# What constitutes a convention?
# Implications for the coexistence of conventions*

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March 2002

Discussion Paper 05/02

**Abstract**

A model of repeated play of a coordination game, where stage games have a location in social space, and players receive noisy signals of the true location of their games, is reviewed. Sugden (1995) suggests that in such a model, there can be a stationary state of convention coexistence only if interaction is non-uniform across social space. This paper shows that an alternative definition of conventions, which links conventions to actions rather than expectations, permits convention coexistence when interaction is uniform.

**Keywords:** Convention; Coordination game; Equilibrium selection

* This paper has benefited greatly from the comments of Bertil Tungodden, Oddvar Kaarbøe, Gaute Torsvik, Robert Sugden, Jörgen Weibull, Alexander Cappelen and Jarle Møen. For any remaining errors, the author is responsible.

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## Introduction

The old adage ”When in Rome, do as the Romans” advises us to adopt the behavioural patterns of the people in whose presence we find ourselves. Whether of necessity or for pleasure, the proverb suggests that it is somehow advantageous to mimic the actions and mannerisms of our social surroundings. The proverb thus prescribes a change in behaviour as we move from one social sphere to another, say, from Rome to Paris, from the cinema to the theatre, from the mail room to the board room and so on. In chameleon-like fashion, we should change our language from Italian to French in the first case, our code of dress from casual wear to formal wear in the second, the formality of our speech from less to more in the third case.

How do the behavioural patterns we observe come into existence in the first place? Why do individuals who find themselves in similar social surroundings often share a common way of doing things; speaking the same language, wearing similar clothes, employing common means of exchange such as money, and using common standards of measure? The evolutionary approach suggests that these *conventions* form through the repeated interaction of individuals. The basic idea is as follows. If there is some advantage to acting in a manner similar to others, and if a shared history of actions is used as a gauge of how others will act, then present actions will reflect past actions, and over time a pattern could form in which one way of acting becomes dominant. According to this line of thinking, then, interdependent individual actions form a collective pattern, a convention, through the indirect observance of precedent. In Rome you speak Italian because it eases communication with others who, based on what you know about Rome, are more likely to speak Italian than any other language.[1]

Evolutionary models of learning adopt the above perspective in one form or another, to explain how agents who adapt to or learn from the actions of their environment, can end up using the same type of action. In the model of Kandori, Mailath and Rob (1993), agents adapt by choosing a best reply to the distribution of actions in the

[1] According to this perspective, agents coordinate by watching the past actions of each other, rather than by communicating with each other. This approach is therefore better suited to situations where communication is costly relative to the cost of switching between different actions. In the current example, we are therefore implicitly assuming that travellers are sufficiently well versed in different languages as to make communication on which language to use a waste.

population in the preceding period. Young (1993, 1998), on the other hand, assumes that agents observe a limited sample of the actions taken in a given number of preceding periods, and choose a best reply to this sample. Both Kandori et al and Young do in fact have even greater ambitions than showing that one convention or another will arise through the adaptation of agents, they also want to establish which convention will be chosen. To this end, they introduce a small probability of error into the strategy implementation of agents, and show that as this error probability grows arbitrarily small, one particular convention might be observed with near certainty in the very long run. For populations playing 2x2 coordination games, both Kandori et al and Young obtain the result that the convention thus selected entails play according to the risk dominant equilibrium of the game, as defined by Harsanyi and Selten (1988).[2]

The models of Kandori et al and Young are global interaction models, where an agent has a positive probability of interacting with any other agent in the population.[3] However, Ellison (1993) shows that the risk dominant equilibrium is also selected in a local interaction model where agents have fixed locations on a circle and adapt to the actions of a limited set of neighbours only, adaptation taking a form similar to Kandori et al. Judging from the results of this model, local interaction seems to leave little room for differences in conventions across locations. These results are, however, due to the persistent errors in the strategy implementation of agents. There exist local interaction models without this particular feature that do permit convention coexistence in simple coordination games. Anderlini and Ianni (1996) assume that errors only occur when agents attempt to use a different strategy than they did in the preceding period, which produces a non-ergodic dynamic process whose absorbing states do in some cases contain different strategies at different locations. In a model without implementation errors, Goyal and Janssen (1997) assume that agents can at some cost choose both strategies, thus always achieving coordination, and show that for intermediate cost levels, convention coexistence can be a stationary state.

