First-best Optimality in Capital Income Taxation

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Abstract

In case of risk, especially aggregate risk which cannot be insured, the literature states that for achieving first-best optimality state-dependent lump-sum taxes are absolutely necessary. However, we show in a two-asset portfolio choice model that a suitably designed capital income tax can ensure a first-best solution without using state-dependent lump-sum taxes. Therefore, taxation must not focus on the single assets' returns but on the prices of commodities, embedded in these assets. Hence, our tax system uses the prices for resource shifting into the future and for incurring risk as tax bases.

JEL-Classification: **H21, G11, D10**

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1 Introduction

Even on pure efficiency grounds, it is not always optimal to use lump-sum taxes instead of distorting tax regimes for maximizing social welfare. In case of uncertainty, this is well-known from the paper of Eaton and Rosen (1980). They consider an economy with uncertainty about the individual wage rate, so called private risk, and show that a proportional wage tax increases welfare compared to a state-independent lump-sum tax. The distortion of the labor supply is overcompensated by the insurance effect, taxation creates by reducing the volatility of labor income.

Similar reasoning takes effect in case of aggregate (or systematic) risk which cannot be insured, as it hits all agents in all economies at the same time in the same manner. Thus, a second-best optimum under risk uses distorting taxes instead of state-independent lump-sum taxes and balances efficiency in resource allocation and optimal risk diversification on different agents or on different types of consumption. A first-best optimum which avoids such trade-offs and ensures ex-ante and ex-post efficiency can only be reached by using state-contingent lump-sum taxes. This is so far the traditional point of view in the literature (see, e.g., Christiansen, 1993, 1995).

In what follows we show that a first-best optimum may be reached by using a proportional capital income tax system, defined appropriately, instead of making use of state-contingent lump-sum taxes. Moreover, we define an optimality rule for taxing risk and show that one cannot conclude from Eaton und Rosen (1980) that a state-independent lump-sum tax, if available, should not be used. Therefore, we use a two-period, two-asset standard portfolio choice model.

Thereby we base our analysis on the literature on risk-taking effects of taxation which was marked by the seminal papers by Mossin (1968), Stiglitz (1969), and Sandmo (1969). In a partial two-asset model they isolated a risk-inducing substitution effect (Musgrave effect) and a wealth-elasticity dependent income effect. Sandmo (1977) generalized the results to the case of several risky assets. An excellent survey on the risk-taking literature can be found in Sandmo (1985). Sandmo (1977, 1985) also derives the Musgrave effect for a net tax; a result we will use later on. In spite of a meanwhile huge literature on positive theory in this area, there are only few articles on normative aspects of optimal portfolio taxation under aggregate risk. Although considering aggregate risk, Richter (1992) collapses to private risk as he assumes risk neutrality in public consumption. Thus, the risk transferred towards the government does not matter and the analysis is more an extension of Eaton and Rosen (1980) and cannot analyze the optimal diversification of risk endogenously. The most important article is Christiansen (1993). He analyzes several tax systems, extends the elasticity rule of Richter (1992) to aggregate (or systematic) risk and shows that the uniform taxation of the risky and the riskless asset is optimal only for special assumptions. Furthermore, Christiansen (1993, 1995) characterizes optimal taxation with state-contingent taxes.

If not using state-contingent lump-sum taxes, all these papers on portfolio choice use either tax systems which tax the return of each asset with one separated tax rate or ones, which introduce restrictions exogenously by taxing all assets with the same rate or setting the tax rate on riskless return equal to zero. The former tax systems create portfolio choice distortions and are moreover not able to reallocate risk without affecting the riskless rate of return. The latter ones are not able to follow optimal risk diversification and efficient resource allocation simultaneously. This is because they do not take notice of households' motivations to invest in assets.

Sandmo (1977) showed that assets can be interpreted as commodities and assets' returns as commodity prices. Thus, optimal taxation of capital income is an analogon to optimal commodity taxation. However, households do not buy assets in order to own these assets; they invest in assets for two reasons. First, households want to shift resources into the future in order to smooth their lifetime consumption. For giving up present consumption they receive a compensation which equals the riskless rate of return. Second, households want to incur risk as long as they are compensated for their disutility. This compensation equals the risk premium and is positive in expected values, but can be negative in bad states of the world. Ex-ante, incurring risk extends therefore the consumption possibility constraint. Taken together, the view of Sandmo (1977) can be reformulated: assets are a way to trade the commodities ”resource shifting” and ”risk,” whereby the riskless rate of return and the risk premium are the appropriate commodity prices. If a government puts weight on efficiency in allocation and optimal risk diversification, the adequate tax bases are not the returns of the assets, but the prices of the commodities, embedded in these assets.

Therefore, we tax the riskless rate of return, which is the price received for postponing present consumption into the future, in both assets with one tax rate. We use another tax rate, applied on the price received for incurring risk. This price equals ex-post the realization of the risk premium in the risky asset. Such a tax system[1] yields enough independent instruments to pursue both risk diversification and resource allocation without creating distortions and can achieve first-best optimality. Our results support that the traditional view of a first best solution being only possible by using state-contingent lump-sum taxes in case of aggregate risk must be handled with care.

In our tax system the government fully participates in households' income risk and we do not need any assumptions whether government deals better with risk than households. Thus, these results are robust against both the Bulow/Summers critic and the Hirshleifer critic.

