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Uncertainty in the Theory of Public Finance

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Abstract

This paper discusses the role that the economics of uncertainty has played in the theory of public finance. From being mostly concerned with its choice-theoretic foundations in the 1950s and '60s, the theory of expected utility maximization and risk averse behaviour has contributed decisively to the development of several areas of the theory of public finance. Three of these have been chosen here to illustrate the general point: Taxation and risk taking, the role of uncertainty in public expenditure and the theory of tax evasion and compliance.

JEL Classification: D80, H24, H26, H40

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1. Introduction

In the early 1960s, when I was first taught the economics of uncertainty by my teacher and mentor Karl Borch, the state of the economics of uncertainty was in some respects a bit curious. At the time, most textbooks in microeconomic theory would include a discussion of uncertainty somewhere in the chapters on consumer theory. But the treatment would usually be limited to an exposition of the expected utility theorem with no applications to real economic decisions in areas like saving and investment - even its application to portfolio decisions would frequently go unmentioned[1]. This was in sharp contrast to the remaining parts of the consumer theory chapters, where the emphasis would be not on the axioms guaranteeing the existence of a utility function but on the implications of utility maximization for commodity demand, labour supply and saving. Like many other economists at the time I was struck by this disparity and set out to exploit the exciting opportunities that were open for someone who was interested not primarily in the axiomatic foundations but in the analysis of economic issues in which uncertainty played an essential part.

From the exploration of the economics of uncertainty in the more specific sense, my interests soon changed in the direction of public finance and the analysis of issues related to public policy. However, my first interest has followed me through the years, leading me to study in particular a number of problems in public finance in which uncertainty plays a significant role. In this lecture I would like to take you on a tour of these areas, discussing some problems in public economics that are of very long standing in the field as well as some that have only more recently begun to attract the attention of researchers. The tour will demonstrate the power of "the economics of uncertainty in action"; the ability of the theories of decision making and market equilibrium under uncertainty to illuminate problems that are both of great practical importance and policy relevance.

Incidentally, when I refer to my field of interest, I shall sometimes refer to it as "public finance" and sometimes as "public economics". Of these, the first is the oldest, while the second was introduced in the 1960s by economists who felt that the reference to finance made

[1] This generalization about the content of the textbooks does not apply to Borch's own innovative work on the economics of insurance. A good example of his own approach is the published version of his Geneva Lecture (Borch 1981).

the field appear too narrow[2]. I have some sympathy with this view; on the other hand, the term "public finance" is a reminder of the fact that there are many issues which are of common interest to economists working in the field of finance as now commonly understood and in public sector economics.

One of these is the topic that I shall start with, viz. the problem of taxation and risk-taking. This is a field where the overlap of problems and methods between economics and finance is especially large, although I will also argue that from a social point of view there may be other aspects of risk-taking that are just as important as those that have been inspired by the finance literature. From taxation I turn to the theory of public expenditure and discuss both the effects of public expenditure on private risk-taking and the problem of evaluating the benefits from public spending when rates of return are uncertain. My third topic concerns the problem of tax administration where uncertainty again plays a crucial role in the theory, particularly as regards the issue of tax compliance.

Let me also note that during the whole of its existence - i.e. from its beginnings in the work of Adam Smith and David Ricardo until the present - public economics or public finance has been concerned with two different approaches to the study of the public sector. One of these is positive, taking up problems like the effects of taxes on saving, labour supply and portfolio choice, the effects of public pensions on the retirement decision etc. The other is normative, studying questions like the optimal design of the tax system, the optimal supply of publicly provided goods and the best choice among alternative social security systems. In the following I will take up elements from each of the two approaches.

2. Taxation and risk-taking

Unlike some other problems in economics, this topic - or at least its formal theory - has historically speaking a well-defined beginning. This is 1944, the year of publication of the seminal paper by Domar and Musgrave (1944) on taxation and risk-taking. Does the taxation of capital income deter or encourage risk-taking? The authors pointed out that previous discussions had tended to suggest that the effect of taxation was negative, primarily because of the adverse effect on the volume of saving. However, they then argue that a more relevant

[2] In the English language literature the book by Leif Johansen (1965) may have been the first to use this term. In France, Serge Kolm began to refer to the field as "économie publique" at about the same time.

specification of the problem relates to the *composition* of saving: Does the taxation of risky capital income lead to more or less investment in risky assets? In a simple model of an individual investor Domar and Musgrave argued that the effect is likely to go in the direction of more risk-taking, although this result depends in an important way on the provisions for loss offset. Suppose that there is full loss offset, so that the government's share of the investor's loss is the same as its share of his gain. Then, under what they argue to be reasonable assumptions the risk-sharing effect dominates the effect of the expected rate of return reduction, and the result follows.

