# The indexing impasse:

## Is «the intersection approach»

## a solution?\*

by

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#### Abstract

Rawls (1971,1993) suggests that interpersonal comparisons of well-being should be based on a primary goods index, but it is well-known that in general this approach is not compatible with the Pareto principle. This is the indexing impasse. Sen (1985,1991) argues that this is partly due to the fact that the approach does not take note of the citizen's orderings of these bundles of valuable objects. He suggests an «intersection approach», which is an incomplete approach to interpersonal comparisons based on judgements that are shared implications of the relevant set of weighting schemes. In this paper, we show that «the intersection approach» does not provide any solution to the indexing impasse. Unless the individuals have identical preferences, «the intersection approach» is incompatible with the Pareto principle.

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## 1. Introduction

Considerations of well-being are essential for any consequensialistic theory of social choice. Within the welfaristic framework, these considerations are all that matters, where well-being is defined by the concept of utility. This approach has been criticised from different perspectives. On the one hand, it has been argued that welfarism contains an unsatisfactory representation of individual advantage. On the other hand, it has been claimed that it is impossible to apply welfarism in practice. We might label these the *fundamental* and *pragmatic* argument against welfarism.

The underlying idea of the pragmatic argument is that in public debates we have to base interpersonal comparisons on «objective features of citizens' social circumstances open to view» (Rawls (1993), p. 181). Welfarism implies that interpersonal comparisons should be based on comparisons of preference satisfaction, which in general is considered to be non-observable. Thus, the welfaristic framework violates the constraint of availability of information to which any practicable normative theory is subject.

The fundamental critique of welfarism is concerned with the substantive claims of this framework. Rawls (1971,1993) argues that utility or well-being is not the relevant feature of states of affairs. Appropriate claims should refer to an idea of rational advantage that is independent of any particular comprehensive doctrine of the good, and for this purpose Rawls suggests a list of primary goods. Sen (1985,1992), on the other hand, defends the focus on well-being in social choices, but argues against the idea of well-being implicit in welfarism. Sen claims that «functionings are *constitutive* of a person's being, and an evaluation of well-being has to take the form of an assessment of these constituent elements» (Sen (1992), p. 39).<sup>1</sup>

The frameworks of Rawls and Sen differ, but formally they are closely related. In order to appeal to these positions in public debates, both theories demand that we identify the relevant elements (functionings or primary goods) and determine how valuable these elements are. In this paper, we presuppose agreement about a list of valuable functionings (or primary goods), and deal with the question about how to weight these relevant elements in social choices. In particular we shall consider the tension between this exercise and the Pareto principle. In the Rawlsian framework, the issue has already been elaborated on by, among others, Gibbard (1979).<sup>2</sup> However, Sen (1994) argues that the well-known *indexing impasse* partly follows

<sup>&</sup>lt;sup>1</sup> We will not deal with the issue of capability in Sen's theory; see Sen (1992).

<sup>&</sup>lt;sup>2</sup> See also Plott (1978), Blair (1988), and Arneson (1990).

from «not allowing sufficiently enriched information within the preferential structure» (Sen (1994), p. 14). In particular, Sen argues that «[i]t would be hopeless to look for an indexing rule...<u>without taking</u> – direct or indirect – note of the citizens' orderings of these bundles» (Sen (1991), p. 15).

Sen argues in favour of an *«intersection approach»*, which *«*articulates only those judgements that are *shared* implications of *all* the possible alternative weights» (Sen (1993), p. 47). Let us name this the unanimity criterion. If we apply this line of reasoning in ordinal interpersonal comparisons, we get the following criterion.

The ordinal unanimity criterion: If the bundle of functionings of person A is considered better than the bundle of functionings of person B by every relevant weighting scheme, then person A is better off than person B.

Surely, in most cases, the unanimity criterion will not be applicable for a wide range of distribution problems, but this does not bother Sen: «The possibility remains that...there may well be substantial incompleteness, which the respective theories of justice would have to take into account» (Sen (1991), p. 21-22).

