The Effects of Different Strategic Variables in Non-Cooperative Fisheries Games

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Abstract

In this paper we use stock size, harvest quantity and fishing effort, respectively, as strategic variables. We model a two agent (nations) non-cooperative fishery game, where the agents harvest a common fish stock. The planning horizon is infinite. The model is solved successively using one instrument at a time as the strategic variable in the game. The net present values of fishing and the escapement stock level from the three different models are compared with each other to show how the choice of variables affects the results. The choice of strategic variable is not a trivial one, as the results are shown to be sensitive to the discounting, the stock rate of growth, and the assumptions about the distribution of the fish in response to being harvested.

1 Introduction

In this paper we will look at the implications of choosing different strategic variables, harvest quantity, stock size, and fishing effort, in non-cooperative fisheries games. We will model a two agent (nations) game, where the agents harvest a common fish stock. The planning horizon is infinite. The model will be solved successively using one instrument at a time as the strategic variable in the game. The net present values of fishing and the escapement stock level from the three different models will be compared with each other to show how the choice of variables affects the results.

The choice of strategic variables, be it fishing effort, harvest rate or stock level, has rarely been discussed in the literature on fisheries and games. The choice of variables seems to be rather *ad hoc*. We have only come across two papers that address the question of choice of strategic variable and which try to analyze what this choice might imply.

Thomas L. Vincent (Vincent 1981) pointed out that different control variables can lead to different game solutions. Vincent (1981) used a prey-predator model due to May, Beddington, Clark, Holt, and Laws (1979) to analyze the vulnerability to extinction by comparing the equilibrium solutions under an effort harvesting and a rate harvesting program. The analysis demonstrated that solutions from a constant harvest quantity strategy will in many cases not secure the species against possible extinction, and an adjustment of the harvest levels may be necessary.

The second paper addressing the choice of strategic variables is by Hämäläinen and Kaitala (1982), where they analyze a fishery divided between two countries. The model is an extension of the harvest game model of Clark (1980) (Kaitala 1986). Each country manage the fishery as a sole-owner within their respective exclusive economic zones. They asked how should the sole-owner fleets choose their policy variables (strategic variables) in the negotiations? The two countries have three options in their choice of policy variables: stock level, harvest rate, and fishing effort. Of the possible steady state Nash equilibria the one where both countries have harvest rate as their policy variable produces the largest joint revenue flows and the largest stock levels. But, since perfect cooperation can not be guaranteed, the Nash solution of the game is that both countries choosing stock level, which has the lowest revenue flow of all the nine possible equilibria and has the lowest stock levels as well.

Choosing the harvest quantity as the strategic variable is comparable to Cournot competition (Tirole 1988). That is to say, each nation, in choosing its current harvest quantity, takes into account the other nations' harvest quantities, as the stock size and growth rate depend on the simultaneous actions of all nations involved in the fishery. Cournot competition here is analogous to Cournot oligopoly. A Cournot game is characterized by quantity constraints; the firms decide on the levels of production simultaneously. Once the level is decided, a firm cannot change its capacity in the short run. The solution each period is a Cournot solution to the game, but the fish stock responds to the quantity harvested by both nations and there may be a change in the size of the fish stock (Levhari and Mirman 1980). Eventually, a steady state in which both harvest quantity and the stock size are in an equilibrium is attained.

While the solution of a game with harvest quantity, or fishing effort, as the control variable is comparable to Cournot competition, choosing the escapement level, on the other hand, as the control variable is analogous to Bertrand competition. In a game with Bertrand competition the firms decide on setting the price rather than production. The production capacity is not constrained, and enables the firms to produce any quantity they choose; a price reduction enables them to sell more of their product.

With the escapement[1] level as the strategic variable, an underlying assumption is that the fishing fleet has a large enough capacity to be able to reduce the stock size from its initial level to the optimal escapement level in just one period of fishing, *i.e.* the initial period. In the following periods the harvest quantity and the escapement level stay constant. The ability to rapidly reduce the stock size, as implied by choosing escapement as strategic variable, makes the competition between the nations fiercer than Cournot competition. Fiercer competition implies that the stock will be depleted further than in a less competitive environment. The ability for a firm to change its price in response to its competitors' price setting makes Bertrand competition fiercer than Cournot competition (Tirole 1988).

How the players' strategy spaces are formulated is also an issue that should be addressed when modelling dynamic games. Two approaches have been adopted. The open loop solution, which assumes that commitment to a strategy extends over the entire future horizon, and the feedback solution, where the assumption is that no commitment at all is possible (Fudenberg and Tirole 1991). This choice can be crucial, and care should be taken

[1]Escapement: The stock left behind after fishing.

to choose a strategy space that is appropriate for the situation in question (Reinganum and Stokey 1985).

In this paper the strategy space is as an open loop solution game, where the strategy chosen at the start of the game is maintained throughout the planning horizon. This is not a very realistic assumption when there is nothing that guarantees that the initial strategy will be the optimal strategy if the optimization should be carried out again in the subsequent periods. However, it has been shown that under certain circumstances the open loop and the feedback solutions coincide (Dockner and Kaitala 1989), and that the quantitative difference between the two, in many cases, is not very large (Eswaran and Lewis 1985).

The situation we have in mind is one where firms may be unable or unwilling to change their strategic variable (effort or harvest) for some period of time. How long that period is will vary from case to case. We will not discuss that further here, but instead look at the stylized case where decisions are made once and for all.

The actual control variable used by managers of fisheries need not be the same as the strategic variable used to analyze the problem. Harvest rate and fishing effort are possible control variables, whereas stock size is not. However, using the stock size as a strategic variable does not require that it is the direct control variable (Kaitala 1986). The desired stock size can be reached by controlling the harvest quantity or fishing effort, *i.e.* harvest quantity and fishing effort are flexible from one period to another, as opposed to when they are fixed once and for all.

The structure of the paper is as follows. In the next section we model a fishery divided between two nations and the problem they face when stock size, harvest quantity, or fishing effort, respectively, is chosen as the strategic variable. We solve the model numerically, successively for the three strategic variables, and perform a sensitivity analysis in section 3. Finally, in section 4 we conclude.