From the present perspective, the Domar-Musgrave analysis of the risk-return trade-off strikes one as being somewhat ad hoc. The risk attached to an investment is represented by the expected value of all negative outcomes of the rate of return, and the investor is assumed to have preferences over the risk and the expected rate of return. There is no reference to expected utility - not surprisingly, since this was two years before the publication of von Neumann and Morgenstern's proof of the expected utility theorem. Moreover, as already mentioned, it did in fact take a long time from the appearance of the proof until expected utility began to be used as a tool of analysis of real economic problems. The mean-variance analysis pioneered by Markowitz (1952) was further developed in Tobin's (1957-58) theory of liquidity preference, but the first modern analyses of the problem of taxation and risk-taking did not appear until the 1960s with articles by Mossin (1968) and Stiglitz (1969).

Let us recall the simplest version of the Mossin formulation, which is especially clear. An investor can allocate his initial wealth, A, among two assets, one with a safe return of zero ("money") and one risky asset with stochastic return x, which can take on both positive and negative values but whose expected outcome is positive. Let m and a be the amounts invested in the two assets and A the initial wealth of the investor, so that

$$m+a=A. \qquad (1)$$

The final wealth of the investor is then

$$Y= m+a[1+x(1-t)]=A+ax(1-t), \qquad (2)$$

where t is the rate of capital income tax and where there is full loss offset. Assume now that the investor maximizes the expected utility of final wealth, $E[U(Y)]$, where U is a concave function, so that there is risk aversion. The first order condition that characterizes the optimal choice of a is

$$E[U'(Y)x(1-t)]=0, \qquad (3)$$

while the second order condition is satisfied by the assumption of concavity. An interior solution with $a>0$ exists by virtue of the fact that we have assumed the expected rate of return on the risky asset to be positive, i.e. $E[x]>0$.

By implicit differentiation we obtain the tax effect as

$$\partial a/\partial t=a/(1-t), \qquad (4)$$

which is positive, confirming the Domar-Musgrave conclusion. The result is remarkable both for its simplicity and for the fact that it is apparently independent of the shape of the utility function. However, it must be kept in mind that the initial allocation between safe and risky assets in the portfolio does indeed depend on the shape of the utility function; in other words, the size of the individual's holding of a depends on the degree of risk aversion as represented by the concavity of the utility function. The explanation for the simplicity of the result can be related to mean-variance analysis: By adjusting the portfolio in the way indicated by the result, the investor is able to keep constant the mean and variance of final wealth that was the result of his initial choice of an optimal portfolio[3].

The result stated in (4) cannot really be taken as a theoretical rejection of what Domar and Musgrave referred to as the conventional view, since it disregards the effect on risk-taking that might occur via a reduction in the amount of saving. The share of risky assets in the portfolio could increase, but if the size of the portfolio were at the same time to shrink, the net result might indeed be a reduction of risk-taking. It was shown in Sandmo (1969), however, that the result (4) continues to hold in a two-period model incorporating both savings and

[3] Let μ and σ be the mean and the standard deviation of the risky rate of return; then, from equation (2), we have that $E[Y] = A + a\mu(1-t)$ and $S[Y] = a(1-t)\sigma$. Taking the differential $dE[Y] = \mu(1-t)da - a\mu dt = 0$, it follows that $da/dt = a/1-t)$. Performing the same operation on the standard deviation $S[Y]$, the same implication follows.

portfolio decisions: Perhaps surprisingly, the effect of the increase in the tax rate leaves the volume of saving unaffected, while the portfolio effect continues to correspond to that which is described by equation (4).

