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# Introduction

In all industrial countries most local governments try to attract firms and people to their cities or region through the economic policies at their disposal. The exact policy instruments available to local governments differ from country to country, but they typically comprise publicly provided goods and services and some tax instruments (property taxes in some countries, local income or sales taxes in others). This thesis is concerned with the effects – on economic geography, industrial structure, factor rewards and private sector productivity – of such policies.

The reason why local governments try to attract more firms and more people is typically that they believe in the benefits of agglomeration. If no-one in a region benefits from an influx of people and firms, there will be no reason for regional governments to pursue such policies. My framework is therefore one in which there are local economic benefits – i.e. pure rents -- from industrial agglomeration. These do not necessarily accrue to the industry itself; the pure rents may just as well accrue to local factor owners as higher regional wages or returns to capital.

Industries for which location does not matter – typically industries with constant returns to scale - will be randomly spread across space. Such businesses might therefore be found in all types of regions. They can coexist with agglomeration industries. If there is unemployment in a region, the establishment of an industrial agglomeration will reduce unemployment and so workers in the region benefit from the establishment. Non-agglomeration industries can also be crowded out, however. If there are no unexploited resources, the establishment of an agglomeration industry leads to increased regional factor prices – i.e. higher regional wages or higher regional returns to capital. Again, this will benefit local factor owners. Whatever the initial situation, therefore, someone in a region will benefit from an industrial agglomeration.

The dissertation builds on the economic literature on new economic geography. An excellent overview of the literature is found in Ottaviano and Puga (1998). Most economic activities are geographically concentrated. An economic agglomeration is a geographic concentration of economic activities. Examples of economic agglomerations are cities and industrial concentrations (i.e. the geographic concentration of firms belonging to an industry).

There are many reasons for concentration of economic activities. Much concentration can be explained by availability of natural resources (the paper industry in Finland, the heavy industry in the Ruhr area in Germany, the emergence of Bergen as a trading centre for Norwegian fish, etc.). Some economic concentrations are due to political circumstances (Copenhagen as a strategic location for collecting taxes from sea traffic to the Baltic Sea) or deliberate political choice (St. Petersburg). Many agglomerations are due to labour immobility (e.g. labour-intensive manufacturing in China).

Such factors cannot, however, explain all economic agglomerations – probably not even the majority. Why is there a heavy concentration of shoe production in the northern Italian regions? Why is the production of furniture in Norway mainly in rural Sunnmøre on the north-western coast? Why is the high-tech industry highly concentrated and mainly located in Silicon Valley? The key words for explaining these agglomerations are economies of scale – internal and/or external.

Internal economies of scale are declining unit costs within a single firm. Whenever this is the case, it is profitable to concentrate production in few, but large, production units.

External economies of scale exist when one economic agent benefits from being located close to other economic agents. Agglomeration is a *means* for extracting these benefits: As long as economic agents benefit from being located close to one another they will choose to do so, and we get agglomerations in the form of many economic agents being located close to one another.

The fact that we get agglomerations whenever there are external economies is thus trivial. The difficult questions are why economic agents benefit from being located close to other agents (i.e. what the sources of external economies are) and what such benefits imply for economic geography. These two questions, the *sources* and *implications* of agglomerations are what the literature on the new economic geography is concerned with. The theory focuses mainly on agglomerations of *firms*, and only on self-reinforcing agglomerations.

The *sources* of external economies are *pure* or *pecuniary* positive externalities. These positive externalities imply that the profitability of each firm is positively related to the number of nearby firms.

The most important *implications* of the existence of agglomerations are:

- Critical mass and coordination failure
- Hysteresis
- Multiple equilibria

The collection of firms must be of a certain size in order to get established in one place. This minimum size is the critical mass of the firm collection. The size of a single firm is usually below this critical mass, and as there generally is no obvious mechanism for coordination of locational choices between firms, the agglomeration may not emerge. This is what is meant by coordination failure.

Hysteresis in this setting means inertia concerning choice of location – once in place, an industry is likely to remain there for a long time; location being insensitive to small changes in factor prices, tax regimes and other local conditions.

Finally, in most of the models the results are unclear in the sense that there are several possible equilibria. *Whether* and *where* there will be agglomerations is unclear.

Regional governments have several ways in which they can try to attract more industrial activity. They might provide public inputs which improve the productivity of capital, lower tax rates, invest in infrastructure, subsidise firms directly, etc. The design of local policies to attract businesses has been the focus of extensive studies within public economics. Keen and Marchand (1997) argue, for example, that the equilibrium pattern of expenditures is inefficient because local governments spend too much on public inputs provision and too little on public goods provision. The reason is that public inputs provision attracts capital whereas public goods provision only benefits residents but does not attract capital. Lowering taxes is another means available to governments. When local taxes are lowered, the costs a firm incurs in the region are lowered, and this makes the place more attractive as a place to locate production. In order to attract firms, regions might successively lower their taxes and engage in so-called tax competition (see e.g. Zodrow and Mieszkowski (1986), Wilson (1999), Oates (1972) and Sinn (1997)). The result of the competition is a "race to the bottom" when it comes to tax rates. Tax rates are set at a lower level than the Pareto optimal level. Following sub-optimal tax levels, public spending and hence welfare becomes lower than the optimal levels.

Common to most of the models within the public economics literature studying what local governments could do to attract firms, is that they rely on the assumption of constant returns to scale and perfect competition. When there are constant returns and perfect competition there are few reasons for industries to be geographically concentrated (the reasons for an uneven distribution are usually trade costs or and uneven distribution of resources). The models are, we might say, without space. This is where new economic geography comes in. It shows that space *is* important. *Where* people and firms locate matter to profits, costs and wages. Geographic agglomeration is beneficial either to firms, to workers or both, as pointed out by Krugman (1991a), Krugman (1991b), Krugman and Venables (1995), Venables (1996) and many others. The fact that geographic agglomeration benefits some economic agents gives us an economic explanation of the formation and sustainability of cities and the concentration of businesses.

Since agglomeration creates hysteresis in location, once production has agglomerated in a region it tends to stay there. A consequence of this is that a marginal change in tax rates is not enough to induce agglomeration firms to relocate. This has immediate implications for the design of local taxes and hence for whether local tax autonomy will result in a race to the bottom.

Empirical studies show that agglomeration forces *are* indeed important for the location of industries. Audretsch and Feldman (1996) show that knowledge spillovers are important and that they tend to be very localized. Botazzi and Peri (2002) show that knowledge spillovers are typically stronger *within* than *between* regions and also stronger *within* than *between* industries. As knowledge spillover is one of the factors that - according to the new economic geography literature - may explain industrial agglomerations, these studies support the importance of agglomeration forces for the location of industrial activities. Other studies also show the existence of localised positive externalities in Europe, e.g. Ciccone (2002) and Combes et al. (2004).

Given the importance of agglomeration forces for the locational choice of industries, it seems natural to incorporate these forces in models which aim at studying competition for industrial activities. Some studies have addressed the issue of *tax competition* in an economic geography framework – see e.g. Ludema and Wooton (2000), Kind et al. (2000) and Andersson and Forslid (1999). These studies, which apply new economic geography models to the

studies of tax competition, find that a race to the bottom regarding tax rates will not necessarily take place. Agglomerations create pure rents which might be taxed.

The new economic geography framework also permits analysis of how the supply of local public goods will affect location and agglomeration. An example is Martin and Rogers (1995), who examine the effects of public infrastructure on industrial location.

The three papers in my dissertation are in this tradition. They all look at local policies (tax or expenditure policies) in a framework with mobility of economic agents (firms or individuals) and local gains from agglomeration (real or pecuniary). I do *not* consider the *sources* of agglomeration gains in any of the papers. My focus is on the *implications* of local policies in settings where there, for some reason or other, *are* gains from geographic concentration of economic activities.

The three papers are

- 1. Agglomeration, tax competition and local public goods supply
- 2. Public goods production and private sector productivity
- 3. The price of decentralisation

The first paper, "Agglomeration, tax competition and local public goods supply", provides a general, two-region framework for studying tax and public service competition between regions when there are both agglomeration gains and fiscal externalities and when local public policy is decided by majority voting. Individuals can move costlessly between the regions, but residential preferences make labour mobility less than perfectly elastic; and the analysis is restricted to the case where migration is sufficiently inelastic (relative to the agglomeration gains) to create viable agglomerations in both regions. The paper compares the equilibrium set of policies that will be pursued by the two regions if they have complete policy autonomy with the set of policies which would give a first-best allocation of resources.

There are two sources of inefficiency with local policy autonomy. One is a cost-ofdemocracy wedge arising because policy in a democracy will reflect the preferences of the median, rather than the average, voter. The other is a policy-competition wedge arising because local policies will be designed to attract more people; so the preferences of the marginal resident will be given greater weight than those of the average one. The paper argues that the combined effect of the two may differ depending on the mobility of individuals between the regions. If most people have relatively weak residential preferences (what the paper calls the "American" case), the two wedges pull in the same direction. If the majority have strong residential preferences (the "European" case), the two wedges pull in opposite directions with the presumption then being that the distortion created by local policy autonomy will be smaller.

The second paper, "Public goods production and private sector productivity", looks at the implications for the private sector of the use of resources in the public sector. It is sometimes argued that local governments should try to foster knowledge-intensive industrial agglomerations by establishing more public jobs for highly educated individuals – the presumption being that this will attract more highly educated people to the region and thus benefit the private sector as well. The paper shows that this line of reasoning is wrong. More public jobs for highly skilled workers will lower their wages and thus make it less attractive for such individuals to move to a region.

Specifically, the paper looks at how the size, structure and productivity in the private sector are affected by the use of resources in public sector production. It shows that the primary effect is a Rybczynski effect: If production in the public sector requires much highly skilled labour, then the size and productivity in knowledge-intensive private industries declines. If, on the other hand, public sector production requires relatively much unskilled (low-skilled) labour, the opposite occurs.

In the years ahead much of the growth in the public sector is expected to be within health care; mainly due to the ageing of the population. As these services are intensive in relatively low-skilled labour, this (i.e. the above mentioned Rybczynski effect) implies that natural growth in the public sector might be the best way in which local governments can contribute to growth in knowledge-intensive agglomerations in the private sector.

The third paper, "The price of decentralisation" addresses the fundamental question of whether a market economy promotes too much or too little centralisation. While many politicians argue that unregulated markets lead to too much centralisation in big cities, the new economic geography literature argues the opposite: With real and pecuniary gains from agglomeration, a market economy will give both too few and too small agglomerations – in other words too few and too small cities. The paper extends and modifies this well-known result in two ways. One is by including local governments as agents; the other is by looking at equilibria with both large and small cities in addition to rural areas.

The fundamental model is one in which individuals have preferences for living either in the city or the countryside. In the cities, there are gains from agglomeration, so income levels there will be higher the larger the city. In rural areas people work in agriculture; and their income levels depend on the total size of the agricultural sector (because the price of food falls with increased production). In the first part of the article this framework is used to look at an economy with one city in which the city government supplies tax-financed local public inputs. One might think that the gains from agglomeration would make the city overprovide public goods in order to attract more people, and that this could offset the standard result that the city is too small. That turns out not to be the case. It is shown that the presence of agglomeration gains will actually make local governments supply less of public services than they would otherwise do; so the result that the city becomes too small and the rural sector too large is, if anything, strengthened.

In the second part of the paper, the model is extended to many regions and many cities. It is shown that the results regarding the number and size of cities are robust as long as people are mobile only within regions. If we allow interregional mobility as well, however, it could be that both large cities as well as rural areas will be overpopulated (relative to the optimum), while smaller cities could be both too few and too small.

# References

Andersson, F. and R. Forslid (2003), "Tax competition and economic geography", *Journal of Public Economic Theory*, 5 (2), 279-303

Audretsch, D. B. and M. Feldman (1996), "R&D spillovers and the geography of innovation and production", *American Economic Review* 86, 630-664

Bottazzi, L. and G. Peri (2002), "Innovation and spillovers in regions: Evidence from European patent data". *IGIER Working Paper* No.215

Ciccone, A. (2002), "Agglomeration effects in Europe", *European Economic Review* 46, 213-277

Combes, P-P., G. Duranton and L. Gobillon (2004), "Spatial wage disparities: Sorting matters!", *CEPR Discussion Paper* No.4240

Keen, M. and M. Marchand (1997), "Fiscal competition and the pattern of public spending", *Journal of Public Economics* 66: 33-53

Kind, H., K. H. Midelfart Knarvik and G. Schjelderup (2000), "Competing for capital in a "lumpy" world", *Journal of Public Economics* 78, 253-274

Krugman, P. R. (1991a), Geography and trade, Cambridge, Massachusetts: MIT Press

Krugman, P. R. and A. J. Venables (1995), "Globalization and the inequality of nations", *Quarterly Journal of Economics* 110, 857-880

Krugman, P. R. (1991b), "Increasing Returns and Economic Geography", *Journal of Political Economy*, vol 99 no.31, 483-499

Martin, P. and C. A. Rogers (1995), "Industrial location and public infrastructure", *Journal of International Economics* 39, 335-351

Oates, W. E. (1972), Fiscal Federalism. New York: Harcourt Brace Jovanovich

Ottaviano, G.I. P. and D. Puga (1998), "Agglomeration in the global economy: A survey of the "new economic geography"", *World Economy* 21(6), August 1998, 707-731

Sinn, H.-W. (1997), "The Selection Principle and Market Failure in Systems Competition", *Journal of Public Economics* 66, 247-74

Venables, A. J. (1996), "Equilibrium location of vertically linked industries", *International Economic Review* 37, 341-359

Wilson, J. (1999), "Theories of tax competition", National Tax Journal LII (2), 269-304

Zodrow, G. and P. Mieszkowski (1986), "Pigou, Tiebout, property taxation and the underprovision of local public goods", *Journal of Urban Economics* 19, 356-370

# Agglomeration, tax competition and local public goods supply<sup>1</sup>

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**Abstract:** In this paper we develop a framework for studying tax competition and local public goods supply in a setting where real and fiscal externalities interact with local democracy. We use the framework (a) to analyse if there is any reason to believe that local autonomy generally will give a tax race to the bottom (there is not), and (b) to look more closely at possible sources of oversupply or undersupply of publicly provided goods in a setting where local democracies compete for people. We identify two potential sources – the relationship between individual mobility and willingness to pay for publicly provided goods, and the mobility distribution of individuals (i.e. the distribution of individuals over residential preferences). The two could reinforce each other in a local democracy if the majority of the residents in a community are relatively mobile (the "American" case), while they would pull in opposite directions if the majority of residents are relatively immobile (the "European" case).

JEL classification: F12, H21, H73, J61

**Keywords:** Tax competition, local public goods, agglomeration, migration, regional economic policy

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## Introduction

The purpose of this paper is to provide a framework for studying local public goods supply and tax competition between jurisdictions in a context where there are gains from geographic agglomeration and where labour is imperfectly mobile. Thus, the paper brings together the literature on local public finance (Tiebout (1956)), Wilson (1986) and the so-called new economic geography literature (Krugman (1991), Krugman and Venables (1995), Venables (1996)), and it does so in a "European" context in which there are strong preferences for place of residence, and correspondingly limited mobility of individuals (Faini et. al. (2000)).

Much of the traditional literature on tax competition focuses on taxation of capital income, and a central result is that local or regional tax autonomy will lead to a tax "race to the bottom" (see Wilson (1999) for a survey). A number of papers in the new-economic-geography tradition have challenged this result, arguing that industrial agglomeration, by generating rents that can be taxed and hysteresis that reduces the effective mobility of capital, could just as easily generate a "race to the top" (e.g. Kind, Midelfart-Knarvik and Schjelderup (2000), Baldwin and Krugman (2003)).

There is a similar, traditional presumption that tax competition will give lower taxes on labour income if individuals are mobile (Sinn (2003), Honkapohja and Turunen-Red (2004)). Again, this could be reversed in the presence of agglomerations. Andersson and Forslid (2003) use a model with immobile and mobile workers to show that there will not be a tax race to the bottom for mobile workers and that taxes on immobile workers will actually be biased upwards.

Our paper brings together the insights from the traditional approach, with its focus on fiscal externalities, and the insights from the agglomeration externalities of the neweconomic-geography literature. Combining the two, we show that local autonomy with respect to taxation and public provision of goods will give too high or too low taxes (compared to a global optimum) depending on whether the willingness to pay for the average publicly provided good increases or decreases with the mobility of the individual, and we show that this result holds even if there are no economies of scale in the publicly provided goods (and thus no fiscal externality); i.e. even if local authorities provide purely private goods produced with constant returns to scale. As most goods provided by local authorities are of that kind, we feel that our model provides a more meaningful framework for understanding the nature of competition between communities than models that focus on purely fiscal externalities. At the same time, we also assume that local decisions are based on majority voting, so that it is the interests of the median local voter which determines taxes and the supply of publicly provided goods. This adds another source of possible bias. We show that if the willingness to pay for publicly provided goods varies systematically with the mobility of the individual, the public-choice bias will reinforce the tax-competition bias if mobility is relatively high (what we call the "American" case), while the public-choice bias will counteract the tax-competition bias if mobility is relatively low (the "European" case). To the extent that the total distortion is smaller if the two pull in opposite directions than if they pull in the same direction, therefore, there should be less reason for concern about possible distortions in the European than in the American case.

We model agglomeration gains in the simplest possible manner, by assuming that individuals consume a bundle of locally produced, differentiated products, produced by monopolistically competitive firms and modelled along Spence-Dixit-Stiglitz lines (Spence (1976), Dixit and Stiglitz (1977)). Because consumers value variety, and the range of products available will be larger the larger the local market, this creates agglomeration gains. These will be reinforced if there are economies of scale in the supply of goods provided by local authorities - i.e. if local authorities provide pure public goods or private goods with scale economies.

The agglomeration forces are counteracted by residential preferences. We assume that individuals differ both as to where they prefer to work and live, and in the degree to which they prefer one place to another. We capture this by an index measuring how highly a consumer values a particular choice. All individuals are assumed to have the same utility function defined over this index, the supply of public goods, and consumption of private, differentiated goods.

In the paper, we use this framework to look at a two-community equilibrium. Labour is the only factor of production in the model, and individuals have to make a joint decision on where to work and live. Equilibrium obtains when the marginal resident has nothing to gain from moving to the other community. There are clearly two possible outcomes. One is agglomeration in one community. That will happen if the agglomeration gains are sufficiently strong relative to the dispersion and intensity of residential preferences. The other possibility, on which we focus, is that the loss in residential surplus that the marginal individual would incur by moving is greater than the marginal gain from agglomeration. In that case, there will be a stable, interior equilibrium - i.e. geographical dispersion. In an interior equilibrium, each community will gain from attracting new residents. Thus, the framework lends itself to the study of competition for residents between communities. The instruments available are publicly provided goods and local tax rates. We assume that no discrimination is possible, so all publicly provided goods are provided in equal quantities to all residents and everyone pays the same tax. If so, a community can only make itself more attractive to new residents if marginal residents differ from non-marginal ones in their willingness to pay for public goods. If potential immigrants are more tax-averse than current residents, a community can attract new residents by reducing the supply of public goods and lowering tax rates; if they value public goods more highly than the natives, immigration will be stimulated by raising taxes and increasing the public goods supply.

The resulting game between the communities will, therefore, be systematically biased towards overprovision of publicly provided goods that the most mobile individuals value more highly than the less mobile ones, and towards underprovision of publicly provided goods with the opposite characteristic.

#### The general model

The model has L individuals, each endowed with one unit of labour, which is the only factor of production. Individuals are mobile between communities, and move to the community where their total utility will be highest.

## Preferences and consumer choice

The utility of an individual depends on three factors: The place of residence, the consumption of publicly provided local goods, and the consumption of private goods.

The utility person h gets when living in community i is

(1) 
$$U_i^h = U(\alpha_i^h, g_i, c_i),$$

where  $\alpha_i^h$  measures the intensity of his preference for living in community *i* (assumed to differ between individuals); and where  $g_i$  and  $c_i$  denote his consumption of publicly provided and private goods, respectively.

We take  $g_i$  to be a single good provided in equal quantities to all residents by the local authority in community *i*. It could be a pure public good or a private good with or without economies of scale in production. Publicly provided goods are financed by local taxes, levied in a non-discriminatory fashion on local residents.