[2] Note that for more complicated games, the models of Kandori et al and Young might differ in their predictions of the very long run outcome, as demonstrated by Jacobsen, Jensen and Sloth (2000).
[3] Sometimes the term uniform, rather than global, is used to describe interaction of this kind, see e.g. Ellison (1993). As the term uniform interaction is used in another context here, I choose the term global interaction to avoid confusion.

On the other hand, we have local interaction models which deem contagion of a particular strategy throughout a population likely. Blume (1995) shows that if there is spatial variation in the initial condition and randomness in the order in which agents revise their strategies, then we get coordination on the risk dominant equilibrium. Lee and Valentinyi (2000) similarly prove that if initially each agent has a positive probability of playing the risk dominant strategy and the population is sufficiently large, the risk dominant equilibrium is realized almost with certainty. In a more general setting, Morris (2000) shows that for any local interaction structure, there exists some contagion threshold, and coexistence is possible if agents do not choose to play according to the risk dominant equilibrium, whenever the probability with which their opponent does so is below this threshold.

A common feature of all the local interaction models discussed above, is that agents have fixed locations in some social space. The proverb "When in Rome, do as the Romans" suggests, however, that there is some manner of local interaction that these models do not properly address. The proverb advises a change in behaviour as we move from place to place, and we therefore need mobile agents to analyze social adaptation of this kind. Sugden (1995) presents a model in which interaction is global in the sense that agents have a chance of meeting all other agents in a population, yet local in the sense that each meeting has a random location in a social space. Agents are matched repeatedly at varying locations to play a coordination game, and adapt to the past history of play at the location at which they find themselves. To make the evolution of play at different locations interdependent, agents are assumed to have an imperfect understanding of their current location. Sugden concludes that in this model, a coexistence of conventions is possible if and only if the frequency of interaction across social space varies in a certain way. If there is no variation, i.e. if interaction is uniform across locations, there can be no coexistence of conventions.

In this paper, I argue that we can expand Sugden’s coexistence result to include the case of uniform interaction without unduly altering the fundamental structure of his model. Specifically, the result that coexistence is impossible under uniform interaction hinges on a definition of conventions that focuses on the expectations rather than the actions of agents. I show that if we adopt a more reasonable definition based on what agents do rather than what they expect others to do, a coexistence of conventions is possible even if there is no spatial variation in the frequency of interaction. With a different and weaker definition of conventions, we thus strengthen the case for coexistence initially made by Sugden.

The paper is organized as follows. In the next section, Sugden's model of convention formation is outlined. In section three, his definition of a convention is reviewed, and his results on coexistence are derived, with a detailed look at why coexistence is impossible when interaction is uniform. In section four, the main reasons for challenging Sugden's definition of a convention are given, and an alternative definition is presented. Section five shows that under this alternative definition, convention coexistence is possible in the case of uniform interaction. A final section raises the important point of robustness of the coexistence outcomes.

## Sugden's spatial model of convention formation

To intuitively understand the model presented by Sugden (1995), let us use a simple example to sketch the situation facing the agents of the model and the manner in which they behave. Suppose you have been invited to a party, and have to decide what to wear. You want to blend in with the other guests, so the first thing you do is form an opinion of who else is likely to come, and what they are likely to wear. You know the identity of your hosts, and who they are likely to invite, but this still leaves you with only an imprecise idea of the mix of people you will face at the party. Suppose that in the past you have observed that the way people dress depends on certain of their personal characteristics, let us say their age. You then combine your imprecise understanding of the average age of the people invited with your expectation of how people of that age will dress, and choose the garment that best matches the resulting estimate.

Now, the way you and others dress at this party, influences the ideas you and others have about what people wear what kind of clothes. So the way you and others dress for the next party with a similar mix of people, will be influenced by what people wear at this party. Moreover, since everyone has an imprecise idea of the mix at this party, they might adapt to different ideas of the average party-goer. The garments normally worn at parties with one mix of people might therefore influence the garments worn at parties with a different mix of people. The kind of question Sugden's model is designed to answer, is whether this will lead to a sitation in which the code of dress is the same for all parties regardless of the age of those invited, or whether we can have a stable situation in which dress codes vary with age.