One simplification that has been made in this analysis is that the rate of return on the safe asset, *r*, is zero, and this assumption naturally dodges the issue of whether the return on the safe asset is taxed or not. Suppose that $r>0$ and that the rate of return is taxed at the same rate as that of the risky asset. Then the simplicity of the result (4) is lost; the effect on the demand for the risky asset can be decomposed into a positive substitution effect which is similar to (4) and a negative income effect[4]. The net effect depends on which of these two partial effects is the largest.

This modification of the initial result was to be expected, but it is interesting to consider the economic intuition behind it. In the case of $r=0$, the taxation of income from capital paradoxically favours the risky asset through the risk sharing effect whereby the government takes on some of the risk that would otherwise have to be born by the investor. With the two rates of return being taxed at the same rate, the discrimination in favour of the risky asset ceases to exist and it is accordingly no longer clear that capital income taxation encourages risk-taking. This interpretation is supported by consideration of the case where the taxation of capital income applies only to the excess return, i.e. to the return in excess of the rate of return on the safe asset. In that case, the tax base is equal to $a(x-r)$ and the result (4) holds without modification. This case shows very clearly that the apparent tax discrimination against the risky asset is in fact a case of discrimination in its favour via the risk-sharing effect.

The two-asset framework is clearly restrictive, and it is important to consider the case of several risky assets[5]. As shown in Sandmo (1977), with an arbitrary number of risky assets and assuming either that $r=0$ or that only the excess return is taxed, the result (4) holds for each and every one of the risky assets. The demand for all risky assets increases and the demand for the safe asset declines. The two asset model therefore provides a good representation of the more general case.

[4] The negativity of the income effect is based on the assumption that the risky asset is a normal good, i.e. that the demand increases with an increase in initial wealth. This in turn follows from the assumption of decreasing absolute risk aversion (DARA).

[5] The case of several safe assets is without interest since only one of these assets would be included in the investor's portfolio.

Another dubious assumption in terms of realism is that of full loss offset. In some cases, no doubt, it is realistic, since the tax laws in many countries allow for losses to be deducted against the positive capital income from other assets. In other cases full loss offset may appear to be an extreme assumption, and modifying it in the direction of only partial loss offset will naturally lead to a qualification of the results. However, it was shown by Mossin (1968) that this change of assumption does not necessarily overthrow the result (4); the deviation from the case of full loss offset has to be sufficiently large for a reversal of the result to occur.

The result that capital income taxation tends to encourage risk-taking is in the nature of an empirical hypothesis. Can it be subjected to empirical verification? Attempts to carry out econometric tests actually encounter a number of difficulties. One of these is the classification of assets into the two categories of safe and risky. Although it may be relatively easy to identify assets that are safe in nominal terms, finding assets that yield a safe rate of return in real terms – which is presumably what we are interested in - is considerably more difficult if not impossible. A reasonable interpretation of the model is that the two assets should be interpreted as being "relatively safe" and "relatively risky", and that the assumption that one of them is completely safe must simply be regarded as an analytical simplification. This raises the question, however, of how an outside observer may be able to identify the two types of assets, given that the distinction between what is relatively safe and relatively risky must in the last instance reflect the personal judgement of the investor. Possibly even more serious is the fact that real-world capital income taxation is characterized by a number of special provisions for the taxation of different types of assets. The returns on assets like bank deposits, shares and housing are in reality taxed at effective rates that tend to vary widely from one asset to another, and the effects of this tax differentiation on portfolio composition tend in an empirical investigation to overshadow whatever effects there may be of capital income taxation as a risk-sharing device[6].

Risk-taking has other aspects than those represented by the theory of financial portfolio composition. While financial wealth can be diversified, the scope for diversification regarding *human* capital accumulation is limited. The effects of taxes on human capital and occupational choice have been studied by Eaton and Rosen (1980) and Kanbur (1981). Eaton