2 The Model

Consider a fish stock where the stock growth depends on the stock size left in the sea after fishing has ceased. That is, the stock size at the beginning of the fishing season (t) is a function of the stock left to grow at the end of the previous season ($t-1$). Ignoring the natural mortality of the fish as long as the fishing season lasts, the seasonal harvest quantity, h_t, will equal the difference between the stock size at beginning of the season, $X(S_{t-1})$, and the stock size at the end of it, S_t. Taking the price of harvest landed, p, as given, the per period revenue is

$$R_t = p[X(S_{t-1}) - S_t]. \tag{1}$$

The instantaneous harvest production function will be specified as $h_t = ES_t^b$, where E stands for fishing effort, and S_t the stock size. The parameter b is the harvest elasticity with respect to the stock size, which takes the value 1 if the stock maintains a uniform distribution, and zero if the stock keeps its density constant when harvested. The total cost becomes $C = cE$, where c is a cost parameter. The instantaneous cost per unit harvested is $c_h = \frac{c}{S_t^b}$.

Total harvest costs can now be expressed as follows[2]

$$C_t = \int_{S_t}^{X(S_{t-1})} \frac{c}{u^b}\, du = \begin{cases} c[\log X(S_{t-1}) - \log S_t] & for \quad b = 1 \\ \frac{c}{1-b}[X(S_{t-1})^{1-b} - S_t^{1-b}] & for \quad 0 < b < 1 \\ c[X(S_{t-1}) - S_t] & for \quad b = 0 \end{cases} \tag{2}$$

where the case $0 < b < 1$ is for the intermediate values of the harvest elasticity with respect to the stock size, and log is the natural logarithm, with the number e as base.

The present value of the profit is

$$V = \sum_{t=0}^{\infty} (R_t - C_t)\delta^t, \tag{3}$$

where $\delta = \frac{1}{1+r}$ is the discount factor, r is the interest rate and X_0 is given.

We let the stock dynamics be described by the discrete variant of the logistic growth function

$$X(S) = S + aS[1 - S], \tag{4}$$

a is the intrinsic rate of stock growth. The carrying capacity usually associated with the logistic growth function is set equal to one.

[2]Since harvest is $H = X - S$, with X given initially in every period, $S \leq X$, $S = X - H$, $S_H < 0$ and $C(S) = C(S(H))$, and $H = EX$, $S = X(1-E)$, $S_E = -X$, the properties of the cost function are $C_H = C_S S_H \geq 0$ and $C_{HH} = -C_{SS} S_H = C_{SS} \geq 0$, and $C_E = -C_S S \geq 0$ (subscripts denote the derivatives).

2.1 Stock Size

By substituting Equation 4 into Equation 3, defining the control variables for nation i and the other nation, respectively, as the escapement level S and $\bar{S}$, the harvest quantity h^i and $\bar{h}$, and the fishing effort E^i and $\bar{E}$, where the bar above Nation Two's controls means that Nation One treats these as constants, we have three objective functions, one for each control variable, which can be maximized with respect to the respective control variable over an infinite planning horizon.

Nation i's problem with respect to the escapement level is

$$\max_S \left\{ \frac{p}{2}\left[X_0 - \bar{S}\right] + p\left[\bar{S} - S\right] - \frac{1}{2}\int_{\bar{S}}^{X_0} \frac{c}{u^b} du - \int_S^{\bar{S}} \frac{c}{u^b} du \right. \\ + \frac{1}{r}\left\{ \frac{p}{2}\left[S + aS[1-S] - \bar{S}\right] + p\left[\bar{S} - S\right] \right. \\ \left. \left. - \frac{1}{2}\int_{\bar{S}}^{S+aS[1-S]} \frac{c}{u^b} du - \int_S^{\bar{S}} \frac{c}{u^b} du \right\} \right\}, \tag{5}$$

with X_0 given.[3]

Each fishing season can be divided into two stages: In the first stage, both nations harvest the stock simultaneously, each catching one half of the total harvest, $X - \bar{S}$, sharing the costs. In the second stage, the assumption is that Nation One's fishermen continue harvesting while Nation Two's don't. Nation One decreases the stock further down to S, making additional profit for itself by doing so. The escapement level, S, should be chosen such that

[3] If the initial stock size is less than the optimal stock size it will be necessary to leave the stock unfished for one or more periods, until $X(S_{t-1}) > S^*$.

it maximizes the net present value of profits over all periods.

The stock size which maximizes nation i's present value of the stock given the other nation's harvest can be found by taking the first derivative of Equation 5 with respect to S. We will show this, and the first order conditions with respect to harvest quantity, and fishing effort as well, in the Appendix.

Both nations' problems are, by the assumption of symmetry, identical.[4] Finding the optimal escapement level for one, and substituting it, as S^* into the other's problem, $\bar{S}$ and S are equal to S^*, and the expression for each nation's net present value simplifies to

$$\begin{aligned} V^i(S^*) = \frac{p}{2}\Big[X_0 - S^*\Big] - \frac{1}{2}\int_{S^*}^{X_0} \frac{c}{u^b} du \\ + \frac{1}{r}\left\{\frac{paS^*[1-S^*]}{2} - \frac{1}{2}\int_{S^*}^{S^*+aS^*[1-S^*]} \frac{c}{u^b} du\right\}, \end{aligned} \tag{6}$$

$i = 1, 2$.

Both nations take an equal share of the total harvest and make the same profit. This is, however, not identical to the nations' objective functions, which are formulated as a non-cooperative game where each nation continues harvesting under the assumption that the other one has stopped, and by unilaterally increasing its catch, making extra profit. Nation Two does the same, so the final escapement level, S^*, is lower than if the two nations

[4]Focus of this analysis is the choice of strategic variable, the complicating cases of asymmetry in the nations costs and time preferences is left out. However, Hannesson (1997) analyzes the case where one nation has lower cost than the others. This could lead the low cost nation to exclude the high cost nations from the fishery altogether.

agreed on sharing the profit equally, which would be equivalent to maximizing Equation 6.

