# Who are the least advantaged?

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January 19, 2005

Discussion Paper 34/2004

Abstract: The difference principle, introduced by Rawls (1971, 1993), is generally interpreted as leximin, but this is not how he intended it. Rawls explicitly states that the difference principle requires that *aggregate benefits* (e.g., average or total) to those in the least advantaged group be given lexical priority over benefits to others, where the least advantaged group includes more than the strictly worst off individuals. We study the implications of adopting different approaches to the definition of the least advantaged group and show that, if acyclicity is required, several seemingly plausible approaches lead to something close to leximin. We then show that significant aggregation is possible, if the least advantaged group is defined as those with those with less benefits than some strictly positive transform of the lowest level of benefits. Finally, we discuss the implications of requiring that, in comparing two alternatives, the cutoff for the least advantaged group of one alternative be the same as that for the other alternative.

## 1. Introduction

The difference principle, introduced by Rawls (1971, 1993), states that the basic institutions of society should promote the social and economic interests of the least advantaged members of society. Most writers have contented themselves with the leximin version of the difference principle, saying that we should assign priority to the interests of the worst off *person* in society (and the second worst off if the worst off is indifferent, and so on).[1] But this is clearly not what Rawls (1971) has in mind. He argues that the persons in the original position should interpret the difference principle as a "*limited aggregative principle* and assess it as such in comparison with other standards. It is not as if they agreed to think of the least advantaged as literally the worst off individual…" (p. 98, our emphasis). In other words, the difference principle accepts a certain trade-off between gains and losses of people that belong to the least advantaged group in society, but assigns absolute priority to the least advantaged in any distribute conflict with the better off members of society.

Rawls (1971) admits that "[t]he serious difficulty [with the difference principle] is how to define the least fortunate group" (p. 98). He sketches some possibilities, for example counting the least advantaged as all those with an income less than half of the median income and wealth or less than the average income and wealth of the unskilled worker, but more generally he suggests that a certain arbitrariness is unavoidable in defining the least advantaged group and thus that the exact formulation of the difference principle has to be done on an ad hoc basis. "In any case we are to aggregate to some degree over the expectations of the worst off, and the figure selected on which to base these computations is to a certain extent ad hoc. Yet we are at some point entitled to plead practical considerations in formulating the difference principle. Sooner or later the capacity of philosophical or other arguments to make finer discriminations is bound to run out" (p. 98).

Even though it is easy to accept that practical constraints will limit the formulation of the difference principle in any particular application, it is hard to accept that the outline of the ideal version of the theory does not need a principled definition of the least advantaged group. The ideal version should guide practical applications of the theory, and thus it is essential to have a clear understanding of the appropriate foundation of the definition of the least advantaged group. Given any particular conception of benefits, should we relate the definition of the least advantaged group to the median benefits, the average benefits, the benefits of the best off person, an independent norm, or some other notion?

The aim of this paper is to study the implications of adopting different approaches to the definition of the least advantaged group in society.[2] In particular, we will study what definitions lead to a version of the difference principle that is significantly different from the leximin principle. We start by showing that certain seemingly plausible conditions on the definition of the least advantaged group leave little room for the difference principle to depart significantly from the leximin principle. We then show that the difference principle can take an *aggregative* form (as intended by Rawls) if the least advantaged group is defined as those having fewer benefits than some strictly positive transform of the lowest benefit level. Finally, we discuss the implications of requiring that, in comparing two alternatives, the cutoff for the least advantaged group of one alternative be the same as that for the other alternative.

## 2. The General Framework

To fully specify an egalitarian theory, one must specify the type of benefits that it seeks to equalize. As is well-known, Rawls defends a focus on some notion of primary goods. Throughout the paper, however, we will leave open the relevant conception of benefit (resources, primary goods, brute luck well-being, etc.) References to a person being worse off than another should be understood in terms of the relevant benefits.

We shall assume, for the sake of argument, that benefits are fully measurable and interpersonally comparable (i.e., there is an natural zero and unit for benefits and these are fully interpersonally comparable). This may seem like a strong assumption, but in the present context it is a very weak assumption. The assumption that benefits are so measurable and comparable does not entail that such information is *relevant* for the moral assessment of options. The assumption is simply that such information is available. This ensures that no definition of the least advantaged group, and hence of the difference principle, is ruled out merely on the grounds that it presupposes that benefits are measurable or comparable in ways that are not possible.

We shall be concerned with the assessment of the justice of alternatives, where alternatives are possible objects of choice (e.g., actions or social policies). Alternatives may have all kinds of features: they generate a certain distribution of benefits, satisfy or violate various rights, involve various intentions, and so on. In what follows, we shall assume that the only relevant information for the assessment of justice is the benefit distribution that an alternative generates. More formally, we shall assume:

**Benefitism:** Alternatives can be identified with (and thus their justice assessed solely on the basis of) their benefit distributions.

Benefitism is a generalization of welfarism (the view that justice supervenes on individual welfare). It holds that justice supervenes on individual benefits whatever they are. If two alternatives generate the same distribution of benefits, then they have the same status with respect to justice. Given Benefitism, we can identify an alternative with the benefit distribution that it generates, and in what follows we shall do so for simplicity. The Difference Principle satisfies Benefitism, since it assesses alternatives solely on the basis of their benefit (e.g., primary good) distributions.

Finally, we assume that the set of distributions generated by the set of possible alternatives is *rich* in the following sense:

**Domain Richness**: For any logically possible benefit distribution X, there is an alternative that generates that distribution.

This condition rules out, for example, the possibility that, where there are just three people, the distribution <3,7,9> (3 to the first person, 7 to the second, 9 to the third) is not one of the alternatives. All logically possible benefit distributions are among the alternatives. This is not to say that all are part of any given *feasible* set (the alternatives that are open to an agent on a given occasion). Of course, there are lots of logically possible benefit distributions that are not feasible on a given occasion. The claim here is about the range of benefit distributions that can be assessed by justice. The condition holds that such judgements can be made for all logically possible distributions. We believe that this is a highly plausible condition. Benefit distributions here play the role of test cases for the theory of justice. All logically possible test cases—assuming, as we shall, a finite population—are admissible. Benefitism and Domain Richness will be assumed throughout the paper, and thus we will not state these conditions explicitly when reporting the results.

