SALMON AQUACULTURE IN NORWAY: AN EMPIRICAL ANALYSIS OF COST AND PRODUCTION PROPERTIES

by

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1. INTRODUCTION

1.1 Objectives

The purpose of this research is to analyze the production process in the Norwegian salmon aquaculture industry. More precisely, we will model the production strategy of a fish farmer and through empirical analysis provide information about cost and production properties for the industry, i.e., economies of scale properties and substitution properties. As a secondary objective, policy implications may be derived. We will focus on three subjects in this respect. First, we will relate our empirical results of the scale elasticity to the Norwegian size regulation of fish farms. The Government's stated objective for regulating farm size, also agreed upon by the fish farmers' organizations, is primarily considerations to have a regional allocation of farms (*NOU 1977:39, NOU 1985:22*). By putting a ceiling on the size, thus having a structure of the industry consisting of relatively small units, one wishes to use it as a distributive device to maintain the pattern of scattered settlement along the Norwegian coast. Thus, if scale economies exist in the industry, size limitations could lead to a high cost industry. Second, by combining the estimated average cost curve for the industry with the results from the demand literature on farmed salmon, we may simulate future cost and revenue patterns of the industry. Thus patterns of possible rent dissipation may be drawn and thus the future levels of expansion in the industry. Furthermore, and as a third point, from the estimated elasticities of substitution, effects of changes in relative prices caused by, for instance, factor subsidies and factor regulation, can be discussed.

To accomplish these objectives we will start by characterizing the decision–making problem of a single grow–out farmer by developing a bioeconomic model for a fish farm. The biology of a single fish and of a year–class will be outlined, from which the options available for a single farmer in determining production strategy can be deduced. When defining the production process of a farm, we stress the differences between commercial fisheries and traditional industrial production. The focus of our analysis is on the use of open sea pen technology during sea growth of Atlantic salmon, which is the dominant mode of production in the Norwegian aquaculture industry. This theoretical framework is general in the sense that it can be used to study also the culture of other species.

Based on the bioeconomic model of optimal harvesting for a single year–class, we formulate both a restricted profit function and a cost function as relevant hypothesis to describe the behaviour of a farmer. Since the available data set is in calendar years, the timing of inputs and output when defining the concepts in the cost and profit function is essential. As a device to express the timing of inputs and output we develop a concept of a "normal" year of production.

A profit function appears to be the preferred function to describe the production strategy of a food fish farmer, *a priori* allowing output to be an endogenous variable. However, it will also be argued that cost minimization with output given exogenously is appropriate in our case due to government regulation of farm size – through capacity regulation – and standardized harvest procedures. Furthermore, since a long run returns to scale measure is of particular interest to us we have to formulate a well–defined long run production process. A long–run cost function will provide us with this measure as opposed to the short–run measure we obtain from a restricted profit function. This is because a short–run formulation with one or more fixed factors from the producer's viewpoint results in a measure of returns to the variable factors which by definition necessarily must be less than one (Weaver, 1983). In addition, with competitive input and output markets and increasing returns to scale, the long run profit function is not well defined since no equilibrium is defined for the producer, and no profit maximum point exists. However, if the output is given to the producer, equilibrium exists for cost minimization under increasing returns to scale (Jorgensen, 1986).

Once the theoretical specification of the decision–making problem has been made, our objective is to obtain parameter estimates of the structural characteristics of the industry. The data set we are using consists of micro–data for fish farms for 1982 and 1983. The Norwegian Directorate of Fisheries supplied the data. From the parameter estimates we will develop measures of elasticities of substitution and elasticity of scale which are consistent with theory and the formulation of the producer behaviour. A relevant measure of scale economies will be obtained from the cost function together with measures of elasticities of substitution. For the restricted profit function, measures of choice such as own price and cross price elasticities of demand and output elasticities with respect to input prices will be provided. These measures will describe the substitution possibilities in the industry. The measures of substitution derived from the restricted profit function are more numerous than those derived from the cost function. This is one reason for specifying a profit function in addition to a cost function. The profit function furthermore could avoid any possible simultaneous equation bias if output is endogenous in the cost function formulation.

Growing international competition in salmon markets as a result of expansion in the aquaculture industry in many countries is another major reason for undertaking an empirical analysis of the Norwegian industry. The revealed structural characteristics may indicate the future competitiveness and the viability of possible further expansion of the industry. When cost measures are coupled with demand functions, information about further expansion appears. Also, when cost studies from the expanding aquaculture industry in other countries are available, a comparative analysis could be undertaken. This, however, must be a subject for further research. Furthermore, it will be possible to provide policy recommendations from the results with regard to the regulations of Norwegian fish farms where size restrictions are paramount.

1.2 Methodology

In order to estimate parameters of the production technology, we make use of the development in duality theory and flexible functional forms. We shall elaborate on this development below.

The dual formulation of production theory has shown that one can characterize a production process by means of both the dual cost function and the profit function by assuming the behavioural hypothesis of price–taking behaviour or cost minimization and profit maximization respectively (Shepard [1953], Diewert [1972], [1982], Lau [1978], McFadden [1978]). This means that parameters of the technology set can either be obtained by estimating the primal formulation of the production function directly, or be recovered from estimating a dual cost or profit function.

By specifying econometric models by the dual formulation of the theory of production, innovations have been made in econometric modelling and testing of the production process (Jorgensen, 1986). The main objective of production theory is to define demand and supply functions to characterize the production process. In empirical research these functions must be stated in explicit form. By imposing strong restrictions on the production function, e.g. additivity and homogeneity in the Cobb–Douglas case, the demand functions for factors and supply function for output can be explicitly deduced from the production function and the associated first–order conditions. By formulating the production problem by means of a dual representation, i.e., a profit or a cost function (both as optimal functions), one avoids solving for the first–order conditions to derive demand and supply functions. One derives explicit demand and supply functions of relative prices by taking first–order derivatives of the cost or profit function by using Shepard's lemma and Hotelling's lemma respectively. By this procedure a consistent relationship between economic theory and the econometric specification of the model is obtained without imposing restrictive constraints on the technology. From the supply and demand functions, additional measures which characterize the production process can be developed, e.g. elasticities of substitution and own—price elasticities. Thus, contrasted to a primal formulation of the production process, i.e., estimating the production function directly, the dual approach has several advantages.

In addition to the above theoretical considerations, estimating a cost or profit function allows the economic behaviour of the agents as cost minimizers and profit maximizers to be taken into account. This behavioural aspect is, of course, not considered in a direct estimate of the production function. Since the scale elasticity is of interest in our case, a related point is of importance. As Hanoch (1975) and later Revier (1987) note, there are two different scale concepts. One concept measures the returns to scale as the relative change in output to a proportionate change in all inputs along a ray from origin, while the other scale concept measures the relative change in output to costs along the expansion path. The first concept forces scale expansions to be possibly price inefficient and will only correspond to the actual expansion path for homothetic technologies. With a dual formulation we get a measure along the expansion path, which is superior to the measure along the ray obtained from a production function.

An additional weakness of the production function approach is that inputs are endogenous right–hand side variables if they are chosen by the producer. Thus, one important assumption is not met for using ordinary least squares regression since the inputs probably are not independent of the error structure. A dual profit function is defined in input and output prices and a cost function in input prices and output, all of which are considered exogenous to the fish farmer.

To take advantage of the development of the dual theory of production, the introduction of flexible functional forms extended the application of duality in econometric work. The flexible functional forms which were first introduced by Diewert (1971) – the generalized quadratic – and by Christensen, Jorgensen and Lau (1973) – the translog – are second–order Taylor series approximations at a point to the underlying functional form. They are generalizations of earlier more restricted forms such as Cobb–Douglas and Constant Elasticity of Substitution (CES); but these newer forms do not impose *a priori* restrictions such as homogeneity, homotheticity, separability and constant substitution elasticities (Lau, 1986).

One can estimate both the translog cost or production function as single equations using ordinary least squares (OLS) since the functions are linear in the parameters and prices are exogenous to the producer under price–taking behaviour in input and output markets. As an alternative approach one can estimate a system of derived demand and supply equations (obtained by differentiating the cost or profit function in prices) to get the coefficients of the cost or profit function. We chose a third possible procedure to estimate a system of the cost function and share equations (and the profit function and share equations) to obtain the most efficient estimates – with the lowest parameter variance – of the associated parameter (Christensen and Greene [1976], Guilkey, Knox Lovell and Sickless [1983]). The chosen functional form is the translog. Since restrictions such as symmetry must be imposed, causing the error structure to correlate across equations, a system estimating procedure is used to take account of this.

1.3 Thesis outline

The outline of the thesis is as follows. Chapter two provides a description of the industry. This includes a characterization of the organizational structure and a description of the production process – both for a single salmon and for the farm. This chapter also describes the restrictions and options of a farmer which are paramount considerations when formulating a farmer's behaviour.

In Chapter three a theoretical model of production is presented. From the optimal time of harvesting one year–class, an analytical profit function is developed for the grow–out farmer. Important here is the concept of a "normal" year of production. Given that the data set is in calendar years and we face an intertemporal problem, the use of "normal" production year allows the consistent use of both input and output data. The alternative behavioural hypothesis for the cost function is formulated. A brief discussion of differences between the two alternative specifications is also made.

In Chapter four the data set will be described and various functional forms will be specified. Exact definitions and summary statistics of the variables will be provided. The translog form of the functions will be presented and the properties discussed. Also, the relevant elasticities will be derived and the desired economic hypothesis will be stated. A stochastic specification will be made and the seemingly–unrelated estimation technique will be presented.

Chapter five presents the empirical results and the tests that were undertaken. A comparison between the results for the two different specifications of the technology will be made. The implications of the results for the industry will be presented with particular emphasis on the scale properties in Chapter six, in addition to a summary of the results.

2. CHARACTERISTICS OF THE SALMON FARMING INDUSTRY IN NORWAY

In this chapter we will characterize the organizational structure and the production process of the industry in order to more realistically formulate the farmer's decision—making problem. Furthermore, this description of the organizational structure and the production process will help to provide more precise definitions of the variables used under empirical estimation.

In order to identify the distinct units of production and the distribution pattern of the farming industry, we will start by presenting a diagram of the industry in Figure 2.1. The focus of our study is the part of the aquaculture industry which rears Atlantic salmon in seawater from smolt size to marketable size for consumption.[1] We will call this producer interchangeably a food fish producer and a grow–out farmer.

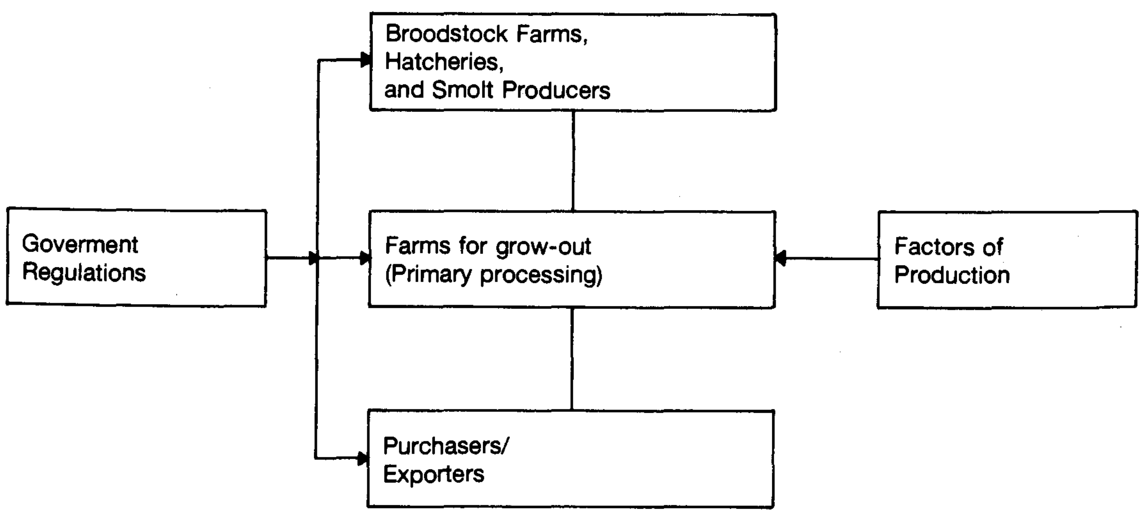


Figure 2.1: Agents of the Norwegian salmon industry.

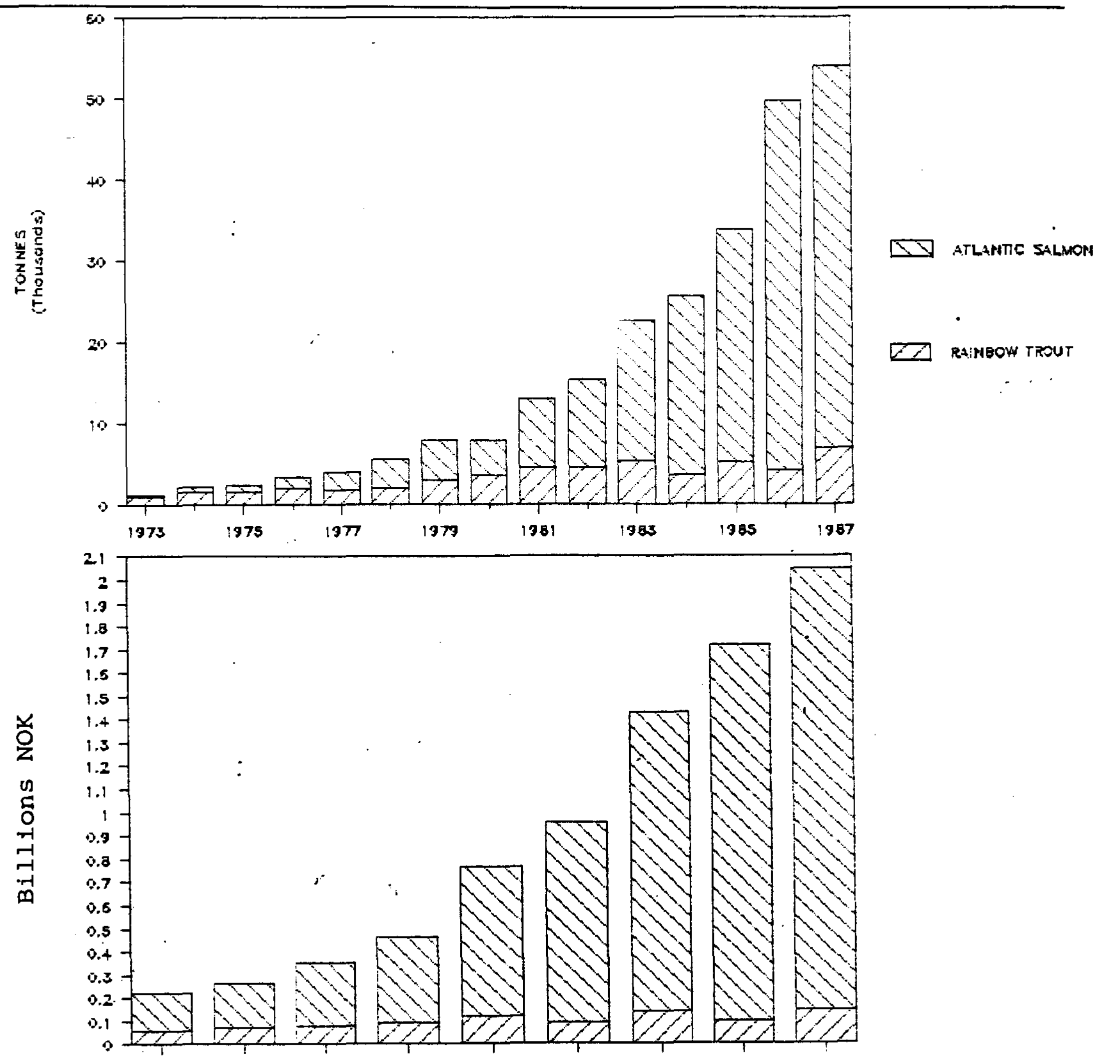
From Figure 2.1 we note that there are three main groups of agents on the input side of food fish production. Smolts and other factors of production, e.g. feed and pens, serve as inputs to the grow–out stage. Smolt producers are subsequently connected to hatcheries and broodstock farms. The government, through regulation, influences the environment of this industry as indicated in Figure 2.1. It regulates the smolt producers on the input side, the grow–out farmers directly and the purchasers/exporters. Salmon is mainly exported. In the distribution channel of salmon, salmon is sold by exporters to an importer/wholesaler and may be handled by a number of other agents before it reaches the consumer. At this market level, competition from other producer countries confronts Norwegian pen–reared salmon sales. Our analysis focuses on modeling and empirically estimating properties of the production level; thus it is appropriate to omit the rest of the distribution channel in our flow diagram.

Using Figure 2.1 as a reference throughout this chapter, we start by presenting some of the socioeconomic characteristics of the industry. Aquaculture in Norway is an export industry and exposed to growing international competition. Hence, we stress the changing circumstances for Norwegian aquaculture due to the expected growth in other countries' salmon output and the consequent pressure on salmon prices and Norwegian market shares. Next, in Section 2.2, we will discuss the market constraints the farmer faces in the factor and output markets. The Norwegian government, through regulations, also acts as an important agent in this industry. Section 2.3 offers a description of the salmon production process. In particular, the biological basics of fish rearing for an individual unit will be outlined. Given this description, the production options open to a single farmer will be deduced. In Section 2.4 we will provide a summary of the relevant points for formulating the economic model in Chapter three.

2.1 Socioeconomic aspects of farming and competition

The rapid growth of the Norwegian fish–farming industry from its start in the early seventies is shown in the diagram in Table 2.1. From 1980, this industry has experienced an annual average growth in quantity of as much as 35–40 percent a year. From 1985 to 1986 growth was more substantial with output rising from 34,000 metric tons to 50,000 tons or a 47 percent increase.

Table 2.1: The quantity (1,000 metric tons) and value (billions NOK) of Atlantic salmon and rainbow trout.



Sources: Central Bureau of Statistics, NOU, 1985:22, and information from "Fish Farmers' Sales Organization."

One attractive feature of salmon farming for a government concerned with employment and regional development is its potential for widely dispersed employment. Since a site location with clean water is essential, fringe areas along the coast can provide good employment opportunities. The aquaculture industry is, however, not a particularly labour–intensive industry. Nonetheless, in contrast to commercial fisheries, it offers year–round and not seasonal employment opportunities. Although not labour intensive, the Norwegian salmon farming industry has still provided a relatively high employment level. As seen from Table I in Appendix A, the industry employed about 4,000 in 1986. In addition, there were 4,000 jobs in secondary activities, such as transportation, feed and equipment production.

Since salmon farming is a net foreign exchange earner, this also enhances its favoured position *vis–à–vis* the government. With about 90 percent of the Norwegian salmon production exported, its relative importance as an export–earner is obvious. In 1985 its share was 25 percent of total fish product export value (Statistical Yearbook, 1986). However, fish exports accounted for only about 6 percent of the value of total Norwegian exports in 1986, implying that the salmon industry is not yet very important to the total economy. On the other hand, in many areas along the coast fish farming is of great importance to the local economy. Since salmon is a commodity traded internationally, the Norwegian salmon farming industry as an export industry is therefore exposed to competition. This situation will probably be maintained since, as indicated in Appendix A, production for 1986 and the 1987–1990 projections depict a large world increase in the production of both Atlantic and Pacific salmon.[2)] These competitive trends in marketing for Atlantic salmon (farmed) and Pacific salmon (wild–caught), are also confirmed by DeVoretz and Salvanes (1987) and Lin and Herrmann (1987).

At this point we summarize some background points and their relevance for the scope of this study. First we note that the salmon aquaculture industry is of increasing importance to the Norwegian economy in terms of employment, net foreign exchange earnings and the rural economy. Second, with 90 percent of production exported and the forecasts presented showing that other salmon producing countries are planning to increase considerably their harvest levels, the industry is exposed to increased competition in foreign markets. A drop in the price of salmon is anticipated from this expected increase in supply.

2.2 The organization of the industry

In this section we will give a description of the important groups of agents in this industry as shown in Figure 2.1, i.e., smolt producers (plus broodstock farms and hatcheries), food fish farmers, purchasers/exporters and the government. Other factors of production will be considered in Section 2.3. In the discussion of the four parts the food fish farmer is emphasized since we are concerned with production. First we will describe government regulations since it is important for the description of the other agents, and hence it will be referred to in the sections to follow.

2.2.1 Government regulations

As briefly mentioned in Chapter one, the fish farming industry is regulated. In this section we will discuss the Fish Farming License Act, and in Section 2.2.3 we will provide a short description of the other aspects of regulation. The licensing act was introduced in 1973 and the present act, from 1985, called for continued regulations (*Stortingsmelding* (Report to Parliament) No. 65 (1986–87)). According to the licensing act the regulation devices for *grow–out farms* may be listed as the following:

i) Barriers to entry (general licensing regulation)

ii) Regulation of regional location

iii) Size restriction (in cubic meters of water volume)

iv) Factor regulation (barriers to entry for smolt producers)

v) Regulation of ownership

vi) Regulation of second–hand transactions of farms

In 1985 the barriers to entry on hatcheries and smolt production were relaxed, except that some environmental criteria must still be fulfilled by the entrants.

Entry for grow–out farming is regulated by the government. The argument posed by the government (also agreed upon by the fish farmer organizations) was that they would limit entry to have a "balanced development" of the industry. Originally this meant the government, through licensing, should adapt the total supply to market demand. Later it was also stated that "balanced development" should be kept with respect to the capacity of veterinary services, education and research.

In addition, the owner structure, the regional location of farms and the size of the farms are controlled. The government desires an owner–operator structure for the industry. Thus, it is only possible for one firm to have a majority interest in one fish farm, and the Directorate of Fisheries must authorize the sale of the majority interests of a farm. Furthermore, one important device to achieve regional dispersion of plants is a government quota on licenses for each region when licenses are awarded.

The size restriction on farms is important, and is a major focus of our interest concerning regulations of the industry. The government's main objective for regulating size is to achieve a regional allocation of farms (NOU 1985:22). By putting a ceiling on the size, which leads to many relatively small units, a distributive mechanism is invoked to maintain the pattern of scattered settlement along the Norwegian coast.

The regulation of the size of a farm is given in cubic meters of water volume to rear fish, or the pen volume. The prevailing limit is 8,000 m^3 of pen volume. The upper size limit of a farm has been changing from the first licensing act of 1973 until today, ranging from 1,000 m^3 to 8,000 m^3. Our sample from 1982/83 includes farms which were restricted to 3,000 m^3 and 5,000 m^3 in addition to 8,000 m^3 farms. A more detailed description of the data in this respect will be given in Chapter four. Moreover, some farms established before 1973 when the law was enacted are included in our sample. These farms have a pen volume considerably above 8,000 m^3.

2.2.2 The broodstock farms, hatcheries and smolt farms

The first three stages of salmon production – egg production, hatcheries, and smolt production – are both distinct units or integrated vertically in varying degrees. The two dominant producers of eggs are research stations supported by the government, the Norwegian Agriculture Research Council's Research Station for Salmonids, and the Fish Farmers Association's Aquaculture Station. The eggs are transported to hatcheries generally where yolk–sack larvae are hatched after two months. One month later feeding of the salmon fry starts. The fry is then sold to smolt producers who undertake the smoltification process in order to make the salmon fit for seawater rearing for market fish producers. Within one year about 80 percent of the young fish grow to 35–50 gram smolt; the rest after two years. The smolt producers hold the young fish in favourable temperatures and feed them a specially formulated diet bought from feed producers. The smolts are transported to food fish farmers by boat or truck.

In general the production of eggs, fry and smolt is very sensitive to small changes in environmental conditions. Hence, high mortality rates may occur if the production is not monitored accurately. These different critical stages in the salmonids life cycle could explain the limited supply of smolts, which has appeared as a problem for our data period (1982/83) until recently. However, another main explanation for the shortage of smolt in aggregate for the industry may be the barriers to entry through government regulation. To verify the impact of the regulation on this factor poses a problem, but one indication of the impact, may be seen from the effect of more liberal rules for entry of smolt producers from 1985 on. The number of licensed smolt producers rose from 152 in 1985 to 565 in 1987, producing 43 million smolts and juveniles of trout in 1987 compared with 27 billions in 1985 (*Stortingsmelding* (Report to Parliament) No. 65 (1986–87)).

Possible implications of the smolt shortage for modeling the behaviour of a food fish farmer will be discussed in Section 2.2.4.

2.2.3 The distribution of salmon

The first–hand sale of farmed salmon is regulated in Norway. A sales cooperative of farmers – the Fish Farmers' Sales Organization – is authorized by the Government to coordinate sales. This sales organization issues purchase licenses to downstream salmon buyers.

All farm sales are transacted through the sales organization. The organization pays the farmer and invoices the purchaser. However, individual brokers interact with individual producers. There are about 75–80 fish buyers now authorized by the sales organization.[3)] Some purchasers only trade salmon in the domestic market.