Private goods are not traded, which means that consumers are limited to the range of locally produced goods. Consumption of private goods,  $c_i$ , is an aggregate of differentiated products. It will be the same for all individuals living at *i*, since they all supply the same amount of labour, pay the same amount of taxes, and face the same prices and product range.

We model product differentiation in the original Spence-Dixit-Stiglitz fashion. Let  $e_{ki}$  be per capita consumption of variety k in community i, and let  $\varphi(e_{ki})$  be the sub-utility from consuming this amount. We make the usual assumptions about  $\varphi(e_{ki})$ ; it is an increasing and concave function ( $\varphi' > 0$ ;  $\varphi'' < 0$ ). The consumption aggregate  $c_i$ , which may be thought of as a quantity index, is defined as

(2) 
$$c_i \equiv \sum_{k=1}^{n_i} \varphi(e_{ki})$$

where  $n_i$  is the number of different varieties produced in community i.

Let  $x_{ki}$  denote total production of variety k in community i. As private goods are not traded, and everyone within the community consumes equal amounts of private goods, per capita consumption of variety k must be

$$(3) \qquad e_{ki} = \frac{x_{ki}}{L_i},$$

where  $L_i$  is the number of consumers in community *i*. Inserting (3) into (2) gives per capita consumption of private differentiated goods as

(4) 
$$c_i = \sum_{k=1}^{n_i} \varphi \left( \frac{x_{ki}}{L_i} \right).$$

The private sector

In the private sector a number of identical firms produce differentiated consumption goods. There are increasing returns to scale in the production of each variety, and these are sufficiently high to ensure that each firm produces only one variety and that each variety is produced by one firm only. The number of firms thus equals the number of different varieties.

Utility maximisation gives the first order conditions for optimal choice of  $e_{ki}$  as

(5) 
$$U_c \varphi'(e_{ki}) = \lambda p_{ki},$$

where  $p_{ki}$  is the price of variety k, and  $\lambda$  the marginal utility of income.

Inserting (3) into (5) and rewriting gives the inverse demand functions

(6) 
$$p_{ki} = \frac{U_c}{\lambda} \varphi' \left( \frac{x_{ki}}{L_i} \right),$$

where  $x_{ki}$  is the output of firm k.

Let  $b(x_{ki})$  be the cost function of firm k. The profits are then

(7) 
$$\pi_{ki} = p_{ki} x_{ki} - b(x_{ki}).$$

We make Chamberlain's large-group assumption that the number of firms is so large that each firm takes the aggregate  $c_i$  as given. From the point of view of an individual firm, the term  $U_c/\lambda$  in equation (6) is then a constant. Inserting (6) into (7) gives the profits of firm k as

(8) 
$$\pi_{ki} = \frac{U_c}{\lambda} \varphi' \left( \frac{x_{ki}}{L_i} \right) x_{ki} - b(x_{ki}).$$

The first order condition for profit maximisation, marginal revenue equals marginal cost, becomes

(9) 
$$p_{ki} + \frac{U_c}{\lambda} \varphi'' \frac{1}{L_i} x_{ki} = b',$$

or, rewriting,

(10) 
$$p_{ki}\left(1+\frac{\varphi''e_{ki}}{\varphi'}\right)=b'.$$

There is free entry and exit in the private sector. New firms will enter until the marginal firm earns zero profits. As firms are identical, the zero-profit condition must hold for all firms in equilibrium,

(11) 
$$\pi_{ki} = p_{ki}x_{ki} - b(x_{ki}) = 0$$
,

which implies

(12) 
$$p_{ki} = \frac{b(x_{ki})}{x_{ki}}$$
.

In equilibrium, both the marginal-revenue-equal-marginal-cost (equation (10)) and the zero-profit condition (equation (12)) must hold, which gives the following equilibrium condition:

(13) 
$$\frac{b'}{1+\frac{\varphi''e_{ki}}{\varphi'}} = \frac{b}{x_{ki}}.$$

Here,  $-\frac{\varphi'}{\varphi'' e_{ki}}$  is the elasticity of substitution between any two varieties of private

goods.

Assume that the elasticity of substitution between any two varieties is constant and equal to  $\sigma$ . Assume also that there are increasing returns to scale in the production of each variety, as represented by the linear labour-requirement function

(14) 
$$A + Bx_{ki}$$
.

Total costs are nominal wages times labour input,

(15) 
$$b(x_{ki}) = w_i(A + Bx_{ki}).$$

Inserting (14) and (15) into (13) gives the following equilibrium condition:

(16) 
$$x_{ki} = \frac{A}{B}(\sigma - 1).$$

We are free to choose units such that

(17) 
$$A \equiv \frac{1}{\sigma}, B \equiv \frac{\sigma - 1}{\sigma}.$$

The supply of each firm is then

(18) 
$$x_{ki} = 1$$
,

and the price of each variety

(19) 
$$p_{ki} = w_i$$
.

Each firm supplies one unit of its exclusive variety, and the price of each variety is equal to the nominal wage rate in the community.

Note that the labour requirement of each firm is (inserting (17) and (18) into (14))

(20) 
$$A + Bx_{ki} = 1$$
.

One unit of labour is needed to produce one unit of each variety. As each firm produces one unit of its exclusive variety, the number of private firms/different varieties equals the number of workers in the private sector; i.e.  $n_i$  denotes both the number of firms and the number of workers in the private sector.

#### The public sector

The residents of each community are provided with some local public goods; pure public goods or publicly provided private goods. Everyone living in a community consumes the same amount,  $g_i$ , of these goods. The production of local public goods is financed by local taxation of the residents of the community. Everyone living in a community pays the same amount of taxes.

Labour is the only factor of production. Let  $h(L_i)g_i$  be the labour requirement function of the public sector. The nature of local public goods, whether they are pure public goods or publicly provided private goods, is reflected in the term  $h(L_i)$ .

If  $h'(L_i) = 0$ , then  $g_i$  is a pure public good, i.e. a good for which there is no rivalry in consumption. If  $h'(L_i) > 0$ ,  $g_i$  is a publicly provided private good in the sense that if one more person is to consume the good, others must reduce their consumption, everything else equal. One reason for the government to supply private goods is that there are increasing returns to scale in the production of these goods. That will be the case when  $h(L_i)/L_i$  is decreasing in  $L_i$ .

#### Population and real income

There are  $L_i$  inhabitants in community *i*, of which  $h(L_i)g_i$  work in the public sector. The number of workers in the private sector is therefore  $L_i - h(L_i)g_i$ . The number of private firms equals the number of workers in the private sector, so the number of private firms must also be  $n_i = L_i - h(L_i)g_i$ .

Inserting for  $n_i$  and  $x_{ki}$  in equation (4), we see that per capita consumption of private goods is

(21) 
$$c_i = \left[L_i - h(L_i)g_i\right]\varphi\left(\frac{1}{L_i}\right) \equiv c^i(g_i, L_i).$$

Note that

(22) 
$$\frac{\partial c^i}{\partial g_i} = -h(L_i)\varphi\left(\frac{1}{L_i}\right) < 0.$$

The effect of increasing the provision of public goods per capita, everything else equal, is that the consumption of differentiated goods per capita is reduced. The production of local public goods is financed by an equal tax on the residents of the community. As the production of public goods increase, so do the costs of public goods production. This leads to increased taxes per capita as long as the number of inhabitants remains unchanged. After-tax income is therefore reduced, leading to reduced consumption of private differentiated goods. The tax effect is equivalent to  $h(L_i)$  units of labour. Because output per firm is given, the entire reduction in private consumption takes the form of a reduction in the number of product varieties available. Increased public employment gives a one-to-one reduction in the number of private firms, and thus in the number of product varieties. This is reflected in the term  $\varphi(1/L_i)$  in (22). Note that this means that the social marginal cost of publicly provided goods is higher than the private marginal cost, which is simply  $h(L_i)$ .

From (21) we also find the relationship between private consumption and the size of the community:

$$\frac{\partial c^{i}}{\partial L_{i}} = \left(L_{i} - hg_{i}\right)\left(-\varphi'\frac{1}{L_{i}}\right) + \varphi\left(1 - h'g_{i}\right)$$

i.e.

(23) 
$$\frac{\partial c^{i}}{\partial L_{i}} = \frac{c_{i}}{L_{i}} \left[ (1 - \beta) + \left( \frac{g_{i} \left[ (h/L_{i}) - h' \right]}{1 - g_{i}(h/L_{i})} \right) \right] \text{ with } \beta \equiv \frac{\varphi' e_{i}}{\varphi}$$

This has an instructive interpretation. The term  $(1 - \beta)$  captures the real, positive externality - i.e. gain from agglomeration: More residents means a larger local market, and thus a wider selection of products. It also means that consumption of each variety is reduced, but the net effect is positive. The second term in brackets captures the fiscal externality. If there are economies of scale in publicly provided goods, the marginal labour requirement will be lower than the average requirement, so the second term will be positive. The economic reason is simply that more people in that case means lower taxes per capita.

Inserting (21) into (1) gives the utility of individual h in community i as

(24) 
$$U_i^h = U(\alpha_i^h, g_i, c^i(g_i, L_i)).$$

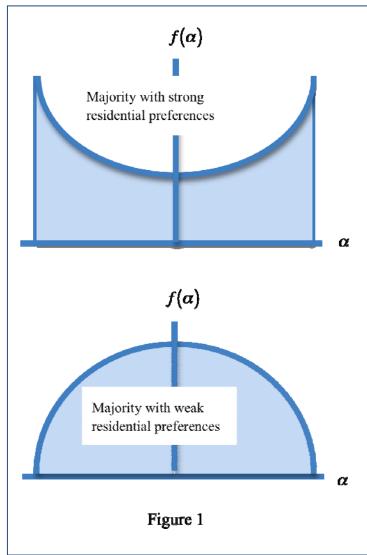
# Migration and geographic equilibrium

Now, consider a country consisting of two communities. Each local community is formally like the one described in the previous section. In each community there are two sectors, a private and a public, producing goods consumed locally. Publicly provided goods are financed by local taxation, whereas the after-tax wage is used for consumption of private differentiated goods. People are mobile between communities,

and settle in the community where their total utility will be highest. Total utility depends on consumption and on the place of living per se. To proceed with the analysis we need to specify these locational preferences in some more detail.

Assume that the utility from living in community 1,  $\alpha_1$ , is distributed on the interval [-(1/2),(1/2)], and that  $\alpha_2 = -\alpha_1$ . A person who very highly values living in community 1 ( $\alpha_1$  is close to 1/2), has an equally strong dislike of living in community 2 ( $\alpha_2$  is close to -1/2). The distribution of  $\alpha_1$  is illustrated in figure 1.  $\alpha_1$  is measured along the horizontal axis, and increases as we move from left to right. (As  $\alpha_2 = -\alpha_1$ ,  $\alpha_2$  is also measured along the horizontal axis, but increases as we move from right to left.) The total number of people in the country, L, is given by the total area under the curve  $f(\alpha_1)$ ; i.e.

$$L = \int_{-\frac{1}{2}}^{\frac{1}{2}} f(\alpha_1) d\alpha_1.$$



We shall be concerned with symmetric equilibria only, so we assume that the distribution is symmetric. We distinguish between two cases - one where there are more people with strong residential preferences than the number of people with weak preferences, in which case the distribution is ushaped; and one where most people have weak residential preferences, in which case the distribution is bellshaped. The two are illustrated in figure 1.

A person settles in community 1 if (and only if)  $U_1^h > U_2^h$ . This can give rise either to an interior equilibrium in which there are residents in both communities, or to complete agglomeration in one community. We focus on the former.

In an interior equilibrium, the utility of the marginal individual must be the same in both communities, so we must have

(25) 
$$U(\alpha_1^M, g_1, c_1) = U(-\alpha_1^M, g_2, c_2)$$

where *M* denotes the marginal inhabitant. Let  $F(\alpha_1^M)$  be the number of people for whom  $\alpha_1 \ge \alpha_1^M$ ; i.e.  $F(\alpha_1^M)$  is the number of inhabitants in community 1. Then

$$L_{1} = F(\alpha_{1}^{M}) = L - \int_{-\frac{1}{2}}^{\alpha_{1}^{M}} f(\alpha_{1}) d\alpha_{1}$$

To find the critical value of  $\alpha_1$ , invert  $F(\alpha_1^M)$ :

$$\alpha_1^M = G(L_1) \equiv F^{-1}(L_1)$$

Inserting for  $\alpha_1^M$  in (25), the equilibrium condition becomes

(26) 
$$U(G(L_1), g_1, c_1) = U(-G(L_1), g_2, c_2).$$

The interior equilibrium is not necessarily stable. If the utility difference  $U_1^M - U_2^M$  increases with  $L_1$ , the equilibrium implied by (26) is unstable in the sense that a small deviation will induce massive immigration or emigration.

Thus, the condition for an interior equilibrium to be stable is that

(27) 
$$\frac{d\left[U(G(L_1), g_1, c^1(g_1, L_1)) - U(-G(L_1), g_2, c^2(g_2, L_2))\right]}{dL_1} < 0,$$

Carrying out the differentiation in (27) gives

(28) 
$$\left(U_{\alpha}^{1}G_{L}+U_{\alpha}^{2}G_{L}\right)+\left(U_{c}^{1}\frac{\partial c^{1}}{\partial L_{1}}+U_{c}^{2}\frac{\partial c^{2}}{\partial L_{2}}\right)<0.$$

Consider a symmetric equilibrium, so  $U_{\alpha}^{1} = U_{\alpha}^{2} \equiv U_{\alpha}$  and  $U_{c}^{1} = U_{c}^{2} \equiv U_{c}$ . Equation (29) then reduces to

(29) 
$$2U_{\alpha}G_{L} + 2U_{c}\frac{\partial c^{i}}{\partial L_{i}} < 0.$$

i.e.

(30) 
$$\frac{\partial c^i}{\partial L_i} < \frac{U_{\alpha}}{U_c} G_L.$$

The term on the left-hand side is the marginal gain from agglomeration (which by (23) is the sum of the real and fiscal externalities). To interpret the right-hand side, note that in the symmetric equilibrium, everyone lives in the community for which they have a residential preference (i.e.  $\alpha_1^M = 0$ ), so if one community is to grow, someone must move from the place they prefer to the place in which they would rather not live. The first term is the compensation necessary to induce one person to move from their preferred location to the other. The stability condition, therefore, is that the necessary compensation must be greater than the marginal gain from agglomeration.

Whether or not a symmetric equilibrium will be stable clearly depends on the size of the agglomeration gains. It is also depends on the intensity of residential preferences  $(U_{\alpha}/U_{c})$  and on the preference distribution of individuals. With an "American" distribution, where most people have weak residential preferences, there are many people with preferences close those of the marginal resident, so  $G_{L}(= d\alpha_{1}/dL_{1})$  is small; with a "European" distribution, it is large. Thus, we are more likely to have a stable, symmetric equilibrium in the latter case.

## Local public finance and tax competition

We now have the framework needed to discuss whether there will be over-, under-, or optimal supply of local public goods in a federal system of competing local communities, and whether the distribution of residents will be optimal.

#### National optimum

Consider first the national optimum. We shall not be concerned with distributional issues, so let us assume an additive national welfare function

(31) 
$$W = \int_{\alpha_1^M}^{\frac{1}{2}} U(\alpha_1, g_1, c_1) f(\alpha_1) d\alpha_1 + \int_{-\frac{1}{2}}^{\alpha_1^M} U(-\alpha_1, g_2, c_2) f(\alpha_1) d\alpha_1$$

The national optimum is found by maximising (31) with respect to  $\alpha_1$ ,  $g_1$  and  $g_2$ , taking into account the effects on private consumption in each region.

Consider first the optimum condition with respect to  $\alpha_1$  - i.e. the optimum size of each community. If the size of the community did not matter for consumption per capita – i.e. if there were no real or fiscal externalities – the first-order condition with respect to  $\alpha_1$  would be

(32) 
$$\frac{\partial W}{\partial \alpha_1} = -U(\alpha_1, g_1, c_1)f(\alpha_1) + U(-\alpha_1, g_2, c_2)f(\alpha_1) = 0$$

i.e. that the utility of the marginal inhabitant should be the same in both communities. With a symmetric distribution this means that each community will have the same number of inhabitants. But if so, a small deviation from (32) will have exactly offsetting effects on welfare in the two communities – per capita consumption in the community which gets an extra individual will rise by exactly the same amount as per capita consumption will fall in the community which loses an individual – so (32) must be the first-order condition for the optimum population distribution with externalities as well.

It is also seen from (32) that the second-order condition for a geographic optimum – and thus the condition for an interior solution – is that the utility differential between the two communities, taking into account the effects on consumption, is declining in  $\alpha_1$ . That is the same condition as the stability condition for a symmetric market equilibrium (condition (30) above). We assume that this condition is satisfied.

The first-order conditions for public goods supplies are

(33) 
$$\frac{\partial W}{\partial g_1} = \int_{\alpha_1^M}^{\frac{1}{2}} \left[ U_g + U_c \frac{\partial c_1}{\partial g_1} \right] f(\alpha_1) d\alpha_1 = 0$$

(34) 
$$\frac{\partial W}{\partial g_2} = \int_{-\frac{1}{2}}^{\alpha_m^M} \left[ U_g + U_c \frac{\partial c_2}{\partial g_2} \right] f(\alpha_1) d\alpha_1 = 0$$

These are the usual first order conditions regarding optimal supply of public goods: The sum of the marginal rates of substitution equals the marginal rate of transformation. Another way of writing (33) and (34) is

$$(33^{\prime}) \quad \frac{U_g^A}{U_c^A} = -\frac{\partial c^1}{\partial g_1},$$

$$(34^{\prime}) \qquad \frac{U_g^A}{U_c^A} = -\frac{\partial c^2}{\partial g_2},$$

where A refers to the average inhabitant. (The sum of the marginal rates of substitution  $(MRS_{g,c})$  equals the number of inhabitants times  $MRS_{g,c}$  of the average inhabitant.)

#### A decentralised equilibrium

In a decentralised equilibrium we assume that the residents of a community decide on taxes and supply of goods from the public sector, and that they do so by majority voting. With single-peaked preferences (which in our case follows from our assumptions about the utility functions and the distribution of individuals over residential preferences), this ensures a unique voting equilibrium, where the amount of local public goods supply is the amount preferred by the median voter.

The maximisation problem that determines taxes and public goods supply in community 1 is therefore

$$\max_{g_1} U(\alpha_1^m, g_1, c_1),$$

with *m* denoting the median voter. The first order condition for optimal choice of  $g_1$  is

(35) 
$$U_g^m + U_c^m \frac{dc_1}{dg_1} = 0$$

Total change in per capita consumption of private differentiated goods due to increased provision of local public goods is

(36) 
$$\frac{dc_1}{dg_1} = \frac{\partial c^1}{\partial g_1} + \frac{\partial c^1}{\partial L_1} \frac{dL_1}{dg_1}$$

The effect on private consumption of an increase in public goods supply may be split in two: The first is the direct effect, as given by equation (22). This is clearly negative. The second is the migration effect. If an increase in  $g_1$  leads to a change in  $U_1^M - U_2^M$ , there will be emigration or immigration. A change in the number of residents leads to a change in per capita consumption of differentiated goods, as given by equation (23). If  $L_1$  increases with increased  $g_1$ , the second term of (36) is positive. If, however,  $L_1$ decreases as  $g_1$  increases, the second term of (36) is negative.

Inserting (36) into (35) gives the first order condition for optimal supply of local public goods in community 1 as

(37) 
$$U_g^m + U_c^m \frac{\partial c_1}{\partial g_1} + U_c^m \frac{\partial c_1}{\partial L_1} \frac{dL_1}{dg_1} = 0$$

The migration effect depends on the direct effect on  $U_1^M$  of an increase in per capita supply of public goods in community 1. Specifically, we must have

(38) 
$$\frac{d(U_1^M - U_2^M)}{dg_1} = \frac{\partial (U_1^M - U_2^M)}{\partial L_1} \frac{dL_1}{dg_1} + U_g^M + U_c^M \frac{\partial c_1}{\partial g_1} = 0.$$

Define

(39) 
$$S \equiv -\frac{\partial (U_1^M - U_2^M)}{\partial L_1}$$

which is positive by the stability condition (equation (28)).

