Efficient (Re-)Scheduling: An Auction Approach[1]

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Abstract

This paper considers one-sided scheduling problems, where a schedule of service is arranged at one location, without regard to other schedules. Typically, such scheduling problems are handled on a first-come-first-serve basis, which is grossly inefficient. The present paper proposes a scheduling mechanism that is a non-standard auction, in which the allocation is ruled by evaluating combinations of bids. The proposed mechanism implements the efficient allocation in dominant strategies and is deficit-free. Since that mechanism is suitable for the scheduling problems at sea-ports, loading or unloading at sea-ports is used as an illustration.

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1 Introduction

In a variety of settings a given capacity has to be rationed among different parties, and a schedule has to be arranged that determines who shall be served when. For example:

- In courts, tort law cases pile up that await processing; should cases that involve a higher damage claim be served first?
- In seaports vessels have to be scheduled for loading and unloading and in airports aircraft has to be scheduled for take-off and landing. On what principles should one assign the available slots?
- In firms composed of quasi-independent subunits, jointly used facilities must be shared. Who gets access at what time, and how can one decentralize the efficient access rule?

First-come-first-served is probably the most frequently used scheduling rule. Seaports employ it to allocate time slots at berth, except for those who have secured access through long-term contracts or by building-up their own facilities. First-come-first-serve is, however, blatantly inefficient. This suggests the search for better scheduling rules. Better rules improve welfare and, give those institutions that employ them a competitive edge.

In this paper we propose an efficient scheduling mechanism for one-sided scheduling problems, where schedules for service at one location can be viewed as independent of other schedules. That mechanism is a non-standard auction that awards slots according to a rule that evaluates combinations of bids. That rule is an adaption of the generalized Clark-Groves-Vickrey rule to a multidimensional framework with buyers and sellers as bidders. It achieves the first-best schedule in dominant strategies, it is deficit-free, and it entails nonnegative prices that are equal to shadow prices. It also allows for forward sales of time slots, and assures that the slots sold in advance are voluntarily brought back to the auction. Moreover, it allows for different service times, and arbitrary distribution of preferences over feasible schedules.

Sea-ports seem to fit reasonably well the pattern of a one-sided scheduling problem. This is why, in the following, we will always refer to the scheduling of vessels for unloading at seaports. It should, however, be understood that our proposal applies to a variety of other scheduling problems.

In order to motivate that sea-port perspective, we mention that sea-ports typically enter into some long-term contracts that allow shipping lines to book time slots in advance. This so called “rendezvous system” allocates part of the port capacity. The remaining capacity is then assigned shortly before vessels are handled or as late as when they have lined up in port. There, the scheduling policy tends to be first-come-first-serve (see Ghosh (2002) and Psaraftis (1998)).

There is a large literature on ports, port investment and pricing mechanisms. The currently used pricing rules across ports are classified as cost based, cost recovery, congestion, strategic or commercial pricing. A review of this literature is found in Strandenes and Marlow (2000). Dasgupta and Ghosh (2000) demonstrate that using prices to regulate queue performance in seaports can make a significant difference, using data from the port of Calcutta. And Ghosh (2002) proposes to adopt a standard English auctions to allocate the next available time slot at berth.

There is also a large literature on priority pricing, and particular scheduling problems, known as the “queuing problem”. Queuing is generally described by stochastic arrival of customers. Due to limited service capacity, customers queue. They all wish to be served as quickly as possible. Therefore, the service provider must set some rules that define priorities of service.

In his seminal paper Marchand (1974) characterizes the optimal allocation in queuing problems, and the use of prices to implement that allocation. He assumes commonly known waiting costs and nonstrategic customers. Naor (1969) shows that that first-come-first-serve entails inefficient entry, because those who join a queue do not take into account how that affects the waiting time of others. Cox and Smith (1961) show that it is optimal to serve members of a queue in descending order of waiting cost. And Balachandran (1972) allows customers to affect the order of service by purchasing priorities.

While the earlier queuing literature assumed that customers’ are not strategic players, because their waiting cost is common knowledge, several recent contributions allowed for private information. Mitra (2002) showed that the efficient queue is implementable in dominant strategies while maintaining a balanced budget if and only if waiting costs are linear.

Despite the similarities between queueing and the present scheduling problem, there are at least two some important differences. Queuing assumes random arrival and impatience (everyone wants to be served as soon as possible). These assumptions are not appropriate for the scheduling issue, because vessel operators plan their arrival time in advance, and thus do not necessarily wish to be served at the first slot available during a given day or week.

