

# The risk sharing effects of social security and the stochastic properties of income growth\*

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Discussion Paper 7/03

## Abstract

Time-series analyses suggest that income shocks are fairly persistent. This has implications for the intergenerational risk-sharing effects of pay-as-you-go (paygo) social security programs. By means of a simple stochastic specification, we derive theoretically how the variance of individuals' lifetime income depends on the degree of persistence in the income shocks and the magnitude of the paygo program. A low or medium degree of persistence ensures that properly scaled paygo programs provide intergenerational diversification of income risk. On the other hand, a somewhat higher degree of persistence may well imply that paygo programs in fact increase the exposure to income shocks. Taking into account that it is hard to reject that income shocks are permanent (and income follows a random walk) in many countries, we can not exclude that many individuals in the OECD area do face a heightened exposure to income risk as a consequence of the actual social security programs financed on a paygo basis.

**JEL Classification:** H55, D91, E32

**Keywords:** Social security, risk sharing, persistence in income shock

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\* I am indebted to Hans Fehr and Egil Matsen for discussion and valuable comments. Financial support from the Norwegian Research Council is gratefully appreciated.

## 1. Introduction

A main conclusion in a couple of seminal papers on the design and effects of social security programs under uncertainty is that paygo financing provides intergenerational diversification of individuals' income risk, see Gordon and Varian (1988) and Enders and Lapan (1982, 1993). The basic intuition is that paygo financing implies that the working generation shares their income shocks with the retired generation through the transfer of parts of their income to the pensioners. Eventually, this working generation becomes the new retired generation and must in return share the subsequent income shocks with the next generation. Ex-ante this leads to a reduced life-income risk for all generations.

A closer look at the analyses leading to the favorable risk sharing conclusion reveals that they rely on specific stochastic specifications of income (or productivity which in turn determines income). The papers of Enders and Lapan as well as those of Gordon and Varian all assume that income shocks have zero persistence and consequently that trend income is deterministic. Taking into account that the research on the stochastic properties of trend output growth and the degree of persistence in output, income and productivity shocks in general has led to fairly ambiguous conclusions – and does not support a zero persistence assumption – we may argue that studies of the risk sharing effects of paygo schemes should be based on a more generalized stochastic specification of income. At the outset we note as an extreme example that a unit-root assumption, which implies that all shocks are permanent, turns out to remove the scope for intergenerational income risk sharing completely. In such a case, no paygo scheme or alternative intergenerational tax-transfer scheme can reduce the exposure of each generation to its own income shock. Consequently, this note analyzes the intergenerational risk sharing effects of paygo schemes in a framework that allows any specification of the degree of persistence in the income shocks.

During the last two decades the testing for stochastic trends in output and other macroeconomic variables has focused on unit-root testing and assessments of the degree of persistence in the data. As surveyed by Balke (1991), the early contributions to this debate mainly concluded in favor of the unit root property, see for example Nelson and Plosser (1982) and Campbell and Mankiw (1989). Thus, shocks seemed to be permanent and the trend stochastic. More recent contributions yield more ambiguous results. A basic insight highlighted by Christiano and Eichenbaum (1990) and also discussed in a useful way by Romer (2001, section 4.8), is that it is nearly impossible to distinguish between a stochastic trend and a deterministic trend in combination with a cyclical component that features a very

high degree of persistence. Thus, while evidence of stochastic trend remains mixed, we may still conclude that data do not support an assumption of zero persistence in output, income and productivity. It seems fair to conjecture that the degree of persistence is significantly above zero but still maybe somewhat below unity (where unity indicates 100 per cent persistence - i.e. permanent shocks) . Such a view seems to be reflected in the literature dealing with calibrated stochastic business cycle models. Nearly “all” such models seem to assume that the degree of persistence in productivity shocks is very high and amounts to values just barely below unity, see for example the 0.95 persistence parameter assumed by Freeman and Kydland (2000).

This note contributes to the literature on social security schemes under uncertainty. The basic income risk sharing conclusion of Gordon and Varian (1988) and Enders and Lapan (1982, 1993) has recently been debated by Thøgersen (1998), Borgman (2002) and Wagener (2003). These papers argue that the income risk sharing conclusion - from an ex-ante perspective (at the time of enactment of the program) - hinges on a fixed contribution rate feature of the actual paygo system. If the system rather features a fixed replacement rate, individuals will face increased income risk. Neither of these papers questions the assumption of zero persistence in the income shocks, however.

