Tax Effects on Educational and Organizational Choice

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Preface

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Contents

Chapter 1

Introduction

In line with the trend in the OECD-area, the Nordic countries carried out base broadening and rate cutting tax reforms in the early nineties. By introducing the dual income tax[1] they went even further and in a different direction than previous reforms in other countries. The dual income tax separates capital income from labor income. In contrast to the global income tax, which levies one tax schedule on the sum of income from all sources, the dual income tax combines a low proportional tax on capital income with a progressive tax on other income, mostly labor income. Later Belgium, France, Italy, and Japan also introduced versions of the dual income tax and have separate tax schedules for labor income and interest income[2]. This constitutes a huge natural experiment which needs to be studied more closely to draw lessons for future tax reforms.

The differential treatment of capital income and labor income under the dual income tax has several justifications[3]. First, the globalization of capital markets limits the scope of national taxation of mobile capital. Typically, labor is less mobile than capital and may be taxed at a higher rate without risking an erosion of the tax base. Second, labor income constitutes the basis for future old age retirement benefits, as well as present health care privileges. Third, capital enters into taxable wealth. The efficient tax rate on capital income should hence be viewed in connection

[1] The dual income tax was introduced in Sweden 1991, Norway 1992, and Finland 1993. The idea originated in Denmark, and was implemented in their 1985 tax reform. Later they introduced a hybrid system, mostly due to redistributive concerns.

[2] See Fuest and Weichenrieder (2002).

[3] See Sørensen (1998) and (2001).

with the wealth tax rate. Fourth, a lower tax on capital income stimulates personal saving. The higher the capital income tax rate, the more the tax system favours inpatient consumers who consume most of their income today. Fifth, in the presence of inflation, a low tax on some forms of capital income compensates for the fact that the tax is levied on the nominal, and not the real return to capital. In addition to this, the justification for keeping the tax on capital income proportional is that a progressive tax on capital income would be highly exposed to avoidance.

The present study analyzes how the dual income tax affects the individuals' optimal investment levels in financial and human capital. It also analyzes how the dual income tax affects small firms' optimal investment levels in real and financial capital, as well as the effect on their choice of organizational form.

One weakness of the dual income tax is the distributional implications of the taxation of small businesses. Income from self-employment and small businesses stems partially from return to the labor effort put in by active owners, and partially from the return to capital invested in the firm. For medium and high income classes, there is a large difference in the marginal tax rates on capital and labor income, providing large incentives for income shifting from labor income to capital income in order to minimize tax payments[4]. Owners of small businesses can easily reduce taxation by reducing their own wage payments and increase dividend payments. To prevent this, the dual income tax countries have implemented different versions of a "split" system of dual income taxation for self-employed entrepreneurs and corporations owned by the employees. Under this split system, one part of firm profits is taxed as capital income and the remaining profits are taxed as labor income. Chapter 1 analyzes how the Norwegian split model encourages small firms to participate in tax minimizing income shifting, which affects both the level of real capital in the firm as well as their preferred type of organizational form.

Higher education can be viewed both as a consumption good for which the individual is willing to pay, and as an investment alternative that yields higher wages later in life. The factors determining the individual's educational choice can be divided into three groups: preferences, returns, and costs. The costs of attending higher education are effort, time and money, both direct monetary outlays and foregone labor income. The return to higher education comes both as pecuniary and

[4] At present, the difference in the top marginal tax rates on labor income and capital income is 37.3 percentage points in Norway, including social security contributions.

non-pecuniary returns. As higher education increases the skill level and thus also the productivity of the individual, he is paid a higher wage in the labor market. Also, higher skilled individuals qualify for different types of jobs than lower skilled individuals. High-skilled jobs often offer various fringe benefits, which are not paid as money, but which all are equivalent to a wage increase. Fringe benefits and the wage premium constitute the pecuniary return to higher education. The individual specific non-pecuniary return to higher education is the consumption value of education, which is defined in chapter 4. The consumption value of education is a tax free return to education. Chapter 2 and 3 analyze how the tax system affects the individual's educational choice in the presence of such a tax exempt consumption value, both regarding the optimal length and type of education.

Chapter 2:

The Sole Proprietor's Income Shifting Under the Dual Income Tax.

The dual income tax provides the sole proprietor with large incentives to participate in tax minimizing income shifting to have more of his income taxed as capital income. The Norwegian split model is designed to remove these incentives, but it contains loopholes. This analysis concludes that the split model counteracts the negative effects of the risk of a technology shock on the sole proprietor's investments in firm specific real capital. It actually induces the sole proprietor to over-invest in less risky real capital. Real capital investment becomes a device for shifting income from the labor income tax base to the capital income tax base and thus reduces total tax payments of the sole proprietor. The incentives to participate in tax minimizing income shifting increase as his income increases. Thus, the incentives to over-invest in firm specific real capital provided by the net risk compensation rate, may increase as the labor income tax rate increases.

In addition, the widely held corporation serves as a tax shelter for high income entrepreneurs. The higher his income, and the larger the difference between the tax rates on labor income and capital income, the larger the incentives to become a widely held corporation in order to escape the split model and reduce total tax payments. Only low-income entrepreneurs have incentives to stay under the split model in order to enjoy the forwarding of negative imputed return to labor and deduct this against future positive imputed return to labor. The prediction of the model is supported by actually observed behavior of sole proprietorships after the introduction of the dual income tax and the split model in Norway in 1992.

Chapter 3:
Does the Tax System Encourage Too Much Education?

This paper provides an efficiency argument in favour of progressive labor income taxation. An individual who faces two investment alternatives, financial capital and human capital, invests in both until their net marginal returns are equal. Foregone labor income is the only cost of getting education. A proportional labor income tax is then a neutral tax on the return to human capital investments and does not alter its marginal return. Nielsen and Sørensen (1997) show that the dual income tax system with proportional tax on capital income and progressive tax on labor income is optimal in a second-best world. With a positive tax on capital income, a proportional tax on labor income leads to an over-investment in human capital. A progressive labor income tax reduces the marginal return to education, and the distortions in the investment market are reduced. If education in addition to being an investment alternative also is a consumption good, this has consequences for the optimal tax policy. A positive consumption value of education is a tax-free return to human capital investments. Hence a proportional labor income tax no longer is a neutral tax on the return to human capital investments. Even when no tax is levied on capital income, a progressive tax on labor income is required to reduce the overinvestment in education.

Chapter 4:
Income Tax, Consumption Value of Education, and the Choice of Educational Type.

Economists have thoroughly discussed how the tax system might affect the individual's educational level. But the question of how the tax system affects the individual's choice of educational type has been mostly ignored. Even if education is mostly treated as homogenous in the economic literature, it is in fact a heterogenous investment alternative and consumption good. Different kinds of education generate different levels of consumption value, as well as different levels of wage return. Depending on their preferences, individuals put different weight on the consumption value when choosing educational type. The return to education is one motivation behind an individual's educational choice. The consumption value is a tax free return to education. This paper finds that a progressive tax system induces the individual to choose more of the educational type with the higher consumption value. This effect is stronger the more weight he puts on the present.

Chapter 5:
Measuring the Consumption Value of Higher Education.

This paper argues for the existence of an individual specific consumption value of education, both during the education and after its completion, and for which the individual is willing to pay. A method for measuring the willingness to pay for the consumption value of education, where the innate ability bias is corrected for, is suggested in a compensating differentials framework. The identification strategy is to compare two individuals who attended teacher's college and business school in Norway during the 1960's. In this period these two types of education required the same minimum average grade level from high school for admittance, but they generated very different wage returns. The wage return from attending business school in this period is used as a benchmark for the potential wage return of the teacher's college graduates.

Using the Norwegian 1970 census, cross section wage profiles are estimated for those business school and teacher's college graduates with different levels of working experience. These wage profiles are interpreted as the expected future wages of the individuals attending business school and teacher's college during the 1960's. The individual who attended teacher's college in Norway during the 1960's expected to start his first job with annual earnings 34.7 % below his potential earnings. The full ex-ante price for the consumption value of teacher's college is estimated to be 38 % of the present value of the individual's potential lifetime income.

Utilizing a full coverage panel data set on the Norwegian population it is estimated that in fact the teacher's college graduates started up their first job earning "only" 20.6 % less than the business school graduates. But these wage differentials increased over time. The ex-post price on the consumption value to teacher's college during the 1960's turned out to be about 46 % of the present value of the individuals' potential lifetime income.

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Chapter 2

The Sole Proprietor's Income Shifting Under the Dual Income Tax*

Abstract

The dual income tax provides the sole proprietor with large incentives to participate in tax minimizing income shifting to have more of his income taxed as capital income. The Norwegian split model is designed to remove these incentives, but it contains loopholes. The present paper concludes that the split model to some extent counteracts the negative effect of technology risk on the level of real capital in the sole proprietorship. But the split model also induces the sole proprietor to over-invest in less risky real capital. In addition, the widely held corporation serves as a tax shelter for the sole proprietor. The higher the business income and the higher the difference between the marginal tax rates on labor and capital, the larger the incentives to incorporate.

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1 Introduction

In line with the trend in the OECD-area, the Nordic countries carried out base broadening and rate cutting tax reforms in the early nineties. By introducing the dual income tax[1] they went even further and in a different direction than previous reforms in other countries. The dual income tax separates capital income from labor income. In contrast to the global income tax, which levies one tax schedule on the sum of income from all sources, the dual income tax combines a low proportional tax on capital income with a progressive tax on other income, mostly labor income. Later Belgium, France, Italy, and Japan also introduced versions of the dual income tax and have separate tax schedules for labor income and interest income[2]. This constitutes a huge natural experiment from which lessons are to be drawn for future tax reforms.

One weakness of the dual income tax is the distributional implications of the taxation of entrepreneurs and small businesses. Income from self-employment and small businesses stems partially from return to the labor effort put in by the active owner, and partially from the return to capital invested in the firm. For medium and high income classes, there is a large difference in the marginal tax rates on capital and labor income[3], providing large incentives for income shifting from labor income to capital income in order to minimize tax payments. Owners of small businesses can easily do this by reducing their own wage payments and increase dividend payments, in order to maximize net income. In the extreme case, all individuals would start own businesses in order to participate in this tax arbitrage. To prevent this, the Nordic countries have implemented different versions of a "split" system of dual income taxation for sole proprietors and closely held corporations. Under this split system, one part of a firm's profits is taxed as capital income and the remaining profits are taxed as labor income.

The Norwegian split model of dual income taxation applies to sole proprietorships and closely held corporations. A corporation is defined as closely held if 2/3 or more

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[3]At present, the difference in the top marginal tax rates on labor income and capital income is 37.3 percentage points in Norway, including social security contributions.

of the shares are held by active[4] owners. A corporation is defined as widely held if less than 2/3 of the shares are held by active owners, and it is then taxed according to corporate tax rules. The split model was introduced at the end of a depression, and a period of strong economic expansion followed. In the years after the tax reform, the number of sole proprietors decreased, while the number of corporations increased. Does this mean that the split model discourages entrepreneurship, or does it mean that the activity of the entrepreneurs is unchanged, while their preferred organizational form has changed[5]? Also, the share of corporations being closely held decreased from 52% in 1992, to 32% in 2000. Which factors make this type of behavior rational? The present paper studies the tax induced distortions in a small firm's investment decision and choice of organizational form in a theoretical model, and three questions are asked. First, which are the sole proprietor's determinants for incorporating? Second, which are the sole proprietors' incentives to invest in risky real capital under the split model? And third, which are the widely held corporations' incentives to invest in risky real capital? But before these questions are answered in the specific case considered in this paper, let us take a closer look at the tax literature.

The tax code's effect on the firm's choice between debt and equity, as well as the choice of whether to retain or distribute earnings are thoroughly discussed in the literature. See for instance Gentry (1994). Different levels of corporate and personal tax rates provide private investors with incentives to use corporations as a tax shelter to save their capital income from high personal tax rates, a point highlighted by Fuest and Weichenrieder (2002).

The combination of a low corporate tax rate and a high personal income tax rate provides managers with incentives to relabel labor income as capital income, effectively reducing their tax on salaries, an effect identified empirically on Norwegian micro data by Fjærli and Lund (2001)[6]. But this income shifting may not be optimal if the individual has a long-term horizon. By receiving wages, he pays

[4] An owner is characterised as active if he works more than 300 hours annually in the firm. Close family members of active owners are not recognized as passive owners by the tax authorities.

[5] Slemrod (2001) states that in many cases, what appear to be real effects of tax changes are in fact only the result of creative re-labelling activity by the individuals, and this needs to be carefully considered when evaluating the effects of a tax reform.

[6] This study utilizes rich micro data from 1991, a year prior to the full implementation of the 1992 tax reform. Hence the split model does not apply here.

higher taxes, but he also becomes entitled to future pension payments from the public sector. Dividends do not entitle him to future pension. If the individual cares about his retirement, it might be optimal to pay more wages than the short-term tax minimization predicts, and Fjærli and Lund also document the presence of this effect.

There is an endogeneity of a firm's tax system: by changing organizational form the firm can experience a shift in the taxes it faces. Gravelle and Kotlikoff (1989, 1993) started a new strand of the literature on the firm's choice of organizational form following a tax reform that altered the relative tax rates on personal and corporate income. If corporate tax rates increase relative to personal tax rates, this reduces the firm's incentives to incorporate, and vice versa. Empirical support for this is presented by Goolsbee (1998), Gordon and MacKie-Mason (1990, 1994), and MacKie-Mason and Gordon (1997).

Non-tax factors also play an important role in the firm's choice of organizational form, as Ayers et al. (1996) thoroughly discuss. Business risk and default risk are factors that work in favor of the corporate organizational form. The sole proprietor carries all risk himself and is personally responsible for all claims. In case of a bankruptcy he may be liable to pay damages beyond the capital he has invested in the firm. In a corporation, the individual shareholder has limited liability and may in case of a bankruptcy lose at most the capital he has invested in the firm. The higher the relative risk of the operation, the more likely the business will be organized as a corporation. Another important factor is the opportunity to raise new capital. A corporation may issue new shares and might more easily raise new capital than the self-employed entrepreneur. Also, size does matter. As firms become large, owners are more likely to hire professional managers and become less directly involved in management decisions. Similarly, the higher the number of owners in a firm, the higher the probability of conflict among them. Then conflicts may be minimized by choosing the corporate form with a more formal ownership structure. The sole proprietor has full control over the activity and strategy of his firm. This might change if he organizes as a corporation with passive shareholders who have strong opinions on how the firm should be run.

The incentives to income shifting under the dual income tax are particularly strong for smaller, often family owned firms. The different Nordic countries have different ways of solving these income shifting problems. Lindhe et al. (2002) analyze the effects of the different Nordic split models on the long-run cost of capital. They find that while in Sweden the cost of capital is the same in closely and widely held corporations, the Finnish system reduces the long-run cost of capital in closely held corporations. The effect of the Norwegian system depends on the size of the imputation rate. Öberg (2003) extends the analysis of Lindhe et al. to find how the cost of capital is affected by the source of finance under the different Nordic split models. Kari (1999) analyzes the effects of mainly the Finnish split model on the splitting of dividend income from a closely held firm into capital and earned income parts. He concludes that the distortions imposed by the split model are very sensitive to the tax system's definition of the capital base of the firm. Risk is not included in any of these three papers. Sannarnes (1995) analyzes how the Norwegian split model in the presence of risk affects the investment behavior of external investors when deciding to invest in a closely or widely held corporation. He concludes that the split model encourages more investments in the closely held corporation.

The analysis in the present paper concludes that the split model counteracts the negative effects of the risk of a technology shock on the sole proprietor's investments in firm specific real capital. It actually induces the sole proprietor to over-invest in less risky real capital, relative to the optimal investment level in the absence of taxation. Real capital investment becomes a device for shifting income from the labor income tax base to the capital income tax base and thus reduces total tax payments of the sole proprietor. The incentives to participate in tax minimizing income shifting increase as his income increases. The net risk compensation rate under the split model is higher the higher the labor income tax rate, and thus the incentives to over-invest in firm specific real capital may increase as the labor income tax rate increases.

In addition, the widely held corporation serves as a tax shelter for high income entrepreneurs. The higher his income, and the larger the difference between the tax rates on labor income and capital income, the larger the incentives to become a widely held corporation in order to escape the split model and reduce total tax payments. Only low-income entrepreneurs have incentives to stay under the split model in order to enjoy the forwarding of negative imputed return to labor and deduct this against future positive imputed return to labor. The prediction of the model is supported by actually observed behavior of sole proprietorships after the introduction of the dual income tax and the split model in Norway in 1992.

Section 2 describes the Norwegian version of the split model of dual income taxation in detail. Section 3 presents the model, and sections 4 and 5 analyze the effect of the split model on the self-employed and the incorporated entrepreneur's investment portfolio. Section 6 compares the two organizational forms, and section 7 presents empirical evidence. Section 8 concludes.

2 The Norwegian split model

The Norwegian tax reform of 1992 implemented the dual income tax in a purer form than all the other Nordic countries. When considering how to solve the problems of a consistent tax treatment of small businesses, the split model of dual income taxation was chosen, separating income from different sources. Under the split model, an imputed return to the capital invested in the firm is calculated by multiplying the value of the capital assets[7] by a fixed rate of return on capital[8]. The imputed return to capital is taxed at the corporate rate, which equals the capital income tax rate at the individual level. Business profits net of imputed capital return[9] are the imputed return to labor, which is taxed as labor income whether the wages are actually paid to the owner or not. This reduces the possibility for the sole proprietor to classify all income as capital income in order to reduce taxes. If imputed labor income is negative, the loss does not offset other income, but may be carried forward to be deducted against future imputed labor income.

By exaggerating the capital assets of the firm, the sole proprietor achieves a reduction in the imputed labor income, and reduces his tax payments. This may be done in several ways, for instance by shifting from leased to owned[10] premises and machinery, by increasing stocks at the end of the year, by increasing and extending customers' trade receivables at the end of the year, and by financing private durable

[7] These assets include physical business capital, acquired good-will and other intangible assets, business inventories, and credit extended to customers net of debt to the firm's supplyers.

[8] This rate of return on capital is set anually by the Parliament on the basis of the average rate of return on government bonds (5% in 2000) pluss a risk premium (5% in 2000).

[9] If the firm has employees in addition to the owner(s), a salary deduction of 12% of the wage bill from taxable wage payments applies before the return to the owner's labor effort is imputed.

[10] There is an offsetting shift of ownership regarding former owners of leased assets. Presumably there will be a clientele effect where assets are owned by sole proprietors and closely held corporations.

goods in the firm. Acquired good-will is very hard for the tax authorities to value, and overstating this and other parts of firm capital reduces the imputed labor income. Also, by letting the firm invest in durable private consumption goods such as boats, cars, holiday homes, etc. the owner increases his consumption and reduces tax payments. Even if the increased wealth tax due on the value of capital assets is taken into account this strategy is lucrative for the sole proprietor[11]. It can even be profitable to borrow in the financial market to invest in business capital. Such debts are private and entitle the borrower to tax allowances.

But the largest loophole in the split system is probably at the margin, the question of whether a firm is subject to the split model at all. By incorporating and selling more than one-third[12] of the shares to passive investors, firms can avoid being taxed according to the split system. The widely held corporation is free to pay its active owners as little wage and as much dividends as it likes. This technique is especially attractive for individuals in "liberal" professions, such as lawyers, medical doctors and dentists. These are typically professions with little capital required to run a business, and the imputed labor income is accordingly high. As a widely held corporation they may take out all the compensation for their own labor effort as dividends.

3 The model

For simplicity, the following analysis abstracts from many of the details discussed above. Consider a utility maximizing entrepreneurial individual who lives for two periods and who is about to start a business. He needs to decide how much to invest in real capital in the firm, which has a stochastic second period return, as well as which organizational form to choose. As a sole proprietor he is taxed under the

[11] Assume that the sole propriertor increases his investments by NOK 100. At the going rate his imputed return to capital increases with NOK 10, which means that the imputed return to labor income is reduced by the same amount. Assuming that he is in the top wage income bracket, this increased investment reduces his personal taxes by NOK 5.2. The increased return to capital is subject to taxation on firm level at 28 per cent. In addition he is subject to a wealth tax of 1.1 per cent on total wealth. His taxes on firm level hence increase by NOK 3.9. Even when the increased wealth tax is taken into consideration, it still pays off to engage in this kind of income shifting.

[12] This is given by the tax code. Widely held corporations are not taxed under teh split model, and these are defined as corporations where passive owners hold more than one third of the shares.

split model. As a widely held corporation he is subject to corporate tax rules, but is required to pay a part of dividends to passive shareholders. Individuals differ in their preferences of which is the preferred organizational form. Here consider the marginal entrepreneur who initially has no intrinsic value of either of the two organizational forms, and who chooses the organizational form that maximizes his utility.

The individual has a given time endowment in both periods, which he spends working in his firm and enjoying leisure. In order to study the individual's investment decision and the choice of organizational form separately from his labor supply decision, assume that total time spent working in the firm is given. The remaining leisure is hence also given. A change of organizational form in order to reduce tax payments is only a re-labelling of the existing nature of the sole proprietor's activity, and he puts in the same amount of labor in the two cases. But the change of organizational form could nevertheless change the return to working, since it affects the net return to entrepreneurial activity in the presence of taxes.

Expected utility. The individual's expected utility function is represented by

$$EU = u(C_1) + E\left[v\left(C_2\right)\right], \tag{1}$$

which has positive and decreasing marginal utilities of both first period consumption, C_1, and second period consumption, C_2, such that

$$u'(C_1) > 0, \quad u''(C_1) < 0, \quad v'\left(C_2\right) > 0, \quad \text{and} \quad v''\left(C_2\right) < 0.$$

The risk averse individual chooses the investment portfolio and organizational form that maximize his lifetime utility.

Investments and income. In the first period he has initial wealth Y, which he allocates to investing in risky real capital K in the firm, and saving B in the financial market. Investments in the financial market yield the exogenously given safe real rate of return r. Savings may be negative, and then the individual borrows in the financial market. Loans are repaid in full in the second period. The gross return to real capital investments is the sales income net of the real capital depreciation, which is represented by the shock-related depreciation rate $\widetilde{\gamma}$ and discussed more closely below. The net of taxes sales income depends on the tax regime and thus on the chosen organizational form. It will be specified separately for each organizational form in the two following sections, as will the expressions for first and second period consumption.

The entrepreneur is the only person employed in the firm, and thus labor as a production factor is fixed. The firm produces one type of product, which is sold in the second period at a given price set to unity, $p = 1$. The production level X varies according to the amount of capital, K, invested in the firm, and sales income is thus given by the production function

$$X = F(K).$$

The production function has a positive and decreasing marginal product of capital; $F_K > 0$ and $F_{KK} < 0$.

Risk. The individual invests in real capital in the first period, and he realizes all his capital in the second period. The second period sales value of the capital stock of the firm depends on the depreciation rate, which is given by the stochastic parameter $\tilde{\gamma}$. There will always be some depreciation, and the maximum loss through depreciation is the initial value of the real capital, such that

$$0 < \tilde{\gamma} < 1.$$

The expected value of the depreciation is positive and given by the ordinary depreciation rate δ:

$$E[\tilde{\gamma}] = \delta > 0. \tag{2}$$

The individual demands a risk premium in order to invest in real capital in the firm. First define Θ as the rate of return to real capital required to compensate the individual for the relative expected second period marginal utility reduction caused by the depreciation. The size of Θ depends on two factors; the individuals preferences regarding risk, as well as the probability of a technology shock changing the real capital depreciation dramatically:

$$\Theta = \frac{E[v'(C_2) \cdot \tilde{\gamma}]}{E[v'(C_2)]} = \delta + \frac{cov[v'(C_2), \tilde{\gamma}]}{E[v'(C_2)]}. \tag{3}$$

A higher probability of a technology shock increases the expected depreciation rate δ. Also, the real capital depreciation reduces the second period consumption. The more risk averse the individual is, the larger is the utility loss from the drop in second period consumption. Thus the covariance of the shock parameter and the second period marginal utility is positive and higher the more risk averse the individual is. Define the risk premium as

$$\lambda \equiv \frac{cov\left[v'\left(C_2\right), \widetilde{\gamma}\right]}{E\left[v'\left(C_2\right)\right]} > 0. \tag{4}$$

Taxes. Let t_w be the proportional tax rate on labor income and t_k the proportional tax rate on capital income. We simplify by assuming that the tax on labor income is proportional, when in fact it is progressive in most countries, including the countries with a dual income tax. But one might think of this tax as the top marginal tax rate on labor income. The progressive labor income tax schedule is then in fact "flat on the top". Assume that the tax rate on labor income is higher than that on capital income, $t_w > t_k$. Total tax payments are given by T. No wealth tax is present in the model.

4 Sole proprietorship

Let the subscript "s" denote the previously described variables when the entrepreneur is a sole proprietor. First period consumption is given as the initial wealth net of investments:

$$C_{1,s} = Y - K_s - B_s. \tag{5}$$

The sole proprietor owns the firm and has full disposal over total sales income. His gross second period income consists of the return to his entrepreneurial investments, which are the sales income $F(K_s)$, as well as the return to his investments in the financial market and the invested capital, $[1+r] \cdot B_s$. Also, the real capital is capitalized in the second period, and the market value is reduced by the stochastic depreciation: $[1-\widetilde{\gamma}] \cdot K_s$. Thus the net of taxes second period income is given by

$$C_{2,s} = F(K_s) + [1-\widetilde{\gamma}] \cdot K_s + [1+r] \cdot B_s - T_s.$$

The imputation rate. The sole proprietor would, if he could and ceteris paribus, have all income taxed as capital income. The tax authorities assign a part of the income as a return to the capital invested, and the residual as a return to labor, which is taxed as labor income. When assigning the part of the income to be taxed at the capital income tax rate, a return to real capital in the firm is imputed at a fixed imputation rate r_i of the total value of the firm real capital at the beginning of the period[13]. The subscript "i" refers to "imputed".

The imputation rate is set by the authorities, and it is the sum of the average return to government bonds, r, and a risk compensation factor, μ, such that $r_i = r + \mu$. The risk compensation factor supposedly acknowledges the fact that the entrepreneur takes a risk by investing in real capital in the firm and hence loses the possibility of risk diversification in the financial market. The government's risk compensation is the same for all types of firms and all types of real capital.

Tax payments and the individual's budget constraint Capital income tax is paid on the imputed return to invested capital, $[r+\mu]\cdot K_s$. Labor income tax is paid on the imputed return to labor, which is the value of the production net of production costs (which are here the ordinary and shock-related depreciation rates) and the imputed return to invested capital[14]. In addition, capital income tax is paid on interest income from the investments in bonds. Total taxes due for the self-employed are thus given by

$$T_s = t_k \cdot [r+\mu]\cdot K_s + t_w \cdot \{F(K_s) - \widetilde{\gamma}\cdot K_s - (r+\mu)\cdot K_s\} + t_k \cdot r \cdot B_s.$$

The second period income of the sole proprietor, $C_{2,s}$, can then be written as:

$$\begin{aligned} C_{2,s} &= [1-t_w]\cdot[F(K_s) - \widetilde{\gamma}\cdot K_s] \\ &\quad + \{1 + (t_w - t_k)\cdot(r+\mu)\}\cdot K_s + [1 + (1-t_k)\cdot r]\cdot B_s \end{aligned} \tag{6}$$

The first part of the right hand side of (6) represents the individual's net of taxes income from his firm if all income were taxed as labor income. But the imputed return to capital is actually taxed as capital income, which increases his net income

[13] When the split model was first introduced, the self-employed individual could choose whether the value at the beginning or at the end of the period should be used in the imputation of the return to firm capital. Later this changed, and at the present, the average of the values of firm capital at the beginning and at the end of the period should be used to impute the return to firm capital. The first specification is chosen for this paper.

[14] If the imputed labor income exceeds a given threshold, which in 1993 was NOK 1.25 Million, the remainder is taxed as capital income. Assume in this analysis that the imputed labor income is always below this threshold.

by a fraction $(t_w - t_k)$ of total imputed return to capital. The larger the difference between the marginal tax rates on labor income and capital income, the more attractive it is to participate in income shifting activities in order to have more of his income taxed as capital income. But this is only relevant if in fact he pays labor income taxes. Thus assume that the sole proprietor at least expects to have positive profits in the firm.

The individual chooses the investment portfolio that maximizes his expected utility.

4.1 The investment portfolio.

The sole proprietor's optimization problem is given by

$$\max_{K_s, B_s} EU_s = u(C_{1,s}) + E\left[v\left(C_{2,s}\right)\right]$$

where $C_{1,s}$ and $C_{2,s}$ are given by equations (5) and (6). The resulting first order conditions are given by

$$FOC_{B_s} \quad : \quad -u'(C_{1,s}) + E\left[v'(C_{2,s}) \cdot \{1 + (1 - t_k) \cdot r\}\right] = 0 \tag{7}$$

$$FOC_{K_s} \quad : \quad -u'(C_{1,s}) + E\left[v'(C_{2,s}) \cdot \left\{ \begin{array}{c} [1 - t_w] \cdot [F_{K_s} - \widetilde{\gamma}] \\ + [t_w - t_k] \cdot [r + \mu] + 1 \end{array} \right\}\right] = 0. \tag{8}$$

The optimal investment condition is found by combining the two first order conditions, as well as applying the definition of the risk premium λ_s :

$$F_{K_s} = r + \delta + \lambda_s - \frac{t_w - t_k}{1 - t_w} \cdot \mu. \tag{9}$$

The sole proprietor invests in real capital in the firm until the value of the marginal product equals the risk adjusted user cost of capital. The higher the expected depreciation rate, and the higher risk premium the individual demands, the higher is the user cost of capital, and the lower is the optimal level of real capital investments in the firm. This effect is counteracted by the risk compensation factor, μ, which isolated considered works as a government subsidy on real capital investments. The total risk compensation under the split model is the relative after tax risk compensation rate, $\frac{t_w - t_k}{1 - t_w} \cdot \mu$. Thus even if the risk compensation factor μ is constant over time, a tax change will change the net risk compensation, and thus also the investment incentives of the sole proprietor. The net risk compensation is larger the bigger the difference between the two marginal tax rates, and the higher the tax rate on labor income.

In the special case that $\lambda_s = \frac{t_w - t_k}{1-t_w} \cdot \mu$ the individual is fully compensated for the risk of investing in real capital in the firm, and he invests in real capital as he would in the absence of both risk and taxes. Then the optimal investment condition reduces to the Fisher condition, $F_{K_s} = r + \delta$.

On the other hand, if $\lambda_s > \frac{t_w - t_k}{1-t_w} \cdot \mu$, the risk compensation under the split model is too small to compensate the individual for the risk he is exposed to by investing in real capital. But the split model still counteracts the negative effect on the level of entrepreneurial investments in the society from the risk of technology shock, and the sole proprietor invests more in real capital in the firm than in the absence of taxes.

The sole proprietor is overcompensated for the risk he is exposed to if $\lambda_s < \frac{t_w - t_k}{1-t_w} \cdot \mu$. In that case the sole proprietor will use real capital investments as a means to shift income from labor income to capital income. The split model induces the sole proprietor to over-invest in less risky types of real capital, in order to minimize tax payments. This effect is larger the less risk averse the sole proprietor is.

In the present model, the net risk compensation rate is constant, as long as none of the parameters is changed. This is due to the simplifying assumption of the labor income tax rate being constant. But under the dual income tax, the marginal tax rate on labor income increases as the income increases, while the capital income tax rate is constant. Thus the net risk compensation rate under the split model increases as the imputed labor income of the sole proprietor increases. This means that high income sole proprietors have greater incentives to participate in this tax minimizing income shifting by increasing real capital investments. In the context of this model, though, only one individual is considered, and the labor income tax rate is assumed to be independent of income level.

4.2 The effect of tax changes on investment behavior.

Tax reforms change the investment incentives of the sole proprietor. Below, the effects of changes in both the labor income tax and the capital income tax rate are analyzed through comparative static analysis of the first order conditions (7) and (8). The effects of tax changes in the sole proprietor's real capital investments can be expressed as a twofold effect, both an income effect and a substitution effect. It can be shown[15] that if $v'''(C_{2,s}) > 0$, then the income effect $\frac{\partial K_s}{\partial Y}$ is positive if $cov\left[v''(C_{2,s}), \tilde{\gamma}\right] \cdot E\left[v'(C_{2,s})\right] < cov\left[v'(C_{2,s}), \tilde{\gamma}\right] \cdot E\left[v''(C_{2,s})\right]$. On the other hand, if $v'''(C_{2,s}) < 0$, then $\frac{\partial K_s}{\partial Y}$ is negative.

