R&D policies, trade and process innovation[*]

Jan I. Haaland[**]
Norwegian School of Economics and Business Administration and CEPR

Hans Jarle Kind[**]
Norwegian School of Economics and Business Administration

November 2004

Abstract:
We set up a simple trade model with two countries hosting one firm each. The firms invest in cost-reducing R&D, and each government may grant R&D subsidies to the domestic firm. We show that it is optimal for a government to provide higher R&D subsidies the lower the level of trade costs, even if the firms are independent monopolies. If firms produce imperfect substitutes, policy competition may become so fierce that only one of the firms survives. International policy harmonization eliminates policy competition and ensures a symmetric outcome. However, it is shown that harmonization is not necessarily welfare maximizing. The optimal coordinated policies may imply an asymmetric outcome with R&D subsidies to only one of the firms.

Keywords: trade, R&D, subsidies, process innovation
JEL classification: F12, F13, F15

1 Introduction

This paper has two main purposes. The first is to explore the relationship between trade costs and R&D investments. We show that increased integration (lower trade costs) may increase both private and social incentives to invest in R&D, and may lead firms to sell more both domestically and abroad. The second purpose is to study the effects of policy competition and cooperation in imperfectly competitive international markets, and in particular to show that R&D subsidies may in fact reduce the number of product varieties in the market. This turns out to be true both if the subsidies are set in a policy game between governments maximizing domestic welfare and if the governments set R&D subsidies cooperatively to maximize aggregate welfare.

These results are developed in a simple two-country model with trade costs, where each country hosts one firm. The firms produce horizontally differentiated goods, and can invest in process-improving R&D to reduce marginal production costs. Freer trade between the countries implies that the size of the market increases, making it profitable to invest more in cost-reducing R&D. Thereby marginal production costs and consumer prices fall. Other things equal, this leads to more export as well as higher sales at home. The latter implies that the social value of any given R&D investment then increases, due to higher domestic consumer surplus. Trade liberalization thus induces the government to increase the subsidy level. It should be noted that the motive for R&D subsidies is not to promote exports per se; the size of the export market is important only because it matters for the choice of R&D investments and hence for consumer surplus at home.[1]

In addition to the consumer-surplus motive for subsidizing R&D, there is also a strategic motive for active R&D policies when firms from different countries produce (imperfect) substitutes. This strategic ('business stealing') motive may give rise to policy competition between the countries. Contrary to many previous studies we find that policy competition does not necessarily result in too high subsidies; it may, however, lead to unstable

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[1] The effect is similar to what Krugman (1984) labelled "import protection as export promotion", in that it focuses on the links between the size of the market and the marginal costs of production. However, while Krugman's focus was on how to promote exports, in our case export is a means to ensure lower costs and higher domestic sales.

or asymmetric equilibria. The determining factor in our model is the degree of product differentiation. If goods are close substitutes, policy competition may be so fierce that it is impossible for both firms to survive in the market. Depending on the degree of product differentiation in the industry, we may thus have a stable symmetric equilibrium, an unstable symmetric equilibrium, or no symmetric equilibria at all. In the latter two cases there may exist stable asymmetric equilibria where one firm monopolizes the market (and the other is inactive), even if the countries and the firms at the outset are completely symmetric.

The outcome of the policy game is inoptimal from a global point of view. Hence, there is a need for R&D policy cooperation that takes into account profit and consumer surplus in both countries, and eliminates policy competition. Coordination of R&D policies may be particularly relevant within closely integrated regions where the use of other policy measures to support domestic industry is already regulated. Based on e.g. actual and proposed tax reforms in the EU, a natural approach could be to require that R&D subsidies are harmonized between the countries. If the countries harmonize their R&D subsidies to a common level in our context, the outcome where one firm monopolizes the market is avoided. Somewhat surprisingly, this is not necessarily welfare maximizing. If the two goods are sufficiently close substitutes, it will not be optimal from society's point of view to invest in process innovation in both firms. Hence, the optimal cooperative R&D policy for the two countries could be to subsidize R&D in one of the countries, but not in the other. In fact, it may even be optimal to tax R&D in the other country. The intuition is that the consumers do not gain very much from having access to different varieties if the goods are close substitutes. So to avoid duplication of the investment costs, the first-best cooperative policy could be to stimulate R&D in one firm and reduce the R&D incentives in the other.

Little research has been done on the links between trade liberalization and R&D policies, and we are not aware of any other studies showing how trade liberalization may increase private and social incentives to invest in R&D and thus lead firms to sell more both domestically and abroad even in absence of strategic interactions. However, starting with Spencer and Brander (1983), there is a large literature focusing on the business-

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stealing motive for subsidizing R&D. This focus can partly be explained by the fact that international agreements prohibit the use of, for instance, pure export subsidies. In such settings Neary and Leahy (2000) emphasize the important point that R&D policies may be a second-best option to support domestic firms in international markets. Moreover, Bagwell and Staiger (1994), Brander (1995) and Leahy and Neary (2001a) have found that R&D subsidies can be a more robust instrument than export policies. It should be noted, though, that these studies typically abstract from consumer-surplus effects and make the simplifying assumption that all production is exported to a third market. This strand of literature thus argues that policy competition tends to result in excessive R&D from the subsidising countries' point of view.[2]

D'Aspremont and Jacquemin (1988) initiated a wave of research that analyzes the consequences of R&D cooperation between firms that compete in the end-user market. Both D'Aspremont and Jacquemin and later studies have found that this kind of cooperation may be welfare improving and increase industry profit.[3] However, Salant and Shaffer (1998 and 1999) and Amir and Wooders (1998) point to the fact that these studies presuppose that the firms choose the same level of R&D and sell the same quantities, while the optimal solution may actually be asymmetric.[4] In particular, Salant and Shaffer (1999) deal with the fact that it may be optimal to treat ex ante identical agents unequally if there is Cournot competition in the product market. Hence, the symmetric equilibria identified in the literature may not represent optimal outcomes. Leahy and Neary (2004) relate the results from Salant and Shaffer to the question of whether the second-order conditions

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[2]Leahy and Neary (2001b) and Haaland and Kind (2004) depart from the simplification of looking only at third-market exports, and focus directly on domestic consumer surplus effects of R&D subsidies. The latter study shows that policy competition gives "wrong" subsidies, but not necessarily too high subsidies, compared to a solution where the countries set R&D subsidies cooperatively. If goods are close substitutes, policy competition implies too high subsidies; if on the other hand, goods are fairly differentiated, a coordinated solution would give higher subsidies than the non-cooperative outcome of the policy competition.

[3]See also Leahy and Neary (1997) for an analysis of similar questions in a more general setting.

[4]Interestingly, Amir and Wooders show that aggregate industry profit may be higher in an asymmetric equilibrium with R&D competition between firms than in a symmetric equilibrium where the firms choose R&D expenditure cooperatively.

for a symmetric equilibrium are satisfied, and discuss more generally how to interpret the results regarding symmetric versus asymmetric outcomes.

Our analysis relates to this recent line of literature in its emphasis on the possibilities of asymmetric equilibria. However, we focus on competition or cooperation at the policy stage, whereas Leahy and Neary (2004) and most of the other studies look at R&D cooperation between firms and abstract from subsidization issues. Moreover, while most of the previous studies assume Cournot competition with homogeneous products, we introduce product differentiation, and show that the degree of differentiation is, indeed, decisive for the type of equilibrium in the market. Hence, our focus is a different one (policy rather than firms' behaviour) and our results are more general, in the sense that we show the importance of product differentiation (and hence competition) for the actual equilibrium in the market.

The rest of the paper is organized as follows: After a brief introduction of the model, we focus on the relationship between trade costs and R&D decisions for the monopoly case in section 2.1. In section 3.1 we show similar effects with Cournot competition between two firms. In section 3.2 policy competition is the focus, and in 3.3 policy cooperation. In both cases the possibilities of unstable and asymmetric equilibria are analysed in some detail. Section 4 draws some conclusions.

2 The model

Demand side

We employ a model with two intrinsically symmetric countries and two firms. Firm 1 is located in and owned by residents of Country 1, while Firm 2 is located in and owned by residents of Country 2. The population size in each country is equal to 1, and the utility function of a representative consumer is given by

$$U_i = \alpha q_{ii} + \alpha q_{ji} - \left(\frac{q_{ii}^2}{2} + \frac{q_{ji}^2}{2} + b q_{ii} q_{ji} \right), \tag{1}$$

where q_{ii} and q_{ji} are consumption of the goods produced by the domestic and the foreign firm, respectively. The first subscript thus indicates in which country the good is produced,

and the second subscript in which country the good is consumed.

