Taxation and Tournaments

by

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Abstract:

This paper analyzes the effects of progressive taxes on labour supply and income distribution in the context of the rank-order tournament model originally developed by Lazear and Rosen (1981). We show conditions under which a more progressive tax schedule will cause so large general equilibrium effects that the inequality in disposable income will actually increase. We also show that a non-zero redistributive tax is always optimal if society's welfare function displays inequality aversion; this result always holds, regardless of behavioral responses and general equilibrium effects.

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1. Introduction

It is a common observation in many countries that the salaries of top executives have increased, relative to those of other employees, in recent decades. In some countries, particularly the United States, this tendency is quite dramatic, while in others it is less clear.[1] One could think of a number of alternative explanations for such a development. However, it has become usual among economists to rely on the tournament model of Lazear and Rosen (1981) for explaining "excessively large" wage differences (i.e., differences that are hard to imagine as being the result of differences in marginal productivities among employees).[2] In fact, a number of empirical studies indicate that the tournament model cannot be rejected as an explanation to observed wage-setting.[3] Widening wage differences could then indicate that the tournament model has become even more relevant then earlier.

While the heated popular debate about increased wage inequality implicitly seems to call for policy intervention, there is not much policy analysis in the scholarly literature. In particular, there is, to the best of our knowledge, no study of how the tax system affects labor supply, wages and income distribution in a tournament setting. The aim of present paper is to fill that gap by applying a simple progressive tax schedule to the simplest possible version of the tournament model. Even within that setup, the model is rich enough to provide a number of interesting results that are significantly different from the competitive model.

The paper is organized as follows. In Section 2, we present the basic model and derive the individual response to an increase in the marginal tax rate. In Sections 3 and 4,

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[1] For data on the U.S. and France, see Piketty and Saez (2001).
[2] See also Lazear (1995) and, for broader surveys of the literature on wage incentives, Prendergast (1999) and Gibbons and Waldman (1999).
[3] See Bognanno (2001) and the works cited therein.

taking account of the effects of taxes on wage formation, we analyze the effects of increased progressivity in the tax schedule when the labor market is monopsonistic and competitive, respectively. In particular, we study whether increased progressivity leads to a lower dispersion of disposable incomes. Section 5 contains a normative analysis of the optimal linear income tax, and Section 7 concludes the paper.

2. The Model

In order to focus on the essential features of our theoretical framework, we adopt the simplest version of the tournament model. There are two workers with identical productivity and preferences. The firm promises a reward W_1 to the worker who wins the tournament, and $W_2 < W_1$ to the looser. With a linear tax schedule, the net-of-tax income to worker *i* is thus $Y_i = W_i(1-t) + k, \quad i = 1, 2$. Worker preferences depend on net income Y_i and effort m_i and are quasi-linear:

$$E[u(Y_i, m_i)] = E[Y_i] - C(m_i), \quad i = 1, 2 .$$

The function $C(.)$ represents the disutility of effort and is a convex function: $C'(m) > 0, C''(m) > 0$. The assumption of quasi-linearity rules out income effects on labor supply, which is obviously restrictive, and the assumption must basically be defended in terms of analytical simplicity. In addition, our interest here lies in the study of tax distortions, and the efficiency implications of these are in any case determined by the compensated effects, i.e. by the tax responses net of income effects.

The worker produces output q_i according to a linear technology: $q_i = m_i + e_i$. Here, e_i is a random disturbance with zero mean and variance s^2, and the disturbance terms are uncorrelated between workers. The firm gives the prize to the worker with the highest output *q*, which means that winning the tournament is the joint result of effort and luck.

A worker's expected utility is now

$$P_i\left[W_1(1-t)+k\right]+(1-P_i)\left[W_2(1-t)+k\right]-C(m_i)\,, \qquad (1)$$

where P_i is the probability of winning the tournament as a function of the worker's own effort. More elaborately, the probability of winning can be written as

$$P_i \equiv prob(q_i > q_j) \equiv prob(m_i - m_j > e_j - e_i) \equiv G(m_i - m_j)\,, \qquad (2)$$

where each worker takes the other's level of effort as given. Each worker chooses m_i to maximize expected utility. This yields the first order condition[4]

$$(W_1 - W_2)(1-t)\frac{\partial P_i}{\partial m_i} = C'(m_i) \qquad (3)$$

We have

$$\frac{\partial P_i}{\partial m_i} \equiv \frac{\partial G_i(m_i - m_j)}{\partial m_i} \equiv g(m_i - m_j)\,. \qquad (4)$$