4.2.1 Labor income tax.

The effect of a labor income tax increase on the level of real capital in the sole proprietorship is given by

(10)

$$\frac{\partial K_s}{\partial t_w} = -\left\{\frac{F(K_s) - (r+\mu) \cdot K_s}{[1+(1-t_k) \cdot r]} + [1+(1-t_k) \cdot r] \cdot \frac{E\left[v''(C_{2,s}) \cdot \tilde{\gamma}\right]}{u''(C_{1,s})} \cdot K_s\right\} \cdot \frac{\partial K_s}{\partial Y}$$

$$+\frac{u''(C_{1,s}) + [1+(1-t_k) \cdot r]^2 \cdot E\left[v''(C_{2,s})\right]}{[1-t_w] \cdot D \cdot E\left[v'(C_{2,s})\right]}$$

$$\cdot \left\{\begin{array}{l} [1-t_w]^2 \cdot K_s \cdot \left\{\begin{array}{c} E\left[v''(C_{2,s}) \cdot \tilde{\gamma} \cdot \tilde{\gamma}\right] \cdot E\left[v'(C_{2,s})\right] \\ -E\left[v'(C_{2,s}) \cdot \tilde{\gamma}\right] \cdot E\left[v''(C_{2,s}) \cdot \tilde{\gamma}\right] \end{array}\right\} \\ \\ -[1-t_k] \cdot \mu \cdot E\left[v'(C_{2,s})\right]^2 \end{array}\right\},$$

where D is positive and defined in the mathematical appendix.

Whether this tax induces the sole proprietor to increase or decrease his investments in firm specific real capital depends on whether the substitution effect or the total income effect is stronger. This again depends on the individual's preferences. The first component of (10) is the total income effect, where the sign is determined by $\frac{\partial K_s}{\partial Y}$. The second component is the substitution effect, where the sign is determined by the expressions in the parenthesis.

Above we stated that if the individual has decreasing absolute risk aversion, then $\frac{\partial K_s}{\partial Y} > 0$, and thus the total income effect is negative. The increased tax on labor income reduces his net income, and thus he is less willing to invest in risky

[15]See the appendix for the formal deduction.

capital. It can be shown[16] that in this case the substitution effect is positive if $cov\left[v''(C_{2,s})\cdot\tilde{\gamma},\tilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] < cov\left[v'(C_{2,s}),\tilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\cdot\tilde{\gamma}\right]$. The higher the tax on labor income, the greater the difference between the two tax rates, and the higher is the private return to shifting income from the labor income tax base to the capital income tax base by increasing the capital stock in the firm. Also, as the net return to these types of investments increase, so does the relative after tax risk compensation rate $\frac{t_w-t_k}{1-t_w}\cdot\mu$ (as defined in equation 9), making the individual more willing to invest in risky firm specific real capital. Therefor, the increased tax on labor income induces the individual to increase his investments in firm specific real capital if the substitution effect dominates the income effect[17].

This whole effect is driven by the fact that the relative after tax risk compensation rate is higher than the risk premium required by the individual to invest in risk real capital. This is the case for less risky types of real capital, and these are typically the types used as means to shift income from the labor income tax base to the capital income tax base. The relative risk compensation rate is the same independently of type of real capital. It depends positively on the difference between the marginal tax rates on labor income and capital income, as well on the risk compensation rate under the split model. In the absence of risk, the risk compensation rate μ ought to be zero, since it otherwise distorts the investment decision of the sole proprietor.

All real capital is owned by the firm in this model, and in order to benefit from the possibility to reduce tax payments through increased investments, the entrepreneur must increase the total level of real capital in the firm. On the other hand, if parts of the real capital were leased, the entrepreneur could purchase this real capital and still have the same level of expenses, just switching from having to pay lease to paying interest on a loan. This manoeuvre would leave the level of firm real capital unchanged, and it would reduce the entrepreneur's tax payments. No wealth tax is present in this model, and in this framework the presence of a wealth tax would not alter the split-model's distortions to the investment portfolio of the entrepreneur. Increased investments in real capital mean reduced investments in financial capital and do not increase the wealth tax liability.

[16] See the mathematical appendix for the proof.

[17] On the other hand, if the individual has increasing absolute risk aversion, the total income effect is positive and the substitution effect is negative. In that case the increased labor income tax rate only induces the individual to increase his investments in risky real capital if the total income effect dominates the substitution effect.

4.2.2 Capital income tax.

The effect of an increase in the capital income tax rate on the level of real capital in the sole proprietorship is unambiguously negative:

$$\begin{aligned}\frac{\partial K_s}{\partial t_k} &= -\left\{\frac{[r+\mu]\cdot K_s + r\cdot B_s}{1+(1-t_k)\cdot r} - r\cdot\frac{E\left[v'(C_{2,s})\right]}{u''(C_{1,s})}\right\}\cdot\frac{\partial K_s}{\partial Y} \\ &\quad + \left\{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,s})\right]\right\}\cdot\frac{\mu\cdot E\left[v'(C_{2,s})\right]}{D}.\end{aligned} \tag{11}$$

When the capital income tax rate increases, the incentives to participate in any kind of income shifting decrease, since the difference $(t_w - t_k)$ decreases, as does the private gain from income shifting. Also, increased capital income tax rate means a decreased net risk compensation rate under the split model. Both factors induce the sole proprietor to invest less in risky real capital in the firm. The higher the sole proprietor's capital income is, the larger share of his total income is affected by the tax increase, and the more is his net income reduced.

4.3 The indirect utility function.

The investment portfolio $\left[\widehat{K}_s, \widehat{B}_s\right]$ maximizes the sole proprietor's expected utility. Thus his maximum achievable level of expected utility, $\widehat{EU}_s$, is given by the indirect utility function:

$$\widehat{EU}_s = u(\widehat{C}_{1,s}) + E\left[v(\widehat{C}_{2,s})\right] \tag{12}$$

where

$$\widehat{C}_{1,s} = Y - \widehat{K}_s - \widehat{B}_s \tag{13}$$

and

$$\begin{aligned}\widehat{C}_{2,s} &= [1-t_w]\cdot\left[F(\widehat{K}_s) - \widetilde{\gamma}\cdot\widehat{K}_s\right] + \{1+[t_w-t_k]\cdot[r+\mu]\}\cdot\widehat{K}_s \\ &\quad + [1+(1-t_k)\cdot r]\cdot\widehat{B}_s.\end{aligned} \tag{14}$$

This will be applied in the analysis of the entrepreneur's choice of organizational form.

5 The widely held corporation

The only reason for the individual to incorporate is to reduce his total tax burden by escaping the split model. A closely held corporation would still be subject to the split model, so in this context he has no incentive for choosing that organizational form. Assume thus that the alternative to being a sole proprietor is to organize as a widely held corporation with the minimum required number of passive[18] shareholders, $(1-\beta)$, where $0 < \beta < 1$ is the active owner's maximum allowed share of ownership as a widely held corporation. The entrepreneurial individual receives revenue from selling shares in his firm to external investors. This can be modelled as a corresponding reduction in the amount of real capital investment required by the individual. The entrepreneurial individual invests the share β of total real capital, and the passive shareholders invest the rest. Assume that the passive shareholder is not more risk averse than the active shareholder, such that the risk premium required by the passive investor is equal to or less than that of the active shareholder.

All shareholders receive dividend payments as a return to their invested capital. The shareholder majority, which here means the entrepreneur, decides what wage to pay the active shareholder as a compensation for his labor effort, as well as how much to pay in dividends. Since an additional pay-roll tax applies to all wage payments made by the corporation, the total tax burden on labor income is higher under the corporate tax regime than under the split model. Hence it is irrational for the tax minimizing entrepreneur to receive any wages as compensation for his own labor efforts. All firm profits are paid as dividends in the second period, of which the entrepreneurial individual receives β and the passive shareholders $(1-\beta)$.

The widely held corporation considered here is typically a smaller, often family owned corporation, whose objective it is to maximize the utility of the active shareholder. This is in contrast to the larger corporations listed on the stock exchange that usually are described in the optimal tax literature, whose goal it is to maximize the stock value of the corporation.

In the following, use the same variables as previously described in the paper, with the subscript "l" denoting the variables when the entrepreneur organizes as a widely held corporation.

[18]In this model all shareholders are passive, except for the entrepreneur.

First and second period consumption. First period consumption is given by

$$C_{1,l} = Y - \beta \cdot K_l - B_l \tag{15}$$

No wages are paid, and thus the net sales income is defined as firm profits, which are taxed at the corporate tax rate t_k at firm level. Then all net profits are distributed tax free to the owners, of which the active shareholder receives β. The firm specific real capital is capitalized in the second period, and the sales value depends on the stochastic depreciation. In addition, the entrepreneurial individual receives the net of taxes return to his investments in the financial market. His second period consumption is given by

$$C_{2,l} = \beta \cdot [1 - t_k] \cdot [F(K_l) - \widetilde{\gamma} \cdot K_l] + \beta \cdot K_l + [1 + (1 - t_k) \cdot r] \cdot B_l. \tag{16}$$

5.1 The optimal investment condition.

The entrepreneur's optimization problem is given by

$$\max_{K_l, B_l} EU_l = u(C_{1,l}) + E\left[v\left(C_{2,l}\right)\right]$$

where $C_{1,l}$ and $C_{2,l}$ are given by equations (15) and (16). The resulting first order conditions are given by

$$FOC_{B_l} \ : \ -u'(C_{1,l}) + [1 + (1 - t_k) \cdot r] \cdot E\left[v'(C_{2,l})\right] = 0 \tag{17}$$

$$FOC_{K_l} \ : \ -\beta \cdot u'(C_{1,l}) + E\left[v'(C_{2,l}) \cdot \{\beta \cdot [1 - t_k] \cdot [F_{K_l} - \widetilde{\gamma}] + \beta\}\right] = 0 \tag{18}$$

Combining the first order conditions yields the optimal investment condition:

$$F_{K_l} = r + \delta + \lambda_l \tag{19}$$

Real capital is invested in the firm until the value of the marginal product equals the risk adjusted cost of capital. As long as external investors are not more risk averse than the active shareholder, and as long as their alternative return is the interest rate r, there will always be sufficient passive shareholders that want to invest in the firm. Everything else equal, the optimal level of real capital in the widely held corporation is lower than in the sole proprietorship. This is due to the fact that the corporation does not experience any risk compensation through the tax system, as the sole proprietor does.

The more risk averse the entrepreneur, and the higher the expected depreciation rate, the less real capital is invested in the firm. Taxes have an indirect effect on the level of real capital in the widely held corporation since only the risk premium is affected through taxes. The extent to which the capital income tax affects the investment level in the firm is studied in detail below. Labor income tax changes have no effect on the investment behavior of the firm, since no wages are paid.

5.2 Effect of increased capital income tax rate

It can be shown that the income $\frac{\partial K_l}{\partial Y}$ effect is positive the individual has decreasing absolute risk aversion, and if $cov\left[v'(C_{2,l}),\widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l})\right] > cov\left[v''(C_{2,l}),\widetilde{\gamma}\right] \cdot E\left[v'(C_{2,l})\right]$.

Now the effect of the level of real capital investments in the widely held corporation can be expressed as the sum of an income and a substitution effect:

$$
\begin{aligned}
K_l'(t_k) &= -\left\{\begin{array}{l} \frac{\beta\cdot F(K_l)+r\cdot B_l}{1+(1-t_k)\cdot r} - r\cdot\frac{E[v'(C_{2,l})]}{u''(C_{1,l})} \\ +\beta\cdot K_l\cdot[1+(1-t_k)\cdot r]\cdot\frac{E[v''(C_{2,l})\cdot\widetilde{\gamma}]}{u''(C_{1,l})} \end{array}\right\}\cdot\frac{\partial K_l}{\partial Y} \\
&\quad +\frac{\beta^2\cdot[1-t_k]\cdot K_l}{F\cdot E\left[v'(C_{2,l})\right]}\cdot\left\{u''(C_{1,l})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,l})\right]\right\} \\
&\quad \cdot\left\{\begin{array}{l} E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right] \\ -E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \end{array}\right\}.
\end{aligned} \tag{20}
$$

The widely held corporation has no income shifting incentives by an increased tax rate on capital income. The increased tax reduces returns to all types of investments at the same rate, and reduces his total income. It is thus likely that the above expression is negative. Nevertheless, the increased tax on capital income induces the individual to increase investments in firm specific real capital if $v'''(C_{2,l}) > 0$, $cov\left[v''(C_{2,l})\cdot\widetilde{\gamma},\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right] < cov\left[v'(C_{2,l}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]$, and if the substitution effect dominates the total income effect.

In the absence of risk, the optimal investment condition reduces to the Fisher condition, and tax changes have no effect on the investment decision in the widely held corporation.

5.3 The indirect utility function.

The indirect utility function of the individual when his firm is organized as a widely held corporation is given by

$$\widehat{EU}_l = u(\widehat{C}_{1,l}) + E\left[v(\widehat{C}_{2,l})\right] \tag{21}$$

where

$$\widehat{C}_{1,l} = Y - \beta \cdot \widehat{K}_l - \widehat{B}_l \tag{22}$$

$$\widehat{C}_{2,l} = \beta \cdot [1-t_k] \cdot \left[F(\widehat{K}_l) - \widetilde{\gamma} \cdot \widehat{K}_l\right] + \beta \cdot \widehat{K}_l + [1+(1-t_k)\cdot r] \cdot \widehat{B}_l \tag{23}$$

This will be used in the analysis of which organizational form to choose.

6 When to incorporate?

The only reason for the sole proprietor to incorporate is assumed to be to reduce tax payments. Under the split model, a part of the firm's income is taxed as labor income, whereas all firm income is taxed as capital income under the corporate tax schedule. But in order to be taxed as a widely held corporation, at least β of profits must be paid as dividends to passive shareholders. Only if the sole proprietor has positive imputed personal income has he incentives to incorporate. Thus assume that the expected imputed personal income of the sole proprietor is positive after he has exhausted the income shifting possibilities inherent in the split model through real capital investments, such that he expects to have a positive imputed return to labor.

For simplicity, let the costs[19] of incorporating be zero. The sole proprietor incor-

[19]This is a simplifying assumption. Still, the actual costs of organizing as a corporation are moderate, and the process is also not that complicated. But corporations are subject to stricter regulations than sole proprietors. For instance, they are obliged to have an accountant.

porates if he achieves the higher maximum achievable expected utility as a widely held corporation:

$$\text{Incorporate if} \quad \widehat{EU}_l - \widehat{EU}_s > 0,$$

where $\widehat{EU}_l$ is defined by the equations (21)-(23) and $\widehat{EU}_l$ is defined by the equations (12)-(14). The larger this difference, the higher the incentives to incorporate in order to reduce total tax payments.

By applying the envelope theorem, let us now study how policy changes affect the incentives to incorporate.

The labor income tax rate. The effect on the incentives to incorporate by an increase in the labor income tax rate is given by:

$$\frac{\partial\left(\widehat{EU}_l - \widehat{EU}_s\right)}{\partial t_w} = \left\{F(\widehat{K}_s) - \left(r + \mu + \delta + \widehat{\lambda}_s\right) \cdot \widehat{K}_s\right\} \cdot E\left[v(\widehat{C}_{2,s})\right]. \tag{24}$$

It is already assumed that the sole proprietor must expect to have a positive imputed return to labor in order to even consider incorporating. From the definition of the risk compensating factor, condition (3), it follows that (24) is positive if the expected imputed return to labor income at least covers the premium in optimum, $\widehat{\lambda}_s$. The higher the expected imputed return to labor income, the larger are the incentives to incorporate. The factor working against this is the fact that the net risk compensation rate under the split model actually increases when the labor income tax rate increases.

The capital income tax rate. The effect of an increase in the capital income tax rate on the incentives to incorporate is given by

$$\begin{aligned}\frac{\partial\left(\widehat{EU}_l - \widehat{EU}_s\right)}{\partial t_k} &= -\left\{\beta \cdot \left[F(\widehat{K}_l) - \delta - \widehat{\lambda}_l\right] + r \cdot \widehat{B}_l\right\} \cdot E\left[v'(\widehat{C}_{2,l})\right] \\ &\quad + \left\{(r + \mu) \cdot \widehat{K}_s + r \cdot \widehat{B}_s\right\} \cdot E\left[v'(\widehat{C}_{2,s})\right],\end{aligned} \tag{25}$$

which most likely is negative. The reason for this is twofold. First, the overall incentives for participating in tax minimizing income shifting decrease when the difference between the marginal tax rates on labor and capital decrease. Second, all income of the entrepreneur is affected by the tax increase when he is organized as a widely held corporation, while only part of the sole proprietor's income is affected by the tax increase.

The risk compensation factor. An increase in the risk compensation factor under the split model reduces the incentives for the sole proprietor to incorporate, as is seen from the below expression:

$$\frac{\partial\left(\widehat{EU}_l - \widehat{EU}_s\right)}{\partial \mu} = -\left[t_w - t_k\right] \cdot \widehat{K}_s \cdot E\left[v'(\widehat{C}_{2,s})\right]. \tag{26}$$

The higher the risk compensation factor, the more of the sole proprietor's income is taxed as capital income, and the less attractive is it to incorporate in order to avoid the split model.

Shares held by the active owner. As long as the expected after tax net business income of the widely held corporation is positive, an increase in the maximum allowed ownership share of the active owner increases the incentives to incorporate:

$$\frac{\partial\left(\widehat{EU}_l - \widehat{EU}_s\right)}{\partial \beta} = \left[1 - t_k\right] \cdot \left\{F(\widehat{K}_l) - \left(r + \widehat{\lambda}_l + \delta\right) \cdot \widehat{K}_l\right\} \cdot E\left[v'(\widehat{C}_{2,l})\right] \tag{27}$$

The effect is greater the lower the tax rate on capital income and the higher the net business income.

7 Empirical observations.

High-income sole proprietors are subject to the top marginal tax rate on the imputed return to labor, and these are expected to take advantage of the income shifting possibilities through increasing their real capital stock. And in fact the Norwegian sole proprietors in the top decile of the income distribution more than doubled the value of their real capital from 1992 to 2000[20], as figure 1 shows. These are aggregate data, and it is not possible to see whether there has been a shift in the type of real capital investments. Unfortunately, there are no available data prior to the 1992-tax reform. Still, it ought to take the firm some time to adjust its investment decision to the new tax rules. As new sole proprietors reach the top marginal tax bracket on labor income, they adapt to the tax minimizing incentives inherent in the split model. Hence one would expect a development towards more real capital in this group over time, rather than a shift to a new investment level directly after the tax reform.

The number of sole proprietors decreased during the 1990's, while the total number of corporations increased by more than the same amount, as is seen in figure 2[21]. Even if part of the decline of sole proprietors is due to a reduction of the primary sector, mostly farming, there was also a reduction in other sectors. At the same time there was a reduction in the number of closely held corporations, as well as an increase in widely held corporations. A strong selection also took place. The closely held corporations mostly have negative imputed return to labor, and their active owners hence do not pay labor income taxes. In 1992, 65% of the closely held corporations had negative imputed return to labor, while this share had increased to 80% in 2000. Also, in 1995, 28% of all one-man corporations were closely held, and already two years later this share had fallen to 20%.

This can be interpreted as an indication of a tax induced shift in organizational form and choice of tax regime. Sole proprietors incorporate in order to escape the split model, and corporations choose to be widely held in order to escape the split model. Only corporations with low profits and thus also low or negative imputed return to labor stay under the split model.

[20] Calculations made on combined survey and register data from Statistics Norway. Annual sample of ca. 4000, but weighted for representability. The primary sector is heavily regulated and subsidized, and self-employed in this sector are excluded from the sample.

[21] Source: Statistics Norway.

Data are unfortunatelly not available for the whole time period in question.

Figure 1: *The value of the sole proprietors' real capital in thousands, 1998-prices.*

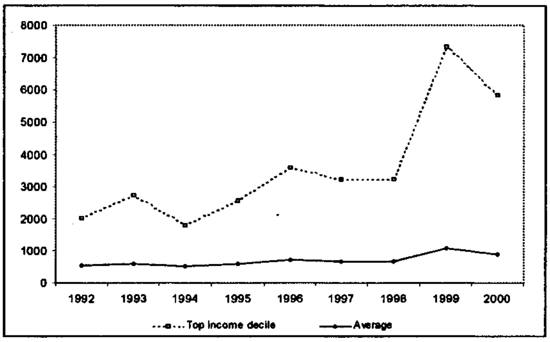
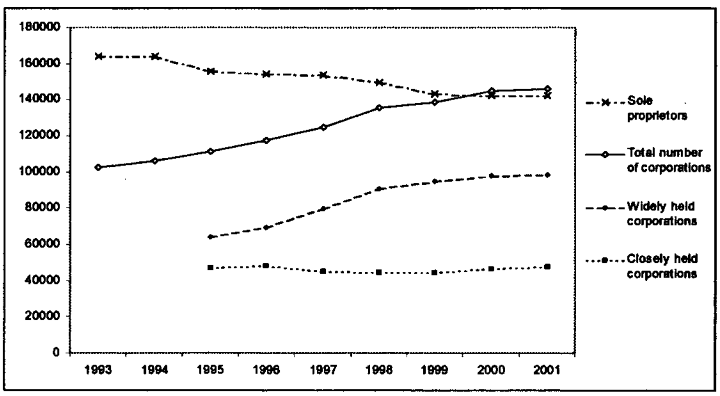


Figure 2: *Number of sole proprietors and corporations over time.*



8 Conclusions.

The above analysis concludes that the split model counteracts the negative effects of the risk of a technology shock on the sole proprietor's investments in firm specific real capital, and it encourages more real capital investments than in the absence of taxes. The split model might actually induce the sole proprietor to over-invest in less risky real capital. Real capital investments are a device for shifting income from the labor income tax base to the capital income tax base in order to reduce the sole proprietor's total tax payments. The incentives to participate in tax minimizing income shifting increase as his income increases. The net risk compensation rate under the split model is higher the higher the labor income tax rate, and thus the incentives to over-invest in firm specific real capital may increase as the labor income tax rate increases.

In addition, the widely held corporation serves as a tax shelter for high income sole proprietors. The higher his income, and the larger the difference between the tax rates on labor income and capital income, the larger the incentives to become a widely held corporation in order to escape the split model and reduce total tax payments. Only low-income entrepreneurs have incentives to stay under the split model in order to deduct the negative imputed labor income against future positive imputed return to labor.

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10 Mathematical appendix

Properties of the utility function: We know that $C_2'(\widetilde{\gamma}) < 0$, $v'(C_2) > 0$ and $v''(C_2) < 0$. It then follows that

$$cov\left[v'(C_2), C_2\right] < 0 \quad \text{and} \quad cov\left[v'(C_2), \widetilde{\gamma}\right] > 0,$$

$$cov\left[v''(C_2), C_2\right] \begin{cases} > 0 \text{ if } v'''(C_2) > 0 \\ < 0 \text{ if } v'''(C_2) < 0 \end{cases} \text{ and } cov\left[v''(C_2), \widetilde{\gamma}\right] \begin{cases} < 0 \text{ if } v'''(C_2) > 0 \\ > 0 \text{ if } v'''(C_2) < 0 \end{cases}$$

The individual has *increasing absolute risk aversion* if $v'''(C_2) < 0$.

The individual has *decreasing absolute risk aversion* if $v'''(C_2) > 0$ and if $v'''(C_2) > \frac{v''(C_2)^2}{v'(C_2)}$.

Developing equation (3):

$$\begin{aligned} \Theta &= \frac{E\left[v'\left(C_2\right) \cdot \widetilde{\gamma}\right]}{E\left[v'\left(C_2\right)\right]} \\ &= \frac{E\left[v'\left(C_2\right)\right] \cdot E\left[\widetilde{\gamma}\right] + cov\left[v'\left(C_2\right), \widetilde{\gamma}\right]}{E\left[v'\left(C_2\right)\right]} \\ &= \delta + \frac{cov\left[v'\left(C_2\right), \widetilde{\gamma}\right]}{E\left[v'\left(C_2\right)\right]} \end{aligned}$$

Developing equation (9): We know from (7) that $u'(C_{1,s}) = \{1 + (1 - t_k) \cdot r\} \cdot E\left[v'(C_{2,s})\right]$. Apply this to equation (8) and rearrange:

$$\begin{aligned} \{1 + (1 - t_k) \cdot r\} \cdot E\left[v'(C_{2,s})\right] &= E\left[v'(C_{2,s}) \cdot \left\{ \begin{array}{c} [1 - t_w] \cdot [F_{K_s} - \widetilde{\gamma}] \\ + [t_w - t_k] \cdot [r + \mu] + 1 \end{array} \right\}\right] \\ &\Downarrow \cdot \frac{1}{E\left[v'(C_{2,s})\right]} \\ 1 + (1 - t_k) \cdot r &= [1 - t_w] \cdot F_{K_s} - \frac{[1 - t_w] \cdot E\left[v'(C_{2,s}) \cdot \widetilde{\gamma}\right]}{E\left[v'(C_{2,s})\right]} \\ &\quad + [t_w - t_k] \cdot [r + \mu] + 1 \\ &\Downarrow \cdot \frac{1}{1 - t_w} \\ F_{K_s} &= r + \frac{E\left[v'(C_{2,s}) \cdot \widetilde{\gamma}\right]}{E\left[v'(C_{2,s})\right]} - \frac{t_w - t_k}{1 - t_w} \cdot \mu \qquad (28) \\ &\Downarrow \text{ Apply equation (3)} \\ F_{K_s} &= r + \delta + \lambda_s - \frac{t_w - t_k}{1 - t_w} \cdot \mu \end{aligned}$$

10.1 Conditions for the existence of a local maximum for the sole proprietor:

$$\begin{aligned} 1) &: \; EU_{BB} < 0 \\ 2) &: \; EU_{KK} < 0 \\ 3) &: \; EU_{BB} \cdot EU_{KK} - (EU_{BK})^2 > 0 \end{aligned}$$

From equation (7) it follows that

$$EU_{BB} = u''(C_{1,s}) + \{1 + (1 - t_k) \cdot r\}^2 \cdot E\left[v''(C_{2,s})\right] < 0.$$

From equation (8) it follows that

$$\begin{aligned} EU_{KK} &= u''(C_{1,s}) + [1 - t_w] \cdot F_{K_s K_s} \cdot E\left[v'(C_{2,s})\right] + A^2 \cdot E\left[v''(C_{2,s})\right] \\ &\quad -2 \cdot A \cdot [1 - t_w] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] + [1 - t_w]^2 \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \\ &< 0 \end{aligned}$$

where

$$A \equiv [1 - t_w] \cdot F_{K_s} + 1 + [t_w - t_k] \cdot [r + \mu] \tag{29}$$

$$\Downarrow \text{ use condition (28)}$$

$$A = [1 + [1 - t_k] \cdot r] + [1 - t_w] \cdot \frac{E\left[v'(C_{2,s}) \cdot \widetilde{\gamma}\right]}{E\left[v'(C_{2,s})\right]} \tag{30}$$

Also, from equation (7) it follows that

$$EU_{BK} = u''(C_{1,s}) + [1 + (1 - t_k) \cdot r] \cdot \{A \cdot E\left[v''(C_{2,s})\right] - [1 - t_w] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right]\}$$

Define

$$D \equiv EU_{BB} \cdot EU_{KK} - (EU_{BK})^2 > 0$$

$$\Downarrow$$

$$
\begin{aligned}
D &= \left\{u''(C_{1,s}) + \{1+(1-t_k)\cdot r\}^2 \cdot E\left[v''(C_{2,s})\right]\right\} \\
&\cdot \left\{ \begin{array}{c} u''(C_{1,s}) + [1-t_w]\cdot F_{K_sK_s}\cdot E\left[v'(C_{2,s})\right] + A^2 \cdot E\left[v''(C_{2,s})\right] \\ -2\cdot A\cdot[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] + [1-t_w]^2\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right] \end{array} \right\} \\
&- \left\{ u''(C_{1,s}) + [1+(1-t_k)\cdot r]\cdot \left\{ \begin{array}{c} A\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array} \right\} \right\}^2 \\
&> 0
\end{aligned}
\tag{31}
$$

10.2 The income effect of the sole proprietor.

From the first order condition (8) it follows that

$$
\begin{aligned}
& u''(C_{1,s})\cdot\left[\frac{\partial K_s}{\partial Y} + \frac{\partial B_s}{\partial Y} - 1\right] + [1-t_w]\cdot F_{K_sK_s}\cdot E\left[v'(C_{2,s})\right] \\
& + A\cdot E\left[v''(C_{2,s})\cdot \left\{ \begin{array}{c} [1-t_w]\cdot(F_{K_s}-\widetilde{\gamma})\cdot\frac{\partial K_s}{\partial Y} \\ +(1+[t_w-t_k]\cdot[r+\mu])\cdot\frac{\partial K_s}{\partial Y} \\ +[1+(1-t_k)\cdot r]\cdot\frac{\partial B_s}{\partial Y} \end{array} \right\} \right] \\
& -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot \left\{ \begin{array}{c} [1-t_w]\cdot(F_{K_s}-\widetilde{\gamma})\cdot\frac{\partial K_s}{\partial Y} \\ +(1+[t_w-t_k]\cdot[r+\mu])\cdot\frac{\partial K_s}{\partial Y} \\ +[1+(1-t_k)\cdot r]\cdot\frac{\partial B_s}{\partial Y} \end{array} \right\} \right] \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
&\Downarrow \\
&\frac{\partial K_s}{\partial Y}\cdot \left\{ \begin{array}{c} u''(C_{1,s}) + [1-t_w]\cdot F_{K_sK_s}\cdot E\left[v'(C_{2,s})\right] + A^2 \cdot E\left[v''(C_{2,s})\right] \\ -2\cdot A\cdot[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] + [1-t_w]^2\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right] \end{array} \right\} \\
&+\frac{\partial B_s}{\partial Y}\cdot \left\{ u''(C_{1,s}) + [1+(1-t_k)\cdot r]\cdot \left\{ \begin{array}{c} A\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array} \right\} \right\} \\
&= u''(C_{1,s})
\end{aligned}
\tag{32}
$$

And from the first order condition (7) it follows that

$$
\begin{aligned}
& u''(C_{1,s})\cdot\left[\frac{\partial K_s}{\partial Y} + \frac{\partial B_s}{\partial Y} - 1\right] \\
& +[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,s})\cdot \left\{ \begin{array}{c} [1-t_w]\cdot(F_{K_s}-\widetilde{\gamma})\cdot\frac{\partial K_s}{\partial Y} \\ +(1+[t_w-t_k]\cdot[r+\mu])\cdot\frac{\partial K_s}{\partial Y} \\ +[1+(1-t_k)\cdot r]\cdot\frac{\partial B_s}{\partial Y} \end{array} \right\} \right] \\
&= 0
\end{aligned}
$$

$$\Downarrow \tag{33}$$

$$\begin{aligned}
&\frac{\partial K_s}{\partial Y}\cdot\left\{u''(C_{1,s})+[1+(1-t_k)\cdot r]\cdot\left\{\begin{array}{c} A\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\end{array}\right\}\right\} \\
&+\frac{\partial B_s}{\partial Y}\cdot\left\{u''(C_{1,s})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,s})\right]\right\} \\
=\;& u''(C_{1,s})
\end{aligned}$$

By Cramer's rule, equations (32) and (33) yield:

$$\frac{\partial K_s}{\partial Y} = -\frac{u''(C_{1,s})}{D}\cdot\left\{\begin{array}{c} u''(C_{1,s})+[1+(1-t_k)\cdot r] \\ \cdot\left\{\begin{array}{c} A\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\end{array}\right\} \\ -\left\{u''(C_{1,s})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,s})\right]\right\}\end{array}\right\}$$

$\Downarrow$ apply definition (30)

$$\frac{\partial K_s}{\partial Y} = -\frac{u''(C_{1,s})}{D}\cdot\left\{\begin{array}{c} u''(C_{1,s})+[1+(1-t_k)\cdot r] \\ \cdot\left\{\begin{array}{c} [1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,s})\right] \\ +[1-t_w]\cdot\frac{E[v'(C_{2,s})\cdot\widetilde{\gamma}]}{E[v'(C_{2,s})]}\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\end{array}\right\} \\ -u''(C_{1,s})-[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,s})\right]\end{array}\right\}$$

$$\Downarrow \tag{34}$$

$$\frac{\partial K_s}{\partial Y} = \frac{u''(C_{1,s})\cdot[1+(1-t_k)\cdot r]\cdot[1-t_w]}{D\cdot E\left[v'(C_{2,s})\right]}\cdot\left\{\begin{array}{c} E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] \\ \\ -E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right]\end{array}\right\}$$

As $\frac{u''(C_{1,s})\cdot[1+(1-t_k)\cdot r]\cdot[1-t_w]}{D\cdot E[v'(C_{2,s})]}<0$, the sign of the income effect is determined by the expressions in the parenthesis. Thus

$$\frac{\partial K_s}{\partial Y}>0 \;\text{ if }\; E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right]-E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right]<0$$

We know that

$$\begin{aligned}
& E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] \\
=\;& \left\{cov\left[v''(C_{2,s}),\widetilde{\gamma}\right]+E\left[v''(C_{2,s})\right]\cdot E\left[\widetilde{\gamma}\right]\right\}\cdot E\left[v'(C_{2,s})\right]
\end{aligned}$$

and

$$\begin{aligned}
& E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right] \\
= & \left\{cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]+E\left[v'(C_{2,s})\right]\cdot E\left[\widetilde{\gamma}\right]\right\}\cdot E\left[v''(C_{2,s})\right]
\end{aligned}$$

This means that

$$\frac{\partial K_s}{\partial Y}>0 \text{ if } cov\left[v''(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right]-cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right]<0.$$

We already know that

$$E\left[v'(C_{2,s})\right]>0,\ cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]>0 \text{ and } E\left[v''(C_{2,s})\right]<0.$$

First, consider the case where $v'''(C_{2,s})>0$.

$$\begin{aligned}
v'''(C_{2,s}) &> 0 \Longrightarrow cov\left[v''(C_{2,s}),\widetilde{\gamma}\right]<0. \\
&\Downarrow \\
\frac{\partial K_s}{\partial Y} &> 0 \text{ if } cov\left[v''(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right]<cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right],
\end{aligned}$$

which are both negative. This means that in order for the above condition to be met, the absolute value of the left hand side must be larger than the absolute value of the right hand side.