The plan of the paper is as follows: Section 2 presents assumptions and the notation. Section 3 introduces the proposed auction mechanism. Section 4 characterizes that mechanism and presents the proofs. Section 5 explains why simpler standard auction mechanisms should not be used. Section 6 discusses the role of a forward market in slots, as a complement to the proposed auction mechanism. The paper closes in Section 7 with a discussion of the limitations of the proposed mechanism.

2 Assumptions and Notation

Suppose there are $n \geq 2$ vessels that need to be queued for unloading at a given port facility, during a given time window. Each ship is denoted by an index $i \in N := \{1, 2, \ldots, n\}$. Vessels may require a different amount of time for unloading (in addition to other resources such as staff, equipment etc.).

Based on the time required for unloading, the port authority computes all feasible time allocations. An allocation, $\alpha^r := (\alpha_1^r, \ldots, \alpha_n^r)$, is a complete schedule; component α_i^r states precisely at which time ship i shall be unloaded, and this in such a way that the sum total of the allocated time slots does not exceed the available time. The set of all feasible allocations is denoted by $A := (\alpha^1, \ldots, \alpha^m)$. Similarly, the restricted set of feasible allocations that apply if time slot s is not available, is denoted by A_{-s}. Of course, feasible allocations may prescribe that some vessels are not unloaded.

3 The Proposed Auction Rule

We propose the following auction mechanism that is an adaptation of the Groves-Clarke-Vickrey mechanism to a multidimensional framework with buyers and sellers as bidders. Without loss of generality, it can be described as a direct revelation mechanism in which bidders are simply asked to report their incremental profits from being served according to each feasible allocation (relative to being served at some given alternative time slot), and the auctioneer is committed to select an allocation and associated prices for unloading, as a function of the complete profile of messages by all n bidders.

We will say that a "bid" by bidder i is a vector $\tilde{\pi}_i$ that states his reported profits for each and every feasible allocation $\alpha \in A$:

$$\tilde{\pi}_i := \left(\tilde{\pi}_i(\alpha_i^1), \tilde{\pi}_i(\alpha_i^2), \ldots, \tilde{\pi}_i(\alpha_i^m)\right) \tag{1}$$

Of course, bidders may not tell the truth; therefore, a bid $\tilde{\pi}_i$ may deviate from true profits, which are denoted by π_i.

The combined "bid vector" of all bidders, $\tilde{\pi} := (\tilde{\pi}_1, \tilde{\pi}_2, \dots, \tilde{\pi}_n)$, uniquely determines which allocation is chosen and how the associated schedules are priced, according to the following rules.

"Allocation rule": For each bid vector, $\tilde{\pi}$, the mechanism selects that allocation, $\alpha^*(\tilde{\pi})$, that maximizes bidders' welfare, which is the sum of their bids:

$$\alpha^*(\tilde{\pi}) := \arg\max_{\alpha \in A} W(\alpha, \tilde{\pi}) \tag{2}$$

$$W(\alpha, \tilde{\pi}) := \sum_{j \in N} \tilde{\pi}_j(\alpha_j). \tag{3}$$

Evidently, this rule selects the efficient allocation, defined as the maximizer of the sum total of true profits, if bidders bid truthfully ($\tilde{\pi}_i = \pi_i$), for all $i \in N$.

A similar allocation rule is applied to two hypothetical circumstances: when bidder i is excluded from participation in the mechanism, and when bidder i is excluded, *and* at the same time slot s is not made available:

$$\alpha^*_{-i}(\tilde{\pi}_{-i}) := \arg\max_{\alpha \in A} W_{-i}(\alpha, \tilde{\pi}_{-i}) \tag{4}$$

$$\alpha^*_{-i,-s}(\tilde{\pi}_{-i}) := \arg\max_{\alpha \in A_{-s}} W_{-i}(\alpha, \tilde{\pi}_{-i}) \tag{5}$$

$$W_{-i}(\alpha, \tilde{\pi}_{-i}) := \sum_{j \neq i} \tilde{\pi}_j(\alpha_j). \tag{6}$$

"Pricing rules": Bidders are required to pay a "service price" for the slot allocated to them, and they receive a "selling price" if they already own a slot and supply it to the auction.

