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Market Power and Storage in Electricity Markets

By

Jostein Skaar

Contents

Acknowledgements

During my work with this thesis I have been very lucky to be able to work together with some of the most outstanding economists in Norway at the Institute of Economics, NHH. This has certainly been a motivating factor in my own work.

Some people have been directly involved in my work with this thesis and I would like to express my gratitude to each of them.

First of all I would like to thank my supervisor, Einar Hope, for his support throughout the process of writing this thesis. Without his encouragement and comments I would not have been able to complete the thesis. I would also like to thank Lars Sørgard as co- author on two of the chapters in my thesis and Lars Mathiesen as co-author on one chapter. It has been highly motivating to work together with these professors. In addition, I would like to thank BKK and Statkraft for financial support.

I would also like to thank my wife, Marie Hauge, for her support and strong belief that I would manage to finish this work.

Bergen May 26, 2004

Jostein Skaar

Introduction

The four chapters of this thesis focus on market power in liberalised electricity markets dominated by hydropower. The purpose of liberalisation of electricity markets is to ensure competitive pricing and thus an efficient production and allocation of electricity. However, liberalisation as such may not be sufficient to ensure competitive pricing. Within the framework of a liberalised market, producers may be able to exert horisontal market power and thus distort pricing and output away from efficient levels.

Since the late 1980's many countries have liberalised their electricity industry and liberalisation is on the way in several other countries.[1] The actual framework of a liberalised electricity market differs between countries in various respects. However, vertical separation between transmission and generation has been a central issue in the liberalisation process in all countries. This separation process has been seen as a basic prerequisite for competition in the supply of electricity. Thus, vertical separation between transmission and generation may be used as an indicator of the degree of liberalisation reached in a specific market.

At the beginning of the 1990's vertical separation between transmission and production of electricity had been established in Britain along with a spot market for electricity. In 1991 an Electricity Act was implemented in Norway. A central part of this Act was the provision to separate generation and transmission into two separate companies, Statkraft and Statnett respectively. In the following years, provisions for liberalisation were undertaken in the other Nordic countries as well and at the beginning of the new millennium an integrated Nordic market for electricity emerged. The liberalisation process in California started in 1998 following several years of debating on the issue. The reform established free access to the network and a spot market for electricity. Also in 1998, electricity market liberalisation reached Spain.

In Europe liberalisation is now pursued through a common European effort. The

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[1]Newberry (1998) refer to a study by Pollitt (1997). Pollitt found that 27 of 62 countries in his study either had implemented vertical separation between production and transmission or were planning to do so.

first effort by the European Union to promote electricity market liberalisation was implemented through the Electricity Directive in 1996. This directive was amended in 2003. According to the amendments, member states are now required to establish legal unbundling of network and supply activities.

There are several studies that analyse how well competition works in an electricity industry. Green and Newbery (1992) used a supply function equilibrium approach to analyse competition in the British electricity market. They showed that the duopoly of Powergen and National Power in the UK market resulted in equilibrium prices well above the competitive level. Their finding was supported by von der Fehr and Harbord (1993), who used an auction model to show that producers would bid above marginal cost. Wolfram (1999) conducted an empirical analysis of the British electricity spot market and found that generators were charging prices above their observed marginal costs. However, the price cost margin was not as high as theoretical models of competition would suggest. The discrepancy was subscribed to threat of entry and possible reactions from the regulator.

Borenstein and Bushnell (1999) and Borenstein, Bushnell and Wolak (2002) are two examples of studies of the Californian electricity industry. Borenstein and Bushnell (1999) use a Cournot model with a competitive fringe to analyse the effects of market power in a deregulated Californian market. The model was implemented for the year 2001 using historical cost data. The Cournot model showed a potential for market power in high demand hours of several months of the year. They also found that the potential for market power was significantly reduced when the elasticity of demand is increased. In 2002, after the crisis in the Californian market in the year 2000, Borenstein, Bushnell and Wolak conducted a second analysis of market power during the period from June 1998 to October 2000. A numerical model was constructed to replicate competitive prices and output in the market, given capacities of all players in the market. The data from this competitive benchmark model was then compared to actual historical prices in the market in order to detect market power. The findings from the 2002 analysis support the predictions made in Borenstein and

Bushnell (1999). In high demand hours during the summer the results indicate that market power had a significant effect on prices.

Hjalmarsson (1999) conducts an econometric analysis based on the Breshnan-Lau model in order to test for market power in the Nordic electricity market. He finds that the hypothesis of no market power cannot be rejected. In another study of market power, Amundsen and Bergman (2002) analyse the effect of cross-ownership in the Nordic market using a numerical model based on Cournot behaviour. The establishment of an integrated electricity market between Norway and Sweden in 1996 reduced the potential problem related to market power. Amundsen and Bergman analyse whether cross-ownership would re-establish this potential. They find that increased cross-ownership contributes to horisontal market power and thus to higher prices in the market.

There are several other studies of competition in the electricity industry in addition to the ones mentioned above. Common to most of these studies is that they are typically based on one-period models with increasing marginal cost in production of electricity. Such models are well suited to analyse a system with thermal production. However, in several countries hydropower has a dominant position. This includes the Nordic market, New Zealand, South American markets like Chile, Colombia and Argentina, Switzerland, and also to some extent markets in the US[2]. An important feature in such a system is that a hydropower producer allocates water resources between different periods by storage in water reservoirs. Thus, the production decision of a hydropower producer is dynamic in nature.

With a few notable exceptions, the large flexibility of hydropower producers to shift production across time is not modelled in the existing literature. Scott and Read (1997) were one of the first to model the inter-temporal aspects of hydropower production within a competition analysis. They constructed a simulation model of the deregulated electricity market in New Zealand, where hydropower plays an important role. The results from their simulation of the New Zealand market suggest a relatively

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[2]See Bushnell (2003).

low efficiency loss due to market power. This result depends largely on the feature of the New Zealand market structure that a large proportion of the generator capacity is sold on long-term contracts at so called "reasonable" prices.

In a hydropower system where producers face a constraint on available energy and where marginal cost in production is assumes to be zero, a monopoly producer would allocate production as to equate marginal revenue between periods. This allocation may differ from the social optimal allocation aimed to equate prices between periods. Johnsen et al. (1999) utilise this difference in allocation pattern to analyse whether generators within specific areas in Norway were able to exercise market power. Their analysis shows some support for the hypothesis that generators withhold some production at times when competitors are constrained by limited transmission capacity. In another study, Johnsen (2001) analyses market power and storage in a situation with limited transmission capacity between two regions connected by a single radial transmission line and when inflow is uncertain. He uses a two period model and a numerical example is provided to illustrate that a monopolist finds it profitable to increase production in the first period when inflow is certain. The monopolist does this to avoid the possibility of becoming export constrained in the second period if high inflow occurs. Thus, storage is concluded to be lower in the monopoly case than in the competitive case.

Crampes and Moreaux (2001) analyse the interaction between hydropower and thermal production of energy. Their analysis includes the social efficient dispatch of the two technologies within a two period model. They also look at the case where a monopoly producer controls both technologies and a situation where thermal and hydropower producers compete in the market. One of the main points of their paper is that even though thermal production of electricity is static in its nature, production based on thermal technologies is affected by demand and costs in other periods through the intertemporal allocation of water between periods.