In order to make sense of this criterion, though, we have to delineate the relevant set of weighting schemes. In much of Sen's writings, he indicates that the set of actual preference structures is a good candidate.

(a) *The ordinal actual unanimity criterion*: If the bundle of functionings of person A is considered better than the bundle of functionings of person B by every person in society, then person A is better off than person B.

However, we might argue that it should be legitimate in a normative debate to appeal to weighting schemes that are not represented by any actual preference structure. In that case, if we allow every weighting scheme that assigns non-negative weights to all the valuable functionings, we end up with a dominance criterion:

(b) *The ordinal dominance criterion*: If person *A* has more of every valuable functioning than person *B*, then person *A* is better off than person *B*.

In the analysis, we will apply the dominance criterion in various settings, because this is sufficient in order to clarify the tension between «the intersection approach» and the Pareto

principle. Of course, the dominance criterion is a minimal version of the unanimity criterion, and hence the same results will be present for any extended version of this principle.

However, before we elaborate on this possible conflict, let us briefly comment on the acceptability of the ordinal dominance criterion. There are two possible objections to this criterion, both related to the issue of preference satisfaction. First, we might argue that person B is better off than person A, because possibly person B gets a larger amount of the functionings which he or she values highly (*the taste argument*). Second, we might argue that person B is better off than person A because possibly the intensity of the preferences of person B is stronger than the intensity of the preferences of person A (*the intensity argument*). The taste argument is the essence of the «extended sympathy» approach developed by Sen (1970) and Harsanyi (1977), which states that any evaluation of a person's well-being should have to take into account both the objective circumstances and the subjective attitudes. The intensity argument is part of a more general problem, stressed by among others Sen (1991, 1992) in his discussion of the Rawlsian framework, namely the problem that different people may have different abilities to convert something into a valuable dimension.

Sen (1992, p. 54) circumvents these objections in his discussion of «the intersection approach» by suggesting the possibility of defining preference satisfaction as one valuable functioning. However, in our view, this does not produce a satisfactory solution to the taste problem, as long as there is a plurality of betterness orderings defined over the extended bundle of functionings which then should have to be taken into account. Moreover, within a pragmatic framework, there is an easier way out. Both objections can be overruled because they demand interpersonal comparisons of preference. Hence, as part of a practicable normative theory, we can read the dominance criterion as follows.

The practicable ordinal dominance criterion: If person A has more of every valuable and observable functioning than person B, then we should consider person A as better off than person B.

In the following, we have the pragmatic version of the dominance criterion in mind, where we assume that preference satisfaction is non-observable. The question is now: Can this criterion be reconciled with the Pareto principle?

## 2. Preliminaries

We assume self regarding preferences, and hence the individual preference relation,  $\frac{R_i}{I_i}$ , is defined as an ordering on  $E^m$ , where  $\frac{P_i}{I_i}$  and  $\frac{I_i}{I_i}$  are the asymmetric and symmetric factors.<sup>3</sup> Moreover, we will restrict the discussion to continuous and monotonic preferences. By continuity, we have that for all  $\underline{i \in N}$  and all  $\underline{a \in E^m}$ , the sets  $A_i^{0+} := \{\forall x \in E^m : xR_ia\}$  and  $A_i^{0-} := \{\forall x \in E^m : aR_ix\}$  are closed; by monotonicity, we have that for all  $\underline{i \in N}$  and all  $x_i, y_i \in X_i$   $[\forall k \in \{1, 2, ..., m\} : x_{ik} > y_{ik}] \Rightarrow x_i P_i y_i$ . Finally, let  $\overline{R} = (R_1, ..., R_n)$  be a profile of orderings, and  $\overline{R}$  be the set of all possible profiles of self regarding, continuous, and monotonic preferences.

Let *R* be the social preference relation. We will consider the possibility for establishing an acyclical social evaluation function  $G: \overline{R} \to \overline{\overline{R}}$ , where  $\overline{\overline{R}}$  is the set of all acyclical social preference relations on *X*. Hence, we will neither demand completeness nor transitivity. In the analysis, we work within a single profile framework, where  $R^* \in \overline{R}$  denotes the actual profile of orderings in society. Moreover, we impose the weak version of the Pareto condition on the social preference relation.

