

Industrial clusters, asymmetric information and policy design*

Linda Orvedal

Norwegian School of Economics and Business Administration
Helleveien 30, 5045 Bergen, Norway
e-mail: Linda.Orvedal@nhh.no

Discussion Paper 25/02

Abstract

Recently there has been some attention on the issue of industrial clusters and industrial policy claiming that there are arguments in favour of subsidising special industries in order to stimulate industrial agglomeration. An important objection against such a selective industrial policy is that the government needs full information about the industries to be able to pick the winners. But often the government does not know whether the industry is a cluster or not. If the policy towards a cluster is more favourable than the policy towards other industries, all industries will have incentives to allege to be clusters. The problem for the government is to identify the true clusters from the ordinary industries. This paper attempts to shed some light on the problem of industrial policy and industrial clusters when there is asymmetric information between the government and the industry. We show that a subsidy contingent of a certain activity level will create a separating equilibrium.

Keywords: Asymmetric information, Industrial clusters, Industrial policy, Scale economies.

JEL: D82, F12, L52, R12

* Thanks to Kåre P. Hagen and Trond Olsen for very valuable discussions and comments. Also thanks to Frøystein Gjesdal, Hans Jarle Kind, Karen Helene Midelfart Knarvik, and Eirik Kristiansen. This research has been financed by the Research Council of Norway.

1. Introduction

Recently there has been some attention on the issue of industrial clusters and industrial policy. Several papers claim that there are arguments in favour of subsidising special industries in order to stimulate industrial clusters (see f. i. Porter (1990), Reve et al. (1992), Reve and Jakobsen (2001) and Norman and Venables (2001)). An important objection against such a selective industrial policy is that the government needs full information about the industries to be able to pick the winners. But often the government does not know whether the industry is a cluster or not. In some cases even the industry themselves does not know to which extent the industry is a cluster, but in most cases the industry does know more about themselves than does the government. If a cluster gets a more favourable policy than other industries, all industries will have incentives to allege to be clusters. The problem for the government is to identify the true clusters from the ordinary industries.

An industrial cluster is an industry with some agglomeration gains. The gains may be due to positive external effects such as knowledge spillovers or other real externalities, or it may be due to pecuniary externalities in industries with economies of scale and imperfect competition. There is a rich literature on industrial clusters, and some papers discuss policy implications¹. But no paper includes asymmetric information between the government and the industry. This paper attempts to shed some light on the problem of industrial policy and industrial clusters when there is asymmetric information between the government and the industry.

The market failures involved are co-ordination failures and increasing return to scale in addition to asymmetric information. It is a complicated issue, and we have to focus on one problem at the time. This paper focuses on the problem of optimal scale when there is increasing return to scale and asymmetric information, leaving the co-ordination problem for further research. The problem is how to design a policy that induces optimal scale of production when there is asymmetric information between the industries and the government. To solve this problem, we add asymmetric information into a model developed by Norman and Venables (2001).

Assume that there are two industries, one with decreasing return to scale and one that alleges to have increasing return to scale at the national level. If the sector has increasing return to scale, it is called a cluster. The input factors are mobile between the sectors, but internationally immobile. All goods can be traded internationally. The countries in the world are identical in the sense that they have identical factor resources and identical preferences.

If there is a sector with increasing return to scale, and it is common knowledge that the sector is a cluster, then in equilibrium some, but not all, countries will produce in the increasing return to scale sector². The national income in those countries hosting a cluster is larger than in those countries without a cluster. Furthermore, the size of the clusters is too small. Therefore, each country will have an incentive to subsidise the clusters.

¹ See Haaland and Wooton (1999), Baldwin and Krugman (2000), Kind, Midelfart-Knarvik and Schjelderup (2000), Ludema and Wooton (2000), Norman and Venables (2001), and Forslid and Knarvik (2002).

² See Norman and Venables (2001).

But what happens if there is asymmetric information between the government and the industries? To be specific, assume that the owners of the input factor know whether the sector is a cluster or not, but that the government does not know. The question is then: How should the government design the policy?

The paper is organised as follows. Section 2 presents the model, while section 3 discusses the various equilibria. Section 4 discusses the national income and industrial policy when there is no asymmetric information. In section 5 asymmetric information and industrial policy are discussed, while conclusions are given in the final section.

2. The model

Assume that there are two sectors in the economy. One sector producing a numeral good, y , and one sector producing a good, x , at a price p . Both sectors use only one input factor, called labour. The total supply of labour in a country is given by L . The employment in the x -sector is given by n , and the rest of the labour, $L-n$, is employed in the y -sector.