Next, consider the case where $v'''(C_{2,s})<0$.

$$\begin{aligned}
v'''(C_{2,s}) &< 0 \Longrightarrow cov\left[v''(C_{2,s}),\widetilde{\gamma}\right]>0. \\
&\Downarrow \\
\frac{\partial K_s}{\partial Y} &< 0.
\end{aligned}$$

10.2.1 The effect on the investment portfolio and risk profile of the sole proprietor by changed tax on labor income.

Differentiate equation (7) with respect to t_w to find that:

$$\begin{aligned}
& -u''(C_{1,s})\cdot\left\{-K'(t_w)-B'(t_w)\right\} \\
& +\left\{1+(1-t_k)\cdot r\right\}\cdot E\left[v''(C_{2,s})\cdot\frac{\partial C_{2,s}}{\partial t_w}\right] \\
= & \ 0
\end{aligned}$$

$$\Downarrow \tag{35}$$

$$\begin{aligned}
& K'(t_w)\cdot\left\{u''(C_{1,s})+[1+(1-t_k)\cdot r]\cdot\left[\begin{array}{c} A\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\end{array}\right]\right\} \\
& +B'(t_w)\cdot\left\{u''(C_{1,s})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,s})\right]\right\} \\
= & \ [1+(1-t_k)\cdot r]\cdot\left\{\begin{array}{c} [F(K_s)-(r+\mu)\cdot K_s]\cdot E\left[v''(C_{2,s})\right] \\ -K_s\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\end{array}\right\}
\end{aligned}$$

For simplicity, name the different parts of the above equation as

$$K'(t_w)\cdot a_{BK}+B'(t_w)\cdot a_{BB}=b_B$$

Next, condition (8) is differentiated:

$$\begin{aligned}
& u''(C_{1,s})\cdot\{K'(t_w)+B'(t_w)\} \\
& +\{-F_{K_s}+[1-t_w]\cdot F_{K_sK_s}\cdot K'(t_w)+r+\mu\}\cdot E\left[v'(C_{2,s})\right] \\
& +A\cdot E\left[v''(C_{2,s})\cdot\frac{\partial C_{2,s}}{\partial t_w}\right] \\
& +E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right] \\
& -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\frac{\partial C_{2,s}}{\partial t_w}\right] \\
= & \ 0
\end{aligned}$$

$$\Downarrow \tag{36}$$

$$\begin{aligned}
& K'(t_w)\cdot\left\{\begin{array}{c} u''(C_{1,s})+[1-t_w]\cdot F_{K_sK_s}\cdot E\left[v'(C_{2,s})\right] \\ +A^2\cdot E\left[v''(C_{2,s})\right]-2\cdot[1-t_w]\cdot A\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \\ +[1-t_w]^2\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\end{array}\right\} \\
& +B'(t_w)\cdot\left\{u''(C_{1,s})+[1+(1-t_k)\cdot r]\cdot\left\{\begin{array}{c} A\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\end{array}\right\}\right\} \\
= & \left\{\begin{array}{c} A\cdot[F(K_s)-(r+\mu)\cdot K_s]\cdot E\left[v''(C_{2,s})\right] \\ +[F_{K_s}-(r+\mu)]\cdot E\left[v'(C_{2,s})\right] \\ -A\cdot K_s\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]-E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right] \\ -[1-t_w]\cdot[F(K_s)-(r+\mu)\cdot K_s]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \\ +[1-t_w]\cdot K_s\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\end{array}\right\}
\end{aligned}$$

Name the parts of the above equation

$$K'(t_w) \cdot a_{KK} + B'(t_w) \cdot a_{KB} = b_K$$

By Cramer's rule equations (35) and (36) yield:

$$K'(t_w) = \frac{1}{-D} \cdot (b_B \cdot a_{KB} - b_k \cdot a_{BB}) \tag{37}$$

where

$$\begin{aligned} & b_B \cdot a_{KB} \\ = & \left\{ u''(C_{1,s}) + \left[\begin{array}{c} [1+(1-t_k)\cdot r] \cdot \left\{ A - [1-t_w] \cdot \frac{E[v''(C_{2,s})\cdot\widetilde{\gamma}]}{E[v''(C_{2,s})]} \right\} \\ \cdot E\left[v''(C_{2,s})\right] \end{array} \right] \right\} \\ & \cdot [1+(1-t_k)\cdot r] \cdot \left\{ \begin{array}{c} [F(K_s) - (r+\mu)\cdot K_s] \\ -K_s \cdot \frac{E[v''(C_{2,s})\cdot\widetilde{\gamma}]}{E[v''(C_{2,s})]} \end{array} \right\} \cdot E\left[v''(C_{2,s})\right] \end{aligned}$$

Apply the definition (30) of A:

$$\begin{aligned} & b_B \cdot a_{KB} \\ = & \left\{ \begin{array}{c} u''(C_{1,s}) + [1+[1-t_k]\cdot r] \cdot E\left[v''(C_{2,s})\right] \\ +\frac{[1+(1-t_k)\cdot r]\cdot[1-t_w]}{E[v'(C_{2,s})]} \cdot \left\{ \begin{array}{c} E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right] \cdot E\left[v''(C_{2,s})\right] \\ -E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \cdot E\left[v'(C_{2,s})\right] \end{array} \right\} \end{array} \right\} \\ & \cdot [1+(1-t_k)\cdot r] \cdot \left\{ \begin{array}{c} [F(K_s) - (r+\mu)\cdot K_s] \\ -K_s \cdot \frac{E[v''(C_{2,s})\cdot\widetilde{\gamma}]}{E[v''(C_{2,s})]} \end{array} \right\} \cdot E\left[v''(C_{2,s})\right] \end{aligned}$$

We know from the income effect (34) that

$$-\frac{\partial K_s}{\partial Y} \cdot \frac{D}{u''(C_{1,s})} = \frac{[1+(1-t_k)\cdot r]\cdot[1-t_w]}{E\left[v'(C_{2,s})\right]} \cdot \left\{ \begin{array}{c} -E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \cdot E\left[v'(C_{2,s})\right] \\ +E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right] \cdot E\left[v''(C_{2,s})\right] \end{array} \right\}$$

$$\Downarrow \tag{38}$$

$$\begin{aligned} b_B \cdot a_{KB} \;=\; & \left\{ \frac{u''(C_{1,s})}{E\left[v''(C_{2,s})\right]} + [1+(1-t_k)\cdot r]^2 \right\} \cdot [1+(1-t_k)\cdot r] \\ & \cdot \left[F(K_s) - \left(r + \mu + \frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v''(C_{2,s})\right]} \right) \cdot K_s \right] \cdot E\left[v''(C_{2,s})\right]^2 \\ & - \frac{D \cdot E\left[v''(C_{2,s})\right]}{u''(C_{1,s})} \cdot \frac{\partial K_s}{\partial Y} \cdot [1+(1-t_k)\cdot r] \\ & \cdot \left[F(K_s) - \left(r + \mu + \frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v''(C_{2,s})\right]} \right) \cdot K_s \right] \end{aligned}$$

Next,

$$b_k \cdot a_{BB} =$$

$$\left\{\begin{array}{c} A \cdot [F(K_s) - (r+\mu) \cdot K_s] \cdot E[v''(C_{2,s})] \\ + [F_{K_s} - (r+\mu)] \cdot E[v'(C_{2,s})] \\ -A \cdot K_s \cdot \frac{E[v''(C_{2,s}) \cdot \widetilde{\gamma}]}{E[v''(C_{2,s})]} \cdot E[v''(C_{2,s})] \\ -\frac{E[v'(C_{2,s}) \cdot \widetilde{\gamma}]}{E[v'(C_{2,s})]} \cdot E[v'(C_{2,s})] \\ - [1-t_w] \cdot [F(K_s) - (r+\mu) \cdot K_s] \\ \cdot \frac{E[v''(C_{2,s}) \cdot \widetilde{\gamma}]}{E[v''(C_{2,s})]} \cdot E[v''(C_{2,s})] \\ \\ + [1-t_w] \cdot K_s \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}] \end{array}\right\}$$

$$\cdot \left\{u''(C_{1,s}) + [1 + (1-t_k) \cdot r]^2 \cdot E[v''(C_{2,s})]\right\}$$

From (28) we know that

$$F_{K_s} - r - \mu = \frac{E[v'(C_{2,s}) \cdot \widetilde{\gamma}]}{E[v'(C_{2,s})]} - \frac{1-t_k}{1-t_w}\mu$$

$\Downarrow$ Also use definition (30) of A:

$$b_k \cdot a_{BB} =$$

$$\left\{\begin{array}{c} [1 + [1-t_k] \cdot r] \cdot [F(K_s) - (r+\mu) \cdot K_s] \cdot E[v''(C_{2,s})] \\ + [1-t_w] \cdot \frac{E[v'(C_{2,s}) \cdot \widetilde{\gamma}]}{E[v'(C_{2,s})]} \cdot [F(K_s) - (r+\mu) \cdot K_s] \cdot E[v''(C_{2,s})] \\ + \left[\frac{E[v'(C_{2,s}) \cdot \widetilde{\gamma}]}{E[v'(C_{2,s})]} - \frac{1-t_k}{1-t_w}\mu\right] \cdot E[v'(C_{2,s})] \\ - [1 + [1-t_k] \cdot r] \cdot K_s \cdot \frac{E[v''(C_{2,s}) \cdot \widetilde{\gamma}]}{E[v''(C_{2,s})]} \cdot E[v''(C_{2,s})] \\ - [1-t_w] \cdot \frac{E[v'(C_{2,s}) \cdot \widetilde{\gamma}]}{E[v'(C_{2,s})]} \cdot K_s \cdot \frac{E[v''(C_{2,s}) \cdot \widetilde{\gamma}]}{E[v''(C_{2,s})]} \cdot E[v''(C_{2,s})] \\ -\frac{E[v'(C_{2,s}) \cdot \widetilde{\gamma}]}{E[v'(C_{2,s})]} \cdot E[v'(C_{2,s})] \\ - [1-t_w] \cdot [F(K_s) - (r+\mu) \cdot K_s] \\ \cdot \frac{E[v''(C_{2,s}) \cdot \widetilde{\gamma}]}{E[v''(C_{2,s})]} \cdot E[v''(C_{2,s})] \\ + [1-t_w] \cdot K_s \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}] \end{array}\right\}$$

$$\cdot \left\{u''(C_{1,s}) + [1 + (1-t_k) \cdot r]^2 \cdot E[v''(C_{2,s})]\right\}$$

$$\Downarrow$$

$$b_k \cdot a_{BB}$$

$$= \left\{ \begin{array}{l} [F(K_s) - (r+\mu)\cdot K_s] \cdot \frac{[1-t_w]}{E[v'(C_{2,s})]} \cdot \left[\begin{array}{c} E[v'(C_{2,s})\cdot\widetilde{\gamma}] \cdot E[v''(C_{2,s})] \\ -E[v''(C_{2,s})\cdot\widetilde{\gamma}] \cdot E[v'(C_{2,s})] \end{array} \right] \\ [F(K_s) - (r+\mu)\cdot K_s] \cdot E[v''(C_{2,s})] \cdot [1 + [1-t_k]\cdot r] \\ -\frac{1-t_k}{1-t_w}\mu \cdot E[v'(C_{2,s})] \\ -E[v''(C_{2,s})\cdot\widetilde{\gamma}] \cdot K_s \cdot \left\{ [1+[1-t_k]\cdot r] + [1-t_w] \cdot \frac{E[v'(C_{2,s})\cdot\widetilde{\gamma}]}{E[v'(C_{2,s})]} \right\} \\ +[1-t_w]\cdot K_s \cdot E[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}] \end{array} \right\}$$

$$\cdot \left\{ u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E[v''(C_{2,s})] \right\}$$

We know from the income effect (34) that

$$\begin{aligned} & -\frac{\partial K_s}{\partial Y} \cdot \frac{D}{[1+(1-t_k)\cdot r]\cdot u''(C_{1,s})} \\ = & \frac{[1-t_w]}{E[v'(C_{2,s})]} \cdot \left\{ \begin{array}{c} -E[v''(C_{2,s})\cdot\widetilde{\gamma}] \cdot E[v'(C_{2,s})] \\ +E[v'(C_{2,s})\cdot\widetilde{\gamma}] \cdot E[v''(C_{2,s})] \end{array} \right\} \end{aligned}$$

$$\Downarrow \tag{39}$$

$$\begin{aligned} & b_k \cdot a_{BB} \\ = & \left\{ u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E[v''(C_{2,s})] \right\} \\ & \cdot \left\{ \begin{array}{l} -\frac{D}{u''(C_{1,s})\cdot[1+(1-t_k)\cdot r]} \cdot \frac{\partial K_s}{\partial Y} \cdot [F(K_s) - (\delta + r + \mu)\cdot K_s] \\ +[1+(1-t_k)\cdot r] \cdot \left[F(K_s) - \left(r + \mu + \frac{E[v''(C_{2,s})\cdot\widetilde{\gamma}]}{E[v''(C_{2,s})]} \right) \cdot K_s \right] \\ \cdot E[v''(C_{2,s})] \\ \\ -\frac{1-t_k}{1-t_w} \cdot \mu \cdot E[v'(C_{2,s})] \\ +[1-t_w]\cdot K_s \cdot \left\{ \begin{array}{c} E[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}] \\ -E[v''(C_{2,s})\cdot\widetilde{\gamma}] \cdot \frac{E[v'(C_{2,s})\cdot\widetilde{\gamma}]}{E[v'(C_{2,s})]} \end{array} \right\} \end{array} \right\} \end{aligned}$$

This yields

$$
\begin{aligned}
K'(t_w) &= \frac{1}{-D} \cdot (b_B \cdot a_{KB} - b_k \cdot a_{BB}) \\
&= -\frac{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right]}{u''(C_{1,s}) \cdot [1+(1-t_k)\cdot r]} \\
&\quad \cdot \frac{\partial K_s}{\partial Y} \cdot [F(K_s) - (r+\mu)\cdot K_s] \\
&\quad + \frac{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right]}{D} \\
&\quad \cdot \left\{ \begin{array}{l} [1+(1-t_k)\cdot r] \cdot E\left[v''(C_{2,s})\right] \\ \cdot \left[F(K_s) - \left(\begin{array}{c} r+\mu \\ + \frac{E[v''(C_{2,s})\cdot\widetilde{\gamma}]}{E[v''(C_{2,s})]} \end{array} \right) \cdot K_s \right] \\ - \frac{1-t_k}{1-t_w} \cdot \mu \cdot E\left[v'(C_{2,s})\right] \\ + [1-t_w] \cdot K_s \cdot \left\{ E\left[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] - \frac{E[v'(C_{2,s})\cdot\widetilde{\gamma}]}{E[v'(C_{2,s})]} \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \right\} \end{array} \right\} \\
&\quad - \frac{1}{D} \cdot \left\{ u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right] \right\} \cdot E\left[v''(C_{2,s})\right] \\
&\quad \cdot [1+(1-t_k)\cdot r] \cdot \left[F(K_s) - \left(r+\mu+\frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v''(C_{2,s})\right]} \right) \cdot K_s \right] \\
&\quad + \frac{E\left[v''(C_{2,s})\right]}{u''(C_{1,s})} \cdot \frac{\partial K_s}{\partial Y} \cdot [1+(1-t_k)\cdot r] \\
&\quad \cdot \left[F(K_s) - \left(r+\mu+\frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v''(C_{2,s})\right]} \right) \cdot K_s \right]
\end{aligned}
$$

This yields condition (10):

$$
\begin{aligned}
K'(t_w) &= -\left\{ \begin{array}{l} \frac{F(K_s)-(r+\mu)\cdot K_s}{[1+(1-t_k)\cdot r]} \\ + [1+(1-t_k)\cdot r] \cdot \frac{E[v''(C_{2,s})\cdot\widetilde{\gamma}]}{u''(C_{1,s})} \cdot K_s \end{array} \right\} \cdot \frac{\partial K_s}{\partial Y} \\
&\quad + \frac{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right]}{[1-t_w] \cdot D \cdot E\left[v'(C_{2,s})\right]} \\
&\quad \cdot \left\{ \begin{array}{l} [1-t_w]^2 \cdot K_s \cdot \left\{ \begin{array}{c} E\left[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,s})\right] \\ -E\left[v'(C_{2,s}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \\ - [1-t_k] \cdot \mu \cdot E\left[v'(C_{2,s})\right]^2 \end{array} \right\}
\end{aligned}
$$

As $F(K_s) - (r+\mu) \cdot K_s > 0$ and $\frac{E[v''(C_{2,s})\cdot\widetilde{\gamma}]}{u''(C_{1,s})} > 0$,then the sign of the first line (which is the total income effect) depends on the sign of $\frac{\partial K_s}{\partial Y}$, which again depends on the sign of $v'''(C_{2,s})$, as previously discussed.

Also, $\frac{u''(C_{1,s})+[1+(1-t_k)\cdot r]^2\cdot E[v''(C_{2,s})]}{[1-t_w]\cdot D\cdot E[v'(C_{2,s})]} < 0$, such that the substitution effect of the tax change depends on whether the last parenthesis is positive or negative.

The substitution effect is positive if

$$\left\{ \begin{array}{c} E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] \\ -E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array} \right\} < \frac{[1-t_k]\cdot\mu\cdot E\left[v'(C_{2,s})\right]^2}{[1-t_w]^2\cdot K_s}$$

We know that

$$\begin{aligned} & E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right]-E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \\ = \; & cov\left[v''(C_{2,s})\cdot\widetilde{\gamma},\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right]-cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{aligned}$$

This means that the substitution effect is positive if

$$cov\left[v''(C_{2,s})\cdot\widetilde{\gamma},\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] < cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] < 0.$$

Thus, a necessary condition for the substitution effect being positive, is that $cov\left[v''(C_{2,s})\cdot\widetilde{\gamma},\widetilde{\gamma}\right] < 0$. This means that $v'''(C_{2,s}) > 0$. Since $0 < \widetilde{\gamma} < 1$, we will still have that $\Delta\widetilde{\gamma} > 0 \Longrightarrow \Delta\left(v''(C_{2,s})\cdot\widetilde{\gamma}\right) < 0$.

The substitution effect is negative if $v'''(C_{2,s}) < 0$.

To summarize:

$$\begin{array}{c} v'''(C_{2,s}) \; > \; 0 \\ \Downarrow \end{array}$$

$$\begin{array}{rcl} \textit{The substitution effect} & : & \\ \text{Positive if } cov\left[v''(C_{2,s})\cdot\widetilde{\gamma},\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] & < & cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \\ & & \text{and} \\ \text{negative if } cov\left[v''(C_{2,s})\cdot\widetilde{\gamma},\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] & > & cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array}$$

and

$$\begin{array}{rcl} \textit{The total income effect} & : & \\ \text{Positive if } cov\left[v''(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] & > & cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right] \\ & & \text{and} \\ \text{negative if } cov\left[v''(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] & < & cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right] \end{array}$$

As $0 < \widetilde{\gamma} < 1$, we know that $E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] > E\left[v''(C_{2,s})\right]$, since $E\left[v''(C_{2,s})\right] < 0$. This means that

$$cov\left[v'(C_{2,s}), \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,s})\right] < cov\left[v'(C_{2,s}), \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \tag{40}$$

Following the same line of reasoning yields that

$$cov\left[v''(C_{2,s}), \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,s})\right] < cov\left[v''(C_{2,s}) \cdot \widetilde{\gamma}, \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,s})\right]. \tag{41}$$

$K'(t_w) > 0$ certainly holds if both the substitution effect and the total income effect are positive, that is if

$$\begin{aligned} cov\left[v''(C_{2,s}) \cdot \widetilde{\gamma}, \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,s})\right] &< cov\left[v'(C_{2,s}), \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \\ &\text{and} \\ cov\left[v''(C_{2,s}), \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,s})\right] &> cov\left[v'(C_{2,s}), \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,s})\right] \end{aligned}$$

But following (40) and (41) this cannot be true. For the same reason, the total income effect and the substitution effects cannot both be negative.

Thus, we have established that the total income effect and the substitution effects have opposite sign.

$$\begin{aligned} K'(t_w) &> 0 \text{ if} \\ 1) &: v'''(C_{2,s}) > 0 \\ &\quad \text{and} \\ 2) &: cov\left[v''(C_{2,s}) \cdot \widetilde{\gamma}, \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,s})\right] < cov\left[v'(C_{2,s}), \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \\ &\quad \text{and} \\ 3) &: \text{The substitution effect dominates the total income effect.} \end{aligned}$$

Next:

$$\begin{aligned} v'''(C_{2,s}) &< 0 \\ &\Downarrow \\ \frac{\partial K_s}{\partial Y} &< 0, \text{ and the total income effect is positive.} \\ &\quad \text{and} \\ &\quad \text{The substitution effect is negative.} \end{aligned}$$

This means that

$$\begin{aligned} K'(t_w) &> 0 \text{ if} \\ 1) &: v'''(C_{2,s}) < 0 \\ & \quad \text{and} \\ 2) &: \text{The total income effect dominates the substitution effect.} \end{aligned}$$

10.2.2 The effect on the investment portfolio and risk profile of the sole proprietor by changed tax on capital income.

Differentiating the first order condition (7) yields

$$\begin{aligned} & -u''(C_{1,s}) \cdot \{-K'(t_k) - B'(t_k)\} - r \cdot E\left[v'(C_{2,s})\right] \\ & + \{1 + (1 - t_k) \cdot r\} \cdot E\left[v''(C_{2,s}) \cdot \frac{\partial C_{2,s}}{\partial t_k}\right] \\ = & \ 0 \end{aligned}$$

$$\Downarrow \tag{42}$$

$$\begin{aligned} & K'(t_k) \cdot \left\{ u''(C_{1,s}) + [1 + (1 - t_k) \cdot r] \cdot \left[\begin{array}{c} A \cdot E\left[v''(C_{2,s})\right] \\ -[1 - t_w] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \end{array} \right] \right\} \\ & + B'(t_k) \cdot \left\{ u''(C_{1,s}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,s})\right] \right\} \\ = & \ r \cdot E\left[v'(C_{2,s})\right] + [1 + (1 - t_k) \cdot r] \cdot \{[r + \mu] \cdot K_s + r \cdot B_s\} \cdot E\left[v''(C_{2,s})\right] \end{aligned}$$

Write this as

$$K'(t_k) \cdot x_{BK} + K'(t_k) \cdot x_{BB} = h_B$$

Next, condition (8) is differentiated:

$$\begin{aligned} & -u''(C_{1,s}) \cdot \{-K'(t_k) - B'(t_k)\} \\ & + \frac{\partial A}{\partial t_k} \cdot E\left[v'(C_{2,s})\right] + A \cdot E\left[v''(C_{2,s}) \cdot \frac{\partial C_{2,s}}{\partial t_k}\right] \\ & - [1 - t_w] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \frac{\partial C_{2,s}}{\partial t_k}\right] \\ = & \ 0 \end{aligned}$$

$$\Downarrow \tag{43}$$

$$K'(t_k)\cdot\left\{\begin{array}{c} u''(C_{1,s})+[1-t_w]\cdot F_{K_sK_s}\cdot E\left[v'(C_{2,s})\right] \\ +A^2\cdot E\left[v''(C_{2,s})\right]-2\cdot A\cdot[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \\ +[1-t_w]^2\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right] \end{array}\right\}$$

$$B'(t_k)\cdot\left\{\begin{array}{c} u''(C_{1,s})+[1+(1-t_k)\cdot r] \\ \cdot\left\{\begin{array}{c} A\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array}\right\} \end{array}\right\}$$

$$=\left\{\begin{array}{c} (r+\mu)\cdot E\left[v'(C_{2,s})\right]+A\cdot([r+\mu]\cdot K_s+r\cdot B_s)\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot([r+\mu]\cdot K_s+r\cdot B_s)\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array}\right\}$$

Write this as

$$K'(t_k)\cdot x_{KK}+K'(t_k)\cdot x_{KB}=h_K$$

By applying Cramer's rule and using the definition of A, equations (42) and (43) yield:

$$K'(t_k)=\frac{h_B\cdot x_{KB}-h_K\cdot x_{BB}}{-D}$$

where

$$\begin{aligned} h_B\cdot x_{KB} &= \left\{\begin{array}{c} r\cdot E\left[v'(C_{2,s})\right] \\ +[1+(1-t_k)\cdot r]\cdot\{[r+\mu]\cdot K_s+r\cdot B_s\}\cdot E\left[v''(C_{2,s})\right] \end{array}\right\} \\ &\cdot\left\{\begin{array}{c} u''(C_{1,s})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,s})\right] \\ +\frac{[1+(1-t_k)\cdot r]\cdot[1-t_w]}{E[v'(C_{2,s})]}\cdot\left\{\begin{array}{c} E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right] \\ -E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] \end{array}\right\} \end{array}\right\} \end{aligned}$$

We know from the income effect (34) that

$$\begin{aligned} -\frac{\partial K_s}{\partial Y}\cdot\frac{D}{u''(C_{1,s})} &= \frac{[1+(1-t_k)\cdot r]\cdot[1-t_w]}{E\left[v'(C_{2,s})\right]} \\ &\cdot\left\{\begin{array}{c} -E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] \\ +E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right] \end{array}\right\} \end{aligned}$$

$$\Downarrow$$

$$h_B \cdot x_{KB} = \left\{ \begin{array}{c} r \cdot E\left[v'(C_{2,s})\right] \\ +\left[1+(1-t_k)\cdot r\right] \cdot \left\{\left[r+\mu\right]\cdot K_s + r \cdot B_s\right\} \cdot E\left[v''(C_{2,s})\right] \end{array} \right\}$$
$$\cdot \left\{ \begin{array}{c} u''(C_{1,s}) + \left[1+(1-t_k)\cdot r\right]^2 \cdot E\left[v''(C_{2,s})\right] \\ -\frac{\partial K_s}{\partial Y} \cdot \frac{D}{u''(C_{1,s})} \end{array} \right\}$$

Now, we have that

$$h_K \cdot x_{BB} = \left\{ \begin{array}{c} (r+\mu)\cdot E\left[v'(C_{2,s})\right] \\ +\left\{ \begin{array}{c} \left[1+(1-t_k)\cdot r\right] \\ +\left[1-t_w\right] \cdot \frac{E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v'(C_{2,s})\right]} \end{array} \right\} \cdot \left(\left[r+\mu\right]\cdot K_s + r \cdot B_s\right) \cdot E\left[v''(C_{2,s})\right] \\ -\left[1-t_w\right] \cdot \left(\left[r+\mu\right]\cdot K_s + r\cdot B_s\right) \cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array} \right\}$$
$$\cdot \left\{u''(C_{1,s}) + \left[1+(1-t_k)\cdot r\right]^2 \cdot E\left[v''(C_{2,s})\right]\right\}$$

$$= \left\{u''(C_{1,s}) + \left[1+(1-t_k)\cdot r\right]^2 \cdot E\left[v''(C_{2,s})\right]\right\}$$
$$\cdot \left\{ \begin{array}{l} (r+\mu)\cdot E\left[v'(C_{2,s})\right] \\ +\left[1+(1-t_k)\cdot r\right] \cdot \left(\left[r+\mu\right]\cdot K_s + r\cdot B_s\right) \cdot E\left[v''(C_{2,s})\right] \\ +\left[1-t_w\right] \cdot \frac{\left[r+\mu\right]\cdot K_s + r\cdot B_s}{E\left[v'(C_{2,s})\right]} \cdot \left\{ \begin{array}{c} E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right] \cdot E\left[v''(C_{2,s})\right] \\ -E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \cdot E\left[v'(C_{2,s})\right] \end{array} \right\} \end{array} \right\}$$

$$\Downarrow$$

$$h_K \cdot x_{BB} = \left\{u''(C_{1,s}) + \left[1+(1-t_k)\cdot r\right]^2 \cdot E\left[v''(C_{2,s})\right]\right\}$$
$$\cdot \left\{ \begin{array}{c} (r+\mu)\cdot E\left[v'(C_{2,s})\right] \\ +\left[1+(1-t_k)\cdot r\right] \cdot \left(\left[r+\mu\right]\cdot K_s + r\cdot B_s\right) \cdot E\left[v''(C_{2,s})\right] \end{array} \right\}$$

$$-\left\{u''(C_{1,s}) + \left[1+(1-t_k)\cdot r\right]^2 \cdot E\left[v''(C_{2,s})\right]\right\}$$
$$\cdot \frac{\partial K_s}{\partial Y} \cdot \frac{D \cdot \left\{\left[r+\mu\right]\cdot K_s + r\cdot B_s\right\}}{\left[1+(1-t_k)\cdot r\right] \cdot u''(C_{1,s})}$$

Thus

$$
\begin{aligned}
K'(t_k) &= \frac{h_B \cdot x_{KB} - h_K \cdot x_{BB}}{-D} \\
&= -\frac{r \cdot E\left[v'(C_{2,s})\right] + [1 + (1-t_k) \cdot r] \cdot \left\{ \begin{array}{c} [r+\mu] \cdot K_s \\ +r \cdot B_s \end{array} \right\} \cdot E\left[v''(C_{2,s})\right]}{D} \\
&\quad \cdot \left\{ u''(C_{1,s}) + [1 + (1-t_k) \cdot r]^2 \cdot E\left[v''(C_{2,s})\right] \right\} \\
&\quad + \left\{ \begin{array}{c} r \cdot E\left[v'(C_{2,s})\right] \\ + [1 + (1-t_k) \cdot r] \cdot \{[r+\mu] \cdot K_s + r \cdot B_s\} \cdot E\left[v''(C_{2,s})\right] \end{array} \right\} \\
&\quad \cdot \frac{\partial K_s}{\partial Y} \cdot \frac{D}{u''(C_{1,s})} \\
&\quad + \frac{u''(C_{1,s}) + [1 + (1-t_k) \cdot r]^2 \cdot E\left[v''(C_{2,s})\right]}{D} \\
&\quad \cdot \left\{ \begin{array}{c} (r+\mu) \cdot E\left[v'(C_{2,s})\right] \\ + [1 + (1-t_k) \cdot r] \cdot ([r+\mu] \cdot K_s + r \cdot B_s) \cdot E\left[v''(C_{2,s})\right] \end{array} \right\} \\
&\quad - \left\{ u''(C_{1,s}) + [1 + (1-t_k) \cdot r]^2 \cdot E\left[v''(C_{2,s})\right] \right\} \\
&\quad \cdot \frac{\partial K_s}{\partial Y} \cdot \frac{[r+\mu] \cdot K_s + r \cdot B_s}{[1 + (1-t_k) \cdot r] \cdot u''(C_{1,s})}
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
K'(t_k) &= \frac{u''(C_{1,s}) + [1 + (1-t_k) \cdot r]^2 \cdot E\left[v''(C_{2,s})\right]}{D} \cdot \left\{ \mu \cdot E\left[v'(C_{2,s})\right] \right\} \\
&\quad + \frac{\partial K_s}{\partial Y} \cdot \frac{1}{[1 + (1-t_k) \cdot r] \cdot u''(C_{1,s})} \\
&\quad \cdot \left\{ \begin{array}{l} -[r+\mu] \cdot K_s \cdot u''(C_{1,s}) - r \cdot B_s \cdot u''(C_{1,s}) \\ -[r+\mu] \cdot K_s \cdot [1 + (1-t_k) \cdot r]^2 \cdot E\left[v''(C_{2,s})\right] \\ -r \cdot B_s \cdot [1 + (1-t_k) \cdot r]^2 \cdot E\left[v''(C_{2,s})\right] \\ +[1 + (1-t_k) \cdot r] \cdot r \cdot E\left[v'(C_{2,s})\right] \\ +[1 + (1-t_k) \cdot r]^2 \cdot [r+\mu] \cdot K_s \cdot E\left[v''(C_{2,s})\right] \\ +[1 + (1-t_k) \cdot r]^2 \cdot r \cdot B_s \cdot E\left[v''(C_{2,s})\right] \end{array} \right\}
\end{aligned}
$$

$$\Downarrow \tag{44}$$

$$
\begin{aligned}
K'(t_k) &= \frac{\partial K_s}{\partial Y} \cdot \left\{ r \cdot \frac{E\left[v'(C_{2,s})\right]}{u''(C_{1,s})} - \left[\frac{[r+\mu] \cdot K_s + r \cdot B_s}{1 + (1-t_k) \cdot r} \right] \right\} \\
&\quad + \left\{ u''(C_{1,s}) + [1 + (1-t_k) \cdot r]^2 \cdot E\left[v''(C_{2,s})\right] \right\} \cdot \frac{\mu \cdot E\left[v'(C_{2,s})\right]}{D} \\
&< 0
\end{aligned}
$$