Sugden frames this basic idea in terms of a model in which agents are repeatedly matched to play a coordination game, where each stage game has a random location in a social space. The players do not know the exact location of their game, instead, they receive a signal of their location which is close to but not necessarily spot on their actual location. The players have a common understanding of the past pattern of play at the various possible signals, and they are able to compute a probability distribution for their opponent’s signal given the signal they themselves have received. Based on this information, each player calculates the probability with which his opponent will choose either strategy, and chooses the strategy which maximizes his expected payoff.

In more formal terms, consider a large population of identical agents. In each period, a pair of agents is drawn at random from the population to play the following game

| | | Player 2 | |
|---|---|---|---|
| | | A | B |
| Player 1 | A | *a*,*a* | *c*,*d* |
| | B | *d*,*c* | *b*,*b* |

(G1)

We assume that $a > d$ and $b > c$, which makes (G1) a coordination game with two Nash equilibria in pure strategies, (A,A) and (B,B). Moreover, we assume that

$a-d>b-c$, which implies that (A,A) is the risk dominant equilibrium as defined by Harsanyi and Selten (1988).[4]

Players choose the strategy that maximizes their expected payoff. From the above payoff matrix, we see that a player is indifferent between strategies A and B if the probability with which his opponent chooses A is $\alpha$, where

$$\alpha \equiv \frac{b-c}{(a-d)+(b-c)} \tag{1}$$

For probabilites greater than $\alpha$, players prefer strategy A. And for probabilities lower than $\alpha$, they prefer strategy B. Note that since strategy A is risk dominant, $\alpha < 0.5$. This implies that players may choose strategy A even if the probability with which their opponent does so is below 50%. In a sense, the players are more easily persuaded to choose strategy A than strategy B, as the former strategy requires a lower probability that their opponent acts similarly.

Each game is assigned a random location in a social space. Social space is continuous, consisting of all points on the real line from 0 to 1. The location of a game is a random variable $y$ in the interval $[0,1]$. The probability that a game is assigned to a location less than or equal to $y$, is represented by $F(y)$. The corresponding density function $f(y)$, which denotes the frequency of interaction at each location, is assumed to be continuous, with $f(y)>0$ for all $y \in [0,1]$. In other words, all points on the real line from 0 to 1, have a positive probability of being host to the game in any given period.

[4] Harsanyi and Selten (1988) define risk dominance in the following way. Consider any 2x2 game with two strict Nash equilibria $U$ and $V$, where the losses to players 1 and 2 from unilaterally deviating from the equilibria are $(u_1, u_2)$ and $(v_1, v_2)$, respectively. $U$ risk dominates $V$ if $u_1 \cdot u_2 > v_1 \cdot v_2$, and V risk dominates U if the opposite inequality holds.

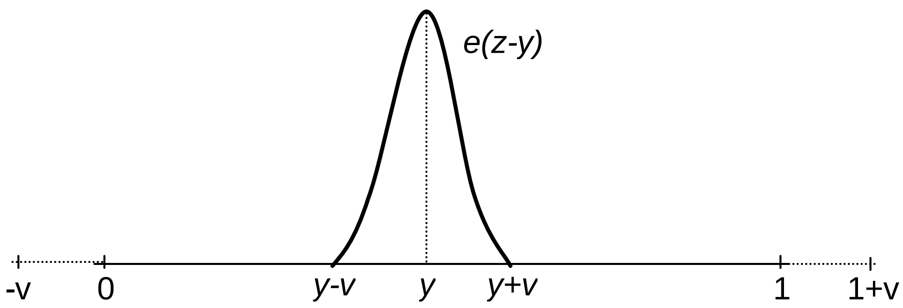


*Figure 1. Social space, location of games and distribution of signals*

Each player receives a signal $z$ of the location of the game. Figure 1 depicts the probability distribution of signals, given the location $y$ of a game. The signal of a player never falls more than a small distance $v$ from the true location $y$ of a game. Signals closer to $y$ do not have lower probabilities than signals further away from $y$, and signals equally far from $y$ are equally probable. Formally, the distance between a signal and the true location of a game, $z-y$, is a random variable with density function $e(z-y)$. For some small positive $v$, $e(z-y)>0$ if and only if $z-y\in(-v,v)$. The density function $e(z-y)$ is continuous, symmetric around a mean of 0, non-decreasing in the interval $[-v,0]$ and non-increasing in the interval $[0,v]$. The signals of the two players are assumed to be stochastically independent. Note that if the game is played at a location less than $v$ from 0 or 1, players may receive signals lower than 0 or higher than 1. The signal space is thus wider than the social space, and contains all points in the interval $[-v,1+v]$.

