Designing social security –

A portfolio choice approach[*]

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Abstract

Public social security systems may provide diversification of risks to individuals' life-time income. Capturing that a pay-as-you-go program (paygo) may be considered as a "quasi-asset", we study the optimal size of the social security program as well as the optimal split between a funded part and a paygo part by means of a theoretical portfolio choice approach. A low-yielding paygo system can benefit individuals if it contributes to hedge other risks to their lifetime resources. Moreover, a funded part of the social security system can be justified by potential imperfections to the individuals' free access to the stock market. Numerical calculations for Sweden, Norway, the US and the UK demonstrate that the optimal size of paygo-part of the pension program varies considerably in response to differences in projected growth rates and the correlation between stock returns and growth. Our calculations suggest that a paygo program has an important role in the three former countries – but not in the U.K.

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1. Introduction

Public social security systems may be justified by paternalism, preferences for redistribution of income and various types of market failures. This paper considers one of the presumably most important types of the latter justification, namely imperfect insurance markets. What we have in mind is the nonmarketability of human capital and also potential limitations in many individuals' access to the stock market. We analyze how such imperfections influence the optimal design of a social security system. Capturing that a pay-as-you-go (paygo) system may be interpreted as a new "quasi-asset" (Persson, 2000), we derive the optimal size of the paygo program as well as the optimal split between funded and unfunded systems by means of a portfolio choice approach.

The main bulk of the recent large literature on social security reforms takes as its point of departure that aging populations weaken the financial viability of social security systems, which mainly are financed on a paygo basis.[1] It is well known that the implicit return of the paygo system is given by the natural rate of economic growth, i.e. the joint effect of productivity growth and growth in the labor supply. Since this implicit return is lower than the real interest rate in a dynamically efficient economy, deterministic models predict that a funded program is always superior to a paygo program in steady-state. The policy challenge is, consequently, to derive a politically feasible and maybe even pareto-optimal transition from a paygo program to a funded program.[2]

The conclusion that the funded program is always superior to a paygo program in steady state is not valid, however, when we take into account that returns on both paygo and funded systems are stochastic. In a stochastic framework, a lower expected rate of return on the paygo system does not necessarily imply that it is an inferior alternative. From the basic theory of portfolio choice, we know that whether an asset should be included in an investor's portfolio depends on the covariance with the return on the rest of the portfolio. Thus, a low-yielding paygo system can benefit individuals if it contributes to hedge other risks to their lifetime resources.

This paper considers three sources of risk to net individual income: i) Technology shocks, which determine the wage rate, ii) fluctuations in the size of the population, which influence the aggregate labor supply, and iii) a stochastic return on stock market investments. Employing a simple theoretical overlapping generations model, we characterize the optimal social security system under various assumptions about individuals' participation in the stock

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[1] See for example Feldstein (1996), Kotlikoff (1996) and Fehr (2000).

[2] A pareto optimal transition from a paygo program to a funded program is only possible if the reform also lowers the excess burdens of the tax-transfer program, see Homburg (1990), Sinn (1999) and Miles (2000).

market. We also present some numerical evidence. The paper focuses exclusively on risk sharing issues in an overlapping generations framework with one representative individual within each generation. We disregard intragenerational redistribution and assume that labor supply is exogenous.[3]

It turns out that the design of optimal pension schemes depends crucially on the relevant risk concept. Referring to a two-period life cycle framework, we consider at the outset an individual who – in the first period of life – participates in the labor market and knows his wage income with certainty. In this case our analysis focuses on the social security system's impact on the sharing of net income risk in the second period of life. We will refer to this as *traditional risk sharing*. In addition to this concept we will also consider the case where no components of the representative individual's net income have been realized yet, i.e. the income risk in the first period of life has not yet been revealed. Following Ball and Mankiw (2001), this can be given a Rawlsian interpretation in the sense that we imagine that all generations are present behind a "veil of ignorance". Clearly, ignorance in this setting refers to uncertainty about whether a given individual is born into a lucky or unlucky generation. We will refer to this concept as *Rawlsian risk sharing*.[4]

This paper adds to the fairly small literature on the design of social security systems under uncertainty. The idea that a paygo system can be considered as an asset, has recently been explicitly highlighted by Persson (2000) and Dutta et al. (2000). Persson provides a brief and verbal discussion of this idea and presents a simple numerical illustration based on Swedish data, which indicates that the paygo system may indeed hedge parts of the risk on a portfolio of stocks and/or bonds. Persson does not offer any formal analysis, however. Dutta et al., on the other hand, do indeed present a formalized analysis based on a portfolio choice approach. Their analysis is based on a static mean-variance set-up, which does not capture several important aspects of public social security systems. For example, there is no explicit modeling of how the paygo system transfers resources between generations. Neither do they capture how the individual's private portfolio decisions are influenced by the public decisions on the design of the social security program. We also note that Dutta et al. do not consider different risk sharing concepts (i.e. their analysis focuses exclusively on what we have defined as traditional risk sharing). Finally, we argue below that mean-variance preferences

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[3] Many papers on social security reforms seem to assume that the tax-benefit link is very weak in a paygo system and fully actuarial (at least marginally) in a funded system. It is quite possible, however, to imagine a tax-benefit link which is close to actuarial in a paygo system and rather weak in a system which is funded in an aggregate sense, see Miles (2000) and Thøgersen (2001). Thus, even if we recognize that the labor supply responses to social security reforms are very important, we will argue that it may be benefical to separate the analysis of this issue from the risk sharing aspects of different social security systems, which are highlighted in the current paper.

[4] Our definitions of respectively traditional and Rawlsian risk sharing have similarities with the distinction between "true-" and "ex-ante risk sharing" made by Hassler and Lindbeck (1997).

combined with a proper dynamic overlapping generations setting imply that the derived optimal social security program is time-inconsistent.

The general insight that a mixed paygo and funded system may be optimal due to diversification dates back to Merton (1983). Merton studies various tax-transfer programs within a theoretical general equilibrium model, and his analysis captures the same stochastic variables as in our paper. He does not address the distinction between various risk concepts, which turns out to be important in the analysis of the current paper. Neither does Merton present any numerical calculations.

Two recent papers, which are fairly closely related to our analysis, are Miles (2000) and Shiller (1999). Using a numerical model, Miles considers the optimal split between funded and paygo programs. Contrary to our paper, he focuses on intragenerational redistribution of various individual risks and disregards risks to aggregate labor income and population growth. Shiller discusses inter- and intragenerational risk sharing as well as international risk sharing by means of social security and alternative institutions. He does not focus on the split between funded and paygo program, however.