10.3 The widely held corporation

The individual's maximization problem when organizing as a widely held organization is given by

$$\max_{K_l,B_l} EU_l = u(C_{1,l}) + E\left[v\left(C_{2,l}\right)\right],$$

where $C_{1,l}$ and $C_{2,l}$ are given by equations (15):

$$C_{1,l} = Y - \beta \cdot K_l - B_l$$

and (16):

$$C_{2,l} = \beta \cdot [1 - t_k] \cdot [F(K_l) - \widetilde{\gamma} \cdot K_l] + \beta \cdot K_l + [1 + (1 - t_k) \cdot r] \cdot B_l.$$

The resulting first order conditions are given by equation (17):

$$FOC_{B_l} : -u'(C_{1,l}) + [1 + (1 - t_k) \cdot r] \cdot E\left[v'(C_{2,l})\right] = 0$$

and (18):

$$FOC_{K_l} : -\beta \cdot u'(C_{1,l}) + E\left[v'(C_{2,l}) \cdot \left\{\beta \cdot [1 - t_k] \cdot [F_{K_l} - \widetilde{\gamma}] + \frac{2}{3}\right\}\right] = 0$$

Combining the first order conditions yields the optimal investment condition (19):

$$F_{K_l} = r + \frac{E\left[v'(C_2) \cdot \widetilde{\gamma}\right]}{E\left[v'(C_2)\right]}$$

Define

$$\begin{aligned} G &\equiv [1 - t_k] \cdot F_{K_l} + 1 \\ &\Downarrow \text{ by (19)} \\ G &= [1 + (1 - t_k) \cdot r] + \frac{E\left[v'(C_2) \cdot \widetilde{\gamma}\right]}{E\left[v'(C_2)\right]} \end{aligned}$$

The conditions for the existence of a maximum of the widely held corporation:

$$EU_{BB} = u''(C_{1,l}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,l})\right] < 0,$$

$$EU_{KK} = \beta \cdot \left\{ \begin{array}{c} [1 - t_k] \cdot F_{K_l K_l} \cdot E\left[v'(C_{2,l})\right] + \beta \cdot u''(C_{1,l}) \\ +\beta \cdot G^2 \cdot E\left[v''(C_{2,l})\right] - 2 \cdot \beta \cdot G \cdot [1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\ +\beta \cdot [1 - t_k]^2 \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \end{array} \right\} < 0$$

$$EU_{BK} = \beta \cdot u''(C_{1,l}) + \beta \cdot [1 + (1 - t_k) \cdot r] \cdot \left\{ \begin{array}{c} G \cdot E\left[v''(C_{2,l})\right] \\ -[1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\}.$$

Thus

$$EU_{BB} \cdot EU_{KK} - (EU_{BK})^2 \equiv F > 0$$

$$(45)$$

$$\begin{aligned} F &= \left\{u''(C_{1,l}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,l})\right]\right\} \cdot \beta \\ &\quad \cdot \left\{ \begin{array}{c} [1 - t_k] \cdot F_{K_l K_l} \cdot E\left[v'(C_{2,l})\right] + \beta \cdot u''(C_{1,l}) \\ +\beta \cdot G^2 \cdot E\left[v''(C_{2,l})\right] - 2 \cdot \beta \cdot G \cdot [1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\ +\beta \cdot [1 - t_k]^2 \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \end{array} \right\} \\ &\quad -\beta^2 \cdot \left\{ u''(C_{1,l}) + [1 + (1 - t_k) \cdot r] \cdot \left\{ \begin{array}{c} G \cdot E\left[v''(C_{2,l})\right] \\ -[1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \right\}^2 \end{aligned}$$

10.4 The income effect in the widely held corporation.

Differentiate (17) with respect to initial income Y :

$$\begin{aligned} &-u''(C_{1,l}) \cdot \{1 - \beta \cdot K_l'(Y) - B_l'(Y)\} \\ &+ [1 + (1 - t_k) \cdot r] \cdot \left\{ \begin{array}{c} \beta \cdot G \cdot E\left[v''(C_{2,l})\right] \cdot K_l'(Y) \\ -\beta \cdot [1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot K_l'(Y) \\ + [1 + (1 - t_k) \cdot r] \cdot E\left[v''(C_{2,l})\right] \cdot B_l'(Y) \end{array} \right\} \\ = &\ 0 \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} &K_l'(Y) \cdot \beta \cdot \left\{ u''(C_{1,l}) + [1 + (1 - t_k) \cdot r] \cdot \left\{ \begin{array}{c} G \cdot E\left[v''(C_{2,l})\right] \\ -[1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \right\} \\ &+B_l'(Y) \cdot \left\{u''(C_{1,l}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,l})\right]\right\} \\ = &\ u''(C_{1,l}) \end{aligned}$$

Define this as

$$K_l'(Y) \cdot o_{BK} + B_l'(Y) \cdot o_{BB} = å_B$$

Next, equation (18) is differentiated:

$$
\begin{aligned}
& -\beta \cdot u''(C_{1,l}) \cdot \{1 - \beta \cdot K_l'(Y) - B_l'(Y)\} \\
& +\beta \cdot [1 - t_k] \cdot F_{K_l K_l} \cdot E\left[v'(C_{2,l})\right] \cdot K_l'(Y) \\
& +\beta \cdot G \cdot \left\{ \begin{array}{c} \beta \cdot G \cdot E\left[v''(C_{2,l})\right] \cdot K_l'(Y) - \\ \beta \cdot [1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot K_l'(Y) \\ + [1 + (1 - t_k) \cdot r] \cdot E\left[v''(C_{2,l})\right] \cdot B_l'(Y) \end{array} \right\} \\
& -\beta \cdot [1 - t_k] \cdot \left\{ \begin{array}{c} \beta \cdot G \cdot E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot K_l'(Y) \\ -\beta \cdot [1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \cdot K_l'(Y) \\ + [1 + (1 - t_k) \cdot r] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot B_l'(Y) \end{array} \right\} \\
= {} & 0
\end{aligned}
$$

$\Downarrow$

$$
\begin{aligned}
& K_l'(Y) \cdot \beta \cdot \left\{ \begin{array}{c} \beta \cdot u''(C_{1,l}) + \beta \cdot [1 - t_k] \cdot F_{K_l K_l} \cdot E\left[v'(C_{2,l})\right] \\ +\beta \cdot G^2 \cdot E\left[v''(C_{2,l})\right] \\ -2 \cdot \beta \cdot G \cdot [1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\ +\beta \cdot [1 - t_k]^2 \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \end{array} \right\} \\
& +B_l'(Y) \cdot \beta \cdot \left\{ u''(C_{1,l}) + [1 + (1 - t_k) \cdot r] \cdot \left\{ \begin{array}{c} G \cdot E\left[v''(C_{2,l})\right] \\ - [1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \right\} \\
= {} & \beta \cdot u''(C_{1,l})
\end{aligned}
$$

Define this as

$$K_l'(Y) \cdot o_{KK} + B_l'(Y) \cdot o_{KB} = å_K$$

By Cramer's rule,

$$K_l'(Y) = \frac{å_B \cdot o_{KB} - å_K \cdot o_{BB}}{o_{BK} \cdot o_{KB} - o_{KK} \cdot o_{BB}} = \frac{å_K \cdot o_{BB} - å_B \cdot o_{KB}}{F}$$

$$
\begin{aligned}
& K_l'(Y) \\
= {} & \frac{\beta \cdot u''(C_{1,l})}{F} \cdot \left\{ \begin{array}{l} \left\{u''(C_{1,l}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,l})\right]\right\} \\ -u''(C_{1,l}) \\ - [1 + (1 - t_k) \cdot r] \cdot \left\{ \begin{array}{c} G \cdot E\left[v''(C_{2,l})\right] \\ - [1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \end{array} \right\}
\end{aligned}
$$

$$
\Downarrow \text{ by the definition of } G
$$

$$
\begin{aligned}
K_l'(Y) &= \frac{\beta \cdot u''(C_{1,l})}{F} \\
&\cdot \left\{ \begin{array}{l} [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,l})\right] \\ -[1+(1-t_k)\cdot r] \\ \cdot \left\{ \begin{array}{c} [1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\right] \\ +(1-t_k)\cdot \frac{E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]}{E\left[v'(C_{2,l})\right]} \cdot E\left[v''(C_{2,l})\right] \\ -[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \end{array} \right\} \end{array} \right\}
\end{aligned}
$$

$$
\begin{aligned}
&\Downarrow \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (46) \\
\frac{\partial K_l}{\partial Y} &= -\beta \cdot \frac{u''(C_{1,l})\cdot[1+(1-t_k)\cdot r]\cdot[1-t_k]}{F\cdot E\left[v'(C_{2,l})\right]} \\
&\cdot \left[\begin{array}{c} E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\right] \\ -E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right] \end{array} \right]
\end{aligned}
$$

$$
\begin{aligned}
&\Downarrow \\
\frac{\partial K_l}{\partial Y} &> 0 \quad \text{if}
\end{aligned}
$$

$$
\begin{aligned}
1) &: \quad v'''(C_{2,l}) > 0 \\
&\quad\ \ \textit{and} \\
2) &: \quad cov\left[v'(C_{2,l}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\right] > cov\left[v''(C_{2,l}),\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right].
\end{aligned}
$$

10.4.1 The effect on real capital investments from increased tax on capital income.

Differentiate (17) with respect to t_k :

$$
\begin{aligned}
& K_l'(t_k)\cdot\beta\cdot\left\{u''(C_{1,l})+[1+(1-t_k)\cdot r]\cdot\left\{\begin{array}{c}+G\cdot E\left[v''(C_{2,l})\right]\\ -[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\}\right\}\\
& +B_l'(t_k)\cdot\left\{u''(C_{1,l})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,l})\right]\right\}\\
= \;& r\cdot E\left[v'(C_{2,l})\right]\\
& +[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\right]\cdot[\beta\cdot F(K_l)+r\cdot B_l]\\
& -\beta\cdot K_l\cdot[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]
\end{aligned}
$$

Define this as

$$K_l'(t_k)\cdot æ_{BK}+B_l'(t_k)\cdot æ_{BB}=j_B$$

Next, condition(17) is differentiated:

$$
\begin{aligned}
& -\beta\cdot u''(C_{1,l})\cdot\{-\beta\cdot K_l'(t_k)-B_l'(t_k)\}\\
& -\beta\cdot F_{K_l}\cdot E\left[v'(C_{2,l})\right]\\
& +\beta\cdot[1-t_k]\cdot F_{K_lK_l}\cdot E\left[v'(C_{2,l})\right]\cdot K_l'(t_k)\\
& +\beta\cdot G\cdot\left\{\begin{array}{c}-\beta\cdot F(K_l)\cdot E\left[v''(C_{2,l})\right]+\beta\cdot K_l\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\\ +\beta\cdot G\cdot E\left[v''(C_{2,l})\right]\cdot K_l'(t_k)\\ -\beta\cdot[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot K_l'(t_k)-r\cdot B_l\cdot E\left[v''(C_{2,l})\right]\\ +[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\right]\cdot B_l'(t_k)\end{array}\right\}\\
& +\beta\cdot E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\\
& -\beta\cdot[1-t_k]\left\{\begin{array}{c}-\beta\cdot F(K_l)\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\\ +\beta\cdot K_l\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\\ +\beta\cdot G\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot K_l'(t_k)\\ -\beta\cdot[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot K_l'(t_k)\\ -r\cdot B_l\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\\ +[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot B_l'(t_k)\end{array}\right\}\\
= \;& 0
\end{aligned}
$$

$\Downarrow$

$$K_l'(t_k)\cdot\beta\cdot\left\{\begin{array}{c}\beta\cdot u''(C_{1,l})+\beta\cdot G^2\cdot E\left[v''(C_{2,l})\right]\\ -2\cdot\beta\cdot G\cdot[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\\ +\beta\cdot[1-t_k]^2\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]+[1-t_k]\cdot F_{K_lK_l}\cdot E\left[v'(C_{2,l})\right]\end{array}\right\}$$

$$+B_l'(t_k)\cdot\beta\cdot\left\{u''(C_{1,l})+[1+(1-t_k)\cdot r]\cdot\left\{\begin{array}{c}G\cdot E\left[v''(C_{2,l})\right]\\ -[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\}\right\}$$

$$=\beta\cdot\left\{\begin{array}{c}F_{K_l}\cdot E\left[v'(C_{2,l})\right]-E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]-\beta\cdot G\cdot K_l\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\\ +\{\beta\cdot F(K_l)+r\cdot B_l\}\cdot\left\{\begin{array}{c}G\cdot E\left[v''(C_{2,l})\right]\\ -[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\}\\ +\beta\cdot[1-t_k]\cdot K_l\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\end{array}\right\}$$

Define this as

$$K_l'(t_k)\cdot æ_{KK}+B_l'(t_k)\cdot æ_{KB}=j_K$$

By Cramer's rule:

$$K_l'(t_k)=\frac{j_B\cdot æ_{KB}-j_K\cdot æ_{BB}}{æ_{BK}\cdot æ_{KB}-æ_{KK}\cdot æ_{BB}}=\frac{j_K\cdot æ_{BB}-j_B\cdot æ_{KB}}{F}$$

$$K_l'(t_k)=\frac{\beta}{F}\cdot\left\{\begin{array}{c}r\cdot E\left[v'(C_{2,l})\right]\\ -\beta\cdot[1+[1-t_k]\cdot r]\cdot K_l\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\\ +\{\beta\cdot F(K_l)+r\cdot B_l\}\cdot E\left[v''(C_{2,l})\right]\cdot[1+(1-t_k)\cdot r]\\ +\frac{1-t_k}{E\left[v'(C_{2,l})\right]}\cdot\{\beta\cdot F(K_l)+r\cdot B_l\}\\ \cdot\left\{\begin{array}{c}E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\right]\\ -E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right]\end{array}\right\}\\ +\beta\cdot\frac{[1-t_k]\cdot K_l}{E\left[v'(C_{2,l})\right]}\cdot\left\{\begin{array}{c}E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right]\\ -E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\}\end{array}\right\}$$

$$\cdot\left\{u''(C_{1,l})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,l})\right]\right\}$$

$$-\left\{\begin{array}{l}r\cdot E\left[v'(C_{2,l})\right]\\ +[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\right]\cdot\{\beta\cdot F(K_l)+r\cdot B_l\}\\ -\beta\cdot K_l\cdot[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\}$$

$$\cdot\frac{\beta}{F}\cdot\left\{\begin{array}{c}u''(C_{1,l})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,l})\right]\\ +\frac{[1+(1-t_k)\cdot r]\cdot[1-t_k]}{E\left[v'(C_{2,l})\right]}\cdot\left\{\begin{array}{c}E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\right]\\ -E\left[v'(C_{2,l})\right]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\}\end{array}\right\}$$

$$\Downarrow$$

$$K_l'(t_k)$$

$$= \frac{\beta}{F}\cdot\left\{\begin{array}{c} -\beta\cdot[1+[1-t_k]\cdot r]\cdot K_l\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \\ +\{\beta\cdot F(K_l)+r\cdot B_l\}\cdot E\left[v''(C_{2,l})\right]\cdot[1+(1-t_k)\cdot r] \\ +\beta\cdot\frac{[1-t_k]\cdot K_l}{E\left[v'(C_{2,l})\right]}\cdot\left\{\begin{array}{c} E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right] \\ -E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\} \\ -\left\{\begin{array}{l} [1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\right]\cdot\{\beta\cdot F(K_l)+r\cdot B_l\} \\ -\beta\cdot K_l\cdot[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\}\end{array}\right\}$$

$$\cdot\left\{u''(C_{1,l})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,l})\right]\right\}$$

$$+\left\{\begin{array}{l} \{\beta\cdot F(K_l)+r\cdot B_l\}\cdot\frac{u''(C_{1,l})}{1+(1-t_k)\cdot r} \\ -r\cdot E\left[v'(C_{2,l})\right] \\ +\beta\cdot K_l\cdot[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\}$$

$$\cdot\frac{\beta}{F}\cdot\frac{[1+(1-t_k)\cdot r]\cdot[1-t_k]}{E\left[v'(C_{2,l})\right]}\cdot\left\{\begin{array}{c} E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\right] \\ -E\left[v'(C_{2,l})\right]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\}$$

Apply the definition of the income effect in equation (46):

$$\begin{aligned} K_l'(t_k) &= -\left\{\begin{array}{l} \frac{\beta\cdot F(K_l)+r\cdot B_l}{1+(1-t_k)\cdot r}-r\cdot\frac{E\left[v'(C_{2,l})\right]}{u''(C_{1,l})} \\ +\beta\cdot K_l\cdot[1+(1-t_k)\cdot r]\cdot\frac{E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]}{u''(C_{1,l})}\end{array}\right\}\cdot\frac{\partial K_l}{\partial Y} \\ &\quad+\frac{\beta^2\cdot[1-t_k]\cdot K_l}{F\cdot E\left[v'(C_{2,l})\right]}\cdot\left\{u''(C_{1,l})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,l})\right]\right\} \\ &\quad\cdot\left\{\begin{array}{c} E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right] \\ -E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\}\end{aligned}$$

As the expression in the first parenthesis is positive, the sign of the total income effect depends on the sign of $\frac{\partial K_l}{\partial Y}$. We also know that $\frac{\beta^2\cdot[1-t_k]\cdot K_l}{F\cdot E\left[v'(C_{2,l})\right]}>0$, and that $\left\{u''(C_{1,l})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,l})\right]\right\}<0$. Thus the sign of the substitution effect depends on the sign of the expression in the last parenthesis, which can be rewritten as $\left\{cov\left[v''(C_{2,l})\cdot\widetilde{\gamma},\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right]-cov\left[v'(C_{2,l}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\right\}$. The signs of both the total income effect and the substitution effect depend on the individual's risk aversion.

$$\begin{aligned}
K_l'(t_k) &> 0 \text{ if} \\
1) &: v'''(C_{2,l}) > 0 \\
&\quad \text{and} \\
2) &: cov\left[v''(C_{2,l}) \cdot \widetilde{\gamma}, \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,l})\right] < cov\left[v'(C_{2,l}), \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\
&\quad \text{and} \\
3) &: \text{The substitution effect dominates the total income effect.}
\end{aligned}$$

This means that

$$\begin{aligned}
K'(t_l) &> 0 \text{ if} \\
1) &: v'''(C_{2,l}) < 0 \\
&\quad \text{and} \\
2) &: \text{The total income effect dominates the substitution effect.}
\end{aligned}$$

10.5 When to incorporate?

$$\widehat{EU}_l - \widehat{EU}_s = u(\widehat{C}_{1,l}) + E\left[v(\widehat{C}_{2,l})\right] - u(\widehat{C}_{1,s}) - E\left[v(\widehat{C}_{2,s})\right],$$

where

$$\begin{aligned}
\widehat{C}_{1,l} &= Y - \beta \cdot \widehat{K}_l - \widehat{B}_l \\
\widehat{C}_{2,l} &= \beta \cdot [1 - t_k] \cdot \left[F(\widehat{K}_l) - \widetilde{\gamma} \cdot \widehat{K}_l\right] + \beta \cdot \widehat{K}_l + [1 + (1 - t_k) \cdot r] \cdot \widehat{B}_l \\
\widehat{C}_{1,s} &= Y - \widehat{K}_s - \widehat{B}_s \\
\widehat{C}_{2,s} &= [1 - t_w] \cdot \left[F(\widehat{K}_s) - \widetilde{\gamma} \cdot \widehat{K}_s\right] + \{1 + [t_w - t_k] \cdot [r + \mu]\} \cdot \widehat{K}_s \\
&\quad + [1 + (1 - t_k) \cdot r] \cdot \widehat{B}_s.
\end{aligned}$$

Effect of increase in the real interest rate

$$\begin{aligned}
&\frac{\partial\left(\widehat{EU}_l - \widehat{EU}_s\right)}{\partial r} \\
=\ & \frac{\partial E\left[v(\widehat{C}_{2,l})\right]}{\partial r} - \frac{\partial E\left[v(\widehat{C}_{2,s})\right]}{\partial r} \\
=\ & (1-t_k)\cdot \widehat{B}_l \cdot E\left[v'(\widehat{C}_{2,l})\right] \\
& -\left\{[t_w - t_k]\cdot \widehat{K}_s + (1-t_k)\cdot \widehat{B}_s\right\}\cdot E\left[v'(\widehat{C}_{2,s})\right]
\end{aligned}$$

Effect of increase in the amount of shares allowed held by the active owner in the widely held corporation:

$$\begin{aligned}
&\frac{\partial\left(\widehat{EU}_l - \widehat{EU}_s\right)}{\partial \beta} \\
=\ & \frac{\partial u(\widehat{C}_{1,l})}{\partial \beta} + \frac{\partial E\left[v(\widehat{C}_{2,l})\right]}{\partial \beta} \\
=\ & -\widehat{K}_l \cdot u'(\widehat{C}_{1,l}) + E\left[\left\{[1-t_k]\cdot\left[F(\widehat{K}_l) - \widetilde{\gamma}\cdot \widehat{K}_l\right] + \widehat{K}_l\right\}\cdot v'(\widehat{C}_{2,l})\right] \\
=\ & \widehat{K}_l \cdot E\left[v'(\widehat{C}_{2,l})\right]\cdot\left\{1 - \frac{u'(\widehat{C}_{1,l})}{E\left[v'(\widehat{C}_{2,l})\right]}\right\} \\
& +\left\{[1-t_k]\cdot F(\widehat{K}_l)\right\}\cdot E\left[v'(\widehat{C}_{2,l})\right] - [1-t_k]\cdot \widehat{K}_l \cdot E\left[v'(\widehat{C}_{2,l})\cdot\widetilde{\gamma}\right] \\
& \Downarrow \ \textit{from } (17)\text{: } \frac{u'(C_{1,l})}{E\left[v'(C_{2,l})\right]} = [1 + (1-t_k)\cdot r] \\
=\ & [1-t_k]\cdot\left\{F(\widehat{K}_l) - r\cdot \widehat{K}_l\right\}\cdot E\left[v'(\widehat{C}_{2,l})\right] \\
& -[1-t_k]\cdot \widehat{K}_l \cdot E\left[v'(\widehat{C}_{2,l})\right]\cdot\left\{\frac{cov\left[v'(\widehat{C}_{2,l}),\widetilde{\gamma}\right]}{E\left[v'(\widehat{C}_{2,l})\right]} + \delta\right\} \\
=\ & [1-t_k]\cdot\left\{F(\widehat{K}_l) - \left(r + \widehat{\lambda}_l + \delta\right)\cdot \widehat{K}_l\right\}\cdot E\left[v'(\widehat{C}_{2,l})\right]
\end{aligned}$$

Effect of increased risk compensation rate under the split model:

$$\frac{\partial\left(\widehat{EU}_l - \widehat{EU}_s\right)}{\partial \mu} = -\frac{\partial E\left[v(\widehat{C}_{2,s})\right]}{\partial \mu} = -\left[t_w - t_k\right]\cdot \widehat{K}_s \cdot E\left[v'(\widehat{C}_{2,s})\right]$$

Effect of increased tax on capital income:

$$
\begin{aligned}
&\frac{\partial\left(\widehat{EU}_l - \widehat{EU}_s\right)}{\partial t_k} \\
= \; & \frac{\partial E\left[v(\widehat{C}_{2,l})\right]}{\partial t_k} - \frac{\partial E\left[v(\widehat{C}_{2,s})\right]}{\partial t_k} \\
= \; & E\left[\left\{-\beta\cdot\left(F(\widehat{K}_l) - \widetilde{\gamma}\cdot\widehat{K}_l\right) - r\cdot\widehat{B}_l\right\}\cdot v'(\widehat{C}_{2,l})\right] \\
& -E\left[\left\{-(r+\mu)\cdot\widehat{K}_s - r\cdot\widehat{B}_s\right\}\cdot v'(\widehat{C}_{2,s})\right] \\
= \; & -\left\{\beta\cdot F(\widehat{K}_l) + r\cdot\widehat{B}_l\right\}\cdot E\left[v'(\widehat{C}_{2,l})\right] + \beta\cdot\widehat{K}_l\cdot E\left[v'(\widehat{C}_{2,l})\cdot\widetilde{\gamma}\right] \\
& +\left\{(r+\mu)\cdot\widehat{K}_s + r\cdot\widehat{B}_s\right\}\cdot E\left[v'(\widehat{C}_{2,s})\right] \\
= \; & -\left\{\beta\cdot\left[F(\widehat{K}_l) - \delta - \widehat{\lambda}_l\right] + r\cdot\widehat{B}_l\right\}\cdot E\left[v'(\widehat{C}_{2,l})\right] \\
& +\left\{(r+\mu)\cdot\widehat{K}_s + r\cdot\widehat{B}_s\right\}\cdot E\left[v'(\widehat{C}_{2,s})\right]
\end{aligned}
$$

Effect of increased tax on labor income:

$$
\begin{aligned}
&\frac{\partial\left(\widehat{EU}_l - \widehat{EU}_s\right)}{\partial t_w} \\
= \; & -\frac{\partial E\left[v(\widehat{C}_{2,s})\right]}{\partial t_w} \\
= \; & -E\left[\left\{-F(\widehat{K}_s) + \widetilde{\gamma}\cdot\widehat{K}_s + [r+\mu]\cdot\widehat{K}_s\right\}\cdot v(\widehat{C}_{2,s})\right] \\
= \; & \left\{F(\widehat{K}_s) - [r+\mu]\cdot\widehat{K}_s\right\}\cdot E\left[v(\widehat{C}_{2,s})\right] - \widehat{K}_s\cdot E\left[v(\widehat{C}_{2,s})\cdot\widetilde{\gamma}\right] \\
= \; & \left\{F(\widehat{K}_s) - [r+\mu]\cdot\widehat{K}_s\right\}\cdot E\left[v(\widehat{C}_{2,s})\right] \\
& -\widehat{K}_s\cdot\left\{\frac{cov\left[v(\widehat{C}_{2,s}),\widetilde{\gamma}\right]}{E\left[v(\widehat{C}_{2,s})\right]} + E\left[\widetilde{\gamma}\right]\right\}\cdot E\left[v(\widehat{C}_{2,s})\right] \\
= \; & \left\{F(\widehat{K}_s) - \left[r+\mu+\delta+\widehat{\lambda}_s\right]\cdot\widehat{K}_s\right\}\cdot E\left[v(\widehat{C}_{2,s})\right]
\end{aligned}
$$

Does the Tax System Encourage Too Much Education?

Annette Alstadsæter*

This paper provides an efficiency argument in favor of progressive labor income taxation. When the consumer faces a trade-off between investments in financial and human capital, a proportional comprehensive income tax tends to discriminate in favor of human capital investments. This effect is strengthened when education no longer is a pure investment, but also holds a direct consumption value.

A comprehensive proportional income tax works as a tax subsidy on human capital investments, and it reduces the price of education as a consumption good. By introducing a progressive labor income tax, the efficiency distortion in the capital market may be partly neutralised.
(JEL: H 21, H 24)

1. Introduction

During the 1980s and 1990s many industrial countries implemented tax reforms that involved broadening the tax base and reducing marginal tax rates. In some countries the top marginal tax rates on labor income dropped from 70–80% to 40–50%, and this meant a sharp reduction in the progressivity of the tax system. These reforms were in line with the Schanz-Haig-Simons principle of comprehensive income taxation with as broad a tax base as possible. Textbook public economics states a trade-off between redistribution and efficiency; the government can reduce the degree of inequality but only at the expense of a larger dead-weight loss; see Stiglitz (1986). This paper suggests the opposite: an efficiency argument in favor of progressive labor income taxation and redistribution. When the individual faces the choice be-

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tween investing in financial or human capital, a proportional comprehensive income tax leads to an overinvestment in human capital. A progressive labor income tax is hence required to correct for the distortions in the investment market. This effect is strengthened when education is not only viewed as an investment, but holds a consumption value as well.

It has long been acknowledged in the literature that the tax treatment of the returns to financial investments can affect the attractiveness of human capital investments, and that the labor income tax system affects the equilibrium level of human capital in the society. Nielsen and Sørensen (1997) investigate the Scandinavian system of dual income taxation, with proportional tax on capital income and progressive tax on labor income. The progressivity of the labor income tax serves to reduce the private return to human capital investment, thereby offsetting the tendency of a proportional comprehensive income tax to discriminate in favor of such investments. Others have come to the opposite conclusion, namely that a proportional tax on labor income discriminates against human capital investments and reduces the educational level in the society below the optimal (Nerlove et al., 1993; Heckman, 1976).

In all of these models, the only reason for the individual to get education, is higher expected wages. This is a plausible explanation for countries like the U.S., where wage differentials are large and the wage return to education is substantial (see Gottschalk, 1997). The Scandinavian countries, and especially Norway, have small wage differentials, and a modest return to education (see Hægeland et al., 1999). According to Moen and Semmingsen (1996), several kinds of higher education even have negative wage return in Norway, compared with having only high school. Still, individuals do choose to get higher education, and the educational levels in the Scandinavian countries are among the highest in the OECD. Hence, there has to be other and possibly more important motives besides higher expected wages behind the individual's decision to invest in human capital.

Additional motivation for higher education is the possibility to enjoy a student life, to learn new things, and to pursue own interests. Higher education increases the chances of getting an interesting job. Flexibility between jobs increases through education, and chances are that if he becomes unemployed, a person with higher education can more easily find a new job. Bishop (1994) found that for most of the OECD-area the larger part of the unemployed in the 1980s had no higher education. The social status connected with higher education is a factor which is not insignificant for the educational decisions. We cannot rule out that expectations and norms in society have an influence on whether or not a young person chooses to get education. In fact, Hægeland et al. (1999) find that the parental educational level has a positive influence on the length of the education their children choose to acquire.

All of these non-wage related motives behind the educational choice may be summarized as the consumption value of education. Education may be viewed as a consumption good, for which the individual is willing to pay. He accepts a lower wage return to human capital investments in order to receive the direct utility gain from education. In this case the consumption value of education is positive. It is also possible that an individual might consider education as a consumption bad. The individual then has to be compensated for his disutility of education in order to invest in human capital, and the consumption value of education is negative.

Little work has been done to estimate the consumption value of education empirically. Lazear (1977) uses data from young men in the U.S. in the period of 1966–1969, and he finds that lower education is in fact a consumption bad. People acquire education below the optimal level, and they are willing to forego wealth in order to avoid the consumption of education. Lazear also finds that at least for individuals with higher levels of education (MA's and PhD's) schooling is considered a good. He does not include taxes in his model.

Few attempts have been made to study the impact of the tax system on the skill formation in a society when education holds a consumption value to the individual. Heckman et al. (1998) mention in a verbal analysis that income tax may have an effect on human capital formation. They argue that proportional taxation is no longer neutral when nonpecuniary costs or benefits are present. If the net financial benefit before taxes is positive, an increased tax rate reduces the investment in human capital. This effect is even stronger with a progressive income tax. Judd (1999) mentions the consumption value of education in his tax analysis, but it is not included in the general optimal tax analysis.

The present paper expands the model of Nielsen and Sørensen (1997) by including the consumption value of education as a motivation behind the educational decision. Optimal taxes are studied in two cases: First when education is a pure investment, and then when education also holds a consumption value. Section 2 presents the general framework and analyzes consumer behavior in the two models. In section 3 the optimal tax analysis is carried out for both cases, and the results are compared. Section 4 summarizes the results.

