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Econometric Modelling of Production Technology under Risk:

The Case of the Norwegian Salmon Aquaculture Industry

by

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A dissertation submitted for the degree of dr. oecon.

PREFACE

My dissertation committee had the following members: Prof. Trond Bjørndal, Norwegian School of Economics and Business Administration (NHH), Prof. James E. Wilen, University of California at Davis, Prof. Daniel V. Gordon, University of Calgary, and Almas Heshmati, University of Gothenburg. Several other people have provided valuable comments, suggestions and encouragement during the research process that lead to this dissertation.

I would like to acknowledge my debt to the following (the order is of no significance):

My main advisor, Professor Trond Bjørndal, played a central role at two critical stages. His careful advises when I wrote my proposal enabled me to narrow down the scope of my work, improve the structure, and make my Ph.D. project more feasible. In the final phase his comments and suggestions contributed to improve the discussion of my empirical results, and to make the dissertation a more polished product.

Almas Heshmati entered the advisory committee at a late stage, but he managed to put his mark on this dissertation. I had an inspiring and highly productive stay as his guest at the University of Gothenburg in the spring of 1997. Working together with him was very instructive. He has particularly influenced the empirical study in Chapter nine.

Jim Wilen was a good discussion partner to have on several occasions. With his experience from natural resource economics, and his understanding of the empirical research, he was a valuable asset for me. Furthermore, he arranged my one-year stay at UC Davis, and gave me several good memories from that period.

Dan Gordon provided suggestions and valuable encouragement in the initial phase of the project.

Professor Kjell G. Salvanes, Norwegian School of Economics and Business Administration, was actually the first to mention the possibility of taking a Ph.D. I have had the pleasure of working under him as research assistant in several projects, and have experienced first-hand his genuine interest in economic research and his frequent moments of inspiration. He also proposed to approach Almas Heshmati, which turned out to be a great idea.

Dr. Frank Asche, my friend and colleague at the Foundation for Research in Economics and Business Administration, once again proved to be a very valuable advisor (as he was when I wrote my master's thesis). He followed my work through the entire process, and his comments and suggestions reflected that he got a very good grip on both theoretical and empirical aspects of my research.

Of course, the above mentioned are not responsible for any deficiencies in my dissertation.

The Foundation for Research in Economics and Business Administration (SNF) was my workplace for most of the period when I worked on this dissertation.

I spent one year at the Department of Agricultural and Resource Economics (Ag.econ.) at UC Davis as a visiting scholar. Ag.econ. provided office, computing resources and other services during this period. It was a friendly place with very accessible faculty members (given their hectic time schedules). I learned to know several of the graduate students, and enjoyed their company. Both Ag.econ. in particular and Davis in general is hereby recommended (I miss Davis!).

The three-year grant from the Norwegian Research Council (NFR), project no. 103145/100, which enabled me to work full time on this dissertation, is gratefully acknowledged. NFR also provided an extra grant for my stay at UC Davis.

The Norwegian Directorate of Fisheries supplied the data set which has been used in the empirical application.

Finally, I am deeply grateful to my wife Janne, who had to endure many week-ends and evenings of dissertation work. Maybe things will get better now?

Ragnar Tveterås

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LIST OF FIGURES[1]

1 Due to space considerations a large number of figures in Chapter 9 were placed in a separate appendix and not listed here. See App. 9.C.

1. INTRODUCTION

This dissertation deals with the econometric specification and estimation of stochastic production technologies when a panel data set is available to the researcher. In biological production sectors it is often the case that not only mean output level is a function of input levels, but also the variance of output. In econometric terminology this means that such production technologies exhibit heteroskedasticity, where the variance of the error term and thus the variance of the dependent variable - the level of output - is related to some explanatory variables.

Since the introduction of flexible functional forms in the 1970s, a vast number of econometric studies of production technology and firm behaviour have been provided in international economic journals. The emphasis of these studies has been the measurement of producers' responses to changes in input and output prices, or the measurement of productivity growth. Most of these studies have, explicitly or implicitly, assumed a deterministic or homoskedastic production technology. A deterministic setting implies that for a given level of inputs the output level is known with certainty, while the assumption of homoskedasticity implies that inputs do not affect output variability. For industries where the level of risk or the magnitude of heteroskedasticity is relatively small such assumptions may be appropriate.

However, if substantial production heteroskedasticity is present, which is the case for many sectors of biological production, this should be accounted for in the econometric model specification. According to the theory of the competitive firm under production risk, the structure of production risk, the firm's *risk preference structure* and the *firm's expectation formation process* influence firm behaviour (see figure 1.1). In general, the competitive firm chooses different input levels and responds differently to price changes under production heteroskedasticity than it would have done under production homoskedasticity or certainty (see Chapter 2). Furthermore, it can be shown that in the presence of production heteroskedasticity and risk aversion, parameter estimates from conventional dual models of the firm generally will be biased.[1] This means that the use of econometric models which assume output homoskedasticity or certainty may provide regulators and policy makers with incorrect inferences with respect to the effects of policy measures which affect input and output prices (Leathers & Quiggin, 1991). Finally, homoskedastic and deterministic econometric models are not able to provide any information on the risk-reducing or risk-increasing effects of inputs.

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[1] Unlike dual models, primal models assuming a deterministic setting provide consistent but inefficient parameter estimates.

We will see in chapters 2 and 3, that dual econometric modelling of the firm under uncertainty generally forces the researcher to account for the firm's risk preference structure and expectation formation process in the model specification. The specification of risk preference structure and expectation formation process in a dual econometric model poses great challenges to the researcher, because of substantial requirements both with respect to theoretical consistency and functional flexibility. Previous econometric studies have to a varying extent been able to find satisfactory solutions to the problems introduced by a stochastic setting. Unlike the standard deterministic theory of production, there is no dual model framework available that is easily tractable for econometric estimation. In this dissertation we have therefore limited ourselves to specifying only primal models of production.

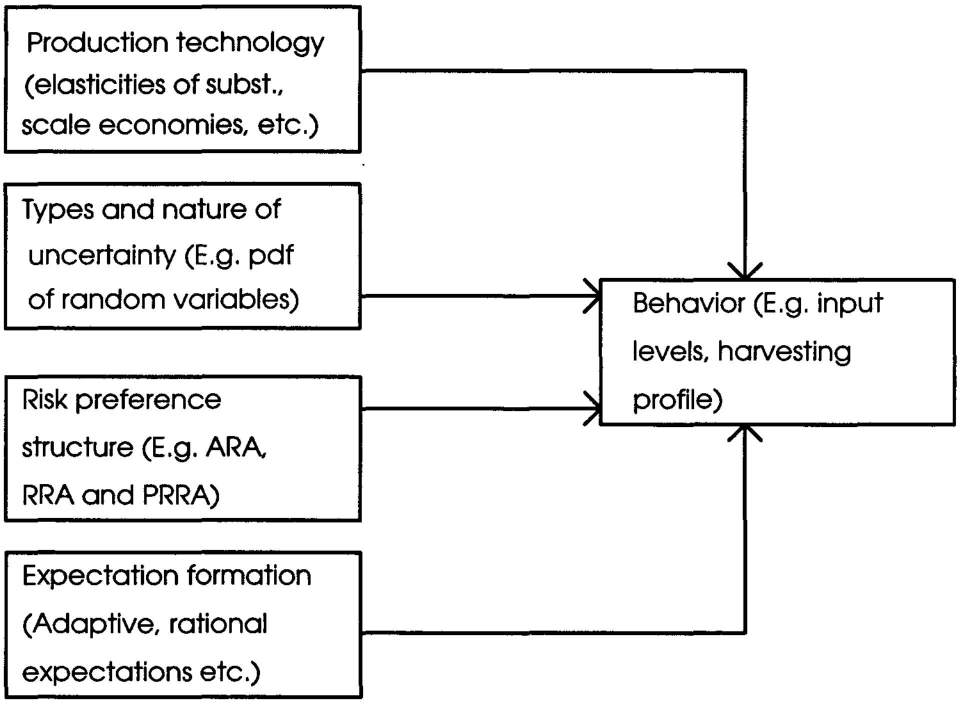

Figure 1.1. Four determinants of producer behaviour under uncertainty[2]

The foundation for the econometric study of heteroskedastic production technologies has to a large extent been provided by Just & Pope (1978). They suggested eight postulates for the stochastic production function which they claim to be reasonable on the basis of *a priori* theorising and observed behaviour. Furthermore, they specified a particular functional form which satisfies the eight postulates, the Just-Pope production function, which is given by (Just & Pope, 1978)

(1.1) $y_{it} = f(\mathbf{x}_{it}; \alpha) + h(\mathbf{x}_{it}; \beta)\varepsilon_{it}$,

[2] See Appendix 2.A for the definitions of ARA, RRA and PRRA.

where y is output level (with firm and time subscripts i and t), $\mathbf{x}$ is a vector of input levels, ε is a stochastic term, and $E[\varepsilon]=0$. The function $f(\cdot)$ is the *mean production function* and $h(\cdot)$ is the *variance production function*. The parameter vectors α and β are the mean and variance function parameters, respectively.

In (1.1) the effect of input changes has been separated into two effects; the effect on mean output and the effect on the variance of output. The Just-Pope production function is a heteroskedastic specification, because the variance of y is a function of the input vector $\mathbf{x}$. An advantage of the Just-Pope model is that it allows us to analyse the effects of changing input levels on mean output and output risk separately. This can be seen by deriving from (1.1) the conditional variance of output

$$\text{var}[y_{it}] = [h(\mathbf{x}_{it}; \beta)]^2 \text{var}[\varepsilon_{it}],$$

and the conditional mean output

$$E[y_{it}] = f(\mathbf{x}_{it}; \alpha).$$

As we shall see in Chapter 3, this particular functional form has been used extensively in econometric analyses of heteroskedastic production technologies.

In Chapter 3 we discuss empirical studies of production risk that have been provided in the literature. The empirical results from this body of studies give strong indications of the presence of heteroskedasticity in biological production processes. On the other hand, the results from individual studies should be interpreted with care, because weaknesses or deficiencies with respect to methodology and data generally characterise the studies. Incorrect, or more precisely, simplistic specification of the functional form has probably given rise to biases in empirical estimates. Empirical studies have tended to use "simple" specifications such as the Cobb-Douglas to facilitate estimation. Estimation has also largely been done by feasible generalised least squares (FGLS), which recently has been criticised (Saha, Havenner, & Talpaz, 1997).

1.1. Heteroskedasticity, Firm Heterogeneity and Econometric Panel Data Techniques

An issue not to be ignored in econometric modelling of production technology and firm behaviour is *firm heterogeneity* with respect to production technology and productivity. Firms which use the same vector of input levels often experience different output levels, and often this can only to some extent be attributed to different outcomes of the stochastic variables in the production process (ε in model (1.1)). Often, some of the productivity differences between firms are of a more persistent nature, which is related to (unobserved) firm characteristics. The findings of a growing body of empirical studies using econometric panel data techniques strongly suggest that firm heterogeneity should be accounted for in production model specifications.[3]

In industries with cross-firm productivity differences, econometric specifications that ignore heterogeneity will provide biased estimates and lead to incorrect inferences. This is particularly the case for heteroskedastic production technologies. Ignoring firm heterogeneity can lead to biased estimates of the parameters of both the mean production function $f(\mathbf{x})$ and the variance production function $h(\mathbf{x})$.

A heteroskedastic panel data model of production, which is an extension of the Just-Pope production function (1.1), can be written as

(1.2) $y_{it} = f(\mathbf{x}_{it}; \alpha) + \eta_i + u_{it}, \qquad i = 1, \ldots, N, \quad t = 1, \ldots, T_i,$

where

$$u_{it} = h(\mathbf{x}_{it}; \beta)\varepsilon_{it},$$

i is the firm subscript, t is the time subscript, and T_i is the number of time periods firm i is observed. If the time-invariant firm-specific η_i's are assumed fixed, we have the fixed effects model (or the least squares dummy variable (LSDV) model), while if they are assumed random we have the random effects model (or one-way error components model). In the random effects model η_i is the *firm-specific error component*, and u_{it} is the *observation-specific error component*.

The use of econometric panel data techniques in empirical analysis of production risk complicates econometric specification and estimation. A fixed effects model may have a large number of firm dummy-variables, depending on the number of firms. A Just-Pope production function is typically estimated by feasible generalised least squares (FGLS) or maximum likelihood (ML) methods, and in practice it may be difficult to find the coefficient values that optimise the objective function with a large number of parameters.

The fixed effects approach is sometimes not to be desired, because the researcher wants to implement time-invariant regressors, such as regional dummies in the model specification. In such cases a random effects model approach can be used. However, the variance-covariance matrix has no longer a simple diagonal structure when one goes to a random effects model, because the firm-specific error component η_i is correlated over time for the observations of firm i (see Chapter 4). The block-diagonal structure of the variance-covariance matrix complicates FGLS and ML estimation.

In the panel data literature the error components η_i and u_{it} are generally assumed to be homoskedastic.[4] We shall see in Chapter 4 that FGLS and ML estimators for random effects

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[3] See chapter 3 for a discussion of empirical studies by Just & Pope (1979), Kumbhakar (1993), Traxler *et al.* (1995), and others.

[4] See Fuller & Battese (1973) and other references cited in Chapter 4.

models with heteroskedastic η_i and/or u_{it} have been suggested, but their performance has been evaluated only to a limited extent through simulation studies. Empirical application of these estimators are even more sparse. It is certainly desirable to explore the performance of heteroskedastic random effect estimators and gain more experience with them in empirical applications.

1.2. The Empirical Application: Norwegian Salmon Aquaculture

The Norwegian salmon aquaculture industry has been chosen as an empirical application in this dissertation to test various hypotheses on the structure of the stochastic production technology. In the following it will be argued that substantial output uncertainty is present in the salmon aquaculture industry. A limited understanding and knowledge of the biophysical environment, combined with the high sensitivity of salmon mortality and growth to contagious diseases, water temperatures and other stochastic environmental variables have resulted in large variations in output at the firm level.

Another reason for choosing the salmon aquaculture industry for empirical application is that there exists a firm level unbalanced panel data set which facilitates estimation of econometric models, and should allow me to test a rich set of hypotheses.

The various types of uncertainty facing economic agents in this industry have only to a limited extent been discussed in previous analyses of the industry, and to an even smaller extent been implemented in formal models. For instance, uncertainty and risk preference structure have not been included in earlier econometric models of salmon production, e.g. Salvanes & Tveterås (1992), Salvanes (1993) and Tveterås (1993). The implicit assumptions of output certainty, risk neutrality and homogeneous production technologies in these papers, may thus have lead to biased estimates of input demand elasticities, output supply elasticities, and productivity measures for the industry.

1.3. Hypotheses to be Tested on Norwegian Salmon Aquaculture

In this dissertation the following hypotheses with respect to the production technology and the nature of production uncertainty in salmon farming will be tested for:

H1. The following factors explain observed cross-firm productivity differences in Norwegian salmon farming in a given year: (1) economies of scale, (2) firm heterogeneity (with respect to the quality of management, labour and capital equipment, etc.,), and (3) "true" randomness in the production process.

H2. The production technology in salmon farming is characterised by

(a) increasing output risk associated with a factor neutral expansion in inputs,
(b) decreasing marginal output risk associated with an increase in the input of capital and labour, and
(c) increasing marginal output risk associated with an increase in the input of fish and fish feed.

H3. During the period 1985-93

(a) the conditional mean output for a given combination of inputs has increased.
(b) the conditional distribution of output in salmon farming has been more condensed, i.e., the level of production risk has decreased.

H4. There are differences in mean productivity and output risk levels between the regions.

The empirical motivation behind hypothesis H1 is that we know little about the relative importance of scale economies, firm heterogeneity and stochastic shocks (e.g. in terms of fish disease outbreaks) for the productivity differences we observe between salmon farms. Hypothesis H2 is motivated by the predictions of theoretical models, which state that a risk averse producer will use less of a risk-increasing input, and use more of a risk-decreasing input, than a risk-neutral producer (Ramaswami, 1992). Hence, if Norwegian salmon farmers are risk averse, the risk properties of inputs are clearly of interest. Hypothesis H3 is concerned with the effects of technology adoption and learning-by-doing on mean output and output risk. If Norwegian salmon farmers are risk averse, then they should not only be concerned about the increase in mean output, but also about output risk properties when they consider adoption of new technologies. Furthermore, learning-by-doing should not only contribute to increase mean productivity, but also reduce the level of output risk, *ceteris paribus*. Hypothesis H4 is motivated by the concern which has always been present regarding the relative productivity of different coastal regions along the north-south axis (Bjørndal & Salvanes, 1995). There are several arguments for productivity differences across regions. The regions have different biophysical conditions in term of temperatures, light conditions and water exchange (tidal currents), etc. The regions also entered the industry at different stages; farms in southern regions tended to enter at an earlier stage than farms in the northern regions. In this dissertation we not only compare mean productivity across regions, but also analyse differences in production risk.

Later chapters define more precisely the implications of the above hypotheses in mathematical terms, and the implications of the above hypotheses for econometric model specification.

1.4. Objectives

The main objectives of this dissertation are:

(1) Specify and estimate primal econometric models of the competitive firm under uncertainty to test hypotheses on firm behaviour and production technology in salmon aquaculture.

(2) Assess biases associated with assuming a deterministic production technology or risk neutrality in comparative static analyses of input demands and output supply in salmon aquaculture.

(3) Specify and assess the performance of estimators for fixed and random effects models with observation-specific error terms which are heteroskedastic in regressors.

The consequences of firm heterogeneity and productivity shifts over time for risk parameter estimates, which have only been addressed to a very limited extent in the empirical literature, will be accounted for using panel data techniques. Consequently, this dissertation should lead to new insight with respect to the quantitative effects of production uncertainty on firm behaviour in the salmon industry. Specifically, we should be able to provide more precise quantitative statements on the effects of risk-reducing measures on input use and output supply. In addition, we should also be able to quantify the effects of different levels of risk aversion on input use and output supply. Since the quantitative empirical evidence so far is somewhat limited due to methodological or data shortcomings of many previous studies, the findings should be of interest beyond a small group of policy-makers and agents in the industry.

The methodological focus will be on the specification of heteroskedastic panel data models, and the assessment of their performance compared with homoskedastic panel data models and heteroskedastic production models that ignore firm heterogeneity. In the context of empirical analysis of production risk an important question is: What can we gain by using panel data techniques in the econometric analysis of risky production technologies? A question of interest in econometric panel data estimation is: What do we gain by using panel data models that account for heteroskedasticity when heteroskedasticity is present in the data set? A related issue that should be explored is which heteroskedastic panel data models are most appropriate, the fixed or the random effects specification?

1.5. Outline

A presentation of underlying postulates and theories of the competitive firm under production risk is provided in Chapter two. Chapter three discusses previous econometric models of production technology and firm behaviour under production risk. Issues in econometric panel data estimation is discussed in Chapter four. This chapter also presents some estimators for heteroskedastic panel data models. Chapter five assesses the performance of different estimators by means of simulation studies on finite samples.

Chapters 6-9 deal with the empirical application: In Chapter six several issues which have consequences for econometric modelling of the production technology of the salmon aquaculture industry are discussed. In Chapter seven a discussion of the nature of risk and risk responses in Norwegian salmon farming is provided. In Chapter eight the Norwegian salmon farm data set is presented. Econometric models that facilitate testing of our hypotheses on Norwegian salmon aquaculture production technology are specified and estimated in Chapter nine. Finally, Chapter ten provides summary and conclusions.

2. THE THEORY OF THE COMPETITIVE FIRM UNDER PRODUCTION RISK

This chapter provides the theoretical motivation for analysing the structure of risk in stochastic production technologies. Furthermore, it motivates the use of a primal approach in econometric productivity analyses in stead of the popular dual approaches. This chapter demonstrates that dual approaches loose much of their attractiveness when production risk is introduced into the neo-classical production function. A primal model framework which is tractable for econometric implementation is also presented here.

In the standard Expected Utility (EU) model the economic agent plays a passive role in the sense that he or she has no possibilities to alter the distribution of the objective function to be maximised. Broadly speaking, the standard EU model is limited to analysing lottery-type decision problems. In the case of voluntary risk, the agent can only decide whether to participate or not. If the agent chooses to participate in the gamble, he can only stand on the sideline and watch the dice roll, without being able to affect its outcome.

In the theory of the firm under uncertainty, the agent (i.e., the firm) has a set of instruments available to affect the probability distribution of his objective function. In addition to deciding whether to participate or not, i.e., to produce or not, the firm is also able to affect the mean and the variance of the objective function through adjustment of input (and thus output) levels. The extension of the EU model to the firm thus makes the decision problem more interesting. But, as we will see in this chapter, the analytical results are also complicated by allowing the firm to affect both the mean and variance of profits (or wealth) through input-choices.

The EU model of the competitive firm is a member of a broad range of maximisation problems that have been considered in the EU theory of choice under uncertainty. Many of these can be fitted into the following general framework:

$$\max_{\alpha} E[U(\phi(\theta,\alpha,W_0))],$$

where $U(\cdot)$ is a von Neumann-Morgenstern utility function, α is a control variable (assumed to take positive values), θ is an economically relevant random variable, W_0 is initial wealth, and $\phi(\cdot)$ is a function mapping actions α and realisations of θ into outcomes, normally taken to be wealth levels.[1] In the theory of the firm the control variable α might be the production level y or a vector of input levels $\mathbf{x}$. The random variable θ might be the production level y or the

[1] Quiggin (1993, pp. 28-31) provides a discussion of the conditions which must be satisfied for a unique optimum to exist for the above EU general control problem. Furthermore, he provides the assumptions with respect to the function $\phi(\cdot)$ which are necessary to obtain clear comparative results.

output price p, or both. The argument of the utility function, $\phi(\cdot)$, might be the profit function plus initial wealth W_0.[2]

In the EU control problem one should distinguish between the *direct* outcome variable $\phi(\cdot)$ and the *indirect* outcome variable θ.[3] In the EU maximisation problem of the competitive firm the indirect outcome variable is output or output price, while the direct outcome variable is profits plus initial wealth. If the direct outcome variable is a positive linear transformation of the indirect outcome variable, which is the case, for example when output price is the only source of uncertainty in the final wealth function, comparative static analysis of changes in the probability density function of θ is relatively easy. However, if the indirect outcome variable enters $\phi(\cdot)$ in a nonlinear fashion, which is generally the case when production uncertainty is present, comparative statics is much more complicated. In the latter case it may be impossible to obtain unambiguous comparative static results.

Output risk is present in most types of agri- and aquacultural production, although the extent of output risk may vary substantially across the various crops and species. In the case of output risk the distributional properties of output has consequences for the optimal input combination and output of the risk averse firm. When a firm alters the level of an input, it may not only change the mean output, but also the variance of output and the skewness of output. For a risk averse firm the optimal quantity of an input will be higher if an increase in the input quantity only leads to a higher expected output, than if an increase leads to both a higher mean and a larger variance of output. Furthermore, the optimal input quantity will be higher if an increase in input does not alter the skewness of the output distribution, than if the increase leads to a more positively skewed output distribution, *ceteris paribus*, because the latter implies that the probability of low output outcomes increases.

A short digression on terminology is also required. The terms 'uncertainty' and 'risk' are frequently used analogously in the literature, e.g. Quiggin (1993, p. 4). However, according to Knight (1921) a situation is said to involve risk if the randomness facing an agent can be expressed in terms of specific (objective or subjective) numerical probabilities to the possible outcomes. Uncertainty is present if the agent cannot (or does not) assign probabilities to the possible outcomes. In a complex world, it is not possible to assign objective probability distributions to random prices or output levels. EU models of the firm generally assume that the firm forms subjective expectations on the probability distribution of random variables. Thus, these are models of risk in the terminology of Knight. However, in the tradition of Sandmo (1971) and other contributions to firm behaviour under uncertainty, we will use the two terms interchangeably in this thesis.

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[2] For an individual decision maker who is the sole owner of a firm, wealth W can be defined as the firm's equity plus the market value of physical assets and bank deposits.

[3] Robison & Barry (1987, pp. 199-211) provide a discussion of indirect and direct outcome variables.

In this chapter we present some theoretical models of the competitive firm when the production function is stochastic. The point of departure will be some requirements for the stochastic production function are presented (section 2.1), the so-called Just-Pope postulates. Then we present different functional forms for output risk, such as the additive, multiplicative and Just-Pope stochastic production functions, and discuss their properties (section 2.2). In particular the conformity of the various production functions with the Just-Pope postulates is assessed. Next, models of the competitive firm under production uncertainty are outlined (section 2.3). In section 2.4 dual models, which, in principle, are tractable for empirical research are presented. Section 2.5 discusses efficiency issues and technology adoption issues in the context of output risk. Finally, the results from theoretical models are summarised and their tractability for empirical research is discussed in section 2.6.

2.1. Just-Pope Postulates for the Stochastic Production Function

The following eight postulates have been proposed by Just & Pope (1978) for the stochastic specification of the production function

$$y=f(\mathbf{x}, \varepsilon),$$

which they claim to be reasonable on the basis of *a priori* theorising and observed behaviour.

P1. Positive production expectations, i.e., $E[y]>0$.

P2. Positive marginal product expectations, i.e.,

$$\frac{\partial E(y)}{\partial x_k} > 0 .$$

P3. Diminishing marginal product expectations, i.e.,

$$\frac{\partial^2 E(y)}{\partial x_k^2} < 0 .$$

P4. A change in the variance for random components in production should not necessarily imply a change in expected output when all production factors are held fixed, i.e.,

$$\frac{\partial E(y)}{\partial \mathrm{var}(\varepsilon)} = 0 \text{ possible.}$$

P5. Increasing, decreasing or constant marginal risk should all be possibilities, i.e.,

$$\frac{\partial \mathrm{var}(y)}{\partial x_k} <=> 0 \text{ possible.}$$

P6. A change in risk should not necessarily lead to a change in factor use for a risk-neutral (profit-maximising) producer, i.e.,

$$\frac{\partial x_k^*}{\partial \mathrm{var}(\varepsilon)} = 0 \text{ possible,}$$

where x_k^* is the optimal input level.

P7. The change in variance of marginal product with respect to a factor change should not be constrained in sign a priori without regard to the nature of the input, i.e.,

$$\frac{\partial \operatorname{var}(\partial y / \partial x_k)}{\partial x_j} <=> 0 \text{ possible.}$$

P8. Constant stochastic returns to scale should be possible, i.e.,

$$f(\theta \mathbf{x}) = \theta f(\mathbf{x}) \text{ possible for scalar } \theta.$$

The postulates P1-P3 and P8 are analogue to postulates for the standard deterministic neoclassical production function (Chambers, 1988, pp. 8-14; Driscoll, McGuirk, & Alwang, 1992). Postulates P4-P7 are concerned with the structure of production risk, and thus represent an extension of the neoclassical postulates. Of particular interest is postulate P5, which states that the specification of the production function should not restrict the effect of a change in the level of an input on the variance of output *a priori*. For an econometric specification this means that for some parameter values var(y) increases in x_k, for some values the input level x_k has no effect on var(y), and for some parameter values var(y) decreases in x_k.[4] Later in this chapter it is demonstrated that the marginal risk properties of inputs have consequences for the optimal input vector of a risk averse firm.

In the following section the conformity of popular stochastic specifications of the production function with the Just-Pope postulates P1-P8 is discussed.

It should also be noted that none of the Just-Pope postulates address the issue of "heteroskewness", or more generally, the possibility that higher moments of the conditional output probability distribution are functions of the input vector **x** (see section 2.2). Both Yassour, Zilberman & Rausser (1981) and Antle (1983, pp. 193-4) have shown that higher moments may affect the optimal input levels of the EU maximising firm.

2.2. Functional Form of Output Risk

Newbery & Stiglitz (1981, p. 65) provide the following general form for the production function under output risk

$$y = f(\mathbf{x}, \varepsilon, \xi),$$

[4] Just & Pope motivate this postulate with examples from agriculture: "For example, agricultural inputs such as land, fertilizer, and chemical thinning practices seem to make a positive contribution to variance of production in some cases. On the other hand, pesticides irrigation, frost protection, disease-resistant seed varieties, and overcapitalization all possibly have a negative effect on the variance of production attributable to weather, insects, and crop diseases..." (Just & Pope, 1978, p. 69).

where **x** is a vector of inputs, ε is the random variable which describes the state of the nature, and ξ is the choice of technique of production (e.g. timing of smolts release, harvesting, etc.). The above form is too general to be tractable for econometric work.

A functional form that has been suggested for output uncertainty is the Just-Pope production function, which is given by (Just & Pope, 1978)

(2.1) $y = f(\mathbf{x}) + h(\mathbf{x})\varepsilon$,

where ε is a stochastic term and $E[\varepsilon]=0$. The effect of input changes has been separated into two effects; the effect on mean and the effect on variance. The function $f(\mathbf{x})$ is the *mean production function* and $h(\mathbf{x})$ is the *variance production function*. The Just-Pope production function is a heteroskedastic specification, because the variance of y is a function of the input vector **x**, i.e., $var(y|\mathbf{x})$. The conditional variance of output is

$$\mathrm{var}[y] = \mathrm{var}[f(\mathbf{x}) + h(\mathbf{x})\varepsilon] = \mathrm{var}[h(\mathbf{x})\varepsilon] = [h(\mathbf{x})]^2\ \mathrm{var}[\varepsilon].$$

Mean output is

$$E[y] = E[f(\mathbf{x}) + h(\mathbf{x})\varepsilon] = f(\mathbf{x}) + h(\mathbf{x})E[\varepsilon] = f(\mathbf{x}).$$

Most important, Just & Pope (1978, p. 85) show that the Just-Pope form satisfies all eight Just-Pope postulates for a stochastic production function. We will later see that this particular form has been extensively used in econometric analyses of stochastic production technologies.

The additive homoskedastic production function

(2.2) $y = f(\mathbf{x}) + \varepsilon,\ E[\varepsilon] = 0,\ var(\varepsilon) = \sigma^2$,

has been used extensively in the literature. For example, in salmon farming homoskedastic output risk implies that a disease reduces the production by a constant quantity, regardless of the size of the production. The homoskedastic production function (2.2) is a special case of the Just-Pope form (2.1), with $h(\mathbf{x}) = 1$. Under homoskedastic risk the mean and variance of output is

$$E[y] = f(\mathbf{x}) + E[\varepsilon] = f(\mathbf{x}) + 0 = f(\mathbf{x}) \text{ and } \mathrm{var}(y) = \mathrm{var}(\varepsilon) = \sigma^2,$$

respectively. Marginal risk is zero for all inputs. Additive homoskedastic risk is difficult to justify for agri-/aquacultural production in general, and for salmon farming in particular, because of the implicit assumption of zero marginal output risk in inputs. It can be shown that the homoskedastic form will always violate the Just-Pope postulates P5, P7 and P8.

Production functions with multiplicative risk of the form

(2.3) $y = f(\mathbf{x})\varepsilon,\ E[\varepsilon] = 1$,

have also been used in the literature (Newbery & Stiglitz, 1981). For salmon farming, for instance, multiplicative risk implies that a disease reduces the production by a constant fraction, regardless of the size of the production. It can be seen that (2.3) is a special case of the Just-Pope form. The mean and variance of output is

$$E[y] = f(\mathbf{x})E[\varepsilon] = f(\mathbf{x}) \qquad \text{and} \qquad \text{var}(y) = [f(\mathbf{x})]^2\,\text{var}(\varepsilon)$$

respectively. Marginal risk is given by

$$\partial\,\text{var}(y)\,/\,\partial x_i = \text{var}(\varepsilon)\partial[f(\mathbf{x})]^2\,/\,\partial x_i = 2f(\mathbf{x})f_i(\mathbf{x})\,\text{var}(\varepsilon) \geq 0\,, \qquad i=1,\ldots,n,$$

if positive marginal product of input is assumed for all inputs. The multiplicative production function is a special case of a stochastically separable production function. It can be shown that the multiplicative form will always violate the Just-Pope postulates P5 and P7.

Another class of stochastic production functions which is discussed by Just & Pope, and which later has been employed in empirical work by Kumbakhar (1993), is

(2.4) $y = f(\mathbf{x})e^{h(\mathbf{x})\varepsilon}$.

In the context of the popular translog parametrization of $f(\mathbf{x})$, this form is more convenient to work with than the Just-Pope, because unlike the Just-Pope, a translog specification can be linearized by taking logarithms on both sides. This facilitates estimation of $f(\mathbf{x})$ by OLS (in the first step).

Assuming $\varepsilon \sim N(0,\sigma)$, the mean and variance of output is

$$E[y] = f(\mathbf{x})e^{h^2(\cdot)\sigma/2} \qquad \text{and} \qquad \text{var}(y) = f^2(\mathbf{x})e^{h^2(\cdot)\sigma}\left[e^{h^2(\cdot)\sigma/2} - 1\right]$$

respectively. Wee see that $E[y] \geq f(\mathbf{x})$. Marginal risk is given by

$$\partial\,\text{var}(y)\,/\,\partial x_i = 2\left[e^{h^2(\mathbf{x})\sigma/2} - 1\right]E[y]\partial E[y]/\partial x_i + E^2[y]\sigma h(\mathbf{x})h_i(\mathbf{x})e^{h^2(\mathbf{x})\sigma},$$

where $\partial E[y]\,/\,\partial x_i = f_i(\mathbf{x})e^{h^2(\mathbf{x})\sigma/2} + f(\mathbf{x})\sigma h(\mathbf{x})h_i(\mathbf{x})e^{h^2(\mathbf{x})\sigma/2}$ and $h_i(\mathbf{x})$ is the partial derivative of $h(\cdot)$ with respect to input i. Production function (2.4) always violates two of the postulates set forth by Just & Pope; postulate P4 of independence between mean output E[y] and the random term ε, and postulate P8 of the possibility of constant stochastic returns to scale (Just & Pope, pp. 83-84). The remaining six postulates may also be violated, depending on the values of the parameters of $f(\mathbf{x})$ and $h(\mathbf{x})$.

A functional form that is popular in econometric productivity analysis is

(2.5) $y = f(\mathbf{x})e^{\varepsilon}$, $E[\varepsilon] = 0$.

It has been common to use Cobb-Douglas or translog specifications and take logarithms on both sides to facilitate use of linear estimation techniques. Specification (2.5) is a special case of (2.4), with $h(\mathbf{x}) = 1$. The mean and variance of y is

$$E[y] = f(\mathbf{x})E[e^{\varepsilon}] \qquad \text{and} \qquad \text{var}(y) = [f(\mathbf{x})]^2\text{var}(e^{\varepsilon}),$$

respectively. The expression for var(y) implies that marginal risks are restricted to be positive for all inputs due to the positive marginal product assumption, $\partial f(\mathbf{x})/x_i$, for production functions. This production function always violate Just-Pope postulates P4-P7.

If the production process is risky, but we have little *a priori* information on the structure of production risk, the Just-Pope form (2.1) is preferable to the specifications (2.2)-(2.5) in empirical work, because it imposes the smallest set of restriction on the stochastic technology. Furthermore, an assessment of different stochastic production functions with respect to conformity with the Just-Pope postulates P1-P8 shows that the Just-Pope form is the only econometric tractable specification that satisfies all eight postulates. This explains to a large extent the popularity of this specification in econometric studies of production risk. In Chapter 3 we will see that most econometric studies of production risk have applied the Just-Pope form.

However, the Just-Pope specification has also been subject to criticism. Although the Just-Pope form is flexible with respect to the effect of input changes on the first two moments of the output distribution, it can be shown that it restricts the effects of inputs **x** across higher moments (Antle, 1983). To see this, note that with $u \equiv h(\mathbf{x})\varepsilon$,

$$E[u_i] = h(\mathbf{x})^i E[\varepsilon^i] \equiv \mu_i.$$

For $i>2$ and $E[\varepsilon^i] \neq 0$ the parameters of the ith moment are directly related to the parameters of the second moment; in particular the elasticity of the ith moment with respect to input k is

$$\eta_{ik} = \frac{\partial \mu_i}{\partial x_k}\frac{x_k}{\mu_i} = i \cdot \frac{\partial h(\mathbf{x})}{\partial x_k}\frac{x_k}{h(\mathbf{x})} = \frac{i}{2} \cdot \eta_{2k}, \qquad i>2. \tag{2.6}$$

Therefore, the elasticity of each higher nonzero moment with respect to an input is directly proportional to the elasticity of the second moment with respect to that input. The restrictions in (2.6) are valid if output conditional on inputs **x** follows a two-parameter distribution, such as the normal distribution, otherwise they are generally not valid.

2.3. Models of the Competitive Firm under Production Uncertainty

This section presents theoretical models of the competitive firm under output risk. The discussion here will serve to illustrate how introduction of output risk complicates comparative statics, thus making it difficult to obtain unambiguous results similar to those we are familiar with from the theory of the competitive firm in the deterministic setting. Since the purpose of the presentation here is not to show mathematically how comparative static results were obtained, the discussion is deliberately kept at a non-technical level, unless when judged necessary to illustrate some particular points.[5]

The firm's EU maximisation problem under production risk of the general form $y = f(\mathbf{x}, \varepsilon)$ and output price certainty is

$$\max_{\mathbf{x}} \mathrm{E}\,U(W(\mathbf{x})) = EU(W_0 + pf(\mathbf{x},\varepsilon) - \mathbf{w}'\,\mathbf{x}),$$

[5] The cited references will provide more rigorous mathematical derivation of results.

where $U(\cdot)$ is a von Neumann-Morgenstern utility function, W_0 and W are initial wealth and final wealth, respectively, p is output price, $\mathbf{x}$ is a vector of input levels, and $\mathbf{w}$ is a vector of corresponding input prices.

Most models of the competitive firm under production risk use profits π as argument of the utility function instead of end-of-period wealth W. Appendix 2.B discusses some issues that surround the argument of the utility function, including the implications of choosing π as argument for $U(\cdot)$.

Pope & Kramer (1979) propose a model of production risk where the competitive firm maximises expected utility in profits. Their model assumes that the firm's risk preferences are characterised by decreasing absolute risk aversion (DARA) in profits, that there are two inputs in the production process, and that the marginal products of the two inputs are positive and diminishing.[6] Comparative statics are provided for two stochastic specifications of the firm's production function; the multiplicative form $y = f(\mathbf{x})g(\varepsilon)$, and the more general Just-Pope form $y = f(\mathbf{x})+h(\mathbf{x})\varepsilon$. As stated earlier in this chapter, the Just-Pope form satisfies the Just-Pope postulates for stochastic specifications of the production function, while this is not the case for the multiplicative form. In particular, the multiplicative form does not allow for decreasing marginal risk in inputs.

Pope & Kramer provide the following results for the Just-Pope specification of the production technology for a mean-preserving increase in risk:[7] If the two inputs are stochastic complements and both inputs marginally increase (reduce) risk, then factor use declines (increases) as risk increases.[8] Furthermore, if the inputs are stochastic substitutes and only one input marginally decreases risk, then the use of the other input decreases in risk.

Pope & Kramer also examine the effects of different levels of absolute risk aversion on input demands in the context of a special case of the Pratt-family of utility functions exhibiting decreasing risk aversion (Pratt, 1964): Under stochastic complementarity and marginally decreasing (increasing) risk for both inputs, the firm with greater risk aversion will utilise larger (smaller) quantities of both inputs. Further, if only one of the inputs marginally reduces risk under stochastic substitution, then increased risk aversion implies an increase in the use of this input. Pope & Kramer also find that input demand curves are downward sloping in own prices if both factors marginally increase (decrease) risk under complementarity (substitution).

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6 See appendix 2.A for a definition of DARA.

7 A mean-preserving increase in risk means a shift in the probability distribution of ε that keeps $E[\varepsilon]$ constant, while $\mathrm{var}[\varepsilon]$ increases.

8 For the Just-Pope function with two inputs stochastic complementarity (substitutability) means that $\partial^2 y/\partial x_1 \partial x_2 = \partial^2 f(\cdot)/\partial x_1 \partial x_2 + \partial^2 h(\cdot)/\partial x_1 \partial x_2 \varepsilon > 0$ (<0). Concavity of production does not imply restrictions on $\partial^2 y/\partial x_1 \partial x_2$.

Pope & Kramer find that it is difficult, within their model framework, to obtain unambiguous results with respect to the effects of input and output price changes on input demands. Additional assumptions on the structure of the production technology have to be imposed in order to obtain unambiguous results. They find that if both factors marginally increase (decrease) risk under stochastic complementarity (substitution), then factor demand curves are downward sloping. Pope & Kramer are unable to sign the effects of an output price change on input demands without imposing several restrictions on the stochastic production function.

Leathers & Quiggin (1991) use the Just-Pope production function and the results of Meyer (1987) to obtain comparative statics results for a risk averse competitive firm. They use the results of Meyer (1987), who showed that the expected utility function $EU(\pi(x))$ can be represented by a mean-standard deviation model $V(\mu_\pi, \sigma_\pi)$, where μ_π and σ_π are mean profits and standard deviation of profits, when the probability distribution of the objective function is a linear transformation of the random variable. An attractive property of Meyer's approach is that, unlike the traditional mean-variance model, it does not require any additional assumptions about the form of the utility function or the distribution of the random variable, ε. The probability density function (pdf) of ε is, for example, allowed to be skewed.

Leathers & Quiggin show that Meyer's condition is actually satisfied when the stochastic production function is of the Just-Pope form. They can therefore utilise the mean-standard deviation approach of Meyer instead of the EU model framework. This makes it possible to obtain comparative static results that are not available in the EU framework. Leathers & Quiggin derive the comparative statics for a single-input production technology, but their approach is also valid in the multi-input case. It is shown that the mean-standard deviation function is consistent with the EU function.

Leathers & Quiggin presents the following analytical results for a risk averse producer and risk-*reducing* input:

(a) Input use decreases in own input price under increasing and constant absolute risk aversion (IARA, CARA).[9]

(b) Input use increases when the output price increases under constant absolute risk aversion (CARA).

(c) Input use increases under a mean-preserving increase in exogenous yield risk under decreasing and constant absolute risk aversion (DARA, CARA).

(d) Input use decreases under a variance-preserving increase in mean yield under decreasing and constant absolute risk aversion (DARA, CARA).

The above results for a risk-reducing input can be summarised under decreasing absolute risk aversion (DARA), which has been established as a stylised fact in the literature. According to

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9 See appendix 2.A for a definition of CARA, DARA, and IARA.

result (a) the effect of an increase in own input price on input demand is ambiguous under DARA. This is also the case for the effect of an increase in the output price. Result (c) states that an exogenous increase in yield risk, holding expected yield constant, will lead to a decrease in input demand under DARA. An increase in the mean of the exogenous production shock holding yield variance constant, will decrease the use of a risk-reducing input by DARA producers.

Furthermore, for a risk averse producer and risk-*increasing* input, Leathers & Quiggin show that:

(a) Input use decreases in own input price under DARA, CARA.
(b) The effect of a change in output price on input use is indeterminate under any assumption on the coefficient of absolute risk aversion.
(c) Input use decreases under a mean-preserving increase in exogenous yield risk under DARA, CARA.
(d) Input use increases under a variance-preserving increase in mean yield under DARA, CARA.

The above results can be summarised as follows for a producer with DARA risk preferences: An increase in own input price leads to a decrease in the demand for the risk-increasing input. The effect of an increase in output price cannot be determined. A mean-preserving increase in exogenous yield risk causes a decrease in the demand for the risk-increasing input. Finally, a variance-preserving increase in exogenous mean yield leads to an increase in input demand.

The ambiguous results from Leathers & Quiggin's model suggest that under production risk, policies aimed at altering output or input levels cannot be based on theory alone. Rather, empirical knowledge of the production function and the risk attitudes of producers is required in order to prescribe policies that obtain the desired objectives.

Ramaswami (1992) examines the impact of production risk on a producer's optimal input decisions, and presents the weakest conditions on the production technology that are sufficient to sign the marginal risk premium for all risk averse preferences. Output y has the conditional cumulative density function $F(y|\,\mathbf{x})$, where F is assumed twice differentiable and partial derivatives F_y, F_{x_i}, F_{yy} and $F_{x_i y}$ exists. Furthermore, F is contained in a compact interval $[y_0, y_m]$, i.e., $F(y_0|\,\mathbf{x}) = 0$ and $F(y_m|\,\mathbf{x}) = 1$ for all $\mathbf{x}$. An increase in input use is assumed to lead to a superior output distribution in the sense of first degree stochastic dominance. The producer maximises

$$\mathrm{E}\,U[\pi(y|\mathbf{x})] = U[\mathrm{E}\,\pi(y|\mathbf{x}) - r(\mathbf{x})],$$

where $\pi(y|\mathbf{x}) = y - \mathbf{w}'\,\mathbf{x}$ is normalised profits, and $r(\mathbf{x})$ is the risk premium the producer is willing to pay in order to eliminate all output risk. The first-order condition for the EU maximisation problem evaluated in EU maximising input levels $\mathbf{x}^*$ is

$$(E\pi)_{x_i} - r_{x_i}(\mathbf{x}^*) = 0,$$

or $$-w_i + \int (y - \mathbf{w}'\mathbf{x}^*) F_{yx_i}(y|\mathbf{x}^*) dy = r_{x_i}(\mathbf{x}^*),$$

or $$\int (y - \mathbf{w}'\mathbf{x}^*) F_{yx_i}(y|\mathbf{x}^*) dy = w_i + r_{x_i}(\mathbf{x}^*).$$

The marginal risk premium r_{x_i} of input i is the wedge between input cost and expected marginal product at the EU maximising level of input use. The sign of the marginal risk premium indicates whether the optimal level of input use is smaller for risk averse producers than risk neutral producers. Ramaswami proves that, for all risk averse producers, the marginal risk premium is positive (negative) if and only if the input is risk-increasing (decreasing). This is an important result, because it means that it is sufficient to obtain information on the marginal risk of an input, e.g. by estimating the parameters of a Just-Pope production function, in order to determine whether a risk averse producer uses less of the input than a risk-neutral producer.

For completeness, a model of the competitive firm by Ratti & Ullah (1976) with a somewhat different structure of production risk is also presented. The model assumes that output is uniquely determined by a given input vector, but the flow of services from the inputs is randomly distributed. More specifically, factor services are given by $K_1 = uK$ and $L_1 = vL$, where K and L are the quantities of capital and labour employed by the firm, and u and v are positive, independently distributed, random variables. Consequently, K_1 and K_2 are the random quantities of factor service actually rendered by capital and labour. As in the previous models the firm maximises the expected utility of profits:

$$\max_{K,L} \mathrm{E}U(\pi) = \mathrm{E}U[(pf(K_1, L_1) - rK - wL],$$

where $f(K_1, L_1)$ is the production function of the firm. Under plausible assumptions with regard to the production function, the first-order conditions of the above maximisation problem provides the following results: (a) Under output uncertainty the risk averse firm demands less of both inputs than the risk neutral firm, and consequently the expected output is smaller for the risk averse firm. (b) The risk averse firm demands less of both inputs than it would under certainty. (c) More interesting, even a risk neutral firm demands less of both inputs than under certainty. The latter result is different from that provided by other models of the competitive firm under uncertainty, in which the input demands of the risk neutral firm is the same under certainty and uncertainty. The reason for Ratti & Ullah's result, is that in their model, profits are a concave function of the random variables u and v.

With respect to changes in the moments of v and u, Ratti & Ullah are only able to obtain determinate analytical results for the risk neutral firm. They show that for the risk neutral firm an increase in the expected flow of labour services from a given level of labour input leads to an increase in the quantity demanded of labour, provided that the marginal product of labour services is greater than minus unity. Furthermore, under certain assumptions on the production technology, an increase in risk leads to a decline in input demand and consequently expected output of the risk neutral firm. In other words, risk aversion is not necessary for changing optimal input levels when the level of risk changes in Ratti & Ullah's model.

Antle (1983, pp. 193-4) presents an EU model of the competitive firm which addresses the effects of output heteroskewness on the optimal input choice. The firm's risk preference structure is assumed to be described by the negative exponential utility function

$$U(\pi) = a - be^{-A\pi},$$

where a, b and A are positive parameters. For simplicity, prices are assumed to be nonstochastic. Normalised profit is defined as

$$\pi = y - \sum_{i=1}^{n} r_i x_i,$$

where r_i is the ith input price divided by the output price. An mth order Taylor series expansion of $U(\pi)$ about expected profit $\bar{\pi}$ gives

$$E[U(\pi)] = a - be^{-A\bar{\pi}} - be^{-A\bar{\pi}} \sum_{i=2}^{m} \frac{(-A)^i}{i!} \mu_i .$$

To further simplify the discussion, consider a third-order expansion of the utility function. The first-order condition for maximisation of expected utility can then be written as

$$\text{(2.7)} \quad \frac{\partial E[U(\pi)]}{\partial x_k} = \frac{\partial \mu_1}{\partial x_k} + \delta^{-1} \frac{(-A)}{2} \frac{\partial \mu_2}{\partial x_k} + \delta^{-1} \frac{(-A)^2}{6} \frac{\partial \mu_3}{\partial x_k} = r_k, \qquad k = 1, \ldots, n,$$

where

$$\delta = 1 + \frac{(-A)^2}{2} \mu_2 + \frac{(-A)^3}{6} \mu_3 .$$

Equation (2.7) can be rewritten as

$$\text{(2.8)} \quad \eta_{1k} + \delta^{-1} \frac{(-A)}{2} \frac{\mu_2}{\mu_1} \eta_{2k} + \delta^{-1} \frac{(-A)^2}{6} \frac{\mu_3}{\mu_1} \eta_{3k} = \frac{r_k x_k}{\mu_1}, \qquad k = 1, \ldots, n,$$

which shows that the firm's behaviour can be expressed in terms of the elasticities of moments with respect to inputs. Equation (2.8) shows that as the coefficient of absolute risk aversion A approaches zero, inputs are chosen such that the mean production elasticity η_{1k} equals the mean factor share $r_k x_k / \mu_1$, as would be the case for a risk-neutral firm. For large positive values of A, (2.8) shows that the equilibrium condition of the risk-neutral firm generally is not satisfied. Of course, the importance of the third order moment μ_3 for the optimal input levels of the firm, depends on to what extent it deviates from zero, i.e., to what extent the pdf of output is skewed. This is an empirical question that can be investigated by econometric analysis of empirical data.

The importance of higher moments is also determined by the structure of the firm's utility function. In Antle's model, the firm's attitudes to risk was represented by the negative exponential utility function, which is a very restrictive and questionable representation of the firm's risk preference structure. For the general utility function $U(W)$, the firm's level of *downside* risk aversion is an important determinant of the effect of the skewness of output pdf on optimal input levels. Menenez, Geiss & Tressler (1980) define downside risk: A distribution is said to have more downside risk than another if it has more dispersion below a specific target or if it is more skewed to the left. An individual is averse to downside risk if he is decreasingly risk averse, i.e., if his utility function has a positive third derivative (equivalently, his marginal utility function is convex).

This section has demonstrated that comparative statics become more ambiguous but richer when production risk is introduced into the model of the competitive firm. The firm's response to price changes now also depends on its risk preference structure and the risk structure of the production function. In general, restrictions have to be imposed on risk preferences and the stochastic production technology in order to obtain unambiguous comparative static results. The results derived from theoretical models of production risk therefore underline the need for econometric model estimates, because these can provide restrictions that make it possible to sign comparative statics.

2.4. Dual Models For Empirical Research

Introduction of production risk means that risk preferences have to be accounted for. This seriously complicates specification of dual functions for empirical work. However, dual model frameworks which represent a step towards empirical tractability have been introduced recently.

Pope & Chavas (1994) characterise cost functions which would be consistent with expected utility maximisation under production uncertainty. When only output price uncertainty prevails, cost minimisation is consistent with maximising expected utility of wealth. However, according to Pope & Chavas the "validity and nature of cost minimisation is much less clear when production is uncertain" (p. 196). Pope & Chavas provide necessary constraints for the firm's cost minimisation problem which are consistent with expected utility maximisation under different types of output level uncertainty. They show that there always exists a nonrandom constraint function for which the *ex ante* cost minimisation problem is consistent with expected utility maximisation. The *ex ante* cost function has the general form

$$c(\mathbf{w}, \mu_y) = \min_{\mathbf{x}} \{ w' x | \mu_y \leq \mathbf{g}(\mathbf{x}, \cdot) \},$$

where μ_y represents the constraints on the relevant moments of output, and $g(\mathbf{x}, \cdot)$ is a vector of moment functions. The structure of the stochastic production technology and the risk preference structure of the firm determines which moments of y are included. Pope & Chavas show that if the stochastic production function takes the multiplicative form $y = f(\mathbf{x})g(\varepsilon)$, cost minimisation subject to a given expected output $\overline{y}$ is consistent with expected utility maximisation. Furthermore, if the stochastic production function takes the Just-Pope form $y = f(\mathbf{x})+h(\mathbf{x})\varepsilon$, then cost minimisation holding mean output $\overline{y}$ and variance of output var(y) constant, is consistent with expected utility maximisation.[10]

A nice property of the *ex ante* cost function derived by Pope & Chavas is that it is devoid of risk preferences. For empirical implementation this means that the researcher does not have to care about the firms' risk preference structure. It is still necessary to make assumptions on the structure of the stochastic production technology. Furthermore, one has to estimate the moments of output, which means that some assumptions have to be made regarding the firm's output expectation formation. A limitation of the cost function approach is of course that it does not explain (endogenize) the firm's output supply decision.

Ex ante cost functions have the same properties in **w** as conventional cost functions; *ex ante* cost functions are nondecreasing in **w**, concave in **w**, positively linearly homogeneous in **w**, and Shephard's lemma holds.

It is important to note that even under risk neutrality, i.e., when the firm maximises expected profit, the appropriate argument of the cost function is expected output $\overline{y}$, not realised output y. This means that the appropriate *ex ante* cost function is $c(\mathbf{w}, \overline{y})$. A cost function that is consistent with expected profit maximisation is obtained by substituting the argument $\overline{y}$ with the distance function, which is the direct dual to the cost function (Pope & Just, 1996). The advantage of embedding the distance function in the cost function is that it is possible to obtain mean output and the cost function parameters simultaneously in an econometric estimation procedure, which is demonstrated by Pope & Just (1996). With this approach it is not necessary to make assumptions on the producer's output expectation formation and estimate the moments of output prior to the estimation of the cost function.

Coyle (1995) presents a dual model which incorporates risk aversion and production risk within the framework of a mean-variance utility function. Utilising the certainty-equivalent version of the mean-variance function, the producer's objective function is

$$U^*(p, \mathbf{w}, W_0, \mathbf{q}) = \max_{\mathbf{x}\geq\mathbf{0}} U(\mathbf{x}) \equiv \{W_0 + p\mathrm{E}y(\mathbf{x}, \mathbf{q}_1)\text{-}\mathbf{wx}$$

$$- \mathrm{A}(W_0 + p\mathrm{E}y(\mathbf{x}, \mathbf{q}_1)\text{-}\mathbf{wx}, p^2\mathrm{V}y(\mathbf{x}, \mathbf{q}_2))/2\, p^2\mathrm{V}y(\mathbf{x}, \mathbf{q}_2)\},$$

where $A(\cdot)$ is the coefficient of absolute risk aversion, $\mathbf{q} = (\mathbf{q}_1, \mathbf{q}_2)'$, and $\mathbf{q}_1$ and $\mathbf{q}_2$ are exogenous vectors of moments influencing mean and variance of output, respectively. Duality between $U^*(\cdot)$ and the primal function $y = f(\mathbf{x}, \varepsilon)$ is established indirectly via the cost function (Coyle, 1995, App. A). A solution to the EU maximising problem in terms of input demands, expected output and variance of output is denoted $\mathbf{x}(p, \mathbf{w}, W_0, \mathbf{q})$, $\mathrm{E}y(p, \mathbf{w}, W_0, \mathbf{q})$ and $\mathrm{V}y(p, \mathbf{w}, W_0, \mathbf{q})$ respectively. These equations are derived from $U^*(\cdot)$. Coyle also presents the properties of $U^*(\cdot)$ and the derived equations.

[10] Under Just-Pope production risk $g_1(\mathbf{x}, \cdot)=f(\mathbf{x})$ and $g_2(\mathbf{x}, \cdot)=h^2(\mathbf{x})\mathrm{var}(\varepsilon)$.

$U^*(\cdot)$ is simplified by assuming constant absolute risk aversion (CARA), or that the primal technology is of the Just-Pope form. Under CARA, which implies that changes in W_0 do not influence utility maximising input choices and that the risk premium in monetary terms is constant, the mean-variance function simplifies to the linear form $U^*(\cdot) = W_0 + E\pi - \alpha(\cdot)/2\ V\pi$ (Coyle, 1995, p. 10). When the production technology is of the Just-Pope form $y = f(\mathbf{x}) + h(\mathbf{x})\varepsilon$, the vector $\mathbf{q}$ of moments of output has the scalar elements $q_1 = E\varepsilon$ and $q_2 = 1/\text{var}(\varepsilon)$ (Coyle, p. 11). Under CARA and Just-Pope assumptions, reduced form equations for $\mathbf{x}(\cdot)$ and $Ey(\cdot)$ are obtained that can generally be estimated by linear methods.

The model is also extended to the case of simultaneous price and production risk. Of course, this complicates the indirect utility function, because moments of the output price and covariance between output price and the output level are now introduced.

In principle, the mean-variance model framework is tractable for empirical research. However, nonlinearities and unobservable variables (e.g. moments of output and output price) will in practice complicate empirical estimation.

A further discussion of the above dual models in the context of empirical research is provided in Chapter 3.

2.5. Technical Efficiency and Technology Adoption

It is natural to ask what implications output risk has for the way efficiency is viewed and measured, and for the process of technology adoption. In this section we take a look at these issues.

2.5.1. Technical and Allocative Efficiency

In the deterministic case efficiency can be represented by profits

$$\pi = p \cdot f(\mathbf{x}) - \mathbf{w} \cdot \mathbf{x},$$

where p is output price, $\mathbf{x}$ is a vector of inputs with associated input price vector $\mathbf{w}$, and $f(\mathbf{x})$ is the (deterministic) production function. Under certainty the firm chooses $\mathbf{x}$ to maximise profits.

Figure 2.1 depicts the profit lines for two technologies. The profit line π^* corresponds to the efficient technology $f^*(\mathbf{x})$, and the profit line π' corresponds to the inefficient technology $f'(\mathbf{x})$.

Two types of inefficiency can be identified: The first type is *technical inefficiency*, which is the difference between the maximum feasible output $y_0^* = f^*(\mathbf{x}^*)$ (point A) and the actual output $y_0' = f'(\mathbf{x}^*)$ (point B) for the chosen input levels $\mathbf{x}^*$. The vertical distance between points A and B is due to technical inefficiency.

The second type of inefficiency is *allocative inefficiency*, which is the difference between profits at profit-maximising input levels and profits at the actual input levels for a given technology $f(\mathbf{x})$, and prices $\mathbf{w}$ and p. In the above figure A represents the allocative efficient point for the technology $f^*(\mathbf{x})$. All other points, including point C, are allocative inefficient.

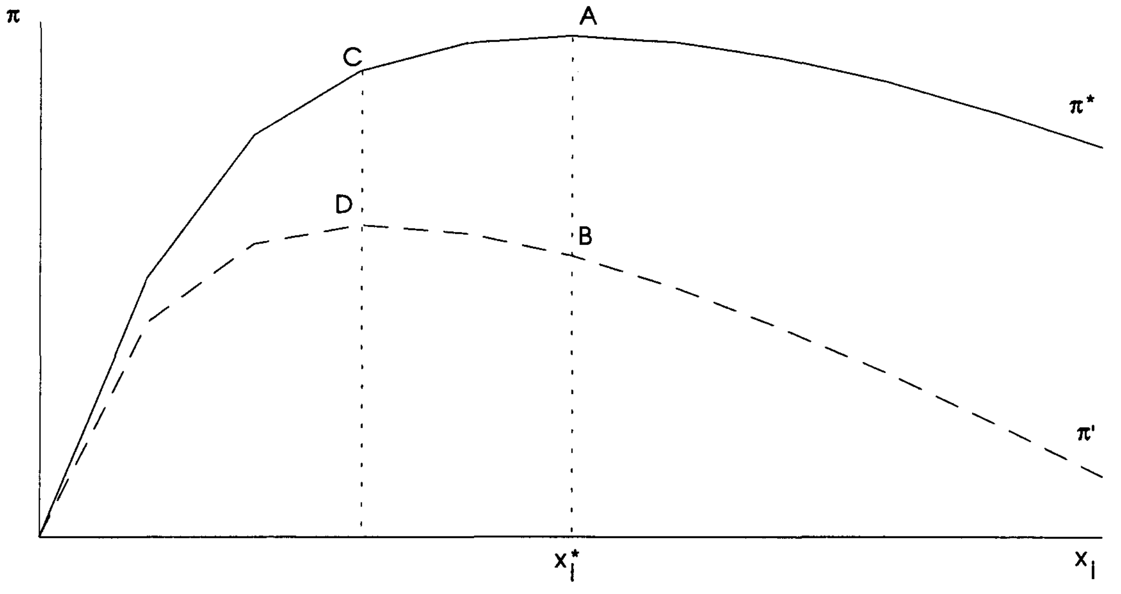

Figure 2.1. Efficiency Analysis under Certainty

Hence, the first-order condition (f.o.c.) for allocative efficiency is the same as the f.o.c. for profit maximisation

$$\frac{\partial \pi}{\partial x_i} = p \cdot \frac{\partial f}{\partial x_i} - w_i = 0.$$

A firm that is both technically efficient and allocatively efficient for the prices (p, $\mathbf{w}$) earns the maximum possible profit, and is called *profit efficient* for (p, $\mathbf{w}$) (Lovell & Scmidt, 1988, p. 7). Assuming that the other inputs are at their profit maximising levels, point A represents profit efficiency.

We now introduce production risk, to see what implications this has for the discussion of efficiency. Parts of the discussion will be in terms of the Just-Pope production function $y = f(\mathbf{x};\alpha) + h(\mathbf{x};\beta)\varepsilon$ and the utility function $U(\pi;\lambda)$, where π is profits and λ is the parameter vector of the utility function.

When production risk is introduced the relevant objective function is no longer profits, but the expected utility derived from profits (or wealth), E[$U(\pi)$]. A dual measure of technical efficiency under output risk accounts for mean output, the variance (and possibly higher moments) of output and the risk preference structure.

An example of a scenario when technical efficiency ranking under production risk may diverge from the deterministic case is the following: Assume that for a given input vector $\mathbf{x}_0$, the Just-Pope production technology 1 has a lower mean output than Just-Pope technology 2, but also a lower output variance, i.e., $E[f^1(\mathbf{x}_0)] > E[f^2(\mathbf{x}_0)]$ and $(h^1(\mathbf{x}_0))^2 > (h^2(\mathbf{x}_0))^2$. Then technology 1 may not necessarily be less technically efficient than technology 2 in terms of expected utility;

this will depend on the mean-variance profit trade-off represented by the producer's utility function. This is illustrated in figures 2.2-2.3.

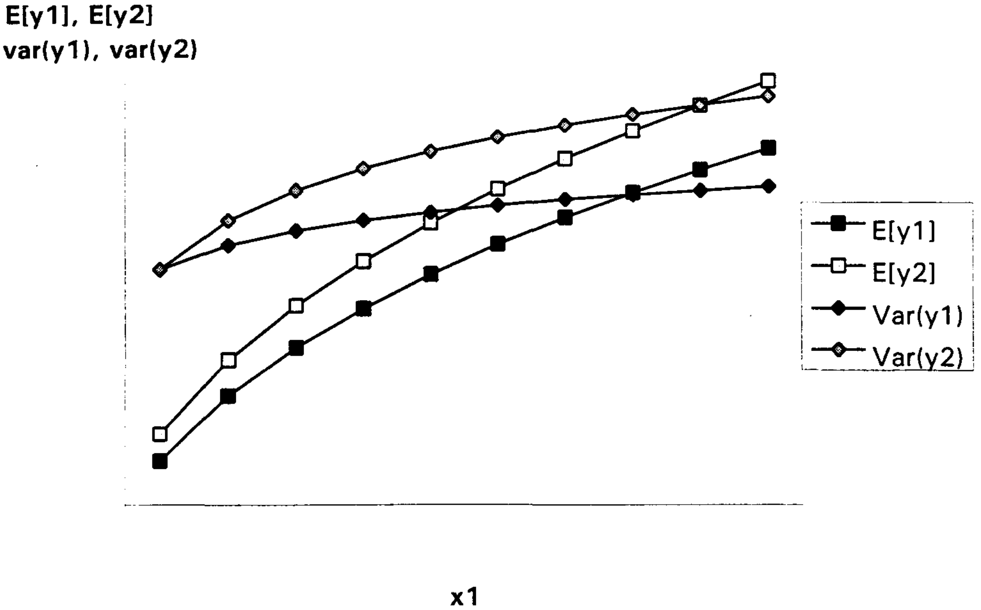

Figure 2.2. Mean and Variance of Just-Pope Production Technology 1 and 2

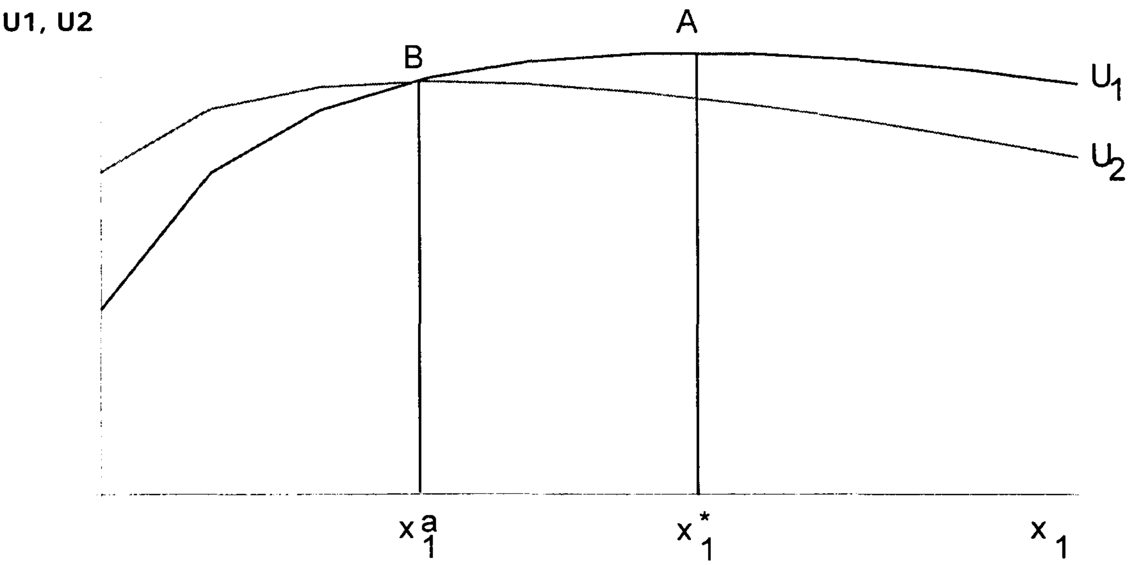

Figure 2.3. Expected Utility Derived from Technology 1 (V_1) and Technology 2 (V_2)

In figure 2.2 we see that technology 1 is characterised by both a smaller mean output and smaller variance of output than technology 2 for all levels of x_1. Figure 2.3 depicts the expected utility of the producer associated with technology 1 (V_1) and technology 2 (V_2) for given utility function parameters λ and prices (p, **w**). For low levels of input x_1 (i.e., $x_1 < x_1^a$), technology 2 provides a higher level of expected utility for the producer than technology 1. This means that the mean effect dominates the variance effect. However, for high levels of x_1 (i.e., $x_1 > x_1^a$), technology 1 gives a higher level of expected utility than technology 2, which means that the variance effect now dominates the mean effect. If the variance of the random term ε had been zero for both technologies, which is equivalent to the certainty case, technology 1 would have provided higher expected utility than technology 2 for all levels of x_1.

In figure 2.3 allocative efficiency is represented by point A on line U_1, corresponding to input level $x_1{}^*$.

The above example assumes homoskewness, i.e., the skewness of the output distribution is not affected by changes in input levels. If the production technology exhibits heteroskewness and the producer is downside risk averse, then the relative technical efficiency of technologies 1 and 2 in terms of expected utility may be different (Antle & Crissman, 1990).

Following Antle & Crissman (1990), a measure of relative technical efficiency (*TE*) which encompasses both the deterministic case and the case of production risk can be defined as

$$TE(\mathbf{x},\delta) = J^l(\mathbf{x},\delta) / J^k(\mathbf{x},\delta),$$

where δ is a vector of parameters. In the certainty case $y = f(\mathbf{x};\alpha)$, the objective function is $J = \pi = p \cdot f(\mathbf{x}) - \mathbf{wx}$, while in the case of Just-Pope production risk $J = E[U(\pi;\lambda)] = E[U(p \cdot (f(\mathbf{x};\alpha) + h(\mathbf{x};\beta)\varepsilon - \mathbf{wx};\lambda)]$. In other words, under certainty the parameters of J are $\delta = (p, \mathbf{w})$, while for the case of production risk $\delta = (p, \mathbf{w}, \alpha, \beta, \lambda)$ Note that *TE* is a function of the input vector $\mathbf{x}$. As $\mathbf{x}$ changes the relative efficiency of technologies k and l could be reversed.

An index of relative allocative efficiency (*AE*) can be defined as

$$AE(\mathbf{x}_i, \mathbf{x}_j, \delta) = J^i(\mathbf{x}_j,\delta) / J^i(\mathbf{x}_i,\delta),$$

where $\mathbf{x}_i$ is the efficient input vector for technology i and $\mathbf{x}_j$ is some other input vector. It is necessary to solve for the efficient input levels to find $\mathbf{x}_i$. Under certainty this means maximising profits, while in the context of production risk expected utility maximising input levels have to be derived. In practice solving for EU-maximising input levels can be a difficult or impossible task (Antle & Crissman, 1990, p. 522).

2.5.2. Technology Adoption

It is a natural extension of the discussion of efficiency under risk to ask what characterises technology adoption under production risk. Often the producer does not have full knowledge of the properties of the stochastic production technology he has adopted, for example, its marginal mean output and marginal output risk in inputs. In such a situation it is important for him to learn about these properties. Experimenting with inputs today, knowingly not optimising in the short run, will provide information about the technology that is useful in optimising behaviour tomorrow (Welch, 1978). Hence, the input decisions in each period become part of a dynamic decision problem, where the producer in each period balances expected utility from short run optimisation in the current period against the gains from experimentation in terms of expected utility in future periods. In the short run, when producers are experimenting and learning about the new technology, they make input decisions that are technically and allocatively inefficient. However, if the producers gain valuable information about the technology, there should be an increase in the measured technical and allocative efficiency over time.

Feder, Just & Zilberman (1985) provide a review and discussion of the theoretical and empirical literature on technology adoption in agriculture. The theoretical literature focuses on the conditions for or degree of adoption of new technology under different assumptions on the structure of production risk, risk preferences, credit constraints, etc. Several of the theoretical models utilise the Just-Pope production function to characterise the structure of production risk (Feder *et al.*, 1985, pp. 258-61).

Another issue is the efficiency of early and late adopters of a new technology. Larger firms, which can exploit economies of scale in information acquisition, may adopt a new technology at an early stage (Feder & O'Mara, 1981). If smaller producers can observe the early adopters, they can skip the experimentation phase and exploit the technology efficiently when they adopt it at later stage.

The condition for adoption of a new technology can be formulated in terms of the firm's utility function when the risk preferences are summarised by

(2.9) $U = U(E[\pi], \mathrm{var}[\pi])$.

Total differentiation of (2.9) yields

$$\mathrm{d}U = U_E \,\mathrm{d}E[\pi] + U_V \,\mathrm{d}\,\mathrm{var}[\pi],$$

where U_E and U_V are the partial derivatives of $U(\cdot)$ with respect to $E[\pi]$ and $\mathrm{var}[\pi]$, respectively. The condition for adoption of a new technology is that $\mathrm{d}U \geq 0$.

By rearranging we obtain (Ghosh, McGuckin, & Kumbhakar, 1994)

(2.10) $$\mathrm{d}E[\pi] \geq -\frac{U_V}{U_E}\mathrm{d}\,\mathrm{var}[\pi],$$

where $-U_V/U_E$ is the Pratt-Arrow coefficient of absolute risk aversion, which is positive under risk aversion (i.e., for concave utility functions). In words, (2.10) states that the firm will adopt a new technology if the increase in mean profits is larger than the increase in the variance of profit weighted by the Pratt-Arrow coefficient of risk aversion. If the firm is risk neutral, then $-U_V/U_E = 0$, which implies that the firm is only concerned about the effect on mean profits of adopting a new technology. Under Just-Pope production risk $y = f(\mathbf{x}) + h(\mathbf{x})\varepsilon$, the mean and variance of profits are $E[\pi] = p \cdot f(\cdot) - \mathbf{w}'\mathbf{x}$ and $\mathrm{var}[\pi] = p^2 \cdot [h(\cdot)]^2 \sigma_\varepsilon^2$, respectively. By substituting these expressions into (2.10), we see that for fixed output price p and input prices $\mathbf{w}$ the firm will adopt a new technology if the increase in mean output is larger than the increase in the variance of output weighted by the Pratt-Arrow coefficient of risk aversion.

2.6. Summary and Conclusions

Although we would like a theoretical model of firm behaviour under production risk to be as general as possible, a minimum set of postulates for the stochastic production technology is required in order to have some conformity with empirical observations and to obtain analytical results. Just and Pope have suggested eight postulates for the stochastic production function which they claim to be reasonable on the basis of *a priori* theorising and observed behaviour. These postulates have been used extensively in subsequent theoretical and empirical research on production uncertainty. The Just-Pope postulates can be viewed as an extension of the postulates suggested for the deterministic production function in the neoclassical production theory.

It has been shown that the Just-Pope form $y = f(\mathbf{x})+h(\mathbf{x})\varepsilon$ satisfies all the Just-Pope postulates. However, other stochastic specifications of the production function which have been employed in theoretical and empirical models of production risk do not perform that well according to the criteria suggested by Just & Pope; all of them violate several of the postulates.

Theoretical models of the competitive firm under production risk use the Expected Utility model framework of von Neumann & Morgenstern. The traditional EU model is extended to allow the firm to alter the probability distribution of the argument of the utility function by changing input levels. The argument of the firm's utility function is usually profit or end-of-period wealth. What complicates the analysis compared with the traditional EU model is that the stochastic production function enters the profit (or end-of-period wealth) function in a nonlinear fashion. This makes it difficult or impossible to obtain unambiguous comparative static results in the general case.

Dual models of producer behaviour under production risk are less tractable for comparative static analysis and econometric implementation than their deterministic counterparts. The dual to the stochastic production function for the competitive EU maximising firm is an indirect utility function which is the solution to the firm's EU maximisation problem, or an *ex ante* cost function which is consistent with EU maximisation. It is generally difficult to derive the indirect utility function, and even if it is recoverable, it is generally difficult to implement econometrically because of nonlinearities and unobservables. The *ex ante* cost function is more tractable for econometric work, partly because it is devoid of utility function parameters. However, for risk averse producers estimation of an *ex ante* cost function requires assumptions on the firms' expectation formation process for the moments of output, unless risk neutrality is assumed. There are also other problems with *ex ante* cost functions which means that the usual appeal of duality is lost in the stochastic production case. In Chapter 3 econometric implementation of dual models will be discussed at greater length.

A more realistic setting for many sectors of biological production, and one which would be appropriate for our empirical application, is the case of simultaneous output price and output uncertainty. The models presented here assume that output risk is the only type of risk present. They are generally not suitable for making predictions on firm behaviour when output price risk is also present; the optimal input levels of the EU maximising firm will generally not be the same in the case when the variance of the output price is zero and in the case when it is larger than zero, even if the mean output price is the same in both cases (Sandmo, 1971). Furthermore, the responsiveness to price changes or changes in output risk will also differ in the two cases. There exists only a limited body of theoretical models which includes both output price and production risk in the literature. Problems are often associated with these models; they may impose very strict restrictions on risk preferences or the stochastic production technology, be intractable for econometric modelling, or preclude testing of certain hypotheses.

We have seen that the introduction of production risk has implications for the way we view technical efficiency. When comparing the technical efficiency of two different production technologies, or measuring technical change over time, the measures of interest are no longer only mean output or profit. A risk averse producer will also be concerned about the riskiness of alternative production technologies, represented by the variance of output. The trade-off between mean, variance and possibly higher moments is represented by the producer's utility function. A risk averse producer may choose a production technology that provides a lower mean output for a given input vector than alternative technologies, when this technology also provides a sufficiently smaller output variance than the alternatives. This has implication for empirical research on productivity in risky production processes. As an illustration, the green revolution has been characterised as the culmination of an era when yield increases were accompanied by higher yield variances (Traxler, Falck-Zepeda, Ortiz-Monasterio R., & Sayre, 1995). Producers that are particularly vulnerable to adverse yield outcomes, such as poor farmers in third-world countries, may have experienced increased income variability and a higher incidence of famine as a consequence of the introduction of new high yielding varieties.

The comparative static results, or the lack of such, from theoretical models presented in the literature strongly suggest that specific assumptions or information on the structure of risk preference and structure of the stochastic production technology are required in order to unambiguously sign the effects of a change in production risk, and changes in input and output prices on input demands and output supply. Unlike the deterministic dual framework, unambiguous results are not available in the general case for the EU maximising firm. An important implication from the theoretical models is that if one wishes to make predictions for a particular industry, a natural first step will be to empirically estimate the parameters of the production technology for that industry. If one can impose restrictions on the production function based on empirical parameter estimates, then one may be able to provide empirical comparative static statements for that industry. The empirical findings may also in a next step be used to simplify a dual model specification.

In the next chapter we will see how the theoretical framework presented here has been used in econometric analyses of production risk. We will see that the extension of the deterministic neoclassical production function to a more flexible form which allow for risk effects of inputs, has complicated specification and estimation of econometric models.

2.A. Appendix: Some Concepts in Expected Utility Theory

Two commonly used risk preference structure measures are the coefficient of absolute risk aversion (ARA) and the coefficient of relative risk aversion (RRA). A third, less used measure, is the coefficient of partial relative risk aversion (PRRA).

For a utility function in risky wealth, $U(W)$, the *coefficient of absolute risk aversion* (ARA), A, is defined as

$$A(W) = -\frac{U''(W)}{U'(W)},$$

and evaluated at some chosen level of final wealth W. It is not a dimensionless measure, and depends on the units in which income is measured.

For a utility function in risky wealth, $U(W)$, the *coefficient of relative risk aversion* (RRA), R, is defined as

$$R(W) = -\frac{U''(W)}{U'(W)}W,$$

and evaluated at some chosen level of income (or wealth) W. As an elasticity it is dimensionless, and hence a very convenient way in which to describe risk aversion. The two measures are related by

$$R(W) = A(W)W.$$

A third measure of risk preference structure, developed by Menenez & Hanson (1970), is the *coefficient of partial relative risk aversion* (PRRA), P, defined as

$$P(W,\pi) = -\frac{U''(W)}{U'(W)}\pi = A(W)\pi.$$

Note that $U''(W)/U'(W)$ is multiplied with profits π instead of W.

Briys & Eeckhoudt (1985) have implicitly shown that the coefficients of absolute, relative and partial relative risk aversion (A, R and P respectively) are related by

$$P(W,\pi) = R(W) - W_0 A(W).$$

Consider a change in profits π. The resulting change in the coefficient of absolute risk aversion, $A(W)$, and the coefficient of relative risk aversion, $R(W)$, is related to the change in the partial relative risk aversion in the following way

$$\frac{dP}{d\pi} = \frac{dR}{d\pi} - W_0 \frac{dA}{d\pi}.$$

The effect of a change in π on P is unambiguous when A and R have opposite signs. When R is increasing (decreasing) and A decreasing (increasing), $dP/d\pi$ is positive (negative).

Most models of producer behaviour under uncertainty apply expected utility in profits instead of wealth. In this case the coefficient of relative risk aversion is identical to the coefficient of partial relative risk aversion.

Having presented the common measures of risk aversion derived from the individual's utility function - ARA, RRA and PRRA - and the risk premium of the individual, we are now in the position to provide some statements about the relationships between them. The ARA, RRA and PRRA measures are useful because each of them provides information on the effect of a certain type of change in *ex ante* final wealth on the individual's risk premium.[11]

The *Markowitz' risk premium*, $r(W_0, \pi)$, is defined as the difference between an individual's expected wealth, given the gamble, and the level of wealth that individual would accept with certainty if the gamble were removed, i.e., his certainty equivalent wealth (Copeland & Weston, 1988, p. 87):

$$r^M(W_0, \pi) = \mathrm{E}[W] - c(W_0, \pi),$$

where $c(W_0, \pi)$ is the certainty equivalent wealth, which is equal to the inverse of the utility function evaluated in expected utility, i.e., $c(W_0, \pi) = U^{-1}(\mathrm{E}[U(W)])$.

The coefficient of absolute risk aversion tells us how an individual will react to a change in initial wealth for a given bet in terms of the risk premium he requires to be indifferent between participating in the gamble and receiving the expected value with certainty. The following relationships exist between $A(W)$ and the effect of a change in initial wealth W_0 on the *risk premium* $r(W_0, \pi)$:

IARA:	Increasing			Increasing	
CARA:	Constant	$A(W)$	===>	Constant	$r(W_0, \pi)$
DARA:	Decreasing			Decreasing	

The following relationships exist between $R(W)$ and the effect of a multiplicative change in W_0 and π, i.e. W_0 and all outcomes of π are multiplied by a constant λ,on the risk premium $r(W_0, \pi)$:

IRRA:	Increasing			Increasing	
CRRA:	Constant	$R(W)$	===>	Constant	$r(W_0, \pi)/\lambda$.
DRRA:	Decreasing			Decreasing	

The following relationships exist between $P(W, \pi)$ and the effect of a multiplicative change in π, i.e. all outcomes are multiplied by a constant λ, on the risk premium $r(W_0, \pi)$:

IPRRA:	Increasing			Increasing	
CPRRA:	Constant	$P(W, \pi)$	===>	Constant	$r(W_0, \pi)/\lambda$
DPRRA:	Decreasing			Decreasing	

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[11] The relationships between ARA, RRA and PRRA, and the risk premium have been more formally discussed by Pratt (1964) and Menenez & Hanson (1970).

2.B. Appendix: The Argument of The Utility Function

The following two issues are important in the specification of utility functions for empirical testing: (1) Should final wealth or profits be used as the argument of the utility function? (2) Can the argument take both negative and positive values? The range of the argument has consequences for the choice of parametric form for the utility function, as some forms are not defined for negative values (e.g. the logarithmic utility function), while others provide "perverse" risk preference measures in terms of ARA or RRA in the case of negative outcomes.

There has been some debate regarding the use of end-of-period wealth or profits as the argument of the firm's utility function.[1] End-of-period wealth is a positive linear function of profits:

$$W = W_0 + \pi ,$$

where W is random wealth at the end of the period, W_0 is certain initial wealth, and π is the flow of profits in the period. End-of-period wealth W increases if realised $\pi>0$, and decreases if realised $\pi<0$.

Katz (1983) pointed out that the measure of relative risk aversion used by Sandmo (1971) and others had profits as argument instead of terminal wealth, i.e., relative risk aversion was defined by

$$R(\pi) = -\frac{U''(\pi)}{U'(\pi)}\pi ,$$

which deviates from the original definition of relative risk aversion provided earlier. In the literature it is generally assumed that terminal wealth is always positive. This is, however, an assumption that not always conforms with empirical observations.[2] If terminal wealth is always positive, the coefficients of absolute and relative risk aversion will always be positive and monotonously decreasing or increasing for the risk averse firm. Profits may be negative in some states of the world, and thus $R(\pi)$ may take both negative and positive values.

Use of profits as the argument of the utility function may imply two things: (a) The special case of initial wealth equal to zero. In this case $W = \pi$. (b) The individual's risk preference structure is such that initial wealth has no influence on the individual's decision. This is the case if the individual is risk neutral, or more generally if the individual's risk preference

1 See Katz's (1983) criticism of Sandmo's (1971) use of profits instead of final wealth as the argument of the firm's utility function, and the subsequent debate (Briys & Eeckhoudt, 1985; Hey, 1985; Katz, 1985).

2 For example, for several years the net wealth of Norwegian salmon farmers, if represented by the equity of the farm, was negative for the industry on average.

structure exhibits constant absolute risk aversion (CARA).[3] In appendix 2.A we saw that CARA implies that the risk premium required by the individual is invariant to changes in initial wealth.

Assumption (b) is contrary to empirical observations, which provide plenty of evidence that the scale of the bet relative to initial wealth to a large extent influences the risk premium required by the individual. If participation in the gamble is voluntary, it determines whether an individual will participate or not. If the magnitude of the bet is such that the individual may loose his entire wealth in the case of an adverse outcome, and he can not buy an insurance, he will probably refrain from participating.

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[3] Under risk neutrality, $U''(W) = 0$, which implies that the coefficient of absolute risk aversion is $A(W) = 0$ =constant.

3. ECONOMETRIC MODELS OF FIRM BEHAVIOUR AND TECHNOLOGY UNDER PRODUCTION RISK

This chapter is concerned with empirical implementation of the theoretical framework provided by models of the competitive firm under production risk provided in Chapter 2. The previous chapter presented theoretical requirements for stochastic production function specifications, the so-called Just-Pope postulates. These postulates have several implications for specification and estimation of econometric models of production. In this chapter we will discuss how earlier empirical studies have dealt with these specification and estimation issues.

In recent years there have been some interesting developments in econometric modelling of producer behaviour under output risk. Not surprisingly, agricultural production has been the subject of most empirical studies, and overall they provide a substantial evidence on the presence of production risk and risk aversion in agricultural decisions. For many empirical studies, however, the methodological approach or the quality of the data may be questionable. Consequently, the empirical results of these studies must be assessed with care. A survey of the literature reveals that there still may be room for methodological improvements, and certainly more empirical studies.

The empirical research on production risk has been overwhelmingly dominated by primal model estimation. To some extent this can be explained by the fact that dual models that in principle may be tractable for empirical application only recently have been provided in the literature. It also turns out that dual models lose some of their attractiveness for empirical work when production risk is introduced, because of unobservables and more complicated functional forms. This chapter will be devoted mainly to discussion of primal specification and estimation issues.

First, in section 3.1, we discuss the problems associated with using traditional neoclassical production function specifications when output heteroskedasticity is present. Just-Pope function approaches are discussed in section 3.2. Section 3.3 presents Kumbhakar's approach to the econometric measurement of a stochastic production technology. Estimation of non-normal production models is discussed in section 3.4. An example of joint estimation of risk preference structure and production technology is presented in section 3.5. Section 3.6 presents studies which deal with econometric measurement of technical change and technical efficiency. Section 3.7 discusses the performance of the FGLS vs. the ML estimator for Just-Pope production function specifications. A discussion of primal versus dual model estimation is undertaken in section 3.8, while section 3.9 provides a summary of the chapter.

3.1. Consequences of Using Traditional Production Function Specifications under Production Heteroskedasticity

Since the introduction of flexible functional forms in the 1970s, a large number of econometric productivity analyses have been presented. Most of these studies have, explicitly or implicitly, assumed a deterministic setting. In terms of production risk, this is probably a relatively safe assumption for a large number of industries, particularly in the manufacturing sector. Production processes in manufacturing are generally characterised by a high degree of control.

However, for many sectors of biological production the assumption of a non-stochastic production technology is more questionable. Variations in temperature, rainfall, diseases and other factors that to a large extent cannot be controlled by producers, are undoubtedly important sources of the substantial yield variations experienced in several sectors. These factors make certain types of production inherently risky, but that does not necessarily mean that the producers are unable to control the level of production risk. Empirical studies indicate that some controllable inputs may increase the level of risk, while other may reduce the level of risk.

While there has been much focus in deterministic productivity analyses on the marginal productivity of inputs, i.e., $\partial y/\partial x_k$, the marginal risks of input, $\partial \text{var}(y)/\partial x_k$, become an issue of interest for the firm when input levels also may affect the level of risk. According the theory of competitive firm under production risk presented in Chapter 2, the firm will take into account both the marginal effect on mean production and on production risk when considering a change in input use. Traditional econometric specifications of the production function are, however, unable to measure the effect of input changes on output risk. Furthermore, they produce biased estimates of the parameters of the mean production function.

These deficiencies of traditional neoclassical production function specifications can be illustrated by the following example: Assume that the "true" production technology is given by the Just-Pope form $y = f(\mathbf{x};\alpha)+h(\mathbf{x};\beta)\varepsilon$, where the mean production function $f(\mathbf{x};\alpha)$ takes the Cobb-Douglas form

$$f(\mathbf{x};\alpha) = \alpha_0 \prod_{k=1}^{n} x_k^{\alpha_k} .$$

In traditional econometric production analyses it has been common to estimate

$$(3.1) \quad y = \alpha_0 \left(\prod_{k=1}^{n} x_k^{\alpha_k} \right) e^{\varepsilon} ,$$

where $\alpha_k > 0$ for all k, ε is an i.i.d. stochastic disturbance with $\text{E}(\varepsilon) = 0$ and $\text{var}(\varepsilon) = \sigma_\varepsilon =$ constant. The conditional variance of output is

$$\text{var}(y) = \alpha_0^2 \left(\prod_{k=1}^{n} x_k^{2\alpha_k} \right) \text{var}(e^{\varepsilon}) ,$$

which gives the marginal risk associated with an increase in input k

$$\frac{\partial \operatorname{var}(y)}{\partial x_k} = \frac{2\alpha_k \alpha_0^2}{x_k}\left(\prod_{k=1}^{n} x_k^{2\alpha_k}\right)\operatorname{var}(e^{\varepsilon}) > 0,$$

assuming that $\alpha_k > 0$. Since the marginal productivity of an input is assumed to be positive, α_k must always be positive. Consequently, the marginal effect of increasing the use of an input will always be to increase the variability of output. However, in agricultural production it is a fact that for several inputs, such as pesticides and irrigation, the marginal effect of increasing their use is to *decrease* the variability of output, at least up to a certain level. Hence, the use of traditional production function estimates in evaluating policies may be questionable, particularly for production processes where risk-reducing inputs are used extensively. Although the above example uses the simple Cobb-Douglas specification, the criticism of traditional stochastic specifications which impose positive marginal risks on all inputs also applies to specifications such as homoskedastic translog specifications. In Chapter two it was shown that the Just-Pope form does not impose such restrictions. For example, both $f(\mathbf{x})$ and $h(\mathbf{x})$ can be specified with the Cobb-Douglas form, but the parameters of $h(\mathbf{x})$ may be negative to allow negative marginal risk.

The econometric specification (3.1) is usually chosen because of econometric tractability; it facilitates use of linear estimation techniques by taking logarithms on both sides. If the correct specification of the Cobb-Douglas technology is not (3.1) but instead a Just-Pope specification, which implies that there is an additive error term instead of a multiplicative error term, then estimates of α will not only be inefficient, but also be biased.

Estimation can be accomplished by rewriting the Just-Pope form for observation i as

(3.2) $\quad y_i = f(\mathbf{x}_i, \alpha) + u_i, \; \mathrm{E}[u_i] = 0, \; \mathrm{E}[u_i u_j] = 0 \text{ for } i \neq j,$

where

$$u_i = h(\mathbf{x}_i, \beta)\varepsilon_i, \; \mathrm{E}[\varepsilon_i] = 0, \; \mathrm{E}[\varepsilon_i \varepsilon_j] = 0 \text{ for } i \neq j.$$

Equation (3.2) can then be considered as a nonlinear, heteroskedastic regression of y on $\mathbf{x}$, and the parameters α can be consistently estimated by nonlinear least squares (NLS) under a broad range of conditions (Just & Pope, 1978). However, there are several shortcomings associated with this approach. First, hypothesis testing of the importance of various variables cannot generally be performed because of misleading estimates of the standard errors due to the heteroskedasticity in the model. The empirical results of Just & Pope (1979) suggest that estimation assuming homoskedasticity can lead to standard error estimates that indicate much greater precision in estimation than is obtained. Second, it is possible to improve the efficiency of the estimates (at least asymptotically) by explicitly taking account of the heteroskedasticity. Just & Pope (1978; 1979) present estimation procedures for obtaining asymptotically efficient parameter estimates when the production technology is of the Just-Pope form.

The problems associated with traditional production function specifications in the presence of production heteroskedasticity in inputs can be summarised as follows:

(i) They do not provide information on the marginal output risks of inputs because of their positive marginal risk restrictions.

(ii) They provide misleading estimates of the standard errors of estimated parameters, and therefore make hypothesis testing difficult.

The subsequent sections will discuss how some studies have dealt with these shortcomings of traditional production function specifications in empirical analyses of risky production technologies.

3.2. Just-Pope Approaches to the Econometric Modelling of the Stochastic Production Technology

Primal approaches in the econometric modelling of the firm under production risk encompass a very heterogeneous group of studies. Table 3.1 presents an almost exhaustive overview of primal approaches. As can be seen from the table, there are different approaches with respect to the functional form of the production function, the probability density function (pdf) of the error term, and method of estimation. Furthermore, most studies estimate only the production function, while a few estimate a system consisting of the production function and the utility function, or functions derived from the utility function. According to table 3.1, Just-Pope production functions have been used in the majority of the empirical studies. Furthermore, the restrictive Cobb-Douglas parametrization dominates in these studies. The most common estimator is a linear or nonlinear feasible generalised least squares. In the following we discuss some of these approaches in more detail.

3.2.1. Econometric Specifications of the Just-Pope Production Function

Following the introduction of the Just-Pope postulates for the stochastic production in Just & Pope (1978), several econometric production models which satisfy these postulates have been presented in the literature. Studies that estimate production models of the Just-Pope form $y_i = f(\mathbf{x}_i; \alpha) + u_i$, $u_i = h(\mathbf{x}_i; \beta)\varepsilon_i$, include Just & Pope (1979), Griffiths & Anderson (1982), Wan & Anderson (1985), Love & Buccola (1991), Saha *et al.* (1994), and Traxler *et al.*(1995). These studies have in common that the effect of input changes has been separated into two effects; the effect on mean and the effect on variance. The majority of the above studies use Cobb-Douglas parametrizations of the mean function $f(\mathbf{x}_i; \alpha)$ and the variance function $h(\mathbf{x}_i; \beta)$. One departure from the Cobb-Douglas

Table 3.1. Primal econometric approaches in the modelling of the firm under production risk

Study	Production function	P.d.f. of error term	Utility function	Time-/firm-specific effects	Method of estimation	Empirical application	Comments
Just & Pope (1978)	Log-linear Just-Pope	Normal under four-stage procedure	No	N.A.	NLS two- and four-stage procedures	None	Both estimation procedures provides consistent parameter estimates, but only 4-stage is efficient, assuming normality
Just & Pope (1979)	Cobb-Douglas and translog Just-Pope w. one explanatory variable	Only iid requirement	No	Time-specific random effects	Three-stage NLS procedure	Cross-section of time series on farm-level corn and oats prod. data	Account for time-specific effects which affect all contemporaneous observations. (e.g. weather) by adding time-specific error term.
Griffiths & Anderson (1982)	Cobb-Douglas Just-Pope w. random time- and firm-effects	Normal	No	Firm and time-specific random effects	6- and 4-stage NLS for hetero- and homoskedastic random effects respectively	Wool production data from a panel of 38 farms for a 10-year period	Explanatory variables: Labour, sheep, water, fencing, plant, land/buildings (the last three as annual real dollar flows).
Antle (1983)	Linear moment model (LMM) for mean, variance and skewness	iid	No	Index for quality of management	Three-stage GLS	Monthly milk production data from nine dairies for a 30-month period	Conditional moments of output assumed to be linear functions of inputs. Expl. vars: Feed, physical capital, animal capital and management. index.
Antle & Goodger (1984)	Quadratic linear moment model for mean, variance and skewness	iid	No	Index for quality of management	Three-stage GLS	Monthly milk production data from nine dairies for a 30-month period	Adjusted for autocorrelation. Additional variables compared with above study: Veterinary services, days open, mastitis, temperature.
Wan & Anderson (1985)	Cobb-Douglas Just-Pope w. random time- and firm-effects	Normal	No	Firm and time-specific random effects	7-stage NLS	Regional aggregate data on Chinese foodgrain production	Expl. vars.: Sown area, fertiliser usage (in total nutrients), irrigated sown area and electricity consumption.
Nelson & Preckel (1989)	Cobb-Douglas	Beta	No	No	Maximum Likelihood	Corn production data from a ten-year panel of 488 farms	Expl. vars.: Soil slope, clay contents, nitrogen, phosphates, potassium.
Antle & Crissman (1990)	Quadratic linear moment model for mean, variance and skewness	iid	No	Firm-specific fixed effects	Three-stage GLS	Philippine rice production data from a five-year panel of 45 farms	Expl. vars.: Labour (three types), land, nitrogen.
Love & Buccola (1991)	Cobb-Douglas Just-Pope form	Normal	Negative exp.	No	Three-stage NLS	Same as Nelson & Preckel	Expl. vars.: Same as Nelson & Preckel Joint estimation of utility function and production function parameters

(Table 3.1 continued on next page...)

Table 3.1. continued

Study	Production function	P.d.f. of error term	Utility function	Time-/firm-specific effects	Method of estimation	Empirical application	Comments
Kumbhakar (1993)	Translog deterministic part and linear stochastic part	Normal	No	Firm and time-specific fixed effects	Three-stage NLS	Dairy production data from a three-year panel of 37 farms	Expl. vars.: Fodder, material, labour, capital, grass fodder, pasture land.
Saha *et al.* (1994)	Just-Pope with Cobb-Douglas mean function and exponential variance function	Weibull	Expo-Power (EP)	No	Combination of numeric integration and NLS	Wheat production data from a four-year panel of 15 farms	Expl. vars.: Capital and materials. Joint estimation of utility function and production function parameters
Traxler *et al.* (1995)	Just-Pope with linear quadratic deterministic and stochastic part	Normal	No	No	Three-stage GLS	Wheat and maize data from experimental trials with different varieties	Tests effects of new varieties on mean and variance of production function. Expl. vars.: Nitrogen input, year of release of variety, year-dummies

specification is Just & Pope (1979), who also estimate a simple translog parametrization - with only one input - of $f(\mathbf{x}_i;\ \alpha)$ and $h(\mathbf{x}_i;\ \beta)$. Saha *et al.* (1994) use an exponential specification for the variance function $h(\mathbf{x})$. Traxler *et al.* (1995) assume a linear quadratic mean function and Harvey's multiplicative heteroskedasticity for the variance function (see Appendix 3.A). The error term ε is generally assumed to be normally distributed. The exception is Saha *et al.* (1994), who assume a Weibull distribution.

According to table 3.1 the dominant estimation procedure is linear/nonlinear feasible generalised least squares (FGLS).[1] Just & Pope (1979), Griffiths & Anderson (1982) and Wan & Anderson (1985) estimate the parameter vectors α and β by a multi-stage nonlinear least squares procedures. The procedures generally involve first estimating $y = f(\mathbf{x};\ \alpha)+u$, where $u=h(\mathbf{x};\ \beta)\varepsilon$, without considering the heteroskedasticity to obtain estimates of α and u. Then the estimated residuals $\hat{u}$ are regressed upon $h(\mathbf{x};\ \beta)$ to obtain estimates of β. The obtained estimates of α and β are consistent but not efficient. Further steps, which include weighted least squares regressions with predicted variances $h(\mathbf{x};\ \hat{\beta})$ as weights, are undertaken to obtain asymptotically efficient estimators for α and β.

Love & Buccola (1991) and Saha *et al.* (1994) estimate the production function together with the utility function, and consequently these studies use other estimation procedures for their econometric models.

3.2.2. Econometric Production Models with Firm- and Time-Specific Effects

One weakness of several econometric studies of production under risk is that firm- and time-specific effects are not accounted for in the model specification. Chapter four provides several arguments why firm heterogeneity and time-specific effects should be accounted for in econometric models, particularly when the objective is to measure the structure of production risk. A few studies of production risk, such as Griffiths & Anderson (1982), Wan & Anderson (1985) and Kumbhakar (1993) have utilised the panel data sets available to them to account for firm heterogeneity.[2]

Griffiths & Anderson (1982) estimate the Just-Pope production model

$$y_{it} = f(\mathbf{x}_{it};\ \alpha)+u_{it}, \qquad i = 1,\ldots, N, \qquad t = 1,\ldots,T,$$

where u_{it} is a heteroskedastic error term, on a balanced panel data set. Two specifications of the variance function is estimated: (1) $u_{it}=h(\mathbf{x}_{it};\ \beta)(\varepsilon_{it}+\eta_i+\lambda_t)$, and (2) $u_{it}=h(\mathbf{x}_{it};\ \beta)\varepsilon_{it}+\eta_i+\lambda_t$,

[1] A GLS estimator assumes that the parameters of the covariance matrix of u_i, $E[\mathbf{uu}'] = \Omega$,is known. In empirical studies the covariance matrix is generally unknown, which means that an estimate of the covariance matrix, $\hat{\Omega}$, is required in order to perform GLS. FGLS is GLS with the estimator $\hat{\Omega}$ used for the covariance matrix.

