Two-Part Tariffs, Consumer Heterogeneity and Cournot Competition*

Sissel JENSEN† Lars SØRGARD‡

Discussion paper 20/2001
October 2001

Abstract

We analyze two-part tariffs in oligopoly, where each firm commits to a certain quantity. The model is an extension of the one introduced in Har (2001). We show that their main results are reversed when the model is extended from one to two types of consumers. In particular, we find that price per unit can exceed marginal costs, and the fixed fee can be below costs. We also show that two-part tariffs may collapse, because each firm would rather commit to a traditional Cournot price system (zero fixed fee). Finally, some numerical examples illustrate that both firms serving both types of consumers can be an equilibrium outcome in duopoly in cases where the monopolist would serve only one type of consumers.

JEL.class.no: L11, L13
Keywords: Oligopoly, Two-part tariff, Cournot

*We are grateful for comments from Petter Osmundsen. We are also indebted to participants at the Nordic Workshop on ICT-related research at Norwegian School of Economics and Business Administration in June 2001, and to participants at the doctoral seminar series at the Norwegian School of Economics and Business Administration for helpful comments.

†Norwegian School of Economics and Business Administration and Telenor Research and Development. Mailing address: Helleveien 30, N-5045 Bergen, Norway, sissel.jensen@nhh.no

‡Norwegian School of Economics and Business Administration. Mailing address: Helleveien 30, N-5045 Bergen, Norway, lars.sorgard@nhh.no

1 Introduction

Nonlinear prices are common in many industries, and have been studied extensively in the economic literature. However, most theoretical studies use a monopoly setting. In contrast, we observe nonlinear prices not only in monopoly markets, but also in other market settings such as oligopoly. The purpose of this article is to help bridging this gap. We analyze two-part tariffs in a Cournot-like setting by extending the seminal model of Har (2001).

Nonlinear pricing may not be sustainable in oligopoly. For example, Mandy (1992) finds that in a traditional Bertrand oligopoly with homogeneous products – where we allow the firms to set nonlinear prices – all prices may collapse to a uniform price. The finding illustrates that, except for some special cases which he explores, some of the assumptions in the traditional Bertrand model have to be relaxed in order to make nonlinear prices sustainable in oligopoly. This has been done in the emerging literature on nonlinear prices. One extension of the traditional Bertrand model is to introduce product differentiation, see Calem and Spulber (1984), Castelli and Leporelli (1993), Economides and Wildman (1995), Shmanske (1991), and Young (1991). Another extension is to introduce capacity constraints, as is done in Har (2001), Oren, Smith and Wilson (1983), Scotchmer (1985a, 1985b) and Wilson (1993). Scotchmer (1985a, 1985b) only considers existence when the number of firms becomes large, while both Oren et al. (1983) and Wilson (1993) assume that the firms predict the market shares of their rivals. In contrast, Har (2001) model quantity as a strategic variable and consider the strategic interaction between a small (or a large) number of firms.

In Har (2001) each firm commits to a certain quantity, as is the case in a traditional Cournot model. In addition, each firm sets its fixed fee while the unit prices are determined endogenously by market forces. The latter is analogous to what is the case in a traditional Cournot model. In their paper, Harrison and Kline also provide some examples where we do observe that fixed fees are less flexible than prices per unit.[1] It is found that in equilibrium price is set equal to marginal costs, and the fixed fee is positive for a given number of firms. Furthermore, it is found that fixed fees extract the entire consumer surplus if the number of firms is sufficiently small. Finally, they found that when the number of firms approaches infinity the fixed fee tends toward zero.

We extend the model introduced in Har (2001) by assuming two instead of one type of consumers. It turns out that none of the conclusions referred to above is robust to such an extension of the model. If both types of consumers are served, we find that price per unit is above marginal costs. Furthermore, fixed fees can be zero or even negative for a finite number of firms. In fact, firms can be better off committing to traditional Cournot competition where the firms can

[1]One example is a consumer club like Costco. The membership fee corresponds to a fixed fee and the price per product a member buy when he visits the store may vary considerably. For more examples, see Har (2001).

only charge a unit price. If the firms can choose whether to serve one or both types of consumers, they may choose to serve only the large consumers. Then the equilibrium outcome replicates the one shown in Har (2001), except there are now some consumers that are not served. However, by using a numerical example we show that there can be multiple equilibria. Moreover, it is shown that both firms serving both types of consumers can be an equilibrium duopoly outcome in cases where the monopolist would have preferred to serve only one type of consumers. The driving force is that the rival, non-deviating firm supplies a given quantity which it is committed to sell, acting as a constraint on the deviating firm's price setting.

The article is organized as follows. In Section 2 we formulate our model, and report optimal pricing strategies given that all firms serve either both types of consumers or only one type. In section 3 we explore the equilibrium outcomes of the model. First, we consider the case with full market coverage, that is, both firms are restricted to sell to both types of consumers. Second, we consider the case where each firm chooses either to sell to one or both types of consumers. Finally, in Section 4 we offer some concluding remarks.

2 The Model

We consider a setup with k identical firms, $k \geq 2$, supplying a homogeneous product. The cost function is characterized by constant returns to scale, $C(Q) = cQ$ where $c > 0$ is the marginal cost and Q is output. For simplicity we omit fixed costs. The number of firms is exogenous and the question of entry is left outside the scope of this paper.[2]

There are two groups of consumers with a total of N. Consumers with taste parameter θ_1 are in proportion λ and consumers with taste parameter θ_2 are in proportion $(1-\lambda)$.[3] Preferences are defined by a quasi-linear utility function

$$\begin{aligned} &V = \begin{cases} u(q,\theta_\ell) - T & \text{if they pay } T \text{ and consume } q \text{ units} \\ 0 & \text{if they do not buy} \end{cases}, \\ &\theta_\ell = \{\theta_1, \theta_2\}, \\ &u(q,\theta_2) \geq u(q,\theta_1), \ \forall q. \end{aligned} \tag{1}$$

The utility function is assumed to be increasing and strictly concave in q, $u(0,\theta) = 0$, $\lim_{q\to 0} u_q(q,\theta) \geq c$, $\lim_{q\to\infty} u(q,\theta) \leq 0$. For any tariff $T = A + pq$, where A is a fixed fee that is paid up-front and p is a unit price, utility maximization yields a downward sloping demand curve for each individual which is independent of income and therefore also of the fixed fee. Indirect utility gross

[2] See Har (2001) on entry in this model.