Knowing the distribution of games in social space, and the distribution of his signal around the true location of a game, a player can compute a probability function for the true location of a game given his own signal. A player is also aware of the distribution of his opponent's signal around the location of the game, and can calculate a probability function for the signal of his opponent given his own signal. Let $H(x|z)$ be a cumulative probability function which states the probability that the signal of his opponent is less than or equal to $x$, given his own signal $z$. The corresponding density function $h(x|z)$ thus represents the probability that the other player receives signal $x$ when a player receives signal $z$. Note that since a player's signal is at most a distance $v$ above or below the location of a game, the signals of two players are at most $2v$

apart. The function $h(x|z)$ is thus positive if and only if $x \in (z - 2v, z + 2v)$, i.e. in an interval of width $4v$.

A player expects his opponent to conform to past behaviour at the signal his opponent has received. There is thus a kind of bounded rationality at play, where players expect others to make simple decisions based on their signals, while they all actually let more complicated evaluations of their opponent's actions determine their own. A state of play function $g_t(z)$ denotes, for all feasible signals $z \in [-v, 1+v]$, the probability that a player receiving signal *z* at time *t* will choose strategy A. This function captures past play in the sense that it increases for signals at which A is chosen, reaching a maximum of 1 after a finite number of periods in which A is played repeatedly. For signals at which B is chosen, the state of play function decreases and reaches a minimum of 0 after a finite number of periods where B is repeatedly chosen. We can thus define the state space $\Omega$ as

$$\Omega = \{g(z) : 0 \le g(z) \le 1, \forall z \in [-v, 1+v]\} \qquad (2)$$

Weighing the probabilities $g(.)$ that strategy A is chosen at different signals with the probabilities $h(.|z)$ that an opponent receives these various signals, a player arrives at a probability that his opponent chooses A given his own signal *z*. Formally, the probability $p(z)$ that your opponent will choose strategy A when you receive signal *z* is

$$p(z) = \int h(x|z) g(x) dx \qquad (3)$$

Maximizing expected payoffs, a player thus chooses strategy A if $p(z) > a$, and B if $p(z) < a$.

The choices of the players in turn feed into the state of play function, and potentially influence play in future periods. We are interested in the stationary states of the system, which can be defined as follows.

DEFINITION 1

*A state* $\overline{g}(z) \in \Omega$ *is a stationary state if and only if the following holds:*

*If* $g_t(z) = \overline{g}(z)$ *then* $g_s(z) = \overline{g}(z)$ *for all* $s > t$ *and all* $z \in [-v, 1+v]$

In other words, we are at a stationary state when the state of play function stays the same forever after we have reached this state.

**Uniform interaction and coexistence**

In the context of the above model, Sugden suggests that a convention is realized at some signal $z$ (or as he puts it, universally followed at $z$), when two conditions are met. Firstly, a player receiving signal $z$ must observe the convention with certainty. Secondly, the opponent of a player receiving signal $z$ must observe the convention with certainty. In other words, we have an A-convention when for some signal $z$, both $g(z)$ and $p(z)$ equal one. Similarly, we have a B-convention when for some signal $z$ both $g(z)$ and $p(z)$ equal zero. Finally, to have a coexistence of conventions we must have an A-convention at some signal, a B-convention at some other signal, and this state of play must be a stationary state.

Interaction is uniform when the frequency of interaction at each location is the same, $f(y) = 1$ for all $y$. In any given period, then, a game has an equal chance of being assigned a location anywhere on the real line from 0 to 1. With uniform interaction, and given the above definition of a convention, no state in which there exist two different conventions can be a stationary state, as implied by the following proposition.

PROPOSITION 1

*Suppose* $f(y)=1$ *for all y.*

*If* $\hat{g}(.)$ *is a state of play function with the following properties for some signals* $z', z'' \in [-v, 1+v]$:

*i)* $\hat{g}(z)=1$ *for all* $z \in (z', z'')$

*ii)* $\hat{g}(z) \neq 1$ *for some* $z \notin (z', z'')$

*iii)* $z''-z' \geq 4v$ *or* $[z''-z' = 2v \text{ and } z' = -v]$ *or* $[z''-z' = 2v \text{ and } z'' = 1+v]$.

