Buying Influence: Aid Fungibility in a Strategic Perspective*

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Abstract

I study equilibria of non-cooperative games between an aid donor and a recipient when there is conflict over the allocation of their combined budgets. The general conclusion is that a donor's influence over outcomes is increasing in the share of the available resources it controls; if this share is large enough, aid is not fungible. The game-theoretic approach to fungibility is contrasted with the traditional non-strategic approach. I argue that the former is superior as it derives final allocations instead of assuming them, making analysis of the sources of influence over outcomes possible.

1 Introduction

The continuing debate over the merits of fiscal federalism illustrates that some of the most complicated issues in public economics arise in inter-jurisdictional fiscal relations. Both matters of efficiency and equity are usually at stake, and the parties involved often evaluate the consequences of different institutional arrangements and policies differently. When the entities belong to different sovereign states, resolving conflicts of interests becomes even harder, since one lacks the framework generated by common political and juridical institutions. In terms of conflict resolution, international law and conventions are rarely perfect substitutes for such national institutions. The prime example of inter-governmental fiscal relations in the international arena is the relationship between a recipient

of foreign aid and its donors. One way to view this relationship is to see aid as donor subsidies for certain projects or programmes, in much the same way as the government of a state (whether federal or unitary) make transfers to sub-national units. Conflicts between the parties to aid transactions over the intended outcomes of their joint efforts are a fact of life, current official rhetoric about "partnerships" notwithstanding. Indeed, most of the history of foreign aid relations might be read as a continual search by the donors to find ways to maximise the returns to their funds as judged by them, with recipients trying to make sure that their spending priorities - which have not always been the same - prevail. Moreover, even though the World Bank now argues for "selectivity" in choosing recipients (see World Bank 1998), i.e., concentrating assistance in countries pursuing policies ajudged to be conducive to economic development, it seems unlikely that disagreements over the allocation of resources will vanish overnight. Indeed, being selective would not be necessary if there was complete agreement among the parties involved about how funds should be spent. Knowledge about what outcomes might be expected will therefore still be helpful in designing aid policies.

An important issue for donors is the extent to which aid is fungible, i.e., can be redirected, partially or completely, from the intended purpose by the recipient if it so wishes. If aid is fungible, the evaluation of its impact is complicated by the difficulty of assessing which activities are ultimately supported by the inflow of funds.[1] In turn, this makes the task of designing optimal aid policies harder. Judging the efficiency of development assistance also becomes more complex. Even though the diversion of funds might improve outcomes from an overall perspective, for example because donors are overly influenced by commercial or strategic interests, in order to make an informed judgment one needs to know into what activities funds leak. Although in the end this is an empirical issue, a solid theoretical understanding of the problem is an essential prerequisite for such investigations. Gaining such insight has taken on added importance with the adoption of the so-called Millennium Development Goals by the international community. The estimates of the additional aid necessary to achieve them runs into tens of billions of US dollars.[2]

The results reported in this paper are derived from first-principles. That is, instead of assuming different degrees of fungibility and discussing their implications, I analyse the degree of influence that recipients and donors have over allocation patterns based on the resources available to them, their preferences, and the manner in which they interact. The game-theoretic approach adopted here differs from the contract-theoretic framework of Pedersen (1995a,b) and Azam and Laffont (2003).[3] These authors assume that donors and recipients can write binding contracts specifying what the former gets in return for the grants and subsidised loans passed on to the latter. This fits with the conditionality approach to aid adopted in the 1980s and 1990s. However, even though

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[1] For a discussion of the issues involved, see e.g. Devarajan and Swaroop (2000).

[2] For example, Devarajan, Miller, and Swanson (2002) estimate that an increase of $40-70 billion per year is needed, which amounts to a doubling of official aid compared to 2000.

[3] Also see Svensson (2000) and Torsvik (2002).

usually agreements between the parties are signed this is not a very fruitful approach to understanding aid impact. Aid "contracts" cannot be enforced in courts, and the generally poor record of conditionality demonstrates that such agreements have not been self-enforcing either.[4]

I prefer, therefore, to study the outcomes of equilibria of non-cooperative games between a donor and a recipient. In section 2, I investigate three different types of equilibria of a simple budgetary game by varying the order in which the players move. Section 3 contains a discussion of aid fungibility in the light of the game-theoretic approach to the issue, contrasting the results with those of the traditional non-strategic approach. In section 4, I show that the pattern of equilibrium outcomes resulting when the budgets of the players are endogenous correspond closely to those derived in section 2 under the assumption that both donor and recipient have a fixed amount of resources to allocate. Finally, in section 5 I summarise and comment on the main results derived in this paper.

2 Modelling the Budgetary Game

2.1 Preferences, Resources, and Order of Moves

Consider the case of a donor agency (D) and a recipient government (R), each with their own fixed budget, interacting to determine the allocation of their combined resources among K goods. The players have the following preferences over the consumption vector $\mathbf{X} = \{x_1, ..., x_k, ..., x_K\}$, in the recipient country:[5]

$$U^p(\mathbf{X}) = \begin{cases} \sum_{k=1}^{K} \beta_k^p \frac{(x_k)^{1-\mu}}{1-\mu}, \mu > 0, \mu \neq 1; \\ \sum_{k=1}^{K} \beta_k^p \ln x_k, \mu = 1 \end{cases}, p = R, D. \qquad (1)$$

Hence, each x_k can be thought of as a collective good for R and D, with different marginal benefits if $\beta_k^D \neq \beta_k^R$.[6]

The resource constraints of the donor and the recipient are

$$\sum_{k=1}^{K} b_k^p \leq B^p, b_k^p \geq 0, \forall k, p = D, R. \qquad (2)$$

That is, neither player can spend more than its total budget B^p. Moreover, the funds allocated to spending on each good must be non-negative. For the donor this assumption is reasonable, as it cannot tax the recipient. For the

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[4]Empirical studies of conditionality include Mosley, Harrigan, and Toye (1991), Killick (1995, 1998), Devarajan, Dollar, and Holmgren (2001), and the World Bank (1998).

[5]In this paper I concentrate on cases where $\mu \neq 1$. See Hagen (2002) for the results for $\mu = 1$ when $K = 2$.

[6]For convenience, I assume $\beta_k^p \in (0, 1)$ $\forall k$ and $\sum_{k=1}^{K} \beta_k^p = 1$.

recipient, it is perhaps a bit more restrictive, but it preserves a certain symmetry between the players. Moreover, it is empirically reasonable: empirical studies of fungibility rarely find that a marginal increase in aid results in lower spending on the activity in question.[7] Given the assumption, one way to interprete aid in this model is as project aid, or, even more precisely, as aid in kind: once the donor has allocated funds for some purpose in the recipient country, these are turned into actual units of goods and services. However, one could easily extend this to programme aid as long as the recipient's ability to tax or transfer resources across budget categories is limited relative to the donor's budget.

For any combination of budgetary allocations by the two parties, the consumption of each good is

$$x_k = \frac{b_k^D + b_k^R}{q_k}, \tag{3}$$

where q_k is the price of good k. All prices are assumed to be constant.[8] Choosing good 1 to be the numeraire, I set $q_1 = 1$.

The "first-best" allocation of each actor - the allocation that it would have chosen if it could dictate how the combined resources of D and R should be spent - is found by maximising $U^p(\mathbf{X})$ subject to $\sum_{k=1}^{K} b_k^p \leq B$, where $B = B^D + B^R$. The result is

$$x_k^{p*} = \frac{\sigma_k^p B}{q_k}, p = D, R, \tag{4}$$

with $\sigma_k^p = \frac{\left(\beta_k^p\right)^{\frac{1}{\mu}}(q_k)^{\frac{\mu-1}{\mu}}}{\sum_{l=1}^{K}\left(\beta_l^p\right)^{\frac{1}{\mu}}(q_l)^{\frac{\mu-1}{\mu}}}$ being the optimal share of the common budget spent on good l from player p's perspective. Let $\mathbf{X}^{p*} = \left\{x_1^{p*}, ..., x_k^{p*}, ..., x_K^{p*}\right\}$ be the vector of optimal levels of provision of the goods for player p.

Of course, if D and R have the same preferences, their common "first-best" allocation will result; when R is a perfect "agent" for D, the latter need not concern itself with how to allocate its budget because in any which way it does so, the very best outcome is realised. Indeed, as noted by Devarajan, Rajkumar, and Swaroop (1999: 1), "[T]he question of what aid ultimately finances is interesting only if the preferences of the donor are different from those of the recipient". This is also the most realistic scenario. To analyse it, I therefore make

Assumption 1 (conflict of interest)
$\frac{\beta_1^R}{\beta_1^D} < ... < \frac{\beta_k^R}{\beta_k^D} < ... < \frac{\beta_K^R}{\beta_K^D} \Leftrightarrow \frac{\sigma_1^R}{\sigma_1^D} < ... < \frac{\sigma_k^R}{\sigma_k^D} < ... < \frac{\sigma_K^R}{\sigma_K^D}$.
Thus, the goods can be ranked in increasing order in terms of the importance R put on them relative to D. Moreover, this ranking is assumed to be strict.

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[7] See e.g. Feyzioglu, Swaroop, and Zhu (1998) and Pack and Pack (1990, 1993).

[8] Hence, in order to focus on the results of the strategic interaction between D and R I disregard well-known phenomena such as the Dutch Disease inflicted by inflows of foreign economic assistance.

This does not preclude the existence of some k for which $\sigma_k^D = \sigma_k^R$, but it does rule out there being more than one such good.[9]

I will analyse three different orders of the timing of moves: D as the Stackelberg-leader (denoted by superscript L), D as the follower (F), and simultaneous moves (N). Much of the traditional aid literature has, at least implicitly, assumed that D is the leader. Conditionality - attaching conditions to the aid transfers - has been a strategy much used by donors in the last couple of decades. One way of viewing conditionality is that donors dictate the terms of the aid relationship.[10] This may be modelled as D having a first-mover advantage in its interaction with the recipient. Most empirical studies conclude, however, that at best conditionality has had a limited impact. Conditions are never fully implemented as specified. Furthermore, at least for altruistic donors, it would be difficult to avoid dynamic inconsistency. If unmet needs are detected in recipient countries, altrustic donors would have a hard time ignoring these even if they are due to the governments of these countries not having implemented conditions previously agreed upon. Therefore, in the literature on the Samaritan's Dilemma (see e.g. Pedersen 1997, 2001 and Svensson 2000), it is assumed that donors are followers. To highlight the differences in outcomes that result, it is common in these works to contrast the cases of donor and recipient leadership. I will do so too. The case of simultaneous moves, where neither party has a first-mover advantage, provides a useful starting point for understanding the basic mechanisms. To keep the exposition as simple as possible while illustrating all aspects of the games, I focus on the case $K = 3$ in the main text. The formal presentation of the results for any $K \geq 2$ and the proofs can be found in appendix A. The special case of $K = 2$ is briefly discussed at the end of this section.

2.2   Simultaneous Moves

In a simultaneous-move game, we are looking for a Nash-equilibrium in which both R and D allocate their budgets optimally given the funding strategy chosen by the other party. The donor will, if possible, choose its aid policy so that the end result is that the consumption of any $x_k \in \mathbf{X}$ is x_k^{D*}. Equating the expression for x_k^{D*} with $x_k = \frac{b_k^D + b_k^R}{q_k}$, we get $b_k^D = q_k x_k^{D*} - b_k^R = \sigma_k^D B - b_k^R$ at an interior solution. That is, as funds from the donor and the recipient are perfect substitutes, the donor would like to add on to whatever the recipient has allocated so that its optimal consumption of the two goods results. In the remainder, I will denote these functions by b_k^{D*} and refer to the set of them as the "first-best" strategy of the donor. The corresponding strategy for the recipient is $b_k^{R*} = \sigma_k^R B - b_k^D$. For the sake of brevity, I will denote these strategies by $\mathbf{b}^{D*}$ and $\mathbf{b}^{R*}$, whereas $\mathbf{b}^D$ and $\mathbf{b}^R$ are used as a general short-hand for the budgetary strategies of the two players.