[6] For a survey of the literature that also discusses empirical work on a wider range of issues than I have covered here see Poterba (2002)

and Rosen analyze the case where the accumulation of human capital increases the wage rate earned by the individual but where the productivity of human capital is uncertain. Earnings taxation may in this case increase investment in human capital for precisely the reason that income taxation increases risk taking in the portfolio model: The government takes on part of the risk that would otherwise have to be born by the individual. They also consider the question of the optimal taxation of earnings and conclude that a positive marginal tax rate on earnings can be justified even under first best conditions. The reason is that the tax sharing effects of the tax partly compensate for the lack of insurance markets for human capital[7]. In the paper by Kanbur (1981) the margin of risk taking is the decision relating to occupational choice: Individuals can decide whether to become workers with a fixed income or entrepreneurs whose income is risky. In equilibrium the entrepreneurs must choose to employ the total number of workers; in addition, the total expected utility of workers and entrepreneurs must be equal in order for the equilibrium to be such that neither workers nor entrepreneurs wish to change their occupation. Kanbur's main interest is in the normative aspects of the model: How should government tax policy be designed in order to promote social welfare? He derives some rules for the shape of the optimal income tax system, but the effects of the marginal tax rate on the amount of risk taking, i.e. on the number of entrepreneurs, are ambiguous and difficult to relate to those that can be derived from the portfolio approach.

3. Uncertainty and public expenditure

When it comes to the analysis of uncertainty and public expenditure there are, as in other areas of public finance, two aspects of the basic theoretical analysis where formal theories in the economics of uncertainty could be applied. One is the question of how public expenditure affects private decisions regarding risk-taking; the other is the issue of the optimal supply of public goods – or, more generally, publicly provided goods – when costs or benefits are uncertain. I begin with the former question, which has not received much attention in the literature but can be illuminated by a simple extension of the Domar-Musgrave-Mossin-Stiglitz framework.

[7] A similar problem of optimal taxation has been analyzed in a saving context by Varian (1980).

We now assume that the utility function of the individual depends both on final wealth – which we may identify with private consumption – and the supply of public goods, *G.* The utility function, which is still assumed to be strictly concave in *Y*, is therefore *U(Y, G).* The individual consumer/investor maximizes expected utility *E[U(Y, G)]*, taking *G* as given, subject to equation (2) above. The first-order condition that characterizes the optimal investment in the risky asset is, with subscripts denoting partial derivatives,

$$E[U_Y x(1-t)]=0. \qquad (5)$$

This looks formally identical to (3), but the difference is of course that (5) now depends on *G.* To see the nature of the dependence, we differentiate (5) with respect to *G* to obtain the comparative statics result

$$\partial a/\partial G = E[U_{YG} x]/-E[U_{YY} x^2](1-t). \qquad (6)$$

The denominator in the expression on the right-hand side is clearly positive from the concavity assumption, so the sign of the whole expression is equal to the sign of $E[U_{YG} x]$.

Clearly, U_{YG} may be of either sign. Note that while in an ordinal utility framework, the sign of a second-order cross partial derivative of the utility function has no meaning (since it is not invariant to monotone transformations of the utility function) in a cardinal von Neumann-Morgenstern context it is indeed meaningful and can be used to classify goods into substitutes ($U_{YG}<0$) and complements ($U_{YG}>0$). Which is the more realistic assumption? A general answer to this question is hardly possible, since it is easy to think of important categories of public expenditure that are likely to be complements (roads) as well as substitutes (child care) for private consumption. Given the ambiguity of this sign, it is obvious that a general statement regarding the sign of the term $E[U_{YG} x]$can hardly be established. However, some insight can be gained by studying the special case where private and public goods are perfect substitutes, which may be reasonably close to capturing the realities public versus private provision of health care and education. With perfect substitutability (linear indifference curves) the utility function becomes simply

$$U = U(Y+G), \qquad (7)$$

and the derivative (6) becomes

$$\partial a/\partial G = E[U''(Y+G)x]/-E[U''(Y+G)x^2](1-t). \quad (8)$$

Making the reasonable assumption that absolute risk aversion (defined as $-U''(Y+G)/U'(Y+G)$) is decreasing, it can be shown that the numerator of this expression is positive, so that an increase in the supply of publicly provided goods increases the willingness to take risks. In fact, in this case an increase in the provision of public or publicly provided goods is equivalent to an increase in initial wealth. By continuity, if private and public goods are close although not perfect substitutes, it must continue to be true that an increased public provision increases the willingness to bear risk.