Garcia et al. (2001) develop an oligopoly model where hydropower producers engage in dynamic price competition when there is uncertainty about future inflow

of water to the reservoirs. A special feature of this model is that demand is perfectly inelastic with a reservation price or a price cap set by authorities. One of the main results from this analysis is that a reduction in the price cap reduces the alternative value for production of electricity today and thus gives a reduced incentive to storage for future production.

Bushnell (2003) constructs a numerical model where producers with hydro and thermal production capacities engage in Cournot competition. The model is tested on data from the Western U.S. electricity market. A central result from the analysis by Bushnell (2003) is that producers by acting strategically may profit considerably by shifting hydro production from peak hours to hours with lower demand. Even though total production over all periods remain fairly unchanged by strategic behaviour the rescheduling of hydro power production would contribute to reduced welfare because thermal producers with unnecessarily high marginal cost in production are used to produce electricity at peak demand hours.

Outline of the thesis

The topic of this thesis relates to the existing literature on competition in hydropower markets discussed briefly in the introduction. Chapter 1 discusses the effects of market power in the context of acquisitions in a situation where transmission capacity is constrained. Chapter 2 and 3 elaborate on the issue of competition and market power when water inflow is uncertain, and finally Chapter 4 focuses on the supply function equilibrium model in the context of a hydropower market. The content of the four chapters of this thesis is outlined in more detail below.

Chapter 1: Temporary Bottlenecks, Hydropower and Acquisitions in Networks

by Jostein Skaar and Lars Sørgard

In this paper we introduce a simple model with hydropower producers that allocate their production between two time periods and two geographical regions. We apply the model to study effects of acquisitions in a situation with temporary bottlenecks. It is shown that an acquisition has an ambiguous effect on welfare. In some instances it would lead to larger differences in prices between different markets, which would lead to an increase in the dead weight loss. In other instances an acquisition would lead to a reduction in price differences between different markets. This may happen if the dominant firm acquires a firm that is active in the market where the dominant firm used to dump its energy capacity before the acquisition took place.

To explain our seemingly counterintuitive results, let us describe our model approach. There are two different regions and two different time periods in our model, implying that there is a potential for four separate sub-markets. Each hydropower producer has a total fixed energy capacity, determined by water available in their

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reservoirs, and allocates its total capacity between the sub-markets.[3] Each producer can shift production in time by storing water in its reservoir, and shift production between regions by exporting through a transmission line. The transmission lines are owned by an independent operator, who acts as an arbitrage player between regions and always exports to the high price region.[4]

We focus on the case with temporary bottlenecks, where transmission lines can be capacity constrained only in one of the two periods. A dominant producer can exploit the potential for bottlenecks strategically. For example, it can withdraw sales in a period so that the capacity constraint is binding. By doing so it is able to increase the price in that particular region in that period, and dump the withheld quantity in the other period where there is no capacity constraint on transmission between the two regions. Although the setting is different, these results are analogous to the ones reached by Borenstein, Bushnell and Stoft (2000).

Given that one of the transmission lines is a binding constraint and there are price differences between sub-markets initially, how would an acquisition influence the market equilibrium? It turns out that an acquisition might lead to a reduction in price differences between sub-markets. This may happen if the dominant firm acquires a firm that is active in the market where the dominant firm used to dump its energy capacity before the acquisition took place.

Two examples, both relevant in the Nordic hydropower market, illustrate how this may happen.[5] First, it can be due to asymmetries in location. Consider the case where one producer has production in both regions (or only in the high price

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[3]We also introduce thermal production in one of the regions, but we show that even then we may have counter-intuitive results.

[4]Our approach is consistent with the institutional setting in the Nordic market, and it is also in line with the "nodal pricing" system first introduced in Schweppe et al. (1988).

[5]There are asymmetries between hydropower producers in this market. Some producers have hydropower production in several regions as well as multiyear reservoirs, while other hydropower producers are located in only one region and have limited or no ability to store water from one year to another.

region), and it acquires a producer that has production only in the low price region. After the acquisition the large producer would sell a lower quantity in the 'dumping' region, thereby increasing the revenues generated from the acquired firm. Second, it can be due to asymmetries in storage. Let us assume that one producer has multiyear reservoirs, and another producer cannot store water from one year to another. They are located in the same region. In one year with large rainfall and large quantities of water in the reservoirs, the producer with no flexibility has to produce in that year despite a low price. The other producer, with large flexibility concerning storage, can dump some water in the season with a low price and store the remaining water for production next year. After an acquisition, the producer with a multiyear reservoir might produce less in the year with a large water inflow, the year the inflexible producer has to produce a large quantity. By doing so the revenue from the acquired firm increases.

Chapter 2: Policy Measures and Storage in a Hydropower System

by Jostein Skaar

In this paper we discuss how three different public policy measures affect water storage controlled by hydropower producing firms. In particular we discuss measures to promote competition, increase transmission capacity and rationing. The analysis is conducted within the framework of an oligopoly model where two hydro producing firms engage in dynamic Bertrand competition. Furthermore, demand is assumed to be perfectly inelastic with a reservation price. Thus, the basic model is identical to the one described by Garcia et al. (2001). We extend this model to be able to analyse how the three policy measures affect storage by hydropower producing firms and focus especially on the probability of hydropower replacing thermal production. These extensions are motivated by the observed energy shortage during the winter 2002/2003 in Scandinavia and the following discussion.

Authorities can promote competition for instance by taking actions to prevent collusion or preventing mergers from taking place. We analyse the effect of promoting competition on the probability of hydropower replacing thermal production in the simplest possible way, by comparing the Bertrand-Nash outcome to the monopoly outcome. We find that competition represented by the Bertrand-Nash outcome implies a higher probability of hydropower replacing thermal production than the monopoly solution.

Another public measure that has been proposed in order to reduce the problem of energy shortage in low inflow situations is increased transmission capacity. The idea is that increased transmission capacity would make it possible to increase production through imports in situations with low inflow and thus reduce the problem of energy shortage. There are several ways this could be modelled. Here, we model transmission between two geographic areas with one hydropower company in each area. In this setting, we find that an increase in transmission capacity leads to a more fierce competition between the two hydro producing companies and thus increases the probability of hydropower replacing thermal production. This effect is similar to a reduction in the price cap as described by Garcia et al (2001).

Finally, we consider the effect on storage by rationing imposed by authorities. Rationing may be thought of as a measure to secure supply of electricity in situations with little or no inflow. We model rationing as an action by authorities to reduce demand in situations when the energy resource is believed to be scarce. Rationing will only affect profits directly in the periods where such rationing is imposed and also affects producers' storage levels. These effects are different from a reduction in the price cap analysed by Garcia et al. We find however, that the effect of rationing is similar to a reduction in the price cap. Increased rationing leads to a more fierce competition when water is plentiful. This again increases the probability of hydropower replacing thermal production of electricity.