2. The Model

2.1. The Individual

A representative consumer lives for two periods. These two periods need not be of the same length. In each period the individual devotes a fixed amount of time to leisure. Consider this leisure to be the time necessary for the individual to eat, rest, and enjoy some social life. Let T_1 and T_2 be the remaining time in each period. The individual has to decide how much of the remaining time in the first period to spend on acquiring education E or working, H_1. The remaining time in the second period is spent working, and hence H_2 is given. The individual's time budget is given by:

$$\text{Period 1}: \quad T_1 = H_1 + E, \tag{1}$$

$$\text{Period 2}: \quad T_2 = \overline{H}_2. \tag{2}$$

Individuals leave no bequests, and there are no government transfers. Thus, labor is the only source of income. The individual has to earn income equivalent to the value of his consumption, and he has spent all of his income by the time he dies. Let w be the real wage which is the basic wage for all workers. This basic labor income is taxed at the rate t_l. In the first period all workers are unskilled and receive the same basic wage if deciding to work. First period consumption is given by

$$C_1 = w(1 - t_l)(T_1 - E) - S, \tag{3}$$

with S being savings in the financial market made in the first period. Savings may be positive or negative, and we assume that there are no liquidity constraints. This implies that the individual may borrow in the financial market to finance his first period consumption.

Education is another kind of savings. If the individual gets education in the first period, his wage in the second period increases. Let the function $g(E)$ represent the return to education, assumed to be increasing and concave in E. The more time he spends on getting education in the first period, the higher his second period wage. We also assume that the individual always gains from acquiring education, i.e. $g(E) > 1$ and $g(0) = 1$. This rules out the possibility that an unskilled worker may get better paid than a skilled worker. The consumer does not face any direct costs of education, and hence the only cost of education is foregone labor income in the first period.

The second period wage is given by $wg(E)$. If the individual chooses not to get any education in the first period, it follows from the definition of the return function to education that he earns the basic wage w for unskilled workers in the second period. On the other hand, if the individual did get some education in the first period, he earns a higher wage in the second period, and the wage increase depends positively on the amount of education. This wage return to education, $[g(E) - 1]$, is taxed at the rate t_h. The basic wage, the wage that the individual receives independently of educational level, is taxed at the rate t_l. If $t_l = t_h$, tax on labor income is proportional, and if

$t_l < t_h$, tax on labor income is progressive[1]. Return to financial investments, the real interest rate r, faces the proportional, exogenously given tax rate τ. This corresponds to a dual system of income taxation, where tax rates on labor income and capital income are set separately and independently of each other.

First period consumption is numeraire, and its price is hence set to one. Second period consumption is given by

$$C_2 = [1 + (1 - \tau)r]S + (1 - t_l)w\overline{H}_2 + (1 - t_h)w[g(E) - 1]\overline{H}_2. \tag{4}$$

Only the wage return to education is taxed at the rate t_h.

By introducing a new notation, we can simplify the representation of consumer behavior both in the case with and in the case without taxes. Define the marginal after tax real wage for an unskilled worker as w_l; the net basic wage

$$w_l \equiv (1 - t_l)w. \tag{5}$$

The marginal after tax real wage for an educated worker is defined as w_h,

$$w_h \equiv (1 - t_h)w. \tag{6}$$

Using this notation, the net wage return to education is given by $w_h[g(E) - 1]\overline{H}_2$.

Define p as the relative price of second period consumption, measured in units of first period consumption,

$$p \equiv \frac{1}{1 + (1 - \tau)r}. \tag{7}$$

The individual's life time budget constraint is found by combining equations (3) and (4):

$$C_1 + pC_2 = w_l[(T_1 - E) + p\overline{H}_2] + pw_h[g(E) - 1]\overline{H}_2. \tag{8}$$

Education is an investment that yields higher wages in the second period. But it might also generate non-pecuniary gains, and it is then viewed as a commodity for which the individual is willing to pay. Hence, education is included in the individual's utility function, together with first and second period consumption. The individual's preferences are represented by the utility function

$$U = U(C_1, C_2, E). \tag{9}$$

All goods are assumed to be normal. The marginal utility of first and second period consumption, U_1 and U_2, are both positive. The marginal utility of

1 If $t_l > t_h$, the tax system is regressive. This case is not politically feasible in egalitarian societies with great focus on redistribution, as in the Scandinavian countries, and this possibility is hence not considered in this paper.

education, U_E, represents the consumption value of education. If U_E is positive, education generates a direct utility gain to the consumer, and holds an intrinsic value. The individual considers education to be a good, for which he is willing to pay. If U_E is negative, education is a bad, and the individual has to be compensated in order to choose education. With U_E being zero, we have the exact situation of the model of Nielsen and Sørensen (1997), where education is a pure investment and does not generate any non-pecuniary gains.

The individual maximizes his utility, given that his life time budget constraint must bind. From the first order conditions of this problem, we find the expressions for the marginal utilities of consumption and education:

$$U_1 = \lambda, \tag{10}$$

$$U_2 = p\lambda, \tag{11}$$

$$U_E = \lambda[w_l - pw_h g'(E)\overline{H}_2]. \tag{12}$$

The marginal utility of income, λ, is positive, and so are the marginal utilities of consumption in both periods. In this paper we consider the specific case where education is a consumption good, and compare it with the situation where education is only an investment alternative. We do not consider the case where education is a bad, and hence the marginal utility of education is assumed to be nonnegative.

Manipulating (12) gives the condition for optimal investment behavior:

$$(1-\tau)r = \frac{1}{1-\frac{U_E}{(1-t_l)w\lambda}}\left(\frac{(1-t_h)}{(1-t_l)}g'(E)\overline{H}_2\right) - 1. \tag{13}$$

The individual invests in financial and human capital until the net marginal returns are equal in the two investment alternatives. The private marginal return to investments in the financial market is the net interest rate, the left hand side of equation (13). The social return to this investment is the real interest rate r. With a positive tax on the return to financial market investments, the social return differs from the private return to the investment, which is the net interest rate $(1-\tau)r$. The size of the return to human capital investments depends on whether education is a pure investment, or if it generates non-pecuniary returns as well. If education is purely an investment that generates a higher wage in the second period, the marginal utility of education is zero, and (13) reduces to

$$(1-\tau)r = \frac{(1-t_h)}{(1-t_l)}g'(E)\overline{H}_2 - 1. \tag{14}$$

Also the private return to human capital investments is reduced by the taxes. The taxes create distortions since they reduce the private return to the investments, which is the basis on which the individual makes his investment decision.

If we compare the two conditions above, we find that the presence of a consumption value of education induces the individual to get education at a lower wage return than if education is only an investment. Compared with (14), the optimal investment condition (13) has an additional fraction on the right hand side: $U_E/[(1-t_l)w\lambda]$. This is the consumer's marginal consumption value of education, measured in net labor income. If the marginal utility of education is positive, this fraction is positive. It is also smaller than one, which can be shown by investigating (12). Due to the positive consumption value of education, the private marginal return to financial investments is higher than the net marginal wage return to human capital investments in optimum. Therefore, due to the decreasing return to education, the educational level of the individual is higher when education has a consumption value than when it is only another investment alternative. If the consumer is to reduce his educational level, he must be compensated for the direct utility reduction. This means that the interest rate now must be higher in order to make the consumer give up one unit of human capital and invest in one extra unit of financial capital instead. The consequences for the optimal tax profile are important, and they will be discussed thoroughly in section 3.

Combined with the budget constraint, the first order conditions give us the Marshallian demand functions. The indirect utility function is obtained by inserting the demand functions into the utility function:

$$V(w_l, w_h, p) \equiv U(C_1(w_l, w_h, p), C_2(w_l, w_h, p), E(w_l, w_h, p)).$$

Using the envelope theorem, we find the first order derivatives of the indirect utility function. These are needed for the optimal tax analysis.

2.2. The Production Sector

The domestic sector produces one good, which is a perfect substitute for the foreign good. The price of the foreign good is exogenously given and normalized to 1. Hence the price of the domestic good also has to be 1. The industry has a standard neoclassical production function of the form

$$Z = F(K, N),$$

where Z is the amount produced, K is the total amount of capital in the industry, and N is total effective labor input. The production function is linear and homogenous of degree one, so that

$$Z = Nf(k), \quad \text{with} \quad k = \frac{K}{N}.$$

In steady-state, when work effort is constant over time, the total effective labor input is given by

$$N = (T_1 - E) + g(\overline{E})\overline{H}_2.$$

At each point in time there are two generations in the economy. $(T_1 - E) = H_1$ is the work effort of the young generation, who also invests in education during the period. The old generation offers $g(\overline{E})\overline{H}_2$ effective units of labor. Their educational choice has already been made in the previous period, and hence their educational level is given.

The industry demand for capital and labor is given by

$$f'(k) = r, \qquad f(k) - rk = w,$$

with k being the capital intensity in the industry (capital per unit effective labor input), and w the real wage per unit of effective labor. From this we see that domestic capital intensity and the real wage are given by the international interest rate, implying that domestic pre-tax factor prices remain unaffected by changes in the domestic tax rates. Saving and labor supply then only need one period to fully adapt to new tax rates. Hence, only one transition period is required if the government introduces new tax rates in the economy.

2.3. The Public Sector

The public sector offers goods and services, and it has a fixed level of expenditure. Public expenditure (G) is financed through an exogenously given tax on financial income (τ), labor income taxes (t), and by issuing debts (D). The government has decided to carry through a tax reform to introduce Pareto-efficient labor income tax rates. In order for such a reform to be politically feasible, the government cannot allow an increased tax burden on the current old generation, who has adopted to the old tax rates. Hence, tax rates are chosen such as to maximize the welfare of the current young generation and all future generations, without reducing the welfare of the current old generation. In order to balance its budget during the transition period, the government adjusts the national debt and keeps this new level of debt constant for all future periods. In each period there are two generations, from which the government receives taxes. With the superscript "0" denoting pre-reform variables, the government budget constraint for the reform period becomes

$$t_l^0 w\overline{H}_2 + t_h^0 w[g(\overline{E}^0) - 1]\overline{H}_2 + \tau r S^0 + t_l w(T_1 - E) + D = G, \tag{15}$$

where $p = p^0$, $D^0 = 0$, and $S^0 = (1 - t_l^0)w\overline{H}_1^0 - C_1^0$.

In the next period, all individuals have fully adapted to the new tax rates. The government may therefore, without problems, tax everybody according to the new Pareto-optimal tax rates. In this period, the governmental budget constraint is

$$t_l w(T_1 - E) + t_l w\overline{H}_2 + t_h w[g(\overline{E}) - 1]\overline{H}_2 + \tau r\overline{S} = G + rD, \tag{16}$$

where $\overline{S} = (1 - t_l)w(T_1 - \overline{E}) - \overline{C}_1$ is the savings of the old generation in the previous period.

By substituting for D from (15) into (16) and manipulating, we find the public budget constraint:

$$(1+r)(w-w_l)(T_1-E)+(w_h-w_l)\overline{H}_2+(w-w_h)g(E)\overline{H}_2 \\ +\tau r(w_l(T_1-E)-C_1)-(1+r)G+rR=0, \qquad (17)$$

where $R = t_l^0 w\overline{H}_2 + t_h^0 w[g(\overline{E}^0) - 1]\overline{H}_2 + \tau r S^0$ is a constant.

3. Optimal Tax Analysis

Taxes on labor income, t_l and t_h, are chosen such as to maximize the welfare of the representative consumer at the least efficiency loss. Consider a tax on capital income, τ, as exogenously given. This corresponds to the system of dual income tax, where tax rates on capital income and labor income are set separately and independently of each other. Two different situations are considered. First, no tax is levied on capital income, $\tau = 0$. Second, a positive tax on capital income exists, $\tau > 0$, and political reasons make it impossible to change this. This situation arises when individuals have income from different sources, some mostly from labor and others mostly from return to capital investments. Taxing only labor income could have severe distributional effects and would not be tolerated by the majority of voters. For both situations we find the optimal labor income tax rates.

In order to compare our results with the results of Nielsen and Sørensen (1997), we are also interested in finding out whether the optimal tax profile changes with the introduction of consumption value of education.

Analytically, we maximize the consumer's indirect utility function with respect to the net wages w_l and w_h subject to the public budget constraint. Using tedious manipulations, the first order conditions of the above maximization problem are developed into the condition deciding the optimal labor income tax rates:

$$\frac{U_E}{w\lambda}\left(\frac{\tau r}{1+(1-\tau)r}\frac{\partial C_1}{\partial Y}-\frac{t_h}{1-t_h}\right)+\left(\frac{1-t_l}{1-t_h}-\frac{1+r}{1+(1-\tau)r}\right)=0, \qquad (18)$$

where $\partial C_1/\partial Y$ is the marginal propensity to consume in the first period[2]. This is the general expression for the optimal tax condition; let us now look closer at two specific cases.

2 See the Appendix for the deduction of this optimal tax condition.

3.1. Case 1: No Consumption Value of Education

With no consumption value of education, $U_E = 0$, education is only of interest to the individual as an investment alternative. In this case, the optimal tax condition (18) reduces to

$$\frac{1-t_l}{1-t_h} = \frac{1+r}{1+(1-\tau)r}. \tag{19}$$

3.1.1. $\tau = 0$

With no tax on capital income, it follows from equation (19) that the Pareto-optimal labor income tax rates are proportional, $t_l = t_h$. A proportional labor income tax does not influence the investment decision of the consumer. It taxes the alternative cost of the investment (foregone labor income in the first period) at the same rate as the return to the investment (higher wages in the second period). A cash-flow tax of this kind levies zero marginal tax on the return to human capital investments[3]. It is thus a neutral tax on the return to human capital investments. Combined with zero taxation of financial income, on the margin the social return on both kinds of investments equals the private return. This is the first-best solution, and the tax does not create any distortions in the capital market. Since leisure is fixed in both periods, labor supply is fixed in the second period. Education is a pure investment that does not generate any non-pecuniary gains, and hence no substitution effect arises from taxing labor income. Therefore the proportional tax on labor income combined with zero tax on capital income is equivalent to a pure consumption tax.

3.1.2. $\tau > 0$

If there exists an exogenously given positive tax on financial income, we find from equation (19) that a progressive tax on labor income[4], $t_l < t_h$, is optimal. To understand the intuition behind this result, let us see why a proportional

3 Sandmo (1979) shows the neutrality of a cash flow tax.

4 Given that the public expenditure is constant, one might expect that the additional tax revenue from taxing capital income would remove the need for taxing labor income. But in order to minimize the distortions in the investment market caused by the taxes, a progressive tax on labor income is optimal. The level of the optimal tax rates might change with the presence of tax on capital income, but this is not treated in this analysis. It is an open question though, whether tax revenue really increases that much from taxing capital income. While getting education, the consumer has a possibility to finance his first period consumption with negative savings. The interest payments in the second period are then tax deductible, and tax revenue decreases. The progressive labor income tax reduces the wage return to human capital investments, and thereby reduces some of the distortions that would arise with a proportional labor income tax.

tax on labor income is not optimal in the presence of a positive tax on capital income. When $\tau > 0$ and $t_l = t_h$, equation (14) reduces to

$$(1-\tau)r = g'(E)\overline{H}_2 - 1. \tag{20}$$

Here we have no marginal tax on the return to human capital investments. This is because the return to the investment faces the same tax rate as the cost, measured in foregone labor income in the first period. Return to financial investments on the other hand, faces a positive marginal tax rate τ. This causes a distortion in the investment market in favor of human capital. We see from the above equation that the consumer invests in human capital at a lower rate of return than in the case with no tax on capital income. Since education has a diminishing rate of return, this means that he invests more in human capital than he would do if there were no taxes. This overinvestment in human capital is counteracted by a progressive labor income tax, reducing the return to education.

Fisher's separation theorem[5] states that the optimal investment decision of the consumer is independent of his preferences. To see this, consider a utility maximizing individual whose utility depends on the level of consumption in the two periods of his life. He wishes to allocate income and consumption over two periods, such as to maximize consumption. He has two means of moving consumption between periods: he might save in financial capital with a fixed return, and in real capital with a diminishing return. Maximizing consumption over his life time is the same as maximizing the present value of his savings portfolio. The optimal solution is to invest in real capital until the marginal return equals the fixed return of financial investments. This is independent of his preferences. Let us now apply this theorem to our context of educational choice.

The present value of the individual's income is given by

$$PV = (1-t_l)w\left(T_1 - E + \frac{\overline{H}_2}{1+(1-\tau)r}\right) + (1-t_h)w[g(E)-1]\frac{\overline{H}_2}{1+(1-\tau)r}. \tag{21}$$

The individual maximizes his life time income with regard to his educational choice, and we hence get a condition for optimal investment decision, independent of his preferences:

$$(1-\tau)r = \frac{(1-t_h)}{(1-t_l)}g'(E)\overline{H}_2 - 1. \tag{22}$$

It is optimal for the individual to invest in education until its net marginal return equals the net interest rate. Equation (22) determines the individual's

5 Carefully described in Fama and Miller (1972).

savings portfolio exclusively by maximizing the present value of this portfolio. And the resulting condition is exactly the same as the optimal investment condition (14) from the individual's utility maximization problem.

The second-best literature states that if a tax-created distortion exists in one market, then trying to achieve efficiency in the other markets is not necessarily optimal. But Diamond and Mirrlees (1971) show that aggregated production efficiency is desirable even though taxation leads to distortions in one market. This result holds for a general equilibrium model. But even though our model is a partial equilibrium model, the intuition of Diamond and Mirrlees may be applied to our context. Substituting for $(1-t_l)/(1-t_h)$ in the optimal investment condition (22) with (19) shows that we might have efficiency in the investment market even in the presence of taxation:

$$r = g'(E)\overline{H}_2 - 1. \tag{23}$$

In spite of taxation, we get a solution where the individual composes his investment portfolio as he would have done in the absence of taxes. But this result depends on equation (19) to hold. From this equation we see that in the case of no tax on capital income, $\tau = 0$, we need proportional tax on labor income, $t_l = t_h$, in order to achieve efficiency in the investment market. But, if for some reason there exists an exogenously given tax on capital income, so that the net return to financial investments is lower than in the absence of taxes, $(1-\tau)r < r$, then a progressive tax, $t_l < t_h$, has to be levied on labor income in order to achieve efficiency in the investment market.

The next step is to investigate the optimal tax rates in the situation where education no longer is a pure investment, but where it also generates a direct utility gain for the individual.

3.2. Case 2: Positive Consumption Value of Education

The individual enjoys the consumption value of education, $U_E > 0$. Education is a good for which the individual is willing to pay. Let us now investigate the implications this has for the optimal labor income tax rates.

3.2.1. $\tau = 0$

First, assume that there is no tax on capital income. The optimal tax condition (18) then reduces to

$$-\frac{t_h}{1-t_h}\frac{U_E}{w\lambda} + \frac{1-t_l}{1-t_h} - 1 = 0. \tag{24}$$

From this it follows that the optimal labor tax profile is characterized by

$$t_h > t_l.$$

We find that *even with no tax on capital income, progressive tax on labor income is optimal.* The intuition behind this can be understood by studying the consumer's optimal investment condition, equation (13).

In case 1, with no consumption value of education, a proportional tax on labor income is a neutral tax on the return to human capital investments and does not distort the individual's investment decision. In case 2, when education is a consumption good for which the individual is willing to pay, a proportional labor income tax is no longer neutral. On the contrary, it discriminates between the two investment alternatives in favor of human capital. To see this clearly, let $\tau = 0$, and $t_h = t_l > 0$ in equation (13), and compare this with the situation with no taxes at all, $\tau = t_h = t_l = 0$. In both situations, the left hand sides of the respective variants of (13) have to equal the real interest wage r. This gives us the following condition:

$$r = \left[\frac{1}{1 - \frac{U_E}{(1-t_l)w\lambda}} g'(E)\overline{H}_2 - 1\right]_{t_h = t_l > 0} = \left[\frac{1}{1 - \frac{U_E}{w\lambda}} g'(E)\overline{H}_2 - 1\right]_{\tau = t_h = t_l = 0}. \tag{25}$$

The proportional tax on labor income reduces the wage return to the investment at the same rate as the alternative cost. But at the same time it also reduces the price on education as a consumption good, measured in foregone first period labor income. Therefore, the proportional labor income tax actually serves as a subsidy on education,

$$\frac{1}{1 - \frac{U_E}{(1-t_l)w\lambda}} > \frac{1}{1 - \frac{U_E}{w\lambda}},$$

and the individual invests in education at a lower wage return than he would have done in the absence of tax on labor income. For equation (25) to hold, at least one of the following has to be true: i) the marginal utility of income is smaller in the presence of labor income tax, and/or ii) the marginal wage return to education is smaller in the presence of labor income tax. Both i) and ii) imply that the educational level is higher in the presence of proportional labor income taxes than in the absence of taxes.

A proportional tax on labor income creates price distortions in favor of human capital investments, and the consumer chooses to get more education than in the case with no taxes. Only the wage return to education is reduced through the income tax, whereas the direct utility return remains unchanged. Hence the total tax rate on return to human capital investments decreases, compared with the case where education is a pure investment. But the alternative cost of investing in human capital, the net basic wage, is the same in the two cases. Put differently, a proportional labor income tax works as a tax subsidy on human capital investments, still creating distortions in the capital market. Return to financial investments must be higher in the case with labor income tax in order to shift investments between financial and human capital. A progressive labor income tax reduces the wage return to education further, inducing the consumer to invest more in financial capital. The progressive labor income tax reduces the distortions in the capital market, and we get a solution closer to the optimum.

3.2.2. $\tau > 0$

Next, look at the case with an already existing positive tax on capital income. Now it is analytically more complicated to characterize the optimal tax rates on labor income. When investigating the optimal tax condition (18), we get the expected result, namely that in the presence of tax on capital income a progressive tax on labor income is optimal[6].

This analysis has been purely qualitative, so we cannot conclude about the optimal degree of progressivity. But intuitively, labor income taxation should be more progressive when capital income is taxed, than when it is not:

$$(t_h - t_l)_{\tau=0} < (t_h - t_l)_{\tau>0}.$$

This follows from the fact that the human capital investments increase when capital income tax is introduced, since this favors human capital investments. Hence the distortions in the investment market increase when tax on capital income is introduced. One would therefore expect that a greater degree of progressivity in the labor income taxation is needed to correct for the distortions in the investment market. From (18) we see that the surtax t_h must be substantially higher than the basic labor income tax t_l for the equation to hold.

But what happens to the nice production efficiency result we obtained when education had no consumption value? Substituting for $U_E/(w\lambda(1-t_h))$ from (12) into the optimal tax condition (18) and rearranging, yields

$$\left(p\tau r\frac{\partial C_1}{\partial Y}+1\right)\frac{U_E}{w\lambda}\frac{1}{p}=r-\{g'(E)\overline{H}_2-1\}>0.$$

This implies that the following has to be true:

$$r > g'(E)\overline{H}_2 - 1, \tag{26}$$

which should be compared to equation (23) in case 1. From (26) we conclude that with a positive consumption value of education, the marginal wage return to human capital investments in optimum is smaller than the marginal return to investments in the financial market. Education is no longer only a production factor, it is a consumption good as well. Thus, the previous efficiency result in the investment market in the presence of taxation breaks down when education has a consumption value. Also, the Fisher result of

6 The analysis is found in the Appendix.

separation from the preferences in the investment decision no longer holds. The consumption value of education adds an additional return to education that is not represented in the present value of the savings portfolio. Hence the only optimal investment decision can be made from the individual's utility maximization.

4. Concluding Remarks

An individual who faces two investment alternatives, financial capital and human capital, invests in both until their net marginal returns are equal. Foregone labor income is the only cost of acquiring education. A proportional labor income tax is then a neutral tax on the return to human capital investment and does not alter its marginal return. Nielsen and Sørensen (1997) show that the dual income tax system with proportional tax on capital income and progressive tax on labor income is optimal in a second-best world. When a positive tax on capital income exists, a proportional tax on labor income leads to over-investment in human capital. A progressive labor income tax reduces the marginal return to education, and the distortions in the investment market are reduced.

If education, in addition to being an investment alternative, is also a consumption good, this has consequences for the optimal tax policy. A positive consumption value of education is a tax-free return to human capital investments. Hence, a proportional labor income tax is no longer a neutral tax on the return to human capital investments. Even when no tax is levied on capital income, a progressive tax on labor income is required in order to reduce the overinvestment in education.

This paper provides an efficiency argument in favor of progressive labor income taxation. It is worth noting though, that the analysis of this paper is purely qualitative and cannot conclude on the optimal level of progressivity in the labor income taxation. Neither does this analysis consider uncertainty, distributional issues, or possible external effects of education. If direct costs of education were to be taken into account, a proportional tax on labor income would no longer be a neutral tax on the return to human capital investments. This would probably reduce the need for progressive labor income taxes to correct for distortions in the investment market.

Education is homogenous in this paper, as in most papers in this strand of the literature. In the model, education generates one rate of wage return and one kind of consumption value. In reality, different kinds of education have varying wage returns, as well as different consumption values to the consumer. As far as I know, no attempt has been made to investigate how the tax system affects the individual's choice of different kinds of education. This is an important question, since the educational choices of today's young generation decide the qualifications of tomorrow's labor force. Hence, today's tax policy might affect future production possibilities and the productivity of the country. This is left for later research.

5. Appendix

5.1. Developing the Expressions for the Marginal Utilities (10), (11), and (12)

$\max_{C_1, C_2, E} U(C_1, C_2, E)$ s.t. the budget constraint (8).

The corresponding Lagrange function is given by

$$L = U(C_1, C_2, E) \\ - \lambda(C_1 + pC_2 - w_l[(T_1 - E) + p\overline{H}_2] - pw_h[g(E) - 1]\overline{H}_2).$$

The resulting first order conditions are:

$$\frac{\partial L}{\partial C_1} = U_1 - \lambda = 0,$$

$$\frac{\partial L}{\partial C_2} = U_2 - p\lambda = 0,$$

$$\frac{\partial L}{\partial E} = U_E - \lambda[w_l - pw_h g'(E)\overline{H}_2] = 0.$$

5.2. Developing the Optimal Investment Condition (13)

From equation (12) we have:

$$U_E = \lambda[w_l - pw_h g'(E)\overline{H}_2]$$

$$\Downarrow$$

$$\frac{1}{p}\frac{U_E}{w_l\lambda} = \frac{1}{p} - \frac{w_h}{w_l}g'(E)\overline{H}_2$$

$$\Downarrow \text{ applying the definitions (5), (6), and (7)}$$

$$1 + (1-\tau)r = \frac{(1-t_h)}{(1-t_l)}g'(E)\overline{H}_2 + [1 + (1-\tau)r]\frac{U_E}{(1-t_l)w\lambda}$$

$$\Downarrow$$

$$(1-\tau)r\left(1 - \frac{U_E}{(1-t_l)w\lambda}\right) = \frac{(1-t_h)}{(1-t_l)}g'(E)\overline{H}_2 - \left(1 - \frac{U_E}{(1-t_l)w\lambda}\right)$$

$$\Downarrow$$

$$(1-\tau)r = \frac{1}{1 - \frac{U_E}{(1-t_l)w\lambda}}\left(\frac{(1-t_h)}{(1-t_l)}g'(E)\overline{H}_2\right) - 1.$$

5.3. The Derivatives of the Indirect Utility Function

Combined with the budget constraint, the first order conditions give us the Marshallian demand functions. The indirect utility function is obtained by inserting the demand functions into the utility function:

$$V(w_l, w_h, p) \equiv U(C_1(w_l, w_h, p), C_2(w_l, w_h, p), E(w_l, w_h, p)).$$

Using the envelope theorem, we find the first order derivatives of the indirect utility function:

$$\frac{\partial V}{\partial w_l} = \lambda\left[(T_1 - E) + p\overline{H}_2\right], \tag{27}$$
$$\frac{\partial V}{\partial w_h} = \lambda p[g(E) - 1]\overline{H}_2.$$

5.4. Developing the Optimal Tax Condition (18)

The government's problem is represented by:

$$\max_{w_l, w_h} V(w_l, w_h, p) \quad \text{s.t. the public budget constraint (17).}$$

The Lagrange function of this problem is given by

$$\begin{aligned} L = {} & V(w_l, w_h, p) \\ & + \mu \left\{ \begin{array}{c} (1+r)(w - w_l)(T_1 - E) + (w_h - w_l)\overline{H}_2 + (w - w_h)g(E)\overline{H}_2 \\ +\tau r(w_l(T_1 - E) - C_1) - (1+r)G + rR \end{array} \right\}. \end{aligned}$$

The corresponding first order conditions are:

$$\begin{aligned} \frac{\partial L}{\partial w_l} &= \frac{\partial V}{\partial w_l} + \mu \left\{ \begin{array}{c} -\overline{H}_2 - (1+r)H_1 + (1+r)(w - w_l)\frac{\partial H_1}{\partial E}\frac{\partial E}{\partial w_l} \\ +(w - w_h)\overline{H}_2 g'(E)\frac{\partial E}{\partial w_l} + \tau r w_l \frac{\partial H_1}{\partial E}\frac{\partial E}{\partial w_l} + \tau r H_1 - \tau r \frac{\partial C_1}{\partial w_l} \end{array} \right\} \\ &= 0, \end{aligned} \tag{28}$$

$$\frac{\partial L}{\partial w_h} = \frac{\partial V}{\partial w_h} + \mu \left\{ \begin{array}{c} \overline{H}_2 + (1+r)(w - w_l)\frac{\partial H_1}{\partial E}\frac{\partial E}{\partial w_h} - g(E)\overline{H}_2 \\ +(w - w_h)\overline{H}_2 g'(E)\frac{\partial E}{\partial w_h} + \tau r w_l \frac{\partial H_1}{\partial E}\frac{\partial E}{\partial w_h} - \tau r \frac{\partial C_1}{\partial w_h} \end{array} \right\} = 0. \tag{29}$$

Substituting for the derivatives of the indirect utility function from (27) yields the following:

$$\begin{aligned} & \lambda\left[H_1 + p\overline{H}_2\right] \\ & + \mu \left\{ \begin{array}{c} -\overline{H}_2 - (1+r)H_1 + (1+r)(w - w_l)\frac{\partial H_1}{\partial E}\frac{\partial E}{\partial w_l} \\ +(w - w_h)\overline{H}_2 g'(E)\frac{\partial E}{\partial w_l} + \tau r w_l \frac{\partial H_1}{\partial E}\frac{\partial E}{\partial w_l} + \tau r H_1 - \tau r \frac{\partial C_1}{\partial w_l} \end{array} \right\} = 0, \end{aligned}$$

$$\lambda p[g(E)-1]\overline{H}_2 + \mu \left\{ \begin{array}{c} \overline{H}_2 + (1+r)(w-w_l)\frac{\partial H_1}{\partial E}\frac{\partial E}{\partial w_h} - g(E)\overline{H}_2 \\ +(w-w_h)\overline{H}_2 g'(E)\frac{\partial E}{\partial w_h} + \tau r w_l \frac{\partial H_1}{\partial E}\frac{\partial E}{\partial w_h} - \tau r \frac{\partial C_1}{\partial w_h} \end{array} \right\} = 0.$$

We know that $\partial H_1/\partial E = -1$. Applying this, as well as combining the two above equations to eliminate the shadow prices μ and λ, gives the following condition:

$$\begin{aligned} &[H_1 + p\overline{H}_2] + \left\{ \begin{array}{c} -[g(E)-1]\overline{H}_2 - \tau r \frac{\partial C_1}{\partial w_h} \\ +\frac{\partial E}{\partial w_h}\left[(w-w_h)\overline{H}_2 g'(E) - (1+r)(w-w_l) - \tau r w_l\right] \end{array} \right\} \\ &\quad - p[g(E)-1]\overline{H}_2 \\ &\quad + \left\{ \begin{array}{c} -\overline{H}_2 - (1+r-\tau r)H_1 - \tau r \frac{\partial C_1}{\partial w_l} \\ +\frac{\partial E}{\partial w_l}\left[(w-w_h)\overline{H}_2 g'(E) - (1+r)(w-w_l) - \tau r w_l\right] \end{array} \right\} \\ &= 0, \end{aligned}$$

which may be reduced to

$$\begin{aligned} &\left\{[H_1 + p\overline{H}_2]\frac{\partial E}{\partial w_h} - p[g(E)-1]\overline{H}_2 \frac{\partial E}{\partial w_l}\right\} \cdot \left\{ \begin{array}{c} (w-w_h)\overline{H}_2 g'(E) \\ -(1+r)(w-w_l) - \tau r w_l \end{array} \right\} \\ &\quad - \tau r \left\{[H_1 + p\overline{H}_2]\frac{\partial C_1}{\partial w_h} - p[g(E)-1]\overline{H}_2 \frac{\partial C_1}{\partial w_l}\right\} \\ &= 0. \end{aligned} \tag{30}$$

From (10) and (11) we know that the marginal rate of substitution between first and second period consumption equals the discount factor p, which is independent of the labor income tax rates. Tax changes have no substitution effects on the consumption in the two periods, only an income effect. The choice of education in the first period is given by (12). Letting Y denote the individual's discounted life time income, the budget constraint (8) may be written as

$$C_1 + pC_2 = Y, \quad \text{where } Y = w_l[H_1 + p\overline{H}_2] + pw_h[g(E)-1]\overline{H}_2.$$

It follows that

$$\frac{\partial C_1}{\partial w_l} = \frac{\partial C_1}{\partial Y}\frac{\partial Y}{\partial w_l} = \frac{\partial C_1}{\partial Y}\left\{H_1 + p\overline{H}_2 + \frac{\partial E}{\partial w_l}(pw_h\overline{H}_2 g'(E) - w_l)\right\}, \tag{31}$$

and

$$\frac{\partial C_1}{\partial w_h} = \frac{\partial C_1}{\partial Y}\frac{\partial Y}{\partial w_h} = \frac{\partial C_1}{\partial Y}\left\{p\overline{H}_2[g(E)-1] + \frac{\partial E}{\partial w_h}(pw_h\overline{H}_2 g'(E) - w_l)\right\}. \tag{32}$$

Inserting (31) and (32) into the optimal tax condition (30) reduces this to

$$\left\{[H_1 + p\overline{H}_2]\frac{\partial E}{\partial w_h} - p[g(E) - 1]\overline{H}_2\frac{\partial E}{\partial w_l}\right\} \cdot \left\{\begin{array}{l}(w - w_h)\overline{H}_2 g'(E) - (1 + r)(w - w_l) \\ -\tau r w_l - \tau r\frac{\partial C_1}{\partial Y}(p w_h \overline{H}_2 g'(E) - w_l)\end{array}\right\} = 0. \quad (33)$$

Increased net top marginal wage for the educated worker affects the educational choice positively through two channels. It increases the wage return to education, and it increases the disposable income, inducing the individual to consume more of all goods, including education. Hence $\partial E/\partial w_h > 0$. Increased net marginal basic wage has two opposite effects on the educational choice. The substitution effect would induce the individual to get less education since the price of education as a consumption good increases, measured in foregone labor income in the first period. But the income effect induces the individual to acquire more education. The total effect depends on the individual's preferences, but it is reasonable to assume that the income effect at the very least naturalizes the substitution effect. Hence $\partial E/\partial w_l \geq 0$. Even if this last effect should be negative, we might assume that for reasonable values of $g(E)$, the first bracket of (33) is positive. For the optimal tax condition (33) to hold, the second bracket of this equation has to equal zero.

