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Discussion paper

Pricing of an Interruptible Service with Financial Compensation and Rational Expectations

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Pricing of an Interruptible Service with Financial Compensation and Rational Expectations.*

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Abstract

This paper proposes a pricing framework that combines the occurrence of supply interruptions with financial compensations. Consumers post *ex ante* demands for a designated period. These demands are met if *ex post* supply capacity is sufficient. However, when supply is inadequate, all *ex ante* demands will be equi-proportionally rationed with compensation being paid for any unserved demand. Consumers posts their demands based on their expectations on the reliability of the supply system. The model is closed by imposing rational expectations. We identify that while a consumer's *ex ante* power demand will be decreasing in the power price and increasing in the compensation rate, it will be increasing when there is a mean-preserving spread in the riskiness of future supplies, provided the consumer is sufficiently prudent, i.e., when his coefficient of relative prudence exceeds two, and his coefficient of interruption aversion exceeds one. We also derive the welfare maximising price and show that when consumers are sufficiently prudent, pessimistic (equilibrium) expectations on the supply reliability warrant a higher price compared with a situation of supply adequacy.

Keywords: public utility pricing, rationing, rational expectations, electric utilities.

JEL-code: D42, D45, D81, H42, L94.

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1 Introduction

The need to restore stability whenever supply is inadequate poses a major challenge for the operators of electric power systems, power utilities and other related enterprises. This is because failure to maintain a consistent supply-demand balance within the power network could result in the breakdown of the entire system, spawning severe economic, social and political consequences.

For example on August 14, 2003, significant portions of the north-eastern United States and Canada were plunged into darkness, leaving three major cities, businesses and several millions of people paralysed by the lack of electric power for up to two days. The incident began with a relatively minor disturbance that generated some instability in a sub-unit of the power system (located in Ohio). The supply-demand balance in the local power system was however not restored in a timely manner, allowing the problem to escalate into a transborder crisis costing millions of dollars. This problem could however have been nipped in the bud if there was an effective mechanism for service interruptions or load management in place.

Load management is characterisable in two main forms. The first is price-based and involves the use of an elevated service price (or a spot price) that is communicated to the consumers in real time, to encourage them to voluntarily reduce their power demand whenever there is inadequate supply. The limitation to this approach is that available information on the price responsiveness of demand during such critical periods may be sketchy, rendering any predictions on the size of the realised demand reductions unreliable. Furthermore, the infrastructure that is required to communicate information on real-time prices to consumers remains non-existent in the majority of power systems.

The second form of load management is quantity-based and involves the involuntary rationing of demand whenever the power supply is inadequate. Such schemes may also involve the payment of financial compensations to consumers to placate them for the losses from being involuntarily rationed. Such an approach may however raise complications when consumers, in the intention of maximising their anticipated compensations, choose to adjust their demands pro-actively.¹

The problem of service pricing when there is supply (or demand) risk along with some form of load management being implemented, is closely re-

¹This would tend to occur in situations where the expected compensation depends on the consumer's demand level.

lated to an extensive literature on public utility pricing and capacity choice with a risky supply or stochastic demand (see for example Brown and Johnson (1969), Panzar and Sibley (1978), Sherman and Visscher (1978) and Coate and Panzar (1989)). A recurrent feature of such analyses is that the service being provided (which is in most cases electric power) is non-storable and the supply-demand balance must be maintained by interrupting (or "managing") the system's load, often involuntarily, to conform with available supply. These analyses differ from the "peak, off-peak" styled analyses in that they focus on load management during a single period, rather than across multiple periods having differing levels of realised demand. As a forerunner of the use of compensation to buttress load management, Serra (1997) examined service pricing under a scheme which required that consumers pay a basic rate for each unit actually consumed, and receive compensation for every unit of demand voluntarily reduced below their normal consumption level during a supply shortage.

Our objective in this paper is to examine service pricing under a simple load management scheme that financially compensates consumers whenever the inadequacy of power supply necessitates interruptions. In the presented set-up, each consumer pre-selects a (notional or *ex ante*) demand level that is fixed for a designated future period, at an assured price. In the event that the available power supply during this period is inadequate, all demands will be equally rationed in proportion to the available supply and compensation is then paid for any unserved demand. The task facing the power utility in this setting is two-fold. The first is to determine the service price and compensation levels that actualise its stated objective, which may be to either maximise expected profit or expected welfare, while the second is to implement supply interruptions whenever required, in order to avoid systemic breakdown.

An interesting question raised whenever supply interruptions are combined with the payment of interruption compensations concerns how consumers would be induced to adapt their consumption behaviours. Would they tend to boost their demands artificially when the possibility of receiving tangible compensations is dangled? Or would any identifiable change in their demands simply be a natural response to an increase in the risk of being interrupted? A related question is how consumer perceptions on the reliability of future power supplies or the extent to which such supplies would be inadequate (thereby necessitating interruptions) would influence demand behaviour and the *actual* reliability of such supplies? Also, how should the service prices be determined, and then what implications would these have for the scheme's implementability?

Intuitively, a stronger perception of the reliability of future supplies or a decrease in the risk of power supplies being inadequate, would tend to induce a higher *ex ante* power demand. Take for example a situation in which positive perceptions about the reliability of such future supplies would persuade consumers to make larger advance purchases of raw materials, to be used as part of a power-intensive production process. Hence, a higher perception of reliable supplies would presumably result in an increasing *ex ante* power demand.²

Our analysis adds precision to this insight by identifying that a consumer's *ex ante* power demand, while decreasing in the power price and increasing in the compensation rate, will also be increasing when there is a mean-preserving spread in the riskiness of future supplies, provided his coefficient of relative prudence is high enough (exceeds two) and his coefficient of interruption aversion exceeds one.³ This is because while an increasing riskiness associated with future supplies would ordinarily make future consumption less attractive and thereby reduce the *ex ante* demand, a high degree of prudence or precautionary behaviour would induce the consumer to increase his demand in order to secure the desired future consumption level. Furthermore, a sufficient condition for the consumer's demand to be increasing in the expected aggregate demand is that the coefficients of relative prudence and interruption aversion exceed three and one respectively.

We also identify that the mark-up of the optimal service price when there is supply uncertainty (with the risk of interruption), expressed as a percentage of the optimal price under capacity adequacy (without the risk of interruption), depends positively on both the degree of relative risk aversion and of relative prudence. An intuitive explanation for this is that a higher degree of prudence underscores the consumer's precautionary motive for boosting his *ex ante* demand when there is supply uncertainty and this 'demand expansion' effect must be mitigated by the mark-up.

We commence the analysis by presenting the model in the next section. Section 3 follows with an analysis of consumer behaviour, while section 4 examines optimal pricing by the power utility under an expected welfare maximisation objective.⁴ Section 5 concludes.

²See Coate and Panzar (1989) for a formalization that supports this line of intuition.

³We will later define a consumer's aversion to being rationed-off by the co-efficient of interruption aversion. The measure of prudence describes the extent to which a consumer's behaviour is influenced by a precautionary motive, or the extent to which he/she when faced with supply uncertainty, would be willing to take advance measures to mitigate the utility loss from being rationed-off.

⁴Profit maximisation is identifiably a special case of the welfare maximisation problem,

2 The Model

Consider a power system or a subset of such a system in which a load-management scheme with compensation payments is to be introduced. The service is provided by a monopolist power utility and no distinction is made between the supply of electric power and usage of the network. The cost of service is a constant b per unit (kilowatt hour).

There is a continuum of identical consumers with size mass normalised to 1. The representative consumer derives benefit from the consumption of electricity and a numéraire commodity and has a lump sum income m from which he pays for power supply that is priced at p per unit. His utility function takes the quasi-linear form:

$$U(x, Y) = u(x) + Y,$$

where x is the amount of electricity consumed and $Y = m - px$ is the income leftover for consumption of the numéraire. It is assumed that $u' > 0$, $u'' < 0$, $u''' > 0$ and $\lim_{x \rightarrow 0} u'(x) = +\infty$. For future reference, we define the coefficients of relative risk aversion and relative prudence w.r.t. electricity consumption as $R_r(x) \stackrel{\text{def}}{=} -\frac{u''(x)x}{u'(x)}$ and $P_r(x) \stackrel{\text{def}}{=} -\frac{u'''(x)x}{u''(x)}$, respectively.