The proceeding of the paper is as follows. In Section 2 we establish the model and characterize the household decision, Section 3 examines the welfare-maximization problem the government has to solve. Finally, we analyze the first-best optimum and show that our tax system can mimic a state-contingent lump-sum tax. The paper closes with some conclusions.

2 The Model and Household's Decision

We apply a two-period model with homogeneous investors and only two assets in order to concentrate on the efficiency considerations and on the risk-shifting effect

[1]The tax system can be named commodity-based capital income taxation. However, it is an analogon to a commodity tax system, not a real one.

of taxing the risk premium. One asset yields a riskless return of $r > 0$. The other asset's return, $\tilde{x} \in [-1, \infty]$, is state-dependent with a probability distribution $F(\tilde{x})$. Further we assume a small open economy with a perfectly integrated capital market. Thus, the assets' returns are given exogenously and independent of taxation. We abstract from uncertain inflation that renders the real rate of return of asset 1 uncertain. This assumption seems reasonable in order to analyze the effects of taxing assets in different risk classes.[2] To keep things as simple as possible, we suppose that Fisher separability is fulfilled, and hence savings can be determined independently of the portfolio choice decision.[3]

Thus, we do not model the saving decision of the household explicitly. The investor has an exogenously given initial wealth which she completely invests in one or in both assets available in the first period. Private consumption in the second period is financed by the principal and the return of the investment. The household consumes also a public good g provided by the government and financed out of tax revenue T in Period 1. The overall population size is normalized to 1 in the aggregate. As we suppose that all households are small, they take the provision of the public good as given and independent of their individual behavior. Thus all prices are treated parametrically. Denote initial wealth as W_0 and the investment in the risky asset as a. The household budget constraint in Period 1 is then

$$\tilde{W}_1 = (\tilde{x} - r) \cdot a + (1 + r) \cdot W_0 - \tilde{T}, \tag{1}$$

whilst the tax structure behind $\tilde{T}$ is to be defined later on. The household maximizes a von Neumann-Morgenstern utility function that is supposed to be additively separable in the consumption of the private and public good. The utility function is written as

$$\Omega = U(\tilde{W}_1) + V(\tilde{g}). \tag{2}$$

Partial derivatives are supposed to fulfill $U_W > 0$, $V_g > 0$, $U_{WW} < 0$, $V_{gg} < 0$.

[2] In a small open economy the riskless asset can be interpreted as an indexed foreign government bond. Nowadays indexed bonds are available on the capital market.

[3] This assumption requires special assumptions concerning the utility function, especially that the marginal rate of substitution between present and future consumption is linear in the latter. See, e.g., Sandmo (1974).

Thus, the investors are risk averse in their private and public consumption. Most preceding papers interpreted this model as a choice between different assets and used the return of each as a separated tax base. Therefore, they tax the riskless return of the first asset with one tax rate and use another one for the complete return of the second, risky asset (see, e.g., Christiansen, 1993). This approach neglects the motivations of buying assets. In fact, the household wants to trade different commodities, being embedded in the assets. The first commodity is future consumption. In order to give up present consumption the investor must be compensated. This compensation equals the riskless rate of return and r can be interpreted as price investors receive for shifting resources into the future. The second commodity in the economy is systematic risk, which cannot be insured. As our investor is risk averse this risk causes disutility. But, if there is a compensation, high enough, the investor will bear some risk according to her risk preferences in order to expand her consumption possibilities. This compensation is normally measured ex-ante as an expected mark up on the riskless return. Thus, this risk premium $\mathrm{E}[\tilde{x}-r]$ can be interpreted as the price, one gets for incurring aggregate risk.

Contrary to the riskless rate of return, we have to remind that the risk premium is not really paid to the investors. After realization of risk, the investor gets ex-post the realization of $\tilde{x}-r$ as additional income out of incurring risk. This realization, we will name realized risk premium or excess return, can be either higher or lower than its expected value. This depends on the state of nature we are in ex-post.

Looking now at the assets, we see that the riskless asset can solely be used for consumption shifting and therefore pays out the riskless rate of return, r, whereas the risky asset combines both commodities. It can be used for shifting resources into the future and with each amount invested in, one receives some risk, too. Therefore, its return $\tilde{x}$ consists of both prices. Ex post, investors get r for consumption shifting and the realization of $\tilde{x}-r$ (and thus the realized risk premium) as return of bearing risk.

We now follow the commodity approach described above and consider a tax system that applies two tax rates, t_0 and t_1. The riskless asset and the riskless part of the random return of the risky asset is taxed by t_0. Therefore, t_0 is a commodity tax on resource shifting and future consumption.[4] The return to the risk component in the risky asset and thus the excess return or realized risk premium, $\tilde{x} - r$, is taxed ex post by t_1.[5] Thereby we assume full loss offset in case the realized risk premium turns negative.

Whereas traditional tax systems need information about the amount and the kind of capital income, our tax system needs information about the amount of capital income, $\triangle W_0$, and the initial wealth (or total savings) W_0. Then, the tax base for taxing the riskless interest return is $TB_0 = r \cdot W_0$.[6] The tax base for taxing the excess return is the difference between total capital income and riskless interest return, $TB_1 = \triangle W_0 - r \cdot W_0$. Thus, there are no additional informational requirements compared to a consumption-orientated income tax system with savings allowances.