[3] We refer to the first group as type 1 consumers or low demand consumers and to the other group as type 2 consumers or high demand consumers.

of the fixed fee is

$$\begin{aligned}
&q_\ell(p) \equiv q(p,\theta_\ell) = \max\nolimits_q u(q,\theta_\ell) - pq - A,\\
&V(p,\theta_\ell) = u(q_\ell(p),\theta_\ell) - pq_\ell(p),\\
&V_p' = -q_\ell(p),\\
&\ell = 1,2.
\end{aligned} \tag{2}$$

With quasilinear utility we can measure the indirect utility in monetary terms. Consumers choose to buy if they obtain a nonnegative net surplus at some firm i, that is, iff $V(p_i,\theta_j) - A_\ell \geq 0$, $i \in \{1,2,...,k\}$ and $\ell = 1,2$. They buy from the firm providing them with the highest surplus, $V(p_i,\theta_\ell) - A_i \geq V(p_j,\theta_\ell) - A_j$, $(i,j \in \{1,2,...,k\}$, $i \neq j$, $\ell = 1,2)$. When the two consumer types are charged the same tariff, a type 2 consumer obtains a surplus that is at least as large as the surplus a type 1 consumer obtains. Thus, if type 1 is able to obtain a nonnegative surplus, type 2 obtains a strictly positive surplus.

Firms act to maximize profit by choosing a strategy $s_i = (Q_i, A_i)$, with $Q_i > 0$ for all $i = 1,2,\ldots,k$, and we assume that firms are able to commit to this strategy. The firm cannot exclude any consumer from buying. In our model, we use the assumption that for a given strategy combination there exists a consumer equilibrium defining a consumer-price profile $((n_1,\ldots,n_k),(p_1,\ldots,p_k))$. This is formally defined in Har (2001). Although we define a firm's strategy in capacity and the fixed fee, from a consumer's point of view he chooses the quantity that maximizes his utility for a given A_i and p_i. The notion behind this reasoning is that it is a competitive equilibrium where a large number of consumers without market power trade, given the fixed fees and quantities from each firm. If all firms leave each consumer with equal and nonnegative surplus, we assume that all firms serve an equal share of each consumer type, $n_i = \lambda N/k + (1-\lambda)N/k$.

If there are at least two active firms, the relevant participation constraints in firm i's optimization problem are given by

$$V(p_i,\theta_\ell) - A_i \geq V(p_j,\theta_\ell) - A_j,\ \ell = 1,2,\ j \in \{1,2..i-1,i+1,...k\}. \tag{3}$$

Profit for firm i is given by

$$\Pi_i = n_i A_i + (p_i - c)\, Q_i. \tag{4}$$

Since the fixed fee is a lump sum transfer from consumers to the firm, the unit price in firm i's tariff is adjusted in such a way that aggregate demand for firm i's product is equal to firm i's supply. Hence, the unit price is independent of the fixed fee. Whenever the fixed fee is positive, consumers will make all or nothing purchases at firm i. When firm i serves a total of n_i consumers, the unit price is adjusted to satisfy the following market clearing condition

$$Q_i = n_i\left[\lambda q_1(p_i) + (1-\lambda)\, q_2(p_i)\right]. \tag{5}$$

In line with Har (2001), let us assume that all firms charge the same fixed fee and the same unit price. Firm i maximizes profit subject to the condition that the unit prices charged by rival firms are adjusted to satisfy the market clearing condition and subject to voluntary participation. When every other firm but i serves both consumer types the unit price p charged by every other firm must satisfy the condition

$$\begin{aligned} Q_{-i} &= (N-n_i)\left[\lambda q_1(p) + (1-\lambda) q_2(p)\right], \\ Q_{-i} &= \textstyle\sum_{j\neq i} Q_j. \end{aligned} \tag{6}$$

When rival firms charge their consumers according to the tariff $T = A + pq$ consumer θ_ℓ is indifferent between buying from firm i and one of the other firms when the participation constraint is binding. If the firm leaves the consumer with additional surplus, it sacrifices profit. We therefore expect

$$V(p_i, \theta_\ell) - A_i = V(p, \theta_\ell) - A, \;\; \ell = 1, 2. \tag{7}$$

Henceforth, superscript 12 denotes that both consumer types are served and superscript 2 denotes that type 1 (the "small" type) is excluded. Let us first suppose that both consumer types are served. Then there is at least one additional active firm where both consumers buy a strictly positive quantity. When the best alternative option for a type 1 consumer is represented by a tariff T^{12}, the relevant participation constraint is given by

$$V\left(p_i^{12}, \theta_1\right) - A_i^{12} = V\left(p^{12}, \theta_1\right) - A^{12}. \tag{8}$$

Taking rival firms' tariffs as given and maximizing profit with respect to p_i^{12} give the following optimality condition for the unit price in a two-part tariff

$$\begin{aligned} p_i^{12} &= c + \frac{(1-\lambda)\left[q_2 - q_1\right]}{-\left[\lambda q_1' + (1-\lambda) q_2'\right]}, \\ q_\ell' &\equiv \frac{dq_\ell}{dp}. \end{aligned} \tag{9}$$

Next, firm i must choose the strategy (Q_i^{12}, A_i^{12}) in such a way that p_i^{12} satisfies the market clearing condition. To attract additional consumers from rival firms, firm i has to adjust the fixed fee. Hence, a marginal increase in market share affects firm i's profit via the fixed fee. Finding the profit maximizing strategy reduces to finding the optimal number of consumers to serve.

The effect on the firm's profit of a marginal increase in market share is

$$\frac{\partial \Pi_i^{12}}{\partial n_i} = A^{12} - \frac{p^{12} q_1}{\varepsilon(Q)} \left(\frac{1}{k-1} - (1-\lambda)\frac{q_2 - q_1}{q_1} \right). \tag{10}$$

If all firms exclude type 1 and serve type 2 alone, the participation constraint when the best alternative option for type 2 consumers is represented by a tariff T^2 becomes

$$V\left(p_i^2, \theta_2\right) - A_i^2 = V\left(p^2, \theta_2\right) - A^2. \tag{11}$$

The optimal tariff is a cost-plus-fixed-fee tariff and firm i chooses a strategy (Q_i^2, A_i^2) in such a way that the market clearing condition is satisfied when $p^2 = c$.

Again, applying symmetry, the effect on the firm's profit of a marginal increase in market share is

$$\frac{\partial \Pi_i^2}{\partial n_i} = (1-\lambda)\left(A^{12} - \frac{1}{k-1}\left[\frac{cq_2(c)}{|\varepsilon(q_2(c))|}\right]\right). \tag{12}$$

Notice that if firm i takes the number of consumers it serves as given, for any tariff charged by rival firms the reservation utility is defined as a constant and will not affect the optimization with respect to unit price. The problem then resembles the monopoly problem, and the marginal price in our model is identical to that in a monopoly.

The following two Lemmas state the pricing strategies in a k-firm oligopoly, given that they either serve both types or exclude type 1.