Power supply is represented by a random variable T with the commonly known cumulative distribution function $F(T)$. The realisation of this variable is exogenous to the consumer. The adequacy of power supplies during the supply period is uncertain and the extent to which the aggregate power demand X^a exceeds the realised supply is the level of supply inadequacy or excess demand. It is commonly known that a positive level of supply inadequacy will result in power consumption being interrupted or rationed-off and a compensation being paid out for the undelivered part. The consumer's anticipation of the power supply being adequate is given by:

$$\Pr(T > X^e) = [1 - F(X^e)],$$

where X^e is the consumer's expectation of the aggregate demand. The assumed rationality of this expectation requires that $X^e = X^a$.

The scheme unfolds as follows: (1) The power utility announces the power price p and the compensation rate c in advance of the period. (2) The consumer chooses an *ex ante* power demand x . (3a) This demand will default as his uninterrupted power consumption if the realised supply is adequate. (3b) If the realised supply is inadequate then his demand will be interrupted

and emerges as the shadow cost of public funds tends to infinity.

using an equi-proportional rationing rule that curtails his power consumption down to $\frac{T}{X^e}x$. (4) The consumer is then compensated for any undelivered portion of his power demand at the rate c .

3 Consumer behaviour

The first question we are interested in answering is how the consumer would behave in choosing his *ex ante* power demand and to what extent this demand would be influenced by the prospect of his being compensated whenever interrupted? The consumer's *ex ante* demand for electricity is the solution to the following utility maximisation problem:

$$\max_x V = \int_0^{X^e} \left[u\left(\frac{T}{X^e}x\right) + m - p\frac{T}{X^e}x + c\left(1 - \frac{T}{X^e}\right)x \right] dF(T) + [u(x) + m - px][1 - F(X^e)]. \quad (1)$$

We will examine the solution under two expectations scenarios. In the first scenario, the consumer anticipates that the realised supply will be adequate and no interruptions will occur. In the second, he anticipates that the realised supply will be inadequate and that rationing with compensation will take place with some positive probability.

3.1 Case A: Adequate power supply

If adequate supply is anticipated then $\Pr(T > X^e) = 1$ meaning $F(X^e) = 0$. The consumer's problem then reduces to

$$\max_x V = u(x) + m - px,$$

where the solution x^* satisfies the necessary condition:

$$u'(x^*) = p. \quad (2)$$

Writing the individual demand as

$$x^* = x^*(p), \quad (3)$$

the aggregate demand, X^a , also amounts to $x^*(p)$ and the rational expectations equilibrium condition requires that $X^e = X^a = x^*(p)$. This can only be an equilibrium if $F[x^*(p)] = 0$.

Comparative statics in this scenario are simple:

$$\frac{\partial x^*}{\partial p} = \frac{1}{u''(x^*)} < 0, \text{ and } \varepsilon = \frac{\partial x^*}{\partial p} \frac{p}{x^*} = -\frac{1}{R_r(x^*)} < 0.$$

3.2 Case B: Inadequate power supply

However if a situation with inadequate supply is anticipated with some positive probability then $\Pr(T > X^e) < 1$ and $F(X^e) > 0$. The power demand \hat{x} that solves problem (1) must now satisfy the necessary condition:

$$\begin{aligned} & u'(\hat{x}) [1 - F(X^e)] + \int_0^{X^e} u' \left(\frac{T}{X^e} \hat{x} \right) \frac{T}{X^e} dF(T) \\ = & p \left\{ [1 - F(X^e)] + \int_0^{X^e} \frac{T}{X^e} dF(T) \right\} - \int_0^{X^e} c \left(1 - \frac{T}{X^e} \right) dF(T). \end{aligned} \quad (4)$$

Applying integration by parts on $\int_0^{X^e} \frac{T}{X^e} dF(T)$ and $\int_0^{X^e} u' \left(\frac{T}{X^e} \hat{x} \right) \frac{T}{X^e}$ allows us to rewrite this first order condition as⁵

$$\begin{aligned} u'(\hat{x}) = & p \left\{ 1 - \int_0^{X^e} \frac{F(T)}{X^e} dT \right\} - c \int_0^{X^e} \frac{F(T)}{X^e} dT \\ & - \int_0^{X^e} \frac{u' \left(\frac{T}{X^e} \hat{x} \right) [R_r \left(\frac{T}{X^e} \hat{x} \right) - 1]}{X^e} F(T) dT. \end{aligned} \quad (5)$$

We may write demand in this scenario as

$$\hat{x} = \hat{x}(p, c, X^e). \quad (6)$$

The second-order condition will be assumed to be satisfied. It is instructive to rewrite it as:⁶

$$SOC_{\hat{x}} = -\frac{\hat{u}'}{\hat{x}} \left\{ \hat{R}_r + \frac{1}{X^e} \int_0^{X^e} \frac{\tilde{u}'}{\tilde{u}'} \tilde{R}_r (\tilde{P}_r - 2) F(T) dT \right\}, \quad (7)$$

where $\hat{}$ above an expression means evaluation at \hat{x} , while $\tilde{}$ means evaluation at $\frac{\hat{x}T}{X^e}$. A sufficiently high relative prudence will thus ensure the second order condition to be verified.

⁵Integration by parts gives:

$$\begin{aligned} \int_0^{X^e} \frac{T}{X^e} dF(T) &= F(X^e) - \int_0^{X^e} \frac{F(T)}{X^e} dT \\ \int_0^{X^e} u' \left(\frac{T}{X^e} \hat{x} \right) \frac{T}{X^e} dF(T) &= u'(\hat{x}) F(X^e) - \int_0^{X^e} \frac{u' \left(\frac{T}{X^e} \hat{x} \right)}{X^e} \left[\frac{u'' \left(\frac{T}{X^e} \hat{x} \right) T}{u' \left(\frac{T}{X^e} \hat{x} \right)} \frac{T}{X^e} \hat{x} + 1 \right] F(T) dT \end{aligned}$$

where it is here assumed that $\lim_{T \rightarrow 0} u' \left(\frac{T}{X^e} \hat{x} \right) \frac{T}{X^e} = 0$.

⁶See equation (A.1) in the appendix for a derivation of the second-order condition.

3.3 Comparing the necessary conditions

Comparing the conditions (2) and (5) allows us to understand the consequences of pessimistic expectations on the *ex ante* power demand. Before doing so, it is useful to rewrite (5) by noting that $\frac{1}{X^e} \int_0^{X^e} \frac{T}{F(X^e)} dF(T) = \frac{E[T|T \leq X^e]}{X^e}$, is the consumer's expectation of the degree of available supply capacity, given that such capacity is regarded as inadequate. We may therefore define the expected degree of supply reliability as

$$r(X^e) \stackrel{\text{def}}{=} F(X^e) \frac{E[T | T \leq X^e]}{X^e} + [1 - F(X^e)],$$

which has the property that $r'(X^e) = -F(X^e) \frac{E[T|T \leq X^e]}{(X^e)^2} < 0$ which intuitively says that the expected degree of supply reliability will be decreasing in the level of expected demand. The first order condition in (5) may now be rewritten as

$$u'(\hat{x}) + \int_0^{X^e} \frac{u'(\frac{T}{X^e} \hat{x}) [R_r(\frac{T}{X^e} \hat{x}) - 1]}{X^e} F(T) dT = r(X^e) \cdot p - [1 - r(X^e)] \cdot c. \quad (8)$$

For a consumer with a logarithmic utility function for electricity, $R_r \equiv 1$ and the second *lhs* term vanish. The optimal demand for electricity then equates the consumer's marginal utility with a net marginal outlay determined by the price, and compensation, both discounted for the reliability rate. Since the reliability rate is less than one under pessimistic expectations, the consumer will boost his *ex ante* order of electricity, both because the expected price to be paid is lower, and because a compensation is paid out for each undelivered unit. This is a first reason for having $\hat{x} > x^*$.