This paper is also related to a small literature that studies the design of social security systems by means of a portfolio choice approach see Merton (1983), Dutta et al. (2000) and Matsen and Thøgersen (2001). These studies interpret a paygo system as a non-marketable asset with an implicit stochastic return equal to the rate of economic growth. It turns out that the analyses in these papers do not capture the zero persistence property of the papers by Gordon and Varian and by Enders and Lapan - but rather resort to the alternative extreme assumption of 100 per cent persistence. Still, this assumption is not discussed in any detail. Because they focus on the optimal split between investments in financial assets (by means of a funded pension system) and implicit investments in the paygo “quasi-asset”, the implications of alternative assumptions regarding the degree of persistence in the income shocks are not highlighted.

The next section presents a simple overlapping generations model, which essentially is a slightly adjusted version of the model suggested by Gordon and Varian (1988) and also utilized by Thøgersen (1998). The innovation in this paper is that the specification of the stochastic income process is altered in order to capture any assumption about the degree of persistence, i.e. an exogenous persistence parameter can capture any value between zero (no persistence) and one (permanent shocks). Section 3 presents our theoretical analysis of the

intergenerational income risk sharing effects of paygo social security systems. We derive closed form theoretical expressions that capture how the variance of individuals' net life income is determined by the size of the paygo system and the degree of persistence in the income shocks. Simple numerical calculations suggest that the paygo pension systems in many OECD economies may well lead to an increase in individuals' exposure to income risk – provided that we accept the view that income shocks are highly persistent. Section 4 offers some final remarks.

## 2. The model

We consider a stylized overlapping generations model that is a version of the model analyzed by Gordon and Varian (1988) and also utilized by Thøgersen (1998) and Borgman (2002). For simplicity both the rate of population growth and the real interest rate are equal to zero. The representative individual in any given generation  $t$  ( $t = 0, 1, 2, \dots$ ) has a two-period life cycle. In the first period of life (period  $t$ ) he supplies inelastically one unit of labor, receives a stochastic gross wage given by  $w_t$  and pays a social security contribution  $\tau_t$ . In the second period of life (period  $t+1$ ), he is retired and receives a public pension benefit  $p_{t+1}$  (see below). There are no bequests.

The development of the gross wage is described by an autoregressive model of the form:

$$(1) \quad w_t = w_0 + \mathbf{b}(w_{t-1} - w_0) + \mathbf{e}_t, \quad \forall t = 1, 2, 3, \dots$$

Here  $w_0$  is the fixed wage in the initial period 0 and the parameter  $\mathbf{b}$  measures the degree of persistence in the wage shocks ( $0 \leq \mathbf{b} \leq 1$ ). The random shock  $\mathbf{e}_t$  is serially uncorrelated and characterized by a zero mean and a variance,  $\mathbf{s}^2$ , that is constant over time. We can imagine that the wage shocks reflect underlying technology shocks.

We observe that the extreme case of zero persistence,  $\mathbf{b} = 0$ , implies that

$$(2) \quad w_t = w_0 + \mathbf{e}_t,$$

which means that the wage is characterized by a fixed (and flat) trend equal to  $w_0$ . In the more general case of  $0 < \mathbf{b} \leq 1$ , we can rewrite (1) as

$$(3) \quad w_t = w_0 + \sum_{s=1}^t \mathbf{b}^{t-s} \mathbf{e}_s .$$

Clearly, as long as  $\mathbf{b} < 1$ , the effect of a given shock gradually dies out. It also follows from (1) that the case of  $\mathbf{b} = 1$ , i.e. the other extreme case, yields a random-walk. This implies that shocks are permanent and equation (3) simplifies to

$$(4) \quad w_t = w_0 + \sum_{s=1}^t \mathbf{e}_s .$$

The net lifetime income of a representative individual in generation  $t$  is

$$(5) \quad y_t = w_t - \mathbf{t}_t + \mathbf{p}_{t+1} .$$

Following Gordon and Varian (1988), Thøgersen (1998), Shiller (1999) and Borgman (2002), we assume that the expected utility of the representative generation  $t$  individual is given by the mean-variance specification

$$(6) \quad U_t = u[E(y_t)] - v[Var(y_t)],$$

where  $u' > 0$  and  $v' < 0$ . This specification allows a fruitful discussion of intergenerational risk sharing, while issues related to consumption smoothing within each generation's life-span are disregarded *per se*. We note that the argument in the utility function could equivalently be specified as consumption - when consumption takes place in the second period only, see Ball and Mankiw (2001) and Gordon and Varian.