Chapter 3: Water With Power: Market Power and Supply Shortage in Dry Years

by Lars Mathiesen, Jostein Skaar and Lars Sørgard

We formulate a model with the purpose to be able to analyse how a producer with market power would distribute his sales between summer and winter. During autumn there will be either heavy rain or little rain. If there is heavy rain, the inflow is so large that some water may be spilled (reservoirs are full). Whether some water is spilled depends on the inflow and the size of the reservoirs. If there is little rain during autumn, all inflow can be stored in reservoirs and used for production in the winter season.

First, we show that even under perfect competition the average price during summer is lower than the average price during winter. The reason is that a high inflow can lead to waste of water (reservoirs are full), and then it would have been better to sell a little more during summer at a low price than to wait and risk a spill of water if there is a large inflow. The implication is that one cannot conclude whether there has been an abuse of market power or not by just observing price differences between summer and winter. In contrast, when there is a zero probability of spill of water we find that absence of market power will lead to identical prices in summer and winter. In such a case a price difference between summer and winter would indicate exertion of market power.

Second, we find that exertion of market power has an ambiguous effect on the distribution of sales between summer and winter (storage). On the one hand, a producer with market power may sell a large quantity during summer in order to constrain his supply and obtain a high price during winter. Or he may choose to do the opposite, selling a low quantity during summer to achieve a higher summer price. In this latter case market power may lead to a more limited difference between prices summer and winter.

Our result contrasts with Garcia et al. (2001), who found that market power always leads to higher prices during summer. The driving force behind their result is the modelling of the demand side. They apply a rectangular demand function, where the price during winter-time is exogenously given. Then a shift of production from summer to winter will have no effect on the winter price. In contrast, in our model there is a trade off. A shift of production from summer to winter would lead to higher prices during summer and lower prices during winter. This explains why we found that market power in some instances can lead to a shift in production from summer to winter, and in other instances to a reallocation of production from winter to summer.

We abstract from the possibility of transmission constraints as we look only at the allocation of water between periods within a single geographic area. In this respect our analysis differs from Johnsen (2001) who analyse a situation with limited transmission capacity between two regions connected by a single radial transmission line. Also, different from Johnsen (2001), we analyse situations where the size of the water reservoir may constrain production and situations where the energy constraint may not be binding. As mentioned above, we find that market power has an ambiguous effect on storage. This is in contrast to Johnsen (2001), who finds that storage is lower in the monopoly case.

Chapter 4: Supply Function Equilibria in a Hydropower Market

by Jostein Skaar

The purpose of this paper is to study how energy constraints affect the performance of the Supply Function Equilibria (SFE) model. In markets dominated by thermal production of electricity, production in one period has limited effect on production in other periods. However, power production in one period is constrained by the installed production capacity. The problem of modelling production constraints within the SFE model framework has been studied in several papers.[6]

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[6]See for instance Baldick and Hogan (2001)

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For hydropower producers the problem is different. Normally, the installed production capacity is so large that production is not constrained by the limit on production capacity. The problem facing hydropower producers is to allocate scarce water resources between different periods. Thus, when we use the SFE model to analyse competition in an electricity market dominated by hydropower the effects of energy constraints should be included in the analysis.[7]

The idea of competition in supply functions origins from the debate on whether firms choose prices or quantities as strategic variables. The idea first outlined by Grossman (1981) was that firms may not be able to set a price or a given quantity for every possible state of the market in advance of trade taking place. Instead, firms may resort to specifying supply functions relating quantity to price. Grossman (1981) studied supply function equilibria in absence of uncertainty. According to Klemperer and Meyer (1989), this approach leads to a vast number of possible Nash equilibria in supply functions. In addition, without uncertainty, there is no reason to choose a more general supply function because firms can maximize profits either by fixing price or quantity.

Klemperer and Meyer (1989) introduced exogenous uncertainty into the supply function framework. They prove that under these conditions it is more profitable for firms to rely on supply functions rather than fixing price or quantity. With uncertainty, a supply function provides valuable flexibility to the firm. Furthermore, they also show that with uncertainty in demand, the number of possible Nash equilibria is dramatically reduced.

The supply function equilibria (SFE) concept developed by Klemperer and Meyer seems to fit quite closely to competition in several markets where firms must commit to bids in advance, including electricity spot markets. Thus, not surprisingly, several papers[8] have used this approach in order to analyze electricity market competition.

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[7]This paper is motivated by a report from the Nordic competition authorities (2003) where a SFE model developed by the Danish system operator Eltra was used to analyse mergers and acquisitions in the Nordic electricity market.

[8]This includes Green and Newbery (1992), Green (1996), Newbery (1998), Baldick et al (2000)

These papers typically focus on competition in a thermal based electricity market. In such a market the focus is on production constraints at a particular time, not on constraints on energy produced over a time period.

This paper is divided in three parts. In the first part we set up the Supply Function Equilibria (SFE) model based on the analysis by Baldick and Hogan (2001) and Green and Newbery (1992). On the basis of this model we develop a simple numerical example. In the second part of the paper we use this example to illustrate how competition in supply functions may be affected both by constraints on power produced at a particular moment in time and constraints related to available energy resources. We illustrate that binding constraints on energy production reduce the number of allowable supply functions. Thus, if the constraint on energy produced is not taken into account when the SFE model is used to analyse competition in electricity markets, the welfare effects of market power might be exaggerated.

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and Baldick and Hogan (2001).

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Temporary Bottlenecks, Hydropower and Acquisitions in Networks[1]

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Abstract: The purpose of this article is to study the effects of an acquisition
in an energy system dominated by hydropower and with temporary bottlenecks. We
apply a model with four markets: two regions and two time periods. It is shown that
an acquisition has an ambiguous effect on welfare. In some instances it would lead to
larger differences in prices between different markets, which would lead to an increase
in the dead weight loss. In other instances an acquisition would lead to a reduction
in price differences between different markets. This may happen if the dominant firm
acquires a firm that is active in the market where the dominant firm used to dump
its energy capacity before the acquisition took place.

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1 Introduction

During the last decade many countries have liberalized their electricity industry. There are several studies that analyse how well competition works in such an industry.[2] These studies are typically using one-period models with increasing marginal costs. Such models are well suited to analyse a system with thermal production. However, in several countries hydropower has a dominant position.[3] An important feature in such a system is that a hydropower producer allocates its water resources between different periods by storage in water reservoirs. With a few notable exceptions, the large flexibility of hydropower producers to shift production across time is not modelled in the existing literature.[4] The purpose of this article is to introduce a simple model with hydropower producers that allocate their production between different time periods and different geographical regions. We apply the model to study effects of acquisitions in a situation with temporary bottlenecks. It is shown that the idiosyncratic characteristics of the hydropower system may reverse the existing results in the literature concerning the consequences of higher concentration.

One main concern which has been raised is that in a hydropower system a dominant producer may exploit its unique flexibility to shift production across time. As shown in Bushnell (2000), this may lead to a further separation of geographically

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[2] Green and Newbery (1992), von der Fehr and Harbord (1993), Green (1996), Newbery (1998) and Wolfram (1999) are all studies that are analysing the British electricity market, while Borenstein and Bushnell (1999) and Borenstein, Bushnell and Wolak (2000) are examples of studies of the Californian electricity industry. Two recent studies of competition in the Nordic electricity market are Hjalmarsson (1999) and Amundsen and Bergman (2002).