Equation (1) is a standard quadratic utility function where the parameter $b \in [0, 1)$ measures the degree of horizontal differentiation between the goods; the goods are completely independent if $b = 0$, while they are identical in the limit $b = 1$. More generally, the two goods are closer substitutes from the consumers' point of view the higher is b.

Letting p_{ii} and p_{ji} denote the end-user prices of the two goods in country i, we may express consumer surplus as $CS_i = U_i - p_{ii}q_{ii} - p_{ji}q_{ji}$. Provided that trade takes place, optimal consumer behaviour implies that $\partial CS_i / \partial q_{ii} = \partial CS_i / \partial q_{ji} = 0$. From this we find that the inverse demand curves are given by

$$p_{ii} = \alpha - (q_{ii} + bq_{ji}) \text{ and } p_{ji} = \alpha - (q_{ji} + bq_{ii}). \tag{2}$$

Supply side

The firm located in country i incurs trade costs $\tau \geq 0$ per unit it exports to country j. We emphasize that trade costs in our setting are exogenously given, and should be interpreted as a synthetic measure of a wide range of barriers to trade including transport costs, costs of frontier formalities, and differing product standards. We do not consider revenue-generating tariffs, as these are typically of limited importance in trade between industrialized countries.

In absence of R&D investments the marginal production cost of firm i is equal to c. In this case the profit margins on domestic sales and exports are given by $(p_{ii} - c)$ and $(p_{ij} - c - \tau)$, respectively. However, each firm may invest in R&D in order to reduce its marginal costs. More specifically, firm i reduces its marginal production costs to $(c - x_i)$ by investing $C(x_i) = x_i^2 + f$ in process innovation, where the parameter $f \geq 0$ represents the fixed costs of setting up an R&D project. We may thus write the profit function of firm i as

$$\pi_i = [p_{ii} - (c - x_i)]q_{ii} + [p_{ij} - (c - x_i) - \tau]q_{ij} - x_i^2 - f + s_i x_i, \tag{3}$$

where s_i is the R&D subsidy level the firm receives from its domestic government.

Clearly, the firms may find it optimal to invest in R&D until marginal costs equal zero if $(\alpha - c)$ is sufficiently large. We shall assume that $(\alpha - c)$ is not so high that this

happens.[5]

Welfare in each country is given by the sum of domestic consumer surplus and profit minus R&D subsidies:

$$W_i = CS_i + \pi_i - s_i x_i. \tag{4}$$

Note that consumer surplus may be written as

$$CS_i = \frac{1}{2} \left(q_{ii}^2 + q_{ji}^2 \right) + b q_{ii} q_{ji}. \tag{5}$$

In the following we consider a two-stage game, where the governments set R&D subsidies at stage 1 and the firms decide quantities and R&D levels at stage 2 (other timing-structures are discussed in section 4).

2.1 Benchmark: Optimal R&D subsidies to a monopoly

As a benchmark we assume that the firms are monopolies in their own market segments, which amounts to setting $b = 0$. This means that there are no strategic interactions between the firms, so that they choose R&D investments and output independent of each other.

Holding R&D investments fixed, profit maximizing output for firm i is found by setting $\partial \pi_i / \partial q_{ii} = \partial \pi_i / \partial q_{ij} = 0$ if there is trade. This yields monopoly outputs

$$q_{ii} = \frac{\alpha - (c - x_i)}{2} \text{ and } q_{ij} = \frac{\alpha - \tau - (c - x_i)}{2}. \tag{6}$$

Suppose f is sufficiently small that the firm chooses to invest in R&D. The cost of increasing R&D investment by one unit is equal to $(2x_i - s_i)$, while the benefit - in terms of reduced marginal production costs - equals $(q_{ii} + q_{ij})$. The benefit is thus increasing in total output. Profit maximizing behaviour implies that $(2x_i - s_i) = (q_{ii} + q_{ij})$, or

$$x_i = \frac{q_{ii} + q_{ij} + s_i}{2}. \tag{7}$$

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[5] A sufficient condition for marginal costs $(c - x_i)$ to be positive, is that $c/\alpha \geq 4/5$. See footnote 17 in the Appendix.

Combining (6) and (7) we find that output equals

$$q_{ii} = (\alpha - c) + \frac{s_i}{2} - \frac{\tau}{4} \text{ and } q_{ij} = (\alpha - c) + \frac{s_i}{2} - \frac{3\tau}{4} \qquad (8)$$

while R&D investment is

$$x_i = (\alpha - c) + s_i - \frac{\tau}{2}. \qquad (9)$$

Not surprisingly, we see that export is decreasing in the level of trade costs. More interestingly, the same is true also for domestic sales and R&D investments. The reason for the latter is that higher trade costs reduce export and thus the firm's willingness to invest in cost reductions. This leads to higher marginal production costs $(c - x_i)$ and therefore lower output also domestically.

It is well known from e.g. Spencer and Brander (1983) that a government may have incentives to grant R&D subsidies to domestic firms in order to improve their competitive position. This has been labelled the "business-stealing effect" in the literature. But there are no strategic interactions between the firms if $b = 0$, and therefore no business-stealing effect. Consequently, the government in country i cannot use R&D subsidies to increase profit net of R&D subsidies for its domestic firm;

$$\frac{\partial (\pi_i - s_i x_i)}{\partial s_i} = -s_i < 0 \text{ for } s_i > 0. \qquad (10)$$

Hence, if R&D subsidies are granted, it must be because of increased consumer surplus. The monopoly charges a lower price the lower the marginal production costs. Therefore consumer surplus is increasing in cost-reducing R&D expenditure. However, the firm does not take this effect into account when deciding how much to invest in R&D. In order to correct for this, the government has incentives to grant R&D subsidies and increase domestic consumer surplus (also foreign consumer surplus increases, but this is irrelevant for the government in country i). Using equations (5) and (8) with $b = 0$ we find

$$\frac{\partial CS_i}{\partial s_i} = \frac{1}{2} q_{ii} > 0. \qquad (11)$$

The consumers gain more from a given price reduction the more they consume of the good. This explains why $\partial CS_i / \partial s_i$ is increasing in q_{ii}. Since output is higher the lower the level of trade costs, equation (11) therefore indicates that the government should

optimally increase the subsidy level if trade costs fall. Formally, setting $\partial W_i/\partial s_i = \partial\left(\pi_i - s_i x_i\right)/\partial s_i + \partial CS_i/\partial s_i = 0$ we have (with superscript M for monopoly):

$$s_i^M = \frac{4\left(\alpha - c\right) - \tau}{6}; \quad \frac{\partial s_i^M}{\partial \tau} < 0. \tag{12}$$

We can now state:

Proposition 1: *Suppose that the firms are monopolists in their own market segments. The governments will then subsidize domestic R&D. Trade liberalization ($d\tau < 0$) makes it optimal to increase the subsidy level.*

As noted above, there are no strategic interactions between the firms (or the governments) if $b = 0$. The mechanisms through which trade makes it optimal for governments to subsidize R&D are therefore qualitatively different from those that have been analyzed in strategic trade policy papers. Indeed, the only reason why the governments increase R&D subsidies when trade costs are reduced in the present context, is that this makes the domestic economy more efficient. The output of the R&D project - here more cost efficient production technologies - is a non-rival good that should be provided in a greater quantity the larger the activity level of the firm. Other things equal, trade liberalization increases total output and therefore makes it optimal to invest more in R&D both from a private and social point of view.

3   R&D policies with possible intra-industry trade

In   the rest of the paper we assume that $b \in (0, 1)$, which means that the two goods are imperfect substitutes. It should be noted that the standard quadratic utility function described by equation (1) has the realistic feature that total market demand is decreasing in b, all else equal.[6] This reflects the common assumption that consumers have convex preferences, so that the size of the market tends to be smaller the less differentiated the goods.

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[6]This is most easily seen by assuming that the goods are sold at a fixed price $\bar{p}$. We then find that consumer demand is given by $q_{ii} = q_{ji} = (\alpha - \bar{p})/(1 + b)$.