From the first order condition (12) we have that

$$\left(w_l - \frac{U_E}{\lambda}\right)\frac{1}{w_h} = -pg'(E)\overline{H}_2,$$

which we use in the manipulation of the second bracket of (33):

$$-(1 + r)(w - w_l) + (w - w_h)\overline{H}_2 g'(E) - \tau r w_l - \tau r\frac{\partial C_1}{\partial Y}(p w_h \overline{H}_2 g'(E) - w_l) = 0$$

$$\Downarrow$$

$$\left(1 + p\tau r\frac{\partial C_1}{\partial Y}\right)(w_l - p w_h \overline{H}_2 g'(E)) + w p \overline{H}_2 g'(E) - p(1 + r)w = 0$$

$$\Downarrow$$

$$\left(1 + p\tau r\frac{\partial C_1}{\partial Y}\right)\frac{U_E}{w\lambda}) + \left(w_l - \frac{U_E}{\lambda}\right)\frac{1}{w_h} - p(1 + r) = 0.$$

Finally, we get the optimal tax condition (18):

$$\frac{U_E}{w\lambda}\left(\frac{\tau r}{1 + (1 - \tau)r}\frac{\partial C_1}{\partial Y} - \frac{t_h}{1 - t_h}\right) + \left(\frac{1 - t_l}{1 - t_h} - \frac{1 + r}{1 + (1 - \tau)r}\right) = 0.$$

5.5. Developing the Optimal Labor Income Tax Rates when $\tau = 0$ and $U_E > 0$

The optimal tax condition is reduced to (24)

$$-\frac{t_h}{1-t_h}\frac{U_E}{w\lambda} + \frac{1-t_l}{1-t_h} - 1 = 0$$

$$\Downarrow$$

$$(1-t_l) - (1-t_h) = t_h\frac{U_E}{w\lambda} > 0$$

$$\Downarrow$$

$$t_h > t_l.$$

5.6. Developing the Optimal Labor Income Tax Rates when $\tau > 0$ and $U_E > 0$

The optimal tax condition (18) has to hold:

$$\left(p\tau r\frac{\partial C_1}{\partial Y} - \frac{t_h}{1-t_h}\right)\frac{U_E}{w\lambda} + \frac{1-t_l}{1-t_h} = \frac{1+r}{1+(1-\tau)r}. \tag{34}$$

We know that

$$\frac{1+r}{1+(1-\tau)r} > 1,$$

implying that the left hand side of (34) is positive and greater than one. Consumption is assumed to be a normal good in both periods. The marginal propensity to consume, $\partial C_1/\partial Y$, is therefore between 0 and 1. A reasonable value of the marginal propensity to consume is 0.5, which is used throughout the analysis.

The capital income tax lies within the interval

$$0 \leq \tau \leq 1.$$

We may assume that it is no larger than 0.5, since, in an open economy, capital flows out of the country if tax rates are too high. In the following analysis, we let $\tau = 0.28$, which is the tax rate on capital income in Norway.

By investigating the expression pr, we find that

$$pr = \frac{r}{1+(1-\tau)r} < 1 \quad \text{if} \quad r < \frac{1}{\tau} = 3.57.$$

That is, the real interest rate must be below 357%, a condition quite likely to be fulfilled. Estimating the real interest rate is difficult, since we have not specified the length of the periods. It is a good approximation to say that the annual real interest rate is 5%, summing up to 165% over a period of twenty years. (Here, we include the compound interest.) Assuming that the individual only has a time span of 20 years when choosing how much to invest in human capital, the above condition is met. In the following $r = 1.65$, i.e. a real interest rate of 165%.

With these values, the first term in the brackets of equation (34) becomes

$$p\tau r\frac{\partial C_1}{\partial Y} = \frac{\tau r}{1+(1-\tau)r}\frac{\partial C_1}{\partial Y} = \frac{1.65 \cdot 0.28}{1+(1-0.28)\cdot 1.65} \cdot 0.5 = 0.106.$$

If the expression in the brackets of (34) is to be positive, the surtax t_h must not exceed a critical value. This threshold value of t_h is:

$$\frac{t_h}{1-t_h} < 0.106 \qquad \Longrightarrow \qquad t_h < 0.096;$$

i.e. t_h must not exceed 9.6%, which is substantially below the current marginal rate of income tax. Therefore, the expression in the brackets of (34) is negative.

We have already stated that the marginal consumption value of education measured in net labor income, $U_E/(w\lambda)$, is positive.

As long as $t_h > 0.096$, the first term on the left hand side of (34) is negative. To make the equality in (34) hold, the following must be true:

$$\frac{1-t_l}{1-t_h} = \frac{1+r}{1+(1-\tau)r} + \left(\frac{t_h}{1-t_h} - p\tau r\frac{\partial C_1}{\partial Y}\right)\frac{U_E}{w\lambda} > \frac{1+r}{1+(1-\tau)r} > 1; \tag{35}$$

i.e.

$$t_l < t_h.$$

These results can be shown to hold for other values of the real interest rate and of the marginal propensity to consume.

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Chapter 4

Income Tax, Consumption Value of Education, and the Choice of Educational Type*

Abstract

The educational choice of today's young generation determines the skill composition of tomorrow's labor force and hence the future production possibilities. The return to education is one motivation behind the individual's educational choice. The consumption value is a tax free return to educatiton. This paper finds that a progressive tax system induces the individual to choose more of the educational type with the higher consumption value. This effect is stronger the more weight the individual puts on the present.

JEL-classifications: H25; I21; I29.

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1 Introduction

The OECD countries as a whole spent 5.8 per cent of their collective GDP on education in 2001, and 12.7 per cent of total public expenditure was devoted to educational institutions[1]. Most of these countries offer publicly financed primary and secondary education, and in many countries tertiary education is also provided by the state at no direct cost for the individual. Part of the justification for publicly funded education is the positive effects of education on general productivity[2]. The government encourages the individuals to get higher education, focusing on the amount of human capital in society and to a great extent ignoring its composition. Different types of education yield different rates of private and social return. It therefore ought to be of great interest to the government to learn more about the mechanisms determining the individual's choice of educational direction, and not only the amount of education. At the very least, one should be aware of which kinds of distortions the income tax system imposes on the educational choice of the individuals. Could it in fact be that the tax system induces the individual to choose other kinds of education than he or she would in the absence of taxes?

The individual's motivation for choosing higher education may be divided into four categories. First, education is an investment that yields higher wages later in life. Individuals invest in education until the expected marginal pecuniary return equals that of other investment alternatives (Nerlove et al. 1993). Second, education is a signal of high abilities of the individual and might correct for information problems in the labor market (Stiglitz 1975). Third, education is insurance against unemployment (Bishop 1994). Fourth, education offers non-pecuniary and non-market types of return, both during the education itself and afterwards (Becker 1964, Lazear 1977). Among these are the joy of learning new things, meeting new people, moving to a new city, enjoying life as a student, in addition to the increased status in society that often comes with studying in particular fields. It is important to remember that even if education is treated as homogeneous in the literature, it is in fact a heterogeneous investment alternative and a consumption good. Thus different kinds of education generate different levels of joy or satisfaction during the educational process. Also, different kinds of education require different levels of effort in order to graduate,

[1]OECD: Education at a Glance.

[2]See Lucas (1988).

a factor the student also considers. After its completion, higher education enables the individual to choose among more interesting jobs. Different educational types offer different degrees of flexibility with respect to working hours and the regional distribution of jobs. Individuals with strong preferences for where to live or for being able to work part time will value these qualities strongly when choosing type of education. Another feature that differs among the different educational directions is the effort required by the student to complete the education, and thus also the amount of leisure available to the student. Let all these non-market and non-pecuniary types of return to education be summarized as the consumption value of education. Depending on their preferences, individuals put different weight on the consumption value when choosing educational type.

Fredriksson (1997) shows on Swedish data that the demand for education responds to economic incentives; more students enrolled at the universities in periods with high expected wage returns or with particularly beneficial student loans arrangements. The link between the income tax system and the length of the individual's education is well studied in the literature. Higher education is considered as an investment alternative in which the individual invests until the expected marginal pecuniary return equals that of other investment alternatives. Taxes on financial income increase the relative pecuniary return to education, and taxes on labor income reduce the return to human capital investments (Boskin 1975, Heckman 1976). The nature of the tax schedule also affects the attractiveness of human capital investments. If no direct costs of acquiring education besides forgone labor income are present, a proportional tax on labor income is a neutral tax on the return to human capital investments. But if a positive tax on capital income exists as well, the comprehensive proportional income tax induces the individual to over-invest in human capital (Nielsen and Sørensen 1997). This effect is even stronger if education has a positive consumption value as well (Alstadsæter 2003). On the other hand, if education requires direct pecuniary investments, a comprehensive proportional income tax discriminates against human capital investments (Trostel 1993).

Monetary return to education no doubt is an important factor in the individual's educational choice, but it is a drawback for the explanatory power of the economic models that the other motives behind the educational choice are mostly ignored. For instance, the Norwegian labor force is among the most highly educated in the

OECD[3], but still has a compressed wage structure and moderate wage return to higher education. Where a country like the US at present has an average wage premium[4] to an additional year of higher education of 10 %, the corresponding rate in Norway is 5,5 %. This is the average wage return over all kinds of education at the same duration. Still, the number of students at universities and regional colleges has more than doubled over the last 20 years[5]. It thus seems like the students are willing to forgo future pecuniary return in order to get the non-pecuniary return to the educational type of their choice. But different types of education do in fact generate different rates of wage return, even if they have the same duration.

The educational choice of today's young generation determines the skill composition and hence the production possibilities of tomorrow's labor force. Small open economies with high wage levels, like many of the European countries, experience a flagging-out of their industrial production to low-cost countries. A consensus exists in these countries that the future economic growth depends on their ability to transfer into knowledge-based industries and innovation production. In order to do this, a highly educated labor force with the required skill combination is essential. Little attention has been given to the link between the country's income tax system and the individual's choice of educational direction. If it is so that the tax system not only affects how much education the individuals choose to get, but also which kind of education they choose, then the tax system does indeed affect future production possibilities.

This paper analyzes how the individual's trade-off between pecuniary and non-pecuniary return in his choice of educational type is affected by the tax system. Depending on the individual's preferences, a progressive tax system might in fact introduce distortions in the individuals's educational choice and induce him to choose more of the educational type with the higher consumption value. If he also puts more weight on the present than on the future, this effect is strengthened further. Section 2 presents the model, and the analysis is carried out in section 3. Section 4 concludes.

[3] OECD: Education at a glance.

[4] Source: Psacharopoulos and Patrinos, 2002.

[5] Hægeland and Møen (2000).

2 The model

The representative individual lives for two periods. He has already decided to spend all available time in the first period on acquiring education. The remaining decision to make is which type of education to choose. This is in order to focus on the choice of educational type and abstract from the decisions of whether or not to get education in the first place and how much education to acquire[6].

Consider the extreme case where the wage return is either low or high, and where the consumption value of the educational type is either negative or positive. No rational individual chooses the educational type with negative consumption value unless he is compensated through a higher wage return. The individual can choose between the two[7] educational types A and B:

Type-A education: Positive consumption value and low wage return.

Type-B education: Negative consumption value and high wage return.

Both types have free admission, and the following model analyzes how taxes affect the individual's choice between these two kinds of education. If the individual chooses to get type-A education he puts more weight on the non-pecuniary return to education and forgoes other consumption since his income is lower than it would have been had he chosen type-B education. No supply effects are considered in the present model; as long as the individual wishes to acquire more of one type of education, he may do so.

The individual chooses the optimal linear combination of the two types of education in the first period. The parameters $0 < E_A < 1$ and $0 < E_B < 1$ denominate the fraction of available time spent on type-A and type-B education, respectively:

$$E_A + E_B = 1. \tag{1}$$

By combining the two educational types in different manners, the individual has a continuum of different kinds of degrees to choose from.

[6]This follows from an simplifying assumption that the choices of how much and which kind of education to acquire are separable. This is analogous to the litterature on saving and portfolio choice, where the savings decision is analyzed separately from the portfolio choice.

[7]There are two more possible combinations of these attributes. No rational individual would choose the educational type with low wage return and negative consumption value. The last possibility is an educational type with high wage return and positive consumption value, but which typically has restricted admission, and therefore is disregarded in the present analysis.

The first and second periods are not restricted to having the same duration, and so the second period may be longer than the first period. Most people spend more of their lifetime working then they do acquiring education. The individual is also assumed to stay in the same job for the whole of the second period. This is the extreme version of the lock-in effect that to some extent exists in the labour market; the individual has full freedom in his choice of educational type, but he has limited possibility to change this choice after the completion of the education[8]. The time spent working, H, is given in the second period and independent of the educational profile chosen in the first period[9].

In each period the consumer enjoys ordinary consumption and education. Education is both a consumption good and an investment alternative. Type-A education yields a direct utility gain through the positive consumption value, and type-B education increases the individual's income and thus consumption possibilities of first and second period ordinary consumption. The individual's preferences are represented by the utility function

$$U = U(C_1, C_2, E_A), \tag{2}$$

which is increasing in all three consumption goods, C_1, C_2, and E_A. First and second period consumption are both assumed to be normal goods, as is type-A education. It follows from equations (1) and (2) that $U(C_1, C_2, E_A) = U(C_1, C_2, 1 - E_B)$, and hence the marginal utility of type-B education is negative.

No tuition fees are paid, but the individual needs to finance his living expenses in the first period. He borrows money in the financial market at a given interest rate r. In the absence of liquidity constraints, he finances all his first period consumption, C_1 , through debt, D. All debt is repaid in the second period. There exist no non-labor income or intergenerational transfers in the model. His first period budget constraint is hence given by:

$$C_1 = D. \tag{3}$$

[8]This is an analogy to the putty-clay hypothesis in production theory (Johansen, 1972), where there ex-ante is full substitution between labor and capital, while the ex-post production coefficients are given when the capital is installed.

[9]Seeing that type-B education leads to a stressful and less enjoyable job that pays better than the alternative, one might also expect that a job requiring type-B qualifications would demand longer hours. That aspect is not considered here. Hence the duration of the second period and the hours worked are independent of the educational profile.

The time spent working in the second period, H, is exogenously given and independent of the educational profile. Second period consumption, C_2, depends crucially on the chosen educational profile. The expected wage w is paid to the individual for all hours he works, independent of educational profile, and it is taxed at the rate t_w. In addition, the individual receives a positive wage return e to type-B education[10]. This additional wage return to education is also taxed at the basic tax rate t_w, but in addition a surtax of t_e applies. His second period consumption is hence given by:

$$\begin{aligned} C_2 = {} & [1 - t_w] \cdot w \cdot E_A \cdot H + [1 - t_w] \cdot w \cdot E_B \cdot H \\ & + [1 - (t_w + t_e)] \cdot e \cdot E_B \cdot H - [1 + r] \cdot D, \end{aligned} \tag{4}$$

If $t_e = 0$, tax on labour income is proportional, and if $t_e > 0$, tax on labour income is progressive. Obviously, if $t_w = t_e = 0$, there is no tax on labour income. Both tax rates are restricted to be larger than or equal to zero, and smaller than one. Regressive income taxation is no option here. r is the exogenously determined net interest rate. A change in the tax rates on labor income hence leaves the tax rate on capital income unaffected[11]. Thus the net interest rate and the discount factor are unaffected by the tax on labor income. By combining the equations (3) and (4), we find the individual's life time budget constraint where type-A education is a consumption good for which the individual is willing to pay:

$$C_1 + \frac{1}{1+r} \cdot C_2 + \frac{[1 - t_w - t_e] \cdot e \cdot H}{1+r} \cdot E_A = \frac{[1 - t_w] \cdot [w + e] - t_e \cdot e}{1+r} \cdot H. \tag{5}$$

The right hand side of (5) represents the individual's full income, which is the maximum achievable income had he chosen only type-B education. The left hand side is the different kinds of consumption. Type-A education is now explicitly viewed as a consumption good with a well defined price, namely the present value of the marginal wage premium by choosing the alternative type-B education. The price of one additional unit of this type-A education is the income he gives up by not

[10]Different types of education have different probabilities of future unemployment, and this affects the expected wage return to education. A higher probability of unemployment for individuals with type-A qualifications would imply a large difference in the expected marginal wage returns to the two kinds of education, with a low w and a high e.

[11]This corresponds to the Scandinavian system of dual income taxation, where tax rates on labour and capital income are set separately.

choosing type-B education. Denote this alternative price of type-A education as p_A :

$$p_A \equiv \frac{[1 - t_w - t_e] \cdot e \cdot H}{1 + r}. \tag{6}$$

The presence of both basic labor income tax t_w and the surtax t_e reduces the price of type-A education as a consumption good, and the substitution effect of taxes induces the individual to get more type-A and less type-B education. This effect is even stronger the higher these tax rates are. The individual makes his consumption and investment decisions for the whole of his life span in the first period. The higher his discount rate, the more weight he puts on the present and the less on the future. That is, the higher the net interest rate r is, the more first period consumption matters relative to second period consumption, and the more type-A education he chooses to consume. The opposite is the result the higher the wage return to type-B education, e, is or the longer the duration of his second period working life, H, is. Then the substitution effect induces the individual to choose less type-A education. Even if the income taxes reduce the price of type-A education as a consumption good, they also reduce total net income. This negative income effect would induce the individual to consume less of all goods, including type-A education. The total effect of the taxes on the individual's educational choice is found in the next section.

This is a partial model that only investigates the individual's educational decision, and hence the government budget constraint is disregarded.

3 The tax analysis.

3.1 The effect of income tax on the educational choice.

In the following, let the prices of first and second period consumption be

$$p_1 \equiv 1, \tag{7}$$

$$p_2 \equiv \frac{1}{1 + r}. \tag{8}$$

This allows us to define the price vector $p = (p_1, p_2, p_A)$. Also, let the individual's full income be defined as y :

$$y \equiv \frac{[1 - t_w] \cdot w + [1 - t_w - t_e] \cdot e}{1 + r} \cdot H. \tag{9}$$

Applying this new notation reduces the individual's life time budget constraint (5) to $p_1C_1 + p_2C_2 + p_AE_A = y$. This new notation simplifies the following development of the response function to a tax change in our particular case.

The individual maximizes his utility under the restriction that his lifetime budget constraint is obeyed. Manipulating the first order conditions and utilizing the first period time constraint, the Marshallian demand functions are found:

$$C_1(p, y), \quad C_2(p, y), \quad \text{and} \quad E_A(p, y).$$

So how does the tax on labor income affect the individual's educational choice? Consider a marginal increase in the tax rates on labour income and investigate how these influence the individual's division of first period time between type-A and type-B education. The effects on the two kinds of education are symmetrical. Since from (1) we have that $E_A + E_B = 1$, it follows that $\Delta E_B = -\Delta E_A$. Hence it is sufficient to investigate the effect of tax changes on type-A education. The effect of a tax change on the demand for education is then given by

$$\frac{\partial E_A}{\partial t_i} = \frac{\partial E_A}{\partial p_A} \cdot \frac{\partial p_A}{\partial t_i} + \frac{\partial E_A}{\partial y} \cdot \frac{\partial y}{\partial t_i}, \qquad i = w, e. \tag{10}$$

As a response function to a tax change, equation (10) is rather unconventional, since the income effect enters twice. A tax increase reduces the price of type-A education as a consumption good. The first element on the right hand side of the equation is this price effect, which consists of the substitution effect and the income effect of a tax increase. But type-A education is also an investment alternative, and the tax reduces the expected return to this investment, measured in expected future wages, and the second element on the right hand side of (10) is this income effect. Thus the tax increase affects the individual's educational choice through two sources; it changes the value of the individual's human capital stock, which in turn determines his income. It also changes the consumption price on education, by which it affects the relative wage return to the two kinds of education. For this reason the second income effect enters the individual's response function.

The first component of the right hand side of (10) reflects how a tax change affects the price of education as a consumption good. This component consists of two factors; the first is the price-effect, which shows how a price change alters the demand for education as a good. The second fraction tells us how much the price of the educational good A is affected by a tax change. The price-effect consists of a substitution effect and an income effect. In this specific case, the Slutsky-equation takes the form

$$\frac{\partial E_A}{\partial p_A} = \frac{\partial E_A}{\partial p_A}|_{\overline{U}} - \frac{\partial E_A}{\partial y} \cdot E_A \tag{11}$$

The total change in the consumption of the educational good A following a price change is given by the substitution effect plus the income effect. The substitution effect states how much a price change affects the individual's consumption of type-A education when his income is adjusted such that he may achieve the same utility level. The price change affects the real income and the purchasing power of the individual. In turn, this affects the achievable consumption bundle of the individual, and this is the income effect.

A tax change also alters the return to education as an investment alternative, namely the second period wage. This is represented by the second component of the right hand side of (10). Increased income induces the individual to consume more of all normal goods, including type-A education, $\frac{\partial E_A}{\partial y} > 0$. But increased taxes reduce total net income, $\frac{\partial y}{\partial t} < 0$. The total of these two effects predicts a negative value on the second component of the right hand side of (10).

Combining all this information, the complete effect of a tax change on the individual's educational decision is given by

$$\frac{\partial E_A}{\partial t_i} = \frac{\partial E_A}{\partial p_A}|_{\overline{U}} \cdot \frac{\partial p_A}{\partial t_i} + \frac{\partial E_A}{\partial y} \cdot \left\{ \frac{\partial y}{\partial t_i} - E_A \cdot \frac{\partial p_A}{\partial t_i} \right\}, \quad i = w, e. \tag{12}$$

Symmetry implies that if the individual chooses less type-A education, he chooses more type-B education. Also, these changes cancel out, such that the total amount of education is the same. Hence we know that

$$\frac{\partial E_B}{\partial t_i} = -\frac{\partial E_A}{\partial t_i}, \quad i = w, e.$$

Equation (12) is the general equation; let us now analyze the two cases $i = w$ and $i = e$ separately.

3.2 The effect of increased top marginal income tax, t_e.

The surtax t_e is levied on the additional wage return e that the individual receives by choosing type-B education. The effect this surtax has on the individual's choice of educational type is found from equation (12) by substituting $i = e$.

$$\frac{\partial E_A}{\partial t_e} = -\frac{e \cdot H}{1+r} \cdot \left\{ \frac{\partial E_A}{\partial p_A} |_{\overline{U}} + \frac{\partial E_A}{\partial y} \cdot E_B \right\}. \tag{13}$$

Type-A education is a normal good, and the substitution effect of a price increase is negative. With increased income the individual consumes more of all goods, and hence the income effect is positive. This tax increase only affects the additional wage return to type-B education, and the basic wage is unaffected by this. The tax reduces the individual's disposable income, but at the same time it reduces the price on type-A education as a consumption good. Whether the increased surtax induces the individual to increase or reduce the amount of type-A education depends entirely on which effect is the dominant, the substitution effect or the income effect.

If the substitution effect dominates the income effect, the individual's preference structure is of a kind that puts great emphasis on the consumption value of type-A education. The tax increase reduces the price on type-A education measured in forgone wage return by not choosing type-B education. Thus the individual changes his educational profile and chooses more of the educational type with the tax free consumption return. Then $\frac{\partial E_A}{\partial t_e} > 0$. The more type-$B$ education the individual has in his original educational portfolio, the stronger is the income effect. An increased top marginal tax rate reduces the return to the education with the less advantageous conditions, and hence the individual chooses less type-B education. This follows from the symmetry assumption in equation (1). The individual experiences a net income reduction through two channels; the tax increase and the reduced investment in type-B education. In order for this to be a sustainable solution, the individual hence reduces his consumption of the other consumption goods, represented by first and second period consumption, C_1 and C_2.

Increased top marginal tax induces the individual to choose less type-A education and more type-B education if the income effect dominates the substitution effect.

The sign of the effect on the individual's educational portfolio of an increase in the surtax depends entirely on the income and substitution effects. But the amplitude of the effect is partly determined by the fraction $\frac{e \cdot H}{1+r}$. The higher the wage return or the length of the second period is, the higher is the return to type-B education, and the larger is the effect of an tax increase.

The importance of the discount rate. The higher the discount rate, the more the individual values consumption and income in the present, and the less he cares about the future income when making his educational choice. The present consumption value of type-A education matters more for the individual than the future expected wages, especially since the price, measured in the present value of future forgone wages, is reduced through this high valuation of the present. A higher discount rate thus dampens the effect of the tax increase. It also alters the relative price between ordinary first period consumption, C_1, and the education good A. The higher the interest rate, the more expensive it is to borrow in the financial market in order to finance first period ordinary consumption, and this reduces the marginal rate of substitution between type-A education and ordinary first period consumption. In our model the discount rate is the net of tax real interest rate. A high tax rate on capital income would thus reduce the discount rate and increase the relative price on type-A education.

Increased uncertainty about the future has the same effect as an increased discount rate. If the future wage return to higher education is uncertain, the expected wage return to type-B education is reduced, and so is the price of type-A education as a consumption good.

3.3 The effect of increased basic labor income tax, t_w.

The tax rate t_w is levied on all wage income earned by an educated worker. From (9), (6) and (12) it follows that

$$\frac{\partial E_A}{\partial t_w} = -\frac{e \cdot H}{1+r} \cdot \left\{ \frac{\partial E_A}{\partial p_A}|_{\overline{U}} + \left[\frac{w}{e} + E_B\right] \cdot \frac{\partial E_A}{\partial y} \right\} \tag{14}$$

which is equivalent to

$$\frac{\partial E_A}{\partial t_w} = \frac{\partial E_A}{\partial t_e} - \frac{w \cdot H}{1+r} \cdot \frac{\partial E_A}{\partial y}. \tag{15}$$

As in the previous case, the effect of this increased tax on the composition of the individual's educational portfolio depends on the individual's preference structure. But, since this tax reduces his disposable income from all sources, and not only the wage return to type-B education, the income effect is more dominant in this case. Even if the income effect and substitution effect would cancel out in equation (13), a tax increase would still induce the individual to consume less type-A education in this case. This is due to the increased importance of the income effect, since the overall return to education is reduced. The importance of the income effect is somewhat neutralized by a high discount factor when the individual values consumption today more than consumption tomorrow.

If the income effect dominates the substitution effect, then the total effect of an increased basic labor income tax is negative. The reduced income level induces the individual to reduce consumption of all goods, including type-A education, and the educational portfolio changes in the direction of less type-A education and more type-B education. This is also true if the income and substitution effects cancel out.

If the individual has very strong preferences for education as a consumption good, he might choose more type-A education when the tax increases. In that case the substitution effect must be so much larger than the income effect as to compensate for the additional weight put on the income effect through the new fraction on the right hand side of equation (15).

These are general results. Now consider a specific utility function as described below, in order to study more closely the importance and sizes of the substitution and income effects.

3.4 A specific utility function.

Let the utility function be given as the Cobb-Douglas function:

$$U = \alpha \cdot \ln E_A + \theta \cdot \ln C_1 + \gamma \cdot \ln C_2, \tag{16}$$

where both first and second period ordinary consumption and type-A education are normal goods ($\alpha > 0$, $\theta > 0$, and $\gamma > 0$)[12]. The individual's lifetime budget constraint is still given by equation (5). The price on type-A education as a consumption good, p_A, and the individual's full income, y, are defined by (6) and (9). In this case the individual's demand function for type-A education is

$$E_A = \frac{\alpha}{\alpha + \theta + \gamma} \cdot \left[1 + \frac{1 - t_w}{1 - t_w - t_e} \cdot \frac{w}{e} \right]. \tag{17}$$

[12]If $\alpha = 0$, the model reduces to the pure human capital model where education is a pure investment alternative and yields no direct consumption value to the individual.

The demand for type-A education depends positively on the weight the individual puts on the consumption value of education, as well as on the relative net wage return to type-A education. The tax system distorts the individual's educational choice only as long as it is progressive. A proportional income tax, $t_e = 0$, does not affect the individual's choice of educational type.

The higher progressivity in the tax system and the tax free consumption value of education induce the individual to choose more type-A education. The size of this effect depends on his relative weighting of this consumption good, $\frac{\alpha}{\alpha+\theta+\gamma}$. The more compressed the wage structure, that is, the higher $\frac{w}{e}$, the larger is the effect of the tax increase. By choosing more type-A and less type-B education his disposable income is reduced, and he thus must reduce his ordinary consumption in both periods. The lower the after-tax wage return to investing in type-B education, the smaller is the income loss by choosing more type-A education, and the less ordinary consumption must he forgo in order to increase the consumption of the educational good.

4 Conclusion.

Economists have thoroughly discussed how the tax system might affect the individual's educational level. But the question of how the tax system affects the individual's choice of educational type has been mostly ignored. The present paper studies this problem in a simple partial model. A progressive tax system distorts the individual's educational choice and induces him to choose more of the educational type with the higher consumption value. The extent of the distortion depends on the individual's preferences.

Since so many effects are present side by side with the consumption motive in the educational choice, it is not possible to draw a uniform policy conclusion from this partial analysis. The main purpose of this paper has been to shed some light on an ignored effect in the literature on taxes, namely the effect on the relative price of different types of education as consumption goods. A natural extension of this model would be to analyze how the presence of uniform and differentiated tuition fees would affect the educational choice of the individual in the presence of taxation when education is considered to be a consumption good.

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6 Mathematical appendix.

The first order conditions :

$$L = U(C_1, C_2, E_A)$$
$$-\lambda\left(C_1 + \frac{C_2}{1+r} + \frac{[1-t_w-t_e]\,eH}{1+r}E_A - \frac{[1-t_w]\cdot[w+e]-t_e\cdot e}{1+r}\cdot H\right)$$

where λ is the marginal utility of income, which is positive.