We concentrate, like Lazear and Rosen (1981) on the symmetric case with identical individuals. Thus $m_i = m_j$, and $g_i(m_i - m_j) \equiv g(0)$. Condition (3) can then be written as

$$(W_1 - W_2)(1-t)g(0) = C'(m)\,. \qquad (5)$$

Equation (5) gives us the worker's optimal effort level as a function of the after-tax wage spread, given that we are in a symmetric equilibrium where $m_i = m_j = m$. The function $g(.)$ is the density of the difference $e_j - e_i$, and $g(0)$ is a measure of the uncertainty attached to the effect of increased effort on the probability of winning the

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[4] The second order condition is satisfied under the additional assumption that $\frac{\partial^2 P_i}{\partial m_i^2} < 0$.

tournament, given that we are in a symmetric equilibrium. Intuitively, if the random element is important (i.e., $\text{var}(e)$ is large), then $g(0)$ is relatively small. It is then uncertain whether an extra effort will result in winning the tournament, and thus the wage dispersion does not stimulate effort so much. If on the other hand $\text{var}(e)$ is small, then $g(0)$ is relatively large, and the wage dispersion will stimulate a relatively higher effort. To put it differently, the more important is the random element in the tournament, the larger must the wage spread be in order to induce a given level of effort.

From (5) it follows immediately that for a given wage dispersion an increase in the tax rate will lead to less effort on the part of the worker; this is the distortionary effect of the income tax in this model. However, when there is a change in the tax rate, there is no reason to expect that the wage dispersion will remain the same. But the analysis of this requires a general equilibrium perspective, and to this we now turn.

The firm chooses W_1 and W_2 to maximize profits $2m-(W_1+W_2)$. The constraint on profit maximization depends on the nature of competition. We have set the product price equal to unity, i.e., we have implicitly assumed perfect competition in the product market. Lazear and Rosen (1981) and Lazear (1995) assume that the firm is a monopsonist in the labour market. This would be the case of, for instance, a large company in a small country, selling its product in the competitive world market and having a considerable influence on the local labor market. In such a case, the firm can drive down wages to the point where the worker's participation constraint is exactly satisfied. Thus, the expected utility of the worker must be equal to some constant, e.g. zero, representing the expected utility of some outside option:[5]

$$\frac{1}{2}\left[W_1(1-t)+k\right]+\frac{1}{2}\left[W_2(1-t)+k\right]-C(m)=0. \tag{6}$$

An alternative approach is to assume that competition in the labor market drives long-run equilibrium profits to zero, so that the relevant constraint becomes

[5] For simplicity we assume that the value of the outside option is independent of the tax rate.

$$2m - (W_1 + W_2) = 0\,. \tag{7}$$

If the information structure of the economy is such that a tournament is an efficient employment contract (which will be the case if it is difficult to observe effort, but easy to observe output), the tournament model is quite compatible with a *monopsonistic competition* model of the labor market. For brevity, we shall refer to this case simply as the competitive case. It is competitive in the sense that profits are driven down to zero, although employers are wage-setters. There is thus a clear similarity with the Chamberlinian model of monopolistic competition.

We now consider the implications of these two alternative approaches.

3. General Equilibrium: The Monopsonistic Case

With a monopsonist firm, and a participation constraint, the firm solves the optimization problem

$$\begin{aligned} &\max_{W_1, W_2} \quad 2m - (W_1 + W_2) \\ &s.t. \quad \frac{1}{2}\left[W_1(1-t) + k\right] + \frac{1}{2}\left[W_2(1-t) + k\right] - C(m) = 0. \end{aligned} \tag{8}$$

This gives us the first-order conditions for the firm:

$$\begin{aligned} &2\frac{\partial m}{\partial W_1} - 1 + l\left[\frac{1}{2}(1-t) - C'(m)\frac{\partial m}{\partial W_1}\right] = 0 \\ &2\frac{\partial m}{\partial W_2} - 1 + l\left[\frac{1}{2}(1-t) - C'(m)\frac{\partial m}{\partial W_2}\right] = 0. \end{aligned}$$

Eliminating l and using the symmetry condition that, by (5), $\partial m / \partial W_1 = -\partial m / \partial W_2$, we can simplify the firm's first-order conditions to

$$(1-t) = C'(m)\,. \tag{9}$$

This condition shows clearly the general equilibrium tax wedge. In the absence of taxes, the marginal cost of effort would be equal to marginal productivity, which is unity. A positive tax rate distorts this connection and leads to a lowering of effort from its first best efficiency level.