2.2 Harvest Quantity

The problem when we choose harvest quantity or fishing effort as the strategic variable follows the same structure as when the escapement level is the strategic variable, the difference is that we need to define the stock levels X, $\bar{S}$ and S as functions of the initial stock size, X_0 and the harvest quantities, h^i and $\bar{h}$ (or the fishing efforts, E^i and $\bar{E}$). The intermediate stock size, $\bar{S}$, is expressed as $X - 2\bar{h}$. $2\bar{h}$ is the total intermediary harvest quantity when both harvest simultaneously. The escapement level of the initial period is $S_0 = X_0 - h_0^i - \bar{h}$, and the stock size when fishing starts next period is $X_1 = S_0 + aS_0[1 - S_0]$, so the escapement level of this period becomes $S_1 = X_1 - h_1^i - \bar{h}$. This goes on until an escapement level is reached when the harvest quantity and the stock size are in equilibrium.

The problem of nation i with respect to the harvest quantity is now

$$\begin{aligned}\max_{h^i} \Bigg\{ ph^i - \frac{1}{2}\int_{X_0-2\bar{h}}^{X_0} \frac{c}{u^b}du - \int_{S_0(h^i)}^{X_0-2\bar{h}} \frac{c}{u^b}du \\ + \sum_{t=1}^{\infty} \delta^t \Bigg\{ ph^i - \frac{1}{2}\int_{X(S_{t-1})-2\bar{h}}^{X(S_{t-1})} \frac{c}{u^b}du \\ - \int_{S_t(h^i)}^{X(S_{t-1})-2\bar{h}} \frac{c}{u^b}du \Bigg\}\Bigg\},\end{aligned} \tag{7}$$

where X_0 is given, and $i = 1, 2$.

S^* is the equilibrium stock size which in this case maximizes the net present value, but before S^* is reached there are several S's that maximizes the present value. The time period when equilibrium is reached is denoted T.

When the optimal harvest quantity, $h^* = h^i = \bar{h}$, $i = 1, 2$, is found and substituted into, say, Nation One's problem, an expression of the nation's net present value simplifies to

$$\begin{aligned} V^i(h^*) = ph^* - \frac{1}{2}\int_{X_0-2h^*}^{X_0} \frac{c}{u^b} du \\ + \delta\left\{ph^* - \frac{1}{2}\int_{X_1-2h^*}^{X_1} \frac{c}{u^b} du\right\} \\ + \delta^2\left\{ph^* - \frac{1}{2}\int_{X_2-2h^*}^{X_2} \frac{c}{u^b} du\right\} \\ + \ldots\ldots\ldots\ldots\ldots\ldots \\ + \frac{\delta^T}{1-\delta}\left\{ph^* - \frac{1}{2}\int_{S^*}^{X(S^*)} \frac{c}{u^b} du\right\}, \end{aligned} \tag{8}$$

$i = 1, 2$.

This is, again, as in the case when the escapement level was the strategic variable. It is not the nation's objective function, but a result of the fact that with the assumption of symmetry, the nations end up choosing the same harvest quantity in equilibrium. Equation 8 is the resulting net present value function when the nations have solved the non-cooperative game.

2.3 Fishing Effort

When using the fishing effort, E,[5] as the strategic variable in the game, the harvest rate is a certain fraction of the stock size at the beginning of the period, $H(E) = EX$. The escapement level, $S(E)$ is $X[1-E]$, and the stock at the beginning of the next season is $X(S(E)) = S(E) + aS(E)[1 - S(E)]$. With two nations competing for the fish the escapement level becomes $S(E) = X[1 - E^i - \bar{E}]$, $i = 1, 2$, where E^i is a single nation's fishing effort, which it can control, and $\bar{E}$ is the fishing effort of the other nation which the first takes as given and treats as a constant in its own objective function.

Nation i's problem with respect to fishing effort is

$$\max_{E^i} \left\{ pE^i X_0 - \frac{1}{2} \int_{X_0[1-2\bar{E}]}^{X_0} \frac{c}{u^b} du - \int_{S_0(E^i)}^{X_0[1-2\bar{E}]} \frac{c}{u^b} du \right.$$
$$+ \sum_{t=1}^{\infty} \delta^t \left\{ pE^i X(S_{t-1}) - \frac{1}{2} \int_{X(S_{t-1})[1-2\bar{E}]}^{X(S_{t-1})} \frac{c}{u^b} du \right. \tag{9}$$
$$\left. \left. - \int_{S_t(E^i)}^{X(S_{t-1})[1-2\bar{E}]} \frac{c}{u^b} du \right\} \right\},$$

where $i = 1, 2$, and X_0 is given.

We find the optimal fishing effort E^* by assuming symmetry between the nations, such that $E^i = \bar{E}$ and substitute this back into Equation 9. Having found E^* we can substitute this into the objective function with respect to

[5] This is not quite what we usually mean by "fishing effort". E here is a fraction of the initial stock size, where $S = X[1 - E]$. With effort, Z, $S = X * exp[-Z]$, such that $Z = -\log[1 - E]$. The reason for formulating the fishing effort as fixed share of the initial stock size, rather than as a fixed fraction of a continuously declining stock size during the fishing season, is that it simplifies the problem.

fishing effort, and the net present value of the fishery for nation i becomes

$$
\begin{aligned}
V^i(E^*) = {} & pX_0E^* - \frac{1}{2}\int_{X_0[1-2E^*]}^{X_0} \frac{c}{u^b}du \\
& + \delta\Big\{pX_1E^* - \frac{1}{2}\int_{X_1[1-2E^*]}^{X_1} \frac{c}{u^b}du\Big\} \\
& + \delta^2\Big\{pX_2E^* - \frac{1}{2}\int_{X_2[1-2E^*]}^{X_2} \frac{c}{u^b}du\Big\} \\
& + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
& + \frac{\delta^T}{1-\delta}\Big\{pX(S(E^*))E^* - \frac{1}{2}\int_{S(E^*)}^{X(S(E^*))} \frac{c}{u^b}du\Big\},
\end{aligned}
\tag{10}
$$

$i = 1, 2$.

Having defined the problem with respect to stock size, harvest quantity and fishing effort, we are able to find numerical solutions to the strategic variables and compare the resulting stock sizes left after fishing has stopped and the net present value of the fishery for the three strategic variables in question.

3 Results

In this section we present the numerical solutions of the problems presented in the previous section. We start by choosing some values of the parameters; price, the initial stock size, the intrinsic rate of stock growth, costs and the discount rate, which we will call the benchmark set. The benchmark set values are shown in Table 1.