The tax structure can now be summarized as

$$\tilde{T} = t_1 \cdot (\tilde{x} - r) \cdot a + t_0 \cdot r \cdot W_0 \tag{3}$$

in which $\tilde{T}$ is a stochastic variable as long as $t_1 \neq 0$.

The advantage of this commodity-based tax system is that we have tax rates tailored to the motives behind the investments. Especially, we get an instrument to influence the risk taking directly without disturbing other prices. Thus, we can get new insights into risk shifting between private and public consumption and we are able to calculate an optimal tax rate that applies solely to the ex-post realization of the risk premium. However, as an investor uses ex ante the expected values, this is equivalent to the risk premium and we can therefore answer the question to what extend – if at all – the risk premium should be taxed.

But, the tax system used has not only an advantage compared to asset-based taxation, it has also one more degree of freedom as a net tax, which corresponds

[4]As we do not model savings decision explicitly, the tax rate t_0 on the riskless return in both assets is in fact equivalent to a lump-sum tax.

[5]After household's optimization we show that, for a representative investor, the tax on the market risk premium is equivalent to taxing her preference-dependent risk premium directly.

[6]The riskless interest rate is public knowledge from the world capital market.

to a consumption tax (see for example Sandmo, 1969, 1977). A net tax uses likewise a tax on the risky component only, but it sets exogenously the tax rate on the safe return to zero. Therefore, the risk allocation and the overall tax level are linked and one cannot be considered without the other. This means, one cannot achieve optimal risk allocation without disturbing the expenditure level, at least in some states of nature, and vice versa.[7]

Our setting implies a three-stage game. First, the government sets the optimal tax rates for a given probability distribution of risky return $F(\tilde{x})$ and an optimal investment behavior of the household. Second, for the resulting expected after-tax returns, the household maximizes utility by choosing her optimal risky investment a. Finally, $\tilde{x}$ is realized, taxes are paid and the household consumes both the private and the public good. We can solve the problem by backward induction.

The household chooses her investment in the risky asset, a, to maximize her expected utility for a given value of $\tilde{g}$ and given tax rates. All private consumption is financed out of second period after-tax wealth and thus her maximization problem is characterized by

$$\max_a \mathrm{E}\left[U(\tilde{W})\right] = \mathrm{E}\left\{U\left((1-t_1)\cdot(\tilde{x}-r)\cdot a + [1+(1-t_0)r]\cdot W_0\right)\right\} + \mathrm{E}[V(\tilde{g})] \quad (4)$$

and the first-order condition gives

$$\mathrm{E}\left[U_W \cdot (\tilde{x}-r)\right] = 0. \quad (5)$$

The second-order condition is always fulfilled, as the assumption $\mathrm{E}[\tilde{x}] > r$ guarantees an interior solution (see Arrow, 1970).

From equation (5) it follows that the representative investor increases a as long as the expected marginal utility of wealth evaluated with the ex-ante risk premium is positive. In the optimum the investor balances risk and reward of the risky asset. To make this point clearer we show that the tax on the market risk premium equals a tax on the risk premium of the representative investor. The FOC can be rewritten as

$$\mathrm{E}\left[U_W \cdot \tilde{x}\right] = \mathrm{E}\left[U_W\right]\cdot r.$$

[7] This linkage is discussed in a similar context in Christiansen (1993, p. 65f).

Applying $\mathrm{E}[x \cdot y] = \mathrm{E}[x] \cdot \mathrm{E}[y] + \mathrm{Cov}(x,y)$ gives

$$\mathrm{E}\left[(\tilde{x}-r)\right] = -\frac{\mathrm{Cov}\left(U_W, \tilde{x}\right)}{\mathrm{E}\left[U_W\right]}. \tag{6}$$

The right-hand side of equation (6) is the risk premium of the representative investor.[8] In the optimum, this risk premium must be equal to the risk premium, ex ante expected to be paid by the capital market. This result shows that taxing the market risk premium is equivalent to taxing the individual preference-dependent risk premium of an investor.

The effect of the tax rate t_1 on the excess return $\tilde{x}-r$ corresponds to the well analyzed effect of a net tax (Sandmo, 1977, 1985) and both equal the Musgrave substitution effect in positive theory of taxation and risk-taking (e.g., Mossin, 1968). Thus, we get[9]

$$\frac{\partial a}{\partial t_1} = \frac{a}{1-t_1}. \tag{7}$$

This effect follows from the preferences of the investor for a certain risk position. Increasing the tax rate on the risk premium distorts this risk position by shifting risk from the private sector to the public sector. However, as the tax does not have an income effect which changes the budget constraint, the former after-tax position is still achievable. To establish her desired risk position again, the investor raises thus her investment in the risky asset.[10] Thus, the representative investor holds pre- and post-tax the same revenue-risk position if $a^* = \frac{a}{1-t_1}$ and her budget constraint remains in fact unchanged. Moreover, the utility of private consumption in each state of nature is independent of the tax rate t_1. This can be easily seen by using a^* in the maximization problem (4). Social risk-taking always increases with the tax rate t_1, whereas private risk-taking is kept constant. The risk tax generates ex ante expected tax revenue of $t_1 \cdot \mathrm{E}[\tilde{x}-r] \cdot a^*$ at no private utility costs, but makes tax revenue riskier and therefore causes higher costs of risk in public consumption.