The firm chooses the wage *dispersion* $(W_1 - W_2)$ such that the workers, by (5), choose an effort m such that (9) is satisfied.[6] The *level* of wages is determined by the participation constraint (6). The full solution of the model then consists of three equations:

a) the worker's first-order condition (5),
b) the participation constraint (6), and
c) the firm's first-order condition (9).

Rearranging, these three equations can be written

$$W_1 = \frac{C(m) - k}{1-t} + \frac{1}{2g(0)} \tag{10}$$

$$W_2 = \frac{C(m) - k}{1-t} - \frac{1}{2g(0)} \tag{11}$$

$$C'(m) = 1 - t . \tag{12}$$

The system is recursive; from (12) we can solve for $m = C'^{-1}(1-t)$ and substitute into (10) and (11) to obtain closed-form solutions for W_1 and W_2.

For the special case of a balanced government budget, we have

$$k = \frac{t \cdot (W_1 + W_2)}{2} .$$

Substituting (10) and (11) into this condition and solving for k, we obtain

$$k = t \cdot C(m) .$$

[6] Note that since we have assumed $C(m)$ to be strictly increasing for all $m > 0$ and defined only for non-negative values of m, (9) makes sense only if $t \le 1$.

Substituting this into (10) and (11) yields a considerably simpler solution of the model:

$$W_1 = C(m) + \frac{1}{2g(0)} \tag{10'}$$

$$W_2 = C(m) - \frac{1}{2g(0)} \tag{11'}$$

$$C'(m) = 1 - t\,. \tag{12'}$$

The case of a balanced budget can be used to study a tax reform; we may e.g. keep the level of government spending constant, and change the degree of progressivity in the tax system. However, the case of an unbalanced budget is not as uninteresting as it may seem at first sight. The tournament model is probably most relevant to a small group of wage earners at the top of the income scale. While the budget must be balanced over all income earners, there may be no reason to impose a balance constraint for a small subgroup of income earners. But the case of a balanced budget for this subgroup may be interesting because it represents a change in the degree of progression facing this particular subgroup, while holding their total net tax payments constant. In the following, we will therefore consider both cases.

The general equilibrium solution can now be used for comparative statics analysis. For example, we see immediately from (12) or (12') that the level of effort m is decreasing in t. And this holds regardless of whether the government budget is balanced or not.

Denoting the average wage rate by $\overline{W}$, we have for the general case, where the government budget is not necessarily balanced, that

$$\frac{\partial W_1}{\partial t} = \frac{\partial W_2}{\partial t} = \frac{\partial \overline{W}}{\partial t} = \frac{\dfrac{C(m)-k}{1-t} - \dfrac{C'(m)}{C''(m)}}{1-t}, \tag{13}$$

the sign of which is undetermined. It is however related to the Laffer curve. Taking the derivative of tax revenue with respect to t, we have

$$\frac{\partial t(W_1 + W_2)}{\partial t} = (W_1 + W_2) + 2t \frac{\partial \overline{W}}{\partial t} . \tag{14}$$

If we are on the peak of the Laffer curve, or to the right of the peak, the expression in (14) must be zero or negative. Since the first term in (14) is positive, this can be the case only if the second term is negative.[7] Thus, for relatively large values of t (by which we mean such values that we are close to, or to the right of, the peak of the Laffer curve), $\partial W_i / \partial t < 0$.

For the special case of a balanced budget, we obtain an unambiguous result; from (10') – (12') it follows that $\overline{W} = C(m)$ and thus that

$$\frac{\partial \overline{W}}{\partial t} < 0 . \tag{15}$$

Thus, a tax reform which increases the degree of progressivity in the income tax schedule, while leaving total government spending constant, will lead to a reduction in the average wage rate.