Assume that the numeral good, y , is produced with a production function given by

$$(1) \quad y = Y(L-n),$$

where $Y'(L-n) > 0$ and $Y''(L-n) < 0$, which implies that there is decreasing return to scale in the y -sector.

Assume that the x -sector alleges to be a sector with increasing return to scale (a cluster). The workers know whether the x -sector is a cluster or not, but the government does not. To make the model as simple as possible, assume that the technology in the x -sector could take one out of two functional forms. If the x -sector is not a cluster, it is common knowledge that the production function is given by $x = X(n)$, where $X'(n) > 0$ and $X''(n) < 0$. If, on the other hand, the x -sector is a cluster, it is common knowledge that each worker produces an output equal to $a(n)$, which is an increasing and concave function, and the total production is given by $x = na(n)$. Thus, summarising the production in the x -sector gives

$$(2) \quad x = \begin{cases} X(n) & \text{if the } x\text{-sector is not a cluster} \\ na(n) & \text{if the } x\text{-sector is a cluster.} \end{cases}$$

The income in country i , m_i , is equal to the sum of the value of the production in the two sectors:

$$(3) \quad m_i = \begin{cases} Y(L-n_i) + pX(n_i) & \text{if the } x\text{-sector is not a cluster} \\ Y(L-n_i) + pn_i a(n_i) & \text{if the } x\text{-sector is a cluster.} \end{cases}$$

The preferences in a country are assumed to be quasi-linear. The utility in country i is given by

$$(4) \quad u_i = v(p) + m_i,$$

where $v(p)$ is the indirect utility function for the x -good, which is a decreasing and convex function. The demand for the x -good is therefore

$$(5) \quad c = -v'(p).$$

If x -sector is a cluster, generally, not all countries will have a x -sector (a cluster). Assume that there are K countries in the world, and k of them have a cluster. The rest of the countries, $K-k$, specialise in the y -sector. The world market for goods will clear when the total demand is equal to the total supply of x -goods, that is

$$(6) \quad -Kv'(p) = \begin{cases} KX(n) & \text{if } x\text{-sector is not a cluster} \\ kna(n) & \text{if } x\text{-sector is a cluster.} \end{cases}$$

If the x -sector is not a cluster, equation (6) implies that the world market price, p , is a function of the employment in the x -sector only. If, on the other hand, the x -sector is a cluster, equation (6) implies that the world market price, p , is a function of both the relative number of countries hosting a cluster and the employment in the clusters. Equation (7) summarises this.

$$(7) \quad p = \begin{cases} P(n) & \text{if the } x\text{-sector is not a cluster} \\ F(k/K, n) & \text{if the } x\text{-sector is a cluster,} \end{cases}$$

where the partial derivative is negative with respect to all arguments (see appendix 1), which implies that if the x -sector is not a cluster, the world market price will fall when the employment, and thereby the production, rises. Furthermore, if the x -sector is a cluster, equation (7) implies that the world market price will fall if the number of clusters in the world rises or if the employment in the clusters increases. In both cases the price falls because the production increases.

3. Equilibrium

Assume that the workers, either as a group or individually, can freely enter or exit a sector. Workers will enter a sector if they get higher income in that sector and they will exit if the income is higher in the other sector. This assumption ensures that we overcome the co-ordination problem.

In the sectors with decreasing return to scale (the y -sector and the x -sector if it is not a cluster) the workers are paid according to the value of the marginal productivity of labour. Equilibrium in the labour market implies that the value of the marginal productivity of labour is equal in the sectors with decreasing return to scale.

If the x -sector is a cluster, each worker is paid according to the value of the average productivity in the sector. Equilibrium in the labour market is given by equality between the value of the marginal productivity in the y -sector and the average productivity in the cluster sector for each country hosting a cluster. Thus, summarising the equilibrium conditions in the labour market for each country that has a cluster gives

$$(8) \quad Y'(L-n) = \begin{cases} pX'(n) & \text{if the } x\text{-sector is not a cluster} \\ pa(n) & \text{if the } x\text{-sector is a cluster.} \end{cases}$$

Equations (7) and (8) give the equilibrium. Equation (7) clears the goods markets and equation (8) clears the labour market. Combining the two gives one equilibrium condition:

$$(9) \quad Y'(L-n) = \begin{cases} P(n)X'(n) & \text{if the } x\text{-sector is not a cluster} \\ F(k/K, n) a(n) & \text{if the } x\text{-sector is a cluster.} \end{cases}$$

In the following, we first analyse the equilibrium when the x -sector is not a cluster. Then we analyse the equilibrium when the x -sector is a cluster.

Equilibrium when the x -sector is not a cluster

When the x -sector is not a cluster, the equilibrium is illustrated in figure 1.