,

Solving (38), we get

(40) 
$$\frac{dL_1}{dg_1} = \frac{1}{S} \left( U_g^M + U_c^M \frac{\partial c_1}{\partial g_1} \right)$$

Inserting (40) into (37) gives

(41) 
$$U_g^m + U_c^m \frac{\partial c_1}{\partial g_1} + U_c^m \frac{\partial c_1}{\partial L_1} \frac{1}{S} \left( U_g^M + U_c^M \frac{\partial c_1}{\partial g_1} \right) = 0,$$

Define

(42) 
$$b \equiv \frac{\partial c^1}{\partial L_1} \frac{1}{S},$$

which is positive.

Manipulating (41) then gives the following first order condition for the local choice of  $g_1$ 

(43) 
$$\left(\frac{U_g^m}{U_c^m} + \frac{\partial c_1}{\partial g_1}\right) + \frac{b}{1+b} \left(\frac{U_g^M}{U_c^M} - \frac{U_g^m}{U_c^m}\right) = 0.$$

 $(U_g^h/U_c^h)$  is the marginal rate of substitution between consumption of publicly provided and private goods of person h, i.e. the marginal willingness to pay for an extra unit of the publicly provided good. Call it  $MRS_{g,c}$ . If  $MRS_{g,c}$  is increasing in  $\alpha_1$ , the median resident has a higher  $MRS_{g,c}$  than the marginal inhabitant. The second term of (44) is then negative, and the first term must be positive for the equality to hold. Conversely, if  $MRS_{g,c}$  is decreasing in  $\alpha_1$ , the second term is positive and the first term must be negative.

### Tax competition or competition in public services?

To interpret (43), consider first what it implies about the nature of competition between communities. Suppose first that  $MRS_{g,c}$  is increasing in  $\alpha_1$ ; i.e. that the marginal resident has a lower willingness to pay for publicly provided goods than the median voter. What will the tax/public-goods reactions functions look like in the two-community equilibrium? The answer is straightforward: If community 2 raises taxes and increases its supply of public goods, marginal residents will move to community 1. That will lower the  $MRS_{g,c}$  of the marginal resident in community 1. It will also lower the  $MRS_{g,c}$  of the median voter in community 1, as the new voters have a lower willingness to pay for public goods than the old ones. Taxes and local supply of public goods in community 1 will therefore unambiguously decrease. Thus, the reactions functions for public goods supply in the two communities must be downward-sloping.

It is equally clear that there will be undersupply of public goods in both communities relative to the preferences of the median voters in equilibrium. To see this, note that the

utility of the median voter in one community is increasing in the supply of public goods in the other community, as higher taxes there will induce people to move away. Thus, the iso-utility curves for the median voters must be as shown in figure 2. It follows that a cooperative solution between the median voters would entail higher taxes and greater supply of public goods in both communities, as indicated by the area I in figure 2. Thus, if  $MRS_{g,c}$  is increasing in  $\alpha_1$ , we shall see tax competition between the communities.

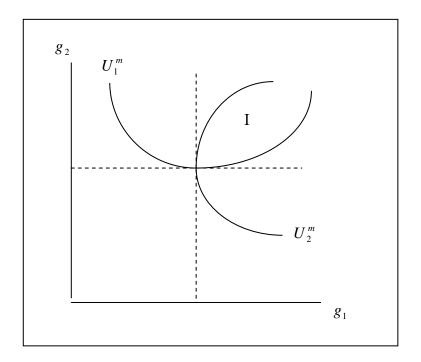


Figure 2

If  $MRS_{g,c}$  is decreasing in  $\alpha_1$ , we get the opposite result; i.e. competition in public services and overprovision of public goods relative to the preferences of the median voters. The verification of this is left to the reader.

## Efficiency

To see how the decentralised equilibrium deviates from the efficient solution, it is instructive to rewrite (43) as

(44) 
$$\left(\frac{U_g^A}{U_c^A} + \frac{\partial c_1}{\partial g_1}\right) + \left(\frac{U_g^m}{U_c^m} - \frac{U_g^A}{U_c^A}\right) + \frac{b}{1+b}\left(\frac{U_g^M}{U_c^M} - \frac{U_g^m}{U_c^m}\right) = 0$$

Recall that the first order condition for efficient supply of local public goods in community 1 is

$$(33^{\prime}) \quad \frac{U_g^A}{U_c^A} = -\frac{\partial c^1}{\partial g_1}.$$

Thus, there are two sources of possible inefficiency. The first is the cost-of-democracy wedge between the willingness to pay for public services of the median and the average voter. The second is the distortion arising because of competition for residents between local authorities. Both wedges could have either sign; so there is no a priori reason to believe that a democratic, decentralized solution will give systematic overprovision or underprovision of publicly provided goods. Nor is there any reason to believe that the two have the same sign. Thus, it could well be that decentralisation counteracts the democratic distortion; but it could equally well be that it magnifies it.

#### The "European" vs the "American" case

Whether the two sources of inefficiency reinforce or counteract each other depends on the distribution of residential preferences – specifically on whether the majority have strong residential preferences (the "European" case) or weak ones (the "American" case). In the latter case, of course, it is more likely that we will have geographic concentration, in which case local tax competition is no longer an issue at all. Barring that outcome, however, it follows from (44) that it is more likely that local autonomy creates serious distortions in the "American" than in the "European" case.

To see that, note from figure 1 (p.11) above that in a symmetric equilibrium, the distribution of people in each community over residential preference will be skewed. In the "American" case there will be overrepresentation in each community of people with  $\alpha$  close to zero; in the "European" case there will be overrepresentation of people with a relatively strong preference for living in the community, i.e. with  $\alpha$  quite different from zero. Thus, in the "American" case, the median voter will have preferences somewhere between those of the marginal resident and the mean preferences, i.e.

# (45) $\alpha_1^M < \alpha_1^m < \alpha_1^A$ The "American" case

while in the "European" case, the mean preferences will be somewhere between those of the marginal resident and the median voter:

If, as many argue, tax aversion increases with mobility, therefore, tax competition to attract more people will be counteracted by the democratic bias in favour of the median voter in the "European" case, while it will be reinforced by the democratic bias in the "American" case. Generally, the two wedges in (44) pull in opposite directions in the "European" case and the same direction in the "American case".

### Conclusions

In this paper we have developed a framework to study tax competition and local public goods supply in a setting where real and fiscal externalities interact with local democracy, and we have used the framework (a) to analyse if there is any reason to believe that local autonomy generally will give a tax race to the bottom (there is not), and (b) to look more closely at possible sources of oversupply or undersupply of publicly provided goods in a setting where local democracies compete in order to attract more people to the area. We have identified two potential sources – the relationship between individual mobility and willingness to pay for publicly provided goods, and the mobility distribution of individuals (i.e. the distribution of individuals over residential preferences). The two could reinforce each other in a local democracy if the majority of the residents in a community are relatively mobile (the "American" case), while they would pull in opposite directions if the majority of residents are relatively immobile (the "European" case).

#### References

Andersson, F. and R. Forslid (2003), "Tax competition and economic geography", *Journal of Public Economic Theory* 5, 279-304.

Faini, R., V. Norman, F. Rouane and P. Seabright (2000), Integration and the Regions of Europe: How the Right Policies Can Prevent Polarization, *Monitoring European Integration*, 10, CEPR.

Baldwin, R. E. and P. R. Krugman (2003), "Agglomeration, integration and tax harmonisation", *European Economic Review* 48, 1-23.

Dixit, A. K. and J. E. Stiglitz (1977), "Monopolistic competition and optimum product diversity", *American Economic Review* 67, 297-308.

Honkapohja, S. and A. Turunen-Red (2004), "Gains and losses from tax competition with migration", *working paper* 2004-01, University of New Orleans, Department of Economics and Finance.

Kind, H. J., K. H. Midelfart-Knarvik. and G. Schjelderup (2000), "Competing for capital in a lumpy world", *Journal of Public Economics* 78, 253-274.

Krugman, P. R. (1991), Geography and trade, MIT Press.

Krugman, P. R. and A. J. Venables (1995), "Globalization and the inequality of nations", *Quarterly Journal of Economics*, 110, nr. 4, 857-880.

Sinn, H.-W. (2003), The New Systems Competition, Basil Blackwell, Oxford.

Spence, A. M. (1976), "Product selection, fixed costs and monopolistic competition", *Review of Economic Studies* 43, 217-235.

Tiebout, C. M. (1956), "A pure theory of local expenditures", *Journal of Political Economy*, 64, 416-424.

Venables, A. J. (1996), "Equilibium locations of vertically linked industries", *International Economic Review* 37, 341-359.

Wilson, J. D. (1986), "A theory of interregional tax competition", *Journal of Urban Economics* 19m 296-315.

Wilson, J. D. (1999), "Theories of tax competition", *National Tax Journal* LII, 269-304.

# Public goods production and private sector productivity

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Abstract: In this paper we study how the use of resources in the public sector affects industrial structure, the size and the productivity in knowledge-intensive clusters in local communities. We also discuss how these considerations should be implemented in costbenefit assessments of local public goods supply. The topics are studied in a setting where there are gains from agglomeration in knowledge-intensive industries, creating clusters of firms in such industries. We find that the primary effect is a Rybczynski effect: If production in the public sector is knowledge-intensive, the size of the knowledge-intensive private industry declines when the public sector increases its production. If, on the other hand, public sector production uses relatively much unskilled labour, increased public goods production leads to higher production in the knowledge-intensive private industries. Private sector productivity is affected in the same way as production: If production in the knowledge-intensive industries, so does its productivity due to agglomeration effects; leading to higher wages for highly skilled labour.

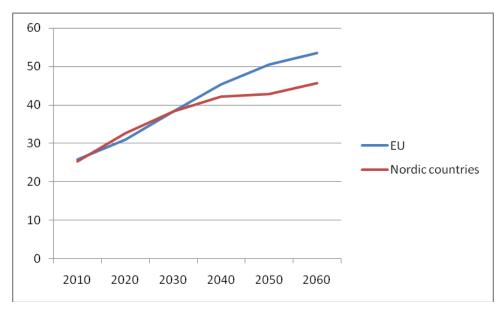
#### JEL classification: D24, H7, R3, R5

**Keywords:** Agglomeration, external economies of scale, firm location, production cost, regional government policies

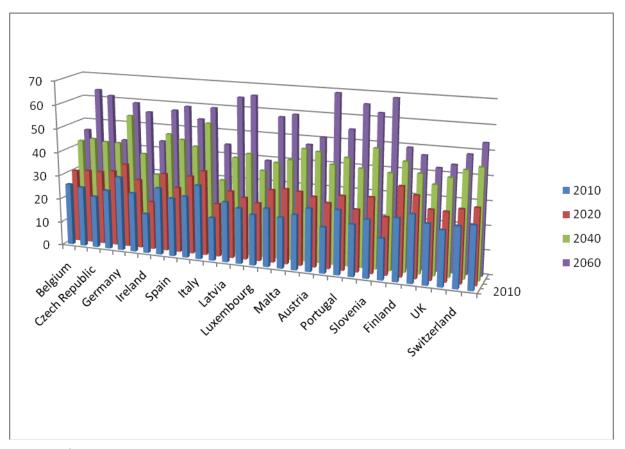
# 1. Introduction

This paper studies how the use of resources for public sector purposes affects the size, structure and productivity in the private sector of the economy. There is concern in many countries and local communities that public sector activities may crowd out important private sector firms and industries. Some observers and policy-makers, however, argue that public sector jobs may be complementary to private-sector employment, and accordingly that a large public sector, properly designed, can have positive effects on the size and productivity of the private sector. The purpose of the paper is to study the interaction between crowding-out and possible complementarities in a setting where there are agglomeration gains in the private sector.

The analysis is relevant in several contexts. One is the effects related to the problems of an ageing population. Many people fear that the ageing of populations that we are witnessing in a range of countries - illustrated by the old-age dependency ratios in figures 1 and 2 - could have a negative effect on industrial productivity. Undoubtedly, as people grow older they require more care and health services. Many, in fact most, of these services are publicly provided and so an aging population implies a larger public sector. Estimations show that public employment in the Norwegian health sector will more than double between 2020 and 2060 (V.O. Nielsen (2008)). Without growth in the labour stock, increased public sector employment leads to decreased private sector employment. If this reduces industrial productivity, an ageing population is indeed a threat to industrial productivity.



*Figure 1: Projected old-age dependency ratio in the EU and the Nordic countries*<sup>1</sup> (*Eurostat*)



*Figure 2<sup>2</sup>: Projected old-age dependency ratio in the EU and the Nordic countries (Eurostat)* 

<sup>&</sup>lt;sup>1</sup> The Nordic countries: Denmark, Finland, Norway and Sweden.

<sup>&</sup>lt;sup>2</sup> From left to right the countries are Belgium, Bulgaria, Czech Republic, Denmark, Germany, Estonia, Ireland, Greece, Spain, France, Italy, Cyprus, Latvia, Lithuania, Luxembourg, Hungary, Malta, the Netherlands, Austria, Poland, Portugal, Romania, Slovenia, Slovakia, Finland, Sweden, UK, Norway, Switzerland, the EU

Another relevant context is that of regional policy. It is sometimes argued<sup>3</sup> that local governments should try to foster knowledge-intensive industrial agglomerations by establishing more public jobs for highly educated individuals – the presumption being that this will attract more highly educated people to the region and thus benefit the private sector as well.

The paper shows that the intuition is wrong in both cases. If care for the elderly is intensive in the use of low-skilled labour, growth in public care will have a positive effect on knowledge-intensive industrial agglomerations and thus on private-sector productivity. Conversely, if local governments try to attract highly educated people to a region by establishing more public jobs for such people, they will fail. More public jobs for highly skilled workers will lower their wage and thus make it less attractive for such individuals to move to a region.

The reason in both cases is that the first-order effect of public sector resource use on the private sector is a Rybczynski effect (Rybczynski (1955)). If the government hires unskilled workers, and thus reduces the supply of such workers to the private sector, the effect will be a reduction in production and employment in low-skilled private firms and growth in production and employment in high-skilled industries. Conversely, public-sector employment of highly skilled workers will have a negative effect on high-skilled private industry and a positive effect on low-skilled firms. This first-order effect is magnified by industrial agglomeration forces, which also leads to effects on private-sector productivity.

The model used in the paper has two factors of production, skilled and unskilled labour. These are used in three production sectors – one public and two private (a knowledgeintensive and a sector which uses unskilled labour intensively). There are constant returns to scale at the firm level<sup>4</sup>, but there are external economies of scale in the knowledge-intensive sector.

In section 2 we develop the basic model. Equilibrium conditions are derived in section 3. In section 3 we also analyse how public sector production affects industrial structure in the private sector. In section 4 we perform a welfare-analysis and discuss how the obtained

<sup>&</sup>lt;sup>3</sup> See e.g. St.meld nr 17 (2002-2003)

<sup>&</sup>lt;sup>4</sup> This implicitly assumes that other factors, such as capital, are freely traded at a fixed price and thus can be netted out.

results should be implemented in cost-benefit analyses of public goods provision. Possible extensions are pointed out in section 5. Section 6 provides a brief summary.

# 2. The basic model

We consider a small, open economy in which there are three sectors of production: Two private and a public sector.

There are two factors of production - skilled and unskilled labour<sup>5</sup>. Every inhabitant is a worker and each worker (whether skilled or unskilled) inelastically supplies one unit of labour. Total labour supply thus equals the total number of inhabitants in the economy and regional labour supply equals the total number of inhabitants in a region.

Firms in the private industries produce homogenous consumption goods sold in perfectly competitive world markets at given prices. One of the industries is knowledge-intensive *and* experiences external economies of scale (see e.g. Audretsch and Feldman (1996) for the importance of external economies in knowledge-intensive industries and Ottaviano and Puga (1998) or Fujita and Thisse (2002) for overviews of the new economic geography literature). The other industry produces subject to constant returns to scale at both firm and industry levels.

In the public sector, locally consumed goods and services are produced.

Our main question is: How does public sector production affect the private sector; in particular, the industrial structure, the size and the productivity in the private sector?

### 2.1. The private sector

In the private sector there are two industries, each composed of a large number of firms. We label the two industries 1 and 2. Firms in both industries produce homogenous consumption goods with constant returns to scale at the firm level. There are two factors of production - skilled and unskilled labour. The two industries differ in their skill-intensiveness<sup>6</sup> of production, with industry 1 being the most skill-intensive industry. In the skill-intensive industry 1 there are external economies of scale in production.

<sup>&</sup>lt;sup>5</sup> We assume that unskilled workers cannot become skilled or vice versa.

<sup>&</sup>lt;sup>6</sup> We use the terms skill-intensive and knowledge-intensive interchangeably

Total production in industry 1,  $x_1$ , is given by the aggregate production function

(1) 
$$x_1 = \varphi(x_1) \alpha(l_1^s, l_1^u),$$

where  $l_1^s$  and  $l_1^u$  are the numbers of skilled and unskilled labour, respectively, used to produce good 1.  $\varphi(x_1)$  captures the external economies of scale. It is an increasing and concave function;  $\varphi'(x_1) > 0$ ,  $\varphi''(x_1) < 0$ .

Total production of good 2,  $x_2$ , is given by the aggregate production function

(2) 
$$x_2 = \beta(l_2^s, l_2^u)$$
.

Both  $\alpha(l_1^s, l_1^u)$  and  $\beta(l_2^s, l_2^u)$  inhibit constant returns to scale.

To simplify the model we assume fixed coefficients in the production of each good, and let the aggregate production functions be

(3) 
$$x_1 = \varphi(x_1) \min(A_1^s l_1^s, A_1^u l_1^u)$$

(4) 
$$x_2 = \min(A_2^s l_2^s, A_2^u l_2^u),$$

where  $A_i^j$  are constants, i = 1,2; j = s, u

Private sector demand for skilled and unskilled labour,  $l_d^s$  and  $l_d^u$  follows as

(5) 
$$l_d^s = \gamma(x_1)a_1^s x_1 + a_2^s x_2,$$

(6) 
$$l_d^u = \gamma(x_1)a_1^u x_1 + a_2^u x_2;$$
  $a_i^j \equiv \frac{1}{A_i^j}; \ \gamma(x_1) \equiv \frac{1}{\varphi(x_1)}; \ i = 1,2; \ j = s, u$ 

Let  $l^s$  and  $l^u$  be the supplies of skilled and unskilled labour facing firms in the private sector. Equality of labour supply and private sector labour demand is given by

(7) 
$$l^s = \gamma(x_1)a_1^s x_1 + a_2^s x_2$$
,

(8) 
$$l^{u} = \gamma(x_{1})a_{1}^{u}x_{1} + a_{2}^{u}x_{2}.$$

Skilled and unskilled labour are the only factors of production, and so total costs of producing good *i* are

(9) 
$$TC_i = w^s l_i^s + w^u l_i^u,$$

where  $w^s$  and  $w^u$  are the wage rates of skilled and unskilled labour, respectively.

Define unit costs of a *single firm* as

(10) 
$$b_i(w^s, w^u) \equiv a_i^s w^s + a_i^u w^u; i=1,2.$$

Unit costs of the entire *industries* are

(11) 
$$\gamma(x_1)b_1(w^s,w^u),$$

(12) 
$$b_2(w^s, w^u)$$

for industry 1 and 2 respectively.

Private firms sell their goods in perfectly competitive world markets with no trade costs. Each firm equates marginal cost to the prevailing market price of the good. There are constant returns to scale at the firm level and hence marginal cost equals unit cost. Equilibria in the goods markets are therefore given by

(13) 
$$\gamma(x_1)b_1(w^s, w^u) = p_1,$$

(14) 
$$b_2(w^s, w^u) = p_2,$$

i.e. unit costs equal the prices of the goods, where  $p_i$  is the price of good *i*.

### 2.2. The public sector

In the public sector locally consumed goods and services are produced by skilled and unskilled labour. The production process is subject to constant returns to scale. We assume constant coefficients in the production process<sup>7</sup>.

Public sector labour demand is

- $(15) \qquad l_g^s = a_g^s g \,,$
- $(16) \qquad l_g^u = a_g^u g ,$

where g is the amount of publicly produced goods and services. The amount of publicly produced goods and services, g, is determined by local governments, and the size of g indirectly determines public sector employment,  $l_g^s$  and  $l_g^u$ .