Is the case where public and private goods are substitutes of particular interest? Above, for brevity, I have sometimes used the term "public goods" to refer to the goods and services provided by the government, but this terminology may in fact be confusing. In the classic treatment by Samuelson (1954), the term public goods was used to refer to goods that are equally available to all, such as "an outdoor circus or national defence". But most goods provided by modern welfare states are not in fact of this type; public education, health services and care for children and for the elderly are private goods that could alternatively have been financed by the individual consumers themselves through the purchase of privately provided substitutes. For a large share of the public expenditure of modern governments, therefore, the provision of public goods – the shorter term being used for the more cumbersome "public goods or publicly provided private goods" – can be reasonably taken to encourage private risk-taking. This is of course on the assumption that the simple portfolio model with one safe and one risky asset can be taken as representative of the wider aspects of risk taking that also have to do with human capital formation and occupational choice.

Taking a broad view of our results concerning the effects of income taxation and public expenditure on private risk taking, it is hard to avoid the conclusion that a government that taxes private risky investments and uses the revenue to provide goods like health, education and care for the elderly and the young encourages private risk-taking behaviour: Through taxation of risky returns with loss offset it provides insurance, and through the provision of "welfare goods" it creates a social safety net. The joint effect of these policies, therefore, is to

increase the willingness of individuals to carry risk. Is this a good thing? Implicit in some of the literature seems to lies a belief that the answer is yes, although the reasons for this are often not spelled out[8]. I believe that there are two main reasons that may be called upon to support this conclusion. One is the incompleteness of risk markets, in particular the incomplete coverage of various risks related to human capital that private insurance - for reasons related to adverse selection and moral hazard - fails to provide. Another is the existence of other disincentives that are entailed both in the tax system and in the pattern of public expenditure. It is likely that some of these tend to discourage labour effort as well as the willingness to save and invest, and the encouragement to risk-taking may partly offset these adverse incentives.

Turning briefly to normative aspects of public expenditure analysis, the literature in this area has paid particular attention to the issue of how public production decisions should take account of risk. Should government production agencies simply maximize expected net benefits or profits, or should their decisions reflect the risk aversion of private agents? The answer to this question depends crucially on the assumptions that one makes about the structure of the economy and about the constraints on the choice of policy instruments. In Sandmo (1972) it is assumed that the various production sectors of the economy are subject to technological risk and that in each sector there are both private and public firms that produce the same kind of commodity subject to the same technological risk. Private firms choose their investments with the objective of maximizing their stock market value. In equilibrium, this value will reflect the investors' average evaluation of the riskiness involved in the investment; in other words, the risk premium will be equalized among investors. As first shown by Diamond (1967) this stock market equilibrium achieves a constrained Pareto optimum. The implication of this for public sector investment is that public firms should imitate private companies by including a risk premium which is the same as that established in the stock market for private investments in the corresponding risk class. This risk premium should reflect the covariance between the marginal utility of private consumption and the risky return, which under natural assumptions would be positive.

[8] Thus, Domar and Musgrave (1944, p. 420) state that the optimal tax rate is the one that leads to a maximum of risk-taking, although they do not present a welfare theoretic justification of this view. A modern treatment is provided by Sinn (1995), who shows how the benefits of the welfare state as an insurance device must be traded off against adverse efficiency effects of redistributive policies.

This conclusion appears to go against the outcome of a classical analysis by Arrow and Lind (1970) who argue that the government ought to maximize expected net benefits, taking no account of private risk aversion. However, some of the assumptions of the Arrow-Lind analysis are significantly different from those outlined above. They assume, first, that public investment takes place in risk classes where there is no private investment. Second, they make the assumption that the government has other instruments of economic policy that can be assumed to stabilize the private consumption of individuals. Then the covariance between project returns and the marginal utility of consumption would be zero, so that no risk premium should be used in the analysis of public investment. Whether this case is of sufficiently wide applicability to provide the foundation for a general conclusion about public investment under uncertainty is a matter of judgement; in any case, it is valuable to have alternative models that can be applied to different economic settings.