To export, one needs to meet the requirements of a general law for exporting fish products with a government export licence. About 300 firms are now active in exporting salmon.[4)] Most purchasers are also exporters.

Transactions between purchasers/exporters and producers are characterized by guaranteed minimum prices to the producers. Prices are negotiated between the Fish Farmers' Sales Organization and the buyers. However, in times of excess demand for salmon, the prices rise above the negotiated prices. Another feature characterizing the relationship between farmers and buyers/exporters is that there were few examples of vertically integrated firms in the period from which we have data (DeVoretz and Salvanes, 1987).

The market description allows one to argue that salmon farmers are price takers in the product market, i.e., the salmon price is exogenous to each farmer. Either the given minimum prices are in effect or the market–clearing prices. Although there exist barriers to entry for the purchasers, the number of buyers is sufficiently large to characterize this market as competitive.

2.2.4 The food fish farmer

Our first point is that the production and reproduction stages, i.e., the grow–out of salmon in seawater and smolt production, are distinct activities in the industry. This implies that it is quite reasonable to analyze the grow–out part separately. This point will be developed further in Section 2.3. Primary processing, i.e., gutting, cleaning and packing, is usually carried out on site and thus may be considered as a part of the grow–out activity of a farm. This is, however, not always the rule, as we indicated in Figure 2.1 by putting primary processing in parenthesis.

Two further aspects of the grow–out farmers' economic behaviour important for our modeling will be considered here. Namely, to what extent are they price takers in the factor markets? Moreover, since both total capacity for the industry and for each individual farm is regulated, some comments on capacity utilization is required. Table 2.2 will be helpful in this respect, showing the development and the number of food fish producers and corresponding licensed and utilized capacity.

Table 2.2: Number of food fish farms, licensed capacity (1,000 m^3) and utilized capacity (m^3).

Year	Number	Licenced[1)] capacity (1,000 m^3)	Utilized capacity(m^3)	Average size(m^3) (working unit)
1973	169	975	899,565	6,472
1980	307	1960	1,508,817	5,149
1982	387	2210	1,998,890	5,165
1984	483	3380	2,440,000	6,052
1985	559	n.a.	3,386,300	6,058

Source: Salmon and sea trout fisheries, Central Bureau of Statistics.
1) *NOU 1985: 22*

We may conclude from Table 2.2 that with 400 farmers in 1982/83 – the two years for which we have data available as mentioned in Chapter one – it is reasonable to consider grow–out farmers as price takers in the factor markets. We refer to them as competitive firms given their numbers in both factor markets particular to this industry; i.e., the markets for feed and specialized capital equipment such as automatic feeders, pens, etc.. Furthermore, we characterize the firms as competitive in the labour market where farmers also compete for workers with other industries.

Concerning capacity utilization, it is clear from columns 3 and 4 in Table 2.2 that the total volume of licensed capacity is indeed larger than total utilized in the period we are considering. This may be for a variety of reasons. Total inactivity of individual licence holders – the licence is tied both to site location and persons – may explain some of the discrepancy.

However, even for farmers actually in operation (working unit) the average level of capacity is quite low. Table 2.2 reports that the average plant's capacity is only about 6,000 m^3. This results partly from the fact that some operations at that time (in 1982/83) did not have the option of using 8,000 m^3, but were restricted to only 3,000 m^3 or 5,000 m^3 as described in Section 2.2.1. This explanation is supported by the fact that for our 1982 and 1983 samples, the percentage of utilization was 94 and 98.5, respectively (see also the summary statistics in *Lønnsomhetsundersøkelser av Fiskeoppdrettsanlegg, 1983*): i.e., the farmers actually in operation were using most of their licensed capacity. Of course, some of the explanation for the relatively low average size for farms actually in operation could also be underutilization in the sense of not using the maximum possible number of pens. This may be due to poor site locations prohibiting full capacity utilization.

On the other hand, underutilization also manifests itself in low pen density. For 1982 and 1983, the average densities of (produced) fish in the pens were respectively 13.6 and 16.1 kg per cubic meter.[5)] This type of underutilization points to an industry bottleneck. That is, smolt production could not match farmers' demand in this period (*NOU 1985:22*). A positive excess demand for smolts by the industry is confirmed by rising smolt prices and the importing of smolts from Scotland and Finland. However, we cannot deduce from this that each individual firm is rationed in smolts. We could have considered it a limiting factor of production for each firm if the agreed upon minimum prices – which will be discussed later – were strictly held, and each firm received a ration of the total production given this price. However, the smolts shortage has been reflected in increasing prices to clear the market. Thus, although smolt appears to be a limited input and a restriction on pen density – the farmer's behaviour should not be modelled exclusively by this consideration as limited in smolts.

From a general description of the market constraints of a grow–out farmer, we will proceed by discussing the production process in salmon aquaculture.

2.3 The production process in fish farming

In Norway the Atlantic salmon *(Salmo Salar)* has become the preferred species for sea pen farming. This species grows larger than rainbow trout *(Salmo Gardneirii)*, which is the other species farmed, and commands a higher price. The production strategy, which has been an important factor for the commercial success of this industry, is to reduce freshwater rearing time and increase seawater rearing time. This is because a more rapid growth is achieved in seawater than in freshwater. The dominant mode of production is the use of floating sea pens for farming salmon and trout during sea growth. We shall present this in the following section.

2.3.1 Definition of Salmon Farming

Aquaculture can be defined as cultivation of sea organisms. The main feature that distinguishes aquaculture – including farming of salmon and trout – from other aquatic production is the degree of human intervention in the process. Mentioned in order of increasing control, we can start with capture fisheries which are controlled only by harvesting. Ocean ranching is a fishery that depends on artificial propagation to enhance harvestable stocks. Traditional aquaculture methods consist of greater intensity and control. This control involves a form of confinement that provides possibilities for supervising factors such as density, protection against predators and enhancing growth via utilization and feeding. Hence, intensive aquaculture production like salmon farming is in this sense conceptually similar to forest and livestock production.

It is possible to divide an aquaculture system into three functional components (Allen *et al.* 1984, p. 18). The system consists of two main elements, the physical and the biological components. Since the biological production takes place in natural environments, a third component – culture environment – also exists.

2.3.2 The Physical System

The physical system consists of facilities for confinement and feeding, as well as operating equipment for the production system. The rearing of salmon and trout from smolt size (40–50g) to marketable size is accomplished by intensive feeding in floating sea pens. These pens have many designs but are generally comprised of a nylon net hung from a floating frame made of steel or plastic. The pens are either anchored in series adjacent to a floating platform or individually anchored. The dominant mode of fish farming in Norway is the use of sea pens attached to a float, but other systems are also employed, particularly the use of natural impoundments. We may call them open production systems in contrast to closed systems.

The closed systems – both land based and floating sea production units – imply that the pens are not made of nets and thus are not directly open to the seawater. This technology was developed during the last few years and is not much used in production, but primarily in pilot projects. The advantage of a closed system – both sea and land based – is better control of factors such as water quality (temperature, oxygen content, etc.), waste products, and disease. In addition, closed systems can be built like race—ways where it is possible for the fish to be more active and hence, be in better condition.

There are many ways of feeding the fish, which in turn depend on what kind of feed the farm is using. The most common is dry feed, and the manner of feeding ranges from hand feeding to automated feeders.

2.3.3 The Biological Process

Atlantic salmon is a species native to Norway. The Norwegian salmon farming industry has adapted the Atlantic salmon for intensive culture. The biological stages of production until the smolts are transported to the food fish farmer have become distinct activities as described in Section 2.2.2.

The biological process and the options available to the food fish farmer may be illustrated by a model which illustrates the changes in the stock of fish over time.

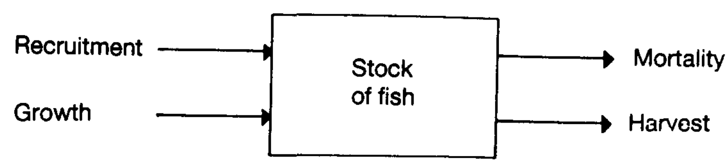


Figure 2.2: Changes of a Stock of Fish over time. Source: Bjørndal *et al.*, 1987.

Due to disease and changes in environmental conditions such as temperature etc., there is natural *mortality*, especially within the first months after the fish recruits are put in the sea. However, the farmer can to a certain degree reduce disease risk and the consequent loss by carefully monitoring the stock of fish and taking steps to prevent the spread of disease. Mortality and harvesting are the factors that reduce the size of the stock. Natural mortality will be directly incorporated in our analysis as output will be defined as production net of mortality.

From this figure describing the dynamics of a stock of fish, there are two biological characteristics which are particularly noteworthy – recruitment, and the dynamic nature of production. We will incorporate these features when we model the farmer's decision problem and specify a profit function for empirical testing.

The reproduction stage of the biological process *(recruitment)* in the production of farmed salmon has become a distinct activity, as mentioned in Section 2.2.4. This implies that we can separate the reproduction part of the biological process from the production of fish for food. Hence, we may use the product from the reproduction stage – in general named juveniles and, in the case of salmon, smolts – as an input factor in the production of salmon for consumption. As argued in Section 2.2.4, it is then reasonable to undertake a separate analysis of the food–producing sector of the salmon industry. Hence, the main dynamic decision problem facing the farmer is when to *harvest*, i.e., in what time period.

To consider the result of the reproduction stage as an input and analyze the food—producing part separately, is a distinctive feature of fish farming in contrast to traditional fisheries. Furthermore, the production of salmon for consumption cannot be looked upon as static or momentary production and hence is different from traditional industry production. Both these features are indications of the similarity of pen reared salmon production to forestry and livestock production.

The economic options available to fish farmers when determining their production strategy are to decide on the quantity of inputs – particularly feed – to influence the *growth* of the salmon stock, and the optimal harvest and sale pattern for salmon. This will be described in more detail first by considering the biology of a single fish and a year–class of Atlantic salmon. The biological basics of a farm are then outlined for what will be defined as a "normal" year of production; the salmon farm is established for continuous production with three distinct year–classes in the pens.

The biology of a single year–class

A typical weight curve of a single Atlantic salmon is sketched in Figure 2.3. Let w(t) represent the weight of the fish of age t. The growth is given by the change in weight over time, dw(t)/dt, and w(t) increases until it reaches a maximum weight for a single fish, w*.

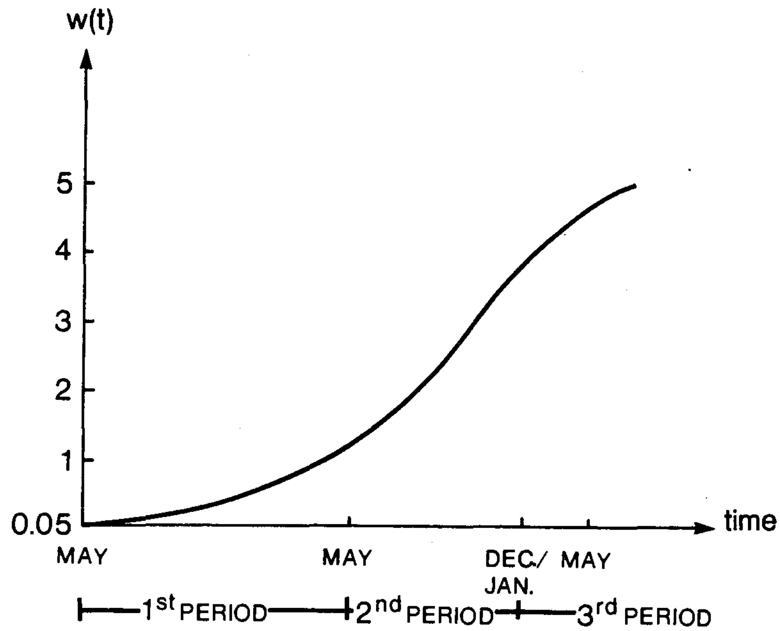


Figure 2.3: Weight curve for Atlantic salmon.
Source: Gjedrem (1981) p. 29.

By assuming that each individual weight curve has the same shape, we get a weight curve for a year–class – the biomass with a similar graph by multiplying the weight for the average fish by the number of fish in the year–class (Clark, Edwards and Friedlander, 1973). We also have to correct for mortality when constructing the weight curve for a year class. Hence, it is reasonable to consider the individual weight curve when discussing the possible harvesting procedures of a food fish farmer. From the weight curve we see that the farmer puts smolts of one year–class into the pen in May (each year). We see that in the first period (May year one to May year two) a fish grows from 40–50 g to 1–1.5 kg and during that period it grows at an increasing rate. After an additional growth season during summer, the fish is about 2–3 kg in August/September. At this time the fish has reached market size and the farmer starts harvesting from this year–class, which continues during fall up to the high demand period before Christmas. Furthermore, we see that, as of spring in the third year, the fish has grown to an average of about 4–5 kg.

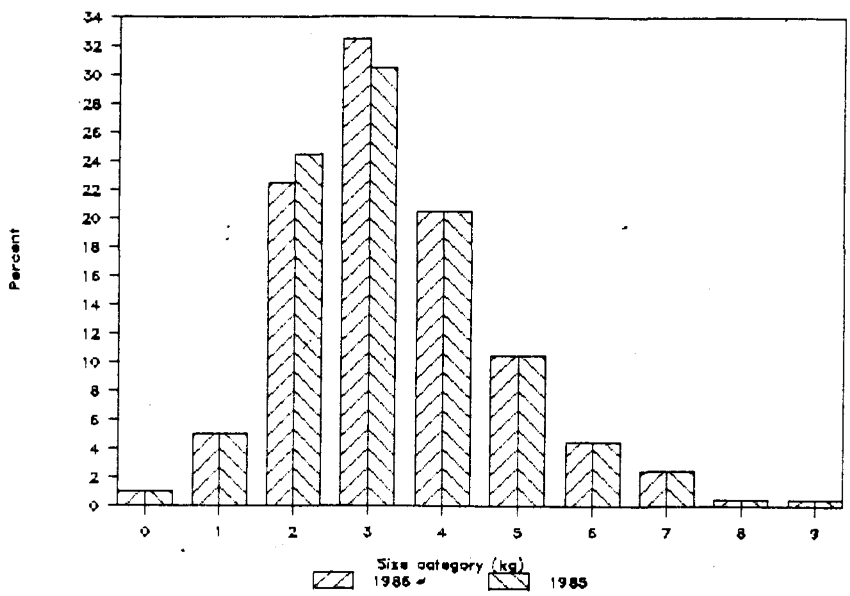
From this discussion of the weight curve – assuming a similar biomass curve for a year–class – we find it reasonable to argue that the farmer has primarily two options concerning when to harvest a single year–class of salmon; either to harvest in autumn in the second period (particularly Christmas) or in spring (peak about Easter) in the third period. This means that a year–class mainly is harvested in autumn of the second year and in spring of the third year. The farmer must compare the change in value of the biomass of that year–class in the second period – autumn – to the costs for feeding, ,mortality, etc. to get increased weight in the third period.[6)] The timing of harvest for the year–class then depends on an optimization rule that incorporates both the growth rate and input prices, and the demand conditions. The latter point reveals when the price is highest due to both seasonal variations and weight–class.

Below we will study actual harvesting procedures of salmon in the Norwegian aquaculture industry to see whether there is any support for our argument of a farmer's harvest procedure. Furthermore, by comparing data for different years of harvesting, we will try to verify if there is a change in the harvesting pattern over time.

The actual harvesting procedures

By comparing figures for the distribution of weight–classes harvested in a calendar year for the salmon industry and the dispersion of harvesting over the year, we can deduce some inferences about the pattern of farmers' harvesting decisions.

Table 2.3: Percentage distribution of weight classes harvested in 1985 and 1986.

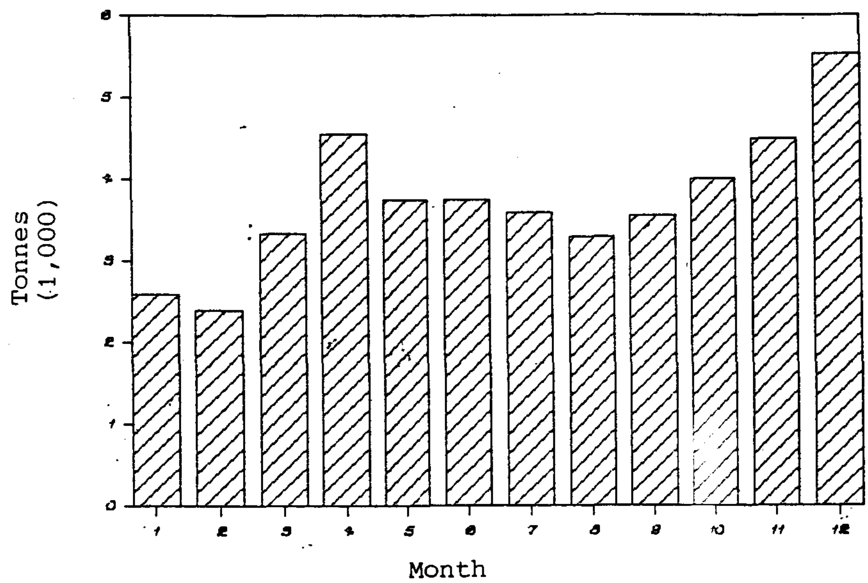


Source: The Fish Farmers' Sales Organization.

The figures over the actual harvest for 1985 and 1986 support our description of harvesting procedures, and we will argue for this by splitting the harvest in calendar year 1986 into year–classes. We do not have these figures over the actual harvest for 82/83, therefore more recent data must be used. Table 2.3 provides us with the information that in 1986 about 50 percent of salmon are harvested as 2 and 3 kg salmon, and more than 40 percent are harvested at 4–6 kg. By assuming the individual weight curve of salmon in Figure 2.3, we may interpret the harvested fish of 2–3 kg as a year–class partly harvested in the second autumn in seawater, and the salmon harvested at 4–6 kg as another year–class partly harvested in the spring of the third year. Hence, it is reasonable from this data to argue that the options available for a farmer to harvest a year–class are to harvest it in the autumn in the second year in seawater or in the spring of the third year. In addition, by comparing 1985 with 1986 we notice that the farmers are using the same pattern of harvesting. This may indicate that the farmers are using standardized harvest procedures.[7)]

Our interpretations of the data in Table 2.3, is confirmed by Table 2.4, where 21,300 metric tonnes out of 45,500 metric tonnes are harvested during the period August to December. Furthermore, we see that the rest are harvested at 4 kg or larger, or during their third year in the sea, i.e., from January till May and even June and July.[8)]

Tablè 2.4: Harvest of salmon per month (tonnes).



Source: The Fish Farmers' Sales Organization.

An additional explanation for this delay is, of course, that there is some dispersion of weights at each time in a year–class. Since growth rates are different between sites – a bad site location with unfavourable conditions may provide poorer growing conditions growth to the third year occurs. Furthermore, differences in genetic factors connected to growth properties may also explain the delay in harvesting.

We can conclude from the above description of the biology of a single fish and of a year–class and the associated figures for actual production, that the farmer has the option of harvesting a year–class both in the fall of the second year and in spring of the third year. Furthermore, the farmer may harvest both in fall and in spring, not in just one of the periods. In addition, although the procedures for harvesting may change from year to year indicating output to be a variable for the farmer, the figures over actual harvest over year–class indicate standardized harvesting procedures.

The biological basics of a farm

From the weight curve and harvesting pattern of a single year–class, we can derive some important properties of the basic biological characteristics of a farm. First, given the growth and harvest pattern we can deduce how many year–classes the farmer needs in order to maintain continuous production. We will define a "normal" year of production when the farm is in equilibrium in this sense. It is this normal year that is our focus and will form the basis for analyzing the fish farming industry. Furthermore, from the description of the harvesting behaviour – allowing harvest to be a choice variable – we can derive a proper measure of production of a single calendar year with continuous production. In a normal year we will be able to show that the farm produces one complete year–class.

Figure 2.4 displays the development of a farm from year one to a normal year with continuous production (third year).

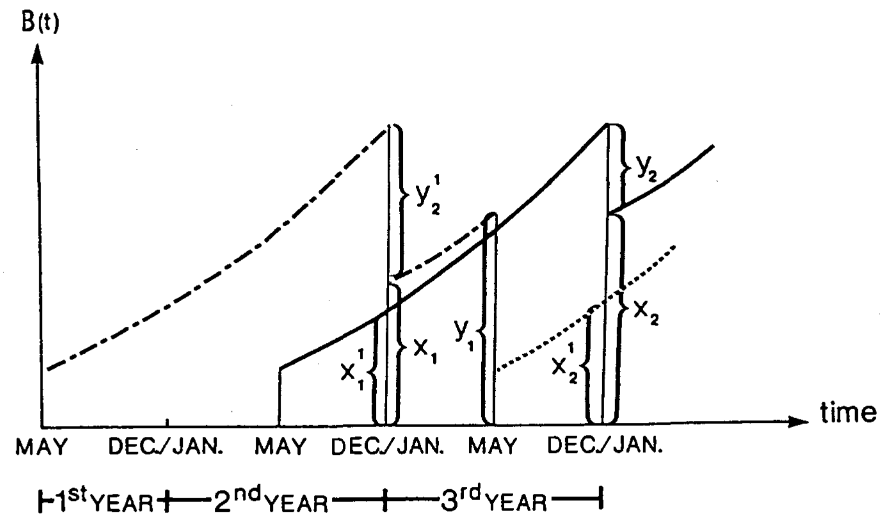


Figure 2.4: The year–classes in a fish farm.

The weight curves are simplified aggregate curves for year–class biomass (B(t)) curves of Atlantic salmon, and we still assume that the smolt is put into the pens once a year in May. The steps in the curves illustrate the harvesting procedures of the year—classes. The pattern of harvesting described above is simplified in Figure 2.4 to a decision to harvest either in May or in December meaning that they harvest in autumn and in spring.

The farm is not fully developed and in equilibrium until three years after establishment. We will call the third year and the years to follow the "normal" year of production. During a normal year there are three year–classes of salmon. In spring the farmer harvests the fraction left from the first year–class (or even extends the date of harvest till fall, which makes no difference since it is in the same period). This is called y_1 in Figure 2.4, and is not a decision variable in this period since it must be harvested in this period. Furthermore, he will decide on harvesting a fraction of the second year–class in the fall (y_2).

Since we consider the harvest behaviour to be a choice variable for the farmer from year to year, y_2 may differ from y_2' which is the comparable harvest size to y_2 in the year before. The third year–class is not harvested. $y_2' + y_1$ is the production of a complete year–class, harvested during two periods. However, since we allow harvesting to be a choice variable, the harvest in a normal year – $y_1 + y_2$ – does not necessarily consist of a year–class. In this case $y_2 < y_2'$, meaning that a smaller fraction is harvested in the first period of the second year–class than of the first year–class in the first year. Hence, we must account for the change in fish stock in the pens when defining output in a normal year. Since the change in stock does not equal zero under changing harvest procedures, production is thus defined as the actual harvest that year ($y_1 + y_2$) plus the change in the stock of fish from the beginning of the year ($x_1 + x_1'$) to the end of the year ($x_2 + x_2'$). In our case the change in harvesting behaviour makes the change in stock; $y_2 < y_2'$ which leads to $x_2 > x_1$. By correcting for this, $y_1 + y_2 + (x_2 - x_1)$ totals up to a complete year–class in a normal year. This measure is identical to the sustainable yield in a steady–state equilibrium when fishing on different year–classes which equals the yield of fishing on one year–class over the complete time period (Beverton and Holt [1957], Andersen, [1979]).

In our definition of a normal year, equilibrium was assumed in the sense that the same amount of juveniles was put in the pens each year to maintain a continuous production. In Figure 2.4 x_2' is set equal to x_1' and the assumption is met. In our sample it is the case that the number of smolts put into the pens increases over the years. This is probably due to the fact that the supply has increased. Then, $x_2' > x_1'$ in Figure 2.4 and this change in the stock also has to be added to the definition of production. This is discussed in more detail in Chapter four.

To conclude this part, we argue that in a normal year the farmer produces one complete year–class. However, the harvest consists of fish from two year–classes. And since harvest procedures may change from one year–class to another, changes in the stock of fish in the pens must be accounted for in defining the annual production which then constitutes a year–class. In addition, the number of smolts released may change from year to year which must be taken into account when defining output.

2.3.4 Culture Environment

The third element of an aquacultural production system – the culture environment of the site location – will interact with the biological process. The growth of the fish will be influenced by the water characteristics of the site. Also the fish themselves contribute to the environment through excretion of waste products. The response of the environment to waste products will in turn influence the biological process. The prevalence of favourable sea temperatures, oxygen content, etc., and good underwater topographical conditions provide an appropriate environment. These factors lead to good conditions for sea pen farming of salmon and trout in Norway. However, since farms have been established all along the coast, a dispersion of conditions may influence economic profitability of the farms. Thus, environmental conditions of the site should be considered in a production analysis.