The supplies of skilled and unskilled labour facing firms in the private sector are

- (17)  $l^s = n^s a_g^s g,$
- $(18) \qquad l^u = n^u a_g^u g \,.$

 $n^{s}$  and  $n^{u}$  denote the total supplies of skilled and unskilled labour in the economy.

<sup>&</sup>lt;sup>7</sup> The assumption is made to simplify the model, and the analytical results hold even if this assumption is relaxed. See the appendix for a geometric "proof".

# 3. Equilibrium

General equilibrium obtains when labour and goods markets clear.

Labour markets clear when equations (7), (8), (15) and (16) hold. Equations (15) and (16) show how much labour the public sector needs in order to be able to produce the desired amount of public goods. Equations (7) and (8) give the conditions for private sector labour market equilibria (i.e. private firms' labour demand equals the labour supply facing private firms).

Goods market equilibrium is obtained when equations (13) and (14) hold.

We perform a two-fold general equilibrium analysis. First, we assume that there is no production in the public sector. This is done so that we may establish the basic general equilibrium conditions. Thereafter, we introduce public goods production in order to study the effects of public sector production on the productivity and production structure in the private sector.

#### 3.1. Equilibrium with no public sector activity

With no public sector activity, there are firms in two industries producing homogenous consumption goods sold in perfectly competitive world markets. General equilibrium requires that the markets for skilled and unskilled labour clear (as given by equations (7) and (8)), and that the goods markets clear (as given by equations (13) and (14)).

Labour supplies are fixed and with no public sector production the supplies of skilled and unskilled workers facing private firms equal the total number of skilled and unskilled workers in the economy,  $l^s$  equals  $n^s$  and  $l^u$  equals  $n^u$ . We may then solve equations (7) and (8) to find the volumes of private goods production

(19) 
$$n^s = \gamma(x_1)a_1^s x_1 + a_2^s x_2$$
 and  $n^u = \gamma(x_1)a_1^u x_1 + a_2^u x_2 \implies (\hat{x}_1, \hat{x}_2)$ .

Goods markets equilibria are given by equations (13) and (14). Inserting for the volumes of private goods production from (19) we solve these to find the wage rates of skilled and unskilled labour

(20) 
$$\gamma(x_1)b_1(w^s, w^u) = p_1 \text{ and } b_2(w^s, w^u) = p_2 \implies w^j(p_1, p_2, \hat{x}_1); j = s, u.$$

The equilibrium conditions are illustrated in figures 3 and 4.

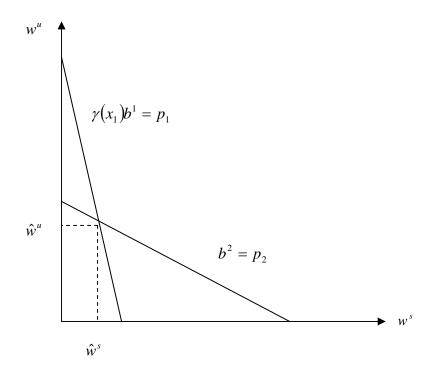


Figure 3: Equilibrium wage rates

In figure 3 we measure the wage rate of skilled workers along the horizontal axis and the wage rate of unskilled workers along the vertical axis. The curves illustrate the conditions for goods market equilibrium; i.e. unit costs equal goods prices. The slope of the unit cost contour of industry *i* is  $-(A_i^u/A_i^s)$ . (The unit cost contours are straight lines due to the assumption of fixed coefficients in production). Equilibrium wage rates,  $\hat{w}^s$  and  $\hat{w}^u$ , are found at the intersection of the two curves.

The location of the unit cost contour of industry 1 depends upon the size of the industry. This contrasts standard Heckscher-Ohlin models and is due to the assumption of external economies of scale in the industry; the larger the industry, the higher the productivity. Thus,

unit costs of the industry as a whole decreases with the size of the industry. The unit cost contour of industry 1 moves to the north-east with increased industry 1 production. We see that the wage rate of skilled labour is an increasing function of the volume of industry 1 production, the wage rate of unskilled labour is a decreasing one.

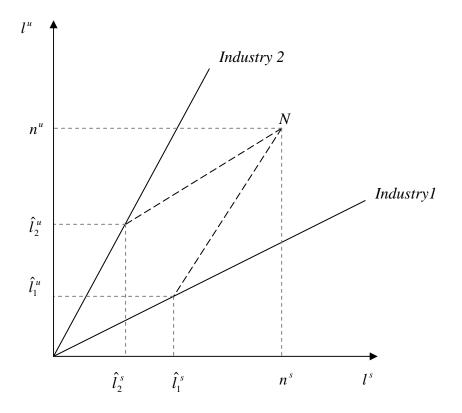


Figure 4: Production structure

In figure 4 we measure the number of skilled workers along the horizontal axis and the number of unskilled workers along the vertical. The curves show the labour input requirements of the two industries. Total labour supply is given by (the point) N. Industry 1 employs  $\hat{l}_1^s$  number of skilled workers and  $\hat{l}_1^u$  number of unskilled workers. Industry 2 employs  $\hat{l}_2^s$  number of skilled workers and  $\hat{l}_2^u$  number of unskilled workers. The size of the two industries (volume of production) follows from the levels of employment.

#### 3.2. Equilibrium with public sector production

In the public sector locally consumed goods are produced by skilled and unskilled labour. We start the analysis by taking the amount of public goods production, g, to be fixed. I.e. we do

 $l^s$ 

not consider how the amount of publicly produced goods and services is determined. Public sector labour employment is then given by the volume of public goods production (the size of g). We will return to the question of optimal public goods supply in section 4. Compared to the analysis of section 2.1, the labour supply facing firms in the private sector is reduced by the level of public sector employment.

Our aim is to study how the structure of production and factor payments (which follow factor productivity) in the private sector are affected by the use of resources for public goods production.

#### **3.2.1. Production structure effects**

In order to make the analysis as simple as possible, we define a new variable,  $z_1$ , defined as

(21) 
$$z_1 \equiv \gamma(x_1) x_1,$$

 $z_1$  shows industry 1 production adjusted for the external economies, i.e. how much firms in industry 1 would have produced if the industry had *not* experienced external economies of scale.

Inserting for  $z_1$  into the equations for labour market equilibrium, equations (7) and (8), gives us

- (22)  $l^s = a_1^s z_1 + a_2^s x_2$ ,
- (23)  $l^u = a_1^u z_1 + a_2^u x_2.$

Manipulating (22) and (23) allow us to express  $z_1$  in terms of production coefficients and labour supply in the private sector

(24) 
$$z_1 = \frac{1}{D} \left( a_2^s l^u - a_2^u l^s \right); \quad D \equiv a_1^u a_2^s - a_1^s a_2^u < 0.$$

Differentiating (24) with respect to g gives

(25) 
$$\frac{dz_1}{dg} = \frac{1}{D} a_g^u a_2^u \left( \frac{a_g^s}{a_g^u} - \frac{a_2^s}{a_2^u} \right).$$

Whether  $(dz_1/dg)$  is positive or negative depends on the skill-intensity in public goods production compared to the skill-intensity in industry 2 production (the least skill-intensive private industry)

(26) 
$$\frac{dz_1}{dg} > 0$$
 iff  $\frac{a_g^s}{a_g^u} < \frac{a_2^s}{a_2^u}$ 

(27) 
$$\frac{dz_1}{dg} < 0 \text{ iff } \frac{a_g^s}{a_g^u} > \frac{a_2^s}{a_2^u}.$$

Next, we differentiate  $z_1$  as given by its definition in equation (21) with respect to the amount of public goods production, g:

(28) 
$$\frac{dz_1}{dg} = \left(\gamma + \gamma' x_1\right) \frac{dx_1}{dg},$$

which gives

(29) 
$$\frac{dx_1}{dg} = \frac{1}{\left(\frac{dz_1}{dg}\right)} \left(\gamma + \gamma' x_1\right)$$

The sign of  $(dx_1/dg)$  is equal to the sign of  $(dz_1/dg)$ :

(30) 
$$\frac{dx_1}{dg} > 0$$
 iff  $\frac{a_g^s}{a_g^u} < \frac{a_2^s}{a_2^u}$ 

(31) 
$$\frac{dx_1}{dg} < 0 \text{ iff } \frac{a_g^s}{a_g^u} > \frac{a_2^s}{a_2^u}$$

The size of industry 1 is an increasing function of the volume of public goods production if and only if public goods production is less knowledge-intensive than industry 2 production.

This is the Rybczynski effect. If public goods production is less skill-intensive than production in industry 2, increased public goods production leads to increased relative supply of highly skilled labour to private firms. The increased relative supply of highly skilled labour leads to increased production in the knowledge-intensive industry, as predicted by the Rybczynski theorem.

The effect of public goods production on the size of industry 2 is found by performing the same kind of mathematical reasoning as done through equations (22) to (31). This leads to the following results

(32) 
$$\frac{dx_2}{dg} > 0$$
 iff  $\frac{a_g^s}{a_g^u} > \frac{a_1^s}{a_1^u}$ 

(33) 
$$\frac{dx_2}{dg} < 0$$
 iff  $\frac{a_g^s}{a_g^u} < \frac{a_1^s}{a_1^u}$ .

The size of industry 2 is an increasing function of the volume of public goods production if and only if public goods production is more skill-intensive than the production process in the most skill-intensive private industry. The Rybczynski effect once again.

When we combine the results given by equations (30) to (33), and bear in mind that industry 1 is more skill-intensive than industry 2, we are left with three possible outcomes regarding the impact of public goods production on the production structure of the private sector, depending on the knowledge-intensity in public sector production.

### Case 1: Public goods production is the least knowledge-intensive production process

If production of public goods is the least knowledge-intensive production process, increasing these activities leads to increased production in industry 1 and decreased production in industry 2.

(34) 
$$\frac{a_1^s}{a_1^u} > \frac{a_2^s}{a_2^u} > \frac{a_g^s}{a_g^u} \implies \frac{dx_1}{dg} > 0, \ \frac{dx_2}{dg} < 0.$$

The effects are illustrated in figure 5. Total labour supply is given by N, which, with no public goods production, is allocated between the two private industries as illustrated in figure 4. With public sector production, the labour supply facing private firms is reduced to N' where the relative supply of unskilled labour facing private firms is reduced. This leads to increased sector 1 production and decreased sector 2 production, as illustrated by the arrows.

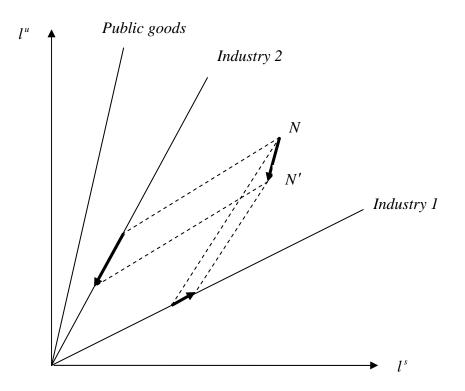


Figure 5: Production structure effects when public sector production is the least skillintensive

### **Case 2: Balanced knowledge-intensity in public sector production**

If public sector production is neither extremely skill-intensive nor extremely intensive in the use of unskilled labour, then increasing public goods production leads to an overall reduction in private sector activities:

$$(35) \quad \frac{a_1^s}{a_1^u} > \frac{a_g^s}{a_g^u} > \frac{a_2^s}{a_2^u} \implies \frac{dx_1}{dg} < 0, \ \frac{dx_2}{dg} < 0.$$

This effect is illustrated in figure 6.

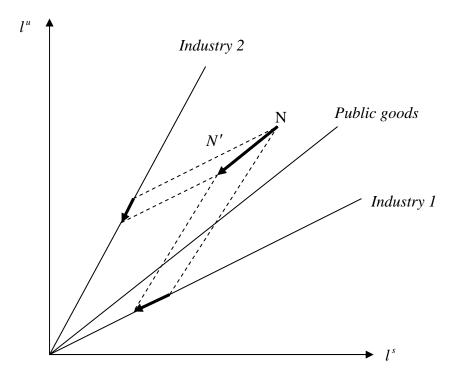


Figure 6: Production structure effects with balanced knowledge-intensity in public goods production

### Case 3: Public sector production is the most skill-intensive production process

Finally, if public sector production is the most knowledge-intensive production process, increased public goods production leads to a smaller knowledge-intensive and a larger unskilled-intensive private industry:

$$(36) \quad \frac{a_g^s}{a_g^u} > \frac{a_1^s}{a_1^u} > \frac{a_2^s}{a_2^u} \implies \frac{dx_1}{dg} < 0, \ \frac{dx_2}{dg} > 0.$$

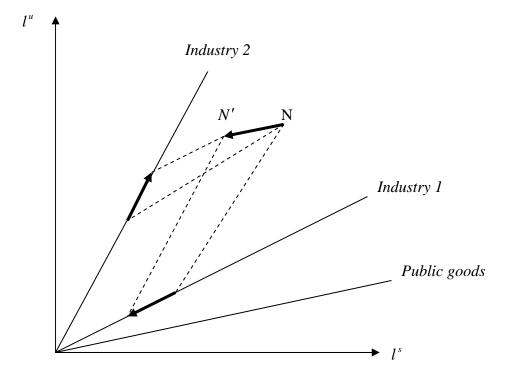


Figure 7: Production structure effects when public sector production is knowledge-intensive

To find the equilibrium wage rates we insert for unit costs into equation (13) and (14), and get

(37) 
$$\gamma(x_1) \left[ w^s a_1^s + w^u a_1^u \right] = p_1$$

(38) 
$$\left[w^{s}a_{2}^{s}+w^{u}a_{2}^{u}\right]=p_{2}$$

Manipulating equations (37) and (38) yields the following expression for the wage rate of skilled labour

(39) 
$$w^{s} = \frac{1}{E} \left[ \frac{p_{2}}{a_{2}^{u}} - \frac{1}{\gamma(x_{1})} \frac{p_{1}}{a_{1}^{u}} \right];$$

Where *E* is defined as the difference between the knowledge-intensiveness in production of the private goods;

$$E \equiv \frac{a_2^s}{a_2^u} - \frac{a_1^s}{a_1^u} < 0$$

Differentiating equation (39) with respect to public sector production, g, yields

(40) 
$$\frac{dw^s}{dg} = \frac{1}{E} \left[ \frac{1}{\gamma^2} \gamma' \frac{p_1}{a_1^u} \frac{dx_1}{dg} \right].$$

We see that

(41) 
$$\frac{dw^s}{dg} > 0 \text{ iff } \frac{dx_1}{dg} > 0$$

The wage rate of skilled labour is an increasing function of the amount of public goods production if and only if the size of the knowledge-intensive industry increases with increased public goods production. This is due to the agglomeration gains. As the knowledge-intensive industry grows, so does productivity and hence payments to the factor used intensively in production – namely highly skilled labour.

(42) 
$$\frac{dw^s}{dg} < 0 \text{ iff } \frac{dx_1}{dg} < 0.$$

Conversely, if the size of the knowledge-intensive industry 1 declines as public goods production increases, the wage rate of skilled labour also declines.

From equations (37) and (38) we find the wage rate of unskilled labour

(43) 
$$w^{u} = \frac{1}{F(x_{1})} \left[ \frac{p_{2}}{a_{2}^{s}} - \frac{1}{\gamma(x_{1})} \frac{p_{1}}{a_{1}^{s}} \right]$$

where  $F(x_1)$  is defined as

$$F(x_1) \equiv \frac{a_2^u}{a_2^s} - \frac{1}{\gamma(x_1)} \frac{a_1^u}{a_1^s} > 0.$$

In order to find the effect of public goods production on the wage rate of unskilled labour, we differentiate equation (43) with respect to g,

(44) 
$$\frac{dw^{u}}{dg} = -\frac{F'}{F^{3}} \left[ \frac{1}{\gamma^{2}} \gamma' \frac{p_{1}}{a_{1}^{s}} \frac{dx_{1}}{dg} \right]; \quad F' = \frac{1}{\gamma^{2}} \gamma' \frac{a_{1}^{u}}{a_{1}^{s}} \frac{dx_{1}}{dg},$$

and find

(45) 
$$\frac{dw^u}{dg} > 0 \text{ iff } \frac{dx_1}{dg} < 0,$$

(46) 
$$\frac{dw^u}{dg} < 0 \text{ iff } \frac{dx_1}{dg} > 0.$$

Combining the results of equations (41), (42), (45) and (46), we find the total effect of public goods production on the wage rates. Two possibilities arise: Either the wage rate of skilled labour increases and the wage rate of unskilled decreases. This happens if public goods production leads to increased production in the skill-intensive industry. Or the wage rate of skilled workers decreases and the wage rate of unskilled workers increases. This is the case if

increased public goods production leads to lower production in the skill-intensive private industry.

#### **3.2.2 Wage effects**

As we have seen, the use of resources in the public sector affects not only production structures but also factor payments. To find the link between knowledge-intensiveness of production and the wage effects we combine the above result with equations (34), (35) and (36), resulting in

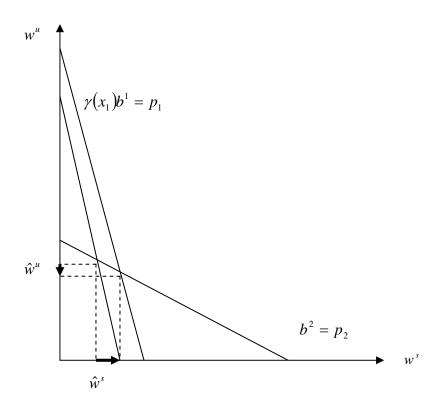
(47) 
$$\frac{a_1^s}{a_1^u} > \frac{a_2^s}{a_2^u} > \frac{a_g^s}{a_g^u} \implies \frac{dw^s}{dg} > 0, \ \frac{dw^u}{dg} < 0$$

$$(48) \qquad \frac{a_1^s}{a_1^u} > \frac{a_g^s}{a_g^u} > \frac{a_2^s}{a_2^u} \lor \quad \frac{a_g^s}{a_g^u} > \frac{a_1^s}{a_1^u} > \frac{a_2^s}{a_2^u} \implies \quad \frac{dw^s}{dg} < 0, \quad \frac{dw^u}{dg} > 0.$$

If and only if the production process in the public sector is the least skill-intensive of all production processes does an increase in public goods production lead to increased wages of skilled labour – as shown by equation (47). Otherwise, public sector activities will lead to reduced wage rates of highly skilled labour and increased wage rates of low-skilled labour – as shown by equation (48).

The two possibilities of public goods production on the wage rates are illustrated in figures 8 and 9. We measure the wage rate of skilled labour along the vertical axis and the wage rate of unskilled labour along the horizontal axis. The unit cost contours of industries 1 and 2 are drawn.

Figure 8 shows the situation in which public goods production leads to increased production in the skill-intensive industry 1. Public goods production is the least skill-intensive of all production processes. As the volume of industry 1 production goes up, so does the productivity in the industry. The increased productivity moves the unit cost contour to the northeast, leading to an increase in the wage rate of skilled labour and a decrease in the wage rate of unskilled labour.



*Figure 8: Wage effects of public sector production which is intensive in the use of unskilled labour* 

Figure 9 illustrates the case in which public goods production leads to a smaller skill-intensive industry 1; which is the case whenever public goods production is more skill-intensive than the least skill-intensive of the private industries. The productivity of industry 1 declines as the volume of production in the industry declines. This leads to a movement of the unit cost contour to the south-west. The wage rate of skilled labour goes down and the wage rate of unskilled labour goes up.

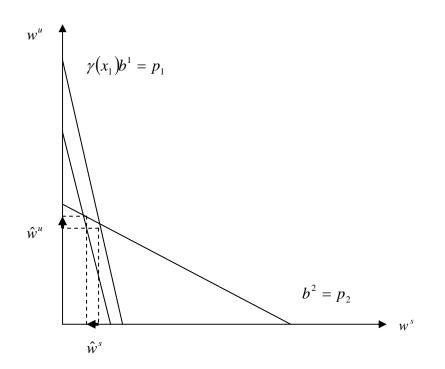


Figure 9: Wage effects of skill-intensive public sector production

### 4. Utility and welfare

Utility derives from the consumption of private and public goods. The utility of individual k is a function of the amounts consumed of the two private goods and the provision of public goods. We assume that everyone shares the same utility *function*. The utility of individual k is

(49) 
$$U^{k} = (c_{1}^{k}, c_{2}^{k}, g),$$

where  $c_i^k$  is individual k's consumption of good i.

