Explicit and Implicit Incentives .

in

Fund Management*

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Abstract

Fund managers compete to attract new investors. Competition and fund management contracts provide implicit and explicit incentives for fund management. I study the combined effect of these two types of incentives on i) investors' search for talented fund managers and on ii) talented fund managers' use of private investment signals. I show that an intermediate level of competition yields less efficient use of private investment signals and a lower average rate of return than in the case of either a high or a low level of competition in the fund managers' use of private information, they may harm new investors' search for talented fund managers. Explicit incentives may improve current performance, but cause prospective performance of the fund industry to deteriorate.

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1 Introduction

Mutual fund managers compete to attract potential investors. This market rivalry provides implicit incentives for fund management. In addition, explicit incentives are provided by contracts between fund managers and investors. I analyze how the combined effect of these two types of incentives influences fund managers' choice of investment strategies and investors' search for talented fund managers.

Implicit incentives exist because many investors believe that previous top-performers are better fund managers than their rivals. Consequently, investors flock to previous winners. In this paper, "flocking to top performers" has a rational explanation. Investors who use Bayes rule and past fund manager performance to update their beliefs about fund managers' talents prefer to invest in the top performer. This tendency creates implicit incentives for fund management. Walter (1999) notes that "despite clear warnings that past performance is no assurance of future results, a rise in the performance rankings often brings in a flood of new investments and management-company revenues, with the individual asset manager compensated commensurately"

Several empirical studies show that there are significant implicit incentives in the fund management industry.¹ More precisely, there is a strong positive relationship between realized return and subsequent inflow of investments for the last years' top performers but a weak and insignificant relationship for the others. For instance, Sirri and Tufano (1998) report that "for funds in the bottom 80th percentile, there is a positive but relatively shallow relationship between realized return and subsequent flows, but no pronounced penalty for extremely poor relative performance. However, there is a market bonus for high realized returns; the performance-flow relationship is very strong for funds whose historic performances place them in the top 20th percentile in the prior year."² Business Week, 28 June 1999, noted that "from January through April, six top performing companies – Janus, Vanguard, Fidelity, Alliance, MFS, and Putnam – accounted for just about all the fund inflows... Most fund companies had no inflows at all". Consequently, the fund managers face a convex or option-like incentive structure that makes risky investment opportunities attractive. I consider investors' optimal provision of explicit incentives, taking the implicit incentives into account.

In the model developed, I assume that some fund managers possess talents or skills that can be used to identify a number of undervalued firms and this allows them to achieve a higher return on the funds they invest than managers with less talent or skills. More precisely, a fund manager has access to two investment opportunities: a) an investment opportunity in which by using his talent the manager can obtain a private signal about the value of the asset

¹Chevalier and Ellison (1997), Ippolito (1992), Goetzmann and Peles (1997) and Sirri and Tufano (1998) have all shown that prior performance is decisive for the growth of a mutual fund.

²See also Ippolito (1992), Gruber (1996), and Chevalier and Ellison (1997) for evidence of this non-linearity.

(informed trade) and b) an investment opportunity in which there is symmetric information about the value of the assets (noise trade). As in Allen and Gorton (1993) and Dow and Gorton (1997), an inefficiently high level of noise trade is due in my model to contracting problems between investors and fund managers.³ Furthermore, I assume that a noise trade is riskier than an informed trade (Allen and Gorton (1993)).⁴

I construct a model in which investors hire fund managers and there are difficulties in writing incentive-compatible efficient compensation contracts due to fund managers' limited liability. After fund management contracts have been signed, each fund manager evaluates an investment opportunity and obtains a private signal about its rate of return. The fund managers decide whether to make a noise trade (ignore the signal) or an informed trade (use the signal). One of the key features in the model is that the quality of the signal depends on the manager's ability or talent. The information structure is as follows. Ex ante, all parties start with common knowledge about the prior distribution of the feasible investment opportunities and the manager's talent.⁵ However, the signal provides new information about the investment opportunities which gives rise to interim asymmetry of information between investors and managers. Ex post, the fund managers' rates of return are observed publicly and the managers are paid according to their fund-management contracts. In a dynamic fund management market with an inflow of new investments and investors searching for highly talented managers, investors use the public data about past performance in updating beliefs about managers' talents and in determining which fund to invest in. Hence, when making a current decision on investment, a fund manager is also concerned about its effect on his market reputation in the future. I focus on how concerns about reputation or implicit incentives influence fund managers' use of private investment signals.

Gompers and Lerner (1999) studied contracts used by 419 United States venture funds and showed that the predictions from a learning model similar to the one used here are more consistent with observed contracts than the predictions from a competing signalling model. Managers appear not to signal their different skills and talents by offering different contracts. Instead, investors learn about the fund managers' talent and skills slowly.⁶

I show that the level of competition – the number of rival fund managers – is decisive for the strength of the implicit incentives and the fund managers' choice of investment strategies.

 $^{^{3}}$ In most of the existing literature the origin of noise trade is not explicitly modeled. Besides trading due to contracting problems between investors and fund managers, noise trade may viewed as trading due to agents' liquidity or hedging motives (Dimond and Verrecchia (1981), Ausubel (1990), and Biais and Hillion (1994)) or trading of irrational traders (Black (1986) and De Long et al. (1990)).

⁴In my model, noise traders have incentives to make their investments risky.

 $^{{}^{5}}Ex$ ante, the talent distribution and the distribution over feasible investment opportunities are assumed to be the same for all fund managers.

⁶Lakonishok et al. (1992) report that contracts are very similar across investors and fund managers. Separating fee structures are not utilized to screen managers of different ability.

Fiercer competition makes it more likely that a fund manager would need a relatively high rate of return to become a top performer and thereby attract a large share of new investments. Hence a more competitive fund management industry may induce fund managers to make risky noise trades instead of safe informed trades which yield a higher expected rate of return.⁷ It follows that fund managers' average performance may decline as the level of competition increases.

This negative relationship between the expected rate of return and the level of competition is non-monotone. If the market rivalry becomes sufficiently fierce, a fund manager's likelihood of becoming a top-performer becomes small and the implicit incentives weaken. As the implicit incentives weaken, fund managers become more responsive to explicit incentives provided by the fund-management contract and, consequently, the share of managers making informed trades increases.

In the latter part of this paper, I study investors' search for talented fund managers. Current investors provide explicit incentives in order to maximize the current net rate of return. They do not take into account that explicit incentives may influence later investors' search for talented managers. Although explicit incentives improve the current rate of return by inducing more managers to make informed trades, I show that explicit incentives may obscure which managers are best suited for future fund management. Fund managers switching from a noise trade to an informed trade decrease their probability of becoming a top performer and, consequently, other fund managers' probabilities of becoming a top performer increase. I show that as a consequence less talented fund managers may improve their win probability sufficiently to cause selection efficiency to deteriorate. Hence, from a social efficiency point of view, fixed-fee contracts, which are common in the fund management industry, might be more efficient than incentive contracts provided that new investors' search for talented managers is taken into account.⁸

Also Dow and Gorton (1997) show that contracting problems may lead to trading activities not reflecting private information. In both their and my model, uninformed trading or noise trading lowers the expected rate of return.⁹ In contrast to their study, I show how implicit incentives may lead to noise trading and how the level of noise trading may depend on the level of competition in the fund industry. I also extend the scope of their analysis by analyzing

⁷James and Isaac (2000) (theory and laboratory experiments) and Karceski (2000) (empirical paper) show that implicit (tournament) incentives can lead to mispricing of financial assets in the absence of private information. Here I show that implicit incentives can lead to inefficient use of private information and hence mispricing.

⁸Golec (1992) reports that only 6 percent (29 of 476) of his sample of fund managerial contracts contain a performance-based fee component.