Obviously, this paper is also related to studies of how a paygo program may contribute to intergenerational sharing of income-risk; see for example Gordon and Varian (1988), Enders and Lapan (1982, 1993), Thøgersen (1998) and Wagener (2001a, 2001b). With an exception of Wagener's contributions, these papers assume only one source of risk and they do not consider a split between funded and paygo programs. We will argue below that the main insight from these papers, namely that a paygo program leads to increased intergenerational income-risk sharing, hinges on specific stochastic properties of the income (or output) path over time. Wagener does capture both stochastic wage growth and stochastic interest rates. Analyzing different versions of paygo systems along similar lines as Thøgersen (1998), his focus is quite different from this paper, however.

The next section presents our theoretical model framework. Section 3 and section 4 study the optimal design of social security systems in the cases of respectively traditional and Rawlsian risk sharing as defined above. We derive the optimal size of the public social security program as well as the optimal split between the funded part of the program and the paygo part. Intuitively, a low or negative correlation between the stock market returns and the natural rate of growth increases the size of the optimal paygo program. Moreover, a funded program must be rationalized by imperfections in the individual's access to the stock market. We demonstrate that the case of traditional risk sharing implies a larger paygo program than in the case of Rawlsian risk sharing if the coefficient of relative risk aversion exceeds one. This reflects that the paygo system contributes to increased wage-income risk as long as the trend wage growth is stochastic – and the exposure to wage risk is higher at the outset in the Rawlsian case.

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Section 5 provides some numerical illustrations for Sweden, Norway, the US and the UK. Our calculations suggest a role for paygo-systems in the three former countries – but not in the UK. Taking limited stock market participation into account, a mixed paygo/funded system is optimal for Sweden, Norway and the US, while a fully funded system is optimal in the UK case. Finally, we offer some concluding remarks in section 6.

2. A simple overlapping generations model

Set-up

Our model framework combines a stylized overlapping generations set-up with the lognormal approach to portfolio choice problems, which recently has been developed by Campbell and Viceira (2001). There are two generations present in any period. The young generation participates in a competitive labor market, while the old generation is retired. We define X_{t+1} as the size of the young generation in period $t+1$ (i.e. generation $t+1$). Population growth from any period t to period $t+1$ is stochastic and given by N_{t+1}, and we have $X_{t+1} = (1 + N_{t+1})X_t$. The representative young individual in any period supplies inelastically one unit of labor and receives a gross wage, which is given by W_{t+1}. The wage growth is stochastic due, presumably, to productivity shocks. We define Λ_{t+1} as the wage growth rate, i.e. $W_{t+1} = (1 + \Lambda_{t+1})W_t$. It is also useful to define G_{t+1} as the growth rate of the aggregate wage income, $G_{t+1} = N_{t+1} + \Lambda_{t+1} + N_{t+1}\Lambda_{t+1}$.

In the same way as Gordon and Varian (1988) and Ball and Mankiw (2001), we assume for simplicity that members of each generation consume only in the second period of life. Consequently, the complete net labor income in the first period of life is saved. Savings are allocated between two types of financial assets. The first option is a risk-free asset with a real rate of return given by R^f. The second option is a risky alternative that yields a stochastic real return given by R_{t+1}. We refer to these assets as bonds and stocks, respectively.

The values of three exogenous stochastic variables are revealed in each period: N_{t+1}, Λ_{t+1} and R_{t+1}. Each of the variables is lognormally, independently and identically distributed over time. Because a product of lognormal random variables is lognormal as well, this implies that also G_t is lognormal. We define $r^f \equiv \log(1 + R^f)$, $r_{t+1} \equiv \log(1 + R_{t+1})$, $n_{t+1} \equiv \log(1 + N_{t+1})$, $l_{t+1} \equiv \log(1 + \Lambda_{t+1})$ and $g_{t+1} \equiv \log(1 + G_{t+1})$. It follows that $g_{t+1} \equiv n_{t+1} + l_{t+1}$. It turns out to be useful to write the expected difference between each of the stochastic variables and the risk free rate in the following way: $E[n_{t+1} - r^f] \equiv m^n$, $E[l_{t+1} - r^f] \equiv m^l$, $E[r_{t+1} - r^f] \equiv m^r$ and $E[g_{t+1} - r^f] \equiv m^g$. We assume that $m^r > m^k$ ($k = g, n, l$) and also note that the possibility of

$m^n > 0$ does not necessarily ruins dynamic efficiency in a stochastic economy, see Bertocchi and Kehagias (1995) and Blanchard and Weil (1991).[5] The variance of a given variable i and the covariance between two variables i and j are denoted by respectively s_i^2 and s_{ij}, $i,j = r$, n, l, g; $i \neq j$. Clearly, the assumed distributional properties imply that these expectations, variances and covariances are constant over time.

We assume that the expected utility of the representative individual in any generation t is given by

$$(1) \qquad U_t = E_t\left[d\, \frac{C_{t+1}^{1-g}}{1-g} \right],$$

where γ is the coefficient of relative risk aversion, which is constant across generations, d is the utility discount factor and C_{t+1} is the consumption level of the representative individual in generation t in period $t+1$. This utility function yields constant portfolio weights over time for the various assets. It turns out that this characteristic simplifies our analysis since it implies that the optimal policy variables are constant over time (see below).

Consumption is given by

$$(2) \qquad C_{t+1} = (1-t)W_t\left(w^p(1+R_{t+1}) + (1-w^p)(1+R^f) \right) + \Pi_{t+1},$$

where w^p is the fraction of private net income invested in stocks, t ($0 \leq t \leq 1$) is a social security contribution rate and Π_{t+1} is a social security benefit, which is determined by the social security formula described below. It is convenient to rewrite (2) as

$$(2') \qquad C_{t+1} = (1-t)W_t\left(1 + R^f + w^p(R_{t+1} - R^f) \right) + \Pi_{t+1}$$

The sole objective of the government is to run a social security program, which may be split between a funded part and a paygo part. The social security contribution in period t is given by tW_tX_t. A share b ($0 \leq b \leq 1$) of this amount is allocated to the funded program, and the remaining share is allocated to the paygo program. In turn, a share w^g of the amount allocated to the funded program is invested in stocks, while the remaining amount is allocated to riskfree bonds. Obviously, the government designs the social security system by means of three choice variables: t, b and w^g. Throughout the paper we assume that the individuals take these variables as given. It is straightforward to show that the social security benefit is given by

$$(3) \qquad \Pi_{t+1} = bt W_{t+1}\left(1 + R^f + w^g(R_{t+1} - R^f) \right) + (1-b)t W_{t+1}(1 + G_{t+1}),$$

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[5] As demonstrated by Bertocchi and Kehagias (1995) and Blanchard and Weil (1991), the conditions for dynamic efficiency in stochastic overlapping generations models are fairly complex. We assume throughout this paper that these conditions are satisfied. See Blanchard and Fisher (1989: 326-329) for an accessible discussion of the major issues involved.

where the first term on the RHS is the proceeds from the funded part of the program (i.e. generation t's own contributions adjusted for returns) and the second term reflects the paygo part (resources transferred from the young generation $t+1$ to the old generation t).