[2] Wan & Anderson (1985) is an empirical application of the model presented by Griffiths & Anderson (1982) on Chinese foodgrain production.

where η_i is the firm-specific random effect and λ_t is the time-specific random effect. In model (1) all three error components, $h(\mathbf{x}_{it}; \beta)\varepsilon_{it}$, $h(\mathbf{x}_{it}; \beta)\eta_i$ and $h(\mathbf{x}_{it}; \beta)\lambda_t$, are heteroskedastic in the sense that their variances depend on input levels. In model (2) the firm- and time effects are homoskedastic. All three random effects ε_i, η_i and λ_t are assumed normally distributed. Furthermore, for both models it is assumed that

$$E[\varepsilon_{it}] = E[\eta_i] = E[\lambda_t] = 0, \quad E[\varepsilon_{it}^2] = \sigma_\varepsilon^2,\ E[\eta_i^2] = \sigma_\eta^2,\ E[\lambda_t^2] = \sigma_\lambda^2,$$

and that the $\mathbf{x}_{it}$, ε_{it}, η_i, λ_t are mutually uncorrelated for all i and t.

Griffiths & Anderson estimate several variants of models (1) and (2) by nonlinear FGLS. They follow the estimation procedure of Just & Pope (1978), with some modifications to allow for the inclusion of error components in the models.

Cobb-Douglas parametrizations of the mean and variance function are used in both Griffiths & Anderson and Wan & Anderson. This specification has largely been abandoned after the introduction of flexible functional forms, because it is regarded as an overly restrictive representation of production technologies.

In both Griffiths & Anderson and Wan & Anderson the estimated variance function parameters are generally insignificant at conventional significance levels. Tentative explanations for the lack of significance, in addition to data problems, can be provided by the theoretical and simulation study findings on FGLS estimators. Just & Pope shows that the FGLS estimator provides asymptotically less efficient estimates of β than the ML estimator, and also performs worse in small sample simulation studies (Saha, Havenner, & Talpaz, 1997).

A potential source of biases in parameter estimates are the random effects assumptions of no correlation among the error components, and between the error components and regressors. It is difficult to know *a priori* how sensitive parameter estimates are to violations of these assumptions.

Wan, Griffiths & Anderson (1992) extend the above model into the seemingly unrelated regressions (SUR) framework, which is relevant when there are several outputs and the disturbances from the production functions corresponding to the different outputs are correlated.

3.3. Kumbhakar's Approach: Translog Production Function with Risk and Technical Efficiency

Kumbhakar (1993) demonstrates a method of measuring both production risk and mean technical efficiency using panel data. A flexible (*translog*) production function is estimated on a data set of Swedish dairy farms. Production risk is specified as a function of the inputs in the following manner:

$$y_{it} = f(\mathbf{x}_{it};\alpha)e^{\eta_i+\lambda_t+h(\mathbf{x}_{it};\beta)\varepsilon_{it}},$$

where η_i is the firm-specific effect, λ_t is the time-specific effect, and ε_{it} is the random exogenous production shock.

In the study η_i and λ_t are treated as fixed, i.e., as dummy variables. Thus the only random component is ε_{it}. Kumbhakar uses a translog function for ln$f(\mathbf{x};\ \alpha)$, and a linear function for $h(\mathbf{x};\ \beta)$:

$$\ln y_{it} = \alpha_0 + \sum_k \alpha_k \ln x_{kit} + \frac{1}{2}\sum_k\sum_j \alpha_{kj} \ln x_{kit} \ln x_{jit} + \eta_i + \lambda_t + \left[\sum_l \beta_l x_{lit}\right]\varepsilon_{it}.$$

In the first step of the estimation procedure $h(\mathbf{x};\ \beta)$ is ignored and the function

$$\ln y_{it} = \alpha_0 + \sum_k \alpha_k \ln x_{kit} + \frac{1}{2}\sum_k\sum_j \alpha_{kj} \ln x_{kit} \ln x_{jit} + \eta_i + \lambda_t,$$

is estimated by OLS. Using the estimators of α, η_i and λ_t, the residuals $\hat{u}_{it}$ are calculated. Then $\hat{u}_{it}$ is regressed on $h(\mathbf{x};\ \beta)$ by non-linear methods to obtain estimates of β. In a third step weighted least squares are performed to obtain asymptotically efficient estimates of α and β (see appendix 9.E).

A nice property of Kumbhakar's specification is that it allows linear estimation of the translog $f(\cdot)$. This is not possible for a translog in the Just-Pope formulation, because the variance function is additively related to the mean function.

The specification is flexible enough to allow both negative and positive marginal risks. Technical efficiency is separated from risk and the usual error term. Thus, the model can be viewed both as an extension of stochastic production frontier models, and as an extension of standard neoclassical production models which include only risk. In Kumbhakar's formulation of the production function, differences in realised output levels may not only be due to different realisations of the production shock ε; it may also be explained by farm-specific and time-specific effects. Kumbhakar finds both negative and positive marginal risks in his empirical application. Furthermore, he finds evidence of variations in mean farm efficiencies over time and across farms within a particular year. However, in the presence of risk aversion Kumbakhar does not measure "overall" technical efficiency, since he does not take into account the variance function when ranking producers (see Chapter two).

A problem with Kumbhakar's specification is that $h(\mathbf{x};\ \beta)$ is not additively separable from $f(\mathbf{x};\ \alpha)$. The variance function interacts with the mean function in a multiplicative manner. As stated in Chapter two, this specification always violates two of the Just-Pope postulates and may also violate the remaining, depending on the values of α and β. Furthermore, unlike the Just-Pope form direct interpretation of the estimated parameters of the variance function is not possible with Kumbhakar's form, as can be seen from the expression for marginal risk presented in section 2.2. The functional form also makes ML estimation, which we later will see can be an attractive alternative to FGLS, very cumbersome.

However, in the presence of firm heterogeneity with respect to mean productivity, the translog has its advantages in the specification of firm-specific effects compared to linear parametrizations of the Just-Pope model. In the usual formulation of translog production function with firm-specific fixed effects,

$$\ln y_{it} = \ln f(\mathbf{x}_{it}) + \eta_i + u_{it} \qquad \text{which implies that} \qquad y_{it} = f(\mathbf{x}_{it}) e^{(\eta_i + u_{it})},$$

where $f(\mathbf{x}) = \exp\left(\alpha_k + \sum_k \alpha_k \ln x_k + 0.5 \sum_k \sum_l \alpha_{kl} \ln x_k \ln x_l\right)$. We see that the firm-specific effect η_i interacts multiplicatively with $f(\mathbf{x}_{it})$. For two input vectors $\mathbf{x}$ and $a\mathbf{x}$ ($a > 1$) and two firms i and j where $\eta_i > \eta_j$, the difference in mean output conditional on $\mathbf{x}$, $E[y_{it} | \mathbf{x}; \eta_i] - E[y_{jt} | \mathbf{x}; \eta_j]$, is smaller than the difference in output conditional on $a\mathbf{x}$, $E[y_{it} | a\mathbf{x}; \eta_i] - E[y_{jt} | a\mathbf{x}; \eta_j]$ with the above formulation. However, the translog violates Just-Pope postulates for stochastic production function because of the multiplicative interaction between the error term u and $f(\mathbf{x})$. A non-logarithmized translog function with additive error term is not tractable for estimation, because of the difficulty of obtaining parameter convergence.

Alternatively, one can employ a Just-Pope parametrization which is linear in parameters α, such as the linear quadratic form

$$y_{it} = \alpha_0 + \sum_k \alpha_k x_{k,it} + 0.5 \sum_k \sum_l \alpha_{kl} x_{k,it} x_{l,it} + \eta_i + u_{it}.$$

With the above specification the firm-specific effect enters the production function additively, which implies that the difference in mean output between two firms i and j is the same for input vectors $\mathbf{x}$ and $a\mathbf{x}$ ($a > 1$).

Intuitively, the multiplicative specification of the fixed effect in the translog model may be considered more appealing than the additive specification in the linear quadratic model. The fixed effects are assumed to represent (often unobservable) factors that are not included as regressors in the model, such as the quality of management. Often unobserved factors are correlated with inputs. It is reasonable to assume that with increasing scale of operation, the difference in output levels between two firms with different unobservable characteristics will not stay constant, but increase.

3.4. Econometric Modelling of the Stochastic Production Technology with Non-Normal Error Terms

There are no *a priori* reasons to assume that the conditional distribution of output is normal (or symmetric); it may be skewed to the right or to the left. Changes in input levels may have effects on third and higher moments. Theoretical models outlined in Chapter two also suggest that changes in third and higher moments affect the optimal input levels of the risk averse firm.

Empirical researchers have responded to the problems associated with assuming normality in two ways; (1) by using other parametric distributions, such as the conditional beta (Nelson & Preckel, 1989) or the conditional Weibull (Saha *et al.*, 1994), (2) by using a non-parametric approach to avoid making any *a priori* distributional assumptions (Antle, 1983; Antle, 1987; Collender & Chalfant, 1986), or (3) estimating a flexible parametric approximation of the cumulative density function (Taylor, 1984). In the following, examples of the first two approaches are presented.

3.4.1. Models with Beta and Weibull Probability Distributions

Nelson & Preckel (1989) propose a conditional beta distribution as a parametric model for the probability distribution of output. If the distributional assumptions are correct, it will produce more efficient estimates than nonparametric approaches, such as Antle's (1983).

For certain parameter values the beta distribution (α>1 and β>1) becomes bell-shaped. The bell-shaped case may exhibit skewness in both directions. The flexible skewness of the bell-shaped beta distribution is not shared by the normal, log-normal, exponential, and gamma distributions.

Crop yield, y, may be distributed as a beta random variable for several reasons. First, crop yields are known to fall in a range from 0 to some maximum possible value. The beta random variable may be defined on an interval (0, y^u), where y^u is a finite upper bound on the random variable. Second, crop yield distributions can be significantly skewed either to the right or to the left. The beta distribution has such flexibility.

The probability density function of an unconditional beta random variable with a range from 0 to y^u can be written as

$$(3.3) \quad p(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \times \frac{y^{(\alpha-1)}(y^u - y)^{(\beta-1)}}{y^{u(\alpha+\beta-1)}}, \qquad 0 \le y \le y^u,$$

where α, β, and y^u are parameters, and $\Gamma(x)$ is the gamma function. The distribution can be conditioned on a vector of inputs, **x**, by expressing the parameters α and β as functions of **x**. The parameter y^u is not expressed as a function of x because regularity conditions for maximum likelihood would be violated.

Implementation of the conditional beta model requires that functional forms for $\alpha(\mathbf{x})$ and $\beta(\mathbf{x})$ are chosen. The functions $\alpha(\mathbf{x})$ and $\beta(\mathbf{x})$ must be consistent with regularity conditions for maximum likelihood estimation. In addition to this requirement, arguments for simplicity and parsimony might justify linear or log-linear functions. After some experimentation, a log-linear functional form was chosen by Nelson & Preckel for their empirical application. They use the following Cobb-Douglas functions for $\alpha(\mathbf{x})$ and $\beta(\mathbf{x})$:

$$(3.4) \quad \alpha(\mathbf{x}) = a_0 \prod_{i=1}^{m} x_i^{a_i} \qquad \text{and} \qquad \beta(\mathbf{x}) = b_0 \prod_{i=1}^{m} x_i^{b_i},$$

where m is the number of inputs and a_i (b_i) is the percentage change in α (β) as x_i rises by 1 %. Nelson & Preckel present expressions for the mean, variance and skewness of the conditional beta distribution.

In order to obtain consistent estimates, the conditional beta distribution is estimated with a two-step procedure. First, maximum likelihood (ML) estimation of the unconditional beta distribution, (3.3), produces consistent, asymptotically normal and efficient estimates of α, β and y^u. ML estimates of α, β and y^u in the unconditional model are obtained by numerically maximising the log-likelihood function of this model. In the second step the functional expressions (3.4) for $\alpha(\mathbf{x})$ and $\beta(\mathbf{x})$ are inserted into (1) for α and β to obtain a conditional beta distribution. ML estimates of a_0, a_1,..., a_m and b_0, b_1,..., b_m are obtained by numerically maximising the log-likelihood function of the conditional beta distribution, using the unconditional ML estimate of y^u obtained in the first step.

ML estimates of the parameters are consistent, asymptotically normal, and asymptotically efficient under weak regularity conditions. In order for the ML estimate of y^u to have these properties β must be greater than two in the first stage of the estimation. Therefore, in the first stage it is necessary to verify that $\beta>2$. The parameters α and β must each be greater than one in order for the beta distribution to be unimodal. If this condition is violated the beta distribution is U or J shaped, implying that an alternative specification is probably needed.

The model was applied on farm-level data sets from five Iowa counties (see table 3.1). Evaluation of the expression (3.3) at the ML estimates and the mean values of the explanatory variables indicated that the yield distribution is negatively skewed in all counties. This implies that a farm using mean values of the explanatory variables would experience above-average yields more frequently than below-average yields in all counties. Nelson & Preckel also undertake likelihood ratio tests to test for the independence of input levels and skewness. The hypothesis is implemented by imposing the restrictions $a_i = b_i$ for all i except 0. For four of the five counties the null hypothesis of independence is rejected.

Saha *et al.* (1994) use the Just-Pope parametrization $y = A\prod x_i^{\alpha i} + \exp\{\Sigma m_i x_i + \varepsilon_i\}$, where ε_i is a Weibull distributed error term. The Weibull distribution does not impose, nor preclude, a symmetric pdf. The domain of a Weibull distributed variable is 0 to $+\infty$, which rules out the possibility of negative output. The approach of Saha *et al.* will be described further in section 3.5. The derivation of ML estimators for the production function parameters by Saha *et al.* clearly shows that the econometric estimation procedure is complicated by assuming a Weibull distribution for the error term instead of a normal distribution. However, the empirical estimates of the parameters of the Weibull distribution indicate that it is appropriate to use such a non-normal error term distribution for Saha *et al*'s empirical application.

3.4.2. Antle's Linear Moment Model Approach

Antle (1983) proposes a flexible moment-based approach to estimate the parameters of the stochastic production technology. The basis for the specification is the moment functions

$$\mu_1(\mathbf{x},\alpha_1)=\int y f(y|\mathbf{x})dy, \qquad \text{and} \qquad \mu_i(\mathbf{x},\alpha_i)=\int (y-\mu^i) f(y|\mathbf{x})dy \qquad i=2,\ldots,m,$$

where μ_i is the ith moment, $\mathbf{x}$ is the vector of inputs, α_i is the vector of parameters relating $\mathbf{x}$ to μ_i, and the model is specified with m moments.

Antle chooses a linear quadratic parametrization of the moment functions, where the ith moment is given by

$$\mu_i=\alpha_{i0}+\sum_k \alpha_{ik}x_k+0.5\sum_k \sum_l \alpha_{ikl}x_k x_l\,.$$

In the linear quadratic moment model there are different parameter vectors α_i for each moment function. The LMM does not impose restrictions on the α_i either within or across equations and is therefore a more general representation of the output distribution than the Just-Pope models presented earlier in this chapter.

Antle presents a three step GLS estimation procedure, where the mean function is estimated first, and then the higher moment functions are estimated by using the estimated residuals from the mean function. Antle shows that the GLS estimators converge in distribution to the true parameter values.

Except for Antle (1983) Antle & Goodger (1984) and Antle & Crissman (1990), Antle's approach has not been applied in this field of research. One reason may be that researchers regard heteroskewness and heterokurtosis to have only limited significance for producer behaviour. Another reason may be that the estimation procedure is very cumbersome. There are also some practical difficulties with the estimation procedure: The estimated variances used in the GLS regressions may be negative, and nonlinear programming methods with nonnegativity restrictions thus have to be employed for estimating the even moments. Although Antle shows that the nonlinear programming methods provide consistent estimates of the parameters of the even moment functions, it makes estimation more cumbersome.

Taylor (1984) criticises Antle's approach for not being

"practical for use in empirical studies that directly require the equation of a pdf or cdf. That is, even though the moments uniquely define (in a theoretical sense) the underlying pdf, the analytical form of this pdf may be difficult, if not impossible to obtain except in special cases. Thus, Antle's method may not be useful in safety-first and stochastic dominance analyses, although it may be practical when the pdf or cdf is not needed per se" (Taylor, 1984, p. 69-70).

3.5. Joint Estimation of Risk Preference Structure and Production Technology

Most econometric models impose rather strict assumptions on the risk preference structure, which may lead to biased risk response estimates. This issue is addressed by Saha (1993), who introduces a new functional form for the utility function, the *Expo-Power* (EP) utility function. This function allows both decreasing, constant or increasing absolute risk aversion (DARA, CARA and IARA) and decreasing or increasing relative risk aversion (DRRA and IRRA), depending on the parameter values of the function. Furthermore, EP can exhibit both risk averse, risk neutral and risk loving preferences. Thus, the EP imposes no *a priori* restrictions on the risk preference structure. The EP utility function is given by

$$U(W) = \theta - \exp(-\beta W^{\alpha}),$$

where W is wealth. Parameter restrictions of the EP utility function are $\theta>1$ and $\alpha\beta>0$. The measures of absolute and relative risk aversion are $A(W) = (1-\alpha+\alpha\beta W^{\alpha}) / W$ and $R(W) = 1-\alpha+\alpha\beta W^{\alpha}$. Under its parameter restrictions, the EP function exhibits DARA if $\alpha<1$, CARA if $\alpha=1$, and IARA if $\alpha>1$. Also, EP exhibits DRRA if $\beta<0$ and IRRA if $\beta>0$. Saha (1993, footnote 4) shows that a utility function can exhibit either CARA or CRRA but never both under finite parameter values. Also, it follows from the EP utility function's parameter restriction $\alpha\beta>0$ that DRRA ($\beta<0$) implies DARA.

Saha *et al.* (1994) proceed to develop a method which allows joint estimation of risk preference structure and production technology, using the expo-power utility function. The model of Saha *et al.* assumes that output level risk is the only source of uncertainty facing the producer. For the production function Saha *et al.* use a parametrization of the general Just-Pope form $y = f(\mathbf{x})+h(\mathbf{x}, \varepsilon)$, where $f(\mathbf{x})$ takes the Cobb-Douglas form and $h(\mathbf{x}, \varepsilon)$ takes the exponential form $\exp\{\Sigma m_i x_i+\varepsilon\}$. If $m_j<0$, then the jth input is risk-reducing. The method is applied on a firm level data set of 15 Kansas wheat farms for four years. According to the parameter estimates, the farmers in the sample exhibit decreasing absolute risk aversion (DARA) and increasing relative risk aversion (IRRA). All parameter estimates for the deterministic part $f(\mathbf{x})$ of the production function are well-behaved and significant. According to the parameter estimates for the stochastic part, capital input exhibits increasing marginal output risk, while materials inputs exhibit decreasing marginal output risk.

Saha *et al.* also estimate the production technology parameters under the hypothesis of CARA and independently of the risk preference structure. Their empirical results show that the parameter estimates are sensitive to alternative assumptions, both with respect to absolute value and sign. Furthermore, they find that combined estimation of the production function and the utility function is more efficient than separate estimation of each in the sense that the standard errors of the estimates are consistently and considerably lower than those under alternative settings.

3.6. Econometric Analysis of Technical Change and Technical Efficiency

An issue that until recently has been ignored in the econometric literature of the firm under production uncertainty, is the effect of learning-by-doing and adoption of new technology on risk. To some extent this can probably be explained by the fact that there has been little discussion in the theoretical literature of the criteria for adoption of new technologies under production risk compared with the deterministic case. In the standard deterministic model of the competitive firm, new technologies are adopted in order to increase productivity. Under production risk the competitive firm should not only adopt new technologies in order to increase mean productivity; it should also be concerned about the effect of introducing a new technology on the variance of productivity (see section 2.5). If a firm can choose between two technologies which are identical with respect to mean productivity for different input vectors, it will choose the technology with the smallest output variance conditional on the input vector. Under risk aversion the firm may adopt a new technology even if it shifts the mean marginal cost curve upwards, provided that the reduction in production risk is sufficiently large to increase the firm's expected utility.

3.6.1. An Econometric Analysis of Production Risk and Innovations

One of the few econometric studies so far to address the issue of production risk effects of technical change is Traxler *et al.* (1995). Traxler *et al.* analyse the effect of introduction of new varieties of wheat for the period 1950-86 on the first two moments of the wheat yield in a Just-Pope model framework. In a stochastic production function of the Just-Pope form, technical change can be accounted for by implementing a trend variable t both in the deterministic and stochastic component in order to decompose the effects on mean productivity and production risk:

$$y = f(\mathbf{x};\, t) + h(\mathbf{x};\, t)\varepsilon.$$

By using a second-order approximation for $h(\mathbf{x};\, t)$, it is possible to analyse if the rate of change in production risk is decreasing or increasing over time. Traxler *et al.*, however, do not use a trend variable to proxy the effects of technological change. Because they have direct information on the particular wheat variety used in each yield observation in the data set, they can use the year of the release of the variety minus a base year as the explanatory variable to analyse the effect on the mean and variance of production. Unlike the conventional trend specification, this makes it possible for them to directly measure the effects of the individual varieties. In general, data sets available to researchers do not provide direct information on particular innovations, e.g. new types of machinery, for which it would be interesting to analyse the productivity effects. Consequently, the researcher is forced to use the conventional trend specification.

Traxler *et al.* use a simple linear quadratic specification for the mean function. It has only one input (nitrogen). For the variance function Harvey's multiplicative heteroskedastic specification

$$\mathrm{var}(u_i) = h(\mathbf{x}_i; \beta) = \exp(\mathbf{x}_i\beta),$$

is used (Harvey, 1976; Judge, Griffiths, Hill, Lütkepohl, & Lee, 1988, pp. 365-66). The function is estimated by a three-stage FGLS procedure. A linear quadratic specification is used as the argument of the exponent.

Traxler *et al.* find that the successive wheat varieties that were introduced until 1970 were characterised by increasing mean yield, but also accompanied by higher yield variances. Thus high yield potential was given higher priority than yield stability. However, for the varieties released from 1971 and onward, yield stability was given much more priority. The post-1970 varieties were characterised by decreasing yield variance, but much slower mean-yield growth. A conclusion to be drawn from the study of Traxler *et al.*, is that there probably exists a mean-variance trade-off also in wheat selection.

3.6.2. Measurement of Technical Efficiency

Antle & Crissman (1990) analyse technical efficiency as farmers adopt new production technologies. The data set is a sample of Philippine rice farmers that use traditional varieties and farms that use modern varieties. Antle & Crissman estimate relative technical efficiency, as defined in Chapter two, by using a two-step procedure. First, the parameters of the production technology are estimated by a linear moment model. This allows the moments of the production technology to be calculated for chosen input levels. Second, the calculated moments and different parameter values for the farmers' attitude towards risk are inserted into an expected utility model. Antle & Crissman postulated a negative exponential utility function expanded with a third-order Taylor series for the Philippine rice farmers:

$$EU = 1 - e^{-\lambda\mu_1} - e^{-\lambda\mu_1}\sum_{i=2}^{3}(-\lambda)^i \mu_i / i!,$$

where λ is the risk aversion parameters and μ_i is the ith moment of farm revenue.[3] Thus, they account for downside risk aversion.

The measure of technical efficiency (*TE*) is defined as

$$TE_{kl} = \frac{EU(\mu(\mathbf{x};\beta_l),\lambda)}{EU(\mu(\mathbf{x};\beta_k),\lambda)},$$

[3] Note that the argument of the EU function is the moments of revenue instead of the moments of profits. Given input levels and input prices, the moments of revenue are equal to the moments of profits, except for mean revenue, which differs from mean profits by the amount of input costs, **wx**.

where β_k and β_l are the parameter estimates for technology k and l, respectively. Antle & Crissman calculated relative technical efficiency for the two different technologies (traditional and modern varieties) at two different stages (1-2 years and 3-5) years for different values of the risk aversion parameter λ. Hence, they were able to compare different technologies accounting for risk characteristics of output and the risk preferences of farmers. Antle & Crissman found that traditional varieties were technically more efficient than modern varieties in the first stage of the adoption process, but that the situation was reversed at the second stage of the adoption process.

3.7. FGLS vs. ML Estimation of Just-Pope Production Functions

As can be seen from table 3.1 the overwhelming majority of previous econometric studies of production risk have used linear or nonlinear FGLS methods in the estimation of Just-Pope production functions.

Since the Just-Pope model is a heteroskedastic specification, we can exploit more general results on the properties of different estimators under heteroskedasticity in the econometric literature. Table 3.2 presents small sample and large sample properties of different estimators which can be used in the estimation of Just-Pope production technologies.

Table 3.2. Theoretical small and large sample properties of different estimators for Just-Pope production technologies

	Mean function (α parameters)	Variance function (β parameters)
Large sample	OLS: Consistent and AND estimate of α, but inconsistent estimate of var(α) GLS: Consistent and AND estimate of α White: Consistent estimates of both α and var(α) (White, 1980) FGLS. Consistent, AND and as. efficient ML: Consistent and as. efficient estimate of α Harvey's two-stage: Consistent and as. efficient	OLS: N.A. GLS: Assumes β already known White: N.A. FGLS: Consistent (except β_0) but as. inefficient estimate of β ML: Consistent and as. efficient estimate of β Harvey's two-stage: Consistent and as. efficient
Small sample	OLS: Unbiased but inefficient estimate of α, inconsistent estimate of var(α) GLS: Unbiased and efficient estimate of α White: Generally unknown FGLS: Unbiased but inefficient estimate of α (Judge *et al.*, 1988, p. 353) ML: Generally unknown, but see Saha *et al.* (1997) Harvey's two-stage: Generally unknown	OLS: N.A. GLS: β already known White: Not estimated FGLS: Biased and inefficient (Saha *et al.*, 1997) ML: Generally unknown, but see Saha *et al.* (1997) Harvey's two-stage: Generally unknown

N.A.: Not applicable AND: Asymptotically Normally Distributed

If the primary interest is on the mean function, then OLS with White-adjusted standard errors provides consistent and asymptotically efficient estimates (White, 1980). On the other hand, if there is substantial heteroskedasticity that can be attributed to production risk, then the variance function also becomes a subject of interest. We have seen in Chapter two that theories of production risk predict that even under risk neutrality optimal input levels will diverge from competitive levels. Therefore one should estimate the variance function, preferably by a method that provides consistent estimates of both the parameters of the variance function, β, and the covariance matrix $\mathbf{var}(\beta)$. According to table 3.2 there are three estimators that provide consistent estimates of β; FGLS, maximum likelihood, and Harvey's two-stage estimator.[4] However, only ML and Harvey's two-stage estimator provide asymptotically efficient estimates of β.

Previous empirical studies applying the Just-Pope framework have only used FGLS estimators. Harvey (1976) has shown for the multiplicative heteroskedastic model, which is a special case of the Just-Pope function with linear mean function and variance function $\text{var}(u_i) = \exp(\mathbf{x}_i\beta)$, that the GLS and ML estimators of β have asymptotic covariance matrices

$$\text{cov}(\hat{\beta}_{GLS}) = 4.9348(\mathbf{X'X})^{-1} \quad \text{and} \quad \text{cov}(\hat{\beta}_{ML}) = 2(\mathbf{X'X})^{-1},$$

respectively, where $\mathbf{X}$ is the matrix of stacked $\mathbf{x}_i$ vectors (Harvey, 1976; Just & Pope, 1978). Just & Pope (1978) have obtained similar results for the special case of the Just-Pope function with both $f(\cdot)$ and $h(\cdot)$ log-linear in parameters, which is the case for the Cobb-Douglas and translog form.[5] They show that the asymptotic covariances are

$$\text{cov}(\hat{\beta}_{GLS}) = 1.2337(\mathbf{X'X})^{-1} \quad \text{and} \quad \text{cov}(\hat{\beta}_{ML}) = 0.5(\mathbf{X'X})^{-1},$$

Comparing the covariance matrix of the ML estimator of β with the GLS covariance matrix we see that both for Harvey's special case and Just & Pope's special case is the ML estimator for β more than twice as efficient asymptotically, or more precisely, by a factor of 2.4674.

The ML estimator involves distributional assumptions for u_i. If the distributional assumption is not correct the ML estimator of α and β may be inconsistent. This is not necessarily so, because ML estimates that are formed on the incorrect assumption of normality, so-called quasi-maximum likelihood estimators, may provide consistent estimates of the population parameters (White, 1982).[6]

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[4] See appendix 3.B for a presentation of Harvey's two-stage estimator for the parameters of the variance function.

[5] With Harvey's formulation of the Just-Pope model, both the mean and variance function can be estimated by linear least squares, while for Just & Pope's special case the mean function has to be estimated by nonlinear least squares.

[6] White (1982) proposes tests for parameter inconsistency under distributional misspecification. See Godfrey (1988) for a discussion of distributional misspecification and testing.

In practice, econometric estimation will usually be undertaken on small samples, and it is therefore maybe more appropriate to be concerned about the finite sample performance of different estimators than their asymptotic properties. According to table 3.2 there are no theoretical findings to support the choice of estimator in finite samples, as long as we are interested in the variance function. But since we have to choose an estimator, it would be useful to obtain some information about the small sample performance of alternative estimators, in particular when underlying assumptions are violated (e.g. departures from normality). The comparative small-sample performance of FGLS and ML estimators under heteroskedasticity is largely unexplored (Fomby, Hill, & Johnson, 1984, p. 201; Judge, Griffiths, Hill, Lütkepohl, & Lee, 1985, p. 455). Harvey (1976, p.464) suggests that if the parameters of the mean function is estimated by FGLS with Harvey's two-stage estimator of β used to generate the weights "...it seems reasonable to suppose that such an estimator will have better small sample properties than..." the ordinary three step FGLS estimator.[7]

A recent paper by Saha, Havenner & Talpaz (1997) examines the small sample performance of FGLS and ML. First, they compare the two estimators through the first order conditions. For the Just-Pope function $y_i = f(\mathbf{x}_i; \alpha) + u_i$, $u_i = h(\mathbf{x}_i; \beta)\varepsilon_i$, with variance $\mathrm{var}(u_i) = h(\mathbf{x}_i; \beta)^2\mathrm{var}(\varepsilon_i)$, the three-step FGLS procedure consists of first estimating the mean function $y_i = f(\mathbf{x}_i; \alpha) + u_i$ by least squares. Then the residuals $\hat{u}_i = y_i - f(\mathbf{x}_i; \hat{\alpha})$ are used in the next step to estimate

$$\ln(\hat{u}_i^2) = \ln(h(\mathbf{x}_i; \beta)^2) + v_i,$$

where $v_i = \ln\varepsilon_i$ (Judge *et al.*, 1988, pp. 367-9). Without loss of generality, the estimation equation in (4) can be rewritten as:

$$\ln(\hat{u}^2) = \beta_0^* + \ln(h(\mathbf{x}_i; \beta)^2) + v_i^*,$$

where $v_i^* = v_i - \mathrm{E}[v_i]$, and $\beta_0^* = \mathrm{E}[v_i] = \mathrm{E}[\ln(\varepsilon_i^2)]$, which implies that $\mathrm{E}[v_i^*] = 0$. Estimation of the above variance function by least squares provides consistent estimates of β, denoted by $\hat{\beta}$. The third step is a weighted least-squares regression of the mean function:

$$y_i^* = f^*(\mathbf{x}_i; \alpha) + u_i^*,$$

where $y_i^* = y_i/h(\mathbf{x}_i; \hat{\beta})$, $f^*(\mathbf{x}_i; \alpha) = f(\mathbf{x}_i; \alpha)/h(\mathbf{x}_i; \hat{\beta})$, and $u_i^* = u_i/h(\mathbf{x}_i; \hat{\beta})$. Jobson & Fuller (1980) have demonstrated that the second stage estimate $\hat{\beta}$, though inefficient, is consistent. This all that is required to ensure that the third stage FGLS estimate $\hat{\alpha}$ is consistent, distributed asymptotically normal. If the errors are normally distributed, the FGLS estimate $\hat{\alpha}$ is asymptotically fully efficient, i.e., its covariance matrix $\mathrm{var}(\hat{\alpha})$ attains the Cramér-Rao lower bound.

[7] However, we have not found any empirical studies or simulation studies which actually compare the small sample performance of Harvey's two-stage estimator versus the FGLS estimator.

In small samples the situation is different. Because each v_i is derived from the same estimate of α in the first stage, the v_i's are serially correlated and heteroskedastic (Amemiya, 1985, pp. 200-7). Amemiya demonstrates that serial correlation is absent in large samples. However, the problem of heteroskedasticity of v_i persists, thus yielding inefficient estimates of the variance function parameters β. In order to adjust var($\hat{\beta}$) White's (1980) consistent estimator of the covariance matrix can be used. This correction provides consistent standard errors of $\hat{\beta}$, but the estimates $\hat{\beta}$ and thus the third stage estimation weights $(h(\mathbf{x}_i;\ \hat{\beta})^{-1}$ remain unchanged. Consequently, the FGLS estimates $\hat{\alpha}$ and their standard errors also remain unaffected. The ML estimator, on the other hand, provides consistent and asymptotically fully efficient estimates of both α and β, in a single stage.

To compare the small-sample performance of the two estimators analytically we provide the criterion functions of the last two stages of FGLS, which for the second stage is

(3.5) $\min_\beta S_2 = \Sigma_i \{\ln(\hat{u}_i^2) - \beta_0^* - \ln(h(\mathbf{x}_i;\ \beta)^2)\}^2$
$= \Sigma_i \{\ln(y_i - f(\mathbf{x}_i;\ \hat{\alpha}))^2 - \beta_0^* - \ln(h(\mathbf{x}_i;\ \beta)^2)\}^2,$

and for the third stage is

(3.6) $\min_\alpha S_3 = \Sigma_i \{(y_i - f(\mathbf{x}_i;\ \alpha))/h(\mathbf{x}_i;\ \hat{\beta})\}^2.$

The log-likelihood function is given by

(3.7) $\max_{\alpha,\beta} \ln L = -1/2[n{\cdot}\ln(2\pi) + \Sigma_i \ln(h(\mathbf{x}_i;\ \beta)^2) + \Sigma_i(y_i - f(\mathbf{x}_i;\ \alpha))^2/h(\mathbf{x}_i;\ \beta)^2].$

By comparing (3.6) and the log-likelihood function, we see that for a given β, the FGLS and ML estimates of α are identical. However, this is not the case for β. For a given value of $\hat{\alpha}$ and attendant $\hat{u}_i$, the ML first order condition for β is

(3.8) $\partial\ln L/\partial\beta = \Sigma_i\{(\hat{u}_i^2/h(\mathbf{x}_i;\ \beta)^2) - 1\}h_\beta(\cdot)/h(\mathbf{x}_i;\ \beta) = 0,$

where $h_\beta(\cdot)$ denotes $\partial h(\cdot)/\partial\beta$. From (3.5) the corresponding FGLS first order condition is

(3.9) $\partial S_2/\partial\beta = -4\ \Sigma_i \{\ln(\hat{u}_i^2) - \beta_0^* - \ln(h(\mathbf{x}_i;\ \beta)^2)\}h_\beta(\cdot)/h(\mathbf{x}_i;\ \beta) = 0.$

The first order conditions (3.8) and (3.9) can never be identical regardless of the functional form of $h(\mathbf{x}_i;\ \beta)$. Furthermore, iteration between the steps of FGLS will not lead to ML estimates.

The question whether one can estimate α and β in a single stage through FGLS, is also addressed by Saha *et al.* (1997). They show that it is not possible to find an interior solution for β in a single stage FGLS optimisation problem. This contrasts with the criterion function in ML, which is well defined, allowing a strictly interior solution for β.

For the single-stage estimation problem the objective function is

(3.10) $\min_{\alpha,\beta} S = \Sigma_i\ \varepsilon_i^2 = \Sigma_i \{(y_i - f(\mathbf{x}_i;\ \alpha))/h(\mathbf{x}_i;\ \beta)\}^2.$

By comparing (3.10) with (3.6) we see that the first-order conditions for α are identical. If an interior solution existed with respect to β, the first-order conditions would be:

(3.11) $\partial S/\partial\beta = \Sigma_i\, 2\{(y_i - f(\mathbf{x}_i;\, \alpha))^2/(h(\mathbf{x}_i;\, \beta))^3\}h_\beta(\cdot) = 0.$

However, it can be shown that such an interior solution is not feasible. Assume for simplicity that β is a scalar. Furthermore, assume that $h_\beta(\cdot) > 0$ (< 0), i.e., that the marginal output risk with respect to x is positive (negative). Since, $h(x;\, \beta)$ is the denominator of (3.10), S is minimised as β goes to plus (minus) infinity. This implies that β will either explode positively or negatively, and that the equality in (3.11) can never be satisfied. Similar arguments apply for the more general case when β is a vector. Hence, single-stage optimisation of an FGLS objective function is not possible.

In contrast, the log-likelihood function allows a strictly interior solution for β, because the term $\ln(h(\mathbf{x}_i;\, \beta)^2)$ has been included in addition to $(y_i - f(\mathbf{x}_i;\, \alpha))^2/h(\mathbf{x}_i;\, \beta)^2$. The conclusion is that only ML allows single-stage estimation; for FGLS the problem of β's unboundedness necessitates a multi-stage estimation procedure.

Since the finite sample bias and inefficiency of FGLS compared to ML estimates cannot be determined *a priori*, Saha *et al.* (1997) undertake Monte Carlo experiments to examine the performance of the two estimators. They use a Cobb-Douglas parametrization of the mean function and a Harvey's multiplicative heteroskedastic parametrization of the variance function (See appendix 3.A). Saha *et al.* find that the standard errors of the ML estimates of β are smaller than the standard errors of the FGLS estimates. Thus, if inferences were drawn from FGLS estimates, one would to a larger extent conclude, incorrectly, that inputs do not have significant risk effects They also find that the ML estimator have considerably smaller mean square error than the FGLS estimator for both the mean function and the variance function.

Saha *et al.* also compared the performance of FGLS and ML when the distribution of the underlying error term ε_i departs from normality. They find that even under pronounced departures from normality ML performs better than FGLS in terms of MSE and power of t-tests. The difference in performance decreases as the departure from normality increases. These findings are important, because they support the use of the more efficient ML estimator even when the underlying error distribution is skewed, which will generally be the case for production risk.

It should be noted, however, that Saha *et al.*'s Monte-Carlo simulations assume homoskewness and homokurtosis. It is difficult to say how the ML estimator performs when the data generating process is characterised by heteroskewness and heterokurtosis. Simulation studies may provide some answers to that question.

The Monte-Carlo study of Saha *et al.* focuses on a homogenous production technology. It is difficult to say to what extent their simulation results apply to heterogeneous technologies and relevant econometric panel data estimators.

3.8. Primal vs. Dual Approaches

The first decision that has to be made in empirical research on firms based on microeconomic theory is whether to use a primal or dual approach. The primal versus dual model specification choice has been discussed by several authors, see for example Pope (1982) for a discussion with relevance to agriculture. However, a discussion that accounts for the most recent advances in dual modelling in the context of production risk has not been provided in the literature.

The empirical research on firm behaviour and productivity has been dominated by applications of duality theory. Estimation of flexible parametrizations of dual functions, particularly translog cost functions, has been very popular the last two decades. Flexible functional forms impose few restrictions on dual functions, and allow the testing of a rich set of hypotheses on the structure of production technology, output supply and input demands. Under production certainty or homoskedasticity the relevant objective functions are the profit function and the cost function. In the case of heteroskedastic production technologies, the relevant objective functions are the indirect utility function, which represents the solution to the EU maximisation problem, and the *ex ante* cost function, which represents the solution to the cost minimisation problem prior to the realisation of stochastic variables.

Several issues are of interest when comparing primal and dual approaches:

- What information can they provide on:
  - the structure of production risk (e.g. marginal risks),
  - output supply and input demand response to changes in prices, and the level of output risk,
  - technical and allocative efficiency, and
  - risk preferences.
- Restrictions on the structure of production technology, risk preferences, and expectation formations.
- Empirical tractability: Data requirements and estimation methods.

In the following these issues will be discussed in relation to the stochastic production function, the *ex ante* cost function and the mean-variance utility function. The two dual functions were presented in section 2.4. For the empirical research on production risk we have the situation that primal models have been applied to a much greater extent than dual models; in fact, papers on dual models have been limited to discussions of how to facilitate empirical implementation, a Monte Carlo simulation and an illustrative application (Coyle, 1995; Pope & Chavas, 1994; Pope & Just, 1996).

Information Derived from Primal and Dual Models

The estimated mean function $f(\mathbf{x};\ \hat{\alpha})$ of a Just-Pope production function provides information on the elasticities of substitution between inputs, marginal productivity of inputs, and the elasticity of scale (Chambers, 1988, Ch. 1). From the estimated variance function $h(\mathbf{x};\ \hat{\beta})$, information on marginal input risk and the elasticity of risk with respect to scale can be derived.

From the estimated *ex ante* cost function $c(\mathbf{w}, \mu_y; \hat{\alpha})$ it is possible to derive input demand functions $\mathbf{x}(\mathbf{w}, \mu_y; \hat{\alpha})$ by Shephard's lemma, own-price and cross-price demand elasticities, average and marginal costs, and elasticity to size.

From the estimated indirect utility function $U^*(p, \mathbf{w}, W_0, \mathbf{q};\ \hat{\alpha})$ it is in principle possible to derive input demand functions $\mathbf{x}(p, \mathbf{w}, W_0, \mathbf{q})$, expected output supply function $\mathrm{E}y(p, \mathbf{w}, W_0, \mathbf{q})$ and output variance function $\mathrm{V}y(p, \mathbf{w}, W_0, \mathbf{q})$.

According to Chapter two, which discussed efficiency concepts under production risk, the relevant function for efficiency measurement is the EU function, because this function accounts for the firm's subjective mean-variance trade-off when it chooses production technology and input vector. By implementing time trend or dummy variables in the primal model it is possible to obtain estimates of shifts in mean output and the variance of output, but in order to perform efficiency ranking it is necessary to have information or make assumptions on risk preferences.

Data Requirements

There are considerable differences in data requirements for the different approaches. Estimation of production function only requires data on input and output levels, all which are in principle observable.

The *ex ante* cost function estimation requires data on input levels and prices, and the moments of output μ_y formed by the producer. The moments of output are unobservable quantities, which means that assumptions have to be made regarding the producer's expectation formation. As demonstrated by Pope & Just (1996), things are simplified if risk neutrality is assumed, because expected output $\mathrm{E}[y|\mathbf{x}]$ is then the only relevant moment and is derived as part of the cost function estimation procedure.

Estimation of the indirect utility function, or its derived demand and supply equations, involves the most comprehensive data requirements. It requires all the data that are necessary for cost function estimation, but also introduces the subjective moments of output price p, initial wealth W_0, and the subjective moments of the exogenous error term ε of the production function as additional information. Both the moments of p and the moments of ε are unobservables, which means that assumptions have to be made regarding the expectation formation of producers.

The Production Function Approach to Obtaining Comparative Static Results

An advantage of production function estimation compared with the dual functions is that it only relies on observables, and does not require assumptions on risk preferences and expectation formation. On the other hand, an estimated production function does not provide information on the input demand and output supply responses of producers to changes in prices p and $\mathbf{w}$.

However, if one has additional information or makes assumptions on the producers' risk preferences, then the estimated production function can be inserted into an EU model and numerical simulation studies can be performed. A Just-Pope specification of the production technology is particularly convenient for simulation studies, because it allows a mean-variance representation of the firm's EU maximisation problem (Leathers & Quiggin, 1991; Meyer, 1987). The firm's maximisation problem can be expressed by the certainty-equivalent version of the mean variance function

(1) $$\max_{\mathbf{x}} U = W_0 + E\pi - A\big(W_0 + E\pi, \mathrm{var}(\pi)\big)/\,2\,\mathrm{var}(\pi),$$

where $A(\cdot)$ is the coefficient of absolute risk aversion. This means that the firm's risk premium, which is the monetary representation of the firm's level of absolute risk aversion, can be calculated as

$$r\big(W_0, \pi^*\big) = E\big(W^*\big) - U^*,$$

where '*' indicates that the variables are evaluated in EU maximising input levels $\mathbf{x}^*$. The calculated risk premium can serve as a means to assess whether the chosen parameter values for the coefficient of absolute risk aversion $A(\cdot)$ are sensible. If one is only measuring marginal (or local) changes in prices p and $\mathbf{w}$, the simplifying assumption of constant absolute risk aversion (CARA) is probably safe to make. Under CARA, $A(\cdot)$ reduces to the constant A.

The most elegant way of obtaining empirical comparative statics based on the estimated production function is to derive analytical expressions for input demand functions $\mathbf{x}(p, \mathbf{w}, W_0, \mathbf{q})$ and expected output supply $Ey(p, \mathbf{w}, W_0, \mathbf{q})$ from the maximisation problem (1), and insert estimated production function parameters into these. In practice this is generally difficult, even under restrictions on $A(\cdot)$ and the stochastic production technology. An alternative is to use nonlinear maximisation techniques to obtain the input vector $\mathbf{x}^*$ that maximises $U(\cdot)$ for different values of A and prices p and $\mathbf{w}$. There are several examples of numerical simulation studies in the empirical literature on production risk, for example, Yassour *et al.* (1981), Antle & Crissman (1990), and Ramaswami (1993). The two latter studies use empirical production function estimates in their simulation models. We have seen earlier in this chapter that Antle &

Crissman calculate technical efficiency indices of different technologies based on the estimated primal model. Ramaswami calculates optimal input use under different levels of insurance coverage.

The Cost Function Approach

When discussing the cost function approach it is useful to recall the decision process that generates the input vectors and moments of output which are used in the estimation of the *ex ante* cost function. The firm maximises its expected utility of profits or end-of-period wealth given the structure of the stochastic production technology and output and input prices. The moments of output are conditional on the EU maximising input vector $\mathbf{x}^*$. Thus, only the indirect utility function derived from the EU maximisation problem endogenizes expected output supply and output variance. Herein lies one of the limitations of the ex ante cost function; it can only provide input demands conditional on the EU maximising output mean and variance.

There are basically two different approaches for addressing the problem of unobservable moments of output in *ex ante* cost function estimation: (i) Assume risk neutrality or that firms have no knowledge on the structure of the variance function, which implies that only mean output is of relevance to the firm, and allows the cost function and mean output to be estimated simultaneously by the procedure suggested by Pope & Just (1996). (ii) First estimate the parameters of the production technology by e.g. a Just-Pope specification, then construct proxies for the output moments based on the estimated primal model for each firm assuming that the firms know the structure of the production technology and have rational expectations, and finally estimate the cost function with the constructed proxies as arguments. For the first approach it can be questioned whether risk neutrality or variance function ignorance is present in the particular industry in question. For the second approach it is a problem that consistent forms for the primal function and the cost function as implied by the primal/dual relationship are difficult to obtain. In the deterministic case only simple specifications such as the Cobb-Douglas and constant-elasticity-of-substitution (CES) functional forms are self-dual, while common flexible functional forms such as the translog are not self-dual. In case (i) the researcher runs the risk of specifying a cost function that is not valid empirically, while in case (ii) the researcher may specify a model that is theoretically inconsistent.

The Mean-Variance Utility Function Approach

From a theoretical point of view the mean-variance approach is a very attractive alternative for analysing the behaviour of the firm under production risk due to the rich set of comparative statics which can be derived. However, it is undoubtedly the most difficult to implement empirically, due to the unobservables it requires and the functional form. Derivation of input demands and expected output supply functions are much more complicated for the indirect utility function than for the objective function under certainty, the profit function. The model specification can be simplified to some extent by assuming constant absolute risk aversion (CARA), but except for small changes in initial wealth the CARA assumption is difficult to defend. It is problematic to assume that optimal input levels $\mathbf{x}^*$ are unaffected by larger changes in initial wealth.

Input Fixity and Availability of Insurance

Both the *ex ante* cost function of Pope & Just (1996) and the indirect utility function of Coyle (1995) assume that all inputs are variable, i.e., they are *long-run* functions. In many industries one or several inputs are fixed or quasi-fixed, thus implying that a long-run specification is inappropriate. Long-run specifications of dual functions may result in biased estimates of economies of scale, economies of substitution, etc. (Braeutigam & Daughety, 1983; Brown & Christensen, 1981; Caves, L.R., & Swanson, 1981; Nelson, 1985).

The indirect utility function of (Coyle, 1995) implicitly assumes that insurance is not available. If insurance is available for some sources of output risk (e.g. diseases), then the conditional probability distribution of revenue is altered (Ramaswami, 1993). Consequently, the EU maximising input levels $\mathbf{x}^*$ are also changed. The presence of insurance does not cause any problem for specification and estimation of production or cost functions. However, an indirect utility function that ignores insurance will provide biased estimates.

Firm Heterogeneity and Panel Data Availability

The assumption of homogeneous firms in the neoclassical production theory is probably not valid for most industries. In recent years, panel data sets that allow the researchers to account for firm heterogeneity have become available. Under risk, producer heterogeneity can operate on several levels: (i) The production process, (ii) risk preferences, and (iii) expectation formation with respect to prices and output.

For production function estimation only heterogeneity with respect to the production process is relevant. This heterogeneity can be captured by implementing firm-specific effects, as discussed in preceding sections. If a cost function approach is chosen, firm-specific effects can be implemented in derived input demand or input share equations. This is a more satisfactory specification of technology heterogeneity if the firm-specific effects are input related, i.e., the productivity of inputs vary across firms. Unless risk neutrality is assumed, technology heterogeneity also has to be accounted for in the construction of moments of output. Firm-specific effects can also be implemented in the input demand and expected output supply equations derived from the indirect utility function. But for a general specification of these equations it is difficult to separate the effects of technology heterogeneity from risk preference heterogeneity on the estimated firm-specific effects.

Concluding Remarks

This section has discussed model choice for econometric estimation at a general level. We have seen that the primal approach has the smallest data requirements, while the largest data requirements are associated with the mean-variance approach. Dual function estimation also forces the researcher to deal with specification issues that can be ignored for the primal approach; risk preferences, expectation formation, fixity of inputs, and insurance possibilities. The mean-variance approach involves the largest set of specification issues, while the cost function approach represents an intermediate case between the mean-variance and the primal approach.

Of course, the model choice decision will depend on data availability and the complexity of specification issues for the particular industry which is the subject of the empirical analysis. Furthermore, it depends on the focus of the study, e.g. whether the primary interest is the structure of the production technology or input demand and output supply elasticities in prices. However, it can be argued that the primal approach generally is a more attractive alternative to dual approaches under production risk than in the standard deterministic case. The fact that empirical implementations of dual functions are almost non-existent, while there are several applications of the primal, supports this argument.

3.9. Summary and Discussion

The article of Just & Pope (1978), in which eight postulates for the stochastic production function were proposed, introduced a theoretical framework for the modelling of production risk. It also seems to have initiated econometric research on the structure of production risk, although some research on this subject had been undertaken earlier (Day, 1965). Just & Pope also proposed econometric estimation procedures that provide consistent and asymptotically efficient estimates of the production function parameters when the production function takes the Just-Pope form.

Most of the subsequent studies to a large extent use the model framework of Just & Pope, but propose modifications of the model specification that may contribute to give a better empirical description of risky production processes. Such modifications include introduction of firm- and time-specific random effects (Griffiths & Anderson, 1982), non-normal error terms such as the beta (Nelson & Preckel, 1989) and Weibull (Saha, et al., 1994), joint estimation of production technology and risk preference parameters (Love & Buccola, 1991; Saha, et al., 1994), more flexible functional forms for the mean production function (Kumbhakar, 1993), and specification of the variance production function in order to allow testing of the effects of technical change on production risk (Traxler, et al., 1995).

The majority of the empirical studies of production risk use the Just-Pope model specification and estimation framework. The most dramatic departures from this framework are Nelson & Preckel (1989), and Antle (1983). Saha *et al.* (1994) use a Just-Pope production function, but estimate it jointly with a utility function. Computational complexity, limited knowledge about the performance of these estimators for different data designs, and a belief that heteroskewness and heterokurtosis may have limited significance for producer behaviour in practice, may explain why the approaches of Nelson & Preckel and Antle have not been adopted by other empirical studies. Similar arguments may apply to Saha *et al.* (1994).

The empirical results of the above studies give strong indications of the presence of heteroskedasticity in biological production processes. Both positive and negative marginal risks associated with changes in input levels are found. Empirical results indicate that input changes also affect the skewness of the output distribution. In other words, for several agricultural sectors, the traditional neoclassical production function specification provides an unsatisfactory description of the production technology. Since the theoretical models of firm behaviour under production risk presented in Chapter 2 predict that the optimal levels of inputs are affected by their effects on the variance and higher moments of output distribution, these empirical findings should be of considerable interest.

However, the results of the individual studies should be interpreted with care, because weaknesses or deficiencies with respect to methodology and data generally characterise the studies. Most of the studies focus on the introduction of a particular methodological improvement, and tend to give less attention to important specification and data issues. Having said that, none of the studies explicitly pretends to have given the final word on the structure of production risk in the particular industry they are analysing.

It is difficult to say something about the magnitude of biases due to errors in variables, because the studies generally provide limited information on data collection and variable construction. Omitted variables bias is probably present in the empirical models in some of the studies listed in table 3.1, because very few inputs are implemented.

Incorrect functional form may also give rise to biases in several studies. Of course, we usually do not know what is the "correct" functional form, but the results from econometric productivity studies which employ second-order flexible functional forms, such as the translog, suggest that the Cobb-Douglas form, which is used frequently in the studies listed in table 3.1, generally provides a poor representation of the underlying production technology.[1] The only study to use a flexible functional form for the mean production function is Kumbhakar (1993).[2] Potential effects of incorrect functional form on estimated residuals have been demonstrated several times, e.g. Gujarati (1988, pp. 407-10); estimating a model that is more restrictive than the "true" model that generated the data will produce residuals that are much larger (in absolute value) than the residuals of the true model. In the context of a Just-Pope production function, this will not only have effects on the parameter estimates of the

1 Remember that the Cobb-Douglas function is a special case of the translog function $\ln y = \alpha_0 + \Sigma_i \alpha_i \ln x_i + \Sigma_i \Sigma_j \alpha_{ij} \ln x_i \ln x_j$, with $\alpha_{ij} = 0$ for all i,j.

2 Just & Pope (1979) also employ the translog for the mean function, but with only one input. Traxler *et al* (1995) employ a linear quadratic mean production function, but again only one input is included in the model.

mean function, but also on the parameter estimates of the variance function. Consequently, this may lead to false inferences regarding the marginal effects of inputs on production risk.[3] Using an incorrect functional form may produce patterns of heteroskedasticity or autocorrelation that in reality do not exist. Based on the large empirical evidence on the significance of second-order terms in flexible functional forms, it can be argued that the results of those studies of production risk that have employed Cobb-Douglas specifications for the mean function must be interpreted with great care. There are good reasons to believe that parameter estimates of the variance function presented in these studies are biased. Again, it should be emphasised that the main objective of previous studies in general probably has not been to provide "correct" estimates of production risk parameters for the particular industry chosen as an empirical application, but rather to demonstrate methodological innovations.

Another potential methodological problem for most studies using the Just-Pope function, is the use of FGLS estimators. The findings of Saha *et al.* (1997) suggest that an ML estimator based on normality of the error term generally outperforms FGLS in finite samples, even when the distribution of the true error departs significantly from normality. This last point is very interesting, considering that normality probably is an unreasonable assumption for agricultural production technologies. The analysis of Saha *et al.* indicates that the insignificance of variance function parameters in several empirical studies may be due to the inefficiency of FGLS, not the absence of production heteroskedasticity in inputs.

This chapter also provided a discussion of primal versus dual approaches. Dual models lose some of their attractiveness under production risk due to presence of unobservables, the difficulty of deriving comparative statics and computational complexity. A simulation approach to obtaining comparative static results based on an estimated production function and postulated risk preferences was outlined. The simulation approach represents an alternative to estimating dual models, which may involve the use of questionable proxies for unobservables and restrictions on technology and risk preferences which lack an empirical basis.

Econometric panel data issues has been given less attention in empirical studies of production risk, although panel data have been available to most of the empirical studies discussed here. There are several issues which deserve to be discussed, for example the specification of the firm-specific effects, the estimation procedures to be used, and sample selection issues. In particular, for the popular random effects model, which has become very popular and is used by Griffiths & Anderson (1982), it is pertinent to ask to what extent the underlying assumptions are appropriate, and what are the consequences for empirical estimates if these assumptions are not valid. Chapter 4 discusses econometric panel data issues, while Chapter 5

[3] If the purpose of a study was to produce "evidence" of substantial production risk, both total and marginal, for an industry production data was available for, the chances for succeeding would be much greater by using a restrictive form such as the Cobb-Douglas rather than the more flexible translog.

provides a simulation study for a Just-Pope technology with firm-specific effects. The simulation study in Chapter 5 can be regarded as an extension to the study of Saha *et al.* (1997).

Finally, Chapter 9 provides a further discussion of the pros and cons of some of the approaches presented in this chapter, particularly with respect to the empirical application in this dissertation.

3.A. Appendix: Efficient Estimation of the Mean Function of Harvey's Multiplicative Heteroskedastic Model

The production function in Harvey's formulation is given by

(3.A1) $y_i = f(\mathbf{x}_i, \alpha) + u_i$,

where $\mathbf{x}_i$ is the $k \times 1$ input vector, and the variance of the error is specified as

(3.A2) $E(u_i^2) = \sigma_i^2 = \exp[\mathbf{z}_i' \beta]$,

where $E(u_i)=0$, $E(u_i, u_j)$ for $i \neq j$, and the first element in the $l \times 1$ vector $\mathbf{z}_i$ is taken as unity ($z_{i1} \equiv 1$). The remaining elements in $\mathbf{z}_i$ may be identical to, a subset of, or functions of the x's in the mean production function. It is easily seen that the conditional variance of output, var(y_i| $\mathbf{x}_i$, $\mathbf{z}_i$), is exp($\mathbf{z}_i'\beta$). The exponentiation of $\mathbf{z}_i'\beta$ in (3.A2) ensures that the variance is always positive. This is not the case for the conditional variance of the general Just-Pope model $y = f(\mathbf{x}; \alpha)+h(\mathbf{x}; \beta)\varepsilon$; in order to allow both negative and positive marginal risks, the β's may take both negative and positive values. Consequently, var(y_i| $\mathbf{x}_i$, $\mathbf{z}_i$) may be negative for certain values of $\mathbf{x}_i$ in the Just-Pope model, depending on the signs and values of the β's. Although it is a nice property of Harvey's formulation that it always yields positive variances, it may be discussed if there are any *a priori* reasons for postulating a natural exponential function for the variance of production, and whether this may be a too restrictive assumption. In the following we discuss the properties of two different specifications of the argument of the exponent, a linear and a log-linear specification, in the context of production risk analysis.

Heteroskedasticity does not destroy the unbiasedness and consistency properties of the OLS estimators. However, OLS estimators are no longer minimum variance or efficient. In this appendix it is shown why the FGLS estimation procedure provides efficient estimates of the mean function parameters in the presence of multiplicative heteroskedasticity of the form (3.A2). Equation (3.A2) can be rewritten as

(3.A3) $\ln \sigma_i^2 = \mathbf{z}_i \beta$.

The σ^2's are unknown, but the least squares residuals $\hat{u}_i = y_i - f(\mathbf{x}_i; \hat{\alpha})$ from (3.A1) can be used to estimate the above equation. Adding $\ln \hat{u}_i^2$ to both sides of (3.A3) yields

$$\ln \hat{u}_i^2 + \ln \sigma_i^2 = \mathbf{z}_i\beta + \ln \hat{u}_i^2,$$

or

(3.A4) $\ln \hat{u}_i^2 = \mathbf{z}_i\beta + v_i$,

where $v_i = \ln \hat{u}_i^2 - \ln \sigma_i^2 = (\ln \hat{u}_i^2 / \ln \sigma_i^2)$. Equation (3.A4) is estimated in the second stage.

It can be shown that (Harvey, 1976)

$E[v_i^*] = -1.2704$

$\text{var}(v_i^*) = E[(v_i^* - E[v_i^*])^2] = 4.9348,$

$\text{cov}(v_i^*, v_j^*) = 0, \text{ for } i \neq j,$

where $v_i^* = (\ln u_i^2/\ln \sigma_i^2)$ is the random variable that $v_i = (\ln \hat{u}_i^2/\ln \sigma_i^2)$ converges in distribution to. Hence, the first element of the estimator $\hat{\beta}$ is inconsistent with an inconsistency of -1.2704, but the remaining elements are all consistent. To make the first element consistent 1.2704 has to be added to it.

In the third stage predicted standard deviations $\exp(\mathbf{z}_i\hat{\beta})^{1/2}$ from equation (3.A3) are used as weights for generating FGLS estimators for the mean production function. In order to show that the use of predicted residuals make the errors in the transformed data homoskedastic, define the constant $\sigma^2=\exp(\beta_1)$, and define $\mathbf{z}_i^{*\prime}=(X_{i1},\ldots,X_{il})$ and $\beta^*=(\beta_2,\ldots,\beta_l)$. The weighted regression equation is given by

$$\frac{y_i}{\exp(\mathbf{z}_i^*\beta^*)^{1/2}} = \frac{\mathbf{x}_i\alpha}{\exp(\mathbf{z}_i^*\beta^*)^{1/2}} + \frac{u_i}{\exp(\mathbf{z}_i^*\beta^*)^{1/2}}$$

$$= \frac{\mathbf{x}_i\alpha}{\exp(\mathbf{z}_i^*\beta^*)^{1/2}} + u_i^*,$$

where

$$u_i^* = \frac{u_i}{\exp(\mathbf{z}_i^*\beta^*)^{1/2}}.$$

The variance of u_i^* is given by

$$E[u_i^{*2}] = E\left[\left(\frac{u_i}{\exp(\mathbf{z}_i^*\beta^*)^{1/2}}\right)^2\right] = \frac{1}{\exp(\mathbf{z}_i^*\beta^*)}E[u_i^2]$$

$$= \frac{1}{\exp(\mathbf{z}_i^*\beta^*)}\exp(\mathbf{z}_i\beta) = \exp(\beta_1) = \sigma^2,$$

i.e., the error term has now become homoskedastic.

3.B. Appendix: A Two-Stage Estimation Procedure for the Variance Function Parameters of the Just-Pope Model

There exists a two-stage estimator with the same asymptotic distribution as the ML estimator for the Just-Pope function. This estimator relies on the use of a consistent estimator of the variance function parameter vector β in the first stage. In fact, this estimator can be applied to a more general class of regression models where least squares yield consistent but not efficient estimates (Cramer, 1986, pp. 71-72).

The least squares estimate of β is consistent, and can thus be used in the first stage. In the second stage β is estimated by (Harvey, 1976, eqn. 15)

$$\hat{\beta}_{(2)} = \hat{\beta}_{(1)} + \phi + 0.2807\left[\sum_{i=1}^{n} \mathbf{z}_i'\mathbf{z}_i\right]^{-1} \sum_{i=1}^{n} \mathbf{z}_i' e^{-\mathbf{z}_i\hat{\beta}_{(1)}} \hat{u}_i^2 ,$$

where $\hat{\beta}_{(1)}$ is the first-stage (least squares) estimate of the $m\times 1$ parameter vector β, $\mathbf{z}_i$ is the $1\times m$ vector of regressors (with first element one), ϕ is a $m\times 1$-vector in which the first element is 0.2704 and the remaining elements are zero. The second-stage estimate $\hat{\beta}_{(2)}$ has the same asymptotic distribution as the ML estimator of β, and is therefore asymptotically efficient.

4. ISSUES IN ECONOMETRIC PANEL DATA ESTIMATION

According to one of the hypotheses presented in Chapter one, substantial firm heterogeneity is present in the salmon farming industry. This heterogeneity should be accounted for in an econometric model. As we have a panel data set available (see Chapter 8), use of econometric panel data techniques to account for heterogeneity is possible. The fact that firm heterogeneity has largely been ignored in empirical productivity studies until recently, can to a large extent be attributed to the underlying theoretical framework. In particular, we saw in Chapter 2 that the theory of the competitive firm under production risk is mainly a theory of the behaviour of the representative firm. In Chapter 3 it was argued that although firm heterogeneity was accounted for in a few studies, discussion of important econometric panel data issues and their implication for empirical models of production risk has been neglected.

In the first sections of this chapter some important panel data issues will be discussed. The discussion will both be at a general level and more specifically in the context of production analysis. In some sections the focus will be on specific problems that are relevant to the empirical application in this dissertation.

Much of the discussion will be in the context of the linear model specification

(4.1) $\quad y_{it} = \mathbf{x}_{it}\alpha + \eta_i + \lambda_t + u_{it}, \qquad i = 1, 2, ..., N, \ \ t = 1, 2, ..., T_i,$

where i refers to units (e.g. firms) and t refers to time periods. The parameter η_i is the group-specific (or firm-specific) effect, while the parameter λ_t is the time-specific effect. Econometric panel data models can be divided into two classes: Fixed effects models and random effects models. The distinction between these two classes pertains to the assumptions on η_i and λ_t. If η_i and λ_t are assumed to be fixed parameters, then (4.1) is the *fixed effects* (FE) model.

Alternatively, if η_i and λ_t are assumed to be random parameters, then (4.1) is the *random effects* (RE) model. In order to allow for an intercept, the vector $\mathbf{x}_{it}$ should then include 1 as the first element. The standard distributional assumptions of the RE specification are $\eta_i \sim$ IID$(0,\sigma_\eta^2)$ and $\lambda_t \sim$ IID$(0,\sigma_\lambda^2)$ and $u_{it} \sim$ IID$(0,\sigma_u^2)$.[1] In other words, the error components η_i, λ_t, and u_{it} are assumed to be homoskedastic, independent of each other and of the regressors $\mathbf{x}_{it}$, see e.g. Baltagi (1992, pp. 87-8). The random effects model is sometimes referred to as the *variance components* or *error components* model (Hsiao, 1986, pp. 33).

In the discussion of RE models in this chapter, we will focus on the panel data model with only firm-specific random effects, η_i. This special case of the RE model is known as the *one-*

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[1] Appendix 4.A1 presents the notation used in this chapter.

way error components model. Of course, time-specific effects λ_t can be included as dummy variables and included in the set of explanatory variables $\mathbf{x}_{it}$ in the RE model. It is probably also more reasonable to treat the time-specific effects as fixed in a data set with a small number of periods, which is generally the case for empirical productivity studies, and for the empirical application in this dissertation in particular.[2]

The observation-specific error term u_{it} is usually assumed homoskedastic with variance $\sigma_u{}^2$ in the panel data literature. For this study it is more appropriate to assume Just-Pope heteroskedasticity for u_{it}, i.e.,

(4.2) $\quad \text{var}(u_{it}) = [h(\mathbf{z}_{it}; \beta)]^2\sigma_\varepsilon^2,$

where σ_ε^2 is the variance of the exogenous error term ε_{it}. Later in this chapter we discuss estimation of FE and RE models with heteroskedasticity of this form.

The disposition of this chapter is as follows: Section 4.1 presents the fixed effects estimator for the case of homoskedastic u_{it}. Section 4.2 discusses advantages associated with using econometric panel data models. Potential pitfalls and limitations of panel data models are provided in section 4.3. Section 4.4 discusses balanced and unbalanced panel data in the context of the particular empirical application in this dissertation. The issue of fixed versus random effects is dealt with in section 4.5. Section 4.6 presents ML estimators for fixed effects models. Estimation of RE models is dealt with in sections 4.7-4.8. Section 4.7 discusses estimation of the homoskedastic random effects model in the case of unbalanced data. A brief survey of previous RE models with different types of heteroskedasticity incorporated is provided in section 4.8. Finally, section 4.9 provides a summary of this chapter.

4.1. The Fixed Effects Model in the Homoskedastic Case

A nice property of the model (4.1) in the fixed-effects context is that the computational procedure does not require use of dummy variables for η_i. In fact, for model (4.1) without the time-effects λ_t, the BLUE estimator of α is given by the so-called *within-estimator*

$$\begin{aligned}\hat{\alpha} &= \left[\sum_{i=1}^{N}\sum_{t=1}^{T_i}(\mathbf{x}_{it}-\bar{\mathbf{x}}_i)(\mathbf{x}_{it}-\bar{\mathbf{x}}_i)'\right]^{-1}\left[\sum_{i=1}^{N}\sum_{t=1}^{T_i}(\mathbf{x}_{it}-\bar{\mathbf{x}}_i)(y_{it}-\bar{y}_i)'\right] \\ &= \left[\sum_{i=1}^{N}\sum_{t=1}^{T_i}\tilde{\mathbf{x}}_{it}\tilde{\mathbf{x}}_{it}'\right]^{-1}\left[\sum_{i=1}^{N}\sum_{t=1}^{T_i}\tilde{\mathbf{x}}_{it}\tilde{y}_{it}\right]\end{aligned},$$

where $\bar{\mathbf{x}}_i$ and $\bar{y}_i$ are the time-series means for the cross-sectional unit i, i.e.,

$$\bar{\mathbf{x}}_i = \frac{1}{T_i}\sum_{t=1}^{T_i}\mathbf{x}_{it} \quad \text{and} \quad \bar{y}_i = \frac{1}{T_i}\sum_{t=1}^{T_i}y_{it},$$

[2] The derivation of estimators is also simplified by using only firm-specific random effects.

and

$$\tilde{\mathbf{x}}_{it} = \mathbf{x}_{it} - \bar{\mathbf{x}}_i \quad \text{and} \quad \tilde{y}_{it} = y_{it} - \bar{y}_i$$

are the *within-transformations* of the independent and dependent variables (Hsiao, 1986).[3] This estimator is also consistent when N or T_i or both tend to infinity. The estimator of the firm-specific effects is

$$\hat{\eta}_i = \bar{y}_i - \bar{\mathbf{x}}_i\alpha\,,\ i = 1, ..., N.$$

The estimator of the firm-specific intercept is BLUE, but is consistent only in T_i (Hsiao, 1986, p. 32). Consequently, increasing the number of firms in the data set will not help in the asymptotic sense. This is also intuitive, since more information on the firm-specific intercept of firm i cannot be obtained by adding observations of other firms $j \neq i$ to the panel.

4.2. Advantages Associated with Using Panel Data Techniques

There are several potential advantages associated with using panel data sets and econometric panel data techniques (Baltagi, 1995, pp. 3-6; Baltagi & Raj, 1992; Hsiao, 1986, pp. 1-5, 213-218).[4] First, if parameter heterogeneity exists among cross-sectional or time-series units, then estimation of a model which ignores such heterogeneity could lead to inconsistent or meaningless estimates of interesting parameters. An example of this is the case of heterogeneous intercepts in the simple linear regression model $y_{it} = \eta_i + x_{it}\alpha + u_{it}$. Estimation of a "pooled" regression, i.e., a regression which assumes homogenous intercepts ($\eta_1 = \eta_2 = ... = \eta_n = \eta$), on a data set generated by a model with heterogeneous intercepts will in general lead to biased estimates of α. Moreover, the direction of the bias cannot be determined *a priori*; it can go either way. Another consequence of particular importance for this empirical study, is that the estimates of the residuals u_{it} will also be biased, which in the next stage will lead to biased estimates of the parameters β of the variance function (4.2).

In empirical studies missing explanatory variables $\mathbf{z}_{it}$ which are correlated with explanatory variables $\mathbf{x}_{it}$ included in the regression model is frequently a problem. The "true" regression model is $y_{it} = \eta + \mathbf{x}_{it}\alpha + \mathbf{z}_{it}\gamma + u_{it}$, but the researcher may estimate $y_{it} = \eta + \mathbf{x}_{it}\alpha + u_{it}$ This leads to biased parameter estimates of the $\mathbf{x}_{it}$ variables included in the model. However, if repeated observations for a group of individuals are available, they may allow us to get rid of the effect of $\mathbf{z}_{it}$. If $\mathbf{z}_{it} = \mathbf{z}_i$ for all t, i.e., $\mathbf{z}$ values are time-invariant for all individuals but vary across individuals, estimation of the regression model on first difference form

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[3] The fixed effects estimator of α with time-specific effects included uses a similar transformation of $\mathbf{x}$ and y, see Baltagi & Raj (1992, pp. 87-88).

[4] For a survey of the state-of-the-art in the econometrics of panel data, see the survey article of Baltagi (1992), and the books by Mátyás & Sevestre (1992) and Baltagi (1995).

$$(y_{it} - y_{it-1}) = (\mathbf{x}_{it} - \mathbf{x}_{it-1})\alpha + (u_{it} - u_{it-1})$$

will provide unbiased and consistent estimates of α. Similarly, if $\mathbf{z}_{it} = \mathbf{z}_t$ for all i, unbiased estimates will also be provided by a regression model where the deviation from the mean across individuals at a given time has been taken for all variables (Hsiao, 1986, pp. 3-4).

A recurrent problem in the application of econometric models on microeconomic data is measurement errors. While it is difficult to find remedies for errors in variables in cross-section or time-series data set, Griliches & Hausman (1986) have shown that availability of panel data improves the possibility of obtaining consistent estimators.

Panel data sets also have advantages when it comes to dynamics, i.e., models with lagged variables; it is not possible to estimate dynamic effects using a cross-sectional data set, while a single time-series data set usually cannot provide precise estimates of dynamic parameters because of multicollinearity. If panel data are available, the individual differences can be utilised to reduce the problem of multicollinearity (Hsiao, 1986, p. 3).

For this particular study it is particularly the possibility of separating the effects of firm heterogeneity and stochastic shocks on productivity that makes the use of panel data attractive. Earlier it was hypothesised that the permanent firm-specific effects η_i and the time-specific effects λ_t are the sources of nonstochastic productivity differences across firm observations, while the error term u_{it} is the source of stochastic (transitory) productivity differences.

4.3. Potential Pitfalls and Limitations of Panel Data

Although panel data provides several advantages compared with cross-section or time-series data, there are also pitfalls and limitations associated with the use of panel data and panel data techniques (Baltagi, 1995, pp. 6-7; Hsiao, 1986, Ch. 9).

For the purpose of making inferences from the sample to the population, the randomness of the sample can often be questioned. When observations are not drawn in a random manner from the population because of the design of the survey or due to other reasons, a selectivity bias may arise in the parameter estimates (Hsiao, 1986, pp. 7-8). There are several reasons for nonrandom selectivity that can be distinguished (Klevmarken, 1989, pp. 526-7): (1) *Self selectivity* is present if some units (e.g. individuals, firms) cannot be included in the sample because they have made decisions which make them unavailable to sampling. For this particular application the decision of some firms to integrate horizontally or vertically, which makes them ineligible for the Directorate of Fisheries' profitability survey of "independent" fish farms, is an example of self selectivity. However, we do not have any information that leads us to believe that these farms are structurally different from the remaining sample with respect to the production technology. (2) *Nonresponse* is another form of selectivity. A designated respondent may choose not to respond to certain questions or decide not to participate at all. For our empirical application this does not constitute a major problem, since fish farms are obliged by law to fill out questionnaires and because the Directorate of Fisheries pursues a rather aggressive policy towards farms that do not respond satisfactorily. (3) *Attrition*, i.e., units dropping out because they die, move or find the response burden to high, is generally a serious problem in panel studies. For this empirical application attrition was particularly present during the period 1990-92, when a substantial number of farms in the Norwegian salmon farming industry went into bankruptcy. (4) Selectivity may also be built into the sampling design, if the selection probabilities depend on economic response variables.

Heckman (1979) partitions the sources of sample selection bias into two groups: (1) self-selection and non-response decisions by the units being investigated or (2) sample selection decisions by survey statisticians/data processors or analysts. To some extent both these sources of selectivity bias are present in this study. However, the magnitude of this bias depends on the definition of the population. The profitability survey on Norwegian fish farms uses data only from farms which are independent in a legal sense, as described in Chapter 8. If the relevant population is defined as the population of independent farms then (2) is not a serious problem. All in all, we do not consider the sample selectivity (or nonrandomness) problem to be of such a magnitude that particular measures should be taken.

In the previous section it was argued that panel data under certain conditions allow the researcher to account for omitted variables in the linear model. For nonlinear models, however, the handling of omitted-variables bias is far more problematic. If the effects of omitted variables stay constant for a given individual through time, or are the same for all individuals in a given time period, the omitted-variables bias can be eliminated for linear regression models by (1) differencing the sample observations, (2) using dummy variables, or (3) postulating a conditional distribution of unobserved variables, given observed exogenous variables. Unfortunately, the results for the linear model are generally not applicable for nonlinear models (Hsiao, 1986, pp. 7-8). In nonlinear models, the FE and the RE approaches yield different estimators.

In the case of an unbalanced panel where some units are observed only once, the use of panel data techniques may force the researcher to drop observations. With a fixed effects specification, two or more observations of each unit is required. Furthermore, most random effects estimators require more than one observation of each cross-sectional unit, although there are exceptions (which we will come back to later). In such cases it must be considered if the heterogeneity among the cross-sectional units is large enough, or if there are other specification problems of such a magnitude that the loss of observations can be defended. For our particular empirical application degrees of freedom are lost when going from a "pooled" regression model to a model with firm-specific effects because firms that are only observed one year have to be dropped. If lagged variables are to be used in the model specification, farms that are observed for two years or less have to be dropped from the estimating sample.

For our empirical application the presence of substantial firm heterogeneity is a compelling reason for constructing a panel data set, even though it will mean a loss of observations, e.g. with a fixed effects approach. We suspect that the biases in estimated parameters associated with using a pooled model specification are larger than the potential bias associated with omitting firms which are observed one period from the sample.

The panel data literature has mainly focused on the regression model with heterogeneous intercepts only, i.e., model (4.1). Models with heterogeneous slopes have been relatively less explored. An example of a model with slope coefficients that vary over individuals is: $y_{it} = \Sigma_k \alpha_{ki} x_{kit} + u_{it} = \Sigma_k(\overline{\alpha}_k + \eta_{ik})x_{kit} + u_{it}$, where $\overline{\alpha}_k$ is the mean coefficient and η_{ik} is the fixed or random firm-specific effect on the coefficient. There is at least three practical explanations for the limited use of this type of model when η_{ik} are assumed fixed: (1) Cross-sectional units are often observed only for a few periods. Unlike the model with heterogeneous intercepts, it is not possible to perform within transformations of the variables; one has to implement dummies for each slope parameter. Consequently, one may soon run into a degrees-of-freedom problem. (2) With a large number of cross-sectional units the computational requirements associated with the estimation of the model may be substantial. The limits of conventional econometric computer software packages may be violated. (3) Multicollinearity makes inference difficult.

These problems can be avoided by assuming that the firm-specific coefficients η_{ik} are random. Several random coefficients models have been proposed in the literature (Judge, Griffiths, Hill, Lütkepohl, & Lee, 1985, pp. 346-58). There are, however, problems associated also with this class of models. First, the variances of the η_{ik}'s are assumed positive, which in the context of production risk analysis implies that the possibility of negative marginal output risks is ruled out. Second, the random coefficients vary around a constant mean $\overline{\alpha}_k$, which may be a questionable assumption for industries with technical change.

Unfortunately, heterogeneous intercept models, such as (4.1), have their own inherent problems. Heterogeneous intercept models may give a poor description of the nature of heterogeneity for many empirical phenomena. In particular, it may provide an unsatisfactory description of the nature of firm heterogeneity. In Chapter three it was suggested that a heterogeneous intercept is problematic in the linear quadratic model because it implies that productivity differences between firms are independent of the level of output. However, this is not the case for log-linear models such as the Cobb-Douglas and translog, where the production function interacts multiplicatively with η_i. In log-linear specifications the productivity difference increases with the scale of operation for two firms i and j with different firm-specific effects η_i and η_j.

The heterogeneous intercepts model (4.1) also implies that there are time-invariant differences in productivity across firms. This restriction may be unrealistic; learning-by-doing and diffusion of innovations may over time contribute to reducing the productivity differentials between the less efficient and the more efficient firms. This is believed to be the case for the particular industry under investigation here. Cornwell *et al.* (1990) have proposed a production function with time-variation in the efficiency levels of the firm of the form

$$\eta_{it} = \theta_{i1} + \theta_{i2}t + \theta_{i3}t^2.$$

This specification is able to capture the effects of learning-by-doing and diffusion of innovations on relative firm efficiencies. They suggest a method for estimating the θ's by regressing the estimated residuals $\hat{u}_{it} = y_{it} - \mathbf{x}_{it}\hat{\alpha}$ on $\theta_{i1} + \theta_{i2}t + \theta_{i3}t^2$. The problem is that the estimates from Cornwell *et al.*'s procedure is only consistent as $T \rightarrow 0$. A large number of time-series observations is not very common for firm data, so this asymptotic requirement is rarely met.

4.4. Balanced and Unbalanced Panels

Unequal time series for the cross-sectional units has primarily consequences for estimation of random effects models. When the panel is incomplete, standard estimation methods cannot be applied for the RE model (Wansbeek & Kapteyn, 1989, p. 355). However, estimators for the RE model with incomplete observations have been derived (Baltagi, 1985; Baltagi & Chang, 1994; Wansbeek & Kapteyn, 1989). Later we will see that the weights used in FGLS estimation of the RE model will depend on the length of the individual time series. Heteroskedasticity is therefore introduced even with homoskedastic error components. Although the computational complexity increases when going from a balanced to an unbalanced panel, this does not constitute a major problem in empirical work.[5]

As we will see in Chapter 8 we have an unbalanced (or incomplete) panel data set: Firms are observed from one to nine years. Furthermore, the first year of observation varies from 1985 to 1993. When data are missing, it has been a common procedure to focus on the subset of individuals for which complete time-series observations are available (Hsiao, 1986, p. 197; Verbeek & Nijman, 1992). In our particular case this may for instance mean including only the firms that are observed throughout the nine year data period, alternatively, constructing a balanced panel data set of firms that for example participate at least seven years using firms that are observed seven, eight and nine years. Both designs will lead to a dramatic reduction in the total number of observations. It will also lead to a loss of information, since the incompletely observed firms contain information about unknown parameters. For example, in the context of the random effects model, Baltagi & Li (1990) have shown that dropping observations to make the panel data set balanced will produce an inferior estimate of the

[5] Econometric software packages such as Limdep has implemented computational procedures for FGLS estimation of homoskedastic RE models with unbalanced panels.

variance components of the two-way random effects model. Simulation results of Baltagi & Chang (1994) also suggests dropping observations to make the panel balanced reduce the performance of RE model estimators compared to those from the entire unbalanced data set.

Omission of variables in order to obtain a balanced panel may be particularly dangerous in the context of empirical modelling of production risk. In order to obtain a balanced panel of length T, firms that are observed only for 1,2,..., T-1 time periods are dropped from the panel. There may be several reasons why a firm is observed less than T periods: (i) the firm entered the industry relatively late, i.e., less than T periods before the last time period of the data set, (ii) the firm dropped out because it was inefficient, (iii) the firm dropped out because it experienced adverse realisations of stochastic variables (ε_{it}, in the context of the variance function (4.2)), or (iv) the firm became ineligible for the survey because it merged with other firms, diversified into other types of production, etc. For our application (ii) and (iii) probably explain in many cases why firms are observed only for a short time period. Consequently, the representativeness of the remaining subset may be questioned, because the firms in the balanced panel subset probably are the most efficient firms and/or the firms that have experienced favourable realisations of the stochastic variables in the population. Both factors will have an effect on the estimates of the error term u_{it} and the estimates of the parameters β of the variance production function (4.2).

4.5. Fixed Versus Random Effects

A question that often arises in the literature is whether the FE or RE model should be used.[6] Some have suggested that the distinction between fixed and random effects models is an erroneous interpretation. Mundlak (1978) argues that we should always treat the individual effects η_i as random. The fixed effects model is simply analysed conditionally on the effects present in the observed sample. The fact is, however, that empirical results indicate that the choice of model can have a large effect on the estimates of α in so-called longitudinal data sets, where N is large and T is small (Hsiao, 1986, pp. 41-42), which is the case for our application.

The random effects model has been criticised because it assumes that there is no correlation between the individual effect η_i and the explanatory variables $\mathbf{x}_{it}$ (Mundlak, 1978). There are reasons to believe that η_i and $\mathbf{x}_{it}$ are correlated in many circumstances. One example which has been mentioned, and is relevant for our empirical study, is the estimation of a production function using data at the firm level. The output of a firm may be affected by its unobservable managerial quality, which is represented by η_i. Managerial quality is often positively correlated with firm size, or the amount of inputs used $\mathbf{x}_{it}$. Larger firms can afford to recruit

6 Both Hsiao (1986, pp. 41-47) and Greene (1991, pp. 494-496) discuss this issue.

highly skilled managers. In this situation η_i and $\mathbf{x}_{it}$ cannot be regarded as independent, and the standard random-effects model will provide biased estimates of α. If the nature of the correlation is known, it should in principle be possible to derive GLS estimators of α in order to obtain unbiased estimates for the linear specification (4.1).[7] However, for dynamic, random-coefficient, and discrete-choice models, it can be shown that the two approaches will never lead to the same estimators, even when one allows for the correlation between η_i and $\mathbf{x}_{it}$ (Hsiao, 1986, p. 45).

It should be noted that when the RE assumptions, such as the assumption of no correlation between the regressors $\mathbf{x}_{it}$ and the individual effect η_i holds, then $\hat{\alpha}_{RE}$ is more efficient than $\hat{\alpha}_{FE}$ (Wansbeek, 1995, p. 38). Degrees of freedom are saved with the RE approach because one only estimates the variance of the firm-specific effect σ^2_η instead of all the N fixed effects $\eta_1, ..., \eta_N$.

There are occasions when only the FE model is applicable. If one of the objectives is to make statements about the particular cross-sectional units in the sample, then an FE approach is required. For example, in productivity studies the researcher often wants to rank the firms in the sample by efficiency (Kumbhakar, 1993). With a RE approach it is not possible to rank the firms; one can only make statements on the relative magnitude of cross-sectional differences in productivity based on the estimate of the variance of the firm-specific error term.[8] An FE approach is also required if one wishes to use Griliches & Hausman's (1986) procedure to correct for errors of measurement.

When time-invariant variables (e.g. region-dummies, year of establishment) are included, the fixed-effect estimators (4.2) cannot be derived, because the time-invariant variables are eliminated when the individual time-series mean is subtracted from the x's. The alternative, estimating a fixed-effect model using dummy variables for the individual effects, is not possible either, because of perfect multicollinearity between the firm-specific dummies and the other time-invariant variables. Time-invariant variables do not constitute a problem for the RE model. Random effects models with four independent error components have also been proposed in the literature (Baltagi & Raj, 1992, p. 98). Such a generalisation is useful when one of the objectives is to separate e.g. regional effects from firm-specific and time-specific effects.

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[7] Mundlak (1978) has presented a GLS estimator for α which provides asymptotically unbiased estimates (when T tends to infinity). However, Mundlak's estimator is based on the assumption that the individual effect is a linear function of the time-series mean of all explanatory variables, which is too restrictive for practical purposes.

[8] For example, an estimated RE model can tell us what per cent of the variance of output, y, can be explained by the regressors (input levels, etc.), what per cent can be explained by the firm-specific effect, and what per cent can be explained by the remaining observation-specific disturbance term.

There are estimators which allow us to include firms that are observed for only one period in the estimating sample when an RE approach is used (Biørn, 1981; Hsiao, 1986, pp. 194-6). This is important if there are structural differences, e.g. with respect to mean productivity and output risk, between firms which are observed for only one period and firms that are observed for longer time periods. It may be the case that firms which are observed for only one period are less productive or more risky than firms that are observed two or more periods.

It should be noted that the assumption of randomness of η_i and λ_t in the RE model does not carry with it the assumption of normality (Hsiao, 1986, p. 47). This assumption is often made for random effects, but it is a separate assumption made subsequent to the randomness assumption. Normality is usually assumed for the purpose of hypothesis testing.

Another issue is the small sample performance of FE and RE estimators in terms of precision and efficiency. Both FE and RE estimators have been derived for unbalanced panel data, but their performance may differ. In subsequent sections we will discuss this further.

The nature of the sample may also be a factor in determining whether an FE or RE model is appropriate. If the sample is not randomly drawn from the population, then the RE approach may be questioned. In other words, if the distribution of the firm-specific effects in the sample departs from the true distribution, then the FE approach is to be preferred. However, also with an FE approach one must be careful with respect to making inferences when sample selection problems are present; the estimated parameter values should only be considered valid for the present sample.

There are tests available to compare the FE and RE model specifications. Hausman's chi-squared test statistic makes it possible to test whether the GLS estimator is an appropriate alternative to the estimator of the homoskedastic, nonautocorrelated FE model (Greene, 1991, pp. 495-496).

If there are no special considerations that clearly make one of the two approaches more appropriate than the other, then one possible approach may be to estimate both the FE and RE models, and then compare the estimates of α to assess to what extent they deviate from each other. If the estimates diverge, then a Hausman test could be undertaken to determine which approach is most appropriate. One should, however, be aware of the assumptions underlying this particular test, and the fact that other model specification errors may also affect the results.

One thing neither the estimated FE model nor the RE model tell us are the *sources* of cross-sectional heterogeneity. Thus, we are still ignorant. But now our ignorance is partly general and partly specific. Significant firm-specific effects or a significant time-invariant variance component tell us that observed productivity differences across firms are not only due to white noise disturbances. Hence, significant firm-specific effects can provide the rationale and constitute the basis of a research programme to identify the underlying sources of productivity differences. The FE model has an advantage if one wishes to undertake a detailed study of selected firms; based on estimated fixed effects one can e.g. pick out the most productive firms and the least productive firms for further study.

4.6. Maximum Likelihood Estimation of Fixed Effects Models

In Chapter three we saw that for Just-Pope production functions, ML provides more efficient estimates of the variance function parameters than FGLS. Furthermore, Monte Carlo studies suggest that in small samples the precision (in terms of MSE) of both mean and variance function parameter estimates are higher when estimated by ML compared with FGLS. However, simulation studies have only been undertaken for homogenous firms, i.e., Just-Pope models devoid of firm-specific effects. It may be the case that the relative performance of ML and FGLS in small samples is changed when firm-specific effects are introduced. In Chapter five the performance of FGLS and ML estimators for Just-Pope models with fixed effects will be assessed in a simulation study.

The FGLS estimator for a Just-Pope model with fixed firm-specific intercepts will be the same as the usual FGLS estimator when the fixed effects are implemented as dummy variables.

This section presents ML estimators for fixed effects models, both the standard homoskedastic specification and a heteroskedastic specification. The fixed effects will be implemented as firm-specific intercepts in the mean function. Both ML and marginal ML (MML) estimators will be provided. The first estimator corresponds to ML with dummy variables for firm-specific effects, which is the most straightforward procedure. However, with a large number of individual effects (e.g. firm dummies), it may be difficult to find ML estimates that converge, because of the problem of multicollinearity among right-hand side variables is aggravated (Baltagi, 1995). Furthermore, it has been shown for the homoskedastic case (i.e., $\sigma_{it}^2 = \sigma_u^2$, $\forall$ i,t), that the ML estimator of α is consistent, but the estimator of the variance σ^2 is inconsistent (Wansbeek, 1995, p. 13).

The MML estimator uses the within transformation of dependent and independent variables, and thus avoids the use of a large number of dummies. Wansbeek (1995, p. 14), when presenting the ML estimator for the fixed effects model in the homoskedastic case, mentions that marginal maximum likelihood (MML) estimation and conditional maximum likelihood (CML) are equivalent to ML estimation with dummy variables. Cornwell & Schmidt (1992) state that for the linear regression model ML, CML and MML provide the same (consistent in N) estimator.

Wansbeek points out that with a fixed effects model one experiences an incidental parameter problem. Because of the individual effects η_i, we have in the fixed effects model a situation where the number of parameters grows as fast as the number of observations (when $N \rightarrow \infty$ and T is fixed). Such parameters are usually called *incidental parameters* or *nuisance parameters* (Neyman & Scott, 1948). The parameters α are the *structural parameters* of the model. For a general pdf $p_i(y_i|\,\alpha, \eta_i)$ of the random variable y_i, the structural parameters are contained in the finite parameter vector α, and η_i is the incidental parameter. Usually, researchers are primarily concerned with the structural parameters. The problem is, however, that the usual ML estimator of the structural parameters α is inconsistent in N, for fixed T (Neyman & Scott, 1948). The objective is therefore to find alternative estimators that lead to consistent estimates of α.[9] In the linear regression model within-transformation of the variables eliminates the incidental parameters problem.

Kalbfleisch & Sprott (1970) discuss elimination of incidental parameters from the likelihood function by the use of CML and MML. They present the conditions that have to be satisfied in order to be able to construct CMLs and MMLs. Chamberlain (1980) is the only who presents a CML estimator for the fixed effects model with balanced panel and homoskedastic observation-specific errors. MML and CML estimators have been provided for other model specifications. Verbeek (1990) proposes a MML estimator for a fixed effects model with sample selectivity.

We begin first with the simple case of a balanced panel and homoskedastic errors, then extend to the unbalanced panel case, and finally introduce heteroskedasticity.

4.6.1. Derivation of the ML-Estimator for the Case of Balanced Data and Homoskedastic Errors

For the special case of a balanced panel (i.e., $T_1 = T_2 = \ldots = T_N$) and homoskedastic observation-specific errors, the log-likelihood function with dummy-variables for the individuals (represented by D) is:

$$\text{(4.3)} \quad \ln L = -\frac{NT}{2}\ln\sigma_u^2 - \frac{1}{2\sigma_u^2}(\mathbf{y} - X\alpha - D\eta)'(\mathbf{y} - X\alpha - D\eta),$$

where $\mathbf{y}$ is a $NT \times 1$ vector, X is a $NT \times k$ matrix, and D is the $NT \times N$ indicator matrix (or individual dummy matrix) defined by $D = \mathbf{l}_T \otimes I_N$, where $\mathbf{l}_T$ is a $T \times 1$ vector of ones (see appendix 4.A3 for an example). The first-order conditions for maximum are

$$\text{(4.4a)} \quad \frac{\partial \ln L}{\partial \alpha} = -\frac{1}{2\sigma_u^2}(-2X'\mathbf{y} + 2X'X\alpha + 2X'D\eta) = 0,$$

$$\text{(4.4b)} \quad \frac{\partial \ln L}{\partial \eta} = -\frac{1}{2\sigma_u^2}(-2D'\mathbf{y} + 2D'X\alpha + 2D'D\eta) = 0,$$

$$\text{(4.4c)} \quad \frac{\partial \ln L}{\partial \sigma_u^2} = -\frac{NT}{2\sigma_u^2} + \frac{1}{4\sigma_u^4}(\mathbf{y} - X\alpha - D\eta)'(\mathbf{y} - X\alpha - D\eta) = 0.$$

First, we solve (4.4b) with respect to η:

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[9] However, when the η_i's are not assumed to be arbitrary constants, but i.i.d. random variables, the ML estimator of α is strongly consistent in N under reasonable regularity conditions (Kiefer, 1956). The ML estimator of the pdf of the η_i's is also consistent.

$$\hat{\eta} = (D'D)^{-1}D'\mathbf{y} - (D'D)^{-1}D'X\hat{\alpha} \quad (4.4b')$$
$$= \bar{\mathbf{y}} - \bar{X}\hat{\alpha},$$

where the $N \times 1$ vector $\bar{\mathbf{y}} = (\bar{\mathbf{y}}_1, \bar{\mathbf{y}}_2, \ldots, \bar{\mathbf{y}}_N)'$ and the $N \times k$ matrix $\bar{X} = (\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_N)'$.

Then (4.4a) is simplified:

(4.4a') $-X'\mathbf{y} + X'X\alpha + X'D\eta = 0$,

Insert (4.4b') for η in (4.4a') and rearrange:

$$-X'\mathbf{y} + X'X\alpha + X'D\left[(D'D)^{-1}D'\mathbf{y} - (D'D)^{-1}D'X\alpha\right] = 0,$$

$$X'X\alpha - X'D(D'D)^{-1}D'X\alpha = X'\mathbf{y} - X'D(D'D)^{-1}D'\mathbf{y},$$

(4.4a'') $\hat{\alpha} = \left[X'X - X'D(D'D)^{-1}D'X\right]^{-1}\left[X'\mathbf{y} - X'D(D'D)^{-1}D'\mathbf{y}\right]$,

which is the ML-estimator for α.

It can be shown that maximisation of ln L with respect to (α, η, σ_u^2) yields the same estimators for α and η as OLS. For the ML estimator of σ_u^2

$$\operatorname*{plim}_{N\to\infty} \hat{\sigma}_u^2 = \frac{T-1}{T}\sigma_u^2,$$

which means that it is inconsistent for finite T (which is usually the case for panel data).

Next, we derive the MML estimator for α, i.e., the ML-estimator when the deviation forms (within transformations) $\tilde{\mathbf{y}} = (\tilde{y}_{11}, \tilde{y}_{12}, \ldots, \tilde{y}_{1T}, \ldots, \tilde{y}_{N1}, \tilde{y}_{N2}, \ldots, \tilde{y}_{NT})'$ and $\tilde{X} = (\tilde{\mathbf{x}}_{11}, \tilde{\mathbf{x}}_{12}, \ldots, \tilde{\mathbf{x}}_{1T}, \ldots, \tilde{\mathbf{x}}_{N1}, \tilde{\mathbf{x}}_{N2}, \ldots, \tilde{\mathbf{x}}_{NT})'$ are used. The log-likelihood function is

(4.5) $\ln L = -\frac{NT}{2}\ln\sigma_u^2 - \frac{1}{2\sigma_u^2}(\tilde{\mathbf{y}} - \tilde{X}\alpha)'(\tilde{\mathbf{y}} - \tilde{X}\alpha)$.

The first-order conditions for maximum are

(4.6a) $\frac{\partial \ln L}{\partial \alpha} = -\frac{1}{2\sigma_u^2}\left(-2\tilde{X}'\tilde{\mathbf{y}} + 2\tilde{X}'\tilde{X}\alpha\right) = 0$,

(4.6b) $\frac{\partial \ln L}{\partial \sigma_u^2} = -\frac{NT}{2\sigma_u^2} + \frac{1}{4\sigma_u^4}(\tilde{\mathbf{y}} - \tilde{X}\alpha)'(\tilde{\mathbf{y}} - \tilde{X}\alpha) = 0$.

By solving (4.6a) with respect to α we obtain the MML estimator

(4.6a') $\hat{\alpha} = \left(\tilde{X}'\tilde{X}\right)^{-1}\tilde{X}'\tilde{\mathbf{y}}$

It can be shown that (Wansbeek, 1995, pp. 11-12)

(4.7) $\tilde{\mathbf{y}} = M\mathbf{y} = (I_{NT} - K)\mathbf{y} = \left(I_{NT} - \frac{1}{T}DD'\right)\mathbf{y}$,

where M is the within-transformation matrix and K is the between-transformation matrix, both of dimension $NT \times NT$.

In order to show that the MML estimator (4.6a') is identical with the ML estimator (4.4a'') we replace $\tilde{\mathbf{y}}$ and $\tilde{X}$ with the within-transformations

$$\hat{\alpha} = \left[X' \left(I_{NT} - \frac{1}{T} DD' \right)' \left(I_{NT} - \frac{1}{T} DD' \right) X \right]^{-1} X' \left(I_{NT} - \frac{1}{T} DD' \right)' \left(I_{NT} - \frac{1}{T} DD' \right) \mathbf{y}$$

$$= \left[\left(X - \frac{1}{T} DD' X \right)' \left(X - \frac{1}{T} DD' X \right) \right]^{-1} \left(X - \frac{1}{T} DD' X \right)' \left(\mathbf{y} - \frac{1}{T} DD' \mathbf{y} \right)$$

(4.6a")
$$\hat{\alpha} = \left[X' X - \frac{2}{T} X' DD' X + \frac{1}{T^2} X' DD' DD' X \right]^{-1} \cdot \left[X' \mathbf{y} - \frac{2}{T} X' DD' \mathbf{y} + \frac{1}{T^2} X' DD' DD' \mathbf{y} \right]$$

It turns out that the above expression can be reduced to

$$\hat{\alpha} = \left[X' X - X' D(D' D)^{-1} D' X \right]^{-1} \left[X' \mathbf{y} - X' D(D' D)^{-1} D' \mathbf{y} \right],$$

which is identical with the expression for the ML estimator (4.4a").

4.6.2. Derivation of the ML-Estimator for the Case of Unbalanced Data and Homoskedastic Errors

We now examine the case when the N individuals are observed in T_i periods, where T_i may vary over individuals i, but the error term is still homoskedastic. The total number of observations is $n = \Sigma_i T_i$.

The expression for the estimator for the dummy-variable specification is identical with the ML estimator (4.4a"), since the indicator matrix D also can describe the structure of an unbalanced data set. The indicator matrix is defined by $D = \text{diag}(\mathbf{l}_{T1}, \mathbf{l}_{T2}, \ldots, \mathbf{l}_{TN})$, where $\mathbf{l}_{Ti}$ is a $T_i \times 1$ vector of ones.