⁹Also Bhattacharya and Pfleiderer (1985) assume that talented fund managers have access to more precise information about an investment's rate of return than less talented managers. Informed trades are low risk trades.

the relationship between the level of noise trading and investors' search for talented fund managers.

Several empirical papers examine how fund managers respond to the flow-performance relationship observed in the mutual fund market. Brown et al. (1996), Roston (1996), and Chevalier and Ellison (1997) all show that implicit incentives have a significant effect on risk taking. In contrast to these papers, I investigate how implicit and explicit incentives may interact and whether this interaction improves or hinders the search for talented fund managers for future fund management.

Chevalier and Ellison (1999) address an important limitation in most of the literature on incentives in fund management. Their empirical study shows that a complete discussion of the incentives facing mutual funds must consider both the agency relationship between the fund company and the fund investors *and* the agency relationship between the fund company and fund management. This requires a model with two layers of agency problems. Instead of developing such a model I have simply assumed that fund companies pass on the incentives it faces to its fund managers.

Most of the previous work on the agency relationship between an investor and his investment manager has assumed a single-period relationship (e.g., Bhattacharya and Pfleiderer (1985), Allen (1990), Stoughton (1993), and Golec (1992)). Dynamics of portfolio management contracts have been examined by Heinkel and Stoughton (1994). These papers focus on motivating managers to acquire information – not on the extent of use of private information. Furthermore, they do not take into account that investment managers face implicit incentives.

The rest of this paper is organized as follows: Section 2 lays out the model used to analyze competition among mutual funds. In Section 3, I show how the interaction of implicit and explicit incentives determines the expected performance of mutual funds. In Section 4, I investigate new investors' search for talented fund managers for future fund management. Section 5 discusses briefly empirical implications of the analysis and related empirical literature. Section 6 points out some implications for public policy. Section 7 concludes. All proofs are relegated to the Appendix.

2 The model

The competition in the mutual fund industry is modeled as a three-stage game involving investors' optimal design of incentive contracts and fund managers' competition for new investors. For the sake of simplicity, both fund managers and investors are assumed to be risk neutral.¹⁰ Figure 1 presents a schematic diagram of the three-stage game.¹¹

Stage 1 Incentive contracts are offered to fund managers.

Stage 2

Fund managers obtain private investment signals (θ) and make either informed trades (use θ) or noise trades (ignore θ).

Stage 3

Rates of return become publicly known and managers are paid according to their contracts. New investors choose a fund manager for future fund management.

Figure 1: Timing.

To keep the notation simple, let a fund manager with talent θ obtain a signal about an investment opportunity yielding $r = \theta$ rate of return. Hence, a fund manager can obtain a certain rate of return by using his private signal. Assume that θ is fixed and at stage 1 incompletely known to the fund managers and the investors. Moreover, the investors and managers share prior beliefs about θ ; this prior is given by distribution function $F(\theta)$. Let $f(\theta)$ denote the corresponding density function. In order to make the model tractable in cases with a large number of competing fund managers, assume that the talent distribution, $F(\theta)$, is uniform on [0, 1].

Instead of applying his investment signal to fund management, a fund manager can choose to make a noise trade (e.g. a random deviation from a given benchmark all fund managers are measured against). A noise trade yields a rate of return given by the probability function G(r) and the corresponding density function g(r). The fund managers' outcomes from noise trades are identical and independently distributed.¹² Note that it is important that a noise

¹⁰If we assumed that investors were risk adverse, the investors would be more severely harmed by managers' excessive risk taking. Stronger explicit incentives would need to be provided to mitigate the consequences of the implicit incentives. However, none of our main results and insights hinges on the assumption that investors are risk neutral.

¹¹Note that investors do not receive private information directly from fund managers but benefit through an extra rate of return. Reasons for the indirect sale of information, i.e. portfolio management, instead of the direct sale of information are discussed in e.g. Admati and Pfleiderer (1990).

¹²Correlated outcomes would reduce the expected outcome of the best noise traders and hence, ceteris paribus, decrease the probability of one of the noise traders outperforming all the fund managers making informed investments. Consequently, if the outcomes from noise trades are correlated, fund managers become less inclined to choose noise trades. However, fund managers will still have inefficiently strong incentives to choose a noise trade instead of an informed trade.

trade yields a riskier outcome than an informed trade. For the sake of simplicity, let the outcome from a noise trade, G(r), be uniform on [0, 1].

All investors have K capital each. At stage 1, investors offer incentive contracts, w(r), to fund managers. The contracts relate a fund manager's payment to the rate of return he has achieved. The fund managers are assumed to have limited liability, $w(r) \ge 0$, and their payment is required to be weakly increasing in rate of return achieved, $w'(r) \ge 0$. If there were a region over which payment and performance were negatively correlated, then this would generate perverse incentives for fund managers to sabotage the performance measure in this region – e.g., inflate the trading costs by making an excessive number of transactions. By assuming that $w'(r) \ge 0$ the analysis is simplified, but the main results do not depend on this assumption. As we will see later, highly talented fund managers need weaker explicit incentives for making an informed trade constant by providing weaker "contract" incentives to highly talented fund managers. This approach would not change the main qualitative results in the paper but it would make it less expensive for investors to provide explicit incentives for low-talented fund managers.

In 1995, only 117 of the 6997 mutual funds in the Morningstar database employed incentive fees. This fact serves as motivation for using commonly observed fixed fee contracts (in which payment increases with the amount of funds under management and is independent of performance) as a benchmark.

Each fund manager is assumed to take care of only one investor's investments. Hence the number of fund managers, n, is the same as the number of investors. For the sake of simplicity, n is treated as a continuous variable. This enables us to ignore possible free-riding and coordination problems among investors in their provision of incentives to fund managers. Possible implications of relaxing this assumption are discussed below.

At stage 3, fund managers are paid according to their fund-management contracts and their performance. New investors search for talented fund managers. Investors expect there to be a positive correlation between the quality of the signals obtained in the past and the quality of future signals (after stage 3). In my model, I show that the top performer is expected to be the most talented and, consequently, the best choice for investors at stage 3 (Proposition 5). Hence, flocking to the top performer is the optimal strategy for new investors.¹³ Instead of modelling fund management after stage 3, I simply assume that the top performer's inflow of new investments is worth $S \cdot \theta$, where S is a positive constant. Hence,

¹³Hence, a fund manager that chooses the strategy that maximizes the probability of becoming top-ranked also maximizes the investors' posterior belief of his ability (given that he wins). In this sense, my model is related to Trueman (1988) who argues that managers choose investment strategies in order to maximize investors' posterior beliefs about abilities.

a talented fund manager expects to receive a larger payment for future fund management than a manager with less talent. This is because e.g. information rent (in a principal-agent framework) increases or talented fund managers charge higher prices because they have more market power (see Heinkel and Stoughton (1994)). It will later become apparent that if the benefits of being top-ranked are independent of talent, my main results are still valid.

Notice that the substance of my results would prevail if there were more than one winner, as long as the number of winners is small compared with the number of competing managers. Each fund manager would consider the probability of becoming one of the winners and still have incentives to choose inefficiently risky investment strategies.¹⁴

3 Competition and noise trade

In this section, I study how the level of competition in the fund management industry may influence managers' use of private investment signals and, consequently, the performance of the funds. Since fixed-fee contracts are frequently used in the mutual fund industry, I use the case with fixed-fee contracts as a benchmark (see Section 3.1 below) to discuss of the case in which incentive contracts (explicit incentives) are provided (Section 3.2). The extensive use of fixed-fee contracts has lead Admati and Pfleiderer (1997) and Heinkel and Stoughton (1994) to suggest that tournament models may be useful for studying competition and performance in the fund management industry.