By setting the price, p, equal to one, we measure the value of the fish

Table 1: The parameters of the model: The benchmark set values

Parameter	Price	Initial stock	Growth rate	Discount rate	Costs
Symbol	p	X_0	a	r	c
Value	1	1	1	0.05	0.5

in the same units as the stock size. The initial stock size, X_0, equal to one means that the stock is in pristine condition when the fishery starts in the initial period. Growth differs from one population to another, and this affects the harvest. In order to account for this we will perform a sensitivity analysis where we solve the models for values of the intrinsic growth rate between one and 0.05. We will also present sensitivity analysis of the interest rate, r, and the cost parameter, c.[6]

3.1 Reference Solutions

Table 2 reports the results from the numerical solutions of the models, where the harvest elasticity with respect to the stock size, b, takes the value 1 and 0.1, respectively, using the benchmark parameter values in Table 1. The variables S, h^i, and E^i are the respective strategic variables of each model. The NPVs are the net present values found by substituting the respective optimal, non-cooperative, values of the strategic variables into Equations 6, 8, and 10. The escapement level refers to the size of the stock left in the sea after the fishing has stopped; harvest quantities and fishing efforts are the equilibrium harvest quantities and the fraction of the initial stock of each period fished, respectively.

[6]The break even stock size, *i.e.* the stock size at which price equals costs, is $[\frac{c}{p}]^{\frac{1}{b}}$, which coincides with the costs, c, when $b \equiv 1$ and approaches zero as b goes to zero.

Table 2: The results from the benchmark case.

	Variables	NPV	Escapement level	Harvest quantity	Fishing effort
	S	0.778	0.592	0.121	0.145
b=1.00	h^i	0.831	0.632	0.116	0.135
	E^i	0.819	0.619	0.118	0.138
	S	0.435	0.065	0.003	0.242
b=0.10	h^i	1.263	0.000	> 0.125	-
	E^i	1.221	0.344	0.113	0.198

From Table 2, for the case where $b = 1$, we see that a constant escapement, S, as the strategic variable in the game, produces the lowest net present value and the lowest escapement level of the three variables.[7] The constant harvest quantity strategy, h, has the highest economic value, as well as the highest escapement level. For the constant effort strategy, E, the NPV and escapement are in between the two others. For the case where $b = 0.1$, the order of the net present values is the same as when $b = 1$. Harvest quantity has the highest NPV, fishing effort the second highest, and the escapement strategy has the lowest NPV. However, the NPV for the escapement strategy is now reduced relative to when $b = 1$, while for harvest quantity and fishing effort it is higher. The escapement strategy's escapement level is very low, close to zero. The constant harvest quantity strategy, for $b = 0.1$, is above the maximum sustainable yield (MSY).[8] Continually harvesting more than MSY will, eventually, lead to the stock's extinction. The harvest quantity strategy, reported in the lower panel of Table 2, is only marginally larger than MSY,

[7] The harvest quantity presented in Table 2 is the individual nation's half of the total harvest quantity.

[8] The MSY is $\max_S\{aS[1-S]\}$, which is satisfied for S_{MSY} equal to 0.5, giving a MSY of 0.25 for an intrinsic growth rate, a, equal to one.

and the associated net present value is only marginally larger than the NPV produced if the harvest rate was identical to MSY. As the harvest elasticity approaches zero, both the escapement strategy and the fixed harvest rate strategy make the stock vulnerable to extinction. However, the constant fishing effort strategy turns out to be the most conservative strategy when the harvest elasticity approaches zero; with a relatively high escapement level and a profitable, sustainable, fishery. This is in accordance with what Vincent (1981) found; namely that an adjustment of the harvest level may be necessary in order to prevent extinction.

In Table 3 we present the results from the global optimization, which is equivalent to maximizing Equation (3) with escapement level, S, harvest quantity, h, or fishing effort, E, respectively, as the strategic variable.[9] However, the NPV, harvest quantity, and fishing effort reported are half of the total value, harvest, and effort, as if the resource is under a single management, as the NPV, harvest, and effort has to be shared between the two nations. This is done to make the comparison between a non-cooperative management (Table 2) and a cooperative management (Table 3) easier.

If the resource is managed as sole owner property and global, long term profits are maximized, the fixed escapement strategy is the most profitable as well as the most conservative strategy with respect to the escapement level. The constant harvest quantity strategy, on the other hand, is the least profitable and less conservative than the other strategies. This, which is true for both $b = 1$ and $b = 0.1$, is the opposite of what we got under a

[9] The global optimization is carried out deciding on a level of the strategic variable and keeping it fixed over the entire planning horizon. Thus, the results from the optimization will depend on the choice of strategic variable.

Table 3: Global optimization: Net present values and escapement levels for the strategic variables, respectively, stock size, harvest quantity, and fishing effort, using the benchmark values in Table 1

	Variables	**NPV**	**Escapement level**	**Harvest quantity**	**Fishing effort**
	S	0.852	0.683	0.108	0.120
b=1.00	h	0.846	0.677	0.109	0.122
	E	0.849	0.679	0.109	0.122
	S	1.310	0.498	0.125	0.167
b=0.10	h	1.263	0.000	> 0.125	-
	E	1.301	0.485	0.1249	0.170

non-cooperative management.

The relatively low NPV, under non-cooperative management and escapement level as strategic variable, may seem somewhat surprising; reaching the steady state after only one period from a pristine stock means that the profit earned in the initial period is high, while the equilibrium stock size is reached after 38 periods, for the harvest quantity strategy, and 21 periods, with fishing effort as strategic variable (for $b = 1$). Harvest rate and fishing effort as control variables we can think of as putting constraints on our decision making, locked by a constant harvest or effort. Profits in every period, except the initial one, are discounted, and even with a high initial profit the net present value from the game played with stock size as the strategic variable is the lowest of all three possible strategic variables.

A comparison between the initial profits from choosing either harvest rate or fishing effort as strategic variable, setting $b = 1$, and the initial profit from the game where stock size is the strategic variable, shows that the initial profit is, respectively 69 and 79 percent of the escapement strategy's initial profit; but from period one onwards the escapement strategy's profit is more than halved, relative to its initial profit. For harvest rate and fishing effort, on the other hand, the reduction in each period's profits is less pronounced, and after a few periods the harvest rate strategy has the highest per period profit. So, even though stock size as a strategic variable gives a high initial profit, the fierce competition implied when stock size is chosen as the strategic variable in the game forces the nations to reduce the stock size to such a low level that the initial gain is offset by the future losses from having to fish the stock at a low level. As the stock size is reduced, the cost of harvesting goes up at an increasing rate. If we are free to chose the optimal levels, a fixed harvest rate or a fixed fishing effort, as opposed to choosing a fixed stock size, does not necessarily mean that we are worse off.