The Weak Pareto Condition (WP): For any 
$$x, y \in X$$
 and  $R^* \in \overline{R}$ :  
 $[x_i P_i^* y_i, \forall i \in N] = [\exists j \in N: x_j P_j y_j] x P y$ .

<sup>&</sup>lt;sup>3</sup> An ordering is a reflexive, transitive and complete binary relation.

#### 3. Analysis

In this section, we will attempt to include «the intersection approach» of Sen in a theory of justice which satsifies the Pareto condition. However, it might be instructive first to consider one formalization of Sen's approach which is in direct conflict with WP.

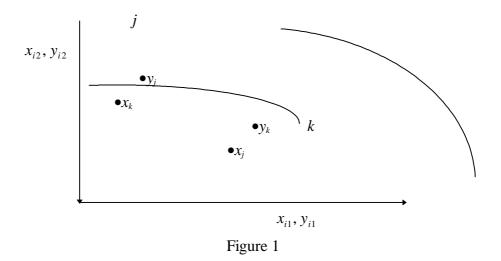
*The Strong Dominance Condition (SDC)*: For any  $x, y \in X$  and  $R^* \in \overline{R}$ :  $[\exists a \text{ permutation } function <math>s: N \to N$  such that  $y_{s(i)} >> x_i, \forall i \in N ] \to yPx$ .

SDC is closely related to the well-known Suppes' principle, but defined in the space of functionings. It states that if for every position in society there is more of every valuable functioning in y than x, then we should prefer y. At the outset, this might seem reasonable. However, as the following observation clarifies, it is in general not possible to reconcile the weak version of the Pareto condition and SDC.

### Observation

If the individuals do not have identical preferences, then SDC is in direct conflict with WP.

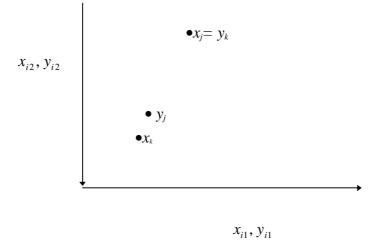
*Proof.* The proof is trivial, and closely related to the discussion in Sen (1970, chap. 9 and 9\*). Hence, we will only illustrate the argument in Figure 1. In this case, we assume that there are only two individuals and that the individual states are two-dimensional.



In Figure 1, SDC implies that yPx, which contradicts WP.

The problem with SDC is of course that it considers a pair by pair increase in all the available bundles of functionings an improvement *irrespective of the ownership of the bundles*. On this basis, SDC concludes that y is better than x. But since the distribution of functionings is more compatible with the individual preferences in x than in y, x is better than y according to the weak version of the Pareto condition .

The tension between SDC and WP does not necessarily imply that «the intersection approach» is in conflict with the weak Pareto condition in all settings. In our view, it simply tells us that SDC introduces too broad a version of «the intersection approach» in social choices. The weak Pareto condition should be considered an appropriate condition when the interests of individuals are aligned. Hence, the purpose of introducing «the intersection approach» should be to find some practicable condition which can solve *cases of conflict*. In this respect, let us consider the following two-person case in Figure 2.





In Figure 2, person j prefers x and person k prefers y. How should we deal with this conflict? Following the «the intersection approach», one possibility might be to argue that we should prefer y to x, because both individuals agree that being individual k in social state x is worse than being individual j in social state y. In this case, we assign absolute priority to the worse off, which is the Rawlsian approach to distributive justice. Within our framework, the Rawlsian idea can be captured by the following condition.