3.1 Market equilibrium

At the last stage the firms simultaneously choose quantities and R&D investments. An equilibrium with intra-industry trade is thus given by $\partial \pi_i / \partial x_i = \partial \pi_i / \partial q_{ii} = \partial \pi_i / \partial q_{ij} = 0$. Holding quantities fixed, we find that $\partial \pi_i / \partial x_i = 0$ implies

$$x_i = \frac{q_{ii} + q_{ij} + s_i}{2}, \tag{13}$$

which is the same expression as we had for the monopoly. The incentives to invest in cost reduction are consequently also in this case increasing in total output and the subsidy level. Solving $\partial \pi_i / \partial q_{ii} = \partial \pi_i / \partial q_{ij} = 0$ when we hold R&D investments fixed we further have

$$
\begin{aligned}
q_{ii} &= \frac{1}{2+b}(\alpha - c) + \frac{b}{4 - b^2}\tau + \frac{2x_i - bx_j}{4 - b^2} \\
q_{ij} &= \frac{1}{2+b}(\alpha - c) - \frac{2}{4 - b^2}\tau + \frac{2x_i - bx_j}{4 - b^2}.
\end{aligned} \tag{14}
$$

Higher trade costs make the home market more protected from foreign competition. For any given R&D investment, we therefore find a positive relationship between domestic sales and trade costs. However, the direct effect of higher trade costs is to reduce export, and it is easily verified that total sales for each firm are decreasing in τ $(\partial (q_{ii} + q_{ij})/\partial \tau < 0)$. Equation (13) therefore shows that higher trade costs lead to less investments in cost-reducing R&D. This effect suggests that also domestic sales may decrease in τ. Indeed, from the analysis above we know that this is true in the monopoly case, and by combining (13) and (14) we find

$$
\begin{aligned}
q_{ii} &= \frac{1}{1+b}(\alpha - c) - \frac{1 - 2b}{2(2-b)(1+b)}\tau + \frac{s_i - bs_j}{2(1 - b^2)} \\
q_{ij} &= \frac{1}{1+b}(\alpha - c) - \frac{3}{2(2-b)(1+b)}\tau + \frac{s_i - bs_j}{2(1 - b^2)}.
\end{aligned} \tag{15}
$$

while

$$x_i = \frac{1}{1+b}(\alpha - c) - \frac{1}{2(1+b)}\tau + \frac{2 - b^2}{2(1 - b^2)}s_i - \frac{b}{2(1 - b^2)}s_j. \tag{16}$$

From (15) and (16) we have the following:

Proposition 2: *Holding subsidies fixed, trade liberalization ($d\tau < 0$) leads to higher domestic output if $b < 1/2$ and to higher export and more R&D investments for all $b \in [0, 1)$.*

For $b > 1/2$ the import-competition effect dominates over the R&D effect; hence, domestic sales go down.

3.2 R&D policy competition

In this section, we assume that the governments non-cooperatively choose subsidy levels so as to maximize domestic welfare at stage 1. We start out by analyzing the first-order conditions, but shall subsequently show that the FOCs do not represent a (unique) equilibrium if the goods are sufficiently close substitutes.

First-order conditions with R&D policy competition

At stage 1 the countries simultaneously solve $\partial W_i / \partial s_i = 0$. This gives rise to a symmetric outcome given by $s_1 = s_2 \equiv s^{PC}$ (superscript PC for policy competition):

$$s^{PC} = \frac{2\left(1 + b^2\right)}{3 + 4b - 3b^2 - 2b^3}(\alpha - c) - \frac{1 - 2b + 3b^2}{(2 - b)\left(3 + 4b - 3b^2 - 2b^3\right)}\tau. \qquad (17)$$

Inserting for (17) into (16) and defining $x_1 = x_2 \equiv x^{PC}$ we further find

$$x^{PC} = \frac{5 - b^2}{3 + 4b - 3b^2 - 2b^3}\left(\alpha - c\right) - \frac{4 - 3b + b^3}{(2 - b)\left(3 + 4b - 3b^2 - 2b^3\right)}\tau. \qquad (18)$$

The subsidy level and R&D investments are thus decreasing in τ, which is what we should expect from the monopoly case. However, differentiating equations (17) and (18) with respect to b, we find that both s^{PC} and x^{PC} are at first decreasing in b and then increasing. This hinges on the fact that there are two opposing effects of a change in b. On the one hand, an increase in b means that the size of the market decreases. In isolation, this market size effect gives rise to smaller subsidies the larger is b. On the other hand, a larger b also means that the consumers perceive the goods to be better substitutes. Thereby demand becomes more price sensitive, giving each country greater incentives to

create a competitive advantage for its home firm by subsidizing cost-reducing R&D. This business-stealing motive is stronger the better substitutes the goods are, and dominates over the market size effect for sufficiently high values of b. Indeed, both s^{PC} and x^{PC} reach a maximum at $b = 1$, even though this is the point where the size of the market is smallest.[7]

The U-shaped relationship between s^{PC} and b is shown in the left-hand side panel of Figure 1. The Figure also illustrates that trade liberalization (trade costs reduced from $\tau = 1/4$ to $\tau = 0$) gives rise to a positive vertical shift in the curve s^{PC}.

The curve labelled $MC(s = 0)$ in the right-hand side panel of Figure 1 shows that marginal costs ($MC = c - x_i$) are increasing in b if the firms do not receive R&D subsidies. This reflects the negative relationship between b and the size of the market. With subsidies, on the other hand, marginal costs are lowest at $b = 1$.[8]
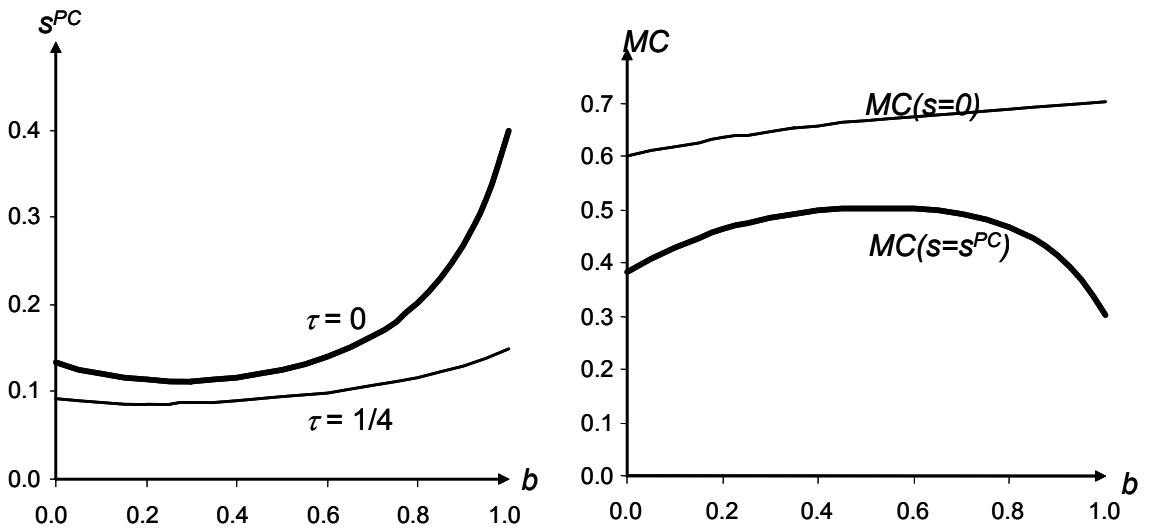
Figure 1: *FOCs for subsidy levels and marginal costs with policy competition.*

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[7]At $b = 0$ we have $s^{PC}_{b=0} = \frac{2}{3}\alpha - \frac{2}{3}c - \frac{1}{6}\tau$ and $x^{PC}_{b=0} = \frac{5}{3}\alpha - \frac{5}{3}c - \frac{2}{3}\tau$, while $s^{PC}_{b=1} = x^{PC}_{b=1} = 2\alpha - 2c - \tau$. Inserting for this into equation (15) we find that $s^{PC}_{b=1} > s^{PC}_{b=0}$ and $x^{PC}_{b=1} > x^{PC}_{b=0}$ whenever τ is so small that trade takes place.

[8]In all the figures we assume that $\alpha = 1$ and $c = 0.8$.

Equilibrium with R&D policy competition

We shall now analyze whether the first-order conditions for subsidies characterize a (unique) equilibrium. To this end we have to check the second-order conditions and the stability of the system. In order to simplify the algebra, we shall in the following assume that $\tau = 0$.

It is straight forward to show that the second-order conditions for the firms' choice of quantities and R&D investments at stage 2 are satisfied. However, when the countries compete in subsidies at stage 1 we find that

$$\frac{\partial^2 W_i}{\partial s_i^2} = -\frac{(3 - b^2)(1 - 2b^2)}{4(1 - b^2)^2},$$

which means that the second-order conditions hold iff $b < b^{SOC} \equiv (1/2)\sqrt{2} \approx 0.707$.

Solving $W_i/\partial s_i = 0$ for the range of b where the second-order conditions are satisfied, we find the reaction function

$$s_i(s_j) = \frac{2(1 - b)(1 + b^2)}{(3 - b^2)(1 - 2b^2)}(\alpha - c) - \frac{(1 + b^2)b}{(3 - b^2)(1 - 2b^2)}s_j. \tag{19}$$

The system is stable if $\left|\frac{\partial s_i(s_j)}{\partial s_j}\right| < 1$. From equation (19) we find that this is satisfied if and only if b is below some critical value b', where $b' \approx 0.591$.