3.1 Benchmark: tournament competition

Fund managers' investment strategies depend on the informed investment available, θ . Denote manager *i*'s investment strategy $I^{i}(\theta)$ and let $I^{i}(\theta) = 1$ and $I^{i}(\theta) = 0$ represent the cases where a manager makes an informed and a noise trade, respectively.

In the tournament case, each fund manager chooses the investment strategy that maximizes the likelihood of his becoming top ranked

$$\operatorname{Max}_{I^{i}(\theta)} \Pr\left(I^{i}, I^{-i}, \theta\right), \tag{1}$$

where $\Pr(I^i, I^{-i}, \theta)$ denotes fund manager *i*'s probability of being top-ranked given that the others follow investment strategy I^{-i} .

Definition 1 Define $H_{z}(r)$ as the probability distribution over rate of return, r, of a random

¹⁴However, if, contrary to shown in empirical papers (e.g. Sirri and Tufano (1998)), a majority of the funds receives significant inflow while a small minority receives no inflow or even an outflow of capital, fund managers would choose inefficiently low risk strategies in order to avoid becoming one of the losers.

fund manager (unknown θ) following strategy

$$I_{z}(\theta) = \begin{cases} 0 & \text{if } \theta < z; \\ 1 & \text{if } \theta \ge z, \end{cases}$$

$$(2)$$

where $z \in [0, 1]$.

Distribution $H_z(r)$ can be expressed using $G(\cdot)$ and $F(\cdot)$.

$$H_{z}(r) = \begin{cases} F(z) \int_{0}^{r} g(x) dx & \text{if } r \in [0, z]; \\ F(z) G(z) + \int_{z}^{r} (F(z) g(x) + f(x)) dx & \text{if } r \in [z, 1]. \end{cases}$$
(3)

Note that a low rate of return $r \in [0, z]$ is achieved if the fund manager has a poor informed investment opportunity (the probability of obtaining poor information is F(z)) and makes a noise trade (the outcome of a noise trade is given by density function g(x)). This explains the first line in equation (3). A high rate of return $r \in [z, 1]$ is achieved if a fund manager either makes a noise trade and obtains a lucky outcome or makes a superior informed trade. The probability of obtaining $r \ge z$ by making a lucky noise trade is F(z)g(x). The probability of obtaining $r \ge z$ by making a superior informed trade is f(x).¹⁵ This explains the second line in equation (3).

Using the assumption that $G(\cdot)$ and $F(\cdot)$ are uniform on [0,1], I have

$$H_{z}(r) = \begin{cases} zr & \text{if } r \in [0, z];\\ (1+z)r - z & \text{if } r \in [z, 1]. \end{cases}$$
(4)

Proposition 1 (Equilibrium strategy)

Fund managers have a unique symmetric equilibrium strategy $I_{z=\widehat{\theta}}(\theta)$, where $\widehat{\theta}$ is defined such that the probability of winning with a noise trade equals the probability of winning with an informed trade:

$$\int_{0}^{1} \left(H_{z=\widehat{\theta}}\left(r\right) \right)^{n-1} dr = \left(H_{z=\widehat{\theta}}\left(\widehat{\theta}\right) \right)^{n-1}.$$
(5)

Proof. See Appendix.

The basic argument is straightforward. If a fund manager has information about an informed investment opportunity yielding $r = \hat{\theta}$, he would be indifferent between making an

$$(1 - F(z)) \frac{f(r)}{1 - F(z)} = f(r).$$

¹⁵Note that given $r \ge z$ (which happens with probability 1 - F(z)) the density function is f(r) / (1 - F(z)). Consequently, the probability of obtaining $r \ge z$ by an informed trade is

informed trade and a noise trade. The probability of becoming top ranked by making an informed trade (given by the right-hand side of equation (5)) is the same as if he makes a noise trade (given by the left-hand side of equation (5)). A better informed trade opportunity, $\theta > \hat{\theta}$, would induce the fund manager to make an informed trade, since this provides the best probability of his becoming top ranked. Poorer informed trade opportunities, $\theta < \hat{\theta}$, induce him to make a noise trade. Note that the cut-off, $\hat{\theta}$, can be interpreted as the expected share of fund managers making noise trades.

A fund manager's expected performance, $r^e(\widehat{\theta})$, depends on the extent to which he makes informed trades instead of noise trades $(\widehat{\theta})$,

$$r^{e}\left(\widehat{\theta}\right) = \int_{0}^{1} x dH_{z=\widehat{\theta}}\left(x\right) = \frac{1}{2}\left(1+\widehat{\theta}-\widehat{\theta}^{2}\right).$$
(6)

From equation (6) it follows that a manager's expected rate of return is maximized if $\hat{\theta} = \frac{1}{2}$. If the informed trade yields a lower return than $\frac{1}{2}$, the expected return from noise trade would exceed the one from the informed trade.¹⁶

In a competitive mutual fund industry, fund managers anticipate that a relatively high rate of return is needed to become a top performer. An informed trade with a certain rate of return slightly above $\frac{1}{2}$ yields a smaller probability of the manager becoming a top performer than a noise trade with larger probabilities for very high and very low outcomes. Consequently, in order to increase their probability of becoming the top performer, fund managers choose $\hat{\theta}$ above $\frac{1}{2}$ (the cut-off which maximizes the rate of return).

As the fund industry becomes more competitive (larger n) the expected rate of return of the top performer increases and the share of managers making informed trades decreases. Hence, fiercer competition decreases the expected rate of return from fund management.

Proposition 2 (Tournament competition)

a) More competition:

$$\frac{d\widehat{\theta}}{dn} > 0 \quad and \quad \frac{dr^e\left(\widehat{\theta}\right)}{dn} < 0.$$

....

b) Perfect competition:

$$\lim_{n \to \infty} \widehat{\theta} = 1,$$

and $r^{e}\left(\widehat{\theta}\right)$ decreases asymptotically towards the expected rate of return from a noise trade. **Proof.** See Appendix.

The relationship between the level of competition and the share of fund managers making noise trades is illustrated in Figure 3 (see the graph denoted B = 0).

¹⁶Recall that the rate of return from a noise trade is assumed to be uniform on [0, 1].

3.2 Incentive-fee contracts

Consider the case in which investors offer incentive contracts, w(r), in order to maximize expected net return on their investment, K,

$$\underset{w(r)}{Max} E\left[K \cdot (1 + r\left(I, \theta\right)) - w\left(r\right)\right],\tag{7}$$

subject to the fund managers' maximization problem,

$$\underset{I^{i}(\theta)}{\operatorname{Max}} E\left[w\left(r\left(I^{i},\theta\right)\right)\right] + \Pr\left(I^{i},I^{-i},\theta\right) \cdot S \cdot \theta,\tag{8}$$

and the two restrictions on feasible contracts,

$$w(r) \ge 0,$$

and

$$w'(r) \ge 0.$$

A fund manager chooses the investment strategy $I^i(\theta)$ that maximizes the expected payment from the investor (explicit incentives) plus the expected benefits of becoming a top performer (implicit incentives). The rate of return achieved, $r(I^i, \theta)$, depends on the fund manager's investment strategy, I^i , and the informed trade, θ , available.

Proposition 3 shows that investors' optimal contract is a simple bonus contract. Only fund managers achieving a rate of return above the cut-off, $\hat{\theta}$, (equation (10)) are rewarded with a bonus *B*. The bonus *B* and the cut-off $\hat{\theta}$ depend on the size of the investment *K*.