The *service price* to be paid by bidder i if he is awarded slot $\alpha^*_i(\tilde{\pi})$ is equal to the negative externality that his presence inflicts upon all other bidders:

$$p_{\alpha^*_i(\tilde{\pi})} := W_{-i}\left(\alpha^*_{-i}(\tilde{\pi}_{-i}), \tilde{\pi}_{-i}\right) - W_{-i}\left(\alpha^*(\tilde{\pi}), \tilde{\pi}_{-i}\right). \tag{7}$$

The *selling price* to be paid to bidder i for a slot s, if he already owns it but supplies it to the auction, is equal to the positive externality that supplying slot s would inflict upon others if he does not participate in the auction as a buyer:

$$P_{si} = W_{-i}\left(\alpha^*_{-i}(\tilde{\pi}_{-i}), \tilde{\pi}_{-i}\right) - W_{-i}\left(\alpha^*_{-i,-s}(\tilde{\pi}_{-i}), \tilde{\pi}_{-i}\right). \tag{8}$$

These rules induce a noncooperative game, which is characterized by the following payoff functions, defined on bidders' messages (which are their only strategies).

The payoff function of a bidder i who does not already own some slot (we may call him a *"buyer"*: b) is:

$$\begin{aligned}\Pi_i^b(\tilde{\pi}) =&\pi_i(\alpha_i^*(\tilde{\pi})) - p_{\alpha_i^*(\tilde{\pi})} \\ =&\pi_i(\alpha_i^*(\tilde{\pi})) + W_{-i}\left(\alpha^*(\tilde{\pi}), \tilde{\pi}_{-i}\right) - W_{-i}\left(\alpha_{-i}^*(\tilde{\pi}_{-i}), \tilde{\pi}_{-i}\right).\end{aligned} \tag{9}$$

Similarly, the payoff function of a bidder i who already owns slot s and made it available to the auction (we may call him *"seller/buyer"*: sb) is

$$\begin{aligned}\Pi_i^{sb}(\tilde{\pi}) =&\pi_i(\alpha_i^*(\tilde{\pi})) - p_{\alpha_i^*(\tilde{\pi})} + P_{si} \\ =&\pi_i(\alpha_i^*(\tilde{\pi})) + W_{-i}\left(\alpha^*(\tilde{\pi}), \tilde{\pi}_{-i}\right) - W_{-i}\left(\alpha_{-i,-s}^*(\tilde{\pi}_{-i}), \tilde{\pi}_{-i}\right).\end{aligned} \tag{10}$$

Of course, if bidder i already owns slot s but does not supply it to the auction (we may call him a *"non-seller"*: ns), his payoff function is only dependent upon π_i:

$$\Pi_i^{ns}(\tilde{\pi}) = \pi_i(s), \tag{11}$$

whereas if he sells at the auction but does not wish to buy there (we may call him a *"pure seller"*: s), his payoff function is only dependent upon $\tilde{\pi}_{-i}$:

$$\Pi_i^s(\tilde{\pi}) = P_{si} = W_{-i}\left(\alpha_{-i}^*(\tilde{\pi}_{-i}), \tilde{\pi}_{-i}\right) - W_{-i}\left(\alpha_{-i,-s}^*(\tilde{\pi}_{-i}), \tilde{\pi}_{-i}\right). \tag{12}$$

4 Properties of the Proposed Auction

THEOREM 1 *1) Truthful bidding is an equilibrium in dominant strategies. 2) That equilibrium has the following properties: a) efficient (re-)scheduling; b) all slots that were sold forward are supplied to the auction; c) nonnegative prices $p_{\alpha_i^*(\pi)}$, P_{si}; d) the price of each slot is equal to its shadow price.*

PROOF 1) Consider a pure buyer. If he tells the truth, his payoff is equal to $\Pi_i^b(\pi, \tilde{\pi}_{-i})$. Whereas, if he lies and reports $\tilde{\pi}_i \neq \pi_i$, and if he thus changes the allocation to $\alpha^k \neq \alpha^*(\pi_i, \tilde{\pi}_{-i})$, his payoff is reduced,

$$\begin{aligned}\Pi_i^b(\pi_i, \tilde{\pi}_{-i}) =&\pi_i(\alpha_i^*(\pi_i, \tilde{\pi}_{-i})) + W_{-i}\left(\alpha^*(\pi_i, \tilde{\pi}_{-i}), \tilde{\pi}_{-i}\right) \\ &- W_{-i}\left(\alpha_{-i}^*(\tilde{\pi}_{-i}), \tilde{\pi}_{-i}\right) \\ \geq&\pi_i(\alpha^k) + W_{-i}\left(\alpha^k, \tilde{\pi}_{-i}\right) - W_{-i}\left(\alpha_{-i}^*(\tilde{\pi}_{-i}), \tilde{\pi}_{-i}\right),\end{aligned}$$

because $\alpha^*(\pi_i, \tilde{\pi}_{-i})$ is the maximizer of the sum

$$\pi_i(\alpha_i^*(\pi_i, \tilde{\pi}_{-i})) + W_{-i}\left(\alpha^*(\pi_i, \tilde{\pi}_{-i}), \tilde{\pi}_{-i}\right),$$

and because adding the constant $W_{-i}\left(\alpha^*_{-i}(\tilde{\pi}_{-i}), \tilde{\pi}_{-i}\right)$ to that sum does not change its maximizer.