The reaction curves $s_1(s_2)$ and $s_2(s_1)$ are illustrated in Figure 2. The left-hand side panel of Figure 2 shows the reaction curves with $b = 0.5 < b'$, in which case the stability conditions are satisfied. If the countries initially have different subsidy levels - $s_1 > s_2$, say - then each country's best response to the other country's subsidy level leads to a convergence where the countries eventually end up with the same subsidies.[9] The stability conditions are, however, not satisfied in the right-hand side panel of Figure 2, where $b = 0.65 > b'$. Here the figure indicates that we eventually end up with a positive subsidy level in Country 1 and zero subsidies in Country 2 if initially $s_1 > s_2$. The reason is that for $b > b'$ the goods are such close substitutes that one of the countries may find it optimal to set sufficiently high subsidy levels that its domestic firm captures the whole market.

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[9]Here we follow the conventions in the literature and use the terms "reaction" and "response" even though the countries set the subsidy levels simultaneously.

Note that it is sufficient for country 1 to set $s_1 = s_1^A$ in order to ensure that Country 2 sets $s_2 = 0$ (where $s_1^A > s^{PC}$).
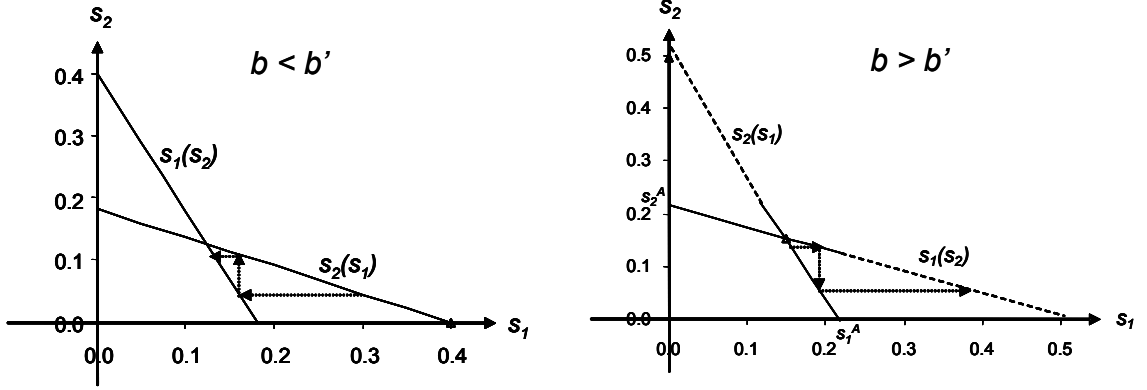
Figure 2: *Stability in R&D competition.*

The right-hand side panel of Figure 2 thus suggests that we have two stable asymmetric equilibria (where only one of the countries grants R&D subsidies) if the symmetric equilibrium is unstable. To verify this, assume that $b \in \left(b', b^{SOC} \right)$, i.e., in the range where the system is unstable but the second-order conditions hold. Suppose Country 1 believes that Country 2 sets $s_2 = 0$. Maximizing welfare in Country 1 with respect to s_1 under the restriction that output and R&D investments in Firm 2 are non-negative, we have (with superscript A for asymmetry)

$$s_1^A = \frac{2(1-b)}{b}(\alpha - c) \tag{20}$$

Inserting for s_1^A and $s_2 = 0$ into equations (15) and (16) we find that Firm 2 will be inactive ($q_{22} = q_{21} = x_2 = 0$). Given that $s_2 = 0$, it is thus optimal for Country 1 to grant such high subsidies that Firm 1 becomes a monopolist. However, comparing with the monopoly subsidy level s_1^M (see equation (12)), we find that $s_1^A - s_1^M = 2(3 - 4b)(\alpha - c)/(3b) > 0$ in the relevant area of b. Country 1 must therefore use a subsidy level which is higher than its first-best choice.[10]

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[10] Country 1 is aware of the fact that the foreign firm at stage 2 invests in R&D and supplies a positive output if $s_2 = 0$ and $s_1 < s_1^A$. As this would have a negative welfare effect in Country 1, it is optimal to set $s_1 = s_1^A > s_1^M$.

Next, suppose that Country 2 believes $s_1 = s_1^A$. We then find

$$\frac{\partial W_2}{\partial s_2} = -\frac{(3 - b^2)(1 - 2b^2)}{4(1 - b^2)^2} s_2 < 0 \text{ for } s_2 > 0 \text{ and } b < 0.707,$$

from which it follows that Country 2's best response to $s_1 = s_1^A$ is $s_2^A = 0$.

We now have:

Proposition 3: *The symmetric equilibrium is stable for $b \in [0, b')$ and unstable for $b \in (b', b^{SOC})$. For $b \in (b', b^{SOC})$ there exist two stable equilibria with $s_i^A = \frac{2(1-b)}{b}(\alpha - c)$ and $s_j^A = 0$ (or vice versa). The subsidy level s_i^A is decreasing in b. Production is equal to zero in the firm that does not receive subsidies.*

The subsidy level s_i^A is decreasing in b because the cost advantage that Country 1 will have to grant its domestic firm in order to foreclose Firm 2 is smaller the less differentiated the consumers perceive the goods to be.

We have now characterized the equilibrium for $b \in [0, b^{SOC})$. For higher values of b there does not exist any equilibrium in pure strategies if the fixed costs f of setting up a research project equal zero. This is due to the fact that the business-stealing effect is then so strong that each country has an incentive to overbid the other in subsidy levels. Indeed, as shown by equation (15), the firms become infinitely sensitive to differences in subsidy levels in the limit $b \to 1$. However, with a fixed cost of setting up research projects, it takes more than a marginal increase in profits to benefit from positive R&D investments, and in the Appendix we show that there exist stable asymmetric equilibria in pure strategies if f is sufficiently high. This equilibrium has the following properties:

Proposition 4: *Assume that $b > b^{SOC}$. There does not exist any equilibrium in pure strategies if $f < (7/9)(\alpha - c)^2$. If $f > (7/9)(\alpha - c)^2$ there exist stable asymmetric equilibria where one country does not provide R&D subsidies ($s_j = 0$) and the other country sets $s_i = s_i^A = \frac{2(1-b)}{b}(\alpha - c)$ for $b \le b'' \equiv 3/4$ and $s_i = s_i^M = 2(\alpha - c)/3$ for $b \in [b'', 1)$. Production is equal to zero in the firm that does not receive subsidies.*

Figure 3 illustrates the equilibrium subsidy levels for the case where f is sufficiently high to ensure the existence of equilibria in pure strategies for all $b \in [0, 1)$. Note in

particular that the subsidy level used by Country 1 is the same if the firms produce independent goods $(b = 0)$ as if $b \in [b'', 1)$. Even if Country 1 could foreclose Firm 2 from the market by setting $s_1 = s_1^A$ in the latter area of b, that would yield a subsidy level lower than the welfare-maximizing one for Country 1.
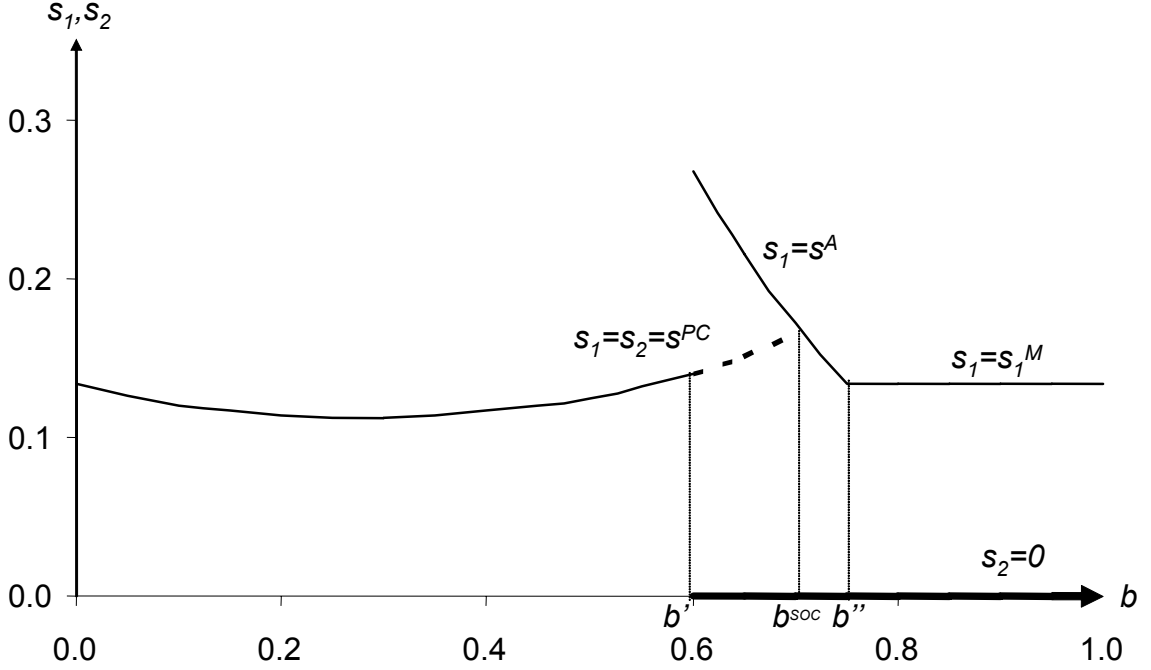
Figure 3: *Equilibrium subsidy levels with policy competition.*