Individuals make optimising choices regarding the consumption of private goods. Utility can therefore be expressed by the indirect utility function

(50) 
$$V^{k} = y^{k} + f^{k}(g); (f^{k})' > 0; (f^{k})'' < 0$$

Preferences are assumed to be quasi-linear.  $f^k(g)$  is the utility person k gets from consuming public goods. Labour income is assumed to be the only source of income, and  $y^k$  is the income net of taxes of individual k. We do not consider the question of how public goods production could or should be financed, but simply assume that it is financed through a lumpsum tax.

The optimal amount of public sector production depends on whether the goods produced are pure public goods or publicly provided private goods. In the following analysis we will assume that the goods are pure public goods with no rivalry in consumption, and in chapter 4 briefly discuss how the results would be altered if the goods produced in the public sector were private goods.

Welfare is the sum of all individuals' utility

(51) 
$$W = \sum_{k=1}^{n} V^{k} = y + \sum_{k=1}^{n} f^{k}(g); y \equiv \sum_{k=1}^{n} y^{k}$$
.

Finding the optimal amount of public goods production amounts to choosing the g which maximises equation (51)

(52) 
$$\max_{g} W = \max_{g} \left[ y + \sum_{k=1}^{n} f^{k}(g) \right].$$

The sum of all individuals' labour income must equal the value of production in the private sector,

$$(53) \qquad y = p_1 x_1 + p_2 x_2.$$

Differentiating y with respect to g gives

(54) 
$$\frac{dy}{dg} = p_1 \frac{dx_1}{dg} + p_2 \frac{dx_2}{dg}.$$

Equilibrium wage rates of skilled and unskilled labour equals the value of the marginal product of skilled and unskilled labour, respectively. From equations (1) and (2) we therefore know that

$$(55) \quad p_1\varphi\alpha'_s=w^s=p_2\beta'_s,$$

$$(56) \quad p_1\varphi\alpha'_u=w^u=p_2\beta'_u,$$

where

$$\alpha'_{j} \equiv \frac{\partial \alpha \left( l_{1}^{s}, l_{1}^{u} \right)}{\partial l_{1}^{j}}; \ j = s, u$$

$$\beta'_{j} \equiv \frac{\partial \beta (l_{2}^{s}, l_{2}^{u})}{\partial l_{2}^{j}}; j = s, u$$

From equation (1) we also find how the production of goods in the knowledge-intensive sector is affected by public goods production,

(57) 
$$\frac{dx_1}{dg} = \alpha \varphi' \frac{dx_1}{dg} + \varphi \alpha'_s \frac{dl_1^s}{dg} + \varphi \alpha'_u \frac{dl_1^u}{dg},$$

which gives

(58) 
$$\frac{dx_1}{dg} = \left(\frac{1}{1 - \alpha \varphi'}\right) \left[\varphi \alpha'_s \frac{dl_1^s}{dg} + \varphi \alpha'_u \frac{dl_1^u}{dg}\right].$$

Similarly, from equation (2) we find how the production of goods in industry 2 is affected by public goods production,

(59) 
$$\frac{dx_2}{dg} = \beta'_s \frac{dl_2^s}{dg} + \beta'_u \frac{dl_2^u}{dg}.$$

Inserting from equations (55), (56), (58) and (59) into (54) gives

(60) 
$$\frac{dy}{dg} = \left[ w^s \left( \frac{dl_1^s}{dg} + \frac{dl_2^s}{dg} \right) + w^u \left( \frac{dl_1^u}{dg} + \frac{dl_2^u}{dg} \right) \right] + \left( \frac{\alpha \varphi'}{1 - \alpha \varphi'} \right) \left[ p_1 \varphi \alpha'_s \frac{dl_1^s}{dg} + p_1 \varphi \alpha'_u \frac{dl_1^u}{dg} \right].$$

The total reduction in private sector labour employment must equal the increased employment in the public sector,

(61) 
$$\frac{dl_g^s}{dg} = -\left(\frac{dl_1^s}{dg} + \frac{dl_2^s}{dg}\right),$$

(62) 
$$\frac{dl_g^u}{dg} = -\left(\frac{dl_1^u}{dg} + \frac{dl_2^u}{dg}\right).$$

Inserting from (61) and (62) into equation (60) gives

(63) 
$$\frac{dy}{dg} = -\left[w^s \frac{dl_g^s}{dg} + w^u \frac{dl_g^u}{dg}\right] + \alpha \varphi' \left\{ \left(\frac{1}{1 - \alpha \varphi'}\right) \left[p_1 \varphi \alpha'_s \frac{dl_1^s}{dg} + p_1 \varphi \alpha'_u \frac{dl_1^u}{dg}\right] \right\}$$

The term in the last parenthesis equals  $p_1(dx_1/dg)$ , i.e. the value of the production effects in the knowledge-intensive industry of increased public goods production. Using this fact, we finally arrive at

(64) 
$$\frac{dy}{dg} = -\left[w^s \frac{dl_g^s}{dg} + w^u \frac{dl_g^u}{dg}\right] + p_1 \alpha \varphi' \frac{dx_1}{dg}$$

Equation (64) shows the marginal cost of public goods production, defined as the amount of foregone consumption of private goods. Total marginal cost is the sum of the direct costs and the indirect costs or cost reductions induced by production of public goods. The direct costs are the wage costs, as shown by the term in parenthesis. The indirect effects on marginal costs arise because of the productivity effects public goods production exert on the private sector. Marginal costs rise if public goods production leads to a smaller knowledge-intensive sector. Whereas they fall if the knowledge-intensive sector grows as public goods production does so. The indirect effect is given by the last term of equation (64).

Now, return to the welfare maximisation problem of equation (52).

The optimal supply of public goods is such that

(65) 
$$\frac{dW}{dg} = 0 \Longrightarrow -\frac{dy}{dg} = \sum_{k=1}^{n} f^{k'}$$

Equation (65) states the (obvious) condition that the optimal g is such that the total marginal cost of public goods production equals the sum of marginal utilities of public goods consumption.

Inserting for (dy/dg) equation (65) becomes

(66) 
$$\left[w^{s}\frac{dl_{g}^{s}}{dg}+w^{u}\frac{dl_{g}^{u}}{dg}\right]-\alpha\varphi'p_{1}\frac{dx_{1}}{dg}=\sum_{k=1}^{n}f^{k'},$$

which is a modified version of the Samuelson rule (Samuelson (1954)). It states that the optimal supply of public goods is such that marginal cost equals the aggregate marginal willingness to pay. Marginal costs are, however, modified to include the indirect costs caused by external economies.

The implications for optimal public goods supply follow: If public goods production leads to decreased productivity in the private sector (the case if knowledge-intensive production is reduced), then the optimal supply of public goods is smaller than in a situation where public goods production has no productivity effect in the private sector. If, on the other hand, public goods production enhances private sector productivity, then optimal public goods supply is larger.

# 5. Possible extensions

The model we have developed is a very simplified model. In this chapter we point to some possible extensions and discuss *one* of these in more detail.

In the model there are two factors of production – skilled and unskilled labour. An obvious extension would be to add one or more factors of production, e.g. capital. We have also assumed that local public goods are *consumption* goods. Many local public goods do, however, have direct or indirect production effects, either because the goods are used directly in the production processes of private firms or because they influence the productivity of other production factors. Analysing the case in which the public goods are *production* goods would be another possible extension of the model. Finally, we have assumed that labour is immobile between communities. We now briefly discuss how the results are modified when we allow for migration; i.e. when labour is mobile between communities.

Take the presented model as a starting point, but assume that highly skilled workers are mobile between communities. The supply of highly skilled labour in a community depends on the utility these workers get when living in that specific community as compared to living in other communities. Utility is a function of the consumption of local public goods and the consumption of private goods. The consumption of private goods depends on income/the wage rate, and so utility is a function of the local public goods supply and the regional wage rate,

$$(67) \qquad U^{s} = U^{s}(w^{s},g).$$

The supply of highly skilled labour in a community, which depends on the difference in utility from living in the specific region compared to living in other regions, may then be written as a function of the regional wage rate and the supply of local public goods:

$$(68) \qquad n^s = n^s \big( w^s, g \big);$$

where  $n_w^s > 0$  and  $n_g^s > 0$ .

/ \

In order to attract more knowledge-intensive clusters, local governments have *one* policy instrument, namely the provision of local public goods. Increased local public goods supply makes the community more attractive as a place to live, and by doing this governments can attract more highly skilled labour. The change in utility of a highly skilled worker that results from increased public goods supply is

(69) 
$$\frac{dU^s}{dg} = U^s_w \frac{dw^s}{dg} + U^s_g$$

The first term shows the indirect effect that results because productivity and hence wages are affected. The second term is the direct effect on utility of higher public goods consumption. The optimal supply of local public goods is such that a small increase in local public goods supply will not alter the utility directly, and so the optimum is characterised by

(70) 
$$U_{g}^{s} = 0$$

Inserting into equation (69) gives

(71) 
$$\frac{dU^s}{dg} = U^s_w \frac{dw^s}{dg}$$

The only effect on utility is the indirect effect which results because wages, and hence private goods consumption, are affected. This means that the results obtained earlier are reinforced.

Assume that public goods production is extremely knowledge-intensive. Then increased public goods production leads to lower wages of highly skilled labour because the productivity in knowledge-intensive private industries falls. The utility of highly skilled workers decreases and some of these will therefore migrate. The total supply of highly skilled labour facing firms in the private industry is further reduced.

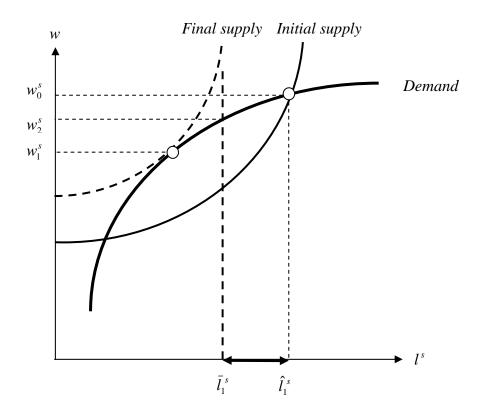


Figure 10: Wage and employment effects of increased public goods production in knowledgeintensive sector when public goods production is knowledge-intensive.

In figure 10 we have illustrated the demand for and supply of highly skilled labour to the knowledge-intensive industry. Demand is an upward-sloping curve due to the agglomeration effects.  $\hat{l}_1^s$  is the initial level of employment with the corresponding wage rate  $w_0^s$ . If local governments decide on increasing public goods production, the supply of highly skilled labour facing the knowledge-intensive industry is reduced to  $\bar{l}_1^s$ . The difference between  $\hat{l}_1^s$  and  $\bar{l}_1^s$  equals the increased need for highly skilled labour in the public sector. If labour supply is given, i.e. labour is immobile, this reduces the wage rate of skilled labour to  $w_2^s$ . But when skilled workers are mobile such a reduction in the wage rate induces migration, and so the supply of highly skilled labour is further reduced, as indicated by the shift of the supply curve (from the initial to the final supply curve). The wage rate of highly skilled labour declines further to  $w_1^s$ .

# 6. Summary

In this paper we address the question: How does the use of resources in the public sector affect private goods production, i.e. how does it affect the production structure and productivity? We analyse these questions in a setting where there are gains from geographic agglomeration in knowledge-intensive private industries.

We find that the effect on the production structure is a pure Rybczynski effect: If production of public goods requires much highly skilled labour, then the production of public goods will come at the expense of production of knowledge-intensive private goods. The size of knowledge-intensive clusters in the private sector are reduced. Due to agglomeration effects, the effect on productivity works in the same direction. As the knowledge-intensive industry declines, productivity also declines because the agglomeration gains cannot be exploited to the same degree. The lower productivity is reflected in factor payments; the wage rate of highly skilled labour goes down.

The fact that production of public goods affects productivity in the private sector has implications for the optimal supply of public goods. The Samuelson rule for optimal supply of public goods – marginal costs equals the sum of marginal utilities of public goods – still applies, but the marginal costs of public goods must be modified to account for the productivity costs or gains.

# 7. References

Audretsch D. B. and M. Feldman (1996). "R&D spillovers and the geography of innovation and production". *American Economic Review* 86, 630-664

Eurostat

Fujita, M. and J.-F. Thisse (2002), *Economics of agglomeration: Cities, industrial location and regional growth*, Cambridge: Cambridge University Press, 2002

Nielsen, V.O., "Utviklingen i offentlige utgifter til helsetjenester mot 2060", Økonomiske analyser 2/2008

Ottaviano, G. I. P. and D. Puga (1998), "Agglomeration in the global economy: A survey of the "new economic geography", *World Economy* 21 (6), August 1998: 707-731

Rybczynski, T. M. (1955), "Factor Endowment and Relative Commodity Prices", *Economica*, November 1955, 336-341

Samuelson, P. A. (1954), "The pure theory of public expenditure" *Review of Economics and Statistics* 36, 387-389

St.meld. nr.17 (2002-2003), "Om statlige tilsyn", p 17

# The price of decentralisation

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Abstract: This paper develops a model for analysing problems related to centralisation and decentralisation. The model is of the new economic geography type, in which there are agglomeration gains in cities but not in rural areas. These gains are counteracted by residential preferences. We show that, even though people have preferences for rural living, an unregulated market economy gives too little centralisation. This result holds even when city governments actively pursue policies to attract economic activities in order to make their city bigger. When allowing for cities of unequal size, a likely outcome is that big cities and rural areas will be overpopulated whereas smaller cities will be too few and too small.

**JEL classification:** R12, R13, H32, H41, R50

**Keywords:** Number of cities, size of cities, external economies, local public inputs, regional competition, agglomeration, welfare

## **1** Introduction

In almost all industrialised countries there has, as shown in table 1, been a strong, long-term trend towards urbanisation and increased centralisation. There are no indications that the trend is abating. In Norway, the share of the population living in urban areas has increased from 72% to 78% during the period from 1990 to 2008. Not only do people move to urban areas – the concentration of people *within* urban areas also increases. In Oslo e.g., the number of people per hectare increased from 37.9 to 42.3 from 2000 to 2009 (Næss et.al. (2009)). The rest of Europe is experiencing a similar development (although there medium-sized cities are growing at the expense not only of rural areas, but also of large cities).

	1800	1850	1910	1980
England	23	45	75	79
France	12	19	38	69
Germany	9	15	49	75
Europe	12	19	41	66
United States	5	14	42	65
Australia	-	8	42	80
Latin America	14	18	22	63
Third World	9	9	10	32

Table 1: Urban share of population 1800-1980 (Crafts and Venables, 2003)

Politically, the trend towards increased centralisation is seen as a problem in many countries. Norway is a case in point:

The overall objective is to ensure equal living conditions throughout the country, maintain the settlement patterns and central features of the potential in all regions. People should have a real, independent choice regarding their place of residence. The Government facilitates economic development in all parts of the country and encourages people to move to rural areas (our italicising).

(Norwegian Ministry of Local Government and Regional Development)

Therefore, to counter centralisation most governments protect or subsidise agriculture, undertake investments in infrastructure in scarcely populated areas, give subsidies or establish favourable tax regimes for firms located in rural areas, or relocate government agencies from the big cities.

An underlying assumption behind such policies is that there is too much centralisation<sup>1</sup>. There are, amongst others, *two* main economic arguments for this view. One is that there are diseconomies of scale in big cities due to e.g. pollution and congestion (see Kanemoto (1997) for a survey of the urban economics literature based on this perspective). The other, discussed e.g. by Martin (1999 a,b) and Puga (2002) within new economic geography models, is that there are negative pecuniary externalities when people move from rural to urban areas – those who leave do not take into account the negative effects on those left behind.

The literature on the new economic geography challenges the presumption that there is too much centralisation. While accepting that there could, at some point, be diseconomies of scale and negative pecuniary externalities, it argues that, under normal circumstances, these are more than offset by the real and pecuniary linkages which create positive external scale economies. The empirical evidence in this regard is strong, as centralisation has been a steady and universal trend for more than two hundred years. Had the negative external effects from centralisation been of the same order of magnitude as the positive scale economies, or had the pecuniary externalities in rural areas been of similar magnitude as the pecuniary externalities in cities, a spontaneous reversal of the centralisation trend would have been expected in at least some countries or over some extended time periods. Since neither are seen, there are good reasons to believe that the agglomeration gains in urban areas dominate any negative externalities in the cities and any positive externalities in rural areas.

If this is the case, the appropriate framework for discussing centralisation is one in which the driving force is the set of linkages which produce agglomeration gains of

<sup>&</sup>lt;sup>1</sup> We define centralisation as a "geographic centralisation of the population" in line with the definition used in the report "Sentraliseringens pris" (2009) written on request by the Ministry of Local Government and Regional Development.

the new economic geography type – see e.g. Fujita, Krugman and Venables (1999) or Ottaviano and Puga (1998) for excellent surveys of the new economic geography literature. During the last decade a large number of new economic geography articles have looked at problems related to decentralisation and centralisation policies (e.g. Matsuyama and Takahashi (1998), Baldwin et.al. (2005), Ulltveit-Moe (2007), Martin (1998, 1999a, 1999b), Martin and Rogers (1995), Puga (2001), Andersson and Forslid (2003), Forslid (2004)).

With real and pecuniary gains from agglomeration, the general presumption is that a market economy will give both too few and too small agglomerations (Norman and Venables (2004)) – in other words too few and too small cities. The purpose of this paper is to develop a framework for examining this presumption and to use it to see whether the presumption holds (a) when allowing for local city governments who actively pursue policies to attract more people in order to make the cities bigger, and (b) when allowing for equilibria with cities of unequal size.

The framework is one in which individuals have preferences for living either in the city or the countryside. Even though more and more people choose to move from rural to urban areas, many nevertheless express a genuine desire for rural living - their reasons being better recreational facilities, neighbourhood qualities, less pollution, less crowding etc. In economic models such non-economic considerations are usually ignored<sup>2</sup>, and job opportunities and wage differences are the only explanatory variables of workers' locational choices. A separate purpose of this paper, therefore, is to incorporate the fact that people value the place of living per se, to see whether it matters for the question of whether or not there is too much centralisation (which it turns out not to do).

In the cities, there are gains from agglomeration, so income levels there will be higher the larger the city. In rural areas, people work in agriculture; and their income levels depend on the total size of the agricultural sector (because the price of food falls with increased production). In the first part of the article, this framework is used to look at

<sup>&</sup>lt;sup>2</sup> An exception is Ludema and Wooton (2000), but their reason for assuming locational preferences is different. They assume locational preferences in order to ease the modeling of an upward-sloping labour demand curve.

an economy with one city in which the city government supplies tax-financed local services. One might think that the gains from agglomeration would make the city over-provide public goods in order to attract more people and that this could offset the standard result that the city is too small. That is not the case. At the margin, it is (by the envelope theorem) not possible to attract more people, so the only effect of expanding the public sector is to crowd out employment in the private sector, which is more costly when there are industrial agglomerations than when there are not. The presence of agglomeration gains will therefore actually make local governments supply *less* public services than they would otherwise do. The result that the city becomes too small and the rural sector too large is, if anything, strengthened.

In the second part of the paper, the model is extended to cover many regions and many cities. It is shown that the results regarding the number and size of cities are robust as long as people are mobile within regions only – in that case there will be at most one city per region; each will be too small; and some regions that ought to have cities will not have any. If we allow for interregional mobility as well (with individual preferences over regions as well as over rural vs. urban life), however, it could be that both big cities and rural areas will be overpopulated (relative to the optimum), while smaller cities could be both too few and too small.

## 2 An informal overview of the model

We consider a closed economy consisting of rural areas and a number of cities. There are three sectors of production; the public, private and agricultural sectors. The public and private sectors are located in cities, the agricultural in rural areas (called the periphery).

There are a fixed number of inhabitants in the economy, each inhabitant supplying one unit of labour inelastically. Total labour supply thus equals the total number of inhabitants.

Workers are perfectly mobile within the economy, and choose location based on a consideration of where their total utility will be highest. Utility derives from the

consumption of private goods and from factors related to the place of residence itself. The utility derived solely from *living* in a specific place differ between individuals, but is exogenous to the model. We assume that people live in the same place as they work. Consumption goods are freely and costlessly traded within the economy. The consumption an individual enjoys therefore depends only upon the local wage rate.