- $$\frac{\partial L}{\partial C_1} = U_1 - \lambda = 0,$$

- $$\frac{\partial L}{\partial C_2} = U_2 - \frac{\lambda}{1+r} = 0$$

- $$\frac{\partial L}{\partial E_A} = U_A - \frac{\lambda\,[1-t_w-t_e]\cdot e\cdot H}{1+r} = U_A - \lambda p_A = 0$$

Finding the Slutsky-equation (11): By substituting for the demand functions in the consumer's utility function, we find the indirect utility function, $V(p,y)$:

$$V(p,y) \equiv U(C_1(p,y), C_2(p,y), E_A(p,y)) \equiv U(C_1, C_2, E_A)$$

It is also the minimum utility level in the dual problem, namely the consumer's expenditure minimization problem. Solving this yields the consumer's expenditure function, $c(p,V)$:

$$c(p,V) \equiv \left\{\min_x p\cdot[C_1, C_2, E_A],\;\; U(C_1,C_2,E_A) \geq V(p,y)\right\}. \tag{18}$$

Since expenditure minimization is equivalent with utility maximization, we get

$$c(\overrightarrow{p}, V) = y. \tag{19}$$

The expenditure function is concave in the prices. By Shepard's Lemma, the Hicksian demand functions are found by differentiating the expenditure function:

$$h_i(p,V) = \frac{\partial c(p,V)}{\partial p_i}, \qquad i = 1, 2, E_A. \tag{20}$$

From the expenditure function and the Marshallian demand function it follows that

$$E_A = E_A(p, c(p,V)) = h_A(p,V) \tag{21}$$

The Slutsky-equation is found by differentiating (21):

$$\frac{\partial E_A}{\partial p_A} + \frac{\partial E_A}{\partial c} \cdot \frac{\partial c}{\partial p_A} = \frac{\partial h_A}{\partial p_A}$$

$$\Downarrow$$

$$\frac{\partial E_A}{\partial p_A} = \frac{\partial E_A}{\partial p_A}|_{\overline{U}} - \frac{\partial E_A}{\partial y} \cdot E_A \tag{22}$$

This means that the total effect on the demand for type-A education from a tax change is given by:

$$\frac{\partial E_A}{\partial t_i} = \frac{\partial E_A}{\partial p_A} \cdot \frac{\partial p_A}{\partial t_i} + \frac{\partial E_A}{\partial y} \cdot \frac{\partial y}{\partial t_i}$$

$$\Downarrow$$

$$\frac{\partial E_A}{\partial t_i} = \left(\frac{\partial E_A}{\partial p_A}|_{\overline{U}} - \frac{\partial E_A}{\partial y} \cdot E_A\right) \cdot \frac{\partial p_A}{\partial t_i} + \frac{\partial E_A}{\partial y} \cdot \frac{\partial y}{\partial t_i}$$

$$\Downarrow$$

$$\frac{\partial E_A}{\partial t_i} = \frac{\partial E_A}{\partial p_A}|_{\overline{U}} \cdot \frac{\partial p_A}{\partial t_i} + \frac{\partial E_A}{\partial y} \cdot \left\{\frac{\partial y}{\partial t_i} - E_A \cdot \frac{\partial p_A}{\partial t_i}\right\}$$

Developing equation (13): From (9) and (6) we know that

$$\frac{\partial p_A}{\partial t_e} = -\frac{e \cdot H}{1+r}, \text{ and } \frac{\partial y}{\partial t_e} = -\frac{e \cdot H}{1+r}.$$

Applying the above results reduces equation (12) to:

$$\frac{\partial E_A}{\partial t_e} = -\frac{\partial E_A}{\partial p_A}|_{\overline{U}} \cdot \frac{e \cdot H}{1+r} + \frac{\partial E_A}{\partial y} \cdot \left\{-\frac{e \cdot H}{1+r} + E_A \cdot \frac{e \cdot H}{1+r}\right\}$$

$$\Downarrow$$

$$\frac{\partial E_A}{\partial t_e} = -\frac{e \cdot H}{1+r} \cdot \left(\frac{\partial E_A}{\partial p_A}|_{\overline{U}} - \frac{\partial E_A}{\partial y} \cdot \{1 - E_A\}\right)$$

$$\Downarrow 1 - E_A = E_B$$

$$\frac{\partial E_A}{\partial t_e} = -\frac{e \cdot H}{1+r} \cdot \left\{\frac{\partial E_A}{\partial p_A}|_{\overline{U}} + \frac{\partial E_A}{\partial y} \cdot E_B\right\}$$

Developing equation (15): From (9) and (6) it follows that

$$\frac{\partial p_A}{\partial t_w} = -\frac{e \cdot H}{1+r}, \text{ and } \frac{\partial y}{\partial t_w} = -\frac{[w+e] \cdot H}{1+r}.$$

Applying the above results in equation (12) with $i = w$ yields

$$\frac{\partial E_A}{\partial t_w} = \frac{\partial E_A}{\partial p_A}|_{\overline{U}} \cdot \frac{\partial p_A}{\partial t_w} + \frac{\partial E_A}{\partial y} \cdot \left\{ \frac{\partial y}{\partial t_w} - E_A \cdot \frac{\partial p_A}{\partial t_w} \right\}$$

$$\Downarrow$$

$$\frac{\partial E_A}{\partial t_w} = -\frac{H}{1+r} \left(e \cdot \frac{\partial E_A}{\partial p_A}|_{\overline{U}} + \frac{\partial E_A}{\partial y} \cdot \{w + e - e \cdot E_A\} \right)$$

$$\Downarrow$$

$$\frac{\partial E_A}{\partial t_w} = -\frac{e \cdot H}{1+r} \cdot \left\{ \frac{\partial E_A}{\partial p_A}|_{\overline{U}} + \left[\frac{w}{e} + E_B \right] \cdot \frac{\partial E_A}{\partial y} \right\}$$

$$\frac{\partial E_A}{\partial t_w} = -\frac{e \cdot H}{1+r} \cdot \left\{ \frac{\partial E_A}{\partial p_A}|_{\overline{U}} + \left[\frac{w}{e} + E_B \right] \cdot \frac{\partial E_A}{\partial y} \right\},$$

$$\Downarrow \frac{\partial E_A}{\partial t_e} = -\frac{e \cdot H}{1+r} \cdot \left\{ \frac{\partial E_A}{\partial p_A} |_{\overline{U}} + \frac{\partial E_A}{\partial y} \cdot E_B \right\}$$

$$\frac{\partial E_A}{\partial t_w} = \frac{\partial E_A}{\partial t_e} - \frac{w \cdot H}{1+r} \cdot \frac{\partial E_A}{\partial y}$$

Developing the demand function (17): The individual maximizes his utility given that his budget constraint binds, and the Lagrange function is then

$$\pounds = \alpha \cdot \ln E_A + \theta \cdot \ln C_1 + \gamma \cdot \ln C_2 - \lambda \cdot \left[C_1 + \frac{1}{1+r} \cdot C_2 + p_A \cdot E_A - y \right]$$

whith the corresponding first order conditions

$$\frac{\partial \pounds}{\partial C_1} = \frac{\theta}{C_1} - \lambda = 0 \quad \Longrightarrow \lambda = \frac{\theta}{C_1} \tag{23}$$

.

$$\frac{\partial \pounds}{\partial C_1} = \frac{\gamma}{C_2} - \frac{\lambda}{1+r} = 0 \quad \Longrightarrow C_2 = C_1 \cdot \frac{\gamma \cdot (1+r)}{\theta} \tag{24}$$

.

$$\frac{\partial \pounds}{\partial E_A} = \frac{\alpha}{E_A} - \lambda \cdot p_A = 0 \quad \Longrightarrow E_A = C_1 \cdot \frac{\alpha}{\theta \cdot p_A} \tag{25}$$

.

$$\frac{\partial \pounds}{\partial \lambda} = -\left[C_1 + \frac{1}{1+r} \cdot C_2 + p_A \cdot E_A - y \right] = 0 \tag{26}$$

The marshallian demand functions are found by combining the first order conditions. From (26) it follows that

$$
\begin{aligned}
&C_1 = y - \frac{1}{1+r} \cdot C_2 - p_A \cdot E_A \\
&\quad \Downarrow \text{(24) and (25)} \\
&C_1 = y - \frac{1}{1+r} \cdot C_1 \cdot \frac{\gamma \cdot (1+r)}{\theta} - p_A \cdot C_1 \cdot \frac{\alpha}{\theta \cdot p_A} \\
&\quad \Downarrow \\
&C_1 = y \cdot \frac{\theta}{\alpha + \theta + \gamma}
\end{aligned}
$$

Applying this expression in (25) yields

$$
\begin{aligned}
&E_A = C_1 \cdot \frac{\alpha}{\theta \cdot p_A} = y \cdot \frac{\theta}{\alpha + \theta + \gamma} \cdot \frac{\alpha}{\theta \cdot p_A} \\
&\Downarrow \\
&E_A = \frac{y \cdot \alpha}{(\alpha + \theta + \gamma) \cdot p_A} = \frac{\alpha}{\alpha + \theta + \gamma} \cdot \left[1 + \cdot \frac{1 - t_w}{1 - t_w - t_e} \cdot \frac{w}{e} \right]
\end{aligned}
$$

The individual can at most achieve the utility level V, and evaluated at this point, the compensated demand function is identical to (17). Thus the substitution effect is given by

$$
\frac{\partial E_A}{\partial p_A}|_{U=V} = -y \cdot \frac{\alpha}{(\alpha + \theta + \gamma) \cdot p_A^2}, \tag{27}
$$

while the income effect is given by

$$
\frac{\partial E_A}{\partial y} \cdot E_A = y \cdot \frac{\alpha}{(\alpha + \theta + \gamma)^2 \cdot p_A^2}. \tag{28}
$$

$$
\frac{\partial E_A}{\partial y} = \frac{\alpha}{(\alpha + \theta + \gamma) \cdot p_A}
$$

$$\frac{\partial E_A}{\partial t_e} = -\frac{e \cdot H}{1+r} \cdot \left\{ \frac{\partial E_A}{\partial p_A}|_{\overline{U}} + \frac{\partial E_A}{\partial y} \cdot (1 - E_A) \right\}$$

$$\Downarrow$$

$$\frac{\partial E_A}{\partial t_e} = -\frac{e \cdot H}{1+r} \cdot \left\{ -\frac{\alpha \cdot y}{(\alpha+\theta+\gamma) \cdot p_A^2} + \frac{\alpha}{(\alpha+\theta+\gamma) \cdot p_A} \cdot \left(1 - \frac{y \cdot \alpha}{(\alpha+\theta+\gamma) \cdot p_A}\right) \right\}$$

$$\Downarrow$$

$$\frac{\partial E_A}{\partial t_e} = \frac{e}{p_A^2} \cdot \frac{H}{1+r} \cdot \frac{\alpha}{(\alpha+\theta+\gamma)} \cdot \left\{ y - p_A + y \cdot \frac{\alpha}{(\alpha+\theta+\gamma)} \right\}$$

$$\Downarrow y = \frac{[1-t_w] \cdot w \cdot H}{1+r} + p_A$$

$$\frac{\partial E_A}{\partial t_e} = \frac{e}{p_A^2} \cdot \frac{H}{1+r} \cdot \frac{\alpha}{(\alpha+\theta+\gamma)} \cdot \left\{ \frac{[1-t_w] \cdot w \cdot H}{1+r} + y \cdot \frac{\alpha}{(\alpha+\theta+\gamma)} \right\} > 0$$

$$y = \frac{[1-t_w] \cdot w + [1-t_w-t_e] \cdot e}{1+r} \cdot H, \qquad p_A = \frac{[1-t_w-t_e] \cdot e}{1+r} \cdot H$$

$$\Downarrow$$

$$\frac{\partial E_A}{\partial t_e} = \frac{1}{[1-t_w-t_e]^2 \cdot e \cdot \frac{H}{1+r}} \cdot \frac{\alpha}{(\alpha+\theta+\gamma)} \cdot \frac{H}{1+r} \cdot \left\{ \begin{array}{c} [1-t_w] \cdot w \\ + [[1-t_w] \cdot w + [1-t_w-t_e] \cdot e] \cdot \frac{\alpha}{(\alpha+\theta+\gamma)} \end{array} \right.$$

$$\Downarrow$$

$$\frac{\partial E_A}{\partial t_e} = \frac{1}{[1-t_w-t_e]} \cdot \frac{\alpha}{(\alpha+\theta+\gamma)} \cdot \left\{ \frac{\alpha}{(\alpha+\theta+\gamma)} + \frac{(1-t_w) \cdot w}{(1-t_w-t_e) \cdot e} \cdot \left[1 + \frac{\alpha}{(\alpha+\theta+\gamma)}\right] \right\}$$

$$\Downarrow$$

$$\frac{\partial E_A}{\partial t_e} = \frac{1}{[1-t_w-t_e]} \cdot \frac{\alpha}{(\alpha+\theta+\gamma)^2} \cdot \left\{ \alpha + \frac{(1-t_w) \cdot w}{(1-t_w-t_e) \cdot e} \cdot (2\alpha+\theta+\gamma) \right\} > 0$$

$$\Downarrow$$

$$\frac{\partial E_A}{\partial t_e} = \frac{\left(\frac{\alpha}{\alpha+\theta+\gamma}\right)^2 + \frac{1-t_w}{1-t_w-t_e} \cdot \frac{w}{e} \cdot \frac{\alpha \cdot (2\alpha+\theta+\gamma)}{(\alpha+\theta+\gamma)^2}}{1-t_w-t_e} > 0$$

$$\frac{\partial E_A}{\partial t_w} = \frac{\partial E_A}{\partial t_e} - \frac{w \cdot H}{1+r} \cdot \frac{\partial E_A}{\partial y}$$

$$\Downarrow$$

$$\frac{\partial E_A}{\partial t_w} = \frac{e}{p_A^2} \cdot \frac{H}{1+r} \cdot \frac{\alpha}{(\alpha+\theta+\gamma)} \cdot \left\{ y - p_A + y \cdot \frac{\alpha}{(\alpha+\theta+\gamma)} \right\} - \frac{w \cdot H}{1+r} \cdot \frac{\alpha}{(\alpha+\theta+\gamma) \cdot p_A}$$

$$\Downarrow$$

$$\frac{\partial E_A}{\partial t_w} = \frac{H}{1+r} \cdot \frac{\alpha \cdot e}{(\alpha+\theta+\gamma) \cdot p_A^2} \cdot \left\{ y - \frac{w+e}{e} \cdot p_A + y \cdot \frac{\alpha}{(\alpha+\theta+\gamma)} \right\}$$

$$\Downarrow$$

$$\frac{\partial E_A}{\partial t_w} = \frac{\alpha}{(\alpha+\theta+\gamma)^2 \cdot [1 - t_w - t_e]} \cdot \left\{ \alpha + \frac{t_e \cdot (2\alpha+\theta+\gamma)}{1 - t_w - t_e} \cdot \frac{w}{e} \right\}$$

$$\Downarrow$$

$$\frac{\partial E_A}{\partial t_w} = \frac{\left(\frac{\alpha}{\alpha+\theta+\gamma}\right)^2 + \frac{t_e}{1-t_w-t_e} \cdot \frac{w}{e} \cdot \frac{\alpha \cdot (2\alpha+\theta+\gamma)}{(\alpha+\theta+\gamma)^2}}{1 - t_w - t_e}$$

Chapter 5

Measuring the Consumption Value of Higher Education*

Abstract

The consumption value of education is an important, but rather ignored factor behind the individual's educational choice. This paper suggests a method for measuring the consumption value of education in a compensating differentials framework when the ability bias is corrected for. As an example, the willingness to pay for the consumption value of attending teacher's college during the 1960's is estimated on unique Norwegian panel data. The ex-ante price of the consumption value of teacher's college is estimated to be 38 % of the present value of the individual's potential lifetime income. The ex-post price of this consumption value is for the same individuals estimated to be about 46 % of the present value of the potential lifetime income.

JEL-classifications: J24; J31; J33; I21; H89.

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1 Introduction.

Higher education can be viewed both as a consumption good for which the individual is willing to pay, and as an investment alternative that yields higher wages later in life. The factors determining the individual's educational choice can be divided into three groups: preferences, returns, and costs. The costs of attending higher education are effort, time and money, both direct monetary outlays and forgone labor income[1]. The return to higher education comes both as pecuniary and non-pecuniary returns. As higher education increases the skill level, and thus also the productivity of the individual, he is paid a higher wage in the labor market. Also, higher skilled individuals qualify for different types of jobs than lower skilled individuals. High-skilled jobs often offer various fringe benefits, which are not paid as money, but which are all equivalent to a wage increase. Fringe benefits[2] and the wage premium constitute the pecuniary return to higher education. The individual specific non-pecuniary return to higher education is the intrinsic or the *consumption value of education*, which is defined in section 3.

This paper suggests a method for measuring the consumption value of education in a compensating differentials framework when the ability bias is corrected for. The identification strategy is to compare two individuals who attended teacher's college and business school respectively in Norway during the 1960's. In this period these two types of education required the same minimum average grade level from high school for admittance, but they generated very different wage returns. The wage return from attending business school in this period is used as a benchmark for the potential wage return of the teacher's college graduates. Using the Norwegian 1970 census, cross section wage profiles are estimated for those business school and teacher's college graduates with different levels of working experience. These wage profiles are interpreted as the expected future wages of the individuals attending business school and teacher's college during the 1960's. The ex-ante price of the consumption value of teacher's college is estimated to be 38 % of the present value of the individual's potential lifetime income. Using unique Norwegian panel data the actual wage profiles for the individuals acquiring their education during the 1960's are estimated. The ex-post price of this consumption value of teacher's college turned

[1]Costs are disregarded in the following analysis.

[2]Fringe benefits are here defined to be benefits with a clear monetary equivalent, such as a company car, free newspaper subscriptions, and a company health insurance.

out to be about 46 % of the present value of their potential lifetime income.

The goal of the paper is not to find the exact value of the willingness to pay for the consumption value of education, but rather to establish as a fact that the consumption value of education does exist and that it is an important factor behind the individual's educational choice. As shown by the example, many individuals are willing to give up substantial future wage returns in order to acquire the educational type of their choice. Therefore, the consumption value of education should not be ignored when modeling the individual's educational choice and estimating the return to education.

The paper is organized as follows: Section 2 gives an historical overview of the debate on the return to education, and section 3 discusses and defines the concept of consumption value of education. The estimation of the monetary value of the consumption value of teacher's college relative to business school is conducted in section 4. Section 5 concludes.

2 Higher education: Investment or consumption?

Prior to the human capital revolution in the 1960's, education was considered to be a consumption good. One shortcoming of this framework was that it ignored the fact that pursuing education actually increases the productivity of the individual and his wages in the next period. Schultz[3] (1960) and Becker (1964) introduced the theory of human capital, where education is an investment that increases the individual's wage in the next period. The individual acquires education until the present value of the expected marginal wage return equals the marginal return of other investment alternatives. The cost of the investment is the sum of the direct costs, such as tuition fees, books and other expenses, and forgone labor income. This theory was highly controversial at the time, since education was considered to be a cultural good. Schultz (1960) stated that "it is held by many to be degrading to man and morally wrong to look upon his education as a way of creating capital. ...For them education is basically cultural and not economic in its purpose, because education serves to develop individuals to become responsible citizens. ...My reply to those who believe

[3]"I propose to treat education as an investment in man and to treat its consequences as a form of capital. Since education becomes a part of the person receiving it, I shall refer to it as *human capital.*" Shultz (1960).

thus is that an analysis that treats education as one of the activities that may add to the stock of human capital in no way denies the validity of their position... Some kind of education may improve the capabilities of a people as they work and manage their affairs, and these improvements may increase the national income."

Mincer (1974)[4] developed the framework which is still the most frequently used in the empirical estimation of the wage return to education. A simplifying assumption in this model is that the wage return to job experience and the wage return to education can be estimated separately. In his model, the log of the individual's earnings, Y, in a period can be decomposed into an additive function of a linear education term and a quadric experience term:

$$\ln Y = \beta_1 + \beta_2 E + \beta_3 X + \beta_4 X^2 + \epsilon, \tag{1}$$

where E is the length of the completed education in years, X represents the number of years of work experience[5] after leaving school, and ϵ is the residual. The parameter β_2 is then the rate of return to an additional year of education. This marginal return to education is assumed to be independent of both type and level of education. β_3 represents the return to experience, which is expected to be concave, and β_4 estimates the extent of this concavity. But there are problems with this approach. It assumes that education increases the individual's wage, but it could also be that this is a result of individuals with high innate income potential choosing to acquire higher education, such that there is an ability bias in the sample. Also, the relationship between occupational choice, earnings, and job attributes is simultaneously determined; the reward structure determines the educational choice, and the educational choice determines the reward structure. Thus the amount of schooling included in the wage equation is not exogenous, and a simultaneity problem exists. Another problem is that there is heterogeneity in the wage return to human capital investments. Willis and Rosen (1979) claim that this induces the individual to choose the type of education for which he has a comparative advantage given his innate abilities. The analysis in the present paper concentrates on solving the selection problem.

Increasingly sophisticated econometric methods have been developed to correct for the above described estimation problems, and this has been the focal point in

[4] See Chiswick (2003) for a retrospective discussion of the importance of Mincer's contribution.

[5] In the absence of direct information on work experience Mincer suggested to use "potential experience", which is the individual's age minus his school starting age minus years in school.

the empirical literature over the last twenty years.[6] The existence of other motives for the individual's educational choice besides higher future wages has been more or less ignored. Instead of analyzing what motivates the individual's educational choice, the effort has been concentrated on analyzing the most easily measurable outcome of this choice, namely the effect on wages.

But economics is the theory of choice, and it deals with the satisfaction of human desires through choice of actions. Human desires are satisfied by human interaction and through economic activity, which is the exchange of goods and services. Plato defined three types of desires; desire for wisdom and knowledge, for honor, fame, and power, and the appetitive desires, which are usually satisfied through spending money. The satisfaction of these desires is motivated and accompanied by pleasure, which is necessary up to a point and harmful when pursued in excess. Marshall distinguished between wants and activities. Wants are satisfied by consumption of services and goods, while activities either contribute to the production of goods and services or are pleasurable in themselves.

Adam Smith was the first to formulate the idea of monetary and non-monetary compensations of a job, an idea later formalized in the compensating differentials literature: "The five following are the principal circumstances which, so far as I have been able to observe, make up for a small pecuniary gain in some employments, and counterbalance a great one in others: first, the agreeableness or disagreeableness of the employments themselves; secondly, the easiness and cheapness, or the difficulty and expense of learning them; thirdly, the constancy or inconstancy of employment in them; fourthly, the small or great trust which must be reposed in those who exercise them; and fifthly, the probability or improbability of success in them. ...Honour makes a great part of the reward of all honorable professions. In point of pecuniary gain, all things considered, they are generally under-recompensed. ...Disgrace has the contrary effect."[7] Later Marshall (1920) stated that "the true reward which an occupation offers the labourer has to be calculated by deducting the money value of all disadvantages from that of all its advantages".

The fact that activities can be pleasurable in themselves and help satisfy certain desires has for a long time been widely ignored in the economic literature. There are a few exceptions, though. Lazaer (1977) finds in his sample of US males that lower levels of higher education are considered a consumption bad by the individual,

[6]See Card (1999) for an extensive literature overview of this field.

[7]The Wealth of Nations, Book 1, Ch. 10, Part1.

while MA's and PhD's are considered to be consumption goods[8]. Oosterbeek and van Ophem (2000) allow the individual to have immediate utility from schooling, and individuals maximize lifetime utility instead of the usual lifetime income approach. They find that the young Dutch individuals in their sample invest too much in education compared with what is optimal from the human capital theory, and they conclude that the consumption motives with regard to schooling are indeed important[9]. Kodde and Ritzen (1984) combine the human capital model and the consumption model and find that the individual demands more education than in the pure human capital model. This could be due to the direct utility gain he experiences through the consumption of education. Oosterbeek and Webbink (1995) find that the integrated model where education is both an investment alternative and a consumption good is the best to explain the educational choices of the young individuals, and that both the consumption motive and the investment motive matter. The shortcomings of the pure human capital model in explaining the individuals' educational choices are also pointed out by Oreopoulos (2003).

More work has been done on identifying different non-pecuniary returns to a particular job, such as pleasant working conditions or status. Examples are Antos and Rosen (1975), Ward and Sloane (2000), and Scott (2001), who all apply the compensating differentials framework described in Rosen (1986).

Stern (1999) considers a sample of postdoctoral biologists who decide where to start working, and who are offered jobs with different job characteristics. The result suggests a strong negative relationship between wages and the opportunity to engage in scientific activity; the biologists have to pay, in forgone wages, to be able to do scientific work. Firms who allow their employees to publish papers based on their results from the job pay on average 25% lower wages than the firms who do not allow their employees to engage in academic activity. This line of reason is also followed by Klette and Møen (2002), who state that academics pay a considerable price for their academic joy, measured in forgone labor income by not working in the private sector.

The literature also mostly ignores that different types of education generate dif-

[8]Gullason (1989) also finds a positive "consumption value" to schooling for US males, where most of this value consisted of avoiding being drafted for the Vietnam war as long as the person was in school.

[9]This idea was already promoted by Schaafsma (1976).

ferent rates of wage return. Education is assumed to be a homogenous good that generates an annual rate of return. One exception is Keane and Wolpin (1997). In a dynamic structural model they consider self-selection in three heterogenic dimensions: schooling, work, and occupational choice, and they find that most of the variance in lifetime utility is explained by inequality in skill endowment. Aakvik et al. (2003) also find on rich Norwegian panel data that the wage return to education is heterogenous among individuals.

Although the existence of non-pecuniary returns to education is acknowledged in the literature, they are seldom included in the formal analysis. The non-pecuniary returns to education are mostly only mentioned anecdotally, and a proper definition of the consumption value of education is to my knowledge missing in the literature. The discussion below aims at correcting for this.

3 Non-pecuniary returns to higher education.

Acquiring higher education has many effects; some serve as incentives for the individual at the time of the educational choice, whereas others are by-products of the educational process. The non-pecuniary return to higher education can from the individual's point of view be divided into two groups; intended and unintended non-pecuniary benefits. The consumption value of higher education is the intended non-pecuniary returns to education; these are the factors the individual is aware of at the time of the educational choice. But there are other non-pecuniary returns to higher education, of which the individual may not be aware at the time of his educational choice. These are the unintended non-pecuniary returns to higher education.

3.1 The consumption value of higher education.

Substantial non-pecuniary advantages and returns to education exist, both during the educational process and after its completion. Duncan (1976) defined the consumption benefits of a job as the positive flow of satisfaction provided by the work situation. This may be enjoyment, interest, challenge, and social relationships, which are all subjective relations of individuals to the job situation. Higher education enables the individual to choose from a broader specter of jobs that are mostly considered more interesting and more challenging (Weisbrod, 1962). Higher education makes the individual more flexible in the type of job he is able to perform, as well as in where to perform it, which provides insurance against unemployment (Bishop, 1994). This flexibility varies between different types of education, and individuals who prefer to live in a particular area or who prefer the option of part-time work will choose educational types that lead to jobs with these attributes.

Different types of education differ in how much effort is required from the individual to complete the education. The effort level required in the jobs available after completed education also varies. Low effort input and thus much leisure are qualities valued by many individuals. There is also a non-dismissable increase in social status from completing a higher education. Dolton et al. (1989) find that among arts and social science graduates it seems like occupational status plays an important role in the educational choice.

The consumption value of education while acquiring it consists among other things of the joy of learning new things, meeting new people, moving to a new city, and participating in campus and student activities, in addition to the increased status in the society that often comes from being a student of particular fields[10]. Nerdrum (1999) discusses this in detail and states that "some people choose to become students mostly to be able to take part in such a way of life. Their aim is principally directed towards immediate consumption, and they consider the other effects, like positive monetary returns, as pure positive by-products".

I summarize all these non-pecuniary returns to education as *the consumption value of education.*

Classification difficulties. Not all non-pecuniary returns to education are straightforward to classify. For instance, Nerdrum (1999) states that memberships in clubs and organizations during their time as a student provide the individuals with a network of people spread over the world, both for professional and private purposes, which often prove to be extremely valuable. If having this network provides the individual with an intrinsic joy, it should be counted as part of the consumption value of education. But if this network furthers his career, it is a kind of investment during the education that yields a future monetary return, and it should not be

[10] Scitovsky (1976) states that as countries get richer and the individuals have more leisure, they need satisfaction to avoid boredom. He also states that education is one such stimulus that increases satisfaction both during and after the educational period if chosen correctly.

regarded as a consumption value of education.

The fact that one educational type requires less effort both during the educational process as well as in the future jobs is above defined as a consumption value, since the individual enjoys having more leisure. But one could also claim that this educational type has lower investment costs, measured in effort.

3.2 Unintended non-pecuniary returns to education.

When making his educational choice, the individual maximizes his ex-ante preferences, and thus the consumption value of education ought to be measured at this point in time. The educational process might change his preferences, such that his ex-post preferences differ from his ex-ante preferences, along with his ex-post valuation of the consumption value of education. These changes in preferences are ex-ante unforeseen. They do not serve as a motivation behind the individual's educational choice, and should thus not be included in the ex-ante consumption value of education. See Sandmo (1983) for a discussion of ex-ante versus ex-post welfare evaluations.

Unintended individual returns. The human capital theory allows for the existence of consumption effects of education, but they are only mentioned anecdotally and consist of factors such as learning to appreciate opera and reading Goethe in the original language[11]. These changes in preferences are unintended, since they are results of influence on the individual during the time of his education. They are not the result of a conscious choice, since he did not treasure these things at the time of the educational choice. Individuals make their educational choice in order to maximize their utility according to their ex-ante preferences. Thus this effect is not part of the consumption value of education as defined in this paper.

If interpreted within a framework similar to the "Rational Addiction"[12] approach of Becker and Murphy (1988), or more generally the "Extended Preference" approach of Becker (1996), the conclusion is the opposite of the one above. These approaches generalize the usual discounted utility model, by letting the instantaneous utility in any given period be a function of past consumption experiences. In the intertemporal

[11]See for instance Judd (2001) and Nerdrum (1999).

[12]See Wangen (2003) for a discussion of this.

optimization problem, the rational consumer takes into account that even if he prefers rock music to classical music in the present period - according to today's instantaneous utility function - he foresees that the educational process changes his future instantaneous utility functions in a way that will make him prefer classical music.

Preferences: Shifts and history dependence. In most economic models individuals' preferences are assumed to be exogenously given and constant over time, when they are in fact influenced and shaped by the surroundings. New information, learning, experience, innovation, and human interactions affect the individual and might induce a shift in his preferences over time. If the individuals were to make their educational choices at the age of five, we would have nations of firemen! This section discusses these preference shifts in more detail.

Croix (2001) claims that intergenerational spillover has taste externalities, as when fear of insects or career aspirations are transmitted from parents to children. Hægeland et al. (1999) find that parental educational level has a positive effect on the length of the education the children acquire. Preferences are also transmitted through the habit formation effect, which reflects the effects of past decisions on the perception of current outcomes. Different aspects of the consumption value of higher education can be subject to history dependence, as stated by Acemoglu (1995). New generations learn from older generations and to some extent inherit established value judgements and attitudes. For instance, what is perceived to give social status and prestige changes over time as the external factors such as political regime, religion, and economics change[13]. This affects who chooses the different occupations and thus also the distribution of talent in the society.

Bowles (1972) argues that "there is considerable evidence that rich, high status parents place a larger value on the non-pecuniary aspects of work and a lower value on monetary returns than poorer, lower status parents". Osterbeek and van Ophem (2000) find support for this; the consumption motive for the educational choice seems to be

[13] Acemoglu (1995) mentions as an example the fall and rise of the merchant's status in the Mediterranean area: "The arrival of Islam in the Mediterranean in the eight century stopped commerce through this sea to a large extent. This lead to the disappearance of merchants. In the twelfth century, the Christian counterattack against Islam started and Europeans took once again control of the Mediterranean. This gradually led to the renewed trade and to the activity organized around towns and merchants."

more important the higher the social background the individual has, and the better skilled he is. They also find that children of highly educated fathers or fathers with higher level occupations have lower discount rates than children of lower educated fathers or fathers with lower level occupations. This means that a child from a poorer family seems to attach lower weight to future earnings than children in richer families do.

As individuals' preferences might change over time, so might their discount rates. Most individuals acquire higher education when they are young. One could claim that young people in general have short time horizons and high discount rates when making their choices. Thus they put more weight on the present consumption value of education than on the future income possibilities when making their educational choice. Later in life they might regret this and have a lower willingness to pay for the consumption value of education (measured in forgone labor income). This type of time inconsistency and hyperbolic discounting is discussed by Ainslie and Haslam (1992). This problem is avoided in the following empirical analysis by applying the individuals' ex-ante preferences in the estimation of the price of the consumption value of education and assuming a constant discount rate.

Social returns to education. The altered preference structures during the educational process have positive effects on the welfare in the society if they induce the individual to take better care of his health and to become a better citizen. Lochner and Moretti (2001) find that education has a causal negative effect on incarceration, Lleras-Muney (2002) finds that education has a causal negative effect on mortality, while Milligan et al. (2003) find that schooling improves civic participation in political processes. Also, higher education has a positive effect on economic growth through technological innovation from increased knowledge spill-overs (Lucas, 1988, and Romer, 1990). These are all reasons why many countries subsidize higher education substantially.

3.3 Uncertainty.

As the individual makes his educational choice based on his expectations of the returns to the investment, both pecuniary and non-pecuniary, there is considerable uncertainty present. It might very well be that he has incomplete information of the content and thus also the consumption value of the education, or that his preferences change during the education process, as discussed above. Also, since there is a substantial lag from when the investment decision is made to when the pecuniary return is generated, he needs to make this investment decision based on his expectations of future wages, job openings, taxes etc. Due to poor information, business cycles, politics, and his future health these expectations are uncertain and very much based on the present situation in the society at the time when he makes his educational choice. When making his educational choice, the individual has a full range of types to choose from, but after the completion of the education he has limited options of which careers to pursue, and this represents a potential lock-in effect.