3.3 Policy cooperation

The above analysis shows that there is a rationale for national governments to subsidize R&D; however, there are at least two reasons why the national subsidies are not optimal from a global point of view. First, national governments do not take costs and benefits for foreign consumers into consideration; second, the business-stealing motive and the accompanying policy competition cannot be optimal in a global sense. Hence, there is a need for international policy cooperation; however, it is not obvious what type of cooperation this should be. A natural approach, motivated by the literature on tax competition, would be to argue for harmonization of subsidies across countries. If R&D subsidies are bound to be at the same level in the two countries, there will be no policy game, and the

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subsidies could be used to correct for the public-goods aspects of R&D. In section 3.3.1 harmonized R&D policies are studied, and the implications of such policies are discussed. Harmonization of R&D subsidies implies a symmetric outcome in the two countries, with the same R&D levels and identical quantities produced and sold. While such a symmetric outcome may seem reasonable given that the countries and the firms are symmetric, it is, in fact, not always welfare maximizing from a global point of view. In section 3.3.2 we study optimal cooperative policies, and show that depending on the degree of product differentiation, the optimal global solution could either be one with the same subsidies to both firms or one where only one of the firms is subsidized.

3.3.1 Optimal harmonized R&D subsidies

Suppose that the countries harmonize their R&D subsidies at a common level $s_1 = s_2 \equiv s$. An optimal harmonization policy requires that the common subsidy level is chosen so as to maximize aggregate welfare, which is given by

$$W = W_1 + W_2. \tag{21}$$

Assuming that the fixed R&D costs f are sufficiently small that both firms perform R&D, we solve $\partial W/\partial s = 0$ to find that the subsidy level is given by (with superscript H for harmonization):[11]

$$s^H = \frac{2}{1 + 3b + b^2}\left(\alpha - c\right); \; \frac{\partial s^H}{\partial b} < 0. \tag{22}$$

The subsidy level is thus monotonically decreasing in b. This is true for two reasons. First, because the size of the market is decreasing in b. Second, because there is stronger competition between the firms the less differentiated goods they produce. All else equal, higher competition implies that output for each firm increases, and thus their incentives to invest in cost reduction. This in turn means that the need to provide R&D subsidies is lower the higher is b.
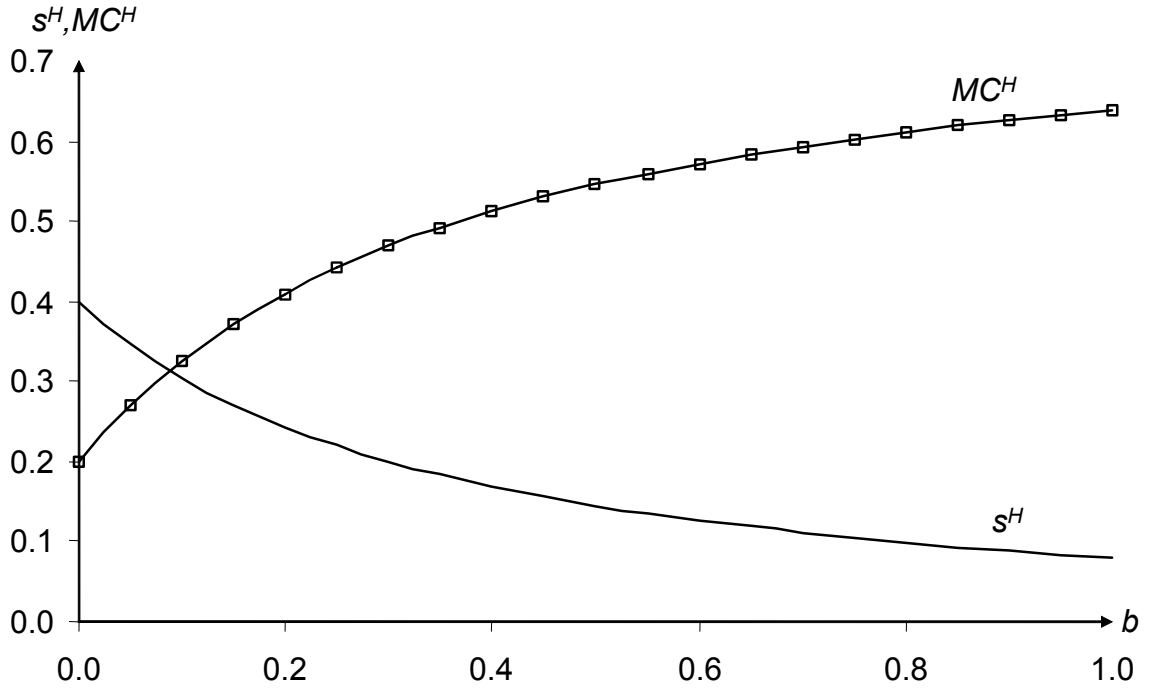
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[11] The second-order condition equals $\frac{\partial^2 W}{\partial s^2} = -\frac{b^2 + 3b + 1}{(1+b)^2}$, and is thus negative for all $b \geq 0$.

Inserting for s^H into equation (15) we have

$$x_i^H = \frac{3 + b}{1 + 3b + b^2} (\alpha - c) ; \quad \frac{\partial x_i^H}{\partial b} < 0.$$

When the countries harmonize their subsidies, we thus see that the larger is b, the lower are subsidy levels and R&D investments. The latter implies that marginal costs are increasing in b, as illustrated in Figure 4.

Figure 4: *Subsidy levels and marginal costs with harmonized subsidies.*

We can now state:

Proposition 5: *Suppose that the countries choose a common subsidy level that maximizes aggregate welfare. Subsidy levels are then lower, and marginal production costs higher, the closer substitutes the consumers perceive the goods to be.*

Using equations (3), (4), (5) and (22) we find that aggregate welfare in this case equals

$$W^H = \frac{2(b + 3)}{b^2 + 3b + 1} (\alpha - c)^2 - 2f. \tag{23}$$

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3.3.2 Optimal cooperative R&D subsidies

The harmonization policy internalizes the business-stealing effect and takes into account the consumer interests in both countries. However, with this policy the countries have only one policy instrument at hand; the common subsidy level s. The number of instruments doubles if the countries allow the subsidy levels s_1 and s_2 to differ, in which case aggregate welfare must be at least as high as if a common subsidy level were chosen. Optimal R&D cooperation between the countries thus solves $\{s_1, s_2\} = \arg\max\{W\}$, where $s_1 \lessgtr s_2$.

Solving $\partial W/\partial s_i = 0$ $(i = 1, 2)$ we find that the first-order conditions still imply $s_i = s^H$, i.e., the same subsidy level as in the case with policy harmonization. However, assume that for some reason only one good is produced. The optimal subsidy level to be granted to this firm is[12]:

$$s_i = s_i^* = 2(\alpha - c) \tag{24}$$

Recall from equation (12) that $s_i = s_i^M = \frac{2}{3}(\alpha - c)$ in a non-cooperative equilibrium for $b = 0$. The intuition for why $s_i^* > s_i^M$, is that the cooperative equilibrium maximizes aggregate welfare, which in particular includes consumer surplus in both countries.

To be specific, suppose that only Firm 1 is active. Letting $s_1 = s_1^*$ and setting $q_{22} = q_{21} = x_2 = 0$ we find that welfare equals:

$$W^* = 3(\alpha - c)^2 - f. \tag{25}$$

Equations (23) and (25) imply that

$$W^* - W^H = \frac{3b^2 + 7b - 3}{b^2 + 3b + 1}(\alpha - c)^2 + f. \tag{26}$$

Clearly, it may be optimal to produce only one good if f is sufficiently high (we abstract from the trivial case where f is so high that it is unprofitable to produce any of the goods). However, equation (26) implies that aggregate welfare is higher with one than with two goods even in the limit case where $f = 0$ if

$$b > \tilde{b}' \equiv \frac{1}{6}(\sqrt{85} - 7) \approx 0.37.$$

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[12]Technically this is found by setting $q_{jj} = q_{ji} = 0$ and recalculating the system of equations. However, an easier approach which yields the same result is simply to set $b = 0$ into equation (22).