Using (2) and (3), we obtain

(4) $\qquad C_{t+1} = W_t(1+R^T_{t+1}), \quad R^T_{t+1} \equiv R^f + a(R_{t+1} - R^f) + b(G_{t+1} - R^f),$

where

(5a) $\qquad a \equiv \boldsymbol{w}^p(1-\boldsymbol{t}) + \boldsymbol{t}\boldsymbol{b}\boldsymbol{w}^g,$

(5b) $\qquad b \equiv \boldsymbol{t}(1-\boldsymbol{b}).$

We interpret R^T_{t+1} as the effective return on the representative individual's total portfolio when the social security system is taken into account. Moreover, we observe that a and b are the effective portfolio shares of gross income, which are invested in stocks and the "paygo-asset", respectively. The share invested in riskfree bonds is of course $1-a-b$. If the social security program is a pure paygo program (i.e. the special case of $\boldsymbol{b}=0$), $\boldsymbol{t}$ can be interpreted as the representative individual's forced portfolio share in the paygo-asset. In the following we generally assume that all portfolio shares are non-negative (i.e. there are no "short" positions) unless otherwise is explicitly stated.

The loglinear approximation

Even the simple portfolio problem in our model has no exact analytical solution when financial markets are not complete. Consequently, we resort to the recently developed loglinear approximation method of John Campbell and Luís Viceira (see Campbell and Viceira, 2001, and the references therein).

Because all underlying stochastic variables in our model are lognormal, portfolio returns are lognormal. Taking logs in equation (4) yields

(6) $\qquad c_{t+1} = w_t + r^T_{t+1},$

where $r^T_{t+1} \equiv \log(1+R^t_{t+1})$, $c_{t+1} \equiv \log C_t$ and $w_t \equiv \log W_t$. The next step is to relate the log portfolio return to the log returns on the individual assets, see (4). Following Campbell and Viceira (2001), a Taylor approximation of (4) yields

(7) $\qquad r^T_{t+1} - r^f = a(r_{t+1} - r^f) + b(g_{t+1} - r^f) + \frac{1}{2}\left(a\boldsymbol{s}^2_r + b\boldsymbol{s}^2_g\right) + \frac{1}{2}\left(a^2\boldsymbol{s}^2_r + b^2\boldsymbol{s}^2_g + 2ab\boldsymbol{s}_{rg}\right).$

Campbell and Viceira discuss the accuracy of this approximation in more detail. Based on work by Barberis (2000) they conclude that the quality of the approximation is good even for long-term portfolio problems provided that returns are i.i.d. This is crucial for our analysis because the portfolio problems related to the design of social security systems are obviously long term.

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3. Traditional risk sharing

Focusing on traditional risk sharing (as defined in the introduction) in this section, we must adopt the perspective of a representative individual whose wage in the first period of life has been realized. Because it turns out that the specified power utility function, see (1), yields constant optimal portfolio shares independent on the level of the realized wage, we derive the optimal social security system by the maximization of the utility of the representative young individual in a given generation t. The optimal system for this generation is also optimal for succeeding generations.[6]

At this stage it is useful for a moment to imagine an alternative mean-variance specification of preferences. Generally, this is not very attractive due to the well-known fact that it implies increasing absolute and relative risk aversion. In turn this implies that we obtain optimal portfolio weights in risky assets as declining functions of initial income. In the current setting this means that the optimal portfolio weights in stocks and the paygo-asset for generation t are not optimal for succeeding generations because wage-income fluctuates, i.e. the optimal social security system is time-inconsistent. As mentioned in the introduction, this problem will occur if we combine the mean-variance portfolio choice set-up of Dutta et al. (2000) with the dynamic overlapping generations framework of this paper. Thus, it seems necessary to consider a power utility function with constant relative risk aversion in this type of social security analyses based on a portfolio approach.

As noted by Campbell and Viceira (2001), maximizing of (1) is equivalent to maximizing the log of the expression in (1). Omitting the scale factor $\boldsymbol{d}/(1-\boldsymbol{g})$ and using that C_{t+1} is lognormally distributed, we write the objective function in the following way:[7]

$$(8) \qquad \log E_t C_{t+1}^{1-\boldsymbol{g}} = (1-\boldsymbol{g})E_t c_{t+1} + \frac{1}{2}(1-\boldsymbol{g})^2 \boldsymbol{s}_c^2,$$

where $c_{t+1} \equiv \log C_{t+1}$ and $\boldsymbol{s}_c^2$ is the variance of c_{t+1}. Dividing by $(1-\boldsymbol{g})$, using (6) and recalling that w_t is known and r^f constant, we may finally write the expected utility function of the representative individual in generation t as,

$$(9) \qquad E_t(u_t) = E_t\left(r_{t+1}^T - r^f\right) + \frac{1}{2}(1-\boldsymbol{g})\boldsymbol{s}_T^2$$

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[6] It follows that our analysis does not capture an optimal transition from a potential initial paygo system to a new and re-designed pension system. An explicit modelling of such a transition combined with the portfolio set-up in the present paper calls for additional research.

[7] We use the following general result for a lognormal stochastic variable Z:

$\log E_t Z_{t+1} = E_t z_{t+1} + \frac{1}{2}\boldsymbol{s}_z^2$, $z_{t+1} \equiv \log Z_{t+1}$, see Campbell and Viceira (2001), p. 20.

where s_T^2 is the variance of r_{t+1}^T.

As a point of departure, we find it useful to consider the benchmark case of no social security system at all, i.e. $t = 0$. It follows from (5a), (5b) and (7) that

$$(10) \qquad r_{t+1}^T - r^f = w^P(r_{t+1} - r^f) + \frac{1}{2}w^P(1 - w^P)s_r^2.$$

This immediately implies that $E_t\left(r_{t+1}^T - r^f\right) = w^P m^r + \frac{1}{2}w^P(1 - w^P)s_r^2$ and $s_T^2 = (w^P)^2 s_r^2$. Substituting these expressions into (9), we obtain a straightforward unconstrained optimization problem in the decision variable w^P. We obtain the solution

$$(11) \qquad w^P = \frac{m^r + \frac{1}{2}s_r^2}{gs_r^2}.$$

Intuitively, the optimal individual portfolio weight in stocks is increasing in m^r and decreasing in respectively γ and s_r^2. Moreover, we note that the numerator in (11) is equal to $\log E_t\left((1 + R_{t+1})/(1 + R^f)\right)$.[8]