The second reason stems from the second *lhs* term. This term accommodates the utility consequences of a marginal ordered unit in those states of the world where supply is insufficient. A benchmark value for relative risk aversion is 1 (see, e.g., Eeckhoudt, Ethner and Schroyen, 2007). If the consumer is highly risk averse w.r.t. electricity consumption ($R_r > 1$), the marginal utility of an extra unit ordered is enhanced, because it will boost the delivery when rationing takes place. This is a second reason for having $\hat{x} > x^*$.

3.4 Comparative statics at the individual level

In this section, we investigate how the consumer who anticipates interruptions will adjust his *ex ante* order of electricity due to marginal changes in

p , c , and X^e , as well as a marginal increase in the uncertainty surrounding the supply capacity.

Simple comparative statics on (8) shows that

$$\frac{\partial \hat{x}}{\partial p} = -\frac{\hat{x} r(X^e)}{\hat{u}' \left\{ \hat{R}_r + \frac{1}{X^e} \int_0^{X^e} \frac{\tilde{u}'}{\tilde{u}} \tilde{R}_r (\tilde{P}_r - 2) F(T) dT \right\}} < 0, \text{ and} \quad (9)$$

$$\frac{\partial \hat{x}}{\partial c} = \frac{\hat{x} [1 - r(X^e)]}{\hat{u}' \left\{ \hat{R}_r + \frac{1}{X^e} \int_0^{X^e} \frac{\tilde{u}'}{\tilde{u}} \tilde{R}_r (\tilde{P}_r - 2) F(T) dT \right\}} > 0. \quad (10)$$

Results (9) and (10) support the intuitive notion that the consumer's power demand will be decreasing in the power price and increasing in the compensation rate. The corresponding elasticity expressions are involved, but with logarithmic utility (for which $R_r(x) \equiv 1$ and $P_r(x) \equiv 2$) they reduce to:

$$\frac{\partial \hat{x}}{\partial p} \frac{p}{\hat{x}} = -\frac{r(X^e) p}{r(X^e) p - [1 - r(X^e)] c} = -r(X^e) p \hat{x} < 0, \text{ and} \quad (11)$$

$$\frac{\partial \hat{x}}{\partial c} \frac{c}{\hat{x}} = \frac{[1 - r(X^e)] c}{r(X^e) p - [1 - r(X^e)] c} = [1 - r(X^e)] c \hat{x} > 0, \quad (12)$$

since (8) reduces to $\frac{1}{\hat{x}} = r(X^e) p - [1 - r(X^e)] c$.

How will the consumer's power demand respond to a small change in the expected aggregate demand? We have

$$\frac{\partial \hat{x}}{\partial X^e} = -\frac{1}{X^e SOC_{\hat{x}}} \left\{ \int_0^{X^e} u'(\tilde{x}) \{ [R_r(\tilde{x}) - 1]^2 - R'_r(\tilde{x}) \tilde{x} \} \frac{F(T)}{X^e} dT - r'(X^e) X^e (p + c) + u'(\hat{x}) [R_r(\hat{x}) - 1] \right\}. \quad (13)$$

Clearly, the curly bracket term can take on any sign. To fix ideas, we may first consider the logarithmic utility function $u(x) = \ln x$. Then (13) reduces to

$$\frac{\partial \hat{x}}{\partial X^e} = \frac{-\hat{x} r'(X^e)(p + c)}{r(X^e) p - [1 - r(X^e)] c} = -\hat{x}^2 r'(X^e)(p + c) \geq 0, \quad (14)$$

where the second equality follows from (8). In elasticity terms, this gives

$$\frac{\partial \hat{x}}{\partial X^e} \frac{X^e}{\hat{x}} = \frac{-X^e r'(X^e)(p + c)}{r(X^e) p - [1 - r(X^e)] c} = \frac{(p + c) [r(X^e) + F(X^e) - 1]}{r(X^e) p - [1 - r(X^e)] c}, \quad (15)$$

so that

$$\frac{\partial \hat{x}}{\partial X^e} \frac{X^e}{\hat{x}} \geq 1 \iff \frac{1 - F(X^e)}{F(X^e)} \leq \frac{c}{p}. \quad (16)$$

Thus with a small compensation rate, the elasticity of individual demand w.r.t. expected aggregate demand will fall short of unity.

Relaxing the logarithmic utility assumption and reverting to the general result means we must now sign the numerator of (13) with the second and third terms present. Re-writing the expression within the curly brackets of the third term as $1 + [P_r(\bar{x}) - 3] R_r(\bar{x})$, it is inferable that a relative prudence larger than 3 and a relative risk aversion exceeding one are sufficient conditions for the consumer to order more *ex ante* when his expectation about the level of aggregate demand increases.

Finally, we examine the effect of a marginal increase in supply uncertainty or in the risk of interruptions on the consumer's demand through a mean preserving spread in the probability distribution for supply. For this purpose, we introduce a new probability distribution for the supply capacity, $G(\cdot)$ that has the same mean as $F(\cdot)$ but second order stochastically dominates the latter. We then define the probability distribution

$$H(T, \theta) \stackrel{\text{def}}{=} (1 - \theta) F(T) + \theta G(T), \quad (17)$$

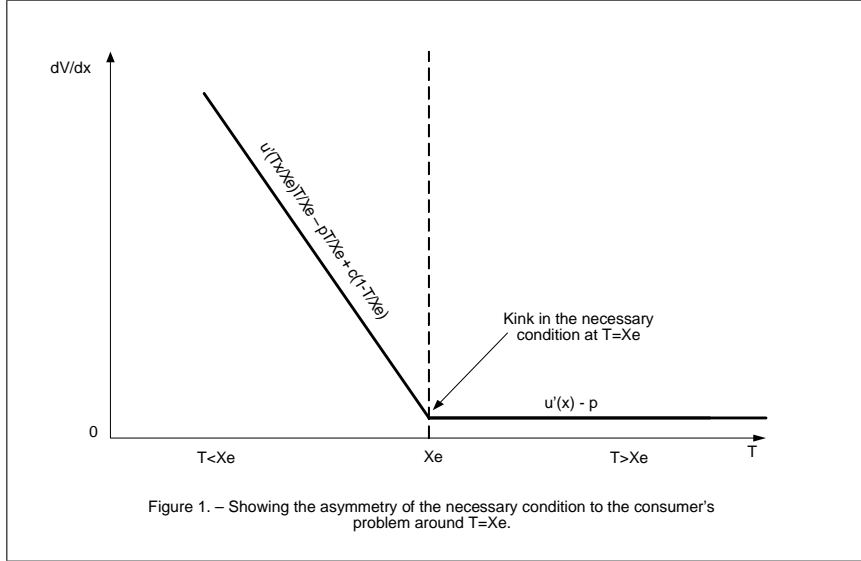
so that a marginal increase in θ can be considered as a marginal increase in risk. By the definition of a mean preserving spread (see Rothschild and Stiglitz (1970)), we must have that

$$\int_0^\infty H_\theta(T, \theta) dT = 0, \text{ and} \quad (18)$$

$$\int_0^z H_\theta(T, \theta) dT \geq 0; \forall z \in [0, \infty). \quad (19)$$

The important observation is that the consumer's expected marginal utility behaves asymmetrically around $T = X^e$ where it displays a kink.⁷ This is intuitive, as in situations with $T > X^e$ no interruption occurs and the necessary condition is simply the marginal utility from power consumption less the power price. But with $T < X^e$, the occurrence of interruptions implies that the necessary condition be adjusted to reflect the rationed demand and compensation payments. A mean preserving spread will thus affect the expected net marginal benefit of an *ex ante* power demand only to the extent that it affects the likelihood of those states in which the consumer anticipates to be rationed. This is presented in the figure below.