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[8] See also Christiansen (1993), S. 59f.

[9] A formal proof is available from the authors upon request.

[10] Obviously in this context the assumptions of $\tilde{x}$, being independent of a, and of unconstrained risky investment are only reasonable for tax rates much smaller than 100 per cent. However, in case of short sale restrictions, $a \leq W_0$, $\frac{\partial a}{\partial t_1}$ will be zero if the risky investment a reaches W_0.

The effect of the tax on riskless return, t_0, on private risk-taking is equivalent to a tax applying onto initial wealth and depends on the risk-wealth elasticity and therefore on assumptions concerning risk aversion (see e.g., Stiglitz, 1969, proposition 1b).

3 Welfare Maximization and Optimal Income Taxation

Now we are able to analyze the optimal income tax policy. We assume that the government must run a balanced budget and chooses the tax rates t_0 and t_1 in order to maximize social welfare. Therefore, the provision level of the public good is determined endogenously in the model. Thus, we can use equation (3) and set $\tilde{T} = \tilde{g}$. As the budget must be balanced in every state of nature, the realization of $\tilde{g}$ varies and depends still on the realization of $\tilde{x}$.

Using $a = a(t_1, t_0)$ as the optimal response of a household to tax rates t_1, t_0, we can state the social maximization problem as follows:

$$\begin{aligned} \max_{t_1,t_0,\tilde{g}} \{\Omega &= \mathrm{E}\left[U\left((1-t_1)\cdot(\tilde{x}-r)\cdot a(t_1,t_0) + [1+r(1-t_0)]\cdot W_0\right)\right] + \mathrm{E}\left[V(\tilde{g})\right] \\ \text{s.t.} \quad & \tilde{g} = t_1\cdot(\tilde{x}-r)\cdot a(t_1,t_0) + t_0\cdot r\cdot W_0\} \end{aligned} \tag{8}$$

Therefore, we get the following FOCs:

$$\frac{d\Omega}{dt_1} = \frac{\partial \mathrm{E}[U(\tilde{W}_1)]}{\partial a}\cdot\frac{\partial a}{\partial t_1} - \mathrm{E}\left[U_W\cdot(\tilde{x}-r)\right]\cdot a + \mathrm{E}\left[V_g\cdot(\tilde{x}-r)\right]\cdot\left(a + t_1\cdot\frac{\partial a}{\partial t_1}\right) = 0 \tag{9}$$

$$\frac{d\Omega}{dt_0} = \frac{\partial \mathrm{E}[U(\tilde{W}_1)]}{\partial a}\cdot\frac{\partial a}{\partial t_0} - \mathrm{E}\left[U_W\right]\cdot rW_0 + \mathrm{E}\left[V_g\cdot(\tilde{x}-r)\right]\cdot t_1\frac{\partial a}{\partial t_0} + \mathrm{E}[V_g]\cdot rW_0 = 0 \tag{10}$$

Recalling that $\frac{\partial \mathrm{E}[U(\tilde{W}_1)]}{\partial a} = 0$ from household portfolio choice, given by (5), and recognizing that $\frac{\partial a}{\partial t_1} = \frac{a}{1-t_1} > 0$ from equation (7) gives

$$\frac{d\Omega}{dt_1} = \mathrm{E}\left[V_g\cdot(\tilde{x}-r)\right] = 0. \tag{11}$$

Substituting equation (11) into (10), results in

$$\left(\mathrm{E}[V_g] - \mathrm{E}[U_W]\right)\cdot rW_0 = 0 \quad \Leftrightarrow \quad \mathrm{E}[V_g] = \mathrm{E}[U_W]. \tag{12}$$

Using again the household optimum (5) and (11), we can write

$$\mathrm{E}\left[U_W \cdot (\tilde{x} - r)\right] = 0 = \mathrm{E}\left[V_g \cdot (\tilde{x} - r)\right] \tag{13}$$

and by using $\mathrm{E}[x \cdot y] = \mathrm{E}[x] \cdot \mathrm{E}[y] + \mathrm{Cov}(x, y)$ and $\mathrm{E}[V_g] = \mathrm{E}[U_W]$ from (12) this equals

$$\mathrm{Cov}(U_W, \tilde{x} - r) = \mathrm{Cov}(V_g, \tilde{x} - r). \tag{14}$$

Hence, social welfare is maximized if the covariances are equalized.

We are now able to conclude:

Proposition 1 The risk tax is used to allocate risk efficiently on private and public consumption, measured by the covariances of marginal utilities and the risky return, whereas the tax on resource shifting guarantees an efficient tax level and equates the ex ante expected marginal utilities of consumption. Moreover, it is never optimal to use solely the lump sum tax t_0 to finance the public good and not to tax the realization of the risk premium with a $t_1 \in (0;1)$.