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[9]The assumption is stricter than is needed, and is made in order to limit the number of equilibrium regions to be characterised.

[10]As noted in the introduction, another is to view conditionality as reflecting a contract between donors and recipients.

Let us start by noting an important implication of assumption 1: namely that, loosely speaking, at the optimal levels of provision for the other player each player has a strict ranking of the marginal benefits from increasing the supply of the various collective goods. Letting superscript $-p$ refer to the other player, e.g. if $p = D$, then $-p = R$. the statement just made may be expressed more precisely in the following way:

Lemma 1

Suppose $x_l = x_l^{-p*}$ and $x_m = x_m^{-p*}$, $m > l$. Then, holding $\mathbf{b}^{-p}$ constant, $\frac{\partial U^p}{\partial b_l^p} > \frac{\partial U^p}{\partial b_m^p}$, $p = D$, and $\frac{\partial U^p}{\partial b_l^p} < \frac{\partial U^p}{\partial b_m^p}$, $p = R$.

For example, when the consumption vector is $\mathbf{X}^{R*}$, for D the highest (lowest) marginal benefit from spending a unit of its budget comes from allocating it to x_1 (x_K). For R, it is the other way around at $\mathbf{X}^{D*}$. However, the result is more general; whenever the expenditure ratio of two goods is optimal according to the preferences of the other player, a player's relative marginal benefit of spending on the goods in question is determined by the index number of the goods.

Another useful result is the following:

Lemma 2

Let $\widetilde{X} \subseteq X$. If for any $x_l, x_m \in \widetilde{X}$, $\frac{\partial U^p}{\partial b_l^p} = \frac{\partial U^p}{\partial b_m^p}$ holding $\mathbf{b}^{-p}$ constant, then

the optimal budgetary strategy of p is $b_n^p = \widetilde{\sigma}_n^p \left(B^p + \sum_{\nu \in \widetilde{X}} b_\nu^{-p} \right) - b_n^{-p} \ \forall n \in \widetilde{X}$,

where $\widetilde{\sigma}_n^p = \dfrac{\sigma_n^p}{\displaystyle\sum_{\nu \in \widetilde{X}} \sigma_\nu^p}$.

I will call this p's second best strategy, denoted by $\mathbf{b}^{p**}$. The important feature of this strategy is that it preserves the first-best expenditure ratios between the sub-set of goods financed by p: $\frac{q_l x_l}{q_m x_m} = \frac{\widetilde{\sigma}_l^p}{\widetilde{\sigma}_m^p} = \frac{\sigma_l^p}{\sigma_m^p}$. This may also be confirmed by noting that when $\widetilde{X} = X$, $\mathbf{b}^{p**} = \mathbf{b}^{p*}$.

After these preliminaries, I first note some obvious results.

Result 1

All collective goods are provided in equilibrium.

This is simply due to the marginal utility of consumption of a goods going to infinity if the good is not supplied. Hence, if one player does not contribute to the provision of a good, the other player will. This result of course extends to sequential games.

It should also be clear that as long as there is conflict over the allocation, it can never be the case that $\{\mathbf{b}^{DN}, \mathbf{b}^{RN}\} = \{\mathbf{b}^{D*}, \mathbf{b}^{R*}\}$. That is, as long as $\sigma_1^D \neq \sigma_1^R$, the first-best strategies of the players cannot constitute a Nash-equilibrium strategy profile. The first-best strategies are constructed such that if they are used by a player, the resulting allocation is the best possible partition of the combined budget from its perspective. When these allocations differ, it is impossible to attain them simultaneously. Hence, we have

Result 2

$\{\mathbf{b}^{D*}, \mathbf{b}^{R*}\}$ cannot be a Nash-equilibrium.

The main issue is therefore under what circumstances one of the players may use its "first-best" strategy. Consider R first. To ask when $\mathbf{b}^{R*}$ is feasible is to ask for which parameter values $b_k^{R*} \in \left[0, B^R\right] \ \forall k$. Denote the share of total resources controlled by the donor by $\alpha = \frac{B^D}{B^D + B^R}$. As will become apparent, in the case of $K = 3$ there are four critical values of this parameter.

When $\alpha \leq \sigma_1^R$, D controls a share of total available resources that is smaller than its optimal budget share for the good it has the strongest preference for in relative terms, x_1. The outcome is then $\mathbf{X}^{R*}$. By lemma 1, at $\mathbf{X}^{R*} \ \frac{\partial U^D}{\partial b_1^D} > \frac{\partial U^D}{\partial b_2^D} > \frac{\partial U^D}{\partial b_3^D}$. It is therefore optimal for D to choose $b_1^D = B^D$. That is, it will only fund the good it attaches the strongest relative priority to. Even so, $\mathbf{b}^{R*}$ is feasible. $b_1^{R*} = \sigma_1^R B - B^D = \left(\sigma_1^R - \alpha\right) B \geq 0$ and $\sum_{k=1}^{3} b_k^{R*} = \left(\sigma_1^R - \alpha\right) B + \left(1 - \sigma_1^R\right) B = (1 - \alpha) B = B^R$.

When R cannot overfund x_3 according to D's preferences even if it devotes its entire budget to this good, it is D that enjoys the best of all possible worlds. More precisely, this is the case when $1 - \alpha \leq \sigma_3^D \Leftrightarrow \alpha \geq 1 - \sigma_3^D$. Hence, $\mathbf{X}^N = \mathbf{X}^{D*}$ and by lemma 2 it will indeed be optimal for R to spend only on x_3. Yet still $b_3^{D*} \geq 0$ and so R is without influence on the final outcome.

For $\alpha \in \left[\sigma_1^R, 1 - \sigma_3^D\right]$, matters are slightly more complex.[11] Consider first values of α slightly higher than σ_1^R. If D chooses $b_1^D = B^D$, b_1^{R*} is not feasible. That is, the budgetary share of x_1 would be sub-optimally high from R's perspective. The recipient therefore spends its resources on the other two goods according to b_2^{R**} and b_3^{R**}, c.f. Lemma 2. On the other hand, D still thinks good 1 is underfunded. It therefore optimally sticks to the "extreme" strategy $\left\{B^D, 0, 0\right\}$. The outcome is then $\mathbf{X}^N = \left\{B^D, \frac{\widetilde{\sigma}_2^R B^R}{q_2}, \frac{\widetilde{\sigma}_3^R B^R}{q_3}\right\}$, where $\widetilde{\sigma}_2^R = \frac{\sigma_2^R}{\sigma_2^R + \sigma_3^R}$ and $\widetilde{\sigma}_3^R = \frac{\sigma_3^R}{\sigma_2^R + \sigma_3^R}$.

If one makes the thought experiment of increasing α from σ_1^R, the equilibrium level of x_1 will increase and those of x_2 and x_3 will decrease. Hence, D's marginal benefit of spending on good 1 drops, while the marginal benefits of spending on the other two goods rises. For high enough values of α it will eventually be the case that given R's strategy D will find it optimal to spend on the other goods. Lemma 1 demonstrates that this will first be the case for good 2. The next critical value of D's share of total available resources is therefore where $\frac{\partial U^D}{\partial b_1^D} = \frac{\partial U^D}{\partial b_2^D}$, given the strategies $\left\{B^D, 0, 0\right\}$ and $\left\{0, b_2^{R**}, b_3^{R**}\right\}$. For values of α above this cut-off rate, x_2 will be jointly funded by the players. The equilibrium in this region therefore has a kind of knife-edge property: a unit increase in α will result in D raising its spending on this good to the same extent while R will be lowering its contribution by one unit. The supply of all goods is therefore constant. This is the only way that one can simultaneously have $\frac{x_1}{q_2 x_2} = \frac{\sigma_1^D}{\sigma_2^D}$ and $\frac{q_2 x_2}{q_3 x_3} = \frac{\sigma_2^R}{\sigma_3^R}$. In sum, equilibrium strategies and outcomes in this region are

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[11]Using assumption 1 and $\sum_{k=1}^{K} \sigma_k^p = 1$, $p = D, R$, one can show that $1 - \sigma_3^D > \sigma_1^R$.

$$\mathbf{b}^{DN} = \left\{ b_1^{D**}, b_2^{D**}, 0 \right\} = \left\{ \widetilde{\sigma}_1^D \alpha_3 B, (\alpha - \alpha_2) B, 0 \right\};$$

$$\mathbf{b}^{RN} = \left\{ 0, b_2^{R**}, b_3^{R**} \right\} = \left\{ 0, (\alpha_3 - \alpha) B, \widetilde{\sigma}_3^R (1 - \alpha_2) B \right\};$$

$$\mathbf{X}^N = \left\{ \widetilde{\sigma}_1^D \alpha_3 B, \frac{(\alpha_3 - \alpha_2) B}{q_2}, \frac{\widetilde{\sigma}_3^R (1 - \alpha_2) B}{q_3} \right\}.$$

α_2 and α_3 are the critical values separating this region from region 2, discussed above, and region 4, which is a mirror image of that region in which D funds goods 1 and 2 whereas R spends on x_3 only. Hence α_3 is the value of α where b_2^{R**} drops to zero, given D's strategy $\left\{ b_1^{D**}, b_2^{D**}, 0 \right\}$. R then controls such a low share of B that it is optimal for it to spend its whole budget on the good for which it has the strongest relative preference in order to keep the equilibrium share of spending on this good as high as possible. It enjoys some success with this strategy until α reaches $\alpha_4 = 1 - \sigma_3^D$; as we have seen, for values of α higher than this D dominates so in terms of relative resources that it is in total control over the outcome.

[Table 1 about here]

The results for this case are illustrated in table 1 and summarised in proposition 1.

Proposition 1

In a simultaneous-move game of budgetary allocations between an aid donor and a recipient with conflicting interests as described by assumption 1, outcomes only depend on the share of the total available resources controlled by D when budgets are exogenous. If the number of collective goods is at least three, there are three different types of equilibria where: i) one of the players is at a corner solution, funding only the good that is most severely underfunded from its perspective, while the other player spends on all goods, thereby controlling the final outcome; ii) each good is financed by only one player, where, if a player funds more than one good, the ratios in which goods are supplied are first-best optimal according to the player providing them; iii) one good is jointly funded even though both players allocates resources to more than one good; in this case the supply of all goods are constant.

What are the effects on the equilibrium budgetary shares of the three goods as the share of total available resources controlled by the donor increase? Let $\eta_k^N (\alpha)$ be the Nash-equilibrium budgetary share of good k as a function of α, i.e., $\eta_k^N (\alpha) = \frac{q_k x_k}{B}$. Given the fact that a higher value of α means greater spending power for the donor and smaller for the recipient, it should not be surprising that in the case of $K = 3$ $\eta_1^N (\alpha)$ is a weakly monotonically increasing function of the share of B controlled by D while $\eta_3^N (\alpha)$ is a weakly monotonically decreasing function of α. This is illustrated in figures 1a and 1c. The figures also demonstrate that $\eta_1^N (\alpha) \in \left[\sigma_1^R, \sigma_1^D \right]$ and $\eta_3^N (\alpha) \in \left[\sigma_3^D, \sigma_3^R \right]$. That is, the equilibrium spending shares of the two players' favourite good always take on values in the closed interval having their respective optimal budgetary shares

as end points. However, this is the case for good 2. Figure 1b shows that not only is $\eta_2^N(\alpha)$ a non-monotonic function; there are regions where $\eta_2^N(\alpha) < Min\{\sigma_2^D, \sigma_2^R\}$.[12] Not being on top of either player's list of priority thus results in spending on good 2 losing out in the battle over resource allocation between R and D to the extent that both players would ideally like to raise the consumption of this collective good. Yet given the budgetary strategy of the other player, none of them has the incentive to do so, as this would mean sacrificing some of the output of their priority goods.

[Figure 1a about here]
[Figure 1b about here]
[Figure 1c about here]

2.3 Sequential Moves

In games concerning economic policy, it is usually an advantage to move first, i.e., to commit to a strategy before one's opponent make its decision. For $K \geq 3$, this is the case in the current setting too. On the other hand, for $K = 2$, outcomes are independent of the order of moves. This interesting special case is explored at the end of this section. First, however, I return to the example of $K = 3$ and show that the change from the simultaneous move game is that there are no interior equilibria with joint financing of a good. The first-mover advantage is thus that one can take the response of the other player into account when evaluating the costs and benefits of spending on a good.