⁷The effect of a kink in the payoff function was first discussed by Kanbur (1982).



We may now rewrite the necessary condition in (5) as:

$$u'(\hat{x}) - p + \int_0^{X^e} \left\{ u' \left(\frac{T}{X^e} \hat{x} \right) \left[R_r \left(\frac{T}{X^e} \hat{x} \right) - 1 \right] + p + c \right\} \frac{H(T, \theta)}{X^e} dT,$$

and then differentiate completely to obtain:

$$\frac{\partial \hat{x}}{\partial \theta} (-SOC_{\hat{x}}) = \int_0^{X^e} \left\{ u' \left(\frac{T}{X^e} \hat{x} \right) \left[R_r \left(\frac{T}{X^e} \hat{x} \right) - 1 \right] + p + c \right\} \frac{H_{\theta}(T, \theta)}{X^e} dT, \quad (20)$$

which upon using partial integration gives

$$\begin{aligned} \frac{\partial \hat{x}}{\partial \theta} (-SOC_{\hat{x}}) &= \{ u'(\hat{x}) [R_r(\hat{x}) - 1] + p + c \} \int_0^{X^e} H_{\theta}(T, \theta) dT \quad (21) \\ &+ \int_0^{X^e} u' \left(\frac{T}{X^e} \hat{x} \right) R_r \left(\frac{T}{X^e} \hat{x} \right) \left[P_r \left(\frac{T}{X^e} \hat{x} \right) - 2 \right] \frac{1}{T} \left(\int_0^T H_{\theta}(S, \theta) dS \right) dT. \end{aligned}$$

The conditions for a mean preserving spread imply that the two underlined terms are strictly positive. The standard effect of an increase in risk on the control variable \hat{x} is given by the second *rhs* term. If the coefficient of relative prudence exceeds 2, the consumer will place a higher order. This result is reminiscent of the analysis of precautionary savings behaviour: if the rate of return to savings becomes more risky, the consumer will increase the amount saved if and only if his relative prudence exceeds 2 (this result dates back to Leland (1968); a modern account is found in Eeckhoudt, Gollier and Schlesinger (2005): 98-99). Prudence needs to be high enough to place a

higher order because on the one hand a more risky distribution makes the consumption of electricity less attractive, but on the other hand, the increase in risk makes the consumer more precautionous.

In addition to the standard effect, there is a second effect at work that is represented by the first *rhs* term of (21). If the consumer were very pessimistic about the adequacy of supply capacity, for example with X^e very large or tending towards $+\infty$, then the underlined term would vanish by condition (18). Otherwise, a sufficient condition for the first term to be positive is that $R_r(\hat{x})$ exceeds 1.

With logarithmic utility, (21) reduces to

$$\frac{\partial \hat{x}}{\partial \theta} = \frac{(p+c) \int_0^{X^e} H_\theta(T, \theta) dT}{r(X^e)p - [1 - r(X^e)]c} > 0, \quad (22)$$

showing that an increase in risk unambiguously increases demand.

3.5 Comparative statics under rational expectations

Prior to now we have treated the expected aggregate demand as an exogenously defined variable. But with rational expectations, these expectations are endogenous and need to be confirmed in equilibrium. Imposing $X^e = \hat{x}(p, c, X^e)$ and the stability condition $|\frac{\partial \hat{x}}{\partial X^e}| < 1$, we have

$$\frac{\partial \hat{x}}{\partial p} \Big|_{\text{eqb}} = \frac{\partial X^e}{\partial p} = \frac{\frac{\partial \hat{x}}{\partial p}}{[1 - \frac{\partial \hat{x}}{\partial X^e}]} \quad \text{and} \quad \frac{\partial \hat{x}}{\partial c} \Big|_{\text{eqb}} = \frac{\partial X^e}{\partial c} = \frac{\frac{\partial \hat{x}}{\partial c}}{[1 - \frac{\partial \hat{x}}{\partial X^e}]} \quad (23)$$

Making use of $\frac{\partial X^e}{\partial p}$, $\frac{\partial X^e}{\partial c}$ and $\frac{\partial \hat{x}}{\partial X^e}$, we obtain

$$\frac{\partial \hat{x}}{\partial p} \Big|_{\text{eqb}} = - \frac{\hat{x} r(\hat{x})}{u'(\hat{x}) \left\{ [1 - F(\hat{x})] [1 - R_r(\hat{x})] - \frac{p}{u'(\hat{x})} [1 - F(\hat{x})] + \frac{c}{u'(\hat{x})} F(\hat{x}) \right\}} < 0, \quad (24)$$

and

$$\frac{\partial \hat{x}}{\partial c} \Big|_{\text{eqb}} = \frac{\hat{x} [1 - r(\hat{x})]}{u'(\hat{x}) \left\{ [1 - F(\hat{x})] [1 - R_r(\hat{x})] - \frac{p}{u'(\hat{x})} [1 - F(\hat{x})] + \frac{c}{u'(\hat{x})} F(\hat{x}) \right\}} > 0. \quad (25)$$

Note that the stability assumption ensures that the denominator is positive. Therefore, also in equilibrium, the price and compensation rates have the expected sign. With logarithmic utility, these marginal effects, in elasticity form, reduce to

$$\left. \frac{\partial \hat{x}}{\partial p} \right|_{\text{eqb}} = - \frac{r(\hat{x})}{[1 - F(\hat{x})] - \frac{c}{p} F(\hat{x})} < 0, \quad (26)$$

and

$$\left. \frac{\partial \hat{x}}{\partial c} \right|_{\text{eqb}} = \frac{c}{p} \frac{[1 - r(\hat{x})]}{[1 - F(\hat{x})] - \frac{c}{p} F(\hat{x})} > 0. \quad (27)$$

In absolute value, the price and compensation elasticities are thus larger in equilibrium than at the individual level. The reason is the multiplier effect of expectations.

4 Welfare maximising pricing

We now study the choice of p and c that maximise social welfare. Social welfare is defined as the sum of expected consumer surplus V and expected profit, while accounting for the fact that any loss which the public firm makes has to be financed through distortionary taxation on other economic activities, or alternatively, that any profit allows for a reduction in such taxation costs. Denoting the shadow cost of public funds by $\lambda > 0$, the problem of the regulator is then

$$\max_{p \geq 0, c \geq 0} W \stackrel{\text{def}}{=} V + (1 + \lambda)E\pi, \quad (28)$$

where V is the consumer's expected utility from (1) and $E\pi$ is the Utility's expected profit defined as follows:

$$E\pi \stackrel{\text{def}}{=} (p - b) \left[x - \int_0^x \left(1 - \frac{T}{x}\right) x dF(T) \right] - \int_0^x c \left(1 - \frac{T}{x}\right) x dF(T). \quad (29)$$

Profit maximisation is a special case of (28) where $\lambda \rightarrow \infty$.

4.1 Case A: Adequate power supply

Recognising that $x^* = x^*(p, X^e)$ means that the Utility's problem becomes:

$$\max_{p \geq 0} W = u(x) + m - px + (1 + \lambda)(p - b)x \quad (30)$$

and with $u'(x^*) = p^*$ the optimal price, p^* , necessarily satisfies

$$\frac{\partial W}{\partial p} = \lambda x(p^*) + (1 + \lambda)(p^* - b)x_p^* = 0. \quad (31)$$

Writing $\varepsilon^* = \frac{d \log x^*}{d \log p}$, this first order condition results in the mark-up rule

$$p^* \left[1 - \frac{\lambda}{1 + \lambda} \frac{1}{\varepsilon^*} \right] = b. \quad (32)$$

Note that as $\lambda \rightarrow +\infty$, (32) reverts to the expected profit maximising result.