*Then* $\hat{g}(.)$ *is not a stationary state.*

A formal proof of the proposition is given in the appendix, as are the proofs of later propositions.

Proposition 1 rules out coexistence in the following way. For an A-convention to exist, there must be some signal $z$ at which a player is certain that his opponent chooses A, i.e. $p(z)=1$ for some $z \in [-v, 1+v]$. From equation (3), we see that this implies that A must be played with certainty, $g(.)=1$, at all signals his opponent has a positive probability of receiving. Since the signal of his opponent can fall anywhere within a distance of $2v$ from his own, this means that A must be played with certainty in a region of width $4v$.[5] However, from proposition 1 we see that if A is played with certainty in a region of this width, we are not at a stationary state if somewhere else A is not played with certainty. With uniform interaction, then, a state in which there is an A-convention somewhere but not everywhere, is not stationary.

There is a simple intuitive reason for this result. Consider a state in which A is played with certainty at all signals between $z'$ and $z''$, where $z'$ and $z''$ are at least $4v$ apart. A player receiving a signal at the edge of the region, say at $z'$, calculates a probability distribution $h(.|z')$ for his opponent's signal which can be illustrated as follows

[5] If we are considering a signal at the edge of signal space, $z=-v$ or $z=1+v$, then the region need only be $2v$ wide. This is reflected in part iii) of the proposition.

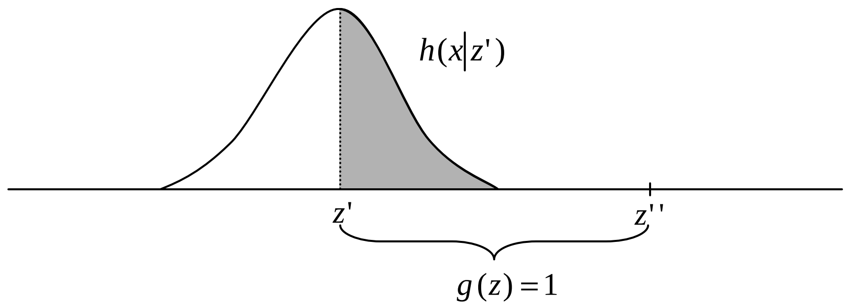


*Figure 2. Probability distribution of opponent's signal at edge of A-region*

When interaction is uniform, $h(.|z')$ has a nice symmetric form around $z'$. For a player receiving signal $z'$, half the bulk of $h(.|z')$ falls within the region in which A is played with certainty. In other words, for a player receiving a signal at the edge of a region where A is played with certainty, the probability that his opponent receives a signal inside the region is 0.5. From equation (3), this means that the probability with which his opponent plays A, $p(z')$, is at least 0.5. Since $a < 0.5$ and thus $p(z') > a$, the player at the edge therefore strictly prefers strategy A. Moreover, by continuity, the same is true for a player receiving a signal ever so slightly to the left of $z'$. The state of play function $g(z)$ therefore increases for signals at the lower edge of the region. A similar argument tells us that players receiving a signal at the upper edge of the region $z''$, also strictly prefer strategy A. The region in which A is played with certainty thus expands in both directions, and keeps doing so until A is played with certainty throughout signal space.

If the distribution of games is uniform, then, the only stationary state which contains an A-convention is a state which contains only an A-convention. Any state in which there is both an A-convention and a B-convention eventually collapses as the space commanded by the A-convention gradually expands. In a sense, the definition of an A-convention used by Sugden, requires a region where A is played which is above the critical size at which conventions are able to coexist when interaction is uniform.

When interaction is not uniform, however, two conventions can stably coexist. Note that if there are variations in the frequency of interaction across locations, then the probability distribution depicted in figure 1 need not be symmetric. If the variations are of a certain order, a player getting a signal at the edge of a region where A is played with certainty, might then calculate the probability of his opponent's getting a signal within the region as being equal to $a$ . In this case, if B is played with certainty to the other side of his signal, the player is indifferent between strategies A and B. Neither region thus expands, and we can have a stationary state with coexistent conventions.