4 A method for measuring the consumption value of higher education.

I apply the compensating differentials framework to measure the consumption value of teacher's college. The model is described below, along with the data and the approach to correct for the innate abilities of the individuals. The results are presented and discussed in the last part of this section.

4.1 Compensating differentials.

Rosen (1986) states that the theory of compensating differentials "refers to observed wage differences required to equalize the total pecuniary and non-pecuniary advantages or disadvantages among work activities and among workers themselves". A modified version of Rosen's model will in the following be applied to measure one particular individual's valuation of the consumption value of *type-A* education when *type-B* education is used as benchmark.

The individual maximizes his utility U, which depends positively on both ordinary consumption C and the consumption value e_i of education E_i :

$$U = u(C, e_i), \qquad i = A, B.$$

e_i is an index of the consumption value of type-i education; the higher the consumption value, the higher the value of e_i. The consumption value index is individual-specific, such that when one individual has higher consumption value of type-A education, another individual may have higher consumption value of type-B education.

Assume that all income is consumed, and that the individual only lives for one period. He acquires education at the beginning of the period and works and consumes in the end of the period. His consumption level thus equals his wage income, $C = w_i$, where the net of tax wage level depends on the type of education chosen. The individual's utility function can be written as

$$U = u(w_i, e_i), \qquad i = A, B. \tag{2}$$

Both variables are continuous, and fringe benefits are not considered. There are no non-wage types of monetary income in the model. This is a one-period model, but w_i and e_i can be viewed as the present values of lifetime income and consumption value that the individual experiences by choosing *type-i* education. At the beginning of the period the individual makes his educational choice, and he may choose between the two educational types A and B, which differ in both consumption value and wage return. For this particular individual, type-A education has the higher consumption value:

$$e_A > e_B. \tag{3}$$

For a given wage return, $\overline{w}$, to both kinds of education, the individual always prefers type-A education, since it holds the higher consumption value to him:

$$u(\overline{w}, e_A) > u(\overline{w}, e_B).$$

The decision is more complicated if the wage return differs between the two types of education. Then the combination of individual preferences, wage return, and consumption value of the educational type determines which is preferred. Let w_B^* be the wage return to type-B education that the individual requires in order to be indifferent between the two educational types when type-A education has the wage return w_A :

$$u(w_A, e_A) = u(w_B^*, e_B). \tag{4}$$

Since type-B education is never preferred to type-A education if they have the same wage return, it follows that

$$w_B^* > w_A. \tag{5}$$

Now define the difference

$$D = w_B^* - w_A$$

as the *individual compensating differential* for type-A education compared with type-B education. The individual compensating differential D is the additional wage return to type-B education necessary to make the individual indifferent between the two educational types at their given consumption values. Thus D *is the wage return that the individual is willing to forgo in order to enjoy the consumption value* e_A *rather than* e_B. This willingness to pay for the consumption value of type-A is individual-specific. For now we only consider one individual, but when we expand the model to consider a group of individuals, D will vary among the individuals choosing type-A education.

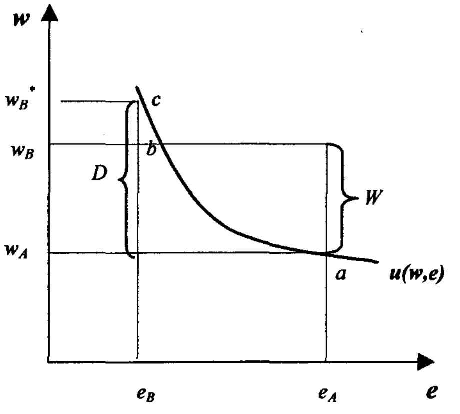
Let W be the *market compensating wage differential,* defined as the difference in the market wage returns to type-B and type-A education.:

$$W = w_B - w_A. \tag{6}$$

The market offers the individual the additional wage return W if he chooses type-B education and forgoes the additional consumption value he could have enjoyed by choosing type-A education. If the individual compensating wage differential is the same as the market compensating wage differential, $D = W$, then the individual is indifferent between the two types of education. If $D < W$, the market offers a greater wage compensation for choosing type-B education than is required by the individual. He chooses type-B education and thus increases his consumption level by more than what is required to compensate for the utility loss by not enjoying the consumption value of type-A education. On the other hand, if $D > W$, the individual chooses type-A education, since the wage premium by choosing type-B education is less than what is required to compensate for the utility loss he experiences by not choosing type-A education.

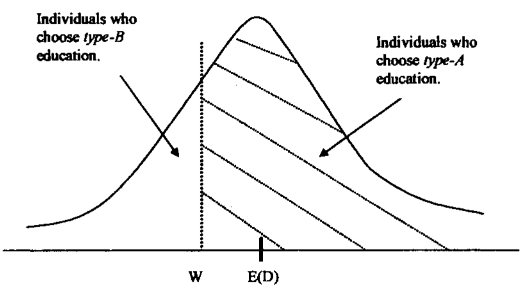
Now consider figure 1, where an example of one particular preference structure is displayed. Type-A education offers the reward structure (e_A, w_A), point a, and type-B education offers the reward structure (e_B, w_B), point b. The individual requires the wage w_B^* in order to be indifferent between the two kinds of education, and here $w_B^* > w_B$. The individual compensating wage differential, D, is given by the vertical distance between points c and a, while the market compensating wage differential, W, is given by the vertical distance between the points b and a. Thus, at this given preference structure and wage structure the individual is under-compensated by the wage return to type-B education for forgoing the consumption value of type-A education, $D > W$, and the individual chooses type-A education.

Figure 1: *Individual and market wage differentials and the choice of educational type.*



The market compensating wage differential, W, is the market price of the consumption value of type-A education, and it is available to all individuals. Still, individuals differ in their preferences, and so does the individual compensating wage differential, D. As an example, let the educational preferences of all individuals be distributed over the individual taste variable D as illustrated in figure 2. Assume that they all have the same level of innate abilities. The average value of the individual compensating wage differential is represented by $E(D)$. As already discussed, the individual's preferences might change due to external influences. This would shift the distribution of preferences and also the average value of the individual compensating wage differential. The market offers the compensation W to the individuals who forgo the consumption value of type-A education and instead choose type-B education. In this specific case $W < E(D)$, and the majority of the individuals choose type-A education, since the forgone labour income by doing so is less than the price they are willing to pay for the consumption value of type-A education, D. As W increases, some individuals are no longer willing to forgo that high a wage return in order to enjoy type-A education, and more individuals choose type-B education. The individuals who choose type-B education have the lowest preferences for the consumption value of type-A education. Since individuals differ in taste, their reservation wage return, D, also differs. This ensures the existence of economic rent in the labour market. Most individuals who choose type-B education receive an economic

Figure 2: *Distribution of individual compensating wage differentials and the choice of educational type.*



rent of the size $W - D$. The marginal individuals earn no economic rent, while most individuals who choose type-A education also receive an economic rent, since their willingness to pay for the consumption value of type-A education is higher than the actual price demanded by the market in the form of the market compensating wage differential. The conclusion from this is that *the market compensating wage differential W serves as a lower bound on the willingness to pay for the consumption value of type-A education among the individuals choosing it.*

Income taxes. In the above model, the market compensating wage differential W is defined in the absence of taxes. Intuitively, one would expect income taxes, T, to reduce the net market compensating wage differential, W_n, available to the individual:

$$W_n = W - T.$$

The consumption value of education is a tax free return to human capital investments. Progressive income taxes reduce the wage return to type-B education relatively more than the wage return to type-A education, and this could reduce the net market compensating wage differential. See Alstadsæter (2003) for a discussion of how the tax system might induce the individuals to choose more of the educational type with the higher consumption value.

But the above discussion implicitly assumes that the gross wage differential is unaffected by taxes, which is usually not the case. As Persson and Sandmo (2002) show in a special case, increased progressivity in the tax schedule might actually lead to higher after tax wage inequality. To say anything about the effects of different tax schedules on the net of taxes wage differentials requires a thorough discussion on the wage determination mechanisms, but this goes beyond the scope of the paper.

Selection problem. The market compensating wage differential might be measured by comparing two types of education with different consumption values. If all individuals had the same level of innate abilities, the difference in the wage return to the two educational types would be the individual's average minimum willingness to pay for the consumption value of the more beneficial educational type. But different individuals have different innate abilities, experiences, and personalities. The wage return to the educational type is now partly endogenous, depending on innate individual ability. The individuals also have different views on which educational type has the higher consumption value.

The selection problem can be accounted for by finding two individuals with the same level of innate ability, but who have different preferences and make different educational and career choices. One possible approach to this is the growing identical-twin study literature (see Ashenfelter and Rouse, 1998). This strand of literature utilizes surveys on identical twins, who are assumed to have the same level of innate abilities. The wage return to one additional year of education is estimated by using the earnings of the other twin as a benchmark for the given ability level. But this method is controversial. Bound and Solon (1999) state that "even monozygonic twins are a little different, and their (often small) differences in abilities and temper may contribute to their (often small) differences in schooling."

This paper proposes an alternative approach. The identification strategy is to compare individuals with approximately the same grade level at high school graduation, but who choose different types of higher education. Grades are here used as an instrument for ability. The individuals who attended teacher's college (type-A education) during the 1960's could have attended business school (type-B) and experienced a much higher wage return but chose the higher consumption value of teacher's college. Thus wage return to business school is the benchmark for their potential future wage return[14]. The educational choice here also implicitly means

[14]This does not mean that business school has a low or negative consumption value for the individuals actually choosing to attend business school. It might very well be that these individuals'

a choice of sector, since most teachers work in the public sector and a majority of business school graduates work in the private sector.

4.2 The consumption value of teacher's college.

Teaching used to be considered a noble profession, and as late as in the 1960's many considered teaching a calling[15], and admission requirements were strict. It is remarkable that teaching was such a popular profession, given that teachers had modest salaries compared with many other jobs available to skilled individuals[16]. One reason for this is that the gender wage discrimination was small among teachers, and that it was a profession easier for women to combine with raising children. Women go in and out of the labor force more frequently than men and, in addition, few women attended business school in the 1960's. Thus only males are considered here.

The remaining explanation for the high popularity of teacher's college is the high consumption value of this educational type. Teacher's college covers a broad range of different subjects, where the students themselves choose which to specialize in, according to their interests. Also, this field of study is considered to be less demanding and time consuming than many others, leaving more time for leisure and extra curricular activities during the education[17]. After completed education, teacher's college graduates can expect to have more leisure time, since teachers have longer holidays. Teachers can get jobs all over the country, and are not bound to live in the larger cities, as are many other of the highly educated individuals, and this might play an important role for individuals planning to live in particular areas.

preferences are such that they have a high consumption value from attending business school. Here we look at the issue from the point of view of the individuals who actually chose teachers' college, even though they could have attended business school and increased their lifetime income substantially (as is shown later in this paper). These individuals most certainly expected a positive consumption value of education that was at least as large as the difference in the expected wage returns to the two kinds of education.

[15] The author's own observations by reading arhived letters to the admission board.

[16] See Aarrestad (1969).

[17] This is here defined as a part of the consumptiton value, since the individual enjoys having a more relaxed life and being able to pursue his other interests. But it might as well be defined as a part of the investment costs, since it means that the teacher's college student needs to invest less effort to graduate than his business school counterpart.

The individuals choosing teacher's college have such a high consumption value of this education that they willingly give up the future wage return they could have achieved by choosing another type of education.

Business school is another field of study[18] that requires a high grade level from high school in order to be admitted. During the 1960's the admission requirements were just as strict for both these fields of study[19], but the wage return to business school was superior to that to teacher's college, as pointed out by Aarrestad (1969, 1972). Even though the teacher's college graduates could have attended business school and had a higher wage return, they still chose to attend teacher's college. Hence they were willing to forgo future wages in order to enjoy the consumption value of teacher's college. Of course, they could have chosen other fields of study as well, but business school is chosen as a benchmark because it has the same admission requirements.

We now apply the model developed in the previous section to calculate W, the lower limit of the teacher's college graduates willingness to pay for the consumption value of teacher's college in the 1960's. A unique Norwegian panel data set provides very complex information on all these individuals. Unfortunately, there is no information on actual working experience for the individuals in question, and thus the potential experience approach of Mincer is applied. Define *potential experience*, X_p, for each year as the age of the individual minus the age at school enrollment minus the duration of the education minus a year for mandatory military service. Both the expected price of the consumption value of teacher's college at the time of the educational choice and the actual price these individuals finally paid are calculated.

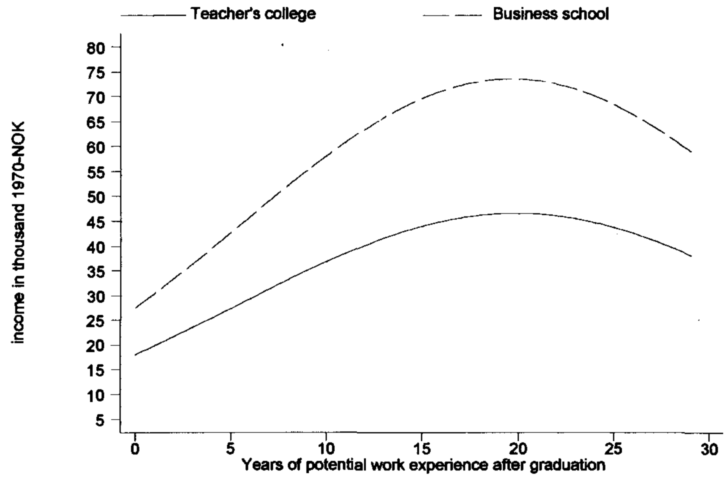
4.3 Measuring the ex-ante price of the consumption value of teacher's college.

The 1970 household census holds information on among other things educational type, gross earnings, and age for all Norwegian adults. Utilizing this information, the earnings by experience profiles for individuals with teacher's college and business school are estimated. These cross-section wage profiles are interpreted as the

[18]Business school was attended directly after high school and had a duration of three years during the 1960's. It was expanded to a four year duration in 1975.

[19]See the appendix for more details.

Figure 3: *1970 gross wage profiles for males with teacher's college and business school, by years of potential experience.*



teacher's college and business school attendants' expected future earnings profiles in the late 1960's.

The estimation approach differs from that of Mincer, since the duration of the education is fixed. The specification below is estimated separately for each group using ordinary least squares:

$$\ln W = \alpha_1 + \alpha_2 X_p + \alpha_3 X_p^2 + \epsilon. \tag{7}$$

From the estimation results (reported in figures 7 and 8 in the appendix) we conclude that even though there are some differences in the return to experience in favor of business school graduates, the major difference is between the constant α_1 in the two groups. Teacher's college graduates actually start their career with gross earnings 34.7 % below that of business school graduates[20].

Smoothed versions of the wage profiles for 29 years of work experience from the 1970 census are shown in figure 3. The teacher's college graduates pay a substantial wage premium in order to enjoy the consumption value of their educational type, and this wage premium increases over their career. The earnings vary more

[20]The estimation results are here transformed to NOK before finding the wage gap between the two groups.

Figure 4: *Present values of 29 years worth of labor experience in thousand 1970-NOK, calculated from the average earnings of males at different levels of working experience in the 1970-cencus. These are the gross* ***expected average lifetime earnings*** *of individuals choosing teacher's college and business school in the 1960's.*

Discount rate:	2%	3%	4%	5%	6%	7%	8%
Business school:	1345	1164	1015	892	789	702	629
Teacher's college:	831	719	627	550	487	433	388
Expected price on teacher's college as a consumption good:	514	445	388	341	302	269	241
Expected price as percentage of potential income:	**38.2**	**38.2**	**38.2**	**38.3**	**38.3**	**38.3**	**38.3**

among business school graduates than among teacher's college graduates. This may to some extent be due to the fact that most teachers work in the public sector where the wage level is set by centralized negotiations. The government is the employer and exercises monopsony power, since the private labor market for teachers is very limited. Business school graduates, on the other hand, mostly work in the private sector, where wage negotiations are local and the wage structure is more flexible.

The earnings of the business school graduates constitute the potential total income for the teacher's college graduates. Hence their minimum willingness to pay for the consumption value of teacher's college is the market compensating wage differential. The start up wage differential is 34.7 %[21] of the teacher's college graduates' potential lifetime income. But as the wage differential increases over the career, one would expect the present value of the life time wage differential to be higher. The exact size of this wage premium is not available directly from this estimation.

By applying the average wage at all levels of experience, the present value of the lifetime income[22] can be calculated for both business school graduates and teacher's

[21] This is in line with Aarrestad's (1969) results from his small sample survey in 1967.

[22] Assume here that the duration of the working period of the individual is 29 years. The reason why this exact period is chosen, is that there are few observations in the sample with longer potential working experience. This is to a great extent due to the early classes of business school being small.

college graduates. The results are shown in the table in figure 4, calculated at different discount rates. Independent of the discount rate, the wage gap between the two groups is substantial. *Teacher's college graduates pay a price for the consumption value of their education in the size of 38 % of the present value of their potential gross lifetime income.*

These are gross wages and, as previously discussed, the presence of a progressive income would tax most likely reduce the wage gap and thus the price on teacher's college as a consumption good. The existence of a substantial willingness to pay for the consumption value of teacher's college is still non-dismissable.

Some objections. Only annual earnings are available in the data. Hence part of the wage gap might be due to differences in hours worked instead of wage differences.

There are no tuition fees at Norwegian universities, but the students still have to finance their living expenses. The existence of publicly provided and subsidized student loans eliminates, or at least reduces, the liquidity constraints that might otherwise be present. For most of the 1960's teacher's college had a two-year duration, while business school had a three-year duration. Thus, the major cost of acquiring higher education, namely forgone labor income, is higher for business school graduates. Therefore part of the wage gap between the two educational types is compensation for the higher investment costs of business school.

The different duration of the two educational types also matters if the individual has a high discount rate. He then wants to start earning money as soon as possible, which might induce him to choose teacher's college rather than business school.

Geographical differences might matter. During the 1960's there were teacher's colleges all over the country, and the individual who disliked moving had a good chance of finding a teacher's college close to home. Business school, on the other hand, for a long time only existed in Bergen (The Norwegian School of Economics and Business Administration), but later another school was founded in Oslo (The Norwegian School of Management). This could also induce the individual to choose teacher's college over business school.

4.4 Measuring the ex-post price of the consumption value of teacher's college.

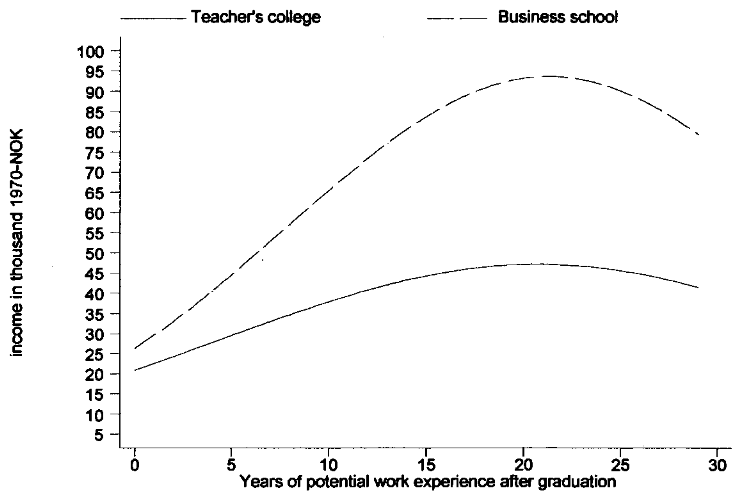
The previous section estimated the ex-ante willingness to pay for the consumption value of teacher's college among the individuals acquiring their education in the 1960's. Did these individuals end up paying a higher or lower price than expected for this consumption value?

We now estimate the actual wage profiles of all individuals attending and graduating from business school and teacher's college during the 1960's. By combining the earnings register and the core administrative register we have information on each individual's income from 1967 to 2000, along with rich information including factors such as educational type, graduation date, and birth date. Each individual now has several entries in the created cross section set of annual earnings per year of potential experience. Apply the same empirical specification as in equation (7), but following Klette and Møen (2002), this time use a random effect regression to estimate the return to potential experience separately for the two educational groups. The estimation results are reported in figures 9 and 10 in the appendix, and smoothed wage profiles for the two groups are drawn in figure 5.

It is clear that the ex-post wage profiles differ quite a lot from the ex-ante wage profiles. It is rather surprising, though, that the wage gap at the beginning of the career is smaller than predicted. The teacher's college graduates started their careers with annual gross earnings 20.6 % below that of the business school graduates, where the corresponding ex-ante wage gap was 34.7 %. But business school graduates experienced rapid wage increases over their careers, relative to the teacher's college graduates, as is clearly seen in figure 5. This would have a large impact on the present value of the two groups' lifetime income.

Since the wage differential increases heavily over the years, the present values of the two groups' actual lifetime income depends on which discount rate is chosen, as shown in the table in figure 6. The more weight the individual puts on future earnings, the lower his discount rate, and the higher the price of the consumption value of teacher's college measured in forgone potential income. The ex-post price on the consumption value of teacher's college is between 45 % and 48 % of the individuals' potential lifetime income, depending on the discount rate. This is substantially higher than the ex-ante price of 38 % of their potential lifetime income. Some of the

Figure 5: *The actual deflated gross wage profiles of individuals attending teacher's college and business school in the 1960's, by years of potential experience after graduation.*



reason for these high wage differentials might be that observations from the 1980's are included in the sample, a period where the private sector enjoyed high wage increases relative to the public sector.

The ex-ante wage profiles were estimated on cross-section data from 1970, while the ex-post wage profiles were estimated as cross-section variation between individuals observed over 29 years. During this time period, the tax system changed several times, both changing the tax base as well as the marginal tax rates. The higher the marginal tax rates in the higher income brackets, the more one would expect the net of taxes wage gap between the two groups to be reduced. Still, even though the marginal tax rates have been reduced over the years, the tax base has been broadened, such that it is not possible to say whether these reduced marginal tax rates increased the net of tax wage differentials or not. A thorough analysis is required to answer this, and that is left for future research. The main objective of this paper is to establish as a fact that individuals have high willingness to pay for the consumption value of education, rather than to find the exact size of this net of taxes willingness to pay.

Figure 6: *Present values of 29 years worth of labor experience in thousand 1970-NOK, calculated at different discount rates. These are the* ***actual average gross lifetime earnings*** *of individuals choosing teacher's college and business school in the 1960's. Data from the earnings and pension registers, only males.*

Discount rate:	2%	3%	4%	5%	6%	7%	8%
Business school:	1734	1486	1283	1116	978	863	767
Teacher's college:	896	777	679	597	529	472	423
Actual price on teacher's college as a consumption good:	**837**	**709**	**604**	**519**	**449**	**391**	**343**
Actual price as percentage of potential income:	**48.3**	**47.7**	**47.1**	**46.5**	**45.9**	**45.3**	**44.8**

Further objections. The estimated wage gaps do not account for the fact that teachers are provided with a public sector retirement insurance, while these retirement insurances vary in both extent and quality in the private sector. If pension benefits had been included in the earnings profiles, it might be that the wage gap between the two groups had been smaller.

We have to some extent controlled for heterogeneity in ability among individuals by comparing two types of educations with the same cut-off grade level requirements from high school. But it is not certain that the upward ability distribution is the same in the two groups, such that some heterogeneity might still exist. Teachers mostly get the same wage independent of performance, while wages are more individual specific in the private sector. Hence the wage incentives to choose business school are higher the more skilled the individual.

Also, the approach in this paper corrects for the level of the innate abilities, but not the difference in types of ability. Willis and Rosen (1979) found that a person chooses the kind and length of education that maximize his income. They only consider monetary income, but the results may also be interpreted to include non-monetary income. This means that a good lawyer would not necessarily have made a good plumber, and that the individuals maximize their income and utility according to their abilities and preferences[23] Both teacher's college and business

[23]This is in contrast to the one-factor-ability-as-IQ literature that says that the best lawyers would also have made the best plumbers.

school are still pretty much all-round types of educations, with a broad range of different subjects. Also, admissions are made based on the average grade level from high school, meaning that the students need good all-round skills.

5 Conclusion.

This paper argues for the existence of an individual specific consumption value of education, both during the education and after its completion, and for which the individual is willing to pay. A method for measuring the willingness to pay for the consumption value of education where the innate ability bias is corrected for is suggested in a compensating differentials framework.

On rich Norwegian cross section data it is estimated that the individuals who attended teacher's college in Norway during the 1960's expected to start their first job with annual earnings 34.7 % below their potential earnings. The full ex-ante price for the consumption value of teacher's college is estimated to be 38 % of the present value of the individual's potential lifetime income.

Utilizing a full coverage panel data set on the Norwegian population it is estimated that the teacher's college graduates in fact started up their first job earning "only" 20.6 % less than the business school graduates. However these wage differentials increased over time. The ex-post price on the consumption value to teacher's college during the 1960's turned out to be about 46 % of the present value of the individuals' potential lifetime income.

The goal of the paper has not been to find an exact value of the willingness to pay for the consumption value of education, but rather to establish as a fact that the consumption value of education does exist and that it is an important factor behind the individual's educational choice. As the example shows, many individuals are willing to give up substantial future wage returns in order to acquire the education of their choice. Therefore, the consumption value of education should not be ignored when modeling the individual's educational choice and estimating the return to education.

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7 Appendix

7.1 Documentation of admission requirements.

It is a general perception that during the 1960's it was just as difficult to be admitted to teacher's college as to business school in Norway. Aarrestad (1969) stated[24] on page 69: "The demand for teacher's college education far exceeds the supply. The minimum requirement for admission has the last years been above 60 grade points (from high school)." Also, on page 75 he states: "The admission requirements for the Norwegian School of Economics and Business Administration are not quite clear. With maximum awarded additional points, it is today possible to be admitted with about 60 grade points from high school."

It proved difficult to find formal evidence for these admittance requirements. In the archives of the Norwegian School of Economics and Business Administration[25] and of the Teacher's Council[26] I found indications that the last student admitted to teacher's college and to business school had about the same grade levels, but no official statistics are available on this issue. Another problem with comparing the two is that the different institutions had different regulations for giving so-called additional points to the applicants, such that their total competitive grade score varied from their high school graduation grade score. Additional points were awarded for previous education and work experience, and for extracurricular activities, but the praxis varied among the institutions.

7.2 Data

The 1970 Household Census covers all Norwegian households and individuals (identified by their personal identification number). The census contains information on among other things on gross income, sex, age, marital status, type and level of education, and personal income.

[24]The following quotations are translated from Norwegian.

[25]For a long time this was the only business school in Norway, but at the and of the 1960's another one was founded as well.

[26]From about 1967 admission to all teacher's colleges in Norway was organized centrally by the Teacher's Council (Lærerutdanningsrådet). Before that time the admission was organized by each school, and the requirements varied from school to school.

The Earnings Register covers all Norwegian adults and contains gross individual earnings based on pension rights earned over the period 1967-2000.

The Core Administrative Register contains information on all Norwegians in the years 1986-2000. It has among many other variables age, sex, marital status, type and length of highest completed education, graduation date. The income history of the individuals can be extended by including the earnings history of the individuals from the earnings register.

The cleaned sample for calculation of the ex-ante wage profiles. Individuals with missing observations on either educational type or income are removed from the sample. Beyond that all individuals who graduated from teacher's college or business school in the period 1941-1970 are included in the sample, in order to estimate the full income profile for 0-29 years of working experience in 1970 for the two groups. Even individuals who for some reason were not active in the labor force are included. When a young person makes his educational choice, the future wage return is uncertain for many reasons, and one of them is that he might become ill and be unable to work. If one type of education leads to more stressful jobs than the other, more individuals will become ill, and the wage level while still at work needs to be higher in order to compensate for this. Hence the income of those not currently in the labor force in 1970, but with potential labor experience between 0 and 29 years, needs to be included to get the full picture.

The full sample of males in the 1970 census counts 2269 business school graduates and 7089 teacher's college graduates.

The cleaned sample for calculation of the ex-post wage profiles. The first challenge was to identify who acquired the two educational types during the 1960's, as well as to find their potential working experience. In principle, I could use the graduation date in the core administrative register to establish when the individual most likely started working, and thus find the potential working experience in years. Unfortunately, all who completed their education prior to November 1970 are listed with this as their graduation date. Therefore I use their date of birth, add 19 years (to complete high-school) to find the time when they most likely started their higher education, and add another 2 or 3 years to find graduation date. Finally I added another year for the mandatory military service (some did this before and others after their education, but most did it before they started working) to find when they most likely started their professional careers.

This procedure identified the individuals acquiring their education during the 1960's, as well as their entry into the labor force. By merging the core administrative data with the earnings register, I got the gross income series for these individuals from 1967 to 2000. From this the earnings history of the individuals from 0 to 29 years of potential experience was extracted.

When the panel was cleaned for entries missing information on annual earnings, the final sample consisted of 465 business school graduates with a total of 13110 observation entries, and 1805 teacher's college graduates with a total of 50153 observation entries.

Figure 7: *Results, ordinary least squares regression, teachers, 1970-census.*

```
      Source |       SS       df       MS                  Number of obs =    7089
-------------+------------------------------               F(  2,  7086) = 1380.21
       Model |  678.480338     2  339.240169               Prob > F      =  0.0000
    Residual |  1741.66185  7086  .245789141               R-squared     =  0.2803
-------------+------------------------------               Adj R-squared =  0.2801
       Total |  2420.14219  7088  .341442183               Root MSE      =  .49577

------------------------------------------------------------------------------
         lnW |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
-------------+----------------------------------------------------------------
        pexp |   .0958324   .0026617     36.00   0.000      .0906146    .1010501
      sqpexp |  -.0024115   .0001008    -23.93   0.000      -.002609    -.002214
       _cons |   9.796686   .0126202    776.27   0.000      9.771947    9.821426
------------------------------------------------------------------------------
```

Figure 8: *Results, ordinary least squares regression, business school graduates, 1970-census.*

```
      Source |       SS       df       MS                  Number of obs =    2269
-------------+------------------------------               F(  2,  2266) =  540.54
       Model |  263.700785     2  131.850392               Prob > F      =  0.0000
    Residual |  552.728836  2266  .243922699               R-squared     =  0.3230
-------------+------------------------------               Adj R-squared =  0.3224
       Total |  816.429621  2268  .359977787               Root MSE      =  .49389

------------------------------------------------------------------------------
         lnW |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
-------------+----------------------------------------------------------------
        pexp |   .0999329   .0043521     22.96   0.000      .0913984    .1084674
      sqpexp |  -.0025426   .0001625    -15.64   0.000     -.0028613   -.0022238
       _cons |   10.22356   .0222558    459.37   0.000      10.17992    10.26721
------------------------------------------------------------------------------
```

Figure 9: *Results, random effects regression, teachers, earnings register.*

```
Random-effects GLS regression                  Number of obs      =     50153
Group variable (i) : pid                       Number of groups   =      1805

R-sq:  within  = 0.2680                        Obs per group: min =         1
       between = 0.0206                                       avg =      27.8
       overall = 0.2021                                       max =        33

Random effects u_i ~ Gaussian                  Wald chi2(2)       =  17749.49
corr(u_i, X)       = 0 (assumed)               Prob > chi2        =    0.0000

------------------------------------------------------------------------------
         lnW |      Coef.   Std. Err.       z     P>|z|       [95% Conf. Interval]
-------------+----------------------------------------------------------------
        pexp |   .0783991   .0007973     98.34   0.000      .0768365    .0799617
      sqpexp |  -.0018876   .0000258    -73.24   0.000     -.0019382   -.0018371
       _cons |    9.94585    .007921   1255.63   0.000      9.930326    9.961375
-------------+----------------------------------------------------------------
     sigma_u |  .25123219
     sigma_e |    .357633
         rho |  .33042589   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

Figure 10: *Results, random effects regression, business school graduates, earnings register.*

```
Random-effects GLS regression                   Number of obs      =     13110
Group variable (i) : pid                        Number of groups   =       465

R-sq:  within  = 0.3778                         Obs per group: min =        13
       between = 0.0593                                        avg =      28.2
       overall = 0.3069                                        max =        30

Random effects u_i ~ Gaussian                   Wald chi2(2)       =   7694.71
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.0000

------------------------------------------------------------------------------
        lnW |      Coef.   Std. Err.       z     P>|z|      [95% Conf. Interval]
------------+-----------------------------------------------------------------
       pexp |   .1192716   .0020061     59.45   0.000      .1153397    .1232035
     sqpexp |  -.0027991   .0000687    -40.75   0.000     -.0029337   -.0026645
      _cons |   10.17645   .0185267    549.28   0.000      10.14014    10.21276
------------+-----------------------------------------------------------------
    sigma_u |  .29900569
    sigma_e |   .4930236
        rho |  .26890429   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```