Optimal social security when capital markets are perfect

Let us first assume that both the government and the representative individual have perfect access to the financial markets for bonds as well as stocks. There are no information asymmetries or transaction costs. Consequently, there is no need for the government to make financial investments on the behalf of the representative individual. We also note that the representative individual may offset any financial position he is exposed to due to the funded part of the social security system provided that "short" positions are allowed. It follows that the optimal social security system in this case may be implemented as a pure paygo system, i.e. we have $b = 0$ at the outset. It then follows from (5a), (5b) and (7) that

$$(12) \qquad \begin{aligned} r_{t+1}^T - r^f &= w^P(1-t)\left[(r_{t+1} - r^f) + \frac{1}{2}s_r^2\right] + t\left[(g_{t+1} - r^f) + \frac{1}{2}s_g^2\right] \\ &\quad - \frac{1}{2}\left[(w^P)^2(1-t)^2 s_r^2 + t^2 s_g^2 + 2w^P t(1-t)s_{rg}\right] \end{aligned}.$$

In turn this implies that

$$(13) \qquad \begin{aligned} E_t\left[r_{t+1}^T - r^f\right] &= w^P(1-t)\left[m^r + \frac{1}{2}s_r^2\right] + t\left[m^g + \frac{1}{2}s_g^2\right] \\ &\quad - \frac{1}{2}\left[(w^P)^2(1-t)^2 s_r^2 + t^2 s_g^2 + 2w^P t(1-t)s_{rg}\right] \end{aligned},$$

and

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[8] This follows from the general result given in footnote 7.

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(14) $\quad s_T^2 = (w^P)^2 (1-t)^2 s_r^2 + t^2 s_g^2 + 2w^P (1-t) t s_{rg}$.

Substituting (13) and (14) into the utility function (9), we derive the following optimal individual portfolio weight in stocks

(15) $\quad (w^P)^* = \dfrac{m^r + \dfrac{1}{2} s_r^2}{(1-t) g s_r^2} - \dfrac{t}{1-t} \dfrac{s_{rg}}{s_r^2}$,

where an asterisk is used in order to denote an optimal value in this "unconstrained case". In order to interpret (15) we assume for a moment that the covariance between the stock market returns and the implicit return on the paygo system is zero, $s_{rg} = 0$. Then we observe from the first term on the RHS of (15) that a larger paygo system (a higher t) increases the individual's portfolio weight in stocks. In the general case of $s_{rg} \neq 0$, this effect is accompanied by the effect of hedging demand, i.e. the last term on the RHS of (15). Because the paygo system introduces a non-tradeable risk caused by stochastic aggregate wage income growth, the individual uses the stock market to hedge this risk. Thus, $s_{rg} < 0$ contributes to a higher $(w^P)^*$, while $s_{rg} > 0$ contributes to a lower $(w^P)^*$. It follows that the paygo system increases $(w^P)^*$ if $s_{rg} < 0$ or if $s_{rg} > 0$ and its magnitude sufficiently small.

The government may derive the optimal size of the paygo system by the maximization of (9) subject to (13), (14) and (15). Solving this problem yields

(16) $\quad t^* = \dfrac{\left(m^g + \dfrac{1}{2} s_g^2 \right) - \left(m^r + \dfrac{1}{2} s_r^2 \right) \dfrac{s_{rg}}{s_r^2}}{g \left[s_g^2 (1 - r_{rg}^2) \right]}$,

where r_{rg} is the coefficient of correlation between stock market returns and returns on the paygo system. The term in the brackets in the denominator of (16) is the unhedgeable, or systematic, risk of the paygo system. As long as $|r_{rg}| < 1$, this term is positive. Looking at the numerator of (16), we first note that $m^g + \frac{1}{2} s_g^2 = \log E_t \left((1 + G_{t+1}) / (1 + R^f) \right)$. Thus, the optimal size of the paygo system is not surprisingly an increasing function of the expected excess return of the paygo system compared to the risk free return. The sign and magnitude of the second term on the RHS depends on s_{rg}. If $m^r + \frac{1}{2} s_r^2 > m^g + \frac{1}{2} s_g^2 > 0$, $s_{rg} < 0$ will

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contribute to a larger paygo system.[9] Moreover, the existence of a paygo system (i.e. $t > 0$) can still be justified if $s_{rg} > 0$ – but that hinges on a not too large magnitude of $s_{rg} \big/ s_r^2$.

Using (5a), (5b), (15) and (16), we may calculate the representative individual's effective portfolio shares of gross income. In the case of no funded part in the public social security system ($b = 0$) – due to perfect access to the capital market for everybody – the effective share in the paygo system is $b = t^*$, see (5b). The effective share invested in stocks, a, may increase or decrease in response to the paygo system. It follows from (5a) and (15) that $\dfrac{da}{dt} = -\dfrac{s_{rg}}{s_r^2}\dfrac{2}{1-t}$. Thus, $s_{rg} < 0$ implies that the paygo system increases a and leads to a lower portfolio share in the riskfree asset $(1 - a - b)$. The case of $s_{rg} > 0$ leads to a lower a, while the effect on the portfolio share in the riskfree asset is ambiguous.

Imperfect access to the stock market

The analysis above assumed that individuals have perfect access to the stock market. Is this really a realistic assumption? In reality most individuals in the OECD area still have only a tiny or zero part of their wealth allocated to the stock market. This is true even in the U.S. According to Poterba (2000), a majority of 80 per cent of U.S. households own only 4.1 per cent of total household stock market wealth including pension claims in 1998. Consequently, it is tempting to assume that most households have limited access to the stock market due to various formal as well as informal transaction costs and information problems.[10]

If we accept the view that the representative individual does not have perfect access to the stock market, it follows that there is indeed a scope for a funded part of the public social security program. In our model context we simply assume in this subsection that the representative individual does not participate in the stock market due to some type of costs or imperfections, i.e. we assume that $w^P = 0$ due to these types of exogenous reasons. In this case it follows that the optimal social security system should be designed in order to replicate the same effective portfolio weigths as derived in the unconstrained case above. We still assume that the government and the representative individual have similar access to risk free

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[9] The condition $m^r + \tfrac{1}{2}s_r^2 > m^g + \tfrac{1}{2}s_g^2 > 0$ is equivalent to

$\log E_t\big((1+R_{t+1})/(1+R^f)\big) > \log E_t\big((1+G_{t+1})/(1+R^f)\big) > 0$.

