{\sigma_S\sqrt{(T-t)}} .$$

The parameter $\overline{Z}$ is defined by equation (55). Note that if we let $\overline{Z}$ be equal to zero, the valuation formula is equal to the first term of the familiar Black and Scholes' option pricing formula with constant proportional dividends.

Table 5.1 contains the estimated value, at the end of September 1996, of a claim maturing at the end of September 2000 with a payoff of one USD or one barrel of oil, depending on the future level of the ICRG composite risk index. The September 1996 levels of the ICRG CR, ψ_t, are listed in the table. For the twenty-four countries included, the hypothesis that the deduced variable $\hat{x}_t$ develops according to an arithmetic Brownian motion could not be rejected in section four, see Table 4.9. The parameters used are based on the estimates for the period 1988 to 1996, reported in section four. When computing the state prices, I use the valuation formulas (54) and (56), where I let $\overline{S}$ be equal to zero. The six and one month Eurodollar rates at end of September 1996 were 0.0567 and 0.0531. I assume that the risk free interest rate is constant and equal to 0.05354 (ln(1.055)). The market premium is set to 0.03, and the Brent Blend oil price at September 1996 was 23.8. The beta for the oil is assumed to be -0.6, which with $\alpha_S = 0.03$ gives $\delta_S = 0.00554$. The volatility of the oil price σ_S is set to 0.25. The processes for the deduced variables are assumed to be as reported in Table 4.11. The sum of the state prices for the claim paying one USD is 0.81, i.e., the present value of one USD discounted with the risk free interest rate for four years. The sum of the claims paying one barrel of oil is USD 23.28 which is equal to the present value of the sales revenue of one barrel of oil at September 2000 given the assumed drift adjustment.

			State Prices(3), in USD									
			$\psi_T<50$		$50\le\psi_T<60$		$60\le\psi_T<70$		$70\le\psi_T<85$		$\psi_T\ge85$	
Country	δ_t(1)	ψ_t(2)	1 USD	1 BL	1 USD	1 BL	1 USD	1 BL	1 USD	1 BL	1 USD	1 BL
Algeria	0.057	57.0	0.17	4.40	0.42	11.96	0.21	6.44	0.01	0.47	0.00	0.00
Angola	0.036	51.0	0.28	7.95	0.38	11.00	0.14	4.07	0.01	0.26	0.00	0.00
Argentina	-0.058	73.0	0.00	0.00	0.00	0.03	0.01	0.62	0.40	13.28	0.39	9.35
Brazil	0.016	65.0	0.00	0.15	0.06	2.73	0.35	11.23	0.39	9.13	0.00	0.04
China	0.008	73.0	0.00	0.09	0.03	0.81	0.14	3.87	0.50	14.56	0.13	3.94
Colombia	0.029	63.0	0.01	0.17	0.14	3.32	0.44	12.38	0.21	7.40	0.00	0.01
Denmark	0.019	88.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.81	23.23
Egypt	0.002	67.5	0.00	0.00	0.00	0.12	0.17	4.57	0.63	18.40	0.01	0.18
Equador	-0.026	63.0	0.01	0.15	0.06	1.24	0.22	5.33	0.46	14.42	0.05	2.13
Gabon	0.042	66.0	0.00	0.00	0.05	1.27	0.53	14.88	0.23	7.13	0.00	0.00
India	-0.012	69.0	0.00	0.13	0.03	1.10	0.15	4.97	0.52	14.64	0.10	2.44
Indonesia	-0.032	72.0	0.00	0.00	0.00	0.01	0.02	0.63	0.60	18.00	0.20	4.63
Iran	-0.081	72.0	0.00	0.00	0.00	0.00	0.01	0.17	0.33	8.23	0.47	14.88
Iraq	0.033	34.0	0.69	19.29	0.08	2.79	0.03	0.99	0.01	0.21	0.00	0.00
Libya	-0.037	64.5	0.00	0.06	0.03	0.78	0.17	4.46	0.53	15.45	0.08	2.52
Malaysia	-0.026	82.0	0.00	0.00	0.00	0.00	0.00	0.00	0.10	4.14	0.71	19.14
Nigeria	0.041	50.0	0.34	8.35	0.38	11.53	0.09	3.28	0.00	0.12	0.00	0.00
Norway	0.036	90.0	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.68	0.79	22.60
Romania	-0.011	66.0	0.00	0.01	0.01	0.50	0.18	6.24	0.60	16.22	0.02	0.31
Syria	-0.036	67.0	0.00	0.00	0.00	0.02	0.06	1.25	0.69	19.73	0.06	2.28
United Kingdom	0.081	81.0	0.00	0.00	0.00	0.00	0.02	0.45	0.77	22.36	0.01	0.47
USA	0.044	85.0	0.00	0.00	0.00	0.00	0.00	0.01	0.35	12.16	0.46	11.10
Venezuela	0.038	65.0	0.06	2.10	0.18	5.38	0.29	8.39	0.26	7.12	0.01	0.29
Vietnam	-0.034	70.5	0.00	0.00	0.00	0.05	0.04	1.22	0.59	17.06	0.17	4.95

General assumptions: The six and one month Eurodollar interest rates at end of September 1996 were 0.0567 and 0.0531. I assume that the risk free interest rate is constant and equal to 0.05354 (ln(1.055)). The market premium is set to 0.03, and the Brent Blend oil price at September 1996 was 23.8. The beta for the oil is assumed to be -0.6, which with α_S=0.03 gives δ_S=0.00554. σ_S is set to 0.25. (1) Drift adjustment, calculated based on general assumptions, estimated betas for the countries, and the other process parameters reported in Table 4.11.C. (2) The September 1996 level of the ICRG composite risk index. (3) Estimated prices at the end of September 1996, time t, of a claims paying either one USD or a barrel (BL) of oil at year 2000, time T, conditioned on level of ψ_T.

Table 5.1 Estimated state prices dependent on the level of the ICRG composite risk index at September 2000, as of September 1996

For Nigeria, which had an index level of 50, the estimated state prices are highest if the future index level is between 50 and 60 at year 2000. For Norway, the highest state prices are for an index level above, or equal to 85. Comparing Norway and Denmark, we see that while the index level is 90 for Norway and 88.5 for Denmark in September 1996, the state prices for a future index value equal to, or higher than 85 is highest for Denmark. From Table 4.11.C we see that the coefficient of correlation between the deduced variable and the oil price is positive for both Denmark and Norway and that the volatility of the deduced variable is higher for Norway than for Denmark. The drift adjustments are positive for both countries, but higher for Norway. Table 5.1 clearly demonstrates that a ranking of countries based on current index levels does not necessarily carry over to some ranking based on state prices.

5.2 Valuation of Oil Investments

Consider an oil investment in a country, where the government will choose between two royalty rates which will leave the investor with a fraction γ_G or γ_B of the sales revenue. The corresponding royalty rates are $(1-\gamma_G)$ or $(1-\gamma_B)$, where $\gamma_i \in [0,1]$, $i \in \{G,B\}$ and $\gamma_B \leq \gamma_G$. Assume that the probability at time T of the government selecting the royalty rate $(1-\gamma_G)$ is p_T. The after-tax expected cash flow from the sale of one barrel of oil, standing at time T just before the royalty rate is announced, is equal to $Z_T^{(C3)}$, where

$$Z_T^{(C3)} = S_T(\gamma_B + p_T(\gamma_G - \gamma_B)) \ . \tag{57}$$

The probability is given by

$$p_T = \mathrm{N}(\ln(Z_T^{(\hat{y})})/\sigma_\epsilon) \tag{58}$$

where $\ln(Z_T^{(\hat{y})}) = \hat{y}_T$. I assume that $\hat{y}_t$ can be written as a linear transform of $\hat{x}_t$, as in equation (19). For this example I assume that the parameters of the transform are: $\beta_0 = 0$ and $\beta_1 = 1$, i.e.,

$$\hat{y}_t = \hat{x}_t + \sigma_\epsilon \ . \tag{59}$$

In this case $\alpha_{\hat{y}}$, $\sigma_{\hat{y}}$, and $\rho_{S,\hat{y}}$ are the same as the parameters for the process of $\hat{x}_t$. The value at time t of this asset is

$$Z_t^{(C3)} = S_t e^{-\delta_S(T-t)}(\gamma_B + \mathrm{N}(c_t)(\gamma_A - \gamma_G)) \ , \tag{60}$$

where

$$c_t = \frac{\ln(Z_t^{(\hat{y})}) + (r - \delta_{\hat{y}} + \rho_{S,\hat{y}}\sigma_{\hat{y}}\sigma_S - \frac{1}{2}\sigma_{\hat{y}}^2)(T-t)}{\sqrt{\sigma_{\hat{y}}^2(T-t) + \sigma_\epsilon^2}} \ .$$

Note that at time T, $\mathrm{N}(c_T) = p_T$, and (60) is equal to (57), as required. I have in Appendix 2 indicated how this formula is derived. Note in particular that when the numerator in c_t is positive, the value of the expected after-tax cash flow from the sale of oil is, ceteris paribus, reduced when the index value today is reduced, when the coefficient of correlation is changed from positive to negative, and when the standard deviation of the noise is increased. When σ_ϵ increases to a high level, c_t will be close to zero and there will be a fifty-fifty chance of a good or bad royalty rate, irrespective of index levels. A positive coefficient of correlation, $\rho_{S,\hat{y}}$, indicates that a situation with low royalty rate and high oil price is more likely to occur than a situation with a high royalty rate and high oil price, which would be the case if the coefficient of correlation was negative.

5.3 Political Risk Insurance

Assume that an investment is made in a country where there is a possibility that the government may expropriate the investment at a future date T. The probability that the investment will not be expropriated at time T is p_T. The expected payoff of an insurance contract paying one dollar at time T, just before the government announces whether to expropriate, is

$$Z_T^{(C4)} = 1(1-p_T) = (1-\mathrm{N}(\ln(Z_T^{(\hat{y})}/\sigma_\epsilon)) \ . \qquad \textbf{(61)}$$

The probability p_T is defined according to equations (58) and (59). The value of the claim at time t, i.e., the insurance premium for insurance covering the loss of one USD due to expropriation at time T, is

$$Z_t^{(C4)} = e^{-r(T-t)}(1-\mathrm{N}(c_t')) \ , \qquad \textbf{(62)}$$

where

$$c'_t = \frac{\ln(Z_t^{(\hat{y})}) + (r-\delta_{\hat{y}}-\frac{1}{2}\sigma_{\hat{y}}^2)(T-t)}{\sqrt{\sigma_{\hat{y}}^2(T-t)+\sigma_\epsilon^2}} \ .$$

Equation (62) is derived in Appendix 2.

Table 5.2 contains the implied level of noise, σ_ϵ, making a one year claim equal to the standard premium rates of MIGA. I use the ICRG composite risk index and the standard deviation of the noise is derived by solving equation (62) with respect to σ_ϵ. For countries where there is no solution, i.e., where the implied value of σ_ϵ^2 is negative, I have used the letters "NO" in the table. The value of the claim is calculated standing at the end of

Country	δ_ϕ [(1)]	ψ_t [(2)]	Natural Resources [(3)] Currency 0.5%	Expropr. 0.9%	War 0.55%	Oil & Gas [(3)] Currency 0.5%	Expropr. 1.25 %	War 0.7%
Algeria	0.057	57.0	NO	NO	NO	NO	NO	NO
Angola	0.036	51.0	NO	NO	NO	NO	NO	NO
Argentina	-0.058	73.0	0.253	0.281	0.257	0.253	0.299	0.268
Brazil	0.016	65.0	0.133	0.150	0.135	0.133	0.162	0.142
China	0.008	73.0	0.208	0.235	0.212	0.208	0.252	0.222
Colombia	0.029	63.0	0.104	0.119	0.106	0.104	0.130	0.112
Denmark	0.019	88.5	0.478	0.522	0.485	0.478	0.551	0.502
Egypt	0.002	67.5	0.179	0.198	0.182	0.179	0.211	0.190
Equador	-0.026	63.0	0.079	0.104	0.083	0.079	0.119	0.093
Gabon	0.042	66.0	0.151	0.167	0.154	0.151	0.178	0.160
India	-0.012	69.0	0.167	0.191	0.170	0.167	0.207	0.180
Indonesia	-0.032	72.0	0.242	0.266	0.245	0.242	0.283	0.255
Iran	-0.081	72.0	0.248	0.275	0.252	0.248	0.294	0.263
Iraq	0.033	34.0	NO	NO	NO	NO	NO	NO
Libya	-0.037	64.5	0.121	0.143	0.124	0.121	0.157	0.133
Malaysia	-0.026	82.0	0.378	0.414	0.383	0.378	0.438	0.397
Nigeria	0.041	50.0	NO	NO	NO	NO	NO	NO
Norway	0.036	90.0	0.499	0.545	0.506	0.499	0.576	0.524
Romania	-0.011	66.0	0.160	0.178	0.162	0.160	0.191	0.170
Syria	-0.036	67.0	0.185	0.205	0.188	0.185	0.219	0.196
United Kingdom	0.081	81.0	0.325	0.355	0.329	0.325	0.375	0.341
USA	0.044	85.0	0.399	0.436	0.404	0.399	0.461	0.419
Venezuela	0.038	65.0	0.048	0.081	0.054	0.048	0.099	0.068
Vietnam	-0.034	70.5	0.217	0.241	0.221	0.217	0.257	0.230

General assumptions: The six and one month Eurodollar interest rates at the end of September 1996 were 0.0567 and 0.0531. I assume that the risk free interest rate is constant and equal to 0.05354 (ln(1.055)). The market premium is set to 0.03. The processes for the deduced variables, $\hat{y}$, are assumed to be as reported in Table 4.11.C. "NO" indicates that there is no rational solution. (1) Drift adjustment, calculated based on general assumptions and estimates reported in Tables 4.11.C. (2) The September 1996 level of the ICRG composite risk index. (3) The MIGA standard premium rates for investments in the natural resource sector for insurance against currency losses, expropriation, and, losses due to war or civil disturbance, are 0.5%, 0.9%, and, 0.55% of the insured amount, respectively. The reported figures are the level of σ_ϵ which makes the calculated price at the end of September 1996, time t, of a claims paying one USD at September 1997, time T, where the probability of expropriation at time T is set to $N(\hat{y}_T/\sigma_\epsilon)$.

Table 5.2 Level of noise, σ_ϵ, which makes the price of claim paying a fixed USD amount in case of expropriation equal to the standard MIGA premium rates

September 1996, and the claim is maturing at the end of September 1997. Note that when the numerical value of σ_ϵ^2 is reduced, $N(c_t^{\prime})$ is increased, provided that the numerator of $c_t^{\prime}$ is positive, and the value of the insurance premium is increased. See Figure 2.1 in section two for the relationship between the index value and the probability of no expropriation when σ_ϵ is less than one. A reason for purchasing an insurance contract may that the buyer considers the insurance premium to be lower than the theoretical market value of the contract. This corresponds to a situation where the investor perceives σ_ϵ as being higher than those reported in Table 5.2. The insurance provider will be willing to sell insurance contracts as long as the perceived standard deviation of the noise is not higher than the level reported in Table 5.2.

6 Summary

In this paper I suggest a method for using risk indices when modeling political uncertainty. The approach is easy to combine with established results from the theory of arbitrage free pricing. I deduce time series for state variables governing the countries' risk indices. For many countries, based on the empirical research presented in the paper, I am not able to reject the hypothesis that these state variables develop according to arithmetic Brownian motions. This approach enables us to find state prices in terms of levels of the risk indices and possibly other state variables, e.g., the spot price of oil. For the majority of the countries we could not reject the hypothesis that the risk measured by the risk indices represents unsystematic risk. The estimated betas for the deduced variables when using the Morgan Stanley Capital World Index as the proxy for the world market portfolio, was not significantly different from zero. State prices in terms of the level of the index can, e.g., be used in capital budgeting when valuing investments for which the cash flow of the investment is contingent upon the level of the index. I present examples involving expropriation and taxation. I have also shown that when the relationship between the probability of an event, e.g., expropriation, and the index can be modeled in a specific way, closed-form valuation formulas may be derived.

Appendix 1 Arbitrage Free Valuation

This appendix serves as a background for section three and for the derivation of pricing formulas presented in section five and Appendix 2. I present the main results for pricing of securities and contingent claims when the pricing is based on the argument of absence of arbitrage. I have used chapter five and six of Duffie (1992) extensively and to some degree chapter zero of Karatzas (1997).

A Model of a Market for Traded Securities

I present a model of a market, M, for traded assets or securities. I take as fixed a Brownian motion $B=(B^{(1)},B^{(2)})$ in $\mathbb{R}^2$, restricted to a time interval $[0,T]$. $B^{(1)}$ and $B^{(2)}$ are independent. B is defined on a complete filtered probability space $(\Omega,\mathcal{F},\mathrm{P},\mathbf{F})$. The filtration is $\mathbf{F}=\{\mathcal{F}_t, 0\leq t\leq T\}$, the sigma algebra generated by B, satisfying the "usual conditions". An *adapted* process X is a function X: $\Omega \times [0,T] \to \mathbb{R}$ such that $X(t)\in\mathcal{F}_t \ \forall \ t\in[0,T]$.

Assume there exists three[1] traded assets. Of these, two are given by the pairs of ex-dividend price processes and cumulate dividend processes[2], $Z=((Z^{(\hat{y})},D^{(\hat{y})}),(Z^{(\pi)},D^{(\pi)}))$. The risk free asset is β. The tradeable securities are thus $A=(\beta,Z)$. The evolutionary equations for the assets are

$$dZ_t^{(\hat{y})} = \alpha_{\hat{y}}(Z_t^{(\hat{y})},t)dt + \sigma_{1\hat{y}}(Z_t^{(\hat{y})},t)dB_t^{(1)} + \sigma_{2\hat{y}}(Z_t^{(\hat{y})},t)dB_t^{(2)} , \tag{1}$$

$$dD_t^{(\hat{y})} = \delta_{\hat{y}}(Z_t^{(\hat{y})},t)dt , \tag{2}$$

---

[1] The number of assets and the sources of risk , B, are chosen so as to get a complete market.

[2] The top script $\hat{y}$ indicates that this asset may be interpeted as a function of the variable $\hat{y}$ deduced from a risk index, while asset π represents the profit, or sales revenue from one unit of production from an investment, e.g., the spot oil price.

$$dZ_t^{(\pi)} = \alpha_\pi(Z_t^{(\pi)},t)dt + \sigma_{1\pi}(Z_t^{(\pi)},t)dB_t^{(1)} + \sigma_{2\pi}(Z_t^{(\pi)},t)dB_t^{(2)} \ , \qquad (3)$$

$$dD_t^{(\pi)} = \delta_\pi(Z_t^{(\pi)},t)dt \ , \qquad (4)$$

and

$$d\beta_t = r_t \beta_t dt \ . \qquad (5)$$

In order for the evolutionary equations to be well specified, we need that the coefficients $\alpha_j(\cdot,\cdot)$, and $\delta_j(\cdot,\cdot)$ belong to the class L^1, and $\sigma_{ij}(\cdot,\cdot)$ belong to class L^2, where $i=\{1,2\}$, $j=\{\pi,\hat{y}\}$ and where

$$L^1 = \{\text{adapted and } \int_0^t |x_s| ds < \infty \text{ a.s. for every } t\} \ .$$

$$L^2 = \{\text{adapted and } \int_0^t x_s^2 ds < \infty \text{ a.s. for every } t\} \ .$$

I have, as implied by the notation, assumed that the coefficients are deterministic functions.

An investor buying an asset j at time zero and holding it until time t, will have a total gain, equal to $G_t^{(j)}$, where

$$G_t^{(j)} = \int_0^t dZ_s^{(j)} + \int_0^t dD_s^{(j)} \ , \qquad (6)$$

The total gain consists of capital appreciation and accumulated dividend payments.

A *dynamic trading strategy* θ is an adapted process. The process $\theta = (\theta^{(\beta)},\theta^{(\hat{y})},\theta^{(\pi)})$ specifies for every t and ω the number of units of the securities A to hold. An *admissible* trading strategy is a dynamic trading strategy θ in $\mathcal{H}^2$, where

$$\mathcal{H}^2 = \{x \in L^2 \; : \; E(\int_0^T x_t^2 dt) < \infty\} \; .$$

Let the value of a portfolio at time t be V_t, where

$$V_t = \theta_t^{(\beta)} \beta_t + \theta_t^{(\mathcal{Y})} Z_t^{(\mathcal{Y})} + \theta_t^{(\pi)} Z_t^{(\pi)} \; . \tag{7}$$

The differential of the portfolio V_t is

$$dV_t = \theta_t^{(\beta)} r_t \beta_t dt + \theta_t^{(\mathcal{Y})} dG_t^{(\mathcal{Y})} + \theta_t^{(\pi)} dG_t^{(\pi)} \; . \tag{8}$$

A *self financing trading strategy* is an admissible trading strategy θ which makes (9) hold.

$$\begin{aligned} \theta_t^{(\beta)} \beta_t + \theta_t^{(\mathcal{Y})} Z_t^{(\mathcal{Y})} + \theta_t^{(\pi)} Z_t^{(\pi)} = \theta_0^{(\beta)} \beta_0 + \theta_0^{(\mathcal{Y})} Z_0^{(\mathcal{Y})} + \theta_0^{(\pi)} Z_0^{(\pi)} + \\ \int_0^t \theta_s^{(\beta)} r_s \beta_s ds + \int_0^t \theta_s^{(\mathcal{Y})} dG_s^{(\mathcal{Y})} + \int_0^t \theta_s^{(\pi)} dG_s^{(\pi)} \end{aligned} \tag{9}$$

By using the self financing trading strategy θ, the value of the holdings of securities at time t is equal to the purchase price of the securities at time zero plus the gains from holding the securities and using strategy θ during the period. Note that this implies that the dividend is reinvested in the portfolio.

An *arbitrage* is a self financing strategy θ where either $\theta_0 A_0 \leq 0$ and $\theta_T A_T > 0$, or $\theta_0 A_0 < 0$ and $\theta_T A_T \geq 0$. The first case represents the situation, where by following the strategy θ, the value today of the portfolio is equal or less than zero, but where the payoff is strictly positive at time T. In the second case, the value of the portfolio today is strictly less than zero but the value at the future date is equal to or larger than zero.

A *deflator* is a strictly positive Ito process. A *regular deflator X* is a deflator for which the admissible strategies for the deflated price process Z, $\mathcal{H}^2(XZ)$, belongs to the same space of admissible strategies for the undeflated price process, $\mathcal{H}^2(Z)$. Let the regular deflator be

$\eta_t = 1/\beta_t$, where $\beta_t = \exp(\int_0^t r_s ds)$. It is assumed that r_t is bounded. I normalize by letting $\eta_0\beta_0 \equiv 1$. When asset $Z^{(j)}$ is deflated by this deflator, it means that ${}^{(\eta)}Z_t^{(j)} = \eta_t Z_t^{(j)} = Z_t^{(j)}/\beta_t$. The deflated tradeable assets are ${}^{(\eta)}A = (1, {}^{(\eta)}Z)$. The deflated value of the portfolio at time t is ${}^{(\eta)}V_t$, where

$$ {}^{(\eta)}V_t = \theta_t^{(\beta)} 1 + \theta_t^{(j)}\, {}^{(\eta)}Z_t^{(j)} + \theta_t^{(\pi)}\, {}^{(\eta)}Z_t^{(\pi)} \; . \tag{10} $$

The differential of this portfolio is

$$ d\,{}^{(\eta)}V_t = \theta_t^{(j)} d\,{}^{(\eta)}G_t^{(j)} + \theta_t^{(\pi)} d\,{}^{(\eta)}G_t^{(\pi)} \; . \tag{11} $$

This means that the trading strategy is self financing for the deflated portfolio if

$$ {}^{(\eta)}V_t = {}^{(\eta)}V_0 + \int_0^t \theta_s^{(j)} d\,{}^{(\eta)}G_s^{(j)} + \int_0^t \theta_s^{(\pi)} d\,{}^{(\eta)}G_s^{(\pi)} \; . \tag{12} $$

Note that the differential of a deflated gains process for an asset j is, by Ito's lemma,

$$ d\,{}^{(\eta)}G_t^{(j)} = \frac{1}{\beta_t}(\alpha_j(Z_t^{(j)},t) + \delta_j(Z_t^{(j)},t) - Z_t^{(j)} r_t)dt + \frac{\sigma_{1j}(Z_t^{(j)},t)}{\beta_t} dB_t^{(1)} + \frac{\sigma_{2j}(Z_t^{(j)},t)}{\beta_t} dB_t^{(2)} \; , \tag{13} $$

where the risk free interest rate is deducted in the dt-term.

Numeraire Invariance Theorem.

Suppose Y is a regular deflator. Then a trading strategy θ is self-financing with respect to X if and only if θ is self financing with respect to ${}^{Y}X$.

A proof is found in Duffie (1992) page 97.

Theorem.
If the gains process ${}^{(\eta)}G^{(j)}$ admits an equivalent martingale measure, then there is no arbitrage.

The proof is found in Duffie (1992), page 101. Besides technical conditions, the proof uses the self financing condition and that ${}^{(\eta)}G^{(j)}$ is a martingale under an equivalent martingale measure (EMM) Q. According to the self financing condition

$$E_t^Q({}^{(\eta)}V_T) = {}^{(\eta)}V_0 + E_t^Q\left(\int_0^T \theta_s^{(\mathcal{Y})} d\,{}^{(\eta)}G_s^{(\mathcal{Y})} + \int_0^T \theta_s^{(\pi)} d\,{}^{(\eta)}G_s^{(\pi)}\right) , \tag{14}$$

or

$${}^{(\eta)}V_0 = E_t^Q({}^{(\eta)}V_T) \tag{15}$$

because ${}^{(\eta)}G$ is a martingale under the Q measure. It therefore follows that the value of an asset at time t paying $Z_T^{(\mathcal{Y})}$ at time T, can be found by using the strategy $\theta = (\hat{\theta}^{(\beta)}=0, \theta^{(\mathcal{Y})}=1, \theta^{(\pi)}=0)$. Inserting this strategy in (14) gives

$$Z_t^{(\mathcal{Y})} = E_t^Q\left[e^{\int_t^T -r_u du} dZ_T^{(\mathcal{Y})} + \int_t^T e^{\int_t^s -r_u du} dD_s^{(\mathcal{Y})}\right] , \tag{16}$$

which states that the value of asset $Z_t^{(\mathcal{Y})}$ is equal to the value of the asset at the future date T and the value of accumulated dividends, discounted by the risk free interest rate.

If an equivialent martingale measure Q can be found, then this is a sufficient condition for using (16) as a valuation equation when the market is arbitrage free. For the existence of an EMM, I use Girsanov's theorem. First I state Novikov's condition. A process $\theta=(\theta^1,\theta^2,\dots,\theta^d)$ in L^2 a.s. satisfies *Novikov's condition* if

$$E\left(\exp\left(\frac{1}{2}\int_0^T \theta_s \theta_s ds\right)\right) < \infty . \tag{17}$$

Girsanov's Theorem.

Let X be an Ito process in $\mathbb{R}^N$ *of the form*

$$X_t = x + \int_0^t \mu_s ds + \int_0^t \sigma_s dB_s,\ 0 \le t \le T\ . \tag{18}$$

Suppose $\nu = (\nu^1, .., \nu^N)$ *is a vector of processes in* L^1 *such that there exists some* θ *satisfying Novikov's condition with*

$$\sigma_t \theta_t = \mu_t - \nu_t,\ 0 \le t \le T\ . \tag{19}$$

Then there exists a probability measure Q equivalent to P such that

$$\hat{B}_t = B_t + \int_0^t \theta_s ds,\ 0 \le t \le T\ . \tag{20}$$

defines a standard Brownian motion $\hat{B}$ *in* $\mathbb{R}^d$ *on* $(\Omega, \mathscr{F}, Q)$ *adapted to the same standard filtration F. The process X defined by (11) is also an Ito process with respect to* $(\Omega, \mathscr{F}, Q, F)$*, and*

$$X_t = x + \int_0^t \nu_s ds + \int_0^t \sigma_s d\hat{B}_s\ ,\ 0 \le t \le T\ . \tag{21}$$

For any random variable W such that $E^Q(|W|) < \infty$*,*

$$E^Q(W) = E^P(W\xi_T)\ , \tag{22}$$

where

$$\xi_t = \exp(-\int_0^t \theta_s dB_s - \frac{1}{2}\int_0^t \theta_s \theta_s ds)\ ,\ t \in [0,T]\ . \tag{23}$$

For the market model $\boldsymbol{M}$, equation (19) is equal to

$$\begin{pmatrix} \dfrac{\sigma_{1\mathcal{Y}}(Z_t^{(\mathcal{Y})},t)}{\beta_t} & \dfrac{\sigma_{2\mathcal{Y}}(Z_t^{(\mathcal{Y})},t)}{\beta_t} \\ \dfrac{\sigma_{1\pi}(Z_t^{(\pi)},t)}{\beta_t} & \dfrac{\sigma_{2\pi}(Z_t^{(\pi)},t)}{\beta_t} \end{pmatrix} \begin{pmatrix} \theta_t^{(\mathcal{Y})} \\ \theta_t^{(\pi)} \end{pmatrix} = \begin{pmatrix} \dfrac{\alpha_{\mathcal{Y}}(Z_t^{(\mathcal{Y})},t)}{\beta_t} + \dfrac{\delta_{\mathcal{Y}}(Z_t^{(\mathcal{Y})},t)}{\beta_t} - \dfrac{Z_t^{(\mathcal{Y})} r_t}{\beta_t} \\ \dfrac{\alpha_{\pi}(Z_t^{(\pi)},t)}{\beta_t} + \dfrac{\delta_{\pi}(Z_t^{(\pi)},t)}{\beta_t} - \dfrac{Z_t^{(\pi)} r_t}{\beta_t} \end{pmatrix} - \begin{pmatrix} \nu_t^1 \\ \nu_t^2 \end{pmatrix} . \qquad \textbf{(24)}$$

Suppose I want to find the value of a new asset, $Z_t^{(C)}$, with a payoff at time T which can be written as a function of the traded asset in $\boldsymbol{M}$. Let the payoff at time T for this new asset be $Z_T^{(C)} = C(\beta_T, Z_T^{(\mathcal{Y})}, Z_T^{(\pi)})$, i.e., a function of the value of the traded assets at time T. If this payoff can be replicated by a trading strategy θ, meaning that $Z_T^{(C)}$ is *redundant*, then

$$Z_T^{(C)} = \theta_T^{(\beta)} \beta_T + \theta_T^{(\mathcal{Y})} Z_T^{(\mathcal{Y})} + \theta_T^{(\pi)} Z_T^{(\pi)} , \qquad \textbf{(25)}$$

or with the deflated assets,

$$Z_T^{(C)} = \beta_T {}^{(\eta)}Z_T^{(C)} = \beta_T(\theta_T + \theta_T^{(\mathcal{Y})} {}^{(\eta)}Z_T^{(\mathcal{Y})} + \theta_T^{(\pi)} {}^{(\eta)}Z_T^{(\pi)}) , \qquad \textbf{(26)}$$

From (15), the value at time t, $t \leq T$, of this contingent claim is

$$Z_t^{(C)} = \mathrm{E}_t^{\mathrm{Q}}[{}^{(\eta)}Z_T^{(C)}] = \mathrm{E}_t^{\mathrm{Q}}[e^{-\int_t^T r_u du} Z_T^{(C)}] . \qquad \textbf{(27)}$$

If r is a constant, equation (27) can be simply be written as

$$Z_t^{(C)} = e^{-r(T-t)} \mathrm{E}_t^{\mathrm{Q}}[Z_T^{(C)}] , \qquad \textbf{(28)}$$

i.e., the value of the contingent claim at time t is equal to the expected value at time T under the Q measure discounted by the risk free interest rate. This is the general pricing principle which I will apply in section five and Appendix 2.

Appendix 2 Deriving Pricing Formulas

In this appendix I show first how the value of the contingent claim $Z_t^{(C2)}$ is derived. The value of the contingent claim $Z_t^{(C1)}$ may be derived by applying the same principles. I then derive the value of the contingent claim $Z_t^{(C4)}$.

A2.1 Deriving the Valuation Formula for $Z_t^{(C2)}$

Standing at time t, $Z_T^{(\hat{y})}$, S_T, and the Radon Nikodym derivative ξ_T are random variables given by[1]

$$Z_T^{(\hat{y})} = Z_t^{(\hat{y})}\exp((\alpha_{\hat{y}} - \frac{1}{2}\sigma_{\hat{y}}^2)(T-t) + \sigma_{\hat{y}}(B_T^{(1)} - B_t^{(1)}))\ , \tag{1}$$

$$S_T = S_t\exp((\alpha_S - \frac{1}{2}\sigma_S^2)(T-t) + \rho\,\sigma_{\hat{y}}(B_T^{(1)} - B_t^{(1)}) + \sigma_S\sqrt{1-\rho^2}(B_T^{(2)} - B_t^{(2)}))\ , \tag{2}$$

and

$$\xi_T = \exp\left(-\left(\frac{\alpha_{\hat{y}}+\delta_{\hat{y}}-r}{\sigma_{\hat{y}}}\right)(B_T^{(1)} - B_t^{(1)}) - \frac{1}{2}\left(\frac{\alpha_S+\delta_S-r}{\sigma_S\sqrt{1-\rho^2}} - \frac{\alpha_{\hat{y}}+\delta_{\hat{y}}-r}{\sigma_{\hat{y}}}\frac{\rho}{\sqrt{1-\rho^2}}\right)(B_T^{(2)} - B_t^{(2)}) \right. \tag{3}$$

$$\left. -\frac{1}{2}\left(\frac{\alpha_{\hat{y}}+\delta_{\hat{y}}-r}{\sigma_{\hat{y}}}\right)^2 (T-t) - \frac{1}{2}\left(\frac{\alpha_S+\delta_S-r}{\sigma_S\sqrt{1-\rho^2}} - \frac{\alpha_{\hat{y}}+\delta_{\hat{y}}-r}{\sigma_{\hat{y}}}\frac{\rho}{\sqrt{1-\rho^2}}\right)^2 (T-t)\right)\ .$$

Define $x = B_T^{(1)} - B_t^{(1)}$ and $u = B_T^{(2)} - B_t^{(2)}$, then x and u are independent random variables, normally distributed with zero mean, and both have a variance of $(T-t)$. I want to find the value at time t of a contingent claim, $Z_t^{(C2)}$, paying S_T at time T if $S_T \geq K$ <u>and</u> $Z_T^{(\hat{y})} \geq A$ where

[1] I have dropped the subscript for the coefficient of correlation ρ in order to simplify the notation.

A and K are constants.