Suppose that D chooses its budgetary strategy before R. In the last stage, the recipient will try to reach $\mathbf{X}^{R*}$. That is, if at all possible, it will use the strategy $\mathbf{b}^{R*}$. This means that if the donor is to move the final allocation away from $\mathbf{X}^{R*}$, it has to ensure that the solution to the recipient's problem is not in the interior of the choice set. In other words, it must make at least one of the non-negativity constraints on R's budgetary policy binding. It should be readily apparent that this is not feasible if $\alpha \leq \sigma_1^R$. Even if D concentrates its resources on increasing the budgetary share of x_1, $b_1^{R*} \geq 0$. Thus, for such parameter values $\mathbf{X}^L = \mathbf{X}^N = \mathbf{X}^{R*}$. Also note that $\alpha_1^L = \alpha_1^N = \sigma_1^R$. The only slight change from the last sub-section is that the strategy of the leader, in this case D, is not unique when it cannot influence the end result in the desired direction. Any allocation of its budget that results in $\mathbf{X}^{R*}$ being the outcome is as good as any other. In the event that D could make one of the other non-negativity constraints on R's budgetary strategy binding, this would only result in x_1 being even lower than x_1^{R*}, a result that is clearly not desirable for the donor. This means that when its financial muscles are weak, the donor must accept the fact that the recipient government is in complete control over the allocation.

Outcomes in region 2 are also the same as in the simultaneous move game. If D sets $b_1^D = B^D$, R will not spend on good 1. Instead it divides its budget optimally between x_2 and x_3. However, now this type of equilibrium exists until

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[12]The figure is drawn for $\sigma_2^D < \sigma_2^R$. If $\sigma_2^D > \sigma_2^R$, the point would be even clearer.

$\alpha = \alpha_3^N$. The reason is that the marginal benefit of spending is lower for D when it takes R's reaction into account. Since R plays $\{0, b_2^{R**}, b_3^{R**}\}$ if $b_1^D = B^D$ for $\alpha \in [\alpha_2^N, \alpha_3^N]$, reallocating a unit of funds from x_1 to, say, x_2 yields the following change in D's objective function

$$\Delta U^D = -\frac{\partial U^D}{\partial x_1} + \frac{\partial U^D}{\partial x_2}\frac{1}{q_2}\left(1 + \frac{\partial b_2^{R**}}{\partial b_2^D}\right) + \frac{\partial U^D}{\partial x_3}\frac{1}{q_3}\frac{\partial b_3^{R**}}{\partial b_2^D} < 0.$$

That is, as long as R is at an interior solution with respect to spending on x_2 and x_3, it will adjust its budgetary allocation to preserve the expenditure ratio $\frac{\tilde{\sigma}_2^R}{\tilde{\sigma}_3^R} = \frac{\sigma_2^R}{\sigma_3^R}$ for these two goods. This means that $\frac{\partial b_2^{R**}}{\partial b_2^D} = -\left(1 - \tilde{\sigma}_2^R\right)$ and $\frac{\partial b_3^{R**}}{\partial b_2^D} = \tilde{\sigma}_3^R$. In this region D could replicate the Nash-equilibrium outcome by choosing its Nash-equilibrium strategy. R's response would then be the same as in the simultaneous move game since as a follower it takes $\mathbf{b}^D$ as given when deciding on its best choice. However, D can do better by leaving the supply of good 2 to the recipient. Then, by Lemma 1, $\frac{\partial U^D}{\partial x_2}\frac{1}{q_2} > \frac{\partial U^D}{\partial x_3}\frac{1}{q_3}$. Since $\tilde{\sigma}_2^R + \tilde{\sigma}_3^R = 1$ the sum of last two terms on the left-hand side of the inequality is therefore less than $\frac{\partial U^D}{\partial x_2}\frac{1}{q_2}$. Moreover, at α_3^N R's best response to $\mathbf{b}^D = \{b_1^{D**}, b_2^{D**}, 0\}$ will be $\mathbf{b}^R = \{0, 0, b_3^{R**}\}$. This will generate the Nash-equilibrium outcome and we know that at this point a marginal reallocation of the kind studied here yields $\Delta U^D = -\frac{\partial U^D}{\partial x_1} + \frac{\partial U^D}{\partial x_2}\frac{1}{q_2} = 0$. But for $\alpha \in \left(\alpha_2^N, \alpha_3^N\right)$, $\Delta U^D = -\frac{\partial U^D}{\partial x_1} + \frac{\partial U^D}{\partial x_2}\frac{1}{q_2}\tilde{\sigma}_2^R + \frac{\partial U^D}{\partial x_3}\frac{1}{q_3}\tilde{\sigma}_3^R < 0$. Hence, in the sub-game perfect equilibrium it is optimal for D to only spend on its favourite good. D will not commence financing x_2 until it can make sure that $\frac{x_1}{q_2 x_2} = \frac{\sigma_1^D}{\sigma_2^D}$, which it realises is not feasible until $\alpha > \alpha_3$ due to the response of R. Only if there is no crowding-out of R's contribution will D finance both x_1 and x_2 when it is the leader in a sequential game.

Joint financing of goods therefore only results for parameter values such that one of the players is without influence over the final allocation. Since good 2 is not on the top of either player's list of priorities, it is never jointly financed. With respect to good 3, joint financing is the outcome for $\alpha > 1 - \sigma_3^D$, as was the case in the simultaneous move game. Then D correctly anticipates that R will choose $\{0, 0, B^R\}$ in response to $\{\sigma_1^D B, \sigma_2^D B, \sigma_3^D B - B^R\}$ and the outcome is thus $\mathbf{X}^{D*}$.

When the donor is a follower, the situation is turned on its head: the recipient will then leave the funding of good 2 entirely in the hands of D for $\alpha \in [\alpha_2^N, \alpha_3^N]$. R will realise that a small contribution by it towards financing the supply of x_2 will be reallocated to suit D's tastes. It will therefore not fund this good until it is certain that D will not do so; only then is the expenditure ratio between goods 1 and 2 right from R's perspective, justifying diverting some of its budget away from its favourite good to increase the supply of x_2. Proposition 2 summarises these results:

Proposition 2
In a sequential game of budgetary allocations between an aid donor and a

recipient with conflicting interests as described by assumption 1, there are two kinds of equilibria in which: i) one of the players is at a corner solution, funding only its favourite good, while the other player spends on all goods, thereby controlling the final outcome; or ii) each good is financed by only one player and the ratios in which goods are supplied are first-best optimal according to the player providing them if at least two goods are funded by it.

Interestingly, the equilibrium budgetary shares for x_1 and x_3 are no longer monotonic functions of α. In fact, neither these nor $\eta_2^N(\alpha)$ are even continuous functions in sequential games. There is a jump in all equilibrium budgetary share functions at $\alpha = \alpha_3^N$ due to the leader taking over the responsibility of financing x_2 from the follower. Figures 2a-c illustrates this for the case when D is the leader.[13] This means that small changes in α around such a critical value could cause large changes in the provision of collective goods, something that might surprise observers disregarding the fact that donors and recipients interact strategically.

[Figure 2a about here]

[Figure 2b about here]

[Figure 2c about here]

From the above, it also follows that there are parameter values for which each player has a strict ranking over the order of moves, as well as values of for which they are indifferent to the type of game being played because outcomes are the same. For $\alpha < \alpha_2^N$ and $\alpha > \alpha_3^N$ $\mathbf{X}^F = \mathbf{X}^L = \mathbf{X}^N$, and so players do not care about the order in which they choose budgetary strategies. However, for $\alpha \in \left[\alpha_2^N, \alpha_3^N\right]$ both players prefer being a leader to playing the simultaneous-move game, with being a follower the least attractive option. It is straightforward to establish that for such parameter values $x_1^L - x_1^N \geq 0 \geq x_1^F - x_1^N$ and $x_3^F - x_3^N \geq 0 \geq x_3^L - x_3^N$, whereas there is a higher level of x_2 in the Nash-equilibrium compared to the equilibria of both sequential games. Since none of the players consider any equilibrium outcome first-best optimal in this range, they obviously prefer a higher level of supply of their favourite good and reduced output of the other player's priority good, a result they can achieve if they are the leader. Lowering x_2 does not totally negate this gain because the higher spending on x_1 or x_3, as the case may be, is partially compensated by the follower spending less on its favourite good and somewhat more on x_2 than in the Nash-equilibrium. Analytically, if we let B_k^g be the equilibrium level of spending on good k in game g, we may exemplify the gain to, say, D from moving from Nash to Stackelberg-leadership as follows

$$\Delta U^D = \frac{\partial U^D}{\partial x_1}\left(B_1^L - B_1^N\right) + \frac{\partial U^D}{\partial x_2}\frac{1}{q_2}\left(B_2^L - B_2^N\right) + \frac{\partial U^D}{\partial x_3}\frac{1}{q_3}\left(B_3^L - B_3^N\right) > 0.$$

Because D finances both x_1 and x_2 but not x_3 in the Nash-equilibrium we are concerned with, $\frac{\partial U^D}{\partial x_1} = \frac{\partial U^D}{\partial x_2}\frac{1}{q_2} > \frac{\partial U^D}{\partial x_3}\frac{1}{q_3}$. Moreover, $B_1^L - B_1^N =$

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[13]The exact location of the functions depend on parameter values, but the jumps are in the direction shown.

$-\left[\left(B_2^L - B_2^N\right) + \left(B_3^L - B_3^N\right)\right]$. That is, we may rewrite the change as $\Delta U^D = -\left(\frac{\partial U^D}{\partial x_1} - \frac{\partial U^D}{\partial x_3}\frac{1}{q_3}\right)\left(B_3^L - B_3^N\right)$, which is positive due to the lower spending on x_3 in the sub-game perfect equilibrium when D is the leader. A corresponding exercise could be performed to show that the higher spending on x_3 more than compensates R for the reduction in spending on the other goods that results from assuming leadership in the budgetary game. So for $K \geq 3$ we have

Proposition 3

For some values of the share of the combined budget controlled by D players are indifferent to the order of moves because outcomes are the same in the Nash-equilibrium and the sub-game perfect equilibria. There are also parameter values such that each player at least weakly prefers being a leader to playing Nash, with being the follower yielding the worst outcomes according to their preferences.

I will now show that Proposition 3 does not apply when $K = 2$.