The Rawlsian Dominance Condition (RDC): For any  $x, y \in X$  and  $R^* \in \overline{R}$ :  $[\exists j, k \in N : x_j, y_j, y_k \in \{a \in E^m : a >> x_k\}, \& x_i = y_i, \forall i \neq j, k] \rightarrow xPy$ . However, we would not like to narrow our discussion to the Rawlsian approach. Our purpose is to see whether there is *any* Parertian theory of justice compatible with «the intersection approach», and thus we would like to consider a much weaker condition than RDC on the social preference relation. The opposite extreme of the Rawlsian position would be to only care about gains and losses in cases of conflict. Within such a «quasi-utilitarian» framework, would it be possible to say anything about the two-person conflict in Figure 2? This framework demands a basis for interpersonal comparisons of gains and losses, which we have not outlined so far. In our view, however, the following version of the dominance criterion should be a straightforward extension of «the intersection approach» to cardinal comparisons:

The cardinal dominance criterion: If person A loses (gains) more of every valuable functioning than person B, then the loss (gain) in well-being of person A is greater than the loss (gain) in well-being of person B.

Let us now reconsider the problem in Figure 2. In this conflict, the loss imposed on person k by choosing x instead of y is greater in every observable dimension than the loss imposed on person j if we make the opposite choice. Thus, according to the cardinal dominance criterion, also a comparison of gains and losses seems to imply that we should prefer y to x in this two-person conflict.<sup>4</sup>

Based on our discussion of Figure 2, it seems reasonable to impose the following very weak condition on the social preference relation.

The Double Dominance Condition (DDC): For any 
$$x, y \in X$$
 and  $R^* \in \overline{R}$ :  
 $[\exists j, k \in N : x_j \ge y_k > x_k > y_j \& x_i = y_i, \forall i \ne j, k] \rightarrow xPy$ .<sup>5</sup>

In our view, *any* reasonable practicable theory of justice based on «the intersection approach» ought to satisfy DDC. DDC is a very conservative condition, saying that if both a comparison of gains and losses and the situation of the worst-off in two-person cases support the choice of one of the social states, then we should prefer this state. However, it turns out that even

<sup>&</sup>lt;sup>4</sup> What about gains and losses in preference satisfaction? In Figure 2, *k* is considered to be in the worst off position in *x*. Hence, since  $x_j = y_k$ , it seems also reasonable to assume that the loss in preference satisfaction is greater for person *k* than for person *j*.

<sup>&</sup>lt;sup>5</sup> Of course, DDC is subsumed under RDC, and hence any tension between DDC and the weak Pareto principle is also present between RDC and WP. Moreover, we could have reported a slightly stronger result by imposing strict inequality between  $x_j$  and  $y_k$  in DDC. However, in order to save notation in the proofs, we content ourselves with discussing this version. The modifications are straightforward.

though DDC is not in direct conflict with the weak Pareto condition, in general the two conditions are incompatible.

#### Proposition

There exists an acyclical social evaluation function G which satisfies WP and DDC if and only if all individuals have identical preferences.

Proof. Se Appendix.

The line of reasoning in the «only if» part of the proof can be illustrated in Figure 3.

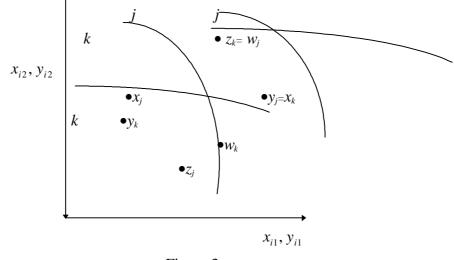


Figure 3

In Figure 3, we have the choice between x, y, z, w. The weak Pareto condition demands  $yPw \wedge zPx$ , DDC implies  $xPy \wedge wPz$ , and hence we have a cycle which contradicts the assumption on *G* in the proposition.

Thus, «the intersection approach» of Sen does not escape the indexing impasse. Even in it's weakest form, this approach is incompatible with any reasonable theory of justice that satisfies the Pareto principle. In this respect, notice that we have worked with a very broad definition of a reasonable theory of justice. We have accepted incompleteness, lack of transitivity, the weakest version of the Pareto principle, and any consequensialistic view on distributive justice. Hence, the reported result should be of concern for anyone interested in «the intersection approach».