Goods produced in the public sector are used as intermediates in the production processes of private firms. Examples of such goods might be infrastructure widely defined og general training experience. We call these goods public inputs. They are financed by local taxes levied on people living in the cities. Private firms produce a homogenous consumption good (which is used as numéraire) with labour and local public inputs as production factors. There are external economies of scale in the private sector, so productivity increases with the size of the sector (i.e. with the total volume of private sector production). The individual firm, however, does not take account of the scale economies because they are external to the firm. As a result workers will be paid the value of their *average* product which is lower than their *marginal* product. Hence there is a market failure in the labour market.

In the periphery agricultural production takes place with labour as the only factor of production.

In the first part of the paper (section 3) we study a single-city economy, i.e. an economy in which there is only *one* city in addition to the periphery. We describe the economy's production structure, and from this we derive labour demand. We continue by studying labour supply, which depends on relative wage rates and residential preferences. Having developed labour demand and supply, we study labour market equilibria and compare these to the efficient outcomes to see whether there will be too much or too little urbanisation. Finally, we find the optimal local supply of public inputs and discuss how any market bias regarding urbanisation is affected by local governments pursuing policies to supply the optimal amount of public inputs.

In the second part of the paper (section 4), we expand the analysis to a multi-region economy with an endogenous number of cities. The context is an economy consisting of several regions, each of which is formally like the one studied in part one, and in which cities will be formed spontaneously so long as they are economically viable and stable. The purpose is to address the question of whether a free market economy produces too much or too little centralisation. In this context we also discuss possible effects of centrally initiated decentralisation policies.

# 3 A single-city economy

We consider an economy consisting of the periphery and *one* city. The economy has n inhabitants, each of whom inelastically supplies one unit of labour. The n inhabitants thus constitute total labour supply. Workers are perfectly mobile between the periphery and the city, and make a joint decision on where to live and work based on where their standard of living will be highest.

In this section we describe the production structure, employment and local public inputs supply in the single-city economy.

### 3.1 Production and labour demand

#### 3.1.1 The private sector

In the private sector a large number of identical firms produce homogenous consumption goods with labour and a local public input as production factors. The aggregate production function is

(1) 
$$x = \varphi(x, z)n^x$$
,

where x is total production in the private sector,  $n^x$  total private sector employment, z the total amount of public inputs, and  $\varphi(x,z)$  is a function capturing labour productivity. The labour-productivity function captures the external economies and any interaction there might be between external scale economies and the supply of local public inputs. Note that we do not model the sources of externalities explicitly; (1) should be interpreted as a reduced form of the market-linkage models developed in the "new economic geography" literature.

We assume that the labour-productivity function is increasing and concave in both arguments:

(2) 
$$\varphi_x \equiv \frac{\partial \varphi(x,z)}{\partial x} > 0, \quad \varphi_{xx} \equiv \frac{\partial^2 \varphi(x,z)}{\partial x^2} < 0,$$

(3) 
$$\varphi_{z} \equiv \frac{\partial \varphi(x,z)}{\partial z} > 0, \ \varphi_{zz} \equiv \frac{\partial^{2} \varphi(x,z)}{\partial z^{2}} < 0.$$

Solving (1) for the externality to express production as a function of labour and public inputs only, obtains

$$(4) x = g(n^x, z).$$

To find the derivatives of this function (the social marginal products of labour and public inputs), we first differentiate equation (1),

(5) 
$$dx = \varphi_x(x, z)n^x dx + \varphi_z(x, z)n^x dz + \varphi(x, z)dn^x,$$

which can be rewritten as

(6) 
$$(1-\varphi_x(x,z)n^x)dx = \varphi(x,z)dn^x + \varphi_z(x,z)n^xdz.$$

Solving this, gives

(7) 
$$g_n \equiv \frac{\partial x}{\partial n^x} = \varphi(x, z) \left( \frac{1}{1 - \varphi_x(x, z) n^x} \right)$$

and

(8) 
$$g_z \equiv \frac{\partial x}{\partial z} = \varphi_z(x, z) n^x \left( \frac{1}{1 - \varphi_x(x, z) n^x} \right),$$

respectively.

The terms in parentheses in equations (7) and (8) represent the external scale economies. With no economies of scale in the private sector, the marginal product of labour would have been  $\varphi$  and the marginal product of public inputs  $\varphi_z n^x$ . Due to external economies of scale, marginal products are higher.

The production function of firm *i* is

(9) 
$$x^{i} = \varphi(x, z)n^{i},$$

where  $x^i$  is firm *i*'s production and  $n^i$  is the number of workers employed by firm *i*. Because each firm is small relative to the entire industry, the effect of  $x^i$  on x is negligible, so the individual firm takes the function  $\varphi(x, z)$  to be constant, i.e. it perceives  $\varphi(x, z)$  as homogenous of degree zero.

The private consumption good is used as numéraire and the price is set equal to one. The inverse labour demand function from the private sector is given by the (firm) *perceived* value of the marginal product of labour:

(10) 
$$w_D^c = \varphi(x, z).$$

Combining equations (4) and (10), private sector labour demand can be expressed as an indirect function of the wage rate and local public inputs supply,

(11) 
$$w_D^c = \varphi \Big( g(n^x, z), z \Big).$$

The private sector labour demand curve is upward-sloping due to external economies:

Productivity and hence the wage rate (which equals the *firm perceived* value of the marginal product of labour) increases with the number of employees in the private sector. The slope of the labour demand curve is found by differentiating equation (11) with respect to  $n^x$ , giving

(12) 
$$\frac{\partial w_D^c}{\partial n^x} = \varphi_x(x,z)g_n = \varphi_x(x,z)\varphi(x,z)\left[\frac{1}{1-\varphi_x(x,z)n^x}\right] > 0.$$

Differentiating equation (11) once more shows that the demand curve is concave<sup>3</sup>:

(13) 
$$\frac{\partial^2 w_D^c}{\partial (n^x)^2} = \varphi_{xx} g_n g_n + \varphi_x g_{nn} = \varphi_{xx} \left( \varphi \left[ \frac{1}{1 - \varphi_x n^x} \right] \right)^2 + \varphi_x g_{nn} < 0$$

Labour demand increases with the volume of public inputs supply. If the provision of public inputs increases, private firms increase production and therefore demand more labour. This is illustrated by the two demand curves in figure 1 (where  $\ddot{z}$  denotes a larger amount of public inputs production than  $\hat{z}$ ).

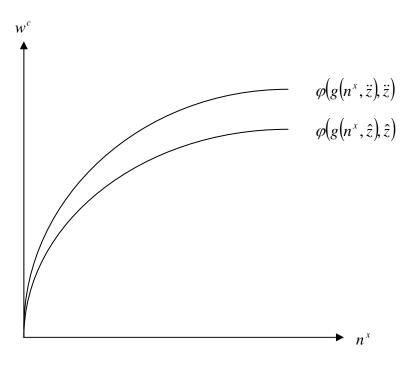


Figure 1: Private sector labour demand

<sup>&</sup>lt;sup>3</sup> Provided that  $g_{nn} < 0$ , i.e. that the production function is concave in *n*.

#### 3.1.2 The public sector

Local public inputs are produced with labour as the only factor of production. Local governments decide on the amount of public inputs production. For the time being we do not consider how regional governments make their decisions, but simply take the chosen amount as given. Optimal public inputs supply is studied in chapter 3.4.

Assume constant coefficients in public inputs production. Public sector labour demand is  $n^z = az$ , where *a* is a positive constant. For simplicity we choose units such that *a* equals one, and then public sector labour demand becomes

$$(14) \qquad n^z = z \,.$$

Total costs of public inputs production are

$$(15) \quad TC^z = w^c n^z,$$

where  $w^c$  is the wage rate in the city.

We assume that the city is self-financed (i.e. does not receive any grants from, or pay taxes to a central government). Public inputs production is financed through a uniform tax on the inhabitants of the city with per capita tax

(16) 
$$t^{c} = \frac{TC^{z}}{n^{c}} = \frac{w^{c}n^{z}}{n^{c}} = w^{c}\left(\frac{n^{z}}{n^{c}}\right),$$

where  $n^c$  is the total number of workers in the city (observe from (16) that the tax decreases with the number of city inhabitants).

#### 3.1.3 The agricultural sector

Agricultural production takes place with labour as the only factor of production according to the production function

$$(17) \qquad y = y(n^{y}),$$

where y is total agricultural production and  $n^{y}$  agricultural sector employment.  $y(n^{y})$  has constant returns to scale. We choose units so that the agricultural sector production function becomes

$$(18) \qquad y = n^y.$$

Agricultural sector labour demand is implicitly given by the value of the marginal product of labour in agricultural production,

(19) 
$$w_D^p = p^y y_n = p^y$$
,

where  $w_D^p$  is the wage rate agricultural "firms" are willing to pay and  $p^y$  is the price of agricultural products.

We look at a closed economy and use the manufacturing output as numeraire. We also assume that demand for agricultural products is completely income-inelastic (derived e.g. from an additively separable utility function which is concave in the agricultural good and linear in the private good). The price of the agricultural good will then depend only (and negatively) on the quantity produced and sold, which in turn is determined solely by total agricultural employment. Thus, the value of the marginal product of labour in the agricultural sector, i.e. the agricultural wage, depends only, and negatively, on agricultural employment.

$$(20) \qquad w^p(n^p); \quad w_n^p < 0$$

The agricultural sector labour demand curve is illustrated in figure 2.

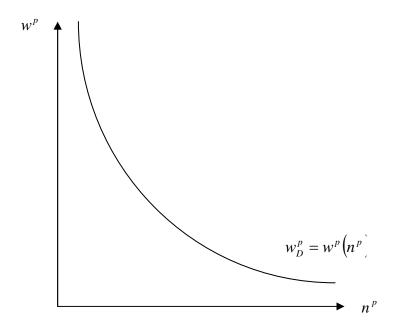


Figure 2: Agricultural sector labour demand

## 3.2 Residential preferences and labour supply

Workers are perfectly mobile between the city and the periphery, and choose to locate where their total utility will be highest. Utility derives from consumption of differentiated and agricultural goods, and from factors related to the place of residence. We assume that consumer goods (agricultural products and goods produced in the private sector in the city) are freely and costlessly traded within the economy, so consumers face the same prices wherever they live. The determinants of individual utility will then only be disposable income and place of residence.

Disposable income in the city is the wage minus the city tax

(21) 
$$\omega^{c} = w^{c} - t^{c} = w^{c} \left(1 - \frac{n^{z}}{n^{c}}\right)$$

while disposable income in the periphery is simply the wage

(22) 
$$\omega^p = w^p$$
,

where c denotes the city and p the periphery.

We do not model *why* people, cet. par., prefer to live in one place rather than another, but simply take this as exogenously given. Residential preferences are modelled by an individual-specific parameter  $\alpha_j$  which shows the additional consumer surplus person *j* gets from living in the periphery as compared to living in the city. The higher the value of  $\alpha_j$ , the higher is his preference for living in the periphery. Note that  $\alpha_j$  might be negative, in which case person *j*, cet.par., prefers to live in the city.

The utility functions of all individuals are assumed additive in consumption and place of residence, and the marginal utility of consumption constant (and equal to one). If person j lives in the city, his utility will therefore be

(23) 
$$u_j^c = u(\omega^c) = \omega^c = w^c - t^c.$$

If he lives in the periphery his utility will be

(24) 
$$u_j^p = u(\omega^p, \alpha_j) = \omega^p + \alpha_j = w^p + \alpha_j.$$

Any distribution of labour compatible with equilibrium must be such that the marginal inhabitant is indifferent between living in the city and in the periphery,

(25) 
$$\omega^c = \omega^p + \alpha_M \Longrightarrow w^c - t^c = w^p + \alpha_M,$$

where M denotes the marginal inhabitant.

Let  $F(\alpha_M)$  be the number of people who values the sheer pleasure of rural living higher than the marginal inhabitant does, i.e. the number of people for whom  $\alpha_j > \alpha_M$ .  $F(\alpha_M)$  is thus the equilibrium number of inhabitants in the periphery,

(26) 
$$n^p = F(\alpha_M).$$

Those who do not live in the periphery, live in the city. Labour supply in the city is therefore

(27) 
$$n^{c} = n - n^{p} = n - F(\alpha_{M}).$$

### 3.3 Equilibrium and efficiency

Labour market equilibrium obtains when (a) the marginal worker is indifferent between working in either sector and (b) the labour market clears, i.e. the sum of employment in the sectors add up to the total labour stock.

We perform a four-step analysis of labour market equilibrium, with the four steps being analyses when there are

- 1. No public inputs production and no residential preferences.
- 2. No public inputs production, but workers have residential preferences.
- 3. Public inputs production, but no residential preferences.
- 4. Public inputs production and workers have residential preferences.

Such step-wise analysis allows for isolation of different effects on the equilibrium conditions. The first case only serves as a benchmark, showing some of the well-known results from the new economic geography literature. Case 2 shows how residential preference affects existence, uniqueness and stability of geographic equilibria. Case 3 enables identification of the conditions under which there will be a local supply of public inputs and what effects that will have on productivity and urbanisation (eller centralisation?). The final case shows how public input supply and residential preferences can interact. It also sets up the complete model used (in section 3.4) to discuss optimal local policy.

#### 3.3.1 No public inputs production, no residential preferences

We begin the equilibrium analysis by assuming that there is no production of public inputs and that workers have no residential preferences. This is the standard case discussed in the literature and can therefore serve as a point of reference.

No production of public inputs implies that there are two sectors of production: The private sector located in the city, and the agricultural sector located in the periphery.

No residential preferences and perfect mobility of workers between sectors imply that workers will enter the sector in which they get the highest income. In the agricultural sector workers are paid the value of their marginal product, as given by equation (20):

$$(20) \qquad w^p(n^p); \quad w_n^p < 0$$

In the private sector workers are paid the value of their average product, as given by equation (11), but where z=0:

(11') 
$$w^c = \varphi(g(n^x)).$$

Workers have no residential preferences. Implicitly, therefore, they have no a priori preferences for working in either of the sectors (they are located at different places). Workers are perfectly mobile between sectors, and this mobility ensures that any labour market equilibrium is such that the wage rate is the same across sectors; i.e. the marginal product of labour in the agricultural sector equals the average product of labour in the private (agglomeration) sector,

(28) 
$$w^c = w^p \Rightarrow \varphi(g(n^x, 0), 0) = w^p$$
.

The labour market must clear, which implies that the sum of employment in the two sectors must add up to the total labour stock,

The two conditions for labour market equilibrium when there is no public input production and workers have no residential preferences are given by equations (28) and (29).

Labour market equilibrium is illustrated in figures 3a and 3b.

In figure 3a we measure labour along the horizontal axis and wages/returns per worker along the vertical axis. The length of the horizontal axis is given by the total labour stock. From left we measure the number of workers in the private sector,  $n^c$ , from right the number of agricultural workers,  $n^p$  (which coincides with the number of inhabitants in the city and in the periphery, respectively). Private sector labour demand is given by the curve  $w^c(n^c)$  and agricultural labour demand by the curve  $w^p(n^p)$ .

As drawn, there are two equilibria satisfying equations (28) and (29), called B and D in figure 3a. These are equilibria because the return per worker is the same in both sectors *and* total employment in the two sectors add up to the total labour stock. Only D, however, is a stable equilibrium. The stability condition is that the slope of the agricultural sector labour demand curve is steeper than the slope of the private sector labour demand curve,

(30) 
$$\left| \varphi_x(x,z)\varphi(x,z) \left[ \frac{1}{1-\varphi_x(x,z)n_x} \right] \right| < \left| w^p \right|.$$

The likelihood of this happening increases with decreased agglomeration gains and the larger the returns to labour in agricultural production.

There is also the possibility that we end up in a situation in which the entire labour stock is employed in the agricultural sector - the point called A in figure 3a. In this situation, no worker will have any incentives to switch to the private sector because the wage rate in the agricultural sector is higher than what they may earn in the private. As the two sectors are located at geographically different places this implies that everyone lives in the periphery.

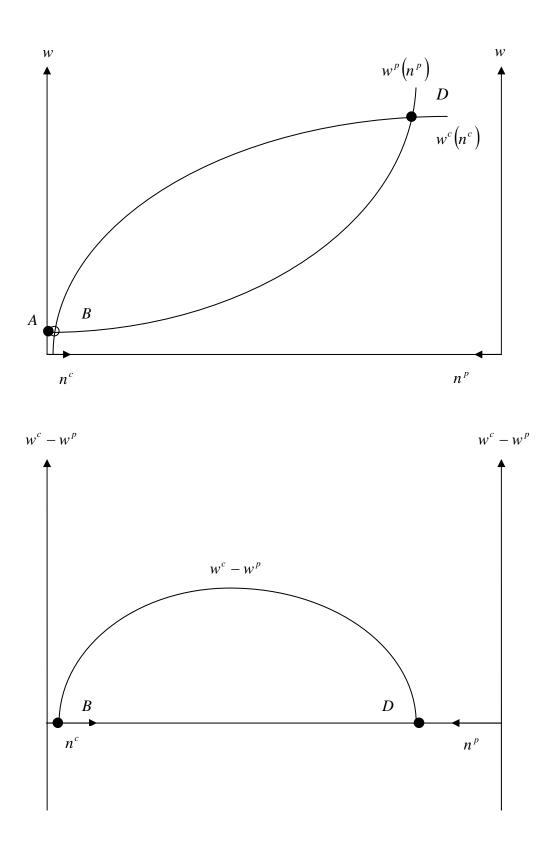
D is a diversified equilibrium, whereas the situation depicted by A is an equilibrium in agricultural production only. B is the critical mass of the city. If, for some reason or other, the size of the city is smaller than this, we end up in A. If the size is larger than this, we end up in the diversified equilibrium.

Labour market equilibrium may alternatively be illustrated as in figure 3b, which is derived from figure 3a. Figure 3b illustrates equilibrium by considering the wage differential between the private and the agricultural sectors. The wage differential,  $(w^c - w^p)$ , is given by the vertical distance between the two curves and in figure 3a. A, B and D in figure 3b correspond to A, B and D in figure 3a.

The stability condition, as given by equation (30), corresponds to a condition saying that the stable equilibrium is at the decreasing part of the wage differential curve,

$$(31) \qquad \frac{d\left(w^c - w^p\right)}{dn^c} < 0$$

At the increasing part of the wage differential curve, the marginal economic gain from increasing the number of city dwellers is larger than the loss in residential surplus of the marginal inhabitant of the periphery. Hence, a stable equilibrium cannot occur at the increasing part of the wage differential curve.



Figures 3a and 3b: Labour market equilibrium, no public inputs production, no residential preferences

These results, the possibility of either a diversified or a concentrated equilibrium, are well-known from the literature on new economic geography - see e.g. Krugman (1991a) and Krugman (1991b). From this literature it is also well-known that equilibrium implies unexploited scale economies. This can be seen in figure 4. D corresponds to the diversified stable equilibrium D in figures 3a and 3b. We have drawn private and agricultural sectors labour demands, i.e. the firm perceived value of the marginal product of labour in private goods production and the value of the marginal product of labour in private goods production is, however, larger than the individual firm perceives. The true value is given by equation (7),

(7) 
$$g_n \equiv \frac{\partial x}{\partial n^x} = \varphi(x, z) \left( \frac{1}{1 - \varphi_x(x, z) n^x} \right)$$

and illustrated by the upper concave curve in figure 4. The efficient equilibrium is E, which implies higher private sector employment and lower agricultural, and hence that the scale economies are more fully exploited.

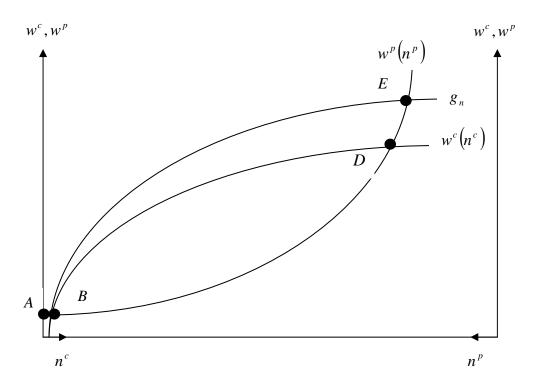


Figure 4: Unexploited scale economies in the stable equilibrium, no public inputs production, no residential preferences

#### 3.3.2 Residential preferences, no public inputs production

When workers *do* have residential preferences, the wage rate alone no longer determines a worker's location. Residential preferences are represented by the individual-specific parameter  $\alpha_j$ . Workers are perfectly mobile and choose location based on where their total returns, including the one derived from place of living per se, will be highest. Mobility of workers ensures that the wage rate in the city equals the sum of the wage rate in the periphery and the marginal inhabitant's residential surplus derived from living in the periphery rather than in the city,

$$(32) \qquad w^c = w^p + \alpha_M$$

Labour market clearing says that total employment in the private and agricultural sectors must add to the total labour stock,

$$(29) \qquad n^c + n^p = n \,.$$

The two conditions for labour market equilibrium when there is no public input production but workers have residential preferences are given by equations (32) and (29).