Looking back on the MML estimator (4.6a"), it may appear to be a problem that it is implicitly assumed that T is identical for all N individuals. However, it can be shown that (4.6a") is identical with

(4.8)
$$\hat{\alpha} = \left[X' X - X' D^* D^{*'} X + X' D^* D^{*'} D^* D^{*'} X \right]^{-1} \cdot \left[X' \mathbf{y} - X' D^* D^{*'} \mathbf{y} + X' D^* D^{*'} D^* D^{*'} \mathbf{y} \right],$$

where D^* is the $n \times N$ matrix

$$D^* = \text{diag}\left(\mathbf{d}_1^*, \ldots, \mathbf{d}_N^* \right),$$

where $\mathbf{d}_i^*$ is a $T_i \times 1$ vector with elements $1/\sqrt{T_i}$. D^* can be viewed upon as a weighted indicator matrix, with $1/\sqrt{T_i}$ as weights.

Furthermore, it can be shown that if D^* is replaced with D in (4.8), then $\hat{\alpha}$ is unchanged, which means that the ML estimator and the MML estimator are identical also in the unbalanced panel case.

4.6.3. Derivation of the ML-Estimator for the Case of Unbalanced Data and Heteroskedastic Errors

We now examine the case when the individuals are not necessarily observed for the same number of periods, T_i, and the error term is heteroskedastic, i.e., var(u_{it}) = σ_{it}^2. The log-likelihood function is now

(4.9) $$\ln L = -\frac{1}{2}\ln|\Omega|^{-1} - (\mathbf{y} - X\alpha - D^*\eta)'\Omega^{-1}(\mathbf{y} - X\alpha - D^*\eta),$$

where $\mathbf{y}$ is a $n \times 1$ vector, X is a $n \times k$ matrix, and D is the $n \times N$ weighted indicator matrix defined by $D^* = \text{diag}(\mathbf{d}_1^*, \ldots, \mathbf{d}_N^*)$, where $\mathbf{d}_i^*$ is a $T_i \times 1$ vector with elements $1/\sqrt{T_i}$. The first-order conditions for maximum are

(4.10a) $$\frac{\partial \ln L}{\partial \alpha} = -\frac{1}{2}(-2X'\Omega^{-1}\mathbf{y} + 2X'\Omega^{-1}X\alpha + 2X'\Omega^{-1}D^*\eta) = 0,$$

(4.10b) $$\frac{\partial \ln L}{\partial \eta} = -\frac{1}{2}(-2D^{*\prime}\Omega^{-1}\mathbf{y} + 2D^{*\prime}\Omega^{-1}X\alpha + 2D^{*\prime}\Omega^{-1}D^*\eta) = 0.$$

Rearrange (4.10b):

$$D^{*\prime}\Omega^{-1}D^*\eta = D^{*\prime}\Omega^{-1}\mathbf{y} - D^{*\prime}\Omega^{-1}X\alpha,$$

(4.10b') $$\eta = (D^{*\prime}\Omega^{-1}D^*)^{-1}[D^{*\prime}\Omega^{-1}\mathbf{y} - D^{*\prime}\Omega^{-1}X\alpha].$$

Rearrange (8a):

(4.10a') $$X'\Omega^{-1}X\alpha = X'\Omega^{-1}\mathbf{y} - X'\Omega^{-1}D^*\eta$$

Substitute (4.10b') for η in (4.10a) and rearrange:

$$X'\Omega^{-1}X\alpha = X'\Omega^{-1}\mathbf{y} - X'\Omega^{-1}D^*(D^{*\prime}\Omega^{-1}D^*)^{-1}D^{*\prime}\Omega^{-1}\mathbf{y} + X'\Omega^{-1}D^*(D^{*\prime}\Omega^{-1}D^*)^{-1}D^{*\prime}\Omega^{-1}X\alpha$$

$$[X'\Omega^{-1}X - X'\Omega^{-1}D^*(D^{*\prime}\Omega^{-1}D^*)^{-1}D^{*\prime}\Omega^{-1}X]\alpha = X'\Omega^{-1}\mathbf{y} - X'\Omega^{-1}D^*(D^{*\prime}\Omega^{-1}D^*)^{-1}D^{*\prime}\Omega^{-1}\mathbf{y}$$

(4.10a'') $$\hat{\alpha} = [X'\Omega^{-1}X - X'\Omega^{-1}D^*(D^{*\prime}\Omega^{-1}D^*)^{-1}D^{*\prime}\Omega^{-1}X]^{-1} \cdot [X'\Omega^{-1}\mathbf{y} - X'\Omega^{-1}D^*(D^{*\prime}\Omega^{-1}D^*)^{-1}D^{*\prime}\Omega^{-1}\mathbf{y}].$$

The covariance matrix Ω is symmetric and positive definite. This means that its inverse can be factored into $\Omega^{-1} = \Omega^{-1/2}(\Omega^{-1/2})'$ (Greene, 1991, p. 385). By substituting this factorisation into (4.10a'') and multiplying into the adjacent matrices, we obtain

$$\hat{\alpha} = [X_P'X_P - X_P'D_P^*(D_P^{*\prime}D_P^*)^{-1}D_P^{*\prime}X_P]^{-1} \cdot [X_P'\mathbf{y}_P - X_P'D_P^*(D_P^{*\prime}D_P^*)^{-1}D_P^{*\prime}\mathbf{y}_P],$$

where $X_P = \Omega^{-1/2}X$, $\mathbf{y}_P = \Omega^{-1/2}\mathbf{y}$, and $D_P^* = \Omega^{-1/2}D^*$. Except for the subscript *P*, this expression is equivalent with the homoskedastic estimator of α. Of course, the covariance matrix Ω may be a function of a vector of parameters β, as in the variance function (4.2). Then, it can be shown that the ML estimator for the fixed effects model will have the same attractive property compared to the FGLS estimator, in that the parameters α and β are estimated in a single stage instead of in separate stages.

The motivation for deriving an MML estimator, as mentioned earlier in this section, is that with a large number of firm dummies it may in practice be difficult to find ML estimates that converge. Hence, also for the heteroskedastic case it will be convenient with an MML estimator that provides estimates which are identical with the ML estimates. However, in the heteroskedastic case one has to rely on asymptotic results, i.e., as the number of firms $N \to \infty$, to show the similarity of the ML and MML estimators of α (Cornwell & Schmidt, 1992). The question is then to what extent the ML and MML estimates diverge in finite samples. We found in a simulation study (cf. Chapter 5), that ML and MML parameter estimates of a Just-Pope model were very similar, and provided elasticity estimates that for all practical purposes were equivalent, even when the number of firms *N* was much smaller than in our empirical application. This finding suggests that MML can be used instead of ML in finite samples even in the heteroskedastic case.

4.7. Estimation of the Homoskedastic Random Effects Model for Unbalanced Data

This section presents a GLS estimator of the homoskedastic one-way error-components model for the case of unbalanced panel data. Because of the relative complexity of GLS estimators for random effects models it is useful to be familiar with the estimators for the homoskedastic specification before proceeding to the heteroskedastic case. Moreover, in the simulation study (Ch. 5) and the empirical analysis (Ch. 9) homoskedastic specifications will also be estimated to assess the relative gain in performance by using heteroskedastic models. In order to gain some *a priori* knowledge about the performance of different homoskedastic estimators for unbalanced panel data, some finite sample simulation results are also presented at the end of this section.

For the homoskedastic random effects model

(4.11) $y_{it} = f(\mathbf{x}_{it}; \alpha) + \eta_i + u_{it}, \qquad i = 1, \ldots, N, \quad t = 1, \ldots, T_i,$

it is assumed that $E[\eta_i] = 0$, $E[u_{it}] = 0$, $E[\eta_i^2] = \sigma_\eta^2$, $E[u_{it}^2] = \sigma_u^2$, $E[\eta_i\eta_j] = 0$ for $i \neq j$, $E[\eta_i u_{jt}] = 0$. For firm *i* the $T_i \times T_i$ covariance matrix of the composite error term $v_{it} = \eta_i + u_{it}$, is

$$(4.12)\quad \Omega_i = \sigma_u^2 I_{Ti} + \sigma_\eta^2 J_{Ti} = \begin{bmatrix} \sigma_\eta^2+\sigma_u^2 & \sigma_\eta^2 & \cdots & \sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2+\sigma_u^2 & \cdots & \sigma_\eta^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_\eta^2 & \sigma_\eta^2 & \cdots & \sigma_\eta^2+\sigma_u^2 \end{bmatrix},$$

where J_{Ti} is a matrix of ones of dimension $T_i \times T_i$. We see that for a given firm i, the correlation between any two disturbances in different time periods is constant. The $n \times n$ covariance matrix for the entire panel data set (where $n = \Sigma_i T_i$) is

$$\Omega = \sigma_u^2 I_n + \sigma_\eta^2 J,$$

where $J = \text{diag}(J_{Ti})_{i=1,N}$ is a $n \times n$ block-diagonal matrix. The off-diagonal elements of Ω are zero if the observations belong to different firms. The non-zero off-diagonal elements are all equal to σ_η^2, while the diagonal elements are all equal to $\sigma_u^2 + \sigma_\eta^2$. It is the existence of non-zero off-diagonal elements in the covariance matrix that makes GLS estimation necessary.

4.7.1. FGLS-Estimation

Examples of FGLS estimators presented in the literature for the error components model in the balanced data case are Fuller & Battese (1973), Hsiao (1986, pp. 34-38) and Greene (1991, pp. 485-94). Wansbeek & Kapteyn (1989) present a GLS (not FGLS) estimator for the case of unbalanced data.

The GLS estimator using the true variance components is

$$\hat{\alpha}_{\text{GLS}} = (X'\Omega^{-1}X)X'\Omega^{-1}\mathbf{y}$$

or

$$\hat{\alpha}_{\text{GLS}} = (X^{*\prime}X^*)X^{*\prime}\mathbf{y}^*,$$

where $X^* = X'\Omega^{-1/2}$ and $\mathbf{y}^* = \mathbf{y}'\Omega^{-1/2}$. Direct GLS estimation requires the inversion of an $n \times n$ matrix, which can be too large in many practical applications. Therefore, it is desirable to find an alternative way to estimate α with reduced computational requirements.

The $T_i \times T_i$ covariance matrix (4.12) for firm i can alternatively be expressed as (Baltagi, 1985)

$$\Omega_i = (T_j\sigma_\eta^2 + \sigma_u^2)J_{Ti}/T_i + \sigma_u^2(I_{Ti} - J_{Ti}/T_i).$$

By multiplying out the expressions in parentheses it can be verified that the above expression reduces to (4.12). Following e.g. Fuller & Battese (1974), it can be shown that

$$\Omega_i^p = (T_j\sigma_\eta^2 + \sigma_u^2)^p J_{Ti}/T_i + (\sigma_u^2)^p(I_{Ti} - J_{Ti}/T_i),$$

where p is any rational number. When p=-1 one obtains the inverse, while p = -1/2 obtains $\Omega_i^{-1/2}$. Let $w_i^2 = T_i\sigma_\eta^2 + \sigma_u^2$. Then

$$\sigma_u\Omega_i^{-1/2} = (\sigma_u^2/w_i)(J_{Ti}/T_i) + (I_{Ti} - J_{Ti}/T_i) = I_{Ti} - \theta_i J_{Ti}/T_i,$$

where

(4.13) $\theta_i = 1 - \sigma_u/w_i$.

It can easily be shown that $\sigma_u\Omega_i^{-1/2}\mathbf{y}_i$ has a typical element $(y_{it} - \theta_i\bar{y}_i)$. This means that GLS can be applied using a weighted least squares (WLS) procedure. Given the true values (or consistent estimates of) variance components σ_η^2 and σ_u^2, one obtains w_i and θ_i. Then WLS-estimation is done by regressing

$$(y_{it} - \theta_i\bar{y}_i) \text{ on } (1 - \theta_i) \text{ and } (\mathbf{x}_{it} - \theta_i\bar{\mathbf{x}}_i).$$

Of course, the true values of the variance components σ_η^2 and σ_u^2 are generally unknown in empirical applications. Hence, consistent estimates of σ_η^2 and σ_u^2 have to be used instead.

The variance of the observation-specific error term, σ_u^2, can be estimated by

$$\hat{\sigma}_u^2 = \Sigma_i\Sigma_t\, \hat{u}^2/(n - N - K - 1),$$

where $\hat{u}$ is the residual from the estimated fixed effects regression model

$$\hat{u} = y_{it} - \hat{\eta}_i - \hat{\alpha}'\mathbf{x}_{it}.$$

For the variance of the firm-specific error component, σ_η^2, at least three estimators have been proposed. An estimator (i) based on residuals from a regression of the N observed firms means $\bar{y}_i$ on a constant and $\bar{\mathbf{x}}_i$, (ii) based on OLS-residuals from the pooled regression $y_{it} = \mu + f(\mathbf{x}_{it}; \alpha) + u_{it}$, (iii) based on the sample variance of the estimated fixed effects $\hat{\mu}_i$.

The variance of the firm mean regression $\bar{y}_i = \mu^* + \hat{\alpha}^{*\prime}\bar{\mathbf{x}}_i + v_i$, is

$$\sigma_{vi}^2 = \text{var}[\eta_i + \Sigma_i u_{it}/T_i] = \sigma_\eta^2 + \sigma_u^2/T_i.$$

We see that the variance is heteroskedastic because T_i is not constant across i's in the unbalanced panel case. It can be shown that the OLS variance estimator in a heteroskedastic regression is a consistent estimator of

$$\bar{\sigma}_v^2 = \text{plim}(1/N)\Sigma_i\sigma_{vi}^2 = \sigma_\eta^2 + \sigma_u^2\text{plim}(1/N)\Sigma_i(1/T_i)$$
$$= \sigma_\eta^2 + \text{plim}\ Q_N^*\sigma_u^2 = \sigma_\eta^2 + \text{plim}\ Q^*\sigma_u^2,$$

assuming that the probability limit exists. The consistency result hangs on increasing N, not T_i, which is taken as fixed. One has to make some assumption about the number of periods firms are observed. We assume that T_i is randomly distributed across individuals with $E[T_i] = T$. If $T_i = T$ for all i, then $Q_N^* = Q^* = Q = 1/T$. Assume that Q_N^* converges to some well defined Q^*. Then, in our sample, the statistic

$$Q = (1/N)\,(1/T_1 + 1/T_2 + \ldots + 1/T_N)$$

is a consistent estimator of Q^* (Greene, 1991, pp. 500-501). A consistent estimator for the variance of the firm-specific error component is then

(4.14) $\hat{\sigma}_\eta^2 = \bar{\sigma}_v^2 - Q\hat{\sigma}_u^2 = \mathbf{v}^{*\prime}\mathbf{v}^*/(N - K) - Q\hat{\sigma}_u^2,$

where $\mathbf{v}$ are the residuals from the group means regression, i.e., $v_i = \bar{y}_i - \hat{\alpha}^{*\prime}\bar{\mathbf{x}}_i$.

A disadvantage of the estimator (4.14) is that it can lead to negative variance estimates.

A consistent estimator of the variance of the firm-specific error component can also be found by estimating the pooled regression $y_{it} = \mu + \alpha'\mathbf{x}_{it} + u_{it}$, and then use the OLS slope estimators α_{OLS} to calculate the residuals $v_i = \bar{y}_i - \hat{\alpha}'_{OLS}\bar{\mathbf{x}}_i$ to be used in (4.14).

The third estimator of the variance of the firm-specific error component is

$$\hat{\sigma}_{\eta}^2 = (1/N)\Sigma_{i=1}^{N}(\Sigma_{i=1}^{N}\hat{\eta}_i/N - \hat{\eta}_i)^2,$$

where $\hat{\eta}_i$, $i = 1, 2, \ldots, N$, are the fixed effects estimators.

An estimator for θ is now

$$\hat{\theta}_i = 1 - \hat{\sigma}_u/(T_i\hat{\sigma}_{\eta}^2 + \hat{\sigma}_u^2)^{1/2},$$

which can be applied in the next step to transform the observations and estimate the model by OLS as shown earlier in this section.

4.7.2. Maximum Likelihood Estimation

Examples of studies which present an ML estimator for the error components model in the case of balanced data are Amemiya (1971), Magnus (1982), Hsiao (1986, pp. 38-41). For the unbalanced data case Wansbeek & Kapteyn (1989, pp. 350-352) and Baltagi & Chang (1994, pp. 74-77) have presented ML estimators. An ML estimation procedure for the unbalanced data case is presented in appendix 4.A4.

4.7.3. The Performance of Homoskedastic Estimators in Simulation Experiments

A few simulation studies have been undertaken in order to compare the performance of different homoskedastic panel data model estimators. Such studies are interesting because they give us an *a priori* idea about which estimators may perform well under different data designs. Performance is usually assessed in terms of mean square error (MSE) of estimated parameters, and the power of (t-) tests. See Chapter 5 for a presentation of these and other performance criteria. In simulation experiments several issues are of interest: (1) The effect on parameter estimates of changing total sample size, n. (2) The effect of changing the degree of unbalancedness. (3) For the error components model: The effect of changing the variance ratio $\rho = \sigma_{\eta}^2/\sigma_u^2$.

Wansbeek & Kapteyn (1989) assess OLS, FE, FGLS and ML estimators of a two-way error components model (i.e., the model has both time-specific and firm-specific effects) with a constant term and one regressor. The error components have the following variances: $\text{var}(\eta_i) = \sigma_{\eta}^2 = 400$, $\text{var}(\lambda_t) = \sigma_{\lambda}^2 = 25$, $\text{var}(u_{it}) = \sigma_u^2 = 25$. They consider three panel cases: Balanced data, random attrition, and rotating panel. For each case 50 runs are made. $T_{MAX} = 5$ and there is a maximum of N = 100 units each year. The OLS estimator clearly has the worst performance of all estimators in terms of the difference between the average coefficient estimate and the true parameter value, and also in terms of the standard error of the coefficient. The FE estimator of the slope coefficient has a slightly smaller average bias than the FGLS and ML estimators, but the average standard error of the FE estimator is higher than the FGLS and ML standard errors. The FGLS and ML estimators give nearly identical results for all three panel cases; this applies to the average values of both coefficients and variance components.

Baltagi & Chang (1994) assess the performance of various estimators for different degrees of unbalancedness and different variance ratios ρ for a one-way error model. The number of units is $N = 30$ in all experiments, but the number of periods firms are observed vary over the cases. In most cases the total number of observations is 210. The units are observed from one to 28 time periods. The performance of the OLS estimator in terms of MSE drops dramatically as the variance ratio increases, i.e., as the magnitude of σ_η^2 increases relative to σ_u^2. Its performance was less affected by the degree of unbalancedness. The ML estimator of the regression coefficient performs remarkably well for all degrees of unbalancedness and variance ratios. According to Baltagi & Chang the ML estimators of the variance components also perform well, although the results are not reported in detail. The performance of FE and FGLS estimators are not assessed by Baltagi & Chang.

Khuri and Sahai (1985) and Baltagi & Chang (1994), also report the performance of other error components model estimators (e.g. MIVQUE) in simulation experiments, but these estimators are outside the scope of this dissertation.

4.8. Heteroskedastic Random Effects Models in the Literature

Random effects models with heteroskedastic firm-specific and/or heteroskedastic observation-specific error component are relatively new. Analyses of the performance of heteroskedastic RE estimators are almost non-existent. Empirical applications of these estimators are also very limited. The surveys of Baltagi & Raj (1992) and Baltagi (1995, pp. 77-81) give a good indication of the status of heteroskedastic RE models.

Most of the estimation procedures suggested for heteroskedastic models are GLS transformations of the variables which change the error terms into classical errors and thus allow OLS estimation.

Mazodier & Trognon (1978) present a stratified two-way error components model for balanced data with heteroskedasticity in the firm-specific error component, η_i, and the time-specific error component, λ_t. Stratified error components models can be relevant when there exist meaningful stratifications of observations, e.g. industrial sectors for firms, or phases of the business cycle. For the firm-specific effect the heteroskedasticity is of the form $\text{var}(\eta_i) = \sigma_r^2$, where $\{r\}$ is a subset of firms in the panel (e.g. firms that belong to industry r). Mazodier & Trognon present GLS and FGLS estimators for this particular case.

In Chapter three we saw that Griffiths & Anderson (1982) have presented FGLS estimators for a nonlinear RE model with variance function $u_{it}=h(\mathbf{x}_{it};\ \beta)(\varepsilon_{it}+\eta_i+\lambda_t)$ or $u_{it}=h(\mathbf{x}_{it};\ \beta)\varepsilon_{it}+\eta_i+\lambda_t$ under a balanced panel design.

Magnus (1982, section 8) presents a two-way error components model for balanced data with a time-specific error component instead of a firm-specific error component. The observation-specific error component exhibits heteroskedasticity of the form $u_{it} \sim$ N(0, $w_i\sigma_u^2$), i.e., the variance differs across firms (not across observations) by a factor of proportionality w_i. Magnus proposes an ML estimator under this assumption which is claimed to be computationally feasible.

Baltagi & Griffin (1988) present a one-way error components model for balanced data which assumes heteroskedasticity in the firm-specific effect. In Baltagi & Griffin's model var(η_i) = σ_i^2, i = 1, 2, . . . , N, which implies that there are now N variances for the firm-specific component rather than one constant variance. Baltagi & Griffin propose two feasible GLS estimators: An iterative estimator based on OLS and within residuals, and a generalisation of the MINQUE estimator. They also note that if normality is assumed on the disturbances, one could derive ML estimators following Magnus (1978, eqn. 9 and 10). Baltagi & Griffin apply their estimators on a panel data set, and reject homoskedasticity of the firm-specific error term based on a Bartlett χ-square test.

Kumbhakar & Heshmati (1996) apply Baltagi & Griffin's model framework in the estimation of alternative cost function specifications with cost share equations by linear and non-linear seemingly unrelated regressions (SURE). They first estimate the variances σ_i^2 and σ_u^2, and then use these estimates to transform the variables to make the composite error term homoskedastic.[10] Then they apply the usual SURE techniques to estimate the set of equations.

Li & Stengos (1994) presents a one-way error components model for balanced data which departs from the standard model by assuming heteroskedasticity of unknown form in the observation-specific error component, i.e., var(u_{it}) = σ_i^2. They propose FGLS estimators both for the random effects model and the fixed effects model. Li & Stengos present Monte Carlo simulation results which show that their proposed estimator performs adequately in finite samples (T = 3 and N = 50, 100). They also propose a modified Breusch & Pagan test for testing the random effects model, and a Hausman type test for testing the random effects model against the fixed effects model.

The above papers all derive estimators for a balanced panel data set and assume that only one of the error components is heteroskedastic. Randolph (1988b), however, proposes a GLS estimator for a one-way error components model for *unbalanced* data with heteroskedasticity in *both* the firm-specific and observation-specific error component. The variances can be

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[10] The transformation Kumbhakar & Heshmati apply is defined in Baltagi (1988, problem 88.2.2).

written as functions of the regressors, the sampling design, or other parameters. Randolph's GLS estimator is presented in appendix 4.A5.

In another paper Randolph (1988a) presents an error components model for unbalanced data with heteroskedastic observation-specific error components. He presents an ML estimator for his model. The model, which is designed for a particular empirical problem, has a somewhat different covariance matrix structure than the error components model presented earlier in this chapter. Randolph's model can be interpreted as a special case of an error components model $y_{it} = \mathbf{x}_{it}\alpha + v_{it}$, $v_{it} = \eta_i + \mathbf{w}_1\mathbf{z}_{it} + \mathbf{w}_2\mathbf{z}_{it}^2 + u_{it}$, $i = 1, \ldots, N$, $t = 1, \ldots, T_i$, where $\mathbf{z}_{it}$ is an $1 \times l$ vector of observed non-stochastic regressors (e.g. input levels) and $\mathbf{w}_1$ and $\mathbf{w}_2$ are $l \times 1$ vectors of stochastic parameters. As usual the term η_i is the firm-specific error, and u_{it} is the observation-specific white-noise error component. The variance of the composite error term is

$$\text{var}(v_{it}) = \sigma_\eta^2 + \Sigma_l (\sigma_{1l}^2 z_{it,l}^2 + \sigma_{2l}^2 z_{it,l}^4) + \sigma_u^2,$$

where σ_1^2 and σ_2^2 are the $l \times 1$ variance vectors of $\mathbf{w}_1$ and $\mathbf{w}_2$, respectively.

The nature of heteroskedasticity is of a form which is somewhat different from that used in econometric studies of production risk. However, as demonstrated by Randolph, this specification of the observation-specific heteroskedasticity facilitates ML estimation. The major problem with this specification in the context of production risk analysis is that the variances σ_{1l}^2 and σ_{2l}^2 are assumed positive, hence precluding negative marginal output risks (i.e., $\partial y_{it}/\partial z_{it,l} < 0$).

Randolph's (1988b) GLS estimator is the only of the above that encompasses the Just-Pope case. Appendix 4.A5 translates his GLS estimator into a FGLS estimator for the variance function chosen for the observation-specific error term in this dissertation. Appendix 4.A5 also outlines a maximum likelihood estimation procedure for a random effects model with Just-Pope heteroskedasticity in the observation-specific error term.

4.9. Summary

This Chapter has mainly discussed issues in econometric panel data estimation that are relevant to our empirical study. The heterogeneous intercepts model (4.1), which has been the focus of this chapter, has its weaknesses, such as the assumptions that there are time-invariant differences in productivity across firms. Furthermore, these productivity differences are independent of input levels (or scale of operation) unless a logarithimic transformation of the variables is used. However, there are so many problems associated with heterogeneous slope coefficients models, both random and fixed, that these do not represent attractive alternatives. We have therefore chosen to use fixed and random specifications of the heterogeneous intercepts model in our empirical modelling.

There are arguments both for using a fixed effects specification and a random effects specification. If the randomness of the sample can be questioned, then a fixed effects approach may be more appropriate. Random effects estimators have some implicit assumptions of uncorrelatedness for the error components η_i and u_{it}. In particular, the degree of correlation between the error components and the regressors $\mathbf{x}_{it}$ may be a critical point of consideration when choosing between the fixed effects and the random effects approach. Ranking of firms by productivity requires a fixed effects approach. If one wishes to include time-invariant regressors, such as region-dummies, then a random effects model is required. Because of the hypotheses we want to test, both random and fixed effects specifications will be used in the empirical study.

Estimators for both homoskedastic and heteroskedastic model specifications have been presented here. It turns out that FGLS and ML estimation of random effects models for unbalanced data are rather complicated, and that introduction of heteroskedasticity on the observation-specific error term u_{it} further complicates estimation. Since estimators for heteroskedastic models are largely untested, their performance should be assessed in simulation studies prior to empirical application. The next chapter will compare the small-sample performance of different fixed effects and random effects estimators for Just-Pope technologies by means of a simulation study.

4.A1. Appendix: Variable List

D $n \times N$ matrix of firm-specific dummies

i subscript for cross-sectional unit (e.g. firm)

k number of parameters in mean function

m number of parameters in variance function $h(\cdot)$

n $(= \Sigma_i T_i)$ total number of observations in panel data set

N number of cross-sectional units (e.g. firms) in data set

t time period

T_i number of observations of firm i

$\mathbf{u}$ $1 \times n$ vector of observations on observation-specific error component

$\mathbf{u}_i$ $1 \times T_i$ vector of observations on observation-specific error component for firm i

$\mathbf{u}_{it}$ $1 \times T_i$ vector of observations on observation-specific error component for firm i in period t

$\mathbf{v}$ $1 \times n$ vector of observations on composite error term

$\mathbf{v}_i$ $1 \times T_i$ vector of observations on composite error term for firm i

$\mathbf{v}_{it}$ $1 \times T_i$ vector of observations on composite error term for firm i in period t

$\mathbf{x}_{it}$ $1 \times k$ vector of observations on independent variables in mean function for firm i in period t

X $n \times k$ matrix of observations on independent variables in mean function

X_i $T_i \times k$ matrix of observations on independent variables in mean function for firm i

$\mathbf{y}$ $1 \times n$ vector of observations on dependent variable

$\mathbf{y}_i$ $1 \times T_i$ vector of observations on dependent variable for firm i

$\mathbf{z}_{it}$ $1 \times m$ vector of observations on independent variables in variance function $h(\cdot)$ for firm i in period t

Z $n \times m$ matrix of observations on independent variables in variance function $h(\cdot)$

Z_i $T_i \times m$ matrix of observations on independent variables in variance function $h(\cdot)$ for firm i

α $k \times 1$ vector of parameters in mean function

β $m \times 1$ vector of parameters in variance function $h(\cdot)$

ε_{it} observation of exogenous error term in variance function $h(\cdot)$ for firm i in period t

η_i observation of firm-specific error component for firm i

ρ ratio of the variance of the firm-specific error component to the variance of the observation-specific component

σ_u^2 variance of the observation-specific error component

σ_v^2 ($= \sigma_\eta^2 + \sigma_u^2$) variance of the composite error term

σ_η^2 variance of the firm-specific error component

Ω $n \times n$ covariance matrix for the entire panel data set

Ω_i $T_i \times T_i$ covariance matrix for firm i

4.A2. Appendix: Some Useful Matrix Rules

This appendix presents some matrix algebra rules that are useful in, for example, derivation of first order conditions of the maximum likelihood functions presented in this chapter.

Determinants

For the block-diagonal matrix $A = \text{diag}(A_1, A_2, \ldots, A_N)$, where A_i has dimension $T_i \times T_i$,

$$|A| = |A_1| \cdot |A_2| \cdot \ldots \cdot |A_N|.$$

Inverses

The inverse of a block-diagonal matrix is equal to the block-diagonal matrix containing the inverses of the blocks. In notation, for a matrix

$$A = \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_n \end{bmatrix} \quad \text{the inverse is } A^{-1} = \begin{bmatrix} A_1^{-1} & 0 & \cdots & 0 \\ 0 & A_2^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_n^{-1} \end{bmatrix}.$$

Matrix differentiation

For every matrix X and Y of appropriate orders (Magnus, 1982, p. 269)

$$\text{d}(XY) = (\text{d}X)Y + X\text{d}Y,$$

and

$$\text{dTr}XY = \text{Tr}(\text{d}X)Y + \text{Tr}X\text{d}Y.$$

For every non-singular X (Magnus, 1982, p. 269),

$$\text{d}X^{-1} = -X^{-1}(\text{d}X)X^{-1},$$

and if $|X| > 0$,

$$\text{d}\log|X| = \text{Tr}X^{-1}\,\text{d}X.$$

Theil (1971, pp. 30-33) and Judge *et al.* (1988, p. 969) present the following rules:

$$\partial \text{tr}A/\partial A = I,$$

$$\partial |A|/\partial A = |A|(A')^{-1},$$

$$\partial \ln|A|/\partial A = (A')^{-1},$$

$$\partial AB/\partial x = A(\partial B/\partial x) + (\partial A/\partial x)B,$$

$$\partial A^{-1}/\partial x = -A^{-1}(\partial A/\partial x)A^{-1},$$

where x is a scalar.

4.A3. Appendix: The Indicator Matrix *D*

For a panel data set with three firms, where firm 1 is observed 2 periods, firm 2 is observed 2 periods and firm 3 is observed 3 periods the indicator matrix D (or matrix of firm-specific dummies) has dimension 7×3. D and DD' are given by

$$D=\begin{bmatrix}1&0&0\\1&0&0\\0&1&0\\0&1&0\\0&0&1\\0&0&1\\0&0&1\end{bmatrix}\qquad DD'=\begin{bmatrix}1&1&0&0&0&0&0\\1&1&0&0&0&0&0\\0&0&1&1&0&0&0\\0&0&1&1&0&0&0\\0&0&0&0&1&1&1\\0&0&0&0&1&1&1\\0&0&0&0&1&1&1\end{bmatrix}.$$

4.A4. Appendix: ML Estimation of a Homoskedastic Random Effects Model in the Unbalanced Panel Data Case

To facilitate ML estimation we first redefine the covariance matrix slightly:

$$\Omega = \sigma_u^2\Phi,$$

where

$$\Phi = I_n + \rho DD',$$

and $\rho = \sigma_\eta^2/\sigma_u^2$, $D = \text{diag}(\mathbf{l}_{Ti})_{i=1,N}$ is an $n \times N$ indicator matrix (or matrix of firm-specific dummies), $\mathbf{l}_{Ti}$ is a vector of ones of dimension T_i. ρ is the ratio of the variance of the firm-specific error component to the variance of the observation-specific error component. The indicator matrix D describes the structure of the panel data set. (See appendix 4.A3 for an example of the matrix D for a particular unbalanced panel).

The log-likelihood function can be written as (Baltagi & Chang, 1994, eqn. 17)

(4.15) $\ln L = -n/2 \ln(2\pi) - n/2 \ln \sigma_u^2 - 1/2 \ln|\Phi| - (\mathbf{y} - X\alpha)'\Phi^{-1}(\mathbf{y} - X\alpha)/2\sigma_u^2$.

The first-order conditions give closed-form solutions for $\hat{\alpha}$ and $\hat{\sigma}_u^2$ conditional on $\hat{\rho}$:

(4.16) $\hat{\alpha} = (X'\hat{\Phi}^{-1}X)^{-1}X'\hat{\Phi}^{-1}\mathbf{y}$,

(4.17) $\hat{\sigma}_u^2 = (\mathbf{y} - X\alpha)'\hat{\Phi}^{-1}(\mathbf{y} - X\alpha)/n$.

Unfortunately, the first-order condition based on ρ is nonlinear in ρ even for known values of α and σ_u^2:

(4.18) $\partial\ln L/\partial\rho = 1/2\ \text{Tr}(D'\Phi^{-1}D) + (\mathbf{y} - X\alpha)'\Phi^{-1}DD'\Phi^{-1}(\mathbf{y} - X\alpha)/2\sigma_u^2 = 0$.

The second derivative of $\ln L$ with respect to ρ is given by

(4.19) $\partial^2\ln L/(\partial\rho\partial\rho) = 1/2\ \text{Tr}\{(D'\Phi^{-1}D)(D'\Phi^{-1}D)\} - (\mathbf{y} - X\alpha)'\Phi^{-1}D(D'\Phi^{-1}D)D'\Phi^{-1}(\mathbf{y} - X\alpha)/\sigma_u^2$.

A numerical solution by means of iterations is necessary for $\hat{\rho}$. Starting with an initial value of $\rho_0(\hat{\Phi}_0)$, $\hat{\alpha}_0$ and $\hat{\sigma}_{u0}^2$ are obtained from equations (4.16) and (4.17). The GLS estimator can provide the initial value $\rho_0(\hat{\Phi}_0)$.

An iterative algorithm is used to update ρ.

The general expression for the updating formula (or algorithm) is

$$\rho_{(j+1)} = \rho_{(j)} - s_{(j)}H_{(j)}\mathbf{g}_{(j)},$$

where $s_{(j)}$ is the step size, $H_{(j)}$ a positive definite matrix, and $\mathbf{g}_{(j)}$ is the vector of first derivatives of the function to be optimised. The product $H_{(j)}\mathbf{g}_{(j)}$ is often called the direction value. Hemmerle & Hartley (1973) use the Newton-Raphson formula, where the step size $s_{(j)} = 1$ for all j, $H_{(j)} = [\partial^2\ln L/(\partial\rho\partial\rho)]^{-1}_{(j)}$, and $\mathbf{g}_{(j)} = [\partial\ln L/\partial\rho]_{(j)}$. Baltagi & Chang (1994) modifies this by letting the step size $s_{(j)}$ be adjusted by step halving. If the updated value is negative, it is replaced by zero and the iteration continues until the convergence criterion is satisfied. Harville (1977, pp. 329-30) discusses several algorithms in the context of variance components estimation, such as the Newton-Rhapson, the method of steepest ascent and the method of scoring.

The iterative procedure is exited at some point when the change in the value of the objective function, parameter values, and gradient is less than a prespecified value (e.g. 0.00001), or when a prespecified upper bound for the number of iterations is attained (e.g. 100 iterations) (Judge *et al.*, 1985, p. 728).

Criteria used to compare different algorithms are the (1) robustness, for example, measured by the number of times the algorithm has converged to the maximum, (2) precision in the solution, (3) the number of function evaluations, and (4) execution time (Judge *et al.*, 1985, p. 744). According to Harville (1977) no algorithm can be said to perform better in general than the others for ML estimation of variance components models.[11]

Unfortunately, the ML estimation procedure involves computation of the $n \times n$ covariance matrix $\mathbf{\Phi}^{-1}$. Hemmerle & Hartley (1973) have, however, provided a transformation that eliminates the need for inverting an $n \times n$ matrix. The so-called *W-transformation*, which is presented in the following, makes the iterative calculations independent of the total number of observations, n, and it is not wedded to a particular iterative algorithm.

4.A4.1 The W-Transformation

For the one-way error components model the transformation matrix W is a $(N + k + 1) \times (N + k + 1)$ matrix with submatrix elements $W(j, k), j, k = 1, 2, 3$, i.e., it is defined as

$$W = \begin{bmatrix} W(1,1) & W(1,2) & W(1,3) \\ W(2,1) & W(2,2) & W(2,3) \\ W(3,1) & W(3,2) & W(3,3) \end{bmatrix}.$$

The submatrices in column 1 are defined as follows:

$W(1, 1) = D'\mathbf{\Phi}^{-1}D,$ (a $N \times N$ matrix),

$W(2, 1) = X'\mathbf{\Phi}^{-1}D,$ (a $k \times N$ matrix),

$W(3, 1) = (W(1, 3))' = \mathbf{y}'\mathbf{\Phi}^{-1}D,$ (a $1 \times N$ matrix).

The submatrices in column 2 are defined as follows:

$W(1, 2) = (W(2, 1))' = D'\mathbf{\Phi}^{-1}X,$ (a $N \times k$ matrix),

$W(2, 2) = X'\mathbf{\Phi}^{-1}X,$ (a $k \times k$ matrix),

$W(3, 2) = \mathbf{y}'\mathbf{\Phi}^{-1}X,$ (a $1 \times k$ matrix).

The submatrices in column 3 are defined as follows:

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[11] A presentation and discussion of computational methods for nonlinear econometric models in general are provided by Judge *et al.* (1985, section 17.4) and Greene (1991, Chapter 12)

$W(1, 3) = (W(3, 1))' = D'\Phi^{-1}\mathbf{y}$, (a $N \times 1$ matrix),

$W(2, 3) = (W(3, 2))' = X'\Phi^{-1}\mathbf{y}$, (a $k \times 1$ matrix),

$W(3, 3) = \mathbf{y}'\Phi^{-1}\mathbf{y}$, (a 1×1 matrix),

However, W does not have to be computed as above. Hemmerle & Hartley show that W can be computed according to the following formula (H&H, eqn. 46):

$$(4.20) \quad W = W_0 - \begin{bmatrix} D' D \\ X' D \\ \mathbf{y}' D \end{bmatrix} Q^{-1} [D'D|D'X|D'\mathbf{y}],$$

where

$$(4.21) \quad Q = F^{-1} + D'D,$$

is a $N \times N$ matrix, and $F = \rho I_N$. The $(N + k + 1) \times (N + k + 1)$ matrix W_0 is the matrix W with $\Phi^{-1} = I_n$.

The determinant of the covariance matrix, Φ, can be calculated by (H&H, eqn. 48)

$$|\Phi| = |F^{-1} + D'D|\,|F|.$$

The closed-form solutions for $\hat{\alpha}$ and $\hat{\sigma}_u^2$ based on the first-order conditions are now:

$$(4.22) \quad \hat{\alpha} = [W(2, 2)]^{-1} W(2, 3),$$

$$(4.23) \quad \hat{\sigma}_u^2 = (1/n)\{W(3, 3) - \hat{\alpha}'W(2,3)\}.$$

The first-order condition based on ρ is:

$$(4.24) \quad \partial \ln L/\partial\rho = -1/2 \operatorname{Tr}\{W(1, 1)\} + (1/2\hat{\sigma}_u^2)P'P = 0,$$

where

$$P' = W(3, 1) - \hat{\alpha}'W(2,1).$$

The second derivative of $\ln L$ with respect to ρ is given by

$$(4.25) \quad \partial^2 \ln L/(\partial\rho\partial\rho) = 1/2 \operatorname{Tr}\{W(1, 1)W(1, 1)\} - P'W(1, 1)P/\sigma_u^2.$$

H&H presents an 8-step iterative procedure to obtain the ML estimates, as provided in table 4.1:

Table 4.1. Eight-step Iterative Procedure to Calculate ML Estimates

Step	Factor computed	Equation
1	Q	4.11 (A GLS estimate can be used as initial value for ρ)
2	W	4.10
3	$\hat{\alpha}_{(j)}$	4.12
4	$\hat{\sigma}_{u}{}^{2}{}_{(j)}$	4.13
5	$(\partial \ln L/\partial \rho)_{(j)}$	4.14
6	$(\partial^2 \ln L/(\partial \rho \partial \rho))_{(j)}$	4.15
7	$\Delta\rho_{(j)}$	$\Delta_{(j)} = [\partial^2 \ln L/(\partial \rho \partial \rho)]^{-1}{}_{(j)}[\partial \ln L/\partial \rho]_{(j)}$
8	$\rho_{(j)}$	$\rho_{(j+1)} = \rho_{(j)} - s_{(j)}\,\Delta\rho_{(j)}$ (or another updating algorithm)

4.A5. Appendix: Estimation of the Heteroskedastic Random Effects Model for Unbalanced Data

This appendix deals with FGLS and ML estimation of two-way error components models for unbalanced data with heteroskedasticity of the Just-Pope form $\text{var}(u_{it}) = h(\mathbf{z}_{it}, \beta)^2\varepsilon_{it}$ for the observation-specific component. It is assumed that the heteroskedasticity is of Harvey's (1976) multiplicative form. Although we are primarily interested in heteroskedasticity in the observation-specific component, we will see below that a GLS estimator is also available for the case of heteroskedasticity in both error components.

4.A5.1. Randolph's GLS Estimator

Randolph (1988b) presents a GLS estimator for the heteroskedastic one-way error components model

$$y_{it} = \mathbf{x}_{it}\alpha + \eta_i + u_{it}, \qquad i = 1, \ldots, N, \;\; t = 1, \ldots, T_i,$$

where $E[\eta_i^2] = \sigma_i^2$, $E[u_{it}^2] = \sigma_{it}^2$. In other words, both the firm-specific error component and the observation-specific component are heteroskedastic. The firm-specific and observation-specific variances are assumed to be functions of regressors, the sampling design or other parameters. The other assumptions on the random components from the homoskedastic model are retained.

For firm i the $T_i \times T_i$ covariance matrix of the composite error term $v_{it} = \eta_i + u_{it}$, is

$$\Omega_i = \sigma_i^2 J_{Ti} + \text{diag}(\sigma_{i1}^2, \sigma_{i2}^2, \ldots, \sigma_{iTi}^2) = \begin{bmatrix} \sigma_i^2 + \sigma_{i1}^2 & \sigma_i^2 & \cdots & \sigma_i^2 \\ \sigma_i^2 & \sigma_i^2 + \sigma_{i2}^2 & \cdots & \sigma_i^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_i^2 & \sigma_i^2 & \cdots & \sigma_i^2 + \sigma_{iTi}^2 \end{bmatrix},$$

where J_{Ti} is a matrix of ones of dimension $T_i \times T_i$. As in the homoskedastic model, for a given firm i, the correlation between any two disturbances in different time periods is constant. The $n \times n$ covariance matrix for the entire panel data set (where $n = \Sigma_i T_i$) is

$$\Omega = \sigma_\eta^2 J + \text{diag}(\sigma_{11}^2, \sigma_{22}^2, \ldots, \sigma_{2Ti}^2, \ldots, \sigma_{N1}^2, \sigma_{N2}^2, \ldots, \sigma_{NT_N}^2),$$

where $J = \text{diag}(J_{Ti})_{i=1,N}$ is a $n \times n$ block-diagonal matrix. The off-diagonal elements of Ω are zero if the observations belong to different firms. The non-zero off-diagonal elements are σ_i^2, while the diagonal elements are $\sigma_i^2 + \sigma_{it}^2$.

As usual the GLS estimator using the true variance components is

$$\hat{\alpha}_{GLS} = (X'\Omega^{-1}X)X'\Omega^{-1}\mathbf{y}.$$

Randolph shows that there exists a $n \times n$ transformation matrix R, such that $X^* = X'R$ and $\mathbf{y}^* = \mathbf{y}'R$, and

$\hat{\alpha}_{GLS} = (X^{*\prime}X^{*})X^{*\prime}\mathbf{y}^{*}$.[12]

For each observation the transformation can be expressed as

(4.26) $\mathbf{x}_{it} = [\mathbf{x}_{it} - \delta_i(\sum_{s=1}^{T_i} w_{is}\mathbf{x}_{is})]\sigma_{it}^{-1}$,

(4.27) $y_{it} = [y_{it} - \delta_i(\sum_{s=1}^{T_i} w_{is}y_{is})]\sigma_{it}^{-1}$,

where

(4.28) $\delta_i = [1 - (1 + \theta_i)^{-1/2}]$,

(4.29) $\theta_i = \sigma_i^2 \sum_{s=1}^{T_i} \sigma_{is}^{-2}$,

(4.30) $w_{it} = \sigma_i^2 / (\theta_i \sigma_{it}^2)$, $\quad \sum_{s=1}^{T_i} w_{is} = 1$.

4.A5.2. An FGLS Estimator for a Special Case of Randolph's Model: Harvey's Multiplicative Heteroskedasticity

It still remains to derive FGLS estimators for special cases of the above model in order to facilitate estimation for empirical purposes. Randolph (1988b) does not provide FGLS estimators for special cases of his one-way error components model. Furthermore, we have not been able to find any FGLS estimators or empirical applications of his model in the literature. The performance of this estimator is therefore an unresolved question.

Randolph mentions Harvey's multiplicative heteroskedastic model as one special case of his heteroskedastic one-way error components model. In the following we present an FGLS estimator for the special case of homoskedastic firm-specific error component and Harvey multiplicative heteroskedasticity for the observation-specific error component. Properties and estimators for Harvey's model in the absence of random firm-specific effects were presented in Chapter 3.

The model is given by

$$y_{it} = \mathbf{x}_{it}\alpha + \eta_i + u_{it}, \qquad i = 1, \ldots, N, \;\; t = 1, \ldots, T_i,$$

where $E[\eta_i^2] = \sigma_\eta^2$ (i.e., constant), $E[u_{it}^2] = \sigma_{it}^2 = \exp(\mathbf{z}_{it}\beta)$, where $\mathbf{z}_{it}$ is a $1 \times m$ - vector of regressors with first element one, and β is a $m \times 1$ vector of parameters. In this case

(4.31) $\theta_i = \sigma_\eta^2 \sum_{s=1}^{T_i} 1/\exp(\mathbf{z}_{it}\beta)$,

(4.32) $w_{it} = \sigma_\eta^2 / (\theta_i \exp(\mathbf{z}_{it}\beta))$.

A consistent estimate of σ_{it}^2 can be found by first estimating the fixed effects model

$$y_{it} = \eta_i - \mathbf{x}_{it}\alpha + u_{it},$$

[12] See the appendix of Randolph's paper for a derivation of this transformation.

and then use the residuals $\hat{u}_{it} = y_{it} - \hat{\eta}_i - \mathbf{x}_{it}\hat{\alpha}$ to estimate the variance function by (Harvey, 1976)

$$\ln(\hat{u}_{it}^2) = \mathbf{z}_{it}\beta + v_{it}.$$

After adding 1.2704 to the estimated intercept $\hat{\beta}_1$, following Harvey (1976), parameter estimates $\hat{\beta}$ are used to provide estimates of the observation-specific variances

$$\hat{\sigma}_{it}^2 = \exp(\mathbf{z}_{it}\hat{\beta}), \qquad i = 1, \ldots, N, \quad t = 1, \ldots, T_i.$$

An estimate of the variance of the firm-specific error component σ_η^2 can be found by using one of the estimators provided in section 4.7. By substituting the estimators $\hat{\beta}$ and $\hat{\sigma}_\eta^2$ in (4.31) and (4.32) for β and σ_η^2, respectively, we have the estimators for θ_i and w_{it}. Next, $\hat{\theta}_i$, $\hat{w}_{it}$ and $\hat{\sigma}_{it}$ is substituted into (4.26) and (4.27) to obtain the transformations of $\mathbf{x}_{it}$ and y_{it} that allow OLS estimation.

4.A5.3. Maximum Likelihood Estimation of Error Component Model with Harvey's Multiplicative Heteroskedasticity

This appendix presents an ML estimator for a one-way error components model for unbalanced data with Harvey's multiplicative heteroskedasticity in the observation-specific error component. The derivation of the ML estimator relies to a large extent on Magnus (1978), who derives an ML estimator for a covariance matrix with a finite number of unknown parameters θ. The covariance matrix of our model can be regarded as a special case of the more general covariance matrix in his model.[13]

The one-way error components model is

$$(4.33) \quad y_{it} = \mathbf{x}_{it}\alpha + v_{it} = \mathbf{x}_{it}\alpha + \eta_i + u_{it}, \qquad i = 1, \ldots, N, \quad t = 1, \ldots, T_i,$$

where $E[\eta_i^2] = \sigma_\eta^2$, $E[u_{it}^2] = \sigma_{it}^2 = \exp(\mathbf{z}_{it}\beta)$, which implies that the variance of the composite error term is $E[v_{it}^2] = \sigma_\eta^2 + \exp(\mathbf{z}_{it}\beta)$.

The $n \times n$ covariance matrix is $\Omega = E[\mathbf{vv}'] = \text{diag}\,(\Omega_i)_{i=1,N}$, where

$$(4.34) \quad \Omega_i = \sigma_\eta^2 J_{Ti} + \text{diag}(\exp(\mathbf{z}_{it}\beta))_{t=1,Ti}$$

$$= \begin{bmatrix} \sigma_\eta^2 + \exp(\mathbf{z}_{i1}\beta) & \sigma_\eta^2 & \cdots & \sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2 + \exp(\mathbf{z}_{i2}\beta) & \cdots & \sigma_\eta^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_\eta^2 & \sigma_\eta^2 & \cdots & \sigma_\eta^2 + \exp(\mathbf{z}_{iTi}\beta) \end{bmatrix},$$

and where J_{Ti} is a matrix of ones of dimension $T_i \times T_i$.

To simplify the notation in the following we first define

[13] Magnus (1978) derives the ML estimators for two special cases of the covariance matrix, autocorrelated errors and Zellner-type regressions.

(4.35) $\theta' = (\sigma_\eta^2, \beta')$

as the $1 \times (m+1)$ parameter vector of the covariance matrix.

The likelihood function is (Magnus, 1978, p. 284)

(4.36) $L = 2\pi^{-n/2} |\Omega|^{-1/2} \exp\{-(1/2)(\mathbf{y} - X\alpha)'\Omega^{-1}(\mathbf{y} - X\alpha)\}$,

which gives the log-likelihood function

(4.37) $\ln L = -n/2 \ln(2\pi) + (1/2) \ln|\Omega^{-1}| - (1/2)(\mathbf{y} - X\alpha)'\Omega^{-1}(\mathbf{y} - X\alpha)$,

The task is now to find the $k + m + 1$ estimators $\hat{\alpha}$ and $\hat{\theta}'=(\hat{\sigma}_\eta^2, \hat{\beta}')$ which maximise the log-likelihood function. A prerequisite for the differentiation of $\ln|\Omega^{-1}|$ is that $|\Omega| > 0$, which is guaranteed when the observation-specific variance σ_{it}^2 is specified as $\exp(\mathbf{z}_{it}\beta)$. Differentiation of ln L, using matrix differentiation rules provided e.g. in Neudecker (1969), leads to the following expression

(4.38) $d \ln L = (1/2) \operatorname{tr} \Omega \, d(\Omega^{-1}) - (\mathbf{y} - X\alpha)'\Omega^{-1} d(\mathbf{y} - X\alpha) - (1/2)(\mathbf{y} - X\alpha)' d(\Omega^{-1})(\mathbf{y} - X\alpha)$,

or, by using the fact that $\mathbf{v} = \mathbf{y} - X\alpha$,

(4.39) $d \ln L = (1/2) \operatorname{tr} \Omega \, d(\Omega^{-1}) - \mathbf{v}'\Omega^{-1} d\mathbf{v} - (1/2)\mathbf{v}' d(\Omega^{-1})\mathbf{v}$

$= \mathbf{v}'\Omega^{-1} X d\alpha + (1/2)\operatorname{tr}(\Omega - \mathbf{v}\mathbf{v}') d(\Omega^{-1})$.

A necessary condition for a maximum is that d ln $L = 0$ for all $d\alpha \neq \mathbf{0}$ and $d\theta \neq \mathbf{0}$. Hence, the $k + m + 1$ first-order conditions for a maximum are

(4.40) $\partial \ln L/\partial\alpha = \mathbf{v}'\Omega^{-1}X = 0$ (an $1 \times k$ matrix of f.o.c.'s)

(4.41) $\partial \ln L/\partial\theta_l = (\partial \ln L/\partial(\Omega^{-1}))(\partial(\Omega^{-1})/\partial\theta_l) = \operatorname{tr}\{(\partial(\Omega^{-1})/\partial\theta_l)\Omega\} - \hat{\mathbf{v}}'(\partial\Omega^{-1}/\partial\theta_l)\hat{\mathbf{v}} = 0$

$l = 1, \ldots, m+1$.

By replacing $\mathbf{v}$ in (4.40) with $\hat{\mathbf{v}} = \mathbf{y} - X\hat{\alpha}$, and solving with respect to $\hat{\alpha}$, we obtain the usual closed-form solution for the parameters of the mean function:

(4.42) $\hat{\alpha} = (X'\hat{\Omega}^{-1}X)^{-1}X'\hat{\Omega}^{-1}\mathbf{y}$.

Unfortunately there exists no closed-form solution for the parameters of the covariance matrix, θ:

(4.43) $\operatorname{tr}\{(\partial\Omega^{-1}/\partial\theta_l)\Omega\} = \hat{\mathbf{v}}'(\partial\Omega^{-1}/\partial\theta_l)\hat{\mathbf{v}}$, $l = 1, \ldots, m+1$.

The estimation of the $T_i \times T_i$ matrix of partial derivatives $\partial(\Omega_i^{-1})/\partial\theta_l$ can be simplified by using the matrix differentiation rule

$$\partial(\Omega_i^{-1})/\partial\theta_l = \Omega_i^{-1}\{\partial\Omega_i/\partial\theta_l\}\Omega_i^{-1}.$$

Furthermore, one can exploit that for a block-diagonal matrix, the inverse can be constructed by inverting the individual blocks.

For our particular model $\partial\Omega_i/\partial\sigma_\eta^2$ is a $T_i \times T_i$ matrix of ones. The matrix $\partial\Omega_i/\partial\beta_0 = \operatorname{diag}(\exp(\mathbf{z}_{i1}\beta), \ldots, \exp(\mathbf{z}_{iTi}\beta))$, while the matrices $\partial\Omega_i/\partial\beta_l = \operatorname{diag}(\exp(\mathbf{z}_{i1}\beta)\mathbf{z}_{i1}, \ldots, \exp(\mathbf{z}_{iTi}\beta)\mathbf{z}_{iTi})$, $l = 1, \ldots, m-1$.

It is convenient to rewrite (4.39) explicitly as a function of $d\alpha$ and $d\theta$. The last term of (4.39) can be transformed as

$$(4.44)\quad \mathrm{tr}(\Omega - \mathbf{vv}')\mathrm{d}(\Omega^{-1}) = \mathrm{vec}(\Omega - \mathbf{vv}')'\mathrm{vecd}(\Omega^{-1})$$

$$= \mathrm{vec}(\Omega - \mathbf{vv}')'(\partial\mathrm{vec}\Omega^{-1}/\partial\theta)'\mathrm{d}\theta.$$

By substituting (4.44) for the last term in (4.39) and do some rearranging we obtain

$$(4.45)\quad \mathrm{d}\ln L = (\mathrm{d}\alpha)'X'\Omega^{-1}\mathbf{v} + (1/2)(\mathrm{d}\theta)'(\partial\mathrm{vec}\Omega^{-1}/\partial\theta)\mathrm{vec}(\Omega - \mathbf{vv}').$$

The Hessian and Information Matrix

Next, we will present the Hessian and information matrix of the log-likelihood function by differentiating the log-likelihood twice with respect to its parameters

$$(4.46)\quad \mathrm{d}^2 \ln L = [(\mathrm{d}\alpha)', (\mathrm{d}\theta)']H\begin{bmatrix} d\beta \\ d\theta \end{bmatrix},$$

where H is a $(k + m + 1) \times (k + m + 1)$ matrix. The structure of H is given by the following theorem (Magnus, 1978, p. 285):

Theorem: Define the symmetric $(m+1) \times (m+1)$ matrices

$$M^{ij} = \partial\Omega_{ij}^{-1}/(\partial\theta\partial\theta'), \qquad i, j = 1, 2, \ldots, n,$$[14]

and let $\Omega = [w_{ij}]$, then the Hessian of the log-likelihood function (11) is

$$H = \begin{bmatrix} H_{11} & H_{12}' \\ H_{12} & H_{22} \end{bmatrix},$$

with

$$H_{11} = -X'\Omega^{-1}X, \qquad \text{(a } k \times k \text{ matrix)}$$

$$H_{12} = (\partial\mathrm{vec}\Omega^{-1}/\partial\theta)(X\otimes\mathbf{v}), \qquad \text{(a } (m+1) \times k \text{ matrix)}$$

$$H_{22} = (1/2)\Sigma_{i,j}(w_{ij} - v_iv_j)M^{ij} - (1/2)(\partial\mathrm{vec}\Omega^{-1}/\partial\theta)(\Omega\otimes\Omega)(\partial\mathrm{vec}\Omega^{-1}/\partial\theta)'$$

$$= (1/2)\Sigma_{i,j}(w_{ij} - v_iv_j)M^{ij} - (1/2)\Psi_\theta, \qquad \text{(a } (m+1) \times (m+1) \text{ matrix)}$$

where Ψ_θ is a symmetric $(m+1) \times (m+1)$ matrix with typical element

$$(\Psi_\theta)_{ij} = \mathrm{tr}\{(\partial\Omega^{-1}/\partial\theta_i)\Omega(\partial\Omega^{-1}/\partial\theta_j)\Omega\}, \qquad i,j = 1, 2, \ldots, m+1.$$

The information matrix of the log-likelihood function, defined as minus the expectation of the Hessian matrix, is (Magnus, 1978, Theorem 3):

$$\Psi = \begin{bmatrix} X'\Omega^{-1}X & \mathbf{0} \\ \mathbf{0} & (1/2)\Psi_\theta \end{bmatrix},$$

---

[14] To simplify notation in the following, the n $(= \Sigma_i T_i)$ observations are assumed ordered by firm and time, and then renumbered from 1 to n. The sub- and superscripts i and j therefore refer to all observations $1, 2, \ldots, n$ in this theorem.

The $m+1$ ($n^2 \times 1$) vectors of partial derivatives $\text{vec}\partial\Omega^{-1}/\partial\theta_1$, $\text{vec}\partial\Omega^{-1}/\partial\theta_2, \ldots, \text{vec}\partial\Omega^{-1}/\partial\theta_{m+1}$ have to be linearly independent in order to ensure that the information matrix is positive definite and that all the parameters of the covariance matrix are identified (Magnus, 1978, pp. 288-9). This condition is satisfied for the particular parametric structure we have postulated for the covariance matrix.[15]

Estimation of the Model

Magnus (pp. 289-90) suggests using the "zig-zag" iterative procedure due to Oberhofer & Kmenta (1974) to find the ML estimates of α and θ. The advantage of this procedure is that it does not involve inversion of the Hessian matrix at each step of the iteration. Unfortunately, the procedure requires that there exists a solution of the $m+1$ first-order conditions $\partial \ln L/\partial\theta_l$. For our particular covariance matrix a procedure that does not need solutions for the first-order conditions has to be used instead.

In the following a Newton-Rhapson estimation procedure for the estimation of α and θ is outlined. The procedure involves computation of matrices of an order that may exceed the capacity of available computer hardware and software. Therefore, methods for reducing the matrix computations are proposed, partly by exploiting the block-diagonal structure of the covariance matrix.[16]

A Newton-Rhapson estimation procedure would involve the following steps:

(1) Choose starting values $\hat{\theta}_0$ for θ, e.g. based on the FGLS estimator derived in section 4.7.

(2) Calculate $\hat{\Omega}_0^{-1} = \Omega^{-1}(\hat{\theta}_0)$, $\hat{\alpha}_0 = (X'\hat{\Omega}_0^{-1}X)^{-1}X'\hat{\Omega}_0^{-1}\mathbf{y}$, and $\hat{\mathbf{v}}_0 = \mathbf{y} - X\hat{\alpha}_0$.

The largest matrix that has to be computed in this step is the $n \times n$ matrix $\hat{\Omega}_0^{-1}$. Since Ω is block-diagonal with diagonal submatrices $\Omega_1, \Omega_2, \ldots, \Omega_N$, one can exploit that the inverse of Ω is equal to the inverses of the diagonal submatrices, i.e., $\Omega^{-1} = \text{diag}(\Omega_1^{-1}, \Omega_2^{-1}, \ldots, \Omega_N^{-1})$. In other words, each Ω_i can be inverted independently.

In the computation of $\hat{\alpha}_0$ the following procedure is equivalent to computing $(X'\hat{\Omega}_0^{-1}X)^{-1}$ directly: First, define the $k \times k$ matrix $R = (X'\hat{\Omega}_0^{-1}X)^{-1} = \mathbf{0}_k$. Then, repeat the following steps for all firms $i = 1, 2, \ldots, N$: (a) Compute the estimate of the covariance matrix of firm i, $\hat{\Omega}_{0i}^{-1}$. (b) Compute $X_i'\hat{\Omega}_{0i}^{-1}X_i$, where X_i is the $T_i \times k$ matrix of observations of the regressors of

[15] Computation of the partial derivatives $\text{vec}\partial\Omega^{-1}/\partial\theta_1$, $\text{vec}\partial\Omega^{-1}/\partial\theta_2, \ldots, \text{vec}\partial\Omega^{-1}/\partial\theta_m$ can be simplified by using the matrix differentiation rule $\partial A^{-1}/\partial x = -A^{-1}(\partial A/\partial x)A^{-1}$, where x is a scalar. However, it is still quite a task to derive these expressions analytically. We verified the linear independence among all the vectors through a simulation study. Because of the block-diagonal structure of Ω, it is sufficient to verify that the condition is satisfied for the observations of one firm, i.e., for any Ω_i.

[16] For instance, the computation of elements of the Hessian matrix involves matrices with dimensions $n^2 \times k$, $n^2 \times m$, and $n^2 \times n^2$. An econometric computer program such as Limdep has a limit of 20,000 values in its matrix work area, and the maximum number of elements in a single matrix is 10,000. This implies that if the total number of observations in the data set, n, is larger than 10, an $n^2 \times n^2$ matrix cannot be constructed.

firm i. (c) Add $X_i'\hat{\Omega}_{0i}^{-1}X_i$ to R, i.e., $R = R + X_i'\hat{\Omega}_{0i}^{-1}X_i$. After N iterations of (a)-(c) we have the matrix $X'\hat{\Omega}_0^{-1}X$, which has to be inverted. An analogue procedure can be used in the computation of $X'\hat{\Omega}_0^{-1}\mathbf{y}$.

(3) Substitute $\hat{\mathbf{v}}_0$ into the m+1 first-order conditions (4.43), i.e., estimate the first derivative functions $(\partial \ln L/\partial\theta_l)_0 = \hat{\mathbf{v}}_0'(\partial\hat{\Omega}_0^{-1}/\partial\theta_{0l})\hat{\mathbf{v}}_0$, $l = 1, \ldots, m+1$.

This step involves the computation of the $n \times n$ matrix of partial derivatives $(\partial\hat{\Omega}_0^{-1}/\partial\theta_{0l})$. To reduce the data storage requirements, repeat the following steps for all firms $i = 1, 2, \ldots, N$: (a) Compute the $T_i \times T_i$ matrix $(\partial\hat{\Omega}_{0i}^{-1}/\partial\theta_{0l})$. (b) Compute the $1 \times T_i$ matrix product $\hat{\mathbf{v}}_{0i}'(\partial\hat{\Omega}_{0i}^{-1}/\partial\theta_{0l})$, where $\hat{\mathbf{v}}_{0i}$ is a $T_i \times 1$ vector. After N iterations of (a)-(c) concatenate the N products to obtain the $1 \times n$ matrix $\hat{\mathbf{v}}_0'(\partial\hat{\Omega}_0^{-1}/\partial\theta_{0l})$. Finally, multiply with $\hat{\mathbf{v}}_0$.

(4) Evaluate the Hessian matrix H for $\hat{\theta}_0$, $\hat{\Omega}_0^{-1}$ and $\hat{\mathbf{v}}_0$ to obtain $\hat{H}_0$.

This step has the largest computational requirements. Separate procedures have to be used for each of the submatrices H_{11}, H_{12}, and H_{22} of the Hessian. For $\hat{H}_{0,11}$ one can use the procedure suggested for $X'\hat{\Omega}_0^{-1}X$ in step 2.

Direct computation of $\hat{H}_{0,12}$ involves computation of $(m+1) \times n^2$ and $n^2 \times k$ matrices. The following procedure reduces the data storage requirements: Define the $(m+1) \times k$ matrix $G = \mathbf{0}$. Repeat the following steps for all firms $i = 1, 2, \ldots, N$: (a) Compute the $n \times k$ matrix $X'_i \otimes \hat{\mathbf{v}}_0$, where X'_i is the ith row of X. (b) Compute the $n \times (m+1)$ matrix of derivatives of column i of $\hat{\Omega}_0^{-1}$, i.e., $\partial\hat{\Omega}_{0i}^{-1}/\partial\theta$. (c) Transpose $\partial\hat{\Omega}_{0i}^{-1}/\partial\theta$ into an $(m+1) \times n$ matrix. (d) Compute the $(m+1) \times k$ matrix $(\partial\hat{\Omega}_{0i}^{-1}/\partial\theta)'(X'_i \otimes \hat{\mathbf{v}}_0)$. (e) Add $(\partial\hat{\Omega}_{0i}^{-1}/\partial\theta)'(X'_i \otimes \hat{\mathbf{v}}_0)$ to G, i.e., $G = G + (\partial\hat{\Omega}_{0i}^{-1}/\partial\theta)'(X'_i \otimes \hat{\mathbf{v}}_0)$. After N iterations of (a)-(e) we have the matrix $\hat{H}_{0,12}$.

××Direct computation of $\hat{H}_{0,22}$ involves computation of $n \times n$ matrices in the last term. The data storage requirements are reduced by the following nested procedure with an outer loop for covariance matrix parameters and an inner loop for firms: For covariance matrix parameters r, $s = 1, 2, \ldots, m+1$: (a) Set the scalar $c = 0$. (b) For all firms $i = 1, 2, \ldots, N$: (i) Compute the $T_i \times T_i$ matrix product $\partial\Omega_i^{-1}/\partial\theta_r)\Omega_i(\partial\Omega_i^{-1}/\partial\theta_s)\Omega_i$. (ii) Compute the trace and add to c, i.e., $c = c + \mathrm{tr}\{\partial\Omega_i^{-1}/\partial\theta_r)\Omega_i(\partial\Omega_i^{-1}/\partial\theta_s)\Omega_i\}$. (c) After having completed N iterations of the steps (i)-(ii) we put the scalar $(\Psi_\theta)_{rs} = c$ into the r,s'th element of $\hat{\Psi}_\theta$. The steps (a)-(c) are repeated until $r = m+1$ and $s = m+1$.

(5) Adjust the parameters of the covariance matrix:

$$\hat{\theta}_1 = \hat{\theta}_0 - [\hat{H}_0]^{-1}[(\partial \ln L/\partial\theta_1)_0, (\partial \ln L/\partial\theta_2)_0, \ldots, (\partial \ln L/\partial\theta_{m+1})_0]'$$

The steps (2) to (5) are repeated until convergence or the maximum number of iterations are obtained.

An alternative to the above procedure is the "method of scoring" (Judge *et al.*, 1985, p. 736), where the Hessian H is replaced by the information matrix Ψ. As seen from the definition of the information matrix, the computational requirements will be somewhat smaller compared with the Newton-Raphson procedure. This is because the off-diagonal matrices $\hat{H}_{12}$ and $\hat{H}_{21}$ have only zero elements, and $\hat{H}_{22}$ reduces to $(1/2)\hat{\Psi}_{\theta}$.

5. SIMULATION STUDY: PERFORMANCE OF ESTIMATORS UNDER HETEROGENEITY AND HETEROSKEDASTICITY IN REGRESSORS

The empirical studies of production risk presented in Chapter 3 relied on a relatively limited number of observations. Usually, only a few hundred observations have been available to the researchers. Studies using the Just-Pope framework have produced rather disappointing results in terms of the significance of risk parameters, and Saha *et al.* (1997) suggest that this is due to the inefficiency of the FGLS estimators, which have predominantly been used in these studies. Instead, Saha *et al.* suggest to use the maximum likelihood (ML) estimator, because the ML estimator provides asymptotically more efficient estimates of the variance function parameters, and also is found to perform better in simulation experiments with small samples (see section 3.7).

Saha *et al.* examined the finite sample performance of FGLS and ML for firms which were assumed to use a simple homogenous Just-Pope production technology $y = f(\mathbf{x}; \alpha) + h(\mathbf{x}; \beta)$. The mean function $f(\cdot)$ had a Cobb-Douglas form, and the variance function $h(\cdot)$ was exponential in a linear function of inputs. Since firms were assumed homogeneous, firm-specific effects were not included.

In this chapter we extend Saha *et al*'s analysis to a more flexible parametrization of the Just-Pope technology. A linear quadratic functional form is chosen for the mean function. This functional form allows the elasticity of scale, substitution elasticities etc. to vary in input levels. The production technology is also characterised by firm heterogeneity in terms of firm-specific effects. We examine the performance of different estimators. Both fixed effects and random effects specifications are estimated. The panel data set we use is unbalanced, which has consequences for the random effects FGLS estimator, as it introduces heteroskedasticity into the covariance matrix of the estimator.

This simulation study does not purport to provide an exhaustive comparison of small sample properties of different estimators for a Just-Pope technology. The reason is that the scope for changes in the design matrix, parameters, etc., which may affect the relative performance of estimators, is almost endless. The results from a simulation study are sensitive to the simulation design. It is difficult to know *a priori* under what changes to the simulation design the findings in this chapter are no longer valid. Hence, one should be careful to generalise the results here too much.

Although we are interested in the small sample performance of competing estimators in the context of Just-Pope production functions, the results obtained in this chapter have relevance for estimation of panel data models with heteroskedasticity in regressors in general. As indicated by the discussion in section 4.8, there is very limited evidence on the performance of different panel data estimators when heteroskedasticity is present.

5.1. Sampling Distribution Properties

In this chapter the sampling distribution properties of competing estimators are compared for small samples. Section 3.7 presented the distributional properties of different estimators in large samples, i.e., as the number of observations approaches infinity. Although knowledge about the large sample properties of an estimator is useful, we will seldom have a data set that is large enough to invoke asymptotic properties when we choose an estimator.

To simplify the discussion, in the following θ denotes any element of the parameter vector ($\alpha_1, \ldots, \alpha_k, \beta_1, \ldots, \beta_m$). The following sampling distribution properties for the estimator $\hat{\theta}$ of θ will be analysed:

(a) The estimated expected value:

$$\bar{\hat{\theta}} = \left(\sum_{i=1}^{r} \hat{\theta}_i\right) \bigg/ r,$$

where $\hat{\theta}_i$ is the parameter estimate in sample i, and r is the total number of repeated samples (e.g. 1,000).

(b) The mean square error (MSE) of $\hat{\theta}$, which is estimated by the average of the squared difference between $\hat{\theta}$ and the true parameter value θ:

$$\text{MSE}(\hat{\theta}) = \left(\sum_{i=1}^{r} (\hat{\theta}_i - \theta)\right)^2 \bigg/ (r-1).$$

The MSE measures how much the estimator $\hat{\theta}$ varies around the true parameter value in r repeated samples.

(c) The probability of rejecting a false null hypothesis H_0: $\theta = 0$, measured by the average estimated t-ratio:

$$\bar{t}_0 = (1/r)\sum_{i=1}^{r} \left(\hat{\theta}_i / SE(\hat{\theta}_i)\right),$$

where SE($\hat{\theta}_i$) is the standard error of the estimator in sample experiment i.

(d) The probability of rejecting a true null hypothesis H_0: θ = actual value (Type I error). This is measured by the t-ratio

$$\bar{t}_\theta = (1/r)\sum_{i=1}^{r} \left\{(\hat{\theta}_i - \theta) / SE(\hat{\theta}_i)\right\}.$$

The ranking of estimators may depend on which of the above criteria is chosen. Often, one cannot find an estimator that is superior according to all criteria. Consequently, an overall assessment, which will involve some degree of subjectivity, has to be made. For example, an estimator may have a smaller average bias than another (measured by the deviation of $\bar{\theta}$ from θ), but at the same time also have a higher MSE than the other estimator. The question is then what performance criteria one should give priority to. There is not a general answer to this. To some extent the choice of estimator will depend on the magnitude of the biases and the magnitude of the MSE's.

5.2. Simulation Design

The "true" model is a Just Pope production function with a linear quadratic mean function and firm specific effects on the intercept. The mean function is

(5.1a) $$y_{it} = f(\mathbf{x};\alpha) + \eta_i + u_{it} = \alpha_0 + \sum_{k=1}^{3} \alpha_k x_{k,it} + 0.5 \sum_{k=1}^{3} \sum_{l=1}^{3} \alpha_{kl} x_{k,it} x_{l,it} + \eta_i + u_{it}.$$

The variance function is specified as

(5.1b) $$\text{var}(u_{it}) = [h(\mathbf{z}_{it})]^2 = \exp(\mathbf{z}_{it}\beta),$$

where $\mathbf{z}_{it} = (1, \ln(x_{1,it}), \ln(x_{2,it}), \ln(x_{3,it}))$.

The variance of the exogenous error term ε_{it} is var(ε) = exp(β_0).

There are two design matrices. Both have three independent variables (x_1, x_2, x_3) which were generated by a uniform distribution with the following[*min*, *max*]-values for (x_1, x_2, x_3); ([0.6, 1.4], [0.2, 1.8], [0.05, 1.95]). The first data set is an unbalanced panel with 250 observations, consisting of 20 firms observed in 10 time periods, and 10 firms observed in 5 periods. The second data set has 1000 observations, with 80 firms observed in 10 time periods and 40 firms observed in 5 time periods. Summary statistics for (x_1, x_2, x_3) are provided in table 5.A1.

Four different model specifications/estimators are compared in the simulation study:

I. Model (5.1a) with η_i's treated as fixed and u_{it} assumed to exhibit heteroskedasticity of unknown form, estimated by OLS and with White-correction of the covariance matrix (White, 1980).

II. Model (5.1) with η_i's treated as fixed, estimated by three-stage FGLS (as described in appendix 3.A).

III. Model (5.1) with η_i's treated as fixed, estimated by ML.[1] The log-likelihood function for the Just-Pope form was presented in section 3.7. A quasi-Newton method is employed for the nonlinear optimisation. The procedure is repeated until the estimates of α and β converge. The *method of scoring* may also be used to obtain the gradients of α and β. (Greene, 1991, pp. 415-6; Judge, Griffiths, Hill, Lütkepohl, & Lee, 1988, pp. 538-41).

IV. Model (5.1) with η_i's treated as random, estimated by FGLS. The estimation procedure is presented in appendix 9.E1.

There is an almost endless list of combination of changes that can be made to a simulation experiment which may affect the relative performance of competing estimators; the values of the parameters α, β and σ_η^2, which includes the variance of the exogenous error term ε_{it} (here: $\text{var}(\varepsilon_{it}) = \exp(\beta_0)$), the degree of unbalancedness of the panel (which affects the random effects estimates), the total number of observations, the functional specification of the mean and variance functions, and the distribution of the errors (normal, chi-squared, etc.). In order not to make the simulation study overwhelmingly large, it is necessary to draw some limits. As mentioned earlier two sample sizes will be used. We will also compare the different specifications/estimators under different values of (α, β, σ_η^2). Different values for α, β and σ_η^2 will produce different values for the sample mean and variance of $f(\mathbf{x};\ \alpha)$, the observation-specific variances $\text{var}(u_{it}) = \exp(\mathbf{z}_{it}\beta)$ that generates the errors u_{it}, and thus the coefficient of variation of the dependent variable, CV(y).

In each of the six experiments 100 samples were generated. In other words, the observation-specific error u_{it} was drawn (from a normal distribution) 100 times for each observation. The parameters were estimated by all of the above estimators for each sample.

The simulation experiments were undertaken for three different parameter sets (α, β, σ_η^2). Since there are two samples, there will be six simulation experiments in all. The parameter values are provided in table 5.A2, together with central sample statistics. Naturally, the sample mean and variance of $f(\mathbf{x};\ \alpha)$ depends on the chosen values for α, and $\text{var}(u_{it}) = \exp(\mathbf{z}_{it}\beta)$ depends on the chosen β's. The coefficient of variation of y conditional on **x**, CV(y|**x**), depends on all the chosen values of (α, β, σ_η^2).

Parameter set 1 is characterised by very low variation in mean output, as measured by var($f(\mathbf{x};\ \alpha)$), across observations. The average $\text{var}(u_{it})$ is small, but highest relative to var($f(\mathbf{x};\ \alpha)$) of

[1] More precisely, it is the marginal maximum likelihood (MML) estimator which is used in this simulation study (cf. section 4.6). The MML is ML on within-transformed variables. As noted in Chapter 4, the equivalency of the ML estimator (with firm dummy variables) and the MML estimator is only asymptotic, as the number of firms $N \to \infty$. However, the ML and MML estimates for our chosen design matrices and parameter values were so similar that the choice had no consequences for all practical purposes. The problem with the ML estimator is that when the number of firms, i.e., firm dummies, is large, it is difficult to find ML estimates that converge.

the three experiments. The value for the variance of the firm-specific effect in set 1 is the smallest among the three sets.

In parameter sets 2 and 3 the mean function parameters and the variance of the firm-specific effect have identical values. Parameter set 2 has the highest average $\mathrm{var}(u_{it})$ due to a high value of β_0. Furthermore, set 2 has the highest average coefficient of variation for the dependent variable y conditional on $\mathbf{x}$. The average $\mathrm{var}(u_{it})$ is smallest for parameter set 3. Consequently, the average coefficient of variation of y conditional on $\mathbf{x}$, is also smallest in set 3.

In all three experiments the variance of the firm-specific effect is small compared to the average variance $\mathrm{var}(u_{it})$. This is also the case in the empirical models which are estimated for the salmon industry in Chapter nine. All in all, parameter set 2 is most similar to the empirical models estimated in Chapter nine in terms of the variances of η_i and u_{it}, and the average coefficient of variation of y. However, the sample variance of $f(\mathbf{x};\alpha)$ is smaller than for the estimated models.

It should be noted that in this simulation study the data generating process (dgp) satisfies the assumptions underlying the random effects model; the error components η_i and u_{it} are independent of each other, and of the regressors $(x_{1,it}, x_{2,it}, x_{3,it})$. This may not be the case in other data generating processes, e.g. the dgp that generated the data set in the empirical application in this dissertation.

It is difficult to have a meaningful economic discussion of the structure of the production technology based on the estimated parameters alone. Here, this is particularly the case for the mean production function. Therefore, in addition to investigating the sample distribution properties of the α- and β-parameters, the estimated sample average and mean squared error of derived elasticities will also be investigated. The elasticities of interest are the output elasticity with respect to input k (e_k), returns to scale (*RTS*), and the *total variance elasticity* (*TVE*). The output elasticity with respect to input k (or input elasticity, for short) is given by

$$e_k(\mathbf{x}) = \frac{\partial f}{\partial x_k}\frac{x_k}{f(\mathbf{x})} = \left[\left(\alpha_k + 0.5\sum_j \alpha_{jk}x_j\right)\frac{x_k}{f(\mathbf{x})}\right].$$

Returns to scale is defined as the sum of the e_k's:

$$RTS(\mathbf{x};\alpha) = \sum\nolimits_k e_k(\mathbf{x}).$$

The total variance elasticity is the variance function counterpart of the RTS, and is given by

$$TVE(\mathbf{x};\beta) = \sum\nolimits_k \frac{\partial h}{\partial x_k}\frac{x_k}{h(\mathbf{x})} = \sum\nolimits_k \beta_k .$$

These elasticity measures are discussed in more detail in Chapter 9.

5.3. Simulation Results

The simulation results are presented in tables 5.A3 to 5.A8.

5.3.1. General Findings

First, we present some findings that are general, in the sense that they apply to all estimators in all experiments. As can be expected the precision of the parameter estimates in terms of MSE is smallest for the parameters associated with the regressor with smallest variation, x_1, while the precision is highest for the parameters associated with x_3, which exhibits the highest variation. The average bias of the estimated parameters can be measured by the deviation of the average estimated parameters from the actual parameter values. In general, the average bias tends to be smaller for the second-order parameters α_{kl} than the first-order parameters α_k. Among the first-order coefficients the average bias is roughly equal for parameter set 1, but higher for α_1 in sets 2 and 3 than α_2 and α_3. The average t-ratios for H_0: $\alpha_k = 0$ tend to be low and insignificant for most coefficients. On the other hand, the average t-ratios for H_0: α_k = actual value are also very low in general.

Although the individual estimated parameters of the mean function may be biased and have high mean squared error in some simulation experiments, the derived elasticities e_k and *RTS*, which are the magnitudes we are primarily interested in for economic analysis, tend to have a smaller bias and MSE. This is particularly the case for the sample with 1000 observations. The largest average biases in the estimates of e_k and their sum, *RTS*, is found in the simulation experiment with parameter set 2 and 250 observations (table 5.A5), where all estimators underestimate e_1 and *RTS*.

The sign of the average estimated parameters of the variance function is equal to the actual sign in all simulation experiments. The average bias of the estimated β's cannot be said to be dramatic. However, in some experiments the precision as measured by the MSE is low, i.e., the estimated β-values tend to vary much from sample to sample in an experiment. The estimated t-ratio associated with H_0: $\beta_k = 0$ is much larger for β_0 than the input coefficients.

For the total variance elasticity (*TVE*) the average bias is significant in most experiments, and the level of precision in terms of MSE is smaller than for the analogue elasticity measure derived from the mean function, *RTS*.

5.3.2. Effect of Increasing the Sample Size

The simulation experiments were undertaken for two sample sizes; 250 and 1000 observations. The effect of adding 750 observations on the average bias of the mean function parameters is best assessed by examining the average estimates of the input elasticities e_k and their sum, *RTS*. Across the estimators the increase in sample size leads to a reduction in the average biases, although the biases in most simulation experiments are not too large for the smallest sample. The most pronounced effect of adding observations is found for parameter set 2, where the average bias in the estimate of e_1 is reduced considerably, and the average estimated *RTS* becomes larger than one for all estimators.

Adding observations leads to a substantial reduction in the MSE of the estimated mean function parameters; usually the average reduction in MSE is around 70-80 %. The increase in sample size also leads to a doubling of the t-ratios associated with H_0: $\alpha_k = 0$ in most cases.

For the estimated variance function parameters we also find that the introduction of new observations leads to a decrease in the average bias across estimators in most cases. The only exception may be parameter set 3. However, for all parameter sets there is a large reduction in MSE associated with increasing the sample size. As for the mean function parameters there is also a doubling of estimated t-ratios. The intercept β_0 is always highly significant, regardless of sample size, but for the input coefficients the increase in sample size has a pronounced effect on the significance of the t-ratio associated with H_0: $\beta_k = 0$.

5.3.3. Comparison of Estimator Performance

It is difficult to find any systematic differences in the average bias of the estimated mean function parameters α between estimators. Hence, we turn to the estimated output elasticities with respect to inputs, which are derived from the estimated α's. Again, it is difficult to see that any estimator is superior or inferior compared with the others. The different estimators tend to provide pretty similar estimates of the input elasticities, e_k. This is also the case for *RTS*, the sum of the input elasticities.

Also for the MSE of the estimated α parameters there are no systematic differences across estimators; in fact, in each experiment the MSEs tend to be pretty similar. This is even more the case for the MSE of the elasticities derived from the mean function, which are almost identical between the estimators.

Usually, the OLS estimator provides somewhat higher t-ratios for H_0: $\alpha_k = 0$ than the other estimators. However, the OLS estimator only has a slight superiority in this respect, and there is no differences in significance; when measured at a given (conventional) confidence level all estimators tend to give the same outcome of the test H_0: $\alpha_k = 0$.

For the variance function we only compare the FGLS and ML estimates from the fixed effects model, since the random effects model provides the same estimates of β as the fixed effects model.

In terms of average bias no estimator seems to dominate the other. The FGLS estimator provides a slightly less biased estimate of β_0 in all simulation experiments, while the ML estimates of the input coefficients (β_1, β_2, β_3) tend to be slightly less biased on average than the FGLS estimates. However, it is not possible to find any systematic difference for TVE, which is the sum of the input coefficients.

There is a pronounced and systematic difference between the ML and FGLS estimators with respect to MSE; over all experiments the MSE's of the ML estimates of (β_1, β_2, β_3) are 40-70 % smaller than the FGLS MSE's. In other words, the distribution of the 100 β-values estimated by ML is more concentrated around the actual values than the FGLS estimates. The superiority of the ML estimator with respect to MSE is smaller for β_0, but here too the MSE is always smaller than for the FGLS estimator.

The ML estimator also dominates with respect to the estimated (asymptotic) t-ratios associated with H_0: $\beta_k = 0$, which are always higher than the FGLS t-ratios. The main reason for this is that the asymptotic covariance matrices are given by $\text{cov}(\hat{\beta}_{FGLS}) = 4.9348(\mathbf{Z'Z})^{-1}$ and $\text{cov}(\hat{\beta}_{ML}) = 2(\mathbf{Z'Z})^{-1}$, i.e., the ML estimator is more than twice as efficient asymptotically.

5.4. Summary and Conclusions

There is very little evidence on the performance of different econometric panel data estimators in the presence of heteroskedasticity, because the standard panel data models in the literature assume homoskedastic errors. This simulation study provides some insight for the case of heteroskedasticity in regressors.

Furthermore, this simulation study extends Saha *et al.*'s study of small sample properties of estimators for Just-Pope technologies to a more flexible specification of the mean function, and also incorporates firm-specific effects in the underlying production technology. A linear quadratic specification is used for the mean function. Unlike the Cobb-Douglas specification in Saha *et al.*, this specification allows common elasticity measures (e.g. input elasticity and RTS) to vary in input levels. We use three fixed effects estimators; an OLS estimator with White-adjusted covariance matrix of the mean function parameters, an FGLS estimator and an ML estimator. In addition a random effects model is estimated by FGLS. If we are unwilling to make any assumptions on the structure of the variance function, or assume that firms are risk neutral, the OLS estimator with White-correction is an alternative to the more cumbersome FGLS and ML estimators, since the OLS estimates of the α's are still consistent under heteroskedasticity.

In this study the biases and mean square errors of the parameter estimates are generally larger than in Saha *et al.*'s simulation study. The performance of all the estimators is a bit disappointing for the mean function, but to some extent this may be caused by the design matrix, e.g. the small variation in the regressors. Despite this, the magnitudes which are of economic interest, the estimated input elasticities and returns to scale (RTS) tends to have smaller biases and MSE's. By increasing the sample size from 250 to 1000 observations the average biases of the mean function parameter estimates are reduced somewhat, and the variation in the estimated parameters from sample to sample is reduced by around 70-80 %, as measured by the MSE. For the mean function parameters it is difficult to find any systematic differences in performance between the estimators. The OLS estimator with White-adjusted standard errors is not inferior to the other estimators, which means that this estimator is a good alternative if one is only interested in the mean function parameters and elasticities.

For the variance function we find that the estimated parameters on average provide the correct sign for the marginal risk effect of inputs. The biases of the estimated β's vary somewhat across experiments, depending on the chosen parameter values and sample size. In some experiments with the smallest sample (i.e., 250 obs.) the estimates of the β's vary much in repeated samples, as measured by the MSE. Adding observations leads to substantial decreases in MSE. When we compare the FGLS and ML estimators for the fixed effects model we find that no estimator strongly dominates the other. However, the ML estimates of the β's vary considerably less from sample to sample than the FGLS estimates; the mean squared errors of the ML estimates are 40-70 % smaller than the FGLS estimates. The estimated t-ratios of the ML estimates are always higher than the FGLS estimates.

The simulation results here suggest that the sample size should be of some concern in empirical studies of production risk. There may be large gains in terms of unbiasedness and efficiency of parameter estimates by increasing sample sizes. Simulation experiments which are not presented here, suggested further gains when increasing the sample size from 1000 to 2000 observations. The empirical application in this dissertation, the Norwegian salmon farm data set, has around 2000 observations.

Based on this simulation study one draws the conclusion that no estimator of Just-Pope technologies is superior if one is primarily interested in the mean function parameters, but that the ML estimator is the preferred estimator if the risk structure is to be analysed. However, the results here do not provide the overwhelming support that Saha *et al.* found for the ML estimator relative to the FGLS estimator.

Again it is important to stress that it is difficult to know *a priori* how sensitive the simulation results are to changes in the simulation design. Some caution should therefore be exercised before one generalises the findings here too much.

Next, we will move on to our empirical application, the salmon aquaculture industry. One of the interesting points is going to be whether the empirical results are similar to the simulation results when one compares different estimators. For example, we will compare the FGLS and ML estimates of the mean and variance function parameters and derived elasticities for the fixed effects specification.

5.A. Appendix A: Simulation Results

Table	**Description**
5.A1	Summary Statistics for Sample with 250 Observations and Sample with 1000 Observations
5.A2	Parameter Sets
5.A3	Simulation Results with Parameter Set 1 and Sample Size n=250
5.A4	Simulation Results with Parameter Set 1 and Sample Size n=1000
5.A5	Simulation Results with Parameter Set 2 and Sample Size n=250
5.A6	Simulation Results with Parameter Set 2 and Sample Size n=1000
5.A7	Simulation Results with Parameter Set 3 and Sample Size n=250
5.A8	Simulation Results with Parameter Set 3 and Sample Size n=1000

Table 5.A1. Summary Statistics for Sample with 250 Observations and Sample with 1000 Observations

250 Observations

	Mean	St.dev.	Min	Max
x_1	0.993	0.230	0.601	1.397
x_2	1.011	0.472	0.208	1.799
x_3	1.029	0.559	0.051	1.945

1000 Observations

	Mean	St.dev.	Min	Max
x_1	1.002	0.233	0.601	1.399
x_2	1.004	0.472	0.202	1.800
x_3	0.999	0.564	0.051	1.947

Table 5.A2. Parameter Sets and Associated Sample Statistics

Parameters	Set 1	Set 2	Set 3
α_1	0.300	0.050	0.050
α_2	0.200	0.100	0.100
α_3	0.300	0.900	0.900
α_{11}	-0.020	-0.002	-0.002
α_{12}	0.030	0.003	0.003
α_{13}	-0.025	-0.002	-0.002
α_{22}	-0.050	-0.005	-0.005
α_{23}	0.030	0.003	0.003
α_{33}	-0.050	-0.005	-0.005
β_1	0.150	0.300	0.500
β_2	0.100	0.100	0.600
β_3	-0.150	-0.200	-0.100
β_0	-3.000	-2.000	-4.000
$\text{Var}(\eta_i)$	0.0036	0.0100	0.0100

Sample Statistics

250 obs.

$\text{Var}(f(\mathbf{x};\alpha))$	0.021741	0.246631	0.246631
$\text{Avg}(\text{Var}(u_{it}))$	0.051208	0.141435	0.018197
$\text{Avg}(\text{CV}(y\mid\mathbf{x}))$	0.435606	0.681075	0.322315

1000 obs.