The sole objective of the government is to run a paygo social security system.<sup>1</sup> Recalling that there is no population growth and disregarding both other public expenditures

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<sup>1</sup> As discussed by Gordon and Varian (1988), we may of course imagine various sorts of complicated income transfer systems between many or "all" future generations. We consider only straightforward paygo systems, however, and do believe that such schemes are the most realistic ones when it comes to implementation.

and any initial public debt, we simply obtain  $t_t = p_t$ , i.e. the social security contribution from the representative member of the young generation in period  $t$  must be equal to the pension benefit to the representative old in the same period. Defining  $g$  as a fixed contribution rate,  $0 \leq g \leq 1$ , we have that

$$(7) \quad t_t = gw_t = p_t.$$

Using (7) and (5), we obtain

$$(8) \quad y_t = (1-g)w_t + gw_{t+1}.$$

### 3. Paygo programs and ex-ante risk sharing

Focusing exclusively on ex-ante intergenerational risk sharing, we imagine that the government in the initial period 0, before any wage shocks have been revealed, implements a paygo program by means of a specific choice of  $g$ . This ex-ante (or “as of the time of enactment of the program”) perspective is common in the literature on the effects of social security on intergenerational risk sharing and adopted in several papers, see for example Gordon and Varian (1988), Enders and Lapan (1982, 1993), Thøgersen (1998), Borgman (2002), Shiller (1999) and Ball and Mankiw (2001).<sup>2</sup> Ball and Mankiw interpret this perspective as “Rawlsian”, i.e. the representative individuals from the various generations are present behind a “veil of ignorance” and do not know whether their generation will experience a favourable or disappointing wage shock.

We immediately observe from (1) that  $E_0(w_t) = w_0$  for all  $t = 1, 2, \dots$ . Thus, it follows from (8) that any  $g$ , even the benchmark case of  $g = 0$  (no program), implies that  $E_0(y_t) = w_0$ . Given the utility function in (8), the period 0 expected welfare effects of any paygo program are therefore solely related to the effect on  $Var_0(y_t)$ . This reflects our assumption of a zero real interest rate as well as a zero population growth rate. Therefore, as of period 0 there is no expected intergenerational re-distribution to the initial generation from

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<sup>2</sup> A useful discussion of the validity of this ex-ante perspective versus an alternative ex-post perspective is provided by Wagener (2002), see also Ball and Mankiw (2001) and Sinn (1996).

future generations as a consequence of the paygo program when the real interest rate exceeds the rate of growth. This effect is well understood and deliberately suppressed here, allowing us to focus exclusively on intergenerational risk sharing effects.

We first consider  $Var_0(y_t)$  in the benchmark case of no paygo program ( $\mathbf{g} = 0$ ). In this case  $y_t^B = w_t$ , see (8), where superscript  $B$  refers to *Benchmark*, i.e. no paygo program. Using (2), (3) and (4), we obtain

$$(9a) \quad Var_0(y_t^B) = \mathbf{s}^2 \quad \text{for } \mathbf{b} = 0,$$

$$(9b) \quad Var_0(y_t^B) = \sum_{s=1}^t \mathbf{b}^{2(t-s)} \mathbf{s}^2 \quad \text{for } 0 < \mathbf{b} \leq 1,$$

$$(9c) \quad Var_0(y_t^B) = \sum_{s=1}^t \mathbf{s}^2 = t\mathbf{s}^2 \quad \text{for } \mathbf{b} = 1.$$