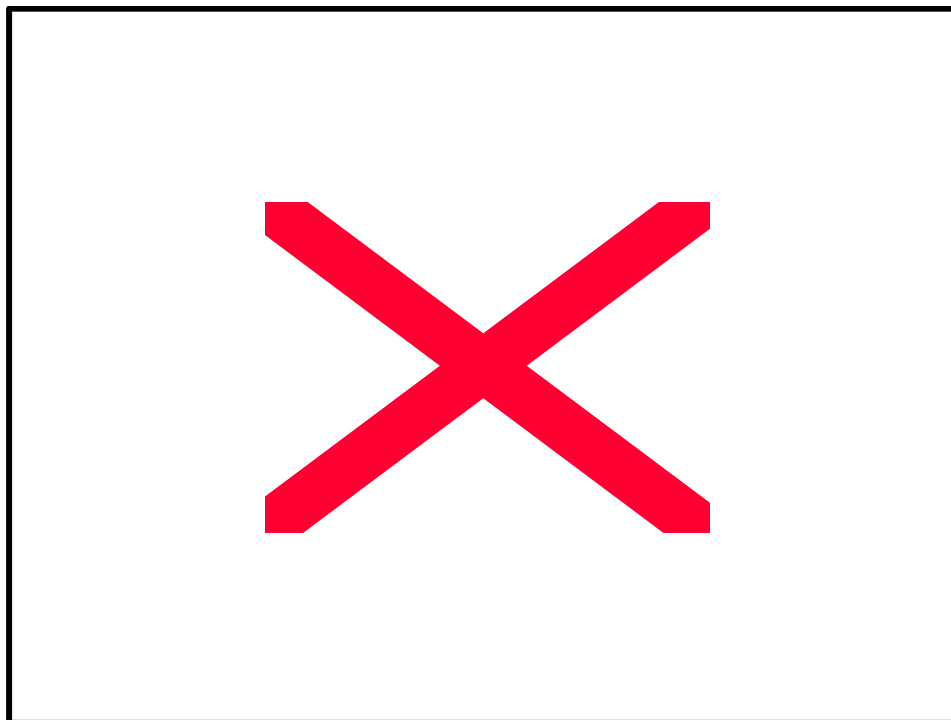


Figure 1: Equilibrium in the labour market when the x -sector is not a cluster

In figure 1, a country's labour force, L , is measured on the horizontal axis, and the return per worker is measured on the vertical axes. The x -sector is measured from the left corner and the y -sector from the right corner. The value of the marginal

productivity in the y -sector is given by the decreasing function $Y'(L-n)$. The value of the marginal productivity in the x -sector is given by the decreasing function $P(n)X'(n)$. Equilibrium is given by equality between the marginal productivity in the two sectors, illustrated by the point E .

Equilibrium when the x -sector is a cluster

The equilibrium in the labour market when the x -sector is a cluster is illustrated in figure 2 for different values of p .