2.4 The Special Case of $K = 2$

With only two goods assumption 1 reduces to $\frac{\sigma_1^R}{\sigma_1^D} < \frac{\sigma_2^R}{\sigma_2^D} = \frac{1-\sigma_1^R}{1-\sigma_1^D} \Leftrightarrow \sigma_1^D > \sigma_1^R$. So R controls the outcome of a simultaneous move game for $\alpha \leq \sigma_1^R$ and D for $\sigma_2^D = 1 - \sigma_1^D \geq 1 - \alpha \Leftrightarrow \alpha \geq \sigma_1^D$. Since there are only priority goods, there are no "interior" equilibria with joint financing of a good. Joint financing of a good only occurs when one of the players is at a corner solution, trying in vain to increase the share of the combined budget being devoted to the good for which it has a stronger relative preference than the other player. For $\alpha \in \left[\sigma_1^R, \sigma_1^D\right]$, both players choose extreme strategies, each spending solely on its favourite good, the result being that the outcome lies between $\mathbf{X}^{D*}$ and $\mathbf{X}^{R*}$, c.f. $\eta_1^N(\alpha) = \alpha$. Switching to a sequential game does not change this fact. Recall from the discussion of $K = 3$ that the only change precipitated by such a switch occurred for values of α where there was joint financing of the non-priority good x_2 in the Nash-equilibrium. As already noted, in the current case there is no such region. Intuitively, for $\alpha \in \left[\sigma_1^R, \sigma_1^D\right]$ D will, if it is the leader, realise that any funds not spent on x_1 by it will be used by R to raise the level of provision of x_2. This is clearly not in D's interest as it would ideally like to have the budgetary share of the former good at $\sigma_1^D > \alpha$. Only when $\alpha \geq \sigma_1^D$ can D "leave money on the table", safe in the knowledge that R will spend its whole budget on x_2. So outcomes follow the pattern established for the simultaneous move game. This is surprising prima facie; as noted above, it is usually an advantage to move first in games of economic policy. Yet the underlying logic of this particular game is that there is a strict conflict over how to split the pie. If possible, each of them will therefore unilaterally make sure that their favourite good is optimally supplied. With only two goods, such an achievement also implies that spending on the other good is optimal according to the preferences of the player in this advantageous position. However, given the resource constraints at most one of them can be in such a position. When none of them has the power to unilaterally achieve optimal spending levels from

their perspective, both D and R try to exploit the incomplete control of the other party over the pie to increase the share allocated to the good they attach the greatest priority to relative to the other player. Since their preferences are strictly opposed, each of them are drawn to extreme positions, spending all of their budget on the good they deem to be undersupplied. Changing the order of moves therefore does not make a difference: either one of the players control a large enough share of the available resources to bring about its first-best allocation or the equilibrium is a stalemate where each uses its budget to ensure that the outcome is as close to this point as possible. Hence, there is no first-mover advantage. It follows that the two possible types of sequential games are mirror-images of each other. The fact that the logic extends to the case where R moves before D means that equilibrium outcomes are completely isomorphic to the order of moves:

Proposition 4

In a budgetary game with conflict over the provision of two public goods as described by assumption 1, outcomes does not depend on the order in which the players move. There are three regions with different equilibria, two of which entails complete control over the outcome by one of the players and one in which each player devotes their budgets to their priority good, and the kind of equilibrium realised depends only on the share of the combined budgets of the players controlled by the donor.

Hence, if one thinks it suffices to aggregate goods into two composites, those that the donor care more strongly for than the recipient and those for which the opposite ranking applies, the model here has the empirical implication that outcomes should not depend on changes in the relative commitment capacity of the players. Whether or not the donor or the recipient is best at tying their hands, it is the amount of resources they are willing to put into the game that matters. If a greater share comes out of the pockets of the donor, one is more likely to observe outcomes that conform to the preferences of the donor and vice versa.

[Figure 3 about here]

With only two goods, it is easier to demonstrate graphically how equilibrium outcomes map out when the comparative statics exercise is in terms of the level of the aid budget, keeping B fixed, so that higher levels of α also means higher levels of the combined budget of the two players. Figure 3 illustrates this situation. Since the objective functions are homothetic, optimal budget shares stay constant as B increases. Hence, instead of studying how the equilibrium changes as α varies for fixed B, the same pattern of outcomes results when B^D is varied holding B^R constant. Note how the bold line marking equilibrium allocations first (i.e., for $B^D \leq \overline{B}^D = \left(\frac{\sigma_1^R}{1-\sigma_1^R}\right) B^R$) follow the expansion path of R. When D starts to have influence, outcomes begin to deviate from this path, moving closer to the donor's expansion path as B^D increases. When $B^D > \overline{\overline{B}}^D = \left(\frac{\sigma_1^D}{1-\sigma_1^D}\right) B^R$, the donor is in complete control, so outcomes move out along its expansion path as the total amount of available resources goes up with B^D.

A final point is the fact that the recipient is always better off playing the aid game. Note that the outcomes always lie northeast of R's optimal allocation when the donor does not transfer any funds. As R's preferences can be represented by indifference curves of the standard type, these outcomes generate a higher value of the recipient's objective function. The reason is simply that at low levels of aid, where one could suspect that the transfer could be inadequate to compensate for any "distortion" in outcomes due to donor influence, D has in fact no leverage. And when D provides resources at a level sufficient to have an impact on outcomes, R is more than compensated by the increase in the budget available for spending on goods 1 and 2.

3   The Issue of Fungibility

It is difficult to define fungibility in a precise way. The definition adopted here corresponds to that of Pedersen (1997), who characterises aid as fungible if it is possible for the recipient to divert resources away from the activity that the donor seeks to finance. As pointed out by him, the possibility of diversion is but a necessary condition for actual diversion; in order to divert funds, the the recipient must also wish to do so. Hence, to explore the importance of diversion, we must investigate how the funding strategies of the recipient and the donor depend on their preferences, their budgets, and the nature of their strategic interaction.

In the literature, the example that is ordinarily used to illustrate the concept is a situation where a donor wants to support a specific activity in the recipient country through an earmarked grant. Aid is then said to be fungible if expenditures on that activity do not rise by the full amount of the grant. Figure 4, adapted from Feyzioglu, Swaroop, and Zhu (1998), is an example of this standard approach.[14]

In Feyzioglu, Swaroop, and Zhu (1998), the donor is assumed not to care about the good or activity x_2. It only wants to support x_1. It does so by donating an amount equal to the distance between points E and B. That is, subject to a restriction to be discussed shortly, the budget line of the recipient is moved out to the extent of the aid given. The donor wants the resulting allocation to be at point F. At that point, x_1 has increased relative to the original allocation by an amount $F - C$, which is equal to $E - B$. Aid is then said to be partially fungible if the recipient can divert part of the grant for x_1 to x_2. It is said to be completely fungible if "the post-aid optimal mix of the two goods, chosen by the country, is an interior solution" (p. 31).

[Figure 4 about here]

Even in this apparently simple setting, however, there are some loose ends. These authors assume that the recipient must spend at least the size of the grant

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[14] A similar illustration appears in Devarajan and Swaroop (2000). Note that Feyzioglu, Swaroop, and Zhu (1998: 33) themselves seem to regard the framework they adopt as less than ideal; they explicitly state that "[w]e take [...] fungibility [...] as given, rather than deriving it from a game-theoretic framework."

on the activity supported by the donor. That is, we must have $x_1 \geq E - B$, so that the new budget constraint has a kink. In figure 4 this occurs at point D, and the assumption of Feyzioglu, Swaroop, and Zhu (1998) means that points between H and D are not accessible to the recipient. This assumption is analogous to the non-negativity constraints that I impose on the recipient's funding choices. The motivation, however, seems to be different; the authors state that the kink indicates aid conditionality, so presumably they believe that the donor will "punish" the recipient if it spends less than this amount. But then why does not the donor punish the recipient if it diverts part of the grant to other activities? Given the problem of punishing straying recipients, as reflected in the generally unimpressing record of conditionality, there is an untold story here that needs elaboration.

A second point to note is that, as long as the objective function of the recipient is homothetic and both goods are normal, it is easy to demonstrate that the assumption $x_1 \geq E - B$ implies that if the grant is "very large", full fungibility is not possible. Moving the point D far enough to the right in figure 3, it will eventually be the case that the expansion path of the recipient lies to the northwest of D. Hence, in this setting as well there is a link between grant size and final allocations, but it is not explored.

A third point is that even if one accepts that the donor only cares about one good (or set of goods), while the recipient wants to spend on some goods not given priority at all by the donor, the latter can always adjust the level of funding. That is, if aid is to some extent fungible, this should be reflected in the size of the grant. The observation just made, namely that under the assumption $x_1 \geq E - B$ full fungibility is impossible if the grant is large enough, makes clear the need to investigate donor and recipient behaviour simultaneously.

In sum, implicit in the standard, non-strategic approach is a naive representation of the donor, particularly if fungibility is indeed an important problem. In the present model, the donor acts strategically, taking into account the possibility of diversion of resources by the recipient. Therefore, it optimally adjusts its aid policy in order to achieve as much as possible. Yet, as the recipient is equally adept at playing the game, the donor's achievements is determined by relative spending power. There is full fungibility if the donor brings only small change to the table, no fungibility if its pockets are sufficiently deep, and partial fungibility in between. One should therefore expect the impact of foreign economic assistance to vary across time and space with the ratio of aid to recipient government budgets. This general conclusion accords well with existing empirical literature, which indeed indicates that the degree to which aid is fungible varies across countries and periods. For example, with respect to Indonesia during 1966-86 Pack and Pack (1990: 193) conclude that "most categorical aid was spent on the purposes for which it was intended by the donors." However, when investigating the same issue in the Dominican Republic for almost the same time period, they report that "[i]n no case does the increase in expenditure nearly equal the increase in categorical aid, indicating substantial diversion away from the intended expenditure patterns" (Pack and Pack 1993: 263).

4 Endogenous Budgets for Collective Goods

Assuming fixed budgets for both the donor and the recipient is a useful benchmark. Tax systems in many developing countries are highly rudimentary and tax administration is notoriously lax, with corruption, tax avoidance, and tax evasion constituting very real constraints on the government's ability to raise revenues. Improving tax capacity takes time. Moreover, many aid recipients, particularly in Africa, lack access to alternative external sources of funds. This is not likely to change over night.

On the donor side, it is noteworthy that aid allocation patterns across countries show a relatively high degree of persistence. One reason for this, is that some donors have favourite recipients, for example due to historical or cultural ties.[15] Even bilateral donors that tend to give aid to the poorest countries often designate some recipients as the main targets for their development assistance. One argument for building long-term relationships is of course that it facilitates the accumulation of country-specific knowledge, which potentially could lead to greater aid efficiency. Thus, over a medium-term horizon, assuming given budget levels for both players is a reasonable approximation to reality.

Still, it is obviously of interest to see whether the results derived so far hold up when budgets are endogenous, especially if the call for aid selectivity is heeded by donors. In this section I show that the same three kinds of equilibria - complete control over the outcome for either player or shared influence - arise in this case in essentially the same way. Specifically, the degree of influence again varies with the share of total available resources controlled by D and the critical values depend on the order of moves. It turns out that even though the donor controls the outcome at a higher relative cost when it is a Stackelberg-follower instead of a leader, it is always better off having the first move. In fact, for the same parameter values being a leader always yields at least as high a pay-off as in the Nash-equilibrium, with the latter in turn is everywhere at least as good as the equilibrium outcome when the donor is a follower in a sequential game. R too, ranks games in this way based on equilibrium outcomes; that is, it would always at least weakly prefer being a leader to playing a simultaneous move game, which in turn is at least weakly preferred to moving last in a sequential game. The reason is that a leader can calculate whether it would be optimal to try to impose its most preferred allocation. If the improvement in the outcome does not generate a benefit at least commensurate with the cost, the leader can always leave provision of one or both goods to the follower. The latter does not have the option of making such a calculation, and therefore cannot be better off than if it were. The simultaneous move game naturally leads to an intermediate constellation of critical parameter values.

In this section then, the preferences of the players are

$$W^p\left(\mathbf{X}, z_p\right) = U^p\left(\mathbf{X}\right) + \beta_p^p \frac{z_p^{1-\mu}}{1-\mu}, p = D, R. \tag{5}$$

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[15] See for example Alesina and Dollar (2000), Boone (1996), Boschini and Olofsgård (2003), Cashel-Cordo and Craig (1997), and Chauvet (2002).

Thus, z_p is a private good that only player p cares for.[16] For example, it might be private consumption in the country in question.[17] As before, $\sum_k b_k^p = B^p$, but now B^D and B^R are determined endogenously taking into account that funding collective goods is costly as it leads to lower levels of consumption of the private good. Assuming convex marginal costs of contributing to the supply of goods in $\mathbf{X}$ is most realistic for R, given the dependence of poor countries on highly distortionary instruments such as trade taxes for a large part of their public revenues. For the donor, constant marginal costs would probably be a better approximation to reality, because most donors are not even fulfilling the UN target of giving at least 0.7% of their GNI in the aggregate.[18] Thus, the total aid budget for a particular recipient is quite small for all donors, and so is unlikely to affect the marginal cost of public funds. In Hagen (2002) I analysed the case of constant costs for both players. I therefore concentrate on the more general case here.