We will be concerned with the *justice relation* of (one alternative) being-at-least-as-just-as (another). Following the standard definitions, (1) an alternative is *more just* than another if and only if it is at least as just and the other is not at least as just as it; and (2) an alternative is *equally as just* as another if and only if it is at least as just and that other is also at least as just as it. We shall assume that the justice relation satisfies the following consistency condition:

**Acyclicity**: If, for alternatives $X_1,\ldots X_n$, $X_1$ is more just than $X_2$, $X_2$ is more just than $X_3$, …. and $X_{n-1}$ is more just than $X_n$, then $X_n$ is not more just than $X_1$.

Acyclicity is much weaker than the better known requirement of transitivity. It applies only to chains of where each alternative is more just than its successor (as opposed to being at least as just) and allows the possibilities of silence (no ranking) and of the first alternative being judged equally just as the last (whereas transitivity requires that it be judged more just in such cases). It is about as close to being uncontroversial as one can get when it comes to consistency requirements on the justice relation.

For some of the results, we will strengthen this condition slightly in two ways. One is to invoke:

**Strong Acyclicity**: If, for alternatives $X_1$,….$X_n$, $X_1$ is at least as just as $X_2$, $X_2$ is at least as just as $X_3$, …. and $X_{n-1}$ is at least as just as $X_n$, then (1) $X_n$ is not more just than $X_1$, and (2) if for some i inclusively between 1 and n-1, $X_i$ is more just than $X_{i+1}$, then $X_n$ is not at least as just as $X_1$.

This strengthens Acyclicity by covering chains where each alternative is at least as just (as opposed to more just) as its successor. If each of the pairs are equally just, then it concludes that the last alternative is *not more just* than the first (as opposed to the silence of Acyclicity). Moreover, if all the relations in this chain are the relations of being more just, then Strong Acyclicity strengthens Acyclicity by requiring that the last alternative *not be at least as just* as the first (as opposed to Acyclicity's requirement that it not be more just). Like Acyclicity, Strong Acyclicity is accepted by almost everyone.

We will also consider the implications of the further strengthening of Acyclicity:

**Transitivity**: If, for alternatives $X_1$, $X_2$, and $X_3$, $X_1$ is at least as just as $X_2$, $X_2$ is at least as just as $X_3$, then $X_1$ is at least as just as $X_3$.

Transitivity is much more controversial than Strong Acyclicity. Indeed, one of us would reject it on the ground that sometimes silence is appropriate concerning the ranking of the first and last alternatives. Fortunately, our core results do not depend on this assumption. We shall merely note how they can be strengthened if one assumes Transitivity as opposed to Strong Acyclicity.

For the record, we state one final condition:

**Completeness:** For any two alternatives, X and Y, either X is at least as just as Y or Y is at least as just as X.

Completeness rules out the possibility of silence in the comparison of the relative justice of any two alternatives. For most of our results, we do not assume Completeness. It is only for a final possibility result that we will invoke it.

## 3. The Least Advantaged Group and the Difference Principle

In order to state precisely the difference principle, we need a definition of the least advantaged group. Formally, we designate, for each alternative X, the least advantaged group as L(X). We assume that this group is defined by a *cutoff function,* c, for which c(X) is the benefit level such that L(X) consists of all and only those individuals with benefits strictly *less* than c(X).[3]

More formally, we have:

**The Least Advantaged Group, L(X):** For any alternative X, an individual i belongs to the least advantaged group, L(X), if and only if i's benefits are strictly less than c(X).

In order to focus our discussion, we need to impose some structure on the cutoff functions. The following condition should be entirely uncontroversial:

**Monotonicity:** For any two alternatives X and Y, if each individual has at least as much benefits in X as in Y, then c(X) ≥ c(Y).

The cutoffs may be either absolute (not dependent on features of the distribution of benefits to which it is applied) or relative (e.g., based on the mean, median, or percentile distribution). If they are absolute, the cutoffs will be the same for all alternatives, and thus Monotonicity will be satisfied. If the cutoffs are relative (e.g., 50% of the average benefits), then they may be different for different alternatives. Where, however, each individual has at least as much benefits in X as in Y, then the cutoffs for X should not be lower than for Y. It would be crazy, for example, for an income cutoff for the least advantaged to be $20,000 in a poor country and $200 in a rich country. If they change, they will be higher for the alternative in which the benefits are higher. Monotonicity is thus highly plausible, and we shall assume throughout.

We shall show that, if the cutoff functions satisfy Monotonicity and certain other conditions, then the difference principle cannot depart significantly from leximin. To start, consider:

**The Full Difference Principle:** For any two alternatives X and Y, if the least advantaged group is strictly better off in X than Y, X is more just than Y.

This principle, as intended by Rawls, requires some way of aggregating benefits in the least advantaged group (so that it can require that such aggregate benefits be maximized). Because our project is to determine what kinds of aggregation are possible, given certain plausible assumptions, we do not want to presuppose any particular method of aggregation. Our point of departure will therefore be the following weaker principle:

**Core Difference Principle:** For any two alternatives X and Y, if (1) each individual who is a member of at least one of L(X) and L(Y) has at least as much benefits in X as in Y, and (2) at least one such member has strictly more benefits in X than Y, then X is more just than Y.

This is entailed by the full difference principle. It says that benefits to those in the least advantaged group (of either X or Y) take absolute priority over benefits to those not in that group. It is silent, however, about how benefits among individuals in the least advantaged group are to be traded off. Our task is to examine what kinds of tradeoffs are permitted by various plausible background conditions. We shall show that, given certain additional conditions to be identified below, Core Difference Principle cannot significantly diverge from leximin. More exactly, we will show that it must agree with the following principle:

**Minimal Maximin**: For any two alternatives X and Y, if there are more benefits in the worst off position in X than Y, then Y is not more just than X.

This is weaker than leximin in several ways. First, it is silent where the benefits for the worst off position are the same for two alternatives (e.g., <2,4,9> vs. <2,3,3>), whereas leximin makes judgements even in such cases (viz., that <2,4,9> is more just). Second, where the benefits for the worst off position in X are greater than those in Y (e.g., <2,3,5> vs. <1,4,6>), Minimal Maximin allows X to be judged *equally just*, and also allows silence about the ranking, whereas leximin requires X to be *more* just.