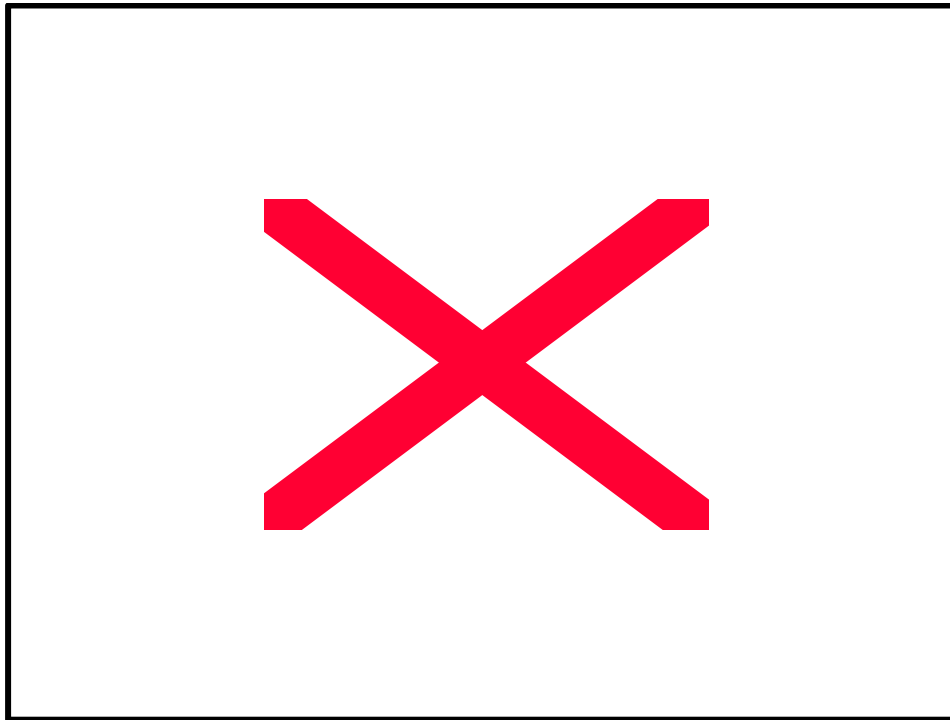


Figure 2: Equilibrium in the labour market when the x -sector is a cluster

In figure 2, a country's labour force, L , is measured on the horizontal axis, and the return per worker on the vertical axes. The x -sector is measured from the left corner and the y -sector from the right corner. The value of the marginal productivity in the y -sector is given by the decreasing function $Y'(L-n)$. The value of the average productivity in the x -sector is given by the increasing function $pa(n)$. The larger p , the higher up in the diagram is the function $pa(n)$.

If the world market price is equal to p' , then there are two equilibria, B and C . The points B and C are equilibria because the return per worker is the same in both the x - and the y -sector, but only point C is a stable equilibrium.

Another point in the diagram, the point A , characterises a situation where the total labour force in a country is employed in the y -sector. In point A no single worker will have any incentives to switch to the x -sector. But a group of workers will have incentives to join the x -sector. A world market price equal to p' will therefore induce an additional country to get a cluster. When the number of clusters in the world increases, the world market price will fall.

Free entry of clusters in the world will ensure that the number of clusters increases until the world market price is equal to p° . The equilibrium is given by the point E . Point E is characterised by a tangency between the value of the average productivity in the x -sector and the marginal productivity in the y -sector, that is

$$(10) \quad Y''(L-n) = pa'(n) \quad \text{if the } x\text{-sector is a cluster.}$$

4. National income and incentives for industrial policy

Assume for a moment that the government knows that the x -sector is a cluster. Comparisons of the national income between the countries show that those countries hosting a cluster have larger income than those without clusters. This is easily seen in figure 3.

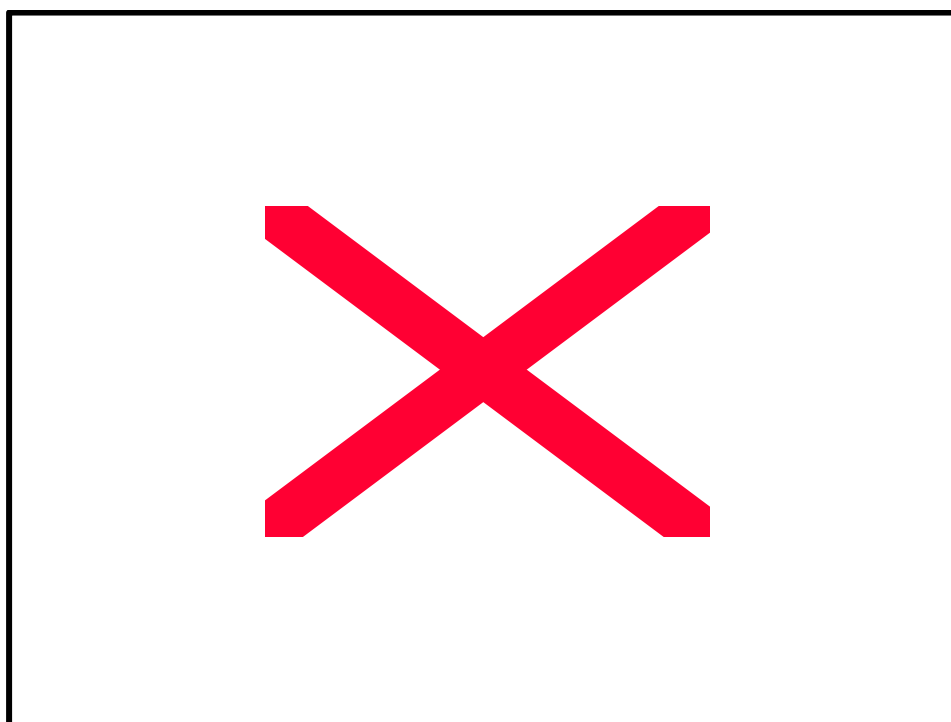


Figure 3: The gains from a cluster

In figure 3, equilibrium is given by point E . The employment in the cluster is given by N , and the employment in the y -sector is given by $L-N$. The value of the production in the cluster is illustrated by the rectangle $ODEN$, which is equal to $Npa(N)$. The value of the production in the y -sector is given by the area below the function $Y'(L-N)$ between L and N . Formally, income in a cluster country is given by equation (3) which implies that

$$(11) \quad m = \int_L^N Y'(L-z)dz + Npa(N) = Y(L-N) + Npa(N) \quad \text{if the } x\text{-sector is a cluster.}$$

Income in those countries without a cluster is equal to

$$(12) \quad m = \int_L^0 Y'(L-z)dz = Y(L),$$

which in figure 2 can be illustrated by the total area below the $Y'(L-n)$ -function. Clearly, this area is smaller than the area generated by the income in a cluster country. The shaded area in figure 2 illustrates the difference. A country with a cluster will therefore have a larger income than a country without a cluster.