The same argument applies to the seller/buyer because his payoff function differs only in the constant.

Finally, the payoff of the pure seller is independent of his message; therefore, he cannot gain from cheating either.

We now use this result in the proof of properties 2a)-2d):

2a) By 1), the mechanism implements the allocation $\alpha^*(\tilde{\pi}) = \alpha^*(\pi)$, which is efficient.

2b) Consider a ship-owner who had purchased a slot forward. Since

$$\begin{aligned}\Pi_i^{sb}(\pi) - \Pi_i^{s}(\pi) =& \pi_i(\alpha_i^*(\pi)) - p_{\alpha_i^*(\pi)} \\ =& \Pi_i^{b}(\pi) \\ \geq& 0,\end{aligned}$$

the policy of selling and buying (sb) payoff dominates that of pure selling (s).

Next, define $(s, \alpha^*_{-i,-s}(\pi_{-i}))$ as that allocation that awards slot s to bidder i and slots $\alpha^*_{-i,-s}(\pi_{-i})$ to the others bidders. Since $\alpha^*(\pi)$ is maximizer of W, and subtracting a constant does not affect that maximizer, it follows that

$$\begin{aligned}\Pi_i^{sb}(\pi) - \Pi_i^{ns}(\pi) =& -\pi_i(s) + \pi_i(\alpha_i^*(\pi)) + W_{-i}\left(\alpha^*(\pi), \pi_{-i}\right) \\ & - W_{-i}\left(\alpha^*_{-i,-s}(\pi_{-i}), \pi_{-i}\right) \\ \geq& -\pi_i(s) + \pi_i(s) + W_{-i}\left(\alpha^*_{-i,-s}(\pi_{-i}), \pi_{-i}\right) \\ & - W_{-i}\left(\alpha^*_{-i,-s}(\pi_{-i}), \pi_{-i}\right) \\ =& 0.\end{aligned}$$

2c) Service prices p are nonnegative, because slots are private goods, and one bidder's participation can only reduce the capacity available to others (see (7)). Similarly, selling prices, P, are nonnegative, because bringing a slot back to the auction, while not participating in the auction, benefits the other bidders (see (8)).

2d) In equilibrium, bidder i is awarded slot $\alpha_i^*(\pi)$. Suppose a replica of that slot is made available to the auction. Then welfare increases by the amount $W_{-i}\left(\alpha^*_{-i}(\pi_{-i}),\pi_{-i}\right) - W_{-i}\left(\alpha^*(\pi),\pi_{-i}\right)$. By (7) this is precisely equal to the service price $p_{\alpha_i^*(\pi)}$. Therefore, services prices can be interpreted as shadow prices. A similar argument applies to selling prices. □

The last property entails that ports are also given the right incentives to invest in port capacity.

5 Why not Use a Standard Auction?

The present scheduling problem is a multi-object auction in which each bidder demands only one object. If the objects for sale were identical, the scheduling problem would be equivalent to a single-unit auction (see Weber (1983)). In that case, one could employ a simple, standard auction rule, in lieu of the more complex scheme proposed here. This raises the question: what would go wrong if one employed a simple, standard auction rule?

If one employed a standard auction procedure one would auction slots sequentially and typically employ an open, ascending-price auction format. For example, one could first auction the early morning slot, then the second late morning slot, and continue in this fashion, down to the last, late evening slot. Like in standard auction, each slot would be allocated to that bidder who values it the most; however, unlike in a standard auction, this allocation is not efficient.

A simple example explains the problem: Suppose three vessels, called A, B, and C, demand to be handled at a port at a given day. Vessel A is considerably larger than B and C, in such a way, that the port can handle either A alone or both B *and* C. Also let A's willingness to pay for immediate port handling, v_A, be considerably higher than that of B and C, v_B, v_C, yet smaller than the sum of the willingness to pay for serving B and C in sequence, v_{BC}. In that case, the standard auction procedure allocates the port capacity to vessel A and neither serves vessel B nor C. This is, however, not efficient, since the value generated by this allocation, v_A, is lower the value that it crowds out, v_{BC}.