Lemma 1 (Two consumer types) *(i) Let us assume that both consumer types are served by all firms. Then the pricing strategy in two-part tariffs in a k-firm oligopoly is given by*

$$\begin{aligned} &A_i^{12} \equiv A_{TT}^{12} = \min\left\{V(p^{12},\theta_1), \tfrac{p^{12}q_1}{\varepsilon(Q^{12})}\left(\tfrac{1}{k-1} - (1-\lambda)\tfrac{q_2-q_1}{q_1}\right)\right\} \\ &p_i^{12} \equiv p_{TT}^{12} = c + \frac{(1-\lambda)[q_2-q_1]}{-(\lambda q_1' + (1-\lambda)q_2')} \\ &Q_i^{12} \equiv Q_{TT}^{12} = \frac{N}{k}(\lambda q_1 + (1-\lambda)q_2) \\ &(q_\ell = q_\ell(p_{TT}^{12}),\ q_\ell' = q_\ell'(p_{TT}^{12}),\ \ell = 1,2) \end{aligned} \tag{13}$$

(ii) If both consumer types are served, the pricing strategy in a traditional Cournot game is given by

$$\begin{aligned} &A_i^{12} \equiv A_{UP}^{12} = 0 \\ &p_i^{12} \equiv p_{UP}^{12} \geq c \\ &Q_i^{12} \equiv Q_{UP}^{12} = \frac{N}{k}(\lambda q_1 + (1-\lambda)q_2) \\ &(q_\ell = q_\ell(p_{UP}^{12}),\ q_\ell' = q_\ell'(p_{UP}^{12}),\ \ell = 1,2) \end{aligned} \tag{14}$$

where p_{UP}^{12} is the (standard) price when both types are served in a Cournot game with k identical firms charging a uniform price.

Lemma 2 (Harrison and Kline) *If one of the consumer types is excluded from purchasing, the pricing strategy in two-part tariffs in a k-firm oligopoly* *is given by*

$$
\begin{array}{l}
A_i^2 \equiv A_{TT}^2 = A = \min \left\{ V(c, \theta_\ell), \dfrac{cq_\ell}{(k-1)\left|\varepsilon\left(q_\ell\right)\right|} \right\} \\
p_i^2 \equiv p_{TT}^2 = c \\
Q_i^2 \equiv Q^2 = \dfrac{N}{k}\lambda_\ell q_\ell \\
\lambda_\ell = \lambda \;\; if \;\; \ell = 1, \;\; \lambda_\ell = (1-\lambda) \;\; if \;\; \ell = 2, \\
\left(q_\ell = q_\ell\left(c\right), \;\; \ell = 1 \;\; or \;\; 2\right)
\end{array} \tag{15}
$$

with $\theta_2 \geq \theta_1$ type 2 will always be served.

Lemma 2 is the result in Har (2001) when the tariffs are symmetric. Lemma 1 is the extension of this to the two-type case, and the proof is given by the previous calculations. According to Lemma 2 the fixed fee in the single-type case converges toward zero as the number of firms approaches infinity. Moreover, note that the price per unit is set equal to marginal costs in the case with one type. As Lemma 1 indicates, these results are reversed when we extend the model from one to two types.

Har (2001) give a thorough treatment of Cournot competition with two-part tariffs and a single consumer type, and they also guide the reader through all proofs in that case. They show that the pricing described in Lemma 2 is a unique Nash equilibrium in pricing strategies for the game. All k firms produce. In addition to the equilibrium with symmetric market, shares there also exist equilibria that are asymmetric in market shares.

In what follows, we consider first the firms' pricing in a symmetric equilibrium when all consumers are served. Next, since low demand types may be excluded we consider the prospects for a unique equilibrium with symmetric pricing in a duopoly with respect to market coverage.

3 Equilibrium outcomes

To illustrate the equilibrium outcomes, we have chosen to focus on a case where consumer preferences are represented by a quadratic utility function. We let the reservation utility be zero for both consumers. $V = \theta_\ell q - \frac{1}{2}q^2 - T$, $\ell = 1, 2$, if they pay T and consume q units, otherwise they obtain zero utility. Each consumer has a linear demand function $q_\ell = \theta_\ell - p$, $\ell = 1, 2$. Letting $\theta \equiv \lambda\theta_1 + (1-\lambda)\theta_2 \geq \theta_1$, expected demand is $\lambda q_1 + (1-\lambda) q_2 = \theta - p$. The indirect utility exclusive of the fixed fee for a consumer paying a unit price of p is $V(p, \theta_\ell) = \frac{1}{2}(\theta_\ell - p)^2$, $\ell = 1, 2$. Because we are interested in how equilibrium strategies are affected by heterogeneity in demand, the example is somewhat simplified by letting $\theta_1 = 1$ and $c = \frac{1}{2}$. Increased demand side heterogeneity is captured by variations in λ and θ_2. Large heterogeneity can then come about either by an increase in the number of type 2 consumers (λ decreases), or because a type 2 consumer has larger willingness to pay relative to a type 1 consumer (θ_2 increases). Hence, increased demand side heterogeneity is captured by an increase in θ. We use Lemmas 1 and 2 to characterize the equilibrium in terms of pricing and expected profit per consumer. All these computations are given in the appendix.

3.1 Market coverage

Let us first consider the case where both types are served by all firms.[4] This could be due to some institutional restrictions, forcing them to provide a universal service. Given such a restriction, which combination of fixed fee and price per unit would each firm choose?

Proposition 1 *Let us assume that both types of consumers are served and each firm sets a two-part tariff. If (i)* $0 \leq \lambda \leq \lambda^* \equiv \frac{4\theta_2 - 5}{4\theta_2 - 4}$*, or (ii)* $k > k^* \equiv \frac{1}{2(\theta_2 - 1)(1-\lambda)}$*, then* $A_{TT}^{12} < 0$ *and* $p^{12} = \lambda + (1-\lambda)\theta_2 - \frac{1}{2} \equiv p_{TT}^{12} > c$*. Otherwise,* $A_{TT}^{12} > 0$ *and* $p^{12} = p_{TT}^{12} > c$*.*

The critical values λ^* and k^* are derived in the appendix. First, we see that each firm would set a price per unit that exceeds marginal costs. In contrast, Har (2001) found that each firm would set a price per unit equal to marginal costs. Obviously, the extension of the model – from one to two types of consumers – explains the change in the result. It is well known from a monopoly model that a firm that serves two types of consumers with one two-part tariff should let the unit price exceed marginal costs, see Oi (1971). By doing so it is able to extract more profits from the high demand consumer, and this outweighs the loss in profit extraction from the low demand consumer as long as the price-cost margin is not above a certain threshold level. The price-cost margin is higher the larger the difference between the consumer types ($\theta_1$versus θ_2), and the larger the proportion of the high demand consumers (λ approaches zero). This is natural, since a large difference between those two groups of consumers would lead to a relatively high price-cost margin to extract profits from the larger group.