The income distribution will be studied by means of two measures. The simplest is the *absolute* wage dispersion

$$A \equiv W_1 - W_2$$

and its corresponding measure for disposable income,

[7] This illustrates an interesting difference between the standard (marginal productivity wages) model and the tournament model. In the former, Laffer curve effects are driven by the elasticity of labor supply at constant wages. In the tournament model, Laffer curve effects are driven by the tax rate affecting effort; this effect, via the relation $\partial W_i / \partial t = C'(m) \partial m / \partial t$, is translated into effects on wage rates which in turn affect tax revenue.

$$A_{disp} \equiv \left(W_1(1-t)+k\right)-\left(W_2(1-t)+k\right)=A(1-t).$$

From (10)-(11) it follows that

$$\frac{\partial A}{\partial t}=0, \quad \frac{\partial A_{disp}}{\partial t}<0. \tag{16}$$

An increase of the marginal tax rate leaves the dispersion of gross wages unaffected, while the spread of disposable incomes is reduced. These results are independent of whether the government budget is balanced or not, since the measures A and A_{disp} do not depend on k.

A richer measure is probably the *relative* dispersion

$$R \equiv \frac{W_1}{W_2}$$

and its corresponding measure for disposable income,

$$R_{disp} \equiv \frac{W_1(1-t)+k}{W_2(1-t)+k}.$$

In the following, we will mainly study the relative dispersion. Since the *absolute* wage dispersion A is independent of the tax rate, the effects of changes in t on the *relative* wage dispersion R will depend on what happens to the average wage rate:

$$\operatorname{sgn}\frac{\partial R}{\partial t}=-\operatorname{sgn}\frac{\partial \overline{W}}{\partial t}. \tag{17}$$

The effect on the average wage rate is in general undetermined by (13), and therefore we might very well have a case where a tax increase leads to an increased (pre-tax) inequality in society. In particular, this will happen for relatively large values of t, i.e., for values of t such that we are close to, or to the right of, the peak of the Laffer curve. For the special case of a balanced government budget, however, (15) implies that

$$\frac{\partial R}{\partial t} > 0 .$$

Thus a tax reform, leading to a higher progressivity in the tax schedule, unambiguously increases the relative dispersion in gross income.

For the relative dispersion of *disposable* income, some calculation yields

$$R_{disp} = \frac{2C(m)g(0)+1-t}{2C(m)g(0)-1+t} .$$

We see that the parameter k does not enter into this expression; thus the relative dispersion of disposable income does not depend on whether the government's budget is balanced or not. Taking the derivative of R_{disp} with respect to t and rearranging, we obtain

$$\operatorname{sgn}\frac{\partial R_{disp}}{\partial t} = \operatorname{sgn}\left[\left(C'(m)\right)^2 - C(m)C''(m)\right]. \tag{18}$$

For the special case of an exponential cost-of-effort function $C(m) = m^{1+a}/(1+a)$, where $a > 0$, we have $\left(C'(m)\right)^2 - C(m)C''(m) = m^{2a}/(1+a) > 0$. Thus a tax increase will lead to an increased dispersion in disposable income whenever the cost function is exponential. We will here have a simple example of a case where an increased progressivity in the tax schedule leads to higher inequality, as measured by the relative dispersion of disposable income.

4. General Equilibrium: The Competitive Case

If the firm operates in a competitive (i.e., monopsonistically competitive) labor market, bidding for the workers implies that the market functions as if the following optimization problem is solved:

$$\begin{aligned}&\max_{W_1, W_2} \quad \frac{1}{2}\left[W_1(1-t)+k\right]+\frac{1}{2}\left[W_2(1-t)+k\right]-C(m) \\ &s.t. \quad 2m-(W_1+W_2)=0\end{aligned} \tag{19}$$

Problem (19) is obviously the dual of problem (8) and has the same first-order condition. The full solution to the competitive model thus consists of three equations:

a) the worker's first-order condition (5),
b) the zero-profit constraint (7), and
c) the firm's first-order condition (9).