4.2 Case B: Inadequate power supply

Recognising x as defined in (6) and the rational expectations equilibrium $\hat{x} = \hat{x}(p, c, \hat{x})$ in (28) gives the necessary conditions:

$$\begin{aligned} \frac{\partial W}{\partial p} &= \left[u'(\hat{x}) \hat{x}_p - \hat{x} - p \hat{x}_p \right] [1 - F(\hat{x})] + c \hat{x}_p F(\hat{x}) - E[T | T \leq \hat{x}] F(\hat{x}) \\ &\quad + (1 + \lambda) \{ \hat{x} r(\hat{x}) + \hat{x}_p [(p - b) [1 - F(\hat{x})] - c F(\hat{x})] \}, \end{aligned} \quad (33)$$

$$\begin{aligned} \frac{\partial W}{\partial c} &= \left[u'(\hat{x}) \hat{x}_c - p \hat{x}_c \right] [1 - F(\hat{x})] + c \hat{x}_c F(\hat{x}) + \hat{x} F(\hat{x}) - E[T] F(\hat{x}) \\ &\quad - (1 + \lambda) \{ (F(\hat{x}) \hat{x} - F(\hat{x}) E[T]) - \hat{x}_c [(p - b) [1 - F(\hat{x})] - c F(\hat{x})] \}, \end{aligned} \quad (34)$$

and using the demand-price derivative from (24) in (33) and then re-arranging gives:

$$u'(\hat{x}) \left[1 - \frac{\lambda}{1 + \lambda} R_r(\hat{x}) \right] = b, \quad (35)$$

which implicitly defines the welfare maximising \hat{p} .

A necessary condition for a finite price \hat{p} to maximise profits is that the square bracket term is positive. This puts an upper bound on the coefficient of relative risk aversion given as:

$$R_r < \frac{1 + \lambda}{\lambda}.$$

When the aim is to maximise profits (i.e. $\lambda \rightarrow +\infty$), R_r is bounded away from 1. If $\lambda = 0.2$ (0.3), then R_r must not exceed 6 ($4\frac{1}{3}$).

Recognising (25) in (34) with $\hat{c} > 0$ yields exactly the same condition as (35). This suggests that one of the two instruments is redundant. The reason is of course the quasi-linear nature of the cardinal utility function. This means that the consumer is risk neutral w.r.t. the *numéraire*. Even though he does not like interruptions in the supply of electricity, it suffices to compensate the consumer for the expected level of interruption, and this

is equally well carried out through the *ex ante* price p . In the remainder of the discussion we will therefore normalise the compensation rate to zero.

Since $\varepsilon(x^*) = \frac{u'(x^*)}{u''(x^*)x^*} = -\frac{1}{R_r(x^*)}$, the necessary conditions in the adequate and inadequate supply cases are identical, meaning that the optimal *ex ante* ordered amount of electricity is the same, no matter whether the supply capacity is regarded as adequate or not. A consequence is that the price \hat{p} needs to be chosen such that it implements the quantity x^* also under an inadequate supply capacity.

Earlier, we concluded that for a given price, the consumer will place a higher *ex ante* order when he anticipates inadequate supply, relative to when he anticipates adequate supply. Hence \hat{p} needs to exceed p^* to choke off the *ex ante* demand, and to equalise the demand in both cases.

Using the consumer's necessary condition (5) with $\hat{c} = 0$, we may ask what the price should be in order for him to place an order x^* under supply inadequacy. The answer is:

$$\hat{p}(x^*) = \frac{u'(x^*) - \int_0^{x^*} u'(T) [1 - R_r(T)] \frac{F(T)}{x^*} dT}{r(x^*)}. \quad (36)$$

To gain a better understanding of this expression, taking a linear expansion of $u'(T) [1 - R_r(T)]$ around $T = x^*$ gives:

$$u'(T) [1 - R_r(T)] \simeq u'(x^*) \left\{ [1 - R_r(x^*)] + R_r(x^*) [2 - P_r(x^*)] \frac{x^* - T}{x^*} \right\}, \quad (37)$$

and allows us to approximate the second numerator term on the RHS of (36) as:

$$\int_0^{x^*} u'(T) [1 - R_r(T)] \frac{F(T)}{\hat{x}} dT \simeq \left\{ u'^* [1 + R_r^*(1 - P_r^*)] \left[1 - \frac{E(T|T < x^*)}{x^*} \right] - R_r^* [2 - P_r^*] \frac{1}{2} \left[1 - \frac{E(T^2|T < x^*)}{x^{*2}} \right] \right\} F^*, \quad (38)$$

where $*$ denotes an evaluation at x^* . And since $u'^* = p^*$, (36) may now be rearranged as Since $u'^* = p^*$, (36) may now be rearranged as

$$\frac{\hat{p}(x^*) - p^*}{p^*} \simeq R_r^*(P_r^* - 1) \frac{(1 - r^*)}{r^*} + \frac{1}{2} R_r^* F^* (P_r^* - 2) \frac{1 - \frac{E(T^2|T < x^*)}{x^{*2}}}{r^*}. \quad (39)$$

Thus the mark-up of the optimal price under supply uncertainty, expressed as a % of the optimal price under capacity adequacy, depends positively on both the degree of relative risk aversion and of relative prudence. Intuitively, a strong degree of prudence underscores the consumer's precautionary motive when ordering electricity. This boosts *ex ante* demand which has to be mitigated through a higher price. In the special case of logarithmic utility, this mark-up reduces to

$$\frac{\hat{p}(x^*) - p^*}{p^*} = \frac{1 - r^*}{r^*}. \quad (40)$$

Thus a perceived reliability of 75% requires a price exceeding base level with 33%.

5 Conclusion

The prime rationale underlying load management has been to increasingly shift the burden of risk associated with random events within a power system to the demand-side.

Problems related to riskiness in the availability of supply have traditionally been viewed as warranting supply-side solutions, notably in the form of additional capacity investment, over the medium to the longer term. It is however evident that with the proper design and implementation of interruption schemes, such matters may be cost effectively addressed by short-term demand-side solutions, provided the incentives given to consumers are sufficiently attractive.

This paper has put forward a relatively simple framework for implementing supply interruptions with financial compensation. In analysing the workings of the proposed scheme, the key issues examined have been: How the service price and interruption compensation should be defined, how consumer demands would respond to the scheme's introduction, and then how such demands would be influenced by the perceived reliability of future power supplies.

Four elements are identified as playing key roles in determining the size of a consumer's *ex ante* demand, notably whether this will expand or contract relative to a benchmark power demand determined in a scenario that is devoid of any anticipated interruptions. These are: the perceived reliability of future power supplies, the size of the expected incentives or compensation payments, the consumer's distaste for interruptions and the strength of the consumer's prudence.

The presented framework has also emphasised the role of expectations in defining consumer behaviour. Although rational expectations and knowledgeable consumers are strong assumptions to make in any realistic setting, they are nevertheless plausible by appealing to schemes in which consumers are well informed about the power system and possess adequate computational capabilities to support rational decision making. Similarly, assuming fixed power demands for a particular period is not unduly restrictive in that provided that consumers face fixed power prices, then their power demands will also tend to be fixed.

A limitation of the current framework is however the difficulty in distinguishing between the optimal policies for the power price and interruption compensation. An explanation for this is the linearity of the consumer's utility in income and the linearity of the power utility's objective in the power price and compensation. These give similar necessary conditions that do not allow for unique policy definitions. Resolving this makes desirable an amendment to the current framework in which the objective functions are allowed to be non-linear in the policy variables and with risk aversion introduced with respect to income.