[3] In New Zealand 80% of production is from hydro, in Chile 70%, Brazil 97% and Norway close to 100%.

[4] Scott and Read (1997), Crampes and Moreaux (2001) and Bushnell (2000) all model a mixed system with hydropower and thermal production. None of them analyse the effects of a more concentrated industry, for example due to acquisitions. von der Fehr and Johnsen (2002) analyse a pure hydropower system, and they compare perfect competition with a situation with market power. In contrast, our main focus is on the effects of an acquisition in a situation where we have imperfect competition both before and after the acquisition.

and temporally distinct markets and an increase in price differences between markets. This suggests that one should carefully watch any market where the production capacity of one large hydropower producer can not be replaced with that of other smaller competitors. Furthermore, a hydropower producer might violate a competitive outcome. It might behave in such a way that it induces a constraint on the transmission line and thereby creates a deviation from a competitive outcome.[5] As shown in Borenstein, Bushnell and Stoft (2000) the producer can by acting like this cause price differences.[6]

We share the concern that a hydropower producer might violate an outcome that otherwise would have been competitive, and that one should watch extra carefully a situation where a hydropower producer is the marginal producer in the marketplace. Indeed, our two first results replicate these two situations. However, in the intermediate situation, where we have an oligopoly situation at the outset, we find that an increase in concentration will not have such a clear-cut welfare deteriorating effect.

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[5]Schmalensee and Golub (1984) pointed at the potential problems associated with congestion on transmission lines. However, with no satisfactory definition of how the scarce transmission resources should be allocated (the pricing issue) and due to lack of area specific data they where forced to define geographic market areas on a more ad hoc basis. Schweppe *et al.* (1988) develops a spot pricing theory where the special features of electric networks are considered. This model, known as "nodal pricing", ensures short-run economic dispatch of load subject to network and generation constraints. Later we have seen several studies of the problems associated with congested transmission lines, such as the pricing of transmission and incentives for investing in transmission lines. See for example Hogan (1992), Oren *et al.* (1995), Bushnell and Stoft (1996), Chao and Peck (1996) and Cardell *et al.* (1997) for analysis of energy systems as networks.

[6]Note, though, that Borenstein, Bushnell and Stoft (2000) applies a model with only thermal production. Whether this phenomena is more profound in a hydropower system than in a thermal system is an open question. Hydropower producers are flexible, and due to this such a producer can easily create a bottleneck. On the other hand, other hydropower producers can react quickly and thereby dampen or even eliminate the attempt to create a bottleneck. In any case, endogenous bottlenecks can emerge in hydropower systems as in other energy systems. Note that von der Fehr and Johnsen (2002) show that strategic behaviour in a hydropower system can lead to larger price differences.

To explain our counterintuitive results, let us describe our model approach. There are two different regions and two different time periods in our model, implying that there is a potential for four separate sub-markets. Each hydropower producer has a total fixed energy capacity, determined by water available in their reservoirs, and allocates its total capacity between the sub-markets.[7] Each producer can shift production in time by storing water in its reservoir, and shift production between regions by exporting through a transmission line. The transmission lines are owned by an independent operator, who acts as an arbitrage player between regions and always exports to the high price region.[8] We focus on the case with temporary bottlenecks, where transmission lines can be capacity constrained only in one of the two periods. A dominant producer can exploit the potential for bottlenecks strategically. For example, it can withdraw sales in a period so that the capacity constraint is binding. By doing so it is able to increase the price in that particular region in that period, and dump the withheld quantity in the other period where there is no capacity constraint on transmission between the two regions. Although the setting is different, these results are analogous to the one found in Borenstein, Bushnell and Stoft (2000).

Given that one of the transmission lines is a binding constraint and there are price differences between sub-markets initially, how would an acquisition influence the market equilibrium? It turns out that an acquisition might lead to a reduction in price differences between sub-markets. This may happen if the dominant firm acquires a firm that is active in the market where the dominant firm used to dump its energy capacity before the acquisition took place.

Two examples, both relevant in the Nordic hydropower market, illustrate how this may happen.[9] First, it can be due to asymmetries in location. Consider the

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[7] We also introduce thermal production in one of the regions, but we show that even then we may have counter-intuitive results.

[8] Our approach is consistent with the institutional setting in the Nordic market, and it is also in line with the "nodal pricing" system first introduced in Schweppe et al. (1988).

[9] There are asymmetries between hydropower producers in this market. Some producers have hydropower production in several regions as well as multiyear reservoirs, while other hydropower

case where one producer has production in both regions (or only in the high price region), and it acquires a producer that has production only in the low price region. After the acquisition the large producer would sell a lower quantity in the 'dumping' region, thereby increasing the revenues generated from the acquired firm. Second, it can be due to asymmetries in storage. Let us assume that one producer has multiyear reservoirs, and another producer cannot store water from one year to another. They are located in the same region. In one year with large rainfall and large amounts of water in the reservoirs, the producer with no flexibility has to produce in that year despite a low price. The other producer, with large flexiblity concerning storage, can dump some water in the season with a low price and store the remaining water for production next year. After an acqusition, the producer with a multiyear reservoir might dump less production in the year with a large water inflow, the year the inflexible producer has to produce a large quantity. By doing so the revenues from the acquired firm increase.

The article is organised as follows. In the next section we introduce our model, and we characterise perfect competition and monopoly, respectively. In section 3 we analyse the effects of acquisitions, and discuss how asymmetries on location and storage as well as the number of producers and the introduction of thermal production may change our results. In section 4 we offer some concluding remarks.

2 The model

Let us consider a market with two different geographical regions, called East (E) and West (W). In addition there are two time periods, called 1 and 2. The combination of geography and time implies that we have four different sub-markets. This set up is illustrated in figure 1. Time can either be interpreted as short run or long run. In the short run each producer decides to produce either at, say, day versus night. In

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producers are located in only one region and have limited or no ability to store water from one year to another.

the long run, producers with multiyear reservoirs have to decide whether to produce this year or to store the water for production next year.
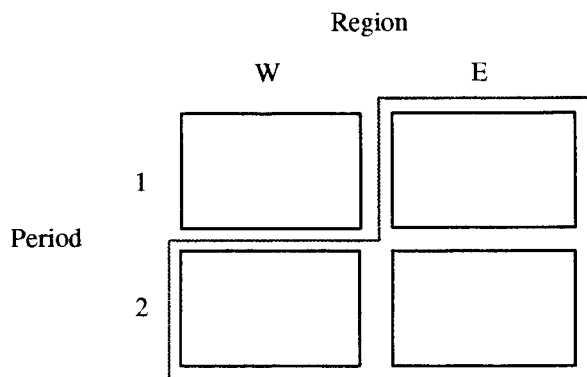
Region

W   E

Period

1

2

Figure 1: There are four different sub-markets depending on geographic location and time period. The dotted line indicates a situation where three of the sub-markets are integrated.