If it is optimal that only Firm 1 produces, the countries should obviously not subsidize R&D in Firm 2. Indeed, it may be necessary to tax R&D in that firm. To see this, suppose that Firm 1 receives the socially optimal subsidy level s_1^*. From equations (15) and (16) we then find that a sufficient condition to prevent Firm 2 from being active is that

$$
\begin{aligned}
s_2 &= -2\left(1 - 2b\right)\left(\alpha - c\right) \ \text{for } b < \tilde{b}'' \\
s_2 &= 0 \ \text{for } b > \tilde{b}'',
\end{aligned}
$$

where $\tilde{b}'' = 1/2$. Thus, if $b < \tilde{b}''$ the countries will have to tax away any incentives that Firm 2 may have to invest in R&D.[13] The required tax level is decreasing in b, reflecting the fact that Firm 2's ability to charge a higher price than Firm 1 is smaller the better substitutes the goods are. If $b \geq \tilde{b}''$ the goods are such close substitutes that $s_2 = 0$ ensures that Firm 2 will not be competitive.

If $f > 0$ welfare is higher with only one good than in the symmetric case with two goods also for $b < \tilde{b}'$. Denote by $\tilde{b}^f$ the value of b which ensures that $W^* > W^H$ for any given value of $f \geq 0$. We can now state:

Proposition 6: *Suppose that $b > \tilde{b}^f$ and that the countries can levy R&D taxes to foreclose one of the firms from the market. The optimal R&D subsidy for the active firm is $s_i^* = 2(\alpha - c)$, and welfare is higher than if both firms are active and receive the optimal harmonized subsidy level s^H. The optimal subsidy level for $b < \tilde{b}^f$ is $s_i = s_j = s^H$.*

By taxing R&D in one of the firms for $b > \tilde{b}^f$, the countries are able to prevent unnecessary duplication of R&D expenses. It may be argued, though, that it is unrealistic to assume that the countries can tax R&D. Moreover, the foreclosure policy lets one of the firms monopolize the market, and this leads to higher consumer prices than if both firms are active. One may therefore conjecture that welfare would be higher if Firm 2 is only partly foreclosed from the market. However, in the Appendix we prove the following:

Proposition 7. *Assume that R&D subsidies must be non-negative and that $b > \tilde{b}^f$.*

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[13]If $f = 0$ the marginal cost of doing the first bit of R&D is zero, in which case it is optimal with some R&D if output is positive. Therefore s_2 must be chosen such that $q_{22} = q_{21} = 0$.

Unless $(s_i, s_j) = (2(\alpha - c), 0)$ completely forecloses one of the firms from the market, welfare is lower than if the countries can tax R&D.

Above, we noted that the symmetric subsidy level $s = s^H$ is optimal for $b < \tilde{b}^f$, while Firm 2 will never be active in the market if $s_2 = 0$ and $s_1 = 2(\alpha - c)$ for $b > \tilde{b}''$.[14] In these cases welfare is the same whether or not we allow for R&D taxation. Otherwise there exists an interval in the range $b \in \left(\tilde{b}^f, \tilde{b}''\right)$ where Firm 2 will be active if the countries cannot tax R&D. There are two main reasons for why this has a negative welfare effect. The first pertains to duplication of R&D costs, as already discussed. The second reason is that Firm 2 - which does not receive R&D subsidies - will have higher marginal production costs than Firm 1. For any given industry output, aggregate production costs are thus minimized if all production takes place in Firm 1.[15] This latter effect is precisely the reason why Salant and Shaffer (1999) argue that it may be welfare improving to grant higher R&D subsidies to one firm than to another even if the firms are intrinsically symmetric.

Figure 5 illustrates the relationship between optimal R&D subsidy levels and b graphically when we allow R&D taxes. For $b \leq \tilde{b}'$ the firms receive the same subsidy level s^H, while we have $s_1 = s_1^*$ for $b > \tilde{b}'$. This is the optimal subsidy when only one good is produced, and is therefore independent of b. R&D taxes ensure that Firm 2 is completely foreclosed from the market if $b \in \left(\tilde{b}', \tilde{b}''\right)$, but the need for R&D taxation in that firm is smaller the less differentiated goods the firms produce. For $b > \tilde{b}''$ (potential) competition between the firms is so fierce that it is unnecessary to tax Firm 2.

Both with policy competition (see Figure 3) and with policy cooperation we thus have that the equilibrium is symmetric for sufficiently low values of b, while only one firm is active for higher values of b. In this sense there are clear similarities between the outcome with policy competition and cooperation, even though the reasons for the asymmetry are fundamentally different.

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[14] This is true even for $f = 0$.

[15] In principle the countries could set s_1 so high that the cost-reducing R&D investments in Firm 1 are large enough to keep Firm 2 out of the market for all $b > \tilde{b}^f$. However, the convexity of the R&D cost function implies that this would be too expensive. See Appendix
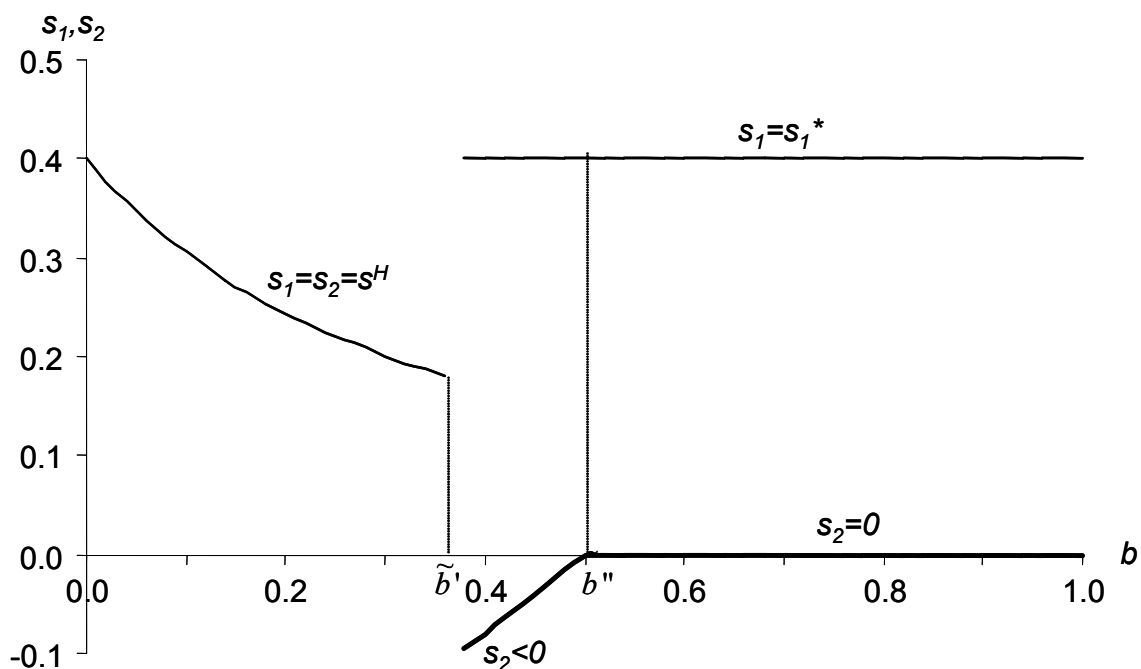
Figure 5. *Equilibrium subsidy and tax levels with optimal policy cooperation.*

4   Concluding remarks

In this paper we have studied optimal industrial R&D investments in an international setting. In a simple model with two countries hosting one firm each, we have looked at the firms' R&D decisions and the governments' incentives to influence R&D levels through subsidies. Both non-cooperative policies and coordinated international policies are studied; for national (non-cooperative) policies there are both a public-goods motive and a business-stealing motive for R&D policies. With coordinated policies, the business-stealing motive disappears, while the public-goods motive is reinforced. A number of interesting conclusions come out of the analysis.

First, it is shown that international trade and trade costs are important for the firms' choice of R&D as well as for the governments' optimal policies towards R&D. Liberalization implies that the firms find it optimal to increase their cost-reducing R&D investments, since the market becomes bigger. And higher R&D implies lower marginal costs, lower

prices and more sales. The government - realizing that the benefits any given level of R&D support then goes up - finds it optimal to raise the subsidy. Freer international trade thus implies more R&D, higher R&D subsidies and more sales, possibly also in the domestic market. The policy effects do not rely on any business-stealing motive; even for a monopoly it would be the case that optimal R&D subsidies and domestic sales increase when trade costs go down.

Second, we study in some detail policy competition between two governments pursuing national interest. Contrary to most of the literature, we explicitly include the effects for domestic consumers in the analysis. We find that the effects of policy competition depend critically on the characteristics of the market. If the goods are poor substitutes, competition between the firms is not very strong, and for the governments the public-good motive for subsidies is more important than the business-stealing one. In such industries there will typically be a symmetric outcome, where both governments subsidize R&D in the domestic firm, and where both firms invest in R&D and sell their products in the two markets. When the goods are close substitutes, on the other hand, the business-stealing motive for subsidies dominates, and competition may become so tough that only one firm survives in the market.