At first sight, it may seem that a group of workers could gain by creating a cluster. But, if an additional cluster appears, the world market price will fall. Individual workers will therefore prefer to work in the y -sector. A new cluster is thus not sustainable without help from the government. The question is whether the government in these countries would have an incentive to stimulate cluster creation.

If the government can force the workers to stay in the cluster, national income will rise. An additional cluster in the world will reduce the world market price marginally. But still, the value of the production in the cluster will be higher than the value of the alternative production in the y -sector. The question is then: how can the government secure that workers stay in the cluster?

Assume that the government offers the workers in the cluster a subsidy, s , financed by a lump sum tax. The workers will stay in the cluster sector if the equilibrium condition

$$(13) \quad Y'(L-n) = spa(n)$$

is fulfilled.

The total productivity in the cluster sector is equal to

$$px = pna(n).$$

The optimal subsidy is set such that the workers get paid according to the value of their marginal productivity in the cluster sector, which implies that

$$(14) \quad s*pa(n) = p[a(n) + na'(n)].$$

Equations (13) and (14) imply that the value of the marginal productivity in the two sectors is equal,

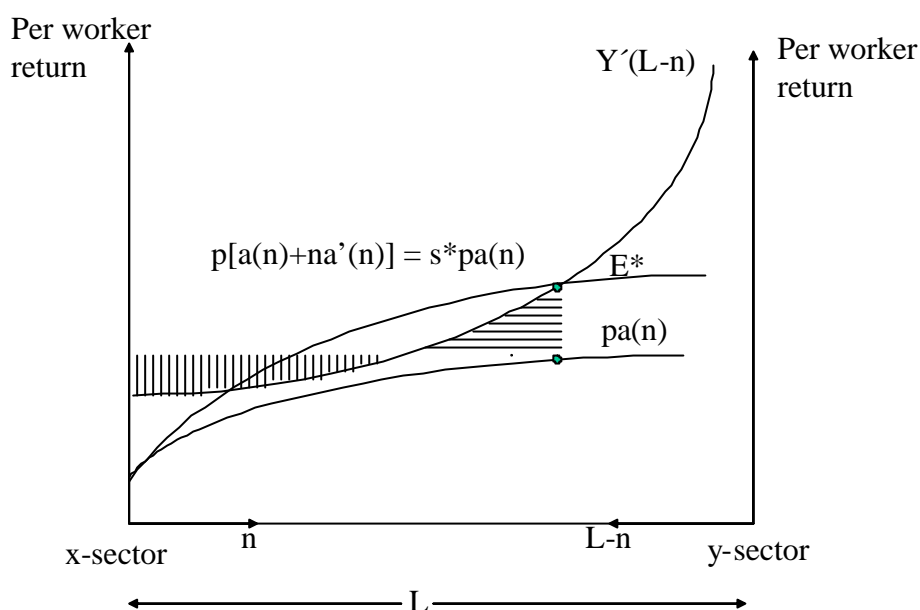
$$(15) \quad Y'(L-n) = p[a(n) + na'(n)].$$

Since the countries are identical, all countries with a cluster have to subsidise the clusters. But how many clusters will there be? New clusters will enter as long as the income in a cluster country is larger than the income in a country without a cluster. When the income in the two groups of countries is identical, no further clusters will enter. This is so when

$$(16) \quad Y(L) = pna(n) + Y(L-n).$$

The three equations (6), (15) and (16) give the policy equilibrium when the government knows that the x -sector is a cluster. Equation (6) gives equilibrium in the

market for goods, equation (15) gives equilibrium in the labour market, and equation (16) is a free entry condition for new cluster countries. The equilibrium is illustrated in figure 4.