Even though Minimal Maximin is weaker than leximin, it nevertheless captures the core of the leximin principle. Like leximin, it gives absolute priority to the benefits of the worst off individuals in that it never judges that things are made more just by reducing the benefits of the worst off in order to increase benefits of others (e.g., it does not judge <1,99,99>) to be more just than <2,3,4>). Hence, in showing below that, given certain assumptions, Core Difference Principle must satisfy Minimal Maximin, we are showing that it cannot significantly diverge from leximin.[4]

**4. Some Initial Results**

The conditions imposed so far do not require Core Difference Principle to satisfy Minimal Maximin. Consider an *absolute benefit level cutoff* function that specifies the same benefit level for all alternatives—for example, the minimum level of benefits compatible with a decent life. The least advantaged group consists of those with benefits lower than the stipulated minimum level. Such a cutoff function satisfies Monotonicity. For a given cutoff (absolute or not), we can appeal to the total *restricted benefits* of an alternative, where these are the total benefits *up to the cutoff* (e.g., for a cutoff of 4, the total restricted benefits of <2,3,5> is 9). Note that the total restricted benefits include everyone's benefits up to the cutoff, and not just those of the least advantaged group. Consider now the following principle scheme:

**Modified Total Restricted Benefits Principle (Scheme)**: For any two alternatives X and Y: X is at least as just as Y if and only if (1) its total restricted benefits are at least as great, or (2) its total restricted benefits are equal and its total (unrestricted) benefits are at least as great.

We shall appeal to this principle throughout the paper. It holds that that ranking of alternatives is determined on the basis of the total restricted benefits, except that in cases of ties, further ranking is done on the basis of the total (unrestricted) benefits. This principle judges one alternative as *more* just if and only if it has greater total restricted benefits or if it has the same total restricted benefits but a greater total benefit.

Given an absolute cutoff function, this principle, it is easy to show, satisfies Core Difference Principle, Transitivity (and indeed even Completeness). Moreover, it is very different from Minimal Maximin. For example, if the cutoff is 10, it judges <1,10,10> as more just than <2,9,9>, whereas Minimal Maximin says that the latter is more just. Unlike Minimal Maximin, this principle allows the benefits of the worst off individual to be significantly traded off against the benefits of other members of the least advantaged group (as opposed to giving absolute priority to the worst off member). So far, then, our conditions allow radical divergence from leximin.

In what follows, however, we shall limit our attention to *relative* cutoff functions, which are cutoff functions that make the definition of the least advantaged group somehow dependent on the individual benefit levels of that distribution (e.g., 50% of the average benefits or 80% of the median benefits). This is appropriate, since for the purposes of the difference principle, the least advantaged group is almost always understood in relative terms.

A natural relative cutoff condition is the following:

**Conditional Best Off Excluded**: If not everyone has the same benefits, then no one who is a best off individual is a member of the least advantaged group.

For distribution <1,1,5>, Conditional Best Off Excluded entails that the third person is not in the least advantaged group.[5] This condition rules out absolute cutoffs. For example, if the cutoff is set at the absolute level of 10, then <1,2,3> violates Conditional Best Off Excluded.

Consider now one further cutoff condition:

**Non-Best Off Includable**: For any alternative, it is possible to increase sufficiently the benefits of the best off individuals so that everyone else becomes a member of the least advantaged group.

This says, for example, that in <1,3,4,n>, for some sufficiently high n, everyone but the best of position is in the least advantaged group. Even the second best off person is in the least advantaged group for some sufficiently great n (e.g., in <1,3,4,999999999999>). After all, for a sufficiently great n, everyone but the best off person will be much closer in benefits to the worst off person than to the best off person and their share of the overall resources in society will be almost negligible. This condition is satisfied, for example, by any cutoff that is based on some positive percentage of the benefits of the best off position, or on some positive percentage of the average benefits (which is sensitive to the benefits of the best off).

We are now ready for our first result:

**Result 1**: Suppose (1) the cutoff function satisfies Monotonicity, Conditional Best Off Excluded, and Non-Best Off Includable, and (2) the justice relation satisfies Core Difference Principle, and consider any two alternatives, X and Y, for which the benefits in the worst off position in X are greater than those in Y. In this case: (a) if the justice relation satisfies Acyclicity, then Y is not more just than X, (b) if the justice relation satisfies Strong Acyclicity, then Y is not at least as just as X, and (c) if the justice relation satisfies Transitivity, then X is more just than Y.

The proofs for all results are in the appendix, but for each result we shall illustrate the force of the result. Suppose that the least advantaged group for an alternative is defined as those who get less than average benefits in that alternative (which satisfies each of the cutoff conditions). Consider the Modified Total Restricted Benefits Principle, which, recall, holds that, for any two alternatives X and Y: X is at least as just as Y if and only if (1) its total restricted benefits are at least as great, or (2) its total restricted benefits are equal and its total (unrestricted) benefits are at least as great. Compare the alternatives <1,2,12> (average of 5) and <1,3,5> (average of 3). The total restricted benefit is 8 in the first alternative and 7 in the second alternative. Thus the Modified Total Restricted Benefits Principle judges <1,2,12> as more just than <1,3,5>. However, this violates Core Difference Principle, and hence this version of the Modified Total Restricted Benefits Principle is not a counterexample to Result 1.

The proof of Result 1 generalizes the intuitions of this illustration. If the conditions of the result are granted, then, the difference principle (which Rawls intended to be aggregative within the least advantaged group) cannot "radically disagree" with leximin (which involves no aggregation). It cannot "radically disagree" in the sense that it cannot judge one alternative to be more just if it gives less benefits to the worst off individual (i.e., cannot violate Minimal Maximin). The core result does not rule out the possibility that the difference principle judges two alternatives equally good, or is silent about their justice ranking, when one gives more benefits to the worst off position. Still, the result places severe limitations on the nature of the difference principle. Moreover, if the highly plausible Strong Acyclicity is assumed, then the two alternatives in this case cannot be judged equally good, and if the more controversial Transitivity is assumed, then the alternative with greater benefits for the worst off individual must be judged more just.

Result 1 depends upon Non-Best Off Includable, but this condition is not entirely uncontroversial. It is violated by relative cutoffs that are based on the benefits obtained at some percentile other than 100%. For example, if the cutoff for the least advantaged group is 90% of the median (i.e., $50^{th}$ percentile) benefits, then, in <1,3,5,7,n>, only those with less than 4.5 (90% of the median of 5) are in the least advantaged group. This cutoff is insensitive to the level of benefits of the best off position (unlike an average benefits cutoff, which is so sensitive). Hence, there does not exist any n such that everyone in <1,3,5,7,n> except the best off position be in the least advantaged group, and thus Non-Best Off Includable is violated.