Table 2 also shows that stock size as the strategic variable has the lowest escapement level, as well as the highest harvest rate and fishing effort in equilibrium. Harvest quantity as the strategic variable, on the other hand, produces the highest escapement level, and has the lowest equilibrium harvest rate and fishing effort. Fishing effort as the strategic variable has an intermediate escapement level, and equilibrium harvest rate and fishing effort, relative to stock size and harvest rate, although not very different from the results with harvest rate as the strategic variable. These results are as expected, given our previous discussion on the analogy between Cournot competition and choosing harvest quantity as the strategic variable, versus Bertrand competition and having stock size as the strategic variable. In addition, the results in Table 2 show another analogy between the different forms of competition known from the literature on industrial organization

(Tirole 1988) and choice of strategic variable in non-cooperative fishery games; choosing the stock size as the strategic variable not only reduces the escapement level relative to harvest quantity and fishing effort, it also reduces the value of the fishery, in spite of high initial profit.

3.2 Sensitivity Analysis

A value of the intrinsic rate of stock growth, a, equal to one is somewhat high for most of the economically important fish stock, it is therefore appropriate to perform a sensitivity analysis where we let the intrinsic growth rate vary between one and zero (in this case between one and 0.05), and report the escapement levels and the net present values from using, respectively, stock size, harvest rate, and fishing effort as the strategic variable. Moreover, the benchmark values of the discount rate and harvest costs, 0.05 and 0.5, respectively, must be said to be picked without any justification in the literature or from empirical evidence and also warrant sensitivity analysis'.

First, we vary the intrinsic growth rate in the non-cooperative solutions. Figure 1, left panel, shows the resulting optimal escapement levels for each of the strategic variables for values of the intrinsic growth rate between 0.05 and one, for the case $b = 1$. The ordering of the escapement levels is shown in Table 2, with the harvest quantity strategy producing the highest escapement level, fishing effort the second largest level and stock size has the lowest escapement level. The ordering is fairly robust to changes in the growth rate. For the escapement strategy, lowering (increasing) the intrinsic growth rate lowers (increases) the escapement level. The escapement levels for harvest

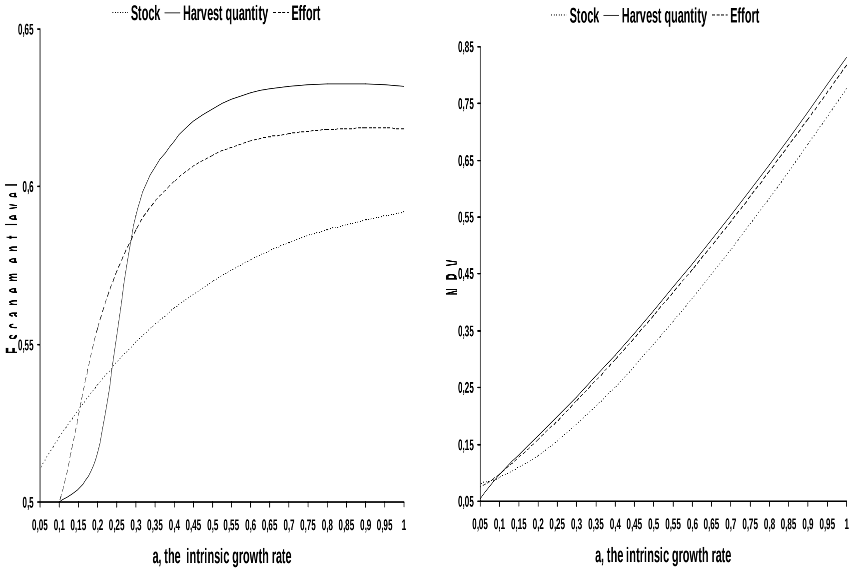

Figure 1: The effects of different values of the intrinsic growth rate, a, on the escapement levels (*left*), and the net present values (*right*)

quantity and fishing effort increase slightly as the growth rate decreases from one, reaching their maxima at values of the intrinsic growth rate equal to, respectively, 0.85 for the harvest quantity and 0.95 for the fishing effort strategy. For growth rates less than the ones producing the "maximum" escapement levels, the optimal, non-cooperative escapement levels decline with decreasing growth rates, giving the escapement level curves for harvest quantity and fishing effort in Figure 1, left panel, a humped shape. As the intrinsic growth rate falls below 0.3, the escapement levels for harvest quantity and fishing effort start to decline more rapidly. At a growth rate of about 0.28 the escapement levels of harvest quantity and fishing effort become equal. The escapement levels continue to fall; harvest quantity has an equal escapement level with stock size when the intrinsic growth rate is about 0.25, and fishing effort and stock size have the same escapement level when a is about 0.15. Both harvest quantity and fishing effort reach the break even escapement level of 0.5 at a growth rate equal to 0.1. Stock size, however, seems to be the most conservative strategic variable at low growth rates, with an escapement level above break even at a growth rate as low as 0.05.

The "paradox" that the harvest quantity and fishing effort strategies become more aggressive, (the escapement levels are lower than the escapement level with stock size as the strategic variable) when a is small, comes from the fact that when a is small, the stock cannot sustain a large, constant catch, without being driven to extinction. In order to avoid extinction, the harvest quantity must be reduced. A smaller harvest quantity means that it will take longer time to reach the final escapement level. The discounting makes the present values of catches caught in the future very low, and to compensate for this a tradeoff is made, setting the harvest quantity as high as possible, so that the break even level will become the escapement level in the long run. Otherwise, the harvest quantity would have be set at a very low level. This is also the case when a constant fishing effort is the strategic variable and the intrinsic growth rate is low; the stock size cannot sustain a high fishing effort level without being driven below the break even level.