Proposition 3 i) The optimal contract type is a bonus contract,

$$w_{\widehat{\theta}}(r) = \begin{cases} 0 & \text{if } r < \widehat{\theta}; \\ B & \text{if } r \ge \widehat{\theta}, \end{cases}$$

$$\tag{9}$$

and

ii) the fund managers' investment strategy is given by $I_{\hat{\theta}}(\theta)$ where $\hat{\theta}$ is defined by the fund managers' incentive constraint,

$$S \cdot \widehat{\theta} \int_{0}^{1} \left(H_{z=\widehat{\theta}} \left(r \right) \right)^{n-1} dr + \left(1 - \widehat{\theta} \right) \cdot B = S \cdot \widehat{\theta} \cdot \left(H_{z=\widehat{\theta}} \left(\widehat{\theta} \right) \right)^{n-1} + B.$$
 (10)

Proof. See Appendix.

The incentive constraint (10) extends the incentive constraint in the tournament case (see equation (5)) to a setting with optimal provision of explicit incentives. If there are no explicit

incentives (B = 0), incentive constraint (10) and (5) would be identical.

The cut-off, $\hat{\theta}$, and bonus, B, are set such that all fund managers having better investment opportunities than the cut-off are induced to make informed trades. Note that as the rate of return from the informed trade increases (i.e. θ increases) the implicit incentive to make an informed trade instead of a noise trade strengthens. It is more difficult to induce fund managers with low θ than high θ to make informed trades. Hence, investors offer a fixed bonus for all performance levels above the cut-off $\hat{\theta}$ (the bonus payment cannot decrease as the performance rises).

The left-hand side of incentive constraint (10) represents the total expected benefits to a manager of making a noise trade. Similarly, the right-hand side represents the total expected benefits of making an informed trade (provided that the rate of return on an informed trade is $\hat{\theta}$). The first items on both sides relate to the implicit incentives and have the same interpretation as in the tournament case (Proposition 1). The second items relate to the explicit incentives. On the left-hand side, $(1 - \hat{\theta})B$ is the expected bonus payment for making a noise trade.¹⁷ On the right-hand side, a fund manager that makes an informed trade (given $\theta = \hat{\theta}$) is certain to obtain bonus B.

The investor's profit-maximizing problem (see equation (7)) can now be simplified to,

$$Max \quad K \cdot \left(1 + \frac{1}{2}\left(1 + \widehat{\theta} - \widehat{\theta}^2\right)\right) - \left(1 - \widehat{\theta}^2\right) \cdot B,\tag{11}$$

subject to the incentive constraint (equation (10)).¹⁸ To obtain (11), I have used equation (6) and that,

$$E[w(r)] = \left(1 - H_{z=\widehat{\theta}}\left(\widehat{\theta}\right)\right) \cdot B = \left(1 - \widehat{\theta}^{2}\right) \cdot B.$$
(12)

The investor's profit-maximizing choice of bonus, B^* , and cut-off, $\hat{\theta}(B^*)$, is given by the fund manager's incentive constraint (10) and the investor's first order condition,

$$K \cdot \left(\frac{1}{2} - \widehat{\theta} + 2 \cdot B \cdot \widehat{\theta}\right) \cdot \frac{\partial \widehat{\theta}}{\partial B} - \left(1 - \widehat{\theta}^2\right) = 0.$$
(13)

Note that $\frac{\partial \hat{\theta}}{\partial B}$ is given by implicit derivation of the fund manager's incentive constraint (equation (10)). By increasing the bonus, it becomes more likely that a fund manager makes informed trades ($\hat{\theta}$ decreases). Hence, the expected rate of return improves. However, such an increase in the bonus payment becomes decreasingly profitable because further decreases in $\hat{\theta}$

¹⁷Since a noise trade yields a rate of return that is assumed to be uniformly distributed on [0, 1], $1 - \hat{\theta}$ is the probability of obtaining $r \ge \hat{\theta}$.

¹⁸Note that the incentive constraint (10) describes the relationship between the cut-off for bonus payments, $\hat{\theta}$, and the size of the bonus, B, which induces all fund managers with $\theta \geq \hat{\theta}$ to make informed trades.

improve r^e at a decreasing rate (see equation (6)).¹⁹ On the other hand, the expected bonus payment increases due both to larger bonus payments to each eligible fund manager and to the increased likelihood that a fund manager will obtain the bonus.

The size of the investor's investment, K, determines the strength of the profit-maximizing explicit incentives (the size of bonus payments, B). An increase in K induces the investors to provide stronger explicit incentives and thereby obtain a higher expected rate of return on a larger investment.

In a setting in which many investors use the same fund manager, an increase in K could be interpreted as improved coordination among the investors in incentive provision (less free-riding in the incentive provision). If investors have a more efficient agreement about how to share the incentive payment to the fund managers, they will provide stronger explicit incentives in order to increase the expected rate of return from their common fund investment.

From the fund managers' incentive constraint (10), it follows that a strengthening of the implicit incentives (e.g. due to a larger inflow of new investments) has a harmful effect on fund managers' use of private investment signals.

Corollary 1 Stronger implicit incentives make the fund manager more inclined to make a noise trade,

$$\frac{d\widehat{\theta}\left(B^{*}\right)}{dS} > 0.$$

Proof. See Appendix.

Stronger implicit incentives make it more attractive for fund managers to choose the investment opportunity with the highest probability of becoming top ranked and less attractive for maximizing expected rate of return.

In contrast to the pure tournament case, the relationship between the level of competition and expected performance becomes non-monotone when investors provide incentive contracts. See Figure $2.^{20}$

Proposition 4 (More competition)

a) There exists a unique number of rivaling fund managers, \overline{n} , such that,

if
$$n \le \overline{n}$$
, then $\frac{d\widehat{\theta}(B^*)}{dn} \ge 0$ and $\frac{dr^e}{dn} \le 0$,
if $n \ge \overline{n}$, then $\frac{d\widehat{\theta}(B^*)}{dn} \le 0$ and $\frac{dr^e}{dn} \ge 0$.

¹⁹Hence straightforward calculations show that the second order condition of the investor's profitmaximization problem is satisfied.

²⁰Figures 2 and 3 are generated using Maple V programs that can be obtained from the author. In Figure 3, the non-monotone graph represents the case where S = 10 and $B^* = \frac{1}{4}$ (for each *n* there exists a *K* that makes $B = \frac{1}{4}$ optimal).

b) \overline{n} decreases if the relative importance of the explicit incentives increases,

$$\frac{d\overline{n}}{d\left(\frac{B}{S}\right)} < 0.$$

c) Perfect competition, $n \to \infty$: The fund managers use all private information signals that increase expected performance, r^e (implies $\hat{\theta} = \frac{1}{2}$).

Proof. See Appendix.

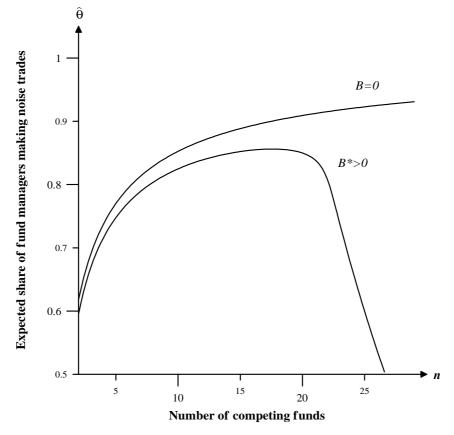


Figure 2: Competition and noise trading.

If there are few fund managers initially, $n \leq \overline{n}$, each fund manager has a relatively large probability of becoming top-ranked. Consequently, the probability of becoming top-ranked is important for the choice between an informed and a noise trade. As the number of rivals increases, the fund managers anticipate that a higher rate of return is needed to win and they become inclined to choose investments with "thick-tailed" probability distributions ($\hat{\theta}$ increases). The competition for "tomorrow's" investors may induce fund managers to be more inclined to make noise trades and hence "today's" expected return decreases.