$\text{Var}(f(\mathbf{x};\alpha))$	0.022126	0.252581	0.252581
$\text{Avg}(\text{Var}(u_{it}))$	0.051432	0.142412	0.018239
$\text{Avg}(\text{CV}(y\mid\mathbf{x}))$	0.44037	0.696212	0.328793

Table 5.A3. Simulation Results with Parameter Set 1 and Sample Size n=250

Parameter		Average estimated Parameters				Mean Squared Error (MSE)				Average t-ratios H_0: $\theta = 0$				Avg. t-ratios H_0: θ=Actual Value			
	Actual Value	FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS*	FE by FGLS	FE by ML	RE by FGLS	FE by OLS*	FE by FGLS	FE by ML	RE by FGLS
α_1	0.300	0.294	0.298	0.301	0.292	0.524	0.530	0.525	0.507	0.455	0.429	0.443	0.436	-0.008	0.001	0.004	-0.010
α_2	0.200	0.208	0.215	0.216	0.212	0.048	0.053	0.048	0.047	0.958	0.914	0.939	0.934	0.025	-0.058	0.063	0.048
α_3	0.300	0.294	0.289	0.290	0.294	0.037	0.038	0.037	0.036	1.704	1.535	1.577	1.662	-0.049	0.062	-0.057	-0.037
α_{11}	-0.020	-0.010	-0.014	-0.014	-0.012	0.111	0.113	0.111	0.108	-0.036	-0.043	-0.048	-0.042	0.030	-0.018	0.015	0.022
α_{12}	0.030	0.025	0.023	0.024	0.027	0.017	0.018	0.017	0.016	0.200	0.166	0.176	0.200	-0.030	0.045	-0.041	-0.020
α_{13}	-0.025	-0.026	-0.023	-0.025	-0.022	0.014	0.013	0.013	0.013	-0.217	-0.178	-0.203	-0.191	0.005	-0.025	0.006	0.028
α_{22}	-0.050	-0.049	-0.052	-0.052	-0.053	0.006	0.007	0.006	0.006	-0.660	-0.641	-0.664	-0.692	0.015	0.015	-0.026	-0.043
α_{23}	0.030	0.030	0.030	0.030	0.031	0.004	0.003	0.003	0.003	0.562	0.509	0.521	0.570	0.003	0.002	-0.004	0.030
α_{33}	-0.050	-0.049	-0.048	-0.048	-0.052	0.004	0.004	0.004	0.003	-0.963	-0.866	-0.875	-0.983	0.012	-0.027	0.038	-0.038
β_1	0.150	N.A.	0.240	0.216	0.240	N.A.	0.376	0.175	0.376	N.A.	0.404	0.571	0.404	N.A.	-0.151	0.174	0.151
β_2	0.100	N.A.	0.118	0.137	0.118	N.A.	0.060	0.034	0.060	N.A.	0.482	0.879	0.482	N.A.	-0.072	0.236	0.072
β_3	-0.150	N.A.	-0.120	-0.103	-0.120	N.A.	0.027	0.017	0.027	N.A.	-0.739	-0.996	-0.739	N.A.	-0.181	0.450	0.181
β_0	-3.000	N.A.	-3.122	-3.149	-3.122	N.A.	0.038	0.032	0.038	N.A.	-20.641	-32.694	-20.641	N.A.	0.810	-1.543	-0.810
Avg. αs						0.085	0.087	0.085	0.082	0.639	0.587	0.605	0.634	0.020	0.028	0.028	0.031
Avg. βs						N.A.	0.125	0.065	0.125	N.A.	5.567	8.785	5.567	N.A.	0.303	0.601	0.303

* White-adjusted t-ratios

Elasticity		Estimated Elasticities				Mean Squared Error (MSE)			
	Actual Value	FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS	FE by FGLS	FE by ML	RE by FGLS
e_1	0.407	0.420	0.417	0.418	0.418	0.010	0.009	0.010	0.009
e_2	0.208	0.216	0.214	0.214	0.211	0.005	0.005	0.005	0.005
e_3	0.257	0.249	0.250	0.251	0.247	0.004	0.004	0.004	0.004
RTS	0.872	0.884	0.882	0.882	0.876	0.017	0.017	0.017	0.017
TVE	0.100	N.A.	0.237	0.249	0.237	N.A.	0.498	0.248	0.498

Table 5.A4. Simulation Results with Parameter Set 1 and Sample Size n=1000

Parameter	Average estimated Parameters					Mean Squared Error (MSE)				Average t-ratios H_0: $\theta = 0$				Avg. t-ratios H_0: θ=Actual Value			
	Actual Value	FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS*	FE by FGLS	FE by ML	RE by FGLS	FE by OLS*	FE by FGLS	FE by ML	RE by FGLS
α_1	0.300	0.319	0.313	0.310	0.326	0.113	0.108	0.107	0.110	0.996	0.926	0.919	1.005	0.059	-0.040	0.028	0.079
α_2	0.200	0.211	0.209	0.209	0.217	0.013	0.013	0.013	0.013	1.976	1.839	1.852	1.978	0.109	-0.082	0.084	0.156
α_3	0.300	0.292	0.290	0.290	0.295	0.008	0.007	0.007	0.007	3.245	3.022	3.038	3.274	-0.087	0.104	-0.110	-0.060
α_{11}	-0.020	-0.030	-0.029	-0.027	-0.031	0.024	0.023	0.023	0.023	-0.200	-0.181	-0.170	-0.202	-0.067	0.055	-0.044	-0.071
α_{12}	0.030	0.028	0.029	0.029	0.025	0.005	0.005	0.005	0.005	0.438	0.419	0.429	0.377	-0.021	0.014	-0.007	-0.074
α_{13}	-0.025	-0.025	-0.023	-0.024	-0.026	0.004	0.004	0.004	0.004	-0.442	-0.387	-0.395	-0.465	0.002	-0.031	0.026	-0.018
α_{22}	-0.050	-0.056	-0.055	-0.055	-0.055	0.002	0.002	0.002	0.002	-1.519	-1.417	-1.432	-1.491	-0.159	0.131	-0.139	-0.146
α_{23}	0.030	0.035	0.035	0.035	0.032	0.001	0.001	0.001	0.001	1.264	1.192	1.200	1.168	0.163	-0.158	0.158	0.068
α_{33}	-0.050	-0.048	-0.048	-0.047	-0.048	0.001	0.001	0.001	0.001	-1.795	-1.688	-1.684	-1.792	0.081	-0.078	0.092	0.090
β_1	0.150	N.A.	0.132	0.146	0.132	N.A.	0.093	0.036	0.093	N.A.	0.453	0.790	0.453	N.A.	0.062	-0.020	-0.062
β_2	0.100	N.A.	0.095	0.103	0.095	N.A.	0.015	0.006	0.015	N.A.	0.789	1.340	0.789	N.A.	0.038	0.041	-0.038
β_3	-0.150	N.A.	-0.151	-0.137	-0.151	N.A.	0.007	0.004	0.007	N.A.	-1.816	-2.587	-1.816	N.A.	0.012	0.247	-0.012
β_0	-3.000	N.A.	-3.134	-3.137	-3.134	N.A.	0.023	0.022	0.023	N.A.	-41.114	-64.664	-41.114	N.A.	1.752	-2.834	-1.752
Avg. αs						0.019	0.018	0.018	0.018	1.319	1.230	1.235	1.306	0.083	0.077	0.076	0.085
Avg. βs						N.A.	0.035	0.017	0.035	N.A.	11.043	17.345	11.043	N.A.	0.466	0.785	0.466

* White-adjusted t-ratios

Elasticity		Estimated Elasticities				Mean Squared Error (MSE)			
	Actual Value	FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS	FE by FGLS	FE by ML	RE by FGLS
e_1	0.404	0.399	0.398	0.398	0.401	0.003	0.003	0.003	0.003
e_2	0.203	0.203	0.204	0.204	0.205	0.001	0.001	0.001	0.001
e_3	0.249	0.252	0.252	0.252	0.251	0.001	0.001	0.001	0.001
RTS	0.857	0.854	0.853	0.853	0.857	0.005	0.005	0.005	0.005
TVE	0.100	N.A.	0.076	0.113	0.076	N.A.	0.120	0.050	0.120

Table 5.A5. Simulation Results with Parameter Set 2 and Sample Size n=250

Para-meter	Average estimated Parameters					Mean Squared Error (MSE)				Average t-ratios $H_0: \theta = 0$				Avg. t-ratios $H_0: \theta$=Actual Value			
	Actual Value	FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS*	FE by FGLS	FE by ML	RE by FGLS	FE by OLS*	FE by FGLS	FE by ML	RE by FGLS
α_1	0.050	0.180	0.167	0.200	0.178	1.051	1.036	1.033	1.003	0.160	0.141	0.171	0.156	0.112	-0.097	0.127	0.110
α_2	0.100	0.045	0.046	0.049	0.044	0.184	0.190	0.190	0.176	0.122	0.117	0.127	0.114	-0.161	0.144	-0.140	-0.156
α_3	0.900	0.873	0.875	0.874	0.864	0.093	0.093	0.092	0.090	3.032	2.808	2.873	2.968	-0.102	0.088	-0.091	-0.130
α_{11}	-0.002	-0.109	-0.103	-0.117	-0.103	0.234	0.234	0.233	0.225	-0.207	-0.187	-0.216	-0.196	-0.203	0.183	-0.212	-0.192
α_{12}	0.003	0.039	0.039	0.036	0.027	0.058	0.060	0.060	0.055	0.193	0.166	0.159	0.118	0.179	-0.154	0.146	0.105
α_{13}	-0.002	0.037	0.037	0.034	0.037	0.048	0.047	0.046	0.046	0.199	0.189	0.181	0.205	0.210	-0.199	0.191	0.215
α_{22}	-0.005	-0.002	-0.002	-0.002	-0.001	0.016	0.017	0.016	0.015	-0.021	-0.013	-0.007	-0.007	0.020	-0.025	0.032	0.033
α_{23}	0.003	-0.002	-0.002	-0.003	0.006	0.009	0.009	0.009	0.009	-0.024	-0.018	-0.027	0.074	-0.057	0.049	-0.058	0.041
α_{33}	-0.005	-0.006	-0.007	-0.005	-0.007	0.008	0.009	0.008	0.008	-0.060	-0.075	-0.056	-0.075	-0.003	0.021	0.000	-0.018
β_1	0.300	N.A.	0.315	0.294	0.315	N.A.	0.343	0.160	0.343	N.A.	0.531	0.778	0.531	N.A.	-0.025	-0.017	0.025
β_2	0.100	N.A.	0.097	0.105	0.097	N.A.	0.056	0.028	0.056	N.A.	0.396	0.677	0.396	N.A.	0.014	0.033	-0.014
β_3	-0.200	N.A.	-0.175	-0.161	-0.175	N.A.	0.027	0.012	0.027	N.A.	-1.075	-1.556	-1.075	N.A.	-0.153	0.372	0.153
β_0	-2.000	N.A.	-2.158	-2.171	-2.158	N.A.	0.051	0.042	0.051	N.A.	-14.265	-22.546	-14.265	N.A.	1.044	-1.778	-1.044
Avg. αs						0.189	0.188	0.188	0.181	0.446	0.413	0.424	0.435	0.116	0.107	0.111	0.111
Avg. βs						N.A.	0.119	0.061	0.119	N.A.	4.067	6.389	4.067	N.A.	0.309	0.550	0.309

* White-adjusted t-ratios

Elasticity		Estimated Elasticities				Mean Squared Error (MSE)			
	Actual Value	FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS	FE by FGLS	FE by ML	RE by FGLS
e_1	0.067	0.016	0.014	0.015	0.014	0.049	0.046	0.048	0.047
e_2	0.131	0.110	0.110	0.111	0.100	0.015	0.015	0.015	0.015
e_3	0.804	0.808	0.808	0.809	0.806	0.002	0.002	0.002	0.002
RTS	1.002	0.935	0.932	0.935	0.920	0.070	0.066	0.066	0.069
TVE	0.200	N.A.	0.236	0.237	0.236	N.A.	0.494	0.223	0.494

Table 5.A6. Simulation Results with Parameter Set 2 and Sample Size n=1000

Parameter	Average estimated Parameters					Mean Squared Error (MSE)				Average t-ratios $H_0: \theta = 0$				Avg. t-ratios $H_0: \theta$=Actual Value			
	Actual Value	FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS*	FE by FGLS	FE by ML	RE by FGLS	FE by OLS*	FE by FGLS	FE by ML	RE by FGLS
α_1	0.050	0.080	0.088	0.085	0.050	0.335	0.331	0.330	0.319	0.157	0.161	0.157	0.097	0.062	-0.072	0.068	0.005
α_2	0.100	0.098	0.094	0.094	0.097	0.037	0.036	0.036	0.035	0.542	0.496	0.501	0.528	-0.016	0.033	-0.032	-0.017
α_3	0.900	0.922	0.921	0.920	0.928	0.034	0.034	0.033	0.033	6.135	5.773	5.800	6.178	0.150	-0.129	0.126	0.186
α_{11}	-0.002	0.001	-0.003	-0.002	0.017	0.074	0.071	0.072	0.072	-0.003	-0.015	-0.012	0.061	0.005	0.008	-0.004	0.069
α_{12}	0.003	-0.006	-0.006	-0.005	-0.001	0.014	0.014	0.013	0.014	-0.048	-0.047	-0.045	-0.005	-0.076	0.073	-0.071	-0.032
α_{13}	-0.002	-0.023	-0.023	-0.022	-0.028	0.012	0.012	0.012	0.011	-0.244	-0.232	-0.225	-0.295	-0.223	0.211	-0.204	-0.274
α_{22}	-0.005	0.002	0.003	0.003	-0.001	0.004	0.004	0.004	0.004	0.032	0.054	0.052	-0.016	0.114	-0.131	0.130	0.064
α_{23}	0.003	0.004	0.004	0.004	0.005	0.002	0.002	0.002	0.002	0.088	0.083	0.079	0.110	0.023	-0.020	0.017	0.044
α_{33}	-0.005	-0.007	-0.006	-0.006	-0.008	0.002	0.002	0.002	0.002	-0.145	-0.125	-0.127	-0.180	-0.034	0.019	-0.020	-0.067
β_1	0.300	N.A.	0.285	0.292	0.285	N.A.	0.071	0.025	0.071	N.A.	0.978	1.577	0.978	N.A.	0.053	-0.042	-0.053
β_2	0.100	N.A.	0.072	0.084	0.072	N.A.	0.014	0.007	0.014	N.A.	0.592	1.088	0.592	N.A.	0.236	-0.211	-0.236
β_3	-0.200	N.A.	-0.168	-0.177	-0.168	N.A.	0.008	0.004	0.008	N.A.	-2.017	-3.353	-2.017	N.A.	-0.389	0.427	0.389
β_0	-2.000	N.A.	-2.119	-2.129	-2.119	N.A.	0.020	0.019	0.020	N.A.	-27.802	-43.886	-27.802	N.A.	1.561	-2.666	-1.561
Avg. αs						0.057	0.056	0.056	0.055	0.822	0.776	0.778	0.830	0.078	0.078	0.075	0.084
Avg. βs						N.A.	0.028	0.013	0.028	N.A.	7.847	12.476	7.847	N.A.	0.560	0.836	0.560

* White-adjusted t-ratios

Elasticity	Actual Value	Estimated Elasticities				Mean Squared Error (MSE)			
		FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS	FE by FGLS	FE by ML	RE by FGLS
e_1	0.071	0.094	0.094	0.094	0.102	0.016	0.015	0.016	0.016
e_2	0.136	0.145	0.146	0.146	0.142	0.004	0.003	0.003	0.004
e_3	0.819	0.818	0.818	0.818	0.817	0.001	0.001	0.001	0.000
RTS	1.026	1.056	1.057	1.057	1.061	0.023	0.022	0.023	0.023
TVE	0.200	N.A.	0.189	0.199	0.189	N.A.	0.085	0.035	0.085

Table 5.A7. Simulation Results with Parameter Set 3 and Sample Size n=250

Parameter	Average estimated Parameters					Mean Squared Error (MSE)				Average t-ratios H_0: $\theta = 0$				Avg. t-ratios H_0: θ=Actual Value			
	Actual Value	FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS*	FE by FGLS	FE by ML	RE by FGLS	FE by OLS*	FE by FGLS	FE by ML	RE by FGLS
α_1	0.050	0.016	0.021	0.023	0.033	0.174	0.167	0.158	0.175	0.019	0.047	0.051	0.067	-0.110	0.080	-0.078	-0.058
α_2	0.100	0.103	0.106	0.105	0.101	0.017	0.017	0.017	0.017	0.808	0.809	0.830	0.746	0.007	-0.037	0.036	0.005
α_3	0.900	0.900	0.896	0.898	0.909	0.010	0.009	0.009	0.010	8.884	8.425	8.632	8.659	-0.007	0.039	-0.016	0.087
α_{11}	-0.002	0.011	0.009	0.007	0.003	0.038	0.036	0.033	0.038	0.076	0.049	0.044	0.031	0.087	-0.060	0.055	0.041
α_{12}	0.003	-0.001	-0.003	-0.002	0.002	0.007	0.007	0.008	0.008	-0.010	-0.026	-0.013	0.032	-0.049	0.063	-0.051	-0.005
α_{13}	-0.002	0.005	0.006	0.006	0.001	0.005	0.005	0.005	0.005	0.088	0.089	0.092	0.032	0.118	-0.118	0.122	0.061
α_{22}	-0.005	-0.003	-0.004	-0.004	-0.003	0.002	0.002	0.002	0.002	-0.055	-0.078	-0.085	-0.065	0.060	-0.033	0.029	0.043
α_{23}	0.003	0.000	0.000	-0.001	-0.002	0.001	0.001	0.001	0.001	-0.014	-0.017	-0.034	-0.050	-0.110	0.109	-0.129	-0.141
α_{33}	-0.005	-0.007	-0.005	-0.007	-0.009	0.001	0.001	0.001	0.001	-0.241	-0.168	-0.209	-0.301	-0.079	0.011	-0.049	-0.141
β_1	0.500	N.A.	0.369	0.449	0.369	N.A.	0.339	0.171	0.339	N.A.	0.622	1.188	0.622	N.A.	0.221	-0.136	-0.221
β_2	0.600	N.A.	0.531	0.569	0.531	N.A.	0.076	0.037	0.076	N.A.	2.175	3.665	2.175	N.A.	0.284	-0.197	-0.284
β_3	-0.100	N.A.	-0.059	-0.078	-0.059	N.A.	0.028	0.011	0.028	N.A.	-0.360	-0.750	-0.360	N.A.	-0.253	0.214	0.253
β_0	-4.000	N.A.	-4.163	-4.180	-4.163	N.A.	0.050	0.045	0.050	N.A.	-27.522	-43.408	-27.522	N.A.	1.080	-1.873	-1.080
Avg. αs						0.028	0.027	0.026	0.029	1.133	1.079	1.110	1.109	0.070	0.061	0.063	0.065
Avg. βs						N.A.	0.123	0.066	0.123	N.A.	7.670	12.253	7.670	N.A.	0.459	0.605	0.459

* White-adjusted t-ratios

Elasticity		Estimated Elasticities				Mean Squared Error (MSE)			
	Actual Value	FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS	FE by FGLS	FE by ML	RE by FGLS
e_1	0.086	0.073	0.071	0.071	0.077	0.011	0.010	0.010	0.011
e_2	0.156	0.158	0.158	0.158	0.159	0.002	0.002	0.002	0.002
e_3	0.903	0.902	0.903	0.902	0.902	0.000	0.000	0.000	0.000
RTS	1.145	1.133	1.131	1.131	1.138	0.014	0.013	0.013	0.014
TVE	1.000	N.A.	0.841	0.940	0.841	N.A.	0.481	0.250	0.481

Table 5.A8. Simulation Results with Parameter Set 3 and Sample Size n=1000

Parameter	Average estimated Parameters					Mean Squared Error (MSE)				Average t-ratios H_0: $\theta = 0$				Avg. t-ratios H_0: θ=Actual Value			
	Actual Value	FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS*	FE by FGLS	FE by ML	RE by FGLS	FE by OLS*	FE by FGLS	FE by ML	RE by FGLS
α_1	0.050	0.086	0.079	0.077	0.089	0.038	0.033	0.033	0.038	0.462	0.415	0.404	0.454	0.194	-0.152	0.140	0.196
α_2	0.100	0.110	0.110	0.110	0.108	0.004	0.004	0.004	0.004	1.776	1.752	1.758	1.643	0.169	-0.164	0.159	0.127
α_3	0.900	0.901	0.900	0.900	0.898	0.004	0.004	0.004	0.004	17.256	16.725	16.839	16.617	0.026	-0.008	0.005	-0.028
α_{11}	-0.002	-0.015	-0.013	-0.012	-0.017	0.008	0.007	0.007	0.008	-0.170	-0.143	-0.128	-0.183	-0.147	0.121	-0.106	-0.161
α_{12}	0.003	-0.006	-0.004	-0.004	-0.004	0.002	0.001	0.002	0.002	-0.159	-0.105	-0.109	-0.113	-0.237	0.182	-0.186	-0.188
α_{13}	-0.002	-0.002	-0.001	-0.002	-0.002	0.002	0.001	0.001	0.002	-0.072	-0.043	-0.053	-0.061	-0.011	-0.016	0.007	-0.001
α_{22}	-0.005	-0.006	-0.007	-0.007	-0.006	0.001	0.000	0.000	0.001	-0.253	-0.308	-0.304	-0.249	-0.024	0.082	-0.076	-0.025
α_{23}	0.003	0.002	0.003	0.003	0.003	0.000	0.000	0.000	0.000	0.128	0.168	0.186	0.155	-0.059	0.016	0.000	-0.028
α_{33}	-0.005	-0.005	-0.005	-0.005	-0.004	0.000	0.000	0.000	0.000	-0.311	-0.330	-0.324	-0.236	0.005	0.020	-0.013	0.078
β_1	0.500	N.A.	0.401	0.416	0.401	N.A.	0.086	0.040	0.086	N.A.	1.377	2.243	1.377	N.A.	0.340	-0.455	-0.340
β_2	0.600	N.A.	0.514	0.525	0.514	N.A.	0.023	0.012	0.023	N.A.	4.249	6.823	4.249	N.A.	0.715	-0.974	-0.715
β_3	-0.100	N.A.	-0.094	-0.088	-0.094	N.A.	0.007	0.003	0.007	N.A.	-1.134	-1.661	-1.134	N.A.	-0.069	0.229	0.069
β_0	-4.000	N.A.	-4.138	-4.140	-4.138	N.A.	0.027	0.023	0.027	N.A.	-54.298	-85.335	-54.298	N.A.	1.815	-2.895	-1.815
Avg. αs						0.007	0.006	0.006	0.006	2.287	2.221	2.234	2.190	0.097	0.085	0.077	0.093
Avg. βs						N.A.	0.036	0.019	0.036	N.A.	15.264	24.015	15.264	N.A.	0.735	1.138	0.735

* White-adjusted t-ratios

Elasticity	Actual Value	Estimated Elasticities				Mean Squared Error (MSE)			
		FE by OLS	FE by FGLS	FE by ML	RE by FGLS	FE by OLS	FE by FGLS	FE by ML	RE by FGLS
e_1	0.087	0.084	0.084	0.084	0.085	0.003	0.003	0.003	0.003
e_2	0.160	0.159	0.158	0.158	0.159	0.001	0.001	0.001	0.001
e_3	0.838	0.838	0.838	0.838	0.838	0.000	0.000	0.000	0.000
RTS	1.085	1.082	1.080	1.080	1.082	0.004	0.004	0.004	0.004
TVE	1.000	N.A.	0.820	0.853	0.820	N.A.	0.135	0.067	0.135

6. THE SALMON AQUACULTURE INDUSTRY: DISCUSSION OF ISSUES WHICH HAVE CONSEQUENCES FOR ECONOMETRIC MODELLING

This chapter discusses some features of the Norwegian salmon aquaculture which will have direct consequences for the specification of econometric models of firm behaviour and production technology in this industry.

6.1. The Production Process in Salmon Aquaculture

The biological production process in salmon aquaculture can be partitioned in the following steps (Bjørndal, 1990, pp. 3-4):

(1) Production of broodstock and roe,

(2) production of fry,

(3) production of smolts,[1] and

(4) production of farmed fish.

These four stages are generally undertaken in distinct plants. The empirical analysis in this dissertation will only be concerned with the production of farmed fish, i.e., the last step in the biological production process.

Prior to the first release of salmon in a newly developed grow-out farm, considerable investment in production facilities have to be undertaken. The farmer generally has to invest in land facilities, such as a building for storage, processing and repairs, pier, road, etc., and the sea-pen system, which consists of gangway, pens, nets, feeding equipment, etc. Initial investments will vary greatly, depending on existing facilities on land and the scale of operation. Typically, investments in capital equipment have been in the range of 4 to 10 million NOK.[2]

A schematic view of the production process in salmon aquaculture is depicted in figure 6.1. Smolts are purchased from a smolt producer and released into the pens, usually in May/June each year.[3] A cohort of salmon is kept in the pens and fed for a period of one to two years

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[1] Smolts are juvenile salmon which are able to adapt to sea water, i.e., it has been through the biological process called the *smoltification* process.

[2] See Bjørndal (1990, pp. 87-8) for a discussion of facility investments in salmon farming and a numerical example.

[3] In recent years, however, it has become more common to have a second release of smolts in late summer or early fall.

before harvesting. The length of the growth period depends to a large extent on the growth rate of the fish, which again is mainly determined by feeding intensity and sea temperatures. In figure 6.1 it is assumed that the last fish of each cohort is harvested after two years in the sea. Due to the long growth period, substantial working capital is required.

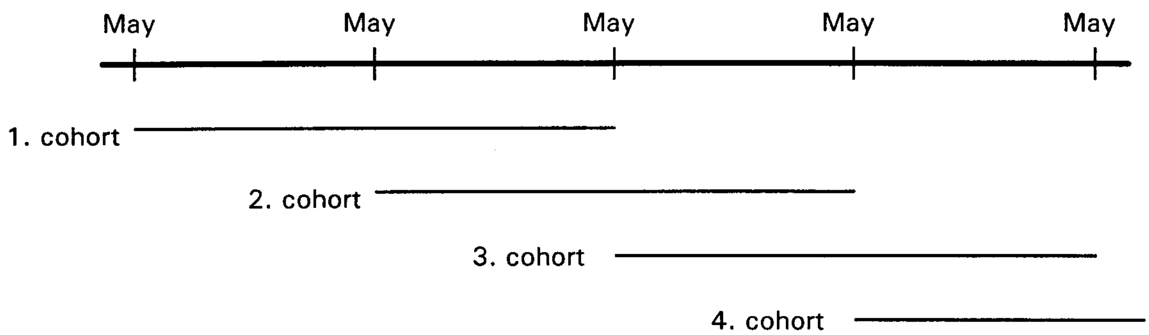

Figure 6.1. The production process in salmon farming

The size of each farm in terms of pen volume is regulated by the government (see section 6.3). Consequently, pen volume is a fixed factor in the production process. The salmon farmer has to take into consideration that pen volume is needed for a new cohort each spring, and he has to form expectations on the prices of salmon in the relevant period when making his harvesting profile decision. In some periods salmon prices have exhibited large short run fluctuations, and consequently the choice of harvesting profile has had significant effects on the profitability of salmon farms.

In terms of cost shares, the most important inputs are feed (about 40 % of total costs), smolts (15 %), capital (10-15%), and labour (10 %) (Fiskeridir., several years).

Both mortality and growth will affect the production of salmon. The stock of salmon in cohort c at the end of month t is defined by,

$$(1+g_{c,t})w_{c,t-1}n_{c,t} = (1-m_{c,t}-h_{c,t})(1+g_{c,t})w_{c,t-1}n_{c,t-1},$$

where $g_{c,t}$ is the growth rate in month t, $m_{c,t}$ is the mortality rate of cohort c in month t, $h_{c,t}$ is the harvest rate in month t (with the restriction $(1- m_{c,t} - h_{c,t}) \geq 0$), $n_{c,t-1}$ is the initial number of fish in cohort c in month t, $w_{c,t-1}$ is the initial average weight of the fish.[4] The production of salmon in cohort c is often defined as

$$y_{c,t} = (1+g_{c,t})w_{c,t-1}(n_{c,t}-n_{c,t-1}) + h_{c,t}(1+g_{c,t})w_{c,t-1}n_{c,t-1}.$$

Thus, production in a given period is defined as the change in total biomass from the beginning to the end of the period plus the harvested biomass in the period. It is interesting to note that, in general, the harvest may not be equal to the level of production. The above

[4] This definition of production implies that salmon in the same cohort is homogenous, i.e., the initial weight and growth rate is identical across all individuals for all periods.

definition also implies that production can be negative, for instance if the mortality rate is extremely high.

The level of smolts input is chosen conditional on information available before production begins. This means that smolt input is a predetermined variable relative to output. The use of other variable inputs, particularly feed, is not predetermined. The quantity of feed input is an important determinant of output. However, the actual quantity of feed used in the production process to some extent also depends on stochastic biophysical variables (or shocks), such as fish diseases and water temperatures. If the survival rate of the fish is lower than expected, e.g. due to unexpected disease losses, the feed consumption will also be lower than expected. If sea temperatures during the year are lower than expected, then feed consumption will also be smaller, since the appetite of the fish depends on sea temperatures (Austreng, Storebakken, & Åsgård, 1987). Thus, realised feed consumption is to some extent endogenously determined. Ideally, this should be accounted for in a model of salmon production. Conventional static EU models of the firm and empirical applications assume that *ex ante* optimal input quantities are not different from *ex post* realised input quantities, because actual input quantities are not affected by the production outcome. Thus inputs are predetermined variables in static EU models. It is a problem, however, that the data set which is available to us, is of a nature that precludes us to account for this. It has only annual observations, and lacks information on diseases and other events which affect feed consumption.

According to EU models presented in Chapter two, risk aversion can explain why a firm employs smaller quantities of inputs than would be optimal for a risk neutral firm. However, this may not be the only explanation. Due to the long time from smolt release to harvesting working capital requirements are substantial in salmon aquaculture. For "normal" production levels the annual operating expenses may range from 5 to 15 million NOK (Fiskeridir., several years). Norwegian salmon farmers may also be subject to credit restrictions due to imperfect capital markets.

6.2. The Regulation of the Norwegian Salmon Aquaculture Industry

The data period for the empirical analysis in this dissertation is from 1985 to 1993. Hence, the discussion of government regulations in this section will primarily be concerned with this period.

Until 1991 the Norwegian salmon farming industry was heavily regulated through the *Fish Farming License Act* (FFLA). The regulation reflected the government's desire for a small-scale, owner operator industry characterised by regional dispersion of production and profits along the entire coast.[5] A license is required to operate a salmon farm, and the government has used this instrument as a device to ensure regional dispersion of farms. The license also restricts the pen volume of the farm. In other words, one of the most important inputs in the production process is, through government regulations, a quasi-fixed factor. Prior to the introduction of the FFLA in 1973 there were no restrictions on pen volume. After 1973 the maximum allowed pen volume for new fish farm licenses changed on several occasions; in 1981 the maximum pen volume was set to 3,000 m^3 for new licenses, in 1983 5,000 m^3, in 1985 8,000 m^3, and in 1989 12,000 m^3. Existing farms with a larger pen volume than the prevailing limit were not forced to reduce their pen volume on these occasions. On the other hand, existing farms with a smaller licensed pen volume could apply for an enlargement of their pen volume. Recently, salmon farms have been allowed to apply for an enlargement of their licensed pen volume to 15,000 m^3.

However, the pen volume regulation did not provide an effective limit on the production of salmon. Firstly, the pen volume was only measured to a depth of five meters. Thus, farmers could acquire pens which were deeper than five meters to avoid the regulation (Møller Committee, 1990). Secondly, during most of the data period 1985-93 there was no limit on the density of fish in the pens. The 1991 revision of the FFLA lead to the introduction of a limit on the maximum production of fish per m^3 pen volume. According to the FFLA, salmon farms are not allowed to produce more than 25 kg fish per m^3 of pen volume, regardless of the biophysical conditions on the farm location. Together with the regulation of the pen volume, this also constitutes a regulation of the total production of the farm. However, during the data period 1985-93, the enforcement of the density regulation was so liberal that it, for all practical purposes, can be ignored.

Ownership was also regulated by the FFLA. Majority ownership interests in two or more farms were prohibited. Consequently, horizontal or vertical integration of firms was not possible. In 1991 the FFLA was somewhat liberalised with respect to ownership interests. This deregulation has lead to a restructuring of the industry into larger operations. It should be noted, however, that firm mergers in the industry have taken the form of physical mergers of farms only to a limited extent.

6.3. Arguments for the Presence of Risk Aversion in Salmon Farming

In this section we provide some arguments, based on available empirical evidence, why agents in the Norwegian salmon industry are risk averse.

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[5] See Salvanes (1989; 1993) and Bjørndal & Salvanes (1995) for a discussion of the various regulations of the fish farming industry, and their effects on efficiency etc.

Production and marketing of farmed salmon are associated with sunk costs in the form of investments in education and training of personnel, capital equipment, market research and advertisement. Alternative use and second-hand markets exist only to a limited extent for such firm- and industry-specific investments, and thus the firm will not be able to recover all investment costs in the event of failure. In the short run, after initial investments have been made, uncertainty also represents a problem for decision makers. Short run production decisions in salmon farms, e.g. smolt, labour and food input decisions and harvesting decisions, are made on the basis of expectations on future output levels and prices. *Ex post* these production decisions may turn out to be suboptimal, i.e., the salmon farmer would have chosen a different intertemporal production profile had he actually known the realised output level and monthly market prices. The consequence may be significantly lower profits than expected even if realised average prices during the period is equal to or higher than expected prices.

As noted earlier in this chapter, salmon farming requires large investments in physical capital. Due to the long period between the release of smolt and harvesting, the operating capital requirements are also substantial. Traditionally Norwegian salmon farmers have acquired a large percentage of the required capital through bank loans. To some extent salmon farmers have used personal assets as security for loans. The debt ratio is higher than for most industries, at least for our data period, and consequently the average salmon farm must pay large regular instalments (Fiskeridir., 1994).

Sunk costs, use of personal assets as security and a high debt ratio should indicate that the salmon farmer is risk averse. This implies for instance that he prefers a set of low future prices with certainty to a set of somewhat higher uncertain prices, provided that the set of low prices gives him a revenue which is sufficient to cover debt service, variable expenses and wages. In other words, uncertainty combined with risk aversion should give the salmon farmer incentives to hedge some of his future income. Furthermore, risk aversion also has implications for the optimal input combination. According to theory presented in Chapter 2, the risk averse salmon farmer should take into account the marginal effects of changes in input use on the level of risk. A risk averse salmon farmer should use more of a risk-decreasing input and less of a risk-increasing input than a risk neutral farmer.

6.4. Firm Heterogeneity in Norwegian Salmon Farming

Different outcomes with respect to profitability, unit costs and output level across firms in the Norwegian salmon aquaculture industry may have several causes. One explanation may be different outcomes of the stochastic variables facing salmon farm operations across firms. Another cause is different restrictions on pen volume across farms. Farms with a large licensed pen volume can to a greater extent exploit economies of scale than farms which are restricted at a relatively small volume. However, there may also be a third explanation for cross-farm differences. According to empirical studies, such as Berge & Blakstad (1989), Grongstad, Blakstad & Kartevoll (1990), Johannessen *et al.* (several years), and Salvanes & Tveterås (1992), there is substantial heterogeneity in the Norwegian salmon farming industry. Previous empirical work suggests that salmon farm operations are heterogeneous with respect to the

- quality of the management, which manifests itself in different management practises with respect to organisation of the work, design of incentive structure for workers, design of quality control systems, etc.,
- quality of the workers,
- technology, e.g. quality of feed, feeding equipment and routines, sea pen system,
- range of operations, e.g. own feed production, own slaughter facilities,
- quality of the location, i.e., the constraints on maximum fish density in the pens and maximum total production defined by the biophysical conditions of the location.

The above sources of heterogeneity may give rise to *persistent* differences in productivity. Stochastic variables in the production process, on the other hand, can only give rise to *transitory* differences. If substantial heterogeneity is present, this heterogeneity may be a more important determinant of cross-firm differences in productivity than different outcomes of stochastic variables in the production process.

In principle, the various sources of cross-firm heterogeneity can be treated as inputs and included in the model of the firm. If these variables are measured "correctly", then they should to a large extent explain cross-firm differences in profitability in a regression model of salmon farm production. In practice, however, it is difficult to measure variables such as the quality of management, the quality of workers or the quality of the production equipment. Furthermore, as is the case for our empirical application, measures of or proxies of the various sources of heterogeneity are often not available to the researcher, or only at a very high cost. Consequently, heterogeneity may give rise to errors-in-variables or omitted-variables biases in econometric parameter estimates. As stated in Chapter one, the working hypothesis in this dissertation - without making any a priori statement on the relative importance - is that both firm heterogeneity and random variables in the production process explain cross-firm productivity differences. The coefficients of the estimated econometric models will give us some indication on what is the most important source of productivity differences.

6.5. Summary

Our empirical analysis will only be concerned with the production of farmed fish, i.e., the last step in the biological production process in salmon production.

Salmon farming requires substantial investments in production facilities prior to the first release of smolt. Initial investments will vary greatly, depending on existing facilities on land

and the scale of operation. Typically, investments in capital equipment are in the range of 4 to 10 millions NOK. Due to the long growth period, substantial working capital is also required. A cohort of salmon is kept in the pens and fed for a period of one to two years before harvesting.

In terms of cost shares, the most important inputs are feed (about 40 % of total costs), smolts (15 %), capital (10-15%), and labour (10 %) (Fiskeridir., several years).

Smolts are inputs which are chosen conditional on information available before production begins. This means that the smolt input is a predetermined variable relative to output. The use fish feed is not entirely predetermined, since biophysical variables such as fish diseases and sea temperatures influence feed consumption. The actual quantity of feed used in the production process to some extent depends on the production outcome.

Production in a given period is usually defined as the change in total biomass from the beginning to the end of the period plus the harvested biomass in the period. In the general case the harvest will not be equal to the level of production. Production can be negative, for instance if the mortality rate is extremely high.

The size of each farm in terms of pen volume is regulated by the government, and pen volume is consequently a quasi-fixed factor in the production process.

Sunk costs, use of personal assets as security and a high debt ratio should indicate that salmon farmers in general are risk averse. This imply that a salmon farmer would like to hedge some of his future income, and that he should take into account the marginal effects of changes in input use on the level of risk.

Previous empirical work suggest that salmon farm operations are heterogeneous with respect to biophysical conditions at the farm location, and the quality of the management, workers, and technology. These sources of heterogeneity may give rise to *persistent* differences in productivity. Stochastic variables in the production process, on the other hand, can only give rise to *transitory* differences. If substantial heterogeneity is present, then this heterogeneity may be a more important determinant of cross-firm differences in productivity than different outcomes of stochastic variables in the production process.

7. THE NATURE OF RISK AND RESPONSES TO RISK IN SALMON AQUACULTURE

From the beginning of the industry, economic risk has been a prominent feature of salmon aquaculture. For example, this risk has for many farms manifested itself in terms of substantial losses due to diseases, a large number of bankruptcies in some periods, and substantial cross-firm variations in profitability. That the economic risk of salmon farming has been of great concern both to the industry itself and to policy makers, has been reflected in the a number of articles in newspapers and trade publications on this issue. The organisation which represents the majority of Norwegian salmon farms, the *Norwegian Fish Farmers' Association* (NFF), has also stated several times that a reduction of the economic risk in fish farming is a high priority task, see e.g. NFF (1990, p. 10).

Table 7.1 serves to illustrate that salmon farming is a risky business. According to table 7.1 the mean profitability exhibited substantial year-to-year fluctuations during the data period 1985-93. Mean profits were positive from 1985 to 1988, and also in 1993, but were negative from 1989 to 1992. The negative profits were accompanied by a negative mean equity from 1989 to 1991, a period which was characterized by a large number of bankruptcies in the industry. The development in profits over time can to a large extent be explained by the development in salmon prices and unit production costs in salmon farming. However, according to the standard deviations (in parenthesis), there was also substantial cross-farm variations in profitability each year. During the same period some farms had large positive profits while others experienced large negative profits.

Table 7.1. Mean and St.Deviation of Profits (before Taxation and Extraordinary Items) and Equity in Norwegian Salmon Farming 1985-93 in Real 1000 NOK (1993=100)*

Year	1985	1986	1987	1988	1989	1990	1991	1992	1993
Profit	376.9	189.9	690.8	788.0	-615.4	-314.1	-1051.2	-10.0	645.5
	(790.6)	(1982.9)	(1990.1)	(2134.6)	(2125.2)	(2219.4)	(2052.8)	(2765.1)	(2331.5)
Equity	2426.8	659.7	1381.5	505.5	-893.1	-1186.8	-775.0	2282.1	2559.1
	(3427.0)	(3835.0)	(4309.2)	(4727.1)	(4826.7)	(5118.0)	(5038.7)	(6131.0)	(5491.7)

*Source: Norwegian Directorate of Fisheries (Fiskeridir., several years).

The question is then what factors explain the substantial cross-sectional differences in profitability. Is it differences in obtained output prices, scale economies, stochastic shocks, firm heterogeneity in terms of the quality of the farm location or the quality of management, or other factors? This dissertation will try to identify the importance of some of these factors for the economic performance of salmon farms, by analyzing the structure of the stochastic production technology in salmon farming.

It is useful to have some information on the structure of risk when we specify a primal model of salmon production. The analysis of the structure of risk in this chapter will be the basis for the specification of risk in our econometric models. First, however, we present some other taxonomies of risk which are useful in order to understand the nature of uncertainty in salmon farming (section 7.1). Section 7.2 discusses the structure of production risk. In section 7.3 the use and characteristics of insurance in salmon farming is discussed. Section 7.4 deals with the effect of innovations and learning-by-doing on the level of production risk in salmon farming. Finally, the discussion in this chapter is summarised in section 7.5.

7.1. A Taxonomy of Risk in Salmon Farming

For analytical purposes it can be useful to partition the risk facing the individual Norwegian salmon farmer in three categories: (1) *Biophysical* risk, (2) *market* risk, and (3) *political* risk.

Biophysical risk encompasses uncertainty regarding sea temperature, the extent of poisonous algae concentrations and fish diseases on the farm location. The outcome of these stochastic variables will affect the growth rate, mortality and quality of the farmed salmon, and thus production volume, average production costs and prices obtained.

Market risk, for lack of a better term, pertains to the performance of different producer groups in the salmon markets and related markets, i.e., suppliers of inputs to the production process and suppliers of substitutes to Norwegian farmed salmon. On the input side, the supply and price of smolts have traditionally exhibited large variations. With regard to substitutes, the supply of wild-caught Pacific salmon has been the most important source of uncertainty.

Political risk encompasses the uncertainty regarding major political events which may affect the GDP, exchange rates or food demand patterns of importing countries (e.g. the Gulf war), trade policy decisions which affect access to markets (e.g. antidumping measures such as tariffs, minimum prices and veterinary restrictions) or actions taken by private groups in importing countries (e.g. boycott of Norwegian salmon by supermarket chains in Germany and elsewhere due to Norwegian whaling, and blockades by French fishermen against truckloads of Norwegian salmon).

Table 7.2 summarises and classifies the most important sources of uncertainty facing Norwegian salmon farmers. The sources of *biophysical* risk mainly give rise to production uncertainty at the firm level, while the sources of *market* risk and *political* risk primarily give rise to output price uncertainty.

Table 7.2. Important sources of uncertainty facing Norwegian salmon farmers

Type of uncertainty	Source of uncertainty	Biophysical (B), market (M), or political (P) risk	Endogenous (E) or exogenous (X) risk at firm level
Production uncertainty	Quality of smolt	B, M	E, X
	Sea water temperature	B	X
	Losses due to fish diseases and poisonous algaes	B	E, X
	Losses due to extreme weather	B	E, X
Output price uncertainty	Supply of wild-caught Pacific salmon	M	X
	Supply of farmed salmon by other countries	M	X
	Antidumping measures by import countries	P	X
	Actions by private groups	P	X

Most of the uncertainty facing salmon farmers can be regarded as *exogenous,* but according to table 7.2 there are also elements of *endogenous* uncertainty. If the outcome of a stochastic variable is not affected by the behaviour of individual agents or the organisation of the market, we have exogenous uncertainty. The sea temperature is an example of this. Endogenous uncertainty is present if outcomes to some extent are affected by the behaviour of individuals or the organisation of the market. Disease losses and smolt quality are examples of random variables which are partly endogenous and partly exogenous. At the firm level only the biophysical sources of risk are partly endogenous, while the sources of market and political risk are purely exogenous.

Relocation of the farm is one means available to the salmon farmer for changing the biophysical risk. By resettling to a sea location with higher recipient capacity, better water exchange, and smaller frequencies of algae growth, the level of biophysical risk can be reduced. However, relocation is only possible if there are available locations within reasonable distance which are not already allocated to other types of use. In addition, the farmer must obtain a permit from the authorities.

This leads to the distinction between *local* and *global* (or micro and macro level) uncertainty in salmon aquaculture. Local uncertainty pertains to random events whose outcomes affect one or a few producers only (e.g. concentration of poisonous algaes), while global uncertainty pertains to random events which to some extent affect all producers (e.g. anti-dumping measures by the EU).

It may also be useful to draw a distinction between *permanent* and *transitory* risk. Salmon farming is a young industry, and to a certain extent the observed volatility of salmon markets can be regarded as an infant industry problem, and hence of a transitory character. There is reason to believe that salmon farmers and exporters through a learning process have acquired a better understanding of important aspects of salmon production and marketing, such as the functioning of the biological production system, the mechanisms of salmon markets and the political environment. In addition to reducing their unit costs, a result of this learning process has been that some elements of uncertainty have been eliminated, i.e., that the probability distribution of output prices and output has been condensed. However, both the supply and demand side have inherent structural characteristics which imply that uncertainty will be present and an important element of suppliers' decisions even after the industry has reached a more mature stage. Regardless how much firms invest in education and training of staff, forecasting models and information collection, they will not be able to eliminate risk completely, i.e., not be able to make "correct" predictions of future prices and output. This uncertainty represents costs both for the individual firm, the sources of financial capital for the industry (e.g. banks), and for society at large.

7.2. Output Risk

There is plenty of empirical evidence that salmon production is more volatile than many other types of biological production, such as livestock production (for example, cattle and chicken meat production). For a given vector of inputs $\mathbf{x}$, output y may vary dramatically. Thus, the general stochastic specification $y=f(\mathbf{x}, \varepsilon)$, where ε is an "error term", is highly appropriate for salmon farming. However, several issues remain to be resolved: What are the signs of the marginal risks associated with inputs such as smolts, feed, labour and capital equipment? How significant are the various marginal risks? What probability distribution provides the best description of the "error term" ε in salmon farming; the normal, the lognormal or another distribution function? What specific stochastic specification of the production function is most appropriate for salmon farming?

In section 2.1 we provided the functional relationship between the mortality and growth rate and the production of salmon. Furthermore, we will shortly discuss the relationship between the quality of salmon and the per-kilo-price obtained for the salmon. When a cohort of smolts is released into the pens, the following factors will determine the mortality and growth rates of the cohort and the quality of the fish:

- The genetic quality of the smolts. Several types of salmon, which originate from different wild stocks of salmon, are used in Norwegian salmon farming. Heterogeneity is present both across stocks and within stocks with respect to robustness and growth potential. Due to genetic improvements, farmed salmon generally grows at a significantly faster rate today than some years ago, and the growth rate is still improving.
- The *bioproductivity* of the marine environment at the farm location, which is determined by *biophysical* variables such as sea temperature, oxygen concentration, salinity, sea currents, topography and concentration of disease bacterias/ viruses/ poisonous algaes. In section 7.2.1 we provide a closer discussion of biophysical factors and their contribution to risk.
- The stocking density of the salmon. Biophysical variables determine the maximum stocking density in the pens. However, this maximum stocking density exhibits variations through the year, and is very difficult to estimate; it may only be learned approximately through a combination of experience and use of the established knowledge made available by the scientific community. If the salmon farmer does not know the correct maximum stocking density, he may under-utilise the capital equipment through low densities, or risk high mortality and reduced growth rates through too high densities.
- Feeding routines and feed quality. Norwegian salmon farmers have determined that the amount of feed and feeding frequencies have a significant effect on salmon growth. Due to the difficulties of finding the appropriate feeding regime, feed quantities, feed qualities and feeding frequencies have been subject to a lot of experimentation in salmon farms. These issues have also been the focus of a considerable body of research. As the quality of diets and the understanding of the salmon's feed requirements have improved, more efficient feeding regimes have been implemented. According to feed input figures, there is substantially less feed waste in Norwegian salmon farming today than five years ago.
- Monitoring and control routines. Monitoring of the fish is important in order to discover diseases early, and thus limit disease losses. In addition, pens, nets and anchoring equipment should be subjected to routine inspections in order to avoid capital equipment and fish losses under extreme weather conditions.

7.2.1. Biophysical Determinants of Salmon Production and Quality

The growth rate, the mortality rate and the quality of salmon are all very sensitive to changes in the marine environment. The most important biophysical determinants of mortality rate, growth rate, and quality are:[6]

- **Oxygen concentration**. With high fish densities in salmon farming, the oxygen requirement of the fish biomass in the pens is substantial. The oxygen concentration in the sea must be above a certain threshold level in order to avoid mortality and to obtain maximum potential growth rates. The oxygen concentration is influenced by several factors:
  - The oxygen concentration decreases with water salinity.
  - The oxygen concentration decreases with increases in water temperature.
  - The oxygen concentration decreases with air pressure.
- **Sea temperature**. The growth rate of the salmon increases with the sea temperature up to a certain level (around 18-20 °C), provided that the values of other biophysical variables are appropriate. However, since the oxygen concentration in the sea decreases with an increase in the sea temperature, high sea temperatures may often be associated with higher mortality.
- **Salinity**. It is important that the sea salinity does not exhibit dramatic variations in order to avoid excessive mortality and obtain maximum growth rates. Salinity variations may be a problem in fjords that receive large amounts of fresh water from the inland.
- **Sea currents**. The high fish densities in salmon farming leads to high water exchange requirements on the farm location. The water exchange on farm locations is driven by the large coastal currents, variations in the sea water level due to tidal water conditions, and meteorological conditions (i.e., wind and air pressure). Some farm locations may experience large fluctuations in the water exchange and periods of insufficient water supply, particularly in the summer months when water exchange requirements are high due to the high oxygen requirements of the salmon.
- **Concentration of disease bacterias/viruses and poisonous algae in the local marine environment**. The risk of disease losses increase with higher concentrations of bacterias or viruses in the marine environment. Although one does not have full knowledge about the spread of fish diseases, it is generally recognised that certain diseases can be carried long distances by currents, wild fish, sea birds, etc. The experiences with fish diseases since the 1980s have also lead to the conclusion that farms which are located close to each other are more susceptible to contagion.

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[6] Tveterås (1993) provides a more thorough discussion of the biophysical determinants of salmon production, and analyses the economic effects of changes in important biophysical variables.

The above biophysical variables determines the *bioproductivity* of the marine environment at the farm location. During the data period 1985-93 the sea temperature exhibited relatively large year-to-year variations. Oxygen concentrations at farm locations have exhibited substantial short-run fluctuations. A large number of salmon farms have also suffered from disease outbreaks or extreme algae concentrations. The volatility of the above listed biophysical factors have thus led to variations in salmon growth and mortality rates from year to year at farm sites.

7.2.2. The Marginal Risks of Important Inputs in Salmon Farming

According to the theory of the competitive firm under production uncertainty, which was outlined in Chapter 2, the risk averse firm is not only concerned with the marginal productivity of inputs when deciding the input combination, but also the marginal risk of inputs. Based on *a priori* knowledge of the production process in salmon aquaculture, the following can be said about the marginal risk of important inputs in salmon farming:

- **Feed**: The salmon is not able to digest all the feed. A fraction of the feed, depending on the quantity and quality of the diets being used, will be released into the environment as feed waste and faeces. The feed requirement of the salmon in a certain period depends on its growth potential during the period, which is again mainly determined by temperature and light conditions, provided sufficient oxygen is available in the water. For a modern salmon feed, a ratio between the quantity of feed supplied and the fish growth (the *feed conversion ratio*) of approximately one is sufficient in order to obtain maximum growth. However, the feed conversion ratio has traditionally been well above one in Norwegian salmon farms.[7] An explanation for the high feed-growth ratios may be the difficulty of estimating the potential growth rate of the fish, assessing the appetite of salmon and the amount of feed that sinks through the cages. When the feed-growth ratio is well above one, some of the feed is not eaten at all, but sinks to the sea bed below the cages. Organic material, such as fish feed and faeces, lead to the consumption of large amounts of oxygen, and hence compete with the salmon for the oxygen in the marine environment. The bacterial decomposition of organic material also leads to the production of ammonia, which is toxic (and lethal) in very small concentrations. If substantial amounts of organic material are deposited at the sea bed under the cages, hydrogen sulphide - an extremely toxic gas - can be produced in the sediment and emitted into the marine environment. Due to the increased consumption of oxygen and production of ammonia and hydrogen

[7] It has been calculated, on the basis of the total amount of commercial salmon feed sold to the industry, and the salmon production figures, that until the beginning of the 1990s, Norwegian salmon farmers had been feeding their fish roughly twice the amount of feed necessary (Wallace, 1993, pp. 139).

sulphide, a marginal increase in the input of feed will increase the level of risk, i.e., the variance of output.

- **Smolts**: The more fish (smolts) that are put into the pens, the bigger is the total oxygen requirement of the fish, the production of oxygen-consuming organic by-products (faeces and feed waste), and the production of toxic by-products such as carbon dioxide and ammonia. Consequently, a marginal increase in the release of smolts will increase the variance of output.
- **Labour**: An increase in labour input, *ceteris paribus*, will increase the ability to monitor the physical condition of the fish, the state of equipment such as pens, nets, feeding equipment, anchoring equipment etc., and important biophysical variables such as temperature, oxygen concentration and alga concentrations. Furthermore, the ability to repair equipment and maintain a high hygienic standard increases with labour input. Consequently, a marginal increase in the input of labour will decrease the variance of output.
- **Pen volume**: An increase in the pen volume, *ceteris paribus*, will lead to lower fish densities. Hence more oxygen will be available per fish and there will be lower concentrations of ammonia and carbon dioxide per m^3 of water. A marginal increase in the input of pen volume will therefore lead to a decrease in the variance of output.
- **Other types of capital equipment**: In general investments in capital equipment such as feeding devices, monitoring equipment, anchoring facilities etc. will decrease the riskiness of production.

Since the theory predicts that marginal risks are important for input decisions, and empirical evidence suggest that both negative and positive marginal risks are present in salmon farming, this should be accounted for in an econometric model of salmon production. The econometric specification should impose no *a priori* restrictions on the marginal risk, so that both negative, zero and positive marginal risks should be possible.

7.2.3. The Effect on Risk of Increasing the Scale of Operation at a Given Farm Site

The exchange of water, sea temperatures and the topography to a large extent determine the availability of oxygen to the fish in the pens. As the production of salmon increases, more oxygen will be consumed by the fish and organic by-products such as faeces and redundant feed. In periods of high temperatures (e.g. 15-20°C) the oxygen concentration in the in-flowing water decreases. However, at the same time the oxygen requirement of fish increases due to increased growth, which again leads to increased production of organic waste. In these circumstances there may be periods when oxygen supply barely equals or is less than the oxygen demand of the salmon. The effect of a short-term oxygen deficit is usually decreased growth and increased mortality among the fish.

Due to limitations in the capacity of a farm site, in terms of the amount of organic by-products it is able to assimilate and the amount of oxygen available to the fish, an increase in the scale of the operation beyond a certain level will increase the level of risk because of adverse effects on growth and mortality rates.

7.2.4. The Probability Density Function of Output

According to Chapter two a few theoretical models of the competitive firm under production uncertainty has emphasised the potential importance of third and fourth order moments of the conditional output probability density function (pdf) for the input behaviour of the firm. Based on the empirical evidence available so far, it is difficult to say whether the pdf in salmon farming is symmetric (e.g. normal) or asymmetric (e.g. beta or Weibull). Several pdf's could be tested in an econometric model, as demonstrated in Chapter 3, due to the limited knowledge we have so far on the distributional properties of salmon production.

7.2.5. Time Series and Cross-Sectional Properties of Output Risk

Previous studies have noted that the time series and cross-sectional properties of the error term $\varepsilon_{i,t}$ of the stochastic production function $y_{i,t}=f(\mathbf{x}_{i,t}, \varepsilon_{i,t})$, have consequences for the econometric model specification. Spatial autocorrelation is present if $\mathrm{cov}(\varepsilon_{i,t}, \varepsilon_{j,t}) \neq 0$ for two farms i and j in period t. One source of spatial autocorrelation are sea temperatures, which is an important determinant of salmon growth. Sea temperatures are strongly correlated along the coast, because they are to a large extent determined by the large currents that flow along the coast. Negative production shocks due to bad weather conditions or diseases are also spatially correlated, at least within regions. Sea temperatures, bad weather and diseases give rise to positive correlation of production shocks across farms.

Time series autocorrelation is present if $\mathrm{cov}(\varepsilon_{i,s}, \varepsilon_{i,t}) \neq 0$ for two time periods s and t for a given farm i. To the extent that time series correlation of production shocks are present in salmon farming, it will most likely be positive. For example, disease outbreaks often tend to come in waves.

Based on our knowledge of the characteristics of the production process, it can be argued that cross-sectional autocorrelation dominates time-series autocorrelation in salmon farming. It is an empirical task to assess the magnitude of production shock correlation along these two dimensions, and consequently how they should be handled in an econometric model framework.

7.3. Insurance in Norwegian Salmon Farming

Some theories of the competitive firm under production risk have suggested that the introduction of insurance generally will alter the optimal input choices of the firm. Since there exists a private insurance market for certain types of production risk in Norwegian salmon farming, it is an important empirical question to what extent this is the case for this industry.

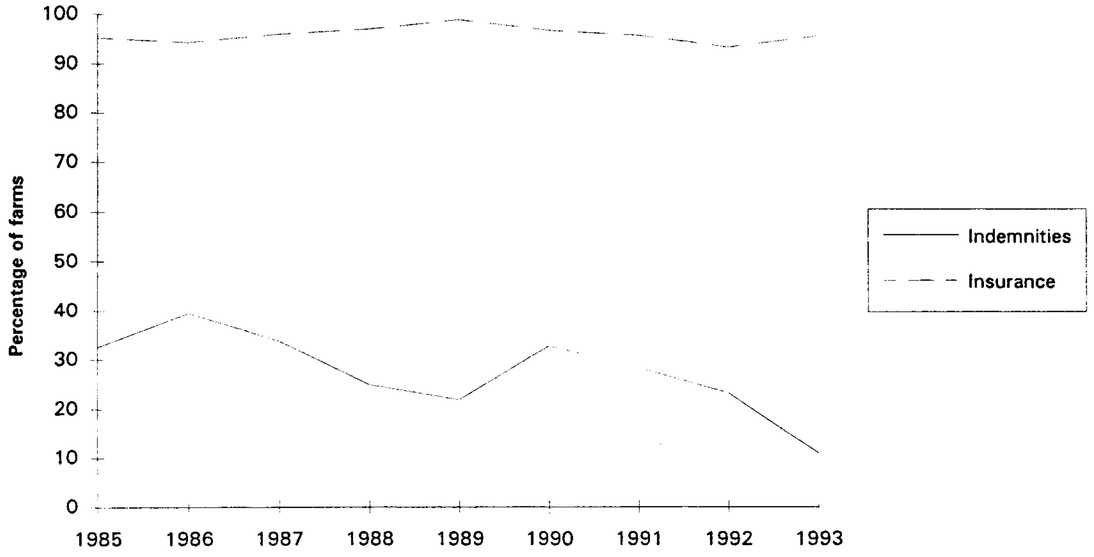

Figure 7.1. Percentage of farms in the profitability survey of Norwegian salmon farming that were insured and received indemnities 1985-93 (Source: The Norwegian Directorate of Fisheries)

As indicated by figure 7.1 most fish farmers are insured against losses due to disease outbreaks or technical accidents caused by extreme weather conditions. In the period 1985-93 between 93 and 99 % of the farms paid insurance premium of some kind each year. To some extent this insurance reduces the economic risk for the farmer. However, the insurance does not provide full coverage; there are several types of risk that the fish farmer cannot insure himself against. The insurance contracts usually cover losses due to diseases, toxic algaes, extreme weather conditions and accidents. Adverse production outcomes caused by lower than expected temperatures are not covered by the insurance. Furthermore, the losses associated with disease outbreaks must be of a certain magnitude in order to obtain indemnities from the insurance company. The fish farmer is generally not fully compensated for losses, because the insurance contract usually states that the fish farmer has to cover some of the losses himself, and because of the insurance companies' valuation principles for the fish lost. Insurance contracts generally state that losses above 20 % of the value of the biomass is to be indemnified. After 1990 losses on fish that have attained a weight of 2 kg or more have generally not been covered by the insurance. For damages on production equipment there is a deductible of 5,000 NOK.

A common procedure for calculating the insurance premium for the biomass in the pens is to multiply the average monthly biomass over the year with an insurance rate that reflects the insurance company's assessment of the risk of the farm. The insurance companies to a large extent use subjective criteria in determining the risk. The subjective assessment includes interviews with the farmer, interviews with other people in the area (e.g. local representative, veterinary), evaluation of the farm site and production facilities, and analysis of account books.

To some extent the insurance companies use the history of the salmon farm when determining the insurance rate for the biomass each year. Insurance companies increase the insurance rate if the farmer tends to be more risky than initially perceived. However, the law prohibits insurance companies from sharing information on insurers. This means that a farmer that experiences an increase in the insurance rate due to an "unfavourable" history, can switch to another insurance company which does not have the information that the original insurer possesses, and pay a lower insurance premium. According to insurance companies this frequently happens.[8]

Insurance contracts also have clauses regarding the fish farmer's responsibility with respect to hygienic precautions, anchoring and maintenance of equipment, and actions to be taken in the event of disease outbreak or bad weather. If the insurance company can prove that the fish farmer did not take the necessary precautions, this will lead to a reduction in indemnities. The percentage of farms that received indemnities of some kind varied from a minimum of 11 % in 1992 to 40 % in 1986 (see figure 7.1).

In general the farmer receives indemnities less than a month after an adverse event covered by the insurance contract has terminated (e.g. a disease outbreak). An insurance company has to pay 12 % interest on the amount of indemnity after one month, so it will have strong incentives to process indemnity cases rapidly.

Figure 7.2 depicts the average insurance costs and indemnities as a percentage of harvest revenues for farms in the annual profitability survey of Norwegian salmon farming. The figure indicates a downward trend in both indemnities and insurance costs relative to harvest revenues from 1986. It should also be noted that because of deductibles and liability clauses in insurance contracts, the indemnities in the below figure probably underestimate the value of the losses caused by diseases, toxic algaes, extreme weather conditions, and accidents.

In the 1980s salmon farming insurance generally was an unprofitable business area for insurance companies. For the average farm in the profitability survey of Norwegian salmon

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[8] Source: Svein Arne Aas, representative of the insurance company UNI Storebrand, personal communication.

farming the cumulative indemnities was larger than the cumulative insurance premium until 1988. In subsequent years, however, the cumulative insurance premium has always been greater than the cumulative indemnities for the average farm. In other words, the insurance companies have in recent years regained their losses from the mid-eighties.

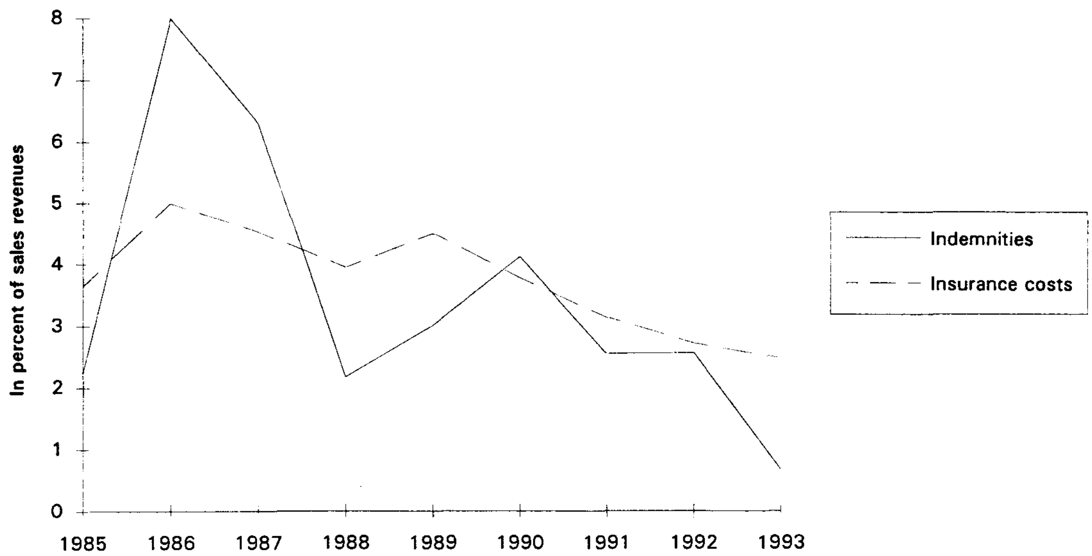

Figure 7.2. Average insurance costs and indemnities in percent of harvest revenues for the farms in the annual profitability survey of Norwegian salmon farming 1985-93 (Source: The Norwegian Directorate of Fisheries)

7.4. Effects of Innovations and Learning-by-Doing on Production Risk

Compared with other sectors of biological production, salmon aquaculture is a young industry. In Norway small-scale salmon farming started as late as the early 1970s. Sea-based rainbow trout farming operations has been active since the early 1960s. To some extent the production risk in salmon farming may be explained by the fact that it is an infant industry. It is infant both in the sense that the industry's product is relatively new, and in the sense that the production technology is new.

The production technology has been subject to frequent innovations during the last 20 years. Innovations have been introduced in several areas: The genetic quality of the salmon, the quality of the fish feed, disease treatment and vaccines, feeding equipment, robustness of the sea-pen system, monitoring of the fish, and hygienic routines. During the 1980s the aquaculture industry received a very large share of government R&D funds. Large research programmes, at least by Norwegian standards, were carried out to improve the understanding of the salmon's biological requirements and the effects of intensive aquaculture on the marine environment, improve the genetic quality of the salmon, develop medication and vaccines, improve the quality of the feed, etc. The generous government funding can to some extent be explained by the fact that aquaculture became a part of the political authorities' regional policy to support fringe areas. The development of aquaculture was regarded as an important element in the struggle to maintain the present pattern of settlements along the coast. However, to some extent the large funding was also a response to the particular problems of this infant industry. Many farms experienced high mortality rates, and during the 80s new diseases were introduced that lead to dramatic losses for some farms. In addition, the marine environment was subjected to substantial emissions of organic material and antibiotics from fish farms in many coastal areas. As manifested in a large number of articles in newspapers and trade publications, these problems were of great concern both to the industry itself and to policy makers. Furthermore, it became clear that they could not be effectively combatted without research on the biology of salmon, and the effects of intensive aquaculture on the marine environment. Undoubtedly, the extensive biological research has substantially contributed to an improved understanding of the biological production system and the requirements of salmon in the fish farmer population. Parallel with the developments on the research frontier, the salmon farmers accumulated valuable production experience. It can be asserted that innovations and learning-by-doing has not only contributed to increased productivity of the average farm, but also to a reduction in production risk at the farm level. Given the general concern among fish farmers with respect to production risk - most farms have experienced at least one incident of substantial losses due to disease or environmental problems - it is reasonable to assume that an important motive at the firm level for adapting new technologies has been to reduce production risk.

The development of a public infrastructure (e.g. public veterinary service) for the industry and the introduction of regulations aimed at promoting diffusion of innovations and reducing the spread of diseases, also played an important part in increasing mean productivity and reducing production risk. Courses in aquaculture were introduced both at the high school and university level during the 1980s. An extensive semi-public veterinary field service was built up. Evaluation of a large number of farm sites was undertaken by marine biologists in order to determine the recipient capacity of the marine environment. Farms were often moved to better locations after such evaluations. Publicly sponsored information campaigns aimed at fish farmers were undertaken. Programmes for training of fish farmers in co-operation with industry organisations were also carried out. Furthermore, regulations with respect to the location of farms, the transportation of smolts, the treatment of dead fish and slaughter waste, and the reporting of disease outbreaks, came into effect.

It is of great interest to measure the effect of adoption of new technology and learning-by-doing on production risk in Norwegian salmon farming. Later we will see that the effect of technical change on production risk is an issue which to a large extent has been ignored in the empirical (and to some extent in the theoretical) literature of the firm under production uncertainty.

7.5. Summary

Since the beginning of the industry's history, economic risk has always been a prominent feature of the Norwegian salmon farming industry. There are several sources of risk in the salmon farming industry. Biophysical factors lead to production uncertainty at the farm level. Actions taken by other economic agents in the salmon market contribute to salmon price uncertainty. Actions by political agents also contribute to the price uncertainty in the industry.

Since both output and price risk is present in salmon farming, specification and estimation of dual models of salmon production are complicated, according to chapters 2 and 3.

Empirical evidence suggests that the marginal risks of the feed and smolt inputs are positive. In other words, feed and smolts increase the variance of the conditional output distribution. The marginal risks of labour, pen volume and other capital equipment inputs are negative. Empirical testing is required in order to verify these *a priori* conjectures on marginal risks.

Empirical evidence also suggests that an increase in the scale of operation at a given site increases output risk. There is no empirical evidence available that can provide us with any information on the pdf of output in salmon farming; whether it is symmetric or skewed, and to what side it is skewed.

Besides adjusting input and output levels, salmon farms have other instruments at its disposition to control total risk, for example insurance. There exists a private insurance market for certain sources of production risk, e.g. diseases, toxic algaes and extreme weather conditions. However, insurance cannot eliminate the economic risk. Consequently, the farmer's input and output choices are still important instruments for controlling the level of risk.

8. THE NORWEGIAN SALMON FARM DATA SET: DATA AND VARIABLE SELECTION ISSUES

In this chapter the features of the Norwegian salmon farm data set, a panel with firm-level observations for the period 1985-93, are discussed. The data set is compiled by the Directorate of Fisheries, which each year collects production, cost and revenue data in an extensive survey of 200-300 farms. In most years the farms included in the data set produced more than fifty percent of the total Norwegian salmon output.

Although the data set provides a very extensive description of the individual farms, there are several problems associated with it. Depending on the chosen model specification, some relevant variables are not available and some variables are probably measured with errors. Consequently, both *omitted-variables* bias and an *errors-in-variables* bias may be present for the parameters of a model estimated on this data set. To understand some of the features of the data set, a description of the data collection process is provided in section 8.1.

The Norwegian salmon farm data set allows the construction of an unbalanced panel data set.[9] A panel data set will allow me to include firm-specific (fixed or random) effects, use lagged firm level variables, detect errors in variables, and to utilise recent techniques for dealing with measurement errors. However, there are also problems associated with the construction of the panel data set, because the data set provides limited information on the owners and location of the farm. Section 8.2 discusses the construction of a panel data set and potential pitfalls. Next, a discussion of the construction of input and output quantities is provided in section 8.3.

8.1. The Norwegian Salmon Farm Data Set

Since 1982 the Norwegian *Directorate of Fisheries* has annually compiled data sets of salmon farm production data for their profitability survey of so-called "independent" Norwegian fish farms (Fiskeridir., several years). Firm level data for the years 1985-93 have been made available for this study.[10] All firms with an aquaculture license receive two detailed questionnaires from the Directorate of Fisheries, which they by law are obliged to complete and return together with their annual accounts.[11] The 1985-1993 samples each year encompass

[9] The panel data set is balanced if all units are observed in all time periods the data set spans; otherwise it is unbalanced.

[10] The 1982 and 1983 data sets have also been acquired, but they lack the farm identification number used by the Directorate of Fisheries in subsequent data sets. The 1982-83 data sets are also somewhat different with respect to cost categories.

[11] Only firms with an *aquaculture license* are allowed to run their own salmon farming operation. This license also specifies the sea location for the farm and the maximum "size" of the farm in terms of m^3 pen volume.

200-293 farms out of a total population of 500-900 producing units. Roughly 80 variables are reported for each farm. Some of the variables included in the 1985-93 data sets are firm identification code, region, costs (feed, labour, smolts, interest on debt, insurance etc.), revenues (harvest, compensations for disease and damages etc.), assets and liabilities, production (in kg), harvest (in kg), stock of fish in pens at the beginning and end of year (in kg), hours of labour, disease dummy, damages dummy, smolts supply problem dummy, licensed pen volume, utilised pen volume etc. A complete list of the variables included in the data set is provided in appendix 8.A.

The returned questionnaires and annual accounts are subjected to a quality assessment process by the Directorate of Fisheries. For instance, for farms that were included in the data set in the previous year, the consistency between the stock figures for December 31st in year *t*-1 and the stock figures for January 1st in year *t* is supposed to be verified. Only farms that have been in production the two preceding years, were in full operation the entire year, and have returned questionnaires and annual accounts of sufficient quality are included in the final data set.[12]

Table 8.1. Share of Farms with Revenues from Other Activities and Average Ratio of Other Revenues to Fish Harvest Revenues

Year	1985	1986	1987	1988	1989	1990	1991	1992	1993
% of farms with other activities	7.17	5.38	9.17	9.61	3.41	9.71	7.88	3.47	14.65
Other rev./Harvest rev. in %									
Only farms with other activities	1.30	5.37	15.65	13.34	18.30	9.24	3.65	14.42	13.51
All farms	0.09	0.29	1.44	1.28	0.62	0.90	0.29	0.50	1.98

The farms included in the sample are "independent" in a legal sense; only farms that deliver separate annual accounts are included in the data set. If the revenue share of the firm's non-aquaculture related business activities, e.g. traditional fisheries, agricultural production, fish processing plant, exceed 10 %, the farm is excluded from the sample. Consequently, mainly "pure" aquaculture operations are included. However, the revenue share of aquaculture-related activities other than the production of farmed fish, such as sale of roe, feed, smolts, packaging of fish etc., are allowed to exceed 10 %. For a few farms in the sample the revenue share of other aquaculture-related activities are well above 10%. If there are not strong reasons to

[12] Information on the data collection and analysis procedures, quality assessment process, etc. was obtained in interviews with consultant Merethe Fauske at the Directorate of Fisheries, who has been responsible for these tasks during the entire data period. The questionnares and guidelines for the questionnaires were also made available.

believe that the use of resources associated with these revenues are insignificant, these farms may be candidates for exclusion from the estimating sample.

The data set includes the dummy variable "Other Activities", which indicates whether the firm is engaged in other business activities. The variable "Other Ordinary Revenues" includes all revenues except from sales of farmed fish, e.g. revenues from sale of roe, feed, smolts, packaging of fish and rental income, and consequently may give an indication of the use of resources in other activities. Table 8.1 shows the percentage shares of farms in the data set that were engaged in other activities, and mean (other revenues)/(harvest revenues)-ratios. It should be noted that the relatively large (other revenues)/(harvest revenues)-ratios observed in most years are due to a few farms with extremely high revenues from other activities.

8.2. Construction of a Panel Data Set

As a large number of farms in the sample participate most of the years, it is possible to construct a balanced or unbalanced panel data set. A panel data set will allows inclusion of firm-specific (fixed or random) effects, use of lagged firm level variables, detection of errors in variables, and utilisation of recent econometric panel data techniques for dealing with measurement errors. Since the data sets cover the 1985-93 period, there are nine annual observations on farms that participate in the sample each year.

The farms in the sample are anonymous; neither the name of the firm nor the aquaculture license code are reported. The variable which allows me to trace a farm over time and construct a panel data set is the *farm identification code*, which consists of a region code followed by a number. However, this farm id. code corresponds to the aquaculture license code. The farm id. code is only changed if the aquaculture license code is changed, and for several types of events the latter code is not changed. It is usually not altered if

(1) the name of the firm is changed but the owner remains the same,

(2) the name of the farm is changed because another firm/person buys the farm,

(3) a new owner takes over the firm but the name of the firm remains the same, or

(4) the farm is moved to another sea location.

One of the reasons for constructing a panel data set is that we want to introduce firm-specific effects into the model framework. These firm-specific effects are assumed to represent unobservable firm characteristics such as the quality of management, the quality of workers and the bioproductivity of the farm site. Furthermore, the econometric panel data model framework assumes that these effects are *constant* over time (Hsiao, 1986, p. 25). If the firm-specific effects are not constant over time, then the standard econometric panel data model framework will be inappropriate for our application. In this context event (1) is unproblematic, but events (2)-(4) require some further discussion, because they may lead to changes in the unobservable firm characteristics that are assumed to be constant.

Events (2) and (3), i.e. the introduction of a new owner, may have consequences for the quality of management, which again will result in changes in efficiency and unit costs. However, there is little reason to believe that a change in ownership has substantial consequences for the other sources of heterogeneity listed in section 6.6, at least in the short run. Event (4), moving to a "better" farm location, may have substantial consequences for maximum fish density in the pens and maximum total production. This may lead to higher productivity and lower unit costs. However, moving to a site with higher bioproductivity generally also means moving to a more exposed site. Consequently, moving the farm may also involve investments in a more robust sea-pen system, which will contribute to reduce the gains with respect to unit costs. Several farms on exposed locations have also experienced dramatic losses, e.g. the loss of the entire stock of fish, due to extraordinary weather conditions during the winter season.

The next question to ask is to what extent events (2) and (3) are present in the data set. That is, to what extent have farms with the same farm id. code changed ownership during the data period? Before going into this issue, a short discussion of some of the characteristics of the salmon farmer population will be useful. The individuals that acquired aquaculture licenses during the 1970s and 1980s were in general not arbitrageurs or speculators that entered the industry to reap short run profits, and then sell their license if they could earn a profit on their investments. Like farmers in traditional agriculture, it can be argued that most salmon farmers have a high degree of attachment to their occupation and farm. This means, as empirical studies have suggested, that a typical salmon farmer must face a severe economic crisis, such as bankruptcy, before he will part with his farm. In addition to the non-economic attachment to his occupation, there are also economic reasons why a salmon farmer must face bankruptcy before he is separated from his farm. The small coastal communities along the Norwegian coast in general offer few alternative employment opportunities. For many salmon farmers this means that there are few alternatives to their present occupation. They may not have skills which are attractive in the service or manufacturing sectors in urban areas. Furthermore, relocation to another place may lead to substantial losses associated with sales of private property, because second-hand markets may be little developed due to few employment opportunities or because the region may experience a local recession.

Legal restrictions should also be taken into account in an assessment of the possible extent of changes in ownership. Prior to 1991 majority ownership interests in more than one salmon farm was prohibited by the Fish Farming License Act, although a few exemptions were granted. This contributed strongly to limit sales of ownership interests in the industry. In 1991 the law was changed, and the ownership restriction was in reality not in effect any longer. As a consequence of this deregulation and the large number of bankruptcies in 1991 and -92, the industry from 1991 onwards has experienced a much higher take-over activity than in the

1980s. This development is also reflected in the salmon farm data set. A substantial number of new firms entered the data set in 1991 and 1992.

According to the Directorate of Fisheries (Merethe Fauske, pers. comm.), farms that were in the sample during the entire data period, did *not* change owners. This may be verified by examining profitability and equity figures for the individual farms. A change of ownership is not very likely for farms with positive profits in preceding years. For farms that experienced negative profits in one or more of the preceding years equity figures should be examined. Farms that eventually went bankrupt, are not included in the data set the year bankruptcy occurred. This is because annual accounts are not produced for that year. If a particular farm id. code is observed in year t-1 and year t+1, but not in year t, examination of profitability and equity figures in the period preceding year t should give an indication whether the farm went bankrupt or was excluded from the sample for other reasons (e.g. poor quality of returned questionnaires or annual accounts). Of course, if lagged variables are included in the econometric model specification, missing observations will represent a problem regardless whether the farm went bankrupt or not.

There are also some instances of mergers of farms in the data set. These are not mergers in the usual *legal* sense, i.e., that one firm buys another. Usually it is two independent firms that decide to join their operations to exploit economies of scale. The original owners maintain control of the assets they originally owned, and are relatively free to dissolve the joint operation if they should so desire. When a joint operation is in effect the two farms will often deliver a joint questionnaire and annual accounts to the Directorate of Fisheries. In the case of such mergers the Directorate of Fisheries sometimes has assigned the farm id. code to one of the firms to the joint operation. This is problematic, since the new operation will have different characteristics than the previous one with the same farm id. code. The absence of mergers can be verified by checking the licensed pen volume and utilised pen volume. If the pen volume increases beyond the maximum allowed pen volume from year t-1 to year t, then a merger may have taken place. However, it should be noted that some farms which were established in the 1970s, prior to the introduction of pen volume restrictions, may have a pen volume above the maximum level.

Identification of farm relocations may represent a bigger problem than identification of ownership changes. According to the Directorate of Fisheries there was a substantial number of relocations of farms during the data period. In the late 1980's and early 90's there was a trend of moving pens to more exposed locations. The problem is that there is no information available in the data set which makes it possible to account for relocation in the empirical model specification. The biophysical factors which defines the bioproductivity of the location can be regarded as fixed inputs in the production process. Since data on the relevant biophysical factors are not available, we will not be able to directly account for changes in the levels of these inputs in the econometric model. Of course, this may bias the parameter estimates of the model.

Before any farms are excluded from the data set due to data problems, 28 farms can be observed for the entire nine-year data period, as shown in table 8.2. Further, 24 farms are observed eight years, 45 farms are observed seven years, etc. There is a total of 2280 observations in the data set.

Table 8.2. Panel Structure of the Norwegian Salmon Farm Data Set

Farms observed...	Number of farms	Number of obs.
9 years	28	252
8 years	24	192
7 years	45	315
6 years	63	378
5 years	59	295
4 years	82	328
3 years	80	240
2 years	101	202
1 years	78	78
Sum	**560**	**2280**

A careful examination of several variables was undertaken for farms that could be observed for 3 years or more in order to determine whether data quality problems were present, or if mergers may have taken place. If such problems were detected the observations considered as problematic were dropped from the sample. Hence, the estimating sample is smaller than the original data set in terms of the number of observations. The estimating sample is presented in Chapter 9.

8.3. Construction of Input and Output Quantities

Previous econometric studies of salmon farming, such as Salvanes (1988; 1993) and Tveterås (1993), have shown possible ways to construct output levels, output prices, and input levels and prices from the salmon farm data set.

This study, unlike previous empirical studies of the industry mentioned here, uses a primal approach. Hence, it is not necessary to construct farm level prices for inputs and output, only input and output quantities. In the empirical analysis in Chapter nine, five inputs will be implemented into the production function: fish feed, initial stock of fish in the pens, labour, capital and materials input. See table 8.3 for a summary of output and input measures.

In previous studies it has been common practice to define output as the harvest of salmon plus the difference between the stock of fish in pens from the beginning of the year to the end of the year. A problem with this output measure is that it allows negative outcomes, and thus violates one of the postulates of the neoclassical production theory for the single-output case. If the reduction in the stock of fish is greater than the sales of fish during the year, then output is negative according to this measure. Examination of the data set reveals that some farms experienced negative output.[13] The common practice has been to drop farms with negative output from the estimating sample. However, this will probably give rise to a sample selection bias, since farms which experienced adverse production shocks (e.g. fish disease outbreaks), and/or used a risky vector of inputs tend to be dropped from the sample, which means that valuable information on the structure of production risk may be lost.

Table 8.3. Output and Input Quantity Measures Used in Empirical Models in This Thesis

Output/input	Unit of measurement	Comments
Salmon output	kg fish	Sales of fish plus stock of fish
Feed input	kg feed	Feed expenditures divided by price (NOK/kg) of feed type "Edel"
Fish input	kg fish	Stock of live fish in pens January 1st
Labour input	hours	Hours of paid and unpaid work
Capital input	real NOK	Real replacement value of capital equipment
Materials input	real NOK	Real expenditures on maintenance and repairs, electricity, office equipment, rent of equipment and buildings, etc.

An alternative way to define output is to set output equal to sales plus the stock of fish at the end of the year. Since these two components of the output measure always will be nonnegative, the problem of negative output outcomes is eliminated with this definition. This definition will be employed in this thesis.

With the chosen definition of output, the stock of fish in the beginning of the year should be included as input, because it will be an important determinant of the output quantity. The stock of fish is measured as the total biomass of live fish in the pens on January 1st.

For feed, only data on total feed expenditure is available. To estimate the quantity of feed used, the relationship that exists between the output of fish and the quantity of feed required, the *feed conversion ratio*, has been utilised in previous studies. The quantity of feed is defined as the product of output and the feed conversion ratio. Aggregate annual data are available for the feed conversion ratio (Seymour & Bergheim, 1991), but not at the firm level. One of the problems with the feed quantity and price proxies used in previous studies is that they assume an *identical* feed conversion ratio across farms. This is a very strong assumption, since studies

[13] Losses due to fish disease and extreme weather are usually the causes of negative output.

indicate substantial cross-farm variations in feed conversion ratios (Berge & Blakstad, 1989; Johannessen, several years). Based on the findings of Johannessen *et al.* and Berge & Blakstad, it is reasonable to assume that the dominant source of the substantial cross-farm variation in the feed price proxy found in Salvanes a.o. is the large variation in feed conversion ratios. Several problems associated with the proxies for feed input and price in a dual econometric model framework are discussed in Tveterås (1993, p. 72).

Each year smolts are also released into the pens, usually in May/June and/or early fall. For smolts, expenditures are available, but not quantities. Consequently, it is not possible to obtain direct measures of the quantity of smolts. However, aggregate regional data on smolts prices can be utilised (CBS, 1985-92). Statistics Norway each year compiles average purchase prices of smolts bought by salmon farms in each county. A proxy for the quantity of smolts can thus be constructed by dividing expenditures on smolts by regional smolts prices. However, smolts prices are reported per piece, not per kilo, which means that further assumptions on the average weight of the smolts are necessary in order to transform into biomass (kilos), which is the measure for output and fish input at the beginning of the year. Another problem with using smolts prices, is that in several years there were substantial differences in the prices farms paid for smolts, even within regions. Previous studies have avoided using smolts input proxies by arguing that in a dual model framework it is possible to eliminate smolts input from the model specification by choosing an appropriate time window for the fish farmer's optimisation problem (Salvanes, 1988). In the one-year period from January 1st to December 31st, the choice of smolts input can be regarded as weakly separable from the other inputs. Given the problem of measurement, and because smolts input may have a limited effect on output the year it is released, smolts are not included as inputs in the empirical models estimated in this thesis. Furthermore, for farms which have been in full operation for two or more years, which is the case for the farms in our data set, there is usually a high degree of correlation between smolts input and the stock of fish in the pens January 1st.

It is possible to construct a proxy for the feed consumption by utilising feed price information available from fish feed manufacturers. The proxy for feed consumption can be obtained by dividing the feed expenditures by the feed price. There is, however, no data available at the farm level on the type of feed used. Then, if the price to some extent reflects the quality of the feed, which has been the case in Norwegian salmon farming, this may not lead to very large biases in the feed input proxy. In the empirical models in this thesis, feed input is measured by the feed expenditures divided by the average annual price of "Edel", a common salmon feed type produced by the largest Norwegian feed manufacturer, Skretting. The price index is reported in table 8.4.

To some extent the fish farms have also been given different discount rates from the fish feed manufacturers, depending on the purchased quantity. This means that there may be a downward bias in the feed consumption proxy for large farms. A particular problem for the first years of the data period is that some farms produced their own feed because they had access to very cheap raw materials, e.g. from the local fish processing industry. This problem diminishes over the time because the fish farms gradually have learned about the superior quality of commercial feed from professional manufacturers. For the farms that made their own feed due to access to cheap raw materials, the value of the feed input proxy may be very low. However, by dividing output quantity by the feed input proxy one obtains a proxy for the feed conversion ratio of the farm and thus an indication of whether the farm used commercial feed or not. Farms with feed conversion ratio proxies less than one should be excluded from the sample, because it was in practice not possible to obtain such low feed conversion ratios with the feed types available in the first half of the data period.

Table 8.4. Price of Salmon Feed "Edel" by Year (in NOK/kg Feed)

Year	1985	1986	1987	1988	1989	1990	1991	1992	1993
Price Edel	6.23	6.29	6.19	6.96	7.61	7.14	7.19	7.13	7.13

The construction of measures for the services of capital input is generally regarded as problematic, since "capital" is not a homogenous input. Capital includes pens, buildings, feeding equipment, etc. In this study capital input is measured by the real replacement value of capital equipment.

Labour input is measured by the hours of paid and unpaid work at the farm by managers/owners and workers.

Materials input is the real expenditures on maintenance and repairs, electricity, office equipment, rent of equipment and buildings, etc. Labour costs associated with maintenance and repairs undertaken by own employees are not included in materials.

For both capital and materials input the nominal figures are deflated by the consumer price index (CPI).

Appendix 8.B provides summary statistics for output and input variables constructed from the Norwegian salmon farm data set, before problematic observations are dropped. Presentation of summary statistics for the estimating samples is postponed to Chapter nine (see appendix 9.D).

8.A. Appendix: List of Variables in Salmon Farm Data Set

No.	Variable	Unit	Comment
1	Year		Observation year
2	Farm_code		Format: x yyy, where x=region (VA, R, H, SF, M, ST, NT, N, T, F) and yyy is a three digit code.
3	Fish species		Values: L=salmon, Ø=trout, B=both.
4	Legal org.		Joint stock company, etc.
5	Parent fish		Values: Yes/No
6	Other activities		Values: Yes/No
7	Full utilisation of license		Values: Yes/No
8	Full utilisation of capacity		Values: Yes/No
9	Damages		Values: Yes/No
10	Lack of smolts		Values: Yes/No
11	Diseases		Values: Yes/No
12	Year of establishment		
13	Licensed pen volume		Maximum pen volume according to license issued by government
14	Actual pen volume		
15	Sales of fish	kg	
16	Production of fish	kg	
17	Stock of salmon Jan. 1st	no.	Fish in pens
18	Stock of salmon Dec. 31st	no.	Fish in pens
19	Stock of trout Jan. 1st	no.	Fish in pens
20	Stock of trout Dec. 31st	no.	Fish in pens
21	Stock of fish Jan. 1st	kg	Fish in pens
22	Stock of fish Dec. 31st	kg	Fish in pens
23	Paid labour input	hours	
24	Unpaid labour input	hours	
25	Total labour input	hours	
26	Credit limit	NOK	
27	Bank deposits	NOK	
28	Short-term outstanding claims	NOK	
29	Value stock of feed	NOK	
30	value stock of fish	NOK	
31	Buildings and equipment	NOK	
32	Long-term outstanding claims	NOK	
33	Debt to vendors	NOK	
34	Bank overdraft	NOK	
35	Other short-term debt	NOK	
36	Long-term debt	NOK	
37	Delayed taxes	NOK	
38	Tax allocations	NOK	
39	Equity	NOK	
40	Sales revenues	NOK	
41	Other revenues	NOK	
42	Compensations	NOK	
43	Income from interest	NOK	
44	Operating revenues	NOK	
45	Smolts costs	NOK	

List of Variables in Salmon Farm Data Set Continued...

No.	Variable	Unit
46	Feed costs	NOK
47	Insurance costs	NOK
48	Labour costs	Nok
49	Calculated salary owner	Nok
50	Harvesting costs	Nok
51	Freight costs	Nok
52	Other operating costs	Nok
53	Losses on outstanding claims	Nok
54	Calculated depreciation historical costs	Nok
55	Calculated depreciation replacement costs	Nok
56	Change in stock	Nok
57	Operating costs	Nok
58	Operating profits	Nok
59	Interest costs	Nok
60	Profit before extraordinary costs	Nok
61	Man-years	No.
62	Utilisation ratio	%
63	Prod./m^3 pen volume	kg
64	Prod./man-year	kg
65	Value of production	Nok
66	Value/man-year	Nok
67	Value/m^3 pen volume	Nok
68	Calculated interest	Nok
69	Calculated depreciation mixed principle	Nok
70	Total costs	Nok
71	Total revenues	Nok
72	Wage means	Nok
73	Wage means/man-year	Nok
74	Profits	Nok

8.B. Appendix: Summary Statistics from the Norwegian Salmon Farm Data Set

Table 8.B1. Overall Summary Statistics (2280 Obs.)

Variable	Mean	St.dev.	Min.	Max.
Output (y) in kg	360 472	244 208	11 050	2 028 801
Materials (M) in real NOK*	1 045 773	1 046 963	0	12 736 623
Feed (F) in kg	344 389	250 803	0	2 479 452
Capital (K) in real NOK*	2 513 372	2 295 081	0	37 212 584
Labour (L) in hours worked	6 979	3 754	0	42 906
Fish (I) in kg	150 265	110 467	0	1 015 800

* Deflated by Consumer Price Index (CPI)

Table 8.B2. Means and St.Deviations (in Parenthesis) of Inputs and Output By Year (2280 obs.)

Year	1985	1986	1987	1988	1989	1990	1991	1992	1993
Output (y)	158 572	170 972	197 558	294 591	387 971	459 260	460 687	454 937	560 094
	(98 535)	(99 296)	(118 685)	(145 648)	(174 098)	(216 417)	(258 124)	(276 602)	(291 053)
Materials (M)	478 706	633 563	688 752	820 503	1 104 390	1 326 852	1 603 479	1 244 831	1 296 716
	(362 278)	(448 332)	(544 681)	(584 896)	(728 547)	(958 509)	(1 525 823)	(1 334 182)	(1 440 422)
Feed (F)	161 244	170 291	191 746	296 084	377 792	422 043	395 678	448 585	540 266
	(106 630)	(99 680)	(127 818)	(162 913)	(186 433)	(233 498)	(247 346)	(300 769)	(326 500)
Capital (K)	1 882 693	2 205 393	2 462 119	2 658 116	2 748 155	2 809 377	2 585 648	2 514 342	2 486 435
	(1 347 634)	(1 628 947)	(1 922 208)	(2 767 025)	(2 164 937)	(2 076 656)	(2 832 661)	(2 578 489)	(2 493 611)
Labour (L)	6 381	6 563	6 269	6 864	7 177	7 474	6 893	7 152	7 681
	(3 447)	(3 636)	(2 861)	(3 277)	(3 440)	(3 684)	(3 587)	(4 492)	(4 687)
Fish (I)	65 912	76 903	82 304	105 837	161 688	226 176	219 169	187 537	189 244
	(55 394)	(51 196)	(60 555)	(60 905)	(76 665)	(111 949)	(130 117)	(129 999)	(110 611)
No. of obs.	192	223	240	281	293	278	241	259	273

9. ECONOMETRIC MODELS OF THE STOCHASTIC PRODUCTION TECHNOLOGY IN SALMON FARMING

In this chapter we specify and estimate econometric models of the stochastic production technology for Norwegian salmon aquaculture in order to test the hypotheses H1-H4 provided in Chapter 1. The objective of the econometric analysis in this chapter is to uncover the structure of the stochastic production technology in salmon farming, particularly the structure of production risk, by utilising information provided by the data set on input and output levels.

A primal approach will be used here. A dual approach have previously been rejected (see Chapter three). Testing of hypotheses H1-H4 only requires that the stochastic production function is specified, so specification of utility functions and expectation formation models is not required.

For sake of convenience the hypotheses are repeated here:

H1. The following factors explain observed cross-firm productivity differences in Norwegian salmon farming in a given year: (1) economies of scale, (2) firm heterogeneity (with respect to the quality of management, labour and capital equipment, etc.,), and (3) "true" randomness in the production process.

H2. The production technology in salmon farming is characterised by

(a) increasing output risk associated with a factor neutral expansion in inputs,

(b) decreasing marginal output risk associated with an increase in the input of capital and labour, and

(c) increasing marginal output risk associated with an increase in the input of fish and fish feed.

H3. During the period 1985-93

(a) the conditional mean output for a given combination of inputs has increased.

(b) the conditional distribution of output in salmon farming has been more condensed, i.e., the level of production risk has decreased.

H4. There are differences in mean productivity and output risk levels between the regions.

The basis for the model specifications will be previous primal model specifications provided in the econometric literature, and the description of the salmon industry presented in previous chapters. In Chapter 3 we presented earlier econometric approaches and discussed important issues in the econometric modelling of the stochastic production technology. In chapters 6-7 the features of the production process in salmon aquaculture were discussed.

Chapter 4 discussed the use of panel data techniques to account for firm heterogeneity. Here, firm heterogeneity will be accounted for by means of various econometric panel data estimators. Both fixed effects and random effects models will be estimated.

According to the theoretical models presented in Chapter 2, information about the production technology is required if one wishes to make predictions about salmon farms' response to changes in input prices or the (expected) output price. The theory predicts that the firm's output supply and input demand responses to price changes are affected by the structure of production risk. Knowledge about the structure of production risk is, however, not sufficient to predict input demand and output supply responses, e.g. to changes in input and output prices. According to the theory of firm behaviour under uncertainty, it is also required that one has knowledge or makes assumptions on the risk preference structure and output price expectations of salmon farmers.

The theoretical models of Chapter two also provided motivation for accounting for production risk in analyses of technical change. Risk averse producers will not only be concerned about the mean productivity when they consider adoption of new technologies (or production practices); they will also be concerned about the change in output risk associated with new technology adoption. Hence, for a risky production process such as salmon farming, analysis of technical change should include the variance function. This will be done in the empirical analysis below.

Continuing the discussion of estimators in chapters three to five, attention will also be given to estimation issues in this chapter. The simulation study in Chapter five did not provide any overwhelming support for any particular estimator, although the maximum likelihood estimator seemed to perform better in the estimation of the variance function parameters. However, the assumptions underlying the data generating process in Chapter five may be violated in the data generating process for the salmon farm data set. It is difficult to say what effects this will have on the relative performance of different estimators. The empirical production models specified here have several characteristics which should make one careful about drawing conclusions based on a single estimator; the linear quadratic parametrization of the mean production function has been used only to a limited extent in the empirical literature: The same can be said for the specification of the output variance function.

Caution should be exercised in particular for random effects models, which have become very popular recently since the efficiency of parameter estimates is increased if the underlying assumptions are valid. The available panel data set is unbalanced. Estimation of random effects specifications on an unbalanced panel with heteroskedasticity of the form chosen here has not been tried before. In addition, it is difficult to know to what extent the random effects assumptions, e.g. the assumption of independence between the firm-specific effects and input levels, are valid.

Given all the above considerations, several estimators should be tried in the estimation of the production models for the salmon industry. Feasible generalised least squares (FGLS) estimation will be undertaken for both fixed and random effects model specifications.[1] Maximum likelihood and FGLS estimates of a fixed effects specification will also be compared. In the assessment of the estimates we are concerned about whether the various estimators provide significantly different estimates of important elasticities derived from the mean and variance functions, and if the estimated t-values of different estimators give different levels of confidence when making predictions. It should be noted that the empirical data set is larger than the data set used in the simulation study in Chapter five. However, it is questionable whether it is of a sufficient size to characterise it as 'large' in the asymptotic sense.

The plan of this chapter is as follows: First, in section 9.1, we discuss important econometric specification issues, both general and specific. Two classes of stochastic production functions will be estimated; Just-Pope production functions and Kumbhakar production functions. In section 9.2 the empirical Just-Pope specifications are presented, while section 9.3 presents the Kumbhakar specifications. Section 9.4 presents summary statistics and the estimators which will be used.

A natural first step in empirical analysis of production risk is to test if heteroskedasticity is present in the data set, particularly if output is heteroskedastic in input levels. Testing of heteroskedasticity is undertaken in section 9.5. Next, in section 9.6, we investigate whether the linear quadratic and translog parametrization of $f(\mathbf{x})$ provides similar elasticity measures. If these two specifications provide similar elasticity estimates, this will increase our confidence in the relatively less used linear quadratic form.

In section 9.7-9.12 empirical results from estimation of Just-Pope models are presented. First, in section 9.7, we compare estimates of models with different assumptions on technical change, the time trend model and the time dummy model. We will assess if the standard time trend specification of technical change is valid for our empirical application, or if a model with separate parameters for each year is required. Next, we examine the effect of different estimators on estimated parameters and elasticities. In chapters three and five we discussed FGLS vs. ML estimation of Just-Pope models. In section 9.8 we will see what effects the choice of estimator has for the empirical estimates for the salmon aquaculture industry. In section 9.9 we investigate the consequences of assuming firm homogeneity for estimated

[1] Note that although we use the term "FGLS estimator" for both the fixed and random effects case, the least squares estimators are different for the two cases since the structure of the covariance matrices are different (cf. Chapter four). However, regardless of the assumptions we make about the firm-specific effects, the least squares estimation procedure is FGLS when we adjust for the heteroskedasticity in the observation-specific error term.

parameters and elasticities of the mean and variance functions. We are particularly interested in what effects the homogeneity assumption has on estimated marginal output risk and estimates of the overall output risk in salmon farming.

Next, in section 9.10, we compare estimates from fixed effects and random effects specifications of the firm-specific effects. If the random effects assumptions of no correlation between error components and between error components and regressors are valid, then it is possible to introduce time-invariant variables, such as region dummies, into the production function. In section 9.11 we analyse the effects of omitting farms which are observed one and two periods by means of a random effects model. In section 9.12 we estimate models with region-specific effects, which enable us to analyse if regional location has any effect on the mean productivity and output risk of salmon farms. Some of the models we estimate are random effects model which allow us to include both firm-specific effects (as random) and region-specific effects (as fixed).

Section 9.13 compares Just-Pope and Khumbakar estimates.

Finally, in section 9.14, the empirical results are summarised, particularly with respect to the hypotheses put forward at the beginning of this dissertation. Based on the empirical findings we also discuss the appropriateness of the different econometric specifications, and assess to what extent our results have implications for the econometric research on production risk in general.

We have chosen to present tables with parameter estimates, tables with estimated elasticities and figures in separate appendices. Estimated parameters are presented in appendix 9.A, estimated elasticities in appendix 9.B, and figures in appendix 9.C. Estimated parameters and elasticities have been presented with three decimals, but this should not be interpreted as an indication of the level of precision of the estimates.

9.1. Discussion of Some Specification Issues

According to the discussion in the previous chapters, several requirements with respect to theoretical consistency, functional flexibility and factual conformity should be accounted for in the specification of the stochastic production model. Undoubtedly, an extensive set of requirements may lead to some very difficult trade-offs when an econometric model specification is to be chosen. In this section we will discuss some specification issues, both general and specific.

Chapter 3, which discussed previous studies in this field of research, presented several specifications of the stochastic production function. The focus will be on two classes of production functions; (1) the Just-Pope form

$$y_{it} = f(x_{it}; \alpha) + \eta_i + h(x_{it}; \beta)\varepsilon_{it},$$

and (2) the Kumbhakar form

$$y_{it} = f(\mathbf{x}_{it}; \alpha)\exp(h(\mathbf{x}_{it}; \beta)(\eta_i + \varepsilon_{it})).$$

For the Just-Pope form a Cobb-Douglas has usually been assumed for $f(\cdot)$ and $h(\cdot)$ in the literature, while Kumbhakar postulated a translog form for $f(\cdot)$ and a linear form for $h(\cdot)$. Econometric parametrizations of these two stochastic production function specifications will be estimated in the empirical study. Both specifications have their advantages and drawbacks, in general and with respect to the particular empirical application. *A priori*, it is therefore far from a trivial question which of the above specifications (or modifications of above specifications) is most appropriate to use. Difficult trade-offs have to be made. Some general guidance has been provided by Lau (1986), who suggests five criteria for choosing a functional form: theoretical consistency, domain of applicability, flexibility, computational facility, and factual conformity.

In terms of *theoretical consistency* the Just-Pope form satisfies the Just-Pope postulates for the stochastic production function. Just & Pope (1978) have shown that Kumbhakar's specification always violates the postulate of independence between the mean function $f(\cdot)$ and the variance of the random term ε (postulate P4), and the requirement that constant stochastic returns to scale should be possible (postulate P8). Depending on the parameter values of α and β it may also violate the other Just-Pope postulates.

In terms of the mean production function $f(\cdot)$ *flexibility* means that the functional form does not impose any *a priori* restrictions on derived elasticities of substitution and scale. The translog function is regarded as flexible in this sense, while the Cobb-Douglas is inflexible.[2] Due to the *a priori* restrictions it imposes on the production technology, the Cobb-Douglas form has been largely abandoned in applied production studies that assumes a deterministic setting. The large body of empirical results from the application of flexible functions strongly suggest that the use of these is warranted in production analysis. However, in econometric studies of production uncertainty the Cobb-Douglas has been widely employed (see table 5.1), although all of these studies have been undertaken recently, after the introduction of flexible forms.

Examples of studies that have used a Cobb-Douglas form are Just & Pope (1979), Griffiths & Anderson (1982) and Saha *et al.* (1994).[3] This must be considered a weakness of these studies, particularly since none provide arguments for rejecting a more flexible specification of the production technology in favour of the Cobb-Douglas. The estimated models probably suffer from an omitted-variables bias which not only affects the estimates of the mean production

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[2] The issue of flexibility, and the properties of the Cobb-Douglas and translog functions are discussed extensively in Chambers (1988).

[3] Just & Pope (1979) also estimate a translog, but this is a very simple version with only one input.

function parameters α, but also translates into the estimates of the errors u_{it} and thus the estimates of the variance function parameters β.[4]

In addition to the general arguments presented against the use of a Cobb-Douglas function, there is also a specific reason for not using it which is related to hypothesis H1. Since the returns to scale for the Cobb-Douglas production function is equal to the sum of the parameters, $\Sigma_j \alpha_j$, it is too restrictive for the testing of the contribution of scale economies to observed productivity differences in hypothesis H1.

On the other hand, there are also problems associated with more flexible functional forms in terms of the domain of applicability. Results from empirical applications and Monte-Carlo studies generally suggest that flexible functional forms are well-behaved around the mean observation, but that the theoretical consistency requirements are violated for observations that lie far from the mean. Due to the inability to approximate the production technology in outlying observations, the accuracy of the predicted error terms is questionable for these observations. Again, this may affect the estimates of the errors ε_i and hence the estimates of β. The performance of different flexible functional forms has been compared in several simulation studies. The results from these studies tend to suggest that the translog is more well-behaved than other functional forms at observations "far" from mean, i.e., it has a larger consistency region than other functional forms.[5] We know little about the performance of the linear quadratic functional form, which will be used in the following, in this respect.

For the salmon farming industry the *factual conformity* criterion, which implies consistency of the functional form with known empirical facts, supports the use of flexible functional forms such as the translog. The empirical results of Salvanes (1989; 1993) a.o. provide solid support for the use of second order approximations to the underlying production technology in salmon farming: The majority of second-order term coefficients in Salvanes' estimated translog cost and profit functions were significant at conventional confidence levels, which strongly suggest that a Cobb-Douglas specification is inappropriate.[6]

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4 In the first step of a FGLS estimation procedure the mean function $y_{it} = f(\mathbf{x}_{it}; \alpha) + u_{it}$ is estimated. If some of the parameters α are omitted, then this will in general lead to biased estimates of u_{it}. In the second step of the estimation procedure the estimates of u_{it} from the first stage are regressed on the variance function $h(\mathbf{x}_{it}; \beta)$, which means that the estimated u_{it}'s provide biased estimates of β unless very strict conditions on the nature of measurement errors in the u_{it}'s are satisfied.

5 Some of the studies that discuss the global properties of the translog and other flexible functional forms are Barnett, Lee & Wolfe (1985), Barnett & Lee (1985), Caves & Christensen (1980), Diewert & Wales (1987), Gallant (1981), Gallant & Golub (1984), Guilkey & Lovell (1980), Guilkey, Lovell & Sickles (1983), Wales (1977), and Westbrook & Buckley (1990).

6 A translog function with all second-order parameters equal to zero reduces to a Cobb-Douglas function.

Computational considerations probably explains the widespread use of the Cobb-Douglas function in econometric analyses of production risk. The problem with using the Just-Pope form in econometric modelling is that the error term does not appear multiplicatively with $f(\cdot)$. In standard deterministic production functions a multiplicative relationship between the production function and the error term of the form $y_{it} = f(\mathbf{x}_{it}; \alpha)\exp(\varepsilon_{it})$ is assumed. For the Cobb-Douglas and translog parametrizations of $f(\cdot)$ this allows one to linearize the function in parameters α by taking logarithms on both sides. However, in the Just-Pope setting the additive error term forces the researcher to use nonlinear least squares or nonlinear maximum likelihood in the estimation of $f(\cdot)$.

One potential problem with nonlinear models is that the obtained parameter estimates may not minimise (maximise) the objective function globally, only locally. Nonlinear methods may therefore provide biased parameter estimates. This is however a much larger problem for the translog function than for the Cobb-Douglas function. The inability to find the parameters that globally optimise the objective function, may it be the residual sum of squares or the likelihood, is probably an important reason for not using the nonlinearized translog.[7] The problems with nonlinear estimation suggest that a flexible functional form which is linear in parameters should be employed.

Kumbhakar's specification allows a Cobb-Douglas or translog $f(\cdot)$ to be linearized in parameters, because the variance function is related to the mean function in a multiplicative exponential manner. The mean function $f(\cdot)$ was specified as a translog by Kumbhakar, and consequently it satisfies Lau's flexibility requirement. The Kumbhakar form allows the use of linear estimation techniques for $f(\cdot)$, but $h(\cdot)$ has to be estimated by nonlinear least squares.

For the variance function both theoretical consistency and flexibility requirements suggest that the econometric specification should allow the conditional variance of output to both increase and decrease in inputs. In this particular application, the testing of hypothesis H2 requires that the production function specification allows both positive and negative marginal effects on risk of changing input levels, i.e., $\partial \text{var}(y)/\partial x_i <=> 0$ is possible for all i.