According to the general pricing formula, equation (28) in Appendix 1, the price of the contingent claim at time t is equal to the expected payoff at time T under the probability measure Q, discounted by the risk free interest rate, i.e.,

$$Z_t^{(C2)} = e^{-r(T-t)} E_t^{Q}[Z_T^{(C2)}] = e^{-r(T-t)} E_t[Z_T^{(C2)} \xi_T] \ . \qquad \textbf{(4)}$$

Note that $Z_T^{(\hat{y})}$ is equal to, or larger than, A if

$$x \geq \frac{1}{\sigma_{\hat{y}}} [\ln(A/Z_t^{\hat{y}}) - (\alpha_{\hat{y}} - \frac{1}{2}\sigma_{\hat{y}}^2)(T-t)] = \bar{x} \ . \qquad \textbf{(5)}$$

In order for both $S_T \geq K$ <u>and</u> $Z_T^{(\hat{y})} \geq A$ we must have that

$$u \geq \frac{1}{\sigma_S \sqrt{1-\rho^2}} (\ln(K/S_t) - (\alpha_S - \frac{1}{2}\sigma_S^2)(T-t)) - \frac{\rho}{\sqrt{1-\rho^2}} \bar{x} = \bar{u} \ . \qquad \textbf{(6)}$$

Because x and u are independent and normally distributed, (4) is equal to

$$Z_t^{(C2)} = e^{-r(T-t)} \int_{\bar{x}}^{\infty}\int_{\bar{u}}^{\infty} \frac{1}{2\pi(T-t)} e^{-\frac{x^2}{2(T-t)} - \frac{u^2}{2(T-t)}} S_T \xi_T \, du\, dx \ . \qquad \textbf{(7)}$$

By inserting for S_T and ξ_T in (7), moving exp(-$r(T$-$t)$) under the integral, multiplying and dividing by

$$\exp(\rho \frac{\sigma_S}{\sigma_{\hat{y}}} (\alpha_{\hat{y}} + \delta_{\hat{y}} - r)(T-t) - \frac{1}{2}\rho^2 \sigma_S^2 (T-t))$$

and arranging terms, we get that (7) is equal to

$$Z_t^{(C2)} = S_t e^{-\delta_S (T-t)} \int_{\bar{x}}^{\infty}\int_{\bar{u}}^{\infty} \frac{1}{2\pi(T-t)} e^{-\frac{(x-\mu_x)^2}{2(T-t)} - \frac{(u-\mu_u)^2}{2(T-t)}} \, du\, dx \ , \qquad \textbf{(8)}$$

where

$$\mu_x = -\frac{(\alpha_{\hat{y}}+\delta_{\hat{y}}-\rho\,\sigma_{\hat{y}}\sigma_S-r)(T-t)}{\sigma_{\hat{y}}} \tag{9}$$

and

$$\mu_u = -\left(\frac{\alpha_S+\delta_S-r-\sigma_S^2}{\sigma_S\sqrt{1-\rho^2}} - \frac{\alpha_{\hat{y}}+\delta_{\hat{y}}-\rho\,\sigma_{\hat{y}}\sigma_S-r}{\sigma_{\hat{y}}}\frac{\rho}{\sqrt{1-\rho^2}}\right)(T-t)\ . \tag{10}$$

By defining new variables $v=\dfrac{x-\mu_x}{\sqrt{(T-t)}}$ and $g=\dfrac{u-\mu_u}{\sqrt{(T-t)}}$, inserting these in (5) and (6), we get

the expressions for $\bar{v}$ and $\bar{g}$. Due to symmetry of the normal distribution, $(1-\mathrm{N}(\bar{v},\bar{g}:c{=}0)) = \mathrm{N}(-\bar{v},-\bar{g}:c{=}0)$ where N(·,·;c) is the bivariate normal distribution with coefficient of correlation c. We can now write (8) as

$$Z_t^{(C2)} = S_t e^{-\delta_S(T-t)}\mathrm{N}(-\bar{v},-\bar{g};c{=}0)\ , \tag{11}$$

where

$$-\bar{v} = \frac{\ln(Z_t^{(\hat{y})}/A)+(r-\delta_{\hat{y}}+\rho\,\sigma_{\hat{y}}\sigma_S-\frac{1}{2}\sigma_{\hat{y}}^2)(T-t)}{\sigma_y\sqrt{(T-t)}}\ , \tag{12}$$

and,

$$-\bar{g} = \frac{1}{\sqrt{1-\rho^2}}\frac{\ln(S_t/K)+(r-\delta_S+\frac{1}{2}\sigma_S^2)(T-t)}{\sigma_S\sqrt{(T-t)}} - \frac{\rho}{\sqrt{1-\rho^2}}\bar{v}\ . \tag{13}$$

Define a new variable $h=\rho v+\sqrt{1-\rho^2}g$, then h and v are normally distributed with coefficient of correlation ρ, and

$$g = \frac{1}{\sqrt{1-\rho^2}} h - \frac{\rho}{\sqrt{1-\rho^2}} v.$$

By using this, we get

$$Z_t^{(C2)} = S_t e^{-\delta_S(T-t)} \mathrm{N}(-\bar{v}, -\bar{h}; c = \rho) \ , \tag{14}$$

where

$$-\bar{h} = \frac{\ln(S_t/K) + (r - \delta_S + \frac{1}{2}\sigma_{\hat{S}}^2)(T-t)}{\sigma_S \sqrt{(T-t)}} \ . \tag{15}$$

■

A2.2 Deriving the Valuation Formula for $Z_t^{(C4)}$

The payoff at time T is given by

$$(1 - p_T) = (1 - \mathrm{N}(\ln(Z_T^{(\hat{y})})/\sigma_\epsilon)) \ . \tag{16}$$

I want the argument in the cumulative distribution function to be equal to the argument in the valuation formula for $Z_T^{(C1)}$ paying one USD, where $\bar{S}$ is zero, maturing at time $T^* > T$, see equation (54) in section five. In order to obtain equality the equation

$$\frac{\ln(Z_T^{(\hat{y})}) - \ln(K) + (r - \delta_{\hat{y}} - \frac{1}{2}\sigma_{\hat{y}}^2)(T^* - T)}{\sigma_{\hat{y}} \sqrt{(T^* - T)}} = \frac{\ln(Z_T^{(\hat{y})})}{\sigma_\epsilon} \tag{17}$$

must be satisfied. By letting

$$\ln(K) = (r - \delta_{\hat{y}} - \frac{1}{2}\sigma_{\hat{y}}^2)(T^* - T) \tag{18}$$

and

$$\sigma_{\hat{y}}\sqrt{(T^{*}-T)}=\sigma_{\epsilon}\ , \tag{19}$$

(17) will be obtained. From (19) we get that $(T^{*}-T)=\sigma_{\epsilon}^{2}/\sigma_{\hat{y}}^{2}$. By inserting (18) and (19) in the formula for the value at time t, maturing at time T^{*}, rearranging terms, and noting that $(T^{*}-t)=((T^{*}-T)+(T-t))$, we get the valuation formula at time t for the contingent claim, i.e.,

$$Z_t^{(C4)}=e^{-r(T-t)}(1-\mathrm{N}(a_t))\ , \tag{20}$$

where

$$a_t=\frac{\ln(Z_t^{(\hat{y})})+(r-\delta_{\hat{y}}-\frac{1}{2}\sigma_{\hat{y}}^{2})(T-t)}{\sqrt{\sigma_{\hat{y}}^{2}(T-t)+\sigma_{\epsilon}^{2}}}\ .$$

■

Appendix 3 List of Symbols

Symbols Related to Indices

ψ_t	Index level at time t
ψ^{MAX}	Maximum level of index
ψ^{MIN}	Minimum level of index
y_t^*	A government/ central planner's net benefit from selecting regime G at time t
y_t	Indicator variable equaling one if regime G is chosen at time t
$\hat{y}_t$	An estimate of y_t^*
ϵ_t	Noise in the estimate $\hat{y}_t$
σ_ϵ	Standard deviation of the noise ϵ_t
p_t	Probability that regime G will be chosen
$\mu_{\hat{y}}$	Drift parameter in stochastic process for $\hat{y}_t$
$\sigma_{\hat{y}}$	Volatility parameter in stochastic process for $\hat{y}_t$
x_t^*	An analyst's "willingness to categorize a government as a no risk government
x_t	Indicator variable equaling one if the government is a no risk government
$\hat{x}_t$	An estimate of x_t, serves as a state variable
ν_t	Noise in the estimate $\hat{x}_t$
σ_ν	Standard deviation of the noise ν_t
$\mu_{\hat{x}}$	Drift parameter in stochastic process for $\hat{x}_t$
$\sigma_{\hat{x}}$	Volatility parameter in stochastic process for $\hat{x}_t$

Symbols Related to Valuation

r	Instantaneous risk free interest rate
$Z_t^{(\hat{x})}$	Price of hypothetical asset which is a function of the state variable $\hat{x}_t$
$\alpha_{\hat{x}}$	Drift parameter in stochastic process for $Z_t^{(\hat{x})}$
$\delta_{\hat{x}}$	Drift adjustment for $Z_t^{(\hat{x})}$
S_t	Oil price at time t
$\alpha_{\hat{S}}$	Drift parameter in stochastic process for S_t

σ_S	Volatility parameter in stochastic process for S_t
δ_S	Drift adjustment for S_t, i.e., rate of return shortfall
$\rho_{S,\mathcal{X}}$	Coefficient of correlation between $Z_t^{(\mathcal{X})}$ and S_t
$Z_t^{(C)}$	Value of a contingent claim at time t

Appendix 4 Statistical Tables

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(In all tables I have used a standard deviation of the "noise" equal to one when deducing the unobserved variables.)

Explanation to Tables:

(1) * and ** indicates whether the estimate is significantly different from zero, using a two sided test and a significance level of 0.05 and 0.01, respectively. (2) The p-value of the Bera-Jarque test of normality, based on the statistic $J = n[(\text{coeff. of skewness})^2/6 + (\text{excess kurtosis})^2/24]$. In case of normality, J is χ^2-distributed with two degrees of freedom. The reported p-value is the probability of observing a J statistic equal to or lower than the sample statistic J. (3) h* and h** indicates that in a normal distribution with n observations, the probability of the observed studentized range being this high is less than 0.05 and 0.01, respectively. Similarly, l* and l** means that in a normal distribution with n observations, the probability of the observed studentized range being this low is less than 0.05 and 0.01. (4) Coefficient of correlation between observations, where one observation is lagged one or two periods.

Country	Mean	t-value(1)	Variance	Coeff. of Skewness	Excess Kurtosis	B-J(2), p-value	Studentized range(3)	ρ_1(1),(4)	ρ_2(1),(4)
Algeria	-0.0017	-0.65	0.0010	-0.958	4.052	0.00 **	7.24 h**	0.007	0.077
Angola	0.0019	0.64	0.0012	2.042	16.708	0.00 **	10.53 h**	0.051	-0.066
Argentina	0.0044	1.72	0.0010	0.934	3.043	0.00 **	7.53 h**	-0.065	-0.038
Australia	-0.0003	-0.12	0.0010	1.676	8.296	0.00 **	8.07 h**	-0.040	0.113
Azerbaijan	NA								
Brazil	0.0015	0.64	0.0009	-0.479	1.267	0.00 **	5.43	0.102	0.139
Brunei	0.0016	1.36	0.0002	3.589	18.74	0.00 **	8.17 h**	-0.014	0.429 **
Cameroon	-0.0002	-0.10	0.0004	-1.767	12.777	0.00 **	10.12 h**	0.068	0.019
Canada	-0.0013	-0.58	0.0008	-0.52	5.254	0.00 **	7.71 h**	-0.057	0.054
China	-0.0002	-0.08	0.0009	0.946	6.641	0.00 **	7.98 h**	0.076	0.171 *
Colombia	-0.0012	-0.55	0.0007	0.367	13.578	0.00 **	11.23 h**	-0.105	-0.014
Congo	0.0002	0.09	0.0005	-3.971	43.507	0.00 **	12.86 h**	0.017	-0.034
Denmark	-0.0023	-0.92	0.0010	0.214	4.671	0.00 **	7.45 h**	-0.073	-0.031
Egypt	0.0002	0.09	0.0005	0.595	8.277	0.00 **	9.93 h**	0.177 *	-0.136
Equador	0.0013	0.50	0.0011	-0.145	3.758	0.00 **	7.05 h**	0.128	0.174 *
Gabon	0.0002	0.10	0.0005	3.507	27.419	0.00 **	10.68 h**	0.013	0.068
India	0.0007	0.20	0.0019	1.147	9.03	0.00 **	9.67 h**	-0.012	0.075
Indonesia	0.0036	1.94	0.0005	1.236	7.436	0.00 **	9.27 h**	0.102	0.165 *
Iran	0.0077	2.67 **	0.0013	1.795	7.856	0.00 **	8.18 h**	0.137	-0.024
Iraq	0.0036	0.88	0.0026	0.504	10.539	0.00 **	10.29 h**	0.197 *	0.042
Kazakhstan	NA								
Kuwait	0.0045	0.75	0.0055	-1.51	32.449	0.00 **	13.23 h**	0.239 **	0.133
Libya	0.0048	1.42	0.0017	4.249	36.553	0.00 **	11.86 h**	-0.049	0.014
Malaysia	0.0002	0.11	0.0006	0.221	3.215	0.00 **	7.22 h**	0.134	0.135
Mexico	-0.0002	-0.06	0.0016	-0.233	19.715	0.00 **	12.55 h**	-0.222 **	0.056
Nigeria	0.0010	0.55	0.0005	-0.353	3.697	0.00 **	7.20 h**	0.058	0.101
Norway	-0.0028	-0.82	0.0018	0.395	13.829	0.00 **	11.07 h**	0.083	-0.236 **
Oman	0.0029	1.60	0.0005	2.676	15.356	0.00 **	9.10 h**	0.048	0.077
Papua New Guinea	0.0005	0.28	0.0005	-0.46	19.12	0.00 **	11.15 h**	0.006	0.019
Peru	0.0025	0.75	0.0017	-1.385	12.239	0.00 **	9.57 h**	-0.073	-0.014
Quatar	0.0029	1.05	0.0011	3.387	19.583	0.00 **	9.16 h**	-0.007	-0.034
Romania	0.0022	1.09	0.0006	1.921	8.602	0.00 **	8.49 h**	0.052	0.226 **
Russian Federation	0.0029	0.62	0.0011	-1.423	9.438	0.00 **	7.48 h**	-0.030	-0.078
Saudi Arabia	0.0022	0.63	0.0018	1.717	9.476	0.00 **	8.64 h**	0.070	-0.110
Syria	0.0071	2.56 *	0.0012	2.531	12.224	0.00 **	8.49 h**	0.068	-0.097
Trinidad & Tobago	0.0016	0.79	0.0006	-0.302	11.553	0.00 **	9.81 h**	-0.190 *	0.130
Tunisia	0.0041	1.67	0.0009	3.356	22.871	0.00 **	10.24 h**	0.082	-0.047
United Arab Emirates	0.0046	1.62	0.0012	6.434	57.905	0.00 **	12.06 h**	0.037	-0.045
United Kingdom	-0.0024	-0.76	0.0015	0.211	2.158	0.00 **	6.69 h**	0.038	0.049
USA	-0.0045	-1.61	0.0012	-0.777	6.376	0.00 **	8.61 h**	0.034	0.056
Uzbekistan	NA								
Venezuela	0.0015	0.50	0.0014	-2.286	24.92	0.00 **	11.92 h**	0.103	0.090
Vietnam	0.0053	2.25 *	0.0007	4.834	40.271	0.00 **	11.54 h**	-0.082	0.130
Yemen	0.0075	1.07	0.0022	0.045	8.088	0.00 **	6.98 h**	-0.021	0.044

The number of observations are 152 for all countries, except for the Russian Federation (53), Yemen (45), Qatar (145), Romania (145), Oman (146), Papua New Guinea (148), Angola (131), Brunei (131), Vietnam (131), China (141), and Congo (137).

Table 1.A Statistics for sample of increments of $\hat{x}_t$ deduced from the ICRG political risk index. Time period: 1984-1996. Monthly observations

Country	Mean	t-value[1]	Variance	Coeff. of Skewness	Excess Kurtosis	B-J[2], p-value	Studentized range[3]	ρ_1 [(1)(4)]	ρ_2 [(1)(4)]
Algeria	-0.0051	-0.59	0.0037	-1.462	3.287	0.00 **	5.10	0.085	-0.055
Angola	0.0058	0.60	0.0041	0.301	2.515	0.00 **	5.50 h*	-0.138	0.319 *
Argentina	0.0130	1.97	0.0022	0.424	0.093	0.47	4.50	0.011	-0.275
Australia	-0.0019	-0.22	0.0038	0.594	1.149	0.06	4.87	-0.042	-0.108
Azerbaijan	NA								
Brazil	0.0047	0.59	0.0031	0.079	-0.36	0.85	4.33	0.038	-0.352 *
Brunei	0.0049	1.37	0.0005	1.759	3.787	0.00 **	4.71	0.418 **	0.039
Cameroon	-0.0005	-0.09	0.0014	-1.837	6.158	0.00 **	6.08 h**	0.072	-0.034
Canada	-0.0032	-0.45	0.0024	-0.905	1.906	0.00 **	5.06	0.131	-0.066
China	-0.0006	-0.08	0.0030	0.27	0.313	0.68	4.41	0.368 *	0.114
Colombia	-0.0031	-0.60	0.0013	-0.906	0.765	0.02 *	4.16	0.312 *	0.256
Congo	0.0000	0.00	0.0017	-2.196	13.205	0.00 **	7.28 h**	0.000	0.123
Denmark	-0.0056	-0.84	0.0022	0.179	1.276	0.16	5.35	0.193	-0.204
Egypt	0.0005	0.08	0.0020	1.209	4.88	0.00 **	6.32 h**	-0.045	-0.093
Equador	0.0040	0.48	0.0035	0.547	2.067	0.00 **	5.94 h**	0.133	-0.083
Gabon	0.0005	0.10	0.0014	1.31	6.671	0.00 **	6.59 h**	0.158	-0.108
India	0.0031	0.28	0.0062	0.919	4.404	0.00 **	6.48 h**	0.246	0.072
Indonesia	0.0108	1.81	0.0018	1.143	2.468	0.00 **	5.43 h*	0.230	0.071
Iran	0.0234	2.70 **	0.0038	1.052	1.146	0.00 **	4.73	0.127	-0.300 *
Iraq	0.0109	0.79	0.0097	-0.512	3.674	0.00 **	6.00 h**	0.062	0.067
Kazakhstan	NA								
Kuwait	0.0137	0.68	0.0205	-3.516	22.794	0.00 **	8.18 h**	0.262	-0.002
Libya	0.0145	1.37	0.0056	1.551	5.951	0.00 **	6.54 h**	-0.014	-0.107
Malaysia	0.0013	0.20	0.0022	0.351	1.188	0.14	5.32	0.172	0.231
Mexico	-0.0028	-0.37	0.0028	-1.284	6.401	0.00 **	7.02 h**	0.056	-0.327 *
Nigeria	0.0050	0.95	0.0014	0.372	1.502	0.05	5.63 h*	-0.123	-0.006
Norway	-0.0086	-0.79	0.0059	1.659	9.028	0.00 **	7.27 h**	-0.101	0.104
Oman	0.0088	1.48	0.0017	1.537	4.065	0.00 **	5.57 h*	0.064	-0.058
Papua New Guinea	-0.0006	-0.10	0.0015	-0.406	6.627	0.00 **	7.23 h**	0.030	0.102
Peru	0.0076	0.74	0.0052	-1.732	6.855	0.00 **	6.11 h**	-0.243	-0.038
Quatar	0.0086	1.09	0.0030	1.926	5.241	0.00 **	5.45 h*	-0.005	0.005
Romania	0.0063	0.88	0.0024	1.473	4.834	0.00 **	6.11 h**	0.228	0.019
Russian Federation	0.0060	0.48	0.0027	-0.84	2.064	0.08	4.46 h*	-0.027	-0.145
Saudi Arabia	0.0067	0.70	0.0046	0.007	0.751	0.56	5.03	-0.026	-0.098
Syria	0.0222	2.86 **	0.0030	1.121	2.787	0.00 **	5.66 h*	0.094	0.191
Trinidad & Tobago	0.0048	0.75	0.0020	0.427	2.736	0.00 **	5.80 h**	-0.233	-0.204
Tunisia	0.0135	1.77	0.0029	1.814	5.797	0.00 **	6.10 h**	0.103	-0.062
United Arab Emirates	0.0140	1.68	0.0035	3.856	19.188	0.00 **	6.63 h**	-0.043	-0.204
United Kingdom	-0.0085	-0.81	0.0055	-0.169	0.354	0.78	4.73	0.004	0.004
USA	-0.0138	-1.69	0.0034	-0.854	1.607	0.00 **	5.00	0.400 **	0.074
Uzbekistan	NA								
Venezuela	0.0036	0.35	0.0053	-2.034	9.96	0.00 **	6.84 h**	0.123	-0.214
Vietnam	0.0155	2.21 *	0.0021	2.627	11.151	0.00 **	6.08 h**	0.030	-0.091
Yemen	0.0226	1.07	0.0067	-0.494	1.904	0.24	2.35	-0.168	-0.066

The number of observations are 50 for all countries, except for the Russian Federation (17), Yemen (15), Qatar (48), Romania (48), Oman (48), Papua New Guinea (49), Angola (43), Brunei (43), Vietnam (43), China (47), and Congo (45).

Table 1.B Statistics for sample of increments of $\hat{x}_t$ deduced from the ICRG political risk index. Time period: 1984-1996. Quarterly observations

Country	Mean	t-value[1]	Variance	Coeff. of Skewness	Excess Kurtosis	B-J[2], p-value	Studentized range[3]	ρ_1[1],[4]	ρ_2[1],[4]
Algeria	-0.0101	-0.62	0.0067	-0.822	0.212	0.24	3.80	0.205	-0.316
Angola	0.0132	0.64	0.0090	-0.214	1.797	0.22	4.85 h**	0.226	0.015
Argentina	0.0260	1.97	0.0044	1.176	1.736	0.01 *	4.40	0.254	-0.082
Australia	-0.0038	-0.20	0.0087	0.402	0.613	0.59	4.51	-0.248	0.213
Azerbaijan	NA								
Brazil	0.0093	0.60	0.0061	-0.503	0.704	0.46	4.36	-0.365	-0.247
Brunei	0.0115	1.31	0.0016	1.584	5.116	0.00 **	5.22 h**	0.029	-0.012
Cameroon	-0.0010	-0.08	0.0035	-1.198	2.281	0.00 **	4.74 h*	-0.124	-0.215
Canada	-0.0063	-0.40	0.0063	-0.347	0.354	0.73	4.05	-0.037	-0.161
China	-0.0025	-0.14	0.0075	0.267	1.788	0.19	4.97 h**	0.295	-0.186
Colombia	-0.0061	-0.56	0.0030	-0.132	-0.327	0.91	4.19	0.315	0.101
Congo	-0.0012	-0.09	0.0034	-1.592	6.292	0.00 **	5.14 h**	0.292	0.119
Denmark	-0.0111	-0.82	0.0046	0.7	0.914	0.23	4.27	-0.125	0.079
Egypt	0.0011	0.08	0.0047	0.595	0.955	0.30	4.53	-0.035	-0.048
Equador	0.0081	0.41	0.0098	0.176	0.156	0.93	4.34	-0.258	-0.004
Gabon	0.0010	0.09	0.0032	0.897	1.578	0.05	4.40	-0.025	-0.320
India	0.0063	0.24	0.0164	0.369	2.281	0.05	4.76 h*	0.051	0.013
Indonesia	0.0216	1.63	0.0044	0.291	-0.08	0.84	3.78	0.193	0.119
Iran	0.0467	2.95 **	0.0062	0.868	0.267	0.20	4.17	0.074	0.210
Iraq	0.0218	0.68	0.0256	0.167	1.368	0.36	4.56	0.029	-0.083
Kazakhstan	NA								
Kuwait	0.0273	0.61	0.0510	-1.639	10.314	0.00 **	6.38 h**	0.141	-0.124
Libya	0.0289	1.40	0.0107	0.399	-0.48	0.64	3.76	-0.081	-0.141
Malaysia	0.0026	0.18	0.0057	-0.477	1.022	0.36	4.52	0.262	0.133
Mexico	-0.0056	-0.33	0.0069	-0.834	2.661	0.01 **	5.04 h**	-0.282	0.194
Nigeria	0.0101	0.93	0.0029	-0.367	-0.67	0.60	3.68	-0.234	-0.239
Norway	-0.0171	-0.96	0.0080	-0.042	-0.459	0.89	3.80	0.384	-0.136
Oman	0.0177	1.46	0.0035	0.487	1.093	0.34	4.55	-0.081	0.166
Papua New Guinea	-0.0011	-0.10	0.0034	-0.3	1.478	0.28	4.77 h*	0.102	0.017
Peru	0.0152	0.93	0.0067	-0.71	1.183	0.17	4.64	-0.234	-0.012
Quatar	0.0173	1.09	0.0061	1.227	0.963	0.03 *	3.85	-0.055	-0.039
Romania	0.0125	0.85	0.0052	0.524	-0.277	0.56	3.90	0.250	0.128
Russian Federation	0.0221	0.84	0.0055	-1.079	1.427	0.33	3.10	-0.340	-0.326
Saudi Arabia	0.0134	0.70	0.0093	0.408	0.568	0.60	4.47	-0.066	-0.039
Syria	0.0444	2.68 *	0.0068	0.739	0.202	0.31	3.99	0.333	0.196
Trinidad & Tobago	0.0095	0.83	0.0033	-0.54	0.506	0.48	4.17	-0.317	0.020
Tunisia	0.0270	1.65	0.0067	1.2	1.63	0.01 *	4.27	-0.023	-0.054
United Arab Emirates	0.0279	1.59	0.0077	2.668	8.618	0.00 **	4.68	-0.245	0.057
United Kingdom	-0.0170	-0.85	0.0100	-0.094	1.145	0.50	4.60	0.105	0.143
USA	-0.0276	-1.28	0.0117	-0.973	2.937	0.00 **	4.91 h*	0.047	0.211
Uzbekistan	NA								
Venezuela	0.0073	0.31	0.0135	-0.714	3.026	0.00 **	5.16 h**	-0.262	0.054
Vietnam	0.0306	1.89	0.0055	2.26	6.741	0.00 **	4.97 h**	-0.215	0.354
Yemen	0.0485	1.26	0.0104	1.099	-0.913	0.44	2.35	-0.251	-0.472

The number of observations are 25 for all countries, except for the Russian Federation (8), Yemen (7), Qatar (24), Romania (24), Oman (24), Papua New Guinea (24), Angola (21), Brunei (21), Vietnam (21), China (23), and Congo (22).

Table 1.C Statistics for sample of increments of $\hat{x}_t$ deduced from the ICRG political risk index. Time period: 1984-1996. Bi-annual observations

Country	Mean	t-value(1)	Variance	Coeff. of Skewness	Excess Kurtosis	B-J(2), p-value	Studentized range(3)	$\rho_1^{(1),(4)}$	$\rho_2^{(1),(4)}$
Algeria	0.0030	0.89	0.0017	0.11	5.79	0.00 **	7.89 h**	-0.145	-0.060
Angola	0.0015	0.56	0.0010	0.36	10.89	0.00 **	10.11 h**	0.041	0.021
Argentina	0.0073	1.76	0.0026	-0.92	6.27	0.00 **	8.43 h**	0.101	0.097
Australia	0.0015	0.51	0.0012	0.18	5.18	0.00 **	6.89 h**	0.039	0.017
Azerbaijan	NA								
Brazil	0.0047	1.49	0.0016	0.18	4.77	0.00 **	7.62 h**	0.064	-0.001
Brunei	-0.0000	-0.00	0.0009	0.00	33.31	0.00 **	13.08 h**	0.241 **	0.000
Cameroon	0.0003	0.17	0.0006	0.34	20.81	0.00 **	12.33 h**	-0.015	-0.136
Canada	0.0015	0.45	0.0017	4.02	42.89	0.00 **	13.40 h**	-0.009	-0.002
China	0.0009	0.23	0.0020	3.24	19.13	0.00 **	8.24 h**	0.099	0.054
Colombia	0.0026	0.85	0.0014	-0.59	6.19	0.00 **	7.79 h**	0.122	0.169 *
Congo	0.0034	1.03	0.0014	2.26	18.26	0.00 **	10.53 h**	0.124	0.023
Denmark	0.0047	1.18	0.0024	6.79	65.86	0.00 **	13.04 h**	-0.028	-0.010
Egypt	0.0013	0.30	0.0031	-5.35	51.75	0.00 **	12.30 h**	0.065	0.074
Equador	0.0060	1.60	0.0022	4.03	41.62	0.00 **	13.14 h**	0.056	0.083
Gabon	0.0023	1.28	0.0005	2.00	21.57	0.00 **	12.04 h**	-0.284 **	-0.046
India	0.0029	1.11	0.0010	0.91	8.20	0.00 **	9.28 h**	0.055	0.171 *
Indonesia	0.0060	1.73	0.0018	1.00	5.43	0.00 **	7.68 h**	0.109	0.359 **
Iran	0.0068	1.89	0.0020	2.50	21.07	0.00 **	11.30 h**	0.117	0.125
Iraq	0.0018	0.24	0.0086	2.43	37.93	0.00 **	13.28 h**	0.168 *	-0.010
Kazakhstan	NA								
Kuwait	0.0045	0.31	0.0327	-6.17	78.13	0.00 **	14.70 h**	0.028	-0.005
Libya	0.0058	1.75	0.0017	3.97	25.37	0.00 **	10.30 h**	-0.060	0.022
Malaysia	0.0047	1.65	0.0012	-0.23	5.04	0.00 **	7.77 h**	0.283 **	0.220 **
Mexico	0.0070	1.70	0.0026	-1.64	13.61	0.00 **	10.04 h**	0.147	0.154
Nigeria	0.0022	0.74	0.0014	-0.48	7.31	0.00 **	8.93 h**	0.039	0.018
Norway	0.0005	0.17	0.0014	2.23	25.04	0.00 **	11.13 h**	0.040	-0.071
Oman	0.0054	1.51	0.0019	7.74	77.08	0.00 **	12.70 h**	0.015	-0.014
Papua New Guinea	-0.0004	-0.15	0.0011	-0.81	15.31	0.00 **	10.05 h**	0.000	0.103
Peru	0.0057	1.03	0.0047	-0.49	11.18	0.00 **	10.47 h**	-0.132	0.118
Quatar	0.0050	1.43	0.0018	7.30	74.51	0.00 **	13.56 h**	-0.003	0.006
Romania	0.0030	0.95	0.0014	2.16	11.26	0.00 **	8.23 h**	-0.131	0.007
Russian Federation	0.0010	0.12	0.0037	2.09	19.51	0.00 **	9.03 h**	-0.305 *	-0.063
Saudi Arabia	0.0044	0.89	0.0037	1.60	28.92	0.00 **	13.46 h**	-0.107	0.090
Syria	0.0057	1.60	0.0019	7.78	79.02	0.00 **	11.92 h**	0.023	-0.037
Trinidad & Tobago	0.0034	1.25	0.0011	-0.50	20.39	0.00 **	11.87 h**	-0.233 **	-0.078
Tunisia	0.0056	1.47	0.0022	8.64	91.54	0.00 **	13.07 h**	0.040	-0.052
United Arab Emirates	0.0038	1.01	0.0022	5.16	45.75	0.00 **	12.32 h**	-0.005	0.144
United Kingdom	-0.0046	-0.58	0.0059	-3.26	22.80	0.00 **	9.51 h**	0.120	-0.006
USA	0.0007	0.13	0.0042	-0.00	21.41	0.00 **	10.81 h**	0.051	0.003
Uzbekistan	NA								
Venezuela	0.0037	0.93	0.0025	-0.85	6.23	0.00 **	8.28 h**	0.111	0.077
Vietnam	0.0039	1.00	0.0020	4.68	34.89	0.00 **	10.45 h**	-0.155	0.055
Yemen	0.0037	1.15	0.0005	3.16	16.08	0.00 **	7.42 h**	-0.031	-0.031

The number of observations are 152 for all countries, except for the Russian Federation (53), Yemen (45), Qatar (145), Romania (145), Oman (146), Papua New Guinea (148), Angola (131), Brunei (131), Vietnam (131), China (141), and Congo (137).

Table 2.A Statistics for sample of increments of $\hat{x}_t$ deduced from the ICRG financial risk index. Time period: 1984-1996. Monthly observations

Country	Mean	t-value(1)	Variance	Coeff. of Skewness	Excess Kurtosis	B-J(2), p-value	Studentized range(3)	ρ_1 (1),(4)	ρ_2 (1),(4)
Algeria	0.0086	0.90	0.0046	-0.10	0.75	0.53	4.73	-0.165	-0.015
Angola	0.0047	0.51	0.0036	0.90	3.90	0.00 **	6.15 h**	0.011	-0.066
Argentina	0.0233	1.59	0.0107	-0.73	1.83	0.00 **	5.22	0.067	0.085
Australia	0.0052	0.59	0.0039	0.01	4.17	0.00 **	6.39 h**	-0.093	0.334 *
Azerbaijan	NA								
Brazil	0.0144	1.33	0.0059	-0.11	0.48	0.74	4.95	0.127	-0.060
Brunei	-0.0000	-0.00	0.0029	0.00	9.72	0.00 **	7.43 h**	0.241	0.000
Cameroon	0.0010	0.19	0.0014	1.20	6.12	0.00 **	6.72 h**	-0.069	0.096
Canada	0.0065	0.66	0.0049	2.40	14.17	0.00 **	7.86 h**	0.113	-0.009
China	0.0026	0.23	0.0063	1.72	4.32	0.00 **	4.92	0.321 *	0.506 **
Colombia	0.0078	0.79	0.0049	0.69	2.09	0.00 **	5.57 h*	0.451 **	0.198
Congo	0.0136	1.32	0.0048	2.89	13.13	0.00 **	6.64 h**	-0.041	0.039
Denmark	0.0151	1.27	0.0071	3.90	21.28	0.00 **	7.61 h**	-0.018	0.124
Egypt	0.0051	0.37	0.0097	-2.89	15.98	0.00 **	7.41 h**	0.105	-0.133
Equador	0.0173	1.46	0.0071	1.89	10.32	0.00 **	7.25 h**	0.078	0.214
Gabon	0.0071	1.35	0.0014	0.99	6.70	0.00 **	7.24 h**	-0.346 *	-0.041
India	0.0083	0.96	0.0038	0.51	1.19	0.08	5.20	0.242	0.007
Indonesia	0.0188	1.46	0.0083	0.95	2.25	0.00 **	5.59 h*	0.508 **	0.456 **
Iran	0.0234	2.21 *	0.0056	1.54	5.81	0.00 **	6.69 h**	0.288 *	0.159
Iraq	0.0044	0.16	0.0358	1.56	14.87	0.00 **	8.51 h**	-0.013	0.072
Kazakhstan	NA								
Kuwait	0.0122	0.28	0.0954	-4.21	28.57	0.00 **	8.39 h**	0.107	-0.256
Libya	0.0171	1.87	0.0042	2.13	5.14	0.00 **	4.95	-0.026	0.027
Malaysia	0.0153	1.41	0.0058	0.08	1.66	0.06	5.38 h*	0.412 **	0.420 **
Mexico	0.0173	1.28	0.0091	-0.87	2.54	0.00 **	5.55 h*	0.256	-0.043
Nigeria	0.0089	0.92	0.0047	-0.40	1.25	0.10	4.81	0.027	0.182
Norway	0.0016	0.16	0.0047	0.78	6.44	0.00 **	6.68 h**	-0.254	0.094
Oman	0.0144	1.33	0.0056	4.43	25.13	0.00 **	7.38 h**	0.004	0.046
Papua New Guinea	-0.0037	-0.49	0.0028	-0.68	4.68	0.00 **	6.19 h**	0.342 *	0.176
Peru	0.0185	1.13	0.0133	-0.17	2.52	0.00 **	5.81 h**	-0.035	-0.086
Quatar	0.0150	1.44	0.0053	4.05	23.46	0.00 **	7.86 h**	-0.002	0.080
Romania	0.0090	0.93	0.0045	0.57	2.29	0.00 **	5.36 h*	0.239	0.310 *
Russian Federation	0.0030	0.21	0.0035	1.01	2.43	0.03 *	4.57 h*	-0.343	0.638 *
Saudi Arabia	0.0134	0.79	0.0143	1.15	10.19	0.00 **	7.95 h**	-0.124	0.204
Syria	0.0182	1.69	0.0058	4.23	23.31	0.00 **	6.84 h**	-0.072	0.007
Trinidad & Tobago	0.0093	1.38	0.0023	0.32	5.39	0.00 **	6.68 h**	-0.271	0.175
Tunisia	0.0172	1.51	0.0064	5.24	32.17	0.00 **	7.11 h**	0.144	-0.122
United Arab Emirates	0.0101	0.77	0.0085	3.09	16.73	0.00 **	7.59 h**	0.108	0.076
United Kingdom	-0.0147	-0.58	0.0190	-1.69	5.44	0.00 **	5.30	0.318	0.425 *
USA	0.0021	0.13	0.0139	0.47	6.53	0.00 **	6.45 h**	0.001	-0.628 **
Uzbekistan	NA								
Venezuela	0.0119	0.88	0.0091	0.29	2.22	0.00 **	5.87 h**	0.180	-0.022
Vietnam	0.0119	1.15	0.0046	1.69	4.75	0.00 **	5.46 h*	0.291	-0.059
Yemen	0.0111	1.16	0.0014	1.46	3.63	0.00 **	4.32 h*	0.047	0.207

The number of observations are 50 for all countries, except for the Russian Federation (17), Yemen (15), Qatar (48), Romania (48), Oman (48), Papua New Guinea (49), Angola (43), Brunei (43), Vietnam (43), China (47), and Congo (45).

Table 2.B Statistics for sample of increments of $\hat{x}_t$ deduced from the ICRG financial risk index. Time period: 1984-1996. Quarterly observations

Country	Mean	t-value[1]	Variance	Coeff. of Skewness	Excess Kurtosis	B-J[2], p-value	Studentized range[3]	ρ_1[1][4]	ρ_2[1][4]
Algeria	0.0173	0.34	0.0080	0.209	-0.282	0.88	3.59	-0.135	0.082
Angola	0.0171	0.28	0.0050	1.086	1.774	0.03 *	4.40	-0.119	0.338
Argentina	0.0466	0.15	0.0248	-0.169	0.868	0.64	4.38	0.030	0.361
Australia	0.0104	0.55	0.0073	-0.277	1.979	0.11	4.92 h*	0.266	0.138
Azerbaijan	NA								
Brazil	0.0288	0.26	0.0157	-0.077	-0.213	0.96	3.99	-0.050	-0.063
Brunei	0.0000	1.00	0.0061	0.000	3.712	0.00 **	5.13 h**	0.269	0.000
Cameroon	0.0020	0.85	0.0027	0.927	2.278	0.01 *	4.85 h*	0.040	0.224
Canada	0.0130	0.52	0.0099	1.565	5.905	0.00 **	5.52 h**	0.105	0.315
China	0.0027	0.92	0.0185	1.310	2.330	0.00 **	4.34	0.530 *	0.134
Colombia	0.0156	0.49	0.0124	1.095	2.518	0.00 **	4.67	0.389	0.049
Congo	0.0232	0.28	0.0096	1.736	5.860	0.00 **	5.10 h**	0.058	-0.097
Denmark	0.0303	0.20	0.0134	2.787	10.580	0.00 **	5.53 h**	0.137	-0.047
Egypt	0.0102	0.75	0.0241	-1.992	8.481	0.00 **	5.73 h**	-0.208	-0.038
Equador	0.0347	0.20	0.0175	1.912	7.273	0.00 **	5.45 h**	0.215	0.303
Gabon	0.0142	0.07	0.0014	0.768	1.120	0.15	4.74 h*	-0.354	0.135
India	0.0166	0.45	0.0116	0.608	0.503	0.41	4.37	-0.032	0.210
Indonesia	0.0377	0.25	0.0257	1.406	2.014	0.00 **	4.24	0.605 **	0.331
Iran	0.0467	0.08	0.0165	1.540	3.296	0.00 **	4.67	0.195	0.320
Iraq	0.0088	0.87	0.0732	1.022	6.684	0.00 **	5.95 h**	0.126	-0.047
Kazakhstan	NA								
Kuwait	0.0244	0.80	0.2170	-2.362	12.069	0.00 **	6.20 h**	-0.147	0.020
Libya	0.0341	0.05	0.0069	1.480	1.693	0.00 **	3.85	0.334	0.216
Malaysia	0.0305	0.25	0.0165	0.211	-0.002	0.91	3.89	0.564 **	0.554 **
Mexico	0.0346	0.28	0.0242	-0.583	0.230	0.48	4.05	0.061	0.042
Nigeria	0.0178	0.38	0.0099	0.449	0.797	0.47	4.41	0.120	0.060
Norway	0.0032	0.87	0.0097	0.517	1.910	0.09	4.68	-0.092	-0.349
Oman	0.0287	0.20	0.0114	2.862	10.420	0.00 **	5.15 h**	0.049	0.421
Papua New Guinea	-0.0076	0.68	0.0081	-0.731	1.661	0.09	4.12	0.412	0.097
Peru	0.0369	0.28	0.0274	-0.408	0.769	0.52	4.35	0.003	-0.308
Quatar	0.0301	0.16	0.0105	2.608	10.202	0.00 **	5.56 h**	0.102	0.390
Romania	0.0180	0.39	0.0100	-0.231	0.091	0.90	4.20	0.545 **	0.658 **
Russian Federation	0.0064	0.78	0.0040	-0.306	0.154	0.94	3.16	0.402	0.005
Saudi Arabia	0.0267	0.43	0.0281	0.594	4.379	0.00 **	5.67 h**	0.084	0.019
Syria	0.0363	0.09	0.0107	2.965	11.832	0.00 **	5.51 h**	0.202	0.054
Trinidad & Tobago	0.0187	0.15	0.0039	0.944	3.495	0.00 **	4.99 h*	0.082	-0.032
Tunisia	0.0343	0.18	0.0154	3.680	15.279	0.00 **	5.00 h*	-0.069	-0.066
United Arab Emirates	0.0201	0.47	0.0184	1.662	6.419	0.00 **	5.53 h**	0.416 *	0.295
United Kingdom	0.0052	0.91	0.0303	-1.551	3.924	0.00 **	4.03	-0.210	-0.096
USA	0.0042	0.90	0.0284	0.302	1.979	0.11	4.51	-0.633 **	0.287
Uzbekistan	NA								
Venezuela	0.0237	0.43	0.0222	0.483	0.724	0.47	4.16	0.022	-0.236
Vietnam	0.0293	0.27	0.0139	1.685	3.504	0.00 **	4.42	-0.042	0.418
Yemen	0.0237	0.35	0.0038	0.755	-0.748	0.66	2.59	0.333	-0.437

The number of observations are 25 for all countries, except for the Russian Federation (8), Yemen (7), Qatar (24), Romania (24), Oman (24), Papua New Guinea (24), Angola (21), Brunei (21), Vietnam (21), China (23), and Congo (22).