It turns out, however, that a significant result can be obtained without assuming Non-Best Off Includable. Consider the following condition:

**Conditional Worst Off Included**: If there is some inequality, then all worst off individuals are in the least advantaged group.

The idea is that, although the least advantaged group may be empty where there is perfect equality, in all other cases the group is not empty and includes at least all worst off persons. This may seem fairly plausible for a relative approach.

**Result 2**: Suppose (1) the cutoff function satisfies Conditional Best Off Excluded, and Conditional Worst Off Included, and (2) the justice relation satisfies Core Difference Principle, and consider any two alternatives, X and Y, for which the benefits in the worst off position in X are greater than those in Y. In this case: (a) if the justice relation satisfies Acyclicity, then Y is not more just than X, (b) if the justice relation satisfies Strong Acyclicity, then Y is not at least as just as X, and (c) if the justice relation satisfies Transitivity, then X is more just than Y.

Adding Conditional Worst Off Included ensures that all conflicts involving worst off individuals take place within the least advantaged group. Given the other conditions, this ensures that absolute priority must be given to the worst off position. Thus, Non-Best Off Includable is not essential for the Minimal Maximin result.

Conditional Worst Off Included is sufficient for the result in the context of the other conditions. It is not, however, as plausible as it may first seem. It rules out, for example, a cutoff of 50% of mean, or median benefits. For example, in <5,6,7> (average and median of 6), the worst off individual has more than the 50% of the average (and median) benefits and is thus excluded from the least advantaged group.

It turns out, however, that a significant—but slightly weaker—result can be obtained without assuming Conditional Worst Off Included (or any replacement). Consider, then:

**Result 3**: Suppose (1) the cutoff function satisfies Monotonicity and Conditional Best Off Excluded, and (2) the justice relation satisfies Core Difference Principle, and consider any two alternatives, X and Y, for which the benefits in the worst off position in X are greater than those in Y *and, for each of X and Y, their respective worst off individuals are in their least advantaged group*. In this case: (a) if the justice relation satisfies Acyclicity, then Y is not more just than X, (b) if the justice relation satisfies Strong Acyclicity, then Y is not at least as just as X, and (c) if the justice relation satisfies Transitivity, then X is more just than Y.

This result shows that if the controversial Non-Best Off Includable is dropped (and the uncontroversial Monotonicity invoked), the difference principle must agree with Minimal

Maximin in those cases where the worst off individuals in each alternative are in their respective least advantaged groups. The result may not hold where the worst off individual is not part of an alternative's least advantaged group. In this case, however, *no one* is a member of that group. The core cases of interest are those where the least advantaged group is not empty. Thus if the conditions of this result are accepted, there are significant limitations on the form that the difference principle can take.

## 5. Conditional Best Off Excluded Reconsidered

The previous results have assumed Conditional Best Off Excluded, which is a relatively weak condition and compatible with, to our knowledge, all the relative definitions of the least advantaged group that are used in practice. Consider, for example, where the cutoff is some percentage, between 0 and 100, of the *mean* benefits. This will satisfy the above condition, since, for any unequal distribution, the best off individuals are above the mean. Conditional Best Off Excluded is also satisfied by any cutoff that is some percentage, between 0 and 100, of the benefits at some specified benefit *percentile* inclusively between 0 and 100. It is satisfied, for example, by the cutoff of 90% of the benefits at the $80^{th}$ percentile.

Nonetheless, it turns out that Conditional Best Off Excluded is not entirely uncontroversial—even for relative cutoff frameworks (as we assume throughout). Consider, for example, <2,2.1,2.1>. Is it obvious that the second and third individuals should be excluded from the least advantaged group? After all, their benefits, we may suppose, are only trivially greater than those of the worst off person. At least sometimes, it seems, best off individuals should be considered part of the least advantaged group.

Consider instead the following condition:

**Non-Worst Off Excludable:** For any alternative and any individual, it is possible to increase sufficiently the benefits of those individuals with greater benefits so that they are not members of the least advantaged group.

This is entailed by Conditional Best Off Excluded since, for any alternative with some inequality (e.g., <2,3,4>), and any given position (e.g., the first person), one can increase the benefits of all the better off positions (if there are any) to be equal to the benefits of the best off position (e.g., to <2,4,4>), and then Conditional Best Off Excluded entails that they are not in the least advantaged group. This condition does not, however, entail Conditional Best Off Excluded, since it does not guarantee that the best off position is excluded from the least advantaged group (even where there is inequality). It can allow, for example, that everyone is in the least advantaged group for <2,2.1,2.1>. It only guarantees that non-worst-off individuals *can be excluded* by sufficiently increasing their benefits. It is thus a much weaker condition than Conditional Best Off Excluded.

If we impose only Monotonicity and Non-Worst Off Excludable (but not Non-Best Off Includable, Worst Off Included, or Conditional Best Off Excluded), then Core Difference Principle can diverge significantly from leximin. This is because these two cutoff conditions are also satisfied by *absolute* cutoff functions, and, as discussed in Section 3, absolute cutoffs can satisfy the background conditions without satisfying Minimal Maximin. Our concern, however, is to explore the possibility of using a *relative* cutoff that diverges from Minimal Maximin. Some of our earlier conditions ruled out absolute cutoffs, but we have now dropped them because they also controversially ruled out certain seemingly plausible relative cutoff functions. The following condition, however, rules out absolute cutoffs without ruling out any minimally plausible relative cutoffs.

**Strict Monotonicity:** For any two alternatives X and Y, (1) if each individual has at least as much benefits in X as in Y, then c(X) ≥ c(Y), and (2) if each individual has more benefits in X as in Y, then c(X) > c(Y).

This is like Monotonicity, except that the second clause has been added. It rules out absolute cutoffs by requiring that the cutoff increase when everyone's benefits increase. All minimally plausible relative cutoffs satisfy this condition.

Result 3 above established that, given Conditional Best Off Excluded and our other conditions, Core Difference Principle must satisfy Minimal Maximin when the worst off individuals of X and Y are in their respective least advantaged groups. We have now dropped Conditional Best Off Excluded and replaced it with the weaker Non-Worst Off Excludable and also strengthened Monotonicity to Strict Monotonicity. We shall now examine whether this broader framework opens up the possibility of Core Difference Principle violating Minimal Maximin.

To start, note that no new possibilities are opened up concerning cutoff functions that set the cutoff: (1) between 0 and 100% of mean benefits, or (2) between 0 and 100 of some percentile benefits (e.g., median benefits). This is because such cutoff functions satisfy Monotonicity and Conditional Best Off Excluded, and thus Result 3 establishes that they cannot significantly diverge from Minimal Maximin.