4

Figure 4: The policy equilibrium when the x -sector is a cluster

In figure 4, a country's labour force, L , is measured on the horizontal axis, and the return per worker on the vertical axes. The x -sector is measured from the left corner and the y -sector from the right corner. The value of the marginal productivity in the y -sector is given by the decreasing function $Y'(L-n)$. The value of the average productivity in the x -sector is given by the increasing function $pa(n)$, and the value of the marginal productivity is given by the function $p[a(n) + na'(n)]$. The workers receive a subsidy s^* so that their average return including the subsidy is equal to the marginal return in both the x - and the y -sector. New countries will enter the x -sector as long as the national income is higher in countries with a cluster. In equilibrium the national income is the same in countries with clusters and in countries without clusters. In figure 4 the equilibrium price is characterised by equality between the two shaded areas.

5. Asymmetric information and industrial policy

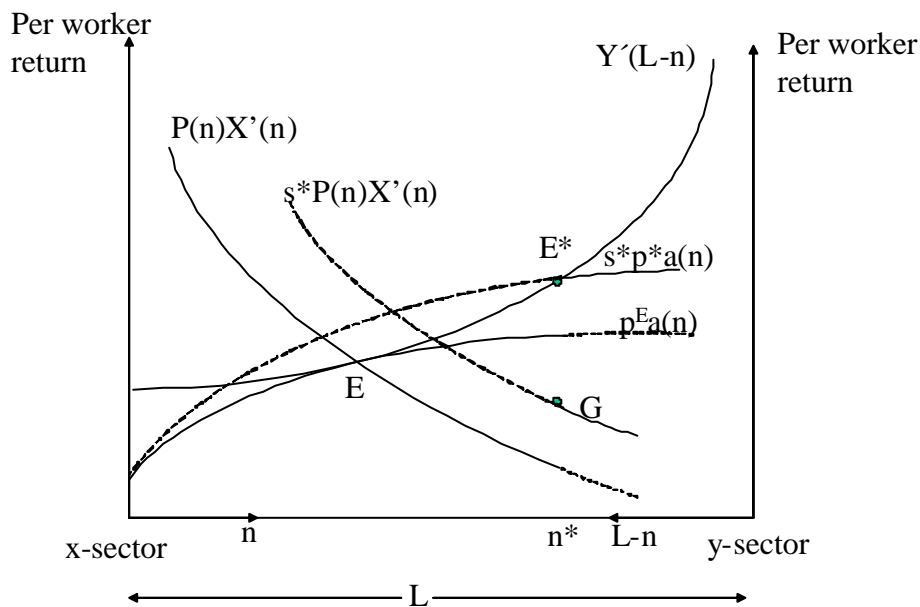
Summarising the results so far we have shown that the government in a country hosting a cluster will have incentives to subsidise the cluster if they know that the x -sector is a cluster. This is the result from the study by Norman and Venables (2001). But if the government does not know whether the x -sector is a cluster or not, how should the government then design its policy?

We have shown that if the x -sector is a cluster, then the optimal policy is a subsidy equal to s^* . If the x -sector is not a cluster, then there is no market failure, and the optimal subsidy is equal to zero. In this simple model, the technology in each sector is not observable, but the employment in each sector is. A subsidy contingent of a

certain employment level will therefore induce the optimal scale of production. The first best policy with asymmetric information is thus

$$(17) \quad s = \begin{cases} s^* & \text{if } n = n^* \\ 1 & \text{if } n < n^*. \end{cases}$$

This gives a separating equilibrium. To prove that equation (17) gives a separating equilibrium, we need to show that a true cluster will choose the policy scheme $(s^*, n=n^*)$. The cluster will obviously choose the combination (s^*, n^*) , which is shown in the previous chapter. If they choose $(1, n \neq n^*)$, the workers will not maximise their income in the cluster sector. In order to prove that equation (17) gives a separating equilibrium, we also have to show that the x -sector will not pretend to be a cluster if they are not so. That is, to show that the x -sector will choose the combination $(1, n < n^*)$ if it has decreasing return to scale. In figure 5 we have illustrated the equilibrium. The point E is the starting equilibrium when there is no industrial policy. Assume the x -sector is not a cluster. If it chooses the combination $(s^*, n=n^*)$ their return per worker is given by the function $s^*P(n)X'(n)$ for $n = n^*$; and if it chooses the combination $(1, n < n^*)$ its return per worker is given by the function $P(n)X'(n)$ for $n < n^*$. The equilibrium will be in point E as long as the point G is below the return in the y -sector (the point E^*).



5

Figure 5: Equilibrium in the labour market with industrial policy and asymmetric information

To prove that E is the only equilibrium if the x -sector is not a cluster, we have to show that point G is below point E^* in figure 5. That is, the return per worker in an alleged

cluster is less than the return per worker in a true cluster when the employment is equal to n^* ,

$$s^*P(n^*)X'(n^*) < s^*F(k^*/K, n^*) a(n^*),$$

which implies that

$$(18) \quad P(n^*)X'(n^*) < p^*a(n^*),$$

where $p^* = F(k^*/K, n^*)$.