Table 2.C Statistics for sample of increments of $\hat{x}_t$ deduced from the ICRG financial risk index. Time period: 1984-1996. Bi-annual observations

Country	Mean	t-value[1]	Variance	Coeff. of Skewness	Excess Kurtosis	B-J[2], p-value	Studentized range[3]	ρ_1[1),(4]	ρ_2[1),(4]
Algeria	0.0005	0.11	0.0030	0.83	5.41	0.00 **	7.46 h**	-0.171 *	-0.062
Angola	-0.0031	-0.43	0.0068	-0.88	5.56	0.00 **	7.40 h**	-0.021	0.037
Argentina	0.0071	1.27	0.0047	-0.47	4.26	0.00 **	7.71 h**	-0.023	-0.025
Australia	-0.0002	-0.08	0.0010	0.38	3.88	0.00 **	8.10 h**	-0.124	-0.012
Azerbaijan	NA								
Brazil	0.0056	1.05	0.0044	0.35	12.87	0.00 **	11.22 h**	-0.158	0.057
Brunei	0.0074	0.87	0.0095	2.95	43.18	0.00 **	13.95 h**	-0.323 **	-0.051
Cameroon	0.0012	0.23	0.0044	1.79	15.29	0.00 **	9.83 h**	-0.054	0.058
Canada	-0.0000	-0.00	0.0006	0.40	4.68	0.00 **	8.02 h**	-0.039	-0.027
China	0.0018	0.28	0.0060	-0.95	6.31	0.00 **	8.03 h**	-0.063	0.009
Colombia	0.0041	0.91	0.0030	0.01	2.64	0.00 **	6.74 h**	-0.111	0.080
Congo	0.0017	0.38	0.0027	0.64	8.01	0.00 **	8.89 h**	-0.108	0.004
Denmark	0.0021	0.74	0.0012	0.51	2.85	0.00 **	7.18 h**	-0.207 *	-0.098
Egypt	0.0044	1.03	0.0027	0.27	1.75	0.00 **	6.14	-0.068	-0.109
Equador	0.0029	0.52	0.0047	-2.75	34.81	0.00 **	13.43 h**	-0.165 **	0.007
Gabon	-0.0031	-0.56	0.0046	-0.86	8.17	0.00 **	8.84 h**	-0.097	-0.056
India	0.0035	1.11	0.0015	0.98	5.43	0.00 **	7.66 h**	0.083	-0.179 *
Indonesia	0.0033	0.90	0.0020	-0.13	9.17	0.00 **	10.25 h**	-0.150	0.048
Iran	0.0029	0.57	0.0039	1.04	6.77	0.00 **	8.18 h**	-0.022	-0.028
Iraq	0.0000	0.00	0.0148	-1.25	18.70	0.00 **	10.35 h**	-0.160	-0.008
Kazakhstan	NA								
Kuwait	-0.0007	-0.06	0.0169	-2.16	14.91	0.00 **	9.32 h**	0.129	-0.047
Libya	0.0037	0.57	0.0063	1.88	19.57	0.00 **	11.49 h**	-0.256 **	0.074
Malaysia	0.0019	0.50	0.0022	1.26	10.76	0.00 **	9.91 h**	-0.069	-0.088
Mexico	0.0025	0.54	0.0033	0.79	4.09	0.00 **	7.98 h**	-0.025	-0.127
Nigeria	-0.0003	-0.05	0.0063	0.69	7.85	0.00 **	9.20 h**	-0.082	-0.003
Norway	0.0030	1.02	0.0013	0.36	2.01	0.00 **	6.40 h*	-0.221 **	0.021
Oman	0.0039	0.60	0.0062	1.86	11.70	0.00 **	9.39 h**	-0.050	0.036
Papua New Guinea	0.0032	0.57	0.0047	-2.85	23.69	0.00 **	10.65 h**	-0.104	-0.109
Peru	0.0044	0.84	0.0041	0.83	6.33	0.00 **	8.23 h**	-0.016	0.174 *
Quatar	0.0009	0.15	0.0044	0.37	7.15	0.00 **	9.15 h**	-0.084	-0.015
Romania	-0.0010	-0.17	0.0054	-1.38	11.73	0.00 **	9.14 h**	-0.061	-0.185 *
Russian Federation	0.0107	0.68	0.0131	-0.08	4.71	0.00 **	6.90 h**	-0.012	-0.006
Saudi Arabia	0.0020	0.28	0.0079	-0.12	9.39	0.00 **	9.36 h**	-0.164 *	0.010
Syria	0.0027	0.62	0.0028	0.59	7.53	0.00 **	8.43 h**	-0.073	0.077
Trinidad & Tobago	0.0006	0.15	0.0023	0.35	5.04	0.00 **	8.19 h**	-0.052	-0.109
Tunisia	0.0021	0.46	0.0032	0.11	16.92	0.00 **	12.09 h**	-0.320 **	0.095
United Arab Emirates	0.0007	0.13	0.0045	1.52	8.85	0.00 **	9.07 h**	-0.231 **	-0.007
United Kingdom	0.0000	0.00	0.0007	-0.23	4.60	0.00 **	8.61 h**	-0.100	0.053
USA	-0.0007	-0.16	0.0029	-0.15	32.84	0.00 **	14.18 h**	-0.395 **	0.002
Uzbekistan	NA								
Venezuela	0.0002	0.03	0.0042	0.92	5.33	0.00 **	7.39 h**	-0.086	0.043
Vietnam	0.0055	1.34	0.0022	5.86	48.56	0.00 **	11.30 h**	-0.147	0.087
Yemen	0.0046	0.23	0.0181	1.09	5.21	0.00 **	6.24 h**	-0.275	0.018

The number of observations are 152 for all countries, except for the Russian Federation (53), Yemen (45), Qatar (145), Romania (145), Oman (146), Papua New Guinea (148), Angola (131), Brunei (131), Vietnam (131), China (141), and Congo (137).

Table 3.A Statistics for sample of increments of $\hat{x}_t$ deduced from the ICRG economic risk index. Time period: 1984-1996. Monthly observations

Country	Mean	t-value[1]	Variance	Coeff. of Skewness	Excess Kurtosis	B-J[2], p-value	Studentized range[3]	ρ_1[1][4]	ρ_2[1][4]
Algeria	0.0010	0.09	0.0067	0.05	0.84	0.47	4.90	-0.197	0.249
Angola	-0.0094	-0.39	0.0245	-2.33	10.50	0.00 **	6.39 h**	-0.111	0.066
Argentina	0.0233	1.31	0.0158	-0.26	1.05	0.24	5.18	-0.394 **	0.272
Australia	-0.0006	-0.09	0.0024	0.76	2.09	0.00 **	5.12	-0.203	-0.092
Azerbaijan	NA								
Brazil	0.0149	1.15	0.0083	0.32	1.52	0.06	5.59 h*	-0.065	-0.231
Brunei	0.0225	1.83	0.0064	1.44	6.02	0.00 **	6.60 h**	0.110	-0.088
Cameroon	0.0022	0.13	0.0131	0.81	4.28	0.00 **	6.11 h**	-0.053	-0.033
Canada	0.0007	0.13	0.0014	-0.00	0.86	0.46	5.07	-0.297 *	-0.158
China	0.0054	0.29	0.0157	-1.11	2.14	0.00 **	5.19	-0.010	-0.048
Colombia	0.0108	0.83	0.0086	0.05	0.08	0.98	4.65	-0.191	-0.316 *
Congo	0.0056	0.45	0.0069	0.06	0.65	0.66	4.80	-0.130	-0.017
Denmark	0.0064	0.89	0.0026	0.68	1.81	0.00 **	5.70 h*	-0.101	-0.013
Egypt	0.0138	1.15	0.0072	0.28	0.59	0.50	4.83	-0.329 *	-0.083
Equador	0.0083	0.67	0.0077	-0.21	0.89	0.37	4.91	0.199	-0.051
Gabon	-0.0076	-0.56	0.0093	-0.43	3.24	0.00 **	6.21 h**	-0.197	0.094
India	0.0097	1.03	0.0045	0.09	0.33	0.86	5.08	-0.015	0.124
Indonesia	0.0100	1.13	0.0039	-0.38	1.60	0.04 *	5.29	0.013	-0.258
Iran	0.0083	0.51	0.0132	1.17	3.03	0.00 **	5.32	-0.351 *	0.067
Iraq	0.0013	0.05	0.0305	-1.67	8.91	0.00 **	6.81 h**	0.009	-0.071
Kazakhstan	NA								
Kuwait	-0.0010	-0.03	0.0631	-3.72	21.56	0.00 **	7.32 h**	-0.105	-0.153
Libya	0.0117	0.81	0.0102	0.67	3.61	0.00 **	5.93 h**	-0.157	-0.159
Malaysia	0.0023	0.21	0.0064	0.48	1.81	0.01 *	5.73 h*	-0.010	-0.017
Mexico	0.0046	0.35	0.0085	0.34	0.47	0.49	4.76	-0.272	0.022
Nigeria	0.0000	0.00	0.0137	0.21	1.02	0.28	5.38 h*	-0.061	-0.122
Norway	0.0098	1.25	0.0031	-0.31	0.09	0.67	4.50	-0.212	0.327 *
Oman	0.0077	0.42	0.0167	1.19	3.25	0.00 **	5.73 h**	-0.064	0.001
Papua New Guinea	0.0097	0.61	0.0125	-1.83	9.34	0.00 **	6.80 h**	-0.092	-0.140
Peru	0.0139	0.80	0.0149	1.06	2.94	0.00 **	5.48 h*	0.139	-0.365 *
Quatar	0.0026	0.17	0.0113	0.19	1.15	0.23	5.45 h*	-0.084	0.022
Romania	-0.0042	-0.26	0.0125	-0.30	2.02	0.01 *	5.28	-0.027	0.026
Russian Federation	0.0409	0.74	0.0517	-0.88	2.11	0.07	4.18	0.113	-0.201
Saudi Arabia	0.0050	0.26	0.0182	-0.50	2.51	0.00 **	6.01 h**	-0.288 *	-0.038
Syria	0.0077	0.61	0.0080	1.02	4.16	0.00 **	6.05 h**	-0.070	0.030
Trinidad & Tobago	0.0018	0.15	0.0072	0.01	1.80	0.03 *	5.89 h**	-0.261	0.136
Tunisia	0.0069	0.75	0.0042	-0.22	-0.38	0.71	4.33	-0.307 *	0.093
United Arab Emirates	0.0029	0.20	0.0106	0.44	0.11	0.44	4.38	-0.329 *	0.090
United Kingdom	-0.0006	-0.09	0.0022	-0.39	3.60	0.00 **	6.81 h**	-0.178	-0.132
USA	-0.0021	-0.34	0.0018	-0.38	1.48	0.06	5.59 h*	-0.113	0.019
Uzbekistan	NA								
Venezuela	0.0026	0.16	0.0135	0.35	0.69	0.37	4.82	0.063	0.043
Vietnam	0.0168	1.40	0.0062	3.54	16.67	0.00 **	6.74 h**	-0.048	0.102
Yemen	0.0138	0.30	0.0320	0.18	-0.85	0.77	3.30	-0.152	-0.254

The number of observations are 50 for all countries, except for the Russian Federation (17), Yemen (15), Qatar (48), Romania (48), Oman (48), Papua New Guinea (49), Angola (43), Brunei (43), Vietnam (43), China (47), and Congo (45).

Table 3.B Statistics for sample of increments of $\hat{x}_t$ deduced from the ICRG economic risk index. Time period: 1984-1996. Quarterly observations

Country	Mean	t-value(1)	Variance	Coeff. of Skewness	Excess Kurtosis	B-J(2), p-value	Studentized range(3)	ρ_1(1),(4)	ρ_2(1),(4)
Algeria	0.0020	0.93	0.0129	0.442	0.920	0.43	4.66	0.021	-0.333
Angola	-0.0193	0.69	0.0477	-3.092	11.797	0.00 **	4.72 h*	-0.098	0.025
Argentina	0.0467	0.17	0.0276	-0.066	-1.201	0.47	3.31 l*	-0.230	-0.481 *
Australia	-0.0012	0.92	0.0036	0.319	-0.624	0.66	3.68	-0.201	0.198
Azerbaijan	NA								
Brazil	0.0298	0.29	0.0189	0.459	-0.352	0.60	3.93	-0.537 **	0.145
Brunei	0.0460	0.10	0.0152	1.150	3.415	0.00 **	5.03 h**	-0.102	-0.069
Cameroon	0.0043	0.89	0.0256	-0.237	1.786	0.17	4.87 h*	-0.198	-0.028
Canada	0.0014	0.90	0.0030	0.283	0.289	0.81	4.20	-0.431	0.348
China	0.0071	0.86	0.0340	-0.712	1.030	0.23	4.51	-0.099	0.217
Colombia	0.0216	0.39	0.0153	-0.097	-0.965	0.60	3.48	-0.387	0.128
Congo	0.0115	0.67	0.0159	0.602	0.161	0.51	3.80	-0.014	-0.226
Denmark	0.0128	0.35	0.0045	1.188	1.785	0.01 *	4.20	0.094	-0.262
Egypt	0.0276	0.15	0.0087	0.343	-0.958	0.49	3.32 l*	-0.542 **	0.141
Equador	0.0166	0.52	0.0159	0.335	0.874	0.53	4.44	0.325	0.041
Gabon	-0.0153	0.49	0.0120	0.354	-0.177	0.76	3.92	-0.154	0.048
India	0.0195	0.30	0.0085	0.154	-0.850	0.65	3.68	0.195	-0.045
Indonesia	0.0199	0.30	0.0087	-0.579	0.418	0.45	4.07	-0.319	-0.036
Iran	0.0166	0.57	0.0209	0.493	0.547	0.52	4.43	-0.036	-0.045
Iraq	0.0026	0.96	0.0619	-1.488	4.392	0.00 **	4.99 h*	-0.025	-0.056
Kazakhstan	NA								
Kuwait	-0.0019	0.98	0.1090	-2.949	13.448	0.00 **	6.00 h**	-0.168	0.048
Libya	0.0233	0.44	0.0225	0.349	0.382	0.72	4.20	-0.239	0.080
Malaysia	0.0047	0.84	0.0138	0.155	1.574	0.26	5.10 h**	-0.209	0.014
Mexico	0.0093	0.64	0.0095	0.138	-0.903	0.63	3.48	-0.252	-0.163
Nigeria	0.0000	1.00	0.0241	0.467	0.431	0.58	4.31	-0.089	-0.105
Norway	0.0196	0.19	0.0052	-0.108	-0.430	0.89	4.02	0.174	0.007
Oman	0.0155	0.70	0.0388	0.655	0.512	0.37	4.01	0.045	-0.272
Papua New Guinea	0.0198	0.52	0.0219	-2.040	6.692	0.00 **	5.06 h**	-0.179	0.160
Peru	0.0278	0.49	0.0393	1.001	0.508	0.11	3.88	-0.374	-0.088
Quatar	0.0051	0.87	0.0223	0.524	0.754	0.43	4.41	0.059	-0.100
Romania	-0.0084	0.81	0.0300	-1.413	3.779	0.00 **	5.03 h**	-0.009	-0.100
Russian Federation	0.0364	0.72	0.0746	-0.994	3.016	0.11	3.55	0.133	0.403
Saudi Arabia	0.0099	0.79	0.0344	0.010	0.462	0.89	4.47	-0.510 *	0.126
Syria	0.0154	0.57	0.0180	0.426	2.955	0.01 **	5.29 h**	-0.015	-0.091
Trinidad & Tobago	0.0035	0.87	0.0118	-0.105	1.159	0.49	4.88 h*	-0.069	-0.241
Tunisia	0.0137	0.40	0.0064	-0.517	-0.247	0.56	3.87	-0.096	-0.131
United Arab Emirates	0.0057	0.82	0.0156	-0.342	-0.058	0.78	3.93	-0.298	0.099
United Kingdom	-0.0012	0.92	0.0032	-0.265	0.769	0.63	4.39	-0.191	-0.064
USA	-0.0041	0.73	0.0035	-0.353	-0.227	0.75	4.06	0.098	0.231
Uzbekistan	NA								
Venezuela	0.0053	0.87	0.0262	-0.590	1.266	0.21	4.45	-0.014	0.161
Vietnam	0.0343	0.17	0.0123	2.253	6.807	0.00 **	4.77 h*	0.069	-0.012
Yemen	0.0187	0.86	0.0703	-0.647	-0.706	0.73	2.75	-0.665	0.008

The number of observations are 25 for all countries, except for the Russian Federation (8), Yemen (7), Qatar (24), Romania (24), Oman (24), Papua New Guinea (24), Angola (21), Brunei (21), Vietnam (21), China (23), and Congo (22).

Table 3.C Statistics for sample of increments of $\hat{x}_t$ deduced from the ICRG economic risk index. Time period: 1984-1996. Bi-annual observations

Country	Mean	t-value$^{(1)}$	Variance	Coeff. of Skewness	Excess Kurtosis	B-J$^{(2)}$, p-value	Studentized range$^{(3)}$	$\rho_1^{(1)(4)}$	$\rho_2^{(1)(4)}$
Algeria	0.0000	0.00	0.0245	-0.55	2.02	0.00 **	6.13	0.060	-0.051
Angola	0.0006	0.24	0.0277	0.10	4.08	0.00 **	7.23 h**	-0.024	-0.040
Argentina	0.0057	2.54 *	0.0277	-0.69	3.87	0.00 **	7.59 h**	0.015	-0.062
Australia	0.0001	0.07	0.0216	0.59	1.93	0.00 **	6.94 h**	0.028	-0.050
Azerbaijan	NA								
Brazil	0.0032	1.57	0.0252	-0.28	1.76	0.00 **	6.75 h**	0.059	0.137
Brunei	0.0024	1.28	0.0213	1.36	24.81	0.00 **	12.69 h**	-0.268 **	0.097
Cameroon	0.0002	0.09	0.0230	0.11	5.20	0.00 **	7.84 h**	-0.063	-0.060
Canada	-0.0004	-0.21	0.0244	0.09	8.01	0.00 **	9.82 h**	-0.152	-0.118
China	0.0005	0.21	0.0299	0.05	6.94	0.00 **	9.36 h**	0.100	0.138
Colombia	0.0010	0.52	0.0242	-0.80	6.68	0.00 **	9.09 h**	-0.083	0.163 *
Congo	0.0003	0.13	0.0248	-2.62	21.62	0.00 **	10.90 h**	-0.032	-0.042
Denmark	0.0005	0.27	0.0220	1.55	11.70	0.00 **	9.10 h**	-0.089	-0.120
Egypt	0.0015	0.72	0.0262	-0.66	5.85	0.00 **	8.78 h**	0.047	0.004
Equador	0.0028	1.34	0.0259	1.61	9.11	0.00 **	8.50 h**	0.209 **	0.157
Gabon	0.0002	0.11	0.0207	0.03	4.84	0.00 **	8.22 h**	-0.019	-0.083
India	0.0019	0.80	0.0298	0.72	6.62	0.00 **	8.72 h**	0.124	0.048
Indonesia	0.0040	2.24 *	0.0220	0.76	3.62	0.00 **	6.81 h**	-0.009	0.182 *
Iran	0.0062	2.52 *	0.0303	2.01	10.32	0.00 **	8.58 h**	0.138	-0.022
Iraq	0.0021	0.58	0.0457	-1.21	10.22	0.00 **	8.98 h**	0.066	0.005
Kazakhstan	NA								
Kuwait	0.0035	0.48	0.0900	-3.50	46.42	0.00 **	13.67 h**	0.252 **	0.047
Libya	0.0046	1.76	0.0324	3.06	21.53	0.00 **	10.18 h**	-0.130	0.056
Malaysia	0.0016	0.87	0.0225	0.25	2.60	0.00 **	6.21 h*	0.037	0.039
Mexico	0.0023	0.92	0.0305	-1.41	10.16	0.00 **	9.50 h**	0.039	0.152
Nigeria	0.0010	0.46	0.0268	-0.07	2.64	0.00 **	7.10 h**	0.044	0.059
Norway	-0.0004	-0.18	0.0261	0.53	6.25	0.00 **	8.80 h**	0.029	-0.144
Oman	0.0037	1.70	0.0262	3.06	16.94	0.00 **	8.80 h**	0.012	-0.072
Papua New Guinea	0.0008	0.41	0.0249	-1.17	9.21	0.00 **	8.83 h**	-0.068	-0.025
Peru	0.0038	1.38	0.0339	-0.52	4.28	0.00 **	8.26 h**	-0.015	0.068
Quatar	0.0029	1.14	0.0305	3.72	26.81	0.00 **	10.50 h**	-0.084	0.008
Romania	0.0015	0.78	0.0237	0.01	3.44	0.00 **	7.59 h**	-0.020	0.142
Russian Federation	0.0043	0.96	0.0328	-0.95	4.26	0.00 **	5.79 h*	-0.173	-0.116
Saudi Arabia	0.0026	0.86	0.0372	3.27	22.31	0.00 **	9.95 h**	-0.047	0.027
Syria	0.0054	2.64 **	0.0254	1.97	9.11	0.00 **	7.88 h**	-0.027	-0.022
Trinidad & Tobago	0.0018	1.05	0.0209	-0.27	5.19	0.00 **	7.66 h**	-0.145	-0.013
Tunisia	0.0040	1.82	0.0273	4.70	41.66	0.00 **	11.36 h**	0.051	-0.056
United Arab Emirates	0.0034	1.30	0.0320	6.27	58.92	0.00 **	11.57 h**	-0.018	0.010
United Kingdom	-0.0013	-0.64	0.0258	-0.01	3.47	0.00 **	8.16 h**	0.042	0.073
USA	-0.0020	-0.82	0.0302	0.23	5.04	0.00 **	7.94 h**	-0.209 *	0.041
Uzbekistan	NA								
Venezuela	0.0017	0.64	0.0327	-1.16	11.48	0.00 **	9.79 h**	0.164 *	0.148
Vietnam	0.0050	2.65 **	0.0215	3.54	17.04	0.00 **	8.38 h**	0.005	0.286 **
Yemen	0.0059	1.07	0.0366	0.41	4.88	0.00 **	6.56 h**	-0.212	0.019

The number of observations are 152 for all countries, except for the Russian Federation (53), Yemen (45), Qatar (145), Romania (145), Oman (146), Papua New Guinea (148), Angola (131), Brunei (131), Vietnam (131), China (141), and Congo (137).

Table 4.A Statistics for sample of increments of $\hat{x}_t$ deduced from the ICRG composite risk index. Time period: 1984-1996. Monthly observations

Country	Mean	t-value[1]	Variance	Coeff. of Skewness	Excess Kurtosis	B-J[2], p-value	Studentized range[3]	ρ_1[1],[4]	ρ_2[1],[4]
Algeria	-0.0005	-0.25	0.0022	-1.42	2.87	0.00 **	4.91	-0.075	0.101
Angola	0.0017	0.35	0.0024	-0.52	1.48	0.05	5.16	-0.025	0.176
Argentina	0.0178	2.18 *	0.0026	-0.02	0.13	0.98	4.86	-0.166	0.111
Australia	0.0004	0.98	0.0015	0.08	-0.12	0.96	4.64	0.021	-0.124
Azerbaijan	NA								
Brazil	0.0092	0.66	0.0019	0.06	-0.58	0.70	4.34	0.150	-0.309 *
Brunei	0.0072	2.02	0.0006	1.05	2.77	0.00 **	5.92 h**	0.162	-0.026
Cameroon	0.0005	0.06	0.0017	0.03	0.56	0.72	4.90	-0.284 *	0.147
Canada	-0.0004	0.00	0.0010	-0.18	1.08	0.26	5.37 h*	0.101	0.077
China	0.0016	0.57	0.0031	-0.04	0.93	0.43	5.01	0.355 *	0.124
Colombia	0.0031	0.59	0.0014	-0.55	1.38	0.04 *	5.55 h*	0.176	0.074
Congo	0.0045	1.10	0.0012	-0.27	3.02	0.00 **	5.77 h**	0.057	0.216
Denmark	0.0024	1.97	0.0011	0.25	1.10	0.22	5.51 h*	-0.046	-0.249
Egypt	0.0021	1.50	0.0026	-2.29	11.90	0.00 **	6.65 h**	-0.055	-0.186
Equador	0.0081	1.53	0.0026	1.28	3.97	0.00 **	6.07 h**	0.238	0.138
Gabon	0.0005	0.43	0.0011	0.24	1.96	0.01 *	5.92 h**	-0.119	0.024
India	0.0059	1.12	0.0032	0.45	2.42	0.00 **	5.80 h**	0.260	0.159
Indonesia	0.0122	2.44 *	0.0015	0.12	0.82	0.47	5.20	0.523 **	0.313 *
Iran	0.0194	2.86 *	0.0029	1.20	1.73	0.00 **	5.18	0.054	-0.095
Iraq	0.0065	0.13	0.0072	-0.80	2.21	0.00 **	5.30	-0.046	0.125
Kazakhstan	NA								
Kuwait	0.0105	0.43	0.0336	-4.70	31.34	0.00 **	8.29 h**	0.121	-0.164
Libya	0.0141	1.66	0.0029	1.41	4.70	0.00 **	5.98 h**	0.068	-0.105
Malaysia	0.0048	2.51	0.0017	0.21	0.68	0.52	4.80	0.245	0.182
Mexico	0.0038	0.69	0.0026	-1.65	6.14	0.00 **	6.06 h**	0.197	-0.083
Nigeria	0.0051	0.24	0.0021	0.18	0.45	0.71	4.61	0.061	-0.103
Norway	-0.0012	0.78	0.0020	1.37	6.46	0.00 **	6.74 h**	-0.015	-0.180
Oman	0.0098	1.66	0.0017	1.37	2.65	0.00 **	4.84	0.010	0.044
Papua New Guinea	0.0009	0.48	0.0017	-1.07	4.16	0.00 **	6.02 h**	0.322 *	0.243
Peru	0.0115	2.23	0.0038	-0.69	2.53	0.00 **	5.68 h*	-0.051	-0.179
Quatar	0.0087	1.33	0.0027	1.69	8.77	0.00 **	7.27 h**	-0.121	0.015
Romania	0.0041	1.72	0.0019	0.70	0.90	0.06	5.06	0.119	0.241
Russian Federation	0.0135	0.83	0.0026	-0.89	0.31	0.32	3.52	0.042	-0.377
Saudi Arabia	0.0074	1.37	0.0037	1.46	6.12	0.00 **	6.11 h**	-0.012	0.179
Syria	0.0171	2.74 **	0.0020	0.83	0.82	0.03 *	4.93	-0.057	0.183
Trinidad & Tobago	0.0049	1.46	0.0015	0.21	2.61	0.00 **	5.99 h**	-0.277	0.051
Tunisia	0.0128	1.78	0.0022	2.76	12.64	0.00 **	6.43 h**	0.068	-0.095
United Arab Emirates	0.0103	1.61	0.0033	3.66	20.00	0.00 **	7.54 h**	-0.142	-0.006
United Kingdom	-0.0040	-1.24	0.0022	-0.53	2.65	0.00 **	5.72 h*	0.037	-0.039
USA	-0.0061	0.28	0.0012	-0.05	0.12	0.98	4.97	0.263	-0.074
Uzbekistan	NA								
Venezuela	0.0052	0.19	0.0043	-0.57	2.00	0.00 **	5.36 h*	0.215	-0.090
Vietnam	0.0149	2.42 *	0.0020	2.78	10.43	0.00 **	6.10 h**	0.112	-0.020
Yemen	0.0176	1.04	0.0034	-1.48	3.43	0.00 **	3.96	-0.195	0.157

The number of observations are 50 for all countries, except for the Russian Federation (17), Yemen (15), Qatar (48), Romania (48), Oman (48), Papua New Guinea (49), Angola (43), Brunei (43), Vietnam (43), China (47), and Congo (45).

Table 4.B Statistics for sample of increments of $\hat{x}_t$ deduced from the ICRG composite risk index. Time period: 1984-1996. Quarterly observations

Country	Mean	t-value[1]	Variance	Coeff. of Skewness	Excess Kurtosis	B-J[2], p-value	Studentized range[3]	ρ_1[1],[4]	ρ_2[1],[4]
Algeria	-0.0010	-0.08	0.0038	-0.404	-0.712	0.55	3.56	0.132	-0.461 *
Angola	0.0060	0.44	0.0039	0.109	0.495	0.88	4.01	0.458 *	-0.171
Argentina	0.0357	2.49 *	0.0051	1.183	2.376	0.00 **	4.60 *	-0.117	-0.053
Australia	0.0008	0.07	0.0034	-0.125	-0.702	0.75	3.58	-0.179	0.202
Azerbaijan	NA								
Brazil	0.0184	1.32	0.0049	-0.059	-0.985	0.60	3.30	-0.342	-0.267
Brunei	0.0157	1.82	0.0016	0.884	1.662	0.08	4.29	-0.135	-0.068
Cameroon	0.0010	0.10	0.0026	0.654	2.085	0.04 *	4.70	-0.154	0.075
Canada	-0.0008	-0.08	0.0023	-0.288	-0.142	0.83	3.96	0.124	-0.055
China	0.0013	0.06	0.0093	-0.310	0.703	0.66	4.36	0.221	0.023
Colombia	0.0062	0.52	0.0036	0.173	-0.105	0.93	4.19	0.099	0.232
Congo	0.0075	0.65	0.0029	-1.402	3.623	0.00 **	4.61	0.177	-0.078
Denmark	0.0048	0.64	0.0014	0.051	-0.564	0.84	3.74	0.069	0.164
Egypt	0.0042	0.29	0.0052	-1.662	6.041	0.00 **	5.26	-0.183	0.027
Equador	0.0161	0.88	0.0085	0.865	1.503	0.06	4.57	0.213	0.075
Gabon	0.0011	0.13	0.0017	0.200	-1.141	0.47	3.41	0.053	-0.282
India	0.0118	0.63	0.0088	0.178	0.782	0.68	4.26	0.150	0.043
Indonesia	0.0243	1.83	0.0044	0.541	0.806	0.39	4.53	0.432 *	0.200
Iran	0.0387	2.43 *	0.0064	1.192	0.673	0.04 *	3.76 *	0.070	0.288
Iraq	0.0131	0.51	0.0161	-0.560	-0.550	0.44	3.31	0.125	-0.199
Kazakhstan	NA								
Kuwait	0.0211	0.38	0.0762	-2.933	14.150	0.00 **	6.20	-0.066	-0.002
Libya	0.0282	1.72	0.0067	0.557	-0.303	0.50	3.79	-0.056	0.099
Malaysia	0.0096	0.69	0.0048	-0.124	0.617	0.79	4.46	0.204	0.189
Mexico	0.0076	0.48	0.0062	-1.602	3.571	0.00 **	4.57	0.076	-0.213
Nigeria	0.0101	0.75	0.0045	0.091	0.083	0.98	4.02	-0.036	-0.027
Norway	-0.0024	-0.19	0.0038	0.049	0.112	0.99	4.06	0.308	-0.220
Oman	0.0197	1.50	0.0041	1.045	1.037	0.07	3.88	-0.037	0.079
Papua New Guinea	0.0018	0.13	0.0045	-1.668	4.188	0.00 **	4.50	0.297	0.079
Peru	0.0230	1.39	0.0068	-0.738	0.124	0.32	3.88	-0.098	-0.067
Quatar	0.0175	1.23	0.0049	0.543	2.175	0.05	5.01	0.000	0.255
Romania	0.0082	0.60	0.0045	-0.022	-1.097	0.55	3.57	0.213	-0.015
Russian Federation	0.0208	0.83	0.0050	-0.833	-0.518	0.60	2.68	-0.595	-0.236
Saudi Arabia	0.0148	0.88	0.0071	1.970	6.817	0.00 **	4.87	0.202	-0.077
Syria	0.0341	2.74 *	0.0039	0.224	-0.491	0.79	3.70 *	0.311	0.367
Trinidad & Tobago	0.0098	0.95	0.0027	0.727	3.026	0.00 **	5.22	-0.144	-0.052
Tunisia	0.0255	1.83	0.0048	1.607	3.785	0.00 **	4.74	-0.103	-0.068
United Arab Emirates	0.0206	1.25	0.0067	2.401	9.945	0.00 **	5.72	-0.064	0.021
United Kingdom	-0.0081	-0.66	0.0037	1.339	3.577	0.00 **	4.58	0.135	0.046
USA	-0.0122	-1.05	0.0034	0.040	-0.256	0.96	4.13	-0.052	0.197
Uzbekistan	NA								
Venezuela	0.0104	0.48	0.0119	-0.307	0.474	0.73	4.04	-0.141	0.078
Vietnam	0.0310	1.93	0.0054	2.102	5.280	0.00 **	4.35	-0.134	0.543 *
Yemen	0.0358	1.04	0.0083	-1.060	0.890	0.46	2.85	-0.231	-0.604

The number of observations are 25 for all countries, except for the Russian Federation (8), Yemen (7), Qatar (24), Romania (24), Oman (24), Papua New Guinea (24), Angola (21), Brunei (21), Vietnam (21), China (23), and Congo (22).

Table 4.C Statistics for sample of increments of $\hat{x}_t$ deduced from the ICRG composite risk index. Time period: 1984-1996. Bi-annual observations

Country	Mean	t-value(1)	Variance	Coeff. of Skewness	Excess Kurtosis	B-J(2), p-value	Studentized range(3)	ρ_1(1),(4)	ρ_2(1),(4)
Algeria	-0.0343	-4.25 **	0.0016	-0.464	0.228	0.62	3.96	0.044	0.177
Angola	-0.0004	-0.04	0.0020	-0.193	-0.005	0.93	4.29	0.083	0.093
Argentina	0.0157	1.38	0.0033	-0.054	-0.052	0.99	4.38	0.623 **	0.527
Australia	-0.0167	-1.90	0.0019	-0.965	0.605	0.12	4.10	0.647 **	0.373
Azerbaijan	NA								
Brazil	0.0091	1.01	0.0020	-0.378	0.604	0.61	4.23	0.360	0.053
Brunei	NA								
Cameroon	-0.0216	-3.55 **	0.0009	0.065	0.578	0.83	4.27	0.473 *	0.382
Canada	-0.0117	-2.04	0.0008	-0.400	-0.132	0.71	3.84	0.307	-0.084
China	-0.0067	-0.71	0.0022	-1.605	3.549	0.00 **	4.46	0.575 **	0.186
Colombia	-0.0024	-0.23	0.0027	-1.209	0.934	0.03 *	3.63	0.383	0.255
Congo	-0.0024	-0.33	0.0013	-0.756	0.172	0.30	3.85	-0.035	0.080
Denmark	0.0142	2.69 *	0.0007	0.155	-0.345	0.89	3.80	0.054	0.229
Egypt	0.0012	0.17	0.0014	-0.577	-0.239	0.49	4.02	0.657 **	0.355
Equador	0.0031	0.33	0.0022	-1.023	0.720	0.09	4.04	0.678 **	0.349
Gabon	-0.0100	-1.37	0.0013	-0.064	-0.121	0.98	4.10	0.501 *	0.116
India	-0.0013	-0.14	0.0021	-1.319	4.501	0.00 **	5.07 h**	0.462	0.229
Indonesia	0.0025	0.47	0.0007	-0.024	1.013	0.59	4.08	0.534 **	0.449
Iran	0.0087	0.96	0.0020	0.322	-0.324	0.76	4.23	0.465 *	0.411
Iraq	-0.0197	-1.39	0.0051	-1.702	4.823	0.00 **	5.06 h**	0.142	0.109
Kazakhstan	0.0041	0.22	0.0029	-1.524	3.773	0.02 *	3.33	-0.795 *	-0.133
Kuwait	-0.0112	-0.46	0.0146	-4.045	18.708	0.00 **	5.62 h**	-0.070	-0.208
Libya	-0.0063	-0.67	0.0022	-0.216	-0.761	0.67	3.60	0.149	0.042
Malaysia	0.0006	0.07	0.0015	-1.097	0.619	0.07	3.87	0.734 **	0.547 **
Mexico	0.0056	0.48	0.0035	-1.324	1.534	0.01 **	3.89	0.511 *	0.363
Nigeria	-0.0229	-3.13 **	0.0013	0.691	0.088	0.37	3.84	0.246	0.471 *
Norway	-0.0044	-0.35	0.0038	0.068	2.524	0.04 *	5.33 h**	-0.103	0.089
Oman	0.0046	0.72	0.0010	-0.295	0.176	0.82	4.08	0.225	0.073
Papua New Guinea	-0.0065	-1.23	0.0007	-0.142	1.308	0.39	4.52	-0.037	0.063
Peru	0.0073	0.53	0.0046	0.038	-1.228	0.45	3.38	0.615 **	0.584 **
Quatar	-0.0028	-0.42	0.0011	-2.417	10.100	0.00 **	5.62 h**	0.102	-0.192
Romania	0.0156	1.49	0.0027	0.134	0.335	0.91	4.41	0.694 **	0.471 *
Russian Federation	-0.0092	-0.48	0.0029	-1.034	1.084	0.40	3.14	0.723	0.545
Saudi Arabia	-0.0184	-2.38 *	0.0015	-0.921	0.931	0.11	4.14	0.315	-0.060
Syria	0.0099	1.52	0.0011	-0.251	1.761	0.17	5.19 h**	-0.033	-0.186
Trinidad & Tobago	-0.0125	-1.41	0.0020	-0.290	0.113	0.83	4.04	0.446 *	0.464 *
Tunisia	0.0003	0.04	0.0016	-0.365	-0.707	0.58	3.79	0.444 *	0.393
United Arab Emirates	0.0003	0.04	0.0017	-1.805	5.326	0.00 **	5.10 h**	0.030	-0.143
United Kingdom	-0.0010	-0.14	0.0014	-0.882	0.817	0.14	4.31	0.014	0.335
USA	-0.0171	-1.37	0.0039	-0.519	-0.972	0.35	3.21 l*	0.427 *	0.078
Uzbekistan	-0.0025	-0.17	0.0017	-1.070	2.384	0.18	3.36	-0.019	0.468
Venezuela	-0.0062	-0.70	0.0019	0.759	0.612	0.25	4.10	0.315	0.122
Vietnam	0.0574	5.20 **	0.0011	-0.491	-1.150	0.65	3.02 l**	0.503	-0.307
Yemen	NA								

The number of observations are 25 for all countries, except for the Russian Federation (8), Kazakhstan (8), Uzbekistan (8), and Vietnam for (9).