Each player now spends out of an endowment of Y^p. Let $Y = Y^D + Y^R$. It is straightforward to derive the "first-best" levels of supply for the players in the current context. They are[19]

$$x_k^{D*} = \frac{\sigma_k^D Y}{q_k}, \forall k, z_D^{D*} = \frac{\sigma_D^D Y}{q_D}, z_R^{D*} = 0; \tag{6a}$$

$$x_k^{R*} = \frac{\sigma_k^R Y}{q_k}, \forall k, z_D^{R*} = 0, z_R^{R*} = \frac{\sigma_R^R Y}{q_R}. \tag{6b}$$

The analogy to the case where total spending by each player on the collective goods is exogenous should be clear. In particular, optimal expenditure shares are of the same form for both collective goods and the private one. One important change, though, is that these allocations are unattainable. Since the marginal benefit from spending on a good goes to infinity as consumption goes to zero, all goods will be supplied in all equilibria. As long as $Y^p > 0$, each player will make sure that $z_p > 0$ even though the other player would prefer that no resources are spent on this good.

The "first-best" budgetary strategies may be derived from $(6a - b)$ as well as the fact that consumption of any good is just the total amount of funds contributed by D and R divided by the price of the good. Denoting spending by p on the two private goods by c_D^p and c_R^p, they may be expressed as

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[16] Note that I now assume $\sum_{k=1}^{K} \beta_k^p + \beta_p^p = 1, p = D, R$.

[17] Potential alternatives are consumption (public and/or private) in other recipient countries for the donor and goods that only the elite in the recipient country benefits from.

[18] Only the Scandinavian countries, the Netherlands, and Luxembourg are currently achieving this target.

[19] I adopt the notational convention that subscripts D and R refer to z_D and z_R, respectively.

$$b_k^{D*} = \sigma_k^D Y - b_k^R, \forall k, c_D^D = \sigma_D^D Y - c_D^R, c_R^D = 0; \tag{7a}$$
$$b_k^{R*} = \sigma_k^R Y - b_k^D, \forall k, c_D^R = 0, c_R^R = \sigma_R^R Y - c_R^D. \tag{7b}$$

The basic assumption of the character of the conflict of interest between R and D is retained. I therefore still assume that $K \geq 2$, otherwise assumption 1 would turn into an assumption about strongest relative preference for private versus collective goods. The simplest case illustrating all the results, which I focus on in the main text, is now $K = 2$. I once again start with a description of the Nash-equilibrium. It turns out to be a straightforward extension of the case analysed in section 2.

Let $\gamma = Y^D/Y$ be the share of total available resources controlled by D. When γ is very high, R will not contribute towards the provision of the collective goods because its consumption of z_R will then be too low from its perspective. The highest critical value, denoted by γ_3^N, is thus found by solving $\frac{\partial W^R}{\partial b_2^R} = \frac{\partial W^R}{\partial c_R^R}$ given D's first-best budgetary strategy as well as $c_R^R = Y^R$. For $\gamma > \gamma_3^N$, $\frac{\partial W^R}{\partial c_R^R} > \frac{\partial W^R}{\partial b_2^R}$, and assumption 1 ensures that $\frac{\partial W^R}{\partial b_2^R}$ is higher than the marginal benefit from spending on x_1. However, for values of γ lower than γ_3^N, it is optimal for R to contribute to the provision of x_2. For $\gamma \in \left[\gamma_2^N, \gamma_3^N\right)$, the equilibria entail joint funding of x_2 in the manner described in section 2. For still lower values of the share of total available resources controlled by D, there is first a region where each player funds one collective good only, then a region with joint funding of x_1. The final region is the mirror image of the one where R only finances z_R: D only spends on z_D, leaving the supply of both collective goods to the recipient.

Thus the only real change is that there are parameter values such that only one player finances $\mathbf{X}$. This is the case when the other player's endowment is so low that it is better off spending it all on its private good, which it must provide on its own. For intermediate values of γ, equilibria switches between regions where each x_k is supplied by only one player to regions of joint funding of a single collective good, with R assuming more and D less of the responsibility for providing $\mathbf{X}$ as γ goes down, in the sense that each switch implies that either D stops funding a collective good or R starts to finance one.

Changing the order of moves also makes a difference when B^D and B^R are endogenous. When one player moves before the other, there are no equilibria with joint funding of a collective good. Intuitively, for the parameter values where it was optimal for the leader to provide funds for x_k in the simultaneous move game even though the follower also contributed, the former is better off by leaving the task to the latter since this means less spending on the follower's private good. We know that whenever the follower finances two or more goods, the ratio in which any two goods are supplied are equal to the ratio of the "first-best" budgetary shares. Hence, if the leader withdraws funding for x_k, the follower reduces its spending on all other goods to which it contributes to compensate for this. Accordingly, the amount of resources allocated to the

private good of the follower is reduced, and the leader may use some of the savings to increase the level of consumption of its private good. Therefore, in sequential games all critical values of γ marks a switch in the identity of a provider of one of the collective goods, and any $x_k \in \mathbf{X}$ is always funded by only one player. Table 2 shows how this maps out for $K = 2$:

[Table 2 about here]

Proposition 5

When the amount of spending by the players on the collective goods is endogenous, there are three types of equilibria: i) if a player controls a sufficiently low share of the combined resources of the players, it only funds its private good, leaving the provision of the collective goods to the other player; ii) when each collective good is funded by only one player, the expenditure ratios of the goods provided by that player (including its private good) are first-best optimal from its perspective; iii) in the simultaneous move game, there are parameter values generating equilibria where both players jointly finance the provision of a collective good. For these levels of the share of total available resources controlled by the donor the leader in a sequential game prefers to have the follower fund the good that is jointly provided in the Nash-equilibrium.

Given the similarities between this case and the case where each player had a fixed total budget to spend on the collective goods, it should not be surprising that the results with respect to equilibrium budgetary shares and preferences over the order of moves are quite similar. The equilibrium budgetary shares of the collective goods are in this case never monotonic functions of the share of total available resources controlled by the donor. In fact, in the sequential versions of the game they are not even contunuous functions of γ. There are parameter values such that equilibrium outcomes are the same in all three games, but where outcomes differ players prefer being a leader to playing Nash, with the latter judged to be better than being a follower.

5   Final remarks

The current version of this paper represents a first attempt to understand the impact of aid on allocation patterns in recipient countries taking into account the fact that whenever spending priorities differ, donors and recipients play a game in which each party tries to use the resources available to it to make sure that the outcome is as good as possible from its perspective. Using a simple framework, I have analysed how the end result of the interaction between a donor and a recipient depends on the preferences and budgets of the players, as well as the order in which they move. Despite the bare-bones approach, some interesting results were derived:

- Fungibility, or influence over outcomes, is a function of relative resource levels. If a player dominates the other player sufficiently in this respect, the equilibrium allocation is first-best according to this player's preferences. Hence, aid will not be fungible if the transfers are large enough compared to the resources at the recipient's command.

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- For some values of the share of total available resources controlled by the donor, there is a first-mover advantage. For other parameter values as well as the special case of only two collective goods and exogenous budgets, equilibrium outcomes are independent of the order in which players move.

- The equilibrium budgetary shares of non-priority goods may be below both players' first-best level.

- In sequential games, equilibrium budgetary shares are discontinuous in the share of the combined budget controlled by the donor. Thus, small changes in this share may lead to large changes in the provision of collective goods.

The main conclusion of this paper, that influence over outcomes is a weakly monotonically increasing function of relative budgets, might be construed as supporting the current emphasis on aid selectivity in the donor community: if greater selectivity is applied for given total aid budgets, some countries must be receiving higher levels of aid. However, the results in Hagen (2003) suggest that selective strategies could backfire by changing the political equilibrium in the recipient countries. That is, if donors have influence, they will necessarily affect the outcome of elections in democratic recipient countries and the change will always be in the direction of reducing the chances of having a government with preferences closer aligned with the donor. Hence, choosing optimal aid strategies from the point of view of the donors require a political economy approach integrating the economics and politics of foreign economic assistance in a setting of strategic interaction.

6 Appendix A: Exogenous Budgets

Proof of Lemma 1

Holding $\mathbf{b}^{-p}$ constant, $\frac{\partial U^p}{\partial b_k^p} = \frac{\partial U^p}{\partial x_k} \frac{1}{q_k}$. Suppose $x_l = x_l^{-p*}$ and $x_m = x_m^{-p*}$. Then $\frac{\partial U^p}{\partial b_l^p} \gtreqqless \frac{\partial U^p}{\partial b_m^p} \Leftrightarrow \frac{\sigma_l^p}{\sigma_l^{-p}} \gtreqqless \frac{\sigma_m^p}{\sigma_m^{-p}}$. By assumption 1, $\frac{\sigma_l^R}{\sigma_l^D} < \frac{\sigma_m^R}{\sigma_m^D}$, $m > l$. Hence $\frac{\partial U^D}{\partial b_l^D} > \frac{\partial U^D}{\partial b_m^D}$ and $\frac{\partial U^R}{\partial b_l^R} < \frac{\partial U^R}{\partial b_m^R}$. QED.

Proof of Lemma 2

Let $\widetilde{K}$ be the number of goods in $\widetilde{X}$. Then lemma 2 follows from solving for optimal budgetary allocations from $\widetilde{K} - 1$ equalities of marginal benefits of spending and the budget constraint of p. QED.

Let $E^{Dg} = \{1, ..., \delta, ..., \Delta\}$ and $E^{Rg} = \{P, ..., \rho, ..., K\}$ be the sets of goods financed by D and R, respectively in the equilibrium of a game of type g, $g = N, F, L$. Whenever D (R) finances more than one good in equilibrium Lemma 2 applies, and so the corresponding optimal budget shares are $\widetilde{\sigma}_d^D = \dfrac{\sigma_d^D}{\displaystyle\sum_{\delta \in E^{Dg}} \sigma_\delta^D}$

$(\widetilde{\sigma}_r^R = \dfrac{\sigma_r^R}{\displaystyle\sum_{\rho \in E^{Rg}} \sigma_\rho^R})$. Let J denote the good jointly financed by the two players, if any. As will be demonstrated, there will be at most one such good.

Proposition A1

In a simultaneous move budgetary game over the allocation of funds to $K \geq 2$ collective goods where the relative preferences of the players are described by assumption 1, there are $2(K-1)$ critical values of the share of the total resources available to the players controlled by D, α, separating K regions where $\Delta = P$ and $K-1$ regions where $\Delta < P = \Delta + 1$. In these regions the Nash-equilibrium strategies and outcomes are as follows

ia) $0 < \alpha < \alpha_1^N = \sigma_1^R : J = 1$
$b_1^{DN} = B^D, b_d^{DN} = 0 \ \forall d > 1; \ b_r^{RN} = \sigma_r^R B - b_r^D \ \forall r; \ \mathbf{X}^N = \mathbf{X}^{R*}$.

ib) $\alpha_{2(K-1)}^N = 1 - \sigma_K^D < \alpha < 1 : J = K$
$b_d^{DN} = \sigma_d^D B - b_d^D \ \forall d; \ b_r^{RN} = 0 \ \forall r < K, b_K^{DN} = B^R; \ \mathbf{X}^N = \mathbf{X}^{D*}$.

ii) $\alpha_{2s-1}^N < \alpha < \alpha_{2s}^N, s = 1, ..., K-1 : \Delta_s = s < P_s = s+1$
$b_{ds}^{DN} = \tilde{\sigma}_{ds}^D B^D - b_d^R \ \forall d \in E_s^{DN}, \ b_{ds}^{DN} = 0 \ \forall d \notin E_s^{DN}; \ b_{rs}^{RN} = 0 \ \forall r \notin E_s^{RN},$
$b_{rs}^{RN} = \tilde{\sigma}_{rs}^R B^R - b_r^D \ \forall r \in E_s^{RN}; \ x_{ks}^N = \frac{\tilde{\sigma}_{ks}^D B^D}{q_k} \ \forall k \in E_s^{DN}, \ x_{ks}^N = \frac{\tilde{\sigma}_{ks}^R B^R}{q_k} \forall k \in E_s^{RN}$.
$\alpha_{2s-1}^N = \frac{\tilde{\sigma}_{\Delta_s}^R}{\tilde{\sigma}_{\Delta_s}^D + \tilde{\sigma}_{\Delta_s}^R (1 - \tilde{\sigma}_{\Delta_s}^D)}, \alpha_{2s}^N = \frac{\tilde{\sigma}_{P_s}^R (1 - \tilde{\sigma}_{P_s}^D)}{\tilde{\sigma}_{P_s}^D + \tilde{\sigma}_{P_s}^R (1 - \tilde{\sigma}_{P_s}^D)}$.