Proof:
The first part of the proposition is obvious. The lump sum tax can ex ante achieve any tax level in expected terms without creating any distortions, but cannot, by definition, transfer risk from private to public consumption. The risk tax, t_1, can instead shift risk without any effect on private consumption. Suppose for the second part the case $t_1 = 0$. Then, government's budget constraint does not depend on the risky return $\tilde{x}$. Thus, g is deterministic and V_g is fixed. Therefore, $\mathrm{Cov}(V_g, \tilde{x} - r) = 0$. From (14) then follows $\mathrm{Cov}(U_W, \tilde{x} - r) = 0$. This is only possible for either $U_W = \mathit{constant}$ and the households being risk neutral in private consumption, which conflicts with our general assumptions, or for $t_1 = 1$, which contradicts the initial assumption $t_1 = 0$. The equivalent reasoning applies to the case $t_1 = 1$ as long as households are risk-averse in public consumption. Thus the optimal tax rate must be in between, $0 < t_1 < 1$, because for $t_1 > 1$ $\mathrm{Cov}(U_W, \tilde{x} - r)$ turns positive whereas still $\mathrm{Cov}(V_g, \tilde{x} - r) < 0$. Hence, $t_1 > 1$ can never be an optimum, as the optimality condition (14) cannot be fulfilled. □

The intuition of this is straightforward. If we use solely the lump sum tax we have a fixed level of g and get ex ante $U_W = V_g$ in *expected terms*. Ex post the actual marginal utility of private consumption depends on the realization of $\tilde{x}$. Then, it is optimal to have a lower level of g in bad states and a higher one in good states. However, this can be efficiently[11] reached by linking public expenditure to the realization of $\tilde{x}$ by taxing the realization of the risk premium, $\tilde{x} - r$, with $t_1 \in (0;1)$. The diversification of aggregate risk on both consumption types depends on the relative strength of the risk aversion in private consumption compared to the one in public consumption. Therefore, the tax rate t_1 depends on this ratio of risk aversion. The higher the relative strength of risk aversion in private consumption, the higher will be the tax rate on the risk premium. Furthermore, only if households are risk-neutral in private consumption and risk averse in public consumption, the risk premium should not be taxed.

That a state-independent lump-sum tax is not optimal in case of risk is well-known. For private risk, which can be insured and gives by the law of great numbers a deterministic tax revenue, Eaton and Rosen (1980) showed that a proportional tax on individually risky wage income is welfare improving. In the optimum there is a trade-off between distorting labor supply and the income insurance effect of the tax. Our setting is equivalent to the Eaton-Rosen approach, if and only if we would assume risk neutrality in public consumption, as then the risk in public consumption does not matter. For the same reason, the result in Richter (1992) is in fact not a tax rule for capital income, being risky in aggregate, but an extension to Eaton and Rosen (1980), although Richter considers aggregate risk.

However, even in such a world the commodity-based tax structure is welfare improving. From the fact that using solely a lump-sum tax is not optimal, we may not conclude, that we should not use a lump-sum tax if available. On the contrary to Richter, for the new tax system the taxation of risk does not create any distortions and we can allocate the whole risk to the public sector. Hence, we do not get the usual trade-off between insurance and efficiency in allocation,

[11] Here, efficiently means in accordance with optimal risk allocation.

which, normally, is supposed to be inevitable. Instead, we reach higher welfare with efficient risk diversification by using the risk tax with $t_1 = 1$, and optimal resource allocation by using the state-dependent lump-sum tax, t_0.

In case of aggregate risk, tax revenue is risky, too, and cannot be returned to households in any deterministic way. In spite of this, income taxation may still have an welfare improving effect, if the government uses the still risky tax revenue in order to provide public goods which cannot be purchased by the private market (see Kaplow, 1994, p. 795.) Now, the results above must be extended to consider additionally the optimal distribution of risk on private and public budgets. The commodity-based tax system achieves this in an again non-distorting manner using the tax on risk.

Our result is general and independent of the Bulow/Summers critic that the government does actually not participate in risk. Bulow and Summers (1984) argue that the risk is incorporated in the economic depreciations but these depreciations are not fully tax deductable. Thus, the corporation tax does not provide any insurance against risk. However, this argument cannot be extended to portfolio investment and income taxation. Unexpected economic depreciations lead to a capital loss and hence the return $\tilde{x}$ of asset 1 will be lower. Therefore, all risk is incorporated in the asset's return.[12] Since in our model the return x is liable to the income tax with tax rates t_0 and t_1 the government actually participates in all risk.

Furthermore the government returns the risk to the households by providing a public good and our result is independent of any assumption, whether the government can deal better with risk than the capital market. The government's ability to deal with risk corresponds to households' risk aversion in public consumption. As we do not have any restrictions on this risk aversion, we do not have any assumption concerning the government's ability to handle risk better than the capital market or not. Thus the Hirshleifer-critic does not apply, either.

[12] Suppose asset 1 as a stock. In this case, its return reflects the variation of the firm value.

4 First-best Optimality and Commodity-based Taxation

In general, the literature states, a first-best optimum which avoids any trade-off between insurance and resource allocation can only be achieved via using state-dependent lump-sum taxes (see, e.g., Christiansen, 1993, 1995). However, taken together our results above, we can show that the commodity-based capital income tax system delivers enough degrees of freedom in order to mimic a state-dependent lump-sum tax. Under certain conditions, we can hence achieve a first-best optimum by taxing the returns to capital invested in an appropriate manner.