Table 5 Statistics for sample of increments of $\hat{x}_t$ deduced from the Institutional Investor's country credit risk index. Time period: 1984-1996. Bi-annual observations

Country	ICRG PR	ICRG FR	ICRG ER	ICRG CR	IICCR
Algeria	✓/✓/-	✓/-/-	✓/-/-	✓/✓/✓	-
Angola	✓/✓/-	✓/✓/✓	✓/✓/✓	✓/-/✓	-
Argentina	✓/-/✓	✓/✓/-	✓/✓/✓	✓/-/✓	✓
Australia	✓/-/-	✓/✓/✓	✓/✓/-	✓/-/-	✓
Azerbaijan	NA	NA	NA	NA	NA
Brazil	-/-/-	✓/-/-	✓/✓/✓	✓/✓/-	-
Brunei	✓/✓/✓	✓/✓/✓	✓/✓/✓	✓/✓/-	NA
Cameroon	✓/✓/✓	✓/✓/✓	✓/✓/✓	✓/✓/✓	✓
Canada	✓/✓/-	✓/✓/✓	✓/✓/-	✓/✓/-	-
China	✓/✓/✓	✓/✓/✓	✓/✓/-	✓/✓/-	✓
Colombia	✓/✓/-	✓/✓/✓	✓/-/-	✓/✓/-	✓
Congo	✓/✓/✓	✓/✓/✓	✓/-/-	✓/✓/-	-
Denmark	✓/-/-	✓/✓/✓	✓/✓/✓	✓/✓/✓	-
Egypt	✓/✓/-	✓/✓/✓	✓/✓/-	✓/✓/-	✓
Equador	✓/✓/-	✓/✓/✓	✓/-/-	✓/✓/-	✓
Gabon	✓/✓/-	✓/✓/✓	✓/✓/-	✓/✓/-	✓
India	✓/✓/✓	✓/✓/✓	✓/-/-	✓/✓/-	✓
Indonesia	✓/✓/-	✓/✓/✓	✓/✓/-	✓/✓/✓	✓
Iran	✓/✓/-	✓/✓/✓	✓/✓/-	✓/✓/✓	✓
Iraq	✓/✓/-	✓/✓/✓	✓/✓/✓	✓/✓/-	✓
Kazakhstan	NA	NA	NA	NA	✓
Kuwait	✓/✓/✓	✓/✓/✓	✓/✓/✓	✓/✓/✓	✓
Libya	✓/✓/-	✓/✓/✓	✓/✓/-	✓/✓/-	-
Malaysia	✓/-/-	✓/✓/✓	✓/✓/✓	✓/-/-	✓
Mexico	✓/✓/✓	✓/✓/-	✓/-/-	✓/✓/✓	✓
Nigeria	✓/✓/-	✓/-/-	✓/✓/-	✓/-/-	✓
Norway	✓/✓/-	✓/✓/-	✓/✓/-	✓/✓/-	✓
Oman	✓/✓/-	✓/✓/✓	✓/✓/-	✓/✓/-	-
Papua New Guinea	✓/✓/✓	✓/✓/-	✓/✓/✓	✓/✓/✓	-
Peru	✓/✓/-	✓/✓/-	✓/✓/-	✓/✓/-	✓
Quatar	✓/✓/✓	✓/✓/✓	✓/✓/-	✓/✓/-	✓
Romania	✓/✓/-	✓/✓/✓	✓/✓/✓	✓/-/-	✓
Russian Federation	✓/✓/-	✓/✓/-	✓/-/-	✓/-/-	-
Saudi Arabia	✓/-/-	✓/✓/✓	✓/✓/✓	✓/✓/✓	-
Syria	✓/✓/-	✓/✓/✓	✓/✓/✓	✓/✓/✓	✓
Trinidad & Tobago	✓/✓/-	✓/✓/✓	✓/✓/✓	✓/✓/✓	✓
Tunisia	✓/✓/✓	✓/✓/✓	✓/✓/-	✓/✓/✓	✓
United Arab Emirates	✓/✓/✓	✓/✓/✓	✓/✓/-	✓/✓/✓	✓
United Kingdom	✓/-/-	NR	✓/✓/-	✓/✓/✓	✓
USA	✓/✓/✓	✓/✓/✓	✓/✓/-	✓/-/-	-
Uzbekistan	NA	NA	NA	NA	✓
Venezuela	✓/✓/✓	✓/✓/-	✓/-/-	✓/✓/-	-
Vietnam	✓/✓/✓	✓/✓/✓	✓/✓/✓	✓/✓/✓	-
Yemen	✓/-/-	✓/✓/-	✓/✓/✓	✓/✓/-	NA
Sum "✓"[(o)]	40/33/15	40/37/29	41/33/16	41/33/17	27
Sum "-"[(o)]	1/8/26	0/3/11	0/8/25	0/8/24	14

A mark "✓" is inserted if the hypothesis of the increments of $\hat{x}_t$ being normally distributed and/or the coefficient of correlation between lagged increments is zero can be rejected based on the tests reported in Tables 1-5 in Appendix ,3 using a significance level of five per cent. The mark "-" is inserted when the hypothesis cannot be rejected. The classification is done according to: monthly data/ quarterly data/ bi-annual data. For the IICCR, only bi-annual data are available. [(o)] Number of countries with "✓"-marks or "-" marks.

Table 6 Summary of results for the tests of whether the increments of the deduced variable $\hat{x}_t$ are normally distributed and uncorrelated. Based on Tables 1-5.

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Chapter 4

The Value of Deferring Investment Decisions for Oil Investments Under Regulatory Uncertainty: A General Valuation Model and Numerical Examples

Abstract

When investing in long-term projects, the operating conditions or the regulations applying to the investment will largely depend on the outcome of events taking place after the investment period is started. This may especially be the case if the host country of the investment is politically unstable. Occasionally situations arise where these future unstable conditions will largely depend on the outcome of events taking place at a fixed future date. Examples of such "watershed events" was the first all-racial elections in South Africa and the hand-over of rule of Hong Kong from UK to China. In this paper I study the investors' incentives to wait for such long-term projects when there is uncertainty regarding future regulations. I develop first a general valuation model. I simplify by allowing only two possible regulatory regimes, a "good" and a "bad" one. I then study situations where either the royalty rate is uncertain or the investment may be expropriated.

1 Introduction

In this paper I examine how political uncertainty influences the decision of whether to invest in a real asset today or to wait before a final investment decision is made. Several authors[1] have analyzed the value of investments and optimal investment policies, either with price uncertainty only, or with price and cost uncertainty. The effects of political uncertainty represented by uncertain regulations on decision making and values have, e.g., been analyzed by Brennan and Schwartz (1982 a and b), Hassett and Metcalf (1994), and Teisberg (1993 and 1994)[2].

It is optimal to wait if the value of the deferred investment opportunity is nonnegative and higher than the value of investing today. It is optimal to invest if the value of investing is nonnegative and not lower than the present value of the investment opportunity at future point(s) in time. In order to determine the optimal investment policy, it is therefore necessary to determine the value of the project if the investment is made today, and today's value of the investment opportunity at later points in time.

The uncertainty regarding future regulation is linked to the possibility of several alternative regulatory regimes during the life time of the investment. I have simplified by allowing only to possible regimes, termed a "good" regime, *G*, and a "bad" regime, *B*. A "good" regulatory regime is assumed to give the investor a higher cash flow from the investment than a "bad" regime. With only two possible regimes, the uncertainty is then not related to the type of regime per se, but to which of the regimes will apply at a given time. The stability in governmental policy is represented by the number of shifts between the two types. At one extreme the government may choose continuously between regime *G* and *B*. At the other extreme the government can select the regime only once. The latter is usually the case for

[1] Brennan and Schwartz (1985), Paddock, Siegel, and Smith (1988), Ekern (1988), Bjerksund and Ekern (1990), Dixit and Pindyck (1994), McDonald and Siegel (1986), and others. For an overview article, see, e.g., Trigeorgis (1993).

[2] See chapter two of this dissertation.

expropriation[3]. Regarding taxation, a more realistic model is that the government selects among the possible regimes at regular fixed intervals, or, e.g., when the profitability of the investment reaches upper or lower bounds, as in Brennan and Schwartz (1982a).

Even though the approach presented here can be applied to many types of regulations, I have chosen to analyze specifically an uncertain royalty rate and the possibility of expropriation. Expropriation may be a real possibility in many emerging markets. In some cases it may be relevant to reinterpret the royalty rate as the payment for use of pipe lines to transport the oil out of the host country, or more generally, as the sum of all taxes and costs caused by government regulations expressed as a fraction of sales revenue. A royalty rate, or sales tax, expresses the fraction of sales revenue that is paid to the government by the investor. The reader should, however, be cautioned that the actual regulation considered may have specific implications on the values of the investment opportunity and hence on the optimal investment decision. The conclusions reached in this chapter are therefore only valid for the specific regulations presented.

In this chapter I focus on situations where the regulatory regime, when determined by the government, will apply for the remaining life time of the project. Even though this is a simplification, I feel that this simplification is relevant when describing many real-world situations. In many instances investors face "watershed events" which are linked to certain fixed calendar dates, and at these dates it may be determined under what type of investment environment the investor will operate. Examples of such "watershed events" was the transferral of rule of Hong Kong from UK to China, the first all-racial elections in South Africa, or the (possible) reelection of Mr. Jeltsin as president in Russia for the second term.

In case of expropriation, it may be plausible to assume that the government will decide whether to expropriate the investment when the investment period is completed and production may start. An expropriation at an earlier date means that the government itself, or a state owned company, would have to complete the investment stage.

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3 In some cases the government may reconsider and give the company back to the initial owners, or pay damages.

I further concentrate on modeling situations where the investor can defer the investment situation to a fixed future date. This means that the valuation of the deferred investment opportunity can be compared to the valuation of a European financial option. This approach enables me to use mainly closed-form valuation formulas when valuing the investment opportunities.

I start by describing the investment opportunity in section two. I then suggest a general valuation model in section three, which is applied when considering the numerical examples in section four.

2 The Investment Opportunity

2.1 The Project

I consider a simplified investment in an oil field. If the investment decision is taken at time t, i.e., the development of the oil field starts at time t, the production starts at time $T_{P|t}$. The length of the development period is therefore $(T_{P|t} - t)$ years. The investment expenditure is I_t, the production costs $K_{T_{P|t}}$, and, the produced quantity will be sold at the prevailing spot price of oil, $S_{T_{P|t}}$. I simplify by letting the production volume be one unit[4], being produced

[4] In the absence of regulatory considerations, this is not a critical assumption. It is straightforward to, e.g., introduce a constant production rate of remaining reserves, as in Bjerksund and Ekern (1990). With a produced quantity at time t equal to q_t, the production discounted at the rate of return shortfall, see sub-section 3.2, is

$A = \int_{T_{P|t}}^{T_{E|t}} q_u e^{-\delta_s (u - T_{P|t})} du$, where $T_{E|t}$ is the date when production ends and δ_S is rate of return shortfall for the oil price. My simplifying assumption can alternatively be restated as A=1. Alternatively, the oil for the whole production period may be sold forward at time $T_{P|t}$ by using the forward identity: $F_{t,T} = S_t e^{-\delta_S (T-t)}$. $F_{t,T}$ is the forward price at time t for one oil to be delivered at time T.

In a similar way, if κ_t is the production cost at time t, then $K_{T_{P|t}} = \int_{T_{P|t}}^{T_{E|t}} \kappa_u e^{-r(u - T_{P|t})} du$, i.e., the present value of production costs discounted to the time when production starts.

If the investment rate at time t is h_t, then

$I_t = \int_{t}^{T_{P|t}} h_u e^{-r(u-t)} du.$

When different regulatory regimes are considered, the assumption of a production volume of one is more critical, see footnote 8.

instantaneously. The before-tax sales revenue if the project is initiated at time t, is then $S_{T_{P|t}}$. I assume further that the investment expenditure is paid in full when the investment is initiated, that the production cost is paid in full when the production occurs, and that there is no uncertainty regarding the investment expenditure or production costs.

The investor can either invest today, at time t, or defer the investment decision to a given future date T_W. I assume further that $(T_{P|t} - t) = (T_{P|T_W} - T_W)$, i.e., the development time if the investment decision is made today is equal to the development time if the investment decision is deferred.

2.2 The Investment Environment

2.2.1 The Economic Investment Climate

The economic investment climate is completely described by the instantaneous risk free interest rate, r, and the spot price of oil. The instantaneous risk free interest rate is assumed constant. The spot price of oil is assumed to develop according to a geometric Brownian motion,

$$dS_t = S_t \alpha_S dt + S_t \sigma_S dB_t^{(S)}, \tag{1}$$

where S_t is the oil price at time t, $dB_t^{(S)}$ is the increment of a standard Brownian motion, and α_S and σ_S are nonnegative constants. The oil price is deterministic if $\sigma_S = 0$.

2.2.2 The Political Investment Climate

The political investment climate is characterized by the diversity in competing regulatory regimes, the uncertainty regarding which regulatory regime will apply, and the degree of expected learning by waiting.

The regulatory regime is completely described by a royalty rate, a "scaling factor" of investment expenditures, $\gamma^{(I)}$, and of production costs, $\gamma^{(K)}$. Under regime I the investor keeps a fraction τ_i, $\tau_i \in [0,1]$, of the revenue and pays $(1 - \tau_i)$ to the government as royalty.

The actual investment expenditure for the investor will be $I\gamma_i^{(I)}$, and the actual production costs will be $K\gamma_i^{(K)}$, where $\gamma_i^{(j)} \in \mathbb{R}$, $j = I,K$. A scaling factor of one means that the governmental policies does not influence on the investment expenditure or the production costs. A scaling factor larger than one means that the investment expenditure or the production costs are increased because of, e.g., stricter environmental requirements involving a more expensive development, special taxes, or requirements for investments in infrastructure not necessarily linked to the development of the oil field. A scaling factor less than one can be caused by subsidizing by the government. A scaling factor of zero may, e.g., be used when modeling expropriation.

There are two possible regimes, or governmental policies, $\pi = \{\pi_G, \pi_B\}$, where the sub script refers to a "good" or a "bad" regime. A regime *I* is a combination of the parameters τ_i, $\gamma_i^{(I)}$, and $\gamma_i^{(K)}$, i.e., $\pi_i = \{\tau_i, \gamma_i^{(I)}, \gamma_i^{(K)}\}$, $i = G,B$. Note that it is the combination of policy parameters which is either termed "good" or "bad", not each parameter individually.

I assume that the risk index, ψ, is of a type presented in section two of chapter three of this dissertation. The underlying state variable governing the risk index, $\hat{x}_t$, develops according to the arithmetic Brownian motion process

$$d\hat{x}_t = (\alpha_{\hat{x}} - \frac{1}{2}\sigma_{\hat{x}}^2)dt + \sigma_{\hat{x}} dB_t^{(\hat{x})}, \tag{2}$$

where $dB_t^{(x)}$ is the increment of a standard Brownian motion and $\alpha_{\hat{x}}$ and $\sigma_{\hat{x}}$ are constants. The Brownian motions $B_t^{(\hat{x})}$ and $B_t^{(S)}$ are not necessarily independent, meaning that $dB_t^{(\hat{x})} dB_t^{(S)} = \rho dt$, where ρ may be interpreted as the coefficient of correlation[5] between the state variable governing the risk index and the spot price of oil.

I introduce a hypothetical asset with price at time *t* equal to Z_t, where[6]

[5] If, as in chapter three, $B_t^{(S)} = \rho B_t^{(1)} + \sqrt{1-\rho^2} B_t^{(2)}$ and $B_t^{(\hat{x})} = B_t^{(1)}$, where $B_t^{(1)}$ and $B_t^{(2)}$ are independent standard Brownian motions, then, by Ito's lemma, $dB_t^{(S)} dB_t^{(\hat{x})} = \rho dt$. The increments of $B_t^{(S)}$ and $B_t^{(\hat{x})}$ between time *T* and *t*, $t \le T$, will be normally distributed with zero mean and variance $T - t$.

[6] Equation (3) is comparable to equation (20) on page 72. In order to simplify the notation, I drop the top script $(\hat{x})$ here.

$$Z_t = e^{\hat{x}_t} , \tag{3}$$

i.e., the exponential of the numerical value of the state variable $\hat{x}_t$. Instead of using the risk index to deduce the variable $\hat{x}_t$, the deduced value of the hypothetical asset can be derived directly by using the formula

$$\psi_t = \mathrm{h}(\ln(Z_t)) = \psi^{MAX}\mathrm{N}[\ln(Z_t)/\sigma_v] , \tag{4}$$

where σ_v is the standard deviation of the "noise", ψ^{MAX} is the maximum numerical value of the index, and $\mathrm{N}[\cdot]$ is the cumulative distribution function for the standard normal variable. In order to obtain equation (4), it is assumed that the minimum level of the index is zero[7].

By applying Ito's lemma to (3), we get that

$$dZ_t = Z_t \alpha_{\hat{x}} dt + Z_t \sigma_{\hat{x}} dB_t^{(\hat{x})} , \tag{5}$$

i.e., the price process of the hypothetical asset is given by an ordinary geometric Brownian motion with constant parameters.

Assume that the regulatory regime for the investment will be determined at time T_D ("D" for disclosure), and that at this time T_D the following events happen in sequence: first the level of the index is observed by the investor and then the government declares whether regime G or B will apply. In order to derive valuation formulas in section three, it is a critical assumption that the investor knows the index level at time T_D before the actual regime is disclosed.

The time of disclosure may be a fixed calendar date or may, e.g., be linked to a given stage of the project. An example is that the government decides the royalty rate when production starts. I assume that $t \le T_D \le T_{P|t}$ and $T_D \le T_{P|T_W}$, i.e., the regulatory regime will not be

[7] Equation (4) is comparable to equation (14) on page 68.

determined after production starts and not before the investment decision has been made today, i.e., at time t. This assumption is made in order to study relevant cases. With an assumed production volume of one unit, a change in policy after the point in time when production occurs will not influence on the project[8]. If the policy regime is already determined, at time t, there will be no uncertainty related to the regulatory regime.

Whether it is possible to wait to a date later than the date of disclosure depends on the actual situation. If the date of disclosure is equal to the date when production occurs, this is not possible. If the date of disclosure is a fixed calendar date, perfect information regarding the regulatory regime may be obtained by waiting.

The probability of the type of regime can be conditioned on the level of the index at time T_D, i.e.,

$$P_{T_D}(\pi_i = \pi_G) = \begin{cases} p_H & \text{if} \quad \psi_{T_D} \geq \overline{\psi} \\ p_L & \text{if} \quad \psi_{T_D} < \overline{\psi} \, . \end{cases} \tag{6}$$

The probability of a good regime conditioned on the fact that the index is equal to, or higher than, a critical level $\overline{\psi}$ is p_H (the sub script H refers to a high level of the index), and the probability of a good policy regime is p_L if the numerical value of the index is below the same critical level[9]. If $p_H = p_L$, then no information regarding the type of regime can be

[8] The assumption about a production volume of one simplifies the exposition considerably. With a production period with a flow of oil, a change in regulatory policy during the production period will affect the *remaining* portion of the revenue and production costs. Even though allowing for $T_D > T_{P|t}$ and $T_D > T_{P|T_w}$ together with a production period is straightforward, the analysis becomes involved and complex. This approach was used in an earlier version of this chapter, but I decided to use the simplifying assumption of a production volume of one unit. The added complexity did not outweigh the additional insights.

The assumption that the investment expenditures are paid in full when the investment decision has been made has similar implications when studying the effects of policy changes during the development period. When all investment expenditures are paid up-front, a change in policy during the development period, e.g., an expropriation, will not influence on I. The assumption of up-front payment of expenditures is not critical when assuming that, e.g., expropriation may take place at the end of the development period.

[9] The critical index level is estimated by the investor together with p_H and p_L. In some cases critical level(s) may be provided by the company producing the index. See, e.g., Table 4.3 on page 83 for the ICRG composite risk index.

obtained by observing the level of the index. If $p_H = 1$ and $p_L = 0$, then there is no remaining uncertainty at time T_D. If so, the type of regime is completely determined by the level of the index.

3 Valuation of Investment Opportunities

3.1 Overview

In this section I develop a general model for valuation of the investment opportunity. I find the value of the project if the investment is made today and today's value of the deferred investment opportunity. At the future date when the investor may reconsider to invest, the regulatory regime applying to the project may be known. This is the case when $T_W \geq T_D$. When $T_W < T_D$ the investor does not know which regime will apply.

I also consider two specific cases, see Figure 3.1. In the first case only the royalty rate is

An Uncertain Royalty Rate Only	**Specific case I** Regime parameters: $\gamma_t^{(I)}=1$, $\gamma_G^{(j)}=\gamma_B^{(j)}=1, j = I, K$ *sub-section 3.3.1*: No learning* before time T_D, i.e., $p_H = p_L = p$ *sub-section 3.3.2*: No remaining uncertainty at time T_D, i.e., $p_H=1, p_L=0$
Possibility of Expropriation Only	**Specific case II** Regime parameters: $\gamma_t^{(I)}=1, \gamma_B^{(I)}=\gamma_B^{(K)}=\tau_B=0$ and $\gamma_G^{(I)}=\gamma_G^{(K)}=\tau_G=1$ *sub-section 3.4.1*: No learning* before time T_D, i.e., $p_H = p_L = p$ *sub-section 3.4.2*: No remaining uncertainty at time T_D, i.e., $p_H=1, p_L=0$

known, i.e., $T_W \geq T_D$ | ***not known, i.e., $T_W < T_D$***

At the future date when the investor may reconsider to invest, the policy regime is -

* "learning" means that the probability of type of regime is updated before the government's announcement at time T_D

Figure 3.1 Overview of specific cases I and II uncertain and in the second case only expropriation is considered. Cases I and II are obtained by selecting a given combination of the regime parameters, see the parameter specifications given in Figure 3.1. For both cases I focus on the situations where either the investor does not obtain any information about the type of regime by observing the level of the risk index, or the situation where the regulatory regime is completely determined by the level of the risk index.

3.2 A General Model

3.2.1 Commitment Value Today, Time *t*

Let the required rate of return from holding an asset with dynamic ex-dividend price behavior given by equation (1) for the next increment of time be $\mu_S dt$. The present value at time t of a payment equal to the oil price at time T, $t \leq T$, is then

$$V_t[S_T] = e^{-\mu_S(T-t)}E_t[S_T] = S_t e^{-(\mu_S-\alpha_S)(T-t)} = S_t e^{-\delta_S(T-t)} \quad . \tag{7}$$

The difference between the required rate of return and the actual drift of the asset is known as the drift adjustment, net convenience yield, or rate of return shortfall, δ_S, where $\delta_S \equiv \mu_S - \alpha_S$. For the case of a common stock, a positive convenience yield corresponds to a constant pay-out rate of dividends proportional to the stock price. If the oil price is deterministic, i.e., $\sigma_S = 0$, the drift adjustment is $\delta_S \equiv r - \alpha_S$. The required gain from holding the asset (sum of capital appreciation and dividend payment) would have been equal to the risk free interest rate. The drift adjustment for the hypothetical asset Z_t, $\delta_{\pounds}$, is defined in exactly the same way, i.e., $\delta_{\pounds} \equiv \mu_{\pounds} - \alpha_{\pounds}$, where $\mu_{\pounds}$ is the required rate of return from holding the hypothetical asset and $\alpha_{\pounds}$ is the drift parameter of the underlying unobservable state variable.

The value at time T_D of the revenue from the project at time $T_{P|t}$, $T_D \leq T_{P|t}$, when the index level is known, but before the regulatory regime is announced, is

$$\begin{array}{ll} S_{T_D} e^{-\delta_S(T_{P|t}-T_D)}[\tau_G p_H + \tau_B(1-p_H)] & \text{if } \psi_{T_D} \geq \overline{\psi} \\ S_{T_D} e^{-\delta_S(T_{P|t}-T_D)}[\tau_G p_L + \tau_B(1-p_L)] & \text{if } \psi_{T_D} < \overline{\psi} \quad . \end{array} \tag{8}$$

The future oil revenue is discounted at the rate of return shortfall for the oil price to time T_D and then weighted by the probability of a good or bad policy regime conditioned on the index level. I assume that the remaining uncertainty, represented by p_H and p_L, is unsystematic and does not require any risk compensation beyond the risk free interest rate. Similarly, the value of the production costs at time T_D when the index level is known, but before the specific policy regime is announced, is

$$\begin{array}{ll} K_{T_{P|t}} e^{-r(T_{P|t}-T_D)}[\gamma_G^{(K)} p_H + \gamma_B^{(K)}(1-p_H)] & \text{if } \psi_{T_D} \geq \overline{\psi} \\ K_{T_{P|t}} e^{-r(T_{P|t}-T_D)}[\gamma_G^{(K)} p_L + \gamma_B^{(K)}(1-p_L)] & \text{if } \psi_{T_D} < \overline{\psi} \end{array} . \tag{9}$$

The value at time t of a claim paying the oil price at time T_D, $t \leq T_D$, if and only if the risk index at time T_D is equal to, or higher than the critical index level $\overline{\psi}$, is $S_t e^{-\delta_S(T_D-t)} N[a(\psi_t, \overline{\psi}, T_D)]$, where[10 11]

$$a(\psi_t, \overline{\psi}, T_D) = \frac{\ln(Z/\overline{Z}) + (r - \delta_£ + \rho\sigma_£\sigma_S - \frac{1}{2}\sigma_£^2)(T_D - t)}{\sigma_£\sqrt{T_D - t}} . \tag{10}$$

The price of the hypothetical asset at time t, Z_t, is found by applying equation (4), i.e., the price is deduced from a risk index by assuming a numerical value of the parameter σ_v. The value of the hypothetical asset making the index equal to the critical level $\overline{\psi}$ is $\overline{Z}$.

The value at time t of a claim paying one unit of money at time T_D if the index is not below the critical index level, and zero otherwise, is $e^{-r(T_D-t)} N[b(\psi_t, \overline{\psi}, T_D)]$, where[10 11]

$$b(\psi_t, \overline{\psi}, T_D) = \frac{\ln(Z/\overline{Z}) + (r - \delta_£ - \frac{1}{2}\sigma_£^2)(T_D - t)}{\sigma_£\sqrt{T_D - t}} . \tag{11}$$

[10] These formulas are derived in Appendix 2 of chapter three in this dissertation.

[11] It is an abuse of notation to use the index level ψ_t on the LHS of the equation while the price of the hypothetical asset, Z_t is used on the RHS. I have settled for this notation in order to emphasize the importance of the level of the directly observable risk index in the pricing formulas.

The commitment value of the investment at time t, C_t, is the value of the investment without any operational flexibility, such as the option to temporarily stop production, and without the option to abandon the investment. The value at time t of the investment commitment is derived by finding the value of the production costs and sales revenue at time T_D given by equations (8) and (9) and by assuming that the scaling of the investment expenditure at time t is $\gamma_t^{(I)}$, i.e.,

$$C_t = -I_t\gamma_t^{(I)} - e^{-r(T_{P|t}-t)}K_{T_{P|t}}\mathrm{G}_t^{(\mathrm{K})}(\psi_t,\overline{\psi},p,\gamma^{(K)}) + S_t e^{-\delta_S(T_{P|t}-t)}\mathrm{F}_t(\psi_t,\overline{\psi},p,\tau) \quad , \tag{12}$$

where

$$\mathrm{F}_t(\psi_t,\overline{\psi},p,\tau) = \tau_B + (\tau_G - \tau_B)p_L + (\tau_G - \tau_B)(p_H - p_L)\mathrm{N}[a(\psi_t,\overline{\psi},T_D)] \tag{13}$$

and

$$\mathrm{G}_t^{(\mathrm{K})}(\psi_t,\overline{\psi},p,\gamma^{(K)}) = \gamma_B^{(K)} + (\gamma_G^{(K)} - \gamma_B^{(K)})p_L + (\gamma_G^{(K)} - \gamma_B^{(K)})(p_H - p_L)\mathrm{N}[b(\psi_t,\overline{\psi},T_D)] \ . \tag{14}$$

The functions F_t and $\mathrm{G}_t^{(\mathrm{K})}$ represent the political investment climate at time t. From equation (12), the break-even spot price of oil is S_t^{BE}, where

$$S_t^{BE} = \frac{I_t\gamma_i^{(I)} + e^{-r(T_{P|t}-t)}K_{T_{P|t}}\mathrm{G}_t^{(\mathrm{K})}(\psi_t,\overline{\psi},p,\gamma^{(K)})}{e^{-\delta_S(T_{P|t}-t)}\mathrm{F}_t(\psi_t,\overline{\psi},p,\tau)} \quad . \tag{15}$$

We see that the break-even spot price is conditioned on the risk index at time t. The investment rule when the choice is whether to invest today, i.e., at time t, or never, is to invest if $C_t \geq 0$. Alternatively, because C_t is monotonically increasing as a function of S_t, the investor will invest if $S_t \geq S_t^{BE}$. When the investor has the opportunity to decide whether to invest today or to defer the investment decision to a given time T_W, the value maximizing investment rule is to invest at time t if the value of investing is at least as high as the value of waiting, i.e., $C_t \geq W_t$, where W_t is the current value of the deferred investment opportunity. If

W_t is an increasing function of the oil price such that $W_t > C_t$ for $S_t < S_t^W$ and $W_t < C_t$ for $S_t > S_t^W$, the optimal investment policy may be reformulated in terms of the oil price. When the oil price reaches a certain level, the value of investing now will be equal to the value of the deferred investment opportunity. The optimal investment policy is then to invest at time t if $S_t \geq S_t^W$ where S_t^W is the break-even price making $C_t = W_t$.

The relative difference between S_t^W and S_t^{BE} is a measure of how much the price of oil must exceed the break-even price for immediate accept/reject in order for investment to take place, where

$$H_t \equiv \frac{S_t^W - S_t^{BE}}{S_t^{BE}} \ . \tag{16}$$

Other terms used in the literature expressing the relationship between S_t^W and S_t^{BE} are "investment threshold", "investment hurdle", or "flexibility factor"[12].

3.2.2 Value Today of the Deferred Investment Opportunity when $T_W \geq T_D$

The investor may now defer the investment decision to a date when the political uncertainty is resolved, i.e., $T_W \geq T_D$. The commitment value at time T_W is then dependent on the type of regulatory regime which has been chosen by the government., i.e., $C_{T_W} \in \{C_{T_W}^G, C_{T_W}^B\}$, where

$$C_{T_W}^i = -I_{T_W}\gamma_i^{(I)} - e^{-r(T_{P|T_W}-T_W)}K_{T_{P|T_W}}\gamma_i^{(K)} + S_{T_W}e^{-\delta_S(T_{P|T_W}-T_W)}\tau_i \ , \quad i = G,B \ . \tag{17}$$

The corresponding break-even price for oil, $S_{T_W}^{BE,i}$, is also dependent on the regime of type I,

$$S_{T_W}{}^{BE,i} = \frac{I_{T_W}\gamma_i^{(I)} + e^{-r(T_{P|T_W}-T_W)}K_{T_{P|T_W}}\gamma_i^{(K)}}{e^{-\delta_S(T_{P|T_W}-T_W)}\tau_i} \ , \quad i = G,B \ . \tag{18}$$

Note that the level of the risk index is not *directly* included in (17) or (18). The investor will

[12] The flexibility factor in Bjerksund and Ekern (1990) p. 74, is equal to S_t^W/S_t^{BE}.

choose to invest at time T_W if the value of the investment commitment is nonnegative, i.e., the value of the investment opportunity at time T_W is

$$W_{T_W}^i = \text{Max}[C_{T_W}^i, 0] \ , \quad i = G,B \ . \tag{19}$$

The value at time t of an asset with value given by (19) is

$$W_t^i = \tau_i e^{-\delta_S(T_{P|T_W} - T_W)} [S_t e^{-\delta_S(T_W - t)} \text{N}[c(S_t, S_{T_W}^{BE,i}, T_W)] - e^{-r(T_W - t)} S_{T_W}^{BE,i} \text{N}[d(S_t, S_{T_W}^{BE,i}, T_W)]], \tag{20}$$

I=*G*,*B*, where

$$c(S_t, S_{T_W}^{BE,i}, T_W) = \frac{\ln(S_t / S_{T_W}^{BE,i}) + (r - \delta_S + \frac{1}{2}\sigma_S^2)(T_W - t)}{\sigma_S \sqrt{T_W - t}} \tag{21}$$

and

$$d(S_t, S_{T_W}^{BE,i}, T_W) = \frac{\ln(S_t / S_{T_W}^{BE,i}) + (r - \delta_S - \frac{1}{2}\sigma_S^2)(T_W - t)}{\sigma_S \sqrt{T_W - t}} \ . \tag{22}$$

The term in brackets on the right hand side (RHS) of (20) is the familiar formula for the value of a European call option with exercise price $S_{T_W}^{BE,i}$ and where the underlying asset is paying a constant proportional rate of dividend.

At time T_D, when the level of the index is known but the type of regime is not announced, the value of the wait-alternative is given by

$$\begin{aligned} W_{T_D} &= W_{T_D}^G p_H + W_{T_D}^B (1 - p_H) \quad \text{if } \psi_{T_D} \geq \overline{\psi} \\ W_{T_D} &= W_{T_D}^G p_L + W_{T_D}^B (1 - p_L) \quad \text{if } \psi_{T_D} < \overline{\psi} \ , \end{aligned} \tag{23}$$

i.e., the value is conditioned on the level of the index. The value at time t of an asset with payoff at time T_D equal to $W_{T_D}^i$ if the index value is not lower than the critical level of the index, and zero otherwise, is

$$W_t^{i|\psi_{T_D}\geq\overline{\psi}} = \tau_i e^{-\delta_S(T_{P|T_W}-T_W)}[S_t e^{-\delta_S(T_W-t)}N_2[a(\psi_t,\overline{\psi},T_D),c(S_t,S_{T_W}^{BE,i},T_W);\lambda] - e^{-r(T_W-t)}S_{T_W}^{BE,i}N_2[b(\psi_t,\overline{\psi},T_D),d(S_t,S_{T_W}^{BE,i},T_W);\lambda]] \ , \ i=G,B \ , \tag{24}$$

where $N_2[\cdot,\cdot;\lambda]$ is the distribution function for the bivariate standard normal distribution with coefficient of correlation λ, given by:

$$\lambda = \rho\sqrt{\frac{T_D-t}{T_W-t}} \ . \tag{25}$$

The valuation formula is derived in Appendix 1. The value of an asset at time t paying $W_{T_D}^i$ at time T_D conditioned on the index being lower than the critical index level $\overline{\psi}$ is, by value additivity,

$$W_t^{i|\psi_{T_D}<\overline{\psi}} = W_t^i - W_t^{i|\psi_{T_D}\geq\overline{\psi}} \ . \tag{26}$$

By combining equations (20), (23), (24), (26), and by rearranging terms, we get the value at time t of the investment opportunity at time T_W:

$$W_t = W_t^B + (W_t^G - W_t^B)p_L + (W_t^{G|\psi_{T_D}\geq\overline{\psi}} - W_t^{B|\psi_{T_D}\geq\overline{\psi}})(p_H - p_L) \ . \tag{27}$$

3.2.3 Value Today of the Deferred Investment Opportunity when $T_W < T_D$

The investor may now defer the investment decision to a date where the political uncertainty is not resolved, i.e., $T_W < T_D$. The commitment value C_{T_W} will be now be given by (12), where T_W replaces t. The value of the wait-alternative is

$$W_{T_W} = \text{Max}[C_{T_W},0] \ , \tag{28}$$

i.e., the value of the deferred investment opportunity is not conditioned on the type of regime (no superscript I appears in (28)). The value today, i.e., at time t, of the alternative to defer the investment decision cannot generally be given by a closed-form valuation formula, because C_{T_W} is determined by two variables: the oil price and the risk index. With a low index level relative to the critical level of the index, i.e., a relatively high probability of a bad regime, the oil price must be high for the commitment value to be nonnegative. If the opposite is the case, i.e., the index level is relatively high, the break-even spot price of oil is reduced. An example is shown in Figure 3.2. If it is certain that the "bad" regulatory regime will apply, the break-even price is S^{**}. With an increasing level of the risk index, a "good" regulatory regime becomes more probable and the break-even price is reduced. When it is certain that the "good" regulatory regime will apply, the break-even price is S^{*}.

Closed-form formulas can, however, be obtained if $p_H = p_L = p$ (the index is uninformative regarding the future regime) or when the oil price is deterministic.

If no information about the type of regime can be obtained by observing the risk index, the closed-form valuation formula can be found by using expected regulation and deriving a formula, which now will be depend only on the oil price.

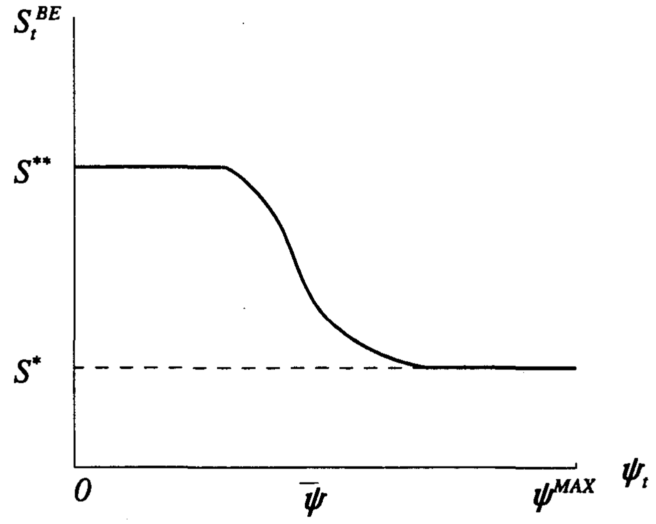

Figure 3.2 Example of the relationship between the risk index and the break-even spot price of oil, p_H=1, p_L=0

If the oil price is deterministic, i.e., $\sigma_S = 0$, the value of the deferred investment opportunity is

$$W_t = -e^{-r(T_W-t)} I_{T_W} \mathrm{N}[b(\psi_t, \hat{\psi}, T_W)] - e^{-r(T_{P|T_W}-t)} K_{T_{P|T_W}} \hat{\mathrm{G}}(\psi_t, \hat{\psi}, \overline{\psi}, p, \gamma^{(K)}) + \\ e^{(\alpha-r)(T_{P|T_W}-t)} S_t \hat{\mathrm{F}}(\psi_t, \hat{\psi}, \overline{\psi}, p, \tau) \ , \tag{29}$$

where

$$b(\psi_t, \hat{\psi}, T_D) = \frac{\ln(Z_t/\hat{Z}) + (r - \delta_{\pounds} - \frac{1}{2}\sigma_{\pounds}^2)(T_D - t)}{\sigma_{\pounds}\sqrt{T_D - t}} \ , \tag{30}$$

$$\hat{\mathrm{F}}_t(\psi_t, \hat{\psi}, \overline{\psi}, p, \tau) = (\tau_B + (\tau_B - \tau_G) p_L) \mathrm{N}[b(\psi_t, \hat{\psi}, T_W)] + \\ (\tau_G - \tau_B)(p_H - p_L) \mathrm{N}_2[b(\psi_t, \overline{\psi}, T_D), b(\psi_t, \hat{\psi}, T_W); \hat{\lambda}] \ , \tag{31}$$

$$\hat{\mathrm{G}}_t(\psi_t, \hat{\psi}, \overline{\psi}, p, \gamma^{(K)}) = (\gamma_B^{(K)} + (\gamma_G^{(K)} - \gamma_B^{(K)}) p_L) \mathrm{N}[b(\psi_t, \hat{\psi}, T_W)] + \\ (\gamma_G^{(K)} - \gamma_B^{(K)})(p_H - p_L) \mathrm{N}_2[b(\psi_t, \overline{\psi}, T_D), b(\psi_t, \hat{\psi}, T_W); \hat{\lambda}] \ , \tag{32}$$

and,

$$\hat{\lambda} = \sqrt{\frac{T_W - t}{T_D - t}} \ . \tag{33}$$

The valuation formula is derived in Appendix 1. The index level which makes the investment commitment nonnegative at time T_W is $\hat{\psi}$. The corresponding value of the hypothetical asset, $\hat{Z}$, is found by applying equation (4).