Figure 1, right panel, shows how the net present value (NPV) is affected as the intrinsic growth rate, a, is gradually lowered from 1 to 0.05, holding the other parameters at the benchmark values in Table 1 and keeping the harvest elasticity with respect to the stock size equal to one. The net present values decline with falling growth rates. The harvest quantity strategy, which has the highest NPV, and the fishing effort strategy, which is number two when it comes to profitability, decline evenly as the intrinsic growth rate is lowered. The difference in NPV between harvest quantity and fishing effort is small compared to the difference in NPV between the escapement strategy and the net present values of harvest quantity and fishing effort. To begin with, the NPV for the escapement strategy declines evenly with falling growth rates, but at growth rates of about 0.3, or lower, the rate of change is reduced, and the net present values for stock, and harvest quantity and fishing effort, start to converge. The net present value for the harvest quantity strategy continues to decline as the growth rate approaches 0.05, the net present values for the escapement strategy and the fishing effort strategy level off.

Figure 2, left panel, shows how the escapement levels change in response to changes in the discount rate, r, between 0.05 and 0.3. At a discount rate of 0.05, which is our benchmark value the harvest quantity strategy has the highest escapement level, followed by fishing effort and the escapement strategy, respectively. The escapement levels fall with increasing discount rates, but the rate of decline varies for the different strategic variables. The escapement strategy, which has the lowest escapement level initially, declines with increasing discount rates at an even rate. So does the escapement for the fishing effort strategy, but at a higher rate, such that the escapement levels for stock and fishing effort are equal at a discount rate of about 18 percent. The escapement level for the harvest quantity strategy declines with increasing discount rates at a increasing rate, however. At $r = 0.15$ harvest quantity and fishing effort have the same escapement level. For discount rates above 20

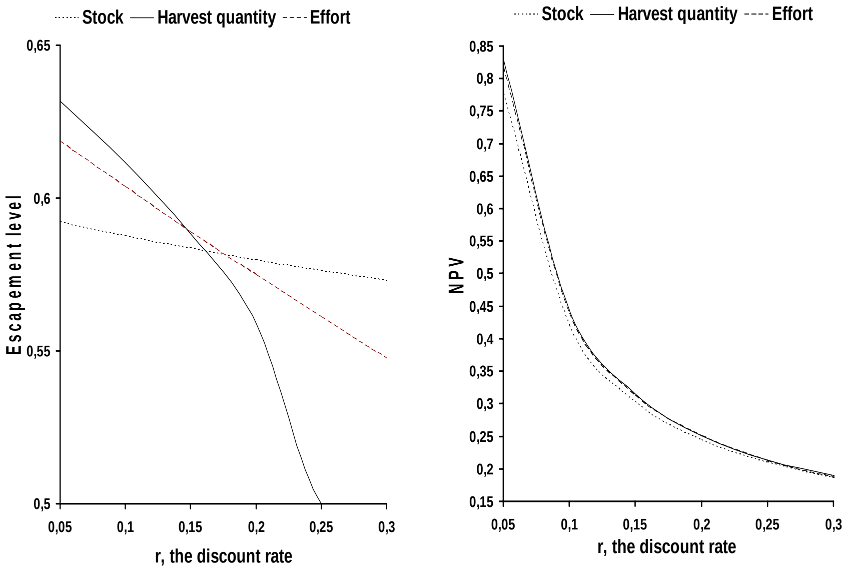

Figure 2: The effects of different values of the interest rate, r, on the escapement levels (*left*), and the net present values (*right*)

percent harvest quantity's escapement level rapidly approaches break even, while the escapement levels for stock and fishing effort continue to gradually decline.

Higher discount rates change the tradeoff between profits earned now, or in the near future, and profits earned further into the future, such that it becomes more attractive for the nations to increase fishing effort and catch, even if it means lower catch values in the future. The escapement level of the fixed harvest quantity strategy is the strategic variables that is most vulnerable to changes in the discount rate. The two other strategic variables, fixed escapement level and constant fishing effort, are more robust. For the escapement strategy this can be explained by the fact that the value of the large initial harvest increases relative to harvest in later periods due to higher discounting.

The comparison of the three strategic variables, escapement level, harvest quantity, and fishing effort, shows that especially the escapement level seems to be sensitive to changes in the intrinsic rate of stock growth, a, and the rate of discount, r. At low values of the intrinsic growth rate, or at high values of the discount rate, the escapement level when using a fixed harvest quantity as the strategic variable is driven down to the break even level, while the escapement level of the fixed escapement level strategy, which had the lowest escapement level of the strategic variable at higher values of a, lower values of r, is now the highest of the three, well above the break even level. Increasing the growth rate means that the productivity of the stock increases relative to the return on other assets, which is equal to r. On the other hand, an increase in the discount rate makes investments in the alternative assets relatively more profitable, and thus the nations would want to increase the catches and invest the earnings from doing so in the assets with the highest returns.

Figure 3 shows the result of varying the cost parameter, c, between 0.3 and 0.9, with the escapement levels and the net present values shown in left and right panel, respectively. Keeping the harvest elasticity with respect to the stock size equal to one, the cost per unit of effort, c, coincides with the break even stock size, is at the stock size where the price per unit equals the cost of catching it. The other parameters are held at their benchmark values.

What Figure 3 shows is that the higher the costs, the less is the difference between the escapement levels and net present values in models using harvest quantity, stock size or fishing effort, respectively, as the strategic variable.

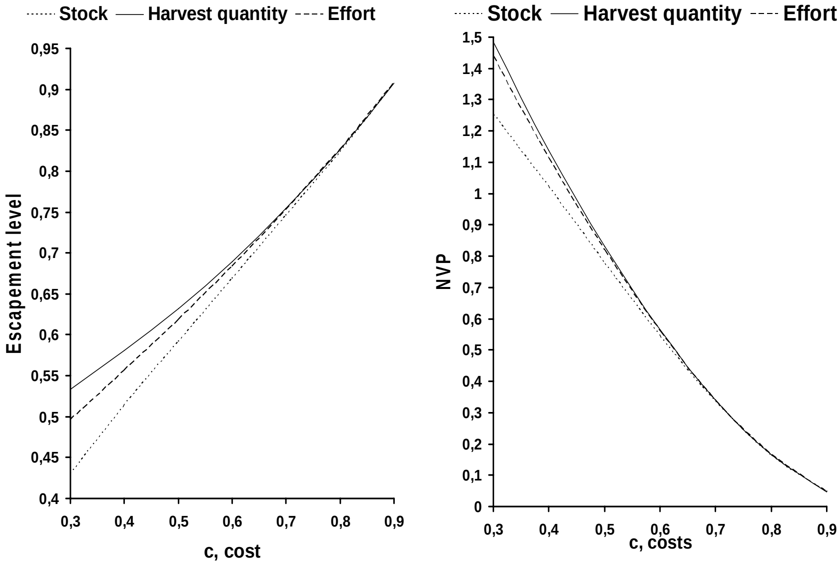

Figure 3: The effects of different cost levels, c, on the escapement levels (*left*), and the net present values (*right*)