We start by characterizing a first-best welfare optimum using state-dependent lump sum taxes T_i.[13] The optimization problem is

$$\max_{T_i, g_i} \sum_i \pi_i \left\{ U\left([x_i - r] \cdot a + [1+r] \cdot W_0 - T_i \right) + V(g_i) \right\} \quad \text{s.t.} \quad T_i = g_i, \tag{15}$$

where π_i is the probability for state i, and results in the well-known Samuelson-condition

$$U_W^i = V_g^i. \tag{16}$$

Hence, the marginal utility in private and in public consumption have to be equalized in each state of the world. Let $z_i = (x_i - r)a + (1+r)W_0$ be overall income in state i. Then, the Samuelson-condition (16) determines private consumption W_{1i} as share of overall income z_i in each state of the world. This results in an optimal risk-sharing function

$$W_{1i}^* = W_{1i}(z_i, \pi) \tag{17}$$

where π is a vector with probabilities for every state. Equivalently, we get for optimal public consumption

$$g_i^* = g_i(z_i, \pi) = z_i - W_{1i}^*. \tag{18}$$

[13]Using state-dependent lump-sum taxes means that for each possible realization x_i of $\tilde{x}$ the government sets ex ante a conditional lump-sum tax T_i. The subscript i then indicates different states. Hence, the tax revenue also depends on $\tilde{x}$.

It can be shown that the sensitivity of the risk-sharing functions to income in state i are given by[14]

$$\frac{\partial W_{1i}^*}{\partial z_i} = \frac{T(\tilde{W}_1)}{\mathrm{E}[T(\tilde{W}_1)]} \quad \text{and} \quad \frac{\partial g_i^*}{\partial z_i} = \frac{T(\tilde{g})}{\mathrm{E}[T(\tilde{g})]}. \tag{19}$$

Hereby, $T(\tilde{W}_1)$ is absolute risk tolerance in private consumption and $\mathrm{E}[T(\tilde{W}_1)]$ is average risk tolerance in overall consumption, which is equal to absolute risk tolerance in overall income, as $\mathrm{E}[T(\tilde{W}_1)] = \mathrm{E}[T(\tilde{g})] = \mathrm{E}[T(\tilde{z})]$. Thus, if overall income in state i, z_i, increases, each type of consumption should be increased by the proportion of absolute tolerance in this consumption type to absolute risk tolerance in overall consumption in order to guarantee an ex post optimum.

However, we can also characterize the first-best solution from an ex ante point of view.[15] Therefore, we differentiate the welfare function totally, and set this equal to zero. Thus the optimum is characterized by a situation where no infinitesimal change of endogenous variables can increase welfare any further.

According to (2) the social welfare function is defined as follows:

$$\Omega = \mathrm{E}\left[U\left((\tilde{x} - r) \cdot a + (1 + r) \cdot W_0 - \tilde{T}\right)\right] + \mathrm{E}\left[V(\tilde{g})\right] \tag{20}$$

Also the budget constraint has to be taken into account:

$$\tilde{g} = \tilde{T}. \tag{21}$$

Differentiating totally, we get:

$$d\Omega = \mathrm{E}\left[U_W \cdot \left((\tilde{x} - r) \cdot da - d\tilde{T}\right)\right] + \mathrm{E}\left[V_g \cdot d\tilde{g}\right] \tag{22}$$

$$d\tilde{g} = d\tilde{T} \tag{23}$$

Substituting (23) in (22) and using the FOC of the household (5), gives for an optimum

$$d\Omega = \mathrm{E}\left[(V_g - U_W) \cdot d\tilde{g}\right] = 0. \tag{24}$$

[14]We skip the derivation of the following results. See for a formal proof and further interpretation i.e., Gollier (2001), pp. 313 and Proposition 80.

[15]See for the following also Christiansen (1993), p. 61.

In an first-best optimum the marginal utilities must be equal ex-ante in expected terms and ex-post in every state of nature.[16] This condition can admittedly be restated and separated into different effects:

$$d\Omega = \left(\mathrm{E}\left[V_g\right] - \mathrm{E}\left[U_W\right]\right) \cdot \mathrm{E}\left[d\tilde{g}\right] + \mathrm{Cov}[V_g, d\tilde{g}] - \mathrm{Cov}[U_W, d\tilde{g}] = 0. \tag{25}$$

Proposition 2 First-best optimality consists out of two effects, which must sum up to zero:

(a) The marginal social net revenue effect of an optimal tax equals the difference between the expected marginal utility of public and private consumption.

(b) Taxes affect the allocation of public and private consumption in every state. Thus a risk-diversification effect between private and public consumption occurs. This diversification is represented by the difference between the covariances.

We can now show that the optimal taxation rule, implied by (12) and (14), approximates ex post optimal risk diversification in a linear manner and therefore gives a linear approximation of a first best Pareto-efficient solution. In each case, ex-ante efficiency is guaranteed by equation (12). Furthermore, if we assume $\tilde{g}^*$ to be a linear function in the risky return, we can additionally show that, compared with the first-best optimum using state-dependent lump sum taxes, the commodity-based capital income taxation implies the same condition for optimal risk allocation as in Proposition 2. To see this, we differentiate government's budget constraint (3) totally and get

$$d\tilde{g} = r \cdot W_0 \cdot dt_0 + (\tilde{x} - r) \cdot a \cdot dt_1 + t_1 \cdot (\tilde{x} - r) \cdot da. \tag{26}$$

This implies

$$\mathrm{Cov}(U_W, d\tilde{g}) = \mathrm{Cov}(U_W, r \cdot W_0 \cdot dt_0 + (a \cdot dt_1 + t_1 \cdot da)(\tilde{x} - r)) \tag{27}$$

[16]First-best optimality requires ex-ante *and* ex-post efficiency. If the marginal utilities are equated only in expected terms, we have ex-ante efficiency, but not first-best optimality. Without ex-post efficiency, $U_W^i = V_g^i \; \forall i$, welfare could be improved by shifting resources after the realization of risk.

which similarly applies to $\mathrm{Cov}(V_g, d\tilde{g})$. It follows that $\mathrm{Cov}(V_g, d\tilde{g}) = \mathrm{Cov}(U_W, d\tilde{g})$ implies $\mathrm{Cov}(U_W, \tilde{x} - r) = \mathrm{Cov}(V_g, \tilde{x} - r)$ and vice versa.