In this empirical study, Harvey's (1976) specification of the variance function is applied. The flexibility of the variance function of Harvey's model may be questioned, because it is given by a linear-in-parameters function which is raised to the power of e. The exponentiation ensures that the variance is always positive. This is not the case for the conditional variance of the general Just-Pope model $y = f(\mathbf{x}; \alpha)+h(\mathbf{x}; \beta)\varepsilon$; in order to allow both negative and positive

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[7] Discussions of nonlinear estimation of translog functions have not been found in the literature. However, preliminary Monte-Carlo results from the estimation of parameters of a nonlinearized, well-behaved translog function indicate that it is very difficult to find estimates that are close to the true parameter values. This is the case even if the true parameter values of the translog function are given as starting values for the estimation procedure.

marginal risks, the β's may take both negative and positive values. Consequently, $var(y_i|\mathbf{x}_i)$ may be negative for certain values of $\mathbf{x}_i$ in the Just-Pope model, depending on the signs and values of the β's. Although it is a nice property of Harvey's formulation that it always yields positive variances, it may be discussed if there are any *a priori* reasons for postulating a natural exponential function for the variance of production.

Firm heterogeneity should be accounted for in the models in order to uncover the 'true' production risk, according to the discussion in chapters 4 and 7. In salmon farming heterogeneity may be a more important determinant of cross-firm productivity differences in a given period than different outcomes of stochastic variables across firms. Ignoring heterogeneity in the econometric model specification may consequently lead to over-estimation of the importance of risk, and biased estimates of the parameters of the production function. Furthermore, hypothesis H1 explicitly requires the implementation of firm-specific effects. In section 9.1 we argued that the Kumbhakar form with translog $f(\cdot)$ provides a more satisfactory representation of firm heterogeneity than the Just-Pope form with a linear quadratic $f(\cdot)$. The former allows the absolute difference in output y_i - y_j between two firms with different firm-specific effects η_i and η_j to increase as the scale of operation increases. In other words, for two input vectors $\mathbf{x}$ and $a\mathbf{x}$ ($a > 1$) and two firms i and j where $\eta_i > \eta_j$, the difference in mean output conditional on $\mathbf{x}$, $E[y_{it}| \mathbf{x}; \eta_i] - E[y_{jt}| \mathbf{x}; \eta_j]$, is smaller than the difference in output conditional on $a\mathbf{x}$, $E[y_{it}| a\mathbf{x}; \eta_i] - E[y_{jt}| a\mathbf{x}; \eta_j]$ for the Kumbhakar form. For the Just-Pope form with linear quadratic parametrization, the firm specific effect interacts additively with $f(\cdot)$, and thus the absolute difference in output y_i - y_j will be constant regardless of input levels $\mathbf{x}$.

In order to test hypothesis H3 of increasing conditional mean output and decreasing production risk in Norwegian salmon farming over time, a trend variable or time-dummies must be implemented in both the mean and variance production function. With a trend variable the general Just-Pope production function is $y = f(\mathbf{x}; t)+h(\mathbf{x}; t)\varepsilon$. By using a second-order approximation for $h(\mathbf{x}; t)$, it is possible to analyse if the rate of change in production risk is decreasing or increasing over time. Flexibility is further increased if time-specific effects, both as separate dummy variables and as interaction terms with input levels, are used instead. The specification of technical change will be discussed further in subsequent sections.

To summarise the above discussion, the Just-Pope form satisfies the theoretical conformity criterion, but violates the flexibility criterion if the restrictive Cobb-Douglas function is used for $f(\cdot)$. Alternatively, if a translog function is used for $f(\cdot)$, then the computational facility criterion is violated, due to the difficulty of finding the nonlinear parameter estimates that optimise the objective function. My solution is to use a linear quadratic specification of $f(\cdot)$ in the Just-Pope model, which is linear in parameters and flexible. Harvey's multiplicative heteroskedastic specification, which is used for the variance function, has the nice property that the variance of output will always take positive values due to the exponentiation of the variance function. Finally, the Kumbhakar form satisfies the flexibility requirement with respect to $f(\cdot)$, but violates some of the Just-Pope postulates set forth in Chapter two. However, it may be more problematic with respect to the computational facility criterion of Lau, because $h(\cdot)$ has to be estimated by nonlinear least squares.

One can thus conclude that none of the above econometric model specifications are able to satisfy all of Lau's criteria simultaneously. What has the literature then to say in general about which requirements should be given most weight? Unfortunately, the answer is that none of Lau's criteria for choosing a functional form can easily be ignored. According to Lau

> "It is however not recommended that one compromises on local theoretical consistency - any algebraic functional form must be capable of satisfying the theoretical consistency restrictions at least in a neighbourhood of the values of the independent variables of interest. It is also not recommended, except as a last resort, to give up computational facility, as the burden of and probability of failure in the estimation of nonlinear-in-parameters models is at least one order of magnitude higher than linear in parameters models and in many instances the statistical theory is less well developed. It is also not advisable to sacrifice flexibility - inflexibility restricts the sensitivity of the parameter estimates to the data and limits a priori what the data are allowed to tell the econometrician. Unless there is strong a priori information on the true functional form, flexibility should be maintained as much as possible. This leaves the domain of applicability as the only area where compromises may be made." (Lau, 1986, p. 1558)

Given the problem of *a priori* determining a superior econometric specification - the Just-Pope form and the Kumbhakar form have different advantages and drawbacks - we choose to estimate several specifications of the stochastic production function. The elasticity of scale, elasticities of substitution, marginal input risks etc. of the estimated models will be evaluated at the mean observation and its neighbourhood in order to determine if the estimated production function is well-behaved. Hopefully, this will also give some information about the sensitivity of the predictions of different functional forms with respect to the characteristics of the production technology.

It should be noted that the neoclassical production function is a frontier function; it gives the *maximum* possible output that can be produced from a given vector of inputs. The production functions we estimate, however, are average functions; they give the *mean* output for a given set of inputs rather than the maximum output. Empirical measurement of production frontiers is outside the scope of this analysis.

9.2. Just-Pope Production Function Specifications

The general Just-Pope production function is given by

(9.1) $y_{it} = f(\mathbf{x}_{it};\alpha) + v_{it}$,

(9.2) $v_{it} = u_{it} + \eta_i = h(\mathbf{z}_{it};\beta)\varepsilon_{it} + \eta_i$

where ε_{it} is an observation-specific i.i.d. random variable representing the exogenous production shock, η_i is a firm-specific effect, and $\mathbf{z}_{it}$ is identical to, a subset of, or functions of the $k\times1$ input vector $\mathbf{x}_{it}$. In (9.1)-(9.2) the Just-Pope model is specified as an error component model, i.e., with random firm-specific effects. The composite variance function for this error components model is given by

$$(9.3)\quad \mathrm{var}(v_{it}) = \mathrm{var}(u_{it}) + \sigma_\eta^2 = [h(\mathbf{z}_{it};\beta)]^2 \sigma_\varepsilon^2 + \sigma_\eta^2,$$

where σ_ε^2 and σ_η^2 are the observation- and firm-specific variances, respectively. If the firm-specific effects are instead assumed to be fixed, η_i is dropped from (9.2) and included in (9.1) to be estimated together with the parameters of $f(\cdot)$.

An important issue is the specification of technical change. In order to be able to test for technical change a deterministic trend-variable t or time-dummies D_t have to be implemented in $f(\cdot)$. In the case of a trend-variable, a quadratic term t^2 should also be included in order to test for increasing or decreasing rate of technical change over time. One question is whether the technical change is embodied in new inputs. The only new inputs introduced during the data period were certain vaccines and medicines, which data are not available for. However, technical change was not independent of inputs. Improvements in the quality of *existing* inputs, such as the fish feed and the genetic quality of the salmon, is believed to have been important sources of productivity increase in the data period. Improvements in the quality of inputs, or factor-augmenting technical change, should thus be accounted for in the model specification.[8] If the trend-variable is allowed to interact multiplicatively with input levels in the econometric model, it is possible to test whether technical change is factor-augmenting or not.

Use of a trend-variable in the production function implies that technical change is assumed to be a continuous monotonic process. Time-dummies, on the other hand, allow the technical change to be discontinuous and non-monotonic. Biophysical productivity shocks of temporary nature, such as year-to-year changes in sea temperature, can thus be captured in the model. Since there is no information available in the data set that allows us to separately identify the effects of technical change and time-specific random biophysical shocks on productivity, time-dummies may be considered to be a more satisfactory approach. This will be analysed in the following.

A note on terminology is also required. The term 'productivity change' will be used analogously to 'technical change' in the following. Since our estimates of technical change will be influenced by biophysical shocks, which we cannot control for due to lack of data, it

[8] See for example Chambers (1988, pp. 210-213) for a discussion of concepts such as embodied technical change and factor-augmenting technical change.

may probably be more appropriate to talk about productivity change. However, the two terms will be used interchangeably in the following.

The inputs that will be included in the model are feed (x_F), labour (x_L), capital (x_K), the stock of fish in the pens in the beginning of the year (x_I), and materials (x_M). Output y is defined as the sales of fish during the year plus the stock of fish in the pens at the end of the year. Unlike the output measure used in previous econometric studies of the industry, e.g. Salvanes (1989; 1993), this measure ensures that the observed values are always non-negative, cf. the discussion in Chapter eight.

A linear quadratic functional form was chosen for the mean function. The linear quadratic allows common elasticity measures, such as returns to scale, to vary in input levels. A problem with the linear quadratic specification is that it does not allow testing of constant returns to scale (Driscoll, McGuirk, & Alwang, 1992). The translog and Leontief production function, on the other hand, allow testing of constant returns to scale. In the previous section we argued why the translog cannot be used in the context of a Just-Pope production function.[9] The use of a linear quadratic form can be defended on empirical grounds; previous econometric studies of the salmon industry strongly suggest nonconstant returns to scale (Salvanes, 1989; Salvanes & Tveterås, 1992a; Tveterås, 1993), which implies that homogeneity flexibility is not an important issue for this particular application.

Two linear quadratic specifications, denoted model JP1 and model JP2, are presented for the mean production function $f(\mathbf{x}; \alpha)$. The first specification, Model JP1, is given by:

$$
\begin{aligned}
& y = f(\mathbf{x}; t, \alpha) + v \\
(9.4) \quad & = \alpha_0 + \sum_{k=F,I,K,L,M} \alpha_k x_k + 0.5 \sum_j \sum_k \alpha_{jk} x_j x_k + \alpha_t t + 0.5 \alpha_{tt} t^2 + \sum_{k=F,I,K,L,M} \alpha_{kt} x_k t + v
\end{aligned}
$$

For notational convenience observation subscripts *it* have been dropped from the above specification.

The second specification, model JP2, is given by

[9] The Leontief production function was dropped for practical reasons; the right-hand side variables exhibited more multicollinearity and the estimated parameters in the Leontief formulation were consequently to a much greater extent insignificant than in the linear quadratic model.

$$
\begin{aligned}
(9.5)\quad & y = f(\mathbf{x}; D_{85},\ldots,D_{92},\alpha) + v \\
& = \alpha_0 + \sum_{k=F,I,K,L,M} \alpha_k x_k + 0.5 \sum_j \sum_k \alpha_{jk} x_j x_k \\
& + \textstyle\sum_{t=85}^{92} \alpha_t D_t + \sum_{t=85}^{92} \alpha_{Ft} D_t x_F + \sum_{t=85}^{92} \alpha_{It} D_t x_I + \sum_{t=85}^{92} \alpha_{Kt} D_t x_K \\
& + \textstyle\sum_{t=85}^{92} \alpha_{Lt} D_t x_L + \sum_{t=85}^{92} \alpha_{Mt} D_t x_M + v
\end{aligned},
$$

where the subscripts of the time-dummies D_t refers to years, with 1993 as base year. In both the above models the input subscripts $j, k = F, I, K, L, M$.

The difference between model JP1 and JP2 lies in the specification of technical change; in JP1 a trend-variable t is used to capture the effect of technical change, while in JP2 time-dummies are used instead.

For the linear quadratic specification with time trend (model JP1) the elasticity of scale, or returns to scale (*RTS*), is given by

$$
(9.6)\quad RTS^{J1}(\mathbf{x},t;\alpha) = \sum_k e_k(\mathbf{x}) = \sum_k \frac{\partial f}{\partial x_k}\frac{x_k}{f(\mathbf{x})} = \sum_k \left[\left(\alpha_k + 0.5\sum_j \alpha_{jk} x_j + \alpha_{kt} t\right)\frac{x_k}{f(\mathbf{x})}\right],
$$

where $e_k(\mathbf{x})$ is the elasticity of output with respect to input k. If the estimate of $RTS(\mathbf{x})$ is greater than, equal to, or less than unity, the returns to scale are increasing, constant, or decreasing, respectively.

For the time dummy model JP2, *RTS* in year t is given by

$$
(9.7)\quad RTS^{J2}(\mathbf{x},t;\alpha) = \sum_k \left[\left(\alpha_k + 0.5\sum_j \alpha_{jk} x_j + \alpha_{kt}\right)\frac{x_k}{f(\mathbf{x})}\right],
$$

where the term $\alpha_{kt} t$ in model JP1 is replaced by the term α_{kt} in model JP2.

For the time trend model (JP1) the elasticity of technical change (*TC*) is given by

$$
(9.8)\quad TC^{J1}(\mathbf{x},t;\alpha) = \frac{\partial \ln f(\mathbf{x},t;\alpha)}{\partial t} = \frac{1}{f(\mathbf{x},t;\alpha)}\left(\alpha_t + \alpha_{tt} t + \sum_{k=1}^{K} \alpha_{kt} x_k\right).
$$

Technical change can be decomposed into two components; *pure* technical change and *non-neutral* technical change. These are given by

$$
(9.9)\quad TCPUR^{J1}(\mathbf{x},t;\alpha) = \frac{1}{f(\mathbf{x},t;\alpha)}\left(\alpha_t + \alpha_{tt} t\right),
$$

and

$$(9.10)\quad TCNON^{J1}(\mathbf{x};t,\alpha)=\frac{1}{f(\mathbf{x},t;\alpha)}\left(\sum_{k=1}^{K}\alpha_{kt}x_k\right),$$

respectively. Note that, unlike the popular translog parametrization, pure technical change (*TCPUR*) in the linear quadratic model is not monotonous, as it also depends on the factor $1/f(\mathbf{x})$.

Elasticities of technical change cannot be derived in the same manner from the model with fixed time-specific effects, JP2, since technical change is formulated as discrete shifts. For a given vector of inputs **x** the absolute difference in productivity between two time periods t and s ($t>s$) *can* be expressed as

$$E[y_t]-E[y_s]=\left(\alpha_t+\sum_{k=1}^{K}\alpha_{kt}x_k\right)-\left(\alpha_s+\sum_{k=1}^{K}\alpha_{ks}x_k\right).$$

Hence, the rate of technical change from period s to period t is given by

$$(9.11)\quad \begin{aligned} TC_{s,t}^{J2}(\mathbf{x};\alpha)= &\left\{f\left(\mathbf{x};\alpha|D_t=1,D_u=0,u\neq t\right)-f\left(\mathbf{x};\alpha|D_s=1,D_u=0,u\neq s\right)\right\}\\ &/f\left(\mathbf{x};\alpha|D_s=1,D_u=0,u\neq s\right)\\ =&\left\{\left(\alpha_t+\sum_{k=1}^{K}\alpha_{kt}x_k\right)-\left(\alpha_s+\sum_{k=1}^{K}\alpha_{ks}x_k\right)\right\}/f\left(\mathbf{x};\alpha|D_s=1,D_u=0,u\neq s\right)\end{aligned}.$$

Pure technical change and non-neutral technical change are given by

$$(9.12)\quad TCPUR_{s,t}^{J2}(\mathbf{x};\alpha)=\{\alpha_t-\alpha_s\}/f\left(\mathbf{x};\alpha|D_s=1,D_u=0,u\neq s\right),$$

and

$$(9.13)\quad TCNON_{s,t}^{J2}(\mathbf{x};\alpha)=\left\{\sum_{k=1}^{K}\alpha_{kt}x_k-\sum_{k=1}^{K}\alpha_{ks}x_k\right\}/f\left(\mathbf{x};\alpha|D_s=1,D_u=0,u\neq s\right),$$

respectively.

We now turn to the specification and estimation of the variance function. Harvey's specification of the variance function will be used for the above Just-Pope specifications. A nice property of the variance function $h(\mathbf{z})=\exp[\mathbf{z}\beta]$ in Harvey's formulation is that positive output variances are always ensured. This is not necessarily the case for other parametrizations of the variance function. The function v_{it} is defined as

$$(9.14)\quad v_{it}=u_{it}+\eta_i=h(\mathbf{x}_{it};\beta)+\eta_i=\left\{\exp\left[\mathbf{z}_{it}^{'}\beta\right]\right\}^{1/2}+\eta_i,$$

with variance

$$(9.15)\quad \mathrm{var}(v_{it})=\exp\left[\mathbf{z}_{it}^{'}\alpha\right]+\sigma_c^2$$

The first element in $\mathbf{z}_i$ will be taken as unity ($z_{i1}\equiv 1$). The other z's could be identical to, a subset of, or functions of, the x's in the mean function.[10]

The subscript c of the variance σ_c^2 of the firm-specific effect indicates that the variance of η_i may be a function of firm characteristics c such as year of entry, type of fish reared, location and licensed pen volume. Both RE models with homoskedastic firm-specific effects (i.e., $\sigma_c^2 = \sigma_\eta^2$ for all i) and RE models with heteroskedastic firm-specific effects will be estimated.

If the firm-specific effects are assumed fixed, then the last term is dropped from the above variance function and η_i ($i = 1, \ldots, N$) is instead estimated together with the parameters of the mean functions JP1 and JP2.

For the variance function $u = h(\mathbf{x})$ two parametric specifications were chosen. Model V1 is specified as

$$(9.16)\quad \ln\left(u^2\right) = \beta_0 + \beta_F x_F + \beta_K x_K + \beta_L x_L + \beta_M x_M + \beta_S x_S + A(t),$$

while Model V2 is given by

$$(9.17)\quad \ln\left(u^2\right) = \beta_0 + \beta_F \ln x_F + \beta_K \ln x_K + \beta_L \ln x_L + \beta_M \ln x_M + \beta_S \ln x_S + A(t).$$

See appendix 9.F for a discussion of some of the properties of these two variance function specifications.

Two parametrizations of the technical change function $A(t)$ will be tried, depending on whether V1 and V2 are estimated with JP1 or JP2; a time-trend variable specification and a time-dummy variable specification. These are given by

$$(9.18)\quad A(t) = \beta_t t + 0.5\beta_{tt} t^2,$$

and

$$(9.19)\quad A(t) = \sum_{t=85}^{92} \beta_t D_t\,,$$

respectively. Since there are two variance function parametrizations with two possible specifications of technical change, there are four different Just-Pope models to be estimated, as illustrated in figure 9.1.

[10] GLS estimation of models with this particular form of multiplicative heteroskedasticity is discussed in Harvey (1976) and Judge *et al.* (1988, pp. 365-9). Testing of models with this particular form of multiplicative heteroskedasticity is discussed in Godfrey (1978). See also Judge *et al.* (pp. 370-1).

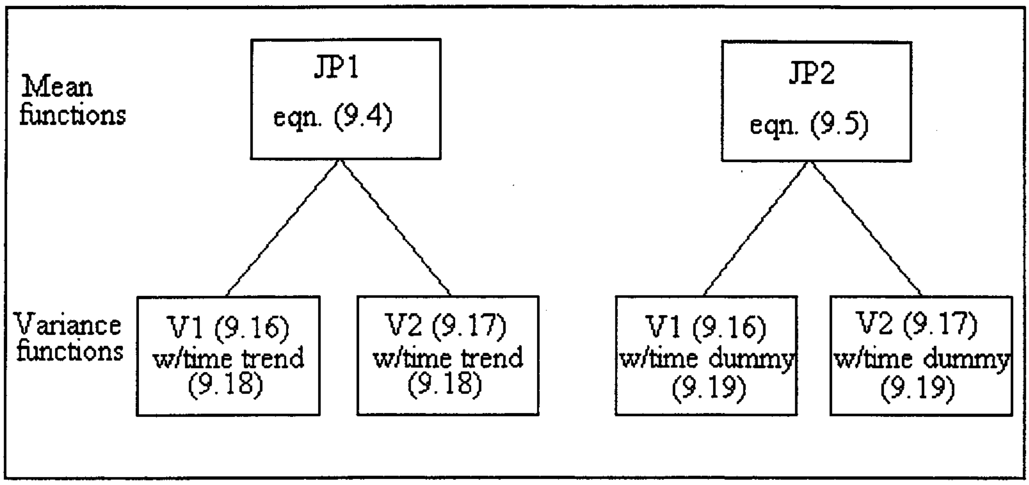

Figure 9.1. The Four Just-Pope Models

In the following we present some elasticity measures for the variance function $h(\cdot)$ which are analogous to the *RTS* and *TC* elasticities for the mean function $f(\cdot)$. The *total output variance elasticity* (*TVE*) in inputs is defined as

$$(9.20)\quad TVE(\mathbf{x};\beta) = \sum_k VE_k(\mathbf{x}) = \sum_k \frac{\partial h}{\partial x_k}\frac{x_k}{h(\mathbf{x})},$$

where VE_K is the output variance elasticity with respect to input k. For the parametric specifications V1 and V2 the total output variance elasticities are given by

$$(9.21)\quad TVE^{V1}(\mathbf{x};\beta) = \sum_k \beta_k x_k\ ,$$

and

$$(9.22)\quad TVE^{V2}(\mathbf{x};\beta) = \sum_k \beta_k\ ,$$

respectively. From the *TVE* expression for variance function V2 we see that the output variance elasticity with respect to input k is equal to the associated coefficient β_k. If input levels are normalised by their sample means this will also be the case for variance function V1 for the mean firm (i.e., $x_k = 1, \forall k$).

The elasticity of technical change for the variance function (*TCV*) with time trend variable is given by

$$(9.23)\quad TCV(\mathbf{x},t;\alpha) = \frac{\partial \ln h(\mathbf{x},t;\alpha)}{\partial t} = \beta_t + \beta_{tt}t$$

for both specification V1 and V2.

If time-specific effects are used instead of the time trend variable, the rate of technical change from period s to period t for the variance functions V1 and V2 are

(9.24) $TCV_{s,t}^{V1}(\mathbf{x};\beta) = \{\exp(\beta_t) - \exp(\beta_s)\} / \exp\left(\beta_0 + \sum_k \beta_k x_{k,s} + \beta_s\right)$

and

(9.25) $TCV_{s,t}^{V2}(\mathbf{x};\beta) = \{\exp(\beta_t) - \exp(\beta_s)\} / \exp\left(\beta_0 + \sum_k \beta_k \ln x_{k,s} + \beta_s\right)$

respectively.

9.3. Kumbhakar Production Function Specifications

In this dissertation the main purpose of estimating Kumbhakar specifications, where $f(\mathbf{x})$ has the well-known translog parametrization, is to use these as means to assess the reliability of estimated elasticities (e.g. returns to scale and technical change) from the linear quadratic parametrization of the Just-Pope model, and to compare marginal risks. Also, as noted earlier, the translog parametrization of the Khumbakar form has a more satisfactory implementation of firm-specific effects, and unlike the linear quadratic is homogeneity flexible (Driscoll *et al.*, 1992). The linear quadratic parametrization has only been used to a limited extent in empirical productivity studies, while the translog parametrization of the Khumbakar model is abundant in empirical studies. If the two parametrizations of $f(\mathbf{x})$ provide similar elasticities, our confidence in the linear quadratic form will increase.

We will also compare the estimated marginal risks from the Just-Pope and Khumbakar production functions, although one should have in mind that the Khumbakar formulation may violate Just-Pope postulates.

The general specification of the Kumbhakar model is given by

(9.26) $y_{it} = f(\mathbf{x}_{it};\alpha)e^{u_{it}} = f(\mathbf{x}_{it};\alpha)e^{h(\mathbf{x}_{it};\beta)\varepsilon_{it}}$,

which implies that $\text{var}(u_{it}) = [h(\mathbf{x}_{it};\alpha)]^2\text{var}(\varepsilon_{it})$.

Assuming $\varepsilon \sim N(0,\sigma_\varepsilon)$, the mean and variance of output is

(9.27) $E[y] = f(\mathbf{x})e^{h^2(\cdot)\sigma_\varepsilon/2}$,

and

(9.28) $\text{var}(y) = f^2(\mathbf{x})e^{h^2(\cdot)\sigma_\varepsilon}\left[e^{h^2(\cdot)\sigma_\varepsilon/2} - 1\right]$

respectively. We see that $E[y] \geq f(\mathbf{x})$ because $e^{h^2(\cdot)\sigma_\varepsilon/2} \geq 1$. We see by the above expression that for the Kumbhakar form it is not appropriate to use the terminology 'mean function' and 'variance function' for $f(\cdot)$ and $h(\cdot)$ respectively.

Marginal risk is given by

(9.29) $\partial \operatorname{var}(y) / \partial x_i = 2\left[e^{h^2(\mathbf{x})\sigma_\varepsilon/2} - 1\right]E[y]\partial E[y]/\partial x_i + E^2[y]\sigma_\varepsilon h(\mathbf{x})h_i(\mathbf{x})e^{h^2(\mathbf{x})\sigma_\varepsilon}$,

where

$$\partial E[y] / \partial x_i = f_i(\mathbf{x})e^{h^2(\mathbf{x})\sigma_\varepsilon/2} + f(\mathbf{x})\sigma_\varepsilon h(\mathbf{x})h_i(\mathbf{x})e^{h^2(\mathbf{x})\sigma_\varepsilon/2}$$

and $h_i(\mathbf{x})$ is the partial derivative of $h(\cdot)$ with respect to input i. An alternative specification of marginal risk is given by:

$$\begin{aligned}\partial \operatorname{var}(y) / \partial x_i &= 2f(\cdot)e^{\sigma_\varepsilon^2 h^2(\cdot)/2}\left(\partial f/\partial x_i\right)\left[e^{\sigma_\varepsilon^2 h^2(\cdot)} - 1\right] \\ &+2f(\cdot)e^{\sigma_\varepsilon^2 h^2(\cdot)/2} f(\cdot)\sigma_\varepsilon^2 h(\cdot)\sigma_\varepsilon^2\left(\partial h/\partial x_i\right)\left[2e^{\sigma_\varepsilon^2 h^2(\cdot)} - 1\right]\end{aligned} \quad (9.30)$$

Random firm-specific effects are implemented by reformulating the model as

(9.31) $y_{it} = f(\mathbf{x}_{it};\alpha)e^{v_{it}} = f(\mathbf{x}_{it};\alpha)e^{\eta_i + h(\mathbf{x}_{it};\beta)\varepsilon_{it}} = f(\mathbf{x}_{it};\alpha)e^{\eta_i}e^{h(\mathbf{x}_{it};\beta)\varepsilon_{it}}$.

Like in Kumbhakar, a translog function is used for $\ln f(\mathbf{x}; \alpha)$. This means that the production function with time trend, which we call Kumbhakar model 1 (K1), can be log-linearized as:

$$\begin{aligned}\ln y_{it} &= \ln f(\mathbf{x}_{it};t,\alpha) + v_{it} \\ &= \alpha_0 + \sum_k \alpha_k \ln x_{kit} + \alpha_t t + \frac{1}{2}\left(\sum_k\sum_j \alpha_{ij} \ln x_{kit} \ln x_{jit} + \alpha_{tt} t^2\right). \\ &+\sum_k \alpha_{tk} \ln x_{kit} t + v_{it}\end{aligned} \quad (9.32)$$

The production function with fixed time-specific effects, hereafter called Kumbhakar model 2 (K2), is specified as

$$\begin{aligned}\ln y_{it} &= \ln f(\mathbf{x}_{it};D_{85},\ldots,D_{92},\alpha) + v_{it} \\ &= \alpha_0 + \sum_k \alpha_k \ln x_{kit} + \frac{1}{2}\sum_k\sum_j \alpha_{ij} \ln x_{kit} \ln x_{jit} + \alpha_t D_t + \sum_k \alpha_{kt} \ln x_{kit} D_t + v_{it},\end{aligned} \quad (9.33)$$

with the same variance function specification as K1.

To compare the empirical results from the linear quadratic and translog specifications of $f(\mathbf{x})$, it is necessary to use dimensionless measures, i.e., elasticities. However, as can be seen from the expression for mean output in (9.27), introduction of input heteroskedasticity leads to much more complicated expressions for returns to scale (*RTS*) and technical change (*TC*). These elasticity estimates may also be sensitive to the nonlinear estimates of the parameters of

$h(\cdot)$. We choose to compare elasticities from the OLS estimates of the translog and linear quadratic, which implies the restriction $h(\cdot) = 1$ for the Kumbhakar model.

For the restricted Kumbhakar model 1 returns to scale (*RTS*) is given by

$$(9.34)\quad RTS^{K1}(\mathbf{x},t;\alpha) = \sum_k e_k(\mathbf{x};t,\alpha) = \sum_k \left(\alpha_k + \sum_j \alpha_{jk} \ln x_j + \alpha_{tk} t\right)$$

where $e_k(\mathbf{x}) = \partial \ln f(\cdot) / \partial \ln x_k$ is the output elasticity with respect to input k.

For model the restricted K2 *RTS* in year t is given by

$$(9.35)\quad RTS^{K2}(\mathbf{x}, D_{85},\ldots,D_{92};\alpha) = \sum_k \left(\alpha_k + 0.5\sum_j \alpha_{jk} \ln x_j + \alpha_{kt}\right)$$

For the restricted time trend model K1 the elasticity of technical change (*TC*) is given by

$$(9.36)\quad TC(\mathbf{x};t,\alpha) = \frac{\partial \ln f(\mathbf{x},t;\alpha)}{\partial t} = \alpha_t + \alpha_{tt} t + \sum_{k=1}^{K} \alpha_{kt} \ln x_k\ ,$$

while pure technical change and non-neutral technical change are given by

$$(9.37)\quad TCPUR(\mathbf{x};t,\alpha) = \alpha_t + \alpha_{tt} t\ ,$$

and

$$(9.38)\quad TCNON(\mathbf{x};t,\alpha) = \sum_{k=1}^{K} \alpha_{kt} \ln x_k\ ,$$

respectively.

For the restricted K2 model technical change is given by

$$(9.39)\quad \begin{aligned} TC_{s,t}^{K2}(\mathbf{x};\alpha) &= \left\{f(\mathbf{x};\alpha | D_t = 1, D_u = 0, u \neq t) - f(\mathbf{x};\alpha | D_s = 1, D_u = 0, u \neq s)\right\} \\ &= \left(\alpha_t + \textstyle\sum_{k=1}^{K} \alpha_{kt} x_k\right) - \left(\alpha_s + \textstyle\sum_{k=1}^{K} \alpha_{ks} x_k\right) \end{aligned}.$$

Pure technical change and non-neutral technical change are given by

$$(9.40)\quad TCPUR_{s,t}^{k2}(\mathbf{x};\alpha) = \alpha_t - \alpha_s,$$

and

$$(9.41)\quad TCNON_{s,t}^{k2}(\mathbf{x};\alpha) = \textstyle\sum_{k=1}^{K} \alpha_{kt} x_k - \sum_{k=1}^{K} \alpha_{ks} x_k,$$

Next, we turn our attention to the random part of the production function. Following Kumbhakar (1993), the composite error term v_{it} is specified with a linear function for $h(\mathbf{x};\ \beta)$:

$$(9.42)\quad v_{it} = u_{it} + \eta_i = h(\mathbf{x}_{it};\beta)\varepsilon_{it} + \eta_i = \left[\sum_l \beta_l x_{lit}\right]\varepsilon_{it} + \eta_i .$$

For the random effects specification the variance of v_{it} is given by

$$(9.43)\quad \mathrm{var}(v_{it}) = \left[h(\mathbf{x}_{it};\beta)\right]^2 \sigma_\varepsilon^2 + \sigma_\eta^2 = \left[\sum_k \beta_k x_{kit}\right]^2 \sigma_\varepsilon^2 + \sigma_\eta^2 .$$

If a fixed effects specification is used instead, σ_η^2 is dropped from the above expression, and the parameters η_i, $i = 1, \ldots, N$, is instead estimated together with the parameters of the mean functions K1 and K2.

Derivation of analogue output variance elasticities with respect to inputs and time, as proposed in section 9.2, leads to much more complicated expressions for the Kumbhakar form than was the case for the Just-Pope form. Here we will limit ourselves to estimate the total output variance elasticity (*TVE*), by exploiting that *TVE* is given by

$$(9.44)\quad TVE(\mathbf{x};\beta) = \sum_k E_k(\mathbf{x}) = \sum_k \frac{\partial\, \mathrm{var}(y)}{\partial x_k}\frac{x_k}{\mathrm{var}(y)},$$

where var(y) is given by (9.28) and $\partial \mathrm{var}(y)/\partial x_k$ is given by (9.29).

9.4. The Estimating Sample and Estimation Procedures

The original sample contained 560 farms observed from one to nine years, and 2280 observations. As discussed in Chapter 8, problematic observations were first dropped from the sample.[11] This reduced the sample to 2238 observations and 555 firms. Due to the estimation of fixed effects models farms that were observed for less than three years were also omitted, which further reduced the estimating sample to 372 farms and 1953 observations. Henceforth, the latter sample (n=1953) is called *sample 1* and the former (n=2238) *sample 2*. Most of the time we will use sample 1, while sample 2 will be utilised in conjunction with a random effects estimator which allows the inclusion of firms which are observed only one period. The panel data structure of the two samples are shown in table 9.D1 in appendix 9.D.

Appendix 9.D presents summary statistics for the estimating samples, both overall statistics (table 9.D1 and 9.D2 for sample 1, and table 9.D8-9.D9 for sample 2) and sample means by farm characteristics (tables 9.D4-9.D7 for sample 1, and tables 9.D10-9.D11 for sample 2).

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[11] Observations with input levels equal to zero were dropped from the sample. Observations with input level equal to zero is particularly problematic in the context of translog production function estimation, because the logarithm of zero is undefined.

According to tables 9.D4 the period 1985-93 was characterised by a large growth in output at the farm level. For the mean farm the production of salmon was three times higher in 1993 than in 1985. The output growth was accompanied by a similar increase in feed input and fish input. Other inputs experienced smaller growth rates, particularly labour input, which only increased by 25 %.

Prior to estimation, input levels are normalised to their sample means. This procedure simplifies analysis of estimated elasticities, particularly for the variance function.

According to table 9.D5 there is some differences in mean output across regions. The farms located in Hordaland had the highest mean output, 20 % above the national average output, while Nord-Trøndelag had the smallest mean output, 21 % below the national average. Mean output was above the national average for the four southernmost regions, while for the four northernmost regions mean production was below the national average.

The Just-Pope models will be estimated with fixed firm-specific effects by feasible generalised least squares (FGLS) and maximum likelihood (ML) methods. Random effects specifications of the Just-Pope models will also be estimated by FGLS. The Kumbhakar model will be estimated with fixed firm-specific effects by FGLS. Chapter three outlined FGLS and ML estimation procedures for Just-Pope models which also apply when fixed effects are introduced, since these can be implemented as dummy variables. Appendix 9.E presents estimators for random effects specifications of Just-Pope models. The FGLS estimation procedure for the Kumbhakar model with fixed effects is also presented in this appendix.

9.5. Empirical Testing for Heteroskedasticity

Prior to estimating Just-Pope models for the salmon industry, we test for the presence of significant marginal output risk in input levels. This is a test of hypothesis H2. In the context of a regression model, H2 states that the variance of the error term is a function of four of the inputs included in the mean production function. Before one proceeds to implement variance functions in the estimating models, it is wise to test that heteroskedasticity is actually present in the data. Several tests for heteroskedasticity, which vary both in generality and power, have been proposed in the literature.

Table 9.1. Goldfeld-Quandt test statistics

Central obs. omitted[a]	Materials	Feed	Capital	Labour	Fish
One	11.202	24.620	0.861	17.279	21.846
1/6	12.626	30.712	0.861	20.250	25.006

[a] The two subsamples which separate regressions were estimated for each have 976 and 814 observations respectively in the two tests.

First, Goldfeld-Quandt (G-Q) tests are undertaken for all five inputs (Goldfeld & Quandt, 1965).[12] The JP2 mean function was estimated under the assumption of firm homogeneity. The Goldfeld-Quandt test involves sorting of data by right-hand variables, splitting the observations in two subsamples, and estimating separate regressions for each subsample. The G-Q test was not undertaken with firm-specific effects included since one risks being left with only one observation of some firms. Two tests were run for each input; in the first test only the central observation was omitted, and in the second test the 1/6 central observations were omitted. Table 9.1 presents the test statistics, which are F distributed with $((n_2\text{-}r\text{-}K_2)/2, (n_1\text{-}r\text{-}K_1)/2)$ degrees of freedom, where r is the number of central observations skipped. For all inputs, except capital, the G-Q test rejects the null hypothesis of homoskedasticity with wide margins at conventional significance levels. For capital the homoskedasticity hypothesis is maintained.

Harvey- tests (Harvey, 1976) were also undertaken for model JP2 both with and without firm-specific fixed effects for variance functions V1 and V2. This test is based on the FGLS estimator. The null hypothesis of the Harvey test is that all coefficients of the multiplicative variance function except the intercept β_0, is zero. The Harvey test statistic is RSS/4.9348, where RSS is the residual sum of squares of the estimated variance function, is asymptotically distributed as a chi-square with degrees of freedom equal to the number of regressors. For the JP2 model without firm-specific effects the Harvey test statistics were 317.645 and 329.828 for variance functions V1 and V2, respectively. This is much higher than the critical chi-squared value of 27.688 (13 df) at the 1 % level. When fixed firm-specific effects were introduced into JP2, the estimated Harvey test statistics decreased to 165.249 and 169.466 for V1 and V2, which still is well above the critical value.

Other conventional tests of heteroskedasticity that were undertaken, but not reported here, also clearly rejected homoskedasticity.[13] All in all, the tests provide evidence of output heteroskedasticity in input levels, and accordingly indicate that output risk is present in salmon farming.

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[12] In the Goldfeld-Quandt test the observations are ranked by the independent variable which the variance is assumed to be a function of. The sample is then divided into two groups with n_1 and n_2 observations, and the model is estimated separately for these two sets of observations. The test statistic is

$$F = s_1^2 / s_2^2,$$

where s^2 is the OLS estimator of the regression variance σ^2.

[13] Other tests included White's general test (White, 1980), the Breusch-Pagan test (Breusch & Pagan, 1979), and Glejser's tests (Glejser, 1969).

9.6. Comparison of Estimates from Linear Quadratic and Translog

Earlier in this dissertation it has been argued that the linear quadratic has been far less used in estimation of production functions than the translog, for which there exists a large body of empirical applications in the literature. Furthermore, the specification of the firm-specific effects differs between the two functional forms; for the linear quadratic the firm-specific intercepts are added to $f(\mathbf{x})$, while for the translog the intercepts are multiplied with $f(\mathbf{x})$.

In the following we compare estimated elasticities from linear quadratic and translog parametrizations of $f(\mathbf{x})$. We use the JP2 form for the linear quadratic and K2 for the translog, and estimate by OLS. We are interested in elasticity measures derived from the OLS estimates of the parameters, which still are unbiased and consistent under heteroskedasticity. A comparison on the basis of OLS estimates is therefore appropriate, even though this dissertation postulates that input heteroskedasticity is present in salmon farming.

Table 9.2. Estimates of Mean Returns to Scale and Elasticity of Technical Change (*RTS/TC*) for Linear Quadratic (JP2) and Translog (K2) Mean Production Functions

Model	Pooled Models	Firm-Specific FE
JP2	0.953 / 0.033	0.896 / 0.044
K2	0.927 / 0.028	0.850 / 0.039

We estimated both pooled models and models with firm-specific fixed effects and pooled models. Parameter estimates are presented in table 9.A1 for the JP2 pooled model and 9.A3 for the K2 pooled model. For the pooled models we find that sample mean returns to scale (*RTS*) are 0.953 for JP2 and 0.927 for K2. According to table 9.2 the sample mean elasticity of technical change (*TC*) is 0.033 and 0.028 for JP2 and K2 (see also tables 9.B1 and 9.B3), respectively. The introduction of firm-specific fixed effects into the models result in an decrease in mean returns to scale for both JP2 and K2 to 0.896 and 0.850, respectively (see tables 9.B2 and 9.B4), and an increase in the mean elasticity of technical change for both models (0.044 and 0.039, respectively).

The introduction of firm-specific fixed effects instead of a common intercept was tested using likelihood ratio tests and F-tests for both models. The null hypothesis is that all firm-specific effects are equal to the common intercept, i.e., H_0: $\alpha_1 = \alpha_2 = ... \alpha_N = \alpha_0$. For the JP2 model the likelihood ratio test statistic is 541.4, while the statistic is 581.5 for the K2 model, which are clearly above the critical chi-square value for both models. F-tests also supported the fixed effects specification at all conventional confidence levels, with F-test statistics of 1.306 and 1.413 for JP2 and K2, respectively.

Analysis of the estimated sample mean elasticities from JP2 and K2 with firm-specific fixed effects reveals that the two models provide very similar values for the input elasticities $e_k(\mathbf{x})$. Feed is the most important input in terms of its output elasticity, with sample mean values of 0.487 and 0.483 for JP2 and K2, respectively, while the stock of fish in the beginning of the year is the second most important input, with sample mean input elasticity values of 0.276 for JP2 and 0.254 for K2, respectively. Materials, labour and capital have much smaller input elasticities in both models.

Although sample mean returns to scale are similar for the two parametrizations, we find some differences when we analyse by year and by regions. Figure 9.C2 shows the development in *RTS* over the data period. We see that for the two first years, 1985-86, the estimates of *RTS* are somewhat higher for the two JP2 models than for the two K2 models. However, in subsequent years the difference seems to be more between the pooled models and the fixed effects models. The pooled JP2 and K2 models provide somewhat higher estimates of *RTS* than the fixed effects models.

Next, we analyse *RTS* by region for the fixed effects specifications of JP2 and K2 (the pooled models provided similar patterns). As shown in figure 9.C4, the two specifications provide very different rankings of regions by *RTS*. The JP2 model tends to give the four northernmost regions a higher than average *RTS*, while the K2 tends to give the four southernmost regions a higher than average *RTS*.

Models JP2 and K2 provide only modest differences in mean year-to-year technical change. Furthermore, the estimated patterns of technical change over the data period are very similar according to figure 9.C1. The homogeneity assumption, i.e., the inclusion of firm-specific fixed effects, seems to be more important for the mean *TC* estimates over time than the functional form choice. The ranking of *TC* by region is also similar, as suggested by figure 9.C3.

Since the linear quadratic and translog are not nested models, we cannot test for structural differences between the two functional forms. One means of assessing to what extent they provide similar estimates of elasticities is to calculate correlation coefficients based on estimates at the observation level. For example, we can calculate the coefficient of correlation between the *RTS* estimates from JP2 and the *RTS* estimates from K2. If the *RTS* estimates are highly correlated, then the correlation coefficient is close to 1.[14]

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[14] The coefficient of correlation between returns to scale estimates from JP2 and K2 is defined as

$$\rho = \frac{\operatorname{cov}(RTS^{JP2}, RTS^{K2})}{\sqrt{\operatorname{var}(RTS^{JP2})\operatorname{var}(RTS^{K2})}}.$$

Table 9.B31 (a)-(c) presents correlation coefficients for the models estimated in this section (see App. 9.B). According to the table functional form is more important for the coefficient of correlation than the homogeneity assumption. We see that the elasticity estimates derived from the two JP2 models, the pooled model and the model with firm-specific fixed effects, are highly correlated; all correlation coefficients are above 0.80. The correlation coefficients for *RTS* and *TC* are 0.97 and 0.90, respectively. To some extent the same applies to the two K2 models, where the correlation coefficients are 0.78 and 0.91 for *RTS* and *TC*, respectively. However, when we compare between functional forms, we find that the correlations are significantly weaker. This is particularly pronounced for returns to scale (*RTS*), with correlation coefficients of 0.24 and 0.31 for the pooled models and the fixed effects models, respectively. The correlation is considerably higher for the rate of technical change (*TC*), with calculated values of 0.71 for the pooled models and 0.67 for the fixed effects models.

Overall, we find that the linear quadratic and translog parametrizations estimated here provide pretty similar overall sample mean elasticities and sample means by year, both for the pooled models and for the models with firm-specific fixed effects, but that *RTS* differ somewhat when broken down by region. At the observation level the differences between the elasticity estimates derived from the linear quadratic and the translog specifications are much larger, as measured by the coefficient of correlation.

According to the estimated models, the homogeneity assumption also has an effect on elasticity estimates. In fact, the homogeneity assumption seems to be more important for the overall sample mean elasticity estimates than the choice of functional form. The pooled models predict higher returns to scale and a lower rate of technical change than the fixed effects models. Tests of the homogeneity assumption suggest that firm-specific effects should be included in the models.

9.7. Comparison of Estimates from Time Trend and Time Dummy Specification of Just-Pope Model

Previously, we have argued that the standard time trend variable model may not be the most appropriate specification of technical change for the salmon farming industry, due to biophysical productivity shocks which we have no data for, and because it is problematic to envision technical change as a smooth process. In the following we compare the elasticity estimates from the time trend model JP1 with the more flexible time dummy model JP2. For mean production function elasticities comparisons are made for OLS estimates and FGLS estimates based on both variance functions V1 and V2.[15]

A comparison of overall sample mean elasticities, cf. table 9.3 below, reveals that estimates of returns to scale and elasticities of technical change based on the time trend model JP1 are very similar over estimators. This is also the case for the time dummy model JP2. The two model specifications provide roughly similar *TC* estimates for the mean firm, but the time dummy model JP2 provides a somewhat higher estimate of *RTS* than the time trend model JP1 for all estimators.[16]

Table 9.3. Estimates of Mean Returns to Scale and Elasticity of Technical Change (*RTS*/*TC*) for Different Time Specifications and Estimators

Model	OLS	FGLS with V1	FGLS with V2
JP1	0.841 / 0.038	0.841 / 0.037	0.840 / 0.037
JP2	0.896 / 0.044	0.896 / 0.041	0.904 / 0.041

However, an examination of the patterns of estimated *TC* and *RTS* over time show substantial differences between the JP1 and the JP2 model. Figure 9.C5 suggests that *TC* exhibit a cyclical pattern in all estimated models, but that the peaks and throughs do not appear in the same year for the JP1 and JP2 models. The cyclical pattern is also much more pronounced in the estimated JP2 models. According to the estimated time dummy models productivity growth was zero or negative from 1985 to 1986. From 1986 to 1988 productivity growth was high, followed by four years of much lower rates of productivity growth. The productivity growth rate increased again from 1992 to 1993.

[15] Within-transformed variables were used instead of firm dummies (LSDV) in the FGLS estimation procedure for the fixed effects models. With a large number of firms the computational requirements are reduced substantially when using the within estimator instead of the LSDV estimator. Within and LSDV estimates are identical when estimated by OLS in the first step, but may diverge in the third step because the firm-specific fixed effects are not weighted. However, the differences in mean function elasticities derived from third stage within estimates and those derived from LSDV estimates were in practice found to be negligible, with differences in sample mean *RTS* and *TC* of less than 1 %.

[16] It should be noted that the empirical models also were estimated with the age of the farms (i.e., years since establishment) implemented as regressors, with the rationale that farm age may be a proxy for learning-by-doing. However, the coefficients of the first and second order age terms were both insignificant. Collinearity with the time trend variable does not explain the insignificance of the age coefficients; the pair-wise correlation coefficients were small, and the age coefficients remained insignificant when the regressors involving time were dropped from the model specification. These empirical results suggest that time-specific effects were more important than firm age effects for productivity changes.

Figure 9.C6 depicts the estimated returns to scale from the estimated JP1 and JP2 models over time. For the JP1 models *RTS* is pretty stable over the data period, fluctuating between 0.81 and 0.88. According to the JP2 models, on the other hand, *RTS* starts around 1.04-1.11 in 1985 and then drops rapidly, until it reaches approximately the same levels as in the JP1 models in 1987 and thereafter fluctuates between 0.78 and 0.92, depending on the year and estimator.

Examination of the sample mean output elasticities with respect to inputs leads to another interesting finding. All the time trend JP1 models (both estimated by OLS and FGLS) provide negative mean output elasticities with respect to labour input, while none of the mean input elasticities are negative for the time-dummy specifications (see tables 9.B5-9.B7 for the JP1 models, and tables 9.B2, 9.B8 and 9.B9 for the JP models). Thus, the estimated time-dummy models provide more sensible results in this respect than the time trend models.

Next, we examine marginal risks and the derived elasticities from the variance functions V1 and V2, the elasticity of technical change of the variance function (*TCV*) and the total variance elasticity (*TVE*). The variance functions that are estimated together with the time trend model JP1 have first- and second-order time variables, while for the variance functions estimated together with JP2 the time trend variables are replaced by time dummies, cf. section 9.2.

Table 9.4 presents the estimated input coefficients for all four estimated models that we compare in this section. For model V2 these coeffcents can also be interpreted as output variance elasticities with respect to inputs. The same applies for V1 if evaluated in mean input levels, since input levels have been normalised prior to estimation. In all four models feed, capital and fish are significantly risk increasing at conventional confidence levels. According to the estimated variance function V1, both when estimated with JP1 and JP2, materials input have no significant risk effects. However, in both estimated V2 models, materials are significantly risk increasing at the 5 % level. Labour is risk decreasing in all four models, but the labour input coefficient is only significantly negative when V1 and V2 are estimated together with JP2. Hence, the time trend model rejects while the time dummy model supports the hypothesis of decreasing marginal output risk for labour.

Table 9.4. FGLS estimates of Input Parameters of Variance Functions V1 and V2

	Materials	Feed	Capital	Labour	Fish
V1 and JP1	-0.027	0.580 [c]	0.096 [a]	-0.030	0.263 [c]
V2 and JP1	0.153 [b]	0.537 [c]	0.230 [c]	-0.104	0.162 [b]
V1 and JP2	-0.061	0.531 [c]	0.112 [a]	-0.189 [a]	0.273 [c]
V2 and JP2	0.145 [b]	0.470 [c]	0.159 [b]	-0.204 [a]	0.144 [b]

[a] Significant at 10 % level (one-tailed test), [b] significant at 5 % level, [c] significant at 1 % level.

Figure 9.C7 plots the elasticities of technical change derived from the variance functions. We see that the TCVs from the variance functions with time trend variables are very stable over time, as implied by the restrictions imposed by the time trend specification. According to the estimated time trend specification of V1, the output variance increased over the data period, while the time trend specification of V2 predicts a decrease in the output variance. On the other hand, the TCVs derived from the time dummy specifications exhibit large year-to-year fluctuations. There are four years of increasing output risk and four years of decreasing output risk, according to the time dummy specifications. If there were substantial year-to-year changes in the biophysical conditions during the data period, this may be reflected in both negative and positive TCVs.

For variance function V2 the total variance elasticity is restricted to be constant both for the time trend and time dummy specification. According to figure 9.C8, *TVE* is 0.979 for V2 with time trend and 0.713 for V2 with time dummies. For variance function V1 *TVE* increases during the data period, which can be attributed to the fact that the marginal input risks are positive for all inputs except labour, and that the level of input use increases during the data period.

An overall assessment of the empirical results from the time trend models and the time dummy models leads to the conclusion that the more flexible time dummy representation of technical change is more appropriate for this empirical application. The time dummy model strongly supports year-to-year fluctuations in technical change both for the mean function and for the variance function. The time trend and time dummy model also provide different results on the marginal risk properties of inputs, particularly for labour.

9.8. FGLS vs. ML Estimation of Just-Pope Models

In this section we compare the three stage FGLS estimates with the iterative ML estimates. Previously, we have discussed the performance of FGLS vs. ML in the context of Just-Pope models. ML estimates of the variance function parameters are asymptotically more efficient than the FGLS estimates. Saha *et al.* (1997) showed that the FGLS and ML first order conditions with respect to the parameters of the variance function can never be identical (see section 3.7), and that iteration of the FGLS steps will not lead to ML estimates. Saha *et al.* also found the ML estimator to be superior in Monte Carlo simulations with Cobb-Douglas parametrization and no firm-specific effects. However, the simulation study in this dissertation, which assumes a data generating process which is more similar to the empirical models estimated here, does not provide the same support for the ML estimator relative to the FGLS estimator, although it suggests that the ML estimator is the preferred estimator for the variance function parameters (see Ch. 5).

The basis for the comparison is the time dummy Just-Pope model JP2. Tables 9.A8 and 9.A9 present FGLS parameter estimates for JP2 with V1 and V2, respectively. The ML estimates are presented in tables 9.A10 and 9.A11. First, we study the estimated mean production functions. We find that there are no dramatic differences between the FGLS and ML first-order input coefficients. Furthermore, when we examine the overall sample mean output elasticities with respect to inputs, we also find very similar values (see tables 9.B8 and 9.B9 for FGLS estimates and tables 9.B10 and 9.B11 for ML estimates). This leads to very similar estimates of returns to scale, as seen from table 9.5 below. According to the same table the overall mean technical change estimates are identical for variance function V1, and only slightly different with V2.

Table 9.5. FGLS and ML Estimates of Mean Returns to Scale and Elasticity of Technical Change (*RTS/TC*) for Just-Pope Model with Firm-Specific Fixed Effects

Model	FGLS	ML
JP2 and V1	0.896 / 0.041	0.899 / 0.041
JP2 and V2	0.904 / 0.041	0.916 / 0.039

If we break down *RTS* and *TC* by year we come to similar conclusions as for the overall means (see figures 9.C9 and 9.C10). In most years mean *RTS* and *TC* are roughly identical for the two estimators.

Table 9.6. FGLS and ML Estimates of Input Parameters of Variance Functions V1 and V2

Coeff.	FGLS		ML	
	V1 and JP2	V2 and JP2	V1 and JP2	V2 and JP2
Materials	-0.061	0.145 [b]	0.126 [c]	0.358 [c]
Feed	0.531 [c]	0.470 [c]	0.532 [c]	0.376 [c]
Capital	0.112 [a]	0.159 [b]	0.159 [c]	0.154 [c]
Labour	-0.189 [a]	-0.204 [a]	-0.122 [a]	-0.028
Fish	0.273 [c]	0.144 [b]	0.350 [c]	0.163 [c]

[a] Significant at 10 % level (one-tailed test), [b] significant at 5 % level, [c] significant at 1 % level.

We find larger differences when we compare the FGLS and ML estimates of variance function parameters and derived elasticities. The input coefficients have the same signs, except for materials in V1, which is negative according to the FGLS estimator and positive according to the ML estimator. Except for labour input in V2, the absolute values of the t-ratios are generally higher for the variance functions estimated by ML, which is largely due to the fact that their asymptotic standard errors are equal to the FGLS standard errors multiplied by the factor $(2/4.9348)^{0.5}$.[17] Based on the estimated t-ratios one will therefore tend to draw inferences with more confidence from the ML estimates than from the FGLS estimates.

Figure 9.C11 plots the elasticities of technical change for the FGLS and ML estimates of the variance functions (*TCV*). One can clearly see that for both V1 and V2, the ML estimator predicts much smaller fluctuations in *TCV* than the FGLS estimator. The pattern of *TCV* over time is largely the same for the two estimators. According to figure 9.C12, the total variance elasticity (*TVE*) based on ML estimates is higher than the *TVE* based on FGLS. For variance function V1 *TVE* follows the same pattern for the two estimators, but the ML estimate is at least 0.2 higher in each year. For V2 the FGLS estimate of *TVE* is 0.713 in each year, while the ML estimate is 1.022.

An overall assessment of the findings in this section leads to the conclusion that the ML and FGLS estimators provide more similar estimates for the mean production function than for the variance function. However, according to the estimated parameters of the variance function the ML estimator seldom provides results that are significantly different from those of the FGLS estimator.

9.9. Effects of Assuming Firm Homogeneity for FGLS and ML Estimates

In section 9.6 we compared estimates from pooled models, i.e., models assuming homogeneous firms, versus models with firm-specific effects in the context of the OLS estimator. Both for the linear quadratic and the translog functional form we found that the homogeneity assumption had significant consequences for the estimated mean function elasticities (*RTS* and *TC*). Likelihood ratio and F-tests based on the (consistent) OLS estimates supported the presence of firm-specific effects, both for the linear quadratic and the translog parametrization of the mean function. In this section we extend the analysis to the FGLS and ML estimators, and examine both the mean and variance functions, with an emphasis on the effect on risk estimates of implementing firm-specific effects.

First, we examine the effects of going from a pooled model to a model with firm-specific effects on the elasticities derived from the mean production function. Table 9.7 presents estimates of means returns to scale (*RTS*) and elasticity of technical change (*TC*) from pooled JP2 models and JP2 with firm-specific fixed effects. The introduction of firm-specific effects leads to a 4-5 % reduction in mean returns to scale. We also see that there are pronounced differences in technical change estimates between the pooled models and the fixed effects models; for the four pooled models mean *TC* ranges between 2.6 % and 2.8 %, while mean *TC*

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[17] See section 3.7.

ranges between 3.9 % and 4.1 % for the fixed effects models. These findings are similar to those based on the OLS estimates of JP2 in a previous section.

Table 9.7. FGLS and ML Estimates of Mean Returns to Scale and Elasticity of Technical Change (*RTS/TC*) for Just-Pope Models without and with Firm-Specific Effects

Model	Pooled	Fixed Effects
JP2 and V1 by FGLS	0.951 / 0.027	0.896 / 0.041
JP2 and V2 by FGLS	0.953 / 0.027	0.904 / 0.041
JP2 and V1 by ML	0.950 / 0.028	0.899 / 0.041
JP2 and V2 by ML	0.954 / 0.026	0.916 / 0.039

The finding that the assumption on the firm-specific effects is more important than the choice of estimator for the mean production function elasticities is reinforced if we break down *TC* and *RTS* by year. According to figure 9.C13 the FGLS and ML estimates of the fixed effects specification of JP2 predicts the same pattern of technical change over time. The same holds for the pooled specification. It is clear from the figure that the conclusions regarding the development in productivity over the data period depend on whether firms are assumed homogenous or not. Figure 9.C14 suggests that similar conclusions can be found for returns to scale.

Next, we examine the effects of going from pooled models to models with firm-specific effects for variance function elasticities. Overall sample mean variance function elasticities are presented in table 9.8. For the FGLS estimator introduction of firm-specific effects leads to an increase in the total variance elasticity (*TVE*). However, the effect on *TVE* is negligible for the ML estimator; mean *TVE* increases slightly when JP2 is estimated with V1, but decreases when JP2 is estimated with V2. When we examine mean technical change of the variance function (*TCV*), we find that the effect of going from the pooled model to firm-specific effects is pretty similar for the FGLS and ML estimators, as *TCV* falls from 18-21 % to 6-8%. Hence, the pooled models predict much larger increase in output variance in salmon farming during the data period than the pooled models.

Table 9.8. FGLS and ML Estimates of Total Variance Elasticity and Technical Change of Variance Function (*TVE/TCV*) for Just-Pope Models without and with Firm-Specific Effects

Model	Pooled	Fixed Effects
JP2 and V1 by FGLS	0.884 / 0.212	0.666 / 0.082
JP2 and V2 by FGLS	0.930 / 0.179	0.713 / 0.066
JP2 and V1 by ML	1.037 / 0.205	1.045 / 0.066
JP2 and V2 by ML	1.043 / 0.197	1.022 / 0.061

A graphic analysis of the technical change of the variance function V1 over time, as shown in figure 9.C15, suggests that both the choice of estimator and the homogeneity assumption is important for the conclusions one draws regarding the development of *TCV*. According to figure 9.C15 the pooled JP2 model estimated by FGLS provides the most erratic pattern for *TCV*, while the fixed effects specification estimated by ML gives the smallest year-to-year changes. For the total variance elasticity (*TVE*) associated with V1, figure 9.C16 suggests that the homogeneity assumption is not so important if the model is estimated by ML, but more important when estimated by FGLS.

Table 9.9 presents input coefficients of the estimated variance functions, which can be interpreted as input elasticities for V2, and also for V1 when evaluated at the normalised sample mean input levels. Implementation of fixed effects has no effect on the signs of input coefficents, regardless of variance function (V1 or V2) or estimator (FGLS or ML). For both the FGLS and ML estimator we see that the feed coefficient is relatively stable when going from the pooled models to the fixed effects specifications. The most dramatic change is found for the FGLS estimates of the labour coefficient, which is negative but not significantly different from zero for the pooled models, but takes much larger negative values and becomes significantly negative both for V1 and V2 when firm-specific effects are implemented. The ML parameter estimates are generally highly significant both with and without firm-specific effects. For the FGLS estimator, on the other hand, there is a tendency that more coefficients become significant at conventional levels when going from the pooled models to models with firm-specific effects, particularly for V1.

Table 9.9. Estimates of Input Parameters of Variance Functions from Pooled JP2 Model and JP2 with Firm-Specific Fixed Effects

Coeff.	Pooled Model JP2				JP2 Model with Fixed Effects			
	FGLS		ML		FGLS		ML	
	V1	V2	V1	V2	V1	V2	V1	V2
Materials	-0.022	0.171 [c]	0.124 [c]	0.356 [c]	-0.061	0.145 [b]	0.126 [c]	0.358 [c]
Feed	0.569 [c]	0.468 [c]	0.509 [c]	0.355 [c]	0.531 [c]	0.470 [c]	0.532 [c]	0.376 [c]
Capital	0.073	0.137 [b]	0.232 [c]	0.171 [c]	0.112 [a]	0.159 [b]	0.159 [c]	0.154 [c]
Labour	-0.037	-0.093	-0.149 [c]	-0.034	-0.189 [a]	-0.204 [a]	-0.122 [a]	-0.028
Fish	0.300 [c]	0.248 [c]	0.322 [c]	0.194 [c]	0.273 [c]	0.144 [b]	0.350 [c]	0.163 [c]

[a] Significant at 10 % level (one-tailed test), [b] significant at 5 % level, [c] significant at 1 % level.

An overall assessment suggests that for mean production function elasticity estimates, the inclusion or exclusion of firm-specific fixed effects is more important for elasticity estimates than the choice of estimator. For the variance function both the inclusion of fixed effects and the choice of estimator have significant influence on elasticity estimates. We also found that for the FGLS estimator, our ability to make statements on marginal risks of inputs increases when fixed effects are introduced, because variance function coefficients become more significant.

9.10. Comparison of Estimates from Fixed Effects and Random Effects Specifications

In this section, we examine if a random effects (RE) specification is appropriate, and what effects the RE assumption has on estimated parameters and derived elasticities. One advantage of the RE approach, if its assumption of no correlation between the regressors $\mathbf{x}_{it}$ and the firm-specific effect η_i is valid, is that efficiency increases, since one only estimates the variance of the firm specific effect instead of all the N fixed effects. The RE approach has become popular in analyses that involve large longitudinal data sets, partly because of computational considerations, because with an RE model it is not necessary to estimate a large number of so-called nuisance parameters (here: the firm-specific effects). Another advantage of the RE approach is that it allows us to introduce other time-invariant variables, such as region dummies, into the model. Finally, there are estimators which allow us to include firms that are observed for only one period in the estimating sample when an RE approach is used. This is important if there are structural differences, e.g. with respect to mean productivity and output risk, between firms which are observed for only one period and firms that are observed for longer time periods.[18]

We continue to focus on the JP2 mean production function, which we estimate with both variance functions V1 and V2. Two different FGLS estimation procedures are used for the random effects specification. Both estimators are presented in appendix 9.E. In the first FGLS estimation procedure (called *RA1*), the fixed effects model is estimated in the first step, and used as basis for estimating the variance function and the variance of the firm-specific effect. This estimator will therefore provide the same parameter estimates for the variance function. Furthermore, unlike other RE estimators, this estimation procedure uses estimates of the firm-specific intercepts, so there is no bias due to correlation between $\mathbf{x}_{it}$ and η_i. However, the *RA1* procedure does not allow the inclusion of firms which are observed only one period.

The second FGLS estimation procedure (called *RA2*), does not involve estimation of a fixed effects model in the first step. It therefore has the advantage that firms which are observed for

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[18] Firms which are observed only one period can be included in the estimating sample with a fixed effects approach if it is possible to group firms in *cohorts* based on observable firm characteristics, such as regional location and year of establishment (Heshmati & Kumbhakar, 1997). The firms must then be assumed to be homogeneous within each cohort, i.e., the firm-specific effect η_i takes the same value for all firms in a cohort.

only one period can be included in the estimating sample. However, the RA2 procedure provides estimates of the observation specific residuals u_{it} that differ from those of the fixed effect model and the estimates from the RA1 estimator, and thus different estimates of the parameters of the variance function. This is because the estimation procedure assumes no correlation between $\mathbf{x}_{it}$, η_i and u_{it}. In the first step a pooled OLS regression is estimated by OLS, and in the second and third steps the variances of the observation-specific error component and the firm-specific error component are calculated under the implicit assumption of uncorrelatedness.

Hausman tests were undertaken to test for appropriateness of the random effects specification RA1 versus the fixed effects specification. The Hausman test statistic is chi-square distributed with degrees of freedom equal to the number of regressors in the model. Large values of the Hausman statistic argue in favour of the fixed effects model over the random effects model. Even if the Hausman test supports the RE specification, the FE specification will still provide consistent parameter estimates. However, the RE estimates will be more efficient, since one estimates one variance parameter instead of N firm-specific effects. First, we estimated the JP2 model with random effects under the assumption of homoskedastic observation-specific effects with the RA1 FGLS estimator, and tested it against the JP2 model with fixed effects estimated by OLS. The Hausman test statistic under homoskedasticity was 51.8, which is outside the 95 % area. Hence, the RE specification is favoured under the homoskedasticity assumption. Next, the JP2 model was estimated with both variance functions. For variance function V1 the Hausman test rejected the RE assumption, with a test statistic of 143.5. For variance function V2, on the other hand, the Hausman test statistic of 63.6 supported the RE specification at conventional confidence levels.

First, in table 9.10, we compare the overall sample mean elasticities derived from the estimated parameters of the mean production function JP2. We find that the *RTS* estimates from both the RA1 and RA2 estimates of the RE model lie between the FGLS estimates from the fixed effects and the pooled JP2 models. The RA2 estimates of the rate of technical change (*TC*) also lie between pooled and fixed effects model estimates, but substantially closer to the latter. However, the RA1 estimates of the rate of technical change, are above both the pooled and fixed effects model estimates, with values of 4.2 % when JP2 is estimated with variance function V1, and 4.6 % when estimated together with V2. Both for *RTS* and *TC* the random effects RA1 estimates are closer to the estimates from the fixed effects specification than those from the pooled models. This also applies to the *TC* estimates from RA2, while the absolute difference between the RA2 estimates of *RTS* and the pooled and FE estimates are roughly equal.

Table 9.10. Estimates of Mean Returns to Scale and Elasticity of Technical Change (*RTS/TC*) under Random Effects and Fixed Effects Assumptions

Model	Pooled	FE	RE (RA1)	RE (RA2)
JP2 and V1 by FGLS	0.951 / 0.027	0.896 / 0.041	0.914 / 0.042	0.930 / 0.039
JP2 and V2 by FGLS	0.953 / 0.027	0.904 / 0.041	0.914 / 0.046	0.934 / 0.039
JP2 and V1 by ML	0.950 / 0.028	0.899 / 0.041	N.A.*	N.A.*
JP2 and V2 by ML	0.954 / 0.026	0.916 / 0.039	N.A.*	N.A.*

* N.A. = Not Available

Next, we examine the predicted time patterns of *TC* and *RTS*. According to figure 9.C17 both RA1 and RA2 estimates of technical change are close to the fixed effects estimates for JP2 estimated together with variance function V1, while there is a more pronounced difference between the former and the pooled FGLS estimates. For *RTS* we come to a similar conclusion (see figure 9.C18).

The variance function parameter estimates are equal for the fixed effects model and the random effects model estimated by RA1, because in both cases the estimated variance functions are based on the consistent first-stage residuals from a fixed effects model estimated by OLS. For RA2, on the other hand, the residuals used in the estimation of the variance function rely on the validity of the random effects assumptions of no correlation between the error components, and between the error components and regressors. In table 9.11 we see that the variance elasticities *TVE* and *TCV* estimated by RA2 differ substantially from both the fixed effects and the pooled model FGLS estimates for variance function V1. For variance function V2 the total variance elasticity estimated by RA2 is an intermediate case between the estimates from the pooled and fixed effects models. But, as for V1, the estimate of the technical change of the variance function is negative, while both the FE and pooled models predict positive TCVs.

Table 9.11. Estimates of Total Variance Elasticity and Technical Change Variance Function (*TVE/TCV*) under Random Effects and Fixed Effects Assumptions

Model	Pooled	FE	RE (RA1)	RE (RA2)
JP2 and V1 by FGLS	0.884 / 0.212	0.666 / 0.082	0.666 / 0.082	1.056 / -0.085
JP2 and V2 by FGLS	0.930 / 0.179	0.713 / 0.066	0.713 / 0.066	0.826 / -0.077
JP2 and V1 by ML	1.037 / 0.205	1.045 / 0.066	N.A.*	N.A.*
JP2 and V2 by ML	1.043 / 0.197	1.022 / 0.061	N.A.*	N.A.*

* N.A. = Not Available

The conclusion that the variance function estimates from RA2 are problematic, is reinforced when we examine the time patterns of *TCV* (figure 9.C19). In several years the RA2 estimates predict a development in *TCV* that is the opposite of the pooled and FE estimates. The time pattern of the total variance elasticity (*TVE*) is more similar to those of the pooled and fixed effects model.

When we compare variance function coefficients from the fixed effects model estimated by FGLS and the random effects model estimated by RA2, we find that the choice of estimator has severe consequences for the conclusions regarding the direction and magnitude of marginal risks. Table 9.12 presents the estimated input parameters of the two models. For V1 four of five input coefficient change sign, although the sign change is not significant according to estimated t-ratios. For V2 two coefficients change sign.

Table 9.12. Fixed and Random Effects Estimates of Input Parameters of Variance Functions V1 and V2

Coeff.	Fixed Effects by FGLS		Random Effects by RA2	
	V1 and JP2	V2 and JP2	V1 and JP2	V2 and JP2
Materials	-0.061	0.145 [b]	0.157 [c]	0.178 [c]
Feed	0.531 [c]	0.470 [c]	0.877 [c]	0.643 [c]
Capital	0.112 [a]	0.159 [b]	-0.020	-0.092
Labour	-0.189 [a]	-0.204 [a]	0.154	0.065
Fish	0.273 [c]	0.144 [b]	-0.111	0.032

[a] Significant at 10 % level (one-tailed test), [b] significant at 5 % level, [c] significant at 1 % level.

When comparing the fixed and random effects approaches, an overall conclusion is that the specification of the firm-specific effects does not have any dramatic consequences for the estimated elasticities derived from the mean function. This applies for both random effects estimators used here. However, use of the random effects FGLS estimator RA2, which relies on the validity of the random effects assumptions, has severe consequences for marginal risk estimates and elasticities derived from the variance function. Hence, it seems like one should be extremely cautious with drawing conclusions regarding the structure of production risk based on FGLS estimators that rely on the validity of the random effects assumptions.

9.11. Estimates from Random Effects Model for the Full Sample

As mentioned in the previous section, the FGLS estimator RA2 allows one to estimate the random effects models also for firms that are observed only one period. Although the RA2 estimator provides different results for the variance function than the other estimator, we would like to assess what effects omission of farms which were observed only one or two periods have on estimated parameters and elasticities. It may be the case that the reason some farms are observed only for a short period, is that they are less productive in terms of mean output, or have higher output risk, or have experienced very adverse outcomes of biophysical stochastic variables, as discussed in Chapter eight.

The number of observations increases by 285, to 2238, when going from sample 1 to sample 2. Summary statistics for sample 2 are presented in appendix 9.D. From the appendix we see that the introduction of new observations does not lead to dramatic changes in overall sample mean output and input levels. However, a majority of observations that are added, are from the latter part of the data period, particularly 1992 and 1993.

Table 9.13. Overall Sample Mean Estimates of Mean Function and Variance Function Elasticities

Mean Production Function Elasticities (*RTS*/*TC*)

Model	FE on sample 1	RE on sample 1	RE on sample 2
JP2 and V1 by FGLS	0.896 / 0.041	0.930 / 0.039	0.962 / 0.027
JP2 and V2 by FGLS	0.904 / 0.041	0.934 / 0.039	0.963 / 0.026
Variance Production Function Elasticities (*TVE*/*TCV*)			
Model	**FE on sample 1**	**RE on sample 1**	**RE on sample 2**
JP2 and V1 by FGLS	0.666 / 0.082	1.056 / -0.085	0.437 / 0.128
JP2 and V2 by FGLS	0.713 / 0.066	0.826 / -0.077	0.228 / 0.141

The coefficients of the RE model JP2 estimated on the full sample by FGLS procedure RA2 are reported in tables 9.A20 and 9.A21. Table 9.13 compares elasticity estimates of the fixed effects model estimated by FGLS on sample 1, and the random effects model estimated by FGLS procedure RA2 on sample 1 and 2. The introduction of new observations leads to a slight increase in returns to scale, around 3 %, when compared with the RA2 estimates on sample 1. There is a pronounced decrease in the rate of technical change when going from sample 1 to sample 2; *TC* is more than a percentage point lower when estimated on the full sample. The time patterns of *RTS* and *TC* (not depicted here) were similar for the two samples.

For the variance function the changes are more dramatic. The overall sample mean estimate of the total variance elasticity (*TVE*) is reduced to less than half for variance function V1, and to less than a third for variance function 2. The *TVE* values are lower than any estimate provided by other specifications and estimators. An explanation for the low *TVE* estimates is the parameter estimates for the variance functions, which are reported in tables 9.A20 and 9.A21. More inputs have negative marginal risks than in any of the previously estimated models. The rate of technical change of the variance function (*TVE*) experiences a large increase through the introduction of new observations. The RA2 estimates of *TVE* on sample 1 was negative, while the RA2 estimates on sample 2 are positive and even larger than the estimates from the fixed effects model.

Based on the above findings one may conclude that if one is primarily interested in the mean production function, the inclusion of firms observed one or two periods in the estimating sample does not lead to significant changes in the empirical findings. For the variance function the impact of adding observations is much more dramatic. However, as stated in the previous section, the uncertainty regarding the validity of the random effects estimator that are used to estimate the variance function, and the substantial differences in estimates we find for sample 1 when compared with all other estimated models, implies that the reliability of the variance function estimates is questionable.

9.12. Models with Region-Specific Effects

Hypothesis H4 asserted that there are differences across regions with respect to mean productivity and output risk levels. This section tests this hypothesis by means of production models which incorporate region-specific effects on mean output and region-specific effects on output variance. The preferred mean production function specification is still JP2. The salmon farms in the data set are located in 10 regions, but we have chosen to group the very few farms in the southernmost county Vest-Agder together with the neighbouring county Rogaland. The two northernmost counties, Troms and Finnmark (T&F) have also been grouped together, which means that there are eight regions in all. See appendix 9.D for regional sample mean output and input levels. In all appendices regions are listed from south to north.

The regions differ in two respects. Firstly, the biophysical conditions in terms of sea temperatures and water exchange (tide) are different. Sea temperatures decrease steadily as one travels along the coast from the southernmost to the northernmost regions. The growth rates of salmon increase with higher sea temperatures. On the other hand, due to tidal currents the water exchange is higher in the northern regions than in the southern regions.

Secondly, the regions differ with respect to the time of entry into the industry. Farms in southern regions tended to enter the industry at an earlier stage than farms in northern regions, and hence may have travelled further down the learning curve.

9.12.1. Region-Specific Effects on Mean Output

It is of interest to investigate whether firm-specific effects or region-specific effects are the most important for mean productivity. If there is substantial inter-firm variation in the biophysical productivity of the farm sites and the quality of management etc. within a region, then firm-specific effects may be the most appropriate. However, if biophysical conditions tend to be similar within the region, and also the quality of management is similar within the region due to diffusion of knowledge etc., then a specification with region-specific effects instead of firm-specific effects could be tested. In order to test whether region-specific effects dominate firm-specific effects with respect to mean productivity, restricted least squares estimation is undertaken.

We first estimate a fixed effects model with region-specific intercepts instead of firm-specific intercepts. Then we estimate the model with firm-specific fixed effects, with the restrictions that the firm-specific intercept is equal to the region-specific intercept estimated in the previous step. For the OLS estimates the F-test statistic is 1.188, which has a p-value of 0.015 (df1=372 and df2=1513). At the 5 % level the null hypothesis of equal intercepts within regions is supported, while at the 1 % level H_0 is rejected. A Wald test rejects the null hypothesis of equal intercepts at all conventional confidence levels, with a chi-square statistic of 442.05.

Despite the lack of support for pooling the firm-specific effects by region, we will say a few words about the empirical results. The OLS and FGLS parameter estimates from the model with region-specific fixed effects are presented in tables 9.A22-9.A24.[19] Table 9.14 presents estimated elasticities from the model with region-specific effects, together with estimates from the pooled model and the model with firm-specific FE.

Table 9.14. Estimates of Mean Function and Variance Function Elasticities

Mean Production Function Elasticities (*RTS/TC*)			
Model	**Pooled**	**Firm FE**	**Region FE**
JP2 and V1 by FGLS	0.951 / 0.027	0.896 / 0.041	0.950 / 0.032
JP2 and V2 by FGLS	0.953 / 0.027	0.904 / 0.041	0.951 / 0.031
Variance Production Function Elasticities (*TVE/TCV*)			
Model	**Pooled**	**Firm FE**	**Region FE**
JP2 and V1 by FGLS	0.884 / 0.212	0.666 / 0.082	0.904 / 0.198
JP2 and V2 by FGLS	0.930 / 0.179	0.713 / 0.066	0.937 / 0.175

For *RTS*, *TC* and *TCV* the estimates from the model with region-specific fixed effects lie between the estimates from the pooled model and the model with firm-specific effects, but generally much closer to the estimates from the pooled model. The only exception is the total variance elasticity, where the estimates of the model with region-specific effects are higher than the others. There are no dramatic differences in the time patterns of the above elasticities between the region-specific and the firm-specific FE models.

According to all three estimators of the model with region-specific FE, both the OLS and two FGLS estimates, farms in Vest-Agder and Rogaland (VA&R) are the most efficient in terms of mean productivity. But the relative differences in productivity are small; for all of the

[19] The regions are listed from south to north.

estimators the farm with sample mean output (i.e., the normalised output $y = 1$) in VA&R is less than 10 % more efficient than the mean farm in the least productive region, which is Sør-Trøndelag (ST) according to all three estimators.

The problem with the fixed effects model is that time-invariant firm-specific and region-specific effects cannot be implemented simultaneously, because of perfect collinearity. A solution is to estimate a random effects model, where the firm-specific effects are assumed to be random, and the region effects are treated as fixed. This was done, with the FGLS *RA2* estimator being used to provide parameter estimates. The FGLS parameter estimates are presented in tables 9.A25-9.A26. According to both FGLS estimates of the region-specific intercepts, Vest-Agder & Rogaland still has the most productive farms in terms of mean output, and Sør-Trøndelag still has least productive farms. However, the introduction of firm effects into the model leads to a reduction in efficiency differences between the regions; the mean farm in VA&R now has only a 5 % higher mean output than the mean farm in ST, according to both estimates. According to the estimated t-ratios of the region-specific intercepts, the differences in mean output between regions are significant at conventional confidence levels.

A likelihood ratio test of the new random effects model with fixed region effects added versus the restricted random effects specification which were estimated in the previous section (all regional intercepts equal) provided chi-square test statistics of 9.41 and 9.35 for the FGLS estimator with variance functions V1 and V2, respectively. These low values support the null hypothesis that the region-specific intercepts are all equal at conventional confidence levels. When we compare with the restricted random effects model estimated in the previous section, we find that the introduction of region-specific effects leads to only small changes in elasticity estimates.

9.12.2. Region-Specific Effects on Output Risk

Next, we analyse if there are structural differences in output risk between regions. First, we estimate the mean production function JP2 with firm-specific fixed effects, and introduce region-specific fixed effects into the variance function. The variance function V1 with time-specific effects now becomes

$$\begin{aligned} \text{var}(u_{it}) &= \exp(\beta_0+\Sigma_k\beta_k x_{k,it}+\Sigma_t\beta_t D_t+\Sigma_r\beta_r D_r) \\ &= \exp(\beta_0)\Pi_k\exp(\beta_k x_{k,it})\Pi_t\exp(\beta_t D_t)\Pi_r\exp(\beta_r D_r), \end{aligned}$$

and similarly for V2 (with x replaced by ln$x$), where r is the region subscript. We see that the region-specific effects interact multiplicatively with the input levels and the year-specific effects. For two regions with different β_r's, the difference in output variance will increase with an increase in the scale of operation.

The FGLS parameter estimates are reported in tables 9.A27 and 9.A28 for variance functions V1 and V2, respectively. Troms & Finnmark (T&F) was chosen as the reference region. According to the estimated region-specific effects of both V1 and V2, Sogn og Fjordane (SF) has the highest output variance, while the output risk is smallest in Nord-Trøndelag (NT). The differences in output risk cannot be said to be dramatic; at sample normalised mean input levels the standard deviation of output is 0.144 in SF and 0.107 in NT for variance function V1 (the corresponding values for V2 are 0.164 and 0.123). The introduction of region-specific effects leads to a reduction in estimated marginal input risks and to a loss of significance in terms of t-ratios. Likelihood ratio tests of the restricted variance function with one common intercept β_0 versus region-specific intercepts were undertaken for both variance functions. The chi-square test statistics were 12.96 and 12.08 for V1 and V2, respectively. These values fall within the 90 % area of the chi-square distribution (critical value 12.02 with 7 df), but outside the 95 % area (critical value 14.07), thus lying in the zone of indecision.

The introduction of region-specific intercepts into the variance functions only lead to marginal changes in overall sample means of the elasticities we have been concerned about in this chapter. The effects on the time patterns of the estimated elasticities were also limited, compared with the restricted model.

In an analysis of structural differences in output risk across regions another possibility is to introduce heteroskedastic firm-specific error components into the random effects model, with region-specific variances of the error component, i.e., var(η_{i_r})=σ_r^2, as suggested in the discussion of the RE specification of the Just-Pope model in section 9.2. This specification differs from the one above not only in the specification of the firm-specific effects, but also that the difference in output variance between two regions is constant, regardless of input levels. We estimated the Just-Pope model by the FGLS procedure *RA1*, with the modification to allow for heteroskedastic variances of the firm-specific random effect (see appendix 9.E). The parameter estimates, including the estimated variances of the firm-specific effect, are presented in tables 9.A29 and 9.A30. Once again, farms in Nord-Trøndelag (NT) has the smallest output risk, *ceteris paribus*. But now Sør-Trøndelag (ST) has the highest output risk.

Based on the models we have estimated in this section, we draw the conclusion that there are regional differences in mean output and output risk for otherwise equal farms, but that regional effects are less important than input levels and unobservable firm-specific characteristics in explaining differences in mean productivity and output risk in salmon farming.

9.13. Comparison of Just-Pope and Khumbakar Estimates

In section 9.6 we compared the estimates from a linear quadratic and translog $f(\mathbf{x})$, focusing on the elasticities of the mean production function. This section provides an analysis of the empirical results on the structure of output risk from the Kumbhakar model K2, which has a translog $f(\mathbf{x})$, against the results from the Just-Pope model JP2. The firm-specific effects are assumed to be fixed.

Appendix 9.E provides a three-stage FGLS estimation procedure for the Kumbhakar model with fixed effects. The first-stage parameter estimates are the OLS estimates for the translog $f(\mathbf{x})$ with fixed effects presented in table 9A.4. In the second step the variance function is estimated by nonlinear least squares, and in the third stage the translog $f(\mathbf{x})$ is estimated by least squares with predicted variances from the second step as weights. The second- and third-stage estimates are presented in table 9.A31. We see that the FGLS estimates of the translog are dramatically different from the OLS estimates. It also turns out, as shown in table 9.15, that the estimates of marginal output risk elasticities (VE_k) and total output variance (TVE) are very different from those derived from the estimated Just-Pope model.

Experimentation with different starting values for the nonlinear estimation of the variance function revealed that the parameter estimates are sensitive to the choice of starting values. The year dummies included in the variance function probably contribute to the sensitivity of parameter estimates to starting values. Kumbhakar (1993) specified a simpler variance function, without time dummies, and this may have lead to more invariant parameter estimates with respect to starting values. The starting values for the estimated variance function parameters presented here are based on OLS estimates.

When the estimation procedure fails to converge to the same parameter values with different starting values, it is recommended that one chooses the parameter values which provide the highest value of the objective function (e.g. log-likelihood function). The parameter estimates presented here were found to give the highest log-likelihood value for the variance function.

Table 9.15. Estimated Marginal Output Risk Elasticities and Total Output Variance from Kumbhakar Model K2.

Elasticity	VE_L	VE_F	VE_I	VE_K	VE_M	TVE
Mean	0.003	0.005	-0.003	-0.004	0.381	0.380
St.dev.	0.010	0.013	0.009	0.012	0.244	0.245
Min.	-0.002	0.000	-0.148	-0.176	0.006	0.005
Max.	0.171	0.200	0.002	0.001	0.986	0.985

Because of the sensitivity of the nonlinear estimates of β to choice of starting values, it seems like caution is required when analysing the structure of output risk with a Kumbhakar model.

9.14. Summary and Conclusions

In this chapter we have tried to shed some light on the structure and magnitude of production risk in Norwegian salmon farming in the period 1985-93 by means of estimation of primal models of production. Since efficiency analysis and analysis of firm behaviour under risk aversion require knowledge about both the conditional mean and variance of output, we have investigated both the mean production function and the variance production function. This has mainly been done through estimation of Just-Pope models.

In the following we will try to summarise the findings from the estimated models. First, we will discuss the effects of model specification choices and estimator choices on empirical results, and give and assessment of the different approaches (section 9.14.1). We have estimated an extensive set of production models, and have found that the elasticity measures we are concerned about to a varying degree are influenced by the choice of functional form, the specification of technical change, the specification of firm-specific effects, and the choice of estimators.

Secondly, we summarise the empirical results for the mean and variance function (section 9.14.2). The findings are related to the theory of the competitive firm under production risk, and also discussed in relation to the information we have about the industry for the data period.

We also discuss the implications of the empirical results for the industry and policy makers (section 9.14.3). Finally, we propose directions for future research on production risk in general and salmon farming in particular (section 9.14.4).

9.14.1. Effects of Specification and Estimator Choices on Results

This chapter has demonstrated the importance of model specification and estimator choice for empirical results. The issue of model specification involves the choice of functional form for the stochastic production function (e.g. linear quadratic or translog), the interaction between the deterministic part and the stochastic part of the production function (linear or additive), and the treatment of firm-specific effects (pooled effects, fixed effects or random effects).

We first examined whether a linear quadratic and a translog functional form for $f(\mathbf{x})$ would provide the same elasticity estimates for the mean function. The overall sample mean returns to scale and technical change elasticities (*RTS* and *TC*) were found to be similar. This was also the case for the time patterns of *RTS* and *TC*. However, the *RTS* estimates derived from the linear quadratic and translog differed somewhat when broken down by region. An analysis of correlation of the elasticity estimates between the two functional forms at the observation level revealed a low correlation for returns to scale. It seems like the choice of a linear quadratic or translog form for $f(\mathbf{x})$ is not critical if one is mainly concerned about elasticities around the sample mean, but the choice of estimator becomes more critical for analysis of observations far from the sample mean. Translog elasticity estimates at observations far from the mean tend to take less extreme values than elasticities derived from the linear quadratic. In other words, the translog seems to be well-behaved in a larger region. This finding suggests that the translog functional form is to be preferred as long as one is only concerned about mean function elasticities.

Two classes of stochastic production functions were estimated when we extended the analysis to production risk; the Just-Pope form and the Kumbhakar form. The Kumbhakar production function can be interpreted as a modification of the standard homoskedastic-in-inputs translog production function, where the variance of the error term is a function of input levels. The FGLS estimates of the Kumbhakar model rely on nonlinear parameter estimates of the variance function, which unfortunately were found to be sensitive to starting values. The FGLS parameter estimates of the translog function were dramatically different from the OLS estimates. Furthermore, we obtained estimates of marginal risks that were very different from those obtained from the Just-Pope models. The sensitivity of the nonlinear estimator of the variance in the Kumbhakar model to starting values, and the violation of Just-Pope postulates, lead us to prefer the Just-Pope model.

In the modelling of technical change both a time trend an a time dummy approach were tried, although we had strong *a priori* reasons to be critical to the validity of the standard time trend approach in the context of salmon farming. The empirical results from the estimated time dummy models strongly indicated large year-to-year fluctuations in the rate of technical change, both for the mean and variance function. Based on this we rejected the time trend model.

The issue of firm heterogeneity specification has also been central here. Inclusion of firm-specific fixed effects had significant impact on elasticity estimates derived from both the mean function and the variance function. This finding applies both to the linear quadratic and the translog parametrization of the mean function, and for different estimators. We found that the pooled models tended to overestimate returns to scale, and underestimate the rate of technical change during the data period.

Several estimators were compared for the Just-Pope model specifications. When we compared FGLS and ML estimators for Just-Pope models with fixed effects, we found that the choice of estimator had greater consequences for variance function estimates than for the mean function estimates. FGLS and ML estimates of sample mean returns to scale and technical change (*RTS* and *TC*) were very similar. However, the total variance elasticity (*TVE*) derived from the estimated variance function was much higher according to the ML estimates. The ML estimates of the variance function parameters generally had higher t-ratios than the FGLS estimates, which means that inference based on ML estimates could be done with greater confidence.

Two variance functions were specified for the Just-Pope models, V1 and V2. They both have their weaknesses; variance function V1 has some undesirable convexity properties for risk-increasing inputs (see appendix 9.F), while V2 restricts the total variance elasticity (*TVE*) to be constant, regardless of input levels. When we estimated the Just-Pope models by FGLS and ML, we found that the mean function parameter estimates were not very sensitive to the choice of variance function. The sample mean function elasticities *RTS* and *TC* were very similar for both estimators. Furthermore, the choice of variance function did not have any significant effect on the sign of the marginal output risk, although the magnitudes of marginal risks were affected. Variance function V1 seems to be the preferred one, because it does not restrict *TVE*, and because we found that the magnitude of the undesirable convexity properties in practice was very small. There is probably also scope for improvement in the specification of variance function for econometric Just-Pope models.

We also compared estimates of the Just-Pope models under the two different assumptions on the nature of the firm-specific effects; the fixed effects and the random effects assumption. The popular random effects approach was found to provide mean function elasticity estimates that did not differ substantially from the fixed effects approach. However, when we used random effects FGLS estimation procedures which rely on the standard random effects assumptions of no correlation between error components and between error components and regressors, we found that the estimated marginal risks and elasticities of the variance function differed dramatically from those of the fixed effects FGLS estimator. Since there are strong reasons to question the random effects assumptions, we tend to give more trust to the fixed effects model estimates.

In addition to comparing sample mean estimates of elasticities from different specifications and estimators, we also compared elasticities at the observation level by means of coefficients of correlation (cf. table 9.B31). For the mean function elasticities *RTS* and *TC*, we found a high degree of correlation when using different estimators for the JP2 model with fixed effects. The correlation coefficients between OLS, FGLS and ML of *RTS* and *TC* were around 0.9 or higher. In particular, the FGLS and ML estimates are highly correlated, with values close to one. We also found high correlation coefficients between FGLS estimates of *RTS* and *TC* based on difference variance functions (V1 and V2). The same applies for the ML estimator. When estimates from pooled models and models with firm-specific fixed effects were compared, we also find a relatively high degree of correlation. The lowest correlations were found between different functional forms (linear quadratic vs. translog), and between different specifications of technical change (time trend model JP1 vs. time dummy model JP2).

For the variance function comparison of total variance elasticities (*TVE*) derived from different specifications and estimators by means of correlation coefficients is less meaningful, due to the restrictions on *TVE*. For example, variance function V2 imposes constant *TVE* across observations. For the technical change of the variance function (*TCV*), we come to similar conclusions as for the mean function elasticities. For the JP2 model with fixed effects, correlation is relatively high between the FGLS and ML estimates (0.72 and 0.79), and also between the two variance functions V1 and V2 (0.84 and 0.98). When we compare the fixed effects model with the pooled model, however, we find much smaller correlations (0.43 and 0.50) for *TCV*. The weakest correlations are found between the time trend and time dummy models (0.02 and 0.18).

Based on the empirical results in this chapter the preferred stochastic production function is the Just-Pope model with time dummy variables and firm-specific fixed effects. The empirical results do not give strong support to any particular estimator for the fixed effects specification, although the marginal risk effects of inputs are more significant according to the ML estimator than the FGLS estimator.

9.14.2. Empirical Results and Implications for the Norwegian Salmon Farming Industry

Next, we summarise our empirical results on the structure of the stochastic production technology in Norwegian salmon farming. We discuss the implications of our findings in relation to the theory of the competitive firm under production risk. We also relate the results to the hypotheses presented in Chapter one, and the information we have about developments in the industry during the data period.

One feature that all estimated Just-Pope models have in common is that the determinant of the Hessian of the mean production function takes very small negative values. This means that the Hessian is negative semidefinite, thus implying diminishing marginal productivity of inputs (Chambers, 1988, pp. 9-10).

The estimated mean output elasticities vary somewhat across model specifications and estimators, but are fairly consistent in terms of the ranking of inputs. In all estimated Just-Pope models feed is the most important input with respect to mean output, i.e., the highest elasticity of mean output with respect to inputs, E_k, is found for feed. The output elasticity with respect to feed ranges from 0.49 to 0.59 across models. The input of fish in the pens at the beginning of the year is the second most important, with output elasticity ranging from 0.23 to 0.31. The third most important input is materials, with output elasticity in the range 0.030-0.053, followed by labour and capital. The ranking and the high elasticity values for feed and fish input were as expected. However, the low output elasticity with respect to capital was somewhat unexpected. The reason may be that our proxy only to some extent reflects the real services of capital. According to the summary statistics for the inputs in appendix 9.D, small variation in capital (as measured by the sample standard deviation) can not explained the limited significance of capital input.

Returns to scale is equal to the sum of all the output elasticities with respect to inputs. The overall sample mean estimated *RTS* varies from 0.840 to 0.957 across the estimated models. *RTS* was highest in the first two years of the data period, when according to most models it was above one, and then stabilised below one. According to the preferred models and estimators, sample mean *RTS* lies in the range 0.89-0.92. An examination of the development of *RTS* over time reveals that except for the first year of the data period, 1985, returns to scale was below one for the mean firm. In other words, the size of the mean firm was sufficient to exhaust economies of scale. This finding is similar to that of Bjørndal & Salvanes (1995), who estimated a cost function for the industry based on data for 1988.

The period 1985-93 was characterised by a large growth in output at the farm level. For the mean farm the production of salmon was three times higher in 1993 than in 1985, according to the summary statistic presented in appendix 9.D. To a large extent this production increase is caused by increases in input levels, but productivity increases is also a factor behind the output growth. The overall sample mean estimated rate of technical change (*TC*) for the mean production function varies from 0.026 to 0.044 across models. According to the preferred specifications and estimators, the overall sample mean rate of technical change is in the range 3.9-4.1 %. We rejected the standard time trend representation of technical change, which seems to be too inflexible to describe technical change in salmon farming.

According to most models, including the translog parametrization of the mean production function, the rate of technical change was high from 1986 to 1988, and from 1992 to 1993. The models also seem to agree that the rate of technical progress was very low, or even negative from 1985 to 1986, and also that the rate of technical progress was substantially smaller in the period 1988-92 than in the peak years.

Since biophysical shocks are unobserved at the farm level, and thus cannot be controlled for, it is difficult to say to what extent our estimates of technical change explain true productivity growth or year-to-year shifts in sea temperatures, diseases, toxic algae concentrations, etc. However, it is interesting to note that the profile of the rate of technical change, as depicted in figure 9.C9 for the preferred models and estimators, to a large extent mirrors the indemnities from the insurance companies to the farms in the sample as plotted in figure 7.2 (Chapter 7). These indemnities cover economic losses due to diseases and extreme weather conditions. Two years of low rates of technical progress, 1986 and 1990, coincide with peaks in indemnities, while two years with high rates of technical progress (1988 and 1993) coincide with low indemnities. It is also interesting to note that periods with low rates of technical progress correspond with periods of major disease outbreaks in Norwegian salmon farming. In 1986 there was a major outbreak of the so-called "Hitra" disease, while in 1990-92 one had the "ILA" (infectious salmon anaemia) disease and furunculosis (Asche, 1997). However, one should be careful in giving diseases a major role, since a number of other factors are important for explaining technical change in the industry.

The rate of technical progress is found to be low in each year from 1989 to 1992. This was also a period with poor profitability and small equity for the industry as a whole, according to table 7.1 in Chapter 7. In each year during this period the mean farm had negative profits.

Furthermore, the mean equity was also negative from 1989 to 1991.[20] Hence, farms did neither have the means to invest in new and improved technologies, nor undertake any on-farm experimentation, during this period. However, at the end of the data period the profitability improved, which too some extent may explain the estimated upswing in technical progress from 1992 to 1993.

Next, we summarise the empirical results for the variance function. We exclude the results from the random effects FGLS RA2 estimator, which we regard as problematic in the context of variance function estimation. First, the findings with respect to marginal risks are summarised. For both variance function specifications, V1 and V2, the input coefficient can be interpreted as the output variance elasticity (or marginal risk elasticity) with respect to the input when evaluated at normalised mean input levels. For the materials input the coefficient varies from -0.64 to 0.358. It is positive in most cases, and never significantly negative. The feed input coefficient ranges from 0.355 to 0.580, and is always significantly positive. Capital also has a risk increasing effect, with estimated parameters ranging from 0.073 to 0.232. For most estimated models the capital input coefficient is significantly positive. The input of fish coefficient ranges from 0.139 to 0.373, and like feed is significantly positive in all estimated models, but at lower confidence levels. Labour input is the only input which always has a negative coefficient (excluding the random effects model estimated by RA2). Its coefficient varies from -0.028 to -0.204 across estimated models, and is significantly negative in several estimated models. To summarise, feed, capital and fish are risk-increasing, with feed having clearly the most significant effect on output variance. Materials input also seems to increase output risk. Labour is the only input which plays a risk-reducing role.

Thus, the models estimated here generally supports our initial hypotheses regarding marginal risks, H2 (b) and (c), for feed, fish and labour. Our null hypothesis of negative marginal output risk in capital is rejected. Still, we believe that an increase in capital services with all other inputs held fixed should lead to a decrease in output risk. However, it may be the case that increases in capital input are strongly correlated with increases in the scale of operation, which we have found is associated with higher levels of output risk, or that our measure of capital input is a poor proxy for the real services of capital.

Following Ramaswami (1993), the marginal risk premium of feed, fish and capital is positive for all risk averse salmon farmers, since these inputs are risk-increasing (see Chapter 2). Furthermore, the marginal risk premium is negative for the risk-reducing input labour. A risk averse salmon farmer will therefore use smaller quantities of feed and fish input than a risk-neutral producer, and use more of labour input.

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[20] The depressed economic conditions for the industry during this period can probably to a large extent be attributed to failure of salmon demand to shift as the total supply of Norwegian salmon increased dramatically at the end of the 1980s (Asche, 1997).

Figures 9.2 and 9.3 illustrate both the marginal risk and the marginal mean productivity properties of feed and labour, respectively, in 1993.[21] The figures are based on FGLS estimates of model V1 and JP2 with firm-specific fixed effects, but other estimated models provide a similar pattern. We see that increases in feed input levels lead to an increase in both mean output and the output risk. An increase in labour input increases mean output, but reduces the level of output risk.