To express the difference in site conditions and biological status of stocks of salmon, we could analyze the weight function for a single fish, w(t,a,b), as being a function of a, an index of parameters which is site specific, and b, an index describing the biological status of the particular stock used by a farm. That is, a is an index describing temperature, oxygen contents, and topological conditions that influence the growth of salmon. When these exact characteristics are not available, varying site conditions can arise due to regional differences in sea temperature, etc., along the coast. Hence, regional differences may be used as a proxy for environmental conditions. However, our data set does not permit us to include such variables in the empirical analysis.

2.4 Summary

To conclude this chapter, we will summarize the relevant points from Sections 2.2 and 2.3 for our economic model construction in Chapter three.

First, we can separate the reproduction part of the biological process from the grow—out stage and hence undertake a separate analysis of the food producing sector. Second, the main dynamic problem for a farmer is when to harvest a year–class. He has primarily two options concerning when to harvest a year–class of salmon; either in autumn of the second year or in spring of the third year the salmon is in the sea. Third, when defining the output in a calendar year, we introduced the concept of a normal year of harvest which consists of a complete year–class. In addition, one has to correct for changes in the number of smolts released from year to year. As the fourth point, although the farmer can change harvesting patterns from year to year making output a variable for the farmer, it is possible to argue for standardized harvesting procedures in the Norwegian salmon industry by inspecting data on actual harvest. The fifth point concerns smolts as an input factor. In the period of our analysis there was a smolts shortage; however, since prices were flexible upwards they cleared the market and the individual farmer was not rationed in smolts. However, there is a period of time from one to two years from a year–class of juveniles is put to sea and before it is harvested. Point seven is that the size of each farm is regulated in cubic meters of pen volume. As the last point, we note that the grow–out farmers can be considered as price takers in the factor and output markets. All these factors are relevant for the specification of economic models of fish farming, and we now turn to this problem.

FOOTNOTES

1) Some of the farmers are rearing both Atlantic salmon and rainbow trout. However, the production process for both species is similar, not requiring a separate analysis of rainbow trout production.

2) The landings of wild–caught salmon have been relatively stable over the last few years both of Atlantic and of the two comparable Pacific species, i.e., coho and chinook (FAO, Fisheries Statistics).

3) Source: The Fish Farmers' Sales Organization, personal communication, summer 1988.

4) Source: The Export Council for Fresh Fish, personal communication, summer 1988.

5) This density is rather low considering the fact that 20–25 kg per cubic meter even in 1982/83 was thought to be possible and is reported by some farmers (*Norwegian Fish Farming* (in Norwegian) no. 7 1980).

6) There exists also the possibility of prolonging the date of the harvest by keeping salmon in the pens for another summer. However, the Atlantic salmon reach sexual maturation 28 months after smoltification. For this reason – and also because market demand is smaller for the larger size classes (6–7 kg) – the bulk of this year–class is harvested before May and a new year–class is put into the sea. However, if we allowed the farmer to prolong the harvest over the third summer, it would not affect the definition of production in that year.

7) One possible criticism might be raised against our interpretations. The figures in Table 2.3 and Table 2.4 are aggregate data for the whole industry, while we draw conclusions for the individual farmer. This means that we might draw biased conclusions about the microlevel from the macrodata. However, in this case it is difficult to find competing hypotheses of harvest behaviour supporting Tables 2.3 and 2.4.

8) It is possible to wait for harvesting after May as farmers are allowed to have extra pens for the new fish even if the total pen volume is regulated.

3. A MODEL FOR A SALMON FARM

Given the industry and farm level information in Chapter two, in this chapter the decision–making problem for a typical food fish producer will be formulated. The data set is in calendar years at the micro unit farm level. The goal of the model construction, as it was outlined in Chapter one, is to investigate empirically the structural properties of scale economies and to measure the various elasticities of substitution between factors of production. Based on three components – the information on farm and industry level, the characteristics of the data set, and the aim of the analysis – we will first develop an analytical profit function for an individual farmer in a normal year. As an alternative behavioural hypothesis of the industry, a cost function based on institutional characteristics will be formulated. Before formulating our models we must study in more detail the optimal pattern of harvesting a year–class given the biological characteristics of Section 2.3.

3.1 The optimal harvest of a year–class

The central intertemporal problem faced by a farmer in a normal year is when to harvest a single year–class. We will focus on this problem in this section in order to define a profit function in a normal year.

The optimal harvest problem is a rotation problem. That is, we use an economic model to seek the optimal rotation period of harvesting a single year–class when we face an infinite number of cycles of releasing smolts to the sea and harvesting. We assume that harvesting and putting smolts to sea occur at the same time. A continuous time model will be used here for simplicity since we are only interested in the qualitative properties of the model. As a preliminary approach we analyze the problem by a single–rotation model, i.e., considering only a single cycle. To do so we need to develop a dynamic bioeconomic model.

The development of the weight of a year–class, B(T), will be elaborated on following the Beverton–Holt modeling of a year–class. B(T) is defined in terms of the weight of an average or a representative fish, w(T), and the number of fish, N(T) (Bjørndal [1988], Clark, Edwards and Friedlander [1973]). This fomulation of the model will provide a clearer connection with the verbal discussion of the biological process in Section 2.3.3, and serve as a mathematical representation of the illustration of the changes of a stock of time given in Figure 2.2.

The time change in the number of fish in a year–class (N(T)) is denoted dN/dT, and can be represented by the following differential equation

$$\frac{dN}{dT} = -MN(T), \quad 0 < T < \mathbf{T}, \tag{3.1}$$

where M is the constant instantaneous natural mortality rate and $\mathbf{T}$ may be interpreted as the time of maturation. The natural mortality reduces the number of fish over time at a constant rate. By integrating equation (3.1) from T = 0, in terms of releasing smolts, we get the number of fish in the year–class by each point in time:

$$N(T) = Re^{-MT} \tag{3.2}$$

Here, R ≡ N(0), i.e., the number of fish initially released in the farm. The mortality rate may vary over time. As a simplification, we will, however, assume it to be constant.

The weight of the average or typical fish may be written in a general form as

$$w(T). \tag{3.3}$$

The derivative of (3.3) with respect to time, dw(T)/dT, is the growth of the individual fish. A typical weight curve of a single Atlantic salmon was given in Figure 2.3. For the case of salmon, we notice that the individual fish will grow at an increasing rate first, then at a decreasing rate until it reaches a maximum. This is a typical development for many species.

By combining equations (3.2) and (3.3) we obtain an expression for the biomass (weight), B(T), of a year–class of fish measured at each point in time determined by the number of fish in the year–class and the weight of an individual fish:

$$B(T) = N(T)\, w(T) \tag{3.4}$$

By taking the derivative of (3.4) with respect to time, and substituting (3.1) and (3.2) into (3.5), we obtain an explicit expression of the components for the development of a year–class over time:

$$\frac{dB}{dT} = \left[\frac{w'(T)}{w(T)} - M \right] B(T) \tag{3.5}$$

w'(T)/w(T) is the relative growth of each individual fish. From (3.5) we note that the development of a year–class, before harvesting is introduced, is a function of the relative growth of each fish, w'(T)/w(T), and natural mortality, i.e., by the change in the number of fish in the year–class as seen from equation (3.2). We will utilize the result in equation (3.5) when we interpret the result of the bioeconomic model for a year–class.

The problem facing a fish farmer is to maximize the net present value (profit) over one rotation period (Hartwick and Olewiler, [1986], Bjørndal [1988]):

$$\pi(T) = V(T)e^{-rT} - cR = [p(T)B(T)]e^{-rT} - cR$$
$$= p(T)N(t)w(T)e^{-rT} \qquad (3.6)$$

where V(T) is the value of a year–class defined as p(T)B(T), P(T) is the price per kilo, (r) is the interest rate and (c) is the unit cost of putting smolts to sea.

The decision variables in this case are the time of harvesting, T, and the number of smolts put into the sea, R. The simultaneous optimization of the optimal harvesting time and the optimal number of recruits to be released is solved by Bjørndal [1988]. We here assume R to be given, which is appropriate in our case, and will concentrate on the first–order solution of the optimal time to harvest and the interpretation of this.[1)]

If we maximize equation (3.6) in its simplest form, $\pi(T) = V(T)e^{-rT} - cR$, with respect to T, we get the first–order condition:

$$V'(T) = rV(T) \qquad (3.7)$$

This condition states that one should harvest when the value of growth, i.e., V'(T), equals the yield of putting the value of the biomass in an alternative asset, V(T)r. This rule is intuitively reasonable. The farmer has two options at time T; either he can harvest the fish and get the biomass value V(T) and deposit the money in a bank account at an instantaneous interest rate of r, or he can wait and keep the fish in the pens where the change in the value of the biomass is given by V'(T). The optimal one–cycle harvesting is obtained when the yield of keeping the fish in the pens in terms of their growth, V'(T), equals the yield on the alternative asset deposit V(T) in a bank, rV(T). An alternative expression for the optimal harvest solution is:

$$\frac{V'(T)}{V(T)} = r, \tag{3.8}$$

which states that one should harvest a year–class when the relative growth of the value of the year–class is equal to the interest rate, i.e., the return on the asset under consideration equals the return of the alternative asset. This rule for optimal harvesting can be interpreted as a Fisher rule common in investment theory, after the economist Irving Fisher (Sandmo, 1968).

By taking the first–order derivative w.r.t. time of equation (3.6) in the form $\pi(T) = [p(T)B(T)]e^{-rT} - cR$, we get the following condition for optimal harvest:

$$\frac{d\pi(T)}{dT} = \left(\frac{dp(T^{*})}{dT} B(T^{*}) + p(T^{*})\frac{dB(T^{*})}{dT}\right)e^{-rT^{*}} + p(T^{*})B(T^{*}))e^{-rT^{*}}(-r) = 0. \tag{3.9}$$

Utilizing equation (3.5) expressing the development over time for the biomass, dB(T)/dT, and rearranging (3.9), yields the condition for optimal harvesting, T^{*}, in an implicit form as:

$$V(T^{*})\left[\frac{p'(T^{*})}{p(T^{*})} + \frac{w'(T^{*})}{w(T^{*})} - M\right] = rV(T^{*}), \tag{3.10}$$

which can be expressed as

$$\frac{p'(T^{*})}{p(T^{*})} + \frac{w'(T^{*})}{w(T^{*})} - M = r \tag{3.11}$$

This may be interpreted in the same way as the simple formula in (3.8), i.e., as a Fisher rule for optimal investment. The relative growth in biomass value consists of three parts: the increase owing to the relative growth of each individual fish, the relative appreciation of price if the price is positively related to time ((dp/dT)>0), and the negative component in the rate of natural mortality, M. The positive relation between price and time may be related to the weight of each individual fish, by making price a positive function of the weight of the fish. This is true for Atlantic salmon as price per kilo is increasing in size (weight) up to a certain limit.

By considering only one single period myopic maximization an important feature is lost. The alternative value of the pens is not taken into account since after harvesting a year–class space is freed for another year–class of smolts. Since the pen volume is scarce – in particular in our case since the size of volume is regulated – this may be an important decision in fish farming. Hence, it is reasonable to extend the above model by considering a chain of cycles of releasing smolts, harvesting, releasing, and so forth. The decision problem can now be seen as maximizing the net value of an infinite series of all future revenues where all rotation periods are equal (T). The rotation periods are equal because all economic and biological parameters are assumed to be constant over time; when one cycle is finished the farmer faces the same optimizing problem in following time periods (Hartwick and Olewiler, 1986).

We will not gain anything further in economic intuition by expressing the biomass explicitly as a function of the number of fish and the weight of a single fish in this framework. The reason for employing the rotation approach is to obtain a new insight into the harvesting problem that this model in particular will provide. Hence, we present the rotation problem in the general form:

$$\pi(t) = [V(t)e^{-rT}]\,[1 + e^{-rT} + e^{(-rT)2} + \dots] \qquad (3.12)$$
$$= [V(T)e^{-rT}](1-e^{-rT})^{-1}$$

In this formulation, release costs have been disregarded. All rotations are identical under the given assumptions, $T_1 = T_2 = T_3$ .... And we have used the formula $[1 + e^{rT} + e^{(-rT)2} + \dots] = (1 - e^{-rT})^{-1}$, to simplify the expression for rotation period in equation (3.12).

The solution of the optimal value of T is given by maximizing (3.12) with respect to T:

$$\pi'(T) = [V'(T)e^{-rT} - rV(T)e^{-rT}]\,(1 - e^{-rT})^{-1}$$
$$- [V(T)e^{-rT}]\,(1 - e^{-rT})^{-2}re^{-rT} = 0 \qquad (3.13)$$
$$= [V'(T)e^{-rT} - rV(T)e^{-rT} - r\pi(T)e^{-rT}]\;\,(1 - e^{rT})^{-1} = 0$$

Which can be reduced to the following (for the interior solution, T°):

$$V'(T^{\circ}) = rV(T^{\circ}) + r\pi(T^{\circ}) \qquad (3.14)$$

This solution has a parallel interpretation as in equation (3.7) in the one–cycle case, only with the exception of an additional factor, $r\pi(T^0)$, on the right side of the formula. The solution requires that any year–class shall be harvested when the value of growth of the year–class, i.e., the left side of the equation, is equal to the opportunity value of the year–class of fish, $rV(T^0)$, plus the opportunity value of the farm (or the pens), $r\pi$. Compared to the optimality condition for the one–period problem given in equations

(3.7) and (3.11), taking the alternative value of the pens into account, $r\pi$, has the effect that the year–class of fish is harvested earlier. Equation (3.14) for the optimal rotation period is a version of the Faustmann formula in the forestry literature (Clark, 1976).

Differences in the harvesting pattern among farmers arise due to different alternative values applicable to each farm such as differences in the growth of salmon and varying site and biological characteristics (see Section 2.3). More specifically, because of different size limits on pen volume, the scarcity and hence the value of pens differ and may cause different lengths of rotation periods.

The figures for harvesting procedures displayed in Section 2.3 indicated that harvesting was spread over time. This may appear to contradict the optimal harvesting rule of equation (3.14), this means that no unique solution is obtained. There are different reasons why this is the case. One reason may be differences in seasonal growth of salmon which will give a more complex harvesting rule rather than harvesting the whole year–class at one time. Furthermore, seasonal variation in market prices for salmon may as well point in the same direction. An additional reason why this result could occur is differences between opportunity values in the pen volume of farms. When harvesting begins for any one cage the scarcity value is high, but this value decreases as the pens empty. Hence, the farmer may stop harvesting and continue to harvest when the scarcity value of the pen has risen again due to the growth of fish in the pen.

3.2 A general profit function for a salmon farmer

In this section we will extend the purely biological production relationship analyzed in Section 3.1 to incorporate the other components of an aquaculture system described in Section 2.3. We will first describe the decision–making problem for a farmer in terms of a *general* profit function before we suggest (in Section 3.3) different approaches in formulating the food fish producer's behaviour based on a discussion of the production function.

In a normal year the farmer faces a two–fold problem. He must determine the optimal combination of inputs and decide on the output level in that period (through the optimal harvest) as analyzed in Section 3.1. This is done under given factor and output prices as described in Section 2.1. Hence, the problem of the farmer is to maximize the net revenues from production with both the input and the output values as decision variables.

This can be presented by a profit function:

$$\pi(p,\mathbf{w}) = \max_{y,\mathbf{q}} (py - \mathbf{wq} \text{ s.t. } f(\mathbf{q}) \geq y) \tag{3.15}$$

where the farmer maximizes net revenues by choosing an output level (y) and a combination of inputs (**q** is a vector of input factors), given the output price (p), the vector of input prices (**w**) and technology under the production function $y = f(\mathbf{q})$. The profit function in (3.15) is extended when compared to the biological production relationship in equation (3.6). First, the biological production system is expanded also to incorporate feed as an input to influence the biomass weight. Second, the physical system is included in the input vector (**q**) comprising factors such as labour and capital and their transformation into output represented by f(**q**). The details of factors we included in the production function will be described in Section 3.3.

For profit maximizing behaviour we require that prices in both input and output markets are given. The number of grow–out farms is sufficient as shown in Table 2.3 to assume a negligible impact on factor prices from any individual farm. The fish farmers are assumed to be price takers in the output market for salmon, either by effective minimum prices or through sufficient buyers at the farm gate sale to ensure a competitive framework.

3.3 The production function

We will proceed by considering the production technology in more detail. The information on industry and farm level in Chapter 2 – especially the description of the production process and of government regulations summarized in Section 2.4 – will be utilized. Given that information we will suggest a more precise formulation of a profit function than the general one formulated in equation (3.15) in addition to an alternative hypothesis of cost minimization. When formulating different functions describing optimizing behaviour for empirical testing, we have to use criteria in addition to the structure of the production process and market constraints as we mentioned in the introduction to this chapter. The goal of our study and the data set available with the limitations it imposes will be used as additional criteria in our case.

The technology for a fish farm can be described by a production function with the usual neoclassical properties:

$$y = f(K,L,F,S), \tag{3.16}$$

where y = output, K = capital, F = feed, L = labour, S = smolts. The precise definitions of inputs and output will be given in Chapter four. Because of the dominant role of feed and the relative unimportance of other intermediate inputs, material inputs other than feed were excluded when modeling the structure of production.[2)]

Five properties of this production function are important and need further discussion. These properties are: (1) the definition of output in a dynamic setting, (2) the corresponding timing of inputs, (3) how to handle government regulation of the size of farms, (4) implications of the presumed standardized harvesting procedures and (5) some discussion of smolt as an input differentiated from other inputs.

We start by examining properties one and two. In the description of the biological basics of a farm in a "normal" year and the harvest strategy outlined in Section 3.1, it was argued that the harvest consists of two year–classes. Furthermore, since the harvest procedures may change from one year to the next, a change in the stock of living fish in the pens must be taken into account in defining annual production. By this definition of production – both actual harvest and the change in the stock of fish – the annual production is a complete year–class. It is important to define the corresponding output concept. Since this definition assumes an equilibrium in terms of smolt release, we have to correct for changes in the number of smolts put to sea from year to year as pointed out in Section 2.3. From Figure 2.4 it is seen that in a "normal" year expenses covering a complete year–class are divided over three successive year–classes at different stages of their biological cycle. Hence, the timing of inputs is straightforward in this dynamic setting: it consists of the annual expenses for capital, labour, feed and materials (including smolts), as depicted in Figure 2.4.

3.3.1 Implications of regulated pen–volume and standardized harvesting procedures

Due to the nature of the size regulation, i.e., each plant is regulated in the use of pen volume measured in cubic meters as described in detail in Section 2.2.1, two possible specifications of a farmer's behaviour may be appropriate. It is possible to consider the regulated water volume as a given factor for a single farmer, and thus the farmer adjusts his production plan given his volume constraint in pen capacity. However, another interpretation of the regulation of the volume of the plants makes it reasonable to argue for cost minimization as a relevant hypothesis to describe the behaviour of a grow–out farmer. First, we will discuss cost minimizing as a behavioural specification for a salmon farmer in Norway compared with that of profit maximizing. Second we will discuss water volume as a fixed factor in both the framework of a cost and profit function.

The case for a cost minimizing hypothesis in contrast to the profit maximization arises both because of the regulation of pen volume (point (3) in Section 3.3) and from another central feature is described in Chapter 2, namely standardized harvest procedures (point (4) in Section 3.3).

According to the Fish Farming Licence Act, pen volume is regulated for each farm. For our data period the volume restriction varies depending on the date of licence issue. Hence, it can be argued that output is exogenous from the point of view of the farmer. Nevertheless, it must be noted that the production level itself is not regulated. Instead, the Act stipulates a limitation on capacity which is given by the the volume of water for fish rearing. We will put forward arguments supporting the view that regulating the volume of water is tantamount to exogenously determining output. Under these conditions, it could be argued that the appropriate behavioural hypothesis should be cost minimization, given (different levels of) output.

With variation in fish density among farms, obviously there is not a one–to–one relationship between production and capacity. Over time fish density in pens has increased considerably (*NOU* 1977:*39*). Still it could be argued that there is a one–to–one relationship between volume of water and output for one or two years – in this case 1982 and 1983. The dispersion of production levels should then be due to different levels of production capacity, which have varied over time, given by the regulating authorities. Comparing the density (production in kilos per cubic meter) across size—groups is inconclusive. The average within and among any group appears to be equal, except for the largest group.

In the first instance, profit maximization appears to be a more general behavioural hypothesis than cost minimization, since output is considered a choice variable and endogenous to the farmer. However, in Section 2.3 it was argued that the *actual* pattern of harvesting the year–classes of salmon was stable; farmers harvested – with the reservation taken in footnote 5 of Chapter 2 for possible aggregation bias in the figures – the same proportion of each year–class in the first period over successive years. Harvesting by "rule of thumb," i.e., harvesting a certain fraction of a year–class when the average fish–size reaches a certain weight, is clearly a decision independent of price or interest rate fluctuations. This harvesting procedure could have arisen since monitoring fish growth on a day–to–day basis and associated parameters for optimal harvesting were not available. There also exist secondary arguments for supporting this "rule of thumb" view. The historic development and organizational structure, i.e., limited linkages in terms of vertical integration between farmers and purchasers/exporters, outlined in Chapter 2 also support this argument. In addition, Norwegian aquaculture industry has until recently been in the fortunate position (and in our data period) of harvesting and selling whatever amount whenever they wanted at relatively high prices.[3)]

To summarize, our line of argument here indicate that a cost function specification with output given for each farmer, is a competing hypothesis to profit maximization describing a grow–out farmer's economic behaviour.

As a second interpretation of government regulation of pen volume both in the cases of specifying a profit function and a cost function, it is possible to consider the regulated water volume as a given factor for a grow–out farmer. Hence, a short run formulation of both a cost function and a profit function utilizing regulated pen volume as the fixed factor could be appropriate. The available data set for 1982/83 limit the opportunity to econometrically test these options. The regulated pen volume in cubic meters is not a continuous variable; it only comprises a few different size groups as described in Chapter 2. Hence, utilizing cubic meters as a variable will not provide enough dispersion as a variable (Salvanes, 1985).[4)]

Since the data set precludes modeling the economic behaviour with a volume constraint in pen capacity, our approach in both the profit and cost functions may then pose a specification error problem. The error in our case is the omission of a relevant explanatory variable (Kmenta, 1986, pp. 445–446). It can be shown that in a context with an omitted variable, the incorrectly specified model will yield biased and inconsistent estimates. Furthermore, the variance of the estimates will be biased. Since we are concerned about the direction and extent of the bias with respect to the cost and profit functions, we will examine the consequences of the specification error in a simple framework which still covers our particular case. Assume we have the true and correct function explaining the underlying process:

$$y = b_1 x_1 + b_2 x_2 + u, \qquad (3.17)$$

where the variation in y is explained by x_1 and x_2. The error term, u, has the usual white noise characteristics. Let x_2 be excluded from the equation, as in our case with no data on one variable, and apply least squares to the equation:

$$y = b_1^* x_1 + u^* \tag{3.18}$$

The specification bias – the difference between the coefficient b_1^* in the misspecified model and b_1 in the correct specification – can be expressed as (See Appendix B):

$$(E(b_1^* - b_1)) = b_2 a_{12} \tag{3.19}$$

That is, the bias in the incorrect specified model depends on the sign of b_2 which, from the correct specification, we expect will be different from zero, and a coefficient, a_{12}, which is shown in Appendix B to be the correlation coefficient or coefficient of regression between the omitted variable and the included variable. From equation (3.19) we notice that the bias of b_1^* will be positive when b_2 and a_{12} have the same sign, and negative otherwise. There will be no bias when b_2 is zero, or the omitted and included variables are orthogonal; when x_1 and x_2 are uncorrelated, x_1 picks up none of the effect of x_2 and no bias occurs.

In our case we need information on the possible sign of the coefficient b_2 in the correctly specified model; the sign of the effect of regulated water volume on total costs and net revenue. Furthermore, we have to speculate about the sign of a_{12}, i.e., the sign of the correlation between our omitted variable – the regulated pen volume – and the included variables in the cost and profit functions.[5)]

Within the framework of a simple and common functional form of the production function, the Cobb–Douglas form, we will analyze the possible effect of omitting capacity given in pen volume as an input factor in our case. We will below specify a cost function and a profit function for empirical estimation of the production process in the fish farming industry, but since the cost and profit function are dual to the production function, analyzing the consequences of omitting an input factor for the primal production function is sufficient. Furthermore, we will focus on the direction of the bias on the elasticity of scale since this is an important concern in the empirical part of the study. To keep the theoretical considerations of the possible specification bias concentrated to one Section, we will rely on the definition of the elasticity of scale which is presented later in this Chapter. The correct specification of the production function with capacity in cubic meters included will be as follows in the Cobb–Douglas case (smolts are here excluded as a factor of production):

$$y = A\,K^{\alpha_K}\,L^{\alpha_L}\,F^{\alpha_F}\,V^{\alpha_V} + u \tag{3.20}$$

where y is output, K,L,F and V are inputs of capital, labour, feed and pen volume, A is a shift parameter, and α_i is the coefficient for marginal productivity of factor i. It is easy to show that we obtain a simple formula for the elasticity of scale – given in equation (3.41) in Section 3.4.1 – in this case

$$\varepsilon = \sum_{i=K,L,F,V} \alpha_i \tag{3.21}$$

When omitting pen capacity the production function is:

$$y = A^*K^{\alpha_K^*}\,L^{\alpha_L^*}\,F^{\alpha_F^*} + u \tag{3.22}$$

and the elasticity of scale becomes,

$$\varepsilon = \sum_{i=K,L,F} \alpha_i^* \tag{3.23}$$

We will now procede by considering each of the coefficients, α_i^*, of the mis-specified production function and compare them to the corresponding coefficients, α_i, in correctly specified production function. Equation (3.19) defining the specification bias will be used. A possible bias for each of the coefficients in equation (3.22) can be expressed as:

$$E(\alpha_K^* - \alpha_K) = \alpha_V a_{KV} \tag{3.24}$$

$$E(\alpha_L^* - \alpha_L) = \alpha_V a_{LV} \tag{3.25}$$

$$E(\alpha_F^* - \alpha_F) = \alpha_V a_{FV} \tag{3.26}$$

If we specified the production function correctly, as in equation (3.20), the sign of the coefficient α_V – the marginal productivity of the pen capacity – will be positive. Thus, the signs of the correlation coefficients between the omitted pen volume and each of the included factors, a_{iv}, will decide the direction of the bias for each factor.