Proposition 3 The commodity-based tax system with a tax on riskless interest return, r, and a tax on the realized price for risk, $\tilde{x} - r$, ensures both efficiency in allocation and efficient risk diversification on private and public consumption simultaneously. The taxation rule implies a non-distorting linear approximation of the first-best optimum without applying to state-contingent lump sum taxes directly.

Proof:
The first-best optimum ist characterized by (25) as

$$d\Omega = (\mathrm{E}[V_g] - \mathrm{E}[U_W]) \cdot \mathrm{E}[d\tilde{g}] + \mathrm{Cov}[V_g, d\tilde{g}] - \mathrm{Cov}[U_W, d\tilde{g}] = 0. \qquad (28)$$

Since from government's optimization in (12), we know that the expected marginal utilities are equalized, $\mathrm{E}[U_W] = \mathrm{E}[V_g]$, all we have to show is that, given the linear approximation of $\tilde{g}^*$, $\mathrm{Cov}(V_g, d\tilde{g}) = \mathrm{Cov}(U_W, d\tilde{g})$ implies $\mathrm{Cov}(U_W, \tilde{x} - r) = \mathrm{Cov}(V_g, \tilde{x} - r)$ and vice versa.

From basic covariance rules, it follows[17]

$$\begin{aligned} \mathrm{Cov}(V_g, d\tilde{g}) &= \mathrm{Cov}(U_W, d\tilde{g}) & (29) \\ \Leftrightarrow \mathrm{Cov}(U_W, rW_0\,dt_0 + (a\,dt_1 + t_1\,da)(\tilde{x} - r)) &= \mathrm{Cov}(V_g, rW_0\,dt_0 + (a\,dt_1 + t_1\,da)(\tilde{x} - r)) & \\ \Leftrightarrow \mathrm{Cov}(U_W, \tilde{x} - r) &= \mathrm{Cov}(V_g, \tilde{x} - r). & (30) \end{aligned}$$

□

Thus, if the lump sum tax on riskless interest income t_0 is used to equate the expected marginal utilities, we get a linear approximation of ex post optimal risk diversification. The reason is that we have no portfolio distortion effect and have an appropriate instrument to influence risk allocation by using the tax on the risk premium. Thus, the combination of a state-independent lump sum tax, t_0, and the optimal tax on the risk premium, t_1, replaces a state-contingent lump sum tax. All

[17] The following transformation assumes that the optimal provision of the public good is a linear function of t_1.

results above hold for *any* utility function, even if it is not additively separable in private and public consumption.

Indeed, we can go one step further by assuming now hyperbolic absolute risk aversion (HARA) in private and public consumption. In this case, risk tolerance is a linear function of state-dependent wealth and we have[18]

$$T(\tilde{W}_1) = \psi_{W_1} + \frac{\tilde{W}_1}{\rho} \quad \text{resp.} \quad T(\tilde{g}) = \psi_g + \frac{\tilde{g}}{\rho}, \tag{31}$$

where ψ_j, $j \in \{W_1, g\}$, and ρ are parameters of the utility function and independent of income. Under the HARA-assumption the optimal risk-sharing functions are linear in income. This can be seen by differentiating the first part of equation (19) for overall income, z_i, again:

$$\begin{aligned}
\frac{\partial^2 W_{1i}^*}{\partial z_i^2} &= \frac{T'(\tilde{W}_1)\cdot\frac{\partial W_{1i}^*}{\partial z_i}\cdot \mathrm{E}[T(\tilde{W}_1)] - T(\tilde{W}_1)\cdot \mathrm{E}\left[T'(\tilde{W}_1)\cdot\frac{\partial W_{1i}^*}{\partial z_i}\right]}{\mathrm{E}[T(\tilde{W}_1)]^2} \\
&= \frac{T'(\tilde{W}_1)\cdot\frac{T(\tilde{W}_1)}{\mathrm{E}[T(\tilde{W}_1)]}\cdot \mathrm{E}[T(\tilde{W}_1)] - T(\tilde{W}_1)\cdot \mathrm{E}\left[T'(\tilde{W}_1)\cdot\frac{T(\tilde{W}_1)}{\mathrm{E}[T(\tilde{W}_1)]}\right]}{\mathrm{E}[T(\tilde{W}_1)]^2} \\
&\overset{\text{HARA}}{=} \frac{T'(\tilde{W}_1)\cdot T(\tilde{W}_1) - T(\tilde{W}_1)\cdot T'(\tilde{W}_1)}{\mathrm{E}[T(\tilde{W}_1)]^2} = 0.
\end{aligned} \tag{32}$$