**What constitutes a convention – actions speak louder than expectations**

The definition of convention existence used by Sugden prevents conventions coexistence when interaction is uniform. A convention only exists if there is some signal where a player can be sure that his opponent observes the conventional strategy. And if there is a signal where a player can be sure his opponent chooses the risk dominant strategy A, then no other convention is stable. The element of certainty in expectations used in the definition of conventions is thus what kills coexistence. It is therefore fitting to ask whether it is reasonable to put so much emphasis on expectations when defining conventions.

Intuitively, the definition used by Sugden seems to include more than a definition of conventions need include. A commonly cited definition of conventions due to Lewis (1969) suggests that "a convention is a pattern of behavior that is customary, expected and self-enforcing". A convention denotes a behavioural pattern, a regularity in the actions taken by a set of agents. The basic units that form a convention are thus the actions of individual agents, not their expectations. Expectations do form a basis on which to choose actions, but it is regularities in the actions chosen that are of interest, not regularities in expectations. Expectations are only of derivative importance, in perpetuating the regularities in actions needed for a convention to persist.

This is certainly the view taken in other parts of the evolutionary literature. Conventions are defined on the basis of state of play vectors, matrices or functions, and expectations are an element of what keeps conventions in place (see e.g. Young, 1993, 1996). A conventional definition of conventions would thus focus on strategies, and impose no stricter requirements on expectations than that they perpetuate strategy choices. In the context of Sugden's model, this means that requiring players to be absolutely certain their opponents choose a particular strategy, is too strict a demand to impose in a definition of conventions. For a player to do what has generally been done at his signal, he need only deem it sufficiently probable that his opponent chooses similarly. A more reasonable definition would thus substitute an idea of sufficient probability in expectations for that of absolute certainty.

At a signal where A is generally played, $g(z)=1$, players need only expect their opponents to play A with probability $p(z)$ above $a$, to keep playing A at this signal. Where B is generally played, $g(z)=0$, we need only $p(z)<a$ for B to continue being played. We thus arrive at an alternative definition of conventions which does not include more than such a definition need include: There is an A-convention if for some signal $z$, A is played with certainty by players at this signal, $g(z)=1$, and A is the optimal choice for a player receiving this signal, $p(z)>a$. Similarly, there is a B-convention if for some signal $z$, $g(z)=0$ and $p(z)<a$. In accordance with Sugden's idea of coexistence, we have a coexistence of conventions when an A-convention exists at some signal, a B-convention exists at another signal, and this state is a stationary state.

## Uniform interaction and coexistence revisited

If we adopt the alternative definition of a convention, states of convention coexistence can be stationary states when interaction is uniform, as the following proposition implies.

PROPOSITION 2

*Suppose* $f(y)=1$ *for all y.*

*If* $\hat{g}(.)$ *is a state of play function with the following properties for some signals* $z', x \in (v, 1-v)$:

*i)* $\hat{g}(z)=1$ *for all* $z \in (z', x)$

*ii)* $\hat{g}(z)=0$ *for all* $z \notin (z', x)$

*Then there exists some signal* $x = z''$ *for which* $\hat{g}(.)$ *is a stationary state.*

The proposition says that with a uniform frequency of games in social space, a state in which strategy A is played with certainty within some region, and B is played with certainty everywhere else, is a stationary state provided the region where A is played is of a certain width. Clearly, such a state meets the requirements of coexistence under the above alternative definition. By using a more reasonable definition of convention coexistence, we thus get a result which is stronger in the sense that it deems coexistence possible even if interaction is uniform.

The intuition behind the proposition is as follows. Imagine that we are in a state $\hat{g}(.)$ where A is played with certainty in some region of signal space $z'$ to $x$, and B is played with certainty at all signals outside this region. Consider a player who receives a signal at the lower end $z'$ of the region where A is played with certainty. The probability distribution of his opponent's signal can be illustrated as follows

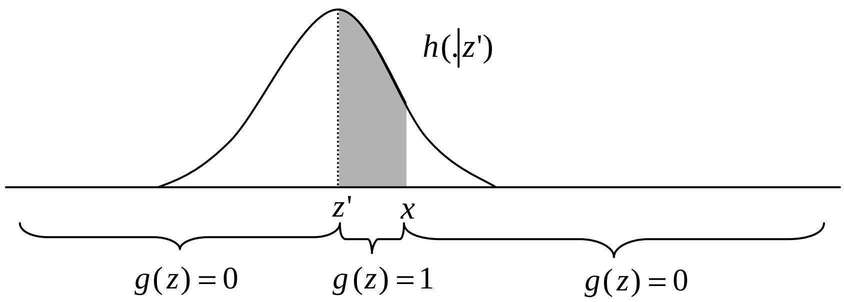


*Figure 3. Probability distribution of opponent's signal at border between A- and B-playing regions.*