For the moment, let us assume that there are four different hydropower producers, $j = U, X, Y, Z$.[10] Except for producer X, each producer has plants (one or several) in only one region. In principle, though, each producer can sell in all four sub-markets. First, reservoirs enables each producer to store water and thereby allocate its total production between the two time periods in the region where the reservoir is located. Second, transmission lines allows each producer to sell in the neighbouring region.

To simplify, let us for the moment assume that at each hydropower plant the producer is able to produce all the available energy at that site in one time period (no binding constraint on effect capacity). However, total production in one region is constrained by the available energy capacity (water in the reservoir). Then each

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[10]Later on we will extend the model by (i) allowing for more hydropower producers and (ii) introducing a thermal producer.

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producer has the following constraint on production in region i:

$$\sum_{t=1}^{2} q_{it}^{j} \leqslant \overline{q}_{i}^{j}, \text{ where } i = W, E \text{ and } j = U, X, Y, Z \tag{1}$$

$\overline{q}_{i}^{j}$ denotes the total energy capacity available to producer j for sale in region i. We assume that all the water that is available is used for production of energy, so that there are no spill of water.[11] Then the energy constraint in (1) holds with equality. We assume that both X and Y are single producers where X has reservoirs in both regions while Y has only capacity available in region E. Furthermore, we interpret U and Z as competitive fringes. It implies that each of them consists of a number of small producers, behaving as price takers. The competitive fringe U is located in region W, while Z is located in region E.

Let us now introduce transmission lines between the two regions. Electricity flows between the regions according to physical laws where regions with demand surplus (high prices) import until the transmission capacity is a binding constraint. In line with the institutional arrangement in the Nordic market, we assume that the transmission lines between regions are operated by independent system operators. At times of congestion the market is divided into different market regions where demand equals supply in each region. When lines are congested the price difference between two regions corresponds to the cost of transmission or the congestion rent. This rent is collected by the grid operator. Thus, we might say that the grid operator behave as

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[11]Whether this assumption is realistic or not is an open question. However, it is often used in the literature (see Johnsen T. A., S. K. Verma and C. Wolfram, 1999 and Crampes, C. and M. Moreaux, 2001). Also, and more important, this assumption has recently been advocated by the Norwegian Competition Authority in relation to the the Authority's evaluation of Statkraft AS's acquisition of shares in Agder Energi AS. In what we could call the Authority's Statement of Objections issued on January 23 2002 the Authority notes on page 26 that [our translation]:

"Producers do not need to renounce production (send water past turbines that are ready for production) in order to exert market power. As mentioned earlier, the low production costs associated with hydro-power production implies that there is a low probability of spill of water taking place."

a competitive arbitrage agent between regions. If we think of the regions as market nodes, we can describe the pricing by the term "nodal pricing". It refers to the term used by Schweppe *et al.* (1988). This pricing regime implies that a seller located in region i will receive the market price in that region, even if its production is exported to the neighbouring region.

We assume there is one market node in each region and one transmission line between these nodes. This line has a capacity of $\overline{K_t}$ and an actual flow of K_t in period t. Prices in the two regions can only differ when the capacity is fully utilized. In this case we would have that $K_t = \overline{K_t}$.

Let S_{it} denote the demand in region i in period t. We can then define the equilibrium condition for the two regions as:

$$S_{Wt} = q_{Wt}^X + q_{Wt}^U + K_t \text{ and } S_{Et} = q_{Et}^X + q_{Et}^Y + q_{Et}^Z - K_t \qquad (2)$$

If $K_t > 0$ and transmission has reached the capacity limit in period t, we have that electricity flows from region E to W and that at period t the price in region W can exceed the price in region E.

We are concerned about the situations where a transmission line becomes a bottleneck and may lead to price differences between different sub-markets. However, the extreme case where transmission lines are binding in both time periods is not of interest. In such a case the two regions are separated, and we could analyse each region in isolation. On the other hand, the case with no binding transmission constraint in any of the two time periods is neither of interest. In this case the two markets can be seen as one integrated market, and the questions concerning bottlenecks are ruled out.[12] More interestingly, we focus on a situation where the lines are congested in just one of the two periods. In such a case the regions are partially integrated or, put another way, the transmission line is temporarily congested (temporary bottlenecks).

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[12]However, this might not always be true. Any change in markets structure, such as an acqusition, may lead to a change in the behaviour so that one or both transmission lines suddenly bites. We will return to this question in our analysis (see Proposition 2).

To analyse a situation with temporary bottlenecks, we assume that in period 2 the regions are integrated with a common price and no congestion on the transmission line. We call this new market $WE2$. Even if the price is the same in both regions we might have transmission on the line between them. However, actual flows (K_2) have to be less than capacity $(\overline{K}_2)$. If not, the prices would differ and result in separate markets. We can now define the equilibrium condition for our new market:

$$S_{W2} + S_{E2} = q_{W2}^X + q_{W2}^U + K_2 + q_{E2}^X + q_{E2}^Y + q_{E2}^Z - K_2 \tag{3}$$

As a benchmark for our analysis of an acquisition, let us contrast perfect competition with monopoly:

Proposition 1 *(i) Perfect competition (all producers are price takers): the prices in all four sub-markets are identical.*

(ii) Monopoly (one owner of all production): If identical price elasticities (e_{it}) in all sub-markets and $|e_{it}| > 1$, then prices are identical in all sub-markets, and identical to prices in a situation with perfect competition. Otherwise, prices in time period 2 are identical while prices in time period 1 is either lower or higher than in period 2.

Proof. (i) The competitive fringes U and Z act as arbitrage players between period 1 and 2 in region W and E, respectively. Since prices are identical in period 2 (no transmission constraint), then all four sub-markets have identical price.

(ii) A monopolist would choose prices across markets it and $\widehat{it} \neq it$ such that:

$$p_{it}[1 - \frac{1}{e_{it}}] = p_{\widehat{it}}[1 - \frac{1}{e_{\widehat{it}}}], \text{ where } i = W, E \text{ and } t = 1, 2.$$

If the price elasticities across all sub-markets are identical and $|e_{it}| > 1$, we then have equal prices across all sub-markets. It is also straight forward to see that if price elasticities differ, then prices between markets would also differ. If $|e_{it}| < 1$, it is well known that the monopolist's second order conditions are not met. Then we have a

corner solution. If negative prices are ruled out, then prices will be equal to zero in one (or several) sub-market(s), and prices are high in one (or several) sub-market(s).
∎

First, note that perfect competition implies that prices in all four sub-markets are identical. If there had been any price differences, then it would lead to shift in sales from one sub-market to another one. For example, a higher price in time period 1 than in time period 2 in region W would imply that both producers in that particular region, which by assumption are price takers, would have incentives to shift production from period 2 to period 1 until prices are identical.

More surprisingly, a monopoly might end up with the same prices as would have been the case with perfect competition. The reason is that total production is by assumption given. It is determined by the total amount of water that is available. Then the monopolist must allocate its production between the four sub-markets. As is well known, a monopolist would discriminate between different market segments according to differences in price elasticities between market segments. Given that price elasticities do not differ between segments, prices are identical in all four sub-markets. Since total production is given, those prices are identical to the prices in a situation with perfect competition.