Weakening Conditional Best Off Excluded to Non-Worst Off Excludable does, however, open up the possibility of a different kind of cutoff function. Consider *worst-off-based cutoff functions*, which are strictly increasing functions of the benefits of the worst off individuals (e.g. some percentage above 100% of the benefit level of the worst off person, or some fixed number of units above that benefit level). Such cutoffs violate Conditional Best Off Excluded, since, wherever they set the cutoff above the worst off position, the best off person might have less than the cutoff but still be better off than the worst off person. In this case, the best off person would be deemed to be in the least advantaged group, and that violates Conditional Best Off Excluded. Moreover, worst-off-based cutoff functions satisfy Strict Monotonicity (trivially) and Non-Worst Off Excludable (since the benefits of all the non-worst off can be increased to be above the specified worst-off-based cutoff).

Of course, it might be that even worst-off-based cutoff functions cannot significantly diverge from Minimal Maximin. Our next result establishes that this is not so. Indeed, it establishes this, even if we add the following standard condition and demand Transitivity:

**Strong Pareto**: For any two alternatives X and Y, if each person has at least as much benefits in X as in Y, then (1) X is as least as just as Y, and (2) if there is at least one person that has more benefits in X than in Y, then X is more just than Y.

Strong Pareto is a weak efficiency condition on the promotion of benefits (much weaker than the utilitarian sum-total conception of efficiency). It requires, for example, that <2,4,6> be judged more just than <1,4,6> and also more just than <2,3,5>. It is silent about whether <2,4,6> is more just than <99,1,6>.

In our results so far we have not needed to assume Completeness or Strong Pareto. The following possibility result, however, holds even if they are assumed.

**Result 4**: Suppose the cutoff function satisfies Strict Monotonicity and Non-Worst Off Excludable. Under these conditions, there exists a justice relation that (1) satisfies Core

Difference Principle, Transitivity, Completeness, and Strong Pareto, and (2) sometimes judges an alternative Y to be more just than an alternative X, where: (a) the benefits in the worst off position in X are greater than those in Y and (b) the worst off individuals of X and Y are in their respective least advantaged groups.

The proof of this result, which we here outline, is constructive in that we define a cutoff function and a justice relation satisfying all the conditions and which violates Minimal Maximin. The justice relation that we appeal to is Modified Total Restricted Benefits Principle with the cutoff as some positive number of units greater than the benefits of the worst off individual. This cutoff straightforwardly satisfies both Strict Monotonicity and Non-Worst Off Excludable. The justice relation generated by applying this cutoff on the Modified Total Restricted Benefits Principle is obviously complete and straightforwardly satisfies Strong Pareto. Moreover, it ensures that Core Difference Principle is satisfied. For example, if the cutoff is 5 units above lowest benefit level, then <2,8,9> (cutoff of 7 and total restricted benefits of 16) is judged more just than <2,3,9> (cutoff of 7 and total restricted benefits of 12), as required by Core Difference Principle. It also ensures that Minimal Maximin is violated. For example, if the cutoff is 5 units above the lowest level, then <1,5,10> (cutoff of 6 and total restricted benefits of 12) is judged more just than <2,2,10> (cutoff of 7 and total restricted benefits of 11). We prove in the appendix that the relation satisfies Transitivity. Hence, this version of the difference principle constitutes a real alternative to leximin.

Thus, if the cutoff function defining the least advantaged group need only satisfy Strict Monotonicity and Non-Worst Off Excludable, the difference principle can significantly diverge from leximin (violate Minimal Maximin)—even if Transitivity, Completeness, and Strong Pareto are required. Of course, there could be additional plausible cutoff conditions that rule out this possibility. We are, however, reasonably optimistic that worst-off-based cutoff functions are the most promising way for the least advantaged group to be defined, and thus doubtful that there are any additional plausible cutoff conditions that would rule out such functions.

There are, however, other ways that the significance of this result can be challenged, and we address them briefly in the next section.

## 6. Indeterminate or Comparison-Relative Cutoffs

So far, we have assumed that there is a determinate (sharp, precise) cutoff that defines the least advantaged group and that the cutoff for a given alternative is *comparison-invariant* in the sense that it is the same no matter what alternative it is compared with. Each of these assumptions could, of course, be questioned. We note below that Results 1-3—the results that show that, given certain assumptions about cutoff functions, the difference principle cannot radically disagree with leximin—remain valid even if these two assumptions are each relaxed. Proofs of these claims are given in Tungodden (2004).

First, the cutoff for the least advantaged group may be allowed to be *indeterminate* (i.e., be in some *range of values* with no particular value in the range being uniquely correct). In this case, the results remain valid even if the core difference principle is weakened to apply only when someone who is *determinately* in the least advantaged group benefits and no one who is determinately or indeterminately in the least advantaged group is made worse off.

A second way of relaxing the assumptions is by allowing cutoffs for the least advantaged group to be *comparison-relative*. This would allow, for example, that the cutoff for the least advantaged group in <98,99,100> need not be the same for the purposes of ranking it with <980,990,1000> as it is for the purposes of ranking it with <8,9,10>. After all, it seems plausible that everyone is in the least advantaged group of <98,99,100> when it is compared with

<980,990,1000>, but no one is in the least advantaged group of <98,99,100> when it is compared with <8,9,10>. A comparison-relative approach might, for example, set the cutoff as 5 units above the lowest value *in either alternative*. This would say that (1) in comparing <98,99,100> with <980,990,1000>, the cutoff is 103, and thus everyone in the former is in its least advantaged group, but (2) in comparing <98,99,100> with <8,9,10>, the cutoff is 13 and no one in the former is in its least advantaged group. Even if the nature of cutoff functions is relaxed in this way, however, Results 1-3 remain valid—even if cutoffs are also allowed to be indeterminate.

Consider now Result 4, which establish that the justice relation can radically diverge from leximin, if the assumptions on the cutoff functions are plausibly relaxed in a certain way—even if the justice relation must satisfy Transitivity, Completeness, and Strong Pareto. We shall now discuss whether this result remains valid in the context of comparison-relative framework. For simplicity, we shall assume that cutoffs are determinate, and will not give the needed reformulations of the various conditions in the comparison-relative framework.