iii) $\alpha_{2t}^N < \alpha < \alpha_{2t+1}^N, t = 1, ..., K-2 : J_t = t+1$

$$b_{dt}^{DN} = \tilde{\sigma}_{dt}^D \left(B^D + \sum_{\delta \in E_t^{DN}} b_\delta^R \right) - b_d^R \ \forall d \in E_t^{DN}, b_{dt}^{DN} = 0 \ \forall d \notin E_t^{DN}; b_{rt}^{RN} = 0$$

$\forall r \notin E_t^{RN}, \ b_{rt}^{RN} = \tilde{\sigma}_{rt}^R \left(B^R + \sum_{\rho \in E_t^{RN}} b_\rho^D \right) - b_r^D \ \forall r \in E_t^{RN}; \ x_{kt}^N = \frac{\tilde{\sigma}_{kt}^D \alpha_{2t+1} B}{q_k}$

$\forall k < J_t, \ x_{J_t}^N = \frac{\tilde{\sigma}_{J_t}^D \alpha_{2t+1} B}{q_{J_t}} = \frac{\tilde{\sigma}_{J_t}^R (1 - \alpha_{2t}) B}{q_{J_t}}, \ x_{kt}^N = \frac{\tilde{\sigma}_{kt}^R (1 - \alpha_{2t}) B}{q_k} \forall k > J_t. \ \alpha_{2t}^N = \frac{\tilde{\sigma}_{J_t}^R (1 - \tilde{\sigma}_{J_t}^D)}{\tilde{\sigma}_{J_t}^D + \tilde{\sigma}_{J_t}^R (1 - \tilde{\sigma}_{J_t}^D)}, \alpha_{2t+1}^N = \frac{\tilde{\sigma}_{J_t}^R}{\tilde{\sigma}_{J_t}^D + \tilde{\sigma}_{J_t}^R (1 - \tilde{\sigma}_{J_t}^D)}$.

Proof:

Regions of type i): a) By lemma 1, at $\mathbf{X}^{R*}$, $\frac{\partial U^D}{\partial b_1} > ... > \frac{\partial U^D}{\partial b_k} > ... > \frac{\partial U^D}{\partial b_K}$. Hence, $b_1^{DN} = B^D = \alpha B$ is the optimal choice for D. The cut-off rate α_1 is thus the value of such that $b_1^{RN} = \sigma_1^R B - \alpha B = 0$. For $\alpha \leq \alpha_1^N = \sigma_1^R$, D is unable to move the equilibrium outcome away from $\mathbf{X}^{R*}$, whereas for $\alpha > \alpha_1$, the donor can move it closer to $\mathbf{X}^{D*}$ by allocating its entire budget to good 1. The proof for region ib) involves the same logic with the roles of R and D reversed. The cut-off rate is thus the value of α for which $1 - \alpha = \sigma_K^D$ so that for $\alpha < 1 - \sigma_K^D \equiv \alpha_{2(K-1)}^N$ R is able to increase the equilibrium budgetary share of good K from σ_K^D by devoting its entire budget to x_K.

Regions of type ii): Each player finances a sub-set of all goods. The optimal budgetary allocation for D is derived from $\frac{\partial U^D}{\partial b_1} = ... = \frac{\partial U^D}{\partial b_\delta} = ... = \frac{\partial U^D}{\partial b_\Delta}$, yielding the "second-best" strategy $b_{ds}^{DN} = \tilde{\sigma}_{ds}^D \left(B^D + \sum_{\delta \in E_s^{DN}} b_\delta^R \right) - b_d^R \ \forall d \in E_s^{DN}$. The optimal strategy for R is found in the same fashion. Since each collective good is financed by only one player, in equilibrium $b_{ds}^{DN} = \tilde{\sigma}_{ds}^D B^D$

$\forall d \in E_s^{DN}$ and $b_{rs}^{RN} = \tilde{\sigma}_{rs}^R B^R \ \forall r \in E_s^{RN}$. The lower cut-off rate is the value of α for which $\frac{\partial U^R}{\partial b_s} = \frac{\partial U^R}{\partial b_{s+1}}$ when D finances goods 1 to s and R funds $s+1$ to K in the way just decribed, whereas α_{2s} is found by calculating when $\frac{\partial U^D}{\partial b_s} = \frac{\partial U^D}{\partial b_{s+1}}$ under the same conditions.

Regions of type iii): Good $t+1$ is jointly financed, i.e., $J_t = t+1$. Then each player sees the resources available for distribution over the goods in E_t^{pN} as B^p plus whatever the other player allocates to J; as long as the good is not oversupplied from p's perspective it may reduce its funding of J and increase the supply of those goods financed solely by itself. Therefore, in equilibrium $b_{dt}^{DN} = \tilde{\sigma}_{dt}^D \left(B^D + b_{J_t}^{RN} \right) - b_d^R \ \forall d \in E_s^{DN}$ and $b_{rt}^{RN} = \tilde{\sigma}_{rt}^R \left(B^R + b_{J_t}^{DN} \right) - b_r^D$ $\forall r \in E_t^{RN}$. Solving for $b_{J_t}^{DN}$ and $b_{J_t}^{RN}$ yields $b_{J_t}^{DN} = \left(\alpha - \frac{\tilde{\sigma}_{J_t}^R \left(1 - \tilde{\sigma}_{J_t}^D \right)}{\tilde{\sigma}_{J_t}^D + \tilde{\sigma}_{J_t}^R \left(1 - \tilde{\sigma}_{J_t}^D \right)} \right) B \equiv$

$\left(\alpha - \alpha_{2t}^N \right) B$ and $b_{J_t}^{RN} = \left(\frac{\tilde{\sigma}_{J_t}^R}{\tilde{\sigma}_{J_t}^D + \tilde{\sigma}_{J_t}^R \left(1 - \tilde{\sigma}_{J_t}^D \right)} - \alpha \right) B \equiv \left(\alpha_{2t+1}^N - \alpha \right) B$. Hence, the lower (upper) critical value is when D (R) optimally starts (stops) contributing to the supply of good J given the optimal funding strategy of R (D). Within the region, any increase in α will result in a corresponding increase (decrease) in $b_{J_t}^{DN}$ $(b_{J_t}^{RN})$ so that the equilibrium supply of x_{J_t} (and all other goods) stays the same. Assumption 1 ensures that at most one good can be jointly funded for the same value of α. For example, assuming two goods, l and m, are jointly funded implies $\frac{\sigma_l^R}{\sigma_l^D} = \frac{\sigma_m^R}{\sigma_m^D}$, which violates this assumption. QED.

Proposition A2

In sequential budgetary games over the allocation of funds to $K \geq 2$ collective goods where the relative preferences of the players are described by assumption 1, there are $K + 1$ regions with different equilibrium outcomes separated by K cut-off rates defined in terms of α. In every region except the first and the last, each good is funded by only one of the players. As α rises above each critical value between α_1 and α_K, the collective good with the lowest index value in E^{Rg} moves to E^{Dg}. In the first (last) region both players spend on x_1 (x_K).

Proof:

In a sequential game, the follower chooses its optimal budgetary strategy given the allocation of funds chosen by the leader. Therefore its equilibrium strategy is the same as in a Nash-equilibrium. The leader, however, has some extra degrees of freedom since it can calculate how the follower will respond to its decision. A slight complication in the game studied here is the fact that one cannot derive optimal strategies for the leader using calculus because it turns out not to be in the leader's interest to let the follower be at an interior solution with respect to the funding of a good that the leader spends on. However, it is easy to demonstrate that outcomes will follow the pattern established for the simultaneous move game with the exception that regions of type iii) do not occur. I concentrate on proving the non-existence of "interior" equilibria with joint financing and leave the rest of the proof to the interested reader. Suppose that D is the leader. The values of α that we are looking at are $\alpha \in \left[\alpha_{2t}^N, \alpha_{2t+1}^N \right]$,

$t = 1, ..., K - 2$. Then Proposition A1 informs us that $J_t = t + 1$ in a Nash-equilibrium and that R plays $b_{rt}^{RN} = \tilde{\sigma}_{rt}^{R} \left(B^R + b_{J_t}^{DN} \right) - b_r^D$ $r = t + 1, ..., K$. α_{2t}^N is defined by $\frac{\partial U^D}{\partial b_t} = \frac{\partial U^D}{\partial b_{t+1}}$ when $\mathbf{b}^D = \left\{ b_1^{D**}, ..., b_d^{D**}, ...b_{t+1}^{D**}, 0, ..., 0 \right\}$ and $\mathbf{b}^R = \left\{ 0, ..., 0, b_{t+1}^{R**}, ..., b_r^{R**}, ..., b_K^{R**}, \right\}$. Thus, it is the value of at which D just finds the budgetary share of good $t + 1$ chosen by R "second-best" optimal and so contributes nothing to the provision of this good. In the same way, α_{2t+1}^N is defined by R's contribution to the financing of x_{t+1} reaching zero when D funds $x_1 - x_{t+1}$ optimally according to its preferences. Hence, for $\alpha = \alpha_{2t+1}^N$ the situation is unchanged by switching to a game where D is the leader: there is no crowding-out of D's contribution to the provision of x_{t+1} as R optimally chooses $b_{t+1}^R = 0$ when D selects $b_{t+1}^D = \tilde{\sigma}_{t+1}^D B^D$. Similarly, there is no reason for D to switch strategy at $\alpha = \alpha_{2t}^N$ since x_{t+1} is provided in a "second-best" optimal manner from its perspective by R. However, for $\alpha \in \left(\alpha_{2t}^N, \alpha_{2t+1}^N \right)$ real-locating a unit of funds from x_t to x_{t+1} now does not increase spending on this good by as much because R is at an interior solution for goods $x_{t+1} - x_K$ and so will reduce its contribution by $1 - \tilde{\sigma}_{t+1}^R$ to increase the provision of goods $x_{t+2} - x_K$ as well. That is, the marginal benefit to D is not $\frac{\partial U^D}{\partial x_{t+1}} \frac{1}{q_{t+1}}$ but

$\sum_{r=t+1}^{K} \frac{\partial U^D}{\partial x_r} \frac{1}{q_r} \tilde{\sigma}_r^R$. By lemma 1 $\frac{\partial U^D}{\partial x_{t+1}} \frac{1}{q_{t+1}} > ... > \frac{\partial U^D}{\partial x_r} \frac{1}{q_r} > ... > \frac{\partial U^D}{\partial x_K} \frac{1}{q_K}$ when the budgetary shares of goods $x_{t+1} - x_K$ are "second-best" optimal as judged by R. Moreover, $\sum_{r=t+1}^{K} \tilde{\sigma}_r^R = 1$. So $\frac{\partial U^D}{\partial x_{t+1}} \frac{1}{q_{t+1}} > \sum_{r=t+1}^{K} \frac{\partial U^D}{\partial x_r} \frac{1}{q_r} \tilde{\sigma}_r^R$. The marginal cost of the reallocation of funds studied is $\frac{\partial U^D}{\partial x_t} \frac{1}{q_t}$. Hence, at the Nash-equilibrium strategy, where $\frac{\partial U^D}{\partial x_t} \frac{1}{q_t} = \frac{\partial U^D}{\partial x_{t+1}} \frac{1}{q_{t+1}}$, $\Delta U^D = -\frac{\partial U^D}{\partial x_t} \frac{1}{q_t} + \sum_{r=t+1}^{K} \frac{\partial U^D}{\partial x_r} \frac{1}{q_r} \tilde{\sigma}_r^R < 0$ in a sequential game and so D should reduce b_{t+1}^D to zero. QED.

Proposition A3

For some values of α players are indifferent to the order of moves because outcomes are the same in the Nash-equilibrium and the sub-game perfect equilibria. There are also values of this parameter such that each player at least weakly prefers being a leader to playing Nash, with being the follower yielding the worst outcomes according to their preferences.