We will make some speculations based on economic theory concerning the signs of the correlation coefficients. One would expect that the correlation coefficients to be positive, i.e., large input of pen volume goes together with high levels of labour, feed and other inputs. Hence, in this case the estimates of the coefficients α_K^*, α_L^* and α_F^* are biased upwards compared to the true coefficients. It is possible to argue that there are differences in the relative use of factors in the Norwegian aquaculture industry because of the regulation of pen size. But the correlation coefficients concern the use of the absolute levels of input factors. Hence, the elasticity of scale is biased upward when pen volume as a factor is not included since the elasticity is obtained by a sum of single coefficients. From this discussion we may suggest that the elasticity of scale will be overestimated in our case – both for the cost and profit function – when pen volume as a factor is omitted. Furthermore, the variance of each coefficient, α_i^*, will be biased upwards. We will obtain a larger variance and a tendency not to reject the zero hypothesis and to draw conservative conclusions about scale economies. We will return to the question of specification bias when results are discussed in Chapter five.

3.3.2 Smolt as an input factor

As earlier stated, reproduction is accomplished by buying a new cohort (year–class) of fish each year. This is a distinct characteristic of fish farming in contrast to wild fisheries. However, at the start of each year when one determines output (through harvesting) and the optimal use of inputs, the corresponding decision on smolts as an input factor has already occurred. There are two ways to view this special feature of smolts as an input factor.

First, smolts as an input can be considered as separable from the group of other factors. In other words, the optimal input decision can occur in two stages. First, the farmer allocates resources between smolts and the group of other factors, then he decides on optimal factor intensities within the subgroup of other factors. This means that smolt input does not affect the elasticities of substitution between other pairs of factors (Berndt and Christensen, 1973). The assumption of separability appears reasonable in our case from what we know about the production process with a sequence of time between smolt release and the decision on output level and the level of the other inputs. However, there exist arguments against considering the input of smolts as separable. For instance in the case of considerable mortality of smolts one would expect the substitution of feed and other factors to be influenced. In the case of strong separability we can express the production function as:

$$y = f(K,L,F) + g(S) \tag{3.27}$$

Thus, the production function in equation (3.27) defines a long run production relationship with no fixed factor, only that the relative input intensities are attained in two stages. The production function approach with separability in smolt implies that a long run production relationship should be appropriate.

Alternatively, we can look at the farmer's economic decision in the beginning of a normal year with a *given* amount of smolt in the pens. The input of smolt is fixed during the production period, but it is variable between production periods. Hence, the farmer must determine the optimal mix of the remaining inputs given smolts in the beginning of the year.

The production function may be written:

$$y = f(K,L,F;S), \tag{3.28}$$

where L,K and F are variable factors and S is considered fixed, with the semicolon implying that S is fixed. Thus, we have a restricted production function. Using this approach with smolts as a given factor implies that a short run model could be appropriate.

From Section 3.3.1 we concluded that a long run cost function could be appropriate due to standardized harvesting procedures, as this represents a possible interpretation of size regulation of farms and data limitations on regulated pen volume. The production function in equation (3.28) with smolts as a separable factor fits into this approach since it requires a long run model specification. Hence, we will formulate a long run cost function as a specification of a food fish farmer's behaviour based on the criteria given above. Furthermore, in this case we also open the opportunity to test long run relationships such as scale properties in the industry. As an alternative hypothesis we will formulate a profit function as discussed in Section 3.2 and in Section 3.2.1. Since a restricted form is required for empirical implementation as described in Chapter one, we will choose the formulation given in equation (3.28) of the production function. Thus, we will formulate a restricted profit function as an alternative description of the grow–out farmer's economic behaviour. We will now proceed to specify a restricted profit function formulation and a cost function.

3.4 A restricted profit function

For the profit function we chose the restricted form to maintain a well–defined function for empirical implementation:

$$\pi(p,\mathbf{w};S) = \max_{y,\mathbf{q}} (py - \mathbf{wq} \text{ s.t. } f(K,L,F;S) \geq y) \tag{3.29}$$

where $\mathbf{q}$ is the vector of inputs chosen in the actual period; i.e., K, L and F. The variable (or restricted) profit function denotes the maximum revenue minus variable expenditures given output price and variable input prices and the fixed factors. This formulation was introduced by Samuelson (1953–54, p. 20), and properties were developed by Gorman (1968) and McFadden (1966). The profit function in equation (3.29) corresponds to the following profit maximization problem:

$$\max \pi = py - \mathbf{wq} \tag{3.30}$$
$$s.t. y = f(K,L,F;S)$$

where the profit maximizing values of input demand (q_i^*) for factors (i=K,L,F) and output supply (y^*):

$$q_i^* = q(p,\mathbf{w};S) \qquad i = K,L,F \tag{3.31}$$
$$y^* = y(p,\mathbf{w};S)$$

are substituted into the expression for profit in equation (3.30), and the restricted profit function can be written in the form given by equation (3.29) or in the following and equal form for ease of exposition:

$$\pi^* = \pi(p,\mathbf{w};S) \tag{3.32}$$

The restricted profit function gives the maximum value of the profit, π^*, (an asterisk denotes that we have a maximum value function) for each set of values of (p,**w**;S).

We now explore additional properties of the variable profit function. There is a duality between the production function and the variable profit function if certain regularity conditions are satisfied.[6)] This means that technology can be equivalently represented by the restricted profit function and the restricted production function. The duality theorem between the production possibility set and variable profit functions is derived in Diewert (1973) or (1982). The neoclassical properties of the production function are:

1) f(·) is continuous in (q;S) (3.33)
2) f(·) is monotonic in (q;S)
3) f(·) is twice differentiable
4) f(·) is strongly concave

5) $f(\cdot)$ is bounded in the range of $(y,\mathbf{q})$

If the firm faces positive output and input prices and fixed factors and given the regularity conditions of the production function (in 3.33), the restricted profit function is specified as:[7)]

$$\pi(p,\mathbf{w};S) = \max_{\mathbf{q},y} (py - \mathbf{wq} \text{ s.t. } f(\mathbf{q};S) \geq y) \qquad (3.34)$$

where q is the vector of variable inputs (K,L,F) with the following properties (Diewert (1973) and (1982)):

(1) Nondecreasing in p. If $p' \geq p$, then $\pi(p',\mathbf{w};S) \geq \pi(p,\mathbf{w};S)$

(2) Nonincreasing in $\mathbf{w}$. If $\mathbf{w}' \geq \mathbf{w}$, then $\pi(p,\mathbf{w}';S) \leq \pi(p,\mathbf{w};S)$

(3) Homogeneous of degree 1 in $(p,\mathbf{w})$, $\pi(\alpha p,\alpha\mathbf{w};S) = \alpha\pi(p,\mathbf{w};S)$ for all $\alpha > 0$.

(4) Convex in input and output prices. Let $\pi(p'',\mathbf{w}'') = (\alpha p + (1-\alpha)p', \alpha\mathbf{w} + (1-\alpha)\mathbf{w}',S)$, then $\pi(p'',\mathbf{w}'';S) \leq \alpha\pi(p,\mathbf{w};S) + (1-\alpha)\pi(p',\mathbf{w}';S)$ for $0 \leq \alpha \leq 1$.

(5) Concave in S. $\pi(p,\mathbf{w};\alpha S + (1-\alpha)S') > \alpha\pi(p,\mathbf{w};S) + (1-\alpha)\ \pi(p,\mathbf{w};S')$ for $0 < \alpha < 1$.

(6) Continuous and twice differentiable in p and $\mathbf{w}$.

These properties lead to the following interpretations. Properties 1) and 2) implies that the profit function is monotonic in input and output prices. Homogeneity of degree 1 in prices imply that the profit maximization behaviour (solution) remains unchanged, i.e., input combinations are invariant if all prices change in the same proportion. The paramount implication of the convexity assumption is that the profit function has a symmetric (Young's theorem) and positive semidefinite Hessian matrix (Varian, 1984).

In sum, duality implies that if we have a function with the properties 1–6 it is a profit function, and that there exists a production function satisfying conditions 1–5 in (3.33) that can be recovered through the profit function. Thus, with an explicit specification of the restricted profit function and without solving for the first–order conditions of profit maximization, the arbitrary function possessing properties 1–6 represents equally the restricted profit function and the underlying production function.

3.4.1 Technological measures and the restricted profit function

Further important technological properties are yielded by the profit function. By Hotelling's Lemma we can state some elasticity measures which will describe the technology. Hotelling's Lemma shows that (Varian, 1984):

$$\frac{\delta\pi(p\,,\mathbf{w};S)}{\delta w_i} = -\,q_i^*(p,\mathbf{w};S) \qquad i = K,L,F \tag{3.35}$$

$$\frac{\delta\pi(p,\mathbf{w};S)}{\delta p} = y^*(p,\mathbf{w};S) \tag{3.36}$$

In other words, the derivatives of the profit function with respect to the output price and input prices yield the firm's supply function as well as the demand functions for factors.

Part of our empirical research will focus on the elasticity of substitution, and using equations (3.35) and (3.36), we can derive own– and cross–price elasticities:

$$\eta_{ii} = \frac{\delta q_i^* \; w_i}{\delta w_i \; q_i^*} = - \frac{\delta^2 \pi \; w_i}{\delta w_i \delta w_i q_i^*} \tag{3.37}$$

$$\eta_{ij} = \frac{\delta q_i^* w_j}{\delta w_j q_i^*} = - \frac{\delta^2 \pi \quad w_j}{\delta w_i \delta w_j q_i^*} \tag{3.38}$$

where $-\delta^2\pi/\delta w_i \delta w_j$ and $-\delta^2\pi/\delta w_i \delta w_i$ are elements in the Hessian matrix of the profit function. The cross–price effects are symmetric $\eta_{ij} = \eta_{ji}$ (Young's theorem) and the own–price effects are negative since the diagonal terms of a negative semidefinite matrix must be nonpositive. Complementarity between pairs of inputs is obtained when $\eta_{ij} < 0$ and substitutability implies that $\eta_{ij} > 0$ (Weaver, 1983).

We are also concerned with the elasticity of demand with respect to the other exogenous variables, output price and the fixed factor. The elasticity of demand with respect to output price (Sidhu and Baanante, 1981):

$$\eta_{ip} = \frac{\delta q_i^* p}{\delta p q_i^*} = \frac{\delta^2 \pi \quad p}{\delta w_i \delta p q_i^*} \tag{3.39}$$

measures the effect on demand for factor i of a change in the price of output. From the sign of this elasticity, we can classify the inputs as superior ($\eta_{ip} > 1$), normal ($1 < \eta_{ip} < 0$) or inferior ($\eta_{ip} < 0$). The elasticity of demand with respect to the smolt as a fixed factor is (Sidhu and Baanante, 1981):

$$\eta_{iS} = \frac{\delta q_i^*}{\delta S}\frac{S}{q_i^*} = \frac{\delta^2 \pi}{\delta w_i \delta S}\frac{S}{q_i^*} \tag{3.40}$$

This elasticity, η_{iS}, measures the effect on the demand for the variable inputs when smolt is expanded.

The standard measure of returns to scale is defined as the elasticity of proportional output response to a proportional change in inputs. Since we are not interested in limiting our study to a constant scale elasticity, i.e., independent of output size, we invoke a *generalization* of Euler's Theorem (to nonhomogeneous functions) to define the scale elasticity where the degree of homogeneity is replaced by the scale elasticity (Førsund, 1974)

$$\varepsilon(y) = \left.\frac{df(kq)}{dk}\frac{q}{f(kq)}\right|_{k=1} = \sum_i \frac{df}{dq_i}\frac{q}{f} = \frac{\sum_i f_i q_i}{y} \tag{3.41}$$

where $y = f(kq)$ is the production function, f_i is the marginal product of factor i and k is the scale factor. Without loss of generality the measure is evaluated at k=1. Hence, we arrive at an explicit measure for the scale elasticity as the sum of the elasticity of output with respect to each *variable* input.

This measure has a limited interpretation since a proportional change in output, resulting only from *proportional* changes in the scale of inputs, is permitted. Thus, we evaluate scale effects only along the ray from the origin (Hanoch [1975], Rivier [1987]). An alternative measure argued to be superior by Hanoch (1975) occurs when the increase in output is allowed to go through the expansion path of the firm. The expansion along the ray corresponds to the expansion path only for homothetic technologies, i.e., when substitution possibilities are independent of output. This alternative and more general concept owing to Hanoch (1975) under the cost minimization case, can also be measured in terms of the profit function (Weaver, 1983).

Using the first–order conditions for profit maximization in equation (3.30) and Hotelling's Lemma (3.35) and (3.36), equation (3.40) can be rewritten as:

$$\varepsilon(y) = \sum_i \frac{f_i q_i}{y} = \sum_i \frac{w_i \; q_i^*}{p \; y} = -\sum_i \frac{\delta\pi^*/w_i}{\delta\pi^*/\delta p} \; \frac{w_i}{p} = -\frac{\sum_i \frac{\delta\pi^*}{\delta w_i} \frac{w_i}{\pi^*}}{\frac{\delta\pi^*}{\delta p} \frac{p}{\pi^*}} = -\frac{\sum_i \frac{\delta \ln\pi^*}{\delta \ln w_i}}{\frac{\delta \ln\pi^*}{\delta \ln p}} \tag{3.42}$$

Since in the scale definition only the variable inputs can be changed – in our case with three variable factors and one fixed factor in the given time period – this measure given by restricted profit function must be interpreted as a measure of *short run* returns to scale. Hence, it is of limited interest in our case since our intention is to get a measure of scale economics along the long run expansion path. This will be accomplished by formulating a well–defined long run cost minimization problem.

3.5 A cost function for a salmon farmer

Given the interpretations in Section 3.3.1 and Section 3.3.2, we will specify the following cost minimization problem for a salmon farmer:

$$\min_{\mathbf{q}} c = \mathbf{qw} \tag{3.43}$$

$$\text{s.t. } y = f(K,L,F) + g(S) \tag{3.44}$$

Equation (3.44) assumes the production function to be strongly separable in smolts. This implies that the smolt is separable in the determination of other input levels.

The first–order conditions of the cost minimization problem yields:

$$q_i^* = q(y,\mathbf{w}) \qquad i = K,L,F \tag{3.45}$$

By substituting cost minimizing input levels into the definition of cost in equation (3.38), we obtain the following cost function:

$$c(y,\mathbf{w}) = c(y,w_K,w_L,w_F) \tag{3.46}$$

which relates total production costs (c) to the level of output (y) and factor prices (w_i). This function assigns to every combination of input prices the minimum costs corresponding to the cost minimizing input levels for any given volume of production.

If factor prices and output level are exogenously determined – which we argued above the theory of duality between cost and production implies that the characteristics of production can be represented by the cost function due to certain regularity conditions of the production function and cost function (Shephard, 1952 and 1970).

The production function properties are, of course, the same as those described in the profit function case equation (3.33), and the cost function must possess the following properties:

(1) Nondecreasing in w. If $\mathbf{w}' \geq \mathbf{w}$, then $c(\mathbf{w}',y) \geq c(\mathbf{w},y)$

(2) Homogeneous of degree one in $\mathbf{w}$. $c(\alpha\mathbf{w},y) = \alpha c(\mathbf{w},y)$ for $\alpha > 0$.

(3) Concave in $\mathbf{w}$. $c(\alpha\mathbf{w} + (1-\alpha)\mathbf{w}',y) \geq \alpha c(\mathbf{w},y) + (1-\alpha)c(\mathbf{w}',y)$ for $\alpha > 0$. (3.47)

(4) Continuous and twice differentiable.

Property number four is assumed for empirical purposes (Fuss, McFadden and Mundlak, 1978). The interpretation of any of the above properties is similar to those associated with the profit function. Monotonicity in factor prices is the rationale behind property one. Homogeneity of degree one in factor prices implies that only relative prices cause re–allocation of factors. Concavity in factor prices has one major implication, namely that the Hessian of second order derivatives is symmetric negative semidefinite.

The assumption of cost minimization yields an explicit set of factor demand equations through Shepard's Lemma:

$$\frac{\delta c}{\delta w_i} = q_i^*(y,\mathbf{w}) \quad i = K,L,F \tag{3.48}$$

This equation states that the first–order derivatives of the cost function yield the factor demand functions for input factors. Taking advantage of this property, the following elasticity measures can be proved. Uzawa has shown that the most widely used measure of input substitutability – the Allen partial elasticity of substitution – may be obtained from the derivatives of a total cost function (Uzawa, 1962). In the case of nonhomogeneous production technology Binswanger (1974) has shown that the Allen – Uzawa partial elasticity of substitution, σ_{ij}, can be expressed as

$$\sigma_{ij} = \frac{c(\cdot)\,\delta^2 c(\cdot)}{\delta w_i\ \delta w_j} \Big/ \frac{\delta c(\cdot)\,\delta c(\cdot)}{\delta w_i\ \delta w_j} \tag{3.49}$$

and from the symmetry assumption, we have that:

$$\sigma_{ij} = \sigma_{ji} \qquad i \neq j \tag{3.50}$$

The cross–elasticity of substitution measure will indicate which pair of inputs are substitutes or complements. Intuitively, we see this clearly using Shepard's Lemma in the elasticity expression in equation (3.50). Thus,

$$\sigma_{ij} = \frac{c(\cdot)\delta q_i / \delta w_j}{q_i \; q_j} \tag{3.51}$$

and a positive value of $\delta q_i/\delta w_j$ is an intuitively reasonable definition for substitutes; when the price of factor j rises the demand for factor i also rises, and a negative value indicates a complementary relationship; when the price of factor j rises the demand for factor i drops.

From equation (3.49) the own–elasticity of factor demand, $\frac{\delta q_i w_i}{\delta w_i q_i}$, can be derived:

$$\eta_{ii} = \frac{\delta q_i \; w_i}{\delta w_i \; q_i} = \frac{q_i \; w_i \; \sigma_{ii}}{\sum_i q_i \; w_i} \tag{3.52}$$

where $q_i w_i / \Sigma q_i w_i$ is the i'th factor share of total costs, and σ_{ii} is the Allen own–elasticity of substitution.

The sign of the own–price elasticity is implied to be negative by economic theory when cost minimizing behaviour is assumed.

For the cost function the returns to scale along the expansion path is developed in a similar fashion as for the profit function. We substitute into the technical definition of scale elasticity in equation (3.41) the first–order conditions for cost minimization, Shepard's Lemma and $\delta c/\delta y = \lambda$ (which is satisfied along the expansion path, (Hanoch [1975]), Revier, [1987]):

$$\varepsilon(y) = \sum_i \frac{f_i q_i}{y} = \sum_i \frac{w_i q_i}{y} = \frac{c}{y\lambda} = \frac{c}{\delta c/\delta y \; y} = \frac{\delta y}{\delta c}\frac{c}{y} = \left[\frac{\delta \ln c}{\delta \ln y}\right]^{-1} \tag{3.53}$$

Equation (3.53) a measure of returns to scale under the cost function approach which is interpreted as the elasticity of output with respect to cost. Unlike the measure in equation (3.42), the elasticities in (3.53) measures the *long run* returns to scale since we have defined the cost function as a function of variable input factors.

Footnotes

1) We do not consider the condition for optimal level of recruits in our model. However, the conditions can be derived as follows

$$\pi(T,R) = (p(T)B(T))e^{-rT} - cR = (p(T)Re^{-rM}w(T))e^{-rT} - cR \qquad \text{(i)}$$

$$\frac{\delta\pi(T,R)}{\delta R} = ((p(T)e^{-MT}w(T))e^{-rT} - c = 0 = (P(T)e^{-MT}w(T))e^{-rT} = c \qquad \text{(ii)}$$

which can be expressed as

$$\frac{\delta V(T)\, e^{-rT}}{\delta R} = c \qquad \text{(iii)}$$

The interpretation of the condition for the optimal level of smolts, R, is straightforward. From (iii) we notice that the optimal number of smolts is obtained when the present value of the marginal revenue w.r.t. the number of smolts equals the marginal costs of putting smolts to sea. The optimal values of T and R are determined simultaneously.

2) Because of a data deficiency on other material inputs, we could not develop an index consisting of all intermediate inputs.

3) In an environment of positive excess demand for their product this has been to the farmers' advantage. It might well change, as there have been some indications of declining excess demand from 1985 on, as market conditions alter.

4) In a different setting the user–cost of capital corresponding to the pen volume was tried as a proxy for regulated pen volume (Salvanes, 1985). Howewer, for the more complete data set utilized here, this information is not available.

5) It can be shown that this framework is sufficiently general to be extended to functions with more than one variable included or excluded (Kmenta, 1986).

6) Varian (1978) provides an introduction and Fuss and McFadden (1978) and Diewert (1974, 1982) provide more advanced treatments.

7) Lau (1976) establishes the duality properties for the normalized restricted profit function and in the corresponding production possibility set. The normalized profit function means that the profit, output and input prices and fixed factors are deflated by the output price. An application of this function for the Cobb—Douglas specification, is found in Lau and Yotopoulos (1973).

4. DEFINITIONS OF VARIABLES AND ECONOMETRIC SPECIFICATION

In this Chapter the data set will be described and alternative functional forms for the profit function and the cost function in Chapter 3 will be specified. A discussion of the data set consisting of two cross–sectional samples of farm–level data from 1982 and 1983, is provided in Section 4.1. The variables used in the econometric estimation, transformed from the original data set, are defined. Since secrecy clauses prohibit reproduction of the data set, summary statistics will be provided. In Sections 4.2 and 4.3 the functional forms applied and stochastic specifications of the restricted profit function and cost function respectively, are presented and discussed. In addition, both the properties of the translog form and the testable hypotheses are derived.

4.1 The data set and variable definitions

4.1.1 The data

We have utilized primary data for fish farms for 91 individual plants for 1982 and for 116 plants in 1983 out of total populations of 382 and 409 for the respective years. However, this is not panel data, i.e., the two samples do not consist of identical firms. The data base was obtained from the Norwegian Directorate of Fisheries and is the same as the one used in its publication on profitability of Norwegian grow–out farms.[1)]

The data set has two important advantages for our research purposes. First, it consists of cross–sectional data at the micro–level which is most appropriate when studying scale properties. In addition, the samples contain observations above the contemporary prevailing upper size limit of 8,000 m^3 since farms established prior to the introduction of licensing in 1973. However, there are relatively few observations above this upper limit. The lack of data on environmental characteristics circumscribed the possibility of testing the impact of environmental conditions of the site in the production process.

The two cross sections of data are two random samples collected by the Norwegian Directorate of Fisheries. Their survey started in 1982 with the aim of collecting data on revenues and costs at the farm level in order to provide information on the profitability of salmon farms.

The survey is constructed such that the observations are selected at random within 23 defined strata, consisting of five–six size groups in each of four regions. This ensured observations for all size groups and regions along the coast. In addition, all farms established before 1973, and thus above the prevailing size limit of 8,000 m^3 as described in Chapter two, are included if data were made available. Farms below 500 m^3 capacity were excluded from the survey.

Tables 4.1 and 4.2 show the dispersion in the sample due to region and size.

Table 4.1: Regional dispersion of the samples (samp) and population (pop) in 1982 and 1983. The size of the farms are given in utilized capacity of pen volume (m³).

	Size	–3,000 m³		3,000–5,000 m³		5,000 m³–		Total	
	Region								
		samp	pop	samp	pop	samp	pop	samp	pop
1982	Finnmark, Troms Nordland	5	43	7	26	8	17	20	86
1983	Finnmark, Troms Nordland	4	38	9	41	13	24	26	103
1982	Nord– and Sør–Trøndelag Møre and Romsdal	6	42	9	28	19	72	34	142
1983	Nord– and Sør–Trøndelag Møre and Romsdal	4	31	1	40	21	77	39	148
1982	Hordaland Sogn og Fjordane	9	45	9	29	15	35	33	109
1983	Hordaland Sogn og Fjordane	6	37	11	42	17	34	34	113
1982	Rogaland Skagerak	7	25	6	12	4	8	17	45
1983	Rogaland Skagerak	6	28	7	9	4	8	17	45

Source: *Lønsemdundersøkingar 1982, 1983* (Directorate of Fisheries, Bergen).