[10] In different contexts Abel (2001) assumes that households face fixed cost of participating in the stock market, while both Abel and Constantinides et al. (1998) assume that young individuals invest "too little" in the stock market due to credit rationing.

lending. Consequently, we consider only stock market investments in the funded part of the social security system, i.e. $0 < b \leq 1$ and $w^g = 1$.[11]

We observe from (5a) and (5b) that the case of $w^g = 1$ and $w^p = 0$ implies that the effective portfolio weigths are respectively $a = tb$ and $b = t(1 - b)$. Using that these weights were $a = (w^p)^*(1 - t^*)$ and $b = t^*$ in the unconstrained case, we easiliy derive the following values of t and w^g, which replicate the optimal effective portfolio weights:

(17a) $\quad t = t^* + (1 - t^*)(w^p)^*,$

(17b) $\quad b = \dfrac{(w^p)^*(1 - t^*)}{t^* + (w^p)^*(1 - t^*)}.$

In the case of no individual access to the stock market we may therefore calculate the optimal size of the social security system and the optimal split between the paygo part and the funded part (in stocks) directly from t^* and $(w^p)^*$, see (15) and (16). Intuitively, we see from (17a) that the increase in the social security contribution rate - compared to the unconstrained case - is exactly sufficient to restore the effective exposure to the stock market and at the same time maintain the size of the paygo system.

4. Rawlsian risk sharing

Turning to the Rawlsian risk sharing concept, we imagine that all the representative individuals of the different generations are present behind "a veil of ignorance" in the sense that they do not know which future generation they will be born into. Thus, at the time of enactment of the social security program, the representative individuals of any future generation do not know the wage income in the first period of life, nor the wage growth and stock market return that determine the net income in the second period of life.

Clearly, we may in principle consider all sorts of sophisticated risk sharing schemes between "all" generations in this Rawlsian case. In, for example, the model of Gordon and Varian (1988) it turns out that a risk sharing scheme, which distributes any income shock between all future generations, is optimal. Such a scheme is obviously hard to implement, however, because it requires a combination of very activistic debt policy and exact knowledge of the stochastic income variable's underlying trend value in each period. Consequently, we will restrict ourself to the design of a straightforward social security program along similar lines as in the previous section. This – in combination with the constant portfolio weights

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[11] An alternative but less interesting way to replicate the optimal effective portfolio weights is to set $t = 1$ and then allocate the resources in the same way as in the unconstrained case.

property of the power utility function – implies that we may derive the optimal Rawlsian program by the maximization of the period $t-1$ expected utility of the representative individual in a given generation t.

In the case of Rawlsian risk sharing we rewrite equation (4) as

(4') $\quad C_{t+1} = W_{t-1}(1+\Lambda_t)(1+R_{t+1}^T), \quad R_{t+1}^T \equiv R^f + a(R_{t+1}-R^f) + b(G_{t+1}-R^f)$,

(where W_{t-1} is known with certainty). In turn the loglinearized version of this equation, equation (6), can be rewritten

(6') $\quad c_{t+1} = w_{t-1} + \boldsymbol{l}_t + r_{t+1}^T$,

where r_{t+1}^T is still given by the Taylor approximation in equation (7). We assume for simplicity that population growth is deterministic and given by $N_{t+1} = N$ for all t. This implies that $g_{t+1} = n + \boldsymbol{l}_{t+1}$ where $n \equiv \log(1+N)$. It follows that $\boldsymbol{s}_g^2 = \boldsymbol{s}_l^2$ and $\boldsymbol{s}_{rg} = \boldsymbol{s}_{rl}$ in (7). We still assume as a benchmark case that the individuals have perfect access to the stock market, i.e. the optimal social security program can be implemented as a pure paygo program ($\boldsymbol{b} = 0$). Consequently, $a = \boldsymbol{w}^P(1-\boldsymbol{t})$ and $b = \boldsymbol{t}$ in (7).

Using that w_{t-1} is known at the time of maximization and r^f is constant, we may write the utility function (8) as

(18) $\quad E_{t-1}(u_t) = E_{t-1}\left\{\boldsymbol{l}_t + (r_{t+1}^T - r^f)\right\} + \dfrac{1}{2}(1-\boldsymbol{g})\boldsymbol{s}_c^2$,

where $\boldsymbol{s}_c^2$ is the variance of c_{t+1}. It follows from (7) that

(19)
$$\boldsymbol{l}_t + r_{t+1}^T - r^f = \boldsymbol{l}_t + \boldsymbol{w}^P(1-\boldsymbol{t})\left[(r_{t+1}-r^f) + \frac{1}{2}\boldsymbol{s}_r^2\right] + \boldsymbol{t}\left[(n + \boldsymbol{l}_{t+1} - r^f) + \frac{1}{2}\boldsymbol{s}_l^2\right]$$
$$- \frac{1}{2}\left[(\boldsymbol{w}^P)^2(1-\boldsymbol{t})^2\boldsymbol{s}_r^2 + \boldsymbol{t}^2\boldsymbol{s}_l^2 + 2\boldsymbol{w}^P\boldsymbol{t}(1-\boldsymbol{t})\boldsymbol{s}_{rl}\right].$$

Using the definition $E\left[\boldsymbol{l}_t - r^f\right] \equiv \boldsymbol{m}^l$ and noting that $\boldsymbol{m}^g = \boldsymbol{m}^l + n$, this implies that

(20)
$$E_t\left[\boldsymbol{l}_t + r_{t+1}^T - r^f\right] = (\boldsymbol{m}^l + r^f) + \boldsymbol{w}^P(1-\boldsymbol{t})\left[\boldsymbol{m}^r + \frac{1}{2}\boldsymbol{s}_r^2\right] + \boldsymbol{t}\left[\boldsymbol{m}^g + \frac{1}{2}\boldsymbol{s}_l^2\right]$$
$$- \frac{1}{2}\left[(\boldsymbol{w}^P)^2(1-\boldsymbol{t})^2\boldsymbol{s}_r^2 + \boldsymbol{t}^2\boldsymbol{s}_l^2 + 2\boldsymbol{w}^P\boldsymbol{t}(1-\boldsymbol{t})\boldsymbol{s}_{rl}\right],$$

and

(21) $\quad \boldsymbol{s}_c^2 = \boldsymbol{s}_l^2 + (\boldsymbol{w}^P)^2(1-\boldsymbol{t})^2\boldsymbol{s}_r^2 + \boldsymbol{t}^2\boldsymbol{s}_l^2 + 2\boldsymbol{w}^P(1-\boldsymbol{t})(1+\boldsymbol{t})\boldsymbol{s}_{rl}$.