On the other hand, if there is fierce competition in the fund business initially, $n \ge \overline{n}$, a further increase in the number of rivals makes it very difficult for a manager to become top-ranked, and, hence, the fund managers become more inclined to choose the investment strategy that provides the highest probability of obtaining the bonus. Hence, the share of fund managers making noise trades decreases as the level of competition increases.

In the case of perfect competition $(n \to \infty)$, becoming a top-ranked fund manager is a zero-probability event and only the explicit incentives direct the fund managers' investment strategies. By offering an infinitely small bonus for performance above the expected outcome of noise trade, investors can induce fund managers to maximize the expected rate of return.

The terms "fixed fees" and "incentive fees" are somewhat misleading. Fixed fees also provide incentives to mutual fund managers to maximize the expected rate of return because they are paid at the end of each period, which implies that a fund manager earns a portion of the initial investment and of the return over the period. From a multi-period perspective, fixed-fee contracts may provide fund managers with considerable incentives to maximize the expected rate of return, because superior returns compound (assuming returns are not paid out to investors) investments and fixed fees grow. Consequently, the non-monotonicity shown by Proposition 4 can arise even if "fixed-fee" contracts are used.

4 Selection efficiency

In this section, I study (stage-3) investors' search for talented fund managers. As above, I use the tournament (fixed-fee) case as a benchmark (Section 4.1) to discuss the case in which incentive contracts are provided (Section 4.2). New investors use past performance to identify talented managers for future fund management. In my setting, investors use information about a fund manager's performance at stage 2 and Bayes rule to update their beliefs about the talent of the manager.

Investors anticipate that only some investment signals are applied at stage 2. More precisely, there is a cut-off, $\hat{\theta}$, separating the fund managers making an informed trade ($\theta \geq \hat{\theta}$) and those making a noise trade ($\theta < \hat{\theta}$) (Proposition 1 and Proposition 3). The expected talent of a fund manager making a noise trade is $\frac{1}{2}\hat{\theta}$ since $F(\theta)$ is uniform. Hence all fund managers achieving $r < \hat{\theta}$ are expected to be of type $\frac{1}{2}\hat{\theta}$. On the other hand, fund managers achieving $r \geq \hat{\theta}$ could either have obtained a lucky outcome from a noise trade or made an informed trade. Conditional on $r \geq \hat{\theta}$, the probability distribution of θ for a given fund manager is

for
$$\theta' \leq \widehat{\theta}$$
: $\Pr\left(\theta' < \widehat{\theta} \mid r \geq \widehat{\theta}\right) = \frac{\Pr\left(\theta' < \widehat{\theta} \land r \geq \widehat{\theta}\right)}{\Pr\left(r \geq \widehat{\theta}\right)} = \frac{\theta'\left(1 - \widehat{\theta}\right)}{\left(1 - \widehat{\theta}\right)\left(1 + \widehat{\theta}\right)} = \frac{\theta'}{1 + \widehat{\theta}}$
(14)

and

$$\Pr\left(\theta' \ge \widehat{\theta} \mid r \ge \widehat{\theta}\right) = \Pr\left(\theta' = r \mid r \ge \widehat{\theta}\right) = \frac{1}{1 + \widehat{\theta}}.$$
(15)

Since the probability that a fund manager's rate of return reflects his talent is constant for all $r > \hat{\theta}$, the expected talent of the top performer exceeds all other fund managers. Proposition 5 sums up this result.²¹

Proposition 5 A fund manager's expected talent is weakly increasing with his rate of return.

Consequently, the empirical tendency to "flock" to the top performing fund is consistent with Bayesian updating of beliefs about fund managers' talent.

Let β denote the expected talent of the top performer. If all fund managers made noise trades ($\hat{\theta} = 1$), β would be the same as the average talent of the pool of competing managers. On the other hand, if all fund managers made informed trades ($\hat{\theta} = 0$), the best fund manager would always be identified.²²

Generally, selection efficiency, β , depends on the extent to which fund managers make informed trades $(\hat{\theta})$ and the number of rivals (n). Given that the expected talent of a fund manager obtaining $r < \hat{\theta}$ is $\frac{1}{2}\hat{\theta}$, and that a fund manager with $r \ge \hat{\theta}$ either has talent $\frac{1}{2}\hat{\theta}$ (an untalented manager with a lucky outcome) or $r = \theta$ (an informed trade), we can calculate the expected talent of the top-performer,

$$\beta\left(\widehat{\theta},n\right) = \int_{0}^{\widehat{\theta}} \frac{1}{2}\widehat{\theta}d\left(H_{z=\widehat{\theta}}(x)\right)^{n} + \int_{\widehat{\theta}}^{1} \left(x\frac{1}{1+\widehat{\theta}} + \frac{1}{2}\widehat{\theta}\frac{\widehat{\theta}}{1+\widehat{\theta}}\right)d\left(H_{z=\widehat{\theta}}(x)\right)^{n}$$

$$= \int_{0}^{\widehat{\theta}} \frac{1}{2}\widehat{\theta}n\widehat{\theta}(x\widehat{\theta})^{n-1}dx$$

$$+ \int_{\widehat{\theta}}^{1} \left(x\frac{1}{1+\widehat{\theta}} + \frac{1}{2}\widehat{\theta}\frac{\widehat{\theta}}{1+\widehat{\theta}}\right)n(1+\widehat{\theta})((1+\widehat{\theta})x - \widehat{\theta})^{n-1}dx$$

$$= \frac{\widehat{\theta}^{2} - \widehat{\theta}^{2n+1}}{2\left(1+\widehat{\theta}\right)} + \frac{(1+n)\widehat{\theta} + n + \widehat{\theta}^{2n+2}}{(1+n)\left(1+\widehat{\theta}\right)^{2}}.$$
(16)

To obtain equation (16), I have used equation (15) and (14) and the definition of H_z (equation

 $^{^{21}}$ Proposition 5 is consistent with Gruber (1996) who examines the performance of some 270 U.S. open-end equity funds over a 10-year period and finds that past performance is related to future performance, and that "sophisticated" investors moving into those funds that have performed well in the past also do well in the future.

²²This follows from the simplifying assumption that all informed trades have sure rates of return. However, one might expect, more generally, that it will be easier to identify highly skilled fund managers when more managers make informed trades.

(4)).

Current investors are assumed to ignore how their incentive contracts influence later investors' ("stage-3 investors") search for a talented fund manager. If I had instead assumed that current and later investors were identical, my main results would still prevail. Each individual investor would not take into account that by altering the incentive contract offered to his fund manager he might improve *all* investors' search for a talented fund manager at stage 3. Selection efficiency is a public good.

From the point of view of social efficiency, it is important that the best investment projects are identified and funded. Consequently, social efficiency depends on fund managers' abilities or talent for identifying good investment projects. In Section 4.1 and Section 4.2, I analyze how selection efficiency is related to the level of competition in the industry and the strength of the explicit incentives. I show that the short-term effect of more competition and stronger explicit incentives may be the converse of the long-term effect. This is due to the fact that, in the short-term, it is important to induce a *given* fund manager to apply investment signals efficiently, while in the long-term it is more important to identify talented managers for future fund management.

4.1 Benchmark: Selection and tournament competition

The level of competition in a tournament is decisive for the information content of past performance. Although fiercer competition may induce more fund managers to make noise trades and thereby reduce current performance, Proposition 6 shows that a more competitive industry may improve selection efficiency and thereby prospective performance.²³

Proposition 6 Selection efficiency improves if the mutual fund industry becomes more competitive $(\Delta n > 0)$,

$$\frac{d\beta}{dn} > 0$$

Proof. See Appendix.