More attention has been given in the literature to technological risk than to uncertainty about benefits although this is often a real issue in the provision of public goods. Rather than going into a detailed survey of the literature that exists on this topic, I will just mention one case where this issue is one of major economic importance. This concerns the problem of global warming that has been discussed in a number of contributions, in particular in the famous Stern Review (Stern 2007). Stern points out that there is considerable uncertainty about the extent of global warming and its effects, although the most likely outcome is that the global temperature will increase to a significant degree as the result of the emission of greenhouse gases. The rational policy to follow in such circumstances, he argues, is to devote some of society's current resources to uses that can be regarded as insurance against the most adverse outcomes of global warming. This is an area where there remains considerable scope for formal analysis, and it is hard to think of a topic that is of greater importance both for individual countries and for the world as a whole.

4. Tax administration and evasion

In much of the literature on taxation one takes a very simple view of tax compliance: Taxpayers simply obey the rules as laid down in the tax legislation. However, the empirical indications that this is not so are very strong, although hard evidence is by the nature of the

case difficult to collect. I will not go into the empirical part of this subject[9] but concentrate on the theory, which contains some interesting applications of the basic theory of behaviour under uncertainty[10].

That the analysis of income tax evasion could be approached using the methods of the economics of uncertainty was first suggested by Allingham and Sandmo (1972). The analysis has a number of features in common with the two-asset portfolio model. We may envisage the taxpayer as he sits down at the end of the tax year to fill in his income tax return. His pre-tax income is given, and the decision he faces is simply how much of his income to report to the government tax collector. The amount that he chooses to report is the equivalent of the safe asset: He knows that it will be taxed at the statutory rate of tax. The amount that he does not report corresponds to the risky asset: His evasion may be discovered, in which case he is liable to pay a penalty, or it may not be discovered, implying that he will not pay any tax at all. Thus, the tax rate on the amount of income evaded is stochastic, corresponding to the uncertain rate of return on the safe asset in the model of portfolio choice. The main features of the model are as follows[11].

Let W be the gross income of the taxpayer. There is a proportional income tax at the rate t. The amount evaded, i.e. the amount of underreporting, is e, so that the reported income is $W-e$. If the tax evasion is not detected by the tax authority, the net income of the taxpayer is accordingly

$$Y=W-t(W-e)=(1-t)W+te. \qquad (9)$$

If, however, it is discovered that the taxpayer has underreported his income, he will pay a penalty rate[12] of tax, θ, on the evaded amount, so that his net income in this case is

$$Z=(1-t)W+te-\theta e=(1-t)W-(\theta-t)e. \qquad (10)$$

[9] Empirical surveys in this area include Andreoni, Erard and Feinstein (1998), Schneider and Enste (2000) and Slemrod and Yitzhaki (2002).
[10] An excellent early survey of the theoretical literature is Cowell (1990).
[11] I use the version of the model contained in my later survey article (Sandmo 2005).
[12] This terminology has become common usage, although it does not correspond to the more everyday meaning of ”penalty rate”, which should rather be identified with $\theta-t$.

It should be kept in mind that an unrealistic simplification in this model is the assumption that all income is equally unknown to the tax collector. This is clearly not the case, and the analysis should therefore be interpreted as applying to that part of his income which the taxpayer can in fact evade without certainty of detection.

The taxpayer's subjective probability of detection is p[13]. He chooses the amount evaded so as to maximize his expected utility, which is

$$E[U]=(1-p)U(Y)+pU(Z). \qquad (11)$$

It is assumed that U is increasing and concave, so that the taxpayer is risk averse. The first-order condition for an interior solution is

$$(1-p)U'(Y)t-pU'(Z)(\theta-t)=0, \qquad (12)$$

or

$$U'(Z)/U'(Y)=(1-p)t/p(\theta-t). \qquad (13)$$

To see the empirical implications of the model one has to differentiate the first order conditions with respect to the exogenous variables W, t, θ and p. It turns out that the signs of the derivatives $\partial e/\partial\theta$ and $\partial e/\partial p$ are both unambiguously negative; a higher penalty rate or a higher probability of detection always tend to discourage tax evasion. Intuitively, this is seen from (13) by noting that the right-hand side of the equation can be interpreted as the relative price of income in the states of detection and non-detection, and this depends negatively on θ and p. When θ or p increases, Z increases relative to Y, which implies that there must be less evasion[14].