Substituting and rearranging, these three equations can be written

$$W_1 = m + \frac{1}{2g(0)}, \tag{20}$$

$$W_2 = m - \frac{1}{2g(0)}, \tag{21}$$

$$C'(m) = 1 - t. \tag{22}$$

Note first that there is no parameter k in (20)-(22); thus the general equilibrium solution for the competitive case is independent of whether we consider a balanced government budget or not. If we compare this solution to the corresponding solutions (10)-(12) in the monopsony case, we see that the absolute wage dispersions A and the effort levels m are the same. This is so because of the quasi-linear utility function, which rules out any income effect on part of the workers; consequently it does not matter whether the surplus ends up with the firms (as in the optimization problem (8)) or with the workers (as in the optimization problem (19)). Further, the average wage level is lower in (10)-(12) than in (20)-(22), because the surplus ends up with the workers in the latter case. This implies that $m > C(m)$, a relation which can also be proved from the participation constraint and the assumption of positive profit in the monopsonistic case.

The comparative statics of the competitive case are simple. Since t only appears in the third equation, all effects from the tax system go via the effect on m. We have that the effort level falls, the absolute pre-tax income dispersion is unaffected, and the average income falls, as t is increased. Further, the absolute disposable income dispersion falls. As for the relative dispersion, we have

$$R = \frac{2mg(0)+1}{2mg(0)-1}$$

and thus

$$\frac{\partial R}{\partial t} > 0 .$$

This means that a higher t will unambiguously increase the relative pre-tax wage dispersion. The relative dispersion in disposable income is somewhat more complicated; some algebra shows that it can be written

$$R_{disp} = \frac{2\left(m(1-t)+k\right)g(0)+(1-t)}{2\left(m(1-t)+k\right)g(0)-(1-t)}, \tag{23}$$

and thus

$$\operatorname{sgn}\frac{\partial R_{disp}}{\partial t} = \operatorname{sgn}\left[\frac{(1-t)\,C'(m)}{C''(m)} - k\right]. \tag{24}$$

The two terms in brackets are of opposite signs, and no firm conclusion can be drawn concerning the tax effects on relative disposable income. On the one hand a tax increase leads to less effort (and more so, the less concave is the function $C(m)$). The effect of this is to leave absolute wage differences the same, but at a lower level; consequently, relative inequality increases. On the other hand, a lower tax bare increases the importance of the transfer k for relative income inequality, and this pulls the inequality index (23) down.

For the special case of a balanced budget, we have $k = mt$ and thus (24) can be written

$$\operatorname{sgn}\frac{\partial R_{disp}}{\partial t} = \operatorname{sgn}\left[\frac{(1-t)^2}{C''(m)} - mt\right] = \operatorname{sgn}\left[h - \frac{t}{1-t}\right], \qquad (24')$$

where

$$h \equiv \frac{\partial m}{\partial(1-t)} \cdot \frac{1-t}{m}.$$

Here we get the intriguing result that if t is close to zero, the first term within the square brackets will dominate, and thus $\partial R_{disp} / \partial t > 0$[8]. This means that if the government introduces a balanced-budget progressive income tax in an economy where there has been no tax before, the relative dispersion in disposable income will unambiguously increase.[9]

5. Social Welfare and Optimum Progressivity.

So far, our approach has been positive. We now turn to the normative aspects of the model and study the design of an optimal linear income tax. Even in a framework where all individuals are identical *ex ante*, there might be a case for progressive taxation, to provide insurance to risk-averse individuals (cf. Varian, 1980). In the present version of the model, however, utilities are quasi-linear, and the individuals thus do not display any risk aversion. This means that the insurance argument does not apply. From the point of view of efficiency, a tax only causes distortion, and

[8] This holds under the assumption that h does not also approach zero as t is lowered.
[9] For the case of an exponential cost-of-effort function and a balanced budget, it is easily shown that $\partial R_{disp} / \partial t > 0$ for $t < 1/(1+a)$ and $\partial R_{disp} / \partial t < 0$ for $t > 1/(1+a)$. This means that if $C(m)$ is almost linear, i.e., a is close to zero, the relative dispersion of disposable income will increase for allmost all values of $t < 1$.

expected utility $E(u_i)$ is maximized for t = 0. It would be interesting to work out the tax rate maximizing expected utility assuming strictly concave utility functions, but this is beyond the scope of the present paper.