This result is modified if we have a price unelastic demand. Then it is well known that there are no solution to the traditional monopolist's pricing problem, since it would always increase profits by reducing its production. In this particular case it implies that the monopolist can find it profitable to sell a large amount in one or several sub-markets so that prices are zero (assuming negative price is ruled out), and then sell the restricted residual production at high prices in the remaining sub-market(s).

Our main topic is the effects of an acquisition. Then, obviously, the right comparison is not between perfect competition and monopoly. A more realistic comparison would be to analyse something inbetween, for example oligopoly both before and after an acquisition. To analyse such a case, we specify a more detailed model.

Demand in the four sub-markets are described by the following linear inverse-demand functions:

$$p_{it} = \alpha_{it} - \beta_{it} S_{it}, \ i = E, W; \ t = 1, 2 \tag{4}$$

We assign the following values to the constant coefficients of the inverse-demand functions above:

$$\alpha_{W1} = 1, \ \alpha_{W2} = \alpha_{E1} = \alpha_{E2} = V \text{ and } \beta_{W1} = \beta_{W2} = 1, \ \beta_{E1} = \beta_{E2} = 1/b$$

If $V = b = 1$, then demand in all four sub-markets are identical. To allow for any possible asymmetry between sub-markets, we assume that both V and b can differ from 1. If $V < 1$, we change the maximum willingness to pay in three of the sub-markets. The linear inverse-demand curve is shifted downwards in all sub-markets except region W in period 1. The willingness to pay in region W in period 1 is then higher than in all the three other sub-markets. If $b > 1$, then the maximum willingness to pay is unchanged while the size of the sub-markets are changed. The demand curves in the two sub-markets in region E are becoming flatter compared to the two sub-markets in region W. The interpretation is that the two sub-markets in region E are of larger size than the two submarkets in region W.

The two sub-markets in period 2 are by assumption integrated (see above). The aggregated linear inverse-demand function for this integrated market becomes:

$$p_{WE2} = V - \frac{1}{1+b}(S_{W2} + S_{E2}) \tag{5}$$

One reason why the transmission lines are only congested in one of the two periods could be that $V < 1$. This implies at least as far as region W is concerned that demand in period 2 is lower than demand in period 1. Given the same transmission capacity in the two periods, less transmission is needed to equate prices in period 2. We assume that the transmission capacity in period 2 is sufficiently large to prevent any incentives to act strategically in order to congest the transmission line in that period.[13]

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[13] See Borenstein, Bushnell and Stoft (2000) for an extensive analysis of producer incentives to induce congestion on transmission lines. We will come back to this situation later on (see Proposition 2).

The fact that the two regions are integrated into one market in period 2 changes the nature of the energy constraint facing producer X. Now X can rely on capacity from both regions when supplying the market in period 2. The new constraint in period 2 becomes:

$$\sum_i q_{i2}^X \leqslant \sum_i \overline{q}_i^X, \text{ where } i = E, W \tag{6}$$

In period 1, where we have the potential for two separate markets, producer X is now able to produce all the available energy capacity within a region in this period;

$$q_{i1}^X \leqslant \overline{q}_i^X \tag{7}$$

and still be able to sell in the same region in period 2 by the use of energy capacity located in the other region. However, these new constraints can not both hold with equality at the same time for positive production levels in both periods and regions. This would result in overall production in excess of available energy capacity. Thus the following must hold:

$$\sum_i \sum_t q_{it}^X \leqslant \sum_i \overline{q}_i^X, \text{ where } i = E, W; \ t = 1, 2 \tag{8}$$

With these new constraints, producer X has gained increased flexibility in production. With four separate markets we had that sales of electricity to customers located in one region was limited by the energy capacity in that region, $q_{i1}^X + q_{i2}^X \leqslant \overline{q}_i^X$. Now with integrated regions (one market) in period 2 this is no longer a limitation on sales and we might very well have that $q_{i1}^X + q_{i2}^X > \overline{q}_i^X$. This implies that producer X can de facto move production from period 1 in region E to period 1 in region W without using the transmission line in period 1 between the two regions. The reason is that the producer is able to reshuffle its sale in period 2, when regions are integrated, in such a way that sales in period 1 is increasing in region W and decreasing in region E.

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Even though the two sub-markets in period 2 are by assumption integrated, we still may have three different sub-markets: region W in period 1, region E in period 1, and the integrated market consisting of both regions in period 2. However, note that we have one competitive fringe in region E and one in region W. Given that the competitive fringes are sufficiently large, they will ensure that there are no price differences between period 1 and 2. For example, let us consider region E. If producer Y (or X) reduces sales in one of the two periods in order to increase the price, the competitive fringe Z would immediately increase sales in this period, giving producer Y no room for such strategic behavior. In a similar manner, the competitive fringe U will ensure that the prices are identical in the two time periods in region W.

3 The effect of acquisitions

The starting point is, as described, that all four sub-markets are integrated. This replicates the perfect competition outcome described in Proposition 1. However, there is a potential for the transmission line in periode 1 to be congested. Then we ask the question of how an acquisitions may change the equilibrium outcome. First, we let X acquire the competitive fringe U. Given such an acquisition, we next consider what happens when X acquires Y.

3.1 An endogenous bottleneck?

If X acquires U, there are no longer any players present that guarantees identical prices in region W in time periode 1 and 2. With potential congestion on the transmission line between the two regions in period 1, producer X has three alternatives. One alternative is that producer X after the acquisition acts so that prices in all four sub-markets are identical, as was the case before the acquisition. Alternatively, producer X might reduce its production in region W in time period 1 in order to cause the line to be congested with full *imports* to region W. By doing so it could achieve a higher price in that sub-market than the price of the three other integrated sub-markets (see figure 1). The third alternative would be to increase production in region W in period 1, causing congestion and full *exports* from region W to E. To check the conditions associated with these three strategies, let us apply the specific model we introduced above. To simplify the exposition, we let production by producer X, denoted q_{it}^X, include production from producer U.

If three of the four sub-markets are integrated, then the aggregated inverse linear demand for this integrated market (market 2) becomes:

$$p_2 = V - \frac{1}{1+2b}(S_{W2} + S_{E1} + S_{E2}) \tag{9}$$

With two markets to analyse, we have the result that both producer Y and the competitive fringe Z only have energy capacity available for production in market 2.