It seems reasonable to assume that a higher gross income will increase evasion if one believes that people become more willing to engage in risky activities as they get richer. This is also

[13] The comparative statics properties of the model are basically unchanged if the probability of detection is assumed to vary (presumably positively) with the amount of evasion. The model makes the simplifying assumption that a tax audit results in certain discovery; for a discussion of the additional complication of uncertainty about the amount detected see Snow and Warren (2005).

[14] Christiansen (1980) shows that if the expected gain from tax evasion is held constant, an increase of the penalty rate combined with a decrease of the probability of detection will always reduce tax evasion.

predicted by the model if one makes the additional assumption of decreasing absolute risk aversion. As regards the effect of the regular marginal tax rate, a notable feature of the original A-S model is that an increase of the tax rate has an ambiguous effect on tax evasion. There is an income effect which is negative; higher taxes make the taxpayer poorer and therefore less willing to take risks. But there is also a substitution effect that works in the direction of increased evasion. In fact, the effect of the marginal tax rate on evasion can be written as

$$\partial e/\partial t=-[(W-e)/(1-t)](\partial e/\partial W)+S. \qquad (14)$$

Here S is the substitution effect, which is positive. The first term on the right is the income effect, and this is negative if evasion increases with gross income. (For proofs of these assertions see Sandmo (2005).)

So far, the analysis has been carried out under the assumption that the taxpayer does engage in some amount of tax evasion, i.e. we have assumed that $e>0$. But the theory should also allow for the case $e = 0$. In order to see when this will be the case, we note first from equations (9) and (10) that $e = 0$ implies $Y = Z = W(1-t)$. If this is to be the optimal solution, the first order condition (12) must be negative at this point, so that

$$(1-p)U'(W(1-t))t - pU'(W(1-t))(\theta-t) < 0.$$

Factoring out $U'(W(1-t))$, we can write this condition as

$$t < p\theta. \qquad (15)$$

As long as the regular tax rate is less than the expected penalty rate, the taxpayer will not engage in any tax evasion. If the reverse inequality holds, however, he will evade some of his income.

There may be a paradox here: Since data on tax rates and the frequency of tax audits suggest that it is in fact the reverse inequality (i.e. $t > p\theta$) which is likely to be satisfied for the large majority of taxpayers in most countries, tax evasion ought according to the theory to be much

more widespread than empirical studies indicate that it is. However, one could imagine a number of explanations for this apparent paradox.

First, a rather obvious point is that not all income can be evaded. In most Western countries, earnings are reported by employers directly to the tax collecting agency, so that any attempt by the taxpayer to underreport his part of his income will necessarily be unsuccessful. For those whose only income is of this type, tax evasion is simply not an option – unless they act in collusion with their employer. This point is supported by the empirical observation which has been frequently made both by tax administrations and researchers that the incidence of tax evasion is significantly higher among the self employed than among wage earners.

Second, there may be widespread misperception among taxpayers of the magnitude of the parameters that enter into the decision of whether or not to evade taxes. In Norway, empirical studies have shown that taxpayers tend to overestimate the probability of discovery by a wide margin, so that even if the assumptions of the model were otherwise to hold true, it matters for its interpretation whether one assumes the taxpayers' probability of detection to correspond to the true frequency of audit or is a reflection of subjective judgement.

Third, taxpayers may in reality take a less cynical view of their own tax evasion than that depicted by the model. In particular, they may attach a negative utility to their own amount of evasion, so that in a sense they impose a "conscience tax" on themselves (Sandmo 2005). The main effect of this additional assumption is to change the condition for when some evasion is desirable. In deciding whether or not to evade taxes, the taxpayer will compare not only the difference in probability-weighted tax rates between the two outcomes of discovery and non-discovery; for evasion to occur, the gap between the two must be sufficiently large to compensate for the utility loss connected with the conscience tax. The implication is that some taxpayers for whom, in a narrow sense, tax evasion would be profitable will nevertheless refrain from underreporting.