With *ex ante* identical and risk-neutral individuals, could there still be a case for progressive taxation? The result that expected utility is maximized for t = 0 gives us the answer to the problem of optimal tax design if society cares only about *equality of opportunity*. But if there is a social concern about the *ex post* distribution of utilities, i.e. about the *equality of outcomes*, we can postulate a social welfare function $j(u_1, u_2)$, where $u_i \equiv W_i(1-t)+k-C(m)$, $i=1, 2$, is the realized utility of individual *i*. If $j(.)$ is strictly increasing, symmetric and strictly concave, it exhibits inequality aversion:

$$u_1 > u_2 \quad \Rightarrow \quad \frac{\partial j(u_1, u_2)}{\partial u_1} < \frac{\partial j(u_1, u_2)}{\partial u_2}. \tag{25}$$

Society now wants to find a tax rate t that maximizes social welfare, subject to the government's budget being balanced, and the wage rates (W_1, W_2) being determined in general equilibrium. In standard microeconomic models (i.e., non-tournament models), some redistributive tax is always optimal if incomes are exogenous to the tax system. But since the tax rate distorts incentives, a non-zero tax is not always called for in a full equilibrium; if the distortions are too large, the optimal tax rate is zero.

By contrast, we will show that for the tournament model, a non-zero redistributive tax is always optimal – even when equilibrium behavioral responses are taken into account. We will first show this for the competitive case of Section 4. By (20)-(21) and the government budget constraint, we have

$$u_1 = W_1(1-t)+k-C(m) = m - C(m) + \frac{1-t}{2g(0)}$$
$$u_2 = W_2(1-t)+k-C(m) = m - C(m) - \frac{1-t}{2g(0)},$$

and some algebra shows that

$$\frac{\partial u_1}{\partial t} = t\frac{\partial m}{\partial t} - \frac{1}{2g(0)}$$
$$\frac{\partial u_2}{\partial t} = t\frac{\partial m}{\partial t} + \frac{1}{2g(0)}.$$

These expressions can be utilized to obtain

$$\frac{\partial j\,(u_1, u_2)}{\partial t} = (j_1 + j_2)\,t\frac{\partial m}{\partial t} - (j_1 - j_2)\frac{1}{2g(0)}.$$

Recalling that $j_1 < j_2$, we have that $\partial j\,/\partial t$ is positive at $t = 0$. Thus, starting out without any tax at all, it is always optimal to introduce some redistributive taxation. Setting the derivative equal to zero gives us the optimal tax rate

$$t^* = \frac{1}{2g(0)} \cdot \frac{j_1 - j_2}{j_1 + j_2} \cdot \frac{1}{\frac{\partial m}{\partial t}}, \qquad (26)$$

which is obviously positive. The optimal tax rate can also be written as an inverse elasticity formula. Let

$$h \equiv \frac{\partial m}{\partial(1-t)} \cdot \frac{1-t}{m}.$$

Substituting, we obtain

$$\frac{t^*}{1-t^*} = -\frac{1}{2g(0)} \cdot \frac{j_1 - j_2}{(j_1 + j_2)m} \cdot \frac{1}{h}. \qquad (26')$$

As we would expect, the tax rate is inversely related to the elasticity of effort supply, which measures the distortionary effect of the tax. Moreover, the tax rate is higher, the higher is the degree of inequality aversion. This is in line with standard insights into the shape of optimal linear tax schedules; see e.g. Dixit and Sandmo (1977). But a novel and interesting aspect of the solution is the role played by the term *g*(0). From (20)-(21) we observe that *g*(0) measures the extent of wage dispersion. If there is no random element in the tournament (which corresponds to $g(0) \to \infty$), there is no inequality and the optimal tax rate is obviously zero. As the random element becomes of increasing importance (which corresponds to $g(0)$ diminishing), wage inequality increases and so does the optimal tax rate.[10] Inequality, which in this model is the result of the random element in the tournament outcome, should be diminished according to our egalitarian social welfare function. However, it should not be eliminated, since this would have an adverse effect on the supply of effort.

Let us also note that there is no contradiction between the result in the present section, which says that with inequality aversion, it is always optimal to impose a non-zero redistributive tax t^*, and the results in Section 4, which say that imposing a redistributive tax may increase (relative) inequality in disposable income. The reason is twofold. First, expression (24') implies that raising the tax rate from zero by a small amount will increase inequality. It does not follow that raising the tax rate somewhat more, for example to t^*, will increase inequality. Second, although imposing a tax may increase inequality in income, the arguments in the social welfare function j are not incomes, but utilities (i.e., disposable income minus the cost of effort $C(m)$).