The third equality is due to linear absolute risk tolerance in $\tilde{W}_1$, which implies the derivation $T'(\tilde{W}_1)$ to be deterministic and constant. From equation (32) follows that the sensitivity of optimal risk sharing towards fluctuations in overall income is constant and therefore optimal risk sharing rules are linear in income z_i. Hence, it can be shown that they take the form[19]

$$W_{1i}^* = M_{W_1} + b_{W_1}\cdot z_i = -M_g + (1-b_g)\cdot z_i, \tag{33}$$

$$g_i^* = M_g + b_g \cdot z_i. \tag{34}$$

In this case, the first best solution to the social planners optimization problem

$$\max_{a, M_g, b_g} \mathrm{E}\left[U(-M_g + (1-b_g\cdot\tilde{z}) + V(M_g + b_g\cdot\tilde{z})\right]$$

[18]Risk tolerance is the inverse of risk aversion. HARA resp. linear absolute risk tolerance contains constant absolute and constant relative risk aversion as special cases. See, e.g., Gollier (2001), pp. 26–29. Meanwhile, HARA-function are often used in capital market theory.

[19]See Gravelle and Rees (1992), pp. 610.

with $\tilde{z} = (\tilde{x} - r)a + (1+r)W_0$ is characterized by

$$\begin{aligned} \mathrm{E}[U_W(\tilde{x}-r)] &= 0 && (35) \\ \mathrm{E}[U_W] &= \mathrm{E}[V_g] && (36) \\ \mathrm{E}[V_g(\tilde{x}-r)] &= 0. && (37) \end{aligned}$$

These conditions are necessary and sufficient for an ex post Pareto efficient solution. From (35) and (37) follows

$$\mathrm{Cov}(U_W, \tilde{x}-r) = \mathrm{Cov}(V_g, \tilde{x}-r). \qquad (38)$$

because $\mathrm{E}[U_W] = \mathrm{E}[V_g]$ from (36).

Obviously, these conditions are the same ones, we get by solving the optimal taxation problem in Section 3. Hence, the commodity-based tax system can achieve first-best optimality without using state-dependent lump-sum taxes. Given optimal risk-sharing parameters M_g^* and b_g^*, the optimal tax rates t_0^* and t_1^* are

$$\begin{aligned} t_0^* &= \frac{M_g^* + b_g^*(1+r) \cdot W_0}{r \cdot W_0} && (39) \\ t_1^* &= b_g^*. && (40) \end{aligned}$$

Proposition 4 If the utility functions belong to the HARA class, first-best optimality can be ensured by using a proportional capital income tax system and requires solely two tax rates. It is not necessary to define a vector of contingent lump-sum taxes which contains a tax payment for every possible state.

This result may seem trivial at first glance, as in our simplified model the same can be achieved by using a state-independent lump-sum tax and a proportional wealth tax. However, this is due to the assumption of exogenous wealth. Our result carries over to a world with endogenous savings and exogenous labor supply. In this case, the optimal taxation rule in Section 3 is preserved mutatis mutandis, if additionally a proportional labor tax is used (Schindler, 2004). Hence, extending our analysis, the special treatment of the risk premium ensures first-best optimality, as long as there is a perfectly inelastic tax base or if a state-independent lump-sum tax is available. On the other hand, a proportional wealth tax looses its optimality properties in such a world, because it distorts intertemporal consumption choice.

To the best of our knowledge, the literature states that, in such a setting, a vector of contingent lump-sum taxes has to be defined ex ante and there is need to decide, which state of the world is realized ex post – even if the utility function fulfills the HARA assumption. Therefore, the information costs are extremely high.

We showed that this point of view has to be handled with care. A commodity-based tax system in capital income taxation can on the contrary be implemented ex ante and at low costs by just deciding over two proportional tax rates.

5 Conclusion

We showed in a standard portfolio-choice model that a positive taxation of the risk premium is optimal, as long as the investor is not risk neutral in private consumption. The resulting risk shifting generates no portfolio distortion. Therefore, the risk allocation is efficient. The tax on the riskless market rate of return in both assets is used to balance the budget and to equate the expected marginal utilities of private and public consumption. These expected marginal utilities determine the optimal provision level of the public good.

Furthermore, the optimal taxation rule linearly approximates a first-best optimum for *any* utility function. Moreover, if the utility function is additive separable and fulfills the HARA assumption, an ex post Pareto-efficient equilibrium can be reached directly without using state-dependent lump sum taxation.

In a nutshell, we can mimic a first-best solution by using a suitably designed capital income tax system, given the assumptions of our model. This tax system is not orientated on the different assets, but the commodities which are embedded in them. These are resource shifting for future consumption and risk. Thus, we can call it a commodity-based tax system. We want to refrain, that our results do not depend on any assumption concerning the government's ability to deal with risk.

Due to the simplicity of our model, further research is needed. Using a more sophisticated model and taking account of an intertemporal consumption decision does not alter our first-best optimality result. Additionally, it is easy to show that using several risky assets does not affect the analysis. Especially in the Markowitz-case, where all households invest in the same portfolio of risky assets, asset 1 in our paper can be interpreted as risky world market portfolio. Our results are transferable as long as there is a lump-sum tax in the model, this means as long as labor supply or savings are exogenous in an intertemporal-consumption model.

However, the interest rates should be endogenized. The assumption that the returns are fixed is acceptable only in a small open economy and an integrated perfect world capital market. Other possibilities to expand the paper are using a model with more than two periods or introducing tax arbitrage considerations, if there is also labor income.

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