Equilibrium is illustrated in figure 5. The  $\alpha$  -curve shows workers in ascending order with regards to preferences for living in the periphery.

In figure 5 there is *one* unique equilibrium, E. To ensure a unique stable equilibrium, residential preferences must not be too weak. More precisely, the wage rate in the periphery must never be high enough compared to the wage rate in the city to induce the person who most highly values urban living to move to the periphery,

$$(33) \qquad \alpha_1 < w^c(0) - w^p(n) ,$$

where 1 denotes the person who values living in the city highest.

If this condition is not fulfilled, there will be three possible equilibria, one unstable and two stable. The stable equilibria are the well-known ones - a concentrated equilibrium where everyone lives in the periphery and a diversified one with settlement in both the city and in the periphery.

This could be illustrated in figure 5, with a less steep residential preference-curve than the one depicted, alternatively a wage differential curve located further down.

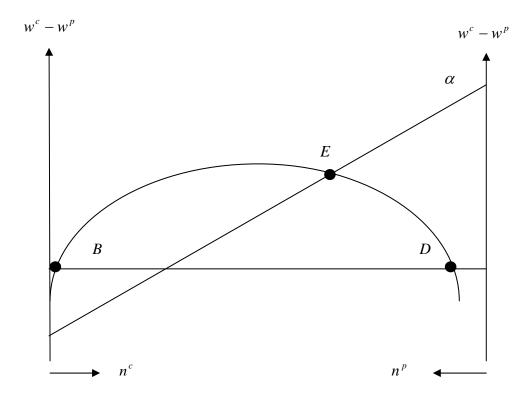


Figure 5: Labour market equilibrium with residential preferences: One unique diversified equilibrium

We see that residential preferences affect equilibria in three different ways.

First, they make it less likely that there are multiple equilibria. As seen from figure 5, provided that residential preferences are not very weak, there will be *one* unique equilibrium, while there would have been three without residential preferences. For the same reason, the equilibrium allocation of people between the centre and periphery will be less sensitive to external shocks – residential preferences reduces

mobility and thus dampens the effects of shocks.

Second, with residential preferences, there will generally be an equilibrium wage gap between the centre and the periphery. If the marginal resident gets a higher "value added" from living in the periphery rather than in the city,  $\alpha_M > 0$ , the equilibrium wage rate in the periphery will be lower than in the city. If the marginal inhabitant values living in the city higher than in the periphery,  $\alpha_M < 0$ , the opposite happens – but empirically this is of little, if any, relevance. (Only if the marginal resident gets the same pleasure solely from living in the periphery and the city,  $\alpha_M = 0$ , will there be no wage gap between the two).

Third, the relative sizes of the city and the periphery differ from the sizes when workers have no residential preferences. If the marginal person gets a higher residential surplus solely from living in the periphery rather than in the city, then the equilibrium size of the city will be smaller than in the no-residential preference case. If, for marginal inhabitant, residential surplus of living in the city is higher than the surplus of living in the periphery, then the opposite happens: The equilibrium size of the city will be smaller than in the no-residential preference case.

#### 3.3.3 Public inputs production, no residential preferences

The third case is one with public input supply, but in which workers do not have residential preferences.

Note first that public inputs production and supply have two opposing effects on the production by private firms: First, a "productivity effect": Public inputs are used directly in the production processes of firms in the private industry, and the provision of public inputs therefore increases private productivity and thereby private firms' labour demand. Second, it has a "crowding-out effect" in the labour market: Workers are required for public inputs production and so part of the labour stock will be publicly employed. The number of workers available to firms in the private sector decreases. This we call the "employment effect".

To see these effects more clearly, recall first that a condition for labour market equilibrium is that the marginal inhabitant is indifferent as to where he lives and works,

(28) 
$$w^{c} - t^{c} = w^{p} \Longrightarrow \varphi(g(n^{x}, 0), 0) - t^{c} = w^{p}.$$

The wage rates must be equal in the two sectors. This is ensured by the mobility of workers.

Secondly, the clearing condition, that the whole labour stock is employed, must be ensured

(35) 
$$n^{z} + n^{x} + n^{p} = n$$
.

I.e. every person in the labour stock must be employed either in the public, the private or in the agricultural sector.

Labour market equilibrium for a given volume of public input production is illustrated in figure 6. With no public input production the diversified equilibrium is  $D_0$ . The employment effect of public inputs production is seen by a reduction in the size of the "bathtub diagram". The length of the horizontal axis measured from the far right corner to the second right corner equals the number of public employees. From the left hand corner we measure the number of private sector workers, from the second right hand corner the number of agricultural sector workers. The isolated employment effect is seen by the horizontal shift, of length  $n^z$ , in the agricultural sector labour demand curve – the movement from equilibrium  $D_0$  to d.

The productivity effect of public inputs supply is seen by an upwards shift in the private sector labour demand curve, which changes the equilibrium from d to  $D_1$ .

The total effect of public inputs production and supply is the sum of the crowding-out and productivity effects. Without external scale economies, the two would pull in the same direction in terms of wages and employment – both would contribute to higher wages in the city; and as a result, more people would move there. *With* external scale economies, however, the two pull in opposite directions. The direct productivity effect contributes to higher wages in the city. Increased public employment, however, does the opposite: By bidding people away from the private sector, it contributes to lower private sector productivity. This could completely offset the direct productivity effect, in which case the urban wage would fall and people would move out of the city.

To see the exact condition, suppose one person, initially employed in the private sector, is hired by the local government to increase production of public inputs. Losing one person reduces private sector productivity by  $\varphi_x g_n$ . In the public sector, the person produces an extra unit of z, which will raise private sector productivity by  $\varphi_z + \varphi_x g_z$ . For the city wage rate to rise, therefore, we must have  $\varphi_z > \varphi_x (g_n - g_z)$ .

It will, of course, never be a rational policy for the city government to produce public inputs (which would also require higher taxes) if the net effect is to lower the wage. If local public goods are supplied, therefore, we can be certain that the direct productivity effect dominates the crowding-out effect at the margin. We can go even further: The city will not increase the supply of public inputs unless the resulting increase in the wage level is at least as high as the necessary increase in the local tax. It follows that local public inputs, if provided, will have a positive effect on the size of the city, and a negative effect on the population in rural areas.

We shall discuss the optimal supply of local public inputs – including the question of whether local policy contributes to excess urbanisation – in greater detail in section 3.4

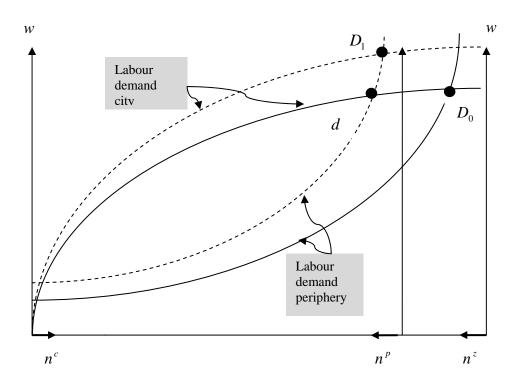


Figure 6: Labour market equilibrium, public inputs production, no residential preferences

### **3.3.4** Public inputs production and residential preferences

Let us finally, before we turn to the normative questions, set up the complete model with both public inputs production and workers with residential preferences.

Generally, labour market equilibrium obtains when the marginal worker is indifferent between working in either sector *and* the labour market clears.

The "indifference"-condition means that the marginal worker is indifferent between working in the public, private or agricultural sectors. The public and private sectors are located in the city whereas the agricultural sector is located in the periphery, and so the "indifference"-condition implies that the marginal worker is indifferent between living in the city or in the periphery. If he lives in the city his utility equals the private goods consumption he enjoys there (which, due to the assumptions that everyone supplies one unit of labour inelastically and that the price of private goods are normalised to one, equals the net wage rate in the city),

$$(36) \qquad u_M^c = w^r - t^c$$

If he lives in the periphery his utility is the sum of private goods consumption there (which equals the wage rate in the periphery) and the residential surplus derived from living in the periphery per se

$$(37) \qquad u_M^p = w^p + \alpha_M \,.$$

The "indifference"-condition becomes,

(38) 
$$u_M^p = u_M^c \Longrightarrow w^p + \alpha_M = w^c - t^c$$

From (38) we find  $\alpha_M$ ,

$$(39) \qquad \alpha_M = w^c - t^c - w^p$$

The second equilibrium condition is that the labour market clears, i.e. the sum of employment in the three sectors equals the total labour stock,

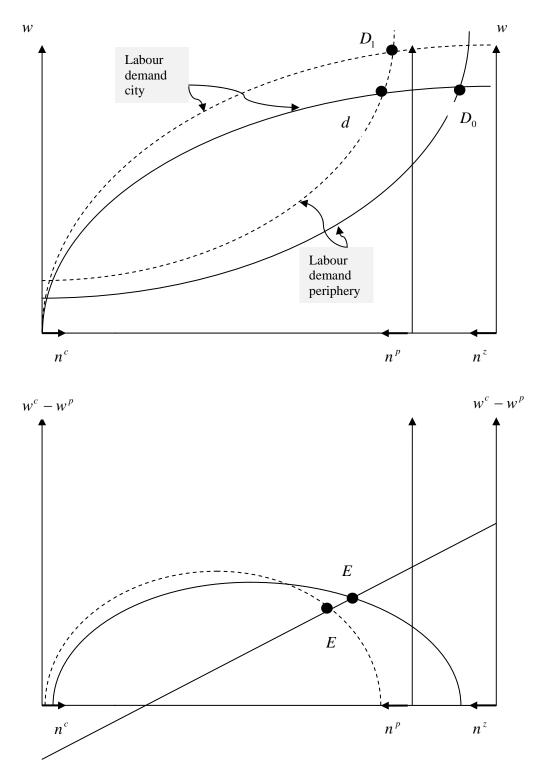
$$(35) \qquad n^z + n^x + n^p = n$$

When equations (39) and (35) hold; labour market equilibrium obtains. This is illustrated in figures 7a and 7b.

Figure 7a is a reproduction of figure 6. Figure 7b shows the wage difference between the city and the periphery, and is derived from figure 6. Without further assumptions, however, we cannot conclude as to whether the wage differential curve will shift downwards or upwards, i.e. whether the wage gap between the centre and the periphery will grow or decline. We know that employment in the agricultural sector

declines, causing a wage increase in the periphery. If the private sector employment also declines, then the wage rate in the city will fall and the wage difference clearly declines causing a downward movement of the curve. If, on the other hand, private sector employment increases then the wage gap may either increase or decrease. Labour market equilibrium is at the point where the residential preference curve crosses the wage differential curve – point E. Provided that residential preferences are not too weak, we get one unique stable equilibrium.

We see that there is one point in the figure that fulfils the two requirements for labour market equilibrium, point E, i.e. there is one unique equilibrium. Provided that the residential preferences are not too weak there will always be one unique equilibrium (see section 3.3.2).



Figures 7a and 7b: Labour market, public inputs production and residential preferences

#### 3.4 Optimal public inputs supply

So far, we have taken the amount of public inputs supply as given, i.e. we have treated z as if it were exogenously given. The amount of local public inputs provision is, however, clearly a political issue. An important question in relation to the overall issue of centralisation is whether local governments will want to use public inputs supply in a way which attracts an excessive number of people to the city.

Local governments choose the amount of public inputs supply so as to maximise the welfare of their citizens. The utility of a representative resident in the city is given by his disposable income, as shown by equation (23),

$$(23) \qquad u^c = w^c - t^c.$$

Inserting for per capita tax,  $t^c$ , from equation (16) and bearing in mind our choice of units such that public sector labour demand equals the amount of public inputs production,  $n^z = z$ , gives us the utility of a city dweller as

$$(40) \qquad u^c = w^c \left(1 - \frac{z}{n^c}\right).$$

We assume welfare is the sum of individual utilities. City welfare is thus

(41) 
$$W^{c} = n^{c} \left[ w^{c} \left( 1 - \frac{z}{n^{c}} \right) \right].$$

Local governments choose the amount of public inputs production so as to maximise the welfare of its current inhabitants. As long as the number of inhabitants is given, this gives the same result as maximising the utility of a representative inhabitant, and so we may write the maximisation problem of local governments as

(42) 
$$\max_{z} u^{c} = \max_{z} \left\{ w \left( 1 - \frac{z}{n^{c}} \right) \right\}.$$

The optimal amount of public inputs supply is such that there is no welfare gain from a marginal increase of public inputs supply:

$$(43) \qquad \frac{du^c}{dz} = 0$$

which gives <sup>4</sup>

(44) 
$$\frac{du^{c}}{dz} = -\frac{w^{c}}{n^{c}} + \left(1 - \frac{z}{n^{c}}\right) \left(\frac{\partial w^{c}}{\partial z} + \frac{\partial w^{c}}{\partial n^{x}}\frac{dn^{x}}{dz}\right) + \frac{\partial u^{c}}{\partial n^{c}}\frac{dn^{c}}{dz} = 0$$

Any optimum is such that the last term of equation (44) equals zero; i.e. there is no utility gain from a marginal increase in public inputs supply. No utility gain implies that there will be no migration either – no one gains from moving to or from the city. So, the optimal amount of public inputs supply is such that

$$(45) \qquad \frac{dn^c}{dz} = 0\,.$$

4

Inserting for from equation (40) gives

$$\frac{du^{c}}{dz} = \frac{d\left(w^{c}\left\{1 - \frac{z}{n^{c}}\right\}\right)}{dz} = 0$$

Carrying out this differentiation gives

$$\frac{du^{c}}{dz} = -\frac{w^{c}}{n^{c}} + \left(1 - \frac{z}{n^{c}}\right) \left(\frac{\partial w^{c}}{\partial z} + \frac{\partial w^{c}}{\partial n^{x}}\frac{dn^{x}}{dz}\right) + \frac{\partial u^{c}}{\partial n^{c}}\frac{dn^{c}}{dz} = 0$$

By setting the last term of equation (44) equal to zero, the condition for optimal public inputs supply becomes

(46) 
$$\frac{du^{c}}{dz} = -\frac{w^{c}}{n^{c}} + \left(1 - \frac{z}{n^{c}}\right) \left(\frac{\partial w^{c}}{\partial z} + \frac{\partial w^{c}}{\partial n^{x}}\frac{dn^{x}}{dz}\right) = 0$$

which gives the optimum condition<sup>5</sup>

(47) 
$$\left(\frac{\partial w^c}{\partial z} - \frac{\partial w^c}{\partial n^x}\right)n^x = w^c$$

i.e. optimal public inputs supply is such that the private sector marginal gain equals the direct costs of public inputs production. The left hand side of equation (47) is the private sector marginal gain from increased public inputs supply. Increased public inputs supply leads to increased private sector production. The value of this is  $w_z n^x$ . In order to produce the public inputs, however, some workers will have to be transferred from the private to the public sector. This reduces private production. The value of this is  $w_n n^x$ . Thus,  $w_z n^x - w_n n^x$  is the net value of a marginal increase in public inputs production. The right hand side is the direct cost of a marginal increase in public inputs production, namely the wage rate.

$$\int \frac{du^{c}}{dz} = -\frac{w^{c}}{n^{c}} + \left(1 - \frac{z}{n^{c}}\right) \left(\frac{\partial w^{c}}{\partial z} + \frac{\partial w^{c}}{\partial n^{x}}(-1)\right) = 0$$
$$\frac{du^{c}}{dz} = -w^{c} + \left(n^{c} - z\right) \left(\frac{\partial w^{c}}{\partial z} - \frac{\partial w^{c}}{\partial n^{x}}\right) = 0$$
$$\left(\frac{\partial w^{c}}{\partial z} - \frac{\partial w^{c}}{\partial n^{x}}\right) n^{x} = w^{c}.$$

#### Rewriting equation (47) gives

$$(48) \qquad w_z n^x = w + w_n n^x \,.$$

Equation (47) is a modified version of the Samuelson rule (Samuelson (1954)) where the left hand side is the sum of marginal values of public input and the right hand side is the social marginal costs of public inputs. The social marginal costs of public inputs equal the sum of the direct and indirect costs. The direct costs are the wage payments, the indirect costs are the private sector productivity costs (caused by reduced labour supply for private firms).

In the earlier discussion of the effects of local public inputs production, it was found that public inputs will not be produced unless they contribute to higher disposable income in the city, and that such production therefore will contribute to a larger city population, and a smaller rural population, than otherwise (see section 3.3.3). In that sense, local production of public inputs contributes to greater urbanisation.

That does not mean that local public inputs production (or, more generally, local public policy) contributes to excessive urbanisation, however. The potential for raising productivity through provision of local public inputs gives rise to a real economic gain which is in the interest of the economy as a whole to realise. To reap the benefits, more people must live in the city. Thus, the urbanisation effect of local public inputs provision does not reflect a market failure.

In fact, from equation (48) it follows that optimal local policies will not contribute to excessive centralisation. If policies are pursued to maximize per capita real income, they cannot, at the margin, have any effect on the size of the local population. The only way in which agglomeration affects optimum policies, therefore, is by raising the opportunity cost of resources used for public production (the second term on the right-hand side of (48)). It follows that the supply of public inputs, *ceteris paribus*, will be smaller, not larger, in the presence of private agglomeration effects.

# 4 A multi-region economy

We now extend the model to a multi-region economy, and assume there are *R* regions. Within each region there is a periphery. *K* regions also has a city, where  $K \le R$ . The periphery and the cities are all formally like those studied in the previous sections. Specifically, there are external returns in private sector production. The number of cities is determined endogenously through a free-entry condition. The idea is that new cities will emerge if they are economically viable, so the equilibrium number of cities will be given by the equilibrium number compatible with a stable labour market equilibrium.

In the single-city economy labour market equilibrium was characterised by two conditions: Labour market clearing *and* the condition that the marginal inhabitant was indifferent as to where he lived and worked.

In a model with several cities, the same two conditions apply for any labour market equilibrium. In addition, there is the free-entry condition: The number of cities must be the maximum number consistent with equilibrium. Thus, labour market equilibrium in a multi-region economy is characterised by

- i. The labour market clears, i.e. the sum of employment in all cities and in the periphery (which equals the sum of private and agricultural sector employment, respectively) add up to the total stock
- ii. The marginal worker is indifferent between working in either sector, which is the same as being indifferent between living in either location
- iii. The number of cities is the maximum number consistent with the two preceding conditions

We begin by looking at these three conditions in some greater detail.

#### *i.* Labour market clearing

The first condition for labour market equilibrium is that the labour market clears in each region i.e. the sum of employment in the city (if any) and the periphery adds up to the total labour supply,

(49) 
$$n = n_i^p + n_i^c$$
,  $i=1,...,R$ 

where *R* is the number of regions, *n* the number of workers in each region,  $n^p$  the number of agricultural workers and  $n^c$  the number of city workers.

If there *is* a city in the region, labour supply in the periphery is given by the number of people who gets a higher residential surplus solely from living in the periphery than the marginal inhabitant does,

(50) 
$$n_i^p = F(\alpha_i^M). \qquad i=1,...,R$$

Inverting equation (50) gives the residential surplus of the marginal inhabitant as a function of the number of residents in the periphery,

(51) 
$$\alpha_i^M = G(n_i^p), \qquad i=1,...,R.$$

Inserting for  $n^p$  from equation (49) gives the residential surplus of the marginal inhabitant as a function of the number of city dwellers,

(52) 
$$\alpha_M(n_i^c) = G(n - n_i^c)$$
.  $i = 1, \dots, R$ .

If there is *no* city in a region, all the inhabitants of the region work in the agricultural sector, so the number of agricultural workers equals the number of residents

(53) 
$$n_i^p = n$$
  $i=K+1,...,R.$ 

The total number of agricultural workers in the economy is the sum of agricultural

workers in the regions with and without a city, i.e.