In order to reach the efficient allocation, the auctioneer must compare not only *individual* bids, but also all feasible *combinations* of bids. Standard auction procedures generally fail to solve scheduling problems precisely because they compare only individual bids, and fail to compare them to all those combinations of bids that are consistent with a feasible schedule. This indicates that a solution of a scheduling problem must employ some combinatorial bidding procedure, like the one proposed here.[1]

6 Forward Sale of Slots

The mechanism assumes that there is both a spot and a forward market in port slots. To create a functioning forward market, an active spot market is needed. Without the spot market, forward transactions can only occur in the form of dedicating facilities to particular users.

At present some liner operators have invested in terminals partly to ascertain specific port slots and partly to secure access to dedicated cargo handling gear. Liner operators that sail their vessels in scheduled operations could secure specific slots without running their own terminal, if they had access to a viable forward market in slots. And even those who maintain their own facilities may improve their utilization by reselling capacity at a functioning spot market.

Even for tramp shipping there is a demand for forward transactions in port slots. Assignments to ship a cargo load between two specific ports is often awarded weeks before the actual loading and a laden trip may last several weeks. A third group of potential bidders in the forward slot auctions are the ship-owners offering contracts of affreightment (COA) who guarantee transport of a fixed volume of cargo at specific intervals within one or more years.

To ports auctioning of slots in a forward market is essentially a financial instrument to hedge exposure to unexpected demand fluctuations.[2] However, the main benefit of offering forward transactions is that it makes the port more attractive to ship-owners, because it admits long-term planning, and guarantees them access at a certain time without investing in their own terminal.

Of course, the schedules arranged in the forward market must be renegotiable in the spot market, as they are in the proposed mechanism, because vessel's arrival is subject unexpected changes. In particular, weather conditions may cause delays. Rerouting of vessels during the laden trip also occurs, for example when the cargo is re-sold while being transported.

[1] Ausubel and Milgrom (2002) advise to use a particular "package bidding" procedures to deal with complex multi-object auction problems. Generally, package bidding is relevant if bidders demand several objects. This is not the case in the present context. Nevertheless, even though our proposed mechanism differs from that by Ausubel and Milgrom, one may view the present proposal as an example of some sort of package bidding, where bidders bid on complete allocations, even though they are only interested in the slot assigned to them and don't care about the slots given to other bidders.

[2] Since the mechanism gives ship-owners an option to resell, the hedging potential for ports is limited, because the income risk of the port is not always reduced by offering forward transactions.

Re-sale of slots will reduce the number of unused slots and diminish ship-owners risk from buying slots forward when they cannot be sure that their vessels will use the specific slots acquired.

The port should only sell some of the slots in the forward market and retain others for sale in the spot market, if the forward market is thin. This may also be necessary in order to avoid predatory blocking of port capacity. But the port could also achieve this by restricting the number of slots sold to each bidder.

The proposed mechanism give ship-owners and incentive to resell a slot in the spot auction, but a second hand market in port slots where ship-owners sell slots among themselves should not be allowed. When a vessel is sold the slots acquired for this vessel belong to the vessel or are brought back to the auction. Alternatively the port may allow the ship-owner selling the vessels to reassign the slots to another vessel in his fleet.

7 Discussion

The present paper has proposed an auction mechanism that solves the problem of scheduling in ports and other applications. This mechanism is an adaptation of the Clark-Groves-Vickrey rule to an auction environment with multidimensional messages where participants are both buyers and sellers. It implements the efficient allocation in dominant strategies, is deficit free, and assures nonnegative slot prices. Thus, the mechanism awards the entire scarcity rent to the port which also entails the right incentives for investment in port capacity.

The proposed mechanism is compatible with forward sales of slots. It assures that the slots acquired in forward markets are always resold in the spot auction. This way, rescheduling occurs whenever it is beneficial.

The proposed mechanism has two limitations. First, it assumes one-sided scheduling problems. This excludes applications in which there are significant interdependencies between schedules arranged at different locations, as in the case of airports, where the rights of take-off and landing must be coordinated across airports (see Rassenti, Smith, and Bulfin (1982)). Second, it assumes stochastic independence of bidders' privately known willingness to pay for slots. While this assumption is appropriate for most port applications, it cannot be applied to the scheduling of feeder service at hub ports, where strong complementarities exist between slots.

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