Second, note that the price-cost margin is *not* influenced by the number of firms. At first glance, this may come as a surprise. Why do they not compete on prices? The reason is that they compete on access prices, not prices per unit. The prices per unit are set to balance the revenues from the two consumer groups, *after* they have competed on fixed fees to attract consumers. Note that our result is in line with the result in Har (2001), where the price per unit is always equal to marginal costs since the unit price in both cases just replicates the monopoly price.

[4] In the next section we show that this can be the equilibrium outcome for a large number of parameter values.

Third, we see that each firm's fixed fee can be set below the fixed cost of serving a consumer (which is normalized to zero in our setting). In contrast, Har (2001) found that the fixed fee is always above costs, but approaches costs when the number of firms approaches infinity. In their setting, as well as in ours, profits approach zero when the number of firms approaches infinity. But the fact that we have a positive price-cost margin, implies that the fixed fee is competed away even for a finite number of firms. In fact, if the demand side heterogeneity is sufficiently large, the fixed fee is competed away even in a duopoly.

Obviously, the existence of many firms would lead to fierce competition on fixed fees. But even with two firms, fixed fees can be negative if the fraction of the high type consumer is large or when the difference in consumers' type is large. In such a case the price per unit is high, to extract profits from the "large" consumer. Then the fixed fee is low even in a monopoly setting, and competed away in a duopoly setting.

An interpretation of a negative fixed fee in our model is that the fixed fee is positive, but below costs. This is what we observe in some cases. In Norway for example, mobile phones have been sold at a price of NOK 1 each, while some retailers have received a payment of approximately NOK 2000 from the producer. The producer then incurs a loss of approximately NOK 2000 for each consumer it captures, and earns revenues on the same consumer from what he pays for the use of the mobile phone.[5] What is labelled *loss leaders* in the grocery sector can be interpreted in a similar way. Grocery stores advertise low prices on certain products in order to attract consumers to the store, and the consumers end up buying both the advertised product as well as other products. It has been shown that the grocery store should then set a price below costs on the advertised products, and a high price-cost margin on other products (see Lal and Matutes (1994)).

In some instances, however, access can be cost free (or close to cost free), for instance joining some kind of club as the examples referred to in Har (2001). Hence, an obvious question is whether the firm would have been better off constraining its tariff policy to uniform pricing. What, then, if the firm sets a fixed fee equal to zero rather than a negative fixed fee? It can then be shown that the following would emerge as equilibrium outcomes

Proposition 2 *Let us assume that both types of consumers are served and each firm can choose either to set a two-part tariff or a uniform price (fixed fee equal to zero). Then each firm chooses a uniform price if the fixed fee in a two-part tariff would be negative (see the previous Proposition), where* $p^{12}_{UP} < p^{12}_{TT}$. *Otherwise, it chooses a two-part tariff with* $A^{12}_{TT} > 0$ *and* $p^{12}_{UP} > p^{12}_{TT}$.

[5] Strictly speaking, the tariff structure is more complicated than the one with a fixed fee and a price per unit. The user pays a fixed fee in addition to a monthly fixed fee and a price per unit. Then the fixed fee is followed by a two-part tariff, not a uniform price as in our model.

First, we see that as long as the fixed fee is above costs in a setting with a two-part tariff, the firm would set a two-part tariff rather than restrict its pricing policy to a uniform price. A uniform price, which equals the traditional Cournot price, would in that case be higher than the unit price in a two-part tariff. This suggests that a firm would find it profitable to deviate from an outcome where both firms set a uniform price. It could deviate by setting a lower price per unit, and extract the gross consumer surplus it generates through a positive fixed fee. Therefore, we would expect that the firms would end up with a two-part tariff with a positive fixed fee.

Second, we see that each firm would choose a uniform price if the alternative is that both firms set a two-part tariff with a negative fixed fee. To understand this, note that in such a case the price per unit in a two-part tariff is higher than the traditional Cournot price (a uniform price). In our model, the firms compete in utility levels. Then if other firms hold a high unit price and generate consumer surplus via a negative fixed fee, it will be profitable to match other firms' offer by restricting the fixed fee to zero and lowering the price per unit, thereby increasing consumer surplus.

Note that competition between the firms leads to a low price per unit: The equilibrium outcome is a uniform (Cournot) price if that price per unit is lower than the price per unit in a two-part tariff, and vice versa. This is illustrated in Figure 1, where the solid lines show the price per unit in equilibrium.

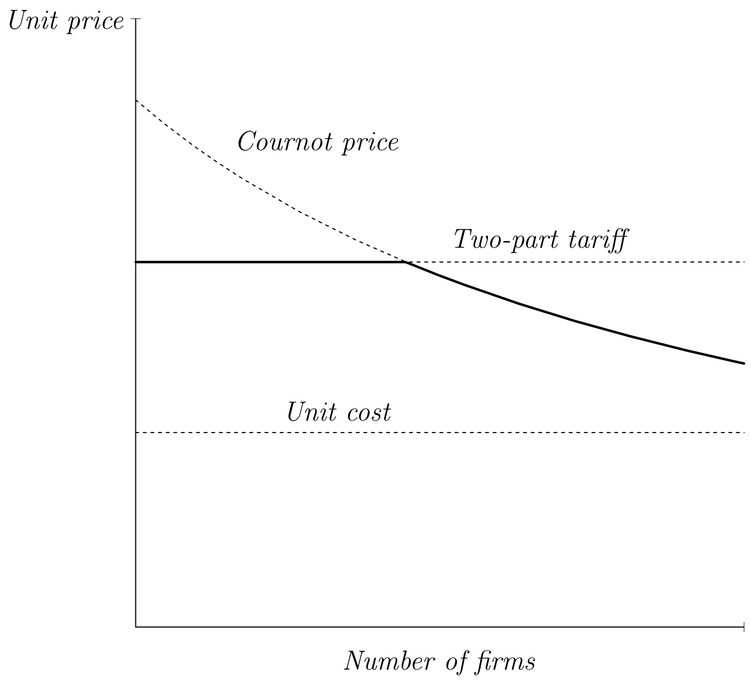

Figure 1: *Price per unit in equilibrium.*

As explained above, in some instances the institutional setting is such that the firms are forced to set a fixed fee. In other instances, though, firms are more flexible. If the choice is either to set a negative fixed fee and a relative high price per unit or a low uniform price, each firm may end up choosing the latter price system because that would generate a larger sale and thereby a larger profit. This suggests that there is no conflict between public policy and private incentives concerning the choice of tariff structure. Each firm has incentive to choose the tariff structure with the lowest price per unit, which is beneficial for consumers and leads to only a limited dead weight loss.