Our task is now to examine how an introduction of a paygo program ( $0 < \mathbf{g} \leq 1$ ) alters  $Var_0(y_t)$ . We consider the extreme cases first. If  $\mathbf{b} = 0$ , (2) and (8) imply

$$(10) \quad y_t^{paygo} = w_0 + (1 - \mathbf{g})\mathbf{e}_t + \mathbf{g}\mathbf{e}_{t+1},$$

where superscript *paygo* refers to the existence of a paygo program. From (10) we derive

$$(11) \quad Var_0(y_t^{paygo}) = (1 - \mathbf{g})^2 \mathbf{s}^2 + \mathbf{g}^2 \mathbf{s}^2.$$

We immediately observe that any  $\mathbf{g} \in \langle 0, 1 \rangle$  implies that  $Var_0(y_t^{paygo}) < Var_0(y_t^B) (= \mathbf{s}^2)$ , see (9a). Hence, a paygo program provides intergenerational income risk sharing in the case of  $\mathbf{b} = 0$  because each income shock is shared between two succeeding generations. This is the essential risk-sharing conclusion in the analyses of Gordon and Varian (1988), Enders and Lapan (1982, 1993) and Thøgersen (1998). It also follows from (11) that  $\mathbf{g} = \frac{1}{2}$  implies a minimum value of  $Var_0(y_t^{paygo})$  equal to  $\frac{1}{2}\mathbf{s}^2$ . Because  $E_0(y_t)$  is not altered by the paygo program,  $\mathbf{g} = \frac{1}{2}$  is also optimal from an ex-ante perspective for all generations.

The alternative extreme case of  $\mathbf{b} = 1$  yields opposite conclusions. From (4) and (8) we obtain

$$(12) \quad y_t^{paygo} = w_0 + \sum_{s=1}^t \mathbf{e}_s + \mathbf{g}\mathbf{e}_{t+1},$$

which in turn implies that

$$(13) \quad \text{Var}_0(y_t^{paygo}) = t\mathbf{s}^2 + \mathbf{g}^2\mathbf{s}^2.$$

Looking at (9c) and (13), we observe that  $\text{Var}_0(y_t^{paygo}) - \text{Var}_0(y_t^B) = \mathbf{g}^2\mathbf{s}^2$ , which means that any paygo program ( $0 < \mathbf{g} \leq 1$ ) raises the exposure to income risk for all generations. The intuition is that an income shock which hits a given generation  $t$ ,  $\mathbf{e}_t$ , also hits the next generation  $t+1$  to a full extent, i.e. we have that  $w_{t+1} = w_{t-1} + \mathbf{e}_t + \mathbf{e}_{t+1}$  when  $\mathbf{b} = 1$ . In effect the paygo program therefore on the one hand taxes away parts of generation  $t$ 's exposure to  $\mathbf{e}_t$  but on the other hand transfers back the same exposure to  $\mathbf{e}_t$ . In addition the paygo program involves an exposure to  $\mathbf{e}_{t+1}$  as well. Consequently, the total exposure to income risk increases. Clearly, no paygo program ( $\mathbf{g} = 0$ ) is optimal from an ex-ante perspective when  $\mathbf{b} = 1$ .

Turning to the general case of  $0 < \mathbf{b} \leq 1$ , we derive from (3) and (8) that

$$(14a) \quad y_t^{paygo} = (1 - \mathbf{g}) \left( w_0 + \sum_{s=1}^t \mathbf{b}^{t-s} \mathbf{e}_s \right) + \mathbf{g} \left( w_0 + \sum_{s=1}^{t+1} \mathbf{b}^{t+1-s} \mathbf{e}_s \right).$$

This expression can be rewritten as

$$(14b) \quad y_t^{paygo} = w_0 + [1 - \mathbf{g}(1 - \mathbf{b})] \sum_{s=1}^t \mathbf{b}^{t-s} \mathbf{e}_s + \mathbf{g}\mathbf{e}_{t+1},$$

and it follows that



$$(15) \quad \text{Var}_0(y_t^{\text{paygo}}) = [1 - \mathbf{g}(1 - \mathbf{b})]^2 \mathbf{s}^2 \sum_{s=1}^t \mathbf{b}^{2(t-s)} + \mathbf{g}^2 \mathbf{s}^2.$$