3.3 Specific Case I: An Uncertain Royalty Rate Only

3.3.1 No Learning Before Time T_D

By inserting for $p_H = p_L = p$ and $\gamma_t^{(j)} = \gamma_G^{(j)} = \gamma_B^{(j)} = 1$ for $j = I,K$ in (12), we find the value of the investment commitment at time t:

$$C_t = -I_t - e^{-r(T_{P|t}-t)} K_{T_{P|t}} + S_t e^{-\delta_S(T_{P|t}-t)} \mathrm{E}[\tau_i] \ , \tag{34}$$

where $\mathrm{E}[\tau_i] = \tau_G p + \tau_B (1-p)$.

When $T_W \geq T_D$, the break-even spot price for oil at time T_W when the royalty rate is known is

$$S_{T_W}^{BE,i} = \frac{I_{T_W} + e^{-r(T_{P|T_W}-T_W)} K_{T_{P|T_W}}}{e^{-\delta_S(T_{P|T_W}-T_W)} \tau_i} \ , \quad i = G,B \tag{35}$$

and the value of the option to wait, from (27) is

$$W_t = W_t^G p + W_t^B (1-p) \ . \tag{36}$$

When $T_W < T_D$, the value of the option to wait is: (comparable to equation (20) with a royalty rate equal to $\mathrm{E}[\tau_i]$)

$$\begin{aligned} W_t = W_t^{\mathrm{E}[\tau_i]} = e^{-\delta_S(T_{P|T_W}-T_W)} \mathrm{E}[\tau_i] \, [e^{-\delta_S(T_W-t)} S_t^{\;W} \mathrm{N}[c(S_t, S_{T_W}^{BE,\mathrm{E}[\tau_i]}, T_W)] - \\ e^{-r(T_W-t)} S_{T_W}^{BE,\mathrm{E}[\tau_i]} \mathrm{N}[d(S_t, S_{T_W}^{BE,\mathrm{E}[\tau_i]}, T_W)] \ , \end{aligned} \tag{37}$$

where (from (34) where T_W replaces t)

$$S_{T_W}^{BE,\mathrm{E}[\tau_i]} = \frac{I_{T_W} + e^{-r(T_{P|T_W}-T_W)} K_{T_{P|T_W}}}{e^{-\delta_S(T_{P|T_W}-T_W)} \mathrm{E}[\tau_i]} \ . \tag{38}$$

Proposition 1 *For the model with no political uncertainty, i.e., $p=0$ or $p=1$, when $T_D \le T_{P|t}$ and $T_D \le T_{P|T_W}$, the investment threshold H_t is not influenced by the level of the royalty rate $(1-\tau_i)$, for $\tau_i \in \langle 0,1]$.*

Proof. Because $p=0$ or $p=1$, it does not matter whether $T_W \ge T_D$ or $T_W < T_D$. Equation (35) is equal to (38) and (36) is equal to (37). Assume that the royalty rate today is $(1-\tau_i)$. By proposition A1 in appendix 2, S_t^W is the unique spot price of oil making the equation

$$e^{-\delta_S(T_{P|T_W}-T_W)}\tau_i[e^{-\delta_S(T_W-t)}S_t^W \mathrm{N}[c(S_t^W,S_{T_W}^{BE,i},T_W)]-e^{-r(T_W-t)}S_{T_W}^{BE}\,\mathrm{N}[d(S_t^W,S_{T_W}^{BE,i},T_W)]]=$$
$$e^{-\delta_S(T_{P|t}-t)}\tau_i S_t^W - I_t - K_t e^{-r(T_{P|t}-t)}$$

hold. The value with waiting on the LHS is given by (36), i.e., by (20) because $p=0$ or $p=1$. The commitment value on the RHS is given by (34). I divide this equation by $e^{-\delta_S(T_{P|t}-t)}\tau_i$, use the assumption that $(T_{P|t}-t)=(T_{P|T_W}-T_W)$, and use the definition of S_t^{BE} given by (15) to get

$$e^{-\delta_S(T_W-t)}S_t^W \mathrm{N}[c(S_t^W,S_{T_W}^{BE,i},T_W)]-e^{-r(T_W-t)}S_{T_W}^{BE}\mathrm{N}[d(S_t^W,S_{T_W}^{BE,i},T_W)]=S_t^W-S_t^{BE}\quad .$$

If a new royalty rate, $\bar{\tau}$, is considered where $\bar{\tau}=\tau_i/k$ for a positive constant *k*, the new break even price at time *t* will be $\bar{S}_t^{BE}=S_t^{BE}k$ and the new break even price at time T_W is $\bar{S}_{T_W}^{BE}=kS_{T_W}^{BE}$:

$$kS_{T_W}^{BE,i}=k\frac{1}{\tau_i}\frac{I_{T_W}+e^{-r(T_{P|T_W}-T_W)}K_{T_{P|T_W}}}{e^{-\delta_S(T_{P|T_W}-T_W)}}=\frac{1}{\frac{1}{k}\tau_i}\frac{I_{T_W}+e^{-r(T_{P|T_W}-T_W)}K_{T_{P|T_W}}}{e^{-\delta_S(T_{P|T_W}-T_W)}}\ .$$

By inspecting the equation where $W_t=C_t$, we see that it is sufficient for this equation to hold that the prices, S_t^W and $S_{T_W}^{BE}$, on the LHS and the RHS is multiplied by *k* (only the relative relationship between S_t^W and $S_{T_W}^{BE}$ matters in $c(\cdot,\cdot,\cdot)$ and $d(\cdot,\cdot,\cdot)$), i.e.,

$$k(e^{-\delta_S(T_W-t)}S_t^W \mathrm{N}[c(kS_t^W, kS_{T_W}^{BE,i}, T_W)] - e^{-r(T_W-t)}S_{T_W}^{BE}\mathrm{N}[d(kS_t^W, kS_{T_W}^{BE,i}, T_W)]] = (S_t^W - S_t^{BE})k \quad .$$

A change in τ_i will of course affect both S_t^{BE} and S_t^W, but H_t will be unchanged, see equation (16).

■

Proposition 2 *For the model with no learning before time T_W and where $T_W \geq T_D$, the investment threshold when there is uncertainty regarding the royalty rate ($p \in \langle 0,1 \rangle$) is not lower than the investment threshold when there is certainty (p=0 or p=1).*

Proof. Assume that $\hat{\tau} = p\tau_G + (1-p)\tau_B$. Let $\nu(S_t, A, T_W)$ be the value at time t of a European call option on the spot price of oil with current oil price equal to S_t, strike equal to A, and maturity at time T_W. Consider now a situation with no uncertainty regarding the future royalty rate. The royalty rate will be $(1-\hat{\tau})$. Let $\hat{S}_t$ be the break-even price making the value of investing now equal to the value of waiting, i.e.,

$$e^{-\delta_S(T_{P|T_W}-T_W)}\hat{\tau}\,\nu(\hat{S}_t, S_{T_W}^{BE,\hat{\tau}}, T_W) = e^{-\delta_S(T_{P|t}-t)}\hat{\tau}\hat{S}_t - I_t - K_{T_{P|t}}e^{-r(T_{P|t}-t)} \tag{39}$$

where $\hat{S}_{T_W}^{BE,\hat{\tau}}$ is the break-even price for the investment opportunity at time T_W. According to proposition one, the investment threshold H_t in this situation is the same as if it were certain that royalty rate $(1-\tau_G)$ or $(1-\tau_B)$ would apply. If the value of waiting in case of uncertainty, i.e., when $p \in \langle 0,1 \rangle$, is strictly higher than the LHS of (39), the implication would be that $\hat{S}_t$ is lower than S_t^W, the break-even price in case of uncertainty. According to equations (36) and (20), the value of the wait-alternative is

$$e^{-\delta_S(T_{P|T_W}-T_W)}[p\tau_G \nu(S_t,, S_{T_W}^{BE,G}, T_W) + (1-p)\tau_B \nu(S_t, S_{T_W}^{BE,B}, T_W)]$$

or

$$\hat{\tau}\, e^{-\delta_S(T_{P|T_W}-T_W)}[q\nu(S_t, S_{T_W}^{BE,G}, T_W) + (1-q)\nu(S_t, S_{T_W}^{BE,B}, T_W)],$$

where $q = p\tau_G / \hat{\tau}$. Note that

$$\hat{\tau}(1-q) = [p\tau_G + (1-p)\tau_B](1 - \frac{p\tau_G}{p\tau_G + (1-p)\tau_B}) = p\tau_G + (1-p)\tau_B - p\tau_G = (1-p)\tau_B.$$

Note further that $S_{T_W}^{BE,\hat{\tau}} = q S_{T_W}^{BE,G} + (1-q) S_{T_W}^{BE,B}$, i.e., the break-even price at time T_W if royalty rate $(1-\hat{\tau})$ applies is a weighted sum of the break-even prices in case of royalty rates $(1-\tau_G)$ or $(1-\tau_B)$, respectively, where the weights are determined by q. We know[13] that

$$\nu(S_t, S_{T_W}^{BE,\hat{\tau}}, T_W) \leq \nu(S_t, S_{T_W}^{BE,G}, T_W) q + \nu(S_t, S_{T_W}^{BE,B}, T_W)(1-q),$$

i.e., the values of three identical call options with three different exercise prices are such that the weighted sum of the values of the two call options with extreme exercise prices is not lower than the value of the call option with the middle exercise price, where the weights q and (1-q) are such that $S_{T_W}^{BE,\hat{\tau}} = q S_{T_W}^{BE,G} + (1-q) S_{T_W}^{BE,B}$.

■

3.3.2 No Remaining Uncertainty at Time T_D

By inserting for $p_H = 1$, $p_L = 0$, and $\gamma_t^{(I)} = \gamma_G^{(j)} = \gamma_B^{(j)} = 1$ for $j = I, K$ in (12), I find the commitment value at time t

$$C_t = -I_t - e^{-r(T_P - t)} K_{T_{P|t}} + S_t e^{-\delta_S(T_{P|t} - t)} [\tau_B + (\tau_G - \tau_B) \mathrm{N}[a(\psi_t, \overline{\psi}, T_D)]] \ . \tag{40}$$

When $T_W \geq T_D$, the value of the option to wait, from (27), is

$$W_t = W_t^B + W_t^{G|\psi_{T_D} \geq \overline{\psi}} - W_t^{B|\psi_{T_D} \geq \overline{\psi}} \ . \tag{41}$$

When $T_W < T_D$, the value of the option to wait is

$$W_t = \mathrm{V}_t[\mathrm{Max}[C_{T_W}, 0]] \ , \tag{42}$$

[13] See proposition 2.c on page 133 in Cox and Rubinstein (1985).

where $V_t[\cdot]$ is the value operator at time *t*. If the oil price is deterministic, i.e., $\sigma_S = 0$, the valuation formula is, from (29),

$$W_t = -e^{-r(T_W-t)} I_{T_W} \mathrm{N}[b(\psi_t, \psi, T_W)] - e^{-r(T_{P|T_W}-t)} K_{T_{P|T_W}} \mathrm{N}_2[b(\psi_t, \psi, T_W)] + \\ S_t e^{(\alpha-r)(T_{P|T_W}-t)} (\tau_B \mathrm{N}[b(\psi_t, \psi, T_W)] + (\tau_G - \tau_B) \mathrm{N}_2[b(\psi_t, \overline{\psi}, T_D, b(\psi_t, \psi, T_W); \hat{\lambda}]) \; . \tag{43}$$

3.4 Specific Case II: Possibility of Expropriation Only

3.4.1 No Learning Before Time T_D

By inserting for $p_H = p_L = p$, $\gamma_B^{(I)} = \gamma_B^{(K)} = \tau_B = 0$, and, $\gamma_t^{(I)} = \gamma_G^{(I)} = \gamma_G^{(K)} = \tau_G = 1$ in (12), we find the value of the investment commitment at time *t*:

$$C_t = -I_t - e^{-r(T_{P|t}-t)} K_{T_{P|t}} p + S_t e^{-\delta_S(T_{P|t}-t)} p \; . \tag{44}$$

When $T_W \geq T_D$, the break-even spot price for oil at time T_W, *if no expropriation occurs*, is

$$S_{T_W}^{BE,G} = \frac{I_{T_W} + e^{-r(T_{P|T_W}-T_W)} K_{T_{P|T_W}}}{e^{-\delta_S(T_{P|T_W}-T_W)}} \tag{45}$$

and the value of the option to wait, from (27), is

$$W_t = W_t^G p \; , \tag{46}$$

i.e., the value of a call option multiplied by the probability p. Note that the value of the investment opportunity *if expropriation occurs* is zero.

For the case when $T_W < T_D$, the value of the deferred investment opportunity is

$$W_t = e^{-\delta_S(T_{P|T_W}-T_W)} p [e^{-\delta_S(T_W-t)} S_t \mathrm{N}[c(S_t, S_{T_W}^{BE}, T_W)] - e^{-r(T_W-t)} S_{T_W}^{BE} \mathrm{N}[d(S_t, S_{T_W}^{BE}, T_W)]] \; , \tag{47}$$

where

$$S_{T_W}^{BE} = \frac{I_{T_W} + e^{-r(T_{P|T_W} - T_W)} K_{T_{P|T_W}} p}{e^{-\delta_S(T_{P|T_W} - T_W)} p} \ . \tag{48}$$

Equation (48) is found by inserting T_W for t in (44), letting C_{T_W} be equal to zero and solving for S_{T_W}.

Proposition 3 *For the case with a possibility of expropriation and no learning before time T_D, if $T_D = T_{P|t}$ $\forall$ t, $I_t = I_{T_W}$, and $K_{T_{P|t}} = K_{T_{P|T_W}}$, the investment threshold H_t is not influenced by the probability of expropriation for $p \in \langle 0,1]$.*

Proof. The proof follows the same line of argument as the proof of proposition one. Uniqueness of S_t^W is given by proposition A3 in appendix 2. Suppose p is reduced such that S_t^{BE} is increased by a multiplicative factor k. The spot price of oil making the value of waiting equal to the value of investing can be increased by a factor equal to k, and the investment threshold, H_t, is therefore not changed.

■

3.4.2 No Remaining Uncertainty at Time T_D

Here I assume that $p_H = 1$ and $p_L = 0$. The commitment value at time t is

$$C_t = -I_t - e^{-r(T_{P|t} - t)} K_{T_{P|t}} \mathrm{N}[b(\psi_t, \overline{\psi}, T_D)] + S_t e^{-\delta_S(T_{P|t} - t)} \mathrm{N}[a(\psi_t, \overline{\psi}, T_D)] \ . \tag{49}$$

When $T_W \geq T_D$, the value of the option to wait, from (27), is

$$W_t = W_t^{G|\psi_{T_D} \geq \overline{\psi}} \ , \tag{50}$$

i.e., the value of a call option conditioned on the future level of the risk index.

When $T_W < T_D$, the value of the option to wait when the oil price is deterministic is

$$W_t = -e^{-r(T_W-t)} I_{T_W} \mathrm{N}[b(\psi_t, \underline{\psi}, T_W)] - e^{-r(T_{P|T_W}-t)} K_{T_{P|T_W}} \mathrm{N}_2[b(\psi_t, \overline{\psi}, T_D), b(\psi_t, \underline{\psi}, T_W); \hat{\lambda}]$$
$$S_t e^{(\alpha - r)(T_{P|T_W}-t)} \mathrm{N}_2[b(\psi_t, \overline{\psi}, T_D), b(\psi_t, \underline{\psi}, T_W); \hat{\lambda}] \ . \qquad \textbf{(51)}$$

4 Numerical Examples

4.1 Overview of Examples

Figure 4.1 summarizes the examples to be considered. I look at the special cases where there is uncertainty regarding the royalty rate and where the investment may be expropriated. In the latter case I only consider the situation where the government's decision to expropriate is made when the investment period is finished. For the cases with an uncertain royalty rate I consider the situation where the royalty rate is determined at a fixed calendar date. I further

	No learning* is possible	No remaining uncertainty at T_D
Uncertain royalty rate	*Sub-section 4.2.1* The date (T_D) when the political uncertainty is resolved is a fixed calendar date	*Sub-section 4.2.2* The date (T_D) when the political uncertainty is resolved is a fixed calendar date
Possibility of expropriation	*Sub-section 4.3.1* The date (T_D) when the political uncertainty is resolved is the date when production starts	*Sub-section 4.3.2* The date (T_D) when the political uncertainty is resolved is the date when production starts

* "learning" means that the probability of type of regime is updated before the government's announcement at time T_D

Figure 4.1 Overview of examples.

Political and oil price uncertainty		Political uncertainty only
$r=0.02$	$\psi^{MAX}=100$	$\sigma_S=0$
$\delta_S=0.04$	$\tau_G=1$	$\alpha_S=0$
$\sigma_S=0.245$	$I=5$	$\mu_S=r=0.02$
$\delta_{\hat{x}}=0.0$	$K=6$	$\delta_S=\mu_S-\alpha_S=0.02$
$\sigma_{\hat{x}}=0.1$	$T_D-t=4$	
$\underline{\psi}=50$	$T_{P\mid t}-t=5$ for all t	

Table 4.1 Assumptions for the reference examples

look at situations where the investor cannot condition the probability of type of regime on the index level, and at situations where the type of regime is completely determined by the future index level. The purpose of the examples is to examine how the presence of political uncertainty influences the investment threshold in some relevant decision situations and to demonstrate the use of the general valuation model developed in section three. I have treated the case with an uncertain royalty rate more comprehensively than the case of a possible expropriation. This is done in order to focus on the effect of "watershed events" taking place at fixed calendar dates.

The assumptions of the examples are listed in Table 4.1. The royalty rate τ_B will be varied. The drift adjustment of the process for the deduced variable, $\delta_{\hat{x}}$, is zero and the critical level of the index is fifty. I have assumed that the standard deviation of the noise, σ_v, is one. A price of the hypothetical asset equal to one will correspond to an index level of fifty[14]. The development time is five years. For the cases where the political uncertainty is resolved at a fixed calendar date, the date is four years ahead, i.e., one year before the development period is completed if development starts today. In some examples I let the volatility of the oil price be equal to zero, i.e., the oil price is deterministic. In these cases I assume that the drift of the oil price is zero, i.e., $\alpha_S=0$, and the rate of return shortfall is then equal to the risk free interest rate.

4.2 Waiting with an Uncertain Royalty Rate Only

4.2.1 No learning before time T_D

[14] From (4): $100N[\ln(1)/1]=50$.

Consider first the situation where the investor waits until the fixed calendar date when the uncertainty regarding the royalty rate is resolved. For the case with a deterministic oil price and no learning before time T_D, the investment threshold as a function of the probability of a "good" policy regime, *p*, is shown in Figure 4.2. We note that the investment threshold is close to zero when *p* is close to zero or one. In these cases the political uncertainty is low.

We also note that for a given *p*, the investment threshold is increasing when the royalty rate in case of regime *B* is increased. When the highest royalty rate of thirty per cent is considered, i.e., $\tau_B = 0.7$, the threshold is increased considerably. The maximum investment threshold in this case is approx. 0.22. The investment threshold is "skewed to the right". One might perhaps expect that the investment threshold would be highest when the uncertainty regarding the type of regime is highest, i.e., when *p* is 0.5.

When the oil price is deterministic, an analytical expression for H_t may be obtained. The break-even price for accepting/reject the project today is

$$S_t^{BE} = \frac{I + e^{-r(T_{P|t}-t)}K}{\mathrm{E}[\tau_i]e^{-\delta_S(T_{P|t}-t)}} \ .$$

The oil price making the equation

$$-I - e^{-r(T_{P|t}-t)}K + \mathrm{E}[\tau_i]e^{-\delta_S(T_{P|t}-t)}S = pe^{-r(T_W-t)}[-I - e^{-r(T_{P|t}-t)}K + \tau_G e^{-\delta_S(T_{P|t}-t)}S]$$

hold, is S_t^W. The LHS of the above equation is the value of investing today and the RHS is today's value of the deferred investment opportunity. Note that the RHS only includes regime *G*, i.e., τ_G. The investor would never defer the investment decision if the value of the investment commitment was nonnegative in case of regime *B*, i.e., if τ_B is applying. By solving the above equation for S, we find

$$S_t^W = \frac{I(1 - pe^{-r(T_W-t)}) + K(e^{-r(T_{P|t}-t)} - pe^{-r(T_{P|T_W}-t)})}{\mathrm{E}[\tau_i]e^{-\delta_S(T_{P|t}-t)} - p\tau_G e^{-\delta_S(T_{P|T_W}-t)}} \ .$$

The investment threshold is

$$H_t = \frac{S_t^W}{S_t^{BE}} - 1 = \frac{I(1-pe^{-r(T_W-t)}) + K(e^{-r(T_{P|t}-t)} - pe^{-r(T_{P|T_W}-t)})}{\mathrm{E}[\tau_i]e^{-\delta_S(T_{P|t}-t)} - p\tau_G e^{-\delta_S(T_{P|T_W}-t)}} \frac{\mathrm{E}[\tau_i]e^{-\delta_S(T_{P|t}-t)}}{(I + e^{-r(T_{P|t}-t)}K)} - 1$$

$$= \frac{(I + e^{-r(T_{P|T_W}-T_W)}K)(1-pe^{-r(T_W-t)})}{\mathrm{E}[\tau_i]e^{-\delta_S(T_{P|t}-t)} - p\tau_G e^{-\delta_S(T_{P|T_W}-t)}} \frac{\mathrm{E}[\tau_i]e^{-\delta_S(T_{P|t}-t)}}{(I + e^{-r(T_{P|t}-t)}K)} - 1$$

which, because $(T_{P|t}-t) = (T_{P|T_W}-T_W)$,

$$= \frac{(1-pe^{-r(T_W-t)})\mathrm{E}[\tau_i]e^{-\delta_S(T_{P|t}-t)}}{\mathrm{E}[\tau_i]e^{-\delta_S(T_{P|t}-t)} - p\tau_G e^{-\delta_S(T_{P|T_W}-t)}} - 1,$$

or

$$= \frac{(1-pe^{-r(T_W-t)})\mathrm{E}[\tau_i]}{\mathrm{E}[\tau_i] - p\tau_G e^{-\delta_S(T_W-t)}} - 1 \ .$$

By noting that $\delta_S = r$ and by rearranging terms, I find the investment threshold for the given example:

$$H_t = \frac{e^{-r(T_W-t)}p(1-p)[\tau_G - \tau_B]}{\mathrm{E}[\tau_i] - e^{-r(T_W-t)}\tau_G p} \ . \tag{52}$$

The numerator is a product of three factors. The first represents the discounting effect of delaying the investment opportunity, $e^{-r(T_W-t)}$, the second is the variance of the indicator variable for regime G, $p(1-p)$, and the last factor in brackets is the range between the fraction the investor will keep under the two types of regimes. The numerator is maximized when p is 0.5, but when considering the effect of the denominator, the investment threshold will be "skewed to the right".

In Figure 4.3, I show the relationship between the time waited and the investment threshold when p is 0.5. The investor cannot learn anything by waiting to a date before T_D, i.e., the investment threshold is zero. When the investor waits until time T_D, we see that the

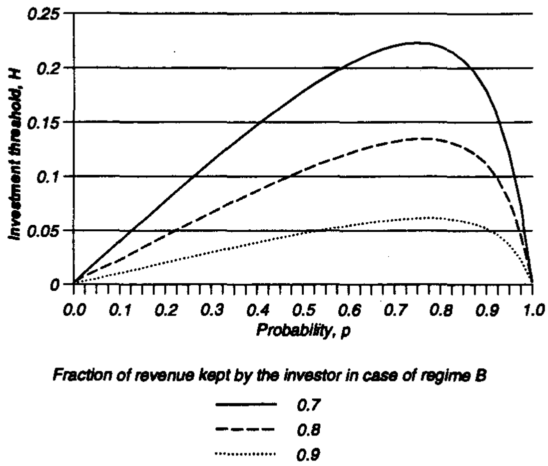

Figure 4.2 The investment threshold, H_r, with an uncertain royalty rate and no learning before time T_D. Deterministic oil price, $T_W = 4$

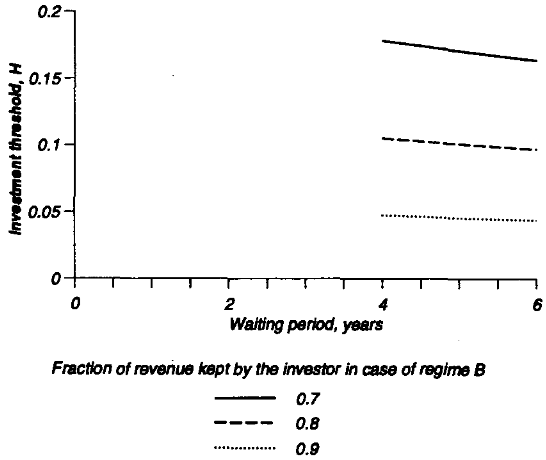

Figure 4.3 The investment threshold, H_r, with an uncertain royalty rate and no learning before time T_D. Deterministic oil price, $p = 0.5$

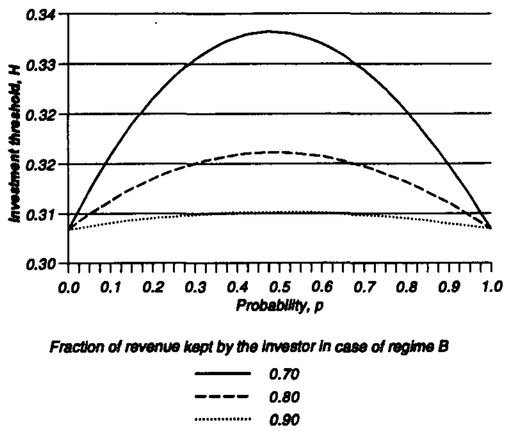

Figure 4.4 The investment threshold, H_t, with an uncertain royalty rate and no learning before time T_D. Stochastic oil price, $T_W = 4$

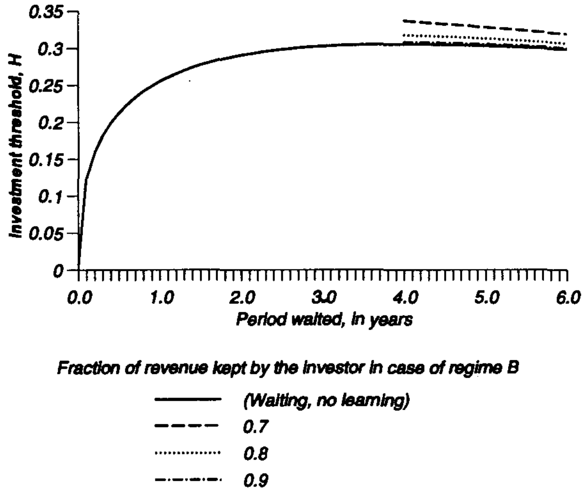

Figure 4.5 The investment threshold, H_t, with an uncertain royalty rate and no learning before time T_D. Stochastic oil price, $p = 0.5$

threshold is highest for the highest royalty rate, i.e., when $\tau_B = 0.7$. If the investor chooses to wait longer than time T_D, we see that the threshold is reduced as compared to the threshold when the investor waits until the date when the political uncertainty is resolved.

For the case with a stochastic oil price and no learning before time T_D, the investment threshold as a function of the probability of a "good" policy regime, *p*, is shown in Figure 4.4. With no political uncertainty, when the investor can defer the investment decision four years the investment threshold, H_t, is[15] 0.306. In order for an investment to take place, the spot price of oil must therefore be 30.6 per cent higher than the break-even oil price for the case when the investor has no possibility to wait.

We note that the investment threshold is close to the investment threshold in case of no political uncertainty when *p* is close to zero or one. We see from the figure that the investment threshold is highest when *p* is approximately 0.5. We also note that for a given *p*, the investment threshold is increasing when the royalty rate in case of regime *B* is increased. When the highest royalty rate of thirty per cent is considered, the investment threshold is increased from 0.306 to approx. 0.335. This is an increase of approx three percentage points. This does not seem to be a very large increase in the investment threshold compared to the situation with no political uncertainty.

In Figure 4.5 I show the relationship between time waited and the investment threshold. The solid line represents the investment threshold when the investor does not update the probability of type of regime at all. Four years ahead, the threshold is 0.306. In the case where the investor learns about the type of regime, the threshold will jump to a higher level exactly at time T_D, which is four years ahead.

For the situations where the uncertainty regarding the type of regime is resolved at the date when production may start, and when the investor cannot update the probability of type of regime, the investment threshold is zero for the case with a deterministic oil price. With a stochastic oil price, the ordinary investment threshold caused by a stochastic oil price only

15 This threshold can, e.g., be found by assuming that $\tau_B = 1$.

will apply. In this latter case the threshold is given by the solid line in Figure 4.5.

4.2.2 No Remaining Uncertainty at Time T_D

I first consider the case with a deterministic oil price. In Figure 4.6 I show the investment threshold as a function of the level of the risk index today, i.e., at time t when the investor can delay the investment decision four years. We see that the investment threshold is higher when the royalty rate is higher, i.e., when τ_B is lower. The critical level of the index is fifty, and the investment threshold is highest when the current index level is close to fifty. With a royalty rate of 0.3 in case of a bad regime, the highest investment threshold is approximately 0.22.

It is important to be aware of the fact that the uncertainty about the type of regime is here a function of the relation between today's level of the index and the critical level of the index, $\psi_t / \overline{\psi}$, the volatility of the deduced variable, $\sigma_{\hat{x}}$, and the time until disclosure, $(T_D - t)$. In Figure 4.7 I show the investment threshold as a function of today's index level for different volatilities of the underlying state variable. According to the "bad news principle" of Bernanke (1983), it is the possibility of receiving future bad news regarding the profitability of the investment which makes the investor wait instead of investing. As Bernanke notes on page 93: "..what irreversible investments is sensitive to is "downside" uncertainty". When the index level today is relatively high, say eighty, a good regime is more likely than a bad regime. When the volatility is increased, it becomes more probable[16] that a bad regime will be chosen by the government. It is therefore more probable that the investor will receive "bad news", i.e., that regime B will apply. The investment threshold will therefore increase.

We see that with a low index level, say thirty, an increase in the volatility increases the investment threshold. With a relatively low index today, i.e., low relative to the critical level of the index, a bad regime is more probable. But when the volatility is increased, the uncertainty about the future regime is also increased and regime G becomes more probable. At the same time the downside risk increases, and so does the investment threshold.

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[16] The probability that regime G will apply is dependent on the current level of the index relative to the critical index level and the change of the index during the period $(T_D - t)$:

$$P_t[\tau_i = \tau_G] = P_t[\psi_{T_D} \geq \overline{\psi}] = P_t[\psi_t + (\psi_{T_D} - \psi_t) \geq \overline{\psi}] = P_t[(\psi_{T_D} - \psi_t) \geq (\overline{\psi} - \psi_t)].$$

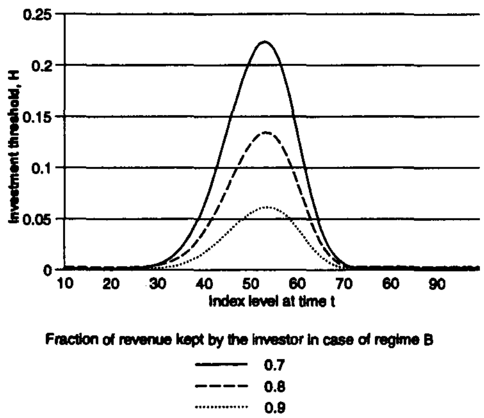

Figure 4.6 The investment threshold, H_t, with an uncertain royalty rate and no learning before time T_D. Deterministic oil price, T_W=4

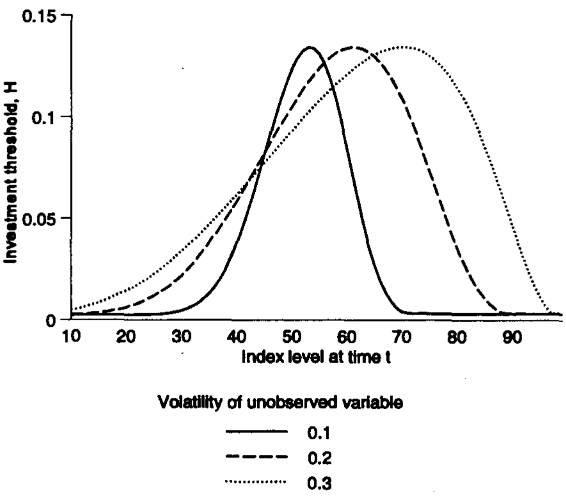

Figure 4.7 The investment threshold, H_t, with an uncertain royalty rate and no learning before time T_D. Deterministic oil price, T_W=4, τ_B=0.8

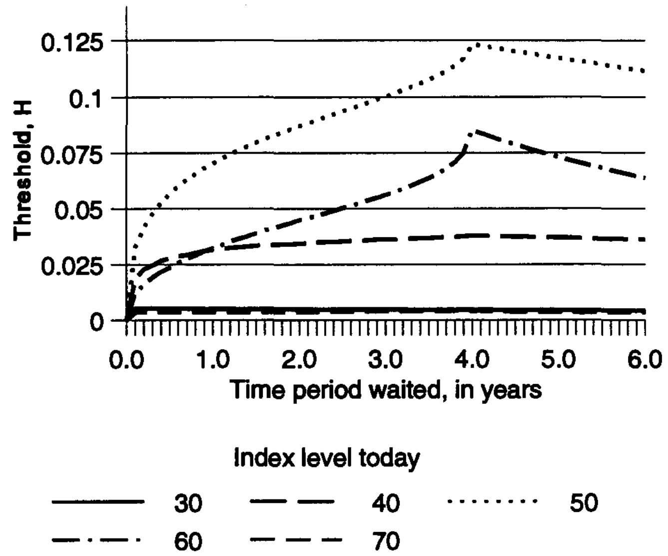

Figure 4.8 The investment threshold, H_t, when there is no remaining uncertainty at time T_D. Deterministic oil price, τ_B=0.8

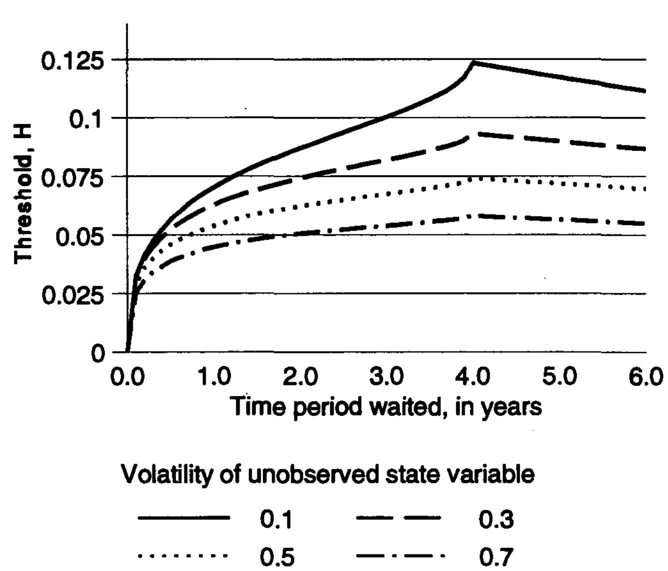

Figure 4.9 The investment threshold, H_t, when there is no remaining uncertainty at time T_D. Deterministic oil price, ψ_t=50, τ_B=0.8

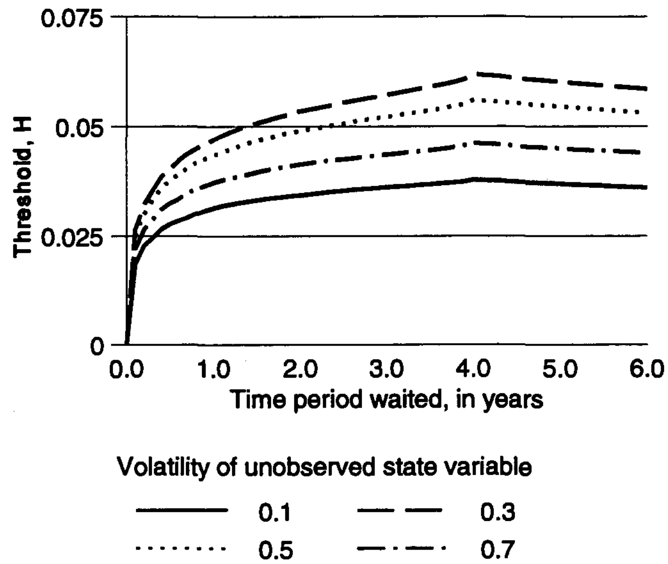

Figure 4.10 The investment threshold, H_t, when there is no remaining uncertainty at time T_D. Deterministic oil price, ψ_t=40, τ_B=0.8

When the index level today is close to the critical level, in this example fifty, the effect of an increase in volatility is opposite to the effect when the index levels today are either high or low. When the index level today is in the neighborhood of fifty, an increase in volatility will reduce the investment threshold. In some sense, an increase in volatility reduces the importance of today's index level for the prediction of the future regime. With an increased volatility, the "expected bad news" regarding the royalty rate is reduced[17], and so is the investment threshold.

In Figure 4.8 I show the relationship between the time waited and the investment threshold for various index levels at time t. When $T_W < T_D$, the remaining political uncertainty is

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[17] The "expected bad news" may be expressed as $(E[\tau_t] - \tau_B)(1-p)$, i.e., the difference between the expected fraction of revenue to be kept by the investor and the worst possible fraction of revenue, multiplied by the probability that the worst possible regime will apply. By rearranging this expression we get: $p(1-p)(\tau_G - \tau_B)$. This expression is maximized for $p=0.5$. The reason why p is reduced, is subtle. An increase in $\sigma_{\hat{x}}$ will reduce the drift of $\hat{x}_t$, see the first term on the RHS of equation (2). When the index level is such that p is approx. 0.5, an increase in the volatility will therefore reduce the drift and thereby p, i.e., $p(1-p)$ is reduced.

reduced the closer the deferred decision date is to the date when the government decides the royalty rate, i.e., $(T_D - T_W)$ gets smaller. This is may cause H_t to increase sharply when $(T_D - T_W)$ goes to zero. See Figure 4.5 for a comparison, where H_t will jump at the point where $T_W = T_D$. If the index levels at time *t* are very high or low, the investor does not expect to revise the probability of regime *G* considerably. When the index level at time *t* is close to the critical level of fifty, a revision of the probability of regime *G* may be expected.

In Figures 4.9 and 4.10 I show the effect of different levels of volatility when today's index levels are, respectively, fifty and forty. When the index level is fifty, an increase in the volatility reduces the investment threshold. When the index level is forty, the effect of the volatility on the threshold is mixed. An increase in volatility does not necessarily imply a higher/lower investment threshold. The reason can be seen from Figure 4.7: the graphs for different volatilities do not intersect in fixed points.