To start, note that the comparison-relative framework can be understood *broadly* so that it allows, but does not require, cutoffs to vary with the comparison being made. In this broad sense, the comparison-relative framework includes the comparison-invariant framework, and thus Result 4 remains fully valid. Alternatively, the comparison-relative framework can be understood *narrowly* in the sense that it *requires* cutoffs to vary, at least sometimes, with the comparisons being made. Any comparison-invariant cutoffs—such as the worst-off-based function that we invoked for the proof of Result 4, are ruled out. Hence, our proof of Result 4 is not valid for this narrow framework.

We are inclined to think that the narrow comparison-relative framework is the appropriate framework. To motivate this view, suppose that we use a cutoff of 2 units above the lowest benefit level, focus on the total restricted benefits (i.e., benefits up to the cutoff), and consider <1,3,3,4,4> and <2,2,2,4,4>. On the comparison-independent approach, the former (with a cutoff of 3) has *less* total restricted benefits (13) than the latter (with a cutoff of 4 and total restricted benefits of 14). On the narrow comparison-relative approach, however, an opposite conclusion can be reached. Suppose that, for this comparison, the cutoff is 2 units higher than the lowest benefit level *in either alternative*. In this case, <1,3,3,4,4> still has a cutoff of 3 and total restricted benefits of 13, but <2,2,2,4,4> now also has a cutoff of 3 (not 4), and total restricted benefits of 12 (not 14). Hence, this comparison-relative version of the total restricted benefits principle favours <1,3,3,4,4> over <2,2,2,4,4>. This seems more plausible than the opposed judgement reached by the comparison-invariant version. In the context of this comparison, it seems quite implausible to ignore the benefits beyond 3 for the first alternative but count them for the second alternative. The same cutoff, it seems, should be used for both.

Suppose, then, that we assume the narrow comparison-relative framework, and that we take the cutoff to be 2 units higher than the lowest benefit level in either compared alternative. Does the Modified Total Restricted Benefits Principle—thus reinterpreted—still satisfy the conditions of Result 4? Unfortunately it does not. It violates Acyclicity (and hence Transitivity). To see this, note that it judges (1) <1,3,3,3,3,3> as more just than <2,2,2,4,4,4> (with a cutoff of 3 and total restricted benefits of 16 and 15 respectively), (2) <2,2,2,4,4,4> as more just than <2,3,3,3,3,3> (with a cutoff of 4 and total restricted benefits of 18 and 17 respectively), and (3) <2,3,3,3,3,3> as more just than <1,3,3,3,3,3> (with a cutoff of 3 and total restricted benefits of 17 and 16 respectively). In sum <1,3,3,3,3,3> is more just than <2,2,2,4,4,4>, which is more just than <2,3,3,3,3,3>, which is more just than <1,3,3,3,3,3>, which violates Acyclicity.

In this example, the problem of Acyclicity arises because the cutoff for a given alternative varies with the comparison being made. <1,3,3,3,3,3> is more just than <2,2,2,4,4,4>

based on a cutoff of 3, and <2,2,2,4,4,4> is more just than <2,3,3,3,3,3> based on a cutoff of 4. The first judgement ignores the benefits above 3 in <2,2,2,4,4,4> whereas the second judgement does not. It is thus not surprising that Acyclicity is violated.

It is, however, possible to satisfy Transitivity (and hence Acyclicity) and all the other conditions of Result 4, if one does not require disagreement with Minimal Maximin. Leximin straightforwardly satisfies Transitivity, Completeness, and Strong Pareto. Moreover, it ensures that Core Difference Principle is satisfied for any definition of the least advantaged group satisfying Strict Monotonicity and Non-Worst Off Excludable (even though it gives no role to the cutoff and thus gives no special role to the least advantaged group so defined).

Thus, in the narrow comparison-relative framework, the conditions of the Result 4 can be satisfied (e.g., by Leximin) as long as one does not require that Minimal Maximin be violated. The crucial question is whether they can be satisfied in a way that does violates Minimal Maximin (i.e., in way that significantly diverges from Leximin). Unfortunately, we do not know the answer to this question. We have tried to find a justice relation that establishes this possibility, and we have tried to prove that it is not possible, but both attempts have been unsuccessful. We must leave this as important open question: Is it possible in the narrow comparison-relative framework to satisfy Strict Monotonicity, Non-Worst Off Excludable, Acyclicity, Strong Pareto, and Core Difference Principle in a way that violates Minimal Maximin (and thus significantly diverges from Leximin)?

## 7. Conclusion

Most interpreters of the Rawls's difference principle have interpreted it as leximin, and thus as requiring that any alternative that gives greater benefits to the worst off position be judged more just. This, however, was not how Rawls intended this principle to be understood. As indicated by the quoted passages at the beginning of the paper, he intended it to require lexical priority to the *aggregative benefits* (e.g., total or average) to the least advantaged group (as opposed to lexical priority to worst off individual) We have explored the possibility of the difference principle significantly diverging from leximin.

As we noted, the difference principle can radically diverge from leximin, if the cutoff for the least advantaged group is a *fixed* number (i.e., absolute cutoff function). In that case, for example, all the conditions that we have invoked other than Minimal Maximin are satisfied by the principle that judges one alternative at least as just as another if and only (1) the sum of the total restricted benefits is greater, or (2) the sum of the total restricted benefits is equal and the total benefits is at least as great. Almost everyone interested in the difference principle, however, is interested in combining it with relative cutoffs (which vary with the level of benefits of individuals and satisfy Strict Monotonicity). Hence, we have focused on this latter approach.

The most common cutoff relative functions specify the cutoff as some percentage, between 0 and 100, of the *mean* benefits, or as some percentage, between 0 and 100, of the benefits at some specified benefit *percentile* inclusively between 0 and 100 (e.g., median benefits). Such cutoff functions satisfy Conditional Best Off Excluded, which requires that the best off individuals not be part of the least advantaged group when not everyone has the same benefits. Results 1-3, however, established that any justice relation satisfying this cutoff condition and the other background conditions must, when the worst off individual is in the least advantaged group, also satisfy Minimal Maximin (which requires that an alternative with lower benefits for the worst off individual not be judged more just). Consequently, any such approach cannot significantly diverge from leximin.

Conditional Best Off Excluded, however, is not uncontroversial. It rules out the possibility of appealing to a worst-off-based cutoff function that sets the cutoff as some fixed number of units above benefit level of the worst off. Such a cutoff violates Conditional Best Off Excluded by including the best off individuals in the least advantaged group when their benefits levels are sufficiently close to those of the worst off individuals. We believe that such cutoffs are especially plausible and in line with the Rawlsian perspective, since they always include the least well off individuals in the least advantaged group, typically include some others who are better off, but do not include those who a sufficiently better off than the worst off individuals.