For a player receiving signal $z'$, the shaded area represents the probability that his opponent gets a signal in the region where A is played with certainty. The location of $x$ determines how large this probability is. The further away $x$ is from $z'$, the larger is this probability, with a maximum of 0.5 if $x$ is a distance $2v$ or more from $z'$. Due to the fact that A is played with probability one between $z'$ and $x$, and probability zero elsewhere, the shaded area also equals $p(z')$, the probability that A is played by the opponent of a player receiving signal $z'$. Now, imagine that we first let $x$ be a distance $2v$ above $z'$, which implies $p(z')=0.5$. If we start sliding $x$ towards $z'$, $p(z')$ decreases, and due to the continuity of $h(.|z')$, at some point $x=z''$, we get

$p(z') = a$. The player at the border $z'$ between two regions where A and B is played, is now indifferent between the two strategies. Due to the fact that $h(.|z')$ is symmetric and the same for all signals when interaction in uniform, the player at the other border $z''$ is also indifferent between A and B.

For a player receiving a signal inside the region $z'$ to $z''$, A is the optimal strategy. The reason is that if we place the centre of the $h(.|z)$-curve anywhere between $z'$ and $z''$, the weight this curve puts on the region in which A is played, is greater than if the curve centred on one of the edges of that region. In other words, the probability $p(z)$ that your opponent plays A is greater for signals inside the region than at its edges, and we thus have $p(z) > a$ for all signals between $z'$ and $z''$. Moreover, a similar argument tells us that $p(z) < a$ for signals ouside this region, and the optimal choice for a player receiving such a signal is strategy B. Consequently, for $x = z''$, state $\hat{g}(.)$ is a stationary state.

## Concluding remarks

Sugden (1995) argues that in a model where agents are matched repeatedly to play a coordination game, where games have a location in a social space, and players do not know the exact location of their game, conventions can coexist only if the frequency of interaction varies across locations. We might interpret this as saying that if everyone acts according to the rule "when in Rome, do as the Romans" or "when at a party, dress the age of the other party-goers", the possibility that over time codes of conduct or dress would remain different in different surroundings, is limited. However, this paper argues that Sugdens's definition of a convention focuses too much on the expectations of the players rather than their actions. If instead we adopt a definition where their actions are the key element, we get the result that coexistence is possible even if interaction is equally frequent at all locations in social space.

The stationary state of coexistence established above is, however, only one type of stationary state. The state in which A is played with certainty across the space of signals, or the one in which B is played with certainty at all signals, are also stationary states. Moreover, as the state of coexistence can crumble if the state of play function is perturbed only slightly, this state might be less robust to different kinds of perturbations than other stationary states. Ideally, we ought to test the different states for robustness. One way to do so is to follow the approach of Young (1993) and Kandori, Mailath and Rob (1993), and introduce a small probability that agents choose their inoptimal strategy. However, the processes studied by Young and Kandori et al are defined on a finite and discrete state space, and as the model studied here has a continuous state space, their algorithm for identifying robust states is not applicable in this case. A different way of assessing robustness which might be more attuned to the present context, is to use the approach of Blume (1995) and see whether variations in initial conditions make some states more likely than others.

Finally, a note on the dispersion of signals in the above model. When the dispersion of signals $v$ is small and interaction uniform, one can have a string of correctly sized segments playing A in a social space where B is otherwise played. As long as these segments are at least $2v$ apart, they do not exert a joint influence strong enough to alter the state of play. A general lesson from the above model is therefore that the more certain players are of their true location, the greater can the variation in conventions across social space be. Conversely, the greater is the confusion about one's correct location, the less variation in conventions is possible. In the extreme, if $v$ is large in comparison to social space, there can be no coexistence of conventions, even by the alternative definition. The impact of the dispersion of signals on the maximum number of regions with different conventions has to do with the influence play in one location has on play in another. The more confusion about true locations, the greater is the range of locations that influences play in any one location.