The relationship between the investment threshold and the country's risk index at time *t* for the case with no remaining uncertainty at time T_D, $T_W = T_D$, and with a stochastic oil price is shown in Figure 4.11. We see that the investment threshold is not visibly changed for high and low levels of the index, i.e., when it is almost certain that the index level at time T_D will be respectively higher or lower than the critical level of the index. When the index level is close to the critical level of fifty, the investment threshold is changed. The relationship between the investment threshold and time waited with a stochastic oil price, a correlation coefficient of 0.5, an index level at time *t* equal fifty, $\tau_B = 0.7$ is shown in Figure 4.12

When the correlation is positive, and when the numerical value of the coefficient of correlation is increased, the investment threshold is also increased. We note from Figure 4.11 that with a coefficient of correlation equal to 0.5, an index level just below fifty, and $\tau_B = 0.8$, the investment threshold is approximately 0.41. This is an increase of approximately 10.5 percentage points compared to the situation with no political uncertainty. This must be regarded as a considerable change in the investment threshold. When the coefficient of correlation is negative, and sufficiently large in numerical value, the investment threshold is reduced. With a coefficient of correlation of -0.5 and an index level

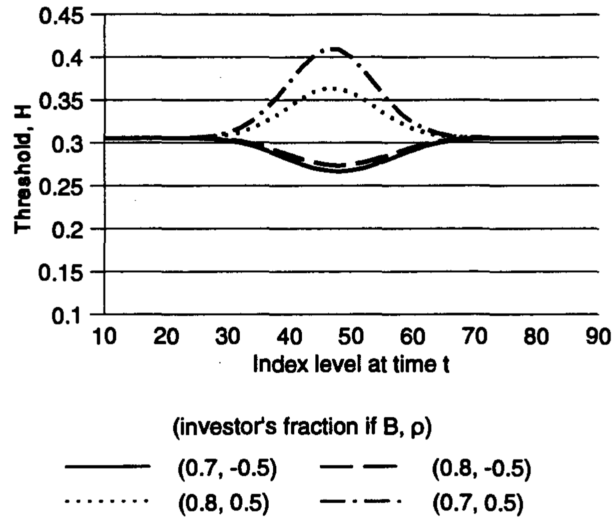

Figure 4.11 The investment threshold, H_t, when there is no remaining uncertainty at time T_D. Stochastic oil price, T_W=4

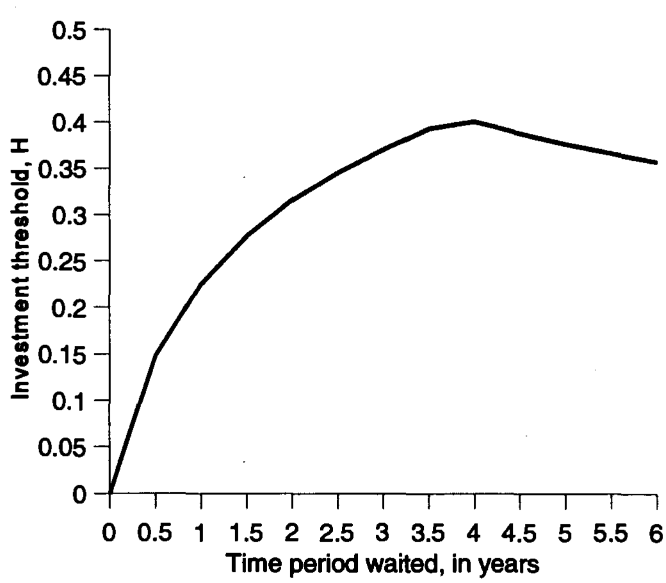

Figure 4.12 The investment threshold, H_t, when there is no remaining uncertainty at time T_D. Stochastic oil price, τ_B=0.7, ρ=0.5, ψ_t=50

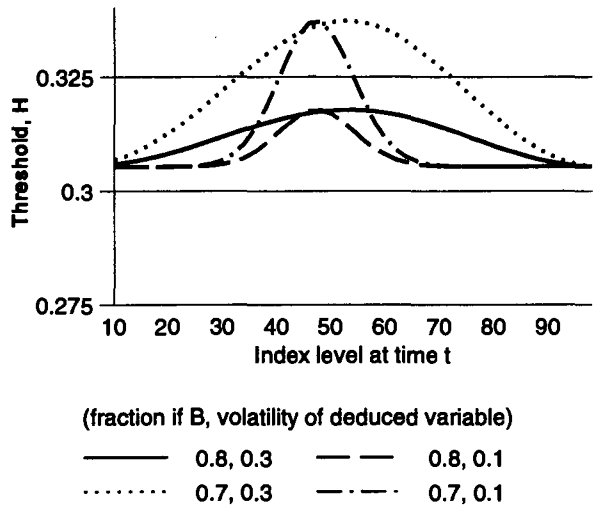

Figure 4.13 The investment threshold, H_t, when there is no remaining uncertainty at time T_D. Stochastic oil price, T_W=4, ρ=0

of fifty, the investment threshold is approximately 0.27. This is a reduction of the threshold by approximately 3.5 percentage points compared to the situation with no political uncertainty.

If the correlation between the risk index and the oil price is positive, it is likely that a reduction in the oil price (decreasing the value of the investment opportunity) will occur together with a decrease in the risk index (which implies that a higher royalty rate is more likely to be announced by the government). The probability of receiving "bad news" is therefore high, as is the corresponding investment threshold. When the coefficient of correlation is negative, the expected bad news is reduced compared to the situation with a zero or positive correlation. In this case, a possible reduction in the oil price is likely to occur together with an increase in the risk index, i.e., the probability of a low royalty rate is increased. The examples shown in Figure 4.11 clearly indicates that increased political uncertainty, as measured by the level of the risk index, does not necessarily imply an increased investment threshold.

When there is no correlation between the index and the oil price, see Figure 4.13, the investment threshold may be increased approximately to the same level as in Figure 4.4. We note that the effect of an increase in the volatility of the deduced variable, $\sigma_{\hat{x}}$, is of the same type as shown in Figure 4.7. The increased volatility may increase or decrease the investment threshold, dependent on the index level at time *t*.

4.3 Waiting with Possibility of Expropriation

The assumptions are as given in Table 4.1. The regime parameters used are those given for "specific case II" in Figure 3.1.

4.3.1 No Learning Before T_D

In the case of a deterministic oil price, nothing can be learned by waiting because the government will only decide whether to expropriate the investment after the development stage is completed. The investment threshold will therefore be zero. When the oil price is stochastic, the investment threshold will be independent of p, $p \in \langle 0,1 \rangle$. The relationship between H_t and the time waited is shown in Figure 4.14. The investment threshold is increasing with the length of the waiting period. With a waiting period of four years, the investment threshold is approximately 0.16.

4.3.2 No Remaining Uncertainty at Time T_D

When the index level was approximately fifty, or lower, my calculations showed that the value of waiting was always higher than the value of investing at time *t*. In these cases the investor will never invest if the investment decision can be deferred. For an explanation, see proposition A4 in Appendix 2. The relationship between the investment threshold and the time waited when the index level is fifty-five and sixty is shown in Figure 4.15. We note from Figure 4.15 that an increase in the volatility of the deduced variable will increase the investment threshold. We note the large effect of a five point difference in index levels on the investment threshold. If the investor can delay the investment decision four years, the difference in investment threshold is approx.15 percentage points. Only a small change in a country's risk index may therefore have a considerable effect on the investor's incentive to wait.

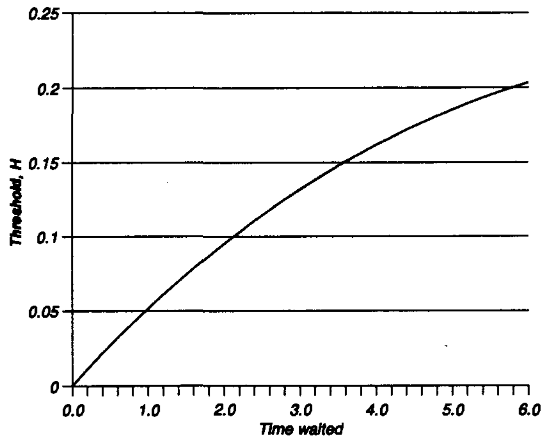

Figure 4.14 The investment threshold, H_t, with possible expropriation and no learning before time T_D. Stochastic oil price

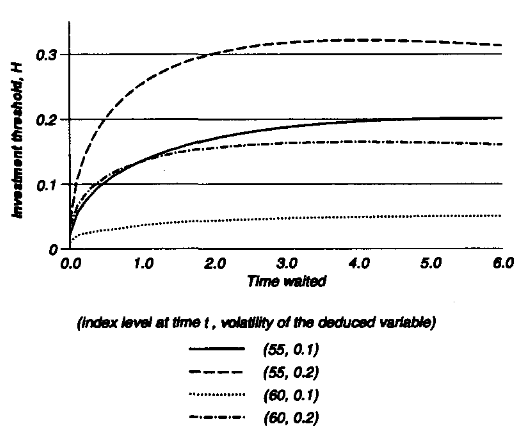

Figure 4.15 The investment threshold, H_t, with possible expropriation and no remaining uncertainty at time T_D. Deterministic oil price

5 Concluding Remarks

In this chapter I have shown how political uncertainty may be included in the evaluation of investment opportunities. This approach enables me to show that increased political uncertainty as measured by the level of a risk index does not necessarily increase the investment threshold for investments, as measured by H_t. For the example in case of an uncertain royalty rate and with no remaining uncertainty at time T_D, the investment threshold was reduced, provided that the coefficient of correlation between the index and the oil price was negative and sufficiently large in numerical value. Even though this result corresponds to the well known "bad news principle", the example clearly demonstrates that the effect of increased political uncertainty on investments and investment thresholds must be analyzed by taking into account the correlation between the probability of a given "political event" and the value of the underlying asset. In many cases it may be too simple just to assume that the future regulatory regime is independent of underlying economic variables, such as the oil price.

Appendix 1 Valuation Formulas

The valuation formulas for the contingent claims in this appendix are derived by discounting the expected future value of the contingent claim at the risk free interest rate, where the expectation is based on an equivalent martingale measure. The approach is identical to the approach used in appendix two of chapter three of the dissertation. The three contingent claims considered may be compared to compound financial options, i.e., "options on options". I consider three points in time, $t \leq T_1 \leq T_2$. The maturity date for the three contingent claims is T_1.

The value of the first contingent claim will at time T_1 be given by

$$Z_{T_1}^{(C1)} = \begin{cases} e^{-\delta_S(T_2-T_1)} S_{T_1} \mathrm{N}[c(S_{T_1}, S_{T_2}^{BE,i}, T_2)] & \text{if } \psi_{T_1} \geq \overline{\psi} \\ 0 & \text{if } \psi_{T_1} < \overline{\psi}\ . \end{cases} \qquad \textbf{(A1.1.)}$$

where

$$c(S_{T_1}, S_{T_2}^{BE,i}, T_2) = \frac{\ln(S_{T_1}/S_{T_2}^{BE,i}) + (r - \delta_S + \frac{1}{2}\sigma_S^2)(T_2 - T_1)}{\sigma_S\sqrt{T_2 - T_1}}\ . \qquad \textbf{(A1.2)}$$

Compared to a financial option, the value of $Z_{T_1}^{(C1)}$, if $\psi_{T_1} \geq \overline{\psi}$, will be equal to the value of an[1] "asset or nothing call" maturing at time T_2 where $S_{T_2}^{BE,i}$ is the contract price. The value of the claim at time t is

$$Z_t^{(C1)} = S_t e^{-\delta_S(T_2-t)} \mathrm{N}_2[a(\psi_t, \overline{\psi}, T_1), c(S_t, S_{T_2}^{BE,i}, T_2); \lambda] \qquad \textbf{(A1.3)}$$

where $\mathrm{N}_2[\cdot,\cdot;\lambda]$ is the distribution function for the bivariate standard normal distribution with coefficient of correlation λ, and where[2]

[1] See, e.g., Hull (1993) page 420.

[2] Regarding notation, see footnote 11 on page 161.

$$a(\psi_t, \overline{\psi}, T_1) = \frac{\ln(Z_t/\overline{Z}) + (r - \delta_{\pounds} + \rho\,\sigma_{\pounds}\sigma_S - \frac{1}{2}\sigma_{\pounds}^2)(T_1 - t)}{\sigma_{\pounds}\sqrt{T_1 - t}} \quad , \qquad \textbf{(A1.4)}$$

and

$$\lambda = \rho\sqrt{\frac{T_1 - t}{T_2 - t}} \ . \qquad \textbf{(A1.5)}$$

The value of the second contingent claim is at time T_1

$$Z_{T_1}^{(C2)} = \begin{cases} e^{-r(T_2 - T_1)} A\,\mathrm{N}[d(S_{T_1}, S_{T_2}^{BE,i}, T_2)] & \text{if } \psi_{T_1} \geq \overline{\psi} \\ 0 & \text{if } \psi_{T_1} < \overline{\psi} \end{cases} \quad , \qquad \textbf{(A1.6)}$$

where A is a constant and

$$d(S_{T_1}, S_{T_2}^{BE,i}, T_2) = \frac{\ln(S_{T_1}/S_{T_2}^{BE,i}) + (r - \delta_S - \frac{1}{2}\sigma_S^2)(T_2 - T_1)}{\sigma_S\sqrt{T_2 - T_1}} \ . \qquad \textbf{(A1.7)}$$

If the index at time T_1 is not below the critical index level, the value of $Z_{T_1}^{(C1)}$ can be compared to the value[1] of a "cash or nothing call" maturing at time T_2. The value of the claim at time t is

$$Z_t^{(C2)} = e^{-r(T_2 - t)} A\,\mathrm{N}_2[b(\psi_t, \overline{\psi}, T_1), d(S_t, S_{T_2}^{BE,i}, T_2); \lambda]] \ , \qquad \textbf{(A1.8)}$$

where

$$b(\psi_t, \overline{\psi}, T_1) = \frac{\ln(Z_t/\overline{Z}) + (r - \delta_{\hat{x}} - \frac{1}{2}\sigma_{\hat{x}}^2)(T_1 - t)}{\sigma_{\hat{x}}\sqrt{T_1 - t}} \qquad \textbf{(A1.9)}$$

and λ is given by (A1.5).

The value of the third contingent claim is at time T_1 given by

$$Z_{T_1}^{(C3)} = \begin{cases} e^{-r(T_2 - T_1)} A N[b(\psi_{T_1}, \overline{\psi}, T_2)] & \text{if } \psi_{T_1} \geq \hat{\psi} \\ 0 & \text{if } \psi_{T_1} < \hat{\psi} \end{cases} . \qquad \textbf{(A1.10)}$$

$\hat{\psi}$ is a given level of the index and may be higher, equal to, or lower than $\overline{\psi}$. The value of the contingent claim at time T_1, provided that index level is not below $\hat{\psi}$, may also be compared to a "cash or nothing call". The value of the claim at time t is

$$Z_t^{(C3)} = e^{-r(T_2 - t)} A N_2[b(\psi_t, \overline{\psi}, T_2), b(\psi_t, \hat{\psi}, T_1); \hat{\lambda}] , \qquad \textbf{(A1.11)}$$

where $b(\psi_t, \overline{\psi}, T_2)$ is given by (A1.9),

$$b(\psi_t, \hat{\psi}, T_1) = \frac{\ln(Z_t/\hat{Z}) + (r - \delta_{\hat{x}} - \frac{1}{2}\sigma_{\hat{x}}^2)(T_1 - t)}{\sigma_{\hat{x}}\sqrt{T_1 - t}} , \qquad \textbf{(A.1.12)}$$

and

$$\hat{\lambda} = \sqrt{\frac{T_1 - t}{T_2 - t}} . \qquad \textbf{(A1.13)}$$

Appendix 2 Uniqueness of Break-Even Prices

Proposition A1 *For the case with an uncertain royalty rate only, no learning before time* T_D, *and* $T_W \geq T_D$, *if the break-even price for oil which makes investment preferable to waiting exists and* $\delta_S > 0$, *then the break-even price is unique.*

Proof. For two continuous functions of S_t, where the derivative of the first function with respect to S_t is always lower than the derivative of the second function with respect to S_t, and the limit as S_t goes to infinity of the derivative of the first function is strictly lower than the limit of derivative of the second function, the graphs of the functions will cross only once, if the derivatives are positive. The derivative of C_t (equation (34)) with respect to S_t is the positive constant $e^{-\delta_s(T_{P|t}-t)}[\tau_G p + \tau_B(1-p)]$. The derivative of W_t (equation (36)) with respect to S_t is always positive and increasing. The highest level of the derivative of W_t is when S_t goes to infinity, where the limit is $e^{-\delta_s(T_W-t)}e^{-\delta_s(T_{P|t}-t)}[\tau_G p + \tau_B(1-p)]$, which is strictly less than the derivative of C_t if $\delta_S > 0$. While W_t is always nonnegative, C_t will be negative if $S_t \leq S_t^{BE}$. C_t and W_t will therefore intercept only once. The spot price for oil where the values of C_t and W_t are equal is therefore unique.

■

Proposition A2 *For the case with an uncertain royalty rate only, no remaining uncertainty at the fixed calendar date* T_D, *and* $T_W \geq T_D$, *if the break-even price for oil which makes investing preferable to waiting exists and* $\delta_S > 0$, *then the break-even price is unique.*

Proof. I use the same line of reasoning as in the proof of proposition A1. For a given level of the index, the derivative of C_t (equation (40)) with respect to S_t is the positive constant $e^{-\delta_s(T_{P|t}-t)}[\tau_B + (\tau_G - \tau_B)\mathrm{N}[a(\psi_t, \overline{\psi}, T_D)]]$. The derivative of W_t (equation (41)) with respect to S_t is always positive and increasing. The limit of the derivative of W_t when S_t goes to infinity is $e^{-\delta_s(T_{P|T_W}-T_W)}(e^{-\delta_s(T_W-t)}[\tau_B + (\tau_G - \tau_B)\mathrm{N}_2[a(\psi_t, \overline{\psi}, T_D), \infty; \lambda]])$, which is strictly less than the derivative of C_t if $\delta_S > 0$, because $\mathrm{N}_2[a(\psi_t, \overline{\psi}, T_D), k; \lambda] \leq \mathrm{N}[a(\psi_t, \overline{\psi}, T_D)]$ for all k.

■

Proposition A3 *For the case with a possible expropriation, no learning before the fixed calendar date T_D, and $T_W \geq T_D$, if the break-even price for oil which makes investment preferable to waiting exists and $\delta_S > 0$, then the break-even price is unique.*

Proof. I continue using the same line of reasoning as in the proof of proposition A1. The derivative of C_t (equation (44)) with respect to S_t is the positive constant $pe^{-\delta_s(T_{P|t}-t)}$. The derivative of W_t (equation (46)) with respect to S_t is always positive and increasing. The highest level of the derivative is when S_t goes to infinity, where the derivative is $e^{-\delta_s(T_W-t)}(pe^{-\delta_s(T_{P|T_W}-T_W)})$, which is strictly less than the derivative of C_t if $\delta_S > 0$. While W_t is always non-negative, C_t will be negative if $S_t \leq S_t^{BE}$. C_t and W_t will therefore intercept only once.

■

Proposition A4 *For the case with a possibility of expropriation, a deterministic oil price, and no remaining uncertainty at T_D, if $T_D = T_{P|t}$ $\forall$ t, $I_t = I_{T_W}$, $K_{T_{P|t}} = K_{T_{P|T_W}}$, and $\alpha_S = 0$ the spot price making investment prefered to waiting may not exist.*

Proof. For a given level of the index, the derivative of C_t (equation (49)) with respect to S_t is the positive constant $e^{-r(T_{P|t}-t)}N[b(\psi_t,\overline{\psi},T_{P|t})]$. The derivative of W_t (equation (51)) with respect to S_t is $e^{-r(T_{P|T_W}-t)}N_2[b(\psi_t,\overline{\psi},T_{P|T_W}),b(\psi_t,\hat{\psi},T_W):\hat{\lambda}]$, which may be strictly higher than the derivative of C_t if $b(\psi_t,\overline{\psi},T_{P|t})<b(\psi_t,\overline{\psi},T_{P|T_W})$.

■

Appendix 3 List of Symbols

Symbols Related to the Investment Opportunity

I_t	Investment expenditure at time t
K_t	Production cost at time t
S_t	Oil price at time t
α_S	Drift parameter in stochastic process for S_t
σ_S	Volatility parameter in stochastic process for S_t
δ_S	Rate of return shortfall for S_t
C_t	Value of the investment commitment at time t
W_t	The value at time t of the deferred investment opportunity
S_t^{BE}	The break-even oil price making $C_t = 0$
S_t^W	The oil price making investing today preferred to the alternative of deferring the investment decision
r	Instantaneous risk free interest rate

Symbols related to the Index

ψ_t	Index level at time t
ψ^{MAX}	Maximum level of the index
$\overline{\psi}$	Critical level of the index
$\underline{\overline{\psi}}$	A given level of the index which makes the investment commitment equal to zero
$\hat{x}_t$	State variable deduced from the index
σ_v	Standard deviation of "noise", used when deducing the state variable $\hat{x}_t$ from the risk index
Z_t	Price of hypothetical asset which is a function of the state variable $\hat{x}_t$
$\alpha_{\hat{x}}$	Drift parameter in stochastic process for Z_t
$\sigma_{\hat{x}}$	Volatility parameter in stochastic process for Z_t
$\delta_{\hat{x}}$	Drift adjustment for $Z_t^{(\hat{x})}$

Dates

t	Either used to indicate "today" or as a general sub script indicating time
T_D	Date when the government announces which regime will apply to the project
T_W	Future date when the investor can reconsider whether to invest in the project if the investment decision is deferred today
$T_{P\|t}$	Date when production starts provided that the investment decision is made at time t
$T_{P\|T_W}$	Date when production starts provided that the investment decision is made at time T_W

Regulatory Regimes

G	Sub script used to indicate a "good" regulatory regime
B	Sub script used to indicate a "bad" regulatory regime
τ_i	Fraction of revenue kept by the investor under regime i, i.e., the royalty rate is $(1-\tau_i)$
$\tau = \{\tau_G, \tau_B\}$	Set of possible fraction of revenue to the investor
$\gamma_i^{(I)}$	Scaling factor of investment expenditure under regime i
$\gamma^{(I)} = \{\gamma_G^{(I)}, \gamma_B^{(I)}\}$	Set of possible scaling factors of investment expenditure
$\gamma_t^{(I)}$	Scaling factor of investment expenditure today, i.e., time t
$\gamma_i^{(K)}$	Scaling factor of production cost under regime i
$\gamma^{(K)} = \{\gamma_G^{(K)}, \gamma_B^{(K)}\}$	Set of possible scaling factors of production cost
$\pi_i = \{\tau_i, \gamma_i^{(I)}, \gamma_i^{(K)}\}$	Regulatory regime i
$\pi = \{\pi_G, \pi_B\}$	Set of possible regulatory regimes

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Chapter 5

Investment and Taxation: A Bargaining Approach with Application to the Oil Industry

Abstract

A government's lack of credibility when promising future taxation and regulation of foreign direct investments, is often regarded as an obstacle to foreign investment. In models with perfect information where the models are solved by backwards induction, the optimal strategy for the investor is not to invest in the country. As shown in this chapter, the total lack of inter-period credibility may not necessarily prevent investment from taking place. If the government in the host country is not able to undertake the investment activity itself, both the government and the investor can benefit from negotiating a series of agreements where the investor gets a share of the revenue generated from previous investments against making new investments. This assumes that intra-period agreements are respected by the parties. In a simple model I show that investment may take place. In a more elaborate model, I allow for a stochastic oil price and study how the decision to "wait" affects the solution.

1 Introduction

When making foreign direct investments, one of the primary concerns of the investor is how the government in the host country will regulate the investment. When it is not known for certain which future regulatory regime will be applicable, the situation is known as a situation with political risk. Central in the analysis of political risk is the question of credibility. The credibility problem can best be described by a simple example, see Figure 1.

In Figure 1. A, an investor, I, can decide at time t_0 to produce[1] a quantity of oil by paying the production cost, K, up front. The produced quantity is sold at time t_1 at a pre tax revenue of R. At time t_1 the government of the country, C, decides the royalty rate, τ. The utility to the investor is a function of the royalty rate, the pre-tax revenue, and the production cost. An increase in after tax revenue will increase utility, while an increase in costs will reduce her utility, ceteris paribus. For the country the production cost may contribute positively to the utility[2] because services may be bought in the host country. This may also result in increased employment. Production will not take place unless the investor's utility from producing is nonnegative. For this to happen, the government must behave suboptimally at time t_1. The government's dominating strategy at time t_1 is to set τ equal to one, i.e., the investor will not receive any of the sales revenue. Knowing this at time t_0, the investor will not produce. In Figure 1.B, the government chooses first the tax rate that will apply at time t_1, and then the investor makes the production decision. The country will determine the sales tax such that the contribution to the investor's utility function[3] from producing is zero. The investor is then indifferent between producing or not, and she might as well produce. In case B it is assumed that the government's promise is credible. This means that after the announcement of the royalty rate, there is no uncertainty at time t_0 regarding which royalty rate will apply at time t_1.

[1] In order to simplify I do not specify an investment, or development, stage preceding a production stage. The term "produce" can be thought of as including both a possible development stage and a production stage.

[2] The term country can be thought of as a central planner, and the utility as the welfare for the people in the country.

[3] Since the utility functions are not necessarily identical in cases A and B, I have used the symbols U and U', respectively.

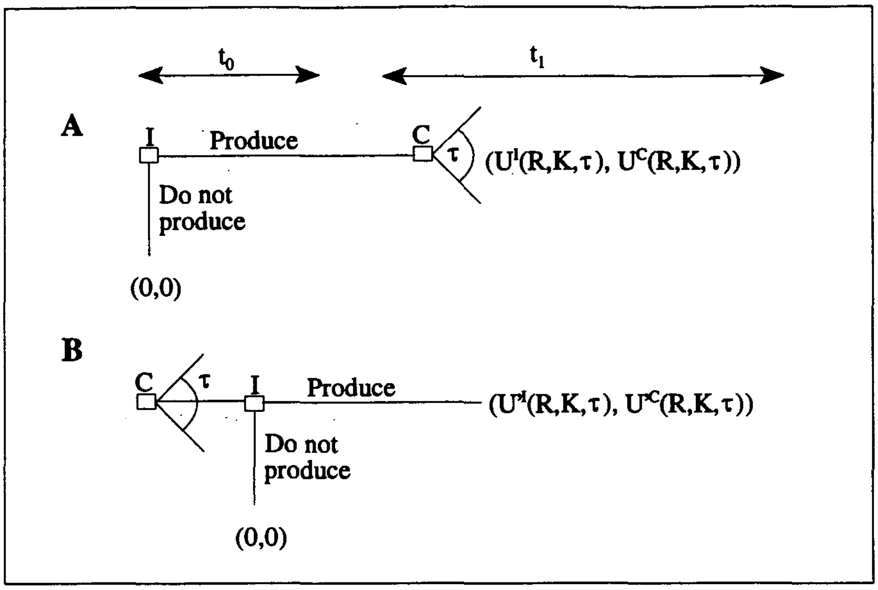

Figure 1 An investment situation without (A), and with (B), commitment from the country regarding taxation.

It is often assumed that the government should be credible. This is a normative point of view, and it is usually based on a specific modeling of investors' behavior based on the following argument. Investors evaluate the probability of a "good" and "bad" policy regime at future points in time. The political risk, or credibility, can thus be measured by the probability of a "bad" policy regime[4]. If increased (foreign) investment is the aim, this may be obtained by a higher credibility (lower probability of a "bad" policy regime) which will increase the expected future cash flow and possibly make the net present value of the investment positive.

Credibility cannot, however, be declared by the government. A government's credibility is determined by the investors' expectations of how the government will act in the future. Consider the situation modeled as a simple signaling game. For an example of such a model, see, e.g., Rodrik (1989b). The government is at the outset either credible or not credible. The credibility can be destroyed (or obtained) by an action from the government revealing its "true type." In this situation the credibility problem arises because the investor does not have perfect information regarding the type of government she is facing.

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[4] An investment operating under a "bad" regime will give a lower cash flow to the investor than an investment operating under a "good"regime. For an example of a model using this approach, see, e.g., Rodrik (1991).

In this chapter I present a model where it is assumed that the government is not able to commit itself to a certain behavior at future points in time, and where the information is assumed to be perfect. The approach is positive and the aim is to show that investment can take place in the absence of inter-time credibility and with perfect information. The parties can commit themselves for the current period, i.e., there is intra-time commitment. In this case it may be possible for the investor and the government to negotiate an agreement which is valid for this period only. The investor will enter the negotiations demanding a lowest possible royalty rate for current sales revenue, while she can provide further production. The country's position will be exactly the opposite. It will demand further production, while it can provide the investor with a lower royalty rate for current sales revenue. In a model with many small investors, it will be impractical for the government to negotiate with each investor. The model captures probably best investment situations where one or a few investors are investing.

In the next section I present a model where the oil price is certain over the period and where the possible outcome of the game played at each point in time is either that the next production quantity is produced, or that the oil field is abandoned. In the real-options literature it is often showed that increased flexibility, e.g., to shut down production temporarily or to delay the decision to invest, increases the value of an investment or investment opportunity. In order to study the effect of "waiting" in a model where negotiations are taking place, I introduce in sub-section three a stochastic oil price and allow the investor to "wait", to abandon the oil field, or to produce the next production quantity.

2 Model with No Oil Price Uncertainty

2.1 The Model

The oil field[5] is characterized by a set, Q, of N production quantities, $Q = \{q_1,..,q_n,...,q_N\}$. There is no uncertainty linked to the size of the total recoverable reserves or each production quantity. When a quantity of oil is produced, it is sold at the prevailing oil price S, which is assumed to be constant[6]. The production time per quantity is one period. The production cost per unit of production, k, is constant[6] and is paid in full at the start of the period. The sales revenue is received at the start of the following period.

The start of the game between the investor, I, and the country, C, is depicted in Figure 2. The game starts at time t_0 at node 1. The investor chooses between the action P_1 of producing the first quantity and E of ending the game by exiting. The "instantanous" or "immediate" utility to the parties from producing quantity q_n is $\mathrm{u}(q_n) = (\mathrm{u}^{\mathrm{I}}(q_n), \mathrm{u}^{\mathrm{C}}(q_n))$, where the investor's utility is

$$\mathrm{u}^{\mathrm{I}}(q_n) = -q_n k \ , \tag{1}$$

i.e., the production cost, and the country's utility is

$$\mathrm{u}^{\mathrm{C}}(q_n) = b q_n k \ , \tag{2}$$

where b is a nonnegative constant. At the start of time t_1, quantity q_1 is ready for sale and production of q_2 may start.

---

[5] If "production" is interpreted to include development too, the corresponding reinterpretation of the term "oil field" is a series of investment in different oil fields. In an earlier version of the chapter I specified the model with an investment, or development, stage preceding the production stages. The results for such a model will be similar to the results obtained for the model presented here.

[6] In order to simplify the exposition, I have assumed that the oil price and the production costs are constants. They can be a function of time, but this will not give any major additional insights.

The sub-game at time t_1, at node 2, starts with a negotiation between the country and the investor. The parties negotiate over the royalty rate for the quantity about to be sold *and* the production of the next quantity. If an agreement is reached, A, the country declares the agreed royalty rate[7] τ_1 applying to q_1. The investor then sells q_1, pays the royalty, and starts production of q_2. I have used the symbol Z_n to describe the investor's action of selling quantity q_n and the subsequent payment of royalty. The parties' "instantaneous" utility from the sale of oil and payment of royalty is $\mathrm{u}(\tau_n) = (\mathrm{u}^I(\tau_n), \mathrm{u}^C(\tau_n))$, where

$$\mathrm{u}^I(\tau_n) = q_n S(1-\tau_n) \tag{3}$$

and

$$\mathrm{u}^C(\tau_n) = a q_n S \tau_n \ , \tag{4}$$

where a is a strictly positive constant[8]. If an agreement is not reached at time t_1, i.e., the parties disagree, D, the country declares a royalty rate for the disagreement situation at node 4. The investor sells q_1 and pays royalty according to the announced royalty rate. At node 8 the investor decides whether to start production of q_2 or to abandon the oil field.

Irrespective of whether the parties agree or disagree, if production of q_2 takes place at time t_1, the parties start at time t_2 negotiating over the royalty rate for q_2 and production of q_3. The outline of the game at time t, $t_1 < t < t_N$, is as for t_1. At time t_N the final production quantity, q_N, is ready for sale and the parties negotiate over the royalty rate only. If an agreement is reached, the country declares the agreed royalty rate, the investor sells the oil, pays royalty, and the game ends. Similarly, if an agreement is not reached, the country declares the royalty rate, the investor sells the oil, pays royalty, and the game ends.

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[7] The country's action may be thought of as announcing the royalty rate which will apply, while τ_n is the numerical value of the announced royalty rate. I simplify by referring to the country's action as τ_n.

[8] Because $a > 0$ and $b \geq 0$, I assume that the country's utility of tax revenue is strictly positive but that production activity does not necessarily contribute positively to the country's utility.

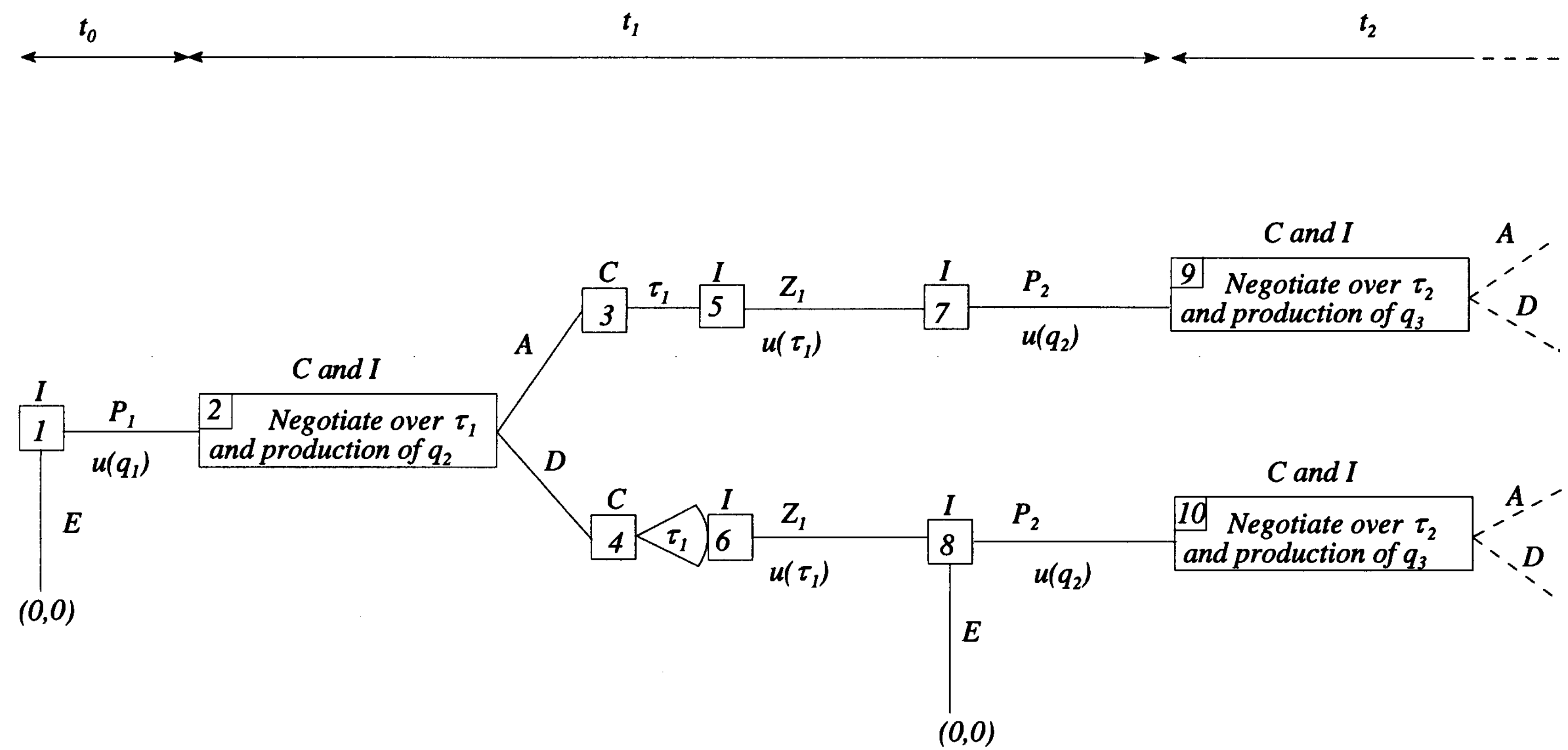

Figure 2 The first stages of the game between the host country, *C*, and the investor, *I*.

For a game in extensive form, it is usually required that only one player moves at a given node in the game tree. In this model we have "negotiation nodes," where both players interact. The set of nodes, X, can be divided in three parts: those where the investor moves, X_I, those where the country moves, X_C, and the "negotiation nodes", $X_{I,C}$. Nodes 2, 9, and 10 in Figure 2 are examples of such "negotiation nodes". In order to give a precise specification of the game, a model is needed to describe the outcome of the bargaining between the country and the investor. Such a model can be thought of as a rule which is applied to the negotiation problem, and where the rule clearly specifies the outcome of the bargaining problem. For now I leave open how this rule is derived, but in the next sub-section I assume that the solution to the bargaining problem corresponds to the Nash bargaining solution.

With a well specified rule solving the negotiation problem, a pure strategy for player j, s_j, is a complete plan for how to play the game, i.e., which actions to choose at every node belonging to X_j. Note that a pure strategy does not specify any action from the player at the negotiation nodes. The set of all pure strategies for player j is Σ_j. I will use the notation (s_I^x, s_C^x) and Σ^x to indicate, respectively, a strategy combination of pure strategies and the set of all pure strategies for the sub-game starting at node x.

When comparing payoffs at different points in time, I assume that the parties apply discount factors $\theta = (\theta_I, \theta_C)$, where $0 \le \theta_j \le 1$, $j = (I,C)$. These discount factors are constants. While the discount factor of the investor probably is influenced by possible other investment alternatives, this may not necessarily be the case for the government. A patient government will have a discount factor close to one and a (very) impatient government will have a discount factor close to zero. The utility to the parties at node x and time t from the strategy combination (s_I^x, s_C^x), where the utility from future time periods is discounted to time t, is $\mathrm{U}(s_I^x, s_C^x) = (\mathrm{U}^{\mathrm{I}}(s_I^x, s_C^x), \mathrm{U}^{\mathrm{C}}(s_I^x, s_C^x))$. The preferences of the country are completely described by the triple (a, b, θ_C). The preferences of the investor are described the discount factor θ_I and the utility functions (1) and (3).

A Nash equilibrium for the game, i.e., the sub-game starting at node 1, is the strategy combination (s_I^{1*}, s_C^{1*}) where

$$\begin{aligned} U^I(s_I^{1*}, s_C^{1*}) &\geq U^I(s_I^1, s_C^{1*}) \text{ for all } s_I^1 \in \Sigma_I^1 \\ U^C(s_I^{1*}, s_C^{1*}) &\geq U^C(s_I^{1*}, s_C^1) \text{ for all } s_C^1 \in \Sigma_C^1 \quad , \end{aligned} \tag{5}$$

i.e., the strategy combination where each players's strategy is a best response to the other player's strategy. In the next sub-section I consider a specific Nash equilibrium.