From (9b) and (15) we now obtain

$$(16) \quad \Delta(t) \equiv \text{Var}_0(y_t^B) - \text{Var}_0(y_t^{\text{paygo}}) = \left[1 - (1 - \mathbf{g}(1 - \mathbf{b}))^2\right] \mathbf{s}^2 \sum_{s=1}^t \mathbf{b}^{2(t-s)} - \mathbf{g}^2 \mathbf{s}^2,$$

and the issue is whether  $\Delta(t) > 0$ , implying that a paygo program leads to intergenerational risk sharing for a given generation  $t$ . We first observe that  $\Delta(t)$  is proportional to  $\mathbf{s}^2$ . More importantly, we observe from (16) that the first term on the RHS is strictly positive because the expression in the brackets is strictly between 0 and 1 when  $0 < \mathbf{g} \leq 1$  and  $0 < \mathbf{b} < 1$ . This captures that the paygo program reduces the exposure of generation  $t$  to the shocks  $\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_t$ . The second term on the RHS of (16) is negative, however, and captures that the paygo program exposes generation  $t$  to  $\mathbf{e}_{t+1}$ .

It follows that the sign of  $\Delta(t)$  is indeterminate and depends on  $t$  as well as on the magnitude of  $\mathbf{g}$  and  $\mathbf{b}$ . The relationship between  $\Delta$  and  $t$  is of course due to the term

$$\sum_{s=1}^t \mathbf{b}^{2(t-s)} \equiv \mathbf{m}(t) \text{ in the first term on the RHS. Because } \frac{\partial \mathbf{m}}{\partial t} > 0, \text{ we have that } \frac{\partial \Delta}{\partial t} > 0.$$

Accordingly, a paygo system is more likely to offer intergenerational income risk sharing for generations in the distant future (a high  $t$ ) than for generations closer to the initial period 0 (for given values of  $\mathbf{g}$  and  $\mathbf{b}$ ). This reflects our ex-ante “as of the time of the enactment of the program” time perspective. When  $t$  increases and  $\mathbf{b} > 0$ , the paygo program removes parts of more shocks (the first term on the RHS of (16)), while the additional risk created by the paygo program is not altered (the last term on the RHS of (16)).

Looking closer at (16), it is also straightforward (but somewhat cumbersome) to verify that  $\frac{\partial \Delta}{\partial \mathbf{b}} < 0$ , see the appendix. Thus, more persistent income shocks monotonically reduce the scope for income risk sharing by means of paygo programs. Having shown above that paygo programs always imply risk sharing for  $\mathbf{b} = 0$  and always increase all generations’ income risk exposure for  $\mathbf{b} = 1$ , we may stress three general insights: i) For fairly low persistence in the income shocks, a paygo program leads to risk sharing for all generations. The gains in terms of a reduced variance as measured by  $\Delta(t)$  increase for later generations. ii) For a very

high degree of persistence, a paygo scheme may increase the risk exposure for all generations. Early generations are worst off. iii) For a medium-high degree of persistence, early generations may face increased exposure to income risk, while later generations may gain. The numerical calculations depicted in Table 1 illustrate these insights. For the given magnitude of the paygo program,  $g = \frac{1}{2}$ , we observe that the finding of an increased income risk exposure (i.e. a negative value of  $\Delta(t)$ ) requires a pretty high  $b$ -value.

**Table 1: Calculation of  $\Delta(t)$  for selected values of  $b$  and  $t$  - when  $g = \frac{1}{2}$  and  $s^2 = 1$ .**

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 10$
$b = 0.1$	0.45	0.45	0.45	0.45	0.45	0.45
$b = 0.2$	0.39	0.42	0.42	0.42	0.42	0.42
$b = 0.5$	0.19	0.30	0.32	0.33	0.33	0.33
$b = 0.8$	-0.06	0.06	0.14	0.19	0.22	0.27
$b = 0.9$	-0.15	-0.07	-0.01	0.04	0.08	0.20
$b = 0.95$	-0.20	-0.16	-0.12	-0.08	-0.05	0.07
$b = 0.99$	-0.24	-0.23	-0.22	-0.21	-0.20	-0.16

Utilizing the formula for a finite geometric sequence, we have that  $m(t) \equiv \sum_{s=1}^t b^{2(t-s)} = \frac{b^{2t} - 1}{b^2 - 1}$ ,

which implies that  $m(t)$  will approach  $\frac{1}{1-b^2}$  as  $t$  increases. Consequently, it follows from (16) that

$$(17) \quad \lim_{t \rightarrow \infty} \Delta(t) = \left[1 - (1 - g(1 - b))^2\right] \frac{1}{1 - b^2} s^2 - g^2 s^2.$$

As illustrated in Table 1,  $\Delta(t)$  approaches its limit fast for low and medium values of  $b$ .