3.2 Market coverage versus exclusivity

In the previous section, we assumed that each firm served both types of consumers. This may not be the equilibrium outcome. As is well known from monopoly, in some cases it is beneficial for a firm to exclude the type with low willingness to pay and in other cases it is preferable to serve both types of consumers. Would the same be true in oligopoly? It turns out to be hard to obtain closed form solutions when we assess the firm's incentive to deviate from an equilibrium with symmetric tariffs and market shares. That is, to decide whether the case where type 1 is served or excluded, respectively, is a stable equilibrium or not. We have therefore chosen to present some numerical examples to illustrate possible equilibrium outcomes.

To simplify, let us consider duopoly. Consider the two equilibrium candidates in pure strategies where the firms announce identical tariffs and serve the same customer base. In the first equilibrium candidate, both consumer types are served with a tariff $(A_{TT}^{12}, p_{TT}^{12})$ and each firm earns a profit per consumer π_{TT}^{12}. In the second equilibrium candidate, low demand consumers are excluded from making purchases and type 2 is served with a tariff (A_{TT}^{2}, c). Each firm earns a profit per consumer π_{TT}^{2}. For now we assume that the firms have equal market shares, *i.e.*, $n_a = n_b = \frac{1}{2}N$. Expected profit in each of the two possible equilibrium outcomes is

$$\Pi_{TT}^{12} = \frac{N}{2}\pi_{TT}^{12} \tag{16}$$

and

$$\Pi_{TT}^{2} = \frac{N}{2}\left(1-\lambda\right)\pi_{TT}^{2}. \tag{17}$$

If demand side heterogeneity is not too large, a duopoly is able to extract all surplus from type 1 when both consumer types are served. They would generate the same profit in each of the symmetric cases when $\Pi_{TT}^{12} = \Pi_{TT}^{2}$, *i.e.*, if

$$\lambda \equiv \lambda^{**} = \frac{1}{2} + \frac{3-4\theta_2+\sqrt{\left(4\theta_2^2-3\right)\left(4\theta_2^2-8\theta_2+5\right)}}{8(\theta_2-1)^2} \tag{18}$$

Since the duopoly extracts all surplus from type 1 provided that $\lambda \geq (2\theta_2 - \sqrt(2) - 1)/(2\theta_2 - 1)$ (which is smaller than λ^{**}), λ^{**} is also the monopolist cutoff value: If $\lambda < \lambda^{**}$ it serves only type 2 consumers, while if $\lambda > \lambda^{**}$ it serves both types of consumers.

Let us use λ^{**} as a reference point for our numerical examples. If $\lambda < \lambda^{**}$, demand side heterogeneity is large and we conjecture that the firms would tend to exclude type 1. Conversely, we conjecture that each firm would tend to serve both types of consumers if $\lambda > \lambda^{**}$. Note, however, that it is not at all obvious that the cutoff point is the same in duopoly as in monopoly. A monopoly can exclude type 1 consumers by designing a tariff they would never accept, while this is not possible in a duopoly. To find the Nash equilibrium, we check for unilateral deviations from each of those two possible equilibrium candidates, for different values of λ. Then we can compare the equilibrium outcome in duopoly with the equilibrium outcome in monopoly.

First, let us consider the equilibrium candidate where both firms serve only type 2 and the firms' tariffs are given by (A^2_{TT}, c). Type 1 is excluded and the firms extract the entire surplus from type 2 via the fixed fee, and $A^2_{TT} = V(c, \theta_2)$. The two firms split the base of type 2 consumers equally, $n_a = n_b = (1-\lambda)N/2$.

Would a unilateral deviation from an outcome where both firms serve only type 2 be profitable? One firm, say firm a, could deviate by setting a tariff that type 1 is just willing to accept and capture all type 1 consumers, λN. However, since low demand consumers derive nonnegative surplus, high demand consumers will derive strictly positive surplus by switching to low demand types' tariff. The deviating firm will then serve a mix of type 1 and type 2 consumers, it will serve all type 1 consumers and more than half of all type 2. Since firm a captures some of the high demand types as well, this tends to make such a deviation profitable. Let the deviating firm choose a strategy $(\tilde{Q}^{12}_{TT}, \tilde{A}^{12}_{TT})$, or equivalently charge a tariff $(\tilde{A}^{12}_{TT}, \tilde{p}^{12}_{TT})$ in order to maximize profit subject to individual rationality and firm b's strategy (Q^2_{TT}, A^2_{TT}). The problem is to maximize

$$\begin{aligned}\tilde{\Pi}^{12}_{TT}|\Pi^2_{TT} = {} & [N - \bar{n}_b]\,\tilde{A}^{12}_{TT} + \\ & \left(\tilde{p}^{12}_{TT} - c\right)[N\lambda\tilde{q}_1 + (N(1-\lambda) - n_b)\,\tilde{q}_2]\end{aligned} \tag{19}$$

subject to

$$V(\tilde{p}^{12}_{TT}, 1) \geq \tilde{A}^{12}_{TT} \tag{20}$$

$$V\left(\tilde{p}^{12}_{TT}, \theta_2\right) - V\left(\tilde{p}^{12}_{TT}, 1\right) = V\left(\bar{p}^2_{TT}, \theta_2\right) - V(c, \theta_2) \tag{21}$$

$$\tfrac{N}{2}(1-\lambda)\,q_2 \geq \bar{n}_b\bar{q}_2 \tag{22}$$

where $\tilde{q}_i = q_i\left(\tilde{p}^{12}_{TT}\right)$, $\ell = 1, 2$, $\bar{q}_2 = q_2\left(\bar{p}^2_{TT}\right)$, and $q_2 = q_2(c)$. Firm b, the non-deviating firm, will then lose type 2 consumers. This leads to a price reduction at firm b in order to restore individual rationality, the unit price falls to $\bar{p}^2_{TT} < c$. Since a unit price reduction in turn leads to an increase in a type 2 consumer's demand, $q_2(\bar{p}^2_{TT}) > q_2(c)$, the capacity supplied by firm b becomes insufficient to serve all type 2 consumers, and $\bar{n}_b < (1-\lambda)N/2$ is adjusted to restore market clearing at firm b. Formally, the individual rationality constraint (21) and the market clearing condition (22) jointly determine firm b's share of type 2 consumers as a function of firm a's strategy, $\bar{n}_b = \bar{n}_b(\bar{p}_2(\tilde{p}^{12}_{TT}))$.

Although firm a obtains lower profit per consumer when it deviates, it expands its market. When λ is low or θ_2 is high, the market expansion effect is less likely to cover the per-consumer-loss in profit. In that case there are few type 1 consumers to serve and expected profit per consumer is significantly lower when firm a deviates. Conversely, we expect that a deviation is profitable when demand side heterogeneity is low. For λ close to λ^{**} the expected revenue per consumer is identical and we therefore conjecture that it is profitable to deviate.