Let us close this section by briefly commenting on the relationship between the proposition reported in this paper and the well-known indexing paradox of Gibbard (1979). Gibbard considered the possibility of making interpersonal comparisons by comparing income sets within a price regime, and established that this approach, within a Rawlsian framework, would produce cycles in the social preference relation if combined with the Pareto principle. This result differs from our proposition in two respects. First, our proposition refers to interpersonal comparisons that rely on shared implications of preference judgements on bundles of valuable objects.<sup>6</sup> Second, our proposition is of more general concern than Gibbard's result, because it covers any view on distributive justice.

#### 5. Concluding Remarks

Do these results imply that «the intersection approach» should be abandoned. In our view, this is too hasty a conclusion. As Rawls (1971) remarked: «All theories are presumably mistaken in places. The real question at any given time is which of the views already proposed is the best approximation overall».<sup>7</sup> The welfaristic approach is in line with the Pareto principle, but (at least) vulnerable to the pragmatic argument. «The intersection approach», defined on some valuable objects (functionings or primary goods) is sometimes in conflict with the Pareto principle, but might still be the best approach to interpersonal comparisons within a pragmatic framework. Why? In our view, it seems implausible that we will ever find a practical public basis for interpersonal comparisons of preference satisfaction. And an inescapable result of this (possible) fact might be that any pragmatic theory of justice sometimes will be in conflict with the Pareto principle.

<sup>&</sup>lt;sup>6</sup> In some sense, Gibbard's framework might also be considered part of «the intersection approach», because the underlying judgements of interpersonal comparisons in Gibbard's analysis can be expressed as shared implications of preference judgements on income sets.

<sup>&</sup>lt;sup>7</sup> Rawls (1971), p. 52

#### **Appendix**

#### Proof of Proposition

Suppose the assumptions of the proposition is satisfied.

(a) We will first establish the following lemma:

#### Lemma

For any  $R^* \in R$ , if the individuals do not have identical preferences, then there exist two individuals  $i, j \in N$ , and six bundles  $a, b, c, d \in E^m$  such that  $bP_j^*c \& aP_k^*d \& fP_ke \& eP_jf \& c \gg a \& d \gg b \& e \gg d \& f \gg c$ .

*Proof.* Define the sets  $A_i^+ := \{ \forall x \in E^m : xP_i a \}$  and  $A_i^- := \{ \forall x \in E^m : aP_i x \}$ . Moreover, define an open neighbourhood of f03000000000 as  $N_e(a) := \{ \forall x \in E^m : d(x,a) < e \}$ , e > 0. If the individuals do not have identical preferences, then there exist bundles  $\underline{a, b' \in E^m}$  and individuals  $\underline{i, j \in N}$  such that;  $\underline{aR_i b' \land b'P_j a}$ .

(i) Let us now prove that there exist two bundles  $b, c \in E^m$  such that  $bP_j^*c$ . By continuity the set  $A_j^{0^-}$  is closed, and hence the set  $A_j^+ = E^m - A_j^{0^-}$  is open. Notice that  $b' \in A_j^+$ . Because  $A_j^+$  is open there exists an open in the neighbourhood of b' which is contained in  $A_j^+$ . The vector  $b = \left( \left( b_1' - \frac{\mathbf{e}}{m} \right), \dots, \left( b_m' - \frac{\mathbf{e}}{m} \right) \right), \mathbf{e} > 0$ , is in the neighbourhood of b', and hence  $bP_ja$ .

By continuity, the set  $B_j^- := \left\{ \forall x \in E^m : bP_j x \right\}$  is open.  $a \in B_j^-$ , and thus there exists an open neighbourhood of f03000000000 a which is contained in  $B_j^-$ . The vector  $c = \left( \left( a_1 + \frac{\bar{e}}{m} \right), \dots, \left( a_m + \frac{\bar{e}}{m} \right) \right)$ , e > 0, is in the neighbourhood of a, and hence  $bP_j c$ .