Conditional Best Off Excluded, we suggested, should be weakened to Non-Worst Off Excludable (which requires that, for any alternative and any individual, it is possible to increase sufficiently the benefits of those individuals with greater benefits so that they are not members of the least advantaged group). Result 4 then establishes that, in a comparison-invariant framework satisfying Strict Monotonicity and Non-Worst Off Excludable, it is possible for Core Difference Principle, Transitivity, Completeness and Strong Pareto to be satisfied in a way that violates Minimal Maximin. Indeed, the result is constructive and appeals to a particularly appealing way of satisfying these conditions: The cutoff for the least advantaged group, for a given comparison, is set at some positive number above the lowest benefit level in either alternative, and then one alternative is judged at least as just as another if and only if (1) its total restricted benefits (i.e., benefits up to the cutoff level) are at least as great, or (2) its total restricted benefits are equal and its total (unrestricted) benefits are at least as great. This is indeed a promising version of the difference principle that clearly diverges significantly from Leximin.

Finally, we noted that the above comparison-invariant framework (in which the cutoff for a given alternative is the same no matter what alternative it is compared with) seemed somewhat questionable. Suppose, for example, that cutoffs are set at 2 units above the lowest level and consider the comparison of <1,3,3,4,4> and <2,2,2,4,4>. The comparison-invariant approach says the cutoff for the former is 3 and the cutoff for the latter is 4. It seems, however, that they should have the same cutoff when they are compared. We therefore briefly discussed what results can be obtained if one moved to a comparison-relative framework in which, for a given comparison, the cutoff is the same for the two alternatives. Unfortunately, we have not been able to determine whether in this framework it is possible to satisfy the basic conditions without entailing Minimal Maximin. This remains an important open question.[6]

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## Appendix 1: Proofs of Results

For any alternative X, let $x_i$ refer to the benefits of person i in X and $x^i$ the benefits in position i in X, where $x^1 \leq x^2 \leq \ldots x^i \leq \ldots \leq x^n$. We may now introduce the formal versions of the various conditions imposed on the cut off function.

**Monotonicity:** For any two alternatives X and Y, if for all individuals i, $x_i \geq y_i$, then $c(X) \geq c(Y)$.

**Conditional Best Off Excluded**: For any alternative X such that $x^1 < x^n$, and any individual i, if $x_i = x^n$, then $x_i \geq c(X)$.

**Non-Best Off Includable**: For any alternative X, there exists another alternative Y such that, for all individuals i, (1) if $x_i = x^n$, then $y_i > x_i$, and (2) if $x_i < x^n$, then $y_i = x_i$ and $y_i < c(Y)$.

**Conditional Worst Off Included**: For any alternative X such that $x^1 < x^n$, and any individual i, if $x_i = x^1$, then $x_i < c(X)$.

**Non-Worst Off Excludable:** For any alternative X and individual k, there exists an alternative Y such that for any individual i (1) if $x_i \leq x_k$, then $y_i = x_i$, and (2) if $x_i > x_k$, then $y_i > x_i$ and $y_i \geq c(Y)$.

**Strict Monotonicity:** For any two alternatives X and Y, (1) if for all individuals i, $x_i \geq y_i$, then $c(X) \geq c(Y)$, and (2) if for all individuals i, $x_i > y_i$, then $c(X) > c(Y)$.

We are now in a position to prove the various results. As in the main text, Benefitism and Domain Richness will not be stated explicitly when reporting the results.

**Result 1**: Suppose (1) the cutoff function satisfies Monotonicity, Conditional Best Off Excluded, and Non-Best Off Includable, and (2) the justice relation satisfies Core Difference Principle, and consider any two alternatives, X and Y, for which the benefits in the worst off position in X are greater than those in Y. In this case: (a) if the justice relation satisfies Acyclicity, then Y is not more just than X, (b) if the justice relation satisfies Strong Acyclicity, then Y is not at least as just as X, and (c) if the justice relation satisfies Transitivity, then X is more just than Y.

**Proof:**

(1) Consider any two alternatives X and Y and persons j,k, where $x_j = x^1 > y_k = y^1$.

(2) By Domain Richness, there exists an alternative Z, for which $x_k \geq x_j > z_k > y_k$ and $x_j > z_i = z_j > z_k$ for all individuals different from k. From Conditional Best Off Excluded, it follows that (at least) all individuals different from k are not in the least advantaged group L(Z).

(3) By Domain Richness, Non-Best Off Includable, and Monotonicity, there exists an alternative W, where $w_i = w^n > y_i$ for all i different from k, $w^n > z_k > w_k > y_k$, and k is a member of L(W). Moreover, from Conditional Best Off Excluded, it follows that no one else is a member of L(W). Finally, from Core Difference Principle, it follows that W is more just than Y.

(4) Suppose that L(X) is empty. Then it follows from Core Difference Principle that X is more just than W (because k is the only member of L(X) and L(W) and $x_k > w_k$ ). By (3) and Acyclicity, it then follows that Y is not more just than X

(5) Suppose that L(X) is non-empty. Then it follows from Core Difference Principle that X is more just than Z. Moreover, in this case, by (2) and (3) and the Core Difference Principle, it

follows that Z is more just than W (because k is the only member of the union of L(Z) and L(W) and $z_k > w_k$) . By (3) and Acyclicity, it then follows that Y is not more just than X.

(6) In both cases, replacing Acyclicity with Strong Acyclicity, ensures that Y is not at least as just as X and replacing Acyclicity with Transitivity, ensures that X is more just than Y.

**Result 2**: Suppose (1) the cutoff function satisfies Conditional Best Off Excluded, and Conditional Worst Off Included, and (2) the justice relation satisfies Core Difference Principle, and consider any two alternatives, X and Y, for which the benefits in the worst off position in X are greater than those in Y. In this case: (a) if the justice relation satisfies Acyclicity, then Y is not more just than X, (b) if the justice relation satisfies Strong Acyclicity, then Y is not at least as just as X, and (c) if the justice relation satisfies Transitivity, then X is more just than Y.

**Proof:**

(1) Consider any two alternatives X and Y and persons j,k, where $x_j = x^1 > y_k = y^1$.