It is also interesting to derive a condition that ensures that the paygo system leads to intergenerational risk sharing, i.e.  $\Delta(t) > 0$ . It follows from (16), or more directly from (A-2) in the appendix, that  $\Delta(t) > 0$  requires that

$$(18) \quad g < \frac{2(b^{2t} - 1)}{(1 - b)(b^{2t} - 1) - (1 + b)}.$$

A quick look at this condition confirms that there is no scope for risk sharing by means of a paygo program when  $\mathbf{b} = 1$  (no  $\mathbf{g}$  in the interval  $\langle 0,1 \rangle$  implies  $\Delta(t) > 0$ ), while any paygo program leads to risk sharing when  $\mathbf{b} = 0$  (any  $\mathbf{g}$  in the interval  $\langle 0,1 \rangle$  implies  $\Delta(t) > 0$ ). More generally, a higher  $\mathbf{b}$  or a lower  $t$  will both reduce the maximum magnitude of a paygo system that implies risk sharing, i.e. the term on the RHS of (18) will approach 0. Not surprisingly, the opposite cases, a lower  $\mathbf{b}$  or a higher  $t$ , will imply that the term on the RHS of (18) approaches 1.

We have seen that the effect of a paygo system on the variance of individuals' lifetime income varies across generations when  $0 < \mathbf{b} < 1$ , i.e.  $\Delta$  is an increasing function of  $t$ . Moreover, it is also straightforward from (16) to verify that  $\frac{\partial \Delta}{\partial \mathbf{g}}$  depends on  $t$ . This implies that it is not possible to derive a socially optimal size of the paygo system without resorting to more specific assumptions about the appropriate weighting of the effects for the present and various future generations. In this context it is interesting to note that the property  $\frac{\partial \Delta}{\partial t} > 0$  implies that the value of  $\mathbf{g}$ , which is optimal for the initial generation  $t = 1$ , actually involves larger gains for the succeeding generations. Consequently, optimization on behalf of generation  $t$  is socially optimal according to a maximin criterion. Adopting this concept of optimality, we note that (16) implies

$$(19) \quad \Delta(1) = \left[ 1 - (1 - \mathbf{g}(1 - \mathbf{b}))^2 \right] \mathbf{s}^2 - \mathbf{g}^2 \mathbf{s}^2,$$

and in turn we derive the optimal magnitude of the paygo system as

$$(20) \quad \mathbf{g}^* = \frac{1 - \mathbf{b}}{1 + (1 - \mathbf{b})^2}.$$

Table 2 illustrates how  $\mathbf{g}^*$ , the magnitude of the paygo system that leads to the largest reduction in the variance of the life-time income of generation  $t = 1$ , is a declining and concave function of  $\mathbf{b}$ .

**Table 2: Calculation of  $g^*$  for selected values of  $b$ .**

	$b = 0$	$b = 0.1$	$b = 0.2$	$b = 0.5$	$b = 0.7$	$b = 0.8$	$b = 0.9$	$b=0.95$	$b=0.99$	$b = 1$
$g^*$ :	0.50	0.50	0.49	0.40	0.28	0.19	0.10	0.05	0.01	0.00

#### 4. Final remarks

Our analysis has demonstrated that the scope for intergenerational income risk sharing by means of a paygo social security program decreases with the degree of persistence in the income shocks. It turned out that the extreme case of permanent shocks actually increases all generations' exposure to income risk, while a fairly high degree of persistence consistent with  $b$  –values in, say, the range of 0.8 – 0.99 puts a severe limit on the size of the paygo programs that provide risk sharing. Taking into account that most OECD economies have large social security programs which are financed on a paygo basis, the potential existence of a high degree of persistence in the income shocks suggests that these programs in fact create additional exposure to income shocks for the individuals.