**Appendix: Proof of propositions**

PROOF OF PROPOSITION 1:

This proposition is proved by reference to theorem 2 in Sugden (1995), which basically establishes the following:

Let $\tilde{g}(.)$ be a state of play function which has the following properties for some signals $z', z'' \in [-v, 1+v]$:

i) $\tilde{g}(z) = 1$ for all $z \in (z', z'')$

ii) $z''-z' \geq 4v$ or $[z''-z' = 2v$ and $z' = -v]$ or $[z''-z' = 2v$ and $z'' = 1+v]$

iii) $1 - H(z'|z') \geq a$

iv) $H(z''|z'') \geq a$

If at any time *t*, the state of play function is $\tilde{g}(.)$, then

a) For all $s > t$, $g_s(z) = \tilde{g}(z) = 1$ for all $z \in (z', z'')$

b) If $1 - H(z'|z') > a$, then for some finite $s > t$, $g_s(z) = 1$ for some $z \in [-v, z')$

c) If $H(z''|z'') > a$, then for some finite $s > t$, $g_s(z) = 1$ for some $z \in (z'', 1+v]$

In state $\tilde{g}(.)$, strategy A is played with certainty in a region of width at least $4v$ (or at least $2v$ at the edges of signal space), and players receiving signals at the edge of this region perceive the chance of their opponent's receiving a signal inside the region as at least $a$. Part a) then says that in all later periods, strategy A will keep being played with certainty at all signals inside the region. Part b) and c) say that if a player receiving a signal at either edge of the region deems the probability of his opponent's getting a signal inside the region as strictly higher than $a$, then the region in which A is played with certainty will expand at this edge.

With uniform interaction, $f(y) = 1$ for all *y*, for a player receiving signal *z*, the probability that the signal of his opponent is above *z* is 0.5, as is the probability that the signal is below *z*. Due to the fact that $a < 0.5$, we thus have

$1 - H(z|z) = H(z|z) > a$ for any signal $z \in [-v, 1+v]$. By iterated application of the above theorem, this means that a region of width 4v in which strategy A is played with certainty, will expand until A is played with certainty at all signals. By definition 1, this implies that a state $\hat{g}(.)$ fitting the description of proposition 1 is not a stationary state if interaction is uniform.?

PROOF OF PROPOSITION 2:

When $f(y) = 1$ for all $y$, $h(x|z)$ has the following properties:

i) $h(x|z)$ is symmetric around mean $z$

ii) $h(x|z)$ is non-decreasing in the interval $[-v, z)$

iii) $h(x|z)$ is non-increasing in the interval $(z, 1+v]$

iv) $h(z'+a|z') = h(z''+a|z'')$ for all $a \in (-2v, 2v)$ and $z', z'' \in (v, 1-v)$

Consider the interval $(z', x)$. Clearly:

$$p(z') = H(x|z') - H(z'|z') \text{ for } x \geq z' \tag{A1}$$

Since $H(x|z)$ is continuous in $x$, $p(z')$ is continuous in $x$, and has a maximum value of 0.5 and a minimum value of 0. There thus exists some point $x = z''$ at which $p(z') = a$ .

Moreover by properties i) and iv):

$$p(z'') = H(z''|z'') - H(z'|z'') = H(z''|z') - H(z|z') = p(z') \tag{A2}$$

Finally, when $f(y)=1$:

$$\frac{\partial p(a)}{\partial a}=\frac{\partial\left[H(z''|a-H\left(z'|a\right)\right]}{\partial a}=h(z'|a)-h(z''|a) \tag{A3}$$

Thus, from properties i), ii) and iii), $p(a)$ is non-decreasing for $a\in\left(-v,\frac{z'+z''}{2}\right)$ and non-increasing for $a\in\left(\frac{z'+z''}{2},1+v\right)$. Which implies:

$$p(a)\geq p(z') \text{ for all } a\in\left(z',z''\right) \tag{A4}$$

$$p(a)\leq p(z') \text{ for all } a\notin\left[z',z''\right] \tag{A5}$$

For a state of play function $\hat{g}(.)$ such as that of proposition 2, agents between $z'$ and $z''$ continue playing A, agents below $z'$ or above $z''$ keep playing B, and agents at $z'$ or $z''$ are indifferent. All of which makes $\hat{g}(.)$ a stationary state by definition 1.?

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