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Appendices

A Comparative statics on the power demand

A.1 The second-order condition

$$\begin{aligned}
 SOC_{\hat{x}} \stackrel{\text{def}}{=} & u''(\hat{x}) + \int_0^{X^e} \left\{ u''\left(\frac{T}{X^e}\hat{x}\right) \frac{T}{X^e} \left[R_r\left(\frac{T}{X^e}\hat{x}\right) - 1 \right] \right. \\
 & \left. + u'\left(\frac{T}{X^e}\hat{x}\right) R'_r\left(\frac{T}{X^e}\hat{x}\right) \frac{T}{X^e} \frac{F(T)}{X^e} \right\} dT < 0
 \end{aligned} \tag{A.1}$$

A.2 The effect of a change in the power price

In the case with adequate supply, differentiating the first order condition in (2) with respect to p and x^* gives:

$$\frac{\partial x^*}{\partial p} = -\frac{\frac{\partial FOC_{x^*}}{\partial p}}{\frac{\partial FOC_{x^*}}{\partial x^*}} = \frac{1}{u''(x^*)} < 0 \tag{A.2}$$

In the case with inadequate supply, differentiating the first order condition in (5) with respect to p and defining $\frac{T}{X^e}\hat{x} \equiv \bar{x}$ gives:

$$\frac{\partial FOC_{\hat{x}}}{\partial p} = \frac{\partial u'(\hat{x})}{\partial p} - \left\{ 1 - \int_0^{X^e} \frac{F(T)}{X^e} dT \right\} + \frac{\partial}{\partial p} \int_0^{X^e} \frac{u'(\bar{x}) [R_r(\bar{x}) - 1] F(T)}{X^e} dT \tag{A.3}$$

where

$$\begin{aligned}
 & \frac{\partial}{\partial p} \int_0^{X^e} \frac{u'(\bar{x}) [R_r(\bar{x}) - 1] F(T)}{X^e} dT \\
 = & \int_0^{X^e} \frac{\partial u'(\bar{x}) [R_r(\bar{x}) - 1] F(T)}{\partial p X^e} dT \\
 = & \int_0^{X^e} \left\{ u''(\bar{x}) \frac{\partial \frac{T\hat{x}}{X^e}}{\partial p} [R_r(\bar{x}) - 1] + u'(\bar{x}) R'_r(\bar{x}) \frac{\partial \frac{T\hat{x}}{X^e}}{\partial p} \right\} \frac{F(T)}{X^e} dT = 0
 \end{aligned} \tag{A.4}$$

meaning that

$$\frac{\partial FOC_{\hat{x}}}{\partial p} = - \left\{ 1 - \int_0^{X^e} \frac{F(T)}{X^e} dT \right\} \tag{A.5}$$

A similar operation with respect to \hat{x} gives

$$\frac{\partial FOC_{\hat{x}}}{\partial \hat{x}} = u''(\hat{x}) + \int_0^{X^e} \frac{\partial u'(\bar{x}) [R_r(\bar{x}) - 1] F(T)}{\partial \hat{x} X^e} dT \tag{A.6}$$

where

$$\begin{aligned}
& \int_0^{X^e} \frac{\partial u'(\bar{x}) [R_r(\bar{x}) - 1]}{\partial \hat{x}} \frac{F(T)}{X^e} dT \\
&= \int_0^{X^e} \left\{ u''(\bar{x}) \frac{\partial \frac{T\bar{x}}{X^e}}{\partial \hat{x}} [R_r(\bar{x}) - 1] + u'(\bar{x}) R_r'(\bar{x}) \frac{\partial \frac{T\bar{x}}{X^e}}{\partial \hat{x}} \right\} \frac{F(T)}{X^e} dT \quad (\text{A.7}) \\
&= \int_0^{X^e} \left\{ u''(\bar{x}) \frac{T}{X^e} [R_r(\bar{x}) - 1] + u'(\bar{x}) R_r'(\bar{x}) \frac{T}{X^e} \right\} \frac{F(T)}{X^e} dT
\end{aligned}$$

meaning that

$$\frac{\partial FOC\hat{x}}{\partial \hat{x}} = u''(\hat{x}) + \int_0^{X^e} \left\{ u''(\bar{x}) \frac{T}{X^e} [R_r(\bar{x}) - 1] + u'(\bar{x}) R_r'(\bar{x}) \frac{T}{X^e} \right\} \frac{F(T)}{X^e} dT \quad (\text{A.8})$$

Combining (A.5) and (A.9) gives

$$\frac{\partial \hat{x}}{\partial p} = -\frac{\frac{\partial FOC\hat{x}}{\partial p}}{\frac{\partial FOC\hat{x}}{\partial \hat{x}}} = \frac{\left\{ 1 - \int_0^{X^e} \frac{F(T)}{X^e} dT \right\}}{u''(\hat{x}) + \int_0^{X^e} \left\{ u''(\bar{x}) \frac{T}{X^e} [R_r(\bar{x}) - 1] + u'(\bar{x}) R_r'(\bar{x}) \frac{T}{X^e} \right\} \frac{F(T)}{X^e} dT} \quad (\text{A.9})$$

Satisfying the second order condition for \hat{x} means that the denominator in (A.9) is required to be weakly negative. It must however be strictly negative for the comparative static to be meaningful. To check the sign on the denominator, we may re-express this as:

$$u''(\hat{x}) + \int_0^{X^e} u'(\bar{x}) \frac{T}{X^e} \left\{ R_a(\bar{x}) [1 - R_r(\bar{x})] + R_r'(\bar{x}) \right\} \frac{F(T)}{X^e} dT \quad (\text{A.10})$$

and the curly bracket term in (A.10) as:

$$\begin{aligned}
& R_a(\bar{x}) [1 - R_r(\bar{x})] + R_r'(\bar{x}) \\
&= \frac{-u''(\bar{x}) [u'(\bar{x}) + u''(\bar{x})\bar{x}]}{(u'(\bar{x}))^2} - \frac{u'(\bar{x}) \{u'''(\bar{x})\bar{x} + u''(\bar{x})\} - (u''(\bar{x}))^2\bar{x}}{(u'(\bar{x}))^2} \\
&= \frac{-u'''(\bar{x})\bar{x}}{u'(\bar{x})} - \frac{2u''(\bar{x})}{u'(\bar{x})} \quad (\text{A.11}) \\
&= -[P_r(\bar{x}) - 2] R_a(\bar{x})
\end{aligned}$$

where $P_r(\bar{x}) \equiv -\frac{u'''(\bar{x})\bar{x}}{u''(\bar{x})}$ is the coefficient of relative prudence, describing the degree of convexity in the consumer's marginal utility from power consumption. Inserting (A.11) into (A.10) gives

$$u''(\hat{x}) - \int_0^{X^e} u'(\bar{x}) \frac{T}{X^e} [P_r(\bar{x}) - 2] R_a(\bar{x}) \frac{F(T)}{X^e} dT \quad (\text{A.12})$$

we will assume that (A.12) is strictly negative. Finally, re-inserting (A.12) into (A.9) gives

$$\frac{\partial \hat{x}}{\partial p} = \frac{\left\{ 1 - \int_0^{X^e} \frac{F(T)}{X^e} dT \right\}}{u''(\hat{x}) - \int_0^{X^e} u'(\bar{x}) \frac{T}{X^e} [P_r(\bar{x}) - 2] R_a(\bar{x}) \frac{F(T)}{X^e} dT} \quad (\text{A.13})$$

A strictly positive numerator in (A.13) ensures that $\frac{\partial \hat{x}}{\partial p}$ will be strictly negative.

A.3 The effect of a change in the compensation rate

Differentiating the foc with respect to c gives

$$\frac{\partial FOC \hat{x}}{\partial c} = \int_0^{X^e} \frac{F(T)}{X^e} dT + \int_0^{X^e} \frac{\partial u'(\bar{x}) [R_r(\bar{x}) - 1] F(T)}{\partial c} \frac{F(T)}{X^e} dT \quad (\text{A.14})$$

as in (A.4), the second term on the RHS of (A.14) will be 0. Combining (A.14) and (A.12) gives

$$\frac{\partial \hat{x}}{\partial c} = - \frac{\frac{\partial FOC \hat{x}}{\partial c}}{\frac{\partial FOC \hat{x}}{\partial \hat{x}}} = - \frac{\int_0^{X^e} \frac{F(T)}{X^e} dT}{u''(\hat{x}) - \int_0^{X^e} u'(\bar{x}) \frac{T}{X^e} [P_r(\bar{x}) - 2] R_a(\bar{x}) \frac{F(T)}{X^e} dT} \quad (\text{A.15})$$

A strictly negative denominator in (A.15) implies that the sign of $\frac{\partial \hat{x}}{\partial X^e}$ will mirror that of the numerator. Having the numerator strictly positive therefore implies that $\frac{\partial \hat{x}}{\partial c}$ will also be strictly positive.