It is not obvious that more competition should lead to a more talented winner. Although a larger number of rivals implies that the expected talent of the best manager improves, it also implies that fewer managers are using their investment signals (Proposition 2). The last effect may harm the search for talented fund managers. Figure 3 illustrates the relationship between the expected talent of the top performer and the level of competition.²⁴

²³Note that the difference between the expected talent of the best manager (highest θ) and the expected talent of the top performer (highest r) may increase with r. From this point of view, selection efficiency may deteriorate with an increase in n.

²⁴In Figure 3, it is taken into account that an increase in *n* changes $\hat{\theta}$ according to the incentive constraint (equation (10)).

The net effect of fiercer competition on social efficiency is ambiguous. Selection efficiency improves, but current performance deteriorates.

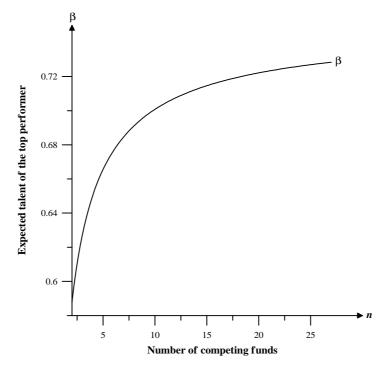


Figure 3: Expected talent of the top performer in the tournament case.

4.2 Selection and performance-fee contracts

Investors offer performance-fee contracts to improve current fund management performance. Performance-fee contracts induce fund managers to use private investment signals more efficiently and thereby increase the expected rate of return. Surprisingly, Proposition 7 shows that better use of talent and private investment signals in current fund management may make it more difficult for later investors to use past performance to identify talented managers.²⁵

Proposition 7 Stronger explicit incentives (due, e.g. to larger current investments, K) have an ambiguous effect on selection efficiency,

$$\frac{d\beta}{dB} \gtrless 0.$$

Proof. See Appendix.

To see the reasoning behind this result, first consider the case without a performance fee (B = 0). The fund managers' investment strategies are solely driven by the (implicit)

 $^{^{25}}$ In a richer model with many investors using the same fund manager, an increase in K has the same consequences as if investors became more able to share the cost of incentive provision efficiently (less free-riding).

incentives to become top-ranked. The introduction of an incentive contract induces some fund managers to switch from noise to informed trades. These fund managers reduce their probability of becoming top-ranked but increase their expected payment today (the bonus). Consequently, the probability of one of the rival fund managers becoming top-ranked increases. The impact on selection efficiency depends on whether it is the more or the less talented fund managers which improve their win probability most. If less talented managers increase their win probability significantly, selection efficiency deteriorates. On the other hand, if more talented managers increase their win probability significantly, selection efficiency improves. Hence, stronger explicit incentives have ambiguous effects on selection efficiency. See Figure 4.²⁶

The intuition for the non-monotonicity result can also be seen by observing that for a given $r > \theta$ the expected talent,

$$E\left(\theta \mid r > \widehat{\theta}\right) = \frac{1}{2}\widehat{\theta}\frac{\widehat{\theta}}{1+\widehat{\theta}} + r\frac{1}{1+\widehat{\theta}} = \frac{\frac{1}{2}\widehat{\theta}^2 + r}{1+\widehat{\theta}},$$

is non-monotone in the cut-off $\hat{\theta}$. Hence, if we keep the expected performance of the top performer, r^{\max} , constant when we introduce a bonus payment (B) and, thereby, reduce $\hat{\theta}$, the expected talent of the top performer may decrease. Note, however, that this cannot serve as a proof of Proposition 7 since r^{\max} changes as $\hat{\theta}$ changes. But for large n, r^{\max} will be close to 1 and a change in $\hat{\theta}$ will only induce a small change in r^{\max} and hence the argument above is approximately correct.

²⁶The end points of the graphs plotted in Figure 4 are given by $\hat{\theta}(n, B = 0)$. Introduction of a performance fee reduces $\hat{\theta}$.

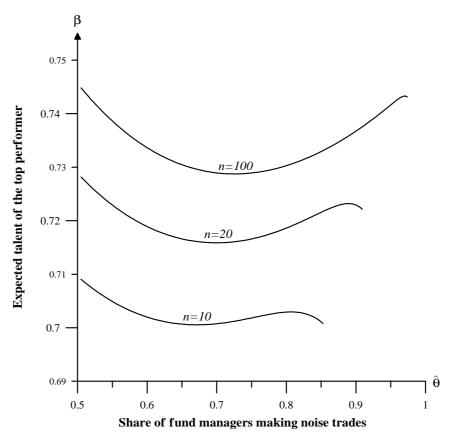


Figure 4: Expected talent of the top performer.

The following example shows that the tournament competition of the type commonly observed in the mutual fund industry might be socially desirable although it deteriorates short-term performance.

Example: Competition and selection efficiency with and without incentive-fee contracts. Let K = 10, S = 7, and n = 30.

The tournament case: It follows from numeric solution of the fund manager's incentive compatibility condition (equation (10)) that $\hat{\theta} = 0.93$. From equation (16) we have $\beta = 0.73$. The incentive-fee case: It follows from numeric solution of the investor's profit-maximization problem (Proposition 2) that $\hat{\theta}^* = 0.70$ and $B^* = 0.14$. From equation (16) we have $\beta = 0.72$.

A comparison of social efficiency in the two cases depends on the importance of selection efficiency. Selection efficiency is best in the tournament case, but expected rate of return is highest in the incentive-fee case.

5 Empirical implications

There are several interesting empirical questions raised by the above analysis.

First, the model points out (Proposition 4 and Corollary 1) the importance of competition and inflow of investments for mutual fund managers' investment strategies. The theoretical analysis indicates that competition for new investments can induce a fund manager to take noise trades, and, consequently, reduce the fund managers' average performance. According to Proposition 4, this tendency to choose noise trade depends on the number of competing fund managers and is most pronounced with an intermediate level of competition. This could, for example, be investigated by comparing fund managers in different industries or countries. Another setting for investigating this would be to compare internal and external fund managers. Since internal fund managers do not compete for investments from outside investors, they may face weaker implicit incentives and, according to my model, choose better investment strategies than outside managers. Hard evidence on the relative performance of internal versus outside fund managers is sparse. Exceptions include a survey among eight hundred corporate and two hundred and fifty public pension plan sponsors in the United States published in Institutional Investor ("In-house afire", April 1996). This survey shows that internally managed funds outperformed those run by outside managers. A report from OECD also points out the same feature for the UK: "Data for the United Kingdom also seem to confirm that the average returns are lowest for external fund managers" (The Impact of institutional investors on OECD financial markets, Financial Market Trends, November 1997, p. 30).

Second, in Section 3 I showed that performance-fee contracts can be used to counteract implicit incentives due to managers' competition for new investments. Since strong implicit incentives lead to a high level of noise trades, investors can offer performance-fee contracts to reduce the level of noise trade. This could be investigated by studying the empirical relationship between inflow of investment and use of performance-fee contracts.²⁷

Third, the model points out that large inflow of investment lead to more noise trade, and, consequently, less performance persistence. There is some empirical evidence consistent with such a relationship. Empirical studies (e.g. Daniel et al. (1997) and Malkiel (1995)) have shown that it is more difficult to discover persistent superior performance in the last two decades than in previous decades. This, together with the fact that the US mutual fund business has experienced an average yearly growth in excess of 20% the last decades, indicates that weak performance persistence might be an effect of a large inflow of investments.

²⁷It should be taken into account that the level of competition, i.e. the number of fund managers, influences the strength of the implicit incentives (Proposition 4).