Table 4.2: Size distribution in terms of utilized capacity (in m³) of the samples (samp) and the population (pop) in 1982 and 1983.

Size		–3,000	3,000–5,000	5,000–7,000	7,000–15,000	15,000–
1982	samp	24	26	17	18	6
	pop	155	95	73	51	8
1983	samp	20	41	28	22	5
	pop	134	132	71	61	11

Source: *Lønsemdundersøkingar 1982, 1983* (Directorate of Fisheries,Bergen)

A problem concerning the use of cross–sectional data when specifying econometric functions demanding factor price dispersion can be considered here. We argued in Chapter two that grow–out farmers could be considered as price takers in the factor markets. One problem may occur when utilizing cross–sectional data in this respect since it could be argued that every single farm faces the same factor price. From Table 4.1 we notice that farms are included from the most northern regions, Finnmark, Troms and Nordland, to the south–west regions of Rogaland and Skagerak. Hence, due to the wide dispersion of farms along the coastline, there exist possibilities for factor—price differences. For instance, factors primarily produced in the southern part of the country such as feed and and some capital equipment, lead to a price dispersion due to transportation costs. In addition, it is also reasonable to argue that separated labour markets exist due to interregional differences in wages.

Two questionnaires were used to collect information on harvest levels, the harvest value to the farmer; capacity in cubic meters, annual expenditures and quantities of inputs, the size of year–classes released; stock of fish in the pens at the beginning and the end of the year; depreciation; the value of the total capital, and the number of hours for self employment and family labour. The summary statistics for the primary variables are given in Table 4.3. All variables are inflated to 1983 values by a general price index from the Norwegian Central Bureau of Statistics (Statistical Yearbook, 1985).[2]

Table 4.3: Summary statistics of primary variables. (in NOK if unless otherwise indicated)

	Mean	Standard Deviation
Feed expenses	939380.0	1137100.0
Smolt expenses	467500.0	526100.0
Transportation	110140.0	572590.0
Material expenses	344820.0	529490.0
Labour expenses	422830.0	501970.0
Hourly wage rate for hired labour	57.61	22.72
Hours of family labour	1475.1	1875.3
Utilized capacity (1,000 m^3)	6064.7	6322.7
Production(kilo)	89296.0	96635.0
Value of production	2970900.0	135030.0

From Table 4.3 we get a picture of the relative importance of the factors of production for the mean sized farm. Feed stands out as the most important input in terms of expenses. However, we notice from the standard deviations that all the variables will vary over the sample.

4.1.2 Definitions of variables

To undertake the empirical estimation of a *restricted profit function* the following variables are necessary: the output price; the sales revenues; expenditures on variable inputs and their unit prices. Finally, we need the initial stock of the restricted input, smolt. The estimation of the cost function requires a definition of the output variable in addition to expenditures for each variable input and its unit price.

The definition of output is especially important for specifying output price and revenues. In Chapters two and three we argued that output in a normal year is defined by the harvest and the change in the stock of living fish in the pens from the beginning of the year to the end. This production constitutes one complete year–class, and it was seen from Figure 2.4 (in Section 2.3) that the expenditures on factors of production for a normal year covered the costs of producing one year–class including expenditures on smolts. In addition, we must correct for the fact that in most of the operations the number of smolts released has increased over time (for some farms, however, the input of smolts decreased). This means that in terms of Figure 2.4 in Section 2.3 that $x_2 > x_1$. We correct for this fact by adding this extra increase in the stock of living fish in pens to the production; i.e., output is $(y_1 + y_2) + (x_2' - x_1') + (x_2 - x_1)$. Further, all three parts of the output can consist of both salmon and trout depending on the production mix for each farm. Thus, we define output as an index of production of salmon and trout in tonnes consisting of tonnes harvested and the change in stock of living fish. The change in stock is the difference between the stock of living fish at the end of the year and the beginning of the year, and calculated in terms of the average weights at the end of the year for the different year–classes. We define an alternative output definition in value terms as the sum of sales and the change in stock evaluated as 70 percent of the sales price, which is the anticipated cost. The change in the stock was evaluated at the anticipated cost by the Directorate of Fisheries, and we utilized the same definitions and data (*Lønsemdundersøkingar 1982,* p. 12).

The revenues are defined as an output index and are based on the value of the harvest to the producer and the value of the change of living fish in the pens. In addition to the value of the harvest, the Directorate of Fisheries provided the values of fish in the pens. They calculated this value by multiplying the number of fish in each size group by the corresponding weight at the end of the year and evaluated it at 70 percent of sales prices, i.e., the anticipated cost price. The Directorate of Fisheries used 70 percent and not 100 percent of the sales value when they evaluated the fish in the pens because 70 percent was considered to be the alternative value of the fish in the water, i.e., the fish was not considered ready for the market. However, it can be argued that one should evaluate the change in living fish in the pens at market prices.

The output price was then calculated by dividing the revenue by the output level. This can be considered an output price index since it is a weighted output price of different year–classes of fish and of two species, Atlantic salmon and rainbow trout.

Five input categories are identified including capital, labour, feed, materials, and smolts, or more generally, juveniles. For the first four we need corresponding expenditures and their price series. For smolts the stock at the beginning of each year is necessary since it is considered fixed to the farmer.

Labour expenses were calculated as the payment for hired labour plus the imputed value for self–employed and unpaid family labour. It was assumed that the self—employed and family workers obtained the same wage rate as the employed worker. The wage rate for hired labour was calculated as the annual expense for hired labour divided by hours of work.

For feed, annual expenditures were provided in the data set (corrected for inventory changes). However, quantities purchased were not available. We therefore defined an index for the feed price by dividing the feed expenditures by output produced. This argument is valid since there exists a relationship between produced output (or growth of the fish) and the quantity of feed used, i.e., the conversion ratio of kg feed to kg fish produced. Of course, this must be considered a proxy for the real feed price since there could exist disturbances in the food conversion ratio. However, there is no reason to believe that there is a systematic error in the ratio between small and big firms, which is of prime importance to us.[3)]

To calculate the capital flows we used one aggregate measure of the different categories of capital. We calculated the flow of services as a user–cost, including depreciation evaluated at current replacement costs and the rate of interest. The amount of depreciation was provided by the Directorate of Fisheries. It was calculated by dividing the different capital subgroups depending on the expected economic durability of capital. The second part of the user–cost of capital was calculated as seven percent annually on total capital. Seven percent was used as an interest rate because this is the discount rate used for public investments in Norway. Total capital is defined as the value of the capital investment in the plant for the actual year. The value of the stock of fish – calculated at the end of the year – is included in the total capital definition since the fish is tied to the farm for a period of time in the same manner as capital equipment. The unit–price of capital was calculated as the capital flow divided by the total capital.

Materials include freight, packing, electricity, insurance, and administration expenses. Material expenditures are available, but neither quantities nor a quantity index. Since these factors are quite unimportant in the total costs, it is reasonable to consider material costs as not affecting the substitution possibilities between any other factors. Thus, we decided to exclude them by assuming strong separability.

According to the above definitions, profit is now defined as revenues minus the expenditures on the variable factors; labour, capital and feed.

The stock of smolts was hypothesized to be fixed within the annual observation interval (or alternatively considered as strongly separable). The number of smolts put into the sea is available by the two actual year–classes (first and second year–class), and thus this definition was used for the restricted input of smolts.

Before proceeding with the econometric specifications of the profit and cost models, we will provide descriptive statistics of some of the more important derived variables for empirical testing. Since our study concentrates on production and cost properties, we will provide information on the mean values and standard deviations on cost shares and unit prices for the factors of production. In Table 4.4. the descriptive statistics are given based on the total sample, and in Table 4.5 the statistics are provided when we split the sample into two groups, small and large, according to output size. Large farms consist of farms above mean size in output terms, and small farms of farms below mean size.

Table 4.4: Cost shares and unit prices (means and standard deviations) for the factors of production based on the total sample (82/83).[1)]

	Cost share		Unit price (index)	
	Mean	St.dev	Mean	St.dev.
Capital	0.16	0.0745	0.11	0.0227
Labour	0.20	0.0738	57.61	22.7200
Feed	0.42	0.1147	11.56	9.2386
Smolt	0.22	0.0897	–	–

1) The number of observations used is 207.

We get a notion of the relative importance of the factors of production from Table 4.4 when we notice that feed is the dominant input factor in the production process of farmed salmon. The unit price for labour is the hourly wage rate for hired labour, i.e., about 58 NOK per hour. The unit price of capital is an index in terms of the annual capital flow divided by the total capital. The feed price is also an index defined as the feed expenditures divided by output produced.

Table 4.5: Cost shares and unit prices (means and standard deviations) for large farms (above mean output size) and small farms (below mean output size).[1]

	Small Farms				Large Farms			
	Cost shares		Unit price		Cost shares		Unit price	
	Mean	St.dev.	Mean	St.dev.	Mean	St.dev.	Mean	St.dev.
Capital	0.16	0.0813	0.11	0.0249	0.17	0.0603	0.11	0.0178
Labour	0.20	0.0797	55.34	24.2470	0.18	0.0596	61.11	19.0840
Feed	0.40	0.1298	12.21	11.0810	0.44	0.0747	10.35	3.9097
Smolt	0.23	0.1028	–	–	0.21	0.0567	–	–

1) The numbers of observations of "small farms" and "large farms" are 130 and 77, respectively.

From Table 4.5 we note that small farms are more intensive in the use of variable factors such as labour and smolts than the large farms, while large farms are more intensive in capital. This general tendency of smaller farms using variable factors more intensively – although not very strongly – has one exception in that feed has a higher variation of the cost share for small farms than for large farms. This conforms with differences in relative factor prices.

We will utilize the descriptive statistics of Table 4.4 and Table 4.5 in the discussion of empirical results in Chapter five.

4.2 Stochastic specification of the behavioural models

4.2.1 The translog restricted profit function

We begin this section by presenting the transcendental logarithmic functional form for the restricted profit function, equation (3.29) in Chapter 3, developed by Christensen, Jorgensen and Lau (1971, 1973).[4] It is a logarithmic Taylor series expansion to the second order for an arbitrarily restricted profit function around the variable levels of ln w = 0, ln p = 0. The translog function is flexible in the sense that it can measure the economic behavioural relationships without prior restrictions. The flexibility is attained from the second–order approach. The restricted profit function in our case is defined in one output, three variable inputs and one restricted factor, is given by:

$$\begin{aligned} \ln\pi(p,\mathbf{w},S) = {} & \alpha_0 + \alpha_P \ln p + 0.5\,\alpha_{PP}(\ln p)^2 \\ & + \sum_{i=K,L,F} \alpha_i \ln w_i + 0.5 \sum_{i=j=K,L,F}\sum \alpha_{ij} \ln w_i \ln w_j \\ & + \sum_{i=K,L,F} \alpha_{iP} \ln p \ln w_i + \alpha_{PS} \ln p \ln S \\ & + \sum_{i=K,L,F} \alpha_{iS} \ln w_i \ln S + \alpha_S \ln S + 0.5\,\alpha_{SS}(\ln S)^2 \end{aligned} \tag{4.1}$$

where $\pi(p,\mathbf{w},S)$ is the restricted profit – total revenue minus total costs of variable inputs; p is the price of output; $\mathbf{w}_i$ (i = K,L,F) are the prices of the variable inputs – capital, labour and feed; and S is the stock of smolts at the beginning of the year.

By applying Hotelling's Lemma (equations (3.35) and (3.36) in Chapter three, and the first–order derivatives of the translog profit function (4.1) with respect to lnp and $\ln w_i$ ($i = K,L,F$), we derive the revenue share and expense shares of profit, respectively:

$$\frac{\delta \ln \pi(p,\mathbf{w},S)}{\delta \ln p} = R = \frac{py}{\pi} \tag{4.2}$$

$$\frac{\delta \ln \pi(p,\mathbf{w},S)}{\delta \ln w_i} = -S_i = \frac{-w_i q_i}{\pi} \qquad i=K,L,F \tag{4.3}$$

We can estimate either the translog restricted profit function alone using OLS, or the share functions in a system to obtain the parameters for the technology, since all the economic information is equivalent. We chose to estimate a set of equations consisting of the restricted profit function and the share equations (Atkinson and Halvorsen, 1976). The profit function estimated alone does not yield very efficient parameter estimates because of possible multicollinearity between many right–hand side arguments. By estimating a joint system one gains more degrees of freedom without adding any coefficients to estimate. The reason for our inclusion of the restricted profit function in the system of share equations is that we obtain a system similar to the set in the cost functions case where the cost function must be included. To get a stochastic system we append a conventional additive disturbance term to each of the shares and the restricted profit function, which has a joint normal distribution with zero mean.[5)]

$$\epsilon_i \sim N(0,\sigma_i^2) \tag{4.4}$$

Furthermore, the vector of errors has non–zero correlations for each firm (off–diagonal elements in the covariance matrix) but, zero correlations across firms, i.e.,

$$E(\epsilon_i, \epsilon_j) = \sigma^2_{ij}$$
$$E(\epsilon_{it}, \epsilon_{js}) = 0 \qquad \forall\, t \neq s \tag{4.5}$$

Thus, we can write our system in stochastic form as:

$$\ln\pi(p,\mathbf{w},S) = \alpha_0 + \alpha_P \ln p + 0.5\alpha_{PP}(\ln p)^2 \tag{4.6}$$

$$+ \sum_{i=K,L,F} \alpha_i \ln w_i + 0.5 \sum\sum_{i=j=K,L,F} \alpha_{ij} \ln w_i \ln w_j$$

$$+ \sum_{i=K,L,F} \alpha_{iP} \ln p \ln w_i + \alpha_{PS} \ln p \ln S$$

$$+ \sum_{i=K,L,F} \alpha_{iS} \ln w_i \ln S + \alpha_S \ln S + 0.5\alpha_{SS}(\ln S)^2 + \epsilon_i$$

$$R = \alpha_P + \alpha_{PP} \ln p + \sum_{i=K,L,F} \alpha_{iP} \ln w_i + \alpha_{PS} \ln S + \epsilon_i, \tag{4.7}$$

$$S_i = -\alpha_i - \sum\sum_{i=j=K,L,F} \alpha_{ij} \ln w_i - \sum_{i=K,L,F} \alpha_{iP} \ln p - \sum_{i=K,L,F} \alpha_{iS} \ln S + \epsilon_i. \tag{4.8}$$

In order for there to exist a dual relationship between the restricted profit function and the underlying production function, the estimated equations must satisfy the regularity conditions outlined in Section 3.4. The translog restricted profit function is an approximation to the underlying production function, and does not necessarily have these properties globally. Symmetry of the cross–price effects of the profit function implies the following constraints on the profit function and share equations:

$$\alpha_{ij} = \alpha_{ji}, \forall\ i,j,\ i \neq j \tag{4.9}$$

Homogeneity of degree one in prices for the profit function means that the share equations are homogeneous of degree zero. These conditions imply the following parameter restrictions on the profit function:

$$\sum_i \alpha_i + \alpha_P = 1$$

$$\sum_i \alpha_{ij} + \sum_i \alpha_i = 0 \qquad j = K,L,F \tag{4.10}$$

$$\sum_i \alpha_{iS} + \alpha_{PS} = 0$$

For the share equations the following restriction should hold instead of the first line of (4.10) to ensure homogeneity of degree zero:

$$\sum_i \alpha_{ij} + \alpha_P = 0 \quad i=K,L,F \tag{4.11}$$

Tests for these restrictions – homogeneity of degree one in prices (4.10) and (4.11) and symmetry (4.9) – should be undertaken before they are imposed on the system of the restricted profit function and share equations. We will perform likelihood ratio tests on the different restrictions. Since minus twice the logarithm of the likelihood ratio – the ratio of the maximum of the likelihood function under the null hypothesis (i.e., with restrictions imposed) to the maximum of the likelihood function under the alternative hypothesis (i.e., the model without restrictions) – is asymptotically Chi–square distributed, significance tests can be performed. The number of degrees of freedom is equal to the restrictions we imposed by the null hypothesis (Theil, 1971).

However, monotonicity and convexity cannot be summarized by linear restrictions on parameters of the equations. Therefore, the consistency of these properties should be tested after estimation. The predicted shares must be negative for the expenditure shares and positive for the revenue share to satisfy monotonicity. For convexity, a necessary and sufficient condition is that the Hessian of the restricted profit function with respect to prices is positive semidefinite.

The Hessian matrix [H] can be conveniently defined in terms of the estimated coefficients in (4.6) and the predicted shares as:

$$[H] = \begin{bmatrix} \alpha_{KK}+S_K(S_K-1) & \alpha_{KL}+S_KS_L & \alpha_{KF}+S_KS_F & \alpha_{KP}+S_KR \\ \alpha_{LK}+S_LS_K & \alpha_{LL}+S_L(S_L-1) & \alpha_{LF}+S_LS_F & \alpha_{LP}+S_LR \\ \alpha_{FK}+S_FS_K & \alpha_{FL}+S_FS_L & \alpha_{FF}+S_F(S_F-1) & \alpha_{FP}+S_FR \\ \alpha_{PK}+RS_K & \alpha_{PL}+RS_L & \alpha_{PF}+RS_F & \alpha_{PP}+R(R-1) \end{bmatrix} \qquad (4.12)$$

Because cross–equation restrictions are imposed – such as symmetry – the equations in (4.6)–(4.8) must be estimated simultaneously by a generalized least squares procedure. The assumptions we made for the error terms satisfy the assumptions of Zellner's Seemingly Unrelated Regression (SUR) Model (Judge et. al., 1985). Therefore, to ensure efficiency SUR was applied. However, because $R + S_i = 1$, the disturbance covariance matrix of the full system (4.6)–(4.8) is singular. To ensure nonsingularity one of the equations was dropped from the system. If the system is iterated until convergence, maximum likelihood estimates obtained (Magnus, 1978). Maximum likelihood estimates are indifferent to which equation is dropped (Barten, 1968). Therefore, we employed the iterating procedure of Zellner's Seemingly Unrelated Regression model to obtain maximum likelihood estimates for the profit function together with three of the four share equations.

When the parameter estimates of equations (4.6), (4.7) and (4.8) have been obtained, the elasticities of demand and scale elasticity can be evaluated at the averages for the shares and at the given price level.

By using the elasticity numbers of equations (3.37), (3.38), (3.39) and (3.40) in Chapter 3, and converting them to logarithms using (4.2) and (4.3), we get the following expressions for substitution and demand elasticities w.r.t. to output price and the fixed factor in terms of the estimated parameters and fitted values for the shares (denoted by ˆ) (Weaver [1983], Sidhu and Baanante [1981]):

$$\hat{\eta}_{ii} = -(\hat{\alpha}_{ii}/\hat{S}_i) - \hat{S}_i - 1 \ , \tag{4.13}$$

$$\hat{\eta}_{ij} = -(\hat{\alpha}_{ij}/\hat{S}_i) - \hat{S}_j \ . \tag{4.14}$$

$$\hat{\eta}_{iP} = \sum_i \hat{S}_i + 1 + \hat{\alpha}_{iP}/\hat{S}_i \ , \tag{4.15}$$

$$\hat{\eta}_{iS} = \sum_i \hat{\alpha}_{iS} \ln w_i + \hat{\alpha}_S - \frac{\hat{\alpha}_{iS}}{\hat{S}_i} \ , \tag{4.16}$$

where we have respectively own–price elasticity of demand for factor i ($\hat{\eta}_{ii}$), the cross–price elasticity ($\hat{\eta}_{ij}$) and the input elasticity with respect to output price ($\hat{\eta}_{iP}$) and the demand elasticity with respect to the fixed factor smolts, ($\hat{\eta}_{iS}$).

From the theoretical measure of returns to scale in equation (3.41) in Chapter 3, we get the following measure for the translog profit function using equations (4.2) and (4.3) and the fitted shares from estimated parameters:

$$\varepsilon(y) = \sum_i \hat{S}_i/\hat{R} = \frac{\sum_i \hat{\alpha}_i + \sum_i \sum_j \hat{\alpha}_{ij} \ln w_j}{p + \sum_i \hat{\alpha}_{iP} \ln w_i} = 1 - (1/\hat{R}), \qquad (4.16)$$

which is dependent on farm size defined as the level of production, but less than one by definition.

4.2.2 The Translog Cost Function

For econometric estimation a specific functional form for $c(y,w_K,w_L,w_F)$ must be used. We postulate a translog cost function for the fish farms. This form was expanded by Christensen and Greene (1976) and Greene (1983) to incorporate the presence of scale economies varying by size. The translog cost function is specified as follows:

$$\ln c(y,\mathbf{w}) = \alpha_0 + \alpha_y \ln y + 0.5\, \alpha_{yy} (\ln y)^2 + \sum_{i=K,L,F} \alpha_i \ln w_i$$

$$+ 0.5 \sum_{i=j=K,L,F} \sum \alpha_{ij} \ln w_i \ln w_j + \sum_{i=K,L,F} \alpha_{yi} \ln y \ln w_i \qquad (4.17)$$

where i,j = K,L,F, c is total cost, y the level of output and wi the prices for inputs. Smolts as an input factor was excluded in this case due to the assumption of strong separability.

We use Zellner's iterative SUR method to estimate a system consisting of the cost function and two of the three cost share equations. One of the cost share equations is dropped since the cost shares sum to unity and the covariance matrix is singular. The results are invariant to which equation is dropped.

Using Shephard's lemma:

$$\frac{\delta c}{\delta w_i} = q_i , \tag{4.18}$$

where q_i is the factor demand function for the input factors, the cost share equations for the translog case can be derived by a logarithmic differentiation of the cost function:

$$\frac{\delta \ln c}{\delta \ln w_i} = \frac{\delta c}{\delta w_i} \frac{w_i}{c} = \frac{w_i q_i}{c} = m_i , \tag{4.19}$$

$$m_i(y,\mathbf{w}) = \alpha_i + \alpha_{yi} \ln y + \sum_{j=K,L,F} \alpha_{ij} \ln w_j , \tag{4.20}$$

where i,j = K,L,F. m_i is the cost share of the ith–factor. We drop arbitrarily the labour equation and thus have a system consisting of the equations for the total cost function and the cost share equations for capital and feed. We make the standard assumption of an additive error structure when transforming the system to an econometric form to fit to the SUR model.

The stochastic system consisting of the cost function and the share equations for capital and feed can be written as:

$$\ln c(y,\mathbf{w}) = \alpha_0 + \alpha_y \ln y + 0.5\, \alpha_{yy}(\ln y)^2$$

$$+ \sum_{i=F,L,F} \alpha_i \ln w_i + 0.5 \sum_{i=j=K,L,F} \sum \alpha_{ij} \ln w_i \ln w_j$$

$$+ \sum_{i=K,L,F} \alpha_{yi} \ln y \ln w_i + \epsilon_i ,$$

$$m_K = \alpha_K + \alpha_{yK} + \sum_{j=K,L,F} \alpha_{Kj} \ln w_j + \epsilon_i , \qquad (4.21)$$

$$m_F = \alpha_F + \alpha_{yF} + \sum_{j=K,L,F} \alpha_{Fj} \ln w_j + \epsilon_i ,$$

where the disturbances have a joint normal distribution, and allowing for non–zero correlations for a particular firm but zero correlation across firms, as in the profit function case.

Economic theory provides a minimum number of restrictions on the cost functions in order to represent a well–behaved technology. The cost function (4.17), which is an approximation of an arbitrary analytical cost function written as a logarithmic Taylor series expansion to the second term, is homogeneous of degree one in prices if the following restrictions on the relationships among the parameters are imposed[6]:

$$\sum_i \alpha_i = 1 ,$$

$$\sum_i \alpha_{ij} = 0 , \qquad (4.22)$$

$$\sum_i \alpha_{yi} = 0 , \qquad \forall\, i,j = K,L,F.$$

For the equality of the cross derivatives, we require the symmetry constraint:

$$\alpha_{ij} = \alpha_{ji}\,, \forall\, i \neq j. \tag{4.23}$$

For symmetry (4.23) and linear homogeneity in prices (4.22), restrictions were imposed throughout. Additional regularity conditions that must be satisfied are monotonicity and concavity in input prices. Monotonicity, i.e., the function must be an increasing function of the input prices, can be shown to be met if the fitted cost shares are positive. Concavity, to represent a convex technology, implies that the Hessian matrix, $(\delta^2 c/\delta w_i \delta w_j)$, is negative semidefinite (Caves, Christensen and Swanson [1980] and Binswanger [1974b]). The Hessian for the translog cost function defined in three prices has the following form (Antle and Aitah, 1983)[7]:

$$[H] = \begin{bmatrix} \alpha_{KK} + (S_K)^2 - S_K & \alpha_{KL} + S_K S_L & \alpha_{KF} + S_K S_F \\ \alpha_{LK} + S_L S_K & \alpha_{LL} + (S_L)^2 - S_L & \alpha_{LK} + S_L S_F \\ \alpha_{FK} + S_F S_K & \alpha_{FL} + S_F S_L & \alpha_{FF} + (S_F)^2 - S_F \end{bmatrix} \tag{4.24}$$

After these regularity conditions for duality have been statistically tested, further testing and evaluation of the underlying production technology using estimated parameters for the cost function can be undertaken. We may use the likelihood ratio test described in Section 4.2 to evaluate the different properties. All the tests of the technical relationships in the industry will be tested against the cost model with symmetry and homogeneity of degree one imposed.