A.4 The effect of a change in the expected aggregate demand

Differentiating the foc wrt X^e gives

$$\frac{\partial FOC \hat{x}}{\partial X^e} = \frac{(p+c)F(X^e)}{X^e} \left\{ 1 - \frac{1}{F(X^e)} \int_0^{X^e} \frac{F(T)}{X^e} dT \right\} + \frac{\partial \int_0^{X^e} \frac{u'(\bar{x}) [R_r(\bar{x}) - 1] F(T)}{X^e} dT}{\partial X^e} \quad (\text{A.16})$$

where

$$\begin{aligned}
& \frac{\partial \int_0^{X^e} \frac{u'(\bar{x})[R_r(\bar{x})-1]}{X^e} F(T) dT}{\partial X^e} \\
&= \frac{u'(\hat{x}) [R_r(\hat{x}) - 1] F(X^e)}{X^e} + \int_0^{X^e} \frac{\partial \frac{u'(\bar{x})[R_r(\bar{x})-1]}{X^e}}{\partial X^e} F(T) dT \\
&= \dots + \int_0^{X^e} \frac{X^e \frac{\partial u'(\bar{x})[R_r(\bar{x})-1]}{\partial X^e} - u'(\bar{x}) [R_r(\bar{x}) - 1]}{(X^e)^2} F(T) dT \tag{A.17} \\
&= \dots + \int_0^{X^e} \left\{ \frac{X^e \frac{\partial T \hat{x}}{\partial X^e} [u''(\bar{x}) [R_r(\bar{x}) - 1] + u'(\bar{x}) R'_r(\bar{x})] - u'(\bar{x}) [R_r(\bar{x}) - 1]}{(X^e)^2} \right\} F(T) dT \\
&= \dots + \int_0^{X^e} \left\{ \frac{-\bar{x} [u''(\bar{x}) [R_r(\bar{x}) - 1] + u'(\bar{x}) R'_r(\bar{x})] - u'(\bar{x}) [R_r(\bar{x}) - 1]}{X^e} \right\} \frac{F(T)}{X^e} dT \\
&= \dots + \int_0^{X^e} \frac{u'(\bar{x})}{X^e} \left\{ [R_r(\bar{x}) - 1]^2 - R'_r(\bar{x}) \bar{x} \right\} \frac{F(T)}{X^e} dT
\end{aligned}$$

meaning that

$$\begin{aligned}
\frac{\partial FOC \hat{x}}{\partial X^e} &= (p+c) \frac{F(X^e)}{X^e} \left\{ 1 - \frac{1}{F(X^e)} \int_0^{X^e} \frac{F(T)}{X^e} dT \right\} \\
&+ u'(\hat{x}) [R_r(\hat{x}) - 1] \frac{F(X^e)}{X^e} \tag{A.18} \\
&+ \int_0^{X^e} u'(\bar{x}) \left\{ [R_r(\bar{x}) - 1]^2 - R'_r(\bar{x}) \bar{x} \right\} \frac{F(T)}{X^{e2}} dT
\end{aligned}$$

Combining (A.18) and (A.12) gives

$$\frac{\partial \hat{x}}{\partial X^e} = - \frac{\frac{\partial FOC \hat{x}}{\partial X^e}}{\frac{\partial FOC \hat{x}}{\partial \hat{x}}} = - \frac{\frac{1}{X^e} \left\{ \begin{aligned} & (p+c) F(X^e) \left\{ 1 - \frac{1}{F(X^e)} \int_0^{X^e} \frac{F(T)}{X^e} dT \right\} \\ & + u'(\hat{x}) [R_r(\hat{x}) - 1] F(X^e) \\ & + \int_0^{X^e} u'(\bar{x}) \left\{ [R_r(\bar{x}) - 1]^2 - R'_r(\bar{x}) \bar{x} \right\} \frac{F(T)}{X^e} dT \end{aligned} \right\}}{u''(\hat{x}) - \int_0^{X^e} u'(\bar{x}) \frac{T}{X^e} [P_r(\bar{x}) - 2] R_a(\bar{x}) \frac{F(T)}{X^e} dT} \tag{A.19}$$

As earlier, a strictly negative denominator implies that the sign of $\frac{\partial \hat{x}}{\partial X^e}$ will mirror that of the numerator. Using (7), the expression within curly brackets in the first term of the numerator is equivalently $\frac{E[T|T < X^e]}{X^e}$, which is positive. The first term must therefore be weakly positive. The second term will also be weakly positive provided the consumer has a normal or high aversion to being interrupted i.e. $R_r(\hat{x}) \geq 1$, but will be negative if otherwise. We may

resolve the term within curly brackets in the third term by writing:

$$\begin{aligned}
& [R_r(\bar{x}) - 1]^2 - R_r'(\bar{x})\bar{x} \\
= & \frac{[u''(\bar{x})\bar{x} + u'(\bar{x})]^2}{(u'(\bar{x}))^2} + \frac{u'(\bar{x})\bar{x}\{u'''(\bar{x})\bar{x} + u''(\bar{x})\} - (u''(\bar{x}))^2\bar{x}^2}{(u'(\bar{x}))^2} \\
= & \frac{3u''(\bar{x})u'(\bar{x})\bar{x} + (u'(\bar{x}))^2 + u'''(\bar{x})u'(\bar{x})\bar{x}^2}{(u'(\bar{x}))^2} \tag{A.20} \\
= & 3\frac{u''(\bar{x})}{u'(\bar{x})}\bar{x} + 1 + \frac{u'''(\bar{x})}{u'(\bar{x})}\bar{x}^2 \\
= & -3\frac{-u''(\bar{x})\bar{x}}{u'(\bar{x})} + 1 + \frac{-u'''(\bar{x})\bar{x} - u''(\bar{x})\bar{x}}{u''(\bar{x})u'(\bar{x})} \\
= & 1 + [P_r(\bar{x}) - 3]R_r(\bar{x})
\end{aligned}$$

re-inserting (A.20) into (A.19) means that

$$\frac{\partial \hat{x}}{\partial X^e} = -\frac{\frac{1}{X^e} \left\{ (p+c) \frac{E[T|T < X^e]}{X^e} F(X^e) + u'(\hat{x}) [R_r(\hat{x}) - 1] F(X^e) + \int_0^{X^e} u'(\bar{x}) \{1 + [P_r(\bar{x}) - 3] R_r(\bar{x})\} \frac{F(T)}{X^e} dT \right\}}{u''(\hat{x}) - \int_0^{X^e} u'(\bar{x}) \frac{T}{X^e} [P_r(\bar{x}) - 2] R_a(\bar{x}) \frac{F(T)}{X^e} dT} \tag{A.21}$$

If the consumer possesses a coefficient of relative prudence that weakly exceeds 3, then the third term will be assuredly positive, otherwise the sign is not obvious. It is thus affirmable that the numerator and therefore $\frac{\partial \hat{x}}{\partial X^e}$ will be weakly positive provided: $R_r(\hat{x}) \geq 1$ and $P_r(\bar{x}) \geq 3$.