Proof:

From propositions A1 and A2 it follows that outcomes are the same except for parameter values such that one of the goods $x_2 - x_{K-1}$ is jointly financed in the Nash-equilibrium. Therefore, for such values of α players are indifferent about the order of moves. For $\alpha \in \left[\alpha_{2t}^N, \alpha_{2t+1}^N \right]$, $t = 1, ..., K - 2$, Stackelberg-leaders could choose their Nash-equilibrium strategy in the sequential game and get the Nash-equilibrium outcome. Since they choose not to, they must be better off. As the funds not spent on good J_t by the leader is spent on goods for which the follower's marginal benefit of spending is lower than for goods J_t to K, the latter must be worse off. QED.

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Proposition A4

In a budgetary game with conflict over the provision of two public goods as described by assumption 1, outcomes does not depend on the order in which the players move. There are three regions with different equilibria, two of which entail complete control over the outcome by one of the players and one in which each player devotes their budgets to their priority good, and the kind of equilibrium realised depends only on α.

Proof:

This involves special cases of the games described in Propositions A1 and A2, and so is left for the interested reader.

7 Appendix B: Endogenous Budgets

This appendix contains the proof for the case where B^D and B^R are endogenous. Redefine E^{pg} as the set of collective goods funded by p in equilibrium. Thus, now $\widetilde{\sigma}_d^D = \dfrac{\sigma_d^D}{\sigma_D^D + \sum\limits_{\delta \in E^{Dg}} \sigma_\delta^D}$, $d = D, ..., \delta, ..., \Delta$, and $\widetilde{\sigma}_r^R = \dfrac{\sigma_r^R}{\sigma_R^R + \sum\limits_{\rho \in E^{Rg}} \sigma_\rho^R}$,
$r = P, ..., \rho, ..., R$, $g = F, L, N$. Since $Lim_{z_p \to 0} \frac{\partial W^p}{\partial z_p} = \infty$ and the other player will never spend money on p's private good, p will always provide z_p.

Proposition B1:

In a simultaneous move game with K collective goods and one private good for each player D and R, there are $2K + 1$ regions separated by $2K$ cut-off rates defined in terms of the share of total available resources controlled by D, γ. The Nash-equilibrium strategies and outcomes are

ia) $0 < \gamma \leq \gamma_0^N = \frac{\sigma_D^D \sigma_1^R}{\sigma_1^D + \sigma_D^D \sigma_1^R} : \Delta = D, P = 1$

$c_D^{DN} = Y^D$, $b_k^{DN} = 0$, $\forall k \in \mathbf{X}$, $c_R^{DN} = 0$; $c_D^{RN} = 0$, $b_k^{RN} = \sigma_k^R Y^R$ $\forall k \in \mathbf{X}$,
$c_R^{RN} = \sigma_R^R Y^R$; $z_D = \frac{Y^D}{q_D}$, $x_k^N = \frac{\sigma_k^R Y^R}{q_k}$ $\forall k \in \mathbf{X}$, $z_R = \frac{\sigma_R^R Y^R}{q_R}$.

ib) $\gamma \geq \gamma_{2K}^N = \frac{\sigma_K^R}{\sigma_K^D \sigma_R^R + \sigma_K^R} : \Delta = K, P = R$

$c_D^{DN} = \sigma_d^D Y^D$, $b_k^{DN} = \sigma_k^D Y^D$ $\forall k \in \mathbf{X}$, $c_R^{DN} = 0$; $c_D^{RN} = 0$, $b_k^{RN} = 0$ $\forall k \in \mathbf{X}$,
$c_R^{RN} = Y^R$; $z_D = \frac{\sigma_d^D Y^D}{q_D}$, $x_k^N = \frac{\sigma_k^D Y^D}{q_k}$ $\forall k \in \mathbf{X}$, $z_R = \frac{Y^R}{q_R}$

ii) $\gamma_{2(t-1)}^N < \gamma \leq \gamma_{2t-1}^N$, $t = 1, ..., K : J_t = t$

$$c_D^{DN} = \widetilde{\sigma}_{Dt}^D Y^D, \ b_{dt}^{DN} = \widetilde{\sigma}_{dt}^D \left(Y^D + \sum_{\delta \in E_t^{DN}} b_\delta^R \right) - b_d^R \ \forall d \in E_t^{DN}, \ b_{dt}^{DN} = 0$$

$\forall d \notin E_t^{DN}, c_R^{DN} = 0$; $c_D^{RN} = 0, b_{rt}^{RN} = 0$ $\forall r \notin E_t^{RN}, b_{rt}^{RN} = \widetilde{\sigma}_{rt}^R \left(Y^R + \sum_{\rho \in E_t^{RN}} b_\rho^D \right) -$

b_r^D $\forall r \in E_t^{RN}$, $c_R^{RN} = \widetilde{\sigma}_{Rt}^R Y^R$; $x_{kt}^N = \frac{\widetilde{\sigma}_{kt}^D \gamma_{2t-1}^N Y}{q_k}$ $\forall k < J_t$, $x_{J_t}^N = \frac{\widetilde{\sigma}_{J_t}^D \gamma_{2t-1}^N Y}{q_{J_t}} = \frac{\widetilde{\sigma}_{J_t}^R (1 - \gamma_{2(t-1)}^N) Y}{q_{J_t}}$, $x_{kt}^N = \frac{\widetilde{\sigma}_{kt}^R (1 - \gamma_{2(t-1)}^N) Y}{q_k} \forall k > J_t$. $\gamma_{2(t-1)}^N = \frac{\widetilde{\sigma}_{J_t}^R (1 - \widetilde{\sigma}_{J_t}^D)}{\widetilde{\sigma}_{J_t}^D + \widetilde{\sigma}_{J_t}^R (1 - \widetilde{\sigma}_{J_t}^D)}, \gamma_{2t-1}^N =$

$\frac{\widetilde{\sigma}^R_{J_t}}{\widetilde{\sigma}^D_{J_t}+\widetilde{\sigma}^R_{J_t}\left(1-\widetilde{\sigma}^D_{J_t}\right)}.$

iii) $\gamma^N_s < \gamma \le \gamma^N_{s+1},\ s = 1,...,2\left(K-1\right)-1 : \Delta_s = s,\ P_s = s+1$

$c^{DN}_D = \widetilde{\sigma}^D_{Ds}Y^D,\ b^{DN}_{ds} = \widetilde{\sigma}^D_{ds}Y^D - b^R_d\ \forall d \in E^{DN}_s,\ b^{DN}_{ds} = 0\ \forall d \notin E^{DN}_s, c^{DN}_R = 0;$ $c^{RN}_D = 0,\ b^{RN}_{rs} = 0\ \forall r \notin E^{RN}_s,\ b^{RN}_{rs} = \widetilde{\sigma}^R_{rs}Y^R - b^D_r\ \forall r \in E^{RN}_s,\ c^{RN}_R = \widetilde{\sigma}^R_{Rt}Y^R;$ $x^N_{ks} = \frac{\widetilde{\sigma}^D_{ks}Y^D}{q_k}\ \forall k \in E^{DN}_s,\ x^N_{ks} = \frac{\widetilde{\sigma}^R_{ks}Y^R}{q_k}\ \forall k \in E^{RN}_s.\ \gamma^N_s = \frac{\widetilde{\sigma}^R_{\Delta_s}}{\widetilde{\sigma}^D_{\Delta_s}+\widetilde{\sigma}^R_{\Delta_s}\left(1-\widetilde{\sigma}^D_{\Delta_s}\right)},\ \gamma^N_{s+1} = $ $\frac{\widetilde{\sigma}^R_{P_s}\left(1-\widetilde{\sigma}^D_{P_s}\right)}{\widetilde{\sigma}^D_{P_s}+\widetilde{\sigma}^R_{P_s}\left(1-\widetilde{\sigma}^D_{P_s}\right)}.$

Proof:

The only change from the proof for Proposition A1 is that the regions with corner solutions have one of the players only spending on its private good. Therefore, the lowest cut-off rate is defined by $\frac{\partial U^D}{\partial c_D} = \frac{\partial U^D}{\partial b_1}$ when $\mathbf{b}^D = \left\{Y^D,...,0,...,0\right\}$ and $\mathbf{b}^R = \mathbf{b}^{R*}$, whereas the highest is now defined by $\frac{\partial U^R}{\partial c_R} = \frac{\partial U^R}{\partial b_K}$ given $\mathbf{b}^D = \mathbf{b}^{D*}$ and $\mathbf{b}^R = \left\{0,...,0,...,Y^R\right\}$. QED.

Proposition B2:

In a sequential game game with K collective goods and one private good for each player D and R, there are $K+1$ regions separated by K cut-off rates defined in terms of γ. In the first region, only R provides the collective goods; in the last, only D funds them. In the other regions each collective good is funded by only one player in such a way that the expenditure ratios between any two goods provided by that player (including the private good) are first-best optimal from its perspective, and each cut-off rate marks the switch in the financing of a collective good from R to D.

Proof:

Corresponds to that of Proposition A2 with the changes noted in the proof of Proposition B1. QED.

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Table 1: Goods financed by each player in Nash-equilibrium, *K=3*

	Region 1	Region 2	Region 3	Region 4	Region 5
x_1	D,R	D	D	D	D
x_2	R	R	D,R	D	D
x_3	R	R	R	R	D,R

Figure 1a: Nash-equilibrium budgetary shares for x_1, *K=3*
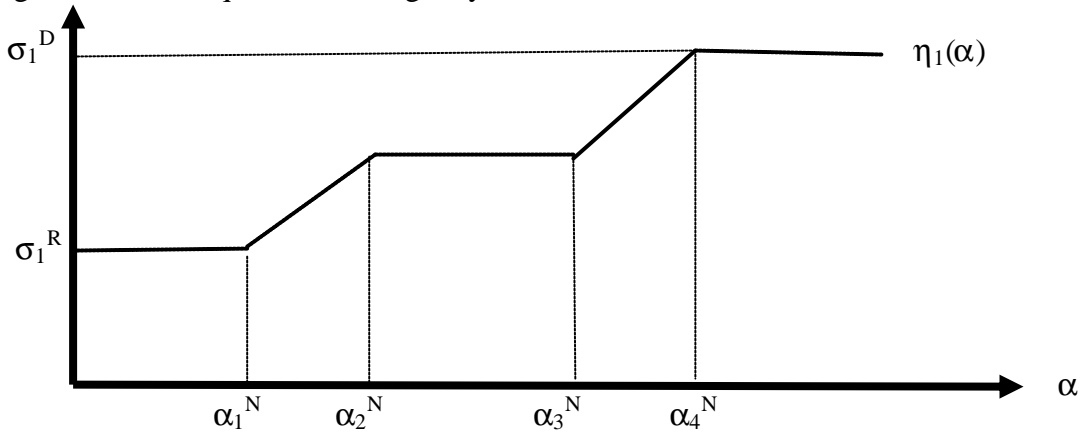
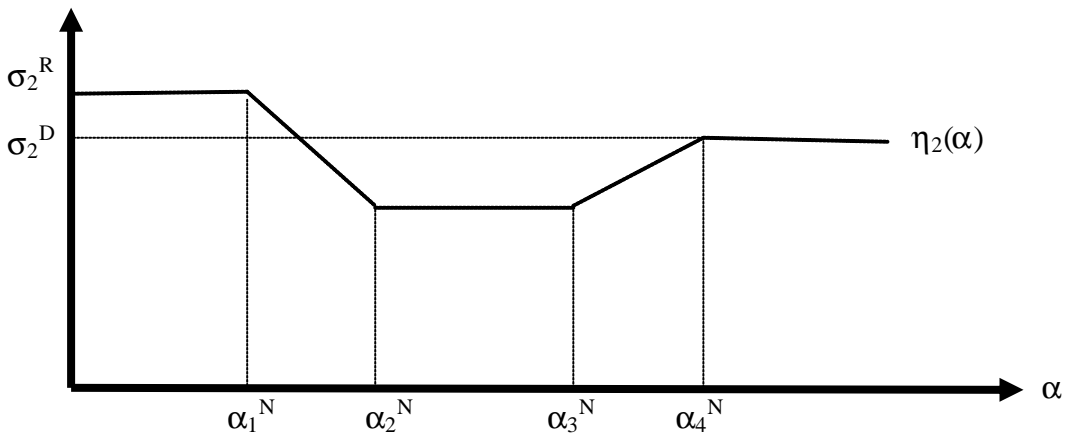Figure 1b: Nash-equilibrium budgetary shares for x_2, *K=3*