2.2 Solution: Nash Bargaining Solution and Backwards Induction

I solve the game by backward induction[9] and I use the Nash bargaining solution to the negotiation, or bargaining, problems at the negotiation nodes. A negotiation problem is characterized by a set of possible allocations of utility among the parties, Y, and a disagreement allocation, d, which obtains if an agreement is not reached. I assume that the royalty rate is nonnegative and not larger than one. A negative royalty rate corresponds to a situation where the country subsidizes the investor and a royalty rate larger than one implies that the investor pays more than the specified project costs. The Nash bargaining solution is a function F where the set of possible payoffs and the disagreement allocation are arguments, i.e., $F(Y,d) = (F^I(Y,d), F^C(Y,d))$, see Appendix 1.

I have in Figure 3 shown different combinations for the set of possible allocations of utility and the disagreement allocation at a given negotiation node x. A solution increasing both parties' utility will be located "north east" of the disagreement allocation. With the set ofpossible allocations Y and disagreement allocation d, a negotiation solution is *feasible*[10]. If the set of possible allocations is Y'' and disagreement allocation d, an agreement is not feasible. This will typically be the situation if current sales revenue is not sufficiently large to make the investor's utility nonnegative even with a royalty rate of zero. If the set of possible

[9] Note that in the finite game with perfect information presented here, the solution to the game found by applying backwards induction is the same as the sub-game perfect equilibrium of the game.

[10] I use the term *feasible* if an agreement is possible which will not make the parties worse off compared to the no-agreement situation. This is done to simplify the presentation. This is, e.g., not the term used by Binmore (1987) page 34, where he uses the term feasible for the axiom stating that both parties should be strictly better off from an agreement compared to the no-agreement situation. Also according to the standard axiom 1, stated in Appendix A1, both parties should be made strictly better off from entering into an agreement.

allocations is Y', and the disagreement allocation is $\bar{d}$, an agreement is feasible. In this case the agreement solution will be the same as the disagreement solution. This will be the situation when the investor will produce the next quantity in case of disagreement, *D*. Consider the bargaining problem at node x involving the royalty rate for sale of quantity q_n and production of quantity q_{n+1}. In order for the investor not to be worse off from an agreement as compared to a disagreement, the following inequality must be satisfied:

$$q_n S(1-\tau_n^A) - q_{n+1}k + \theta_I U^I(s_I^{h*}, s_C^{h*}) \geq q_n S(1-\tau_n^D) + U^I(s_I^{g*}, s_C^{g*}) \ . \qquad \textbf{(6)}$$

The top scripts *A* and *D* indicates whether the royalty rate is a part of an agreement or determined by the government in case of a disagreement. Node h is the node[11] where negotiations start at the next point in time following an agreement, *A*, this time period and node g is the node[11] where the investor decides whether to produce the next quantity or to exit the game. The left hand side (LHS) of inequality (6) is the investor's utility at node x if an agreement is made, while the right hand side (RHS) of the inequality is the investor's utility at node x in case of disagreement.

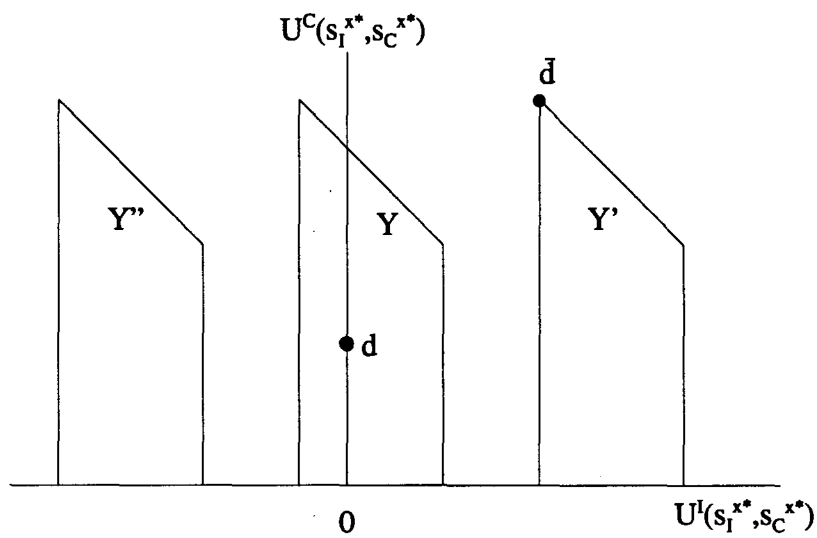

Figure 3 Examples of sets of possible allocations of utility and disagreement allocations

[11] If, e.g., x is node 2 in Figure 2, node h corresponds to node 9. Formally, with a predecessor node function, $\sigma(\cdot)$: $x = \sigma(\sigma(\sigma(h)))$. Comparing with Figure 2, node g corresponds to node 8 and $x = \sigma(\sigma(g))$.

When solved by backward induction, it will be optimal for the country to set the royalty rate equal to one in case of disagreement, i.e., $\tau_n^D = 1$ for all n. By inserting $\tau_n^D = 1$ in (6) and rearranging terms we find the investor's after tax revenue, i.e., the after tax revenue which does not make the investor worse off from an agreement:

$$q_n S(1-\tau_n^A) \geq -[-q_{n+1}k + \theta_I \mathrm{U}^{\mathrm{I}}(s_I^{h*}, s_C^{h*})] + \mathrm{U}^{\mathrm{I}}(s_I^{g*}, s_C^{g*}) \ . \qquad \textbf{(7)}$$

Let N^I be equal to the RHS of inequality (7), i.e., the lowest amount the investor is willing to accept in after tax revenue from sale of quantity q_n in order to make an agreement.

Similarly, in order for the country not to be worse off from an agreement as compared to a disagreement at node x, where the parties bargain over the royalty rate for sale of quantity q_n and production of quantity q_{n+1}, the following inequality must be satisfied:

$$aq_n S\tau_n^A + bq_{n+1}k + \theta_C \mathrm{U}^{\mathrm{C}}(s_I^{h*}, s_C^{h*}) \geq aq_n S\tau_n^D + \mathrm{U}^{\mathrm{C}}(s_I^{g*}, s_C^{g*}) \ . \qquad \textbf{(8)}$$

The LHS of inequality (8) is the country's utility if an agreement is made and the RHS is the country's utility in case of disagreement. By inserting $\tau_n^D = 1$ in (8) and rearranging terms we find the amount the country is willing to let the investor keep in order to enter into an agreement:

$$q_n S(1-\tau_n^A) \leq [bq_{n+1}k + \theta_C \mathrm{U}^{\mathrm{C}}(s_I^{h*}, s_C^{h*})]/a - \mathrm{U}^{\mathrm{C}}(s_I^{g*}, s_C^{g*})/a \ . \qquad \textbf{(9)}$$

The RHS of inequality (9), N^C, is the highest amount the country is willing to give the investor in order to achieve an agreement solution.

In order for an agreement not to make both parties worse off as compared to the disagreement situation, the inequalities (7) and (9) must both be satisfied, i.e.,

$$N^I \leq q_n S(1-\tau_n^A) \leq N^C \ . \qquad \textbf{(10)}$$

Because $\tau_n^A \in [0,1]$, we see that N^C must be nonnegative and N^I cannot be larger than $q_n S$. Define N^N as the difference between the highest amount the country is willing to give the investor and the lowest amount the investor is willing to accept, i.e., $N^N \equiv N^C - N^I$. The numerical value of N^N indicates what the parties are bargaining over, measured in units of money. For a negotiation solution involving the royalty rate for sale of quantity q_n and production of quantity q_{n+1} to be feasible, conditions C1, C2, and C3 must all be satisfied, where

C1: $N^N \geq 0$

C2: $N^I \leq q_n S$

C3: $N^C \geq 0$

The actual part of the revenue received by the investor according to the Nash bargaining solution is

$$q_n S(1-\tau_n^A) = \mathrm{Max}[0, \mathrm{Min}[N^I + N^N/2, q_n S]] \ , \tag{11}$$

see Appendix A1. The agreed royalty rate can be determined by calculating the RHS of (11) and then solving for τ_n^A.

Proposition 1 *For the game with no oil price uncertainty, the amount the country is willing to give the investor to obtain an agreement solution is strictly positive, i.e.,* $N^C > 0$, *if, and only if, the investor abandons the oil field in case of disagreement and* $b > 0$ *and/or* $\theta_C > 0$.

Proof. Consider first the case where the investor produces quantity q_{n+1} even if no agreement is reached. In this case

$$N^C = [bq_{n+1}k + \theta_C \mathrm{U}^C(s_I^{h*}, s_C^{h*})]/a - [bq_{n+1}k + \theta_C \mathrm{U}^C(s_I^{v*}, s_C^{v*})]/a \ ,$$

where v is the negotiation node[12] at the next point in time, t_{n+1}. Note that both node h and node v are located at the same point in time. The sub-game starting at node h is identical to the sub-game starting at node v. When solving these sub-games with backward induction, the country's discounted utility at node v is equal to the discounted utility at node h, i.e., $N^C = 0$.

The other possible action by the investor in case of disagreement is to abandon the oil field. In case of abandonment:

$$N^C = [bq_{n+1}k + \theta_C \mathrm{U}^C(s_I^{h*}, s_C^{h*})]/a \ .$$

Because it is assumed that $a > 0$ and because the country's lowest possible utility at node h is the utility corresponding to a royalty rate equal to one and an abandonment of the oil field, i.e., $\mathrm{U}^C(s_I^{h*}, s_C^{h*}) \geq aSq_{n+1} > 0$, we see that N^C will be strictly positive because $b > 0$ and/or $\theta_C > 0$.

■

Proposition 2 *In the game with no oil price uncertainty, the lowest amount the investor is willing to accept in order to enter into an agreement is strictly positive, i.e., $N^I > 0$, if, and only if, the investor abandons the oil field in case of disagreement.*

Proof. Consider first the case where the investor produces the next production quantity in case of disagreement. Then:

$$N^I = -[-q_{n+1}k + \theta_I \mathrm{U}^I(s_I^{h*}, s_C^{h*})] + [-q_{n+1}k + \theta_I \mathrm{U}^I(s_I^{v*}, s_C^{v*})] \ ,$$

where node v is the negotiation node at the next point in time[12]. Note that both node h and node v are located at the same point in time, t_{n+1}, and that the investor's discounted utility at

[12] Comparing with Figure 2, see footnote 11, node v corresponds to node 10.

nodes h and v are equal because the sub-games starting at these nodes are identical. The implication is that $N^I = 0$.

If the investor abandons the oil field in case of disagreement, the lowest amount the investor is willing to accept in order to enter into an agreement is given by

$$N^I = -[-q_{n+1}k + \theta_I U^I(s_I^{h*}, s_C^{h*})] \ .$$

If the expression in brackets is negative, then $N^I > 0$. This will be the case because the utility to the investor at node h is equal to the utility at node v, and it is assumed that the investor will abandon the oil field in case of disagreement.

■

Proposition 3 *For the game with no oil price uncertainty, an agreement covering royalty rate* τ_n *and production of* q_{n+1} *is feasible if, and only if,* $N^N \geq 0$ *and* $N^I \leq q_n S$*. If an agreement is feasible, the amount to the investor according to the Nash bargaining solution is*

$$q_n S(1-\tau_n^A) = \mathrm{Min}[N^I + N^N/2, q_n S] \ . \qquad \mathbf{(12)}$$

Proof. The Nash bargaining solution is given by (11). I only need to show that $N^I \geq 0$. If $N^I \geq 0$ and $N^N \geq 0$, then $N^C \geq 0$, and condition C3 will always be satisfied. In the first part of the proof of Proposition 2 I showed that N^I is equal to zero if the investor produces the next production quantity in case of disagreement and, according to Proposition 2, N^I is strictly positive if the investor abandons the oil field in case of disagreement. N^I will therefore be nonnegative.

■

Proposition 4 *If the initial production quantity* q_1 *is produced in the game with no oil price uncertainty,* $q_n S \geq q_{n+1}k$*, and* $\theta_C q_{n+1} S \geq q_{n+1}k(1-b/a)$ *for* $1 \leq n \leq N-1$*, then all* *remaining quantities* $q_2, \ldots\ldots, q_N$ *will also be produced*.

Proof. If the investor chooses to produce in the case of disagreement, no restrictions are necessary for production to take place. In the case where the investor will abandon the oil field in case of disagreement, an agreement involving production of q_{n+1} will be obtained, according to Proposition 3, if conditions C1 and C2 are satisfied. The highest possible level of N^I is $q_{n+1}k$, and $N^I \le q_n S$ will be satisfied if $q_{n+1}k \le q_n S$, i.e., if the current sales revenue is not lower than the current production costs. For the situation where the investor abandons the oil field in case of disagreement we get

$$N^N = [bq_{n+1}k + \theta_C \mathrm{U}^{\mathrm{C}}(s_I^{h*}, s_C^{h*})]/a - [q_{n+1}k - \theta_I \mathrm{U}^{\mathrm{I}}(s_I^{h*}, s_C^{h*})]\ .$$

The country's lowest possible level of utility at the next point in time will be the utility corresponding to full taxation of the sales revenue and no further production, i.e., $\mathrm{U}^{\mathrm{C}}(s_I^{h*}, s_C^{h*}) \ge aq_{n+1}S$. Because $\mathrm{U}^{\mathrm{I}}(s_I^{h*}, s_C^{h*}) \ge 0$, it is sufficient for $N^N \ge 0$ that

$$\frac{b}{a} q_{n+1}k + \theta_C q_{n+1}S - q_{n+1}k \ \ge\ 0$$

or

$$\theta_C q_{n+1}S \ \ge\ q_{n+1}k(1 - \frac{b}{a}).$$

■

2.2.1 Example 1

The oil field consists of two production quantities of equal size 10, the production cost is 6 per unit, and the oil price is 18. Table 1 summarizes the assumptions for Example 1. The discount factors are arbitrarily chosen. Because $\theta_C < \theta_I$, the government is more impatient than the investor. We also note that the government's instantaneous utility from one unit of money in tax revenue is twice the instantaneous utility from one unit of money spent on production costs. In order to simplify the exposition I use U_x^{j*} to denote the discounted utility for player *j* at node *x* when the sub-game starting at node *x* is solved with backwards induction and application of the Nash bargaining solution, i.e., $U_x^{j*} \equiv U^j(s_I^{x*}, s_C^{x*})$.

Time t_2 (q_2 is ready for sale)

At this point, no further production will occur. The parties will agree, A, and the royalty rate is equal to one. The parties' utility at nodes 9 and 10 is $U_9^* = U_{10}^* = (0, 180)$, see Figure 4.

Time t_1 (q_1 is ready for sale and production of q_2 may start)

If an agreement is not reached, the investor will produce q_2 if

$$0 \leq u^I(q_2) + \theta_I U_{10}^{I*} .$$

The RHS of the inequality measures the utility of producing, while the LHS measures the utility of abandoning the oil field. The production cost is 60, but because the investor will have to pay a royalty rate of one on the sale of quantity q_2, the best alternative is to exit. It will then be optimal for the country to set $\tau_1{}^D = 1$. This rate maximizes the level of utility for the country, and the utility in case of disagreement, D, is therefore $U_4^* = (0,180)$.

Verifying if a negotiation solution is feasible

From equation (9), we know that the country is willing to give the investor a part of the revenue, $q_1 S(1-\tau_1{}^A)$, such that

$$q_1 S(1-\tau_1^A) \leq [bq_2 k + \theta_C U_9^{C*}]/a - U_8^{C*}/a.$$

By inserting for $U_9^{C*} = \theta_C a q_2 S$ and $U_8^{C*} = 0$, we find the highest amount the country is willing to give to the investor to obtain an agreement, i.e.,

Q = {10,10}	a = 1
S = 18	b = 0.5
k = 6	$\theta_C = 0.9$
	$\theta_I = 0.95$

Table 1 Assumptions Example 1

$$N^C = \frac{1}{a}[bq_2k + \theta_C aq_2S] = 0.5x10x6 + 0.9x1x10x18 = 30 + 162 = 192 \ .$$

In order for the investor to produce she needs a part of the revenue such that, see equation (7),

$$q_1S(1-\tau_1^A) \geq -[-q_2k + \theta_I U_9^{I*}] + U_8^{I*} \ .$$

By inserting for $U_9^{I*} = U_8^{I*} = 0$, we find the lowest amount the investor is willing to accept in order to enter into an agreement:

$$N^I = q_2k = 10x6 = 60.$$

The numerical values of conditions C1-C3 are:

C1:	$N^N = 132 \geq 0$
C2:	$N^I = 60 \leq 180$
C3:	$N^C = 192 \geq 0$

The conditions are satisfied and an agreement involving production of q_2 is therefore feasible.

Finding the bargaining solution

By inserting for N^I and N^N, in (12), we find the negotiated revenue to the investor,

$$180(1-\tau_1{}^A) = \text{Min}[60 + 132/2, 180] = 126,$$

which implies that $\tau_1{}^A = 0.3$. The parties will accept an agreement, and the utility will be $U_2^* = (126-60, (180-126) + 30 + 0.9x180) = (66,246)$.

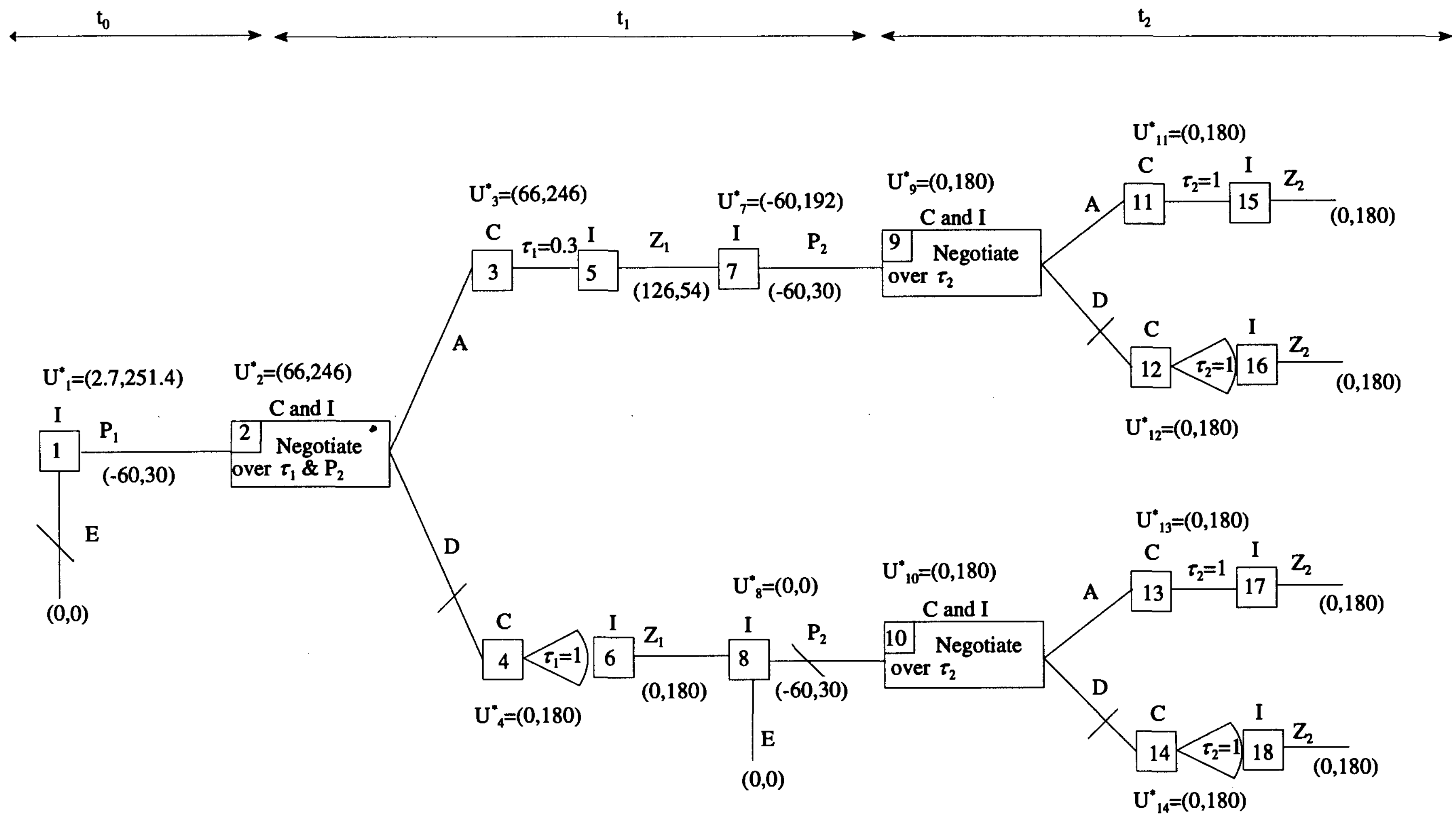

Figure 4 Example 1

Time t_0 (production of q_1 may start)

The investor will produce q_1 if

$$-q_1 k + \theta_I U_2^{I*} \geq 0$$

By inserting the numbers, we find that

$$-60 + 0.95 \times 66 = 2.7 > 0$$

This means that the investor will produce, and the utility to the parties is $U_1^* = (2.7, 0.5x10x6 + 0.9x246) = (2.7, 251.4)$.

2.2.2 Example 2

The assumptions for Example 2 are exactly as for example 1, except that one more quantity is produced. See Table 2. The first part of the game is pictured in Figure 5. Note that the sub-games starting at nodes 9 and 10 in Figure 5 are identical to the sub-game starting at node 2 in Figure 4. When the second production quanitity is produced in Example 2, there is one remaining quantity which may be produced. This corresponds exactly to the situation in Example 1 when the first production quantity is produced. Note further that the sub-game starting at node 8 in Figure 5 is identical to the sub-game starting at node 1 in Figure 4. At both these nodes, the investor decides *independently*, i.e., without an agreement, whether to produce the first of two remaining production quantities. When sub-games are identical, the solution of the sub-games will also be identical. The parties' discounted utility at these nodes will therefore be identical, as can bee seen by comparing Figure 5 and Figure 4.

Time t_1 (q_1 is ready for sale and production of q_2 may start)

If an agreement is not reached, the investor will produce q_2 (standing at node 8) if

Q = {10,10,10}	a = 1
S = 18	b = 0.5
k = 6	$\theta_C = 0.9$
	$\theta_I = 0.95$

Table 2 Assumptions for Example 2

$$0 \leq u^{I}(q_2) + \theta_I U_{10}^{I*}$$

The RHS of the inequality measures the value of producing, while the LHS is the utility of abandoning the oil field. The production cost is 60 and the discounted utility from next period is 0.95x66 = 62.7. The best alternative is therefore to produce. It will be optimal for the country to set $\tau_1^{\ D} = 1$, and the utility in case of disagreement, D, is $U_4^* = (62.7 - 60, 180 + 251.4) = (2.7, 431.4)$.

Verifying if a negotiation solution is feasible

The country is willing to give the investor a part of the revenue, from (9), such that

$$q_1 S(1 - \tau_1^A) \leq [bq_1 k + \theta_C U_9^{C*}]/a - U_8^{C*}/a$$

By inserting for $U_8^{C*} = bq_1 k + \theta_C U_{10}^{C*}$ and noting that $U_9^{C*} = U_{10}^{C*}$, we find that the highest amount the country is willing to give to the investor in order to obtain an agreement solution is zero.

In order for the investor to produce, she is willing to accept a part of the revenue given by:

$$q_1 S(1 - \tau_1^A) \geq -[-q_2 k + \theta_I U_9^{I*}] + U_8^{I*}.$$

By inserting for $U_8^{I*} = -q_1 k + \theta_C U_{10}^{I*}$ and noting that $U_9^{I*} = U_{10}^{I*}$, we conclude that $N^I = 0$.

We compute the conditions C1-C3:

C1:	$N^N = 0$
C2:	$N^I = 0 \leq 180$
C3:	$N^C = 0$

The conditions are satisfied, and an agreement solution is therefore feasible. By inserting for N^I and N^N, in (12), we find the negotiated revenue to the investor,

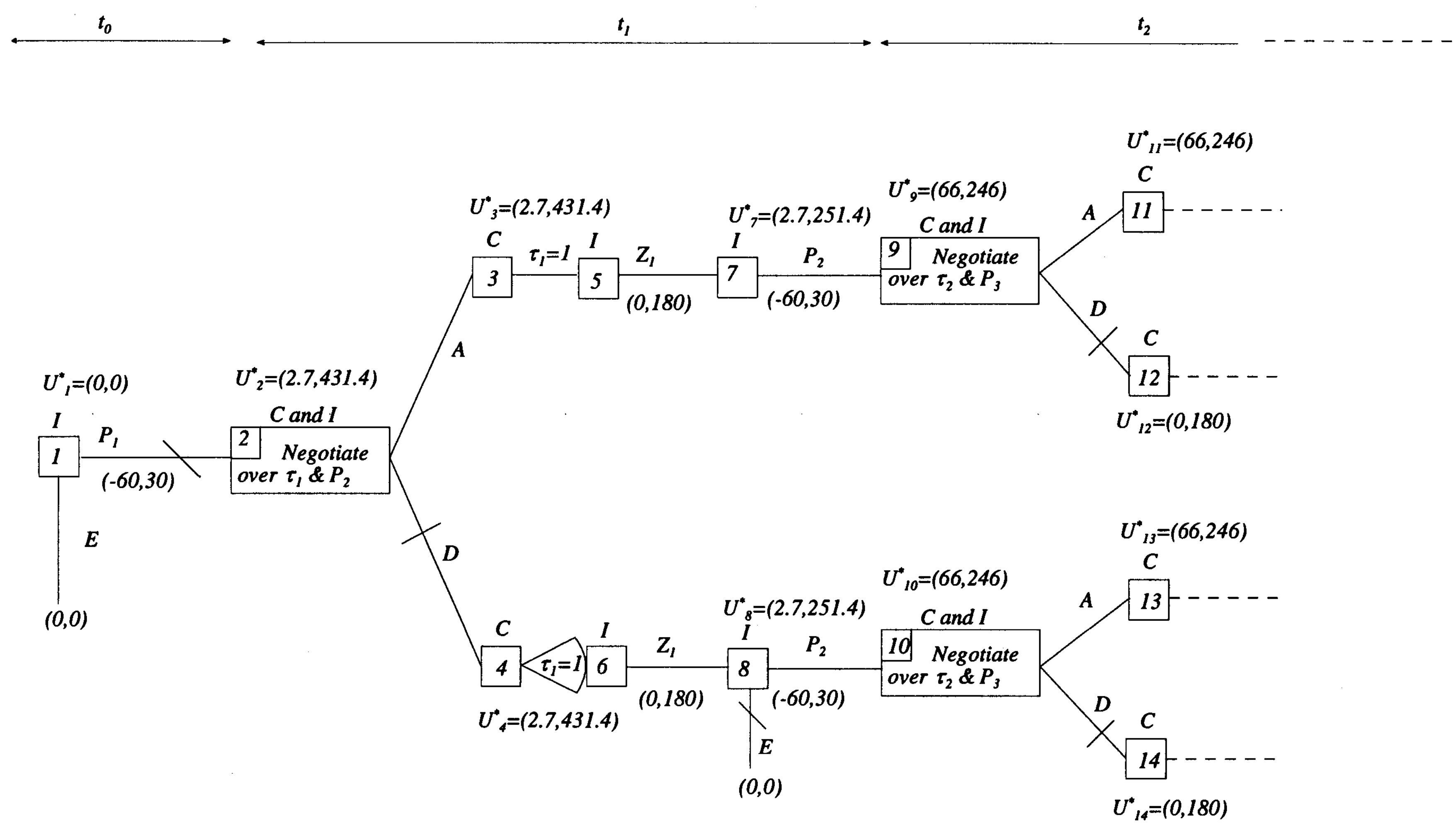

Figure 5 Example 2

$$180(1-\tau_1{}^A) = \text{Min}[0+0/2, 180] = 0,$$

which implies that $\tau_1{}^A = 1$. The utility will be
$U_2^* = (-60+0.95\text{x}66, 1\text{x}10\text{x}18+0.5\text{x}10\text{x}6+0.9\text{x}246) = (2.7,431.4)$.

Time t_0 (production of q_1 may start)

The investor will produce q_1 if

$$-q_1 k + \theta_I U_2^{I*} \geq 0$$

By inserting the numbers, we find that the LHS is given by

$$-60 + 0.95 \text{ x } 2.7 = -57.435 < 0 .$$

This means that the investor will not produce, and the utility to the parties is $U_1^* = (0,0)$.

Table 3 summarizes Example 2 for different oil prices. The table contains key variables for the model with intra-period credibility. In addition I consider a model with inter-period credibility, where the country can commit itself for the life time of the investment. In this latter model, the country declares at time t_0 the royalty rate that will apply for the life time of the oil field. With constant production quantities q, the investor will produce if

$$q[\theta_I S(1-\tau)-k] \geq 0 , \qquad \textbf{(13)}$$

i.e., if the discounted after-royalty oil price covers the unit production cost

Listed in Table 3 is first the investor's decision of whether to produce the first production quantity, P_1, or not, E, and the parties utility at node 1. I then report the actual royalty rates for quantity n, τ_n, and whether the game ends, E, or production occurs, P_n, at time t_1 and t_2. Note that the royalty rate for quantity q_3 is one.

	Intra-period credibility						Inter-period credibility	
	t_0		t_1		t_2			
Oil Price	P_1/E	U^*_1	τ_1	P_2/E	τ_2	P_3/E	τ^{BE}	$U_{t_0}(\tau^{BE})$
10	E	(0,0)	0.108	P_2	0.100	P_3	0.368	(0,171.2)
15	P_1	(36.0,256.0)	0.259	P_2	0.250	P_3	0.579	(0,293.1)
18	E	(0,0)	1.000	P_2	0.300	P_3	0.649	(0,366.3)
20	E	(0,0)	1.000	P_2	0.325	P_3	0.684	(0,415.1)
25	E	(0,0)	1.000	P_2	0.370	P_3	0.747	(0,537.0)
30	E	(0,0)	1.000	P_2	0.400	P_3	0.789	(0,659.0)
35	P_1	(11.6,710.9)	1.000	P_2	0.421	P_3	0.820	(0,780.2)
40	P_1	(31.9,874.7)	1.000	P_2	0.438	P_3	0.842	(0,902.9)

Table 3 Summary of Example 2 for different oil prices with intra-period and inter-period credibility

In Figure 6 I show the value of the oil field for different levels of the oil price and Figure 6 corresponds to the figures in Table 3. For the case with inter-period credibility I report the highest royalty rate which makes the investor willing to produce, τ^{BE}, and the parties's utility discounted to time t_0 when this royalty rate is applied, $U_{t_0}^C(\tau^{BE})$. τ^{BE} is the royalty rate which makes (13) hold with equality. The investor's discounted utility at time t_0, when the royalty rate τ^{BE} is applied, is zero.

The following observations may be made based on the figures in Table 3:

1. The country's utility at time t_0 is highest for all levels of oil prices if there is inter-period credibility. This may indicate that the country should strive to obtain inter-period credibility.
2. For all levels of the oil price, the investor's level of utility at time t_0 in the model with intra-credibility only is always equal to, or higher than, the level of

utility in the model with inter-period credibility. This implies that the investor may be better off investing in a country where there is no inter-period credibility, ceteris paribus. For a country with credibility, it is sufficient to offer the investor a tax regime such that the investor's utility of producing is nonnegative. The country with intra-period credibility only may have to negotiate a tax regime in order to make the investor produce. This tax regime, even though it is changing from one period to the other, may give the investor a higher level of utility at time t_0.

3. For the model with intra-period credibility, a higher oil price does not necessarily imply that the investor's utility at time t_0 is higher. According to Table 3, the investor's level of utility is higher for an oil price of fifteen compared to an oil price of eighteen. The implication is that an investment in a country with no inter-period credibility may take place in a situation with low oil prices but not necessarily in a situation with high oil prices. The reason is that with high oil prices the investor may continue to produce even if the royalty rate for the oil about to be sold is one. This may however reduce the utility of producing at earlier points in time.

In Figure 7 I show the investor's discounted utility at time zero for different numerical values of the country's discount factor. This utility may be interpreted as the value of the oil field if production takes place at time zero. As seen from the figure, the result is contradictory to conventional wisdom due to the endogenous tax policy. A government which is more "investor friendly" than another, i.e., a country having a higher degree of patience (higher θ_C) may cause a lower after tax value of the investment to the investor. This means that if, e.g., a new government is elected and this government is seen as more investor friendly than the old one, the result may be that foreign investments are reduced. A government with a high degree of patience will in a given negotiation be willing to give the investor lower taxes in order to make the investor produce as compared to a government with a low degree of patience. Thi investor does, however, see this before she has made the initial investment. She then knows that when she first has invested, she will continue to produce even if the

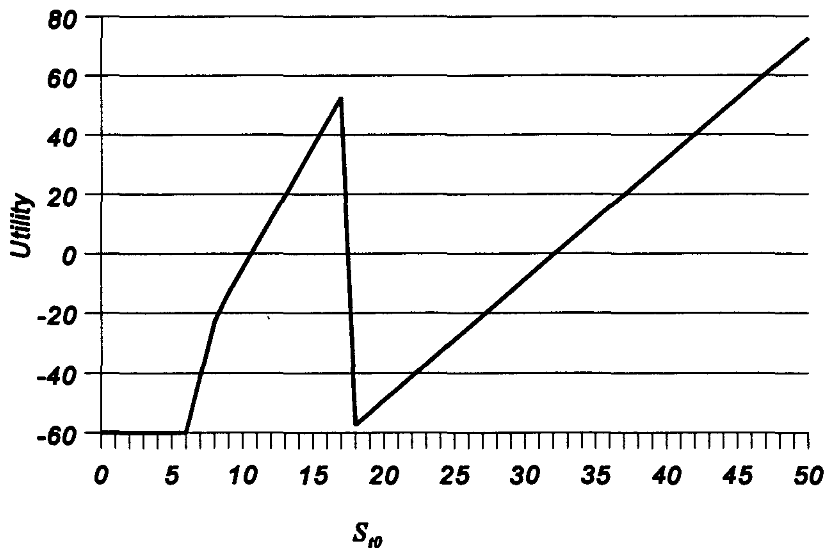

Figure 6 Investor's utility at time zero if production occurs. Based on Example 2.

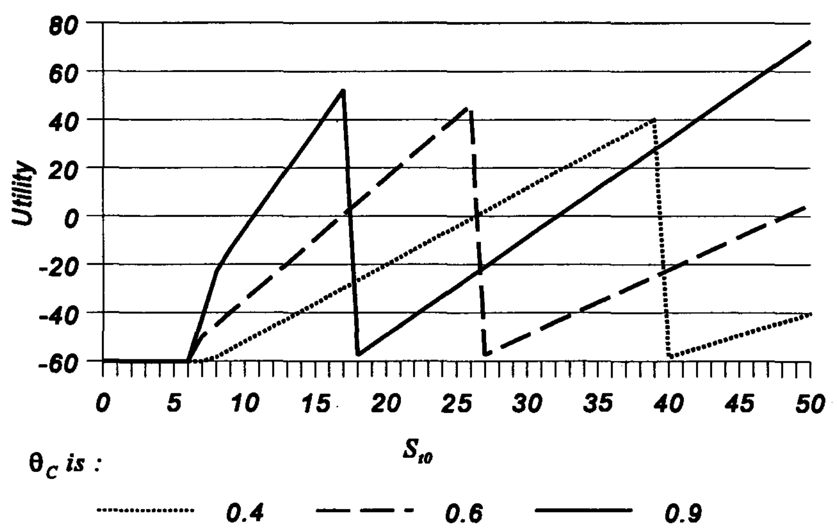

Figure 7 Investor's utility at time zero for different numerical values of the country's discount factor, θ_C. Based on Example 2

royalty rate is set to one for the oil about to be sould. This may cause an abandonment of the oil field at earlier points in time, or an "exit" at node one in the game.

2.3 Comments

In earlier versions of this chapter I specified models with increasing/decreasing production quantities and with a larger number of quantities than presented in this version. A more elaborate model is of course implementable, but at a cost of simplicity. The three observations based on Table 3 would, however, not be qualitatively changed by introducing such a detailed, or expanded, model.

The model presented in this section may explain why investment takes place in countries where the government cannot commit itself to a future regulatory regime for the investment. At every point in time the parties act in self interest, taking into account their actions today and optimal future actions. The model may be reinterpreted by considering each production quantity as an investment in an oil field. With this interpretation, the investor should not evaluate each oil field separately, but as a part of the whole investment programme in the country. A critical condition underlying the model is that the country does not itself produce i.e., there is no national oil company which can extract and sell the oil. If the country would be able to extract oil without any (foreign) investors, the bargaining solution would be affected: the country would be less willing to give lower taxes in order to make production happen. However, many countries may not be able to extract oil due to lack of financial strength and knowledge. A lack of credibility may also make it difficult for the country to borrow funds to invest in its natural resource sector.

3 Model with an Uncertain Oil Price and Possibility to "Wait"

3.1 The Model

I assume that the oil field, cost structure, and payment dates are as described in the previous section. The oil price is now assumed to develop according to a multiplicative binomial model

$$S_{t+1} = \begin{cases} mS_t & \text{with probability } p \\ \Delta S_t & \text{with probability } (1-p) \end{cases}, \tag{14}$$

for positive constants m and Δ, where $m > 1$ and $0 < \Delta < 1$. The start of the game is shown in Figure 8. The game starts at time t_0. The investor decides whether to produce the first quantity, P_1, wait one period, W, or end the game, E. If the investor chooses to wait, the cost per period waited is a constant[6] k^W. The parties' immediate utility from the decision to wait is $u(W) = (u^I(W), u^C(W))$

$$u^I(W) = -k^W, \tag{15}$$

i.e., the cost of waiting, and

$$u^C(W) = ck^W, \tag{16}$$

where c is a nonnegative constant. The parties' immediate utility from production and sales revenue is as in section two.