The substitution possibilities in factor space, and scale effects, i.e., the spatial location of the isoquants, are two issues of interest in production technology. Empirical implementation of production and cost functions by traditional forms, i.e., Cobb–Douglas and Constant Elasticity of Substitution (CES), *a priori* impose restrictions on substitution and scale possibilities. This means that one is only capable of estimating a subset of the possible range for factor substitution and scale elasticities. The flexible functional forms such as the translog do not constrain the production technology in this sense (see Caves and Christensen [1980] and Guilkey, Lovell and Sickles [1983], for properties and performance of other flexible forms). Restrictions on elasticities can instead be tested statistically by imposing linear restrictions on the parameters of the cost function and the share equations.

Now consider the substitution possibilities. Unitary elasticity of substitution can be tested by the restriction:

$$\alpha_{ij} = 0 , \qquad \forall\, i,j \tag{4.25}$$

The Allen partial elasticity of substitution defined in equation (3.49) in Section 3.5, has the following form for the translog cost function (Berndt and Wood [1975], Binswanger [1974a], [1974b]):

$$\hat{\sigma}_{ij} = \frac{\hat{\alpha}_{ij}}{\hat{S}_i \hat{S}_j} + 1 \tag{4.26}$$

where $\hat{\sigma}_{ij}$ is a combination of parameter estimates ($\hat{\alpha}_{ij}$) and factor cost shares ($\hat{S}_i$). The Allen elasticity measure is evaluated at the means for actual (observed) cost shares (Binswanger, 1974a). The own–price elasticity has the following form:

$$\hat{\eta}_{ii} = \hat{\alpha}_{ii}/\hat{S}_i + \hat{S}_i - 1 \tag{4.27}$$

Since the single parameter estimates that constitute equations (4.26) and (4.27), have no economic meaning, statistical inferences for substitution possibilities must be based on estimated elasticity measures. Since the elasticity measures are nonlinear combinations of parameter estimates and cost shares, statistical properties of the elasticities are not well known. One way to approach this is to construct asymptotic variances or standard deviations for the elasticity estimators. However, since little is known about the real properties of the estimators, only point estimates for the parameters and standard deviations (or only parameter estimates) are reported. (Binswanger [1974b], Greene and Christiansen [1976], Antle and Aitah [1983]). Thus, one makes inferences regarding the values of substitution possibilities in the industry based only on the point estimates of the elasticities. Straightforward significance tests from the constructed asymptotic variances (although asymptotic parameters of the cost function are obtained by iterating SUR), cannot be undertaken since the derivation of the variances of σ_{ij} and σ_{ij} require that the cost shares be nonstochastic. However, the formulation of the stochastic model for estimation where cost shares were included required the assumption that they were stochastic (Kopp and Smith, 1981).

There have been some notable new developments regarding the statistical properties of translog elasticity of substitution estimators. Using Monte Carlo experiments, Anderson and Thursby (1986) evaluated properties of elasticity estimators with different assumptions of factor cost shares with different assumptions of the cost shares and distributions. They conclude that a normal distribution for the Allen elasticity estimator is appropriate if the estimator uses means of actual (observed) shares. Furthermore, their examination of empirical results suggests that inferences regarding values of substitution elasticities can not be derived from point estimates alone; the proper confidence intervals that they define contain both negative and positive values of the elasticities. Hence, the upper and lower bounds of the confidence interval give conflicting information whether factors are substitutes (positive values) or complements (negative values).

Anderson and Thursby's approach appears to be preferable given a translog context, and we will adopt their procedure for constructing confidence intervals. Hence, we can test whether the confidence intervals support or contradict the calculated estimates of the substitution elasticities given in equation (4.26).

The focus of our study is to provide information on the scale properties of the fish farming industry. Thus, we are interested in testing restrictions such as homotheticity and homogeneity. If the technology is homothetic in the inputs, the scale elasticity will be independent of the marginal rates of substitution. It implies that all isoquants have the same shape as the unit isoquant. This implies that output is expanding in a Hicks neutral manner, i.e., at a constant factor ratio. The general cost function can then be written as a separable function in output and factor prices. From equation (4.17) we see that if:

$$\alpha_{yi} = 0 , \qquad \forall\, i, \tag{4.28}$$

the translog function can be decomposed in this manner, and (4.28) is then the restriction to test for homotheticity.

The technology is homogeneous in all inputs if the returns to scale are independent of the output level. Homogeneity requires then that the following restrictions are imposed in terms of equation (4.17):

$$\alpha_{yi} = 0 , \quad \forall\, i, \quad \alpha_{yy} = 0. \tag{4.29}$$

Given that our aim is to analyze scale properties, the following three models will be considered:

(i) Model I. No restrictions imposed, i.e., a complete translog function. In terms of scale economies we have a nonhomothetic function.

(ii) Model II. Restriction on returns to scale to a homothetic function ($\alpha_{yi} = 0, \forall\, i$).

(iii) Model III. Restrictions on return to scale to a homogeneous function ($\alpha_{yi} = 0, \forall\, i, \alpha_{yy} = 0$).

The respective formulas for the elasticity of scale of the three models as defined by the relationship between total cost and output along the expansion path are (Hanoch, 1975):

$$\text{(i)}\quad \varepsilon_1(y) = 1/\frac{\text{dlnc}}{\text{dlny}} = (\alpha_y + \alpha_{yy}\text{lny} + \sum_i \alpha_{iy}\text{lnw}_i)^{-1},$$

$$\text{(ii)}\quad \varepsilon_2(y) = 1/\frac{\text{dlnc}}{\text{dlny}} = (\alpha_y + \alpha_{yy}\text{lny})^{-1}, \tag{4.30}$$

$$\text{(iii)}\quad \varepsilon_3 = 1/\frac{\text{dlnc}}{\text{dlny}} = \frac{1}{\alpha_y},$$

where i = K,L,F.

We notice that the models from (iii)–(i) are in increasing order of complexity according to returns to scale. The homogeneous function may indicate scale economies if it is greater than one. The next model is more interesting in this respect since we are able to show the possible variation in the scale elasticity with the size of the farms, and to detect if the scale economies are exhausted, i.e., if $\varepsilon(y)$ approaches unity within our sample.

Model one allows the opportunity to test whether expansion in this industry is non—neutral in character. Thus, the input ratios are dependent on the level of output.

A priori we expect a better fit to the underlying data the more flexible the models. Also, the additional information gained by defining the level of output in value terms as was properly defined in Section 4.1.2, may provide a better specification. The two empirical models and different elasticities has now been specified. We now turn to the presentation of results from the estimated models.

FOOTNOTES

1) The publication is: *Lønsemdundersøkingar for fiskeoppdrettsanlegg 1982 og 1983* (in Norwegian). The survey, furthermore, provides information for the Norwegian Central Bureau of Statistics for its annual publications *Statistical Yearbook* and *Salmon and Trout Fisheries in Norway.*

2) We could have used specific indices for different inputs and outputs, but for those items for which the specific indices were available differed very little.

3) Since output is a part of the unit feed price, multicolinearity could be introduced in regressions of the cost functions where output is a right–hand size variable. However, in the context of the translog flexible functional form we are not interested in the separate effects of each factor, only elasticities consisting of combinations of parameters.

4) Other flexible functional forms are, for instance, generalized quadratic (Denny, 1974) and generalized Leontief (Diewert,1971). In Lovell, Guilkey and Sickless (1983), these three functional forms are evaluated by Monte Carlo techniques. Their results showed that translog was preferred in possessing the properties of the underlying technology in many cases, and this is one of the reasons for applying this functional form in our case.

5) Instead of this conventional procedure for stochastic specification, a new method has been suggested lately by McElroy (1987). She suggests that instead of first specifying a deterministic production model and then the stochastic system of cost functions and share equations by adding error terms, that the error specification should be an integral part of the optimization problem. She refers to this general procedure as a general error model (GEM). By estimating earlier work through the use of GEM specification, McElroy states that the new method is preferred.

6) See Christensen et al. (1973) for derivation of these restrictions.

7) This matrix is equivalent to the matrix given in Antle and Aitah (1983) with the exception that we had to include the shares since we did not define that data matrix in mean values.

5. EMPIRICAL RESULTS

In this chapter, estimated parameters are reported for the models specified in Sections (4.2.1) and (4.2.2). In Section 5.1 the empirical results for the translog cost function are presented. To determine the technical structure of the Norwegian salmon industry, statistical tests for scale economies and substitution possibilities are undertaken. Furthermore, estimates for both scale and substitution elasticities are presented and discussed.

In Section 5.2, estimates using the restricted translog profit function are presented and discussed. Measures of elasticities of substitution and returns to variable factors are provided. Section 5.3 compares the overall empirical fit to the underlying data for the profit function and the cost function. The elasticities of choice and scale are also contrasted and discussed. Moreover, the extent of the possible specification bias is discussed.

5.1 Empirical results of the translog cost function

The estimated parameters for the three specifications of the translog cost function using Zellner's seemingly unrelated procedure (SUR) are reported in Table 5.1. The models were estimated by imposing constraints for linear homogeneity in factor prices and symmetry. We present two versions of the complete translog cost function: Model I.1, where output is defined in physical units, and Model I, where output is defined in value terms to correct for the fact that many farms produce both salmon and trout. In models II and III, output is defined in value terms.

Theory imposes no *a priori* restrictions on the parameter estimates of the first– and second–order logarithmic derivatives of the cost function. Thus, the validation of the model must rest on overall goodness–of–fit; significance of parameter estimates, whether the estimated cost function conforms to regulatory conditions of duality, and whether own–price elasticities given in equation (4.27) have the correct sign.

Table 5.1: Cost function estimates based on 1982 and 1983 data.[1) 2)]

	Model			
Parameters	I.1	I	II	III
α_0	2.561 (2.04)	1.788 (0.41)	2.818 (2.61)	1.009 (2.62)
α_Y	0.796 (5.82)	0.407 (1.55)	0.177 (0.58)	0.848 (49.04)
α_{YY}	0.026 (3.02)	0.054 (2.40)	0.047 (2.19)	–
α_K	0.324 (1.54)	–0.092 (–0.13)	0.665 (7.98)	0.663 (7.90)
α_L	0.784 (4.31)	0.966 (1.51)	0.178 (1.71)	0.169 (1.62)
α_F	–0.108 (–0.84)	0.122 (0.31)	0.157 (2.48)	0.167 (2.64)
α_{KK}	0.073 (5.32)	0.070 (4.89)	0.087 (7.07)	0.089 (7.20)
α_{KL}	–0.012 (–1.10)	–0.025 (2.18)	–0.016 (–1.39)	–0.017 (–1.48)
α_{KF}	–0.061 (–6.81)	–0.045 (–4.47)	–0.071 (–7.36)	–0.072 (–7.42)
α_{LL}	0.115 (9.30)	0.106 (8.26)	0.031 (1.39)	0.033 (1.48)
α_{LF}	–0.103 (–12.10)	–0.081 (–8.59)	–0.016 (–1.39)	–0.017 (–1.48)
α_{FF}	0.164 (16.21)	0.126 (9.68)	0.087 (7.07)	0.089 (7.20)
α_{YK}	0.015 (0.84)	0.048 (1.51)	–	–
α_{YL}	–0.074 (–4.67)	–0.068 (–1.51)	–	–
α_{YF}	0.059 (6.01)	0.019 (0.71)	–	–
$\bar{R}^2$	0.92	0.93	0.91	0.91

1) Asymptotic t–ratios are given in parenthesis.

2) $\bar{R}^2$ is the multiple correlation coefficient for the cost function equation corrected for degrees of freedom.

The parameter estimates reported in Table 5.1 for Model I and Model II needed six iterations for convergence. The asymptotic t–values are given in the parenthesis. About eighty per cent of the parameter estimates in the different model specifications were significantly different from zero at the 95 percent confidence level. The multiple correlation coefficient, corrected for degrees of freedom, ranges from 0.91 to 0.93, and provide a strong goodness–of–fit measure given cross–sectional data. We used a Chow test for testing the structural stability over the sample period, i.e., between 1982 and 1983. The calculated F–statistics (F = 0.42) indicate that the null hypothesis of structural difference can be rejected.[1] This implies the structure of production is stable over the sample period and permits pooling of the data for the two years.

In addition to the statistical results, which provide a good validation of the translog cost function for the Norwegian fish farming industry, we must ensure that the conditions for duality as outlined in Sections (3.4) and (4.3) are satisfied. The symmetry constraints in (4.23) and the homogeneity constraints in (4.22) can be tested by performing likelihood ratio tests as described in Section 4. However, we could not invert the data matrix for the complete unconstrained model, and hence could not test the symmetry condition. Multicollinearity among right–hand side variables – all defined as combinations of factor prices either in logarithms of single factor prices of the first or second term, cross prices, or output and prices – precluded an inverse of the matrix. However, we tested homogeneity of degree one against the model with symmetry imposed. The hypothesis that the cost function is homogeneous of degree one in prices was not rejected (the likelihood ratio is 5.59 and the critical χ^2 value is 9.2 at the 1% level). Hence, we imposed the homogeneity and symmetry restrictions on the cost model for further testing.[2]

Monotonicity and convexity cannot be summarized by linear restrictions on the parameters. Instead, these properties have to be tested on the estimated functions. The predicted share equations were positive at every observation point, i.e., the monotonicity condition is satisfied. The Hessian matrix was negative semidefinite at the point of expansion of the translog cost functions, implying convex technology. We used the principal minors of the Hessian matrix to determine the definiteness of matrix. The necessary and sufficient conditions for a three by three matrix to be negative semidefinite are (Chiang, 1982, p. 336)

$$D_1 \leq 0 ,$$
$$D_2 \geq 0 \text{ (given that } D_1 \leq 0 \text{ already) ,}$$
$$D_3 \leq 0 \text{ (given that } D_2 \geq 0 \text{ already) ,}$$

where D_i, i = 1,2,3, refers to the principal minors. In Table 5.2 the elements of the Hessian are reported as calculated by using equation (4.24) in Chapter 4.

Table 5.2: Elements of the Hessian matrix for the cost function.

	Capital	Labour	Feed
Capital	–0.876	0.106	–0.005
Labour	0.106	–0.609	0.029
Feed	–0.005	0.029	–0.774
	Principal minors		
$D_1 = -0.87601$	$D_2 = 0.5220$		$D_3 = -0.4035$

Many farms produce both salmon and trout. Hence, on *a priori* grounds we prefer to define the level of output in value terms to correct for this fact. We present the results for the cost function where output is defined in physical units in Model I.1 and in value terms in Model I. The results are very similar; however, Model I provides a slightly better overall fit as the $\bar{R}^2$ has risen to 0.93.

This improved fit for Model I, and also the fact that it provides a higher scale elasticity, may indicate the presence of economies of scope, i.e., the ability of firms producing two products (salmon and trout) to pool inputs and produce more efficiently. However, since some of the firms produce only salmon and others only trout, and if the tendency to produce only one of these species is correlated with size, this can also yield the above result if there are differences in the costs for producing these two species.

We performed likelihood ratio tests to test our different restrictions. The number of degrees of freedom is equal to the restrictions imposed by the null hypothesis (Theil, 1971). Table 5.3 below shows the likelihood ratio statistics for the hypothesis of homogeneity and homotheticity for Models I and I.1. In addition, we tested for unitary elasticity of substitution.

Table 5.3: Test statistics for various restrictions.

	Homotheticity	Homogeneity	Unitary Elasticities of Substitution
Model I	27.10	31.81	109.01
Model I.1	80.30	87.99	224.76
Critical χ^2 values (1%)	9.2	11.34	11.34
Number of restriction	2	3	3

Table 5.3 reports that the null hypothesis of unitary elasticities of substitution in both models is rejected at the 1% level. That is, there are substitution possibilities beyond those implied by a Cobb–Douglas technology in fish farming at the firm level, and modeling the production structure should allow for these possibilities. The estimated Chi–square values corresponding to the null hypothesis of homogeneity and homotheticity are both significant, and hence the null hypotheses are rejected for Models I.1

and I. From these tests we can conclude that a nonhomothetic and thus nonhomogeneous cost function best corresponds to the production structure for Norwegian salmon farming.

5.1.1 Factor demand properties

Table 5.4 below reports estimates of factor demand with respect to own–price. The mean values of the actual cost shares in the industry are used when computing the measures defined by equation (4.27).

Table 5.4: Estimated demand elasticities based on translog cost function.[1]

	Capital	Labour	Feed
Elasticities of factor demand	–0.456	–0.318	–0.227

1) Elasticities are calculated at mean values of the actual shares.

The demand elasticities have the correct sign suggested *a priori* from economic theory. Thus, the model is also valid in the sense that the signs of the own–price demand elasticity are consistent with the implications of cost minimization and market behaviour. However, we see that the absolute values are less than one, implying that the farmers are relatively insensitive with respect to price changes. It is interesting to note that this is particularly true for the feed input, indicating a very stable pattern of feeding by farmers.

In Table 5.5 the point estimates for the Allen elasticities of substitution are reported. They were calculated by employing equation (4.26). In addition, the confidence intervals were derived from Anderson and Thursby's study.[3]

Table 5.5: Point estimates and confidence intervals for Allen partial elasticities of substitution.[1]

	Estimated value	Interval lower	upper
Capital–labour	0.43	0.38	0.48
Capital–feed	0.61	0.43	0.79
Labour–feed	0.37	–0.98	1.71

1) Elasticity values have been calculated from mean actual cost shares.

From Table 5.5 we notice that in this model specification, capital–labour, capital–feed and labour–feed are all substitutes. *A priori* we would expect capital and labour to be substitutes, i.e., we would expect an increase in the wage rate to provide an incentive for the fish farmer to substitute capital for labour. To be specific, if the relative price of automatic feeders dropped compared to labour, we expect that it should be possible for a farmer to employ a higher proportion of automatic feeders. Furthermore, we would expect capital and feed to be substitutes *a priori* since it may be argued that automatic feeders probably allows conservation of feed also. The results of the cost function estimation support this relationship. However, one would expect labour and feed to be complements and not substitutes as indicated from the estimated value in Table 5.5. This is an implication of the above discussion that capital in the form of automatic feeders both could save labour and feed for the farmer. Given the confidence interval (in Table 5.5) around the point estimate for labour–feed, we see that both substitution and complementary properties exist in this range. One rationale for this result is that one method of husbandry in the industry has not yet been established and different techniques and combinations of methods for feeding are still utilized. Automatic feeding combined with complementary hand feeding might be one method of husbandry reflected by the results given in Table 5.5. Furthermore, from Table 4.5 in

Chapter four we note also that a high cost share of labour did not necessarily imply a high cost share of feed.

5.1.2 The Scale Properties

Nonhomogeneity

In order to investigate the scale properties in more detail in addition to Model I, we also report the results for Models I and II, using output defined in value terms in Table 5.1.

The estimated values for Model III, i.e., the homogeneous case, indicate significant increasing returns to scale with $\varepsilon = 1.18$ (Nerlove, 1963). When scale is allowed to vary with output in Models I and II, more interesting results appear.

A convenient way to summarize the scale properties of the nonhomogeneity property of the fish farming industry found in the three model specifications, is to graph the returns to scale functions for the models. That is, we graph the scale elasticity within the range of the observed production values (see equation 4.30) using the following functions:

(i) Model I

$$\varepsilon_3(y) = (\alpha_y + \alpha_{yy}\ln y + \sum_i \alpha_{yi}\ln w_i)^{-1}$$
$$= (0.407 + 0.054\ln y + \sum_i \alpha_{yi}\ln w_i)^{-1}$$

(ii) Model II

$$\varepsilon_2(y) = (\alpha_y + \alpha_{yy}\ln y)^{-1} = (0.177 + 0.0471\ \ln y)^{-1} - 1$$

(iii) Model III[4]

$$\varepsilon_1(y) = (\alpha_y)^{-1} = (0.848)^{-1}$$

where i = K,L,F.

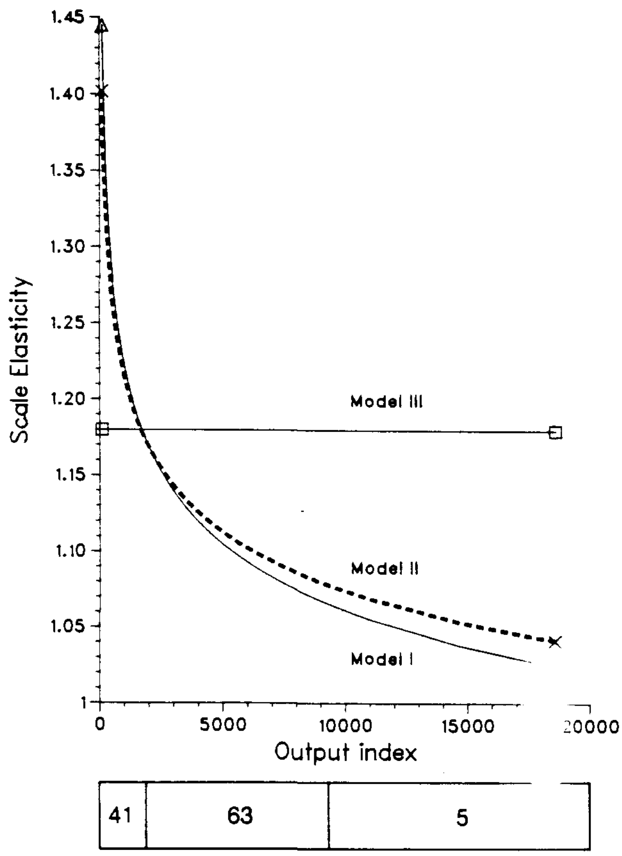


Figure 5.1: The returns to scale function for Models I,II and III, and the size distribution of firms in 1983.

The output index in Figure 5.1 is output in value terms as defined in Section 4.1.2. Figure 5.1 shows that the scale elasticity for the homogeneous case, which can be interpreted as the "average" level of scale elasticity, disguises the variation in the returns to scale over the sampled firms. We also note that Models I and II, which allow for variations in scale, graph approximately the same curve for scale economies. These results appear to confirm our *a priori* expectations of first increasing returns to scale, followed by a portion of the curve which is flat and approaching $\varepsilon(y) = 1$ with no statistically significant returns to scale. Where the scale elasticity has the size of

$\varepsilon = 1.07$ we can no longer reject the null hypothesis of constant returns to scale ($t = 2.10$) at the 95 percent confidence level with a two–tailed test. Hence, the flat portion of the curve indicates that the scale economies in the Norwegian fish farming industry are exhausted within the range of our sample. However, we are not able to detect any diseconomies of scale where the scale elasticity is below one.

To illustrate the potential of unexploited economies of scale, we present the size distribution of firms in 1983 beneath the returns to scale curves. This comparison reveals that a large portion of the plants are operating in the size interval where the returns to scale are significantly greater than one. Hence, gains exist from expanding production as interpreted by lower average production costs. However, our results also indicate a limit to these gains since the flat portion of the curve indicates constant returns to scale within the range of our sample size. Although no data are available, it is likely that location is the restricting factor providing constant returns to scale for the largest farms.

Nonhomotheticity

The nonhomotheticity hypothesis is supported by the test statistics in Table 5.3. This implies that, for given factor price ratios, an increase in output changes the ratio of factor inputs (or factor shares). Hence, there is a non–neutral expansion with scale increases, implying differences in relative factor intensities between small and large firms. This translog result provides a great deal of information on the structure of Norwegian salmon farming and has a strong, intuitive economic appeal.

We will put forward two hypotheses to explain the existence of nonhomotheticity in the Norwegian fish farming industry. Due to indivisibilities in capital and the high fixed costs for specialized harvesting and feeding equipment, some processes are only feasible for firms with high output levels. This leads to rising capital intensity with volume. An indication of this is seen from an inspection of Table 4.5 in Chapter four, where large farms (an average of farms above the mean size of the total sample) have a slightly higher capital–labour ratio than small farms (an average of farms below mean size). However, by splitting the sample into further subgroups, a stronger impression of difference in relative use of inputs is revealed. The capital–labour ratios for the average of the ten largest and ten smallest farms are presented in Table 5.6.

Table 5.6 : Capital–labour ratio for size distribution of farms.