6 Market efficiency and public policy

Fee structure: In some countries, the legal and regulatory structure has placed constraints on how fund managers are paid. For instance, permissible fee structures in the United States mutual fund industry are laid out in the 1970 Amendment to the Investment Company Act of 1940. The regulation allows mutual funds and their investment advisers to enter into performance-based compensation contracts only if they are of the "fulcrum" variety.²⁸ The fee must be symmetric around a chosen benchmark or index, decreasing for underperforming the benchmark in the same way as it increases for outperforming it. It has been claimed that this regulation is misdirected. For instance, Baumol et al. (1990) suggest the elimination of all regulation of fee structures in the mutual fund industry because "such pricing rules often induce the imposition of prices other than those that the forces of competition would have yielded. Consequently, the resulting industry prices will not be those that serve the public interest" (see also Das and Sundaram (1998)). I have shown that competition among funds does not necessarily lead to an efficient outcome. Current investors do not pay sufficient attention to selection efficiency (which can be considered a public good) when incentive fees are set. Investors may choose fee structures which impede late investors' search for talented fund managers. Although is an open question whether government regulation can improve selection efficiency, questions regarding selection efficiency should not be ignored.

Switching costs: Costs associated with changing investment funds can impede competition. Examples of switching costs include possible capital tax liability due to changing of funds, low fund share liquidity, sales load and redemption fees. These switching costs were quite high at the time of the enactment of the Investment Company Act of 1940 and a common fear that investment funds may abuse market power motivated the regulation of the fee structure. The widespread reduction in switching costs in the last decades suggests that fee regulation should be revised or removed. However, I have shown that switching costs, which serve as an imperfect commitment to using the current fund manager in the future, can improve (short-term) fund performance.²⁹ Furthermore, my analysis shed some light on investors' growing demand for removal of fee regulation. Fee regulation impedes the use of powerful incentives on fund management. It is commonly claimed that competition removes

²⁸A 1972 Securities and Exchange Commission (SEC) study shows that performance-based compensation contracts were becoming common before 1970. In 1968 and 1969, roughly 40 percent of all new funds used performance fees. After the 1970 Amendment to the Investment Company Act of 1940 the use of performance fees were significantly reduced. By 1972, performance fees were used in only 10 percent of funds.

²⁹Investors might be better off if they committed not to flock to the top performer. This would reduce the implicit incentives and improve fund management. However, as shown by Proposition 5 it is expost optimal to use the last period's top-performer. Consequently, it might be difficult for the investors to commit credibly to using an inferior fund manager at a later stage.

the need to use explicit contracts. Baumol et al. (1990), for instance, argue that "explicit contracts are likely to be unnecessary to enforce transaction in markets that are highly competitive because, with low transaction costs, market forces can be depended on to provide the necessary discipline."³⁰ I show that competition is not a good substitute for contracts.

From a social efficiency perspective, the net benefit of a decrease in the switching costs depends on whether the long-term benefit of directing a larger share of total investments to the fund manager expected to be best, exceeds the cost of reduced current fund performance due to stronger implicit incentives.

7 Concluding remarks

Competition as well as contracts influences fund managers' choice of investment strategy. I show that competition for new investors creates implicit incentives which may result in an inefficient use of managers' private investment signals. On the other hand, explicit incentives provided by contracts is shown to counteract harmful implicit incentives and improve the fund managers' performance. The interaction of implicit and explicit incentives results in a non-monotone relationship between the level of competition (number of rivaling funds) and expected performance of fund managers. An intermediate level of competition induces a less efficient use of private information than would be the case with either a high or low level of competition. Furthermore, I show that explicit incentives provided by current investors may impede new investors in their search for talented fund managers for prospective fund management. Although explicit incentives improve current fund managers' performance, it may reduce selection efficiency and thereby negatively affect prospective performance.

My analysis may shed some light on other settings in which implicit and explicit incentives interact. In organizations, workers often face both explicit incentives provided by employment contracts as well as implicit incentives associated with their career concerns.³¹ Similarly to the discussion in my paper, Gibbons and Murphy (1992), argue that the principal – the investor in my case – has to pay attention to total incentives; the combination of implicit and explicit incentives. The focus in this branch of the literature has been on the workers' choice of effort, not choice of project or task. However, if workers are delegated the authority to choose among tasks or projects that differ in risk and the worker obtaining the best outcome is promoted, then the setting resembles the one discussed here.³² My analysis can provide

³⁰Baumol et al. (1990) suggest that funds' capacity to impose costs on investors who want to change funds should be restricted. Such restrictions may strenghten the harmful effects of implicit incentives on fund management from.

 $^{^{31}}$ See the seminal papers Fama (1980) and Holmstrom (1999). Gibbons (1996) provides a recent survey of incentive pay and careers in organizations.

³²The fact that performance-fee or performance-wage contracts can harm the search for talented workers can shed some light on the empirical evidence showing that low-power incentives commonly are used in

insights about the link between competition for promotion, workers' choice of tasks, incentive pay, and the expected talent of the promoted worker.

Apart from the empirical issues raised in Section 5, the analysis presented here could be extended in several directions. A straightforward extension of the model presented would involve studying the impact of implicit and explicit incentives on managers' efforts to gather information. One would expect that fund managers who seldom make informed trades would have weak incentives to spend resources on acquiring information. Consequently, investors will suffer both from inefficient use of information as well as inefficient incentives to acquire information. A more demanding extension of the model would be to assume that although managers slowly learn about their talents, they have some private information about their talent which they can signal to investors through their choice of contract.

organizations (see e.g. Jensen and Murphy (1990)).

Appendix

Proof of Proposition 1: Using the definition of H_z (equation (4)) it follows that

$$\int_{0}^{1} \left(H_{z=\widehat{\theta}}\left(x\right)\right)^{n-1} dx = \int_{0}^{\widehat{\theta}} \left(\widehat{\theta}x\right)^{n-1} dx + \int_{\widehat{\theta}}^{1} \left(x\left(1+\widehat{\theta}\right)-\widehat{\theta}\right)^{n-1} dx$$
$$= \frac{1}{n} \widehat{\theta}^{2n-1} + \frac{1-\widehat{\theta}^{2n}}{\left(1+\widehat{\theta}\right)n}$$

and

$$\left(H_{z=\widehat{\theta}}\left(\widehat{\theta}\right)\right)^{n-1} = \widehat{\theta}^{2n-2}.$$

Hence, equation (5) can be simplified to

$$\frac{1}{n}\widehat{\theta}^{2n-1} + \frac{1-\widehat{\theta}^{2n}}{\left(1+\widehat{\theta}\right)n} = \widehat{\theta}^{2n-2}$$
(17)

Note that equation (17) has a unique solution, $\hat{\theta}$: Define $k(\hat{\theta}) \equiv n\left(\hat{\theta}^{2n} + \hat{\theta}^{2n-1}\right)$ and $l(\hat{\theta}) \equiv \hat{\theta}\left(1 + \hat{\theta}^{2n}\right)$. Note that $k(\hat{\theta}) - l(\hat{\theta}) = 0$ is identical to equation (17). Since $k(0) \leq l(0), k(1) > l(1), k_{\hat{\theta}}(\hat{\theta}), l_{\hat{\theta}}(\hat{\theta}) > 0$ and $l_{\hat{\theta}\hat{\theta}}(\hat{\theta}) < k_{\hat{\theta}\hat{\theta}}(\hat{\theta})$, it follows that $k(\hat{\theta})$ and $l(\hat{\theta})$ intersect only once. Hence $\hat{\theta}$ is unique.

Furthermore, since an increase in θ improves the win probability resulting from choosing an informed trade while keeping the win probability resulting from choosing a noise trade constant, only fund managers of type $\theta > \hat{\theta}$ make an informed trade. $I_{\hat{\theta}}(\theta)$ is a unique symmetric pure strategy equilibrium. Q.E.D.