In Table 1 we have reported some numerical examples for $N = 100$ and $c = \frac{1}{2}$. Hence, $p^2_{TT} = \frac{1}{2}$ and $A^2_{TT} = V(c, \theta_2)$. The results in Table 1 confirm our conjecture. Note that when $\lambda < \lambda^{**}$, the monopolist would serve only type 2 consumers. This particular case therefore suggests that a Nash equilibrium in a duopoly where both firms serve only one type of consumers to a large extent coincides with the case where a monopolist prefers to serve only one type of consumers.

Table 1: *Deviation from a symmetric equilibrium where type 1 is excluded.*

θ_2	λ	λ^{**}	$\tilde{p}^{12}_{TT}$	$\tilde{A}^{12}_{TT}$	$\bar{p}^2_{TT}$	$\bar{n}_b$	$\bar{n}_b/[N(1-\lambda)]$	Π^2_{TT}	$\tilde{\Pi}^{12}_{TT}$
1.2	.2	.467	.618	.073	.374	33.9	.42	**9.8**	8.9
1.2	.4	.467	.580	.088	.365	25.1	.42	7.4	**9.7**
1.2	.47	.467	.568	.093	.362	22.1	.42	6.5	**10.0**
1.2	.8	.467	.522	.114	.351	8.2	.41	2.5	**11.5**
1.5	.4	.732	.714	.041	.261	24.2	.40	**15.0**	11.6
1.5	.7	.732	.597	.081	.214	11.7	.39	7.5	**11.5**
1.5	.74	.732	.583	.087	.209	10.1	.39	6.5	**11.6**
1.5	.9	.732	.531	.110	.189	3.8	.38	2.5	**12.1**
2	.4	.883	.927	.003	.156	24.4	.41	**33.8**	17.8
2	.7	.883	.699	.045	.037	11.5	.38	**16.9**	13.0
2	.8	.883	.630	.068	.003	7.5	.38	11.3	**12.4**
2	.9	.883	.564	.095	0	3.7	.37	5.6	**12.3**
3	.8	.959	.762	.028	0	7.5	.37	**31.3**	15.0
3	.92	.959	.603	.079	0	2.9	.36	12.5	**12.7**
3	.97	.959	.593	.107	0	1.1	.36	4.7	**12.4**

Second, let us consider the equilibrium candidate where both firms serve both types of consumers, where the firms' tariffs are given by $(A^{12}_{TT}, p^{12}_{TT})\,(> (0, c))$. Then type 2 enjoys positive surplus, and type 1 receives his reservation utility. Again, assume that the firms have equal market shares so that they each serve

$N/2$. Consider, again, a unilateral deviation by firm a, and keep the strategy for firm b fixed $(Q^{12}_{TT}, A^{12}_{TT})$.

In this case firm a can deviate by using one of two strategies. Firm a can aim for all type two consumers $N(1-\lambda)$, but leave them a positive surplus, hence setting $\tilde{A}^2_{TT} < V(c, \theta_2)$. Or, knowing that firm b has a limited capacity, firm a could act as a monopoly on any residual demand. He will then serve less than the pool of type 2 consumers $N(1-\lambda)$ but extract all surplus $\tilde{A}^2_{TT} = V(c, \theta_2)$.

Consider the first strategy. Firm a announces a tariff $(\tilde{A}^2_{TT}, c)$ that is strictly preferred by type 2 consumers. It will extract as much as possible from type 2 consumers via the fixed fee and will maximize

$$\tilde{\Pi}^2_{TT}|\Pi^{12}_{TT} = N\left(1-\lambda\right)\tilde{A}^2_{TT} \tag{23}$$

subject to

$$V\left(c, \theta_2\right) - \tilde{A}^2_{TT} \geq V\left(\bar{p}^{12}_{TT}, \theta_2\right) - A^{12}_{TT} \tag{24}$$

$$\frac{N}{2}\left(\lambda q_1 + \left(1-\lambda\right) q_2\right) \geq N\lambda\bar{q}_1 \tag{25}$$

where $q_i = q_i\left(p^{12}_{TT}\right)$, or $q_i = q_i\left(p^{12}_{UP}\right)$ if $A^{12}_{TT} = 0, (\ \ell = 1, 2)$, and $\bar{q}_1 = q_1\left(\bar{p}^{12}_{TT}\right)$. The unit price $\bar{p}^{12}_{TT}$ is adjusted to account for the fact that firm b is now left with only type 1 consumers instead of a mix of type 1 and type 2. Given that type 1 consumers receive exactly their reservation utility, the unit price that clears the market at firm b cannot exceed p^{12}_{TT}, (instead, type 1 consumers are rationed at firm b). Hence, $0 < \bar{p}^{12}_{TT} < \min\{p^{12}_{TT}, p^{12}_{UP}\}$. This restricts the fixed fee in (24), which in turn will restrict the profitability earned on type 2 consumers.

From (23) it would seem that a deviation is profitable when λ is small. However, when λ is small, $\bar{p}^{12}_{TT}$ is low as well in order to restore market clearing at firm b. Hence, $\tilde{A}^2_{TT}$ is also low in this case. The more intensely firms compete, either via a low fixed fee or a low unit price, the more binding is the restriction on $\tilde{A}^2_{TT}$. This suggests that in duopoly an outcome where both firms serve both types of consumers can be an equilibrium outcome in situations where a monopolist would have preferred to serve only one type of consumers. In our numerical example, the second effect always dominates the first and a deviation is never profitable. In Table 2 we have reported some numerical examples, again using $N = 100$, and $c = \frac{1}{2}$, hence $p^{12}_{TT} > c$, $A^{12}_{TT} = V\left(p^{12}_{TT}, 1\right)$.