We assume that the representative individuals still make their optimal portfolio decisions after the magnitude of their wage income has been realized. This implies that the optimal individual portfolio weight in stocks is still given by (15) when we recall that

$s_{rg} = s_{rl}$ in the current case. The government may therefore derive the optimal t by the maximization of (18) subject to (15), (20) and (21). This yields

$$(22) \qquad t^* = \frac{\left(m^g + \frac{1}{2}s_l^2\right) - \left(m^r + \frac{1}{2}s_r^2\right)\dfrac{s_{rl}}{s_r^2} + (1-g)\dfrac{(s_{rl})^2}{s_r^2}}{g\left[s_l^2(1-r_{rl}^2)\right]}.$$

Comparing (22) to (16), when $s_g^2 = s_l^2$, $s_{rg} = s_{rl}$ and $r_{rg} = r_{rl}$ in the latter case, we immediately observe that the difference between the optimal size of the paygo system in the case of Rawlsian risk sharing versus the case of traditional risk sharing is given only by the last term in the numerator in (22). In order to interpret this difference we first note that the case of Rawlsian risk sharing captures an additional source of risk facing a given generation $t+1$, namely l_t. We also note from the utility function (18) that a partial increase in s_c^2 actually increases utility. This may seem strange at first sight. We recall, however, that s_c^2 is the variance of log consumption and, using the mathematical result given in footnote 6, this implies that the utility function (18) can be rewritten as

$$(23) \qquad E_{t-1}(u_t) = \log E_{t-1}\left\{(1+\Lambda_t)(1+R_{t+1}^T)W_{t-1}\right\} - \frac{1}{2}gs_c^2 - r^f - w_{t-1}.$$

According to (23) the representative individual trades off the log of the expected arithmetic consumption level versus the variance of the log of consumption. It follows from (18) and (23) that an increase in s_c^2 leads to a higher value of $\log E_{t-1}\left\{(1+\Lambda_t)(1+R_{t+1}^T)W_{t-1}\right\}$ for a given value of $E_{t-1}(w_{t-1} + l_t + r_{t+1}^T)$ in the case of $g < 1$.

We are now able to give the intuition for the difference between Rawlsian and traditional risk sharing. Rawlsian risk sharing considers an additional source of risk, which implies that the ex-ante risk related to a given amount invested in the paygo-asset increases (compared to traditional risk sharing). In the case of $g > 1$, this leads to a smaller paygo program in the Rawlsian case. In the following empirical part of this paper we will indeeed argue that this is the most likely case. The interpretation is simply that the government on behalf of the representative individual offsets parts of this increased exposure to wage risk by means of a smaller investment in the paygo-asset. The opposite respons follows from $g < 1$. In this case a higher exposure to wage risk contributes to an increase in $\log E_{t-1}\left\{(1+\Lambda_t)(1+R_{t+1}^T)W_{t-1}\right\}$, which dominates the effects of a higher s_c^2, see (23). Clearly, $g = 1$ yields a boundary case characterized by similar portfolio weigths as in the case of traditional risk sharing for the the paygo-asset as well as for the other assets.

Looking at other papers on the risk sharing implications of paygo program, a common conclusion is that a paygo program contributes to increased intergenerational

income-risk sharing in the cases where the risk sharing concept is analogous to what we have called Rawlsian risk sharing, see Gordon and Varian (1988), Enders and Lapan (1982, 1993) and Thøgersen (1998). In our analysis the paygo program contributes to a higher exposure to wage income risk, however. The discrepancy is due to different assumptions regarding the stochastic properties of income (or output) growth our time. The cited series of papers all assume that the trend growth of income (output) is deterministic and even zero. In that case a paygo program leads to increased intergenerational risk sharing because each independent income shock is shared between the young and the old generation.[12] In the present paper we assume that the (trend) income *growth rate* is stochastic, however. Then it is impossible for the representative individual in a given generation t to avoid the full exposure to the income shock in period t by means of a paygo program. The reason is that this individual will receive a paygo pension benefit equal to $t(1 + \Lambda_t)W_{t-1}(1 + \Lambda_{t+1})(1 + N)$. On the other hand, if trend growth was deterministic (and the shocks still independently and identically distributed), the pension benefit would only be subject to the income shock in period $t+1$ and not to the income shock in period t.

Throughout our analysis of Rawlsian risk sharing we have assumed that the representative individuals in all generations have perfect access to the stock market. If the representative indivividuals do not participate in the stock market, the government may, however, maintain the optimal exposure to the stock market and the paygo-asset by adjusting t and introducing a funded part of the social security system. This can be done in exactly the same way as derived in the last part of section 3, i.e. the equations (17a) and (17b) are still valid – but the relevant t^* is of course given by (22) in the case of Rawlsian risk sharing.

5. Numerical illustrations

This section attempts to provide numerical estimates of the optimal size of the paygo system and the portfolio weights derived above for four countries: The US, the UK, Sweden and Norway. The variances and covariances used in the calculations are estimated from historical data series. These series are described in more detail in the data-appendix. Stock market returns are calculated from total value indices for each individual country. The implicit return

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[12] In order to see this imagine a representative individual in a given generation t, who works and receive a stochastic income w_t in the first period of life and is retired and receives only a pension benefit in the final second period of life. Let $w_t=w+e_t$ where w is the constant trend income and e_t is an i.i.d. stochastic shock. Let $\text{Var}(e_t)=\sigma^2$, let the real interest rate be zero and disregard population growth. Then it is straightforward to show that the variance of the net life income of this individual is equal to $t^2\sigma^2 + (1-t^2)\sigma^2$, where t is the contribution rate of a paygo system. Clearly, the existence of a paygo system ($0<t<1$) reduces this variance below unity due to intergenerational risk sharing.

of the paygo system, g_t, is calculated exactly (based on data for labor force growth and real wage growth) in the Norwegian case and approximated by using data for real GDP growth in the other countries. We also estimate the mean expected stock market returns from the historical data. Recogizing that future growth rates for g_t are expected to decline in all the countries in our sample due mainly to a stagnation in the labor supply, we use growth projections from recent government white papers in order to specify $E(g_t)$. As an estimate of the riskfree real interest rate, we use $r^f = 2$ per cent for all countries. This is in line with the real returns on short-term money market instruments reported by Campbell (2001, table 1) for several countries.[13] Finally, we assume that the coefficient of relative risk aversion is given by $\boldsymbol{g} = 5$ for all countries.

Table 1 reports the key statistics from our historical data, the projected mean growth rates and our assumptions for r^f and $\boldsymbol{g}$. We observe from Table 1 that equity returns have been high on average and volatile in all countries for our sample periods. It is also interesting to note that the contemporaneous annual correlation between equity returns and economic growth (g_t) is low for all countries, and even negative for Norway.

Traditional risk sharing

Adopting the traditional risk sharing concept, we first consider the benchmark case of no public pension system. Panel A in Table 2 gives the optimal individual allocation to stocks in this case (calculated from equation (11)). Riskfree savings range from about 60 per cent of the representative individual's portfolio in Sweden to about 85 per cent in Norway (which had a rather poor stock market performance in our sample period).