If the investor produces q_1, the oil price S_{t_1} is then determined in the spot market for crude oil, *M*. The investor and the country negotiate over the royalty rate for q_1, *and* production of q_2. If the parties disagree, *D*, the country determines the royalty rate and the investor sells the oil and pays revenue to the country. She then decides whether to produce the second quantity, wait one period, or abandon the oil field. When the investor decides to wait with the production of a quantity, the next time period starts with the market

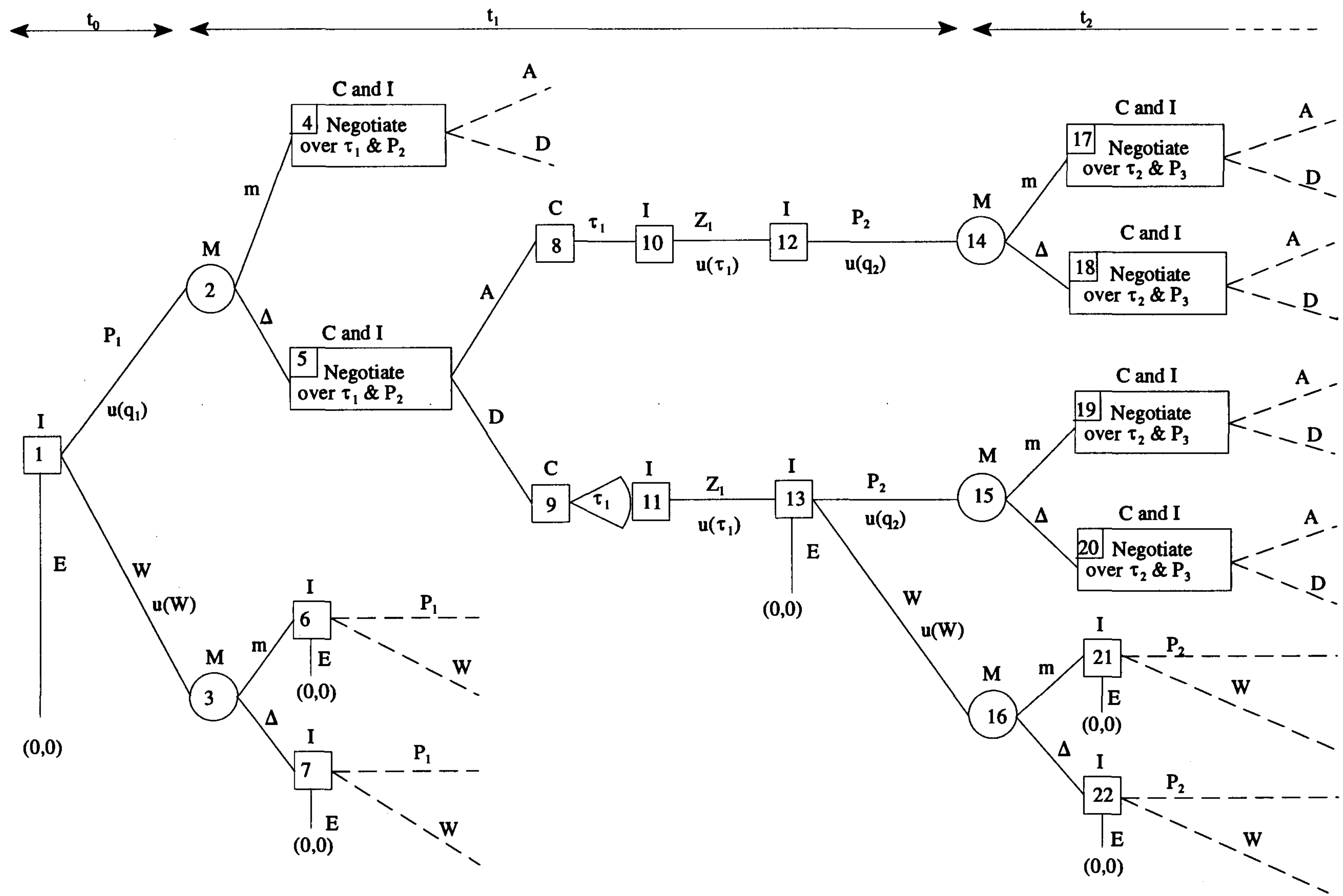

Figure 8 The first stages of the game between the host country, *C*, and the investor, *I*.

determining the oil price and the investor deciding whether to produce, continue to wait, or abandon the oil field. The number of periods the investor is allowed to wait, T^W, may be restricted. When the last production quantity is produced, the parties negotiate over the royalty rate only. Irrespective of whether an agreement is reached, the game ends after the royalty has been paid.

When comparing payoffs at different points in time, I assume that the country applies a discount factor, θ_C, where $0 \leq \theta_C \leq 1$. This discount factor is used when discounting *expected* future utility, where expectations are based on the probability p in equation (14).

I assume that the investor uses a value operator, $V[\cdot]$, based on state prices found by applying the principle of absence of arbitrage when valuing future payoff. If a financial asset exists, possibly hypothetically, where the ex-dividend price is given by (14), and where the dividend at time $t+1$ is δS_{t+1}, the value at time t of a claim, $C_t(1|m)$, paying one USD if the price rises the next period and zero if the price goes down, will be

$$C_t(1|m) = \frac{1+r-\Delta-\delta}{m-\Delta}\frac{1}{1+r} , \tag{17}$$

where r is the risk free interest rate[13]. The value of a claim paying one USD if the price goes down, and zero if not, is

$$C_t(1|\Delta) = \frac{1}{1+r} - C_t(1|m) . \tag{18}$$

As in section two, a pure strategy s_j for player j is a complete plan for how to play the game, i.e., choose an action at every node where the player moves. The notation for strategies and utility is as in section two. The preferences of the country is completely described by the quadruple (a,b,c,θ_C). The preferences of the investor is described by the utility functions (1), (3), (15), and the value operator $V[\cdot]$.

[13] This is a standard result. See, e.g., Cox and Rubinstein (1985) for valuation of options when binomial price processes are used. For equation (17) it is assumed that $(1+r) \geq (\Delta+\delta)$.

3.2 Solution: Nash Bargaining Solution and Backward Induction

As in the previous section, I solve the game by backward induction and the Nash bargaining solution is used at the negotiation nodes. The game will in principle be played as for the model in section two. The only difference is that with an uncertain oil price, the investor may choose W instead of P_n or E. Another consequence of the uncertainty caused by a random oil price, is that the calendar dates when the nodes are reached will depend on the path of the oil price. The time when production occurs will therefore be random.

Consider the case for node x and time t, where the parties negotiate over production of the royalty rate for sale of quantity q_n and production of quantity q_{n+1}. In order for the investor not to be worse off from entering into an agreement, the following inequality must be satisfied:

$$q_n S_t(1-\tau_n^A) - q_{n+1}k + \mathrm{V}_t[\mathrm{U}^{\mathrm{I}}(s_I^{\bar{h}*}, s_C^{\bar{h}*})] + \mathrm{V}_t[\mathrm{U}^{\mathrm{I}}(s_I^{\underline{h}*}, s_C^{\underline{h}*})] \geq q_n S_t(1-\tau_n^D) + \mathrm{U}^{\mathrm{I}}(s_I^{g*}, s_C^{g*}), \quad \textbf{(19)}$$

where nodes $\bar{h}$ (oil price increases) and $\underline{h}$ (oil price decreases) are the nodes where negotiations start in the next time period following an agreement this time period[14] and g is the node where the investor chooses between P_{n+1}, W, and E in case of disagreement. In case of disagreement it will be optimal for the country to set $\tau_n^D = 1$. By inserting $\tau_n^D = 1$ in (19) and rearranging terms, we find that in order for the investor to prefer an agreement to a disagreement, the investor's after tax sales revenue must satisfy:

$$q_n S_t(1-\tau_n^A) \geq -[-q_{n+1}k + \mathrm{V}_t[\mathrm{U}^{\mathrm{I}}(s_I^{\bar{h}*}, s_C^{\bar{h}*})] + \mathrm{V}_t[\mathrm{U}^{\mathrm{I}}(s_I^{\underline{h}*}, s_C^{\underline{h}*})]] + \mathrm{U}^{\mathrm{I}}(s_I^{g*}, s_C^{g*}) \ . \quad \textbf{(20)}$$

As in section two, N^I is equal to the LHS of this inequality.

The inequality applying to the country, corresponding to (19), is

$$aq_n S_t \tau_n^A + bq_{n+1}k + \theta_C \mathrm{E}_t[\mathrm{U}^{\mathrm{C}}(s_I^{\bar{h}*}, s_C^{\bar{h}*}) + \mathrm{U}^{\mathrm{C}}(s_I^{\underline{h}*}, s_C^{\underline{h}*})] \geq aq_n S_t \tau_n^D + \mathrm{U}^{\mathrm{C}}(s_I^{g*}, s_C^{g*}) \ , \quad \textbf{(21)}$$

---

[14] Compare with Figure 8. If, e.g., node x is 5, node $\bar{h}$ corresponds to node 17, node $\underline{h}$ to node 18, and node g corresponds to node 13.

where the LHS is the country's utility in case of an agreement and the RHS is the country's utility in case of disagreement. We insert for $\tau_i^D = 1$ in (21), rearrange terms, and find the inequality to be satisfied by the investor's after tax revenue:

$$q_n S_t(1-\tau_n^A) \leq [bq_{n+1}k + \theta_C \mathrm{E}_t[\mathrm{U}^{\mathrm{C}}(s_I^{\bar{h}*}, s_C^{\bar{h}*}) + \mathrm{U}^{\mathrm{C}}(s_I^{\underline{h}*}, s_C^{\underline{h}*})]]/a - \mathrm{U}^{\mathrm{C}}(s_I^{g*}, s_C^{g*})/a \ . \qquad \textbf{(22)}$$

The RHS of (22) is N^C.

In order for an agreement to be feasible, the after tax sales revenue to the investor must satisfy the condition

$$N^I \leq q_n S(1-\tau_n) \leq N^C , \qquad \textbf{(23)}$$

which is equal to (10). For a negotiation solution involving sale of quantity n and production of quantity q_{n+1} to be feasible, conditions C1, C2, and C3 must be satisfied as in section 2.2. The negotiation solution is given by equation (11).

Proposition 5 *For the game with an uncertain oil price, if the investor in case of disagreement produces the next production quantity, then the amount the country is willing to pay to the investor in order to obtain an agreement involving further production is zero, i.e.,* $N^C = 0$. *If the investor abandons the oil field in case of disagreement and* $b > 0$ *and/or* $\theta_C > 0$, *then* $N^C > 0$.

Proof. If the investor produces the next production quantity in case of a disagreement, then

$$N^C = [bq_{n+1}k + \theta_C \mathrm{E}_t[\mathrm{U}^{\mathrm{C}}(s_I^{\bar{h}*}, s_C^{\bar{h}*}) + \mathrm{U}^{\mathrm{C}}(s_I^{\underline{h}*}, s_C^{\underline{h}*})]]/a -$$
$$[bq_{n+1}k + \theta_C \mathrm{E}_t[\mathrm{U}^{\mathrm{C}}(s_I^{\bar{v}*}, s_C^{\bar{v}*}) + \mathrm{U}^{\mathrm{C}}(s_I^{\underline{v}*}, s_C^{\underline{v}*}]]/a \quad ,$$

where $\bar{v}$ and $\underline{v}$ are the nodes[15] where negotiations start at time t+1. N^C will be zero, because the country's discounted utility at nodes $\bar{v}$ and $\bar{h}$ and the discounted utility at nodes $\underline{v}$ and $\underline{h}$

[15] Compare with Figure 8: node $\bar{v}$ corresponds to node 19 and node $\underline{v}$ to node 20. See footnote 14.

are equal.

For the case where the investor abandons the oil field in case of disagreement, the country can accept that the investor receives an after tax sales revenue equal to

$$N^C = [bq_{n+1}k + \theta_C E_t[U^C(s_I^{\bar{h}*}, s_C^{\bar{h}*})] + U^C(s_I^{\underline{h}*}, s_C^{\underline{h}*})]]/a$$

in order to enter into an agreement. We see that N^C will always be strictly positive if $b > 0$ abd/or $\theta_C > 0$, because the lowest possible utility for the country at nodes $\bar{h}$ and $\underline{h}$ is, respectively, $aS_t mq_{n+1} > 0$ and $aS_t \Delta q_{n+1} > 0$. The utility at these nodes corresponds to full taxation and abandonment of the oil field.

■

Proposition 6 *For the game with an uncertain oil price, if the investor decides to wait in case of disagreement,* $bq_{n+1}k > ck^W$*, and* $\theta_C = 0$*, then the amount the country is willing to give the investor in order to obtain an agreement is strictly positive.*

Proof. If the investor chooses to wait in case of disagreement, the amount the country is willing to give to the investor to obtain an agreement is

$$N^C = [bq_{n+1}k^P + \theta_C E_t[U^C(s_I^{\bar{h}*}, s_C^{\bar{h}*}) + U^C(s_I^{\underline{h}*}, s_C^{\underline{h}*})]]/a - [ck^W + \theta_C E[U^C(s_I^{\bar{w}*}, s_C^{\bar{w}*}) + U^C(s_I^{\underline{w}*}, s_C^{\underline{w}*})]]/a$$

where $\bar{w}$ and $\underline{w}$ is the investor's decision node[16] at time t+1 following the investor's decision to wait at time t. The RHS of this inequality is strictly positive if $bq_{n+1}k > ck^W$ and $\theta_C = 0$.

■

[16] Compare with Figure 8: node $\bar{w}$ corresponds to node 21 and node $\underline{w}$ corresponds to node 22. See footnotes 14 and 15.

Proposition 7 *For the game with an uncertain oil price, if the investor chooses to produce in case of disagreement, then the lowest amount the investor is willing to accept in order to enter into an agreement,* N^I*, is zero. If the investor abandons the oil field, then* $N^I > 0$*. If the investor chooses to wait, then* $N^I \geq 0$.

Proof. If the investor produces in case of disagreement, then

$$N^I = -[-q_{n+1}k + \mathrm{V}_t[\mathrm{U}^{\mathrm{I}}(s_I^{\bar{h}*}, s_C^{\bar{h}*})] + \mathrm{V}_t[\mathrm{U}^{\mathrm{I}}(s_I^{\underline{h}*}, s_C^{\underline{h}*})]] + [-q_{n+1}k + \mathrm{V}_t[\mathrm{U}^{\mathrm{I}}(s_I^{\bar{v}*}, s_C^{\bar{v}*})] + \mathrm{V}_t[\mathrm{U}^{\mathrm{I}}(s_I^{\underline{v}*}, s_C^{\underline{v}*})]] \ .$$

Because the sub-games starting at nodes $\bar{h}$ and $\bar{v}$ and the sub games starting at nodes $\underline{h}$ and $\underline{v}$ are identical, the investor's utility of the sub-games at these nodes will be identical, and N^I will be zero.

If the investor abandons the oil field in case of disagreement, the amount received by the investor making her willing to accept an agreement is

$$N^I = -[-q_{n+1}k + \mathrm{V}_t[\mathrm{U}^{\mathrm{I}}(s_I^{\bar{h}*}, s_C^{\bar{h}*})] + \mathrm{V}_t[\mathrm{U}^{\mathrm{I}}(s_I^{\underline{h}*}, s_C^{\underline{h}*})]] \cdot$$

N^I will be strictly positive if the expression in brackets is negative. This will be the case because it is assumed that the investor will abandon the oil field in case of disagreement and because the sub-games at nodes $\bar{h}$ and $\bar{v}$ and the sub-games at nodes $\underline{h}$ and $\underline{v}$ are identical.

If the investor waits in case of disagreement, then

$$N^I = -[-q_{n+1}k + \mathrm{V}_t[\mathrm{U}^{\mathrm{I}}(s_I^{\bar{h}*}, s_C^{\bar{h}*})] + \mathrm{V}_t[\mathrm{U}^{\mathrm{I}}(s_I^{\underline{h}*}, s_C^{\underline{h}*})]] + [-k^W + \mathrm{V}_t[\mathrm{U}^{\mathrm{I}}(s_I^{\bar{w}*}, s_C^{\bar{w}*})] + \mathrm{V}_t[\mathrm{U}^{\mathrm{I}}(s_I^{\underline{w}*}, s_C^{\underline{w}*})]] \ .$$

The first expression in brackets on the RHS corresponds to the utility when the investor produces, because the utility at nodes $\bar{h}$ and $\bar{v}$ and nodes $\underline{h}$ and $\underline{v}$ is the same. The last expression corresponds to the investor's utility of waiting. Waiting is preferred to production if the utility of waiting is larger than the utility of producing and nonnegative. This implies that N^I will be nonnegative.

■

3.2.1 Example 3

Table 4 summarizes the assumptions for Example 3. The expected return of an asset, possible hypothetical, with an ex-dividend price process equal to the oil price process (14) and with the parameters given in Table 4, is 7.3 per cent per period, included a drift adjustment, δ, of two per cent. The factors m and Δ are chosen so that the grid of oil prices is recombining, meaning that if the oil price goes down on period and up the next, the oil price will be exactly the same in numerical value.

In Table 5 I report the investor's utility at time zero for the alternatives when the investor either produces the first production quantity or waits, for different lengths of allowed waiting time, T^W. The investor's utility of producing the first quantity when the oil price is thirteen is lower if waiting is allowed compared to the utility when no waiting is allowed. This is caused by a change in the negotiation solution involving the royalty rate for the first production quantity, i.e., negotiation of τ_1^A. If no waiting is allowed, the investor will abandon the oil field. If waiting is allowed, the investor will choose to wait. This changes the negotiated royalty rate, and for this example, to a higher royalty rate. I have in Figure 10 shown the investor's utility from producing the first quantity at time zero for $T^W = 0$ and $T^W = 1$. We see from Table 5 that the investor's utility of producing now is not affected by increasing the length of allowed waiting time from one to two or three periods.

The shaded areas in Table 5 indicate when waiting is preferred to producing at time zero.

$q = \{10,10,10\}$	$a = 1$
$k = 6$	$b = 0.5$
$k^W = 5$	$c = 0.4$
$m = 1.2$	$\theta_C = 0.9$
$\Delta = 1/1.2$	$r = 0.05$
$p = 0.6$	$\delta = 0.02$

Table 4 Assumptions for Example 3

The investor's utility of producing or waiting when $T^W = 1$ is shown in Figure 9.

The country's expected utility at time zero for different lengths of allowed waiting time is reported in Table 6 and shown in Figure 11.

	T^W=0	T^W=1		T^W=2		T^W=3	
Oil Price	Produce	Produce	Wait	Produce	Wait	Produce	Wait
5	-56.87	-56.87	-5.00	-56.87	-5.00	-56.87	-5.00
6	-47.10	-47.10	-5.00	-47.10	-5.00	-47.10	-5.00
7	-35.41	-35.41	-5.00	-35.41	-5.00	-35.41	-5.00
8	-21.70	-21.70	-5.00	-21.70	-2.95	-21.70	-2.95
9	-10.04	-10.04	0.64	-10.04	0.64	-10.04	0.64
10	1.77	1.77	6.28	1.77	6.28	1.77	6.28
11	12.87	12.87	-5.00	12.87	-3.95	12.87	-3.95
12	22.07	22.07	-2.75	22.07	0.20	22.07	0.20
13	31.23	-7.25	0.01	-7.25	0.01	-7.25	0.01
14	-22.83	-0.02	3.39	-0.02	3.39	-0.02	3.39
15	-16.56	-16.56	7.93	-16.56	7.93	-16.56	7.93
16	-10.28	-10.28	-5.00	-10.28	-4.50	-10.28	-4.50
17	-4.00	-4.00	-4.48	-4.00	-3.25	-4.00	-3.25
18	2.27	2.27	-5.00	2.27	-1.50	2.27	-1.50
19	8.55	-25.03	-5.00	-25.03	-5.00	-25.03	-5.00
20	-39.53	-19.89	-5.00	-19.89	-5.00	-19.89	-5.00
21	-34.96	-34.96	-5.00	-34.96	-5.00	-34.96	-4.22
22	-30.40	-30.40	-5.00	-30.40	-3.97	-30.40	-3.17
23	-25.84	-25.84	-5.00	-25.84	-2.25	-25.84	-2.12
24	-21.28	-21.28	-4.69	-21.28	-0.54	-21.28	-0.54
25	-16.72	-16.72	-1.89	-16.72	1.18	-16.72	1.18
26	-12.16	-12.16	0.90	-12.16	2.89	-12.16	3.09
27	-7.60	-7.60	3.70	-7.60	4.60	-7.60	5.44
28	-3.04	-3.04	6.50	-3.04	6.50	-3.04	7.78
29	1.53	1.53	9.29	1.53	9.29	1.53	10.13
30	6.09	6.09	12.09	6.09	12.09	6.09	12.99

Table 5 The investor's utility at time zero for Example 3 when the investor either produces or waits, for different lengths of allowed waiting time, T^W. The shaded areas indicate the oil prices for which the investor will wait.

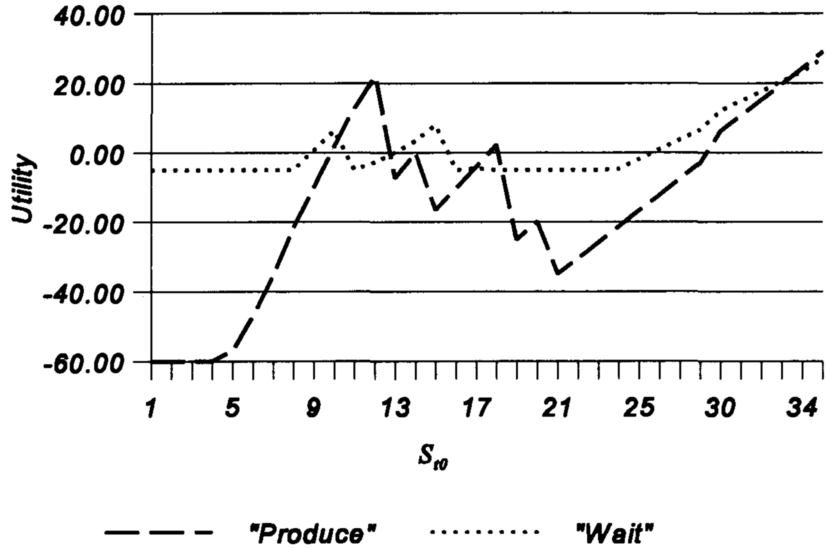

Figure 9 Investor's utility at time zero, for Example 3, for the alternatives to produce and to wait when T^W=1

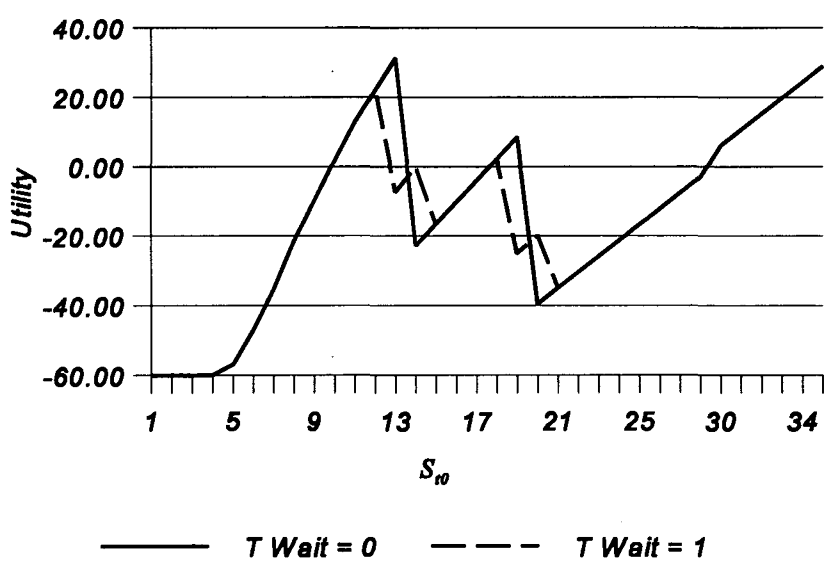

Figure 10 Investor's utility at time zero from producing the first quantity when there is no allowed waiting time, and when the allowed waiting time is one period. Based on Example 3

Oil Price	T^W=0	T^W=1	T^W=2	T^W=3
5	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	0.00
9	0.00	110.88	110.88	110.88
10	188.50	122.78	122.78	122.78
11	205.31	205.31	205.31	205.31
12	223.67	223.67	223.67	223.67
13	241.99	74.81	74.81	74.81
14	0.00	80.32	80.32	80.32
15	0.00	308.53	308.53	308.53
16	0.00	0.00	0.00	0.00
17	0.00	0.00	0.00	0.00
18	412.42	412.42	412.42	412.42
19	433.75	0.00	0.00	0.00
20	0.00	0.00	0.00	0.00
21	0.00	0.00	0.00	0.00
22	0.00	0.00	0.00	0.00
23	0.00	0.00	0.00	0.00
24	0.00	0.00	0.00	0.00
25	0.00	0.00	254.74	254.74
26	0.00	409.12	264.28	343.81
27	0.00	423.85	273.82	356.22
28	0.00	438.58	438.58	368.62
29	703.93	453.30	453.30	381.03
30	726.66	468.03	468.03	485.14

Table 6 The country's expected utility in Example 3 for different lengths of allowed waiting time, T^W. The shaded area indicates when the situation of no waiting time results in the highest expected utility

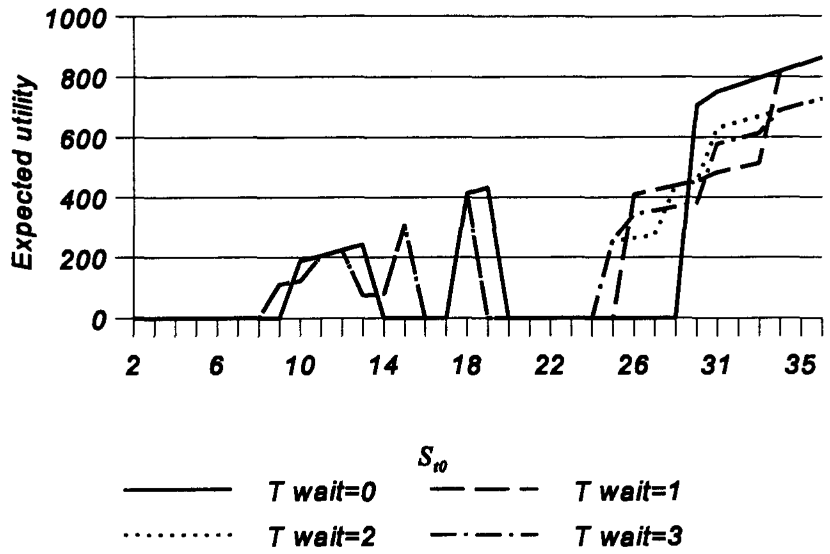

Figure 11 The country's expected utility at time zero for different levels of oil price and allowed waiting time, T^W

Based on the computations of Example 3, I conclude that the effect on the investor's and the country's utility at time zero of increasing the allowed waiting time is inconclusive. The effect of increasing the length of allowed waiting time must be analyzed given the specific assumptions about the oil field, the type of government, and the other parameters of the model.

3.3 Comments

It is straightforward to include a stochastic oil price and the option to wait in the model presented in sub-section two. We see that the investor's option to wait does not result in a unique oil price S^*, where waiting is preferred for oil prices lower than S^* and producing is preferred for oil prices higher than S^*, as is often the case with a constant royalty rate. This could not be expected because the non-uniqueness of the break-even spot price when waiting is not allowed.

4 Concluding Remarks

If negotiation of tax rates, here represented by a royalty rate, at different points in time is a reasonable description of how governments and oil companies interact, this paper shows that analyses based on fixed and exogenously specified royalty rates may lead to wrong conclusions regarding when to invest, wait, or abandon the oil field. An observer studying investor behavior without properly taking into account the endogenous nature of government regulations will face trouble when trying to understand actual investor behavior and investment flows. The models presented in this paper, even though they are rather simple, may be used when evaluating and analyzing investments in countries with high political uncertainty and lack of credibility.

I see several possible extensions of this approach. The first one is to investigate how taxation actually is changing over time, and try to explain the changes by using simple models of the types presented here. The second extension is to explain the use and composition of investment syndicates in the oil industry. Investment syndicates consisting of many oil companies may cause a credible threat of abandonment if the negotiations for lower taxation fails. This may lead to lower negotiated royalty rates, which may increase the value of the oil field to the investors. The third extension, which is linked to the previous one, is to investigate the government's preferences for composing investment syndicates. It might be optimal for the country to compose a strong syndicate which can cause a threat of abandonment. Investments may take place with a "strong" syndicate, but not necessarily with a "weak" investment syndicate.

Appendices

A1 The Nash Bargaining Solution

Given a set of assumptions about the negotiation problem (Y,d), and a set of axioms that a solution to the problem must satisfy, the axiomatic approach predicts a unique solution (y^{I*},y^{C*}) to the bargaining problem. The assumptions the bargaining problem must satisfy are:

A1. Y is a convex set.

A2. Y is compact.

A3. $d \in Y$

We see that A1 and A2 are satisfied for the case with linear utility functions.

Let Σ be the set of all (Y,d) which satisfies A1.-A3. A bargaining solution is a function $F: \Sigma \rightarrow \mathbb{R}^2$ where $F(Y,d) \in Y \ \forall \ (S,d) \in \Sigma$.

The axioms are:

Axiom 1. Individual rationality.

The outcome of the bargaining problem $(y^{I*},y^{C*}) = F(Y,d)$ shall be strictly better for both parties than the no agreement payoff, i.e., $d^I < y^{I*}$ and $d^C < y^{C*}$.

Axiom 2. Pareto optimality.

Compared to the chosen solution, no other bargaining solution exists such that both parties can be made strictly better off.

$PO(Y) = \{u \in Y \mid u' > u \Rightarrow u' \notin Y\}$. $F(Y,d) \in PO(Y,d) \forall (Y,d) \in \Sigma$.

Axiom 3. Invariance.

For all (Y,d) and (Y',d') in Σ, if there exist a positive affine transformation $\lambda: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, such that $Y' = \lambda(Y)$ and $d' = \lambda(d)$, then $F(Y',d') = \lambda(F(Y,d))$.

Axiom 4. Independence of irrelevant alternatives.
If $F(Y,d) = u$ and $u \in Y' \subset Y$, then $F(Y',d) = u$.

Axiom 5. Symmetry.
If Y and d are symmetric, then $F^I(Y,d) = F^C(Y,d)$.

Theorem

If a bargaining solution F satisfies axioms 1-4, there exists $\beta \in (0,1)$ *such that* $F(Y,d) = N(Y,d) \ \forall \ (Y,d) \in \Sigma$ *where* $N(Y,d) = \text{argmax}\{(u^I - d^I)^{\beta}(u^C - d^C)^{(1-\beta)}\}$. *If the solution also satisfies axiom 5, then* $\beta = 0.5$.

A proof of the theorem can be found, e.g., in Eichberger (1993) p. 255. When symmetry holds, the solution is known as the Nash bargaining solution.

For the models in section two and three, the parties' utility functions are linear in the division of the of sales revenue. The parties bargaining problem can therefore be studied in terms of the amount, N^N, measured in units of money, that the parties are bargaining over. The Pareto optimal allocation of N^N is such that $X^I + X^C = N^N$, where X^I and X^C are the investor's and the country's part of N^N, see Figure A1. The curve *a* represents combinations of X^I and X^C where their product is a constant *K*, i.e., $X^I X^C = K$. The highest *K* is obtained for $X^I = X^C$, provided that the current sales revenue, $q_i S$, is higher than this allocation. This combination maximizes the function $N(\cdot,\cdot)$ in the Theorem. If the current sales revenue is lower than this allocation, $q_i S'$ in Figure A1, the optimal *K* will be obtained for $X^I = q_i S'$ and $X^C = N^N - q_i S'$. This means that the solution to the bargaining problem can be written as

$$q_i(1 - \tau_i^A) = \text{Min}(N^L + N/2, q_i S) \ . \tag{1}$$

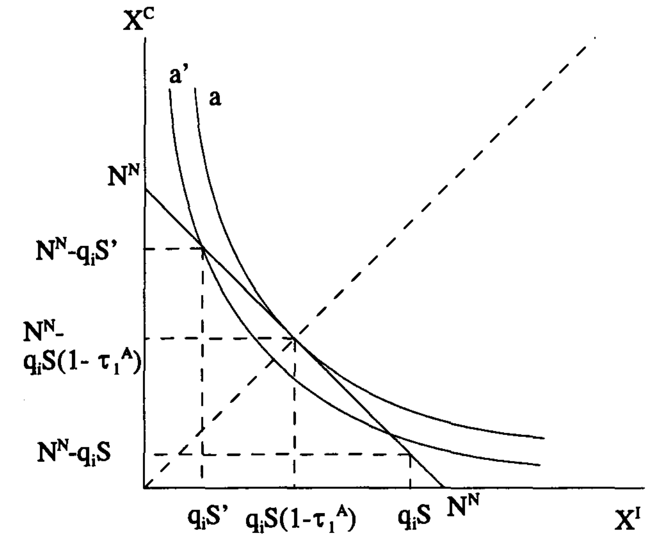

Figure A1

A2 An Equivalent Method for Computing the Solution to the Bargaining Problem, Example 1

Time t_2 (q_2 is ready for sale)

At this point, no further production will occur. The parties will agree and the royalty rate is equal to one. The parties' utility is $U_9^* = U_{10}^* = (0, 180)$.

Time t_1 (q_1 is ready for sale and production of q_2 may start)

Because quantity q_2 will be taxed at a royalty rate $\tau_2 = 1$, the investor will select E if an agreement is not reached. This gives a utility of zero. If an agreement is not reached, it will be optimal for the country to tax production quantity q_1 with a royalty rate $\tau_1^D = 1$.

Verifying if a negotiation solution is feasible

The country's utility when an agreement is made is

$$\mathrm{U}^{C}(s_I^{2*},s_C^{2*}|A) = aq_1S\tau_1^A + bq_2k + \theta_C q_2 S \ , \tag{2}$$

and the investor's utility is

$$\mathrm{U}^{I}(s_I^{2*},s_C^{2*}|A) = q_1S(1-\tau_1^A) - q_2k \tag{3}$$

By combining (2) and (3) we get the set of possible utility allocations involving an agreement,

$$\begin{aligned} Y = \{ & (\mathrm{U}^{I}(s_I^{2*},s_C^{2*}|A), \mathrm{U}^{C}(s_I^{2*},s_C^{2*}|A)) \in \mathbb{R}^2 \,| \\ & \mathrm{U}^{C}(s_I^{2*},s_C^{2*}|A) = +B - a\mathrm{U}^{I}(s_I^{2*},s_C^{2*}|A) \ , \\ & -q_2k \le \mathrm{U}^{I}(s_I^{2*},s_C^{2*}|A) \le q_1S - q_2k \} \ , \end{aligned} \tag{4}$$

where $B = aq_1S + (b-a)q_2k + \theta_C q_2 S$.

The disagreement allocation for the investor is zero.

By inserting $d^I = \mathrm{U}^{I}(s_I^{2*},s_C^{2*}|D) = \mathrm{U}^{I}(s_I^{2*},s_C^{2*}|A) = 0$ in (4), we find that the implied allocation to the country on the Pareto frontier is higher than the disagreement allocation for the country, i.e.,

$$\mathrm{U}^{C}(s_I^{2*},s_C^{2*}|A) = B = aq_1S + (b-a)q_2 + \theta_C q_2 S \ge d^C = aq_1S .$$

because $(b-a)q_2k + \theta_C q_2 S > 0$. The actual numbers are: $B = 312$ and $aq_2S = 180$. A negotiation solution involving P_2 is therefore feasible.

Finding the bargaining solution

The negotiation problem (Y,d) is given by

$$\begin{aligned} Y = \{ & (\mathrm{U}^{I}(s_I^{2*},s_C^{2*}|A), \mathrm{U}^{C}(s_I^{2*},s_C^{2*}|A)) \in \mathbb{R}^2 \,| \\ & \mathrm{U}^{C}(s_I^{2*},s_C^{2*}|A) = +B - a\mathrm{U}^{I}(s_I^{2*},s_C^{2*}|A) \ , \\ & -q_2k \le \mathrm{U}^{I}(s_I^{2*},s_C^{2*}|A) \le q_1S - q_2k \} \ , \end{aligned} \tag{5}$$

where , a=1 and $B = aq_1S + (b-a)q_2k + \theta_C q_2S$ and $d = (0,180)$. Consider the negotiation problem (X,(0,0)), where $x_I + x_C \leq 1$. This is a negotiation problem over the division of one unit (e.g., one unit of revenue) between the parties, where the disagreement allocation is zero to both parties. If the set of possible payoffs given by (5) is a positive affine transformation of the negotiation problem over the unit, i.e., $U^I(s_I^{2*}, s_C^{2*}|A) = a_1 + b_1x_I$ and $U^C(s_I^{2*}, s_C^{2*}|A) = a_2 + b_2x_C$, then the solutions (x_I^*, x_C^*) and $(U^{I*}(s_I^{2*}, s_C^{2*}|A), U^{C*}(s_I^{2*}, s_C^{2*}|A))$ are related . We have that $U^{I*}(s_I^{2*} s_C^{2*}|A) = a_1 + b_1x_I^*$ and $U^{C*}(s_I^{2*}, s_C^{2*}|A) = a_2 + b_2x_C^*$, by axiom 3 in A1. The parameters of the affine transformation are given by (6)-(9):

$$a_1 = d^I . \quad (6)$$

$$a_2 = d^C . \quad (7)$$

$$b_1 = \frac{B}{a} - \frac{a_2}{a} - a_1 . \quad (8)$$

$$b_2 = B - a a_1 - a_2 . \quad (9)$$

For Example 1 we get that $a_1 = 0, a_2 = 180$, and $b_1 = b_2 = 180$. The Nash bargaining solution stipulates that the parties get half of what they negotiate over (see Theorem in A1). In this case, the solution is $U^{I*}(s_I^{2*}, s_C^{2*}|A) = 132 \text{x} 0.5 = 66$ and $U^{C*}(s_I^{2*}, s_C^{2*}|A) = 180 + 132 \text{x} 0.5 = 246$. By inserting $U^{C*}(s_I^{2*}, s_C^{2*}|A)$ in (2), or $U^{I*}(s_I^{2*}, s_C^{2*}|A)$ in (3), and solving for the royalty rate, we find that $\tau_1^A = 0.3$. This is exactly the royalty rate computed in section two for Example 1.

A3 List of Main Symbols

Symbols Related to the Project

$Q = \{q_1, \ldots, q_n, \ldots, q_N\}$	Set of N production quantities
k	Production cost per barrel of oil
k^W	Waiting costs
S	Oil price, possibly with a sub script indicating time
τ_n	Royalty rate for revenue generated from sale of quantity n

Symbols Related to the Player's Actions and Negotiations

I	Investor
C	Country
P_n	Production of quantity n
Z_n	The sale of quantity n, and the subsequent payment of royalty from the sale of quantity n
W	Deferring the production decision one period
A	The parties agree
D	The parties disagree
τ_n^A	Royalty rate for quantity n declared by the government following an agreement, identical to the numerical value of the royalty rate
τ_n^D	Royalty rate for quantity n declared by the government following a disagreement, identical to the numerical value of the royalty rate
N^N	What the parties are negotiating over, measured in units of money
N^I	The lowest amount I is willing to accept in after-tax revenue from an agreement involving the production of the next quantity
N^C	The highest amount C is willing to give I in order to obtain an agreement solution involving the production of the next quantity

Symbols Related to Utility

a	Constant in C's utility function, used in connection with tax revenue
b	Constant in C's utility function, used in connection with production

	costs
c	Constant in *C*'s utility function, used in connection with waiting costs
$\mathrm{u}^{\mathrm{i}}(q_n)$, $i = I, C$	Instantaneous utility from the production of quantity *n*
$\mathrm{u}^{\mathrm{i}}(\tau_n)$, $i = I, C$	Instantaneous utility from the taxation of revenue generated from the sale of quantity *n*
$\mathrm{u}^{\mathrm{i}}(W)$, $i = I, C$	Instantaneous utility from deferring the production one period involving payment of waiting costs
θ_i, $i = I, C$	Discount factor
$\mathrm{U}^{\mathrm{i}}(s_I^x, s_C^x)$, $i = I, C$	The utility at node *x* from the strategy combination (s_I^x, s_C^x), where the instant utility from future time periods is discounted to the time where node *x* appears in the game
$\mathrm{U}_{\mathrm{x}}^{*}(\cdot,\cdot)$	Shorthand for the parties' discounted utility to node *x* when the sub-game at node *x* is solved with backwards induction and application of the Nash bargaining solution

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