(54) 
$$n^p = (R - K)n + \sum_{i=1}^{K} n_i^p$$

where  $n^p$  is the total number of agricultural workers in the economy.

#### ii. "Indifference" condition

The second condition for labour market equilibrium is that the marginal inhabitant is indifferent between living in a city and in the periphery. This condition applies, naturally, only for those regions in which there *is* a city. The "indifference" condition is fulfilled when the utility of the marginal inhabitant is equal in the city and in the periphery i.e. when the wage rate in the city equals the sum of the wage rate and the residential surplus of the marginal inhabitant in the periphery,

(55) 
$$w_i^c = w_i^p + \alpha_i^M \qquad i=1,...,K$$

When this indifference condition is fulfilled, the residential surplus of the marginal inhabitant equals the wage gap between the city and the periphery,

$$(56) \qquad \alpha_i^M = w_i^c - w_i^p$$

The city wage rate equals the value of the average product of labour in private production, as given by equation (11'), p.16

(11'') 
$$w^c(n_i^c) = \varphi(g(n_i^c)).$$

Equation (11'') is a modified version of (11') where we have specified that we look at a single region i. The wage rate in the periphery is given by the value of the marginal product of labour in agricultural production,

(57) 
$$w_i^p = p^{y}\left(\sum_{i=1}^R n_i^p\right).$$

Inserting for the number of inhabitants in the periphery,  $n^{p}$ , from equation (54), gives the wage rate in the periphery as a function of the number of city dwellers,

(58) 
$$w_i^p = p^y \left( (R-K)n + \sum_{i=1}^K (n-n_i^c) \right) = p^y \left( Rn - \sum_{i=1}^K n_i^c \right).$$

Labour market clearing and "indifference" conditions Labour market equilibrium may now be summarised by

(59) 
$$\alpha_i^M = w^c \left( n_i^c \right) - p^y \left( Rn - \sum_{i=1}^K n_i^c \right).$$

Equation (59) incorporates both the "clearing" and "indifference" conditions. The market clearing condition is fulfilled by saying that the number of residents in the periphery is given by the number of workers not employed in the cities (we have set  $n^p = n - Kn^c$ ) and the indifference condition is fulfilled by saying that the residential surplus of the marginal inhabitant is given by the wage gap between any city and the periphery.

Note from (59) that the city population (and thus also the agricultural population) must be the same in all regions with cities. In the following, therefore, let  $n^c$  and  $\alpha^M$  denote the common values for all regions with cities.

## iii. Free-entry condition

The third condition for equilibrium is the free-entry condition. Before considering it formally, consider first a different, but related question: How many cities are viable within any one region? As the inhabitants of a particular region would be indifferent between cities within the region (they have preferences for urban vs. rural life; they also have preferences for living in their particular region; but they have no preferences over different cities within the region), the answer is straightforward: Any equilibrium with more than one city would be unstable, in the sense that any difference in size between the cities – however small – would make everyone move to the largest city. It is clear, therefore, that there can be at most one city in each region.<sup>6</sup>

The free-entry question, therefore, is the maximum number of cities – in different regions – which is consistent with equilibrium. Since the urban population must be the same in all regions with cities, the question is the maximum K consistent with equation (59), i.e. the maximum consistent with

(59') 
$$\alpha^{M} = w^{c} \left( n^{c} \right) - p^{y} \left( Rn - Kn^{c} \right)$$

which says that the number of cities will be the maximum number consistent with equilibrium i.e. K is the maximum number such that (58') is fulfilled. This maximum number is the K for which

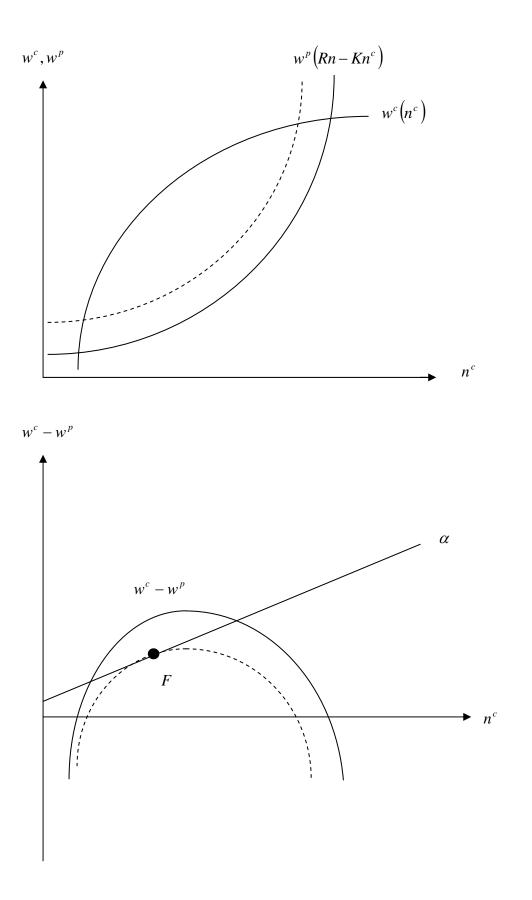
(60) 
$$\frac{d\alpha^{M}}{dn^{c}} = \frac{d\left[w^{c}\left(n^{c}\right) - \left\{p^{y}\left(Rn - Kn^{c}\right)\right\}\right]}{dn^{c}}$$

i.e. the equilibrium must be such that a marginal increase in the number of city dwellers (an external person) does not change the wage gap between any city and the periphery. Solving equation (60) we find

$$K = -\frac{\left(\frac{dw^{c}}{dn^{c}}\right)}{\left(\frac{p_{n}^{y}}{dn^{p}} + \frac{dG}{dn^{p}}\right)}$$

Labour market equilibrium in a region in which there *is* a city is illustrated in figures 8a and 8b.

<sup>&</sup>lt;sup>6</sup> I want to thank Kjetil Bjorvatn for pointing at this.



*Figures 8a and 8b: Labour market equilibrium in a representative region with a city in a multi-city economy* 

In figure 8a we measure the number of city dwellers along the horizontal axis, and the wage rate along the vertical axis. Labour demand in the city is shown by  $w^c(n^c)$ , and labour demand in the periphery by  $w^p(Rn - Kn^c)$ . Labour demand in the periphery is decreasing in the number of workers in the periphery, and hence increasing in the number of city workers (the more people live in the city, the fewer live in the periphery).

Increasing the number of cities causes an upward shift in the labour demand curve in the periphery, as illustrated by the movement of  $w^p (Rn - Kn^c)$  from the solid to the dotted curve. In order to understand the mechanism behind this shift, the following thought experiment might be useful: Imagine that a new city is established in a region in which there is no city in the first place. Some inhabitants (of this region) will move to the new city. Holding the number of city dwellers in all other regions constant, the total number of agricultural workers decreases as a result of the new city establishment. As a result, total agricultural production declines causing an increase in the price of agricultural products. This price increase induces an increase in the value of the marginal product of labour in agricultural production, as reflected in an upward shift of the agricultural labour demand curve in any region.

Figure 8b is derived from figure 8a, and hence also applies to a representative region in which there *is* a city. We measure the number of workers in the city along the horizontal axis and the wage differential between the city and the periphery along the vertical axis. Labour market equilibrium is a.o. characterised by the "indifference condition"; i.e. the marginal inhabitant is indifferent between living in the periphery or in the city. The "indifference condition" is fulfilled when the residential surplus of the marginal inhabitant equals the wage differential between the city and the periphery. In figure 8b, at a point in which the two curves cross. An increase in the number of cities leads to higher prices of agricultural products, inducing a wage increase in the agricultural sector. The wage differential between the city and the periphery thus decreases with the number of cities, causing a downward movement of the wage differential curve – as illustrated by the movement of the wage differential curve from the solid to the dotted curve in figure 8b. New cities will emerge until the point in which the wage differential and the residential preference curves are tangent. The maximum number of cities compatible with equilibrium is thus the one in which the wage differential curve and the residential preference curve are tangent – point F in figure 8b.

Several features of equilibrium are worth noting:

First, there will typically be cities (all of equal size) in some, but not all, regions. There will also be wage (and income) difference between regions in which there *is* a city and regions in which there is no city. This can be seen most clearly from figure 8a. The wage rates in regions in which there is a city will be  $w^c$  and  $w^p$  in the city and in the periphery, respectively. In regions where there is no city the wage rate will be  $w^{no \, city}$ .

Second, there is very little a central government can do to eliminate these income differences. Since the number of cities is the maximum number compatible with equilibrium, attempts at establishing urban agglomerations in more regions will at best mean that an existing city in another region is no longer viable. Similarly, if a central government attempts to eliminate the income differences through subsidies to agriculture, the only effect will be to reduce the equilibrium number of cities – but the income differences between regions with and without cities will remain.

Third, the equilibrium implies that the scale economies are not fully utilised. Equilibrium is at a point in which the wage differential curve is rising. This means that if the volume of private sector production were to increase ( $n^c$  increases) then the wage rate in the cities would increase due to higher productivity. In other words, the equilibrium implies that the cities are too small, and thus confirms the neweconomic-geography presumption that there is too little, not too much, urbanisation.

## 5 Extension: Interregional mobility

The multi-region model in the previous section has a very rigid structure in that people are assumed to be perfectly mobile within regions, but not between them. As an extension, we shall in this section consider a model with both interregional and intraregional mobility. We assume that people have residential preferences along two dimensions. First, they have regional preferences, which we model as a preference for living in a particular region (but indifference between all the others). Second, people have preferences about rural vs. urban living. These rural vs. urban preferences are modelled in exactly the same way as residential preferences were modelled in the previous sections. We do not develop the model fully, but sketch it in sufficient detail to see possible outcomes and discuss economic policy.

Let  $\beta$  denote a person's preference (i.e. willingness to pay) for living in the region they come from. We assume that everyone is indifferent about all other regions than the one they come from. So, each person has a preference  $\beta$  for living in the region they come from in addition to the preference  $\alpha$  for urban vs. rural living. The utility functions are

(61) 
$$U = u(y) + x + \alpha + \beta.$$

Let  $H(\alpha, \beta)$  be the cumulative density function over rural and regional preferences. I.e.  $H(\alpha, \beta)$  is the number of persons in a region with a rural preference parameter equal to or larger than  $\alpha$  and a regional preference parameter equal to or larger than  $\beta$ .

Workers are *perfectly* mobile both within and between regions. There will be a number of conceivable equilibria: A perfectly symmetric equilibrium with cities in all regions and no inter-regional movements, equilibria where some regions do not exist at all, equilibria with complete specialisation (complete urbanisation in some regions and agricultural production only in others), etc.

Our focus is on policy rather than on the pure theory of spatial equilibrium.

Therefore, we do not make an attempt to trace all of these equilibria, but instead focus on the ones we think are empirically most likely: An equilibrium with cities in some (but not all) regions, with workers moving from purely rural regions and with an asymmetry between the urban regions which results in one large and a number of smaller cities.

Why is this equilibrium possible and likely? An equilibrium with no inter-regional mobility implies that there are cities in some, but not in all, regions. People living in the periphery in a region and who have a low regional preference (low  $\beta$ ) and a low preference for living in the periphery (low  $\alpha$ ) will have a lower utility than they would get if they could move to a city in another region. If mobility is possible, therefore, they will move.

Where will they move? They are indifferent to other regions, so they will move to the city which offers the highest wage. Initially, i.e. with no mobility, all cities offer the same wage rate and we may therefore expect the migrants to be evenly distributed between cities. An even distribution is, however, not stable. If, for some reason, one city gets one more immigrant than the others, the wage rate would become slightly higher there than elsewhere. This would make all the migrants leave the other cities in favour of the higher-wage city.

Migrants from purely rural regions (regions with no city) will therefore move to *one* city only. Other cities will, however, continue to exist (inhabited by people with preferences for city-life *and* strong regional preferences), but they will have no inhabitants from other regions. Compared to the situation when people do not move between regions, the other cities will be smaller because some people (those with low regional preferences) will move from the smaller to the large city.

A likely outcome, therefore, is a rural-urban hierarchy where some regions are purely rural, some have small cities and one region has a large city. This is the outcome that will be our focus and the one we use as the basis for policy analysis. In order to analyse policy we need to formalise this equilibrium in more detail. Let subscript A denote the purely rural/agricultural regions, S the regions with small cities and L the

region with a large city.

Consider first the purely agricultural regions. A person from such a region will, if he stays in the region, get utility

$$(62) \qquad U_A = p^y + \alpha + \beta \,.$$

If this person moves to the large city, his utility will be  $w_L$ . The marginal resident of a purely agricultural region will be given by

(63) 
$$(\alpha + \beta)_A^M = w_L - p^y.$$

The number of inhabitants in each of the purely rural regions will be the number of people for which  $(\alpha + \beta) > (\alpha + \beta)_A^M$ .

Next, let us consider the regions with small cities. Some people in these regions will choose to live in rural areas and work in the agricultural sector. They get utility

$$(64) \qquad U_s^y = p^y + \alpha + \beta \,.$$

Some will choose to live in the small city. Their utility will be

$$(65) \qquad U_{s}^{x} = w_{s} + \beta \,.$$

The last group are those who prefer to move to the large city, which gives them utility  $w_L$ .

Thus, there are two marginal person(s) in the regions with a small city: Those who are indifferent between living in the periphery or the city within the region, *and* those who are indifferent between staying in the region or moving to the large city. We assume that the preferences are such that only the "city-lovers" are potential migrants to the large city i.e. we assume that there is a sufficiently strong correlation between

rural and regional preferences to prevent migration from the agricultural sector in small-city regions to the large city.

With this assumption, the marginal agricultural worker is the one with rural preferences,  $\alpha_s^M$ , equal to

$$(66) \qquad \alpha_s^M = w_s - p^y$$

and the marginal migrant from the small-city regions is the one with regional preferences,  $\beta_s^M$ , given by

$$(67) \qquad \beta_S^M = w_L - w_S \,.$$

The number of agricultural workers in each of these regions is the number of people for whom  $\alpha > \alpha_s^M$ . The number of emigrants is the number of people for whom  $\beta < \beta_s^M$ . The rest will be city residents.

Finally, let us consider the region with a large city. The population in this region consists of three groups: First, local people with a low rural preference; second, immigrants from small-city regions with a low rural preference. And third, immigrants from purely rural regions with weak regional or rural preferences.

The rural population are those people for whom  $\alpha > \alpha_L^M$ , where

$$(68) \qquad \alpha_L^M = w_L - p^y.$$

The city population is the rest of the local population plus those from purely rural regions with  $(\alpha + \beta) < (\alpha + \beta)_A^M$  plus those from small-city regions with  $\beta < \beta_S^M$ .

The exact expressions for the number of people are not possible to asses without further assumptions regarding  $H(\alpha, \beta)$ . For our analysis, however, we do not need to

know the exact number.

Consider in this setting the effects of regional policy. Suppose that central governments want to promote decentralisation i.e. they want to encourage rural preference-people to move to rural areas, and do so by subsidising agriculture. The purpose of this chapter is to briefly describe how such a policy may be analysed within the theoretical framework developed in this paper.

Subsidising agricultural production leads to higher profitability in the agricultural sector, and an upwards shift in the inverse demand function for labour in the agricultural sector. Migration from purely rural regions to the large city will definitely be reduced. As a result of the reduced immigration from rural regions, the large-city wage rate declines.

The agricultural subsidy also leads to higher agricultural employment in the regions with a small city, which might lead to a reduction in the number of viable small cities. If this happens, migration to the large city from small-city regions could increase. (Some of those originally living in small cities no longer have any city in their preferred region. If they love living in cities they probably choose to move to the large city instead of into the periphery in their "home-region".)

This last effect, increased immigration to the large city from previously small-city regions, might be so large that the net outcome is both increased agricultural employment and a bigger large city. The losers would be the small-city regions.

## **6** Conclusions

In this paper we have developed a model for analysing centralisation and decentralisation policies. The model is of the new economic geography type, in which there are gains from agglomeration in cities - acting as a centrifugal force. These agglomeration gains are counteracted by residential preferences – acting as a centripetal force. In the basic model, we establish the equilibrium conditions. We find that – in contrast to the well-established new economic geography models – there will be one unique stable equilibrium provided that residential preferences are not too weak. Generally, there will also be a wage gap between the city and the periphery.

Having developed the formal model, we first use this framework for studying the policies of a city government providing a locally tax-financed public input. The aim of the policy is to attract economic activity to the city. We find that the presence of agglomeration gains makes the government *undersupply* local public inputs. The reason is that public inputs production – due to the agglomeration effects - *raises* the opportunity costs of resources used for public production. This contrasts with previous results from new economic geography models studying public policies. In this literature findings show that there are reasons to believe in a "race to the top" regarding local taxes (and hence the supply of publicly provided goods and services) because agglomeration industries create pure rents which might be taxed (see e.g. Andersson and Forslid (2003), Baldwin and Krugman (2004)).

In the second part of the paper we extend the model to a multi-region economy, i.e. allowing for several regions, of which some have a city whereas others do not. The number of cities is determined endogenously. We find that there will be too few and too small cities as long as there is no interregional mobility. If people are interregionally mobile, the result might be that there will be too many large cities and too many people living in rural regions, whereas the number and size of smaller cities will be too low.

## 7 References

"Sentraliseringens pris. Er sentralisering et problem?" (2009) NIBR-rapport 2009:5

Andersson, F. and R. Forslid, 2003, "Tax competition and economic geography", *Journal of Public Economic Theory*, 5 (2): 279-303

Baldwin, R., R. Forslid, P. Martin, G. Ottaviano and F. Robert-Nicoud (2005), *"Economic geography and public policy"*, Princeton University Press

Baldwin, R. E. and P. Krugman (2004), "Agglomeration, integration and tax harmonization", (2004), European Economic Review 48: 1-23

Forslid, R. (2004), "Regional policy, integration and the location of industry in a multiregion framework", *CEPR Discussion Paper no.4630*, September 2004

Fujita M., P. Krugman and A. J. Venables, (1999) *The spatial economy. Cities, regions and international trade,* MIT Press, Cambridge MA

Kanemoto, Y. (1997), "Externalities in space", in Miyao, T. and Kanemoto, Y. (eds.), *Urban Dynamics and Urban Externalities* (Chur: Harwood Academic Publishers)

Kind, H., K. H. Midelfart Knarvik and G. Schjelderup (2000), "Competing for capital in a "lumpy" world", *Journal of Public Economics* 78: 253-274

Krugman, P. R. (1991a) Geography and trade, Cambridge, Massachusetts: MIT Press

Krugman, P. R. (1991b), "Increasing Returns and Economic Geography", *Journal of Political Economy*, vol 99 no.31: 483-499

Ludema, R. D. and I. Wooton (2000), "Economic geography and the fiscal effects of regional integration", *Journal of International Economics* 52: 331-357

Martin, P. (1998), "Can regional policies affect growth and geography in Europe?",

World Economy vol.21, no. 6: 757-774

Martin, P. (1999a), "Are European regional policies delivering?", *European Investment Bank Papers* 4(2): 10-23

Martin, P. (1999b), "Public policies, regional inequalities and growth", *Journal of Public Economics*, 73(1): 85-105

Martin, P. and C. A. Rogers (1995), "Industrial location and public infrastructure", *Journal of International Economics* 39: 335-351

Matsuyama, K. and T. Takahashi (1998), "Self-defeating regional concentration", *Review of Economic Studies* 65: 211-234

Norman, V. and A. J. Venables (2004), "Industrial clusters: Equilibrium, welfare and policy", *Economica* 71: 543-558

Næss, P., T. Næss and A. Strand (2009), "The challenge of sustainable mobility in urban planning and development in Oslo Metropolitan Area", TØI report 2024/2009

Ottaviano, G. I. P. and D. Puga (1998), "Agglomeration in the global economy: A survey of the "new economic geography"", *World Economy* 21(6): 707-731

Puga, D. (2002), "European regional policy in light of recent location theories", *Journal of Economic Geography* 2(4), October 2002: 373-406

Samuelson, P. A. (1954), "The pure theory of public expenditure" *Review of Economics and Statistics* 36: 387-389

Ulltveit-Moe, K. H. (2007), "Regional policy design: An analysis of relocation, efficiency and equity", *European Economic Review* 51: 1443-1467