(ii) In this step, we will prove that there exists a bundle  $d \in E^m$  such that  $aP_k^*d$ . By assumption,  $aR_k^* \stackrel{b'}{\longrightarrow}$ , and hence it is easily seen from (i) that monotonicity and transitivity of preferences implies  $\frac{aP_ib}{m}$ .  $b \in A_i^-$ , and thus there exists an open neighbourhood of  $f_{03000000000} \stackrel{b}{\longrightarrow}$  which is contained in  $A_k^-$ . The vector  $d = \left( \left( b_1 + \frac{\overline{e}}{m} \right), \dots, \left( b_m + \frac{\overline{e}}{m} \right) \right), e > 0$ , is in the neighbourhood of b, and hence  $aP_k^*d$ .

(iii) By construction in (i) and (ii),  $c \gg a \& d \gg b$ .

(iv) In this step, we will prove that there exist bundles  $e, f \in E^m$  such that  $fP_k^*e \& eP_i^*f$ .

From (ii) and (iii) we have that  $aP_k^*d \& c \gg a$ . Define  $d = \left( \left( b_1 + \frac{\overline{e}}{m} \right), \dots, \left( b_m + \frac{\overline{e}}{m} \right) \right), e > 0$ . Monotonicity and transitivity of preferences implies that  $fP_k^*d$ . By continuity, the set  $B_j^- := \left\{ \forall x \in E^m : bP_j x \right\}$  is open.  $a \in B_j^-$ , and thus there exists an open neighbourhood of d which is contained in  $\frac{B_j^-}{m}$ . The vector  $d = \left( \left( b_1 + \frac{\overline{e}}{m} \right), \dots, \left( b_m + \frac{\overline{e}}{m} \right) \right), e > 0$ , is in the neighbourhood of d, and hence  $\frac{bP_jc}{m}$ .

From (i) and (iii), we have that  $bP_j^*c \& d \gg b$ . Moreover, we know from the first part of this step that  $e \gg d$ . Thus, monotonicity and transitivity of preferences implies that  $eP_j^*c$ . By continuity, the set  $B_j^- := \{\forall x \in E^m : bP_j x\}$  is open.  $a \in B_j^-$ , and thus there exists an open neighbourhood of c which is contained in  $E_j^-$ . But from the first part of this step it follows that f is in this neighbourhood of c, and hence  $\frac{bP_jc}{c}$ .

(v) By construction in (iv),  $e \gg d \& f \gg c$ .

Hence, the lemma is proven.

(b) We will now prove the «only if» part of the proposition.

Consider the social states  $x, y, z, w \in X$ , which are defined by;  $x:=(x_i = e, x_j = c)$ ,  $y:=(y_i = a, y_j = e)$ ,  $z:=(z_i = e, z_j = b)$ ,  $w:=(w_i = d, w_j = e)$ , and  $x_k = y_k = z_k = w_k = e$ . By the lemma and DDC, we have:  $xPy \wedge wPz$ . By the lemma and WP we have:  $yPw \wedge zPx$ . Hence, we have the cycle xPy, yPw, wPz, zPx, which contradicts the assumption on *G*.

(c) Finally, we will show that when individuals have identical preferences, a version of the leximin rule satisfies WP and DDC.

Let  $\underline{R_k}$  be the ordering of a representative individual. For all social states,  $\underline{x \in X}$ , define a permutation function,  $\underline{s_x: N \to N}$ , as follows;  $x_i P_k x_j \Rightarrow s_x(i) > s_x(j)$  and  $x_i I_k^* x_j \to s_x(i) > s_x(j) \lor s_x(i) < s_x(j)$ . Define the inverse of the permutation function by;  $\underline{r_x(i)} = \underline{s_x}^{-1}(i)$ . Hence,  $\underline{r_x(1)}$  is the individual who is ranked as worst off in social state x. Define the following leximin rule:  $\left[\exists j \in N : y_{r_y(i)} I_k^* x_{p_x(i)}, \forall i < j \land y_{r_y(j)} P_k^* x_{p_x(j)}\right] \to yPx$ ; otherwise yIx.

It is straightforward to show that the leximin rule above is acyclical and satisfies both WP and DDC when the preferences are identical.ð

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