Panel B reports the values of the key variables when capital markets are perfect and the optimal pension system is a pure paygo system. Given the data and assumptions in Table 1, the attractiveness of a paygo program varies widely across the four countries. In order to explain the differences in the optimal contribution rates (i.e. the effective portfolio share in the paygo asset), we first look at Norway and the UK. In both these countries the expected excess return on the paygo asset is negative ($\boldsymbol{m}^g + \frac{1}{2}\boldsymbol{s}_g{}^2 < 0$). Hence, a necessary condition for the optimal contribution rate to be strictly positive is that $\boldsymbol{s}_{rg} < 0$, see equation (16). This is not fulfilled for the UK (table 1), and consequently we obtain $\boldsymbol{t}^* = 0$ in this case.[14]

In Norway the projected value of $\boldsymbol{m}^g$ $(= E(g) - r^f)$ is even lower than for the UK, but still the negative correlation between stock market returns and growth implies that the contribution rate should be positive (albeit low in this example). Thus, it is the hedging

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[13] As noted by Campbell (2001), the returns on short-term t-bills are not completely risk-free, but we ignore this complication here.

Table 1: Baseline values of key variables and parameters

	Norway	Sweden	UK	USA
Panel A: Values from historical data				
Period	1970-99	1920-98	1919-98	1891-1998
$\bar{r}$	5.07 %	7.07 %	7.41 %	6.93 %
s_g	3.57 %	6.28 %	3.50 %	5.77 %
s_r	34.85 %	18.65 %	21.69 %	18.67 %
r_{gr}	−0.298	0.055	0.087	0.112
Panel B: Projected and assumed values				
$E[g]$	1.69 %	2.20 %	1.88 %	2.96 %
r_f	2.00 %	2.00 %	2.00 %	2.00 %
g	5	5	5	5

Note: $\bar{r}$ denotes the historical mean stock market return.

Table 2: Private and public allocation rules under traditional risk sharing

	Norway	Sweden	UK	USA
Panel A: Optimal allocation to stocks, no pension system ($t = 0$)				
$a = w^p$	15.06 %	39.16 %	32.99 %	38.28 %
Panel B: Allocation with pure paygo ($b = 0$), perfect capital markets				
t	4.94 %	13.56 %	0	54.32 %
w^p	16.00 %	45.01 %	32.99 %	79.68 %
A	15.21 %	38.91 %	32.99 %	36.40 %
$b = t$	4.94 %	13.56 %	0	54.32 %
Panel C: Allocation with non-participation in stock-markets ($w^p = 0$)				
t	20.16 %	52.46 %	32.99 %	90.71 %
b	75.48 %	74.15 %	100 %	40.12 %
a	15.21 %	38.91 %	32.99 %	36.40 %
b	4.94 %	13.56 %	0	54.32 %

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[14] The optimal t is actually negative for the UK, but we rule out negative contribution rates per definition.

properties of the paygo asset that is attractive in the Norwegian case. For Sweden and the US, on the other hand, the combination of comparatively high growth rates (higher than r^f) and low values of the correlation r_{rg} imply positive contribution rates. In the US case, the projected growth rate is so much higher than r^f that the portfolio weight in the paygo-asset should be larger than 50 per cent.

Comparing the second row in panel B to panel A, we see that private individual risk-taking increases for the countries with positive contribution rates. For Sweden and the US, this occurs despite a negative hedging demand ($r_{rg} > 0$). However, the total effective portfolio weight in stocks (given by $a = w(1-t)$ in this case, see eq. (5a)) decreases due to this positive correlation. For Norway we have $r_{rg} < 0$. This induces positive hedging demand, which increases total effective risk taking in the stock market. In all countries except the UK, the total portfolio weight in risky assets ($a + b$) is much higher than in the benchmark case of no public pension program.

In panel C of Table 2 we report the key variables in the case of no individual participation in the stock market ($w^p = 0$). In Norway, the rather modest stock market performance implies that the optimal contribution rate should be increased with approximately 15 percentage points. In the other countries, the contribution rate increases with approximately 35 percentage points compared to the case with perfect capital markets. Moreover, the optimal UK system will now be fully funded (with all contributions invested in stocks) and characterized by a contribution rate equal to the portfolio weight in stocks in the case of perfect capital markets. In the other three countries the pension system is partially funded. It follows that the effective allocation of gross income to the different assets replicates that in panel B.

A possible objection to our analysis is that the various variances and correlations in Table 1 are calculated from annual data, while pension saving typically has much longer investment horizons. In particular, one may be concerned that the correlation between stock returns and aggregate wage growth may be substantially higher for longer horizons, and that the risk sharing properties embedded in a paygo system thus weakens (Jermann, 1999). Figure 1 plots the correlation coefficient between GDP growth and stock returns (based on overlapping growth rates) for Sweden, the UK and the US, varying the horizon between 1 and 30 years (Norway is excluded due to the short data series for Norwegian stock returns). No general pattern emerges from this Figure. While the US correlation between growth and stock returns clearly increases for horizons longer than 20 years (as Jermann, 1999, demonstrates), the UK data show a low and decreasing correlation for long horizons, and the Swedish correlation is remarkably stable. Moreover, the correlation reaches its maximum for short horizons (2-3 years) for both Sweden and the UK. Consequently, our data do not in

general support the claim that a long investment horizon diminishes the importance of paygo systems in retirement savings.

Table 3: Ralwsian vs. traditional risk sharing

	Norway	Sweden	UK	USA
Panel A: Ralwsian risk sharing, perfect capital markets				
t	0	13.32 %	0	53.30 %
w^p	15.06 %	48.56 %	32.99 %	78.02 %
a	15.06 %	42.10 %	32.99 %	36.43 %
$t(\text{trad}) - t(\text{R})$	4.94 %	0.24 %	0	1.01 %
Panel B: Ralwsian risk sharing, non-participation in stocks markets				
t	15.06 %	55.41 %	32.99 %	89.74 %
b	100 %	75.97 %	100 %	40.60 %

Figure 1: Correlation between GDP growth and stock returns over different horizons.
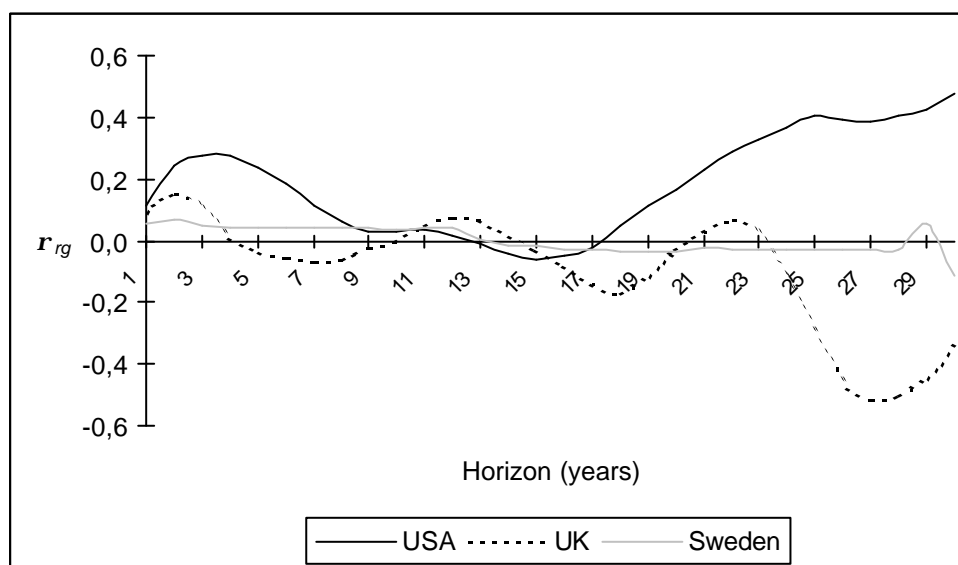
Rawlsian risk sharing