To what extent the theoretical insights of this paper are crucial for the design of social security programs, depends on the actual stochastic properties regarding income persistence. Unfortunately, it is fair to say that available empirical evidence is inconclusive. This partly reflects that the main bulk of empirical research in this area deals with tests of the unit root property and estimates of the degree of persistence in aggregate output measured as GDP (or GNP). Aggregate output per capita is a proxy for the income variable in our stylized theoretical model and it might have been better to consider the stochastic properties of the underlying productivity growth rate or to focus on the development of real wages. Still, we conjecture that aggregate output per capita is a good proxy for individual income when we recall the long run nature of our social security context.

More important is probably the fact the empirical evidence in this field is mainly carried out in a short run business cycle setting. Based on annual (or quarterly) data it is, as discussed in the introduction, nearly impossible to distinguish between a stochastic trend (i.e. permanent shocks) and a deterministic trend in combination with a high degree of persistence in the income shocks. Quantitatively and in terms of our autoregressive specification of income, see (1), this means that annual data imply a  $b$  –parameter which is either equal to 1 or alternatively barely below 1 (in approximately the range 0.90 – 0.99). As highlighted by

Romer (2001, section 4.8), data over somewhat longer horizons yield estimates that are not very precise. These findings are crucial for the interpretation of our theoretical results. Our social security setting implies that one period in our stylized overlapping generations model corresponds to approximately 25 years. Taking into account that a 0.95 persistence parameter in an annual setting implies “only” a  $0.95^{25} = 0.28$  parameter in our overlapping generations setting (while a 0.99 annual persistence parameter implies a 0.78 parameter in our setting), it is obviously hard to reach a final verdict regarding the scope for intergenerational income risk sharing by means of paygo programs. On the one hand we must therefore conclude that the possibility of permanent or alternatively highly persistent income shock questions the validity of the traditional view that paygo programs provide intergenerational income risk sharing. On the other hand, current empirical evidence is also ambiguous about the possibility that the degree of income shocks is so high that it implies that paygo programs in fact create additional risk exposure.

## Appendix

In order to prove that  $\frac{\partial \Delta}{\partial \mathbf{b}} < 0$ , we first rewrite (16) as

$$(A-1) \quad \frac{\Delta(t)}{\mathbf{s}^2 \mathbf{g}} = 2(1 - \mathbf{b})\mathbf{m}(t) - \mathbf{g}(1 - \mathbf{b})^2 \mathbf{m}(t) - \mathbf{g},$$

which in turn implies that

$$(A-2) \quad \frac{\Delta(t)}{\mathbf{s}^2 \mathbf{g}} = -2\hat{\mathbf{m}}(t) + \mathbf{g}(1 - \mathbf{b})\hat{\mathbf{m}}(t) - \mathbf{g}, \quad \hat{\mathbf{m}}(t) = \frac{\mathbf{b}^{2t} - 1}{1 + \mathbf{b}}.$$

We then obtain

$$(A-3) \quad \frac{\partial \Delta}{\partial \mathbf{b}} = \mathbf{s}^2 \mathbf{g} \left[ -\mathbf{g}\hat{\mathbf{m}} - (2 - \mathbf{g}(1 - \mathbf{b})) \frac{\partial \hat{\mathbf{m}}}{\partial \mathbf{b}} \right].$$

Using the definition  $\hat{\mathbf{m}}(t) = \frac{\mathbf{b}^{2t} - 1}{1 + \mathbf{b}}$ , see (A-2), we have

$$(A-4) \quad \frac{\partial \hat{\mathbf{m}}}{\partial \mathbf{b}} = \frac{(2t\mathbf{b}^{2t-1})(1 + \mathbf{b}) - (\mathbf{b}^{2t} - 1)}{(1 + \mathbf{b})^2}.$$

From (A-3) and (A-4) we obtain

$$(A-5) \quad \frac{\partial \Delta}{\partial \mathbf{b}} = -(1 + \mathbf{b})[2 - \mathbf{g}(1 - \mathbf{b})]2t\mathbf{b}^{2t-1} - (1 - \mathbf{b}^{2t})(2 - 2\mathbf{g}).$$

Both terms on the RHS of (A-5) are strictly negative for  $0 < \mathbf{g} \leq 1$  and  $0 < \mathbf{b} < 1$ . Hence, we conclude that  $\frac{\partial \Delta}{\partial \mathbf{b}} < 0$ .

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