The other possible deviation strategy in this situation was for firm a to act as a monopoly on any residual demand from type 2. This time, consider a deviation where firm a announces a tariff that extracts all surplus from type 2, $\left(V\left(c, \theta_2\right), c\right)$. Type 2 enjoys positive surplus by switching to firm b's tariff. Hence, type 2 consumers will crowd out type 1 consumers at firm b since capacity at firm 1 is insufficient to meet all demand. Firm a earns monopoly profit on each type 2 consumer it serves and aggregate profit is given by

$$\tilde{\Pi}^2_{TT}|\Pi^{12}_{TT} = \left[N\left(1-\lambda\right) - \bar{n}_b\right] V\left(c, \theta_2\right) \tag{26}$$

Table 2: *Deviation from a symmetric equilibrium* $(Q^{12}_{TT}, A^{12}_{TT})$*, the fixed fee in type 2's tariff is restricted.*

θ_2	λ	λ^{**}	p^{12}_{TT}	A^{12}_{TT}	$\tilde{p}^{2}_{TT}$	$\tilde{A}^{2}_{TT}$	$V(c,\theta_2)$	$\bar{p}^{12}_{TT}$	Π^{12}_{TT}	$\tilde{\Pi}^{2}_{TT}$
1.2	.2	.467	.66	.058	.5	-.417	.245	0	**6.9**	-33.4
1.2	.4	.467	.62	.072	.5	-.023	.245	.38	**6.6**	-1.4
1.2	.47	.467	.61	.078	.5	.055	.245	.47	**6.5**	2.9
1.2	.8	.467	.54	.106	.5	.133	.245	.54	**6.3**	2.7
1.5	.4	.732	.8	.020	.5	-.183	.5	.38	**7.1**	-11.0
1.5	.7	.732	.65	.061	.5	.194	.5	.64	**6.8**	5.8
1.5	.74	.732	.63	.069	.5	.190	.5	.63	**6.7**	4.9
1.5	.9	.732	.55	.101	.5	.150	.5	.55	**6.3**	1.5
2	.7	.883	.8	.020	.5	.154	1.125	.64	**7.1**	4.6
2	.8	.883	.7	.045	.5	.309	1.125	.69	**7.3**	6.2
2	.9	.883	.60	.080	.5	.225	1.125	.60	**6.5**	2.3
3	.8	.959	.9	.005	.5	.301	3.125	.69	**9.0**	6.0
3	.92	.959	.66	.058	.5	.445	3.125	.66	**6.9**	3.6
3	.97	.959	.56	.097	.5	.245	3.125	.56	**6.3**	0.7

where $\bar{n}_b$ is the number of type 2 consumers that can be served by firm b. Type 2 is indifferent between the two firms' tariffs when he receives zero surplus. Hence, the unit price in firm b's tariff must be adjusted in order to restore individual rationality for type 2, $\bar{p}^2_{TT}$.

$$V\left(\bar{p}^2_{TT}, \theta_2\right) - A^{12}_{TT} \geq 0 \tag{27}$$

$$\frac{N}{2}\left(\lambda q_1 + (1-\lambda)\, q_2\right) \geq \bar{n}_b \bar{q}_2 \tag{28}$$

$$N\,(1-\lambda) \geq \bar{n}_b \tag{29}$$

This time, firm b is left with type 2 consumers only, instead of with a mix of type 1 and type 2. Again, we would have thought it is profitable to deviate when λ is small. But now, when λ is small, the fixed fee A^{12}_{TT} is low. And therefore, type 2 consumers will gain considerably if they switch to firm b. Hence, the unit price $\bar{p}^2_{TT}$ is high and demand from type 2 is restricted. This means that $\bar{q}^2_{TT}$ is low and that $\bar{n}_b$ is large in order to restore market clearing.

In Table 3 we report some numerical examples, still using $N = 100$ and $c = \frac{1}{2}$. As shown, we find no examples where such a deviation is profitable. Again, the fact that the non-deviating firm has committed itself to sell a certain quantity acts as a constraint on the deviating firm's behaviour. If there are few type 2 consumers, the non-deviating firm would serve them all and the deviating firm would have no residual demand. If there are many type 2 consumers, the price per unit would be close to marginal costs. If so, there is a limited scope for the deviating firm to generate additional consumer surplus from type 2 by setting price per unit equal to marginal costs.

Table 3: *Deviation from a symmetric equilibrium* $(Q^{12}_{TT}, A^{12}_{TT})$*, acting as a monopoly on the residual demand from type 2.*

θ_2	λ	λ^{**}	p^{12}_{TT}	A^{12}_{TT}	$\bar{p}^{2}_{TT}$	$\hat{A}^{2}_{TT}$	$\bar{n}_b$	$N(1-\lambda)$	Π^{12}_{TT}	$\hat{\Pi}^{2}_{TT}$
1.005	.02	.196	.505	.123	.510	.128	50	98	**6.25**	6.1
1.05	.17	.168	.542	.105	.592	.151	55	83	**6.29**	4.3
1.1	.3	.292	.570	.093	.670	.180	58	70	**6.37**	2.1
1.2	.2	.467	.660	.058	.860	.245	74	80	**6.89**	1.6
1.2	.4	.467	.620	.072	.820	.245	60	60	**6.61**	0
1.2	.47	.467	.606	.078	.806	.245	53	53	**6.53**	0
1.2	.8	.467	.540	.106	.740	.245	20	20	**6.29**	0
1.5	.4	.732	.800	.020	1.3	.500	60	60	**8.5**	0
1.5	.7	.732	.650	.061	1.15	.500	30	30	**6.81**	0
1.5	.74	.732	.630	.069	1.13	.500	26	26	**6.67**	0
1.5	.9	.732	.550	.101	1.05	.500	10	10	**6.31**	0
2	.7	.883	.800	.020	1.80	1.125	30	30	**8.5**	0
2	.8	.883	.700	.045	1.70	1.125	20	20	**7.25**	0
2	.9	.883	.600	.080	1.60	1.125	10	10	**6.5**	0
3	.8	.959	.900	.005	2.90	3.125	20	20	**10.25**	0
3	.92	.959	.660	.058	2.66	3.125	8	8	**6.89**	0
3	.97	.959	.560	.097	2.56	3.125	3	3	**6.34**	0

4 Concluding remarks

Har (2001) have shown how we can extend the traditional Cournot model to a setting with not only a unit price, but also a fixed fee. They found that each firm sets a price per unit equal to marginal costs, and a positive fixed fee that approaches zero when the number of firms becomes large. Thus, we extend their model from one to two types of consumers. It turns out that the conclusions in Har (2001) are not robust to such an extension. Let us assume that both types are served. We then find that price per unit exceeds marginal costs and the fixed fee can be negative. If the firms can choose between a traditional Cournot pricing (a uniform price) and a two-part tariff, they may choose a uniform price.

We have also explored the case where the firms can choose whether to serve both types of consumers or only one type. It turns out that this case is difficult to solve analytically. We have therefore chosen to illustrate the possible equilibrium outcomes with numerical examples. The examples suggest that there might be multiple Nash equilibria. First, both firms serving only one type of consumers can be an equilibrium outcome. The numerical examples suggest that this equilibrium outcome to a large extent coincides with the cases where the monopolist chooses to serve only one type of consumers. Second, we find that both firms serving both types of consumers can be an equilibrium outcome for a large number of parameter values. In fact, we find no examples where the firms would deviate from such an outcome. The intuition is that the rival, non-deviating firm's given quantity acts as a constraint on the deviating firm's behavior. Although this is just a numerical example, it illustrates that there are instances where a duopoly serves both types of consumers while the monopoly would prefer to serve only one type.