B Comparative statics on the expected aggregate demand

B.1 The effect of a change in the power price

In a rational expectations equilibrium with $N = 1$, the expected aggregate demand would be $X^e = \hat{x}(p, c, X^e)$. Differentiating this completely with respect to p gives:

$$\frac{\partial X^e}{\partial p} = \left[\frac{\partial \hat{x}}{\partial p} + \frac{\partial \hat{x}}{\partial X^e} \frac{\partial X^e}{\partial p} \right] \tag{B.1}$$

and then re-arranging to obtain

$$\frac{\partial X^e}{\partial p} = \frac{\frac{\partial \hat{x}}{\partial p}}{\left[1 - \frac{\partial \hat{x}}{\partial X^e} \right]} \tag{B.2}$$

Note that the stability of this equilibrium requires $\frac{\partial \hat{x}}{\partial X^e} < 1$. Rewriting the equilibrium second-order condition to the consumer's problem (which is also the denominator in the comparative static results $\frac{\partial \hat{x}}{\partial p}$, $\frac{\partial \hat{x}}{\partial c}$ and $\frac{\partial \hat{x}}{\partial X^e}$) as:

$$SOC_{\text{in eqb.}} = \frac{u'(\hat{x})}{\hat{x}} \left\{ -R_r(\hat{x}) - \int_0^{\hat{x}} \frac{u'(T)}{u'(\hat{x})} [P_r(T) - 2] R_r(T) \frac{F(T)}{\hat{x}} dT \right\} \quad (\text{B.3})$$

using (13) means that

$$\left[1 - \frac{\partial \hat{x}}{\partial X^e} \right]_{\text{in eqb.}} = \frac{-\frac{u'(\hat{x})}{\hat{x}} R_r(\hat{x}) - \int_0^{\hat{x}} \frac{u'(T)}{\hat{x}^2} \{R_r(T) - 1\} F(T) dT + (p+c) \frac{E[T|T < X^e]}{\hat{x}^2} F(\hat{x}) + \frac{1}{\hat{x}} u'(\hat{x}) [R_r(\hat{x}) - 1] F(\hat{x})}{\frac{u'(\hat{x})}{\hat{x}} \left\{ -R_r(\hat{x}) - \int_0^{\hat{x}} \frac{u'(T)}{u'(\hat{x})} [P_r(T) - 2] R_r(T) \frac{F(T)}{\hat{x}} dT \right\}} \quad (\text{B.4})$$

from the consumer's first-order condition in (5) we will have in equilibrium that

$$\frac{u'(\hat{x}) - p + (p+c) F(\hat{x})}{\hat{x}} = - \int_0^{\hat{x}} \frac{u'(T)}{\hat{x}^2} [R_r(T) - 1] F(T) dT + (p+c) F(\hat{x}) \frac{E[T | T < X^e]}{\hat{x}^2} \quad (\text{B.5})$$

recognising (B.5) in (B.4) gives

$$\left[1 - \frac{\partial \hat{x}}{\partial X^e} \right]_{\text{in eqb.}} = \frac{\frac{u'(\hat{x})}{\hat{x}} \left\{ [1 - F(\hat{x})] [1 - R_r(\hat{x})] - \frac{p[1-F(\hat{x})]}{u'(\hat{x})} + \frac{cF(\hat{x})}{u'(\hat{x})} \right\}}{\frac{u'(\hat{x})}{\hat{x}} \left\{ -R_r(\hat{x}) - \int_0^{\hat{x}} \frac{u'(T)}{u'(\hat{x})} [P_r(T) - 2] R_r(T) \frac{F(T)}{\hat{x}} dT \right\}} \quad (\text{B.6})$$

and with $\frac{\partial \hat{x}}{\partial p}$ means that

$$\begin{aligned} \frac{\partial X^e}{\partial p} \Big|_{\text{in eqb.}} &= \left[\frac{\frac{\partial \hat{x}}{\partial p}}{1 - \frac{\partial \hat{x}}{\partial X^e}} \right]_{\text{in eqb.}} \quad (\text{B.7}) \\ &= \frac{\hat{x} \left\{ 1 - F(\hat{x}) \frac{(\hat{x} - E[T|T < \hat{x}])}{\hat{x}} \right\}}{u'(\hat{x}) \left\{ [1 - F(\hat{x})] [1 - R_r(\hat{x})] - \frac{p}{u'(\hat{x})} [1 - F(\hat{x})] + \frac{c}{u'(\hat{x})} F(\hat{x}) \right\}} \end{aligned}$$

Evaluating (B.7) with logarithmic utility gives⁸

$$\frac{\partial X^e}{\partial p} = - \frac{\hat{x} \left\{ 1 - F(\hat{x}) \frac{(\hat{x} - E[T|T < \hat{x}])}{\hat{x}} \right\}}{p [1 - F(\hat{x})] - cF(\hat{x})} \quad (\text{B.8})$$

⁸It is helpful to re-collect that having $u(x) = \ln x$ implies: $u'(x) = \frac{1}{x}$, $u''(x) = -\frac{1}{x^2}$, $u'''(x) = \frac{2}{x^3}$ with $R_a(x) = \frac{1}{x}$, $R_r(x) = 1$ and $P_r(x) = 2$. Also that $\bar{x} = \frac{T}{X^e} \hat{x}$.

B.2 The effect of a change in the compensation rate

Differentiating $X^e \equiv 1.\hat{x}(p, c, X^e)$ completely with respect to c gives

$$\frac{\partial X^e}{\partial c} = \frac{\frac{\partial \hat{x}}{\partial c}}{\left[1 - \frac{\partial \hat{x}}{\partial X^e}\right]} \quad (\text{B.9})$$

as in (B.7) we will have that

$$\begin{aligned} \frac{\partial X^e}{\partial c} \Big|_{\text{in eqb}} &= \left[\frac{\frac{\partial \hat{x}}{\partial c}}{1 - \frac{\partial \hat{x}}{\partial X^e}} \right]_{\text{in eqb.}} \quad (\text{B.10}) \\ &= \frac{-\hat{x}F(\hat{x}) \frac{(\hat{x} - E[T|T < \hat{x}])}{\hat{x}}}{u'(\hat{x}) \left\{ [1 - F(\hat{x})] [1 - R_r(\hat{x})] - \frac{p}{u'(\hat{x})} [1 - F(\hat{x})] + \frac{c}{u'(\hat{x})} F(\hat{x}) \right\}} \end{aligned}$$

which with logarithmic utility becomes

$$\frac{\partial X^e}{\partial c} \Big|_{\text{in eqb}} = \left[\frac{\frac{\partial \hat{x}}{\partial c}}{1 - \frac{\partial \hat{x}}{\partial X^e}} \right]_{\text{in eqb.}} = \frac{\hat{x}F(\hat{x}) \frac{(\hat{x} - E[T|T < \hat{x}])}{\hat{x}}}{p[1 - F(\hat{x})] - cF(\hat{x})} \quad (\text{B.11})$$

C Pricing with consumer welfare maximisation and a budget restriction

C.1 Case B: Inadequate power supply

$$V = [u(\hat{x}) + m - p\hat{x}] [1 - F(\hat{x})] + \int_0^{\hat{x}} [u(T) + m - pT + c(\hat{x} - T)] dF(T) \quad (\text{C.1})$$

$$E\pi = \left\{ (p - b) \left[\hat{x} - \int_0^{\hat{x}} (\hat{x} - T) dF(T) \right] - \int_0^{\hat{x}} c(\hat{x} - T) dF(T) \right\} \quad (\text{C.2})$$

$$V_p = \left[u'(\hat{x}) \hat{x}_p - \hat{x} - p\hat{x}_p \right] [1 - F(\hat{x})] + c\hat{x}_p F(\hat{x}) - E[T | T \leq \hat{x}] F(\hat{x}) \quad (\text{C.3})$$

$$E\pi_p = \hat{x} \left\{ 1 - F(\hat{x}) \frac{(\hat{x} - E[T | T \leq \hat{x}])}{\hat{x}} \right\} + \hat{x}_p [(p - b) [1 - F(\hat{x})] - cF(\hat{x})] \quad (\text{C.4})$$

$$V_c = \left[u'(\hat{x}) \hat{x}_c - p\hat{x}_c \right] [1 - F(\hat{x})] + c\hat{x}_c F(\hat{x}) + \hat{x}F(\hat{x}) - E[T] F(\hat{x}) \quad (\text{C.5})$$

$$E\pi_c = - (F(\hat{x}) \hat{x} - F(\hat{x}) E[T]) + \hat{x}_c [(p - b) [1 - F(\hat{x})] - cF(\hat{x})] \quad (\text{C.6})$$



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