(2) By Domain Richness, there exists an alternative Z, where $x_k \geq x_j > z_k > y_k$ and $z_i = x_j$ for all individuals different from k. From Conditional Best Off Excluded, it follows that all individuals different from k are not in the least advantaged group L(Z). From Conditional Worst Off Included, it follows that k is in the least advantaged group L(Z). From Core Difference Principle, it follows that X is more just than Z.

(3) By Domain Richness and Conditional Worst Off Included, there exists an alternative W, where $w_i = w^n > y_i$ for all i different from k, $w^n > z_k > w_k > y_k$, and k is a member of L(W). Moreover, from Conditional Best Off Excluded, it follows that no one else is a member of L(W). Finally, from Core Difference Principle, it follows that W is more just than Y.

(4) By (2) and (3) and the Core Difference Principle, it follows that Z is more just than W

(because k is the only member of the union of L(Z) and L(W) and $z_k > w_k$) . By (2) and (3) and Acyclicity, it then follows that Y is not more just than X. Replacing Acyclicity with Strong Acyclicity, ensures that Y is not at least as just as X. Replacing Acyclicity with Transitivity, ensures that X is more just than Y.

**Result 3**: Suppose (1) the cutoff function satisfies Monotonicity and Conditional Best Off Excluded, and (2) the justice relation satisfies Core Difference Principle, and consider any two alternatives, X and Y, for which the benefits in the worst off position in X are greater than those in Y *and, for each of X and Y, their respective worst off individuals are in their least advantaged group*. In this case: (a) if the justice relation satisfies Acyclicity, then Y is not more just than X, (b) if the justice relation satisfies Strong Acyclicity, then Y is not at least as just as X, and (c) if the justice relation satisfies Transitivity, then X is more just than Y.

**Proof:**

(1) Consider any two alternatives X and Y and persons j,k, where $x_j = x^1 > y_k = y^1$ and j is in L(X) and k in L(Y).

(2) By Domain Richness, there exists an alternative Z, where $x_k \geq x_j > z_k > y_k$ and $x_j > z_i = z_j > z_k$ for all individuals different from k. From Conditional Best Off Excluded, it follows that all individuals different from k are not in the least advantaged group L(Z). From Core Difference Principle, taking into account that j is in L(X) (from (1)), it follows that X is more just than Z.

(3) By Domain Richness, Monotonicity, and (1), there exists an alternative W, where $w_i = w^n > y_i$ for all i different from k, $w^n > z_k > w_k > y_k$, and k is a member of L(W). Moreover, from Conditional Best Off Excluded, it follows that no one else is a member of L(W). Finally, from Core Difference Principle, it follows that W is more just than Y.

(4) By (2) and (3) and the Core Difference Principle, it follows that Z is more just than W (because k is the only member of the union of L(Z) and L(W) and $z_k > w_k$) . By (2) and (3) and Acyclicity, it then follows that Y is not more just than X. Replacing Acyclicity with Strong Acyclicity, ensures that Y is not at least as just as X. Replacing Acyclicity with Transitivity, ensures that X is more just than Y.

**Result 4**: Suppose the cutoff function satisfies Strict Monotonicity and Non-Worst Off Excludable. Under these conditions, there exists a justice relation that (1) satisfies Core Difference Principle, Transitivity, Completeness, and Strong Pareto, and (2) sometimes judges an alternative Y to be more just than an alternative X, where: (a) the benefits in the worst off position in X are greater than those in Y and (b) the worst off individuals of X and Y are in their respective least advantaged groups.

**Proof**:

As indicated in the text, Modified Total Restricted Benefits Principle satisfies the various stated conditions. We here give the proof that it is transitive. The rest of the proof is straightforward and indicated in the test.

Transitivity: We will show that for any three distinct alternatives X, Y, Z, if the Modified Total Restricted Benefits Principle judges X as at least as just as Y and Y as at least as just as Z, then it judges X at least as just as Z. Given that Modified Total Restricted Benefits Principle judges one alternative at least as just as another if and only if one of two clauses is satisfied, there are four possible cases to consider (two ways that X can be judged at least as just as Y, and two ways that Y can be judge at least as just as Z).

(1) Suppose clause (1) of the Modified Total Restricted Benefits Principle applies both to the comparison of X and Y and to the comparison of Y and Z. It then follows straightforwardly that it also applies to a comparison of X and Z, where the total restricted benefits are at least as great in X as in Z.
(2) Suppose clause (2) of the Modified Total Restricted Benefits Principle applies both to the comparison of X and Y and to the comparison of Y and Z. It then follows straightforwardly that it also applies to a comparison of X and Z, where the total (unrestricted benefits) are at least as great in X as in Z.
(3) Suppose clause (1) of the Modified Total Restricted Benefits Principle applies to the comparison of X and Y and clause (2) to the comparison of Y and Z. It then follows straightforwardly that clause (1) applies to a comparison of X and Z, where the total restricted benefits are at least as great in X as in Z.
(4) Suppose clause (2) of the Modified Total Restricted Benefits Principle applies to the comparison of X and Y and clause (1) to the comparison of Y and Z. It then follows straightforwardly that clause (1) applies to a comparison of X and Z, where the total restricted benefits are at least as great in X as in Z.

Given (1) – (4), it follows that Transitivity is satisfied.

Notes

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[1] Pogge (1989) is an exception. He explicitly defines the least advantaged group as those below a certain percentile of the distribution.

[2] See also Tungodden (1999).

[3] Notice that the introduction of a cutoff function excludes a definition of the least advantaged group that is common in practice, to wit as the bottom n-th (e.g., 20th) percentile of the population. To see this, consider <1,1,2,3,4>. The bottom 20th percentile here consists of just the first person, but any cutoff function will treat the first person the same as the second (since they have the same benefits). Defining the least advantaged group in terms of percentiles may be useful in practice, but the fact that it treats individuals with the same benefits differently shows that it is theoretically unsound. Note, however, that the appeal to cutoffs does not preclude defining the least advantaged group as those who have less benefits than those at some percentile. In the above example, setting the cutoff at the benefits of the 60th percentile (i.e., 3 units) has the result that the first two individuals are in the least advantaged group.

[4] For a further discussion of the relationship between the leximin principle and the Core Difference Principle, see Tungodden (1999).

[5] Note that Conditional Best Off Excluded is silent for <1,1,1> and all other cases of perfect equality.

[6] For helpful comments, we thank Nils Holtug, Karsten Klint Jensen, and the participants at the University of Copenhagen Conference on Equality (2004).