Fourth, there is the possibility that expected utility maximization does not adequately capture the motivation of taxpayers regarding tax evasion. A common objection to this type of modelling is in fact that individuals that engage in criminal behaviour are not as rational as expected utility would have us to believe, although this argument must be assumed to carry less weight in the case of tax evasion than when applied to violent crime. Moreover, as the

previous argument shows, expected utility theory can easily be reconciled with the adoption of a richer set of assumptions than that contained in the simplest version of the model. It should also be kept in mind that if one decides to discard the rationality implied by expected utility maximization one has to adopt an alternative definition of rationality or consistency if prediction of individual behaviour is to be at all possible.

The setting in which this analysis has been carried out is obviously very much simplified and needs to be extended in a number of directions. A public finance economist might first of all emphasize the point that the assumption of a given pre-tax income fails to capture much of the policy concern about the development of the hidden economy. Tax evasion is in fact closely connected with the growth of the hidden economy and the activities in the "unofficial" part of the labour market, and the determinants of tax evasion therefore needs to be studied in the context of models of variable labour supply and occupational choice. Various aspects of this problem have been studied by Baldry (1979), Sandmo (1981), Pestieau and Possen (1991) and Jung, Snow and Trandel (1994).

On the normative side, an interesting theoretical challenge lies in the combination of the theory of tax evasion and unofficial labour markets with the literature on optimal taxation. The theory of optimal taxation is concerned with the question of how to mitigate the effects of tax distortions of the market mechanism, and tax evasion obviously has the effect of lowering the effective tax rates in a number of sub-sectors of the labour market. Lower effective tax rates are usually taken to imply social gains in the form of increased efficiency. So is it a good idea to allow tax evasion? On the one hand, it should be kept in mind that the loss of revenue through tax evasion will have to be compensated, provided that it is desirable to hold tax revenue constant, by higher taxes on non-evaded income, and this will result in larger distortions and efficiency losses. The two types of effects must be balanced against each other, so that one may argue that some degree of toleration of tax evasion might in fact be socially optimal, which in fact also seems to be the attitude of policy makers in this area[15].

An additional consideration concerns the distributive effects of tax evasion. The choice of policy instruments to control tax evasion should, in addition to efficiency considerations, take

[15] Weiss (1976) has argued that under specific assumptions about the nature of risk aversion a random rate of tax might be preferable to a certain one. Some tolerance of tax evasion is the most natural implementation of this idea.

account of the effects on the personal distribution of income and welfare and reflect the attitudes of policy makers towards inequality and social justice. In the case of earnings in the black labour market, the distributive effects will presumably be quite different depending on whether control and punishment are directed towards the supply side or the demand side of these markets.

This application of optimal tax theory suggests some questions of a more fundamental nature for normative economics. The notion of social welfare that lies at the bottom of optimal tax analysis is derived from individual utilities – their expected utilities, in this case. But should the expected utility of tax evaders count in terms of social welfare in exactly the same way as the utility of law-abiding individuals? Suppose the expected utility of a tax evader increases as a result of a lowering of the probability of detection. Should this, *caeteris paribus,* be counted as a gain in terms of social welfare? Reflections on this issue may lead one to become sceptical about the "welfarist" approach to the normative analysis of tax compliance, since it is obviously possible to argue that the enforcement of public morality should take precedence over pure efficiency calculations. This raises an intriguing set of issues that are still in need of further theoretical clarification.

5. Concluding remarks

The theory of public finance offers a wide and interesting field of application for specialists in the economics of uncertainty. The topics that I have discussed in this survey have been taken from different areas of the subject: Tax theory, public expenditure analysis and issues relating to the costs of administration and compliance. The applications cover both positive issues of individual adjustment to the parameters of public policy and the normative questions of how best to design economic policy in the face of uncertainty. All areas and applications have involved an extensive use of concepts and theories from the economics of uncertainty. The hypothesis of expected utility maximization and the application of measures of risk aversion have been used to illuminate a range of interesting questions of great policy relevance in public finance.

In spite of its wide scope, the survey has in fact been highly selective. I have disregarded both some important methodological approaches like the economics of information and game theory which also belong to a broader view of the economics of uncertainty. I have also

neglected the analysis of important areas of economic policy like social security, social insurance and pensions and barely touched on some others like environmental policy and income redistribution. In spite of these omissions, I believe to have established that the interaction between the economics of uncertainty and the theory of public finance has been a very fruitful one over the last half century, and I expect that it will continue to be so in the future.

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