Equation (18) requires that when $n=n^*$, the return per worker is larger if the x -sector is a cluster. We can easily argue that this must be true. We have that

$$(19) \quad P(n^E)X'(n^E) = p^E a(n^E),$$

where $p^E = F(k^E/K, n^E)$.

We also have that

$$(20) \quad P(n^*)X'(n^*) < P(n^E)X'(n^E),$$

since $P'(n) < 0$, $X''(n) < 0$, and $n^* > n^E$.

Combining equations (19) and (20) gives

$$(21) \quad P(n^*)X'(n^*) < p^E a(n^E).$$

Assume that the production in the y -sector is given by

$$(A1) \quad Y = \mathbf{a} + \mathbf{b}(L-n) - \mathbf{g}(L-n)^2/2$$

so that

$$Y' = \mathbf{b} - \mathbf{g}(L-n) \quad \text{and} \quad Y'' = -\mathbf{g}$$

Furthermore, assume that the production in the x -sector is given by

$$(A2) \quad a(n) = n^q, \quad \mathbf{q} \hat{\mathbf{I}} (0,1),$$

and that the price is given by

$$(A3) \quad p = [n^{q+1} k/K]^{-1}.$$

Appendix 2 shows that the assumptions (A1) – (A3) imply that

$$(22) \quad p^E a(n^E) = p^* a(n^*).$$

Combining equations (21) and (22) gives

$$(23) \quad P(n^*)X'(n^*) < p^* a(n^*),$$

which proves equation (18).

We have now shown that if the x -sector is a cluster, it will choose to receive the subsidy and employ n^* workers. If the x -sector is not a cluster, it will choose not to receive the subsidy. Thus, equation (17) gives a separating equilibrium.

6. Conclusions

We have shown that in the presence of industrial clusters and asymmetric information there exists a policy that creates a separating equilibrium. The model which is used is very simple – it has only one input factor, which is immobile between the countries, all countries are identical, and the alternative production functions are common knowledge. All these assumptions imply that the model is very simple, and not very realistic. However, we have chosen this simple model to be able to focus on one problem at the time. And from this model, we can draw the following lesson: If the government wishes to subsidise an alleged cluster that can overcome the co-ordination problem, then the subsidy should be made contingent of a certain activity level.

A lot of further research needs to be done. This paper may be a starting point. In addition to analysing how critical the simplifying assumptions are for the optimal scale problem, further research must also solve the co-ordination problem.

References

Baldwin, R. E. and P. Krugman (2000): “Agglomeration, integration and tax harmonization”, *CEPR Discussion Paper* no 2630.

Forslid, R. and K. H. Midelfart-Knarvik (2002): “Globalization, industrial policy and clusters”, *CEPR Discussion Paper* no 3129.

Haaland, J. I. and I. Wooton (1999): “International competition for multinational investment”, *Scandinavian Journal of Economics* 101(4), 631 – 649.

Kind, H. J., K. H. Midelfart-Knarvik and G. Schjelderup (2000): “Competing for capital in a ‘lumpy’ world”, *Journal of Public Economics* 78, 253 – 274.

Ludema, R. D. and I. Wooton (2000): “Economic geography and the fiscal effects of regional integration”, *Journal of International Economics* 52, 331 – 357.

Norman, V. D. and A. J. Venables (2001): “Industrial clusters: equilibrium, welfare and policy.” *Discussion paper, Norwegian School of Economics and Business Administration*, no 26/2001; and *CEPR Discussion Paper No 3004, October 2001*.

Porter, M. E. (1990): *Competitive Advantage of Nations*. Macmillan. London.

Reve, T. and E. W. Jakobsen (2001): *Et verdiskapende Norge*. Universitetsforlaget, Oslo.

Reve, T., T. Lensberg and K. Grønhaug (1992): *Et konkurransedyktig Norge*. Tano, Oslo.

Appendix 1

This appendix shows that the partial derivatives of p are negative.

Total differentiation of equation (6) implies that

$$-v''(p)dp = \begin{cases} X(n)dn & \text{if the } x\text{-sector is not a cluster} \\ na(n)d(k/K) + (k/K)[a(n) + na'(n)]dn & \text{if the } x\text{-sector is a cluster.} \end{cases}$$

If the x -sector is not a cluster, then

$$\frac{\partial P}{\partial n} = - \frac{X'(n)}{v''(p)} < 0.$$

+ +

If the x -sector is a cluster, then

$$\frac{\partial F}{\partial (k/K)} = - \frac{na(n)}{v''(p)} < 0$$

+ +

and

$$\frac{\partial F}{\partial n} = - \frac{(k/K)[a(n) + na'(n)]}{v''(p)} < 0.$$

+ + + +

Appendix 2

The following example shows that the value of the average productivity in the x -sector if the x -sector is a cluster, is equal in the equilibrium and in the optimum, that is

$$(22) \quad p^E a(n^E) = p^* a(n^*),$$

where $p^E = F(k^E/K, n^E)$, and $p^* = F(k^*/K, n^*)$.