So far, we have only dealt with optimal taxation in the context of a competitive labor market (i.e., the model of Section 4). Working out the corresponding optimization for the case of a monopsonistic labor market (the model of Section 3), we have from (10')-(12') and the government's budget constraint that

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[10] It can be shown that for the special case of a quadratic cost-of-effort function $C(m) = \frac{1}{2}m^2$, and an exponential social welfare function $j\,(u_1, u_2) = u_1^a u_2^a$, with 0 < *a* < 1, the optimal tax rate turns out to be

$$t^* = \frac{\sqrt{2 + g(0)^2}}{2g(0)} - \frac{1}{2}.$$

Thus, in this case, t^* is independent of a and monotonically increasing in $g(0)$. It takes the values zero for *g*(0) = 0, and approaches unity as $g(0)$ approaches ½.

$$u_1 = \frac{1-t}{2g(0)}$$
$$u_2 = -\frac{1-t}{2g(0)}. \qquad (27)$$

(Note that no significance should be attached to the possibility of a negative utility, since utility is only unique up to a linear transformation) We then have

$$\frac{\partial u_1}{\partial t} = -\frac{1}{2g(0)}$$
$$\frac{\partial u_2}{\partial t} = \frac{1}{2g(0)},$$

which yields

$$\frac{\partial j\,(u_1, u_2)}{\partial t} = -\frac{1}{2g(0)}(j_1 - j_2) > 0.$$

Thus no interior optimum t^* exists for the monopsonistic case, and it is therefore optimal to set *t* at its upper limit, $t^* = 1$. The economic interpretation of this is straightforward. In the monopsonistic case, the participation constraint (6) says that $u_1 + u_2 = 0$. Since utilities then are given by (27), a society with inequality aversion obviously maximizes welfare by choosing $t^* = 1$, thereby obtaining $u_1 = u_2 = 0$.

This could be interpreted in two ways. Either we should not take the optimal tax problem seriously; it is not a suitable representation of society's policy choice. Or we *should* take it seriously; in that case, a monopsonistic labor market will lead to policy decisions implying higher and higher marginal tax rates, and more and more redistribution, while the endogenous wage rates respond with higher and higher inequality. Whether the tournament model is a good representation of reality – and, if so, whether the competititive case or the monopsonistic case is the most realistic one – is of course an empirical question. But if there is at least some realism in the monopsonistic case, it can help us understand some tendencies in the general development of tax systems in the Western world during the post-World War II period.

6. Concluding Comments

The tournament model could of course be extended in many directions. For instance, assuming a concave (rather than a quasi-linear) utility function would introduce income effects on individual behavior, and assuming that individuals are different with respect to preferences and/or productivity would add new dimensions to the analysis. Nevertheless, even the simplest version of the model seems to yield a number of interesting results that are potentially important for our understanding of public policy. For example, the model implies that a redistributive tax can cause more inequality in society, once general equilibrium effects are taken into account. Still, with a social welfare function displaying inequality aversion, a non-zero redistributive tax is always optimal. If there are monopsonistic tendencies in the labor market, this optimal tax rate may even be very high.

Working out some of the more obvious extensions of the simple version of the model seems to be a fruitful avenue for future research. Among other potentially interesting issues, one can mention the intriguing property of the Mirrlees (1971) model that the optimal marginal tax rate is zero at the top; under what circumstances this also holds for the tournament model is an unresolved issue. Also, questions regarding signaling and incentive compatibility seem particularly relevant in a rank-order environment. There is a large literature on special problems arising if the policy maker has imperfect information about preferences and ability on part of the agents. In a tournament context, where there are two policy makers (the firm, deciding about the prize, and the government, deciding about the tax rate) the signaling game becomes even more involved.

It has long been realized in the labor economics literature that the labor market occupies a special position among markets. Not only is it, for most people, the most important market in terms of the determinants of their personal welfare. It is also a market which deviates in a number of important respects from the standard competitive model. The latter insight has so far had little effects on the theory of taxation, which still relies heavily, both for positive and normative studies, on competitive assumptions. The present study encourages us to believe that there may be further gains to be made from studies of taxes and incentives in a non-competitive setting.

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