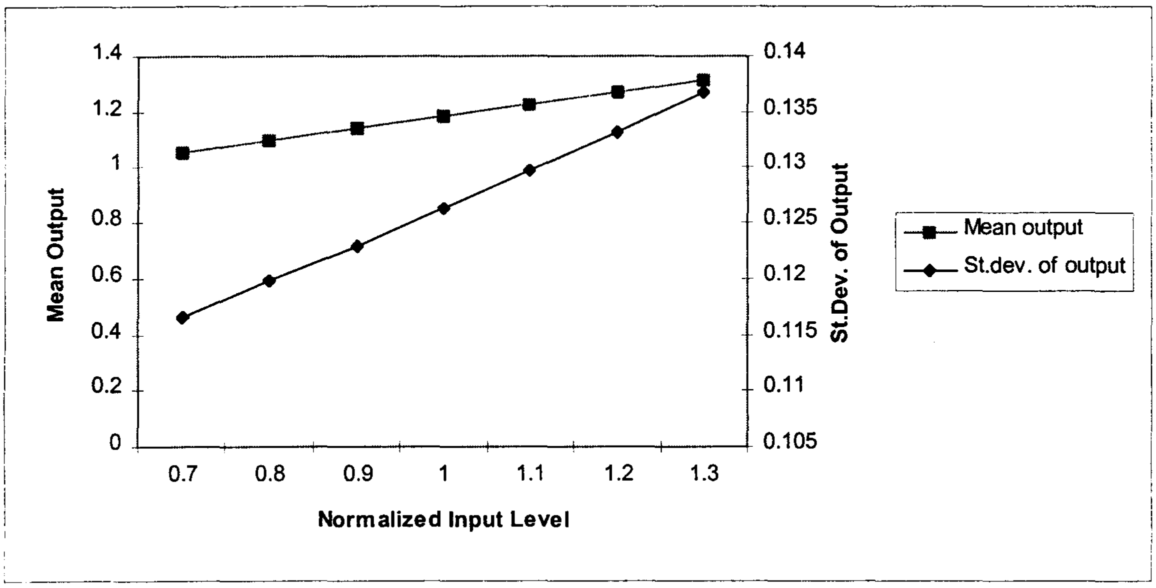

Figure 9.2. **Mean Output and Standard Deviation of Output for Different Levels of Feed Input in 1993**

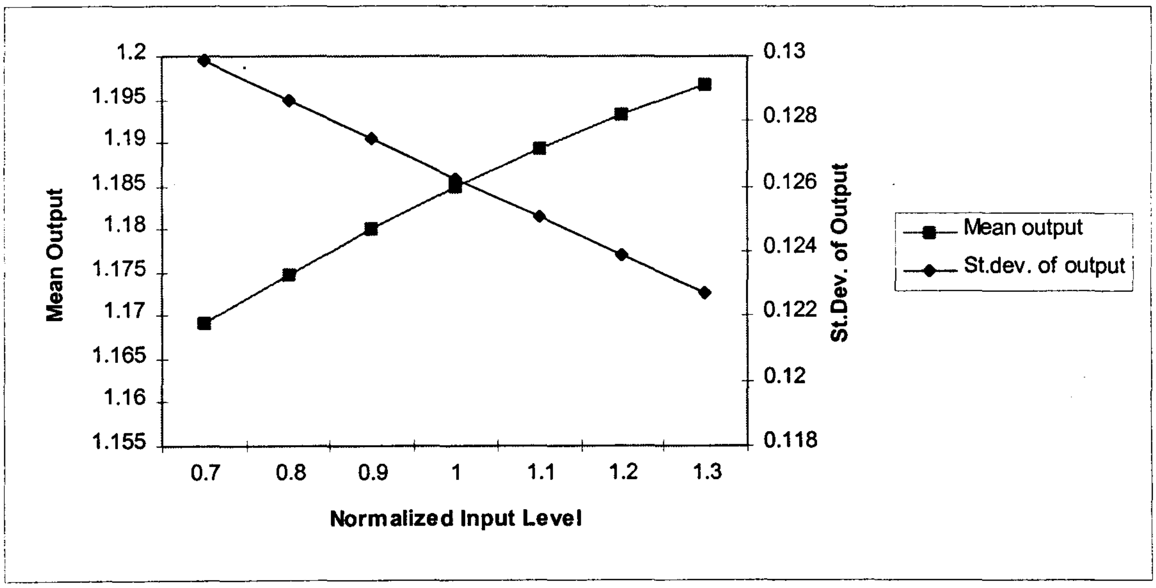

Figure 9.3. **Mean Output and Standard Deviation of Output for Different Levels of Labour Input in 1993**

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[21] The level of the other inputs are held fixed at the sample mean values (i.e., 1).

The overall sample mean of the output variance elasticity (*TVE*), which is equal to the sum of the input coefficients at all input levels for variance function V2 and at the normalised sample mean input levels for V1, ranges from 0.635 to 1.045 across the estimated models. However, it is below 1 for most models. These results imply that hypothesis H2(a) of increasing output risk associated with a factor neutral expansion in inputs cannot be rejected, since *TVE* is positive for V2 regardless of initial input combination, and *TVE* will only be negative for V1 under very high levels of labour input relative to the other inputs.[22] When we analyse the development in *TVE* over time, we have to focus on the variance function V1, because V2 restricts *TVE* to be constant. For all estimated models *TVE* increases during the data period. This can be explained by two factors; the risk-increasing inputs dominate in the production process, and the scale of operation (i.e., input levels) increases over time.

The overall sample mean rate of technical change of the variance function (*TCV*) varies from -0.035 to 0.212. However, it is only the time trend model JP1 estimated together with variance function V2 that produces a negative overall sample mean. For the preferred models and estimators, the JP2 models with fixed effects estimated by FGLS and ML, sample mean *TCV* lies between 6 % and 9 %. An analysis of *TCV* over time reveals large year-to-year fluctuations for all models, except the time trend model, which restricts the flexibility of *TCV*. All estimated time dummy models agree that *TCV* was positive from 1988 to 1989 and from 1990 to 1991, which means that for constant input levels output risk increased during these two periods. Furthermore, according to all time dummy models *TCV* was negative between 1989 and 1990, i.e., the variance of output decreased for constant input levels. As for the estimated technical change of the mean production function, year-to-year shifts in biophysical variables probably influence the *TCV* estimates. However, since we have no information that gives us reason to believe that biophysical conditions have deteriorated during the data period, leading to higher exogenous output risk, the empirical results suggest that output risk actually has increased from 1985 to 1993.

In Chapter two we argued that risk averse producers are not only concerned about mean productivity in their learning process or when they adopt new technologies. They should also be concerned about output risk. Consequently, if salmon farmers are risk averse, new technology adoption and adjustment of production practices should lead to decreases in output risk over time, particularly since the industry was very young at the beginning of the data period and therefore should have a potential for risk reduction through these means.

One explanation for the counterintuitive empirical results is that the agents who are responsible for most of the research and development activities related to salmon farming, e.g. the Norwegian government and the Norwegian Research Council, are less concerned about the

[22] For V1 and JP2 with fixed effects estimated by FGLS there is actually one farm observation for which *TVE* is negative.

risk properties of a new technology than the effect on mean output. If exogenous biophysical shocks are only weakly correlated across salmon farms, and there is a large number of farms, then a social planner may mainly be concerned about funding research and development which has the potential to increase mean productivity.

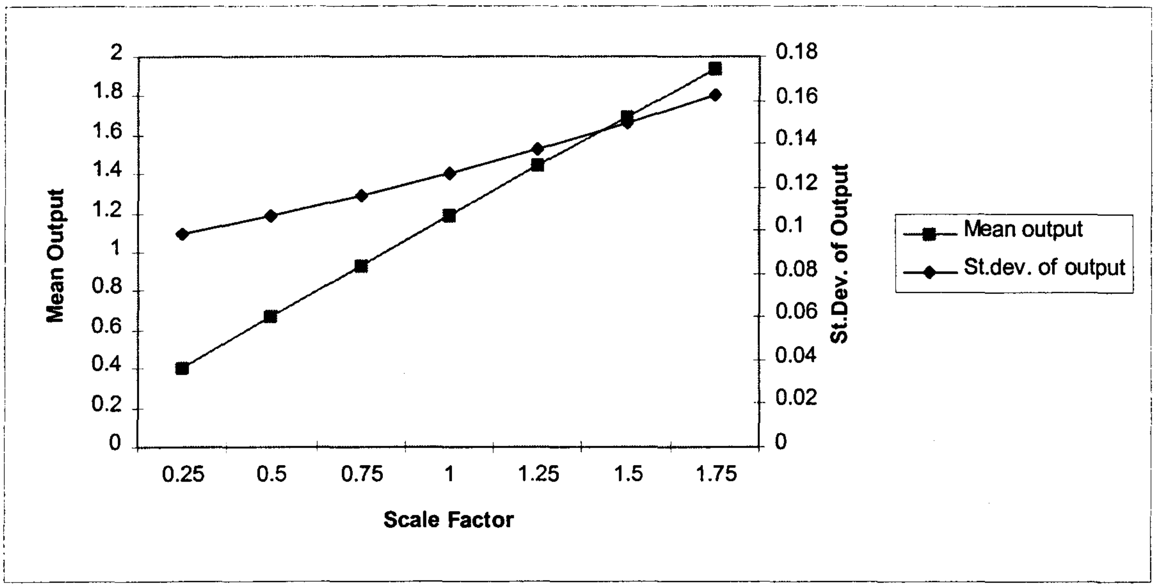

Figure 9.4. Mean Output and Standard Deviation of Output for Factor Neutral Changes in Input Levels from Normalised Sample Mean in 1993

Until now mean output and output risk have been discussed separately. In the following we will analyse these together. Figure 9.4 depicts mean output and the standard deviation of output when the normalised sample mean input vector is multiplied by different scale factors.

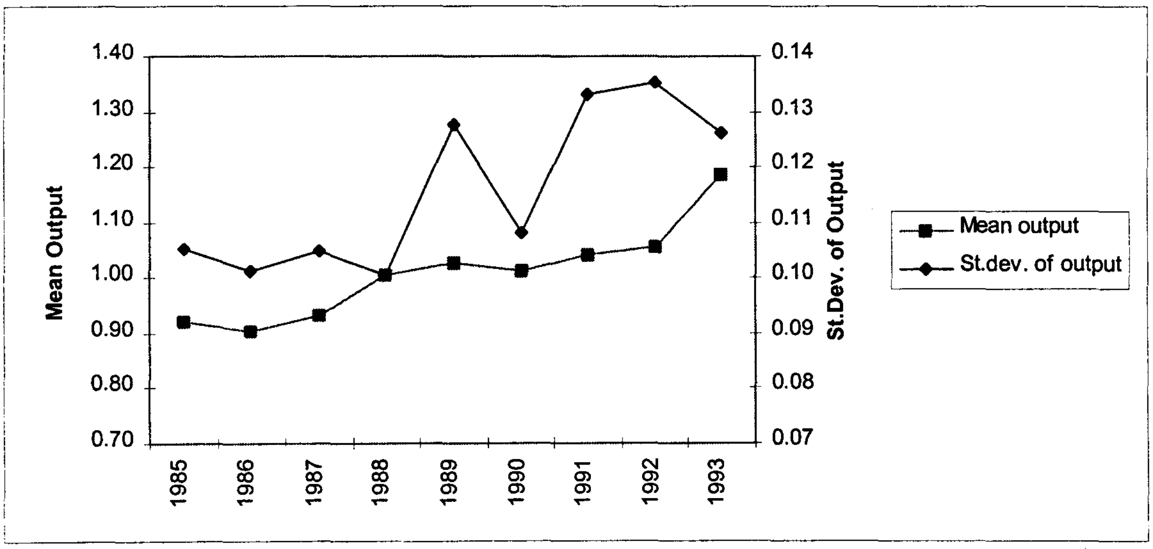

Figure 9.5. Development in Mean Output and Standard Deviation of Output at Normalised Sample Mean Input Levels during the Data Period

Hypothesis H3 in Chapter 1 proposed that for a given level of inputs (a) mean output increased during the period 1985-93, while (b) output risk decreased during the same period. Figure 9.5, plots the development in mean output and the standard deviation of output for the normalised sample mean input levels over the data period. The figure is based on model V1 and JP2 with firm-specific fixed effects estimated by FGLS, but other preferred estimated models provide a similar pattern. We see that output risk exhibits a much more erratic pattern than mean output, but seems to experience an upward trend from 1985 to 1993. Thus, figure 9.5 supports hypothesis H3(a), but rejects hypothesis H3(b).

Hypothesis H4 asserted that there was differences in mean productivity and output risk between regions. The rationale for this is different biophysical conditions with respect to temperatures, water exchange, etc. Furthermore, there may be diffusion of improved technologies and production practices which is mainly within regions, through the regional fish farmer organisations or other more informal networks. We find regional differences with respect to both mean output and production risk, although not very large. For mean input levels the difference in mean output between the most productive region (Vest-Agder & Rogaland) and the least productive region (Sør-Trøndelag) is between 5 and 10 %, depending on model specification and estimator. When evaluated at mean input levels, output risk measured by the standard deviation of output is around 33-34% higher in the riskiest region (Sogn og Fjordane) than in the region with smallest output risk (Nord-Trøndelag) for the estimator we have most confidence in.

9.14.3. Implications for Industry and Policy Makers

It is difficult to say to what extent the risk properties of inputs have influenced the farms' input choices. This depends on the risk preference structure of the decision makers in Norwegian salmon farms, which we did not measure here, and have very limited knowledge about. During the data period Norwegian salmon farms have increased their scale of operation, which has served to increase both mean output and the level of output risk. There are several possible explanations for this development in mean output and output risk, assuming that salmon farmers are optimising agents. One explanation is that salmon farmers are risk neutral, in which case they are only concerned about mean profit, i.e., mean output. A second explanation is that even if salmon farmers are risk averse, their risk preference structure is such that the increase in mean output (profit) associated with the increase in the scale of operation is sufficient to more than compensate for the increased output (profit) risk, and thus provide them with a higher level of utility. The third explanation is that salmon farmers have limited knowledge about the structure of production risk, which means that they know little about the effects of altering input levels on output risk.

The findings here should be of interest to the industry, since this study is the first of its kind to examine the structure of production risk in salmon farming. Furthermore, the data set used here is the most extensive in productivity studies of this industry so far, both with respect to the length of the time period and the number of firms. For an individual salmon farmer it is difficult to estimate the effects of changing input levels on output risk only based on his own production history. This study provides information on the structure of production risk based on a sample with around 2000 observations, which means that conclusions can be drawn with greater confidence than if one have to rely on observations from an individual farm only. However, some caution is required in the interpretation of the results due to the quality of the data (see Chapter 8).

According to the empirical results the level of output risk increases as the scale of operation is increased. This finding suggests that risk averse salmon farmers instead of increasing the scale of operation at the existing farm site, should consider to produce some of the salmon at another farm location. During the first half of the 1990s the industry in fact experienced an increasing degree of multi-site operations, probably because the biophysical capacity of existing farm sites precluded further increases in production. The decentralisation decision involves a mean-variance trade-off, because it has effects on both the mean and variance of output. Decentralisation of the salmon farming operation leads to a lower mean output and profits, due to the increased logistical requirements, and the necessity for undertaking investments in certain types of capital equipment which would have been redundant with only one farm location. On the other hand, the variance of output is decreased, since the salmon may be subject to less stress, and the potential for loss is reduced if a fish disease outbreak occurs at a site.

The structure of production risk is also relevant with respect to horizontal integration of salmon farms. Since the liberalisation of the Fish Farming License Act with respect to ownership interests, the industry has experienced an increasing degree of horizontal integration. This may be because increasing output variance did not encourage firms to undertake further expansion of production at existing farm sites. Acquisition of other farms represents an alternative in such a situation. Furthermore, if stochastic shocks, such as disease outbreaks and extreme weather conditions, to some extent are correlated across adjacent farm sites, then a salmon firm may reduce the total level of output risk by acquiring farms in different regions instead of increasing the production at existing neighbouring sites. Reduced output variance associated with such regional diversification may also make the firm a more reliable supplier of salmon, and increase the possibility for long-term production planning.

According to the theoretical results of Leathers & Quiggin (1991), policies aimed at altering output supply and input demands cannot be based on theory alone when production risk is present (see Chapter 2). Thus, when a tax on for example fish feed is considered, policy makers should also take into account the risk properties of this input.

For public research programmes aimed at salmon farming, an implication of the empirical results here is that one should be concerned about both mean and risk properties in research on new technologies. The results suggest that technical progress has contributed more to increasing mean output than to reduce the level of risk. However, it is an open question to what extent this development has been driven by the farmers or the government-sponsored research and development.

9.14.4. Limitations and Future Research

Based on the findings here, it is believed that this empirical study has increased our knowledge about the structure of the stochastic production technology in salmon farming in particular, and also has contributed to the discussion of model specification and estimator choice for empirical modelling of risky production technologies in general. However, this study has its limitations. In the course of the research work several interesting paths were not followed up, mainly because the scope of the analysis would otherwise be too large.

This empirical study of production risk only used a primal approach. In the absence of information on risk preferences and expectation formations, a primal model is not sufficient to provide predictions on the effects of price change on input demands and output supply. In principle, dual approaches provide input demands and output supply elasticities under risk. However, econometric implementation necessitates restrictions on risk preferences, expectation formation, etc., which means that in practice one may not get any further with a dual approach. At the current stage the primal approach is still regarded as fruitful compared to a dual approach in the context of production risk.

In future research on production risk within the Just-Pope framework, other parametrizations of the mean function, such as the generalised Leontief, should be tried. The emphasis should be on flexibility, global properties and effects on variance function estimates. The linear quadratic seems to have a rather limited consistency region, i.e., estimated elasticities take extreme values as one moves from the mean observation. If a functional form is ill-behaved at data points far from mean, then this may also have consequences for variance function estimates. It is difficult to say what effects outliers may have had on variance function estimates, and this should be investigated in future studies. Even though the generalised Leontief may provide less significant mean function parameter estimates due to high multicollinearity, it may be more well-behaved far from mean than e.g. the linear quadratic. In addition, it is more flexible than the linear quadratic in some respects (Driscoll, et al., 1992).

For the variance function more flexible forms than those used here should be tried. Second-order approximations are natural candidates. Tentative experimentation with these suggests, not unexpectedly, that first-order parameters become less significant. On the other hand, the variance function elasticity measures presented here become more flexible with a second-order functional form. The effects of different variance function specifications on (third-stage FGLS or ML) mean function estimates should also be investigated.

For the empirical application here, the salmon aquaculture industry, it would be interesting to adjust estimates of technical change for the effect of diseases, damages due to extreme weather conditions, and sea temperatures. Disease outbreaks, which tend to be concentrated in certain period, can be regarded as noise when we are interested in the underlying technical change in the production technology. Sea temperatures, which influence the growth rate of salmon, have been found to exhibit cyclical movements to some extent.[23] By implementing indicators for diseases and damages caused by extreme weather conditions into the production function specification, for example indemnity payments or disease and damages dummy variables, one should be able to obtain estimates of technical change which to some extent are devoid of these factors. For sea temperatures there are regional data available which have been utilized by Tveterås (1993).

Another empirical issue worth investigating is the autocorrelation properties of stochastic shocks in salmon farming, both within firms over time, and across firms in a region (see section 7.2). Panel data estimators which facilitates estimation of model with different autocorrelation structures have recently become available (Baltagi & Raj, 1992). It should be interesting to learn to what extent stochastic shocks carry over to the next period for a firm, and to what extent stochastic shocks are correlated between firms within a region.

[23] For example, during the winter 1990-91 temperatures above the seasonal mean lead to an increase in total Norwegian salmon production by 10.000 tonnes, which was a substantial fraction of the production during this period (Salvanes & Tveterås, 1992.

9.A. Appendix A: Estimated Parameters

Table	Description
9.A1	JP2 pooled estimated by OLS
9.A2	JP2 with firm-specific fixed effects estimated by OLS
9.A3	K2 pooled estimated by OLS
9.A4	K2 with firm-specific fixed effects estimated by OLS
9.A5	JP1 with firm-specific fixed effects estimated by OLS
9.A6	V1 and JP1 with firm-specific fixed effects estimated by FGLS
9.A7	V2 and JP1 with firm-specific fixed effects estimated by FGLS
9.A8	V1 and JP2 with firm-specific fixed effects estimated by FGLS
9.A9	V2 and JP2 with firm-specific fixed effects estimated by FGLS
9.A10	V1 and JP2 with firm-specific fixed effects estimated by ML
9.A11	V2 and JP2 with firm-specific fixed effects estimated by ML
9.A12	V1 and JP2 pooled estimated by FGLS
9.A13	V2 and JP2 pooled estimated by FGLS
9.A14	V1 and JP2 pooled estimated by ML
9.A15	V2 and JP2 pooled estimated by ML
9.A16	V1 and JP2 with firm-specific random effects estimated by FGLS proc. RA1
9.A17	V2 and JP2 with firm-specific random effects estimated by FGLS proc. RA1
9.A18	V1 and JP2 with firm-specific RE estimated by FGLS proc. RA2
9.A19	V2 and JP2 with firm-specific RE estimated by FGLS proc. RA2
9.A20	V1 and JP2 with firm-specific RE estimated by FGLS proc. RA2 on sample 2
9.A21	V2 and JP2 with firm-specific RE estimated by FGLS proc. RA2 on sample 2
9.A22	JP2 with Region-Specific Fixed Effects Estimated by OLS
9.A23	V1 and JP2 with Region-Specific Fixed Effects Estimated by FGLS
9.A24	V2 and JP2 with Region-Specific Fixed Effects Estimated by FGLS
9.A25	V1 and JP2 with Firm-Specific RE and Region-Specific FE Estimated by FGLS Procedure RA2
9.A26	V2 and JP2 with Firm-Specific RE and Region-Specific FE Estimated by FGLS Procedure RA2
9.A27	V1 with Region-Specific Effects and JP2 with Firm-Specific Fixed Effects Estimated by FGLS
9.A28	V2 with Region-Specific Effects and JP2 with Firm-Specific Fixed Effects Estimated by FGLS
9.A29	V1 and JP2 with Firm-Specific Region-Heteroskedastic Random Effects Estimated by FGLS Procedure RA1
9.A30	V2 and JP2 with Firm-Specific Region-Heteroskedastic Random Effects Estimated by FGLS Procedure RA1
9.A31	Kumbhakar model K2 with firm-specific fixed effects estimated by FGLS

Table 9.A1 JP2 Pooled Estimated by OLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.095	0.027	3.448	α_{F85}	-0.005	0.059	-0.093
α_F	0.520	0.062	8.405	α_{F86}	0.046	0.063	0.734
α_K	0.071	0.039	1.791	α_{F87}	0.052	0.065	0.795
α_L	0.068	0.054	1.261	α_{F88}	0.012	0.075	0.162
α_I	0.393	0.056	7.006	α_{F89}	0.100	0.070	1.439
α_{MM}	0.002	0.004	0.533	α_{F90}	-0.037	0.077	-0.475
α_{ML}	0.035	0.019	1.888	α_{F91}	0.012	0.074	0.158
α_{MI}	-0.013	0.015	-0.886	α_{F92}	0.036	0.059	0.601
α_{MF}	-0.038	0.018	-2.067	α_{L85}	0.053	0.052	1.019
α_{MK}	-0.004	0.011	-0.334	α_{L86}	-0.026	0.050	-0.523
α_{FF}	-0.019	0.018	-1.038	α_{L87}	-0.024	0.051	-0.461
α_{FK}	0.008	0.017	0.438	α_{L88}	-0.003	0.053	-0.056
α_{LF}	-0.006	0.041	-0.135	α_{L89}	0.029	0.048	0.611
α_{IF}	0.084	0.031	2.706	α_{L90}	0.014	0.050	0.274
α_{KK}	0.003	0.004	0.628	α_{L91}	-0.038	0.064	-0.589
α_{LK}	-0.019	0.018	-1.057	α_{L92}	0.011	0.050	0.210
α_{IK}	0.001	0.017	0.055	α_{I85}	-0.029	0.059	-0.487
α_{LL}	-0.016	0.013	-1.234	α_{I86}	-0.016	0.075	-0.212
α_{LI}	-0.034	0.032	-1.058	α_{I87}	-0.091	0.060	-1.529
α_{II}	-0.024	0.016	-1.498	α_{I88}	-0.041	0.067	-0.615
α_{85}	-0.097	0.049	-1.973	α_{I89}	-0.150	0.064	-2.324
α_{86}	-0.063	0.048	-1.314	α_{I90}	-0.087	0.065	-1.332
α_{87}	-0.001	0.050	-0.018	α_{I91}	-0.068	0.063	-1.073
α_{88}	-0.011	0.058	-0.180	α_{I92}	-0.125	0.056	-2.232
α_{89}	-0.010	0.055	-0.182	α_{K85}	-0.067	0.039	-1.697
α_{90}	0.005	0.054	0.091	α_{K86}	-0.025	0.039	-0.623
α_{91}	-0.007	0.055	-0.134	α_{K87}	-0.069	0.039	-1.773
α_{92}	0.011	0.056	0.199	α_{K88}	-0.040	0.040	-1.012
α_{M85}	-0.073	0.039	-1.850	α_{K89}	-0.064	0.042	-1.514
α_{M86}	-0.132	0.037	-3.532	α_{K90}	-0.074	0.040	-1.828
α_{M87}	-0.082	0.030	-2.752	α_{K91}	-0.027	0.043	-0.627
α_{M88}	-0.063	0.031	-2.059	α_{K92}	-0.019	0.038	-0.494
α_{M89}	-0.056	0.031	-1.802	α_0	0.050	0.050	1.005
α_{M90}	0.016	0.030	0.522				
α_{M91}	-0.008	0.028	-0.284	Log-likel.	704.049		
α_{M92}	-0.032	0.026	-1.217	R^2-adj.	0.933		

Table 9.A2 JP2 with Firm-Specific Fixed Effects Estimated by OLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.098	0.026	3.827	α_{F85}	-0.022	0.070	-0.313
α_F	0.429	0.059	7.333	α_{F86}	-0.039	0.069	-0.566
α_K	0.095	0.037	2.580	α_{F87}	0.024	0.064	0.368
α_L	0.098	0.055	1.769	α_{F88}	0.033	0.072	0.458
α_I	0.340	0.051	6.619	α_{F89}	0.098	0.063	1.568
α_{MM}	-0.001	0.003	-0.287	α_{F90}	-0.056	0.067	-0.832
α_{ML}	0.045	0.018	2.589	α_{F91}	0.053	0.070	0.763
α_{MI}	-0.014	0.014	-1.025	α_{F92}	0.020	0.054	0.373
α_{MF}	-0.037	0.017	-2.222	α_{L85}	0.063	0.054	1.163
α_{MK}	0.000	0.012	0.031	α_{L86}	-0.020	0.050	-0.396
α_{FF}	-0.008	0.019	-0.431	α_{L87}	-0.057	0.050	-1.134
α_{FK}	0.000	0.018	-0.001	α_{L88}	-0.025	0.049	-0.504
α_{LF}	0.013	0.037	0.355	α_{L89}	0.039	0.042	0.937
α_{IF}	0.077	0.029	2.617	α_{L90}	0.019	0.045	0.428
α_{KK}	0.004	0.005	0.905	α_{L91}	-0.012	0.058	-0.212
α_{LK}	-0.037	0.017	-2.225	α_{L92}	-0.010	0.043	-0.228
α_{IK}	0.003	0.016	0.204	α_{I85}	0.025	0.062	0.397
α_{LL}	-0.030	0.015	-1.951	α_{I86}	0.025	0.076	0.324
α_{LI}	-0.017	0.031	-0.542	α_{I87}	-0.058	0.055	-1.050
α_{II}	-0.020	0.015	-1.287	α_{I88}	-0.070	0.062	-1.126
α_{85}	-0.168	0.047	-3.562	α_{I89}	-0.165	0.059	-2.808
α_{86}	-0.117	0.047	-2.500	α_{I90}	-0.093	0.057	-1.619
α_{87}	-0.028	0.048	-0.574	α_{I91}	-0.112	0.057	-1.968
α_{88}	-0.026	0.056	-0.454	α_{I92}	-0.123	0.052	-2.354
α_{89}	-0.017	0.051	-0.344	α_{K85}	-0.052	0.038	-1.368
α_{90}	0.018	0.046	0.398	α_{K86}	-0.015	0.036	-0.402
α_{91}	-0.015	0.049	-0.308	α_{K87}	-0.056	0.036	-1.545
α_{92}	0.016	0.046	0.359	α_{K88}	-0.039	0.036	-1.085
α_{M85}	-0.094	0.050	-1.892	α_{K89}	-0.061	0.037	-1.664
α_{M86}	-0.109	0.037	-2.986	α_{K90}	-0.078	0.037	-2.128
α_{M87}	-0.084	0.034	-2.506	α_{K91}	-0.024	0.038	-0.621
α_{M88}	-0.055	0.029	-1.905	α_{K92}	-0.014	0.032	-0.445
α_{M89}	-0.052	0.029	-1.788				
α_{M90}	0.024	0.027	0.888				
α_{M91}	-0.020	0.026	-0.760	Log-likel.	974.771		
α_{M92}	-0.015	0.023	-0.639	R^2-adj.	0.937		

Table 9.A3 K2 Pooled Estimated by OLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.107	0.023	4.734	α_{F85}	0.154	0.078	1.971
α_F	0.396	0.045	8.722	α_{F86}	0.258	0.064	4.029
α_K	0.017	0.025	0.660	α_{F87}	0.228	0.061	3.768
α_L	0.048	0.034	1.440	α_{F88}	0.211	0.063	3.352
α_I	0.340	0.032	10.494	α_{F89}	0.294	0.064	4.590
α_{MM}	0.015	0.005	2.791	α_{F90}	0.070	0.059	1.170
α_{ML}	0.049	0.018	2.729	α_{F91}	0.112	0.065	1.725
α_{MI}	-0.032	0.018	-1.758	α_{F92}	0.150	0.056	2.713
α_{MF}	-0.055	0.019	-2.934	α_{L85}	0.007	0.066	0.106
α_{MK}	0.003	0.013	0.231	α_{L86}	-0.120	0.055	-2.181
α_{FF}	0.096	0.012	8.312	α_{L87}	-0.047	0.052	-0.909
α_{FK}	-0.010	0.025	-0.410	α_{L88}	-0.065	0.052	-1.256
α_{LF}	-0.036	0.038	-0.930	α_{L89}	-0.021	0.042	-0.495
α_{IF}	-0.043	0.021	-2.073	α_{L90}	-0.022	0.045	-0.494
α_{KK}	0.006	0.007	0.930	α_{L91}	-0.069	0.050	-1.363
α_{LK}	-0.028	0.020	-1.409	α_{L92}	-0.019	0.044	-0.432
α_{IK}	0.025	0.013	1.946	α_{I85}	0.047	0.050	0.931
α_{LL}	0.008	0.018	0.476	α_{I86}	0.028	0.056	0.492
α_{LI}	-0.057	0.027	-2.126	α_{I87}	-0.003	0.043	-0.079
α_{II}	0.052	0.005	10.908	α_{I88}	-0.035	0.043	-0.813
α_{85}	-0.244	0.039	-6.280	α_{I89}	-0.124	0.052	-2.403
α_{86}	-0.238	0.036	-6.715	α_{I90}	-0.003	0.048	-0.068
α_{87}	-0.221	0.028	-7.827	α_{I91}	-0.028	0.041	-0.683
α_{88}	-0.167	0.025	-6.585	α_{I92}	-0.127	0.042	-3.012
α_{89}	-0.180	0.025	-7.281	α_{K85}	0.013	0.038	0.342
α_{90}	-0.182	0.027	-6.696	α_{K86}	0.036	0.039	0.929
α_{91}	-0.141	0.026	-5.499	α_{K87}	-0.023	0.038	-0.588
α_{92}	-0.123	0.026	-4.757	α_{K88}	0.026	0.036	0.717
α_{M85}	-0.111	0.044	-2.544	α_{K89}	-0.002	0.034	-0.059
α_{M86}	-0.149	0.041	-3.624	α_{K90}	-0.031	0.030	-1.041
α_{M87}	-0.123	0.036	-3.429	α_{K91}	0.029	0.035	0.825
α_{M88}	-0.099	0.033	-3.028	α_{K92}	-0.001	0.030	-0.038
α_{M89}	-0.110	0.031	-3.517	α_0	0.164	0.021	7.705
α_{M90}	-0.035	0.029	-1.210				
α_{M91}	-0.039	0.029	-1.348	Log-likel.	570.438		
α_{M92}	-0.033	0.028	-1.194	R^2-adj.	0.931		

Table 9.A4 K2 with Firm-Specific Fixed Effects Estimated by OLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.098	0.020	4.933	α_{F85}	0.089	0.067	1.324
α_F	0.370	0.037	9.905	α_{F86}	0.190	0.053	3.562
α_K	0.039	0.025	1.573	α_{F87}	0.191	0.053	3.602
α_L	0.064	0.033	1.919	α_{F88}	0.189	0.056	3.348
α_I	0.291	0.031	9.449	α_{F89}	0.233	0.057	4.096
α_{MM}	0.013	0.006	2.280	α_{F90}	0.021	0.051	0.420
α_{ML}	0.035	0.018	1.884	α_{F91}	0.073	0.056	1.306
α_{MI}	-0.038	0.016	-2.344	α_{F92}	0.105	0.044	2.377
α_{MF}	-0.030	0.018	-1.634	α_{L85}	0.058	0.067	0.866
α_{MK}	0.008	0.012	0.683	α_{L86}	-0.126	0.056	-2.274
α_{FF}	0.095	0.012	8.270	α_{L87}	-0.082	0.053	-1.561
α_{FK}	-0.032	0.024	-1.316	α_{L88}	-0.095	0.047	-2.030
α_{LF}	-0.033	0.037	-0.912	α_{L89}	0.010	0.041	0.235
α_{IF}	-0.045	0.022	-2.014	α_{L90}	0.007	0.045	0.145
α_{KK}	0.021	0.008	2.782	α_{L91}	-0.005	0.048	-0.097
α_{LK}	-0.036	0.022	-1.674	α_{L92}	-0.009	0.040	-0.220
α_{IK}	0.027	0.011	2.400	α_{I85}	0.097	0.046	2.109
α_{LL}	0.011	0.019	0.567	α_{I86}	0.061	0.050	1.223
α_{LI}	-0.044	0.027	-1.614	α_{I87}	0.020	0.041	0.498
α_{II}	0.052	0.004	11.736	α_{I88}	-0.062	0.039	-1.577
α_{85}	-0.283	0.041	-6.871	α_{I89}	-0.167	0.048	-3.506
α_{86}	-0.281	0.034	-8.300	α_{I90}	-0.009	0.047	-0.181
α_{87}	-0.262	0.027	-9.585	α_{I91}	-0.031	0.038	-0.813
α_{88}	-0.194	0.023	-8.555	α_{I92}	-0.112	0.037	-3.012
α_{89}	-0.164	0.022	-7.520	α_{K85}	-0.013	0.035	-0.362
α_{90}	-0.167	0.025	-6.632	α_{K86}	0.018	0.034	0.517
α_{91}	-0.136	0.021	-6.332	α_{K87}	-0.047	0.034	-1.379
α_{92}	-0.119	0.022	-5.515	α_{K88}	0.007	0.029	0.230
α_{M85}	-0.092	0.041	-2.258	α_{K89}	-0.016	0.029	-0.546
α_{M86}	-0.102	0.037	-2.783	α_{K90}	-0.068	0.026	-2.600
α_{M87}	-0.098	0.032	-3.013	α_{K91}	0.008	0.029	0.279
α_{M88}	-0.070	0.028	-2.545	α_{K92}	-0.018	0.025	-0.712
α_{M89}	-0.080	0.028	-2.858				
α_{M90}	-0.003	0.026	-0.125				
α_{M91}	-0.038	0.026	-1.468	Log-likel.	861.177		
α_{M92}	-0.007	0.024	-0.284	R^2-adj.	0.936		

Table 9.A5 JP1 with Firm-Specific Fixed Effects Estimated by OLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_{M}	0.008	0.029	0.290	α_{KK}	0.008	0.008	1.060
α_{F}	0.423	0.044	9.502	α_{LK}	-0.072	0.030	-2.354
α_{K}	0.029	0.027	1.086	α_{IK}	-0.004	0.021	-0.178
α_{L}	0.116	0.043	2.685	α_{LL}	-0.053	0.028	-1.873
α_{I}	0.301	0.038	7.963	α_{LI}	-0.047	0.046	-1.014
α_{MM}	-0.009	0.006	-1.361	α_{II}	-0.073	0.020	-3.662
α_{ML}	0.060	0.031	1.945	α_{T}	0.003	0.010	0.257
α_{MI}	0.002	0.021	0.108	α_{TT}	0.002	0.001	1.543
α_{MF}	-0.071	0.022	-3.202	α_{MT}	0.011	0.004	2.777
α_{MK}	0.001	0.018	0.029	α_{FT}	0.004	0.006	0.716
α_{FF}	-0.037	0.021	-1.741	α_{KT}	0.003	0.004	0.849
α_{FK}	0.018	0.023	0.798	α_{LT}	-0.003	0.006	-0.572
α_{LF}	0.073	0.043	1.702	α_{IT}	-0.004	0.006	-0.680
α_{IF}	0.166	0.033	5.048				
Log-likel.	889.045						
R^2-adj.	0.873						

Table 9.A6 V1 and JP1 with Firm-Specific Fixed Effects Estimated by FGLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	-0.027	0.073	-0.374	β_I	0.263	0.111	2.381
β_F	0.580	0.124	4.689	β_T	0.114	0.095	1.201
β_K	0.096	0.074	1.305	β_{TT}	-0.003	0.009	-0.348
β_L	-0.030	0.126	-0.236	β_0	-5.595	0.226	-24.784
Log-likel.	-4432.21						
R^2-adj.	0.083						

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	-0.024	0.026	-0.924	α_{KK}	-0.001	0.009	-0.139
α_F	0.465	0.042	11.114	α_{LK}	-0.050	0.034	-1.493
α_K	0.029	0.023	1.276	α_{IK}	0.009	0.032	0.273
α_L	0.093	0.036	2.578	α_{LL}	-0.045	0.026	-1.705
α_I	0.347	0.035	9.800	α_{LI}	-0.117	0.052	-2.249
α_{MM}	-0.001	0.008	-0.166	α_{II}	-0.056	0.026	-2.134
α_{ML}	0.078	0.035	2.245	α_T	0.009	0.008	1.180
α_{MI}	0.013	0.026	0.490	α_{TT}	0.001	0.001	1.005
α_{MF}	-0.132	0.026	-5.019	α_{MT}	0.015	0.004	4.039
α_{MK}	0.015	0.021	0.702	α_{FT}	0.001	0.006	0.147
α_{FF}	0.005	0.031	0.152	α_{KT}	-0.001	0.004	-0.227
α_{FK}	0.025	0.031	0.786	α_{LT}	0.000	0.005	-0.045
α_{LF}	0.066	0.054	1.237	α_{IT}	-0.006	0.005	-1.125
α_{IF}	0.140	0.048	2.946				
Log-likel.	1135.70						
R^2-adj.	0.881						

Table 9.A7 V2 and JP1 with Firm-Specific Fixed Effects Estimated by FGLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	0.153	0.080	1.908	β_I	0.162	0.090	1.807
β_F	0.537	0.113	4.773	β_T	-0.069	0.100	-0.689
β_K	0.230	0.082	2.824	β_{TT}	0.013	0.009	1.448
β_L	-0.104	0.140	-0.746	β_0	-4.032	0.254	-15.894
Log-likel.	-4419.85						
R^2-adj.	0.095						

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	-0.022	0.027	-0.820	α_{KK}	-0.001	0.009	-0.169
α_F	0.445	0.039	11.337	α_{LK}	-0.036	0.033	-1.071
α_K	0.020	0.023	0.882	α_{IK}	0.009	0.028	0.322
α_L	0.107	0.035	3.095	α_{LL}	-0.057	0.027	-2.132
α_I	0.363	0.033	10.882	α_{LI}	-0.140	0.053	-2.659
α_{MM}	0.009	0.008	1.183	α_{II}	-0.060	0.025	-2.450
α_{ML}	0.063	0.036	1.740	α_T	0.006	0.007	0.855
α_{MI}	0.024	0.025	0.930	α_{TT}	0.001	0.001	1.366
α_{MF}	-0.162	0.023	-7.184	α_{MT}	0.016	0.004	4.215
α_{MK}	0.010	0.021	0.447	α_{FT}	0.000	0.006	0.052
α_{FF}	0.013	0.027	0.494	α_{KT}	0.000	0.004	0.095
α_{FK}	0.019	0.029	0.668	α_{LT}	0.002	0.005	0.286
α_{LF}	0.095	0.051	1.847	α_{IT}	-0.008	0.005	-1.610
α_{IF}	0.153	0.044	3.497				
Log-likel.	1142.43						
R^2-adj.	0.892						

Table 9.A8 V1 and JP2 with Firm-Specific Fixed Effects Estimated by FGLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	-0.061	0.074	-0.824	β_{87}	-0.375	0.236	-1.590
β_F	0.531	0.128	4.139	β_{88}	-0.459	0.221	-2.082
β_K	0.112	0.074	1.519	β_{89}	0.024	0.217	0.112
β_L	-0.189	0.127	-1.496	β_{90}	-0.311	0.220	-1.414
β_I	0.273	0.117	2.332	β_{91}	0.105	0.232	0.453
β_{85}	-0.367	0.250	-1.468	β_{92}	0.138	0.222	0.624
β_{86}	-0.443	0.243	-1.819	β_0	-4.805	0.211	-22.822
Log-likel.	-4493.69						
R^2-adj.	0.060						

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.110	0.027	4.119	α_{F85}	-0.063	0.065	-0.970
α_F	0.400	0.045	8.907	α_{F86}	0.008	0.063	0.130
α_K	0.046	0.034	1.357	α_{F87}	0.075	0.057	1.307
α_L	0.101	0.051	1.988	α_{F88}	0.134	0.051	2.647
α_I	0.411	0.041	10.056	α_{F89}	0.177	0.050	3.544
α_{MM}	0.001	0.004	0.287	α_{F90}	0.017	0.046	0.383
α_{ML}	0.045	0.017	2.726	α_{F91}	0.043	0.053	0.801
α_{MI}	-0.004	0.014	-0.258	α_{F92}	0.088	0.044	1.985
α_{MF}	-0.057	0.014	-3.952	α_{L85}	0.032	0.051	0.627
α_{MK}	0.001	0.010	0.134	α_{L86}	-0.036	0.046	-0.771
α_{FF}	0.012	0.016	0.763	α_{L87}	-0.055	0.049	-1.131
α_{FK}	-0.009	0.016	-0.603	α_{L88}	-0.053	0.045	-1.189
α_{LF}	0.021	0.027	0.803	α_{L89}	0.009	0.044	0.195
α_{IF}	0.056	0.025	2.249	α_{L90}	-0.007	0.046	-0.159
α_{KK}	0.001	0.004	0.161	α_{L91}	-0.032	0.054	-0.591
α_{LK}	-0.032	0.016	-2.043	α_{L92}	-0.023	0.046	-0.502
α_{IK}	0.032	0.016	1.950	α_{I85}	-0.017	0.053	-0.325
α_{LL}	-0.021	0.013	-1.639	α_{I86}	-0.065	0.055	-1.194
α_{LI}	-0.048	0.028	-1.737	α_{I87}	-0.118	0.051	-2.320
α_{II}	-0.021	0.014	-1.502	α_{I88}	-0.162	0.047	-3.497
α_{85}	-0.127	0.047	-2.740	α_{I89}	-0.249	0.047	-5.345
α_{86}	-0.088	0.047	-1.871	α_{I90}	-0.119	0.039	-3.036
α_{87}	-0.023	0.048	-0.473	α_{I91}	-0.118	0.042	-2.820
α_{88}	-0.028	0.047	-0.588	α_{I92}	-0.160	0.043	-3.756
α_{89}	-0.008	0.050	-0.160	α_{K85}	-0.015	0.038	-0.398
α_{90}	0.006	0.047	0.130	α_{K86}	0.014	0.034	0.420
α_{91}	-0.022	0.050	-0.450	α_{K87}	-0.024	0.034	-0.698
α_{92}	0.032	0.047	0.679	α_{K88}	0.002	0.033	0.067
α_{M85}	-0.074	0.046	-1.617	α_{K89}	-0.019	0.034	-0.551
α_{M86}	-0.116	0.035	-3.332	α_{K90}	-0.064	0.033	-1.903
α_{M87}	-0.108	0.031	-3.466	α_{K91}	0.015	0.036	0.416
α_{M88}	-0.075	0.029	-2.582	α_{K92}	-0.034	0.036	-0.958
α_{M89}	-0.068	0.029	-2.357				
α_{M90}	-0.005	0.028	-0.166				
α_{M91}	-0.027	0.026	-1.059	Log-likel.	1208.75		
α_{M92}	-0.033	0.026	-1.308	R^2-adj.	0.886		

Table 9.A9 V2 and JP2 with Firm-Specific Fixed Effects Estimated by FGLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	0.145	0.081	1.786	β_{87}	-0.405	0.239	-1.694
β_F	0.470	0.116	4.042	β_{88}	-0.602	0.218	-2.761
β_K	0.159	0.082	1.937	β_{89}	-0.148	0.214	-0.692
β_L	-0.204	0.140	-1.453	β_{90}	-0.423	0.214	-1.980
β_I	0.144	0.092	1.568	β_{91}	0.024	0.226	0.108
β_{85}	-0.238	0.261	-0.911	β_{92}	0.122	0.220	0.556
β_{86}	-0.430	0.249	-1.729	β_0	-3.863	0.162	-23.834
Log-likel.	-4491.91						
R^2-adj.	0.062						

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.123	0.028	4.359	α_{F85}	-0.056	0.064	-0.871
α_F	0.389	0.043	8.958	α_{F86}	0.003	0.063	0.046
α_K	0.060	0.034	1.771	α_{F87}	0.068	0.057	1.198
α_L	0.116	0.053	2.212	α_{F88}	0.124	0.050	2.502
α_I	0.391	0.039	9.976	α_{F89}	0.181	0.049	3.714
α_{MM}	0.003	0.004	0.732	α_{F90}	0.011	0.045	0.251
α_{ML}	0.041	0.018	2.290	α_{F91}	0.033	0.052	0.643
α_{MI}	-0.004	0.014	-0.308	α_{F92}	0.077	0.043	1.781
α_{MF}	-0.062	0.014	-4.469	α_{L85}	0.011	0.054	0.204
α_{MK}	0.004	0.011	0.422	α_{L86}	-0.041	0.048	-0.845
α_{FF}	0.010	0.015	0.657	α_{L87}	-0.044	0.051	-0.865
α_{FK}	-0.015	0.014	-1.059	α_{L88}	-0.046	0.047	-0.986
α_{LF}	0.030	0.026	1.147	α_{L89}	0.008	0.046	0.170
α_{IF}	0.071	0.023	3.097	α_{L90}	-0.004	0.049	-0.084
α_{KK}	-0.001	0.004	-0.339	α_{L91}	-0.029	0.056	-0.512
α_{LK}	-0.020	0.016	-1.218	α_{L92}	-0.028	0.050	-0.573
α_{IK}	0.025	0.014	1.752	α_{I85}	0.020	0.052	0.383
α_{LL}	-0.026	0.013	-1.932	α_{I86}	-0.048	0.055	-0.881
α_{LI}	-0.057	0.029	-1.992	α_{I87}	-0.090	0.051	-1.789
α_{II}	-0.023	0.013	-1.784	α_{I88}	-0.129	0.046	-2.822
α_{85}	-0.122	0.045	-2.681	α_{I89}	-0.214	0.046	-4.638
α_{86}	-0.083	0.045	-1.833	α_{I90}	-0.103	0.039	-2.666
α_{87}	-0.042	0.046	-0.915	α_{I91}	-0.092	0.042	-2.190
α_{88}	-0.040	0.045	-0.889	α_{I92}	-0.143	0.042	-3.398
α_{89}	-0.025	0.048	-0.514	α_{K85}	-0.029	0.038	-0.768
α_{90}	0.008	0.045	0.174	α_{K86}	-0.005	0.035	-0.150
α_{91}	-0.026	0.048	-0.532	α_{K87}	-0.037	0.034	-1.086
α_{92}	0.026	0.045	0.588	α_{K88}	-0.015	0.032	-0.449
α_{M85}	-0.085	0.050	-1.694	α_{K89}	-0.035	0.034	-1.021
α_{M86}	-0.124	0.037	-3.381	α_{K90}	-0.075	0.034	-2.196
α_{M87}	-0.119	0.034	-3.482	α_{K91}	-0.013	0.036	-0.356
α_{M88}	-0.087	0.031	-2.832	α_{K92}	-0.027	0.035	-0.769
α_{M89}	-0.084	0.030	-2.749				
α_{M90}	-0.011	0.029	-0.390				
α_{M91}	-0.023	0.028	-0.840	Log-likel.	1209.53		
α_{M92}	-0.037	0.028	-1.325	R^2-adj.	0.893		

Table 9.A10 V1 and JP2 with Firm-Specific Fixed Effects Estimated by ML

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	0.126	0.047	2.694	β_{87}	-0.360	0.150	-2.396
β_F	0.532	0.082	6.514	β_{88}	-0.356	0.141	-2.535
β_K	0.159	0.047	3.374	β_{89}	-0.007	0.138	-0.052
β_L	-0.122	0.081	-1.516	β_{90}	-0.096	0.140	-0.682
β_I	0.350	0.075	4.692	β_{91}	-0.053	0.148	-0.360
β_{85}	-0.401	0.159	-2.517	β_{92}	0.070	0.141	0.496
β_{86}	-0.510	0.155	-3.289	β_0	-4.958	0.134	-36.985

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.111	0.029	3.809	α_{F85}	-0.072	0.061	-1.193
α_F	0.410	0.044	9.355	α_{F86}	0.012	0.059	0.197
α_K	0.041	0.034	1.194	α_{F87}	0.067	0.055	1.215
α_L	0.125	0.052	2.429	α_{F88}	0.129	0.051	2.546
α_I	0.403	0.040	10.026	α_{F89}	0.172	0.049	3.484
α_{MM}	-0.001	0.005	-0.109	α_{F90}	0.009	0.048	0.191
α_{ML}	0.049	0.018	2.698	α_{F91}	0.029	0.053	0.545
α_{MI}	-0.001	0.015	-0.082	α_{F92}	0.086	0.044	1.973
α_{MF}	-0.060	0.016	-3.736	α_{L85}	0.009	0.051	0.184
α_{MK}	0.004	0.011	0.352	α_{L86}	-0.054	0.046	-1.169
α_{FF}	0.014	0.017	0.832	α_{L87}	-0.066	0.049	-1.329
α_{FK}	-0.007	0.017	-0.411	α_{L88}	-0.066	0.046	-1.425
α_{LF}	0.011	0.028	0.402	α_{L89}	-0.004	0.045	-0.079
α_{IF}	0.058	0.026	2.253	α_{L90}	-0.019	0.050	-0.386
α_{KK}	-0.002	0.005	-0.505	α_{L91}	-0.040	0.055	-0.734
α_{LK}	-0.026	0.017	-1.560	α_{L92}	-0.031	0.048	-0.658
α_{IK}	0.033	0.017	1.911	α_{I85}	0.003	0.050	0.063
α_{LL}	-0.021	0.013	-1.662	α_{I86}	-0.058	0.051	-1.139
α_{LI}	-0.061	0.029	-2.102	α_{I87}	-0.103	0.050	-2.082
α_{II}	-0.018	0.014	-1.294	α_{I88}	-0.150	0.046	-3.258
α_{85}	-0.108	0.045	-2.395	α_{I89}	-0.237	0.046	-5.217
α_{86}	-0.072	0.045	-1.602	α_{I90}	-0.104	0.040	-2.588
α_{87}	-0.016	0.046	-0.347	α_{I91}	-0.094	0.041	-2.323
α_{88}	-0.023	0.046	-0.491	α_{I92}	-0.153	0.041	-3.700
α_{89}	0.003	0.048	0.061	α_{K85}	-0.015	0.037	-0.399
α_{90}	0.017	0.047	0.365	α_{K86}	0.013	0.034	0.376
α_{91}	-0.021	0.047	-0.440	α_{K87}	-0.023	0.034	-0.689
α_{92}	0.039	0.045	0.857	α_{K88}	0.007	0.033	0.205
α_{M85}	-0.084	0.045	-1.870	α_{K89}	-0.012	0.034	-0.356
α_{M86}	-0.120	0.035	-3.438	α_{K90}	-0.062	0.035	-1.769
α_{M87}	-0.111	0.033	-3.332	α_{K91}	0.015	0.036	0.420
α_{M88}	-0.079	0.031	-2.545	α_{K92}	-0.038	0.037	-1.026
α_{M89}	-0.081	0.031	-2.646				
α_{M90}	-0.013	0.031	-0.412				
α_{M91}	-0.032	0.028	-1.156				
α_{M92}	-0.033	0.029	-1.151	Log-likel.	1231.195		

Table 9.A11 V2 and JP2 with Firm-Specific Fixed Effects Estimated by ML

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	0.358	0.052	6.929	β_{87}	-0.590	0.152	-3.872
β_F	0.376	0.074	5.079	β_{88}	-0.515	0.139	-3.712
β_K	0.154	0.052	2.951	β_{89}	-0.239	0.136	-1.754
β_L	-0.028	0.089	-0.315	β_{90}	-0.258	0.136	-1.897
β_I	0.163	0.058	2.794	β_{91}	-0.147	0.144	-1.021
β_{85}	-0.336	0.166	-2.026	β_{92}	0.003	0.140	0.021
β_{86}	-0.648	0.158	-4.095	β_0	-3.502	0.103	-33.948

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.118	0.029	4.108	α_{F85}	-0.057	0.060	-0.950
α_F	0.407	0.042	9.616	α_{F86}	0.005	0.058	0.085
α_K	0.060	0.034	1.762	α_{F87}	0.068	0.054	1.258
α_L	0.136	0.052	2.592	α_{F88}	0.124	0.050	2.466
α_I	0.395	0.038	10.292	α_{F89}	0.177	0.048	3.704
α_{MM}	0.002	0.005	0.385	α_{F90}	0.003	0.047	0.073
α_{ML}	0.047	0.019	2.540	α_{F91}	0.021	0.051	0.399
α_{MI}	-0.005	0.015	-0.373	α_{F92}	0.068	0.042	1.614
α_{MF}	-0.062	0.015	-4.224	α_{L85}	-0.006	0.053	-0.106
α_{MK}	0.007	0.011	0.595	α_{L86}	-0.062	0.048	-1.289
α_{FF}	0.011	0.015	0.698	α_{L87}	-0.048	0.050	-0.950
α_{FK}	-0.014	0.015	-0.923	α_{L88}	-0.050	0.047	-1.054
α_{LF}	0.014	0.027	0.493	α_{L89}	-0.009	0.046	-0.187
α_{IF}	0.075	0.024	3.168	α_{L90}	-0.019	0.052	-0.371
α_{KK}	-0.004	0.004	-0.879	α_{L91}	-0.043	0.056	-0.769
α_{LK}	-0.017	0.016	-1.023	α_{L92}	-0.038	0.051	-0.746
α_{IK}	0.027	0.015	1.800	α_{I85}	0.025	0.049	0.511
α_{LL}	-0.025	0.013	-1.909	α_{I86}	-0.055	0.050	-1.087
α_{LI}	-0.061	0.029	-2.093	α_{I87}	-0.093	0.048	-1.934
α_{II}	-0.024	0.013	-1.885	α_{I88}	-0.130	0.046	-2.831
α_{85}	-0.098	0.043	-2.248	α_{I89}	-0.204	0.045	-4.563
α_{86}	-0.055	0.043	-1.291	α_{I90}	-0.098	0.040	-2.481
α_{87}	-0.033	0.044	-0.758	α_{I91}	-0.074	0.041	-1.827
α_{88}	-0.034	0.044	-0.786	α_{I92}	-0.139	0.041	-3.423
α_{89}	-0.012	0.046	-0.257	α_{K85}	-0.026	0.037	-0.704
α_{90}	0.021	0.045	0.474	α_{K86}	-0.007	0.034	-0.196
α_{91}	-0.017	0.046	-0.382	α_{K87}	-0.036	0.033	-1.066
α_{92}	0.040	0.043	0.932	α_{K88}	-0.006	0.033	-0.188
α_{M85}	-0.094	0.048	-1.947	α_{K89}	-0.028	0.034	-0.832
α_{M86}	-0.120	0.036	-3.379	α_{K90}	-0.070	0.036	-1.974
α_{M87}	-0.115	0.034	-3.397	α_{K91}	-0.005	0.036	-0.130
α_{M88}	-0.093	0.032	-2.931	α_{K92}	-0.030	0.036	-0.848
α_{M89}	-0.090	0.031	-2.928				
α_{M90}	-0.008	0.031	-0.274				
α_{M91}	-0.025	0.028	-0.903				
α_{M92}	-0.029	0.028	-1.027	Log-likel.	1235.41		

Table 9.A12 V1 and JP2 Pooled Estimated by FGLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	-0.022	0.074	-0.296	β_{87}	-0.445	0.236	-1.887
β_F	0.569	0.128	4.441	β_{88}	-0.215	0.221	-0.972
β_K	0.073	0.074	0.987	β_{89}	0.127	0.217	0.588
β_L	-0.037	0.127	-0.293	β_{90}	-0.292	0.220	-1.326
β_I	0.300	0.117	2.560	β_{91}	-0.029	0.232	-0.126
β_{85}	-1.143	0.250	-4.570	β_{92}	0.253	0.222	1.139
β_{86}	-1.121	0.243	-4.606	β_0	-4.740	0.211	-22.514
Log-likel.	-4450.71						
R^2-adj.	0.1198						

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.106	0.025	4.180	α_{F85}	-0.025	0.050	-0.506
α_F	0.456	0.045	10.055	α_{F86}	0.109	0.051	2.162
α_K	0.029	0.032	0.895	α_{F87}	0.130	0.053	2.443
α_L	0.074	0.044	1.676	α_{F88}	0.128	0.052	2.447
α_I	0.475	0.040	11.928	α_{F89}	0.201	0.052	3.868
α_{MM}	0.002	0.004	0.388	α_{F90}	0.029	0.045	0.645
α_{ML}	0.020	0.016	1.245	α_{F91}	0.040	0.052	0.769
α_{MI}	-0.006	0.013	-0.422	α_{F92}	0.102	0.048	2.138
α_{MF}	-0.053	0.014	-3.680	α_{L85}	-0.001	0.044	-0.032
α_{MK}	0.018	0.010	1.788	α_{L86}	-0.067	0.043	-1.558
α_{FF}	0.027	0.017	1.525	α_{L87}	-0.044	0.047	-0.929
α_{FK}	-0.008	0.017	-0.496	α_{L88}	-0.048	0.046	-1.057
α_{LF}	0.013	0.026	0.474	α_{L89}	-0.027	0.047	-0.575
α_{IF}	0.016	0.026	0.636	α_{L90}	-0.021	0.048	-0.431
α_{KK}	-0.003	0.004	-0.874	α_{L91}	-0.047	0.055	-0.844
α_{LK}	-0.018	0.014	-1.355	α_{L92}	-0.027	0.051	-0.531
α_{IK}	0.021	0.016	1.337	α_{I85}	-0.077	0.040	-1.943
α_{LL}	0.004	0.010	0.366	α_{I86}	-0.094	0.043	-2.204
α_{LI}	-0.057	0.025	-2.259	α_{I87}	-0.126	0.047	-2.700
α_{II}	-0.013	0.013	-0.988	α_{I88}	-0.115	0.047	-2.456
α_{85}	-0.064	0.041	-1.563	α_{I89}	-0.197	0.047	-4.216
α_{86}	-0.049	0.042	-1.169	α_{I90}	-0.101	0.038	-2.618
α_{87}	-0.019	0.044	-0.439	α_{I91}	-0.091	0.040	-2.292
α_{88}	-0.026	0.045	-0.587	α_{I92}	-0.166	0.045	-3.742
α_{89}	-0.024	0.050	-0.485	α_{K85}	-0.017	0.033	-0.531
α_{90}	-0.003	0.046	-0.068	α_{K86}	0.008	0.031	0.270
α_{91}	-0.036	0.048	-0.752	α_{K87}	-0.034	0.033	-1.043
α_{92}	0.034	0.049	0.687	α_{K88}	-0.009	0.033	-0.262
α_{M85}	-0.063	0.033	-1.907	α_{K89}	-0.024	0.035	-0.702
α_{M86}	-0.133	0.028	-4.795	α_{K90}	-0.056	0.033	-1.692
α_{M87}	-0.111	0.029	-3.875	α_{K91}	0.004	0.035	0.103
α_{M88}	-0.077	0.029	-2.655	α_{K92}	-0.030	0.038	-0.779
α_{M89}	-0.083	0.029	-2.832	α_0	0.056	0.041	1.360
α_{M90}	-0.020	0.027	-0.745				
α_{M91}	-0.014	0.025	-0.575	Log-likel.	1050.60		
α_{M92}	-0.041	0.027	-1.520	R^2-adj.	0.9212		

Table 9.A13 V2 and JP2 Pooled Estimated by FGLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	0.171	0.081	2.110	β_{87}	-0.461	0.239	-1.928
β_F	0.468	0.116	4.027	β_{88}	-0.354	0.218	-1.627
β_K	0.137	0.082	1.668	β_{89}	-0.070	0.214	-0.329
β_L	-0.093	0.140	-0.666	β_{90}	-0.440	0.214	-2.060
β_I	0.248	0.092	2.705	β_{91}	-0.137	0.226	-0.607
β_{85}	-0.953	0.261	-3.654	β_{92}	0.226	0.220	1.028
β_{86}	-1.087	0.249	-4.374	β_0	-3.533	0.162	-21.798
Log-likel.	-4445.32						
R^2-adj.	0.1247						

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.141	0.027	5.254	α_{F85}	0.008	0.047	0.169
α_F	0.444	0.041	10.732	α_{F86}	0.124	0.048	2.574
α_K	0.035	0.031	1.119	α_{F87}	0.132	0.052	2.547
α_L	0.090	0.044	2.042	α_{F88}	0.131	0.050	2.599
α_I	0.418	0.037	11.276	α_{F89}	0.207	0.050	4.162
α_{MM}	0.007	0.004	1.583	α_{F90}	0.044	0.044	1.003
α_{ML}	0.023	0.017	1.362	α_{F91}	0.055	0.050	1.114
α_{MI}	-0.004	0.013	-0.322	α_{F92}	0.109	0.046	2.376
α_{MF}	-0.070	0.013	-5.354	α_{L85}	-0.023	0.045	-0.510
α_{MK}	0.014	0.010	1.390	α_{L86}	-0.073	0.043	-1.685
α_{FF}	0.019	0.014	1.334	α_{L87}	-0.041	0.047	-0.857
α_{FK}	-0.011	0.014	-0.815	α_{L88}	-0.051	0.046	-1.110
α_{LF}	0.014	0.025	0.550	α_{L89}	-0.031	0.047	-0.662
α_{IF}	0.046	0.022	2.077	α_{L90}	-0.021	0.048	-0.440
α_{KK}	-0.003	0.004	-0.762	α_{L91}	-0.065	0.055	-1.175
α_{LK}	-0.011	0.014	-0.779	α_{L92}	-0.039	0.053	-0.733
α_{IK}	0.014	0.014	0.965	α_{I85}	-0.036	0.039	-0.928
α_{LL}	-0.002	0.011	-0.160	α_{I86}	-0.057	0.042	-1.334
α_{LI}	-0.062	0.025	-2.449	α_{I87}	-0.079	0.046	-1.717
α_{II}	-0.012	0.012	-1.045	α_{I88}	-0.065	0.046	-1.428
α_{85}	-0.090	0.039	-2.332	α_{I89}	-0.146	0.046	-3.148
α_{86}	-0.076	0.039	-1.978	α_{I90}	-0.085	0.038	-2.242
α_{87}	-0.059	0.040	-1.467	α_{I91}	-0.057	0.040	-1.446
α_{88}	-0.060	0.042	-1.450	α_{I92}	-0.131	0.043	-3.033
α_{89}	-0.053	0.047	-1.122	α_{K85}	-0.016	0.032	-0.491
α_{90}	-0.017	0.043	-0.405	α_{K86}	-0.002	0.031	-0.076
α_{91}	-0.055	0.045	-1.241	α_{K87}	-0.039	0.033	-1.185
α_{92}	0.000	0.044	0.006	α_{K88}	-0.013	0.033	-0.394
α_{M85}	-0.095	0.036	-2.617	α_{K89}	-0.031	0.035	-0.893
α_{M86}	-0.157	0.030	-5.325	α_{K90}	-0.055	0.034	-1.620
α_{M87}	-0.137	0.032	-4.338	α_{K91}	0.000	0.035	-0.010
α_{M88}	-0.105	0.031	-3.372	α_{K92}	-0.014	0.038	-0.382
α_{M89}	-0.117	0.031	-3.795	α_0	0.083	0.038	2.156
α_{M90}	-0.046	0.028	-1.607				
α_{M91}	-0.039	0.027	-1.435	Log-likel.	1054.29		
α_{M92}	-0.066	0.029	-2.290	R^2-adj.	0.9381		

Table 9.A14 V1 and JP2 Pooled Estimated by ML

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	0.124	0.047	2.632	β_{87}	-0.798	0.150	-5.312
β_F	0.509	0.082	6.233	β_{88}	-0.486	0.141	-3.456
β_K	0.232	0.047	4.939	β_{89}	-0.082	0.138	-0.593
β_L	-0.149	0.081	-1.851	β_{90}	-0.327	0.140	-2.333
β_I	0.322	0.075	4.318	β_{91}	-0.274	0.148	-1.852
β_{85}	-1.331	0.159	-8.360	β_{92}	-0.182	0.141	-1.291
β_{86}	-1.178	0.155	-7.606	β_0	-4.479	0.134	-33.417

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.115	0.030	3.880	α_{F85}	-0.021	0.050	-0.413
α_F	0.463	0.047	9.929	α_{F86}	0.114	0.052	2.207
α_K	0.038	0.036	1.041	α_{F87}	0.122	0.052	2.332
α_L	0.081	0.045	1.787	α_{F88}	0.142	0.052	2.743
α_I	0.457	0.041	11.124	α_{F89}	0.190	0.053	3.607
α_{MM}	0.000	0.005	-0.074	α_{F90}	0.025	0.048	0.518
α_{ML}	0.027	0.017	1.576	α_{F91}	0.036	0.053	0.681
α_{MI}	-0.008	0.015	-0.556	α_{F92}	0.100	0.045	2.192
α_{MF}	-0.050	0.016	-3.218	α_{L85}	-0.011	0.045	-0.237
α_{MK}	0.018	0.011	1.614	α_{L86}	-0.071	0.044	-1.612
α_{FF}	0.024	0.018	1.339	α_{L87}	-0.049	0.046	-1.063
α_{FK}	0.000	0.019	0.013	α_{L88}	-0.057	0.046	-1.249
α_{LF}	-0.002	0.026	-0.083	α_{L89}	-0.038	0.047	-0.811
α_{IF}	0.018	0.026	0.686	α_{L90}	-0.021	0.050	-0.425
α_{KK}	-0.008	0.005	-1.626	α_{L91}	-0.040	0.055	-0.728
α_{LK}	-0.015	0.014	-1.051	α_{L92}	-0.038	0.049	-0.786
α_{IK}	0.016	0.017	0.943	α_{I85}	-0.060	0.040	-1.482
α_{LL}	0.003	0.010	0.259	α_{I86}	-0.088	0.045	-1.975
α_{LI}	-0.053	0.025	-2.127	α_{I87}	-0.103	0.046	-2.255
α_{II}	-0.009	0.013	-0.706	α_{I88}	-0.109	0.046	-2.355
α_{85}	-0.062	0.042	-1.456	α_{I89}	-0.177	0.048	-3.727
α_{86}	-0.047	0.043	-1.087	α_{I90}	-0.091	0.041	-2.232
α_{87}	-0.021	0.044	-0.475	α_{I91}	-0.079	0.041	-1.933
α_{88}	-0.030	0.045	-0.650	α_{I92}	-0.155	0.042	-3.652
α_{89}	-0.019	0.050	-0.385	α_{K85}	-0.022	0.036	-0.602
α_{90}	0.003	0.048	0.059	α_{K86}	0.002	0.035	0.070
α_{91}	-0.039	0.049	-0.809	α_{K87}	-0.037	0.036	-1.035
α_{92}	0.035	0.047	0.750	α_{K88}	-0.011	0.035	-0.317
α_{M85}	-0.078	0.035	-2.211	α_{K89}	-0.018	0.038	-0.473
α_{M86}	-0.143	0.031	-4.581	α_{K90}	-0.055	0.037	-1.466
α_{M87}	-0.117	0.031	-3.761	α_{K91}	0.000	0.039	0.009
α_{M88}	-0.087	0.031	-2.782	α_{K92}	-0.030	0.040	-0.761
α_{M89}	-0.096	0.032	-2.986	α_0	0.052	0.042	1.234
α_{M90}	-0.035	0.031	-1.115				
α_{M91}	-0.025	0.029	-0.873				
α_{M92}	-0.043	0.030	-1.465	Log-likel.	1079.91		

Table 9.A15 V2 and JP2 Pooled Estimated by ML

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	0.356	0.052	6.897	β_{87}	-0.959	0.152	-6.296
β_F	0.355	0.074	4.800	β_{88}	-0.634	0.139	-4.567
β_K	0.171	0.052	3.285	β_{89}	-0.296	0.136	-2.175
β_L	-0.034	0.089	-0.376	β_{90}	-0.472	0.136	-3.466
β_I	0.194	0.058	3.321	β_{91}	-0.382	0.144	-2.653
β_{85}	-1.293	0.166	-7.785	β_{92}	-0.218	0.140	-1.554
β_{86}	-1.321	0.158	-8.345	β_0	-3.037	0.103	-29.435

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.140	0.030	4.743	α_{F85}	-0.001	0.047	-0.017
α_F	0.469	0.043	11.003	α_{F86}	0.116	0.049	2.338
α_K	0.041	0.034	1.228	α_{F87}	0.122	0.050	2.426
α_L	0.094	0.046	2.021	α_{F88}	0.132	0.051	2.603
α_I	0.411	0.038	10.705	α_{F89}	0.183	0.051	3.619
α_{MM}	0.004	0.004	0.962	α_{F90}	0.021	0.047	0.446
α_{ML}	0.032	0.018	1.826	α_{F91}	0.039	0.051	0.753
α_{MI}	-0.013	0.014	-0.926	α_{F92}	0.096	0.044	2.184
α_{MF}	-0.064	0.014	-4.534	α_{L85}	-0.025	0.047	-0.524
α_{MK}	0.018	0.010	1.772	α_{L86}	-0.078	0.046	-1.701
α_{FF}	0.015	0.015	1.028	α_{L87}	-0.043	0.048	-0.901
α_{FK}	-0.013	0.014	-0.863	α_{L88}	-0.058	0.048	-1.223
α_{LF}	0.000	0.025	-0.004	α_{L89}	-0.034	0.050	-0.686
α_{IF}	0.049	0.022	2.213	α_{L90}	-0.021	0.052	-0.398
α_{KK}	-0.004	0.004	-1.083	α_{L91}	-0.060	0.057	-1.055
α_{LK}	-0.010	0.014	-0.710	α_{L92}	-0.046	0.052	-0.876
α_{IK}	0.011	0.014	0.794	α_{I85}	-0.025	0.040	-0.638
α_{LL}	-0.002	0.011	-0.187	α_{I86}	-0.055	0.043	-1.255
α_{LI}	-0.060	0.025	-2.387	α_{I87}	-0.073	0.045	-1.645
α_{II}	-0.009	0.012	-0.752	α_{I88}	-0.070	0.046	-1.507
α_{85}	-0.067	0.040	-1.660	α_{I89}	-0.125	0.047	-2.667
α_{86}	-0.051	0.040	-1.277	α_{I90}	-0.080	0.041	-1.962
α_{87}	-0.037	0.041	-0.902	α_{I91}	-0.051	0.041	-1.237
α_{88}	-0.038	0.043	-0.881	α_{I92}	-0.124	0.042	-2.992
α_{89}	-0.028	0.048	-0.594	α_{K85}	-0.024	0.034	-0.711
α_{90}	0.007	0.046	0.151	α_{K86}	-0.011	0.033	-0.321
α_{91}	-0.036	0.046	-0.779	α_{K87}	-0.045	0.034	-1.319
α_{92}	0.020	0.044	0.446	α_{K88}	-0.015	0.035	-0.435
α_{M85}	-0.107	0.037	-2.891	α_{K89}	-0.033	0.037	-0.905
α_{M86}	-0.159	0.032	-4.937	α_{K90}	-0.057	0.037	-1.550
α_{M87}	-0.140	0.033	-4.300	α_{K91}	-0.005	0.038	-0.127
α_{M88}	-0.112	0.033	-3.408	α_{K92}	-0.021	0.038	-0.549
α_{M89}	-0.127	0.033	-3.841	α_0	0.057	0.040	1.430
α_{M90}	-0.046	0.032	-1.444				
α_{M91}	-0.042	0.030	-1.407				
α_{M92}	-0.062	0.030	-2.064	Log-likel.	1075.57		

Table 9.A16. V1 and JP2 with Firm-Specific Random Effects Estimated by FGLS proc. RA1*

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	-0.061	0.074	-0.824	β_{87}	-0.375	0.236	-1.590
β_F	0.531	0.128	4.139	β_{88}	-0.460	0.221	-2.082
β_K	0.112	0.074	1.519	β_{89}	0.024	0.217	0.112
β_L	-0.189	0.127	-1.496	β_{90}	-0.311	0.220	-1.414
β_I	0.273	0.117	2.332	β_{91}	0.105	0.232	0.453
β_{85}	-0.367	0.250	-1.468	β_{92}	0.138	0.222	0.624
β_{86}	-0.443	0.243	-1.819	β_0	-4.805	0.211	-22.822
Log-likel.	-4493.69			R^2-adj.	0.0605		

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.102	0.022	4.611	α_{F85}	-0.018	0.064	-0.284
α_F	0.453	0.036	12.518	α_{F86}	-0.041	0.071	-0.574
α_K	0.095	0.027	3.518	α_{F87}	0.014	0.057	0.238
α_L	0.092	0.047	1.967	α_{F88}	0.017	0.048	0.354
α_I	0.348	0.032	10.880	α_{F89}	0.093	0.040	2.340
α_{MM}	0.000	0.003	0.126	α_{F90}	-0.057	0.038	-1.513
α_{ML}	0.048	0.014	3.385	α_{F91}	0.044	0.042	1.051
α_{MI}	-0.017	0.011	-1.631	α_{F92}	0.017	0.032	0.532
α_{MF}	-0.039	0.011	-3.499	α_{L85}	0.060	0.054	1.119
α_{MK}	-0.005	0.008	-0.577	α_{L86}	-0.018	0.047	-0.377
α_{FF}	-0.013	0.010	-1.233	α_{L87}	-0.051	0.050	-1.013
α_{FK}	0.005	0.011	0.479	α_{L88}	-0.020	0.044	-0.462
α_{LF}	0.004	0.022	0.202	α_{L89}	0.041	0.039	1.040
α_{IF}	0.089	0.017	5.210	α_{L90}	0.018	0.042	0.425
α_{KK}	0.004	0.004	1.187	α_{L91}	-0.024	0.044	-0.541
α_{LK}	-0.034	0.014	-2.405	α_{L92}	-0.002	0.038	-0.047
α_{IK}	0.003	0.011	0.318	α_{I85}	0.022	0.054	0.408
α_{LL}	-0.027	0.013	-2.119	α_{I86}	0.020	0.061	0.334
α_{LI}	-0.026	0.023	-1.116	α_{I87}	-0.063	0.051	-1.235
α_{II}	-0.021	0.010	-2.207	α_{I88}	-0.065	0.045	-1.445
α_{85}	-0.146	0.046	-3.189	α_{I89}	-0.167	0.040	-4.232
α_{86}	-0.097	0.047	-2.062	α_{I90}	-0.093	0.033	-2.857
α_{87}	-0.009	0.047	-0.197	α_{I91}	-0.097	0.034	-2.872
α_{88}	-0.012	0.046	-0.261	α_{I92}	-0.122	0.032	-3.771
α_{89}	-0.006	0.045	-0.144	α_{K85}	-0.057	0.039	-1.479
α_{90}	0.020	0.043	0.458	α_{K86}	-0.017	0.033	-0.529
α_{91}	-0.008	0.044	-0.173	α_{K87}	-0.059	0.031	-1.879
α_{92}	0.021	0.038	0.550	α_{K88}	-0.039	0.029	-1.339
α_{M85}	-0.093	0.055	-1.701	α_{K89}	-0.063	0.028	-2.248
α_{M86}	-0.111	0.041	-2.740	α_{K90}	-0.079	0.028	-2.831
α_{M87}	-0.083	0.032	-2.601	α_{K91}	-0.026	0.027	-0.984
α_{M88}	-0.054	0.029	-1.894	α_{K92}	-0.016	0.024	-0.670
α_{M89}	-0.052	0.024	-2.145	α_0	0.1187	0.0438	2.7127
α_{M90}	0.029	0.023	1.221	$\hat{\sigma}^2_\pi$	0.009		
α_{M91}	-0.020	0.021	-0.938	Log-likel.	913.730		
α_{M92}	-0.019	0.020	-0.950	R^2-adj.	0.9328		

* The parameter estimates of the variance function are the same as those of the FE model.

Table 9.A17. V2 and JP2 with Firm-Specific Random Effects Estimated by FGLS proc. RA1*

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	0.145	0.081	1.786	β_{87}	-0.405	0.239	-1.694
β_F	0.470	0.116	4.042	β_{88}	-0.602	0.218	-2.761
β_K	0.159	0.082	1.937	β_{89}	-0.148	0.214	-0.692
β_L	-0.204	0.140	-1.453	β_{90}	-0.423	0.214	-1.980
β_I	0.144	0.092	1.568	β_{91}	0.024	0.226	0.108
β_{85}	-0.238	0.261	-0.911	β_{92}	0.122	0.220	0.556
β_{86}	-0.430	0.249	-1.729	β_0	-3.863	0.162	-23.835
Log-likel.	-4491.91			R^2-adj.	0.0622		

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.097	0.022	4.418	α_{F85}	-0.017	0.064	-0.269
α_F	0.451	0.037	12.350	α_{F86}	-0.038	0.071	-0.538
α_K	0.095	0.027	3.538	α_{F87}	0.019	0.057	0.327
α_L	0.091	0.047	1.945	α_{F88}	0.022	0.048	0.459
α_I	0.354	0.032	11.055	α_{F89}	0.097	0.040	2.440
α_{MM}	0.000	0.003	-0.073	α_{F90}	-0.055	0.038	-1.456
α_{ML}	0.046	0.014	3.255	α_{F91}	0.045	0.042	1.091
α_{MI}	-0.015	0.011	-1.403	α_{F92}	0.020	0.032	0.617
α_{MF}	-0.037	0.011	-3.322	α_{L85}	0.065	0.054	1.206
α_{MK}	-0.003	0.008	-0.377	α_{L86}	-0.016	0.047	-0.335
α_{FF}	-0.011	0.011	-1.043	α_{L87}	-0.050	0.050	-0.997
α_{FK}	0.003	0.011	0.305	α_{L88}	-0.018	0.044	-0.410
α_{LF}	0.007	0.022	0.327	α_{L89}	0.040	0.039	1.029
α_{IF}	0.082	0.017	4.762	α_{L90}	0.020	0.042	0.488
α_{KK}	0.004	0.004	1.159	α_{L91}	-0.020	0.044	-0.467
α_{LK}	-0.034	0.014	-2.479	α_{L92}	-0.002	0.038	-0.043
α_{IK}	0.003	0.011	0.318	α_{I85}	0.015	0.054	0.275
α_{LL}	-0.027	0.012	-2.145	α_{I86}	0.017	0.061	0.277
α_{LI}	-0.024	0.024	-1.029	α_{I87}	-0.066	0.051	-1.298
α_{II}	-0.021	0.010	-2.174	α_{I88}	-0.070	0.045	-1.562
α_{85}	-0.148	0.046	-3.241	α_{I89}	-0.171	0.040	-4.325
α_{86}	-0.098	0.047	-2.099	α_{I90}	-0.096	0.033	-2.933
α_{87}	-0.012	0.047	-0.264	α_{I91}	-0.102	0.034	-3.015
α_{88}	-0.015	0.046	-0.316	α_{I92}	-0.125	0.032	-3.856
α_{89}	-0.010	0.045	-0.214	α_{K85}	-0.058	0.039	-1.511
α_{90}	0.019	0.043	0.438	α_{K86}	-0.018	0.033	-0.546
α_{91}	-0.009	0.044	-0.209	α_{K87}	-0.059	0.031	-1.878
α_{92}	0.019	0.038	0.508	α_{K88}	-0.041	0.029	-1.389
α_{M85}	-0.090	0.054	-1.665	α_{K89}	-0.063	0.028	-2.260
α_{M86}	-0.110	0.040	-2.735	α_{K90}	-0.080	0.028	-2.861
α_{M87}	-0.081	0.031	-2.567	α_{K91}	-0.027	0.027	-0.998
α_{M88}	-0.053	0.029	-1.841	α_{K92}	-0.017	0.024	-0.692
α_{M89}	-0.050	0.024	-2.047	α_0	0.118	0.044	2.697
α_{M90}	0.028	0.023	1.194	σ^2_a	0.009		
α_{M91}	-0.017	0.021	-0.831	Log-likel.	920.856		
α_{M92}	-0.017	0.020	-0.864	R^2-adj.	0.9220		

* The parameter estimates of the variance function are the same as those of the FE model.

Table 9.A18. V1 and JP2 with Firm-Specific Random Effects Estimated by FGLS proc. RA2

	Coeff.	St.error	t-value
β_M	0.157	0.074	2.130
β_F	0.877	0.128	6.837
β_K	-0.020	0.074	-0.273
β_L	0.154	0.127	1.214
β_I	-0.111	0.117	-0.947
β_{85}	0.963	0.250	3.852
β_{86}	0.879	0.243	3.613
Log-likel.	-4307.73		

	Coeff.	St.error	t-value
β_{87}	0.629	0.236	2.667
β_{88}	0.183	0.221	0.828
β_{89}	-0.026	0.217	-0.121
β_{90}	0.348	0.220	1.582
β_{91}	0.196	0.232	0.843
β_{92}	-0.040	0.222	-0.181
β_0	-2.848	0.211	-13.526
R^2-adj.	0.0712		

	Coeff.	St.error	t-value
α_M	0.096	0.022	4.372
α_F	0.480	0.036	13.212
α_K	0.097	0.027	3.649
α_L	0.070	0.046	1.521
α_I	0.361	0.032	11.163
α_{MM}	0.001	0.003	0.204
α_{ML}	0.043	0.014	3.042
α_{MI}	-0.015	0.011	-1.382
α_{MF}	-0.036	0.011	-3.271
α_{MK}	-0.005	0.008	-0.628
α_{FF}	-0.016	0.010	-1.555
α_{FK}	0.004	0.011	0.409
α_{LF}	0.002	0.021	0.116
α_{IF}	0.089	0.017	5.159
α_{KK}	0.004	0.004	1.217
α_{LK}	-0.029	0.014	-2.108
α_{IK}	0.001	0.011	0.077
α_{LL}	-0.021	0.012	-1.830
α_{LI}	-0.027	0.023	-1.157
α_{II}	-0.023	0.010	-2.382
α_{85}	-0.136	0.046	-2.994
α_{86}	-0.092	0.046	-1.970
α_{87}	-0.013	0.047	-0.271
α_{88}	-0.018	0.046	-0.386
α_{89}	-0.009	0.046	-0.192
α_{90}	0.010	0.044	0.237
α_{91}	-0.008	0.045	-0.177
α_{92}	0.019	0.039	0.487
α_{M85}	-0.085	0.054	-1.574
α_{M86}	-0.121	0.040	-3.042
α_{M87}	-0.080	0.031	-2.533
α_{M88}	-0.056	0.029	-1.946
α_{M89}	-0.047	0.025	-1.906
α_{M90}	0.028	0.023	1.183
α_{M91}	-0.014	0.021	-0.671
α_{M92}	-0.021	0.020	-1.029

	Coeff.	St.error	t-value
α_{F85}	-0.008	0.063	-0.121
α_{F86}	0.008	0.070	0.108
α_{F87}	0.028	0.057	0.500
α_{F88}	0.018	0.048	0.365
α_{F89}	0.096	0.040	2.387
α_{F90}	-0.043	0.038	-1.140
α_{F91}	0.031	0.042	0.742
α_{F92}	0.016	0.033	0.497
α_{L85}	0.066	0.053	1.236
α_{L86}	-0.015	0.047	-0.327
α_{L87}	-0.034	0.050	-0.681
α_{L88}	-0.007	0.044	-0.158
α_{L89}	0.041	0.040	1.027
α_{L90}	0.021	0.042	0.502
α_{L91}	-0.024	0.045	-0.542
α_{L92}	0.008	0.039	0.220
α_{I85}	0.009	0.053	0.170
α_{I86}	0.007	0.061	0.115
α_{I87}	-0.068	0.051	-1.325
α_{I88}	-0.056	0.046	-1.220
α_{I89}	-0.169	0.040	-4.190
α_{I90}	-0.095	0.033	-2.882
α_{I91}	-0.089	0.035	-2.576
α_{I92}	-0.123	0.033	-3.750
α_{K85}	-0.070	0.038	-1.810
α_{K86}	-0.027	0.033	-0.826
α_{K87}	-0.070	0.031	-2.247
α_{K88}	-0.046	0.029	-1.557
α_{K89}	-0.065	0.028	-2.317
α_{K90}	-0.083	0.028	-2.954
α_{K91}	-0.027	0.027	-0.996
α_{K92}	-0.017	0.025	-0.674
α_0	0.0939	0.0419	2.2393
$\hat{\sigma}^2_{\pi}$	0.029		
Log-likel.	846.395		
R^2-adj.	0.9389		

Table 9.A19. V2 and JP2 with Firm-Specific Random Effects Estimated by FGLS proc. RA2

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	0.178	0.081	2.199	β_{87}	0.493	0.239	2.061
β_F	0.643	0.116	5.533	β_{88}	-0.027	0.218	-0.125
β_K	-0.092	0.082	-1.119	β_{89}	-0.260	0.214	-1.215
β_L	0.065	0.140	0.461	β_{90}	0.112	0.214	0.524
β_I	0.032	0.092	0.346	β_{91}	0.030	0.226	0.133
β_{85}	0.912	0.261	3.495	β_{92}	-0.139	0.220	-0.630
β_{86}	0.779	0.249	3.136	β_0	-1.445	0.162	-8.913
Log-likel.	-4337.15			R^2-adj.	0.0428		

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.094	0.022	4.257	α_{F85}	-0.005	0.063	-0.075
α_F	0.487	0.036	13.389	α_{F86}	0.014	0.070	0.203
α_K	0.094	0.026	3.567	α_{F87}	0.036	0.057	0.624
α_L	0.072	0.045	1.585	α_{F88}	0.022	0.048	0.454
α_I	0.369	0.032	11.520	α_{F89}	0.099	0.041	2.442
α_{MM}	0.001	0.003	0.236	α_{F90}	-0.044	0.038	-1.167
α_{ML}	0.042	0.014	2.980	α_{F91}	0.030	0.042	0.709
α_{MI}	-0.015	0.011	-1.376	α_{F92}	0.019	0.033	0.592
α_{MF}	-0.036	0.011	-3.238	α_{L85}	0.062	0.053	1.158
α_{MK}	-0.004	0.008	-0.435	α_{L86}	-0.019	0.047	-0.408
α_{FF}	-0.015	0.011	-1.401	α_{L87}	-0.034	0.050	-0.691
α_{FK}	0.004	0.011	0.329	α_{L88}	-0.009	0.044	-0.208
α_{LF}	0.002	0.022	0.075	α_{L89}	0.037	0.040	0.928
α_{IF}	0.084	0.017	4.821	α_{L90}	0.019	0.042	0.452
α_{KK}	0.004	0.004	1.182	α_{L91}	-0.026	0.045	-0.582
α_{LK}	-0.028	0.014	-2.089	α_{L92}	0.006	0.039	0.144
α_{IK}	0.001	0.011	0.090	α_{I85}	0.000	0.052	-0.006
α_{LL}	-0.020	0.012	-1.697	α_{I86}	0.005	0.060	0.076
α_{LI}	-0.029	0.023	-1.250	α_{I87}	-0.070	0.051	-1.368
α_{II}	-0.022	0.010	-2.241	α_{I88}	-0.056	0.046	-1.232
α_{85}	-0.128	0.045	-2.819	α_{I89}	-0.165	0.040	-4.120
α_{86}	-0.084	0.046	-1.805	α_{I90}	-0.090	0.033	-2.757
α_{87}	-0.010	0.047	-0.213	α_{I91}	-0.085	0.034	-2.483
α_{88}	-0.015	0.046	-0.325	α_{I92}	-0.123	0.033	-3.749
α_{89}	-0.009	0.046	-0.199	α_{K85}	-0.070	0.038	-1.816
α_{90}	0.010	0.044	0.236	α_{K86}	-0.028	0.033	-0.846
α_{91}	-0.009	0.045	-0.192	α_{K87}	-0.071	0.031	-2.285
α_{92}	0.020	0.039	0.497	α_{K88}	-0.047	0.029	-1.600
α_{M85}	-0.082	0.054	-1.531	α_{K89}	-0.066	0.028	-2.325
α_{M86}	-0.125	0.040	-3.134	α_{K90}	-0.082	0.028	-2.938
α_{M87}	-0.080	0.031	-2.552	α_{K91}	-0.027	0.027	-0.997
α_{M88}	-0.056	0.029	-1.954	α_{K92}	-0.018	0.025	-0.707
α_{M89}	-0.049	0.025	-1.972	α_0	0.083	0.041	2.019
α_{M90}	0.025	0.023	1.078	σ_η^2	0.019		
α_{M91}	-0.014	0.021	-0.650	Log-likel.	831.238		
α_{M92}	-0.021	0.020	-1.033	R^2-adj.	0.9325		

Table 9.A20. V1 and JP2 with Firm-Specific Random Effects Estimated by FGLS proc. RA2 on Sample 2

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	0.099	0.063	1.566	β_{87}	-0.858	0.217	-3.946
β_F	0.458	0.117	3.920	β_{88}	-1.061	0.202	-5.254
β_K	0.100	0.067	1.491	β_{89}	-0.512	0.194	-2.631
β_L	-0.087	0.116	-0.748	β_{90}	-0.541	0.200	-2.704
β_I	-0.134	0.106	-1.262	β_{91}	-0.285	0.212	-1.346
β_{85}	-0.409	0.232	-1.763	β_{92}	-0.551	0.199	-2.770
β_{86}	-0.567	0.223	-2.548	β_0	-1.191	0.188	-6.322
Log-likel.	-4756.55			R^2-adj.	0.0435		

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.028	0.016	1.720	α_{F85}	-0.132	0.075	-1.753
α_F	0.589	0.027	21.836	α_{F86}	-0.053	0.076	-0.696
α_K	0.139	0.020	7.089	α_{F87}	0.023	0.061	0.384
α_L	0.002	0.036	0.059	α_{F88}	-0.028	0.046	-0.606
α_I	0.298	0.023	13.241	α_{F89}	0.089	0.034	2.586
α_{MM}	0.002	0.002	0.840	α_{F90}	0.002	0.030	0.058
α_{ML}	0.026	0.011	2.412	α_{F91}	0.007	0.042	0.168
α_{MI}	-0.003	0.008	-0.344	α_{F92}	-0.059	0.029	-2.022
α_{MF}	-0.009	0.010	-0.949	α_{L85}	0.128	0.057	2.255
α_{MK}	0.003	0.007	0.468	α_{L86}	0.027	0.042	0.651
α_{FF}	0.042	0.009	4.862	α_{L87}	0.006	0.050	0.122
α_{FK}	-0.046	0.010	-4.535	α_{L88}	0.096	0.041	2.331
α_{LF}	-0.007	0.021	-0.336	α_{L89}	0.140	0.031	4.529
α_{IF}	-0.035	0.017	-2.105	α_{L90}	0.044	0.042	1.051
α_{KK}	0.000	0.003	-0.049	α_{L91}	0.022	0.046	0.482
α_{LK}	0.006	0.011	0.504	α_{L92}	0.103	0.032	3.240
α_{IK}	0.019	0.010	1.999	α_{I85}	0.069	0.058	1.185
α_{LL}	-0.028	0.011	-2.638	α_{I86}	0.065	0.061	1.080
α_{LI}	-0.005	0.018	-0.288	α_{I87}	0.001	0.051	0.020
α_{II}	0.009	0.009	1.055	α_{I88}	0.133	0.039	3.428
α_{85}	-0.191	0.044	-4.351	α_{I89}	-0.013	0.026	-0.499
α_{86}	-0.166	0.035	-4.744	α_{I90}	-0.048	0.029	-1.652
α_{87}	-0.101	0.041	-2.456	α_{I91}	0.020	0.031	0.629
α_{88}	-0.147	0.035	-4.180	α_{I92}	0.072	0.024	2.959
α_{89}	-0.137	0.031	-4.408	α_{K85}	-0.135	0.042	-3.208
α_{90}	-0.069	0.041	-1.686	α_{K86}	-0.119	0.030	-3.979
α_{91}	-0.115	0.035	-3.251	α_{K87}	-0.119	0.024	-4.925
α_{92}	-0.092	0.031	-2.955	α_{K88}	-0.138	0.024	-5.804
α_{M85}	-0.008	0.065	-0.120	α_{K89}	-0.179	0.020	-8.754
α_{M86}	-0.022	0.029	-0.764	α_{K90}	-0.120	0.025	-4.786
α_{M87}	-0.057	0.034	-1.675	α_{K91}	-0.101	0.024	-4.195
α_{M88}	-0.068	0.030	-2.244	α_{K92}	-0.193	0.019	-10.023
α_{M89}	-0.085	0.019	-4.556	α_0	0.1570	0.0297	5.2918
α_{M90}	0.009	0.022	0.389	$\hat{}^{2}$	0.039		
α_{M91}	-0.006	0.020	-0.305	Log-likel.	461.091		
α_{M92}	-0.018	0.015	-1.181	R^2-adj.	0.9813		

Table 9.A21. V2 and JP2 with Firm-Specific Random Effects Estimated by FGLS proc. RA2 on Sample 2

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	0.104	0.074	1.413	β_{87}	-0.935	0.222	-4.206
β_F	0.299	0.108	2.761	β_{88}	-1.158	0.200	-5.787
β_K	-0.033	0.074	-0.442	β_{89}	-0.625	0.192	-3.254
β_L	-0.130	0.126	-1.032	β_{90}	-0.668	0.196	-3.417
β_I	-0.013	0.085	-0.155	β_{91}	-0.381	0.207	-1.839
β_{85}	-0.479	0.242	-1.978	β_{92}	-0.606	0.198	-3.064
β_{86}	-0.641	0.230	-2.790	β_0	-0.594	0.144	-4.117
Log-likel.	-4770.31			R^2-adj.	0.0312		

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.024	0.016	1.494	α_{F85}	-0.136	0.076	-1.791
α_F	0.591	0.027	22.002	α_{F86}	-0.058	0.076	-0.763
α_K	0.146	0.020	7.432	α_{F87}	0.022	0.061	0.363
α_L	-0.007	0.036	-0.200	α_{F88}	-0.029	0.046	-0.630
α_I	0.302	0.022	13.494	α_{F89}	0.085	0.034	2.495
α_{MM}	0.002	0.002	0.687	α_{F90}	-0.006	0.030	-0.195
α_{ML}	0.026	0.011	2.329	α_{F91}	0.003	0.043	0.073
α_{MI}	-0.004	0.008	-0.502	α_{F92}	-0.063	0.029	-2.172
α_{MF}	-0.007	0.010	-0.736	α_{L85}	0.134	0.057	2.355
α_{MK}	0.005	0.007	0.666	α_{L86}	0.034	0.042	0.807
α_{FF}	0.044	0.009	5.116	α_{L87}	0.013	0.050	0.253
α_{FK}	-0.050	0.010	-4.801	α_{L88}	0.102	0.041	2.481
α_{LF}	-0.004	0.022	-0.189	α_{L89}	0.143	0.031	4.654
α_{IF}	-0.042	0.017	-2.458	α_{L90}	0.045	0.042	1.079
α_{KK}	0.000	0.003	-0.034	α_{L91}	0.030	0.046	0.650
α_{LK}	0.003	0.011	0.237	α_{L92}	0.107	0.032	3.374
α_{IK}	0.020	0.010	2.048	α_{I85}	0.063	0.058	1.090
α_{LL}	-0.026	0.011	-2.458	α_{I86}	0.063	0.060	1.043
α_{LI}	-0.004	0.018	-0.211	α_{I87}	0.000	0.051	0.001
α_{II}	0.011	0.009	1.264	α_{I88}	0.131	0.039	3.412
α_{85}	-0.190	0.044	-4.339	α_{I89}	-0.012	0.025	-0.483
α_{86}	-0.167	0.035	-4.789	α_{I90}	-0.047	0.029	-1.629
α_{87}	-0.103	0.041	-2.536	α_{I91}	0.018	0.031	0.588
α_{88}	-0.148	0.035	-4.267	α_{I92}	0.074	0.024	3.061
α_{89}	-0.140	0.031	-4.523	α_{K85}	-0.139	0.042	-3.287
α_{90}	-0.066	0.041	-1.605	α_{K86}	-0.123	0.030	-4.128
α_{91}	-0.117	0.035	-3.337	α_{K87}	-0.123	0.024	-5.067
α_{92}	-0.094	0.031	-3.033	α_{K88}	-0.142	0.024	-5.984
α_{M85}	-0.002	0.065	-0.030	α_{K89}	-0.179	0.020	-8.823
α_{M86}	-0.018	0.029	-0.613	α_{K90}	-0.124	0.025	-4.930
α_{M87}	-0.053	0.034	-1.550	α_{K91}	-0.104	0.024	-4.367
α_{M88}	-0.065	0.030	-2.137	α_{K92}	-0.197	0.019	-10.291
α_{M89}	-0.082	0.019	-4.402	α_0	0.157	0.029	5.338
α_{M90}	0.014	0.022	0.639	$\hat{}^2$	0.037		
α_{M91}	-0.002	0.020	-0.090	Log-likel.	456.824		
α_{M92}	-0.014	0.015	-0.906	R^2-adj.	0.9817		

Table 9.A22. JP2 with Region-Specific Fixed Effects Estimated by OLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_{M}	0.099	0.022	4.506	α_{M85}	-0.070	0.055	-1.277	α_{I85}	-0.016	0.053	-0.308
α_{F}	0.510	0.036	14.125	α_{M86}	-0.116	0.040	-2.876	α_{I86}	-0.003	0.061	-0.048
α_{K}	0.077	0.025	3.038	α_{M87}	-0.079	0.032	-2.483	α_{I87}	-0.088	0.051	-1.714
α_{L}	0.086	0.044	1.958	α_{M88}	-0.059	0.029	-2.044	α_{I88}	-0.058	0.046	-1.280
α_{I}	0.386	0.032	12.193	α_{M89}	-0.050	0.025	-2.010	α_{I89}	-0.160	0.040	-4.021
α_{MM}	0.002	0.003	0.564	α_{M90}	0.019	0.024	0.808	α_{I90}	-0.082	0.033	-2.456
α_{ML}	0.036	0.014	2.577	α_{M91}	-0.012	0.021	-0.571	α_{I91}	-0.065	0.035	-1.873
α_{MI}	-0.014	0.011	-1.286	α_{M92}	-0.033	0.021	-1.563	α_{I92}	-0.126	0.033	-3.788
α_{MF}	-0.037	0.011	-3.262	α_{F85}	-0.018	0.065	-0.284	α_{K85}	-0.070	0.039	-1.814
α_{MK}	-0.006	0.008	-0.737	α_{F86}	0.013	0.071	0.186	α_{K86}	-0.031	0.033	-0.928
α_{FF}	-0.019	0.011	-1.811	α_{F87}	0.023	0.058	0.403	α_{K87}	-0.067	0.031	-2.159
α_{FK}	0.008	0.011	0.722	α_{F88}	0.007	0.048	0.151	α_{K88}	-0.039	0.029	-1.334
α_{LF}	-0.003	0.022	-0.137	α_{F89}	0.091	0.041	2.227	α_{K89}	-0.063	0.028	-2.204
α_{IF}	0.088	0.018	4.965	α_{F90}	-0.047	0.039	-1.201	α_{K90}	-0.077	0.028	-2.692
α_{KK}	0.004	0.003	1.082	α_{F91}	0.012	0.043	0.286	α_{K91}	-0.030	0.028	-1.081
α_{LK}	-0.022	0.013	-1.650	α_{F92}	0.035	0.033	1.039	α_{K92}	-0.020	0.026	-0.787
α_{IK}	0.002	0.011	0.213	α_{L85}	0.048	0.054	0.890	$\alpha_{VA\&R}$	0.081	0.040	2.019
α_{LL}	-0.016	0.011	-1.480	α_{L86}	-0.031	0.047	-0.663	α_{H}	0.050	0.040	1.251
α_{LI}	-0.037	0.023	-1.613	α_{L87}	-0.027	0.050	-0.545	α_{SF}	0.077	0.041	1.896
α_{II}	-0.024	0.010	-2.471	α_{L88}	-0.009	0.044	-0.193	α_{MR}	0.025	0.040	0.634
α_{85}	-0.093	0.045	-2.063	α_{L89}	0.031	0.040	0.756	α_{ST}	-0.007	0.041	-0.169
α_{86}	-0.055	0.046	-1.193	α_{L90}	0.017	0.043	0.408	α_{NT}	0.014	0.041	0.336
α_{87}	0.012	0.046	0.255	α_{L91}	-0.034	0.046	-0.737	α_{N}	0.045	0.040	1.104
α_{88}	0.004	0.046	0.093	α_{L92}	0.015	0.040	0.368	$\alpha_{T\&F}$	0.032	0.041	0.783
α_{89}	0.000	0.046	0.006								
α_{90}	0.006	0.045	0.131								
α_{91}	-0.005	0.046	-0.118								
α_{92}	0.011	0.041	0.274	Log-likel.		724.472		R^2-adj.		0.9341	