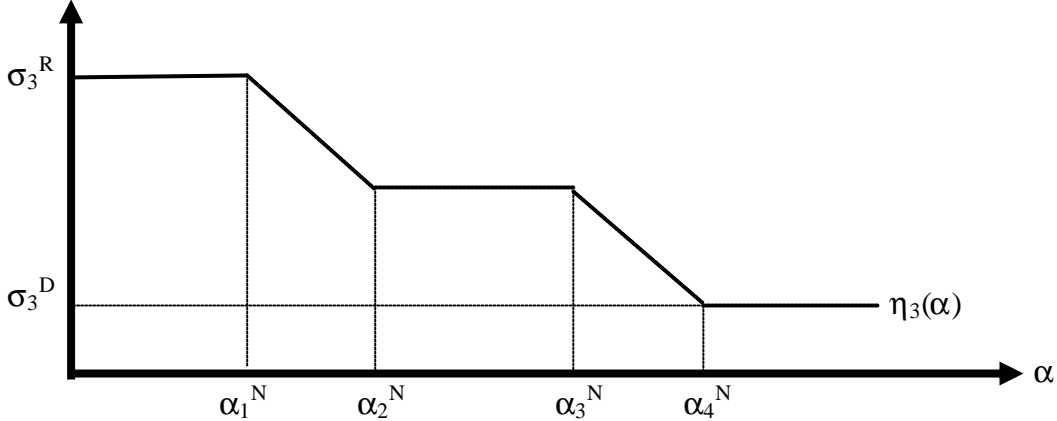Figure 1c: Nash-equilibrium budgetary shares for x_3, $K=3$
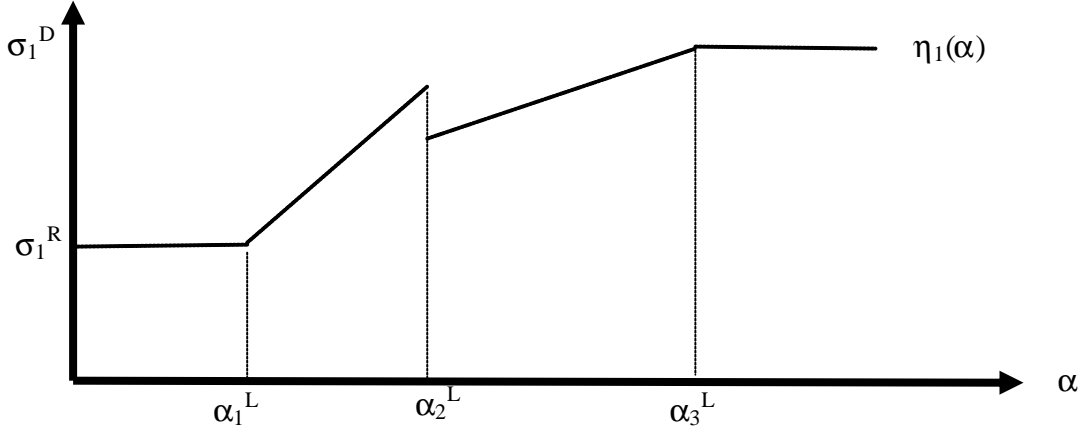

Figure 2a: Sub-game perfect equilibrium budgetary shares for x_1, $K=3$

Figure 2b: Sub-game perfect equilibrium budgetary shares for x_2, $K=3$
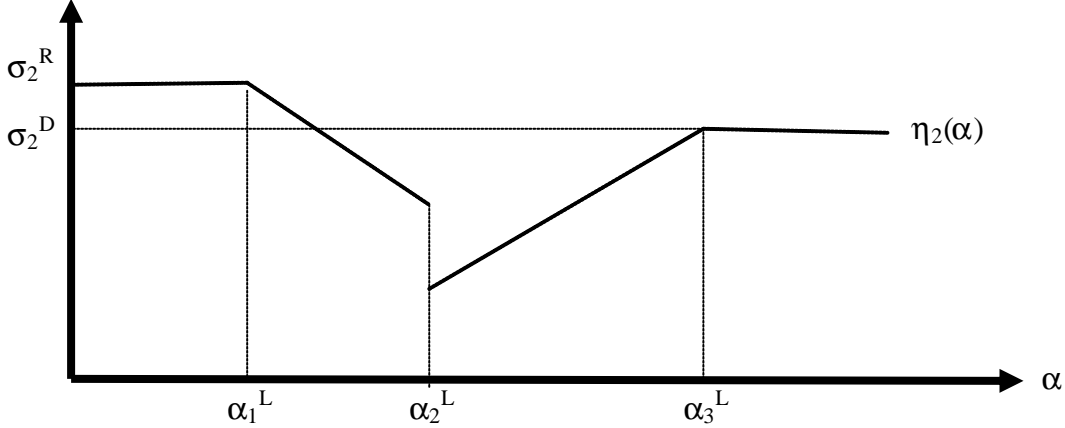
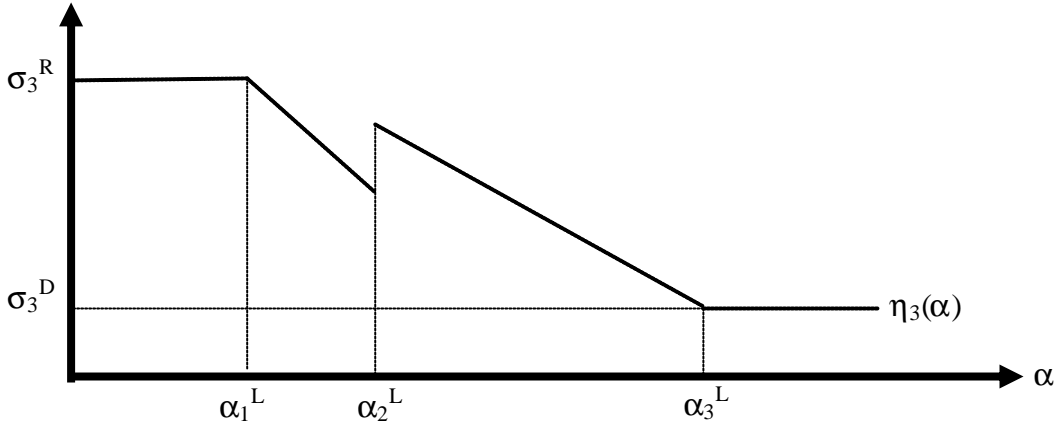Figure 2c: Sub-game perfect equilibrium budgetary shares for x_3, $K=3$

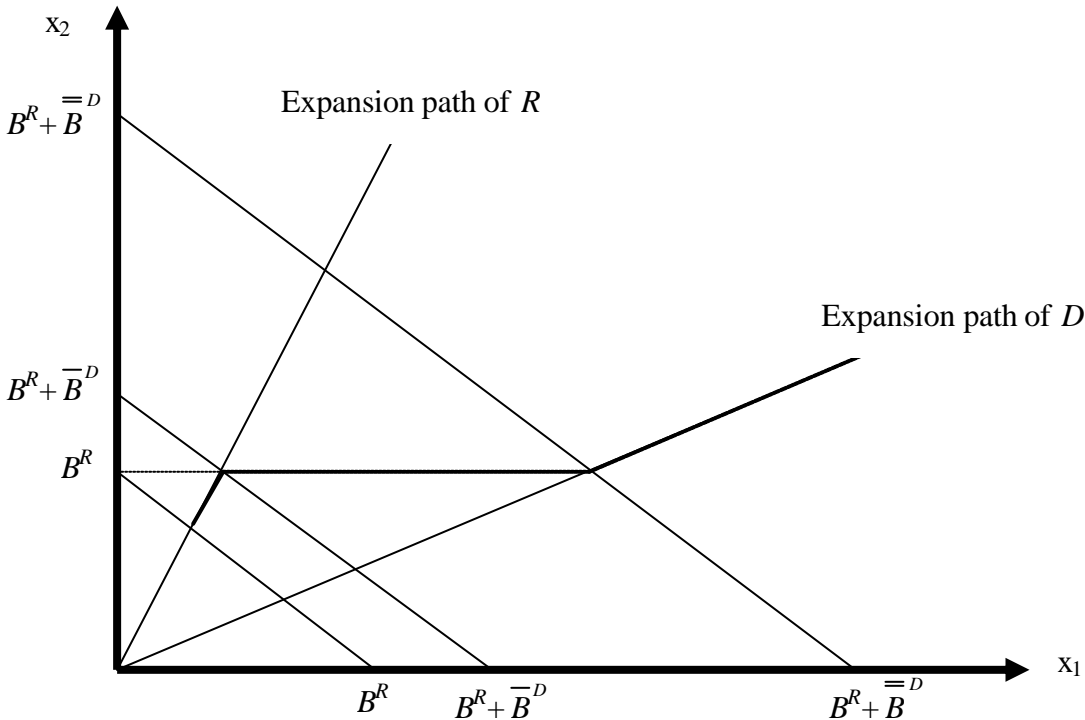Figure 3: Equilibrium outcomes as functions of B^D, $K=2$
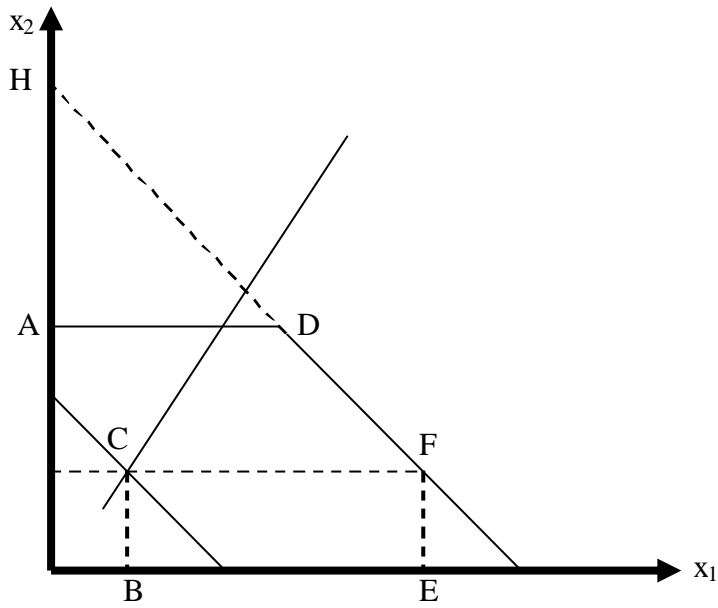
Figure 4: The non‑strategic approach to fungibility

x_2

H

A   D

C   F

B   E x_1

Table 2: Goods financed by each player in equilibrium when budgets are endogenous

$g/\mathbf{g}$		$(0, \mathbf{g}_0^N]$	$(\mathbf{g}_0^N, \mathbf{g}_1^N]$	$(\mathbf{g}_1^N, \mathbf{g}_2^N]$	$(\mathbf{g}_2^N, \mathbf{g}_3^N]$	$> \mathbf{g}_3^N$
$N:$	D	z_D	z_D, x_1	z_D, x_1	z_D, x_1, x_2	z_D, x_1, x_2
	R	x_1, x_2, z_R	x_1, x_2, z_R	x_2, z_R	x_2, z_R	z_R
L	D	z_D	z_D	z_D, x_1	z_D, x_1	z_D, x_1, x_2
	R	x_1, x_2, z_R	x_1, x_2, z_R	x_2, z_R	x_2, z_R	z_R
F	D	z_D	z_D, x_1	z_D, x_1	z_D, x_1, x_2	z_D, x_1, x_2
	R	x_1, x_2, z_R	x_2, z_R	x_2, z_R	z_R	z_R