Producer X, however, can produce in both markets. Given that all the water is used to produce energy, we can express production in market 2 by:

$$q_2^X = \overline{q}_W^X + \overline{q}_E^X - q_{W1}^X \tag{10}$$

The production in market 2 consists of the energy capacity available in region E and the difference between capacity in region W and production in the same region in period 1 (market $W1$). If producer X reduces production in market $W1$ enough to create congestion, we know that $p_{W1} > p_2$. Then we can find the level of production from producer X in sub-market $W1$ corresponding to separate markets, where $W1$ is the high price market. In a similar manner we can find the production levels corresponding to the integrated market case when all four sub-markets are integrated and the case where sub-market 2 is the high price market, respectively:

$$
\begin{bmatrix}
p_{W1} > p_2 & \text{if} & q_{W1}^X < \frac{1}{2+2b}Q - (V-1)(\frac{1+2b}{2+2b}) - \overline{K}_1 \\
p_{W1} = p_2 & \text{if} & -\overline{K}_1 < q_{W1}^X - \frac{1}{2+2b}Q + (V-1)(\frac{1+2b}{2+2b}) < \overline{K}_1 \\
p_{W1} < p_2 & \text{if} & q_{W1}^X > \frac{1}{2+2b}Q - (V-1)(\frac{1+2b}{2+2b}) + \overline{K}_1
\end{bmatrix}
\tag{11}
$$

where $Q = \overline{q}_W^X + \overline{q}_E^X + \overline{q}_E^Y + \overline{q}_E^Z$. We can observe from (11) that the production range (q_{W1}^X) for which we have integrated markets increases with higher transmission capacity in place between the two sub-markets. Remember that before X's acquisition of U all sub-markets are by assumption integrated ($p_{W1} = p_2$), because U acted as a competitive fringe. We let p denote the price of the integrated market.

After the acquisition producer X would face different profit maximation problems depending on whether the markets are separated or not. The producer maximizes profit by choosing production in both sub-markets subject to the constraints on energy production in the two markets; $q_{W1}^X \leq \overline{q}_W^X$ and $q_2^X \leq \overline{q}_W^X + \overline{q}_E^X$[14]. When the two sub-

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[14]In the situation $p_{W1} - p_2 > 0$, we have two possibilities. First, we may have a situation where one (both can not bind at the same time) of these two constraints are binding before the acquisition.

markets are integrated producer X receive the price p for all the available energy. Thus the profit function (π^{XI}) becomes:

$$\pi^{XI} = p(\overline{q}_W^X + \overline{q}_E^X) \tag{12}$$

In a similar manner, we can define the profit functions corresponding to the case where production in market $W1$ is reduced sufficiently to create congestion and full import to $W1$ $(\pi^{XM} = p_{W1}(q_{W1}^X) + p_2(q_2^X))$ and full export from $W1$ $(\pi^{XL} = p_{W1}(q_{W1}^X) + p_2(q_2^X))$. Thus we have that:

$$\begin{bmatrix}
\max_{q_{W1}^X} \pi^{XM} & \text{if} & q_{W1}^X < \frac{1}{2+2b}Q - (V-1)(\frac{1+2b}{2+2b}) - \overline{K}_1 \\[2mm]
\pi^{XI} & \text{if} & -\overline{K}_1 < q_{W1}^X - \frac{1}{2+2b}Q + (V-1)(\frac{1+2b}{2+2b}) < \overline{K}_1 \\[2mm]
\max_{q_{W1}^X} \pi^{XL} & \text{if} & q_{W1}^X > \frac{1}{2+2b}Q - (V-1)(\frac{1+2b}{2+2b}) + \overline{K}_1
\end{bmatrix} \tag{13}$$

We note that in the case of integrated markets, producer X's profit is the same regardless of how production is allocated between the two sub-markets. The price p in the integrated market is determined by the total amount of energy available, Q. Thus, producer X's allocation of energy between regions and periods have no effect on the price as long as the sub-markets are integrated.

We can now state our proposition 2:

Proposition 2 *If X acquires U and if the profit maximation level of q_{W1}^X is positive but low enough to cause congestion on the line between the two regions $(0 < q_{W1}^X < \frac{1}{2+2b}Q - (V-1)(\frac{1+2b}{2+2b}) - \overline{K}_1)$, then after the acquisition we have that $p_{W1} - p_2 > 0$.*

If one of these constraints are binding we have a corner solution. Second, we may have a situation where all the energy is used and none of the two constraints are binding, implying that producer X in equilibrium sells in both markets. If we have a corner solution before the acquisiton takes place, this will constrain producer X from behaving differently after the acquisition. Furthermore, if one of the constraints are only binding on the solution after the acquisition this will limit producer X's behaviour. In the following we shall for simplicity assume internal solutions both before and after the acquisition.

Assuming that $p_{W1} - p_2 > 0$, we can find the exact price difference after the acquisition of U by inserting the solution to producer X's maximation problem (q_{W1}^X) into the two inverse demand functions,

$$\Delta p \equiv p_{W1} - p_2 = \frac{1}{2} \frac{-(K_1 + V - 1)(1 + 2b) - \overline{K}_1 + \overline{q}_E^Y + \overline{q}_E^Z}{1 + 2b}. \tag{14}$$

Proposition 2 can be illustrated by a numerical example (see figure 2). Let us assume that $V = 1$, $\overline{K}_1 = (\frac{1}{32})$, $b = 0.5$ and $\sum_{ji} \overline{q}_i^j = 1$ with $\overline{q}_W^X = \frac{15}{32}$ and $\overline{q}_E^X = \frac{1}{32}$. It can then be shown that $\pi^{XI} = 0.333$ (profits if integrated markets) and $\pi^{XU} = 0.336$ (maximum profits if high price in region 1 in period W. The latter case corresponds to a production level $q_{W1}^X = 0.24$, which is low enogh to ensure that $p_{W1} - p_2 > 0$. In the choice between creating an import constraint on the transmission line in period 1 and letting the markets be integrated, producer X would choose to induce congestion. If we look at the possible range of production corresponding to $p_{W1} - p_2 < 0$, there is no production level resulting in profits higher than in the integrated market case.[15] After the acquistion producer X would therefore find it profitable to reduce production in sub-market $W1$ which, in turn, leads to congestion and higher prices in this market.

Our result replicates the result found in Borenstein, Bushnell and Stoft (2000). After the acquisition, the firm can find it profitable to induce a congestion on a transmission line. By reducing production in region W in period 1, producer X can act as a monopoly firm on the residual demand in that sub-market; total demand in that sub-market deducted the imports through the transmission line. Strategic behaviour has in such a case led to a temporary bottleneck on transmission.

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[15]The maximum profit from inducing congestion and lower prices in sub-market $W1$ is even higher, $\pi^{XL} = 0.344$. This corresponds to a production level $q_{W1}^X < \frac{1}{2+2b}Q - \overline{K}_1$ implying that $p_{W1} - p_2 > 0$. This is a contradiction, so therefore not attainable.