Third, we analyze policy cooperation, and look at the optimal R&D policy from a global point of view. Given the potentially harmful effects of policy competition, it is not difficult to see why there is a need for policy coordination. However, contrary to what one might expect, policy cooperation does not necessarily lead to a harmonization of the subsidies to the two firms. In fact, our analysis shows that when goods are fairly close substitutes, an optimal cooperative policy may imply that only one of the firms receives R&D subsidies, and that the other firm ceases to produce. Hence, the surprising result is that both with policy competition and policy coordination we may end up with an asymmetric equilibrium where one firm monopolizes the market.

In the model we have assumed a two-stage game where the firms at the second stage determine R&D and output simultaneously. Many of the contributions to the literature assume three stages, such that the firms at the second stage (i.e. after the subsidies are set) determine the R&D investments, and at the third stage produce and sell the goods.

This assumption is not critical for our main results. With a three-stage game, there would be strategic motives for the firms' R&D decisions in addition to the cost-minimizing ones, but that would not change our results qualitatively. A second assumption to discuss, is the specific cost function for R&D. In the analysis it was shown that the fixed costs of an R&D project could be important for the existence of asymmetric equilibria. The same applies with respect to the convexity of the cost function. In particular, the existence of a stable, symmetric equilibrium is more likely the more convex the cost function (see also Leahy and Neary, 2004). Hence, the exact outcomes that we find may depend on the specific cost function. However, the main conclusions regarding the effects of trade liberalization and the possibilities of asymmetric as well as symmetric equilibria remain valid also with more general R&D functions.

5 Appendix

Proof of Proposition 4:

Assume that $b > b^{SOC}$, in which case $\partial^2 W_i / \partial s_i^2 > 0$. If neither country grants subsidies we find that

$$W_i^{s=0} = \frac{2+b}{(1+b)^2} (\alpha - c)^2 - f.$$

This is an equilibrium if none of the countries has incentives to depart from zero subsidies. However, with $s_2 = 0$ we find

$$\frac{\partial W_1}{\partial s_1} = \frac{(1+b^2)}{2(1+b)^2(1-b)}(\alpha - c) + \frac{(3-b^2)(2b^2-1)}{4(1-b^2)^2}s_1 > 0, \tag{27}$$

which means that welfare in Country 1 is monotonically increasing in s_1. This implies that Country 1 will choose a subsidy level which is so high that Firm 2 is foreclosed from the market. The first-best subsidy level for Country 1 if Firm 2 is foreclosed, is the monopoly subsidy level $s_1^M = 2(\alpha - c)/3$. However, Firm 2 will not be foreclosed as long as $s_1^A = \frac{2(1-b)}{b}(\alpha - c) > s_1^M$, which is true for $b < b''$.

We will now analyze the cases $b \in \left[b^{SOC}, b''\right]$ and $b \in [b'', 1]$ separately.

Case A: $b \in \left[b^{SOC}, b''\right]$.

In order to ensure $q_{22} = q_{21} = 0$ at stage 2 for $b \in \left[b^{SOC}, b''\right]$, Country 1's best response to $s_2 = 0$ is $s_1^A = \frac{2(1-b)}{b}(\alpha - c)$ (the same as in the range $b \in \left(b', b^{SOC}\right)$. This subsidy level is higher than Country 1's first-best subsidy, but the lowest which forecloses the foreign firm at stage 2.

With $(s_1, s_2) = \left(s_1^A, 0\right)$ we find that welfare in the two countries equals

$$W_1^A = \frac{8b - 2b^2 - 3}{2b^2}(\alpha - c)^2 - f \text{ and } W_2^A = \frac{(\alpha - c)^2}{2b^2}.$$

Since

$$W_1^A > W_1^{s=0},$$

it follows that Country 1's best response to $s_2 = 0$ is $s_1 = s_1^A$ also for $b \in \left[b^{SOC}, b''\right]$.

What is Country 2's best response to $s_1 = s_1^A$? Setting $s_1 = s_1^A$ we have

$$\frac{\partial W_2}{\partial s_2} = \frac{\left(3 - b^2\right)\left(2b^2 - 1\right)}{4\left(1 - b^2\right)^2} s_2 > 0 \text{ for } s_2 > 0.$$

This means that if Firm 2 performs R&D, then it will be optimal for Country 2 to grant subsidies which foreclose Firm 1 from the market (in which case Country 1's belief that $s_2 = 0$ and that Firm 2 is foreclosed from the market is wrong). Solving $q_{11} = q_{12} = 0$ with respect to s_2 for $s_1 = s_1^A$ we find $s_2' = \frac{2\left(1 - b^2\right)}{b^2}\left(\alpha - c\right)$ and

$$W_2' = \frac{8b^2 - 2b^4 - 3}{2b^4}\left(\alpha - c\right)^2 - f.$$

Given that $s_1 = s_1^A$, it is not profitable for Country 2 to grant subsidies if $W_2^A > W_2'$. This inequality holds if

$$f > f' \equiv \frac{\left(3 - b^2\right)\left(2b^2 - 1\right)}{2b^4}\left(\alpha - c\right)^2,$$

which reaches a maximum at $b = b''$, where $f' = (13/27)\left(\alpha - c\right)^2$. We can therefore conclude that there exists an asymmetric equilibrium $(s_i, s_j) = \left(\frac{2(1-b)}{b}\left(\alpha - c\right), 0\right)$ for $b \in \left[b^{SOC}, b''\right]$ if $f > (13/27)\left(\alpha - c\right)^2$.[16]

Case B: $b \in [b'', 1)$

Given that $s_2 = 0$ and $b \in [b'', 1)$, Country 1 will use its first-best subsidy level $s_1 = s_1^M$ to foreclose Firm 2 from the market. Welfare in the two countries is then equal to

$$W_1^B = \frac{15}{9}\left(\alpha - c\right)^2 - f \text{ and}$$
$$W_2^B = \frac{8}{9}\left(\alpha - c\right)^2.$$

Using the same procedure as above, we find that Country 2's best response to $s_1 = s_1^M$ is $s_2 = \frac{2(4 - 3b)}{3b}\left(\alpha - c\right)$ or $s_2 = 0$, depending on the size of the fixed costs. With the subsidy levels $\left(s_1, s_2''\right) = \left(s_1^M, \frac{2(4 - 3b)}{3b}\left(\alpha - c\right)\right)$ we have

$$W_2'' = \frac{16b - 3b^2 - 8}{3b^2}\left(\alpha - c\right)^2 - f.$$

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[16] Comparing welfare in the two countries we find $W_1^A - W_2^A > 0$ if $f < f^{crit} \equiv \frac{4b - b^2 - 2}{b^2}\left(\alpha - c\right)^2$.

Subtracting $W_2^B - W_2''$ we find that $s_2 = 0$ is Country 2's best response to $s_1 = s_1^M$ for all $b \in [b'', 1)$ if

$$f > \frac{7}{9} (\alpha - c)^2 .$$

Q.E.D.

Proof of Proposition 7

The second-order conditions for optimal subsidies when the subsidy levels may differ are

$$\frac{\partial^2 W}{\partial s_i^2} = -\frac{1 - 4b^2 + b^4}{2(1 - b^2)^2} < 0 \text{ for } b < \frac{1}{2} \left(\sqrt{6} - \sqrt{2} \right) \approx 0.52$$

$$\left(\frac{\partial^2 W}{\partial s_i^2} \right) \left(\frac{\partial^2 W}{\partial s_j^2} \right) - \left(\frac{\partial^2 W}{\partial s_i \partial s_j} \right)^2 = \frac{1 - 7b^2 + b^4}{4(1 - b^2)^2} > 0 \text{ for } b < \frac{1}{2} \left(3 - \sqrt{5} \right) = 0.38,$$

which means that the SOCs are satisfied only if $b < \hat{b}^{SOC} \equiv \frac{1}{2}(3 - \sqrt{5}) = 0.38$. For $b < \hat{b}^{SOC}$ we therefore have a local optimum with symmetric subsidies (this corresponds to what Leahy and Neary (2004) label Restricted Cooperative Substitutability). However, this is not necessarily a global optimum, since we know from equation (26) that $(W^* - W^H) > 0$ if $b > \tilde{b}^f$, where $\partial \tilde{b}^f / \partial f < 0$ and $\tilde{b}^f = \tilde{b}' \equiv \frac{1}{6} \left(\sqrt{85} - 7 \right) \approx 0.37$ for $f = 0$.