Table 9.A23. V1 and JP2 with Region-Specific Fixed Effects Estimated by FGLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	-0.064	0.074	-0.863	β_{87}	-0.882	0.236	-3.740
β_F	0.552	0.128	4.304	β_{88}	-0.454	0.221	-2.059
β_K	0.111	0.074	1.510	β_{89}	0.013	0.217	0.058
β_L	-0.069	0.127	-0.544	β_{90}	-0.496	0.220	-2.255
β_I	0.373	0.117	3.184	β_{91}	-0.101	0.232	-0.435
β_{85}	-1.105	0.250	-4.419	β_{92}	-0.020	0.222	-0.089
β_{86}	-1.071	0.243	-4.399	β_0	-4.659	0.211	-22.129
Log-likel.	-4432.40			R^2-adj.	0.1232		

	Coeff.	St.error	t-value		Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.105	0.025	4.157	α_{M85}	-0.049	0.034	-1.431	α_{I85}	-0.071	0.043	-1.663
α_F	0.427	0.047	9.084	α_{M86}	-0.116	0.029	-4.030	α_{I86}	-0.085	0.046	-1.857
α_K	0.041	0.034	1.219	α_{M87}	-0.100	0.027	-3.731	α_{I87}	-0.131	0.045	-2.890
α_L	0.095	0.046	2.066	α_{M88}	-0.071	0.028	-2.492	α_{I88}	-0.137	0.047	-2.922
α_I	0.469	0.042	11.291	α_{M89}	-0.070	0.029	-2.380	α_{I89}	-0.213	0.049	-4.392
α_{MM}	0.001	0.004	0.266	α_{M90}	-0.012	0.027	-0.440	α_{I90}	-0.097	0.040	-2.447
α_{ML}	0.018	0.016	1.146	α_{M91}	-0.011	0.025	-0.423	α_{I91}	-0.092	0.042	-2.174
α_{MI}	-0.004	0.013	-0.288	α_{M92}	-0.036	0.026	-1.385	α_{I92}	-0.174	0.045	-3.913
α_{MF}	-0.050	0.014	-3.624	α_{F85}	-0.038	0.053	-0.727	α_{K85}	-0.027	0.035	-0.789
α_{MK}	0.015	0.010	1.511	α_{F86}	0.080	0.054	1.489	α_{K86}	-0.002	0.033	-0.050
α_{FF}	0.029	0.018	1.652	α_{F87}	0.102	0.051	1.983	α_{K87}	-0.039	0.033	-1.182
α_{FK}	-0.010	0.017	-0.618	α_{F88}	0.128	0.052	2.488	α_{K88}	-0.015	0.034	-0.434
α_{LF}	0.020	0.026	0.747	α_{F89}	0.194	0.053	3.643	α_{K89}	-0.030	0.036	-0.842
α_{IF}	0.022	0.026	0.843	α_{F90}	0.019	0.046	0.418	α_{K90}	-0.064	0.034	-1.867
α_{KK}	-0.001	0.004	-0.333	α_{F91}	0.035	0.054	0.661	α_{K91}	-0.001	0.037	-0.023
α_{LK}	-0.022	0.014	-1.631	α_{F92}	0.107	0.047	2.279	α_{K92}	-0.033	0.038	-0.868
α_{IK}	0.022	0.016	1.364	α_{L85}	0.004	0.046	0.091	$\alpha_{VA\&R}$	0.094	0.043	2.171
α_{LL}	-0.003	0.011	-0.250	α_{L86}	-0.062	0.045	-1.390	α_H	0.070	0.043	1.617
α_{LI}	-0.054	0.025	-2.145	α_{L87}	-0.038	0.046	-0.820	α_{SF}	0.077	0.043	1.771
α_{II}	-0.016	0.014	-1.183	α_{L88}	-0.045	0.046	-0.970	α_{MR}	0.055	0.043	1.276
α_{85}	-0.073	0.043	-1.699	α_{L89}	-0.019	0.048	-0.399	α_{ST}	0.016	0.043	0.362
α_{86}	-0.053	0.044	-1.210	α_{L90}	-0.014	0.048	-0.292	α_{NT}	0.035	0.044	0.802
α_{87}	-0.014	0.044	-0.315	α_{L91}	-0.040	0.057	-0.696	α_N	0.060	0.043	1.382
α_{88}	-0.019	0.046	-0.416	α_{L92}	-0.019	0.050	-0.387	$\alpha_{T\&F}$	0.037	0.043	0.851
α_{89}	-0.017	0.052	-0.327								
α_{90}	-0.005	0.047	-0.105								
α_{91}	-0.039	0.050	-0.780								
α_{92}	0.026	0.049	0.535	Log-likel.		1078.42		R^2-adj.		0.9202	

Table 9.A24. V2 and JP2 with Region-Specific Fixed Effects Estimated by FGLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	0.161	0.081	1.987	β_{87}	-0.988	0.239	-4.129
β_F	0.379	0.116	3.256	β_{88}	-0.651	0.218	-2.989
β_K	0.227	0.082	2.768	β_{89}	-0.221	0.214	-1.032
β_L	-0.099	0.140	-0.708	β_{90}	-0.661	0.214	-3.093
β_I	0.270	0.092	2.950	β_{91}	-0.238	0.226	-1.053
β_{85}	-0.998	0.261	-3.824	β_{92}	-0.058	0.220	-0.261
β_{86}	-1.131	0.249	-4.552	β_0	-3.379	0.162	-20.853
Log-likel.	-4429.66			R^2-adj.	0.1257		

	Coeff.	St.error	t-value		Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.137	0.028	4.963	α_{M85}	-0.082	0.038	-2.142	α_{I85}	-0.025	0.041	-0.617
α_F	0.429	0.043	9.957	α_{M86}	-0.148	0.031	-4.773	α_{I86}	-0.040	0.045	-0.882
α_K	0.044	0.033	1.314	α_{M87}	-0.125	0.030	-4.134	α_{I87}	-0.077	0.044	-1.756
α_L	0.125	0.046	2.714	α_{M88}	-0.099	0.031	-3.224	α_{I88}	-0.080	0.046	-1.753
α_I	0.403	0.038	10.491	α_{M89}	-0.106	0.032	-3.346	α_{I89}	-0.154	0.048	-3.235
α_{MM}	0.004	0.004	0.846	α_{M90}	-0.036	0.029	-1.232	α_{I90}	-0.080	0.039	-2.067
α_{ML}	0.026	0.017	1.542	α_{M91}	-0.034	0.028	-1.231	α_{I91}	-0.054	0.041	-1.315
α_{MI}	-0.006	0.013	-0.439	α_{M92}	-0.059	0.029	-2.054	α_{I92}	-0.132	0.043	-3.080
α_{MF}	-0.060	0.013	-4.468	α_{F85}	0.006	0.050	0.128	α_{K85}	-0.028	0.035	-0.822
α_{MK}	0.011	0.010	1.111	α_{F86}	0.098	0.052	1.885	α_{K86}	-0.009	0.033	-0.284
α_{FF}	0.016	0.014	1.121	α_{F87}	0.108	0.050	2.157	α_{K87}	-0.041	0.033	-1.236
α_{FK}	-0.013	0.014	-0.896	α_{F88}	0.135	0.050	2.703	α_{K88}	-0.018	0.034	-0.516
α_{LF}	0.011	0.025	0.451	α_{F89}	0.199	0.051	3.891	α_{K89}	-0.032	0.036	-0.884
α_{IF}	0.050	0.022	2.267	α_{F90}	0.032	0.045	0.707	α_{K90}	-0.059	0.035	-1.678
α_{KK}	-0.001	0.004	-0.354	α_{F91}	0.048	0.052	0.934	α_{K91}	-0.006	0.037	-0.154
α_{LK}	-0.012	0.014	-0.884	α_{F92}	0.111	0.045	2.466	α_{K92}	-0.016	0.038	-0.433
α_{IK}	0.014	0.014	0.978	α_{L85}	-0.033	0.047	-0.691	$\alpha_{VA\&R}$	0.115	0.041	2.829
α_{LL}	-0.010	0.011	-0.868	α_{L86}	-0.081	0.045	-1.789	α_H	0.085	0.040	2.107
α_{LI}	-0.058	0.025	-2.310	α_{L87}	-0.052	0.047	-1.102	α_{SF}	0.097	0.041	2.385
α_{II}	-0.012	0.012	-1.008	α_{L88}	-0.060	0.046	-1.299	α_{MR}	0.077	0.041	1.894
α_{85}	-0.094	0.040	-2.324	α_{L89}	-0.036	0.049	-0.743	α_{ST}	0.047	0.041	1.140
α_{86}	-0.075	0.040	-1.866	α_{L90}	-0.027	0.049	-0.546	α_{NT}	0.058	0.041	1.409
α_{87}	-0.052	0.041	-1.253	α_{L91}	-0.061	0.057	-1.069	α_N	0.085	0.041	2.093
α_{88}	-0.053	0.042	-1.253	α_{L92}	-0.047	0.052	-0.900	$\alpha_{T\&F}$	0.055	0.041	1.347
α_{89}	-0.044	0.048	-0.918								
α_{90}	-0.013	0.044	-0.292								
α_{91}	-0.055	0.046	-1.189								
α_{92}	0.000	0.045	0.006	Log-likel.		1082.42		R^2-adj.		0.9381	

Table 9.A25. V1 and JP2 with Firm-Specific RE and Region-Specific FE Estimated by FGLS Procedure RA2

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	0.153	0.074	2.077	β_{87}	0.607	0.236	2.575
β_F	0.852	0.128	6.644	β_{88}	0.203	0.221	0.918
β_K	-0.017	0.074	-0.224	β_{89}	-0.011	0.217	-0.052
β_L	0.179	0.127	1.416	β_{90}	0.352	0.220	1.597
β_I	-0.103	0.117	-0.876	β_{91}	0.215	0.232	0.925
β_{85}	0.968	0.250	3.872	β_{92}	-0.028	0.222	-0.127
β_{86}	0.887	0.243	3.646	β_0	-2.864	0.211	-13.603
Log-likel.	-4303.22			R^2-adj.	0.0706		

	Coeff.	St.error	t-value		Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.098	0.022	4.469	α_{M85}	-0.084	0.054	-1.556	α_{I85}	0.014	0.053	0.270
α_F	0.477	0.036	13.152	α_{M86}	-0.115	0.040	-2.886	α_{I86}	0.013	0.061	0.210
α_K	0.097	0.027	3.627	α_{M87}	-0.079	0.031	-2.517	α_{I87}	-0.066	0.051	-1.290
α_L	0.079	0.046	1.712	α_{M88}	-0.055	0.029	-1.901	α_{I88}	-0.062	0.046	-1.354
α_I	0.358	0.032	11.079	α_{M89}	-0.046	0.025	-1.854	α_{I89}	-0.171	0.040	-4.235
α_{MM}	0.001	0.003	0.208	α_{M90}	0.028	0.023	1.198	α_{I90}	-0.091	0.033	-2.758
α_{ML}	0.043	0.014	3.050	α_{M91}	-0.016	0.021	-0.772	α_{I91}	-0.086	0.035	-2.483
α_{MI}	-0.015	0.011	-1.399	α_{M92}	-0.021	0.020	-1.050	α_{I92}	-0.123	0.033	-3.754
α_{MF}	-0.036	0.011	-3.273	α_{F85}	-0.015	0.063	-0.243	α_{K85}	-0.070	0.038	-1.834
α_{MK}	-0.006	0.008	-0.731	α_{F86}	-0.007	0.070	-0.104	α_{K86}	-0.029	0.033	-0.888
α_{FF}	-0.017	0.010	-1.602	α_{F87}	0.017	0.057	0.293	α_{K87}	-0.069	0.031	-2.214
α_{FK}	0.005	0.011	0.446	α_{F88}	0.014	0.048	0.287	α_{K88}	-0.044	0.029	-1.496
α_{LF}	0.004	0.021	0.172	α_{F89}	0.092	0.040	2.269	α_{K89}	-0.064	0.028	-2.285
α_{IF}	0.090	0.017	5.230	α_{F90}	-0.048	0.038	-1.271	α_{K90}	-0.083	0.028	-2.986
α_{KK}	0.005	0.004	1.340	α_{F91}	0.030	0.042	0.714	α_{K91}	-0.028	0.027	-1.037
α_{LK}	-0.030	0.014	-2.206	α_{F92}	0.016	0.033	0.488	α_{K92}	-0.018	0.025	-0.724
α_{IK}	0.002	0.011	0.139	α_{L85}	0.064	0.053	1.192	$\alpha_{VA\&R}$	0.017	0.014	1.242
α_{LL}	-0.022	0.012	-1.864	α_{L86}	-0.018	0.047	-0.381	α_H	0.003	0.012	0.273
α_{LI}	-0.029	0.023	-1.244	α_{L87}	-0.035	0.050	-0.706	α_{SF}	0.014	0.013	1.081
α_{II}	-0.023	0.010	-2.374	α_{L88}	-0.009	0.044	-0.202	α_{MR}	-0.012	0.013	-0.948
α_{85}	-0.132	0.046	-2.911	α_{L89}	0.041	0.040	1.041	α_{ST}	-0.032	0.014	-2.279
α_{86}	-0.086	0.046	-1.855	α_{L90}	0.023	0.042	0.543	α_{NT}	-0.022	0.013	-1.670
α_{87}	-0.006	0.047	-0.125	α_{L91}	-0.024	0.044	-0.531	α_N	-0.003	0.011	-0.228
α_{88}	-0.011	0.046	-0.236	α_{L92}	0.011	0.039	0.281	α_0	0.092	0.044	2.099
α_{89}	-0.004	0.046	-0.090					$\hat{\sigma}^2_\pi$	0.027		
α_{90}	0.011	0.044	0.248								
α_{91}	-0.007	0.045	-0.149								
α_{92}	0.018	0.039	0.471	Log-likel.		851.099		R^2-adj.		0.9392	

Table 9.A26. V2 and JP2 with Firm-Specific RE and Region-Specific FE Estimated by FGLS Procedure RA2

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	0.175	0.081	2.163	β_{87}	0.480	0.239	2.005
β_F	0.634	0.116	5.457	β_{88}	0.000	0.218	0.000
β_K	-0.089	0.082	-1.092	β_{89}	-0.238	0.214	-1.113
β_L	0.081	0.140	0.579	β_{90}	0.124	0.214	0.580
β_I	0.032	0.092	0.345	β_{91}	0.056	0.226	0.249
β_{85}	0.924	0.261	3.544	β_{92}	-0.123	0.220	-0.557
β_{86}	0.797	0.249	3.205	β_0	-1.459	0.162	-9.004
Log-likel.	-4332.15			R^2-adj.	0.0426		

	Coeff.	St.error	t-value		Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.096	0.022	4.379	α_{M85}	-0.081	0.054	-1.504	α_{I85}	0.006	0.052	0.109
α_F	0.484	0.036	13.338	α_{M86}	-0.117	0.040	-2.934	α_{I86}	0.011	0.060	0.189
α_K	0.095	0.026	3.621	α_{M87}	-0.079	0.031	-2.525	α_{I87}	-0.068	0.051	-1.343
α_L	0.082	0.046	1.805	α_{M88}	-0.054	0.029	-1.895	α_{I88}	-0.064	0.045	-1.404
α_I	0.366	0.032	11.443	α_{M89}	-0.046	0.025	-1.884	α_{I89}	-0.169	0.040	-4.206
α_{MM}	0.001	0.003	0.235	α_{M90}	0.026	0.023	1.117	α_{I90}	-0.086	0.033	-2.629
α_{ML}	0.042	0.014	2.986	α_{M91}	-0.016	0.021	-0.759	α_{I91}	-0.082	0.034	-2.380
α_{MI}	-0.015	0.011	-1.403	α_{M92}	-0.022	0.020	-1.066	α_{I92}	-0.124	0.033	-3.761
α_{MF}	-0.036	0.011	-3.234	α_{F85}	-0.014	0.063	-0.222	α_{K85}	-0.071	0.038	-1.855
α_{MK}	-0.005	0.008	-0.562	α_{F86}	-0.004	0.070	-0.058	α_{K86}	-0.031	0.033	-0.943
α_{FF}	-0.016	0.011	-1.471	α_{F87}	0.021	0.057	0.366	α_{K87}	-0.070	0.031	-2.262
α_{FK}	0.004	0.011	0.357	α_{F88}	0.017	0.048	0.358	α_{K88}	-0.046	0.029	-1.549
α_{LF}	0.003	0.022	0.134	α_{F89}	0.094	0.041	2.311	α_{K89}	-0.065	0.028	-2.299
α_{IF}	0.086	0.017	4.925	α_{F90}	-0.050	0.038	-1.318	α_{K90}	-0.084	0.028	-2.990
α_{KK}	0.005	0.004	1.308	α_{F91}	0.029	0.042	0.673	α_{K91}	-0.029	0.027	-1.053
α_{LK}	-0.030	0.014	-2.192	α_{F92}	0.019	0.033	0.586	α_{K92}	-0.019	0.025	-0.771
α_{IK}	0.002	0.011	0.159	α_{L85}	0.059	0.053	1.102	$\alpha_{VA\&R}$	0.023	0.015	1.540
α_{LL}	-0.020	0.012	-1.730	α_{L86}	-0.022	0.047	-0.472	α_H	0.005	0.013	0.414
α_{LI}	-0.031	0.023	-1.353	α_{L87}	-0.036	0.050	-0.716	α_{SF}	0.019	0.014	1.360
α_{II}	-0.022	0.010	-2.246	α_{L88}	-0.011	0.044	-0.251	α_{MR}	-0.012	0.013	-0.909
α_{85}	-0.122	0.045	-2.700	α_{L89}	0.038	0.040	0.950	α_{ST}	-0.034	0.015	-2.295
α_{86}	-0.076	0.046	-1.652	α_{L90}	0.021	0.042	0.504	α_{NT}	-0.022	0.014	-1.624
α_{87}	-0.001	0.047	-0.026	α_{L91}	-0.025	0.045	-0.561	α_N	-0.001	0.012	-0.072
α_{88}	-0.007	0.046	-0.142	α_{L92}	0.009	0.039	0.223	α_0	0.0769	0.0430	1.7878
α_{89}	-0.004	0.046	-0.082					$\hat{\sigma}_u^2$	0.017		
α_{90}	0.011	0.044	0.246								
α_{91}	-0.007	0.045	-0.163								
α_{92}	0.019	0.039	0.485	Log-likel.	835.914			R^2-adj.		0.9333	

Table 9.A27. V1 with Region-Specific Effects and JP2 with Firm-Specific Fixed Effects Estimated by FGLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	-0.057	0.074	-0.769	β_{87}	-0.423	0.237	-1.788	β_H	0.384	0.214	1.798
β_F	0.470	0.132	3.554	β_{88}	-0.491	0.221	-2.217	β_{SF}	0.501	0.227	2.201
β_K	0.086	0.075	1.150	β_{89}	0.007	0.217	0.031	β_{MR}	0.409	0.221	1.854
β_L	-0.133	0.131	-1.015	β_{90}	-0.317	0.221	-1.434	β_{ST}	0.275	0.238	1.157
β_I	0.268	0.118	2.268	β_{91}	0.092	0.233	0.396	β_{NT}	-0.093	0.234	-0.397
β_{85}	-0.459	0.252	-1.820	β_{92}	0.132	0.222	0.596	β_N	0.343	0.213	1.607
β_{86}	-0.509	0.245	-2.079	$\beta_{VA\&R}$	-0.017	0.238	-0.070	β_0	-5.007	0.259	-19.306
Log-likel.		-4487.2		R^2-adj.		0.0634					

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.113	0.027	4.238	α_{F85}	-0.073	0.064	-1.138
α_F	0.427	0.045	9.476	α_{F86}	-0.006	0.062	-0.093
α_K	0.047	0.033	1.426	α_{F87}	0.048	0.057	0.833
α_L	0.095	0.052	1.841	α_{F88}	0.117	0.051	2.311
α_I	0.391	0.041	9.438	α_{F89}	0.156	0.050	3.114
α_{MM}	0.000	0.004	0.008	α_{F90}	-0.007	0.045	-0.163
α_{ML}	0.043	0.017	2.579	α_{F91}	0.021	0.053	0.396
α_{MI}	-0.005	0.014	-0.391	α_{F92}	0.070	0.044	1.587
α_{MF}	-0.053	0.015	-3.634	α_{L85}	0.039	0.051	0.753
α_{MK}	0.003	0.010	0.247	α_{L86}	-0.037	0.047	-0.799
α_{FF}	0.003	0.016	0.175	α_{L87}	-0.055	0.049	-1.125
α_{FK}	-0.008	0.015	-0.495	α_{L88}	-0.052	0.045	-1.157
α_{LF}	0.014	0.027	0.542	α_{L89}	0.008	0.044	0.188
α_{IF}	0.071	0.025	2.880	α_{L90}	-0.009	0.046	-0.199
α_{KK}	0.000	0.004	-0.084	α_{L91}	-0.033	0.054	-0.606
α_{LK}	-0.030	0.015	-1.916	α_{L92}	-0.031	0.046	-0.675
α_{IK}	0.030	0.016	1.871	α_{I85}	-0.010	0.053	-0.197
α_{LL}	-0.020	0.012	-1.571	α_{I86}	-0.054	0.055	-0.985
α_{LI}	-0.040	0.028	-1.445	α_{I87}	-0.094	0.051	-1.865
α_{II}	-0.027	0.014	-1.895	α_{I88}	-0.146	0.047	-3.128
α_{85}	-0.119	0.046	-2.594	α_{I89}	-0.234	0.047	-5.011
α_{86}	-0.074	0.046	-1.603	α_{I90}	-0.107	0.040	-2.695
α_{87}	-0.016	0.047	-0.348	α_{I91}	-0.102	0.042	-2.435
α_{88}	-0.024	0.046	-0.528	α_{I92}	-0.150	0.043	-3.525
α_{89}	-0.001	0.049	-0.027	α_{K85}	-0.015	0.036	-0.407
α_{90}	0.020	0.046	0.433	α_{K86}	0.015	0.033	0.458
α_{91}	-0.012	0.049	-0.248	α_{K87}	-0.023	0.033	-0.694
α_{92}	0.044	0.045	0.981	α_{K88}	0.003	0.031	0.085
α_{M85}	-0.088	0.045	-1.939	α_{K89}	-0.017	0.033	-0.519
α_{M86}	-0.123	0.035	-3.527	α_{K90}	-0.057	0.033	-1.742
α_{M87}	-0.110	0.031	-3.551	α_{K91}	0.011	0.035	0.312
α_{M88}	-0.077	0.029	-2.657	α_{K92}	-0.029	0.035	-0.843
α_{M89}	-0.067	0.029	-2.337				
α_{M90}	-0.007	0.028	-0.240				
α_{M91}	-0.027	0.026	-1.052	Log-likel.	1228.99		
α_{M92}	-0.031	0.025	-1.200	R^2-adj.	0.8902		

Table 9.A28. V2 with Region-Specific Effects and JP2 with Firm-Specific Fixed Effects Estimated by FGLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	0.151	0.082	1.846	β_{87}	-0.455	0.241	-1.890		0.389	0.216	1.804
β_F	0.405	0.121	3.333	β_{88}	-0.622	0.219	-2.846		0.490	0.229	2.138
β_K	0.113	0.084	1.345	β_{89}	-0.152	0.214	-0.708		0.418	0.222	1.879
β_L	-0.123	0.148	-0.831	β_{90}	-0.420	0.214	-1.958		0.267	0.237	1.123
β_I	0.139	0.093	1.498	β_{91}	0.013	0.226	0.058		-0.080	0.234	-0.340
β_{85}	-0.343	0.264	-1.299	β_{92}	0.120	0.220	0.545		0.297	0.215	1.385
β_{86}	-0.504	0.251	-2.006	β_0	-0.018	0.241	-0.073		-4.110	0.225	-18.247
Log-likel.		-4485.9		R^2-adj.		0.0646					

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.122	0.028	4.328	α_{F85}	-0.065	0.064	-1.016
α_F	0.418	0.044	9.579	α_{F86}	-0.011	0.062	-0.172
α_K	0.058	0.033	1.775	α_{F87}	0.040	0.057	0.699
α_L	0.107	0.053	2.024	α_{F88}	0.108	0.050	2.186
α_I	0.374	0.040	9.398	α_{F89}	0.159	0.049	3.263
α_{MM}	0.001	0.004	0.265	α_{F90}	-0.013	0.045	-0.283
α_{ML}	0.042	0.018	2.334	α_{F91}	0.010	0.052	0.187
α_{MI}	-0.006	0.014	-0.453	α_{F92}	0.057	0.043	1.338
α_{MF}	-0.056	0.014	-3.925	α_{L85}	0.024	0.054	0.445
α_{MK}	0.004	0.011	0.395	α_{L86}	-0.040	0.048	-0.821
α_{FF}	0.001	0.014	0.043	α_{L87}	-0.047	0.051	-0.927
α_{FK}	-0.012	0.014	-0.850	α_{L88}	-0.046	0.047	-0.980
α_{LF}	0.020	0.026	0.749	α_{L89}	0.008	0.045	0.178
α_{IF}	0.083	0.023	3.633	α_{L90}	-0.006	0.048	-0.123
α_{KK}	-0.002	0.004	-0.511	α_{L91}	-0.025	0.055	-0.459
α_{LK}	-0.018	0.016	-1.181	α_{L92}	-0.036	0.048	-0.735
α_{IK}	0.024	0.014	1.686	α_{I85}	0.021	0.052	0.402
α_{LL}	-0.024	0.013	-1.846	α_{I86}	-0.041	0.055	-0.745
α_{LI}	-0.047	0.028	-1.659	α_{I87}	-0.073	0.050	-1.463
α_{II}	-0.027	0.013	-2.101	α_{I88}	-0.118	0.046	-2.574
α_{85}	-0.116	0.045	-2.599	α_{I89}	-0.204	0.046	-4.421
α_{86}	-0.073	0.044	-1.635	α_{I90}	-0.096	0.039	-2.447
α_{87}	-0.031	0.046	-0.683	α_{I91}	-0.082	0.042	-1.956
α_{88}	-0.035	0.045	-0.785	α_{I92}	-0.135	0.042	-3.219
α_{89}	-0.016	0.048	-0.328	α_{K85}	-0.025	0.037	-0.682
α_{90}	0.022	0.045	0.503	α_{K86}	-0.001	0.034	-0.018
α_{91}	-0.015	0.047	-0.306	α_{K87}	-0.033	0.033	-1.007
α_{92}	0.041	0.044	0.932	α_{K88}	-0.011	0.031	-0.347
α_{M85}	-0.102	0.049	-2.073	α_{K89}	-0.031	0.033	-0.929
α_{M86}	-0.127	0.037	-3.472	α_{K90}	-0.065	0.033	-1.957
α_{M87}	-0.117	0.034	-3.474	α_{K91}	-0.014	0.035	-0.387
α_{M88}	-0.087	0.031	-2.853	α_{K92}	-0.022	0.034	-0.634
α_{M89}	-0.080	0.030	-2.658				
α_{M90}	-0.011	0.029	-0.395				
α_{M91}	-0.021	0.027	-0.763	Log-likel.	1231.99		
α_{M92}	-0.031	0.027	-1.131	R^2-adj.	0.8959		

Table 9.A29. V1 and JP2 with Firm-Specific Region-Heteroskedastic Random Effects Estimated by FGLS Procedure RA1*

	Coeff.	St.error	t-value		Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_{M}	0.102	0.022	4.616	α_{M85}	-0.093	0.055	-1.694	α_{I85}	0.021	0.054	0.384
α_{F}	0.454	0.036	12.559	α_{M86}	-0.110	0.041	-2.717	α_{I86}	0.018	0.061	0.291
α_{K}	0.096	0.027	3.554	α_{M87}	-0.082	0.032	-2.573	α_{I87}	-0.064	0.051	-1.262
α_{L}	0.090	0.047	1.925	α_{M88}	-0.054	0.029	-1.898	α_{I88}	-0.066	0.045	-1.467
α_{I}	0.350	0.032	10.959	α_{M89}	-0.052	0.024	-2.115	α_{I89}	-0.167	0.040	-4.233
α_{MM}	0.000	0.003	0.126	α_{M90}	0.028	0.023	1.210	α_{I90}	-0.095	0.033	-2.903
α_{ML}	0.048	0.014	3.350	α_{M91}	-0.019	0.021	-0.909	α_{I91}	-0.097	0.034	-2.846
α_{MI}	-0.018	0.011	-1.649	α_{M92}	-0.019	0.020	-0.939	α_{I92}	-0.121	0.032	-3.754
α_{MF}	-0.039	0.011	-3.481	α_{F85}	-0.016	0.064	-0.253	α_{K85}	-0.057	0.039	-1.482
α_{MK}	-0.005	0.008	-0.559	α_{F86}	-0.037	0.071	-0.520	α_{K86}	-0.018	0.033	-0.538
α_{FF}	-0.013	0.010	-1.281	α_{F87}	0.014	0.057	0.251	α_{K87}	-0.059	0.031	-1.880
α_{FK}	0.005	0.011	0.502	α_{F88}	0.019	0.048	0.394	α_{K88}	-0.040	0.029	-1.378
α_{LF}	0.005	0.022	0.227	α_{F89}	0.093	0.040	2.352	α_{K89}	-0.062	0.028	-2.224
α_{IF}	0.089	0.017	5.220	α_{F90}	-0.055	0.038	-1.472	α_{K90}	-0.078	0.028	-2.810
α_{KK}	0.004	0.004	1.198	α_{F91}	0.042	0.042	1.018	α_{K91}	-0.025	0.027	-0.946
α_{LK}	-0.034	0.014	-2.444	α_{F92}	0.016	0.032	0.512	α_{K92}	-0.016	0.024	-0.644
α_{IK}	0.003	0.011	0.261	α_{L85}	0.061	0.054	1.122	α_{0}	0.1150	0.0431	2.6695
α_{LL}	-0.026	0.012	-2.145	α_{L86}	-0.017	0.047	-0.364				
α_{LI}	-0.025	0.023	-1.059	α_{L87}	-0.049	0.050	-0.983	$\sigma^2_{VA\&R}$	0.006		
α_{II}	-0.022	0.010	-2.259	α_{L88}	-0.019	0.044	-0.432	σ^2_{H}	0.010		
α_{85}	-0.146	0.046	-3.192	α_{L89}	0.040	0.039	1.004	σ^2_{SF}	0.007		
α_{86}	-0.098	0.047	-2.085	α_{L90}	0.017	0.042	0.407	σ^2_{MR}	0.008		
α_{87}	-0.010	0.047	-0.220	α_{L91}	-0.025	0.044	-0.559	σ^2_{ST}	0.013		
α_{88}	-0.013	0.046	-0.275	α_{L92}	-0.003	0.038	-0.078	σ^2_{NT}	0.003		
α_{89}	-0.007	0.045	-0.155					σ^2_{N}	0.008		
α_{90}	0.020	0.043	0.467					$\sigma^2_{T\&F}$	0.005		
α_{91}	-0.009	0.044	-0.194								
α_{92}	0.021	0.038	0.559	Log-likel.		909.830		R^2-adj.		0.9336	

* Parameter estimates for the variance function are identical with the fixed effects estimates presented earlier.

Table 9.A30. V2 and JP2 with Firm-Specific Region-Heteroskedastic Random Effects Estimated by FGLS Procedure RA1*

	Coeff.	St.error	t-value		Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.097	0.022	4.429	α_{M85}	-0.090	0.054	-1.661	α_{I85}	0.014	0.054	0.255
α_F	0.452	0.036	12.400	α_{M86}	-0.109	0.040	-2.707	α_{I86}	0.014	0.061	0.237
α_K	0.096	0.027	3.564	α_{M87}	-0.080	0.031	-2.541	α_{I87}	-0.067	0.051	-1.325
α_L	0.088	0.046	1.904	α_{M88}	-0.053	0.029	-1.842	α_{I88}	-0.072	0.045	-1.588
α_I	0.356	0.032	11.129	α_{M89}	-0.049	0.024	-2.014	α_{I89}	-0.171	0.040	-4.330
α_{MM}	0.000	0.003	-0.078	α_{M90}	0.028	0.023	1.179	α_{I90}	-0.097	0.033	-2.982
α_{ML}	0.045	0.014	3.214	α_{M91}	-0.017	0.021	-0.802	α_{I91}	-0.101	0.034	-2.992
α_{MI}	-0.015	0.011	-1.415	α_{M92}	-0.017	0.020	-0.852	α_{I92}	-0.124	0.032	-3.836
α_{MF}	-0.037	0.011	-3.306	α_{F85}	-0.016	0.064	-0.245	α_{K85}	-0.058	0.039	-1.511
α_{MK}	-0.003	0.008	-0.356	α_{F86}	-0.036	0.071	-0.500	α_{K86}	-0.018	0.033	-0.554
α_{FF}	-0.012	0.011	-1.120	α_{F87}	0.019	0.057	0.333	α_{K87}	-0.059	0.031	-1.875
α_{FK}	0.004	0.011	0.338	α_{F88}	0.024	0.048	0.493	α_{K88}	-0.042	0.029	-1.422
α_{LF}	0.008	0.022	0.357	α_{F89}	0.098	0.040	2.449	α_{K89}	-0.062	0.028	-2.231
α_{IF}	0.083	0.017	4.786	α_{F90}	-0.053	0.038	-1.423	α_{K90}	-0.079	0.028	-2.833
α_{KK}	0.004	0.004	1.160	α_{F91}	0.044	0.042	1.046	α_{K91}	-0.025	0.027	-0.955
α_{LK}	-0.035	0.014	-2.513	α_{F92}	0.019	0.032	0.595	α_{K92}	-0.016	0.024	-0.667
α_{IK}	0.003	0.011	0.255	α_{L85}	0.065	0.054	1.209	α_0	0.1153	0.0432	2.6672
α_{LL}	-0.027	0.012	-2.189	α_{L86}	-0.015	0.047	-0.321				
α_{LI}	-0.023	0.024	-0.963	α_{L87}	-0.048	0.050	-0.970	$\sigma^2_{VA\&R}$	0.006		
α_{II}	-0.022	0.010	-2.239	α_{L88}	-0.017	0.044	-0.384	σ^2_H	0.010		
α_{85}	-0.148	0.046	-3.243	α_{L89}	0.039	0.039	0.989	σ^2_{SF}	0.007		
α_{86}	-0.099	0.047	-2.113	α_{L90}	0.019	0.042	0.465	σ^2_{MR}	0.008		
α_{87}	-0.013	0.047	-0.280	α_{L91}	-0.021	0.044	-0.485	σ^2_{ST}	0.013		
α_{88}	-0.015	0.046	-0.324	α_{L92}	-0.003	0.038	-0.077	σ^2_{NT}	0.003		
α_{89}	-0.010	0.045	-0.224					σ^2_N	0.008		
α_{90}	0.020	0.043	0.459					$\sigma^2_{T\&F}$	0.005		
α_{91}	-0.010	0.044	-0.219								
α_{92}	0.020	0.038	0.519	Log-likel.		917.108		R^2-adj.		0.9232	

* Parameter estimates for the variance function are identical with the fixed effects estimates presented earlier.

Table 9.A31. Kumbhakar Model K2 with Firm-Specific Fixed Effects Estimated by FGLS

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
β_M	-0.288	0.037	-7.810	β_{87}	0.141	0.084	1.682
β_F	-0.233	0.061	-3.832	β_{88}	0.278	0.081	3.441
β_K	0.224	0.037	6.123	β_{89}	0.290	0.084	3.466
β_L	-0.136	0.062	-2.203	β_{90}	0.520	0.089	5.862
β_I	0.147	0.058	2.522	β_{91}	0.330	0.095	3.485
β_{85}	0.164	0.092	1.786	β_{92}	0.263	0.091	2.888
β_{86}	0.320	0.089	3.617				
Log-likel.	-5056.661						

	Coeff.	St.error	t-value		Coeff.	St.error	t-value
α_M	0.384	0.044	8.752	α_{F85}	0.340	0.067	5.052
α_F	0.220	0.059	3.750	α_{F86}	0.303	0.069	4.408
α_K	-0.219	0.041	-5.365	α_{F87}	0.313	0.066	4.752
α_L	0.537	0.066	8.094	α_{F88}	0.461	0.073	6.296
α_I	0.178	0.046	3.904	α_{F89}	0.155	0.078	1.987
α_{MM}	0.038	0.012	3.134	α_{F90}	0.211	0.077	2.759
α_{ML}	-0.005	0.029	-0.179	α_{F91}	0.007	0.073	0.101
α_{MI}	-0.156	0.019	-8.140	α_{F92}	0.221	0.073	3.015
α_{MF}	0.021	0.024	0.874	α_{L85}	-0.257	0.080	-3.226
α_{MK}	0.025	0.015	1.638	α_{L86}	-0.433	0.085	-5.102
α_{FF}	0.194	0.017	11.371	α_{L87}	-0.688	0.074	-9.249
α_{FK}	0.045	0.020	2.279	α_{L88}	-0.711	0.081	-8.827
α_{LF}	-0.362	0.042	-8.562	α_{L89}	-0.307	0.073	-4.215
α_{IF}	-0.155	0.019	-8.289	α_{L90}	-0.501	0.083	-6.002
α_{KK}	-0.030	0.008	-3.801	α_{L91}	-0.321	0.082	-3.918
α_{LK}	0.052	0.031	1.712	α_{L92}	-0.474	0.073	-6.468
α_{IK}	0.007	0.014	0.521	α_{I85}	0.215	0.054	4.020
α_{LL}	0.126	0.032	4.000	α_{I86}	0.167	0.055	3.023
α_{LI}	0.173	0.030	5.700	α_{I87}	0.115	0.048	2.411
α_{II}	0.092	0.006	16.110	α_{I88}	-0.068	0.051	-1.332
α_{85}	-0.477	0.049	-9.807	α_{I89}	-0.052	0.053	-0.972
α_{86}	-0.507	0.042	-11.983	α_{I90}	0.097	0.067	1.449
α_{87}	-0.562	0.043	-13.123	α_{I91}	0.365	0.059	6.229
α_{88}	-0.423	0.039	-10.898	α_{I92}	0.183	0.052	3.500
α_{89}	-0.258	0.034	-7.534	α_{K85}	0.130	0.047	2.783
α_{90}	-0.400	0.042	-9.533	α_{K86}	0.164	0.050	3.297
α_{91}	-0.250	0.035	-7.167	α_{K87}	0.248	0.047	5.284
α_{92}	-0.292	0.038	-7.601	α_{K88}	0.097	0.041	2.377
α_{M85}	-0.446	0.056	-8.019	α_{K89}	0.226	0.052	4.396
α_{M86}	-0.534	0.061	-8.703	α_{K90}	0.006	0.047	0.121
α_{M87}	-0.591	0.052	-11.322	α_{K91}	0.314	0.053	5.949
α_{M88}	-0.390	0.058	-6.705	α_{K92}	0.181	0.043	4.205
α_{M89}	-0.369	0.053	-7.043				
α_{M90}	-0.087	0.058	-1.487				
α_{M91}	-0.498	0.056	-8.893	Log-likel.	-1462.08		
α_{M92}	-0.366	0.047	-7.800	R^2-adj.	0.9719		

9.B. Appendix B: Estimated Elasticities

Table	Description
9.B1	JP2 pooled estimated by OLS
9.B2	JP2 with firm-specific fixed effects estimated by OLS
9.B3	K2 pooled estimated by OLS
9.B4	K2 with firm-specific fixed effects estimated by OLS
9.B5	JP1 with firm-specific fixed effects estimated by OLS
9.B6	V1 and JP1 with firm-specific fixed effects estimated by FGLS
9.B7	V2 and JP1 with firm-specific fixed effects estimated by FGLS
9.B8	V1 and JP2 with firm-specific fixed effects estimated by FGLS
9.B9	V2 and JP2 with firm-specific fixed effects estimated by FGLS
9.B10	V1 and JP2 with firm-specific fixed effects estimated by ML
9.B11	V2 and JP2 with firm-specific fixed effects estimated by ML
9.B12	V1 and JP2 pooled estimated by FGLS
9.B13	V2 and JP2 pooled estimated by FGLS
9.B14	V1 and JP2 pooled estimated by ML
9.B15	V2 and JP2 pooled estimated by ML
9.B16	V1 and JP2 with firm-specific random effects estimated by FGLS proc. RA1
9.B17	V2 and JP2 with firm-specific random effects estimated by FGLS proc. RA1
9.B18	V1 and JP2 with firm-specific random effects estimated by FGLS proc. RA2
9.B19	V2 and JP2 with firm-specific random effects estimated by FGLS proc. RA2
9.B20	V1 and JP2 with firm-specific random effects estimated by FGLS proc. RA2 on sample 2
9.B21	V2 and JP2 with firm-specific random effects estimated by FGLS proc. RA2 on sample 2
9.B22	JP2 with Region-Specific Fixed Effects Estimated by OLS
9.B23	V1 and JP2 with Region-Specific Fixed Effects Estimated by FGLS
9.B24	V2 and JP2 with Region-Specific Fixed Effects Estimated by FGLS
9.B25	V1 and JP2 with Firm-Specific RE and Region-Specific FE Estimated by FGLS Procedure RA2
9.B26	V2 and JP2 with Firm-Specific RE and Region-Specific FE Estimated by FGLS Procedure RA2
9.B27	V1 with Region-Specific Effects and JP2 with Firm-Specific Fixed Effects Estimated by FGLS

Table	Description
9.B28	V2 with Region-Specific Effects and JP2 with Firm-Specific Fixed Effects Estimated by FGLS
9.B29	V1 and JP2 with Firm-Specific Region-Heteroskedastic Random Effects Estimated by FGLS Procedure RA1
9.B30	V2 and JP2 with Firm-Specific Region-Heteroskedastic Random Effects Estimated by FGLS Procedure RA1
9.B31	Coefficients of Correlation Between Estimated Elasticities Derived from Different Model Specifications and Estimators

Table 9.B1 JP2 Pooled Estimated by OLS

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
Mean	0.026	0.564	0.307	0.019	0.037	0.953	0.033	0.030	0.002
St.dev.	0.083	0.160	0.111	0.039	0.062	0.213	0.081	0.080	0.093
Min.	-0.533	0.002	0.000	-0.113	-0.223	0.380	-0.490	-0.153	-0.892
Max.	0.982	3.204	1.073	0.293	0.694	4.507	1.216	1.259	0.295

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
1985	0.192	0.578	0.320	-0.011	0.035	1.114	.	.	.
1986	-0.004	0.593	0.385	0.069	-0.041	1.002	-0.011	0.103	-0.114
1987	0.003	0.583	0.280	-0.013	0.024	0.878	0.072	0.152	-0.080
1988	0.020	0.563	0.294	0.029	0.030	0.936	0.070	-0.015	0.085
1989	0.041	0.649	0.236	0.000	0.025	0.950	-0.003	0.000	-0.004
1990	0.014	0.498	0.330	-0.010	0.092	0.925	0.009	0.015	-0.005
1991	-0.020	0.507	0.338	0.032	0.080	0.937	0.021	-0.013	0.034
1992	0.014	0.576	0.262	0.038	0.035	0.925	0.014	0.019	-0.006
1993	0.001	0.528	0.335	0.043	0.043	0.950	0.086	-0.009	0.095

Mean Elasticities by Region*

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
VA&R	0.022	0.535	0.312	0.018	0.029	0.915	0.021	0.017	0.004
H	0.015	0.589	0.297	0.022	0.026	0.949	0.032	0.023	0.009
SF	0.024	0.556	0.302	0.019	0.030	0.930	0.029	0.027	0.002
MR	0.025	0.568	0.328	0.020	0.031	0.973	0.028	0.025	0.003
ST	0.023	0.581	0.317	0.025	0.049	0.995	0.034	0.043	-0.010
NT	0.039	0.555	0.304	0.014	0.048	0.960	0.051	0.050	0.001
N	0.033	0.561	0.292	0.013	0.039	0.937	0.031	0.029	0.001
T&F	0.039	0.542	0.317	0.023	0.059	0.979	0.041	0.037	0.004

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B2 JP2 with Firm-Specific Effects Estimated by OLS

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
Mean	0.045	0.487	0.276	0.037	0.051	0.896	0.044	0.049	-0.006
St.dev.	0.111	0.141	0.111	0.054	0.061	0.217	0.096	0.116	0.109
Min.	-1.054	0.002	0.000	-0.196	-0.128	0.126	-0.395	-0.201	-0.933
Max.	1.230	2.600	1.021	0.541	0.615	4.345	1.781	1.827	0.289

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
1985	0.237	0.478	0.337	0.035	0.024	1.111	.	.	.
1986	0.028	0.429	0.392	0.106	0.008	0.964	0.002	0.153	-0.151
1987	-0.042	0.482	0.279	0.031	0.035	0.784	0.089	0.220	-0.132
1988	0.008	0.515	0.241	0.038	0.051	0.853	0.099	0.003	0.096
1989	0.076	0.582	0.187	0.009	0.041	0.895	0.026	0.009	0.017
1990	0.049	0.419	0.289	-0.008	0.111	0.861	0.017	0.035	-0.017
1991	0.033	0.487	0.252	0.043	0.071	0.887	0.009	-0.035	0.043
1992	0.024	0.502	0.233	0.048	0.057	0.863	0.004	0.033	-0.029
1993	0.028	0.478	0.308	0.044	0.050	0.907	0.095	-0.013	0.108

Mean Elasticities by Region*

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
VA&R	0.047	0.465	0.269	0.031	0.039	0.851	0.029	0.029	0.000
H	0.035	0.512	0.266	0.036	0.039	0.888	0.041	0.038	0.003
SF	0.044	0.477	0.272	0.037	0.041	0.872	0.035	0.044	-0.009
MR	0.052	0.492	0.297	0.038	0.043	0.921	0.037	0.040	-0.003
ST	0.026	0.501	0.291	0.050	0.074	0.942	0.055	0.070	-0.015
NT	0.056	0.477	0.277	0.031	0.068	0.910	0.072	0.081	-0.009
N	0.049	0.482	0.264	0.030	0.054	0.879	0.042	0.048	-0.007
T&F	0.053	0.468	0.284	0.047	0.073	0.925	0.049	0.060	-0.011

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B3 K2 Pooled Estimated by OLS

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
Mean	0.024	0.553	0.300	0.015	0.036	0.927	0.028	0.030	-0.002
St.dev.	0.068	0.115	0.081	0.028	0.049	0.039	0.068	0.040	0.065
Min.	-0.239	-0.518	-0.362	-0.156	-0.109	0.694	-0.258	-0.014	-0.310
Max.	0.478	0.880	0.529	0.118	0.353	1.051	0.345	0.123	0.327

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
1985	0.123	0.494	0.326	0.004	0.043	0.991	.	.	.
1986	-0.019	0.574	0.340	0.039	0.000	0.935	-0.027	0.005	-0.032
1987	0.050	0.560	0.307	-0.018	0.021	0.920	0.053	0.017	0.035
1988	0.007	0.608	0.274	0.032	0.021	0.942	0.058	0.055	0.003
1989	0.027	0.703	0.215	0.013	-0.009	0.948	-0.007	-0.014	0.007
1990	0.013	0.469	0.359	-0.009	0.059	0.891	0.021	-0.002	0.023
1991	-0.020	0.488	0.332	0.052	0.063	0.915	0.025	0.041	-0.015
1992	0.015	0.584	0.221	0.015	0.053	0.889	0.019	0.018	0.000
1993	0.031	0.468	0.335	0.012	0.080	0.925	0.079	0.123	-0.044

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
VA&R	0.010	0.567	0.315	0.020	0.021	0.934	0.021	0.031	-0.010
H	0.007	0.587	0.304	0.019	0.023	0.939	0.023	0.030	-0.007
SF	0.012	0.561	0.312	0.019	0.028	0.932	0.026	0.030	-0.005
MR	0.012	0.555	0.317	0.020	0.029	0.933	0.019	0.030	-0.011
ST	0.036	0.524	0.289	0.014	0.053	0.916	0.036	0.029	0.007
NT	0.050	0.528	0.279	0.005	0.052	0.914	0.040	0.024	0.015
N	0.037	0.553	0.285	0.010	0.041	0.926	0.031	0.029	0.002
T&F	0.038	0.507	0.293	0.012	0.058	0.909	0.036	0.033	0.003

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B4 K2 with Firm-Specific Fixed Effects Estimated by OLS

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
Mean	0.053	0.483	0.254	0.012	0.048	0.850	0.039	0.035	0.004
St.dev.	0.073	0.110	0.093	0.042	0.040	0.050	0.076	0.037	0.072
Min.	-0.142	-0.531	-0.345	-0.214	-0.065	0.651	-0.296	-0.003	-0.347
Max.	0.461	0.773	0.523	0.231	0.307	1.037	0.317	0.119	0.315

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
1985	0.190	0.394	0.332	-0.004	0.043	0.956	.	.	.
1986	-0.010	0.474	0.327	0.044	0.026	0.862	-0.024	0.002	-0.026
1987	0.028	0.491	0.285	-0.018	0.028	0.814	0.060	0.019	0.041
1988	-0.008	0.557	0.200	0.031	0.040	0.819	0.090	0.068	0.022
1989	0.075	0.616	0.122	0.016	0.013	0.843	0.032	0.030	0.002
1990	0.062	0.397	0.303	-0.029	0.081	0.814	0.034	-0.003	0.038
1991	0.062	0.430	0.278	0.050	0.052	0.871	0.006	0.031	-0.026
1992	0.048	0.511	0.187	0.009	0.074	0.829	0.015	0.017	-0.002
1993	0.053	0.443	0.285	0.018	0.077	0.876	0.088	0.119	-0.031

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
VA&R	0.047	0.500	0.263	0.011	0.034	0.854	0.030	0.037	-0.007
H	0.036	0.514	0.257	0.017	0.041	0.865	0.032	0.036	-0.004
SF	0.041	0.487	0.267	0.018	0.042	0.855	0.034	0.036	-0.001
MR	0.045	0.483	0.272	0.019	0.042	0.861	0.031	0.035	-0.004
ST	0.059	0.455	0.245	0.018	0.061	0.838	0.053	0.035	0.018
NT	0.076	0.463	0.234	-0.003	0.059	0.829	0.058	0.031	0.027
N	0.063	0.486	0.240	0.004	0.052	0.846	0.044	0.036	0.008
T&F	0.068	0.440	0.249	0.013	0.063	0.832	0.042	0.038	0.004

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B5 JP1 with Firm-Specific Fixed Effects Estimated by OLS

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
Mean	-0.006	0.563	0.237	0.004	0.042	0.841	0.038	0.025	0.013
St.dev.	0.147	0.162	0.080	0.052	0.065	0.240	0.023	0.019	0.009
Min.	-2.143	0.002	-0.244	-0.559	-0.660	-1.021	0.003	0.003	-0.004
Max.	0.838	2.829	0.760	0.846	0.432	3.806	0.295	0.290	0.101

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
1985	0.017	0.565	0.238	-0.017	0.038	0.841	0.033	0.020	0.013
1986	-0.021	0.542	0.269	-0.017	0.060	0.833	0.043	0.027	0.016
1987	-0.014	0.529	0.248	0.001	0.059	0.823	0.046	0.030	0.016
1988	-0.014	0.558	0.230	0.003	0.042	0.821	0.037	0.024	0.013
1989	-0.006	0.575	0.250	0.009	0.035	0.863	0.033	0.020	0.013
1990	-0.014	0.590	0.230	0.008	0.036	0.850	0.033	0.021	0.012
1991	0.005	0.542	0.217	0.016	0.043	0.824	0.040	0.026	0.014
1992	-0.006	0.590	0.232	0.017	0.039	0.874	0.040	0.029	0.011
1993	0.003	0.575	0.223	0.014	0.024	0.840	0.034	0.025	0.009

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
VA&R	0.024	0.534	0.237	0.013	0.032	0.840	0.033	0.023	0.010
H	-0.006	0.589	0.243	0.015	0.017	0.857	0.035	0.023	0.013
SF	0.003	0.550	0.229	0.006	0.033	0.822	0.034	0.022	0.012
MR	0.004	0.579	0.247	0.009	0.028	0.867	0.035	0.023	0.012
ST	-0.061	0.588	0.242	0.001	0.072	0.843	0.047	0.029	0.017
NT	0.001	0.549	0.232	-0.007	0.068	0.842	0.042	0.029	0.013
N	-0.007	0.554	0.226	-0.007	0.047	0.813	0.036	0.023	0.013
T&F	-0.017	0.542	0.242	-0.001	0.069	0.835	0.045	0.031	0.014

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B6 V1 and JP1 with Firm-Specific Fixed Effects Estimated by FGLS

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	-0.030	0.588	0.236	0.014	0.032	0.841	0.882	0.037	0.026	0.010	0.106
St.dev.	0.138	0.192	0.091	0.038	0.090	0.240	0.595	0.028	0.024	0.011	0.004
Min.	-1.560	-0.372	-0.196	-0.634	-0.677	-0.849	0.025	0.002	0.004	-0.011	0.100
Max.	0.669	3.170	0.848	0.227	0.878	4.028	5.997	0.442	0.410	0.133	0.112

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	-0.009	0.610	0.238	0.004	0.018	0.861	0.419	0.048	0.038	0.011	0.112
1986	-0.039	0.569	0.267	0.001	0.048	0.846	0.477	0.053	0.038	0.014	0.111
1987	-0.024	0.554	0.249	0.007	0.051	0.838	0.528	0.049	0.035	0.013	0.109
1988	-0.021	0.594	0.224	0.009	0.029	0.834	0.754	0.036	0.025	0.011	0.107
1989	-0.032	0.601	0.249	0.019	0.021	0.857	0.974	0.030	0.020	0.010	0.106
1990	-0.057	0.598	0.238	0.023	0.030	0.831	1.132	0.028	0.019	0.009	0.104
1991	-0.028	0.535	0.231	0.026	0.049	0.813	1.098	0.034	0.022	0.012	0.103
1992	-0.033	0.617	0.226	0.020	0.034	0.863	1.151	0.031	0.024	0.008	0.101
1993	-0.019	0.618	0.206	0.018	0.006	0.829	1.314	0.026	0.020	0.007	0.100

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	-0.009	0.559	0.248	0.019	0.018	0.835	0.871	0.03	0.023	0.007	0.105
H	-0.028	0.629	0.243	0.025	-0.001	0.867	1.083	0.032	0.023	0.009	0.106
SF	-0.023	0.581	0.232	0.018	0.024	0.832	0.88	0.032	0.024	0.009	0.106
MR	-0.03	0.603	0.251	0.022	0.014	0.861	0.987	0.034	0.025	0.009	0.106
ST	-0.066	0.605	0.236	0.001	0.076	0.852	0.84	0.047	0.032	0.015	0.106
NT	-0.023	0.566	0.223	0.005	0.06	0.832	0.69	0.044	0.032	0.012	0.106
N	-0.029	0.572	0.221	0.008	0.038	0.809	0.782	0.036	0.025	0.011	0.106
T&F	-0.036	0.559	0.236	0.004	0.067	0.829	0.751	0.044	0.032	0.012	0.105

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B7 V2 and JP1 with Firm-Specific Fixed Effects Estimated by FGLS

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	-0.027	0.588	0.232	0.016	0.030	0.840	0.979	0.037	0.025	0.013	-0.035
St.dev.	0.150	0.207	0.097	0.029	0.106	0.253	0.000	0.027	0.021	0.014	0.016
Min.	-1.694	-0.603	-0.272	-0.512	-0.657	-1.012	0.979	-0.002	0.004	-0.012	-0.062
Max.	0.767	3.115	0.865	0.189	1.133	4.062	0.979	0.397	0.345	0.149	-0.009

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	-0.004	0.609	0.239	0.004	0.006	0.854	0.979	0.045	0.029	0.016	-0.062
1986	-0.041	0.566	0.266	0.004	0.041	0.835	0.979	0.052	0.032	0.019	-0.055
1987	-0.017	0.549	0.248	0.010	0.046	0.836	0.979	0.049	0.032	0.018	-0.049
1988	-0.013	0.593	0.223	0.012	0.019	0.833	0.979	0.037	0.024	0.013	-0.042
1989	-0.031	0.601	0.246	0.020	0.018	0.855	0.979	0.031	0.019	0.012	-0.036
1990	-0.062	0.599	0.230	0.024	0.033	0.824	0.979	0.029	0.019	0.010	-0.029
1991	-0.031	0.528	0.225	0.026	0.072	0.820	0.979	0.035	0.023	0.013	-0.022
1992	-0.028	0.622	0.217	0.022	0.039	0.872	0.979	0.034	0.025	0.008	-0.016
1993	-0.007	0.627	0.198	0.020	0.001	0.839	0.979	0.028	0.021	0.007	-0.009

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	-0.004	0.559	0.244	0.019	0.019	0.838	0.979	0.030	0.022	0.008	-0.032
H	-0.020	0.629	0.242	0.025	-0.008	0.868	0.979	0.033	0.022	0.011	-0.034
SF	-0.018	0.581	0.228	0.019	0.023	0.834	0.979	0.032	0.022	0.010	-0.036
MR	-0.026	0.604	0.247	0.022	0.013	0.861	0.979	0.034	0.023	0.011	-0.035
ST	-0.064	0.603	0.230	0.008	0.073	0.850	0.979	0.048	0.030	0.019	-0.036
NT	-0.026	0.566	0.217	0.009	0.056	0.822	0.979	0.045	0.030	0.016	-0.037
N	-0.031	0.571	0.217	0.011	0.039	0.807	0.979	0.037	0.023	0.014	-0.037
T&F	-0.033	0.557	0.229	0.009	0.070	0.832	0.979	0.046	0.030	0.015	-0.032

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B8 V1 and JP2 with Firm-Specific Fixed Effects Estimated by FGLS

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.034	0.504	0.283	0.026	0.049	0.896	0.666	0.041	0.037	0.004	0.082
St.dev.	0.096	0.161	0.120	0.042	0.064	0.201	0.518	0.084	0.090	0.101	0.303
Min.	-0.897	0.001	0.000	-0.180	-0.175	0.181	-0.121	-0.334	-0.173	-0.873	-0.285
Max.	1.099	2.207	0.919	0.302	0.678	3.823	4.693	1.316	1.326	0.432	0.622

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.202	0.402	0.352	0.022	0.060	1.038	0.254	.	.	.	.
1986	0.027	0.446	0.360	0.082	0.015	0.929	0.299	-0.016	0.120	-0.135	-0.073
1987	-0.012	0.502	0.283	0.012	0.018	0.803	0.353	0.079	0.160	-0.081	0.070
1988	-0.009	0.596	0.212	0.038	0.038	0.875	0.546	0.089	-0.008	0.097	-0.081
1989	0.052	0.634	0.158	0.020	0.033	0.896	0.745	0.026	0.022	0.005	0.622
1990	0.021	0.455	0.323	-0.015	0.093	0.878	0.892	0.025	0.014	0.011	-0.285
1991	0.012	0.440	0.313	0.056	0.080	0.902	0.862	0.014	-0.030	0.044	0.517
1992	0.014	0.546	0.246	0.007	0.048	0.860	0.914	0.021	0.057	-0.036	0.034
1993	0.032	0.466	0.342	0.024	0.050	0.914	1.047	0.082	-0.025	0.107	-0.129

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.031	0.485	0.278	0.026	0.036	0.856	0.690	0.031	0.023	0.008	0.101
H	0.025	0.531	0.276	0.024	0.032	0.889	0.856	0.038	0.029	0.008	0.086
SF	0.031	0.497	0.279	0.028	0.040	0.876	0.676	0.038	0.034	0.004	0.072
MR	0.035	0.503	0.308	0.031	0.041	0.918	0.769	0.035	0.030	0.005	0.081
ST	0.022	0.517	0.305	0.030	0.072	0.946	0.605	0.049	0.053	-0.004	0.089
NT	0.046	0.500	0.276	0.021	0.063	0.906	0.473	0.061	0.059	0.001	0.062
N	0.040	0.498	0.264	0.023	0.052	0.876	0.561	0.041	0.036	0.006	0.080
T&F	0.043	0.480	0.293	0.028	0.073	0.917	0.538	0.045	0.045	0.000	0.083

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B9 V2 and JP2 with Firm-Specific Fixed Effects Estimated by FGLS

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.049	0.495	0.281	0.026	0.053	0.904	0.713	0.041	0.033	0.008	0.066
St.dev.	0.087	0.159	0.115	0.037	0.073	0.200	0.000	0.075	0.073	0.092	0.310
Min.	-0.667	0.001	-0.001	-0.253	-0.177	0.201	0.713	-0.282	-0.204	-0.768	-0.240
Max.	1.061	2.175	0.900	0.278	0.851	3.786	0.713	0.977	1.254	0.353	0.574

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.183	0.402	0.361	0.028	0.060	1.034	0.713	.	.	.	.
1986	0.043	0.433	0.347	0.078	0.021	0.922	0.713	-0.023	0.118	-0.140	-0.175
1987	0.036	0.487	0.282	0.017	0.021	0.842	0.713	0.066	0.099	-0.034	0.025
1988	0.020	0.577	0.218	0.035	0.037	0.888	0.713	0.091	0.003	0.088	-0.178
1989	0.063	0.634	0.169	0.015	0.028	0.909	0.713	0.027	0.017	0.010	0.574
1990	0.033	0.449	0.311	-0.016	0.098	0.875	0.713	0.027	0.032	-0.005	-0.240
1991	0.023	0.430	0.314	0.041	0.103	0.910	0.713	0.017	-0.035	0.052	0.564
1992	0.020	0.533	0.240	0.021	0.055	0.869	0.713	0.023	0.055	-0.032	0.103
1993	0.043	0.462	0.326	0.030	0.057	0.918	0.713	0.085	-0.021	0.106	-0.115

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.041	0.478	0.278	0.024	0.040	0.862	0.713	0.031	0.022	0.009	0.093
H	0.040	0.520	0.275	0.021	0.037	0.893	0.713	0.038	0.027	0.011	0.074
SF	0.045	0.488	0.277	0.028	0.044	0.882	0.713	0.037	0.030	0.006	0.054
MR	0.047	0.495	0.306	0.030	0.046	0.922	0.713	0.033	0.027	0.007	0.070
ST	0.046	0.506	0.298	0.030	0.079	0.960	0.713	0.048	0.045	0.003	0.068
NT	0.063	0.491	0.272	0.024	0.066	0.916	0.713	0.058	0.051	0.006	0.038
N	0.053	0.489	0.262	0.025	0.056	0.886	0.713	0.040	0.030	0.011	0.056
T&F	0.061	0.471	0.288	0.033	0.080	0.933	0.713	0.045	0.037	0.008	0.075

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B10 V1 and JP2 with Firm-Specific Fixed Effects Estimated by ML

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.035	0.504	0.287	0.026	0.046	0.899	1.045	0.041	0.033	0.009	0.066
St.dev.	0.097	0.163	0.122	0.040	0.062	0.201	0.717	0.081	0.083	0.098	0.166
Min.	-0.760	0.001	-0.021	-0.418	-0.216	0.054	0.024	-0.319	-0.232	-0.826	-0.103
Max.	1.104	2.186	0.909	0.234	0.610	3.764	7.579	1.14	1.146	0.44	0.418

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.196	0.396	0.359	0.019	0.054	1.023	0.471	.	.	.	.
1986	0.030	0.453	0.353	0.074	0.016	0.926	0.563	-0.015	0.107	-0.123	-0.103
1987	0.006	0.499	0.286	0.007	0.020	0.818	0.631	0.074	0.138	-0.064	0.162
1988	-0.004	0.596	0.213	0.041	0.038	0.884	0.879	0.088	-0.010	0.099	0.003
1989	0.050	0.635	0.162	0.026	0.022	0.895	1.152	0.026	0.028	-0.003	0.418
1990	0.016	0.453	0.334	-0.013	0.088	0.878	1.373	0.026	0.014	0.012	-0.085
1991	0.012	0.433	0.335	0.057	0.076	0.913	1.379	0.016	-0.040	0.055	0.043
1992	0.012	0.551	0.246	0.004	0.050	0.863	1.343	0.023	0.063	-0.039	0.131
1993	0.035	0.473	0.337	0.025	0.051	0.920	1.495	0.081	-0.031	0.112	-0.068

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.030	0.486	0.285	0.026	0.034	0.861	1.018	0.031	0.020	0.011	0.073
H	0.024	0.533	0.281	0.024	0.030	0.892	1.276	0.037	0.026	0.011	0.066
SF	0.031	0.498	0.283	0.029	0.038	0.879	1.036	0.038	0.030	0.008	0.064
MR	0.033	0.503	0.314	0.032	0.038	0.920	1.172	0.034	0.026	0.008	0.057
ST	0.030	0.516	0.307	0.025	0.071	0.949	1.019	0.050	0.047	0.004	0.071
NT	0.051	0.498	0.277	0.022	0.061	0.909	0.813	0.059	0.052	0.006	0.062
N	0.042	0.496	0.267	0.024	0.049	0.878	0.928	0.042	0.031	0.011	0.070
T&F	0.050	0.479	0.295	0.026	0.071	0.921	0.898	0.045	0.038	0.007	0.065

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B11 V2 and JP2 with Firm-Specific Fixed Effects Estimated by ML

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.049	0.500	0.285	0.031	0.051	0.916	1.022	0.039	0.027	0.012	0.061
St.dev.	0.090	0.160	0.115	0.037	0.071	0.203	0.000	0.072	0.072	0.094	0.155
Min.	-0.676	0.001	-0.003	-0.517	-0.180	0.080	1.022	-0.249	-0.237	-0.804	-0.268
Max.	1.059	2.241	0.918	0.250	0.765	3.839	1.022	1.068	1.372	0.370	0.318

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.183	0.409	0.367	0.034	0.049	1.041	1.022	.	.	.	.
1986	0.035	0.443	0.341	0.075	0.027	0.919	1.022	-0.019	0.129	-0.148	-0.268
1987	0.060	0.495	0.281	0.018	0.024	0.877	1.022	0.058	0.054	0.004	0.060
1988	0.032	0.585	0.219	0.045	0.030	0.910	1.022	0.084	-0.002	0.086	0.078
1989	0.056	0.639	0.180	0.022	0.020	0.917	1.022	0.028	0.025	0.003	0.318
1990	0.025	0.452	0.316	-0.009	0.099	0.882	1.022	0.027	0.033	-0.005	-0.019
1991	0.019	0.428	0.333	0.050	0.095	0.924	1.022	0.018	-0.041	0.059	0.118
1992	0.016	0.534	0.244	0.020	0.059	0.874	1.022	0.022	0.060	-0.038	0.162
1993	0.043	0.471	0.327	0.031	0.056	0.929	1.022	0.084	-0.032	0.116	-0.003

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.040	0.485	0.285	0.029	0.037	0.875	1.022	0.031	0.019	0.012	0.080
H	0.036	0.527	0.280	0.026	0.035	0.903	1.022	0.036	0.024	0.013	0.064
SF	0.044	0.494	0.281	0.033	0.042	0.894	1.022	0.035	0.026	0.009	0.058
MR	0.044	0.501	0.311	0.034	0.043	0.933	1.022	0.031	0.023	0.009	0.054
ST	0.053	0.509	0.302	0.032	0.078	0.974	1.022	0.049	0.038	0.010	0.063
NT	0.067	0.494	0.275	0.029	0.065	0.930	1.022	0.055	0.044	0.011	0.047
N	0.055	0.494	0.265	0.030	0.053	0.898	1.022	0.040	0.024	0.016	0.056
T&F	0.068	0.475	0.292	0.036	0.078	0.949	1.022	0.042	0.029	0.014	0.067

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B12 V1 and JP2 Pooled Estimated by FGLS

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.019	0.574	0.312	0.013	0.033	0.951	0.884	0.027	0.017	0.010	0.212
St.dev.	0.064	0.187	0.125	0.037	0.064	0.200	0.601	0.074	0.052	0.086	0.385
Min.	-0.424	0.001	0.000	-0.331	-0.213	0.404	0.025	-0.656	-0.202	-0.791	-0.343
Max.	0.734	2.845	1.014	0.257	0.695	3.992	5.793	0.610	0.599	0.359	0.966

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.144	0.501	0.330	0.002	0.058	1.036	0.409	.	.	.	.
1986	-0.020	0.600	0.365	0.057	-0.026	0.976	0.467	-0.029	0.048	-0.077	0.022
1987	0.024	0.608	0.307	-0.022	0.002	0.919	0.518	0.057	0.072	-0.015	0.966
1988	0.009	0.642	0.269	0.017	0.025	0.962	0.744	0.057	-0.011	0.069	0.259
1989	0.014	0.703	0.228	0.008	0.007	0.961	0.975	-0.006	0.002	-0.008	0.408
1990	0.002	0.496	0.365	-0.012	0.068	0.918	1.148	0.012	0.021	-0.009	-0.343
1991	-0.015	0.467	0.366	0.044	0.088	0.951	1.115	0.023	-0.035	0.057	0.300
1992	0.006	0.601	0.251	0.005	0.034	0.897	1.161	0.025	0.073	-0.048	0.326
1993	0.029	0.517	0.343	0.022	0.045	0.955	1.320	0.078	-0.027	0.104	-0.223

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.010	0.541	0.320	0.014	0.025	0.909	0.882	0.021	0.013	0.008	0.196
H	0.011	0.608	0.294	0.013	0.025	0.951	1.081	0.028	0.016	0.012	0.198
SF	0.014	0.569	0.307	0.015	0.028	0.933	0.882	0.029	0.017	0.012	0.215
MR	0.013	0.569	0.333	0.018	0.030	0.962	0.992	0.023	0.014	0.009	0.189
ST	0.020	0.591	0.330	0.008	0.052	1.001	0.832	0.024	0.023	0.001	0.246
NT	0.032	0.572	0.310	0.010	0.037	0.960	0.690	0.034	0.026	0.008	0.216
N	0.028	0.571	0.295	0.011	0.031	0.936	0.785	0.028	0.015	0.014	0.236
T&F	0.031	0.545	0.329	0.009	0.054	0.968	0.751	0.034	0.021	0.013	0.212

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B13 V2 and JP2 Pooled Estimated by FGLS

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.025	0.565	0.310	0.013	0.041	0.953	0.930	0.027	0.018	0.009	0.179
St.dev.	0.059	0.181	0.118	0.032	0.078	0.202	0.000	0.069	0.039	0.078	0.368
Min.	-0.287	0.001	0.000	-0.276	-0.200	0.445	0.930	-0.570	-0.230	-0.685	-0.309
Max.	0.663	2.920	0.993	0.173	0.940	4.069	0.930	0.440	0.461	0.312	0.870

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.119	0.518	0.321	0.020	0.059	1.038	0.930	.	.	.	.
1986	-0.009	0.596	0.351	0.050	-0.011	0.977	0.930	-0.037	0.041	-0.078	-0.125
1987	0.053	0.589	0.305	-0.019	0.013	0.940	0.930	0.048	0.042	0.007	0.870
1988	0.021	0.621	0.277	0.017	0.029	0.965	0.930	0.062	-0.001	0.063	0.113
1989	0.021	0.689	0.246	0.001	0.004	0.960	0.930	-0.007	0.008	-0.015	0.329
1990	0.010	0.498	0.348	-0.014	0.074	0.915	0.930	0.011	0.034	-0.023	-0.309
1991	-0.021	0.466	0.370	0.037	0.104	0.955	0.930	0.021	-0.040	0.060	0.354
1992	0.004	0.591	0.260	0.017	0.039	0.910	0.930	0.024	0.058	-0.034	0.439
1993	0.036	0.492	0.327	0.020	0.062	0.937	0.930	0.085	0.000	0.086	-0.203

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.013	0.538	0.320	0.013	0.030	0.914	0.930	0.021	0.015	0.006	0.175
H	0.016	0.595	0.297	0.011	0.027	0.947	0.930	0.026	0.017	0.010	0.169
SF	0.019	0.561	0.305	0.015	0.035	0.934	0.930	0.026	0.017	0.009	0.180
MR	0.016	0.562	0.333	0.017	0.036	0.964	0.930	0.022	0.015	0.007	0.163
ST	0.031	0.578	0.323	0.011	0.064	1.007	0.930	0.025	0.022	0.003	0.205
NT	0.039	0.562	0.303	0.011	0.046	0.961	0.930	0.032	0.024	0.007	0.169
N	0.033	0.562	0.293	0.013	0.041	0.941	0.930	0.028	0.015	0.013	0.190
T&F	0.038	0.536	0.321	0.014	0.067	0.976	0.930	0.034	0.021	0.013	0.192

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B14. V1 and JP2 Pooled Estimated by ML

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.018	0.572	0.312	0.013	0.035	0.950	1.037	0.028	0.016	0.011	0.205
St.dev.	0.061	0.185	0.125	0.049	0.062	0.202	0.717	0.071	0.052	0.085	0.229
Min.	-0.323	0.001	0.000	-1.171	-0.255	-0.104	0.022	-0.615	-0.257	-0.743	-0.217
Max.	0.678	2.884	1.016	0.207	0.628	4.009	8.141	0.541	0.618	0.353	0.498

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.135	0.505	0.332	0.004	0.056	1.032	0.477	.	.	.	.
1986	-0.013	0.605	0.355	0.051	-0.023	0.976	0.573	-0.027	0.045	-0.073	0.164
1987	0.026	0.599	0.314	-0.024	0.010	0.925	0.645	0.057	0.063	-0.006	0.463
1988	0.002	0.657	0.262	0.015	0.029	0.964	0.886	0.058	-0.013	0.071	0.366
1989	0.007	0.690	0.236	0.016	0.005	0.954	1.144	-0.005	0.011	-0.017	0.498
1990	0.006	0.491	0.365	-0.010	0.063	0.914	1.352	0.009	0.022	-0.013	-0.217
1991	-0.003	0.464	0.368	0.042	0.084	0.954	1.360	0.024	-0.044	0.068	0.055
1992	-0.002	0.596	0.253	0.008	0.040	0.895	1.320	0.025	0.078	-0.053	0.096
1993	0.027	0.511	0.335	0.026	0.053	0.952	1.463	0.079	-0.028	0.107	0.200

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.008	0.539	0.321	0.017	0.026	0.911	0.997	0.021	0.013	0.009	0.193
H	0.009	0.607	0.293	0.013	0.026	0.948	1.276	0.028	0.015	0.013	0.197
SF	0.012	0.568	0.307	0.018	0.029	0.933	1.030	0.029	0.016	0.013	0.210
MR	0.012	0.567	0.333	0.019	0.030	0.961	1.163	0.023	0.013	0.011	0.185
ST	0.022	0.591	0.328	-0.002	0.055	0.994	1.033	0.027	0.021	0.006	0.223
NT	0.031	0.568	0.309	0.012	0.040	0.960	0.799	0.032	0.024	0.008	0.208
N	0.026	0.568	0.295	0.015	0.032	0.936	0.913	0.029	0.014	0.016	0.230
T&F	0.032	0.542	0.327	0.010	0.056	0.968	0.890	0.035	0.019	0.016	0.197

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B15. V2 and JP2 Pooled Estimated by ML

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.024	0.566	0.311	0.013	0.041	0.954	1.043	0.026	0.015	0.011	0.197
St.dev.	0.061	0.180	0.119	0.034	0.075	0.202	0.000	0.067	0.042	0.079	0.207
Min.	-0.283	0.001	0.000	-0.401	-0.183	0.452	1.043	-0.508	-0.260	-0.649	-0.161
Max.	0.655	2.974	0.969	0.173	0.838	4.074	1.043	0.416	0.490	0.308	0.436

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.122	0.523	0.325	0.014	0.053	1.037	1.043	.	.	.	.
1986	-0.011	0.601	0.345	0.044	-0.004	0.976	1.043	-0.033	0.046	-0.078	-0.027
1987	0.055	0.592	0.303	-0.022	0.017	0.945	1.043	0.048	0.035	0.014	0.436
1988	0.013	0.635	0.268	0.020	0.029	0.964	1.043	0.061	0.000	0.062	0.384
1989	0.019	0.676	0.261	0.003	-0.003	0.957	1.043	-0.004	0.010	-0.014	0.401
1990	0.013	0.488	0.349	-0.012	0.075	0.912	1.043	0.007	0.035	-0.028	-0.161
1991	-0.013	0.464	0.371	0.038	0.098	0.957	1.043	0.022	-0.045	0.067	0.094
1992	-0.002	0.588	0.264	0.015	0.044	0.909	1.043	0.022	0.058	-0.036	0.178
1993	0.033	0.500	0.325	0.023	0.067	0.946	1.043	0.084	-0.016	0.100	0.243

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.012	0.540	0.323	0.013	0.029	0.916	1.043	0.020	0.012	0.008	0.196
H	0.014	0.594	0.298	0.010	0.029	0.945	1.043	0.026	0.015	0.012	0.191
SF	0.017	0.562	0.307	0.014	0.035	0.934	1.043	0.025	0.015	0.010	0.201
MR	0.015	0.563	0.335	0.016	0.035	0.964	1.043	0.022	0.012	0.009	0.178
ST	0.033	0.578	0.322	0.009	0.067	1.009	1.043	0.027	0.019	0.008	0.212
NT	0.039	0.563	0.303	0.012	0.048	0.963	1.043	0.031	0.023	0.008	0.190
N	0.032	0.563	0.293	0.013	0.040	0.941	1.043	0.028	0.013	0.016	0.213
T&F	0.039	0.538	0.320	0.013	0.068	0.978	1.043	0.033	0.017	0.016	0.198

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B16. V1 and JP2 with Firm-Specific Random Effects Estimated by FGLS proc. RA1

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.038	0.502	0.281	0.040	0.053	0.914	0.666	0.042	0.046	-0.004	0.082
St.dev.	0.107	0.144	0.111	0.054	0.065	0.220	0.518	0.094	0.114	0.108	0.303
Min.	-0.944	0.002	0.000	-0.170	-0.141	0.224	-0.121	-0.406	-0.167	-0.909	-0.285
Max.	1.179	2.742	1.005	0.600	0.643	4.443	4.693	1.732	1.780	0.270	0.622

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.225	0.504	0.337	0.034	0.028	1.127	0.254	.	.	.	.
1986	0.027	0.448	0.389	0.108	0.009	0.981	0.299	-0.001	0.150	-0.151	-0.073
1987	-0.036	0.492	0.277	0.032	0.039	0.804	0.353	0.086	0.215	-0.128	0.070
1988	0.007	0.518	0.249	0.045	0.053	0.871	0.546	0.093	-0.004	0.098	-0.081
1989	0.068	0.597	0.190	0.014	0.040	0.909	0.745	0.022	0.006	0.015	0.622
1990	0.036	0.441	0.294	-0.004	0.114	0.881	0.892	0.018	0.026	-0.008	-0.285
1991	0.012	0.500	0.273	0.044	0.073	0.902	0.862	0.010	-0.029	0.038	0.517
1992	0.020	0.519	0.241	0.051	0.053	0.883	0.914	0.007	0.030	-0.023	0.034
1993	0.016	0.494	0.316	0.049	0.049	0.924	1.047	0.093	-0.017	0.109	-0.129

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.038	0.481	0.278	0.033	0.039	0.870	0.690	0.027	0.026	0.002	0.101
H	0.027	0.528	0.273	0.040	0.038	0.907	0.856	0.039	0.035	0.004	0.086
SF	0.036	0.493	0.278	0.040	0.042	0.889	0.676	0.034	0.041	-0.008	0.072
MR	0.042	0.509	0.304	0.041	0.043	0.938	0.769	0.035	0.037	-0.002	0.081
ST	0.024	0.516	0.294	0.054	0.074	0.962	0.605	0.053	0.066	-0.013	0.089
NT	0.051	0.490	0.279	0.034	0.071	0.925	0.473	0.070	0.076	-0.005	0.062
N	0.043	0.496	0.268	0.032	0.057	0.896	0.561	0.040	0.045	-0.005	0.080
T&F	0.048	0.483	0.288	0.050	0.076	0.944	0.538	0.047	0.055	-0.008	0.083

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B17. V2 and JP2 with Firm-Specific Random Effects Estimated by FGLS proc. RA1

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.040	0.503	0.282	0.038	0.051	0.914	0.713	0.042	0.046	-0.004	0.066
St.dev.	0.108	0.144	0.111	0.053	0.062	0.219	0.000	0.093	0.113	0.107	0.310
Min.	-0.968	0.002	0.000	-0.178	-0.130	0.201	0.713	-0.388	-0.171	-0.925	-0.240
Max.	1.206	2.748	1.018	0.577	0.618	4.454	0.713	1.703	1.748	0.270	0.574

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.232	0.504	0.335	0.030	0.026	1.128	0.713	.	.	.	.
1986	0.029	0.450	0.392	0.106	0.004	0.980	0.713	0.000	0.151	-0.152	-0.175
1987	-0.037	0.496	0.279	0.031	0.036	0.806	0.713	0.086	0.211	-0.125	0.025
1988	0.009	0.522	0.248	0.042	0.051	0.872	0.713	0.092	-0.003	0.096	-0.178
1989	0.067	0.600	0.190	0.012	0.040	0.909	0.713	0.021	0.006	0.016	0.574
1990	0.039	0.439	0.295	-0.005	0.112	0.879	0.713	0.018	0.028	-0.010	-0.240
1991	0.015	0.498	0.273	0.044	0.073	0.902	0.713	0.011	-0.029	0.040	0.564
1992	0.021	0.519	0.240	0.049	0.053	0.882	0.713	0.006	0.030	-0.024	0.103
1993	0.017	0.493	0.316	0.048	0.048	0.922	0.713	0.092	-0.016	0.108	-0.115

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.040	0.481	0.277	0.032	0.039	0.869	0.713	0.027	0.026	0.001	0.093
H	0.029	0.529	0.272	0.038	0.037	0.906	0.713	0.039	0.035	0.004	0.074
SF	0.038	0.494	0.278	0.038	0.041	0.889	0.713	0.034	0.041	-0.007	0.054
MR	0.044	0.509	0.303	0.039	0.042	0.938	0.713	0.035	0.037	-0.002	0.070
ST	0.024	0.517	0.296	0.052	0.072	0.960	0.713	0.052	0.066	-0.014	0.068
NT	0.052	0.492	0.280	0.032	0.068	0.926	0.713	0.070	0.076	-0.006	0.038
N	0.045	0.498	0.269	0.031	0.054	0.896	0.713	0.039	0.045	-0.005	0.056
T&F	0.049	0.484	0.290	0.048	0.073	0.943	0.713	0.047	0.055	-0.008	0.075

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B18. V1 and JP2 with Firm-Specific Random Effects Estimated by FGLS proc. RA2

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.028	0.531	0.288	0.036	0.046	0.930	1.056	0.039	0.041	-0.002	-0.085
St.dev.	0.095	0.149	0.110	0.052	0.063	0.217	0.696	0.089	0.103	0.103	0.243
Min.	-0.793	0.002	0.000	-0.150	-0.176	0.330	0.030	-0.387	-0.112	-0.939	-0.360
Max.	1.066	2.961	0.996	0.616	0.636	4.480	8.032	1.556	1.606	0.278	0.454

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.206	0.541	0.334	0.019	0.028	1.127	0.565	.	.	.	.
1986	0.004	0.522	0.385	0.099	-0.018	0.991	0.620	-0.004	0.136	-0.140	-0.080
1987	-0.030	0.530	0.281	0.019	0.033	0.833	0.675	0.081	0.194	-0.113	-0.221
1988	0.007	0.540	0.265	0.043	0.043	0.897	0.968	0.084	-0.008	0.092	-0.360
1989	0.053	0.620	0.197	0.014	0.040	0.924	1.169	0.015	0.010	0.005	-0.189
1990	0.025	0.472	0.301	-0.005	0.108	0.900	1.237	0.016	0.019	-0.003	0.454
1991	-0.003	0.506	0.292	0.046	0.074	0.914	1.233	0.014	-0.019	0.034	-0.141
1992	0.016	0.535	0.247	0.053	0.047	0.897	1.335	0.010	0.028	-0.019	-0.210
1993	0.005	0.509	0.322	0.051	0.046	0.933	1.613	0.090	-0.015	0.105	0.041

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.028	0.506	0.287	0.030	0.036	0.887	0.970	0.025	0.024	0.001	-0.081
H	0.019	0.556	0.280	0.037	0.033	0.925	1.290	0.037	0.032	0.005	-0.076
SF	0.027	0.522	0.284	0.036	0.037	0.905	1.026	0.032	0.037	-0.005	-0.094
MR	0.031	0.537	0.310	0.037	0.038	0.953	1.133	0.033	0.034	-0.001	-0.071
ST	0.016	0.546	0.300	0.051	0.063	0.976	1.026	0.048	0.060	-0.012	-0.104
NT	0.039	0.521	0.286	0.030	0.062	0.937	0.894	0.063	0.068	-0.005	-0.074
N	0.033	0.527	0.274	0.028	0.049	0.912	0.985	0.037	0.041	-0.004	-0.099
T&F	0.036	0.510	0.296	0.046	0.068	0.956	0.924	0.045	0.050	-0.005	-0.087

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B19. V2 and JP2 with Firm-Specific Random Effects Estimated by FGLS proc. RA2

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.029	0.537	0.293	0.032	0.044	0.934	0.826	0.038	0.039	-0.001	-0.077
St.dev.	0.094	0.152	0.111	0.049	0.062	0.216	0.000	0.087	0.098	0.101	0.259
Min.	-0.778	0.002	0.000	-0.151	-0.196	0.337	0.826	-0.415	-0.116	-0.931	-0.406
Max.	1.059	3.024	1.013	0.560	0.625	4.507	0.826	1.456	1.500	0.273	0.451

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.205	0.550	0.330	0.013	0.029	1.126	0.826	.	.	.	.
1986	0.004	0.533	0.388	0.092	-0.025	0.992	0.826	-0.003	0.134	-0.137	-0.124
1987	-0.024	0.541	0.285	0.012	0.030	0.844	0.826	0.078	0.181	-0.103	-0.249
1988	0.009	0.549	0.267	0.038	0.041	0.904	0.826	0.080	-0.008	0.088	-0.406
1989	0.051	0.626	0.204	0.011	0.036	0.929	0.826	0.011	0.006	0.005	-0.208
1990	0.024	0.472	0.310	-0.007	0.104	0.904	0.826	0.015	0.019	-0.004	0.451
1991	-0.004	0.506	0.301	0.044	0.073	0.920	0.826	0.016	-0.020	0.036	-0.079
1992	0.015	0.541	0.250	0.050	0.046	0.901	0.826	0.010	0.030	-0.020	-0.155
1993	0.006	0.513	0.323	0.049	0.045	0.936	0.826	0.089	-0.016	0.105	0.149

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.028	0.511	0.292	0.028	0.034	0.892	0.826	0.024	0.023	0.002	-0.067
H	0.019	0.562	0.283	0.034	0.032	0.929	0.826	0.036	0.030	0.006	-0.065
SF	0.027	0.528	0.288	0.032	0.035	0.910	0.826	0.032	0.036	-0.004	-0.086
MR	0.031	0.542	0.314	0.033	0.036	0.956	0.826	0.032	0.032	0.000	-0.058
ST	0.017	0.553	0.304	0.046	0.060	0.980	0.826	0.045	0.057	-0.011	-0.100
NT	0.040	0.528	0.290	0.026	0.058	0.943	0.826	0.061	0.065	-0.004	-0.076
N	0.034	0.534	0.278	0.025	0.047	0.917	0.826	0.035	0.038	-0.003	-0.097
T&F	0.038	0.516	0.301	0.041	0.065	0.961	0.826	0.044	0.048	-0.004	-0.070

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B20. V1 and JP2 with Firm-Specific Random Effects Estimated by FGLS proc. RA2 on Sample 2

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.034	0.577	0.330	-0.002	0.023	0.962	0.437	0.027	0.037	-0.010	0.128
St.dev.	0.099	0.190	0.153	0.060	0.052	0.228	0.355	0.112	0.099	0.125	0.400
Min.	-0.568	-0.088	0.000	-0.616	-0.164	0.247	-0.049	-0.743	-0.745	-1.190	-0.252
Max.	1.058	2.941	2.212	0.521	0.372	4.287	4.280	1.317	1.361	0.740	0.734

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.216	0.489	0.355	-0.005	0.048	1.104	0.197	.	.	.	.
1986	-0.028	0.546	0.404	0.034	0.046	1.003	0.223	-0.072	0.083	-0.155	-0.147
1987	-0.056	0.594	0.318	0.029	-0.006	0.877	0.266	0.095	0.174	-0.079	-0.252
1988	0.077	0.583	0.378	-0.022	-0.016	0.999	0.398	0.074	-0.075	0.149	-0.184
1989	0.104	0.683	0.291	-0.062	-0.036	0.980	0.483	-0.034	0.011	-0.045	0.732
1990	0.004	0.554	0.297	0.003	0.062	0.920	0.505	0.003	0.066	-0.064	-0.029
1991	-0.005	0.529	0.363	0.020	0.055	0.963	0.508	0.014	-0.049	0.063	0.292
1992	0.056	0.547	0.362	-0.068	0.034	0.930	0.566	-0.009	0.024	-0.033	-0.233
1993	-0.037	0.621	0.231	0.067	0.039	0.920	0.679	0.137	0.075	0.062	0.734

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.031	0.546	0.331	-0.002	0.016	0.923	0.396	0.030	0.030	0.000	0.157
H	0.027	0.608	0.301	-0.015	0.021	0.943	0.576	0.033	0.032	0.001	0.142
SF	0.034	0.569	0.326	-0.006	0.019	0.942	0.438	0.027	0.032	-0.005	0.120
MR	0.041	0.579	0.346	-0.007	0.022	0.982	0.488	0.021	0.030	-0.009	0.142
ST	0.041	0.585	0.359	0.000	0.031	1.016	0.418	0.025	0.048	-0.023	0.095
NT	0.034	0.581	0.327	0.005	0.020	0.968	0.327	0.032	0.048	-0.016	0.100
N	0.034	0.575	0.318	0.000	0.021	0.947	0.394	0.020	0.033	-0.014	0.129
T&F	0.035	0.549	0.362	0.021	0.036	1.001	0.345	0.027	0.049	-0.022	0.125

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B21. V2 and JP2 with Firm-Specific Random Effects Estimated by FGLS proc. RA2 on Sample 2

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.033	0.576	0.332	-0.003	0.023	0.963	0.228	0.026	0.036	-0.010	0.141
St.dev.	0.098	0.192	0.155	0.061	0.052	0.228	0.000	0.112	0.097	0.125	0.415
Min.	-0.552	-0.120	0.000	-0.568	-0.170	0.238	0.228	-0.766	-0.727	-1.198	-0.254
Max.	1.036	2.941	2.237	0.494	0.365	4.285	0.228	1.265	1.303	0.734	0.832

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.214	0.488	0.353	-0.004	0.051	1.102	0.228	.	.	.	.
1986	-0.030	0.544	0.406	0.033	0.047	1.001	0.228	-0.074	0.080	-0.154	-0.150
1987	-0.058	0.595	0.321	0.029	-0.005	0.882	0.228	0.094	0.166	-0.072	-0.254
1988	0.077	0.585	0.379	-0.023	-0.016	1.001	0.228	0.073	-0.073	0.147	-0.200
1989	0.102	0.682	0.294	-0.061	-0.036	0.981	0.228	-0.035	0.010	-0.046	0.704
1990	0.001	0.547	0.301	0.001	0.064	0.915	0.228	0.001	0.072	-0.070	-0.042
1991	-0.002	0.527	0.365	0.019	0.056	0.964	0.228	0.013	-0.055	0.068	0.332
1992	0.056	0.546	0.366	-0.070	0.034	0.932	0.228	-0.009	0.025	-0.033	-0.201
1993	-0.039	0.626	0.231	0.067	0.037	0.921	0.228	0.137	0.076	0.061	0.832

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.031	0.545	0.334	-0.002	0.016	0.924	0.228	0.029	0.029	0.000	0.173
H	0.027	0.607	0.302	-0.016	0.022	0.942	0.228	0.033	0.032	0.001	0.156
SF	0.033	0.568	0.328	-0.006	0.020	0.943	0.228	0.027	0.031	-0.005	0.134
MR	0.040	0.578	0.348	-0.007	0.022	0.981	0.228	0.021	0.030	-0.009	0.154
ST	0.038	0.584	0.361	-0.001	0.033	1.015	0.228	0.024	0.046	-0.022	0.103
NT	0.033	0.582	0.330	0.004	0.021	0.970	0.228	0.031	0.047	-0.016	0.105
N	0.033	0.575	0.320	0.000	0.021	0.949	0.228	0.019	0.033	-0.014	0.138
T&F	0.032	0.548	0.365	0.021	0.036	1.001	0.228	0.026	0.048	-0.022	0.143

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B22. JP2 with Region-Specific Fixed Effects Estimated by OLS

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
Mean	0.041	0.548	0.302	0.023	0.044	0.957	0.036	0.031	0.005
St.dev.	0.090	0.154	0.112	0.041	0.060	0.217	0.082	0.088	0.093
Min.	-0.604	0.002	0.000	-0.123	-0.156	0.399	-0.435	-0.120	-0.889
Max.	1.096	3.062	1.018	0.406	0.688	4.520	1.297	1.355	0.291

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
1985	0.214	0.556	0.325	-0.009	0.042	1.128	0.000	0.000	0.000
1986	0.015	0.553	0.390	0.066	-0.014	1.010	-0.006	0.115	-0.121
1987	0.020	0.548	0.277	0.000	0.031	0.876	0.078	0.163	-0.085
1988	0.030	0.551	0.274	0.037	0.036	0.929	0.079	-0.012	0.091
1989	0.055	0.634	0.219	0.006	0.035	0.949	0.004	-0.004	0.009
1990	0.028	0.486	0.329	-0.008	0.098	0.932	0.013	0.005	0.008
1991	-0.007	0.505	0.334	0.034	0.077	0.942	0.019	-0.012	0.031
1992	0.029	0.571	0.256	0.041	0.036	0.932	0.011	0.017	-0.006
1993	0.011	0.524	0.331	0.046	0.045	0.956	0.086	-0.009	0.095

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*	*TCPUR*	*TCNON*
VA&R	0.034	0.522	0.305	0.021	0.034	0.915	0.023	0.017	0.006
H	0.027	0.572	0.293	0.026	0.031	0.949	0.034	0.023	0.011
SF	0.037	0.539	0.296	0.023	0.035	0.930	0.031	0.028	0.003
MR	0.038	0.553	0.324	0.024	0.037	0.975	0.030	0.025	0.005
ST	0.039	0.564	0.312	0.033	0.059	1.007	0.042	0.046	-0.004
NT	0.058	0.538	0.299	0.017	0.058	0.969	0.058	0.053	0.005
N	0.048	0.543	0.286	0.016	0.047	0.940	0.035	0.031	0.004
T&F	0.057	0.527	0.310	0.028	0.066	0.988	0.044	0.039	0.005

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B23. V1 and JP2 with Region-Specific Fixed Effects Estimated by FGLS

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.038	0.552	0.307	0.015	0.039	0.950	0.904	0.032	0.021	0.012	0.198
St.dev.	0.072	0.182	0.126	0.034	0.060	0.202	0.625	0.076	0.059	0.088	0.324
Min.	-0.551	0.001	0.000	-0.209	-0.154	0.404	0.007	-0.607	-0.212	-0.787	-0.399
Max.	0.909	2.633	1.035	0.241	0.696	4.008	5.876	0.756	0.765	0.361	0.595

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.177	0.466	0.335	0.002	0.071	1.051	0.413	.	.	.	.
1986	0.005	0.552	0.375	0.057	-0.008	0.982	0.474	-0.025	0.064	-0.088	0.035
1987	0.050	0.563	0.300	-0.012	0.010	0.910	0.528	0.067	0.092	-0.026	0.208
1988	0.024	0.628	0.250	0.022	0.027	0.951	0.755	0.070	-0.009	0.078	0.534
1989	0.033	0.683	0.212	0.011	0.013	0.951	0.997	0.003	0.004	-0.001	0.595
1990	0.015	0.476	0.367	-0.013	0.073	0.918	1.191	0.015	0.010	0.005	-0.399
1991	0.003	0.453	0.362	0.044	0.091	0.952	1.149	0.020	-0.036	0.056	0.485
1992	0.022	0.602	0.236	0.007	0.036	0.903	1.193	0.027	0.071	-0.044	0.085
1993	0.039	0.509	0.342	0.026	0.042	0.958	1.341	0.076	-0.022	0.098	0.020

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.027	0.521	0.311	0.016	0.030	0.905	0.912	0.024	0.015	0.009	0.208
H	0.027	0.587	0.288	0.014	0.029	0.945	1.109	0.031	0.017	0.014	0.193
SF	0.032	0.547	0.301	0.017	0.033	0.929	0.908	0.033	0.020	0.013	0.198
MR	0.032	0.548	0.327	0.019	0.035	0.960	1.023	0.026	0.016	0.010	0.181
ST	0.039	0.566	0.326	0.016	0.058	1.005	0.852	0.032	0.028	0.003	0.220
NT	0.055	0.547	0.308	0.012	0.044	0.966	0.694	0.043	0.032	0.011	0.183
N	0.047	0.547	0.290	0.013	0.038	0.935	0.793	0.033	0.019	0.015	0.212
T&F	0.054	0.523	0.324	0.015	0.059	0.976	0.764	0.040	0.025	0.015	0.189

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B24. V2 and JP2 with Region-Specific Fixed Effects Estimated by FGLS

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.042	0.547	0.303	0.013	0.045	0.951	0.937	0.031	0.021	0.010	0.175
St.dev.	0.072	0.174	0.118	0.027	0.069	0.204	0.000	0.071	0.046	0.079	0.301
Min.	-0.404	0.001	0.000	-0.166	-0.153	0.446	0.937	-0.562	-0.275	-0.706	-0.356
Max.	0.851	2.788	0.973	0.169	0.848	4.120	0.937	0.496	0.545	0.305	0.538

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.153	0.496	0.320	0.006	0.075	1.050	0.937	.	.	.	.
1986	0.023	0.556	0.358	0.048	-0.002	0.982	0.937	-0.034	0.051	-0.085	-0.125
1987	0.073	0.552	0.297	-0.011	0.023	0.933	0.937	0.058	0.060	-0.002	0.155
1988	0.034	0.614	0.259	0.018	0.030	0.954	0.937	0.071	0.001	0.070	0.400
1989	0.034	0.671	0.232	0.006	0.007	0.950	0.937	0.001	0.011	-0.010	0.538
1990	0.021	0.479	0.345	-0.013	0.079	0.910	0.937	0.014	0.031	-0.017	-0.356
1991	0.005	0.454	0.360	0.033	0.105	0.956	0.937	0.019	-0.047	0.066	0.526
1992	0.010	0.592	0.250	0.018	0.039	0.910	0.937	0.026	0.060	-0.033	0.198
1993	0.047	0.480	0.327	0.025	0.060	0.939	0.937	0.085	0.001	0.084	0.059

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.030	0.521	0.312	0.013	0.035	0.910	0.937	0.023	0.016	0.007	0.197
H	0.029	0.578	0.291	0.012	0.033	0.942	0.937	0.029	0.018	0.011	0.175
SF	0.035	0.542	0.299	0.015	0.039	0.929	0.937	0.030	0.020	0.010	0.174
MR	0.034	0.545	0.327	0.016	0.040	0.962	0.937	0.025	0.017	0.008	0.164
ST	0.051	0.559	0.316	0.015	0.067	1.007	0.937	0.031	0.026	0.004	0.191
NT	0.061	0.541	0.297	0.011	0.052	0.963	0.937	0.040	0.029	0.010	0.147
N	0.050	0.542	0.286	0.012	0.046	0.936	0.937	0.032	0.019	0.014	0.178
T&F	0.063	0.519	0.313	0.015	0.071	0.981	0.937	0.041	0.025	0.016	0.180

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B25. V1 and JP2 with Firm-Specific RE and Region-Specific FE Estimated by FGLS Procedure RA2

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.036	0.525	0.287	0.036	0.048	0.932	1.065	0.040	0.041	-0.001	-0.087
St.dev.	0.098	0.147	0.111	0.052	0.062	0.218	0.690	0.089	0.105	0.103	0.235
Min.	-0.823	0.002	0.000	-0.155	-0.153	0.307	0.031	-0.363	-0.107	-0.935	-0.333
Max.	1.125	2.898	1.001	0.670	0.635	4.481	8.050	1.580	1.635	0.283	0.437