Average of the ten largest farms	Mean farm	Average of the ten smallest farms
1.28	0.84	0.57

The inputs of capital and labour are measured as annual costs, as defined in Chapter four. From Table 5.6 we notice there is a difference in relative use of inputs, indicating nonhomotheticity, i.e., the capital–labour ratio is higher for large firms.

Another and perhaps more interesting explanation is the possible relationship between the current measured nonhomotheticity and the difference in the relative factor prices for big and small firms.[5)] That is, the differences in the relative factor intensities – nonhomotheticity – may also be explained by a change in relative factor prices that firms faced as they came into existence. More importantly, this substitution allowed non–neutral expansion as the scale varied. The two explanations for nonhomothetic production technology are illustrated in Figure 5.2 below.

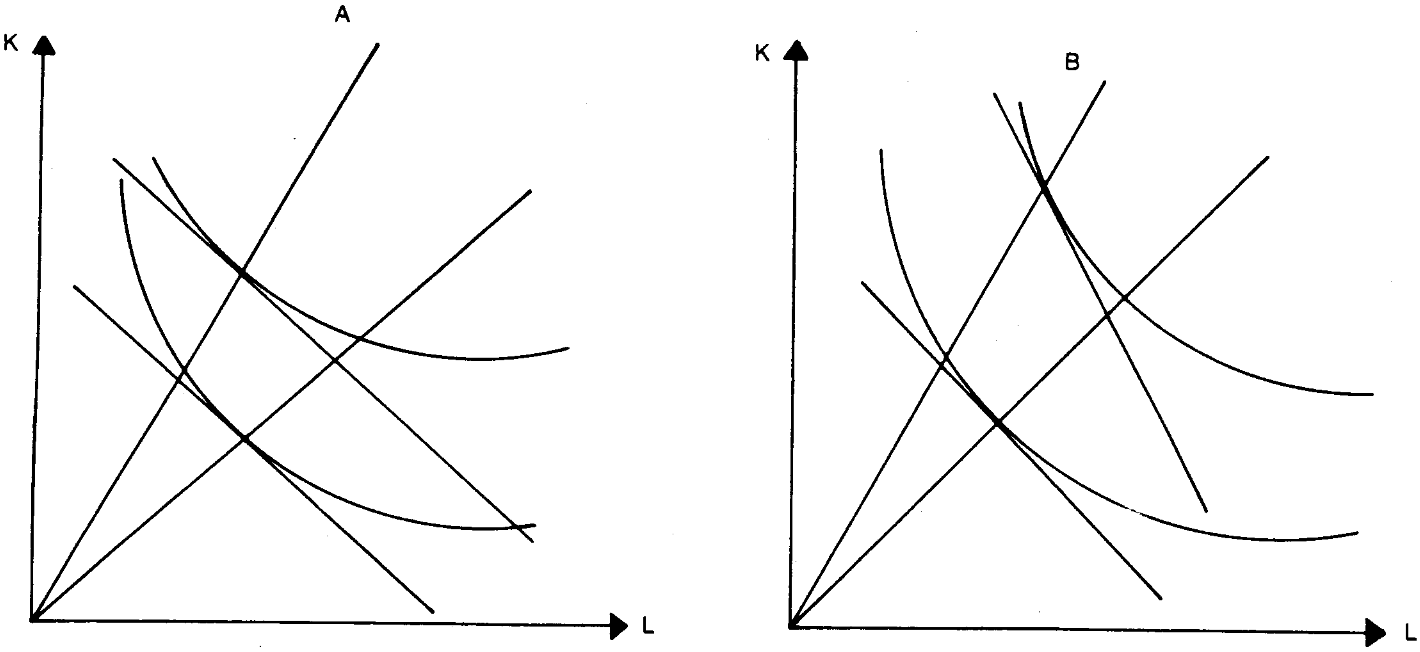


Figure 5.2: Nonhomotheticity for the Norwegian Fish Farming Industry explained by indivisibilities (A) and difference in relative factor prices between big and small plants (B).

Since we use cross–sectional data and therefore expect every plant to confront the same factor prices if the markets are perfect, the only source of variation in relative prices is separated factor markets. Therefore, to explain difference in relative factor prices one must investigate the institutional framework of Norwegian salmon farming. Imperfections exist in the labour market and in the market (or nonexistence of markets) for production capacity, i.e., "salmon–rearing water." The second imperfection is due to the size regulation.

The imperfection in the labour market is in terms of a dual labour market. Smaller firms are family–based production units where family members work for a wage lower than the market rate. Larger fish farms must hire labour in the ordinary labour market. Hence, bigger firms face a higher relative price for labour and may substitute capital, e.g. automatic feeding, alarm systems, and harvesting equipment, for labour as size expands. Thus, the necessary factor price disparities exist to allow substitution possibilities in a manner which, in our model, appear as nonhomotheticity.

The argument for the relative scarcity of production capacity for small farms compared to bigger farms due to regulations, points in the same direction as above. It is intuitively reasonable to argue that small farms, being restricted in production capacity in terms of regulated pen volume, tend to use more inputs which they are free to hire, for instance labour.[6)]

This finding, that small fish farms using variable factors (particularly labour) more intensively than bigger farms, is also an important characteristic for the traditional farming sector in developing countries (Khan, 1977). In that case land, is the scarce factor for small farms, and farmers compensate by using fertilizer and labour. The similarity with our results is obvious and is due to the small farms' relative scarcity of production capacity in terms of pen volume, and points to the generality of our results.

5.2 Estimates for the restricted translog profit function

In Table 5.7 the estimated parameters of the restricted translog profit function are given. The coefficients represent the maintained hypothesis of symmetry and homogeneity of degree one in prices. Thirteen iterations were necessary for convergence. The adjusted multiple correlation coefficient for the profit function equalled 0.68, which is a good fit in estimations based on cross–sectional data.

Table 5.7: Seemingly unrelated regression estimates of the translog profit function.[1) 2)]

Parameters		Parameters	
α_0	3.753 (1.91)		
α_P	5.736 (5.42)	α_{FF}	-1.055 (-7.63)
α_{PP}	-2.439 (-5.59)	α_{KP}	0.419 (3.63)
α_K	-2.021 (-2.42)	α_{LP}	0.555 (2.84)
α_L	-2.805 (-2.00)	α_{FP}	1.466 (6.50)
α_F	-7.485 (-1.68)	α_S	0.312 (-1.05)
α_{KK}	-0.1536 (-1.90)	α_{SS}	0.030 (0.60)
α_{KL}	-0.818 (-1.35)	α_{PS}	-0.100 (0.68)
α_{KF}	-0.184 (3.04)	α_{KS}	0.044 (-1.31)
α_{LL}	-0.246 (-2.12)	α_{LS}	0.063 (0.98)
α_{LF}	-0.228 (-1.99)	α_{FS}	-0.119 (1.37)

$\bar{R}^2 = 0.68$

1) Asymptotic t–ratios are given in the parenthesis.
2) $\bar{R}^2$ is the multiple correlation coefficient for the profit function corrected for the degrees of freedom.

We notice that when the initial stock of smolts variable is included, that only half of the estimated coefficients are significantly different from zero. However, in terms of statistical significance of price parameters, the econometric profit function specification is supported by the fact that 78 percent of the parameters where prices are involved are significant at the 95 percent confidence level. The same Chow test used in the cost function case was conducted to test for structural stability between the two periods. For the profit function, a stable production structure between 1982 and 1983 also was supported. The calculated F–value was F = 0.40, which again permitted pooling of the data for the two years.

We continue to evaluate the restricted translog profit function in terms of the regularity conditions for duality as in Section 4.2. As with the translog cost function, we could not test for the symmetry condition since estimation of the unrestricted model, i.e., with symmetry not maintained, could not be undertaken due to difficulties in inverting the data matrix. However, homogeneity of degree one in prices (and of degree zero for the share equations) was tested against the profit function with symmetry imposed.

Table 5.8: Test statistics for various restrictions on the translog profit function.

	Homogeneity of degree one in prices	Homotheticity	Homogeneity
Observed χ^2 values	6.30	29.42	110.0
Critical χ^2 values (1%)	9.2	9.2	11.34
Number of restrictions	2	2	3

From Table 5.8 we notice that the reported likelihood ratio test does not reject the null hypothesis of homogeneity of degree one in prices for the profit function. Hence, for further testing, homogeneity of degree one and symmetry were imposed on the profit function.

When we tested the monotonicity condition, the predicted expenditure shares were negative and the revenue share was positive. Hence, the monotonicity condition is satisfied. By employing the principal minors of the Hessian matrix given by equation (4.12), we tested to determine whether the profit function is convex in prices. Table 5.9 reports the elements of the Hessian matrix at the point of expansion, and the corresponding principal minors.

$D_1 \geq 0$

$D_2 \geq 0$ (given that $D_1 \geq 0$ already)

$D_3 \geq 0$ (given that $D_2 \geq 0$ already)

$D_4 \geq 0$ (given that $D_3 \geq 0$ already)

Table 5.9: Elements of the Hessian matrix of the restricted translog profit function.

	Capital	Labour	Feed	Output
Capital	2.22	1.38	6.18	–10.53
Labour	1.38	5.57	10.93	–18.64
Feed	6.18	10.93	36.95	–54.13
Output	–10.57	–18.64	–54.13	83.42

Principal minors

$D_1 = 2.22$	$D_2 = 10.46$	$D_3 = 74.25$	$D_4 = 562.83$

From the values of the calculated principal minors, we conclude that the Hessian of the restricted profit function is positive semidefinite (Chiang, 1984, p. 336).

Before proceeding to evaluate the scale and factor demand properties, we will conclude this section by stating that the validation of the restricted profit function for the fish farming industry is reasonably good in terms of general goodness of fit, significance of parameter estimates and regulatory conditions of duality. However, none of the coefficients where smolt as a fixed factor were involved is significantly different from zero.

5.2.1 Factor demand properties

Table 5.10 reports elasticities for factor demand. We used the sample means of the fitted revenue and cost shares when calculating the elasticities. Furthermore, since the elasticities are nonlinear combinations of the estimated parameters and the fitted shares, and their properties are not well known or tested as in the translog cost function case, we do not report the standard errors (Weaver,1983).[7]

Table 5.10: Input demand elasticities based on restricted translog profit function.[1]

	Inputs		
Prices Smolt input	Capital	Labour	Feed
Capital	–1.98	1.54	1.15
Labour	2.69	–2.80	2.00
Feed	5.81	5.76	–6.49
Output	10.14	10.05	10.02
Smolt input	0.81	0.96	0.91

1) Elasticities are calculated at the means of fitted shares.

The following properties of the elasticities can be seen: the own–price input demand elasticities (i.e., the diagonal elements) have the correct sign according to profit maximization behaviour, and they are nonincreasing in their own prices. Furthermore, all the own–price demand elasticities are greater than one in absolute value, implying that they are price elastic and thus quite responsive to changes in input prices.

We find considerable substitutability among the variable inputs. From the cross–price elasticities we notice that capital–labour, capital–feed and labour–feed are substitutes, given their positive elasticities. From the values of the elasticities, we conclude that a one percent increase in the price of labour increases the demand for capital by 2.69 percent. Also a one percent increase in the price of capital increases demand for labour by 1.54 percent. Hence, the salmon industry for 1982/1983 was moving towards more capital–intensive farms. This is a tendency which has been observed in the industry.

From the results of the demand elasticity of inputs with respect to output price changes, we can classify the inputs as superior ($\eta_{ip} > 1$), normal ($1 < \eta_{ip} < 0$) or inferior ($\eta_{ip} < 0$), according to the value of this elasticity. Our results indicate that capital, labour and feed are all superior inputs for Norwegian salmon farms.

Furthermore, we notice that the fixed variable, smolts, appears to have an impact on the demand for the variable factors. Expansion of smolt increases the demand for all variable inputs in a uniform way since the elasticities are close in value. However, as seen from Table 5.7, where the results of the profit function are presented, all the coefficients where input of smolts are included are not significantly different from zero. Hence, since these coefficients are important in the definition of the elasticity measure of the effect of smolt on the demand for variable inputs, as seen from equation (4.16) in Chapter four, we are not able to draw strong conclusions in this case.

5.2.2 Scale properties

From test statistics of Table 5.8 we notice that the null hypothesis of homotheticity and homogeneity are both rejected at the one percent significance level. Hence, the short–run returns to scale measure will vary over the production surface. By using the following parameter estimates and share equations and inserting them into equation (4.16), we get the following scale function:

$$\varepsilon(y) = 1 - (1/\hat{R})$$

In Table 5.11 we report the distribution of the "returns to scale" measure which is dependent on firm size (i.e., output level) calculated at the mean farm size in addition to the minimum and maximum farm size.

Table 5.11: Short–run returns to scale based on the translog restricted profit function

Mean	Minimum	Maximum
0.88	0.83	0.91

We note from Table 5.11 that short–run returns to scale measure for the variable factors less than one as expected from the theory, and are quite close in values for minimum mean and maximum size of the farm. However, the reported elasticity measure will vary as a function of farm size since the estimated profit function is non-homogeneous. From these results we may conclude that profit cannot be increased by expanding the level of production in the short run when smolt is a fixed factor. The result is actually obvious since it would be difficult to image an increase in profits from producing fish when the number of fish is given.

5.3 Comparison of the cost function and profit function approach in terms of general fit

The results for both the cost function and the profit function specifications are satisfactory in terms of fit and in terms of satisfying the duality properties. Thus, the models fit well given the technology and economic performance of the fish farming industry as revealed by the underlying data. Furthermore, it appears that the cost function approach is the preferred specification for the reasons outlined below. First, the cost function provides a better general fit to the data in terms of a higher $\bar{R}^2$ (corrected for degrees of freedom). Second, the price elasticities derived from the parameter estimates of the translog cost function compared to the comparable elasticities from the profit function are more reasonable in value. Furthermore, the cost function provides us with a long–run measure for the scale elasticity. Finally, given our objective of analyzing the effects of size regulations, this scale measure derived from the long run cost function approach is highly desirable.

We conclude this chapter by addressing the possible specification bias inherent in the cost and profit function specifications. When omitting pen volume as a given factor of the production function (due to data limitations), the mathematical formulation of the specification bias in our case indicated that the elasticity of scale may be overestimated. This finding derived from the production function should also be valid for the cost and profit functions, since they represent the production process of fish farming in the dual forms to the production function. We will evaluate this bias only in terms of the cost function approach here, since the cost function yielded the long–run scale elasticity measure. First, we will illustrate the direction of the specification bias in the cost function case. Then, the extent of the bias will be discussed by contrasting the result of scale elasticity given in Table 5.1 to results from a cost function estimation where data on capacity restriction were available.

The origins of the specification bias can be stated as follows. Due to inherent pen volume limits caused by different levels of size regulation depending on the time of receiving licence, it is possible to argue that we do not have a proper long–run cost curve as an envelope to successive short–run curves. In other words, given a volume constraint, individual farms will adjust along their short–run cost curves as they expand their quantity of production. Firms will continue this expansion along their short–run cost curves until each size–group equalizes its marginal cost and marginal revenue. This can be illustrated in Figure 5.3 below.

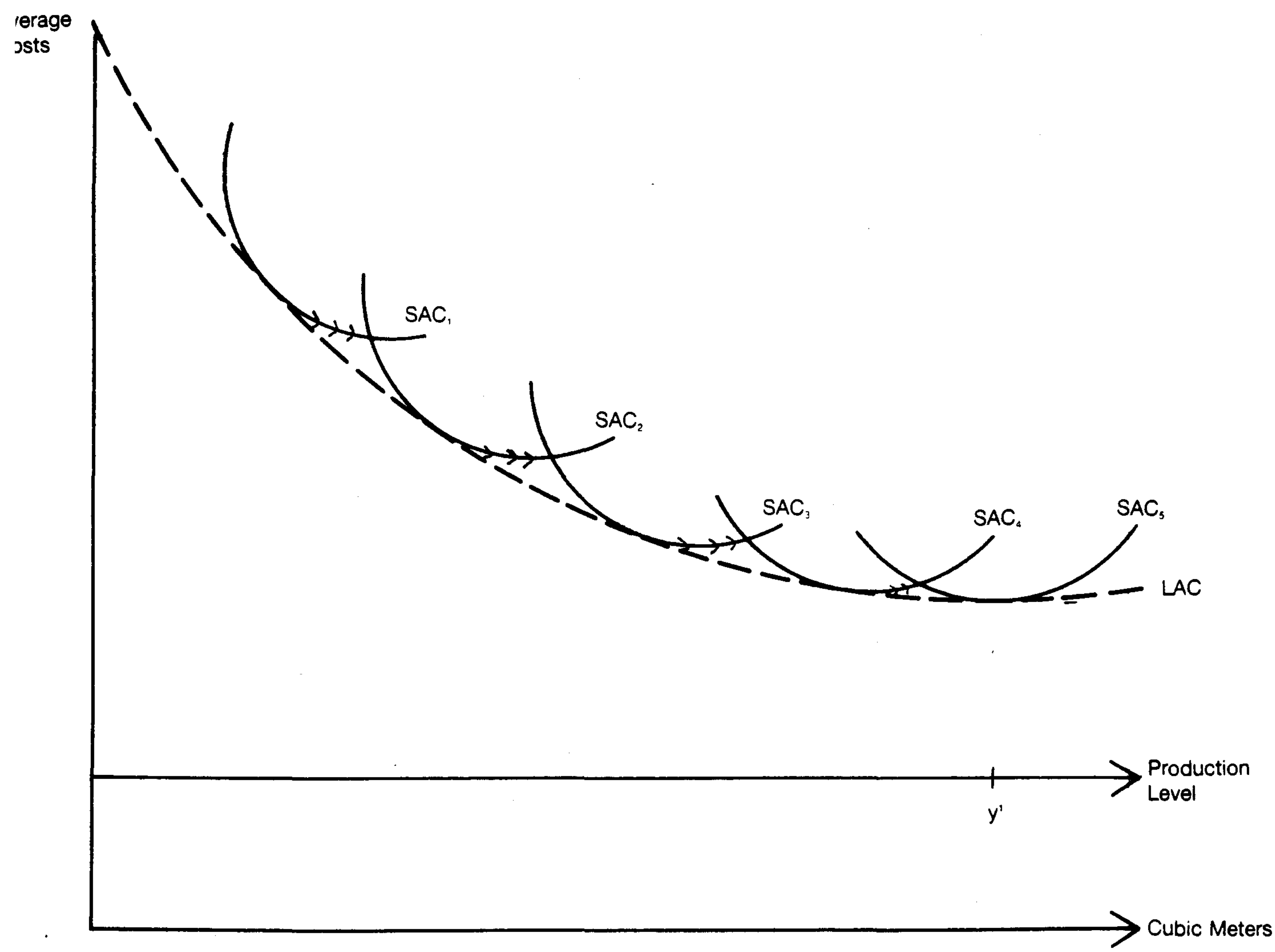


Figure 5.3: Illustration of possible estimation bias in the long–run cost function.

Given plant size, firms are adjusting along the denoted arrow portions of short–run curves (SAC) and of course different curves appear for each sub–group due to different given pen volumes. Thus, when estimating the long–run cost curve (LAC) we actually fit a line which lies everywhere above the LAC curve for the plants smaller than the minimum of the long–run average cost in y'. Hence, to the extent that firms in scales 1–4 operate to the right side of the tangency points of the SAC curves, we get a steeper average cost curve thus overestimating the scale effect. The scale elasticity has an upward bias and we overestimate the scale economies.

As long as some input is not infinitely divisible we will in every industry observe firms adjusting along their short–run cost curves. However, unless they are regulated in the use of one factor as in our case, the firms will switch from one short–run curve to another where the short–run curves intersect. In our particular case the farms might adjust even beyond the switching points (intersections of the SAC curves) since the use of pen volume is restricted. The extent to which this happens would be possible to measure if we had data to calculate the short–run cost curves for each size group in pen volume or production level. However, an indication that firms actually adjust beyond the switching points is that they have high capacity utilization in terms of percentage use of licensed pen volume; 94 and 98.5 percent for the 1982 and 1983 samples, respectively.

The explanation given above of the fish farms' adjustment when the pen size is restricted, points to a different interpretation of the difference in average costs between small and large farms. Some of the difference in cost efficiency is due to scale economies as interpreted in the concept of scale elasticity. However, in our case some of the difference in production costs between small and large farms is also due to the regulation of pen volume as an input factor. The regulation of pen volume causes the restricted farms to adjust along their short–run cost curves instead of along the most cost efficient long–run cost curve. Hence, our result concerning increasing returns to scale – which is an overestimate of the pure measure of scale economies – is probably a result of the combined effects of size regulation of the industry and of scale economies. However, the form of the data set precludes separating the different aspects of cost difference due to scale.

Furthermore, the next question to pose is: How important is this bias? In an earlier work, using only the 1982 data set, capacity data was available. Thus, we estimated a Cobb–Douglas cost function introducing the capacity restriction in two different ways (Salvanes, 1985). First we estimated segments of the long–run cost function by splitting the sample into subsamples due to regulated volume constraints. Then, we estimated a restricted short–run cost function utilizing size capacity as a given factor. From this specification we were able to derive and test the long–run elasticity of scale measure since we used the simple Cobb–Douglas functional form. In both cases we obtained results which could not be significantly distinguished from a an unrestricted Cobb–Douglas cost function (Salvanes, 1985). This comparison of the restricted and unrestricted results from different specified cost functions should not be taken as proof of the unimportance of the bias since we are of course contrasting two different models. However, it gives an indication of the extent of the bias in our case.

FOOTNOTES

1) The critical value is $F = 2.04$ for $v_1 = 15$ and $v_2 = 171$ degrees of freedom at the 95 percent confidence level.

2) It is not unusual for problems to arise in testing some of the duality properties. The reason may be difficulties in inverting the data matrix as in our case or too few degrees of freedom to estimate the unconstrained model (Weaver [1983], Binswanger [1974b]). Since these properties must be assumed for a well–defined technology – and a translog cost function is an approximation to the true function and thus does not necessarily globally satisfy these properties – one has to impose them for further testing (Weaver, 1983).

3) We used the following formulas developed in Anderson and Thursby (1986) for deriving the confidence intervals:

$$\hat{\sigma}_{ij} \pm A/B \qquad \text{where}$$

$$\hat{\sigma}_{ij} = 1 + \hat{a}_{ij}/\bar{S}_i\bar{S}_j \quad \text{(from equation (4.))}$$

$$A = Z_0(v^2\hat{\sigma}^2_{ij} - 2vs_\alpha r_1\hat{\sigma}_{ij} + s^2_\alpha)^{1/2}$$

$$B = \bar{S}_i\bar{S}_j + r_{ij}s_is_j/T$$

$$v^2 = \left[\bar{S}^2_i s^2_j + \bar{S}^2_j s^2_i + 2\bar{S}_i\bar{S}_j s_i s_j r_{ij} + (1 + r_{ij})s^2_i s^2_j\right]/T$$

Z_0 is the critical value for the standard normal distribution (the confidence interval estimator is normal distributed); $\bar{S}_k$ and s_k are sample mean and standard deviation of S_k $(k = i,j)$; s_α is the estimated standard error of $\hat{\alpha}_{ij}$, r_1 is the sample correlation between $\hat{\alpha}_{ij}$ and $\bar{S}_i\bar{S}_j$; r_{ij} is the sample correlation between S_i and S_j and T is the sample size.

The required statistics for the calculation were obtained using the econometric computer program (SHAZAM). r_1 was set to zero (Anderson and Thursby, 1986).

4) We used the mean values of the factor prices when calculating the scale variation dependent on output for model III. It may be interpreted as the return to scale function for the mean firm.

5) Josin and Fairchild (1984) use historical changing factor price ratios leading to a bias in technological development to account for measured nonhomotheticity in an industry. To test such a hypothesis in the Norwegian fish farming industry we need time–series data which is not available since the Norwegian fish farming industry has only been developed during the last few years.

6) Possible imperfections in the capital market may also partly explain nonhomotheticity in the industry. The possible imperfection in the capital market could be due to a screening process undertaken by the banks when providing loans to farms. For instance, one could argue that large farms could easier obtain loans from the banks since they are considered more reliable customers than smaller farms. However, such an argument, if true, does not necessarily indicate that large farms should be more capital intensive than small farms. This type of screening process would rather support the argument that large farms could more easily expand in general by financing their demand for all factors. Furthermore, in the period for which we have data, 1982/83, apparently there were few restrictions in financing fish farms in Norway.

7) In the literature, both actual and fitted shares are used when computing elasticities from the estimates of translog profit functions. Weaver (1983) and Gordon (1986) use fitted shares, while Antle and Aitah (1986) and Atkinson and Halvorsen (1976) employ actual shares. Some of the studies also report standard deviations of the elasticity measures, thus assuming linear relationships among the parameter estimates (Antle and Aitah, 1986).