Suppose that $f = 0$. For $b > \tilde{b}'$ we then have to look for corner solutions. It is straight forward to show that it is inoptimal to set $s_1 = s_2 = 0$. This leaves us with the following candidates for optimum:

I) Set $s_2 = 0$ and choose a welfare maximizing level of s_1, possibly without foreclosing Firm 2 (alternatively, choose s_2 optimally, given that $s_1 = 0$)

II) Set $s_2 = 0$, and choose s_1 such that Firm 2 is completely foreclosed (alternatively, set $s_1 = 0$, and choose s_2 such that Firm 1 is completely foreclosed)

III) Set s_1 at the optimal level, given that only Firm 1 is present in the market (alternatively, set s_2 at the optimal level, given that only Firm 2 is present in the market).

IV) Set s_1 optimally, given that Firm 2 is foreclosed by setting $s_2 \leq 0$.

Case I:

Setting $s_2 = 0$ we have

$$\frac{\partial^2 W}{\partial s_1^2} = -\frac{1}{2}\frac{b^4 - 4b^2 + 1}{(b-1)^2(1+b)^2}$$
$$< 0 \text{ for } b < \frac{1}{2}\left(\sqrt{6} - \sqrt{2}\right) \approx 0.52.$$

Provided that all non-negativity constraints are satisfied, we can solve $\partial W/\partial s_1 = 0$ to find (with superscript to signify Case I):

$$s_1^I = \frac{2(1-b)^2}{b^4 - 4b^2 + 1}(\alpha - c); \quad \frac{\partial s_1^I}{\partial b} > 0. \tag{28}$$

Inserting for s_1 and s_2 into equation (16) we find

$$x_1^I = \frac{3 - 5b + b^3}{1 - 4b^2 + b^4}(\alpha - c)$$

and[17]

$$x_2^I = \frac{b^3 - b^2 - 2b + 1}{1 - 4b^2 + b^4}(\alpha - c).$$

We now have $x_2^I > 0$ for $b < \hat{b}' \equiv 0.44$, in which case welfare is given by

$$W^I = \frac{2b^3 - 10b + 5}{1 - 4b^2 + b^4}(\alpha - c)^2. \tag{29}$$

Case II:

Setting $s_2 = 0$ we find that $q_{22} = q_{21} = x_2 = 0$ if

$$s_1^{II} = s_1^A = 2\frac{1-b}{b}(\alpha - c). \tag{30}$$

Equations (16) and (30) yield

$$x_1^{II} = \frac{2-b}{b}(\alpha - c)$$

and

$$W^{II} = \frac{4b - b^2 - 1}{b^2}(\alpha - c)^2. \tag{31}$$

---

[17] From this we find that a sufficient condition for $c - x_1 > 0$ is that $c/\alpha > 0.778$.

Case III:

Given that Firm 2 does not produce, the optimal subsidy level to Firm 1 equals[18]

$$s_1^{III} = 2(\alpha - c) = s_1^*, \tag{32}$$

from which it follows that

$$x_1^{III} = 3(\alpha - c).$$

This yields the welfare level

$$W^{III} = 3(\alpha - c)^2. \tag{33}$$

Case III is relevant only if $s_2 = 0$ and $s_1 = s_1^{III}$ do not lead Firm 2 to invest in R&D and produce at stage 2. Using equations (15) and (16) we find that this holds for $b \geq \tilde{b}''$.

Case IV:

Suppose the countries set $s_1 = s_1^{III}$. From equation (15) we find that $(q_{22} + q_{21}) = 0$ if

$$s_2^{IV} = -2(1 - 2b)(\alpha - c) < 0 \text{ for } b < 1/2. \tag{34}$$

In this case Firm 2 will not produce or invest in R&D, and we therefore have

$$W^{IV} = 3(\alpha - c)^2. \tag{35}$$

Proof that welfare is highest with complete foreclosure of Firm 2 if $b > \tilde{b}'$:

Comparing equations (29), (31) and (35) we find that welfare is highest in Case IV, where R&D taxes imply that Firm 2 is inactive. It can further be shown that welfare is higher by choosing $s_2 = s_2^{IV}$ than by setting s_2 such that $x_2 = 0$; in the latter case Firm 2 would have positive output for $b < 0.47$ with an optimal choice of s_1 (even though it does not invest in R&D). If we allow R&D taxes, we thus see that the countries would prefer to completely foreclose Firm 2 from the market for $b > \tilde{b}'$. Q.E.D.

Equilibrium if R&D taxes are not available:

Suppose that we require $s_i \geq 0$. Comparing equations (29) and (31) we then have that welfare is highest if Firm 2 is not completely foreclosed from the market for $b < \hat{b}'$. We

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[18] This is most easily found by setting $b = 0$ in equation (22).

further find from equations (31) and (33) that welfare is higher with $s_1 = s_1^{III}$ than with $s_1 = s_1^{II}$, which is feasible for $b > \tilde{b}''$.

Figure A1 shows the relationship between R&D subsidies and b if R&D taxes are not available and $f = 0$. For $b \in \left(b^{SOC}, \hat{b}' \right)$ the optimal non-negative subsidies are $s_2 = 0$ and $s_1 = s_1^I$. In this area Firm 2 will produce and invest in R&D; complete foreclosure of Firm 2 through granting sufficiently high subsidies to Firm 1 would be too expensive. This is due to the convexity of the R&D cost function and the relatively low competitive pressure between the firms when b is 'small'. Additionally, there is also a gain for the consumers of having access to both varieties. This advantage is smaller, though, the less differentiated the goods are. Therefore s_1^A is increasing in b.

For $b \in \left(\hat{b}', \tilde{b}'' \right)$ we have $s_1 = s_1^{II}$. The goods are then such close substitutes that it is beneficial for the countries to completely foreclose Firm 2 from the market by providing relatively high R&D subsidies to Firm 1. However, given that there is only one good in the market, this subsidy level is higher than the first-best subsidy level $s = s_1^{III} = s_1^*$. The latter is obtainable only for $b > \tilde{b}''$, in which case the goods are sufficiently close substitutes to make Firm 2 uncompetitive with $s_2 = 0$ and $s = s_1^{III}$.
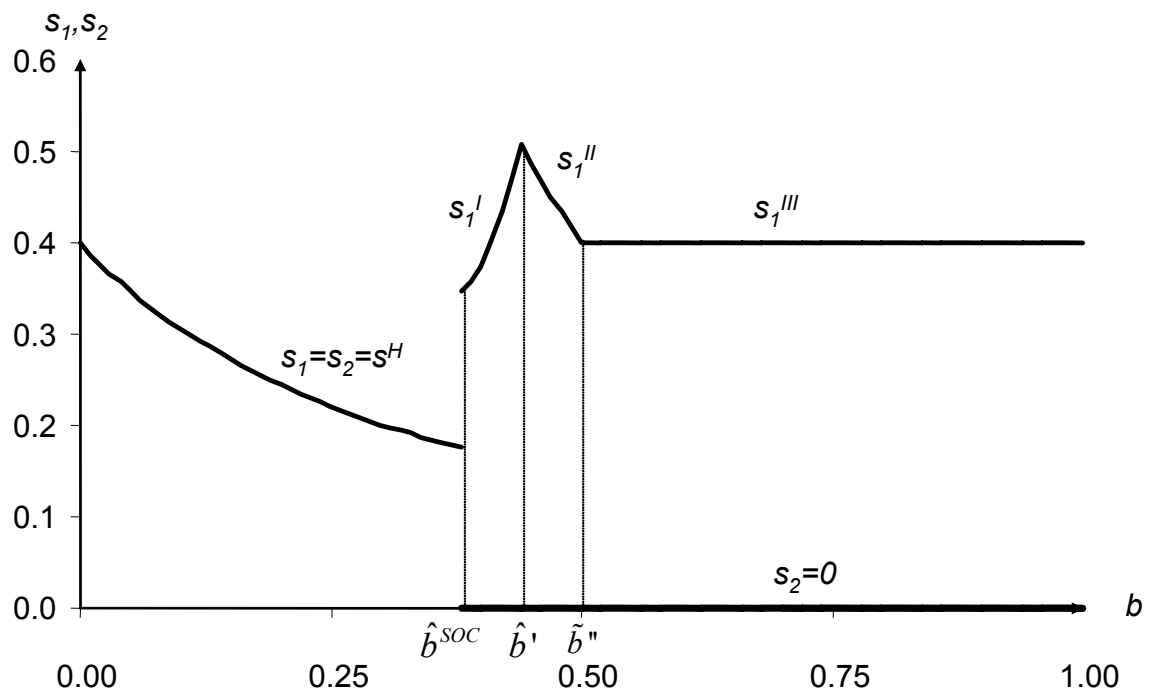
Figure A1: *Equilibrium subsidies with policy cooperation when R&D taxes are not available.*

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