Turning to the Ralwsian risk sharing concept, we recall that we derived the optimal contribution rate, given in (22), under the assumption of deterministic population growth. To provide a meaningful comparison between the contribution-rates under traditional and Rawlsian risk sharing we now assume that all fluctuations in g_t is due to fluctuations in productivity (and real wage) growth. This is, of course, a very crude approximation, which

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nevertheless allows us to get a quantitative impression of the significance of the Rawlsian risk sharing concept. Referring to equation (22), we assume that $s_l = s_g$ and $r_{rl} = r_{rg}$.

Panel B of Table 3 reports the key variables under Ralwsian risk sharing. As we know from section 4, the optimal contribution rates are lower under this risk sharing concept because we have $g > 1$, see (22). The optimal contribution rate is therefore still zero for the UK. For Sweden and the US, the difference in contribution rates under the two risk sharing concepts turns out to be very small. The reason can easily be seen from the last term in the nominator of equation (22). We observe that the difference in contribution rates between the two risk sharing regimes is lower the closer r_{rl} is to 0. As we saw in Table 1 (recalling that $r_{rl} = r_{rg}$), the absolute value of this correlation coefficient is indeed low in the annual data for Sweden and the US. This implies that the term s_{rl}^2 in (22) is very small. Even if we imagine for a moment that $g = 30$, the difference between the contribution rates under the two regimes is just 1.23 percentage-points in the US case. For Norway, the absolute value of r_{gr} is approximately 0.3, and this is sufficient to create a substantial difference between the contribution rates under the two risk sharing concepts. In fact, the optimal contribution rate falls to 0 when the concept is changed from traditional to Rawlsian in the Norwegian case.

Finally, panel B of Table 3 demonstrates that the effect of non-participation in stocks markets is analogous to what we saw under traditional risk sharing (compare to the difference between the panels B and C of Table 2).

6. Final remarks

During the recent years a large part of the literature on social security systems has dealt with comparisons between funded and paygo program as well as the design of potential transitions from paygo financing to funded systems. Adopting a portfolio choice approach, this paper has provided a different perspective on the design of social security systems. Interpreting the paygo system as a "quasi-asset" along the lines of Persson (2000), the analysis has focused on the optimal size of the paygo system and the optimal split between the paygo part of pension savings and the funded part. Clearly, the funded part of the pension savings can – from a representative individual's point of view – be handled individually if access to the stock market is perfect, or by the government if this access is imperfect.

The general insight from our analysis is that a low-yielding paygo system can benefit the representative individuals if the correlation between the implicit return on the paygo program and the stock market returns is low or negative. We have derived analytical formulas for the optimal size of the paygo system and the optimal magnitude of the funded pension

saving in the stock market. The optimal size of the paygo program depends on the risk concept. It turns out that the optimal paygo system is smaller under "Rawlsian risk sharing" (which captures all risks to the net lifetime income of the representative individual at birth) than under "traditional risk sharing" (which capture only risks which are realized in the representative individual's second period of life). The reason is that the paygo system increases the exposure to wage income risk.

We provide numerical illustrations for the USA, the UK, Sweden and Norway. It turns out that the paygo-asset should play a role in pension savings in all of these countries except in the UK. Not surprisingly, the size of the paygo system is rather sensitive to the estimated mean implicit return on the paygo system. The Norwegian case illustrates, however, that a negative correlation between equity returns and growth may justify a paygo program even if the mean expected return on the paygo asset is very low. With limited stock market participation, our calculations suggest that a mixed paygo/ funded system is optimal (except in the UK case). Finally, we note that the Rawlsian risk sharing concept reduces the scope for paygo programs marginally in the US and Sweden, and more significantly in Norway.

Looking at the social security reform agenda in many OECD countries, we note that attempts are made to introduce funded parts in systems, which have been entirely paygo financed at the outset. Assuming that the representative individual does not have perfect access to the stock market, our analysis clearly suggests that such developments would improve the risk sharing properties of the social security program.

Data appendix

Stock market data
Norway: The Morgan Stanley Capital International Gross Return Index in local currency. Year-end quotes from 1969 to 1999. Available at www.msci.com.
Sweden, UK and US: The annual data from Campbell (1999) updated through 1998. They are described in detail in the appendix to Campbell (1999).

GDP / calculation of g_t
Norway: Data on labor force growth are collected from OECD Labor Force Statistics, while data on wage growth are collected from the Hourly Earnings Index in IMF's International Financial Statistics, 2000.
Sweden, UK and US: Real GDP data are collected Maddison (1991) for the period up to and including 1989 and then from IFS, 2000 for 1990-1998.

Growth projections

<u>Norway:</u> The latest Government Long Term Program (Government White Paper "Stortingsmelding 30, 2001/2002") assumes mean annual GDP growth rates equal to 1.7 per cent over the 1999-2010 period. Thus, $E[g] = \ln(1.017) = 1.69$ per cent.

<u>Sweden:</u> The latest Government Long Term Program ("Government White Paper SOU 2000:7") assumes mean annual GDP growth rates equal to 2.5 per cent over the 1999-2010 period and 1.8 per cent for the period 2005-2008. We use the average of these numbers (2.22 per cent), which implies $E[g] = \ln(1.0222) = 2.20$ per cent.

<u>UK:</u> Long run projections in the "Economic and Financial Strategy Report" of the Government Budget for 2001 assume annual growth rates amounting to 2.25 per cent for 2000-2010 and 1.75 per cent for 2011-2030. We use $E[g] = \ln(1.019) = 1.88$ per cent.

<u>USA:</u> The "2001 – Annual Report of the Council of Economic Advisers" assumes annual growth rates equal to 3.1 per cent over the 2000-2008 period and 2.9 per cent from 2009 and onwards. We use an average of these projections; $E[g] = \ln(1.03) = 2.96$ per cent.

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