Proof of Proposition 2: Part a): Define

$$\Phi^{0} \equiv (1-n)\widehat{\theta}^{2n-1} + 1 - n\widehat{\theta}^{2n-2}$$

Note that equation (5) (see also equation (17)) is identical to $\Phi^0 = 0$. From implicit differentiation of the fund manager's incentive constraint it follows that:

$$\frac{d\widehat{\theta}}{dn} = -\frac{\Phi_n^0}{\Phi_{\widehat{\theta}}^0}$$

Straightforward calculations show that $\Phi_{\hat{\theta}}^0 < 0$. Hence $sign\left(\frac{d\hat{\theta}}{dn}\right) = sign\left(\Phi_n^0\right)$. First note that

$$\Phi_n^0 = \widehat{\theta}^{2n-2} \left(2\widehat{\theta} \ln \widehat{\theta} - \widehat{\theta} - 2\widehat{\theta} n \ln \widehat{\theta} - 1 - 2n \ln \widehat{\theta} \right) = 0$$

if n = n':

$$n' = -\frac{\widehat{\theta} + 1 - 2\widehat{\theta}^2 \ln \widehat{\theta}}{2\left(1 + \widehat{\theta}\right) \ln \widehat{\theta}}.$$

However, since θ satisfying $\Phi^0(n',\theta) = 0$ is not in [0,1], Φ_n^0 cannot change sign. It is now sufficient to show that $\Phi_n^0 > 0$ for one feasible $(n,\hat{\theta})$ -pair (that satisfies equation (17)): E.g. $(n = 3, \hat{\theta} = 0.69)$ implies $\Phi_n^0 = 0.35 > 0$. For the proof of Proposition 4 it is useful to note that Φ_n^0 is monotonically decreasing in n and $\lim_{n\to\infty} \Phi_n^0 = 0$.

Part b) follows from the fact that the expected rate of return of the top performer approaches 1 as $n \to \infty$. Q.E.D.

Proof of Proposition 3 (sketch of the proof): First, note that the implicit incentives inducing the fund manager to choose an informed trade increase with an increase in θ (see the proof of Proposition 1). Second, note that the implicit benefits to investors of choosing an informed trade instead of a noise trade increase with an increase in θ (see equation (6)) Consequently, an increase in θ implies that weaker or no explicit incentives are required to induce the fund manager to choose the informed trade opportunity. It follows that an investor will not induce a fund manager with high θ to make a noise trade while a fund manager with low θ is induced to make an informed trade (in fact, it is impossible to induce such behavior given that $w'(r) \geq 0$). Hence, the investors' optimal provision of explicit incentives induces the fund managers to follow a strategy of type I_z (see equation (2)).

By paying no bonus to fund managers achieving r < z (these managers have definitely made noise trades) and by paying a positive bonus to fund managers achieving $r \ge z$ (these managers may have made informed trades) investors maximize a " $\theta = z$ -fund manager's" expected difference in payment due to their making an informed trade instead of a noise trade. Because the implicit incentives for making an informed trade increase with an increase in θ , the bonus should be constant for $\forall r \in [z, 1]$ (recall that payment is required for an increase in performance, $w'(r) \ge 0$). Such an incentive contract is described by equation (9). For a given $B, z = \hat{\theta}$, defined by incentive constraint (10), is the smallest feasible z which induces managers with $\theta \ge z = \hat{\theta}$ to make informed trades and managers with $\theta < z = \hat{\theta}$ to make noise trades. A manager of type $\hat{\theta}$ is indifferent with respect to making an informed or a noise trade. Hence equation (9) and the incentive constraint (10) describe the optimal incentive contract. Q.E.D.

Proof of Corollary 1:Define

$$\Phi \equiv S\left[\left(\frac{1}{n}\widehat{\theta}^{2n-1} + \frac{1-\widehat{\theta}^{2n}}{\left(1+\widehat{\theta}\right)n}\right) - \left(\widehat{\theta}^{2n-2}\right)\right] - B^*$$
(18)

Note that the fund manager's incentive constraint (10) is identical to $\Phi = 0$ (see also equation

(17) and equation (10)). From implicit derivation it follows that

$$\frac{d\widehat{\theta}}{dS} = -\frac{\Phi_S}{\Phi_{\widehat{\theta}}}$$

Note that according to the envelope theorem, the optimal bonus payment (B^*) does not change (no first-order effect) as a result of marginal change in one of the parameters (e.g. S). Straightforward calculations show that $\Phi_{\hat{\theta}} < 0$ and $\Phi_S > 0$. Q.E.D.

Proof of Proposition 4: Part a): From implicit differentiation of equation (18) it follows that

$$\frac{d\widehat{\theta}}{dn} = -\frac{\Phi_n}{\Phi_{\widehat{\theta}}}$$

Straightforward calculations show that $\Phi_{\hat{\theta}} < 0$ (as before).

Note that equation (10) can be written as

$$\Phi^+ \equiv \Phi^0 - \left(1 + \widehat{\theta}\right) n \frac{B}{S} = 0$$

As before $\Phi_{\hat{\theta}}^+ < 0$ and $sign\left(\frac{d\hat{\theta}}{dn}\right) = sign\left(\Phi_n^+\right)$.

$$\Phi_n^+ = \Phi_n^0 - \left(1 + \widehat{\theta}\right) \frac{B}{S}$$

Since Φ_n^0 is monotonically decreasing in n and $\lim_{n\to\infty} \Phi_n^0 = 0$ (see the proof of Proposition 2) and $\Phi_n^0 > 0$, we can conclude that there exists a unique \overline{n} such that $\Phi_n^+ > 0$ for $n < \overline{n}$ and $\Phi_n^+ < 0$ for $n > \overline{n}$. From equation (6), it follows that $\frac{dr^e}{d\theta} < 0$, $\forall \theta \in [\frac{1}{2}, 1]$. That completes the proof of part a).

Part b) follows from the fact that Φ_n^+ is decreasing in $\frac{B}{S}$.

Part c): Recall that $\hat{\theta} = \frac{1}{2}$ maximizes (gross) rate of return. From the fund manager's incentive constraint (equation (17)) B^M induces the fund managers to set $\hat{\theta} = \frac{1}{2}$

$$B^{M} = S\left[\frac{1}{n}\left(\frac{1}{2}\right)^{2n-1} + \frac{1-\left(\frac{1}{2}\right)^{2n}}{\left(1+\left(\frac{1}{2}\right)\right)n} - \left(\frac{1}{2}\right)^{2n-2}\right]$$

Furthermore, note that $\lim_{n\to\infty} B^M = 0$. Since the cost to the investor of inducing the fund manager to follow a profit-maximizing investment strategy $(\hat{\theta} = \frac{1}{2})$ approaches 0 as the number of competing fund managers approach infinity, investors will induce $\hat{\theta} = \frac{1}{2}$ and hence maximize r^e . Q.E.D.

Proof of Proposition 6: Differentiate the incentive constraint (17) and obtain $\frac{d\theta}{dn}$ which can be used to show that $\frac{d\beta}{dn} = \frac{\partial\beta}{\partial\theta}\frac{d\theta}{dn} + \frac{\partial\beta}{\partial n}$ (differentiation of equation (16)) is positive as illustrated in Figure 3. Q.E.D.

Proof of Proposition 7: First note that $\hat{\theta}(B)$ is decreasing in B (which follows from Corollary 1 and equation (10)). Second, two examples can be used to show that $\beta\left(\hat{\theta},n\right)$ is non-monotone in $\hat{\theta}$: Example 1: Suppose $\hat{\theta} = 0.8$ and n = 20 (for different Ks, all $\hat{\theta}$ in $\langle \frac{1}{2}, 1 \rangle$ are feasible outcomes from an optimal incentive contract) then $\frac{\partial\beta}{\partial\hat{\theta}} = 0.05$. Example 2: Suppose n = 20 and $\hat{\theta} = 0.6$ then $\frac{\partial\beta}{\partial\hat{\theta}} = -0.06$. Q.E.D.

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