In our benchmark case, with cost equal to 0.5, harvest quantity had the highest escapement level and NPV, followed by fishing effort and stock size had the lowest escapement level and NPV. This ordering is not changed by changing the cost, but escapement levels and net present values converge with increasing costs and diverge with lower costs. Since c is identical to the break even level, the stock size where price equals costs, when $b = 1$, this indicates that the choice of strategic variable is more important when we are dealing with fish stocks that are less protected from extinction economically. An example of this is fish species with schooling behaviour, where the increase in costs of fishing is less than proportional to reduction in the stock level. Therefore, the break even level becomes $[\frac{c}{p}]^{\frac{1}{b}}$, where $0 \leq b < 1$ is a measure of how sensitive the costs are to changes in the stock size.

4 Conclusions

The choice of strategic variable is not a trivial one, as the results obtained from using stock size, harvest quantity, and fishing effort, respectively, as strategic variables are shown to be sensitive to the discounting, and the stock rate of growth.

In a competitive environment the fixed escapement strategy becomes the least profitable, and it is the strategy with the lowest escapement level. The constant harvest quantity, on the other hand, is now the strategy that has the highest net present value and the highest escapement level. The net present values and the escapement levels are lower with non-cooperation than with full cooperation, and the fixed escapement strategy and the constant harvest quantity strategy management makes the stock vulnerable to extinction at lower stock elasticities.

The effects of the choice of strategic variable are to some extent sensitive to the level of the intrinsic growth rate and discounting. At lower growth rates the fixed escapement strategy becomes the strategy with the highest escapement level, while the escapement levels of harvest quantity and fishing effort tend towards the break even level. A high discount rate also increases the escapement strategy's escapement level, relative to harvest quantity and fishing effort.

The ranking with respect to economic value, however, seems to be less sensitive to changes in the intrinsic growth rate and discount. The strategy of a fixed harvest quantity has the highest net present value except at very low levels of the intrinsic growth rate, where the net present values of the strategic variables of fixed escapement and fixed fishing effort are higher than the net present value of the harvest quantity strategy. At higher rates of discount the differences in economic value are decreasing.

Changing the cost per unit of effort does not affect the ranking with respect to the escapement levels or the net present values. The escapement levels are increased, and the net present values are reduced, by higher costs. However, both the escapement levels and the net present values converge as the cost rises.

The analogies, suggested in the introduction, between, on the one hand, the Cournot oligopoly and a fixed harvest quantity strategy, or a fixed fishing effort, as the strategic variable, and Bertrand competition and constant escapement as the strategic variable, on the other, are confirmed by the comparison of escapement levels, and the net present values in this paper. The fiercer competition when stock size is the strategic variable of the game leads to a lower escapement level, and a lower net present value, relative to the escapement levels and net present values for harvest quantity and fishing effort as the strategic variables.

The assumptions about the distribution of fish in the sea, associated with its response to being harvested, are crucial. As the tendency to a uniform distribution is reduced and the harvest elasticity with respect to the stock size approaches zero the stock becomes more vulnerable to extinction. At stock elasticities close to zero a fishing effort strategy is the only strategic variable that sustain a profitable stock size in the long run.

APPENDIX

This shows the solution of the first order necessary conditions for the problems in Equations (5), (7), and (9), and gives a description how solutions for the strategic variables can be found.

The escapement level, S should be chosen such that it maximizes the net present value of profits over all periods. The first order necessary condition for this is

$$-p+\frac{c}{S}+\frac{1}{r}\left\{\frac{p}{2}\Big[1+a(1-2S)\Big]-p-\frac{c\Big[1+a(1-2S)\Big]}{2\Big[S+aS(1-S)\Big]}+\frac{c}{S}\right\}=0. \quad \text{(A1)}$$

Equation (A1) is a function of the parameters of the model and the escapement level, S, only, and independent of the initial stock size X_0 and the intermediate stock size $\bar{S}$. Equation (A1) can be solved for the optimal escapement level S^*.

In order to find the first order necessary condition that solves the problem when harvest quantity is the strategic variable (Equation (7)) we will have to take the derivative of the objective function with respect to the harvest quantity, h^i for all periods $t = 0, 1, 2, ..., T$, where T is the period where the stock size reaches its optimal level.

The first order necessary condition with respect to the harvest quantity

$$
\begin{aligned}
&p-\frac{c}{S_0}+\sum_{t=1}^{T-1}\left\{p-\frac{c}{S_t}\right\}\delta^t+\left\{p-\frac{c}{S_T}\right\}\sum_{t=T}^{\infty}\delta^t\\
&+\frac{cX_1'(S_0)}{2}\left[\frac{2}{S_1}-\frac{1}{X_1}-\frac{1}{X_1-2\bar{h}}\right]\left\{\frac{dS_0}{dh^i}\right\}\delta\\
&+\frac{cX_2'(S_1)}{2}\left[\frac{2}{S_2}-\frac{1}{X_2}-\frac{1}{X_2-2\bar{h}}\right]\left\{\frac{dS_1}{dh^i}\right\}\delta^2\\
&+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(A2)}\\
&+\frac{cX_{T-1}'(S_{T-2})}{2}\left[\frac{2}{S_{T-1}}-\frac{1}{X_{T-1}}-\frac{1}{X_{T-1}-2\bar{h}}\right]\left\{\frac{dS_{T-2}}{dh^i}\right\}\delta^{T-1}\\
&+\frac{cX_T'(S_{T-1})}{2}\left[\frac{2}{S_T}-\frac{1}{X(S_T)}-\frac{1}{X(S_T)-2\bar{h}}\right]\left\{\frac{dS_{T-1}}{dh^i}\right\}\sum_{t=T}^{\infty}\delta^t=0,
\end{aligned}
$$

where $i = 1, 2$.