Assume that the production in the y -sector is given by

$$(A1) \quad Y = \mathbf{a} + \mathbf{b}(L-n) - \mathbf{g}(L-n)^2/2$$

so that

$$Y' = \mathbf{b} - \mathbf{g}(L-n) \quad \text{and} \quad Y'' = -\mathbf{g}$$

Assume that the production in the x -sector is given by

$$(A2) \quad a(n) = n^q, \quad \mathbf{q} \hat{\mathbf{I}} (0,1).$$

and that the price is given by

$$(A3) \quad p = [n^{q+1} k/K]^{-1}.$$

We find the optimal n^* by dividing equation (16) by equation (15), that is

$$(23) \quad [Y(L) - Y(L-n)] / Y'(L-n) = [n a(n)] / [a(n) + n a'(n)].$$

With the specific functions (A1) and (A2), equation (23) becomes

$$(24) \quad [\mathbf{b}n - \frac{1}{2} \mathbf{g}n(2L-n)] / [\mathbf{b} - \mathbf{g}(L-n)] = n / (1 + \mathbf{q}),$$

which implies that

$$(25) \quad n^* = 2\mathbf{q}(\mathbf{b} - \mathbf{g}L) / [\mathbf{g}(1 - \mathbf{q})].$$

The equilibrium allocation of labour, n^E , is found by dividing equation (8) by equation (10), which gives

$$(26) \quad -[Y'(L-n) / Y''(L-n)] = a(n) / a'(n).$$

With the specific functions (A1) and (A2), equation (26) becomes

$$(27) \quad [\mathbf{b} - \mathbf{g}(L-n)] / \mathbf{g} = n^q / \mathbf{q}n^{q-1},$$

which implies that

$$(28) \quad n^E = \mathbf{q}(\mathbf{b} - \mathbf{g}L) / [\mathbf{g}(1 - \mathbf{q})].$$

The ratio between the optimal and the equilibrium employment in the cluster is found by dividing equation (28) by equation (25), which gives

$$(29) \quad n^E = 1/2 n^*.$$

The ratio between the optimal and the equilibrium employment in the cluster is independent of the demand-function. However, the number of clusters depends on the demand function. Equation (15) gives the optimal number of clusters, k^* . Substituting the specific functions (A1) – (A3) into equation (15), gives

$$(30) \quad \mathbf{b} - \mathbf{g}(L-n^*) = [(n^*)^{q-1} k/K]^{-1} [(n^*)^q + \mathbf{q}(n^*)^q].$$

Substituting n^* from equation (25) into equation (30) gives the optimal number of clusters,

$$(31) \quad k^* = (\mathbf{g}K/2\mathbf{q}) [(1-\mathbf{q})/(\mathbf{b}-\mathbf{g}L)]^2.$$

The equilibrium number of clusters, k^E , is found from equation (8). Substituting the specific functions (A1) – (A3) into equation (8) gives

$$(32) \quad \mathbf{b} - \mathbf{g}(L-n^E) = [(n^E)^{q-1} k/K]^{-1} (n^E)^q.$$

Substituting n^E from equation (28) into equation (32) gives the equilibrium number of clusters,

$$(33) \quad k^E = (\mathbf{g}K/\mathbf{q}) [(1-\mathbf{q})/(\mathbf{b}-\mathbf{g}L)]^2.$$

Dividing equation (31) into equation (33), we find the ratio between the equilibrium number of clusters and the optimal number of clusters,

$$(34) \quad k^E = 2k^*.$$

With the specific functions (A2) and (A3), the value of the average productivity in optimum is given by

$$(35) \quad p^* a(n^*) = [(n^*)^{q+1} k^*/K]^{-1} (n^*)^q = K / (n^* k^*).$$

The value of the average productivity in equilibrium is given by

$$(36) \quad p^E a(n^E) = [(n^E)^{q+1} k^E/K]^{-1} (n^E)^q = K / (n^E k^E).$$

Substituting equations (29) and (34) into equation (36) gives

$$(37) \quad p^E a(n^E) = K / (n^* k^*).$$

From equations (35) and (37) it is easily seen that the value of the average productivity in equilibrium and in optimum is equal. Thus, we have shown that equation (22) holds for the specific functions (A1) – (A3).