Appendix

Calculation of pricing and profit

In the following we derive the firms' pricing in the case when they announce identical tariffs, as given in Lemma 1 and Lemma 2. Superscript 12 is used when both types are served (superscript 2 when type 1 is excluded) and k is an argument used to describe the number of active firms.

A.1 Both consumers are served

Pricing is given by Lemma 1. With two active firms we have

$$A^{12}(2) = \begin{cases} \frac{1}{8}(3-2\theta)^2 & 1 \leq \theta < \frac{1}{2}\left(\sqrt{2}+1\right) \\ \frac{5}{4}-\theta & \frac{1}{2}\left(\sqrt{2}+1\right) \leq \theta < \frac{5}{4} \\ 0 & \theta \geq \frac{5}{4} \end{cases} \tag{30}$$

$$p^{12}(2) = \begin{cases} \theta - \frac{1}{2} & 1 \leq \theta < \frac{5}{4} \\ \frac{1}{3}(\theta+1) & \theta \geq \frac{5}{4} \end{cases} \tag{31}$$

$$\pi^{12}(2) = \begin{cases} \frac{1}{8}+\frac{1}{2}(\theta-1)^2 & 1 \leq \theta < \frac{1}{2}\left(\sqrt{2}+1\right) \\ \frac{1}{2}\left(\frac{3}{2}-\theta\right) & \frac{1}{2}\left(\sqrt{2}+1\right) \leq \theta < \frac{5}{4} \\ \frac{1}{18}(2\theta-1)^2 & \theta \geq \frac{5}{4} \end{cases} \tag{32}$$

With three active firms we have

$$A^{12}(3) = \begin{cases} \frac{1}{4}\left(\frac{7}{2}-3\theta\right) & 1 \leq \theta < \frac{7}{6} \\ 0 & \theta \geq \frac{7}{6} \end{cases} \tag{33}$$

$$p^{12}(3) = \begin{cases} \theta - \frac{1}{2} & 1 \leq \theta < \frac{7}{6} \\ \frac{1}{4}\theta + \frac{3}{8} & \theta \geq \frac{7}{6} \end{cases} \tag{34}$$

$$\pi^{12}(3) = \begin{cases} \frac{3}{8} - \frac{1}{4}\theta & 1 \leq \theta < \frac{7}{6} \\ \frac{3}{64}(2\theta-1)^2 & \theta \geq \frac{7}{6} \end{cases} \tag{35}$$

With more than three firms we have

$$A^{12}(k) = \begin{cases} \dfrac{1-2k(\theta-1)}{4(k-1)} & 3 < k < \frac{1}{2(\theta-1)} \\ 0 & k \geq \frac{1}{2(\theta-1)} \end{cases} \tag{36}$$

$$p^{12}(k) = \begin{cases} \theta - \frac{1}{2} & 3 < k < \frac{1}{2(\theta-1)} \\ \dfrac{2\theta + k}{2(k+1)} & k \geq \frac{1}{2(\theta-1)} \end{cases} \tag{37}$$

$$\pi^{12}(k) = \begin{cases} \dfrac{3 - 2\theta}{4(k-1)} & 3 < k < \frac{1}{2(\theta-1)} \\ k\dfrac{(2\theta - 1)^2}{4(k+1)^2} & k \geq \frac{1}{2(\theta-1)} \end{cases} \tag{38}$$

In Proposition 1 the critical value λ^* solves the inequality $\frac{5}{4} - \lambda - (1-\lambda)\theta_2 \leq 0$ from (30). k^* solves the inequality $\frac{1-2k(\lambda+(1-\lambda)\theta_2-1)}{4(k-1)} \leq 0$ from (36).

A.2 Only type 2 is served

Pricing is given by Lemma 2. The unit price is always equal to marginal price, $p^2(2) = p^2(3) = c$, and the firms' profit per consumer is whatever they manage to capture via the fixed fee $A^2(k)$. With less than 3 active firms we have

$$A^2(2) = \pi^2(2) = A^2(3) = \pi^2(3) = \tfrac{1}{8}(2\theta_2 - 1)^2 \tag{39}$$

With more than 3 firms we have

$$A^2(k) = \pi^2(k) = \frac{(2\theta_2 - 1)^2}{4(k-1)} \tag{40}$$

A.3 Uniform Cournot price

When both types are served in a k-firm oligopoly and all firms charge a uniform price, we have

$$p^{12}_{UP}(k) = \frac{2\theta + k}{2(k+1)} \tag{41}$$

and

$$\pi^{12}_{UP}(k) = k\frac{(2\theta - 1)^2}{4(k+1)^2} \tag{42}$$

Proposition 2 can be verified by comparing the firms' profit in the two relevant cases. When A^{12}_{TT} is negative $\pi^{12}(k)$ (from (37)) is equal to or greater than $\pi^{12}_{UP}(k)$ (from (42)).

The monopolist's cut-off rate λ^{**} solves the equality $\pi^{12}(2) = \pi^2(2)$ in (32) and (39) respectively, given that the duopoly extracts all surplus from type 1 when both types are served.

References

(2001). "Quantity competition with access fees." *International Journal of Industrial Organization*, **19**, 345–373.

Calem, P. S. and D. F. Spulber (1984). "Multiproduct two part tariffs." *International Journal of Industrial Organization*, **2**, 105–115.

Castelli, F. and C. Leporelli (1993). "Critical mass of users versus mass of services in a multiproduct information service system." *Information Economics and Policy*, **5**, 331–355.

Economides, N. and S. Wildman (1995). "Monopolistic competition with two-part tariffs." Working Paper EC-95-10, New York University, Stern School of Business.

Lal, R. and C. Matutes (1994). "Retail pricing and advertising strategies." *Journal of Business*, **67**, 345–370.

Mandy, D. M. (1992). "Nonuniform Bertrand competition." *Econometrica*, **60**(6), 1293–1330.

Oi, W. Y. (1971). "A Disneyland dilemma: Two-part tariffs for a Mickey Mouse monopoly." *Quarterly Journal of Economics*, **85**, 77–96.

Oren, S., S. Smith and R. Wilson (1983). "Competitive nonlinear tariffs." *Journal of Economic Theory*, **29**, 49–71.

Scotchmer, S. (1985a). "Profit-maximizing clubs." *Journal of Public Economics*, **27**, 25–45.

Scotchmer, S. (1985b). "Two tier pricing of shared facilities in a free entry equilibrium." *RAND Journal of Economics*, **16**(4), 456–486.

Shmanske, S. (1991). "Two-part pricing in differentiated duopoly." *Journal of Institutional and Theoretical Economics*, **147**, 441–458.

Wilson, R. (1993). *Nonlinear Pricing.* Oxford University Press, Oxford.

Young, A. R. (1991). "Transaction costs, two-part tariffs, and collusion." *Economic Inquiry*, **29**, 581–590.