The term $\{p - \frac{c}{S_t}\}\delta^t$ is the present value of the instantaneous marginal benefit obtained by increasing the harvest quantity, h_t^i, by one unit. The other term, $\frac{cX'(S_{t-1})}{2}[\frac{2}{S_t} - \frac{1}{X(S_{t-1})} - \frac{1}{X(S_{t-1})-2\bar{h}}]\{\frac{dS_{t-1}}{dh^i}\}\delta^t$, is the present value of the extra future marginal costs incurred by reducing the stock size by one unit.

From period T onwards, the stock size, $S_T = S_{T-1}$, and the harvest quantity, h^i, is in equilibrium and all the expressions in Equation (A2) can be treated as constants, for all $t \geq T$. The marginal benefits and costs terms are clearly constant for all $t \geq T$, and since $\frac{dS_{t-1}}{dh^i}$, is equal for all $t \geq T$, we can consider $\frac{dS_{T-1}}{dh^i}$ as a constant as well for all $t \geq T$. This leaves us with $\sum_{t=T}^{\infty} \delta^t$, which is an infinite geometric series, and since $0 < \delta < 1$ this series converges to $\frac{\delta^T}{1-\delta}$, which is a constant scalar.

The first order necessary conditions, Equation (A2), is a function of the initial stock size, X_0, and the others' harvest quantity, $\bar{h}$, as well as the parameters and T, and, thus, we cannot solve for h^i analytically. However, the optimal harvest quantity, h^*, can be solved numerically for a given initial stock size and making use of the fact that the terms in Equation (A2) are constant for $t \geq T$, and assuming symmetry between the nations.

Find the first order necessary conditions with respect to the fishing effort, the problem in Equation (9), by taking the derivative of nation one's fishing effort at every period $t = 0......T$. From period T onwards the stock and fishing effort are in equilibrium, and, thus, the harvest quantity stays constant.

The first order necessary condition with respect to fishing effort

$$
\begin{aligned}
& pX_0 - \frac{c}{1 - E^i - \bar{E}} \\
& + \sum_{t=1}^{T-1} \left\{ pX(S_{t-1}) - \frac{c}{1 - E^i - \bar{E}} \right\} \delta^t \\
& + \left\{ pX_T(S_{T-1}) - \frac{c}{1 - E^i - \bar{E}} \right\} \frac{\delta^T}{1 - \delta} \\
& + pE^i X_1'(S_0) \left\{ \frac{dS_0}{dE^i} \right\} \delta \\
& + pE^i X_2'(S_1) \left\{ \frac{dS_1}{dE^i} \right\} \delta^2 \\
& + ... \\
& + pE^i X_{T-1}'(S_{T-2}) \left\{ \frac{dS_{T-2}}{dE^i} \right\} \delta^{T-1} \\
& + pE^i X_T'(S_{T-1}) \left\{ \frac{dS_{T-1}}{dE^i} \right\} \frac{\delta^T}{1 - \delta} = 0,
\end{aligned}
\tag{A3}
$$

$i = 1, 2$.

The term $\{pX(S_{t-1}) - \frac{c}{1-E^i-\bar{E}}\}\delta^t$ is the present value of marginal benefit obtained by increasing the fishing effort E^i by one unit. The other, $pE^iX_t'(S_{t-1})\{\frac{dS_{t-1}}{dE^i}\}\delta^t$, is the net present value of the change in the marginal revenue from a one unit change in E^i.

Equation (9) is a function of X_0, $\bar{E}$, T and the parameters of the model, and is difficult to solve analytically. However, as was the case with the first order necessary condition for the harvest quantity strategy, the terms for $t \geq T$ are constants, which we can exploit to help us find numerical solution for E^i. The structure of solution method is similar to the one used solving for the harvest quantity.

References

Clark, C. W. (1980): “Restricted Access to Common-Property Fishery Resources: A Game-Theoretic Analysis,” in *Dynamic Optimization and Mathematical Economics*, ed. by P.-T. Liu, chap. 7, pp. 117–132. Plenum Press, New York, London.

Dockner, E. J., and V. Kaitala (1989): “On Efficient Equilibrium Solutions in Dynamic Games of Resource Management,” *Resources and Energy*, 11, 23–34.

Eswaran, M., and T. Lewis (1985): “Exhaustible Resources and Alternative Equilibrium Concept,” *Canadian Journal of Economics*, 18(3), 459–473.

Fudenberg, D., and J. Tirole (1991): *Game Theory*. The MIT Press, Cambridge, Massachusetts; London, England.

Hämäläinen, R. P., and V. Kaitala (1982): “A Game on the Choice of Policy Variables in a Dynamic Resource Management Game,” in *Proceedings of the 21st IEEE Conference on Decision and Control, Orlando, Florida, December 8-10, 1982*, ed. by I. C. on Decision, and Control, pp. 181–185, New York. IEEE, IEEE.

Hannesson, R. (1997): “Fishing as a Supergame,” *Journal of Enviromental Economics and Management*, 32, 309–322.

Kaitala, V. (1986): “Game Theory Models of Fisheries Management - A Survey,” in *Dynamic Games and Applications in Economics*, ed. by

T. Basar, chap. 10, pp. 252–266. Springer Verlag, Berlin, Heidelberg, New York, Tokyo.

Levhari, D., and L. J. Mirman (1980): "The Great Fish War: An example using a Dynamic Cournot-Nash Solution," *Bell Journal of Economics*, 11, 322–344.

May, R. M., J. R. Beddington, C. W. Clark, S. J. Holt, and R. M. Laws (1979): "Management of Multispecies Fisheries," *Science*, 205, 267–277.

Reinganum, J. F., and N. L. Stokey (1985): "Oligopoly Extraction of a Common Property Natural Resource: The Importance of the Period of Commitment in Dynamic Games," *International Economic Review*, 26, 161–173.

Tirole, J. (1988): *The Theory of Industrial Organization.* The MIT Press, Cambridge, Massachusetts; London, England.

Vincent, T. L. (1981): "Vulnerability of a Prey-Predator Model under Harvesting," in *Renewable Resource Management. Proceedings of a Workshop on Control Theory Applied to Renewable Resource Management and Ecology in Christchurch, New Zealand, January 7-11, 1980*, ed. by T. L. Vincent, and J. M. Skowronski, pp. 112–132, Berlin, Heidelberg, New York. Springer Verlag.