6. SUMMARY AND CONCLUSIONS

In this chapter we will provide a summary of the main results. First, we briefly explain the models utilized when specifying a food fish farmer's production relationship. Next, the empirical results concerning production and cost properties are summarized. Furthermore, possible implications following from the empirical results will be illustrated and discussed according to the secondary goals of the thesis formulated in Chapter one. Given the parameter estimates of the translog cost function, we can derive an average cost curve for the industry. By using the average cost curve we will discuss the implications resulting from the regulation of farm size in the industry. Then, utilizing results from the demand literature on farmed salmon and employing future supply estimates for salmon, we can give some indications on the possible dissipation of economic rent and thus the future expansion of the industry. In addition, given the estimated substitution elasticities in the industry, the effects of changes in relative prices caused by subsidies and regulations can be discussed. In the concluding paragraph we suggest topics for further research.

6.1 Summary of results

The two–fold aim of this thesis is to model the production process of the Norwegian fish farming industry and to undertake an empirical analysis of production and thus provide estimates of the scale elasticity and substitution properties. When representing the primary mode of production for the industry we emphasized two main features. First, juvenile production could be separated out as a distinct activity. This is a feature of fish farming different from commercial fisheries, and enables us to undertake a separate analysis of the food fish segment of the aquaculture industry. Second, we argued that in a "normal" year the farmer produces one complete year–class.

However, the harvest quantities may change from one year to another. By defining the annual production as consisting both of actual harvest and the change in the stock of fish in pens, the measure of annual production constitutes a year–class. Hence, the dynamic optimization problem can be reasonably approximated by a static profit or cost function. This realistic simplification was required since our data set is given in calendar years.

Based on the two characteristics of the production process, we developed an analytical profit function for a farmer in a "normal" year. A restricted form of the profit function was formulated in order to have a well–defined profit function for empirical testing. The given factor was smolts, since the input of smolts can be considered as fixed during the production period. An alternative behavioural hypothesis for a grow–out farmer was hypothesized by defining a cost function. The cost function approach was chosen because (1) regulation of farm volume, and (2) standardized harvest procedures are two central characteristics of the Norwegian industry. In the long run cost function specification smolts is argued to be a separable factor. This view is consistent with one realistic interpretation of smolts as an input. Moreover, the cost function approach allowed an estimate of the long–run measure for scale elasticity.

However, both the restricted profit function and cost function models may be misspecified. The omission of a pen volume variable due to a lack of data caused this problem. The resulting possible bias with respect to the interpretation of the scale economies was discussed.

Both the cost and the profit functions were specified in the translog flexible functional form and they were estimated using Zellner's seemingly unrelated equations technique. The results for both specifications were satisfactory in terms of two criteria, i.e., high multiple correlation coefficients and t–statistics. Also the own–price demand elasticities obtained the correct signs both under cost minimization and profit maximization models. Furthermore, duality properties of linear homogeneity in prices, monotonicity and convexity were satisfied. The symmetry property could not be tested without prior restrictions on the functions. The cost function rather than the profit function approach appears to be the preferred specification both in terms of a better general fit and by the fact that it provides the desired long–run measure for scale elasticity. Furthermore, the price elasticities appears to be more reasonable for the cost function specification.

The empirical findings of paramount importance regarding scale properties can be summarized by the following three points:

i) Scale economies existed in the industry circa 1982/83 in the sense that the homogeneous version of the cost function shows statistically significant increasing returns to scale. This implies that there exist potential benefits from increasing output. However, estimates of a nonhomogenous cost function indicate that the biggest plants within our sample have exhausted their scale economies. However, we note that our measure of the cost efficiency of large farms compared to small farms probably incorporates the combined effect of regulation and scale economies.

ii) The findings also indicate that there a non–neutral scale expansion as factor intensity alters between small and large plants. The result arose under the nonhomothetic cost function.

iii) In case of the restricted profit function, returns to the variable factors were found to be less than one.

With respect to the behaviour of input, four major results can be highlighted:

i) The own–price elasticities for input demand all obtained the correct sign, i.e., nonincreasing in own price, which confirms rational behaviour.

ii) Both capital–labour and capital–feed input combinations are found to be substitutes for both the profit and the cost specifications. The labour—capital relationship is, however, ambiguous: the cost function specification yields a relationship indicating that these inputs could be both complements and substitutes.

iii) Input demand with respect to output price indicates that all three inputs were superior for the production process under the restricted profit function.

iv) The demand for inputs w.r.t. smolts as a fixed factor in the restricted profit function all obtained positive signs and were similar in value.

6.2 Implications of the scale properties

The implications derived from the empirical results concerning scale properties in the Norwegian fish farming industry, will be discussed according to the objectives stated in the introduction. We will illustrate the implications by computing an industry average cost curve utilizing our parameter estimates. We relate our scale economy results and the inherent unexploited scale economies to size regulations given in Chapter five.

From the returns to scale function and the distribution of plants in the Norwegian salmon farming industry, we concluded that there exist potential benefits from increasing the production of the plants. In fact, Figure 5.1 in Chapter five indicates that almost all plants, including the average sized farm, are smaller than the plant size where the elasticity of scale is not significantly different from one and thus the most cost efficient size. The benefits from scale economies in terms of lower production costs can be achieved through larger production at the plant level. This is because our results concern cost advantages due to scale economies in *production* and not benefits for bigger enterprises due to possible advantages in sales, marketing, purchase of inputs etc.

We will now proceed by relating our results of unexploited economies of scale to the size regulation of the industry. Thus, we ask if a more liberal regulation act would have encouraged better resource use in the industry.

We will try to shed some light on this query by pointing out a change in the regulation act since 1982/1983 which may affect our interpretation of unexploited economies of scale. As described in Chapter two, in 1985 the licensing act was liberalized, so that every farm that had been in operation for three years or more – and thus were limited to 3,000 or 5,000 cubic meters – and any new entrants were allowed to use 8,000 cubic meters in pen volume. Hence the picture presented by the distribution of farms below the scale function in Figure 5.1 for 1982/83, may be biased. The average farm today is probably larger both in terms of pen size and produced quantity. Therefore, some of the potential benefits available in 1982/83 are probably now exploited in that the average size has increased. However, Figure 5.1 established that most of the farms were below the efficient farm size in terms of low average production costs. Thus, the question still persists whether size limitations of 8,000 m^3 can lead to a high cost industry by not fully exploiting the benefits of scale economies.

We will illustrate this by computing the average cost function in the Norwegian fish farming industry using the translog cost function.[1] Figure 6.1 presents the average cost of producing one kilo of fish as a function of plant size. The 1983 value of the cost function was inflated to 1986 values by a general price index for investment goods (*Statistical Yearbook, 1986*).[2] The output index in Figure 6.1 is an index in kg of both salmon and trout.

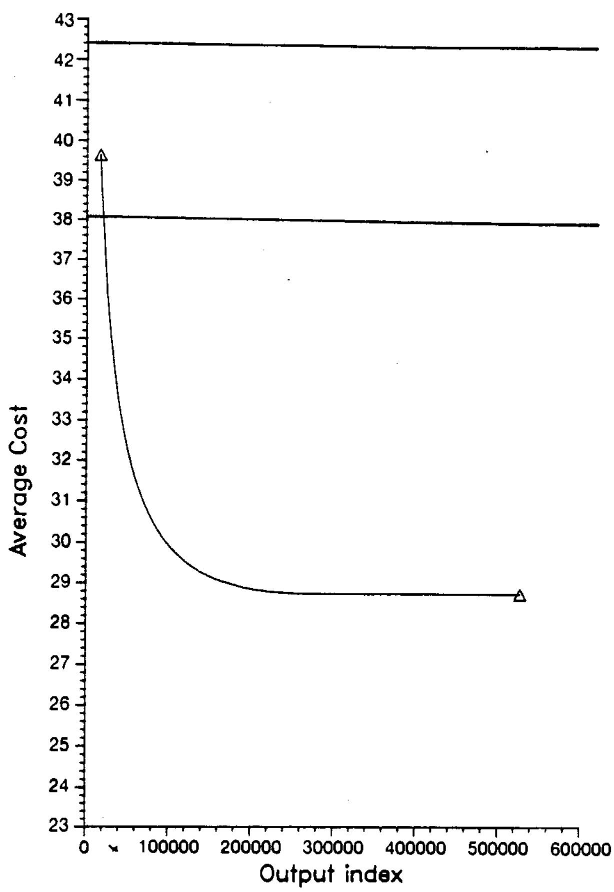


Figure 6.1: Average cost curve in the Norwegian salmon farming industry for 1986 based on translog cost function estimates. Average world export price (from Norway) for fresh Atlantic salmon for 1986 is used together with a predicted price for 1990 based on expected increase in supply and estimated own price demand elasticity.

First, we note the shape of the average cost curve. It provides the same information as the scale function regarding the scale properties in the industry.[3] That is, there exist scale economies illustrated by the decreasing portion of the average cost function. In

Appendix C the optimal size of a fish farm – the size of the farm when the scale elasticity is one and hence the lowest average cost – is computed using the parameter estimates from the cost function with an output range between 142 and 192 tonnes in production a year. Since the size of the farms is given in cubic meters of water volume and not in production in tonnes, we will try to convert optimal size in tonnes into a water volume dimension. More precisely, we illustrate this by presenting a table of actual farms along the two size dimensions. Table 6.1 below displays the combination of the two size distributions.

Table 6.1: Size distribution of farms along the two dimensions of size, annual production (tonnes) and cubic meters of pen volume respectively.

Volume / Production	$8.000\ m^3 \leq V < 15.000\ m^3$		$V > 15.000\ m^3$	
Year	1982	1983	1982	1983
$y < 100$	7	10	–	1
$100 < y \leq 130$	3	5	–	–
$130 < y \leq 190$	2	2	3	–
$190 < y$	4	3	2	4
Total	16	20	5	5

From Table 6.1 we notice that a regulated farm of 8,000 m^3 may be able to produce between 142 and 192 tonnes. This is represented by rows one and two in Table 6.1 consisting of farms producing between 130 and 190 tonnes; a finer subdivision if this data would not provide more information and would, furthermore, pose a problem of revealing the identity of the farms. On the other hand, Table 6.1 indicates that farms with a capacity above 15,000 m^3 (columns three and four) – which is almost twice the size of the ceiling put on the regulated farms – have a greater fraction of optimal sized farms than the group of farms from 8,000 m^3 up to 15,000 m^3 of pen volume (columns one and two). Or in other words, there is a greater probability of finding optimal farms in the group consisting of farms with a capacity above 15,000 m^3, than in the group with a lower capacity. Furthermore, we notice that the group of farms from 8,000 m^3 to 15,000 m^3 (columns one and two) has a considerable number of plants that are clearly suboptimal. The latter result may be partly due to the fact that in 1982/83 newly established farms would not be fully developed in terms of capacity. This concerns mainly the smaller farms in this group, since farms above 15,000 m^3 were established earlier.

These results are also confirmed by an earlier study utilizing a Cobb–Douglas functional form estimating segments of the cost function by splitting it into subgroups of farms for the 1982 sample grouped according to pen volume size in cubic meters (Salvanes, 1985). It appeared that the scale elasticity decreased in value up to the group consisting of plants above 15,000 m^3 where the scale elasticity was not significantly different from one, i.e., indicating that the optimal size of a farm is in this group.

It is necessary for us to slightly modify the interpretations given above, since as indicated by the Table 6.1, there is a significant dispersion of production level for all the farms of 8,000 m^3 and above. This is an important reservation to make in the interpretation of the results particularly because we have relatively few observations for big farms. Nevertheless, this may provide us with some further information. First, the dispersion of farms can indicate that other factors – which we were not able to actually test due to lack of data – may be important in influencing the costs of the industry. This may indicate that an optimal plant size can depend on the quality of site location. Perhaps also a certain period of learning for fish farmers may explain the dispersion of results. In fact, when this hypothesis was tested – this information was available for 1982 only– showed that farms in operation for five years or more more cost efficient than the rest of the sample (Salvanes, 1985). In Rusdal (1987) the hypothesis of a period of learning was tested utilizing a data set from 1985, and the result was not confirmed.

We conclude this section by stating that the empirical analysis indicated an industrial structure characterized by suboptimal plants for the 1982/1983 study period. The cost disadvantage due to size in 1982/83 can be connected to size limitations in that the regulation act was more restrictive in that time period with size regulated at 3,000 m^3 or 5,000 m^3 for many farms, making the average farm size relatively small. However, we cannot state any conclusion concerning the current size limit of 8,000 m^3 since there is a great dispersion on production levels for all farms above 8,000 m^3, and it appears to be possible for a regulated farm to reach a level of production in the same range as a farm that is not regulated. Since all farms today (1988) probably are fully developed in terms of capacity utilization, and the relative factor prices have changed since 1982/83, further research utilizing a more recent data set might provide better information concerning the effect on cost efficiency of the size limit of 8,000 m^3.

6.3 Implications of factor demand elasticities

In both the cost and profit function case, the elasticity measures indicated that labour and capital are substitutes. Furthermore, these cross price elasticities under the profit function specification indicated a stronger tendency towards greater capitalization. This trend is expected for a developing industry as automatization of feeders and harvesting equipment occurs. However, given government subsidies of capital, one main government goal for regional development, i.e., creating jobs in fringe areas, will be thwarted as capital–labour substitution is accelerated. One example of subsidizing capital in the fringe areas of Norway, is through a particular tax act for firms in rural areas which taxes investments in capital at a lower tax rate than for urban areas and through direct subsidies of capital investments in certain areas. It can be proved that some of these devices for regional development result in a lower unit price of capital (Serck–Hansen, 1977).

On the other hand, the industry is characterized by a nonhomothetic production technology. Thus, the government regulation on farm size has a rationale, since larger farms are more capital–intensive than smaller farms. Hence, many small firms will create more jobs than will fewer and bigger farms. But the government's capital subsidy policy coupled with a strong substitution effect between labour and capital partially offsets the desired employment effects from size restrictions. We must stress that this favourable analysis of the employment effects of nonhomotheticity in the Norwegian salmon farming industry is made without considering the effect of underutilization of scale economies especially if prices decline in the future. As discussed in Section 6.2 these are serious arguments against the continuing size regulation for the industry. As farms become more capital–intensive, a greater potential for scale economies probably exists in the industry, making size regulation even more difficult to rationalize.

As noted, capital and feed are also substitutes in both functional specifications. This relationship is easily interpreted since the effect of automatic feeders is to save on both labour and feed. Moreover, the absolute values for these cross elasticities indicate another source for greater capitalization. Given that a one percent increase in feed price leads to a 5.81 percent increase in the use of capital, and a one percent increase in capital price leads to only a 1.15 percent increase in the use of feed, rising relative prices for non–subsidized feed will increase capital intensity.

The input relationship between labour and feed is more ambiguous. The profit function estimates indicate a substitution relationship, but for the cost function the elasticity range indicates the possibilities of both a complementary and substitution relationship. This ambiguous result probably reflects a wide variety of methods of husbandry in the fish farming industry. For instance, some farmers will simultaneously use complementary hand–feeding, which is very labour intensive, and automatic feeding.

6.4 Concluding remarks

Our concluding remarks will focus on areas of further research. To achieve better model specifications, different profit and cost functions – especially with regulated capacity in pen volume as a fixed factor – should be an area of future research. Such a study would require a more complete data set, and in the cost function case we would have to trade off a long run scale elasticity for an *a priori* better model specification. A more recent data set, and thus an updating of the results, could be appropriate when using different model specifications.

Another area for further research is to investigate the implications of the size regulations of farms. Of particular interest would be to utilize a more recent data set to further investigate empirically if the 8,000 m^3 limit on pen volume is a binding constraint. Furthermore, a dynamic counterpart to the results regarding size regulation could also be investigated using a newer data set. We will outline the rationale for this possible implication and illustrate it by using the results already available. This point is connected to the underlying principle of survivor phenomenon of firms (Stigler [1958], Chandler [1977]). Over time only the most efficient firms will survive from the competition among firms of different sizes. The invisible hand allows only the most efficient in terms of costs to survive. In our case this principle of survival can be described in the following way. To date, Norwegian salmon farms have been very profitable as a partial consequence of high prices. However, this picture will probably change in the next few years. Above–normal profits were initially possible with limited output and high prices. As a consequence of this high profitability other countries are expanding or planning to invest in this industry. Downward pressure on price is inevitable given the demand elasticities found in recent research (DeVoretz [1987], DeVoretz and Salvanes [1987], Lin and Herrmann [1987], Kabir and Ridler [1984]). Thus, size restrictions, which seem to preclude exploiting scale advantages – although desirable for the rural economy – may restrain the further expansion of the most cost efficient segment of the industry in Norway. Once again we will utilize Figure 6.1 to illustrate our argument.

From 1982 to 1986 the annual output growth averaged over 40 percent, with production increasing from 10.7 thousand metric tons to 45.5 thousand metric tons (see Table 2.1). The importance of a cost efficient industry becomes obvious when one incorporates the effects of a large increase in supply, which is expected in about 1990 (see Appendix A). Using a conservative estimate for output expansion, i.e., doubling production from 1986 to 1990, and combining it with the known own–price elasticity (–10.0), a price for 1990 is obtained (DeVoretz and Salvanes, 1988). The own–price elasticity is in accordance also with results in Kabir and Ridler (1984) and Lin and Poor (1986). The dotted price line in Figure 6.1 indicate this computed 1990 price. This real price drop between 1986 to 1990, when compared with the real average cost curve, seems to indicate little rent to the industry.

It is difficult to judge how strong this rent dissipation effect will be in the future. First, we have to use a more recent data set than the 1982/83 data set we have available now. This is because the costs by 1986/87 may have changed also because we would expect the relative factor prices to have changed. For instance from 1986/87 on the price of smolts has dropped relative to other input prices. Second, we based our estimated price change only on the actual markets for Norwegian export today, and did not consider possible new markets in the future. Moreover, our results from the cost function estimation also clearly indicate a limit to scale economies within our sample of Norwegian firms. This means that possible cost advantages of other countries' farms may be limited once the average Norwegian farm reaches its capacity limit as determined by the government. Thus, a comparative study between Norway and other producing countries such as Scotland, Chile and Canada (British Columbia) would also be of interest and could especially isolate locational effects on costs. Furthermore, location is probably the limiting factor in production which ultimately gives decreasing returns, and differences with respect to this factor may cause different cost structures among different countries. Therefore, if possible, by using a more complete data set location should be included either via cross–classification in the sample or included directly in the production function as an index variable.

An additional point we will mention concerns the method of estimation. McElroy's (1987) suggestion of specifying a stochastic production model adapted to the decision problem in question instead of adding a stochastic term to an already deterministic model appears to be a relevant extension of empirical work using duality theory and flexible functional forms.

FOOTNOTES

1) Since strong separability was assumed for smolt in the cost function we estimated, and thus expenses for smolt were not included – material expenses were also excluded due to lack of data – this function is not fit for representing the average total cost curve. To accomplish this we estimated a translog cost function where we did not span out the substitution possibilities, and hence used the total costs including smolt and material since the associated prices were not required in this specification. The results are presented in Appendix C.

2) When we inflate the cost function for 1982/83 to the 1986 price level for salmon with the average cost level, we implicitly assume constant relative factor prices. This may not be true; particularly the price of smolts has probably changed relative to other factors.

3) The slightly upward–bending part of the curve is not significantly different from the flat portion.

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APPENDIX A

Table I: Employment in the Norwegian fish farming industry.

Year	Total	Full–time	Part–time	Food–Fish Farms	Hatcheries	Other (Research)
1985	2,948	1,943	1,005	2,184	588	175
1986[1)]	4,000	–	–	–	–	–

Source: See Directorate of Fisheries and the Directorate of Labour, 1985 (*Undersøkelser av sysselsettingen innen oppdrettsnæringens primæraktiviteter pr. 1.–7. December 1985*).

1) This projection is based on interviews with farmers about their expected demand, plus expected new entrants.

Table II: Norwegian and world production of farmed salmon; 1986 and 1987 and 1990 predicted (1,000 tonnes).

Country	Atlantic salmon		Pacific salmon	
	1986	1990	1986	1990
Norway	45,494	100–120,000[4]		
Scotland	10,338	25,000		
Faroe Islands[1]	1,370	6,000		
Ireland	1,250	10,000		
Iceland	200	2,000		
North America	na	3,000		
Chile	na	na	1,600[2]	10,000–20,000
Canada	na	na	1,000[3]	8,000–10,000
Others	na	7,000	na	na
Total	58,652	153–173,000	2,600	18–30,000

Sources: Data compiled by *Norwegian Fish Farming*, the official journal of the Norwegian Fish Farmers Association (in Norwegian), and in *Stortingsmelding* (Report to Parliament) *No. 65, 1986–87. Om havbruk* (On Fishfarming).

1) From *Fish Farming International*, volume 14.
2) Estimated for the harvest 1986–87.
3) Estimated figures.
4) This is the prognosis given by the Norwegian Fish Farmers Association, and is based on smolt production for 1987 and expected smolt production for 1988.

Table II provides an overview over the production in 1986 of farmed salmon and estimated figures for 1990. We notice that Norway in 1986 was the dominant producer of farmed salmon with a share of 78 percent of total Atlantic salmon. From the forecast a great expansion is expected to 1990. Since the figures are planned production levels for each country, the projections are uncertain. However, the table indicate that the total supply will increase by about 200 percent by 1990. The Norwegian share of Atlantic farmed salmon would have dropped to 65 percent. Moreover, the planned increase in production of Pacific salmon particularly from Chile and Canada (British Columbia) re–enforces the impression of increased competition.

APPENDIX B

The difference and bias between b_1^* and b_1 may be found as follows. Apply OLS to the equation (3.19) in Chapter three and the estimated slope parameter is (This model is a simplification of the model given in Kmenta (1986), p. 442).

$$b_1^* = \frac{\Sigma y\, x_1}{\Sigma x_1^2} \qquad \text{(i)}$$

The normal equations for the correctly specified model are:

$$\begin{aligned} \Sigma y x_1 &= b_1 \Sigma x_1^2 + b_2 \Sigma x_1\, x_2 \\ \Sigma y x_2 &= b_1 \Sigma x_1\, x_2 + b_2 \Sigma x_2^2 \end{aligned} \qquad \text{(ii)}$$

By dividing the first equation through by Σx_1^2 we get:

$$\frac{\Sigma y\, x_1}{\Sigma x_1^2} = b_1 + b_2 \frac{\Sigma x_1 x_2}{\Sigma x_1^2} \qquad \text{(iii)}$$

Substituting equation (i) on the left hand side of (ii), we obtain:

$$b_1^* = b_1 + \frac{b_2 \Sigma x_1 x_2}{\Sigma x_1^2} \qquad \text{(iv)}$$

Hence, $\Sigma x_1 x_2 / \Sigma x_1^2$ is the regression coefficient – and thus the correlation coefficient – associated with the regression of x_2 on x_1.

We define $a_{12} = \Sigma x_1 x_2 / \Sigma x_1^2$, and from the above discussion we may write, using the OLS procedure:

$$E(b_1^*) = b_1 + b_2 a_1 \qquad \text{(v)}$$

or the bias of b_1^* is:

$$(E(b^*) - b_1) = b_2 \, a_1 \qquad \text{(vi)}$$

APPENDIX C

In this appendix we will present the results of estimating the translog function used for deriving the average cost curve in Chapter six, i.e., when total costs also include expenses for smolts and materials. The translog cost function has the following form:

$$\ln c = \beta_0 + \beta_y \ln y + \beta_{yy} (\ln y)^2 \qquad \text{(i)}$$

where c – total costs – in this case consists of expenses for capital, labour, feed, smolts and materials. Output y is defined in both volume and value terms as discussed in Chapter four. By utilizing parameter estimates from this function estimated by the Ordinary Least Squares (OLS) procedure, the average cost function for the Norwegian fish farming industry was derived and plotted as in Figure 6.1. Table I below presents the results of estimating (i) in volume and value terms of output, respectively.

Table I: Cost function estimates based on 1982/83 data using a simple translog function.

Model	β_0	β_y	β_{yy}	$\bar{R}^2$
i.a (Output in volume)	14.41 (5.54)	-0.878 (-1.83)	0.158 (3.56)	0.75
i.b (Output in value)	2.02 (3.52)	0.559 (-1.17)	0.099 (2.99)	0.85

Also in this case the best fit is provided when output is defined in value terms. Hence, specification (i.b) is preferred.

Optimal size of a plant is given when the elasticity of scale equals one; and in this case the scale elasticity has the following form

$$\varepsilon(y) = 1/\frac{\text{dlnc}}{\text{dlny}} = (\beta_y + \beta_{yy}\text{lny})^{-1} \qquad \text{(ii)}$$

Thus, by setting equation (ii) equal to one, optimal size of a farm is given by

$$y_{\varepsilon}=1 = \exp\left[\frac{1-\beta_y}{\beta_{yy}}\right] \qquad \text{(iii)}$$

When employing the parameter estimates from Table I for (i.a) and (i.b), i.e., output defined in volume (tonnes) and value (NOK), the optimal size given by equation (iii) is 143 tonnes and 192 tonnes in annual production. As noted earlier, the cost function when specified in value terms provides the best general fit and thus these results concerning optimal size should be more reliable.