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.218	0.531	0.335	0.018	0.030	1.131	0.581	.	.	.	.
1986	0.014	0.505	0.387	0.095	-0.007	0.994	0.636	-0.002	0.140	-0.142	-0.078
1987	-0.019	0.516	0.280	0.022	0.035	0.833	0.689	0.084	0.197	-0.113	-0.244
1988	0.014	0.535	0.257	0.046	0.045	0.896	0.978	0.087	-0.008	0.095	-0.333
1989	0.060	0.615	0.192	0.016	0.042	0.924	1.177	0.017	0.007	0.010	-0.193
1990	0.031	0.467	0.302	-0.006	0.109	0.904	1.246	0.017	0.015	0.002	0.437
1991	0.002	0.504	0.293	0.045	0.073	0.917	1.239	0.014	-0.018	0.032	-0.128
1992	0.023	0.534	0.244	0.052	0.047	0.901	1.338	0.008	0.026	-0.018	-0.216
1993	0.010	0.508	0.320	0.051	0.047	0.936	1.611	0.090	-0.015	0.105	0.029

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.034	0.501	0.284	0.031	0.037	0.887	0.975	0.026	0.023	0.003	-0.083
H	0.025	0.550	0.278	0.038	0.035	0.925	1.293	0.037	0.031	0.006	-0.078
SF	0.034	0.515	0.282	0.036	0.039	0.906	1.034	0.033	0.037	-0.004	-0.096
MR	0.038	0.531	0.309	0.037	0.040	0.954	1.140	0.034	0.034	0.000	-0.073
ST	0.025	0.540	0.298	0.052	0.066	0.981	1.041	0.051	0.060	-0.009	-0.106
NT	0.049	0.514	0.284	0.030	0.065	0.942	0.907	0.066	0.069	-0.003	-0.076
N	0.042	0.520	0.272	0.028	0.052	0.914	0.996	0.038	0.041	-0.003	-0.101
T&F	0.046	0.504	0.294	0.046	0.070	0.961	0.937	0.046	0.051	-0.004	-0.090

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B26. V2 and JP2 with Firm-Specific RE and Region-Specific FE Estimated by FGLS Procedure RA2

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.038	0.529	0.290	0.034	0.047	0.938	0.833	0.039	0.039	0.000	-0.080
St.dev.	0.096	0.149	0.112	0.050	0.062	0.218	0.000	0.087	0.101	0.101	0.251
Min.	-0.809	0.002	0.000	-0.156	-0.159	0.313	0.833	-0.387	-0.111	-0.923	-0.381
Max.	1.123	2.950	1.020	0.632	0.625	4.514	0.833	1.481	1.532	0.278	0.436

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.218	0.538	0.332	0.014	0.032	1.133	0.833	.	.	.	.
1986	0.015	0.513	0.391	0.089	-0.012	0.997	0.833	0.000	0.139	-0.139	-0.120
1987	-0.011	0.525	0.282	0.017	0.033	0.846	0.833	0.082	0.185	-0.103	-0.272
1988	0.017	0.543	0.258	0.042	0.044	0.904	0.833	0.083	-0.008	0.092	-0.381
1989	0.060	0.620	0.197	0.014	0.040	0.931	0.833	0.014	0.003	0.011	-0.212
1990	0.032	0.466	0.311	-0.007	0.106	0.909	0.833	0.016	0.014	0.002	0.436
1991	0.003	0.504	0.302	0.044	0.072	0.924	0.833	0.015	-0.019	0.034	-0.065
1992	0.023	0.540	0.247	0.050	0.046	0.906	0.833	0.009	0.028	-0.019	-0.164
1993	0.012	0.512	0.321	0.050	0.046	0.940	0.833	0.089	-0.015	0.105	0.130

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.035	0.505	0.290	0.029	0.036	0.894	0.833	0.025	0.022	0.003	-0.070
H	0.026	0.554	0.281	0.035	0.034	0.930	0.833	0.036	0.030	0.007	-0.069
SF	0.035	0.520	0.286	0.033	0.038	0.912	0.833	0.032	0.035	-0.003	-0.089
MR	0.039	0.535	0.313	0.034	0.039	0.959	0.833	0.033	0.032	0.001	-0.062
ST	0.028	0.545	0.302	0.048	0.065	0.988	0.833	0.049	0.057	-0.008	-0.102
NT	0.052	0.519	0.288	0.027	0.063	0.950	0.833	0.065	0.065	0.000	-0.079
N	0.044	0.525	0.276	0.026	0.050	0.921	0.833	0.037	0.038	-0.001	-0.100
T&F	0.049	0.509	0.298	0.042	0.069	0.968	0.833	0.045	0.048	-0.003	-0.074

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B27. V1 with Region-Specific Effects and JP2 with Firm-Specific Fixed Effects Estimated by FGLS

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.030	0.507	0.283	0.030	0.047	0.897	0.635	0.042	0.036	0.006	0.094
St.dev.	0.095	0.157	0.115	0.041	0.062	0.200	0.476	0.081	0.091	0.103	0.303
Min.	-0.830	0.001	0.000	-0.184	-0.163	0.172	-0.058	-0.286	-0.194	-0.935	-0.276
Max.	1.099	2.262	0.887	0.303	0.655	3.852	4.261	1.163	1.452	0.375	0.645

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.209	0.414	0.346	0.026	0.045	1.040	0.256	.	.	.	.
1986	0.017	0.454	0.356	0.087	0.007	0.921	0.298	-0.005	0.136	-0.141	-0.049
1987	-0.018	0.496	0.293	0.016	0.018	0.805	0.345	0.068	0.141	-0.073	0.090
1988	-0.013	0.597	0.217	0.041	0.038	0.880	0.521	0.090	-0.012	0.102	-0.065
1989	0.049	0.633	0.161	0.023	0.035	0.901	0.707	0.028	0.025	0.002	0.645
1990	0.019	0.455	0.319	-0.007	0.091	0.877	0.848	0.025	0.021	0.004	-0.276
1991	0.011	0.444	0.312	0.054	0.080	0.900	0.818	0.013	-0.033	0.046	0.505
1992	0.005	0.547	0.244	0.013	0.051	0.859	0.863	0.025	0.059	-0.034	0.041
1993	0.029	0.478	0.336	0.025	0.052	0.921	0.983	0.082	-0.036	0.118	-0.124

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.028	0.489	0.276	0.028	0.036	0.857	0.655	0.031	0.022	0.009	0.111
H	0.022	0.532	0.277	0.028	0.032	0.891	0.802	0.039	0.029	0.009	0.098
SF	0.028	0.500	0.278	0.032	0.038	0.876	0.642	0.039	0.034	0.005	0.084
MR	0.033	0.507	0.307	0.035	0.039	0.920	0.728	0.036	0.030	0.006	0.092
ST	0.018	0.520	0.305	0.034	0.069	0.946	0.580	0.051	0.052	-0.001	0.101
NT	0.041	0.502	0.276	0.026	0.061	0.904	0.463	0.061	0.059	0.002	0.075
N	0.036	0.501	0.264	0.027	0.049	0.877	0.543	0.041	0.034	0.007	0.094
T&F	0.039	0.484	0.292	0.033	0.071	0.919	0.520	0.044	0.042	0.003	0.093

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B28. V2 with Region-Specific Effects and JP2 with Firm-Specific Fixed Effects Estimated by FGLS

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.044	0.498	0.281	0.031	0.050	0.903	0.684	0.042	0.033	0.008	0.078
St.dev.	0.088	0.155	0.111	0.036	0.068	0.200	0.000	0.074	0.081	0.096	0.306
Min.	-0.622	0.001	0.000	-0.329	-0.158	0.189	0.684	-0.257	-0.225	-0.842	-0.235
Max.	1.085	2.251	0.878	0.300	0.791	3.846	0.684	1.098	1.407	0.302	0.601

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.199	0.415	0.354	0.035	0.039	1.041	0.684	.	.	.	.
1986	0.034	0.442	0.345	0.085	0.014	0.920	0.684	-0.011	0.132	-0.143	-0.148
1987	0.019	0.481	0.290	0.022	0.020	0.832	0.684	0.060	0.102	-0.043	0.050
1988	0.012	0.580	0.222	0.039	0.036	0.890	0.684	0.092	-0.006	0.098	-0.154
1989	0.059	0.632	0.171	0.019	0.030	0.911	0.684	0.029	0.022	0.007	0.601
1990	0.030	0.449	0.307	-0.007	0.095	0.874	0.684	0.027	0.037	-0.010	-0.235
1991	0.024	0.432	0.311	0.039	0.099	0.906	0.684	0.015	-0.039	0.053	0.541
1992	0.009	0.532	0.241	0.026	0.058	0.865	0.684	0.027	0.058	-0.031	0.113
1993	0.037	0.474	0.324	0.030	0.057	0.923	0.684	0.085	-0.033	0.117	-0.113

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.037	0.482	0.276	0.027	0.039	0.862	0.684	0.032	0.022	0.010	0.104
H	0.035	0.521	0.277	0.026	0.035	0.894	0.684	0.039	0.028	0.011	0.086
SF	0.040	0.491	0.276	0.033	0.042	0.881	0.684	0.038	0.031	0.007	0.067
MR	0.044	0.499	0.305	0.034	0.043	0.924	0.684	0.035	0.027	0.007	0.081
ST	0.041	0.509	0.299	0.034	0.074	0.957	0.684	0.051	0.047	0.005	0.082
NT	0.056	0.493	0.272	0.029	0.063	0.912	0.684	0.058	0.054	0.005	0.053
N	0.048	0.493	0.263	0.029	0.052	0.885	0.684	0.041	0.030	0.011	0.070
T&F	0.056	0.476	0.288	0.037	0.077	0.933	0.684	0.046	0.037	0.009	0.086

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B29. V1 and JP2 with Firm-Specific Region-Heteroskedastic Random Effects Estimated by FGLS Procedure RA1

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.036	0.504	0.282	0.040	0.052	0.915	0.666	0.042	0.045	-0.004	0.082
St.dev.	0.106	0.144	0.111	0.055	0.064	0.220	0.518	0.093	0.113	0.107	0.303
Min.	-0.951	0.002	0.000	-0.171	-0.140	0.214	-0.121	-0.403	-0.175	-0.903	-0.285
Max.	1.160	2.760	1.003	0.615	0.640	4.446	4.693	1.728	1.777	0.265	0.622

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.221	0.507	0.337	0.034	0.028	1.128	0.254	.	.	.	.
1986	0.025	0.453	0.388	0.108	0.009	0.984	0.299	-0.002	0.147	-0.149	-0.073
1987	-0.036	0.494	0.277	0.032	0.040	0.807	0.353	0.086	0.214	-0.128	0.070
1988	0.006	0.521	0.249	0.044	0.052	0.873	0.546	0.093	-0.004	0.097	-0.081
1989	0.065	0.599	0.191	0.014	0.041	0.910	0.745	0.021	0.006	0.015	0.622
1990	0.035	0.443	0.293	-0.003	0.113	0.881	0.892	0.018	0.027	-0.009	-0.285
1991	0.011	0.499	0.276	0.045	0.073	0.904	0.862	0.010	-0.030	0.040	0.517
1992	0.018	0.519	0.243	0.051	0.053	0.884	0.914	0.007	0.031	-0.024	0.034
1993	0.016	0.494	0.317	0.049	0.049	0.925	1.047	0.093	-0.017	0.110	-0.129

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.037	0.483	0.279	0.033	0.039	0.871	0.690	0.027	0.026	0.002	0.101
H	0.026	0.530	0.274	0.041	0.038	0.908	0.856	0.039	0.035	0.004	0.086
SF	0.035	0.495	0.278	0.040	0.042	0.890	0.676	0.033	0.041	-0.007	0.072
MR	0.041	0.511	0.304	0.041	0.043	0.940	0.769	0.035	0.037	-0.002	0.081
ST	0.022	0.518	0.295	0.055	0.074	0.963	0.605	0.052	0.065	-0.013	0.089
NT	0.049	0.492	0.280	0.034	0.071	0.926	0.473	0.070	0.075	-0.005	0.062
N	0.041	0.498	0.268	0.032	0.057	0.897	0.561	0.039	0.044	-0.005	0.080
T&F	0.046	0.485	0.289	0.051	0.076	0.945	0.538	0.047	0.055	-0.008	0.083

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B30. V2 and JP2 with Firm-Specific Region-Heteroskedastic Random Effects Estimated by FGLS Procedure RA1

Summary Statistics Estimated Elasticities

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
Mean	0.038	0.505	0.283	0.038	0.051	0.914	0.713	0.042	0.045	-0.004	0.066
St.dev.	0.107	0.144	0.111	0.053	0.062	0.219	0.000	0.093	0.112	0.107	0.310
Min.	-0.974	0.002	0.000	-0.179	-0.128	0.185	0.713	-0.383	-0.179	-0.919	-0.240
Max.	1.187	2.763	1.016	0.588	0.614	4.453	0.713	1.698	1.743	0.267	0.574

Mean Elasticities by Year

Year	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
1985	0.228	0.507	0.336	0.030	0.026	1.128	0.713	.	.	.	.
1986	0.026	0.454	0.391	0.106	0.005	0.981	0.713	-0.001	0.149	-0.150	-0.175
1987	-0.038	0.497	0.280	0.031	0.037	0.808	0.713	0.086	0.210	-0.124	0.025
1988	0.008	0.525	0.248	0.041	0.050	0.873	0.713	0.092	-0.003	0.095	-0.178
1989	0.065	0.601	0.191	0.013	0.041	0.910	0.713	0.021	0.005	0.015	0.574
1990	0.038	0.441	0.294	-0.005	0.111	0.879	0.713	0.018	0.029	-0.012	-0.240
1991	0.014	0.497	0.274	0.044	0.073	0.903	0.713	0.011	-0.031	0.042	0.564
1992	0.019	0.519	0.242	0.050	0.053	0.882	0.713	0.007	0.031	-0.024	0.103
1993	0.017	0.493	0.317	0.048	0.048	0.923	0.713	0.092	-0.016	0.108	-0.115

Mean Elasticities by Region

Region	E_L	E_F	E_I	E_K	E_M	*RTS*	*TVE*	*TC*	*TCPUR*	*TCNON*	*TCV*
VA&R	0.039	0.482	0.278	0.032	0.039	0.870	0.713	0.027	0.026	0.001	0.093
H	0.028	0.530	0.273	0.039	0.037	0.907	0.713	0.039	0.035	0.004	0.074
SF	0.037	0.495	0.279	0.038	0.041	0.890	0.713	0.034	0.041	-0.007	0.054
MR	0.043	0.510	0.304	0.039	0.042	0.938	0.713	0.035	0.037	-0.002	0.070
ST	0.022	0.519	0.296	0.052	0.072	0.961	0.713	0.052	0.065	-0.013	0.068
NT	0.050	0.494	0.281	0.032	0.068	0.926	0.713	0.069	0.075	-0.006	0.038
N	0.043	0.499	0.269	0.031	0.054	0.897	0.713	0.039	0.044	-0.005	0.056
T&F	0.047	0.485	0.291	0.048	0.073	0.944	0.713	0.047	0.055	-0.008	0.075

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.B31. Coefficients of Correlation Between Estimated Elasticities Derived from Different Model Specifications and Estimators

(a) JP2 Pooled vs. JP2 with Fixed Effects (Both Estimated by OLS)

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*
Corr. Coef.	0.93	0.96	0.95	0.84	0.93	0.97	0.90

(b) K2 Pooled vs. K2 with Fixed Effects (Both Estimated by OLS)

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*
Corr. Coef.	0.88	0.97	0.95	0.88	0.94	0.78	0.91

(c) Linear Quadratic vs. Translog: JP2 vs. K2 (Both Estimated by OLS)

Elasticity	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*
Pooled Models	0.71	0.55	0.69	0.48	0.67	0.24	0.71
Models w. Fixed Effects	0.67	0.58	0.71	0.49	0.67	0.31	0.67

(d) Time Trend vs Time Dummy Model: JP1 with Fixed Effects and JP2 with Fixed Effects (Both Estimated by FGLS)

Elasticity	*RTS*	*TVE*	*TC*	*TCV*
w. Variance Function V1	0.84	0.99	0.27	0.02
w. Variance Function V2	0.82	0.00	0.24	0.18

(e) OLS Estimates vs FGLS Estimates: JP2 with Fixed Effects

Elasticity	*RTS*	*TVE*	*TC*	*TCV*
w. Variance Function V1	0.98	N.A.	0.92	N.A.
w. Variance Function V2	0.97	N.A.	0.89	N.A.

(f) Pooled vs. Fixed Effects Specification: JP2 Estimated by FGLS

Elasticity	*RTS*	*TVE*	*TC*	*TCV*
w. Variance Function V1	0.95	0.99	0.85	0.50
w. Variance Function V2	0.96	0.00	0.86	0.43

(g) FGLS vs. ML Estimates: JP2 with Fixed Effects

Elasticity	*RTS*	*TVE*	*TC*	*TCV*
w. Variance Function V1	1.00	0.97	0.99	0.79
w. Variance Function V2	1.00	0.00	0.98	0.72

(h) Different Variance Functions (V1 vs. V2): JP2 with Fixed Effects

Elasticity	*RTS*	*TVE*	*TC*	*TCV*
FGLS Estimates	0.99	0.00	0.97	0.98
ML Estimates	0.99	0.00	0.94	0.84

9.C. Appendix C: Figures

Figure	Description
9.C1	Elasticities of Technical Change by Year from Models JP2 and K2 Estimated by OLS
9.C2	Returns to Scale by Year from Models JP2 and K2 Estimated by OLS
9.C3	Elasticities of Technical Change by Region from Models JP2 and K2 with Firm-Specific Fixed Effects Estimated by OLS
9.C4.	Returns to Scale by Region from Models JP2 and K2 with Firm-Specific Fixed Effects Estimated by OLS
9.C5.	Elasticities of Technical Change by Year from Models JP1 and JP2 with Firm-specific Fixed Effects
9.C6.	Returns to Scale by Year from Models JP1 and JP2 with Firm-Specific Fixed Effects
9.C7.	Elasticities of Technical Change for Variance Function by Year from Models JP1 and JP2 with Firm-specific Fixed Effects
9.C8.	Total Variance Elasticity by Year from Models JP1 and JP2 with Firm-specific Fixed Effects
9.C9	Elasticities of Technical Change by Year from Model JP2 with Firm-specific Fixed Effects
9.C10	Returns to Scale by Year from Model JP2 with Firm-Specific Fixed Effects
9.C11	Elasticities of Technical Change for Variance Function by Year from Model JP2 with Firm-specific Fixed Effects
9.C12	Total Variance Elasticity by Year from Model JP2 with Firm-specific Fixed Effects
9.C13	Elasticities of Technical Change by Year from JP2 with/without FE and V1
9.C14	Returns to Scale by Year from JP2 with/without FE and V1
9.C15	Elasticities of Technical Change for Variance Function by Year from JP2 with/without FE and V1
9.C16	Total Variance Elasticity by Year from JP2 with/without FE and V1
9.C17	Elasticities of Technical Change from Pooled, Fixed Effects and Random Effects Specifications of JP2 and V1
9.C18	Returns to Scale from Pooled, Fixed Effects and Random Effects Specifications of JP2 and V1
9.C19	Elasticities of Technical Change for Variance Function from Pooled, Fixed Effects and Random Effects Specifications of JP2 and V1
9.C20	Total Variance Elasticity from Pooled, Fixed Effects and Random Effects Specifications of JP2 and V1

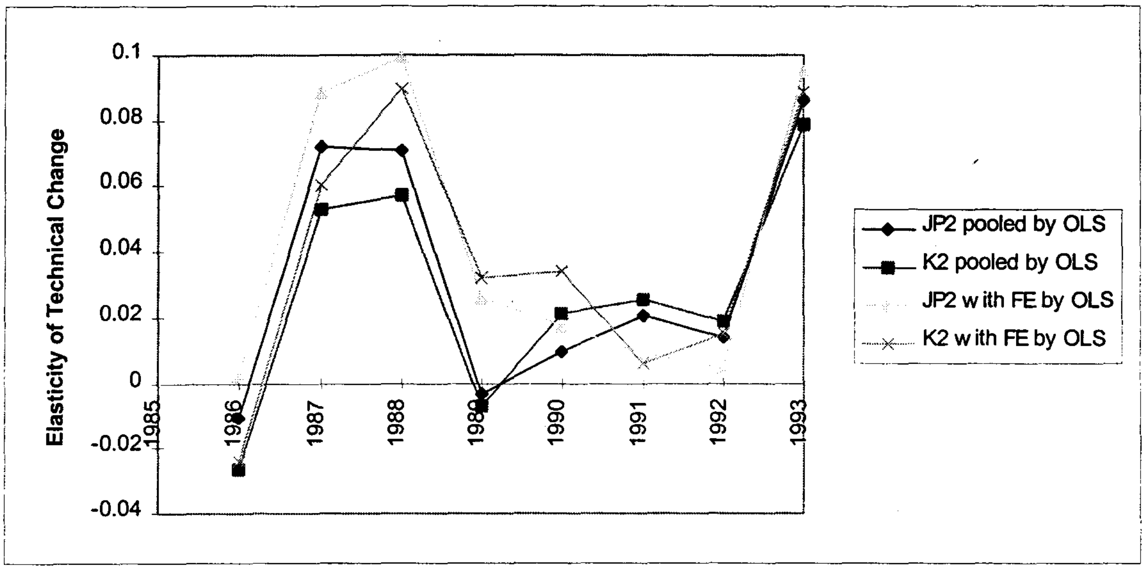

Figure 9.C1. Elasticities of Technical Change by Year from Models JP2 and K2 Estimated by OLS

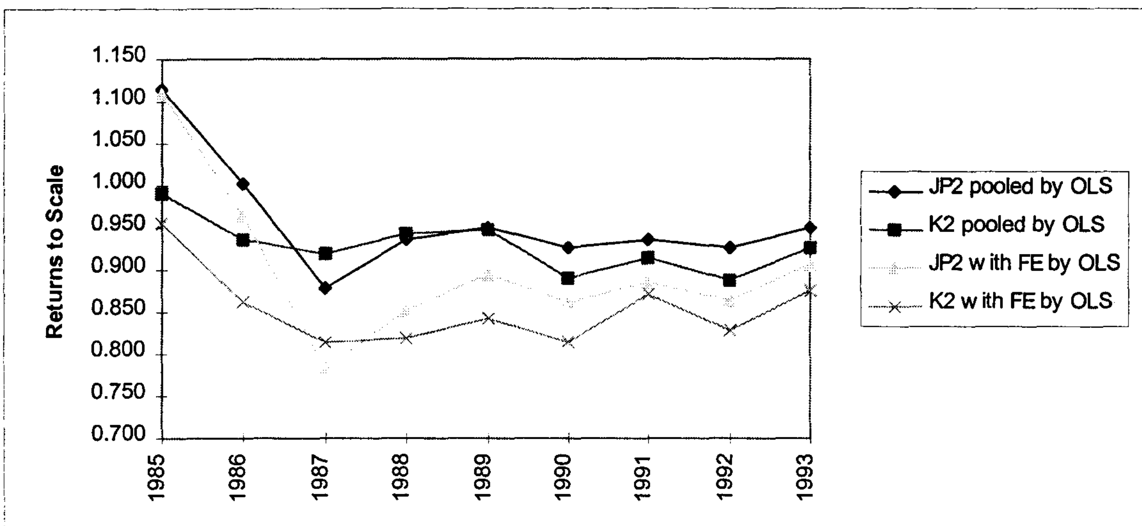

Figure 9.C2. Returns to Scale by Year from Models JP2 and K2 with Firm-Specific Fixed Effects Estimated by OLS

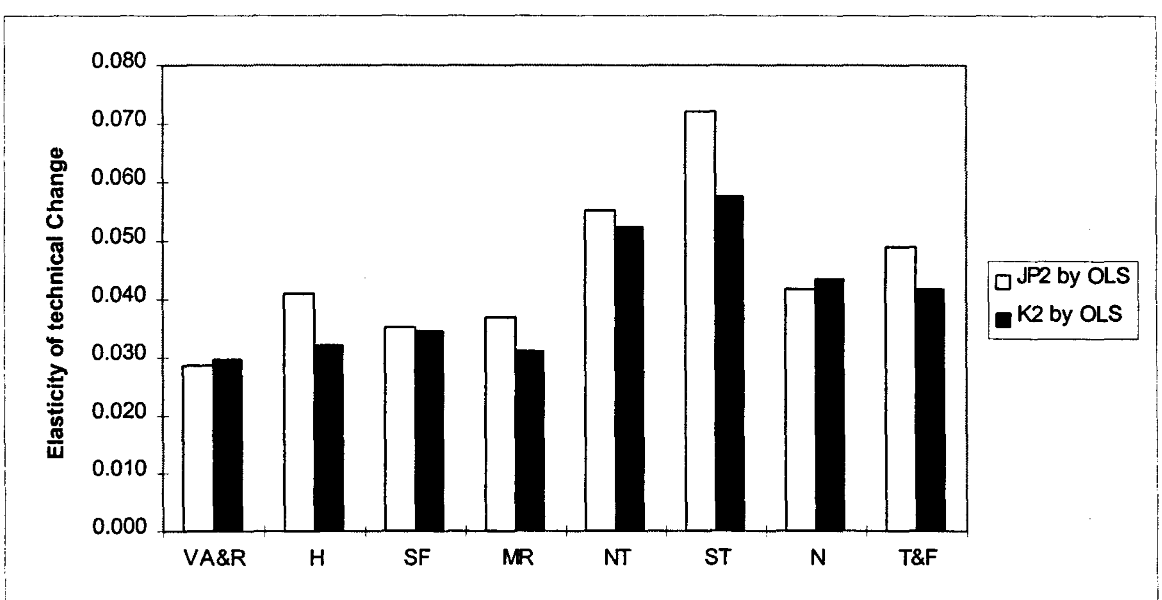

Figure 9.C3. Elasticities of Technical Change by Region from Models JP2 and K2 with Firm-Specific Fixed Effects Estimated by OLS

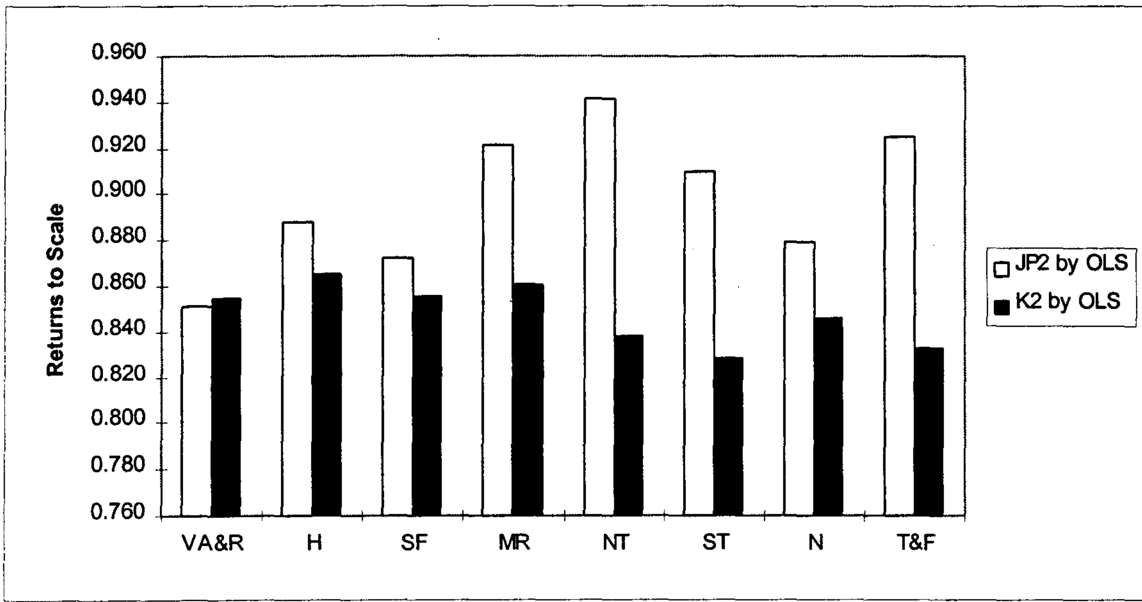

Figure 9.C4. Returns to Scale by Region from Models JP2 and K2 with Firm-Specific Fixed Effects Estimated by OLS

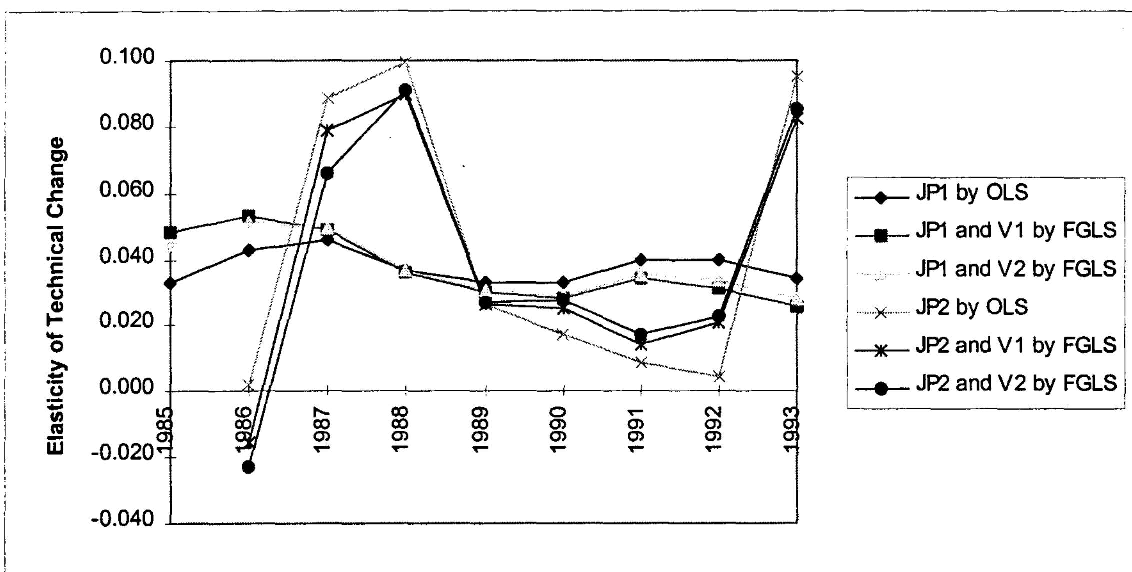

Figure 9.C5. Elasticities of Technical Change by Year from Models JP1 and JP2 with Firm-specific Fixed Effects

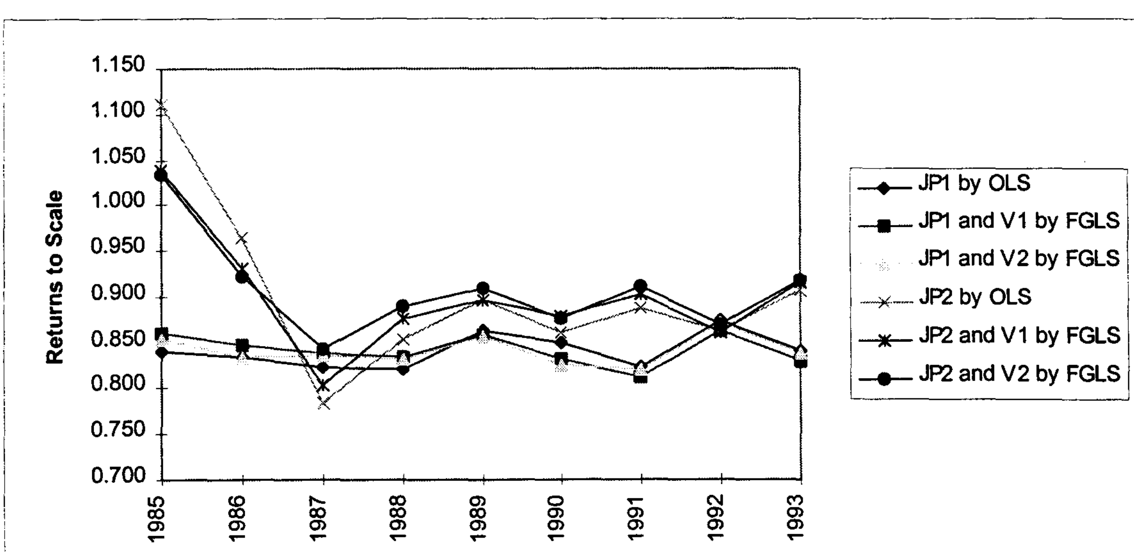

Figure 9.C6. Returns to Scale by Year from Models JP1 and JP2 with Firm-Specific Fixed Effects

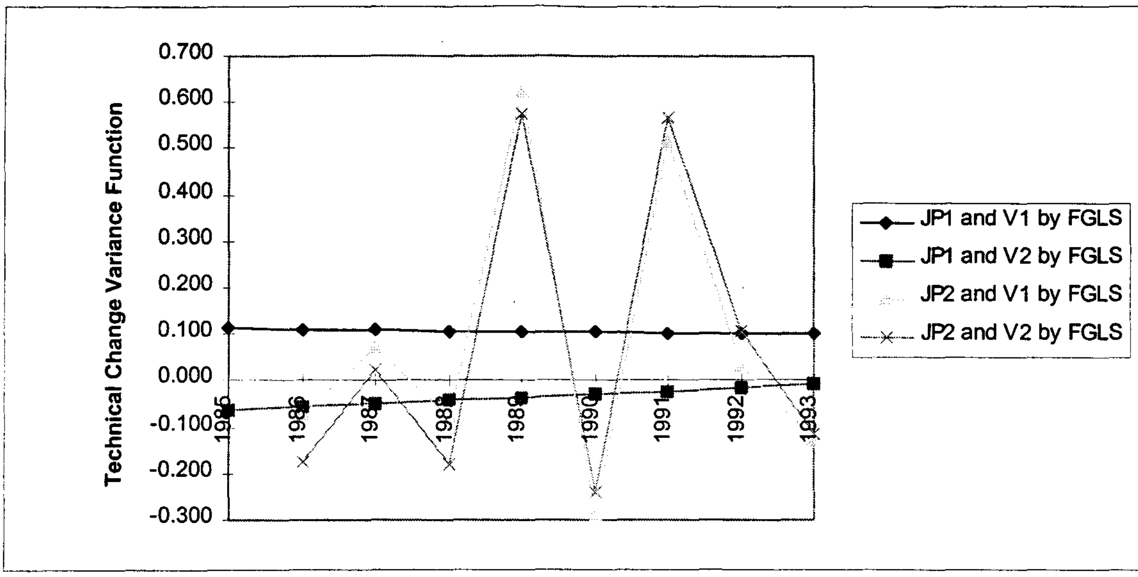

Figure 9.C7. Elasticities of Technical Change for Variance Function by Year from Models JP1 and JP2 with Firm-specific Fixed Effects

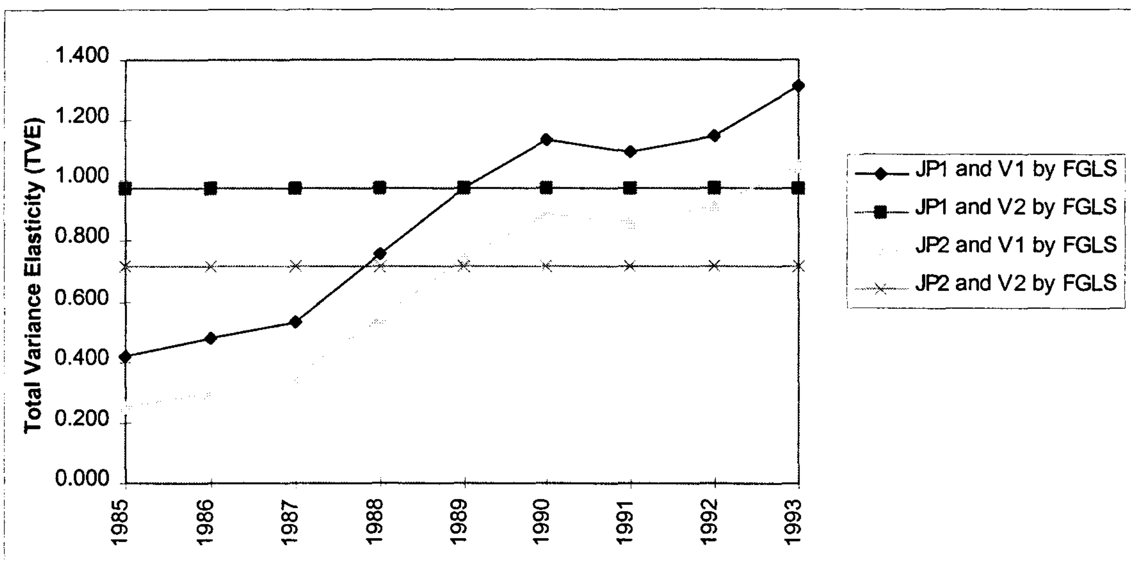

Figure 9.C8. Total Variance Elasticity by Year from Models JP1 and JP2 with Firm-specific Fixed Effects

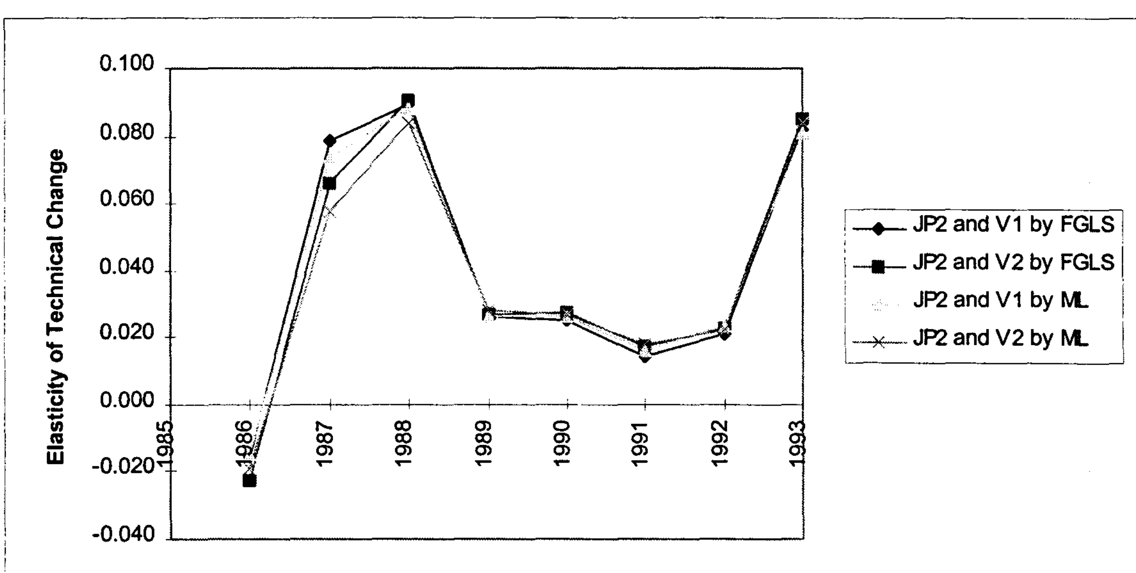

Figure 9.C9. Elasticities of Technical Change by Year from Model JP2 with Firm-specific Fixed Effects

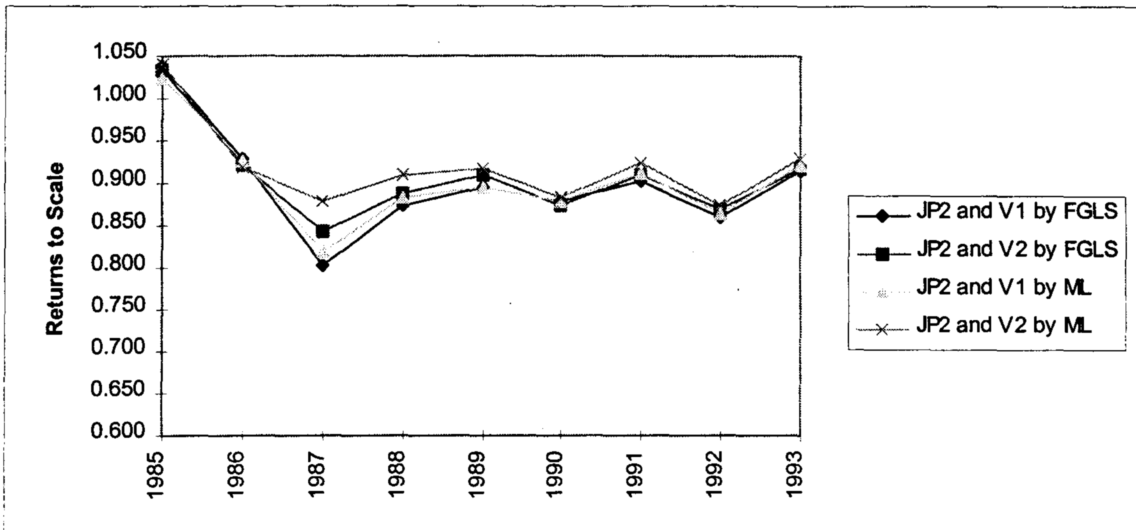

Figure 9.C10. Returns to Scale by Year from Model JP2 with Firm-Specific Fixed Effects

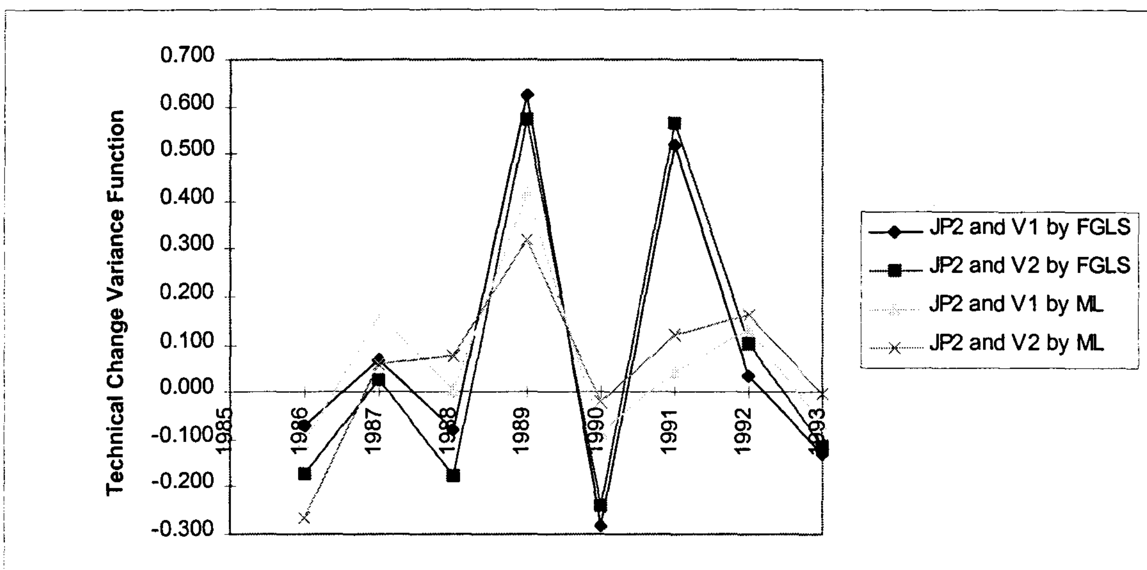

Figure 9.C11. Elasticities of Technical Change for Variance Function by Year from Model JP2 with Firm-specific Fixed Effects

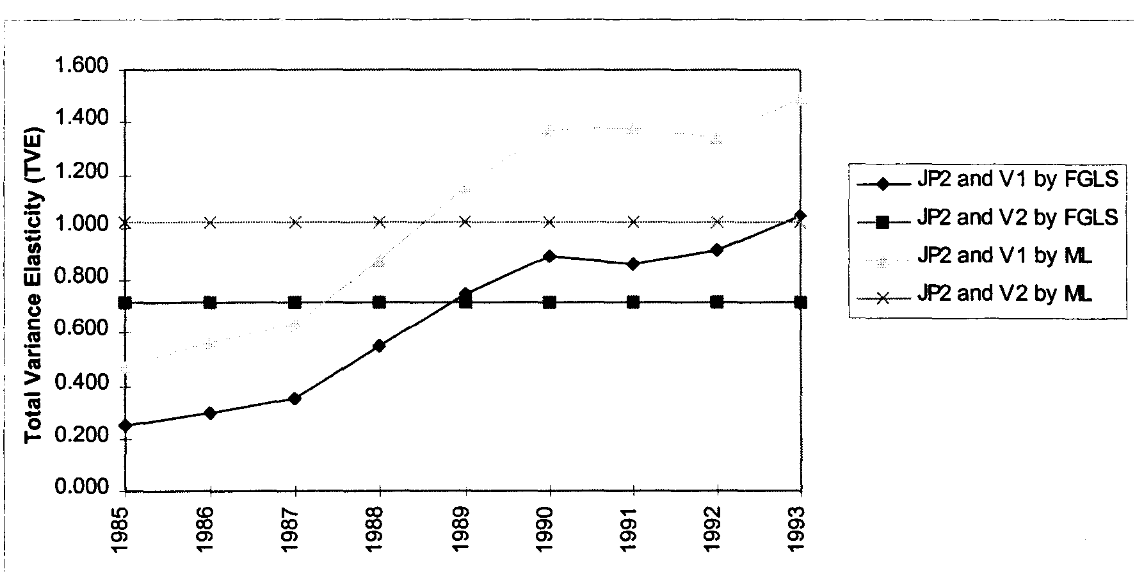

Figure 9.C12. Total Variance Elasticity by Year from Model JP2 with Firm-specific Fixed Effects

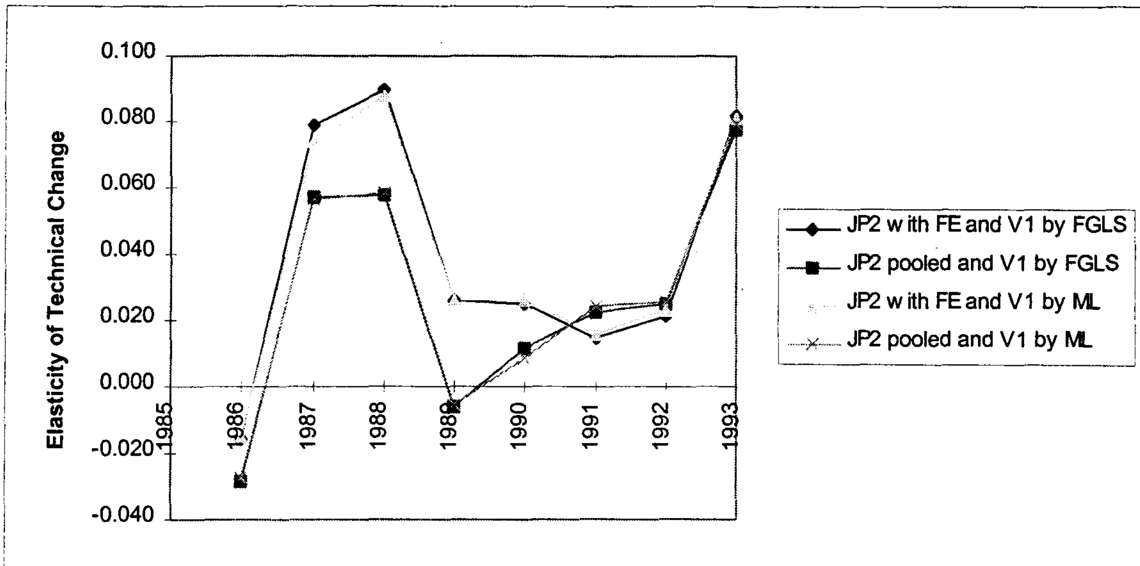

Figure 9.C13. Elasticities of Technical Change by Year from JP2 with/without FE and V1

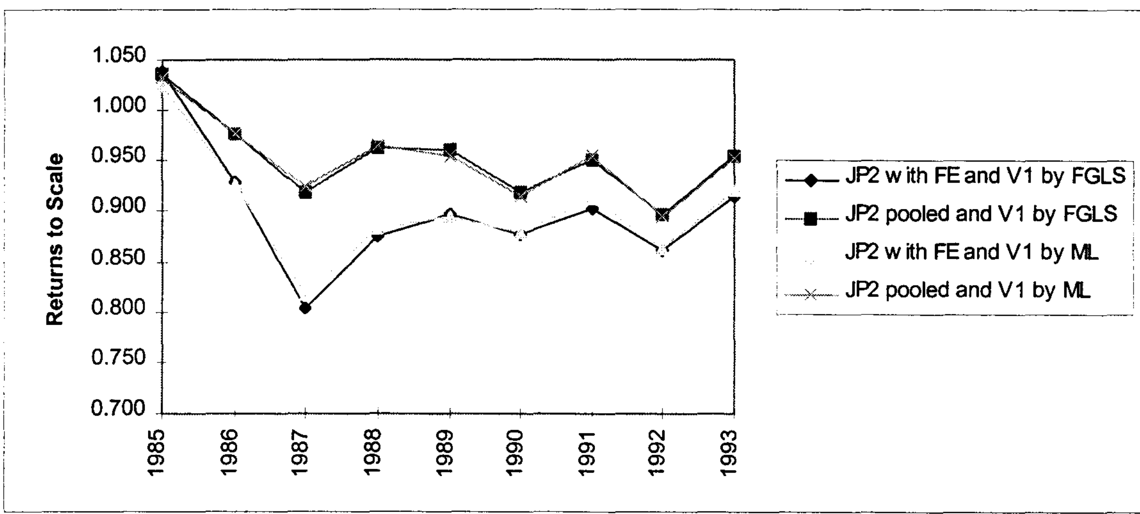

Figure 9.C14. Returns to Scale by Year from JP2 with/without FE and V1

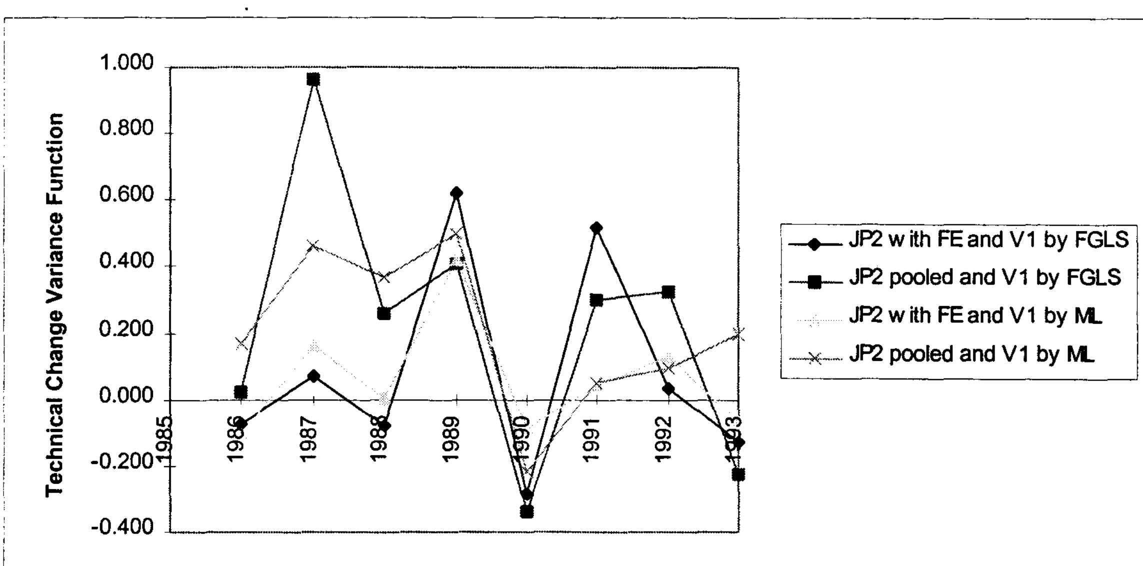

Figure 9.C15. Elasticities of Technical Change for Variance Function by Year from JP2 with/without FE and V1

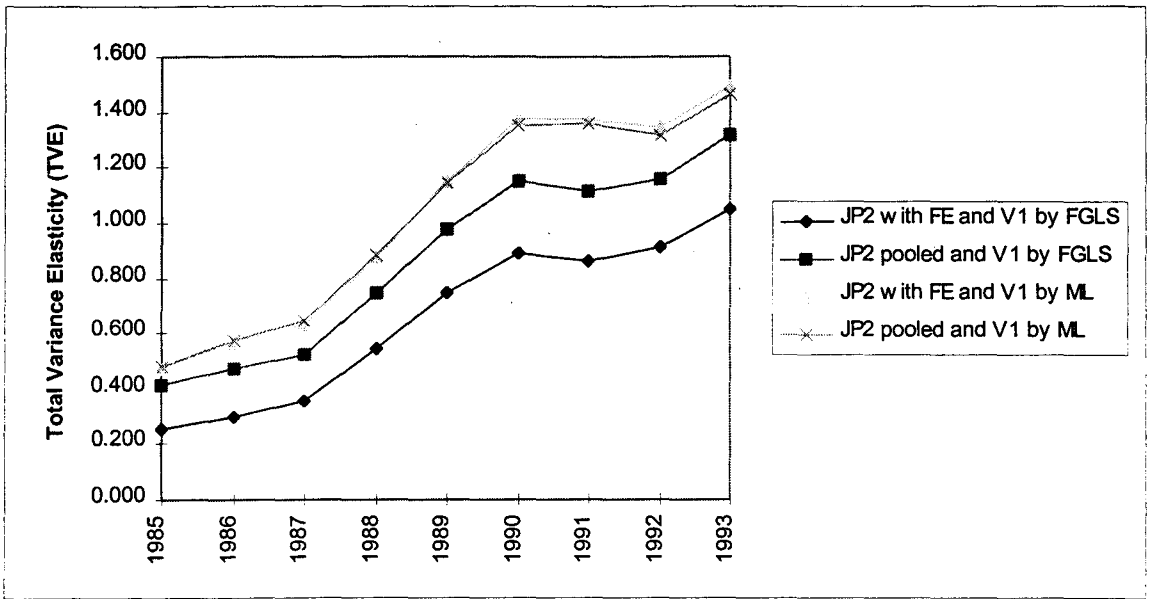

Figure 9.C16. Total Variance Elasticity by Year from JP2 with/without FE and V1

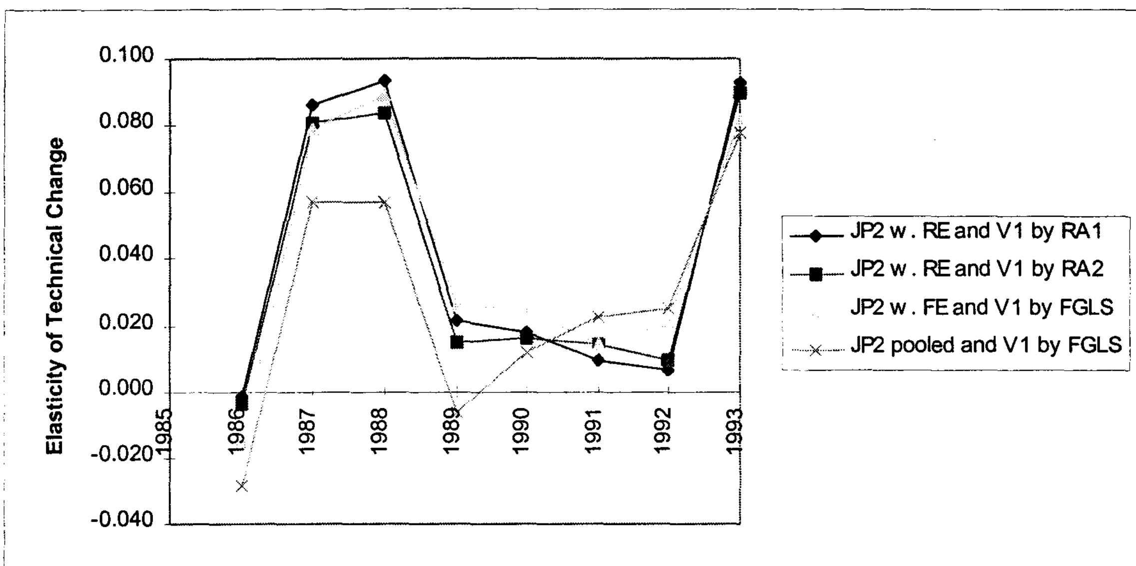

Figure 9.C17. Elasticities of Technical Change from Pooled, Fixed Effects and Random Effects Specifications of JP2 and V1

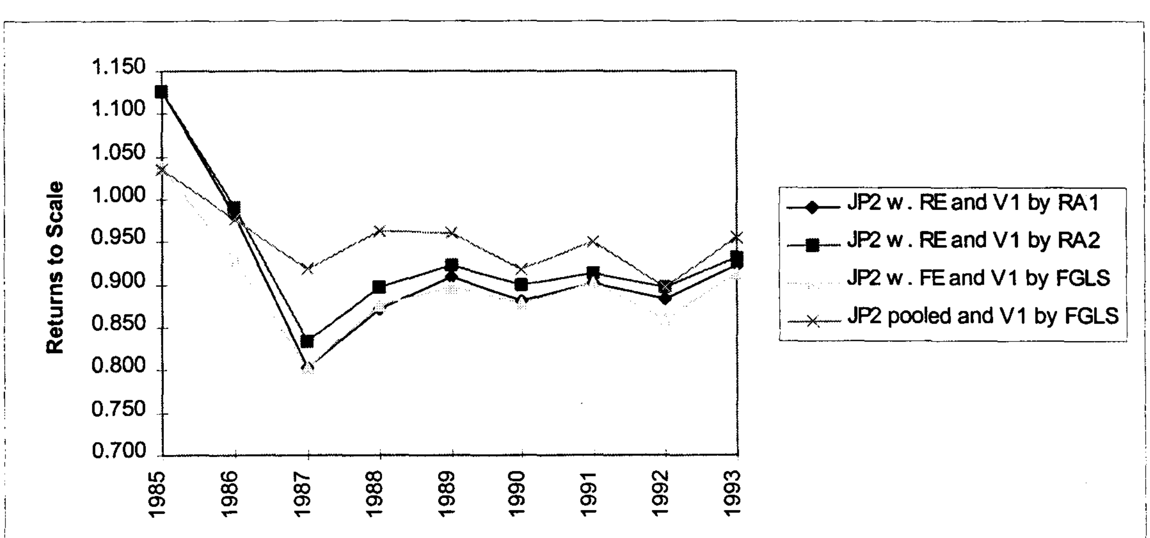

Figure 9.C18. Returns to Scale from Pooled, Fixed Effects and Random Effects Specifications of JP2 and V1

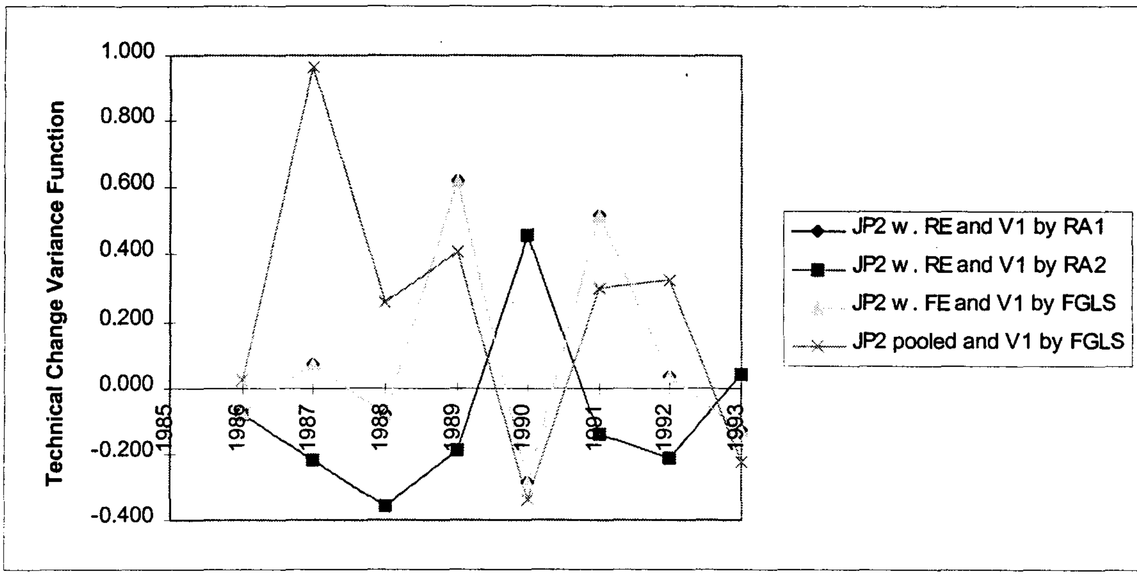

Figure 9.C19. Elasticities of Technical Change for Variance Function from Pooled, Fixed Effects and Random Effects Specifications of JP2 and V1

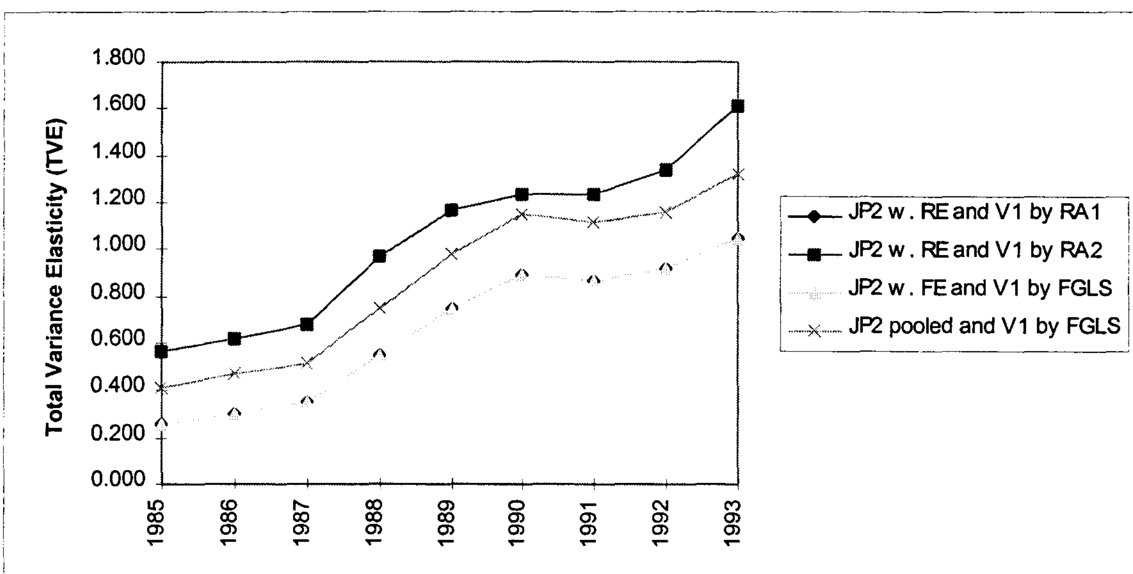

Figure 9.C20. Total Variance Elasticity from Pooled, Fixed Effects and Random Effects Specifications of JP2 and V1

9.D. Appendix D: Summary Statistics from the Estimating Sample

Table 9.D1. Structure of the Estimating Unbalanced Panel of Salmon Firms

Farms observed...	Number of farms	Number of obs.
9 years	27	243
8 years	22	176
7 years	46	322
6 years	64	384
5 years	53	265
4 years	83	332
3 years	77	231
Sum sample 1	**372**	**1953**
2 years	102	204
1 years	81	81
Sum sample 2	**555**	**2238**

Table 9.D2. Overall Summary Non-Normalized Variables Statistics Sample 1 (n=1953 obs.)

Variable	Mean	St.dev.	Min.	Max.
Output (y) in kg	355982.3	236220.9	11050	2014140
Materials (M) in real NOK*	998381.0	907280.9	9657	9636125
Feed (F) in kg	340299.1	242741.0	2358	2479452
Capital (K) in real NOK*	2572787.1	2277836.4	4707	37212584
Labor (L) in hours worked	7034.5	3694.1	250	42906
Fish (I) in kg	150379.0	109897.5	50	1015800

* Deflated by Consumer Price Index (CPI)

Table 9.D3. Overall Summary Statistics Sample 1 (n=1953 obs.)

Variable	Mean	St.dev.	Min.	Max.
Output (y)	1.000	0.664	0.031	5.658
Materials (M)	1.000	0.909	0.010	9.652
Feed (F)	1.000	0.713	0.007	7.286
Capital (K)	1.000	0.885	0.002	14.464
Labor (L)	1.000	0.525	0.036	6.099
Fish (I)	1.000	0.731	0.000	6.755
Time (t)	5.053	2.478	1	9
Year dummy 1985 (D_{85})	0.091	0.287	0	1
Year dummy 1986 (D_{86})	0.102	0.303	0	1
Year dummy 1987 (D_{87})	0.113	0.317	0	1
Year dummy 1988 (D_{88})	0.129	0.335	0	1
Year dummy 1989 (D_{89})	0.123	0.328	0	1
Year dummy 1990 (D_{90})	0.127	0.333	0	1
Year dummy 1991 (D_{91})	0.105	0.307	0	1
Year dummy 1992 (D_{92})	0.104	0.306	0	1

Table 9.D4. Sample 1 Means and St.Deviations (in Parenthesis) By Year (n=1953 obs.)

Year	1985	1986	1987	1988	1989	1990	1991	1992	1993
Output (*y*)	0.443	0.500	0.569	0.849	1.098	1.257	1.282	1.304	1.596
	(0.272)	(0.277)	(0.336)	(0.406)	(0.475)	(0.533)	(0.699)	(0.775)	(0.824)
Materials (*M*)	0.478	0.636	0.695	0.855	1.089	1.296	1.523	1.126	1.197
	(0.363)	(0.432)	(0.546)	(0.602)	(0.682)	(0.907)	(1.292)	(1.159)	(1.157)
Feed (*F*)	0.474	0.515	0.580	0.890	1.118	1.206	1.169	1.330	1.618
	(0.314)	(0.293)	(0.380)	(0.477)	(0.522)	(0.576)	(0.727)	(0.874)	(0.986)
Capital (*K*)	0.742	0.872	0.938	1.057	1.074	1.100	1.090	1.019	1.028
	(0.534)	(0.614)	(0.683)	(1.114)	(0.740)	(0.754)	(1.141)	(1.012)	(1.043)
Labor (*L*)	0.893	0.941	0.900	1.001	1.037	1.064	0.997	1.025	1.112
	(0.482)	(0.513)	(0.395)	(0.476)	(0.490)	(0.521)	(0.518)	(0.586)	(0.682)
Fish (*I*)	0.430	0.531	0.560	0.719	1.075	1.495	1.467	1.306	1.302
	(0.351)	(0.335)	(0.412)	(0.408)	(0.489)	(0.736)	(0.865)	(0.878)	(0.732)
Time (*t*)	1	2	3	4	5	6	7	8	9
No. of obs.	177	199	221	251	240	248	205	204	208

Table 9.D5. Sample 1 Means By Regional Location (n=1953 obs.)*

Region	VA&R	H	SF	MR	ST	NT	N	T&F
Output (*y*)	1.035	1.196	1.026	1.087	0.899	0.792	0.911	0.880
Materials (*M*)	0.832	1.123	0.923	1.054	1.089	0.908	0.997	0.958
Feed (*F*)	0.970	1.264	0.990	1.101	0.921	0.791	0.897	0.837
Capital (*K*)	0.757	1.224	0.983	1.069	1.305	0.787	0.843	0.912
Labor (*L*)	0.807	0.993	0.939	0.989	1.151	1.058	1.042	1.023
Fish (*I*)	1.074	1.111	1.005	1.156	0.929	0.805	0.910	0.891
Time (*t*)	5.528	5.179	4.996	5.079	4.894	4.723	4.728	5.461
Year dummy 1985 (D_{85})	0.056	0.087	0.099	0.111	0.090	0.079	0.109	0.072
Year dummy 1986 (D_{86})	0.067	0.095	0.107	0.108	0.101	0.129	0.116	0.084
Year dummy 1987 (D_{87})	0.087	0.103	0.112	0.097	0.133	0.134	0.131	0.114
Year dummy 1988 (D_{88})	0.133	0.119	0.137	0.104	0.149	0.144	0.141	0.108
Year dummy 1989 (D_{89})	0.138	0.127	0.116	0.108	0.122	0.129	0.131	0.108
Year dummy 1990 (D_{90})	0.138	0.136	0.116	0.133	0.112	0.158	0.109	0.114
Year dummy 1991 (D_{91})	0.133	0.114	0.090	0.122	0.106	0.079	0.081	0.120
Year dummy 1992 (D_{92})	0.128	0.106	0.116	0.108	0.096	0.074	0.081	0.144
No. of obs.	195	369	233	279	188	202	320	167

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

Table 9.D6. Sample 1 Means By Licensed Pen Volume (n=1953 obs.)*

Size group	Small	Medium	Large
Output (y)	0.426	0.731	1.375
Materials (M)	0.459	0.785	1.316
Feed (F)	0.436	0.736	1.368
Capital (K)	0.601	0.910	1.166
Labor (L)	0.748	0.941	1.107
Fish (I)	0.427	0.710	1.395
Time (t)	2.341	3.636	6.969
Year dummy 1985 (D_{85})	0.358	0.124	0.006
Year dummy 1986 (D_{86})	0.256	0.170	0.006
Year dummy 1987 (D_{87})	0.205	0.202	0.009
Year dummy 1988 (D_{88})	0.108	0.254	0.010
Year dummy 1989 (D_{89})	0.034	0.104	0.159
Year dummy 1990 (D_{90})	0.023	0.059	0.214
Year dummy 1991 (D_{91})	0.017	0.042	0.184
Year dummy 1992 (D_{92})	0.000	0.027	0.200
No. of obs.	176	878	899

* Size groups: Small: <=5000 m^3. Medium: >5000 m^3 and <=8000 m^3. Large: >8000 m^3.

Table 9.D7. Sample 1 Means By Year of Establishment (n=1953 obs.)

Year group	1*	2*	3*
Output (y)	0.954	0.885	1.310
Materials (M)	1.094	0.836	1.155
Feed (F)	0.966	0.884	1.292
Capital (K)	1.213	0.872	0.869
Labor (L)	1.062	0.955	0.977
Fish (I)	0.971	0.876	1.297
Time (t)	4.629	4.461	6.993
Year dummy 1985 (D_{85})	0.117	0.113	0.000
Year dummy 1986 (D_{86})	0.121	0.136	0.000
Year dummy 1987 (D_{87})	0.141	0.141	0.007
Year dummy 1988 (D_{88})	0.136	0.146	0.081
Year dummy 1989 (D_{89})	0.121	0.128	0.118
Year dummy 1990 (D_{90})	0.111	0.121	0.167
Year dummy 1991 (D_{91})	0.088	0.081	0.184
Year dummy 1992 (D_{92})	0.080	0.068	0.221
No. of obs.	738	807	408

* Year groups: 1: <1980. 2: >=1980 and <1986. 3: >=1986.

Table 9.D8. Overall Summary Non-Normalized Variables Statistics Sample 2 (n=2238 obs.)

Variable	Mean	St.dev.	Min.	Max.
Output (y) in kg	360886	243874.8	11050	2028801
Materials (M) in real NOK*	1035541	993135.2	9656.869	11721380
Feed (F) in kg	345561.4	250772.1	2357.997	2479452
Capital (K) in real NOK*	2555590	2294108	4706.767	37212584
Labor (L) in hours worked	7019.342	3744.157	50	42906
Fish (I) in kg	150701	110595.9	50	1015800

* Deflated by Consumer Price Index (CPI)

Table 9.D9. Overall Summary Statistics Sample 2 (n=2238 obs.)

Variable	Mean	St.dev.	Min.	Max.
Output (y)	1.000	0.676	0.031	5.622
Materials (M)	1.000	0.959	0.009	11.319
Feed (F)	1.000	0.726	0.007	7.175
Capital (K)	1.000	0.898	0.002	14.561
Labor (L)	1.000	0.533	0.007	6.113
Fish (I)	1.000	0.734	0.000	6.741
Time (t)	5.156	2.486	1	9
Year dummy 1985 (D_{85})	0.084	0.277	0	1
Year dummy 1986 (D_{86})	0.100	0.300	0	1
Year dummy 1987 (D_{87})	0.107	0.309	0	1
Year dummy 1988 (D_{88})	0.125	0.330	0	1
Year dummy 1989 (D_{89})	0.130	0.337	0	1
Year dummy 1990 (D_{90})	0.124	0.329	0	1
Year dummy 1991 (D_{91})	0.102	0.303	0	1
Year dummy 1992 (D_{92})	0.111	0.314	0	1

Table 9.D10. Sample 2 Means By Year (n=2238 obs.)

Year	1985	1986	1987	1988	1989	1990	1991	1992	1993
Output (y)	0.445	0.474	0.547	0.821	1.077	1.275	1.281	1.274	1.569
Materials (M)	0.465	0.612	0.665	0.797	1.069	1.285	1.504	1.203	1.226
Feed (F)	0.470	0.493	0.555	0.861	1.095	1.224	1.162	1.318	1.581
Capital (K)	0.743	0.863	0.963	1.047	1.079	1.103	1.069	1.021	1.007
Labor (L)	0.914	0.935	0.893	0.982	1.025	1.068	0.984	1.038	1.112
Fish (I)	0.447	0.510	0.546	0.706	1.075	1.503	1.459	1.262	1.278
Time (t)	1	2	3	4	5	6	7	8	9
No. of obs.	188	223	240	279	292	277	228	248	263

Table 9.D11. Sample 2 Means By Regional Location (n=2238 obs.)

Region	VA&R	H	SF	MR	ST	NT	N	T&F
Output (y)	1.023	1.198	1.053	1.097	0.859	0.786	0.946	0.849
Materials (M)	0.862	1.132	0.901	1.104	1.043	0.864	1.004	0.949
Feed (F)	0.968	1.258	1.024	1.121	0.884	0.785	0.930	0.797
Capital (K)	0.781	1.224	1.001	1.076	1.272	0.794	0.849	0.896
Labor (L)	0.817	1.000	0.948	1.001	1.127	1.051	1.042	1.002
Fish (I)	1.045	1.109	1.032	1.163	0.894	0.798	0.944	0.877
Time (t)	5.647	5.333	5.174	5.134	4.842	4.809	4.824	5.533
Year dummy 1985 (D_{85})	0.054	0.081	0.093	0.102	0.086	0.073	0.103	0.061
Year dummy 1986 (D_{86})	0.068	0.088	0.097	0.102	0.120	0.118	0.117	0.087
Year dummy 1987 (D_{87})	0.086	0.098	0.104	0.090	0.124	0.132	0.117	0.118
Year dummy 1988 (D_{88})	0.127	0.115	0.135	0.099	0.148	0.145	0.136	0.105
Year dummy 1989 (D_{89})	0.127	0.130	0.120	0.137	0.134	0.145	0.146	0.096
Year dummy 1990 (D_{90})	0.131	0.130	0.108	0.130	0.105	0.155	0.111	0.122
Year dummy 1991 (D_{91})	0.131	0.108	0.089	0.121	0.096	0.073	0.079	0.122
Year dummy 1992 (D_{92})	0.131	0.122	0.127	0.112	0.100	0.073	0.084	0.140
No. of obs.	221	409	259	322	209	220	369	229

* VA&R = Vest-Agder & Rogaland, H = Hordaland, SF = Sogn og Fjordane, MR = Møre og Romsdal, ST = Sør-Trøndelag, NT = Nord-Trøndelag, N = Nordland, T&F = Troms & Finnmark.

9.E. Appendix E: Estimation Procedures

This appendix presents estimation procedures for Just-Pope and Kumbhakar models.

9.E1. FGLS Estimator *RA1* for Just-Pope Model with Random Effects

The Just-Pope random effects model is given by

$$y_{it} = \mathbf{x}_{it}\alpha + v_{it}, \qquad v_{it} = \eta_i + u_{it}, \; i = 1,\ldots, N, \qquad t = 1, \ldots, T_i,$$

where $\mathbf{x}_{it}$ is a $1\times k$ vector of transformations of input levels (e.g., quadratic terms) and possibly other variables (e.g., region dummies), α is a $k\times 1$ vector of parameters, η_i is a firm-specific effect with variance $\text{var}(\eta_i) = \sigma_\eta^2$, and u_{it} is an observation specific error term with variance $\text{var}(u_{it}) = [h(\mathbf{x}_{it};\beta)]^2\sigma_\varepsilon^2$, where $\sigma_\varepsilon^2 = \text{var}(\varepsilon_{it})$. for which Harvey's multiplicative heteroskedasticity is chosen here.

The FGLS estimation procedure for the above Just-Pope model which is presented here requires that firms are observed at least two periods, since a least squares dummy variable (LSDV) model is estimated in the first step.

Step 1. Estimate the mean production function with fixed effects by OLS (with fixed effects implemented as firm dummies):

$$y_{it} = \mathbf{x}_{it}\alpha + \eta_i + u_{it}.$$

Step 2. Estimate the variance of the firm-specific effects

$$\hat{\sigma}_\eta^2 = \frac{1}{n-1}\sum_i \left\{T_i(\hat{\eta}_i - \bar{\hat{\eta}})^2\right\},$$

where $n = \Sigma_i T_i$ is the total number of observations, and

$$\bar{\hat{\eta}} = \frac{1}{N-1}\sum_{i=1}^{N}\hat{\eta}_i,$$

i.e., the sample mean firm-specific effect.

Alternatively, the firm-specific error term can be heteroskedastic of the form $\text{var}(\eta_i) = \sigma_g^2$ for firm $i \in g$, $g = 1, \ldots, G$, and $\text{var}(u_{it}) = \sigma_u^2$. The subscript g denotes a firm characteristic, e.g., regional location. Then, instead of estimating a single variance, one estimates the G variances of the firm-specific effects

$$\hat{\sigma}_{\eta_g}^2 = \frac{1}{n_g - 1}\sum_{i\in g}\left\{T_{i,g}\left(\hat{\eta}_i - \bar{\hat{\eta}}_g\right)^2\right\}, \qquad g = 1,\ldots, G,$$

where

$$\bar{\hat{\eta}}_g = \frac{1}{n_g - 1} \sum_{i \in g} T_{i,g} \hat{\eta}_i \text{, (i.e., the sample mean firm-specific effect),}$$

$$n_g = \sum_{i \in g} T_{i,g} \text{ is the total number of observations belonging to group g,}$$

and $T_{i,g}$ is the number of time periods firm i belongs to group g. From the formulation of the variance we see that $T_{i,g}$ is used as a weight.

Step 3. Estimate the variance function based on the estimated residuals from the first step (Harvey, 1976)

$$\ln \hat{u}_{it}^2 = \mathbf{z}_{it}\beta + w_{it},$$

where the first element in $\mathbf{z}_{it}$ is 1. After adding 1.2704 to the estimated intercept $\hat{\beta}_1$, because $E[w_{it}]$ = -1.2704 (Harvey, 1976), parameter estimates $\hat{\beta}$ are used to provide estimates of the observation-specific variances

$$\hat{\sigma}^2_{u,it} = \exp(\mathbf{z}_{it}\hat{\beta}), \qquad i = 1, \ldots, N, \; t = 1, \ldots, T_i.$$

Step 4. Estimate the n total variances

$$\hat{\sigma}^2_{v,it} = \bar{T}_i \cdot \hat{\sigma}^2_{\eta} + \hat{\sigma}^2_{u,it},$$

Step 5. Estimate the FGLS weight for each observation

$$\hat{\theta}_{it} = 1 - \frac{\hat{\sigma}_{u,it}}{\hat{\sigma}_{v,it}}, \text{if } T_i > 1.$$

Step 6. Estimate the RE model by the FGLS regression

$$y_{it} - \hat{\theta}_{it}\bar{y}_i = (1 - \hat{\theta}_{it})\alpha_0^* + (\mathbf{x}_{it} - \hat{\theta}_{it}\bar{\mathbf{x}}_i)\alpha^* + v_{it}^*.$$

9.E2. FGLS Estimator *RA2* for Just-Pope Model with Random Effects

This section presents an alternative FGLS estimator for the Just-Pope model with random effects. The estimation procedure follows Hsiao (1986, p. 194-6), with a modification for the estimation of the variance of the observation-specific error term.

Step 1. Estimate the pooled OLS regression

$$y_{it} = \alpha_0 + \mathbf{x}_{it}\alpha + v_{it},$$

to obtain parameter estimates $\hat{\alpha}$.

Step 2. Estimate the variance function with Harvey's parametrization, based on the estimated residuals $\hat{u}_{it} = (y_{it} - \bar{y}_i) - \hat{\alpha}(\mathbf{x}_{it} - \bar{\mathbf{x}}_i)$, using only observations from firms observed more than one period

$$\ln \hat{u}_{it}^2 = \mathbf{z}_{it}\beta + w_{it},$$

where the first element in $\mathbf{z}_{it}$ is 1. After adding 1.2704 to the estimated intercept $\hat{\beta}_1$, because $E[w_{it}] = -1.2704$ (Harvey, 1976), parameter estimates $\hat{\beta}$ are used to provide estimates of the observation-specific variances

$$\hat{\sigma}^2_{u,it} = \exp(\mathbf{z}_{it}\hat{\beta}), \qquad i = 1, \ldots, N, \quad t = t_i, \ldots, T_i.$$

Alternatively, if homoskedasticity is assumed for the observation-specific error term, the variance can be estimated as proposed in Hsiao (p. 196)

$$\hat{\sigma}_u^2 = \frac{1}{N^*}\sum_{i\in\theta}\sum_{t=t_i}^{T_i}\left[(y_{it} - \bar{y}_t) - \hat{\alpha}(\mathbf{x}_{it} - \bar{\mathbf{x}}_i)\right]^2,$$

where q_i is the number of periods firm i is observed, $\theta = (i \mid q_i > 1)$ is the set of firms observed more than one period, N^* is the number of firms observed more than one period, $\bar{y}_t$ is the mean over all firms observed in period t, and $\bar{\mathbf{x}}_i$ is the vector of means over all q_i observations of firm i.

Step 3. Estimate the variance of the firm-specific effects, using all observations,

$$\hat{\sigma}_\eta^2 = \frac{1}{N}\sum_{i=1}^{N}\left[(\bar{y}_i - \hat{\alpha}\bar{\mathbf{x}}_i)^2 - \frac{1}{q_i}\hat{\sigma}_u^2\right].$$

Step 4. Estimate the total variance

$$\hat{\sigma}^2_{v,it} = q_i \cdot \hat{\sigma}^2_\eta + \hat{\sigma}^2_{u,it}.$$

Step 5. Estimate the FGLS weights (Hsiao, 1986, p. 195)

$$\hat{\theta}_{it} = 1 - \frac{\hat{\sigma}_{u,it}}{\hat{\sigma}_{v,it}}, \text{if } q_i > 1, \text{ or}$$

$$\hat{\theta}_{it} = 1 / \sqrt{\hat{\sigma}^2_\eta + \hat{\sigma}^2_{u,it}}, \text{ if } q_i = 1.$$

Step 6. Undertake the FGLS transformation

$$y^*_{it} = y_{it} - \hat{\theta}_{it}\bar{y}_i \quad \text{and} \quad \mathbf{x}^*_{it} = \mathbf{x}_{it} - \hat{\theta}_{it}\bar{\mathbf{x}}_i, \qquad \text{if } q_i > 1, \text{ or}$$

$$y^*_{it} = \hat{\theta}_{it} y_{it} \quad \text{and} \quad \mathbf{x}^*_{it} = \hat{\theta}_{it}\mathbf{x}_{it}, \qquad \text{if } q_i = 1.$$

Step 7. Estimate the RE model by the FGLS regression

$$y^*_{it} = \alpha^*_0 + \mathbf{x}^*_{it}\alpha^* + v^*_{it},$$

where the intercept associated with α_o^* has been transformed similar to the y's and x's.

9.E.3. FGLS Estimation Procedure for Fixed Effects Specification of Kumbhakar Model

For the Khumbhakar form estimation procedures which are somewhat different from the above Just-Pope specifications have to be used.

If the firm- and time-specific effects are treated as fixed, these are estimated together with $\ln f(\mathbf{x};\alpha)$. The estimation procedure for the fixed effects specification consists of the following steps:

Step 1. Least squares estimation is undertaken for the function

$$\ln y_{it} = \ln f(\mathbf{x}_{it};\alpha) + \eta_i + u_{it}$$
$$= \alpha_0 + \sum_k \alpha_k \ln x_{kit} + \frac{1}{2}\sum_k \sum_j \alpha_{kj} \ln x_{kit} \ln x_{jit} + \eta_i + u_{it},$$

where the η's are treated as fixed. One can either use dummy variables or a within transformation in step 1.

Step 2. Using the estimators of α and η_i from the first stage, the residuals $\hat{u}_{it} = \ln y - \ln f(\mathbf{x}_{it};\hat{\alpha}) + \hat{\eta}_i$ are calculated, and then the variance function

$$\ln\left(\hat{u}_{it}^2\right) = \ln\left[\left(\sum_l \beta_l x_{lit}\right)^2\right] + \ln \varepsilon_{it}^2$$

is estimated by nonlinear least squares (NLS). The error $\hat{\varepsilon}_{it}$ converges in distribution to ε_{it},which is distributed as a chi-squared random variable with one degree of freedom under the assumption that ε_{it} is a standard normal distributed variable. The mean and variance of $\ln \varepsilon_{it}^2$ are -1.2704 and 4.9348, respectively (Just & Pope, 1978). In the NLS estimation of the variance function one must adjust for this.

Step 3. Weighted least squares (WLS) are performed by dividing left- and right-hand variables by predicted standard deviations

$$\hat{\sigma}_{it} = h(\mathbf{x}_{it};\hat{\beta})\hat{\sigma}_{\varepsilon} = \left[\sum_l \hat{\beta}_l x_{lit}\right]\hat{\sigma}_{\varepsilon}.$$

Since the variance of ε_{it}, σ_{ε}^2, is assumed identical across all observations, it will not affect the relative weight given to each observation in the WLS estimation, and can thus be replaced by one in the above expression.

9.F. Appendix F: Properties of Harvey's Multiplicative Heteroskedastic Model

9.F.1. Linear Variance Function

First, I examine the marginal risk properties of specification V1 of Harvey's formulation, where $\text{var}(y) = [h(\mathbf{x};\, \beta)]^2 = \exp(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_l x_l)$. The marginal risk of output with respect to input j is

$$\frac{\partial\, \text{var}(y)}{\partial x_j} = \exp(\beta_0 + \beta_1 x_1 + \ldots + \beta_l x_l)\beta_j\,.$$

Since the exponentiated term is always positive, β_j will determine the sign of the above expression. The marginal risk is positive (negative) if β_j is positive (negative).

Note also that

$$\frac{\partial^2\, \text{var}(y)}{(\partial x_j)^2} = \exp(\beta_0 + \beta_1 x_1 + \ldots + \beta_l x_l)(\beta_j)^2 \geq 0\,,$$

i.e., the variance of y is always convex in x_j. This means that if marginal risk is positive in x_j ($\beta_j > 0$), the variance of output will increase at faster and faster rate, and eventually approach infinity. If marginal risk is negative in x_j ($\beta_j < 0$) the variance of output will decrease at a slower and slower rate in x_j. For fixed and finite values of the other x's, var(y) will converge towards zero as x_j increases. The fact that the variance of output diverge in x_j for positive β_j is unsatisfactory restriction from an empirical point of view. Of course, the rate at which the variance diverge depends on the actual value of β_j. Thus, after estimation of the model has been undertaken, it may be a good idea to carefully investigate the behavior of the variance function for different levels of the x's.

9.F.2. Log-Linear Variance Function

Next, I will examine the marginal risk properties of specification V2 of Harvey's formulation, where $\text{var}(y) = [h(\mathbf{x};\, \beta)]^2 = \exp(\beta_0 + \beta_1 \ln x_1 + \beta_2 \ln x_2 + \ldots + \beta_l \ln x_l)$. The marginal risk of output with respect to input j is

$$\frac{\partial\, \text{var}(y)}{\partial x_j} = \exp(\beta_0 + \beta_1 \ln x_1 + \ldots + \beta_l \ln x_l)\beta_j \frac{1}{x_j}\,.$$

Since the exponentiated terms are always positive, and $1/x_j$ is positive because input levels are constrained to be positive, β_j will determine the sign of the above expression. The marginal risk is positive (negative) if β_j is positive (negative).

The second derivative of the variance function with respect to x_j is

$$\frac{\partial^2 \text{var}(y)}{(\partial x_j)^2} = \exp(\beta_0 + \beta_1 \ln x_1 + \ldots + \beta_l \ln x_l)\beta_j^2 \frac{1}{x_j^2} - \exp(\beta_0 + \beta_1 \ln x_1 + \ldots + \beta_l \ln x_l)\beta_j \frac{1}{x_j^2},$$

The first term on the right hand side of the equation sign is always positive. The sign of the last term depends on the sign of β_j; if $\beta_j < 0$ ($\beta_j > 0$) then the last term is positive (negative). In the case when $\beta_j > 0$, the sign of the second derivative of the variance function depends on whether $\beta_j <=> 1$. If $\beta_j > 1$ then $\partial^2\text{var}(y)/\partial x_j > 0$, i.e., the variance function is convex in x_j; if $0 < \beta_j < 1$ then $\partial^2\text{var}(y)/\partial x_j < 0$, i.e., the variance function is concave in x_j.

We can summarize the properties of the output variance function for the log-linear Harvey model specification as follows:

(i) $\beta_j < 0$: var(y) decreases in x_j, is convex in x_j, and var(y) $\rightarrow 0$ as $x_j \rightarrow \infty$ when other x's are fixed and finite.

(ii) $0 < \beta_j < 1$: var(y) increases in x_j, is concave in x_j.

(iii) $\beta_j > 1$: var(y) increases in x_j, is convex in x_j, and var(y) $\rightarrow \infty$ as $x_j \rightarrow \infty$ when other x's are fixed and finite.

A useful transformation of the log-linear variance function is

$$\begin{aligned}\text{var}(y) &= \exp(\beta_0 + \beta_1 \ln x_1 + \ldots + \beta_k \ln x_k) = \exp(\beta_0)\exp(\beta_1 \ln x_1) \cdot \ldots \cdot \exp(\beta_k \ln x_k) \\ &= \exp(\beta_0) x_1^{\beta_1} \cdot \ldots \cdot x_k^{\beta_k}\end{aligned}.$$

We see that the transformed variance function now has the familiar Cobb-Douglas form. If the input levels **x** are increased by the same factor a, then the variance of output increases by a factor of $a^{(\beta_1+\beta_2+\ldots+\beta_k)}$. The properties of the output variance function with respect to an increase in input levels **x** by the same factor a can be summarized as follows:

(i) If $\beta_1 + \beta_2 + \ldots + \beta_k < 0$, then var($y$) will decrease as input levels **x** are increased by the same factor a.

(ii) If $0 < \beta_1 + \beta_2 + \ldots + \beta_k < 1$, then var($y$) will also increase as input levels **x** are increased by the same factor a, but at a slower rate.

(iii) If $\beta_1 + \beta_2 + \ldots + \beta_k > 1$, then var($y$) will increase as input levels **x** are increased by the same factor a, but at a faster rate.

The log-linear form seems to have more desirable marginal risk properties than the linear form. Contrary to the linear form presented above, it allows the variance of output to be both increasing and concave in inputs.

10. SUMMARY AND CONCLUSIONS

This dissertation has provided empirical evidence on the structure of production risk in Norwegian salmon farming. It is the first study to provide knowledge on the risk effects of inputs based on a comprehensive panel data set spanning over a nine year and including over 300 farms. The methodological contribution of this study, both through the empirical study and a simulation study, has been to increase our understanding of the performance of competing model specifications and estimators which can be used in econometric studies of production risk. In general, the relative performance of different estimators for econometric models with firm-specific effects and heteroskedasticity in regressors is largely unknown. It is our belief that the results from this study using different specifications and estimators for models of production risk will be useful for future empirical studies in this field of research.

Chapter two was concerned with the theoretical foundations for analysis of the competitive firm under production risk. The Just-Pope postulates for the stochastic production function were presented. These postulates can be regarded as an extension of the postulates suggested for the deterministic production function in the neoclassical production theory. It turns out that most popular specifications of the production function violate some or all of the Just-Pope postulates. The exception is the Just-Pope production function $y = f(\mathbf{x})+h(\mathbf{x})\varepsilon$. It was demonstrated that with the introduction of production risk, comparative static analysis became richer but more ambiguous than in the standard deterministic case. Furthermore, the dual approach to comparative static analysis is complicated, particularly if output price risk is also present, because expectation formation and risk preferences generally have to be accounted for in dual models.

Chapter two also demonstrated that the introduction of production risk has implications for efficiency analysis. A risk averse producer will be concerned about both mean output and the variance of output when considering alternative production technologies. This mean-variance trade-off is represented by the producer's utility function.

An important implication from the theory, given the intractability of the dual approach, is that estimation of primal models that account for production risk is a natural approach to empirical analysis of firm behaviour and productivity. In a further step, the restrictions which can be imposed on the stochastic production technology, can be used to simplify a dual model approach.

Chapter three provided a discussion of the empirical work on production risk which has been presented in the literature. Most of the studies are primal approaches, that utilise the Just-Pope framework. The empirical results from these studies must be judged with care, because of methodological weaknesses or poor data quality. This is because most of the studies focus on the introduction of a particular methodological improvement, and tend to give less attention to important specification and data issues. Omitted variable bias is probably present in several studies, due to the very few inputs included in the production function. Although the studies were undertaken from the late 1970s, they generally employ very restrictive functional forms, such as the Cobb-Douglas. Furthermore, some employ estimation procedures that are relatively unexplored, are sensitive to starting values, or imply strong assumptions on the data generating process.

Chapter three also discussed the performance of two competing estimators, the feasible generalised least squares (FGLS) and the maximum likelihood (ML) estimator. For the Just-Pope production function the ML provides more efficient estimates of the variance function parameters. According to a simulation study of Saha *et al.* (1997), the ML estimator also outperforms the FGLS estimator in small samples, even when the distribution of the error term departs significantly from normality. The simulation results of Saha *et al.* were, however, based on a simple Just-Pope technology with Cobb-Douglas parametrization of the mean function, and no firm-specific effects. It can be questioned to what extent their findings are valid for more flexible functional forms and heterogeneous firms.

Chapter four discussed issues in econometric panel data estimation which are relevant for this study. Since firm heterogeneity is believed to be an important characteristic of the salmon farming industry, an econometric model specification which incorporates firm-specific effects is clearly warranted. The firm-specific effects can be specified as fixed or random. The random effects approach, which is popular for longitudinal data sets, relies on assumptions that probably are violated for the empirical application in this dissertation. It is difficult to know *a priori* how sensitive parameter estimates are to violation of the random effects assumptions. The fixed effects approach, on the other hand, precludes inclusion of time-invariant regressors such as region-dummy variables. According to Chapter four, the performance of econometric panel data models under heteroskedasticity, particularly of the Just-Pope form, is relatively unexplored, since standard panel data models assume homoskedastic error components.

Chapter five presented a simulation study of the small-sample performance of different panel data estimators for Just-Pope technologies. A linear quadratic functional form was used for the mean function. Different values were tried for the mean and variance function parameters, and for the variance of the firm-specific error component. The simulation design was such that it could be argued to be relevant in the context of the empirical application. We found that all competing estimators performed similarly with respect to the mean function parameters. This implies that if one is only concerned about the properties of the mean function, the OLS estimator with White-adjusted standard errors for the fixed effects model is a good alternative. Unlike Saha *et al.*'s simulation study of a simpler Just-Pope technology, our study did not provide the same overwhelming support for the ML estimator relative to the FGLS estimator, although the precision and efficiency of the ML estimates were somewhat higher for the variance function estimates.

Chapters 6-8 discussed our empirical application, the Norwegian salmon aquaculture industry.

Chapter six provided a description of the production process. Salmon aquaculture is characterised by a rather long production period from the juvenile salmon, the smolts, are released into the pens until grown-out salmon are ready to be harvested. Smolts are released once or twice a year, and depending on sea temperatures and feeding intensity, the salmon are kept in the pens from one to two years. A farm in a normal mode of operation will thus have at least two cohorts of salmon in the pens. In terms of cost shares, the most important inputs are feed (about 40 % of total costs), smolts (15 %), capital (10-15 %) and labour (10 %).

Salmon farms are heterogeneous with respect to the biophysical conditions they operate under. Sea temperatures, organic recipient capacity, oxygen availability and the frequency of fish disease outbreaks have been found to vary much between farm locations. Another source of heterogeneity is differences in the quality of management and workers, and technology. The entrepreneurs who established salmon farms during the 1970s and 1980s also had very different skills. Furthermore, there has been a high degree of innovation, and an almost continuous learning-by-doing process, since salmon farming is a young industry.

Chapter seven discussed the structure of risk in salmon aquaculture. Salmon farmers face both production risk and output price risk. In terms of specification and estimation of econometric models of production, this means that dual model approaches loose much of their attractiveness compared to a primal approach. It was argued that the level of output risk increases in feed and fish input, but decreases in labour and pen volume. Furthermore, it was argued that increasing the scale of operation on a given location, will lead to an increase in the level of output risk.

Chapter eight provides a discussion of the Norwegian salmon farm data set, an unbalanced panel for the period 1985-93 with 560 farms and 2280 observations. The farms are identified by an identification code. Since econometric panel data models assume that firm-specific effects are time-invariant, we discuss whether the unobservable characteristics associated with the farm identification code can be assumed to be constant. Due to ownership changes and relocation, which are not observed, this is not always the case. Chapter eight also presents the output and input measures used in the empirical analysis. For an important input such as feed, a proxy measure has to be used, since the quantity of feed input is not directly available.

Chapter nine provides the empirical model specifications and the empirical results. It is recognised that difficult trade-offs have to be made in the specification of the empirical models. The chosen model is a Just-Pope production function with linear quadratic parametrization of the mean function and Harvey's parametrization for the mean function. The linear quadratic is found to provide estimates that are similar to translog estimates for returns to scale (*RTS*) and the rate of technical change (*TC*) around the sample mean observation.

Empirical results from two competing specifications of technical change, the time trend model and the time dummy model, suggest that the standard time trend specification is too restrictive. According to the time dummy model, the rate of technical change exhibits substantial year-to-year fluctuations. Furthermore, the time trend model provides implausible input elasticity estimates.

Inclusion of firm-specific effects had profound effects on elasticity estimates. Pooled models overestimated *RTS* and underestimated *TC*, and also provided very different estimates of variance function elasticities.

The empirical estimates of *RTS* and *TC* were relatively invariant to the choice of estimator around the mean observation. However, variance function estimates were found to be sensitive to estimator choice. For the fixed effects model the ML and FGLS estimators provided somewhat different estimates of the magnitudes of marginal output risk in inputs, although the signs of marginal risk tended to be the same. The most dramatic differences were found when we compared random effects estimators which relied on the random effects assumptions with fixed effects estimators. The random effects and fixed effects estimators predicted different signs for the marginal risks for several inputs. Since we have good *a priori* reasons to question the validity of the random effects assumptions, our preferred specification is the fixed effects model.

According to the preferred models, feed is clearly the most important input in terms of the output elasticity. The input of fish is the second most important for mean output, while the other inputs have much smaller output elasticities. For the mean farm in the sample, returns to scale lies around 0.9. This means that the average farm was of a sufficient size to exhaust scale economies.

The mean rate of technical change for the entire data period is around 4 %. From 1985 to 1986, a period with large losses due to fish diseases, the rate of technical change was negative. The years 1987 and 1988 were characterised by high rates of technical progress. However, from 1989 to 1992 the rate of technical progress was low. This may be attributed to low profitability in salmon farms during the period, which precluded investments in new and improved technologies. In the last year of the data set, 1993, technical progress made an upswing, as improved economic conditions at the end of the data period allowed farms to acquire new technologies.

The empirical models provide evidence of significant marginal output risk in inputs in salmon farming. In other words, output risk is a function of inputs in the industry, and input levels can thus be used as instruments to control the level of risk. Feed, fish, capital and materials input were found to increase the level of output risk, while labour input reduces output risk. Feed input is important for both mean output and the variance of output. The main role of labour input seems to be risk reduction. However, according to the empirical results, the technical change during the data period did not lead to a reduction in the level of output risk. This implies that the risk preference structure of salmon farmers is such that improvement in mean productivity is more important than risk reduction, or that salmon farmers have limited knowledge about the structure of production risk. It may also be the case that government-sponsored research programmes have focused less on reducing output risk than improving mean output.

Chapter nine discussed implications of the empirical results for the industry and policy makers. Given that salmon farmers have little knowledge about the structure of production risk, since they have to rely mainly on observations from their own farm, the findings here should be of interest for future production decisions. The results also have implications for site diversification and horizontal integration decisions, since they suggest that site decentralisation may be a strategy for reducing the level of risk. Furthermore, the results suggest that in research programmes aimed at salmon production, one should be concerned about both mean and risk properties of new technologies.

Chapter nine also proposed directions for future research on risky production technologies, both methodological and for the salmon industry in particular. Other functional forms for the mean production function should be tried, with an emphasis on global properties, because these have influence also on risk parameter estimates. Furthermore, it is desirable to test more flexible functional forms for the variance function. For the empirical application, the salmon aquaculture industry, it would be interesting to implement biophysical shocks in the model, in order to obtain estimates of technical change that are devoid of these. It is obvious that there is still room for methodological improvements, and certainly a need for more empirical knowledge in this research area.

Given that output risk is an inherent feature of the production process in many sectors of biological production, and given the limited empirical evidence we have so far, it is certainly desirable that more work is done in this field of research, both methodologically and empirically.

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