$\pi(q^X_{W1})$

High price regime in
region W in period 1

Inte-
gration
regime

Low price
regime in
region W
in period 1

0.345

0.34

$p_{W1} > p_2$

$p_{W1} = p_2$

0.335

0.33

0.325

$p_{W1} < p_2$

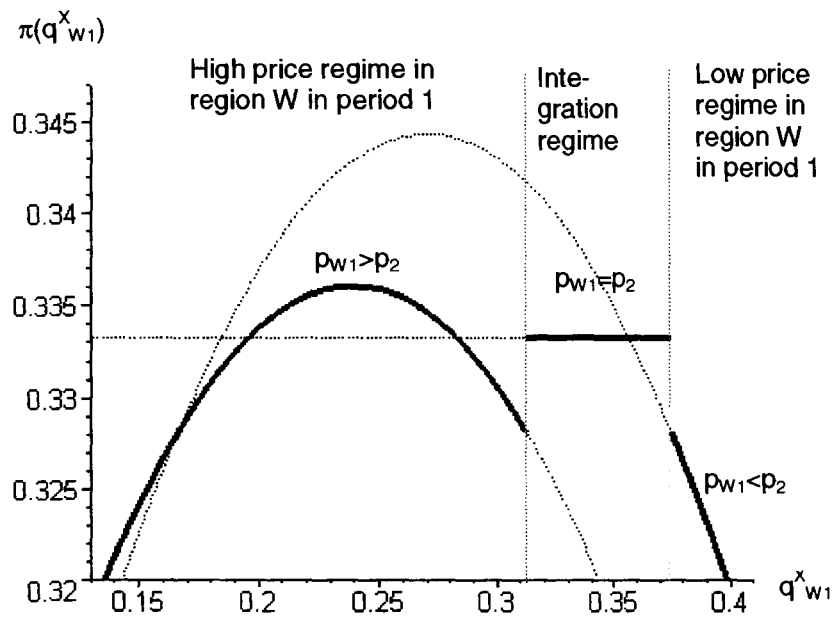0.32

0.15   0.2   0.25   0.3   0.35   0.4 q^X_{W1}

Figure 2: Profit functions for producer X under the three different price regimes ($p_{W1} > p_2$, $p_{W1} = p_2$ and $p_{W1} < p_2$). Attainable profit levels as a function of q^X_{W1} are represented by the solid line.

3.2 Asymmetry concerning location

Let us now assume that X has acquired U, and that it has led to price differences as described in Proposition 2. The next question is what will happen to this price difference when producer X acquires producer Y. Is the price difference increasing as a result of the concentration?[16]

After this second acquisition producer X controls the energy capacity of producer Y located in region E. Assuming positive production in both markets also after the merger, production by X in market 2 can now be expressed as follows:

$$q_2^X = \overline{q}_W^X + \overline{q}_E^X + \overline{q}_E^Y - q_{W1}^X \tag{15}$$

Producer X solves the same maximization problem as defined in equation (12) subject to the following par of constraints;

$$q_{W1}^X \leq \overline{q}_W^X \text{ and } q_2^X \leq \overline{q}_W^X + \overline{q}_E^X + \overline{q}_E^Y \tag{16}$$

where we observe that producer X now has more energy available for production in market 2. Again, we have two possible outcomes within our framework. The producer may in equilibrium choose to produce some of the energy available in region W in period 2, thus selling electricity in market 2. As long as production in region W in period 1 also is positive, then we have an internal solution to our problem where none of the two constraints (16) above are binding. We assume this is the case and describe the solution.

As before we find the solution by solving the producer's first order condition and inserting this solution into the inverse demand functions (4 and 9). The new expression defining the price difference between the two markets then becomes:

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[16]One possibility would of cause be that a merger changes the incentives so significantly that the price difference dissappers or turns negative. This implies that transmission changes direction. Unless otherwise stated, we shall disregard such effects and confine the analysis to whether the positive price difference becomes larger or less positive.

$$\Delta\widehat{p} \equiv \widehat{p}_{W1} - \widehat{p}_2 = \frac{1}{2}\frac{-(K_1 + V - 1)(1 + 2b) - \overline{K}_1 + \bar{q}_E^Z}{1 + 2b} \tag{17}$$

If the acquisition leads to higher price difference this implies increased welfare loss. Consumption is shifted from consumers in market $W1$ with high willingness to pay for electricity to consumers with lower willingness to pay in market 2. However, in our case we see that the price difference is reduced and thus leading to an increase in welfare:[17]

$$\Delta p - \Delta\widehat{p} = \frac{\bar{q}_E^Y}{2(1 + 2b)} > 0 \tag{18}$$

The reduction in price difference follows directly from change in producer X's incentives following the acquisition. After the acquisition producer X takes into account the price effect on energy previously controlled by producer Y. This energy is located in region E and offered for sale in market 2. A reduction in sales in market 2 would make a larger contribution to producer X's profit through the price effect after the acquisition simply because producer X now controls more of the energy sold.

We can summarize our results as follows:

Proposition 3 *Given that X has already acquired U and resulted in $p_{W1} - p_2 > 0$, then X acquiring Y would result in a smaller price difference or have no price effects*

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[17]One alternative is that we have an internal solution to our problem before the aquisition but that this changes to a corner solution after the aquisition. Producer X's incentives to transfer energy from market 2 to $W1$ may be limited by the constraint on production in region W. This would not change the conclusion. The price difference would still decrease. However, the reduction may be limited by the production constraint.

Another possibility is that we face a corner solution before the acquisition. That would be the case if one of the production constraints in equation (16) are binding. Given the assumption that $W1$ is the high price market, we know that producer X would always produce some electricity in this market. Let us therefore consider the case where $q_{W1}^X = \bar{q}_W^X$ before the acquisition, the only relevant constraint. After the acquisition we know that producer X has incentives to transfer energy to market $W1$. However, because of the binding production constraint this is not possible and the acquisition would have no effect on the price difference.

at all. If the price changes due to the acquisition of Y, then the price difference is reduced by $\frac{\bar{q}_E^Y}{2(1+2b)}$ or less.

Note that the change in the price difference only depends on two parameter values: the relative market size (region W versus region E) and the size of the acquired producer ($\bar{q}_E^Y$). Both effects are quite intuitive. The larger the size of the acquired firm, the larger the price change following the acquisition; the larger the size of the integrated market, the smaller the price change following the acquisition.

More surprisingly, though, is that the size and distribution of firm's production, except for the acquired firm's total production, does not matter for the price change after the acquisition. For example, it does not matter how much production producer X has in each of the two regions, or how large its total production is. The intuition is that such differences in size and distribution of production is incorporated in the equilibrium price as long as we have an interior solution at the outset. Due to this, such factors do not have any effect on price changes following an acquisition.

Let us now extend the model by introducing more than one producer that have production in both regions. We assume that there are now n producers that have production in both regions. Then it can be shown the following result:

Proposition 4 *If there are n producers that have production in both regions and $p_{W1} - p_2 > 0$, then X_i acquiring Y would, if any effect at all, reduce the price difference by $\frac{\bar{q}_E^Y}{(n+1)(1+2b)}$ or less.*

Proof. See the Appendix A ■

We see that the result is parallel to the one obtained in the situation referred to in the last Proposition where we had just one hydropower producer with capacity in both markets. After the acquisition producer X_i takes into account price effects on energy previously controlled by producer Y, and shifts production from the integrated market in period 2 to sales in region W in period 1. Now, however, a reduction of sales in market 2 would induce the other producers that are active in both regions

to increase sales in this market. The price effect of reducing sales in market 2 is therefore dampened compared to the situation where we had one supplier with flexible production. The larger the number of producers active in both regions, the more limited is the price effect of an acquisition.

3.3 Asymmetry concerning storage and location

So far we have assumed that all producers are flexible concerning storage and production, since they all can shift all its production from period 1 to period 2 or vice versa and there is no limit on production. This might not be the case in real electricity markets. With a given level of energy capacity available over the periods there are two different relevant constraints, one short run and on long run constraint. In the short run some producers may face a constraint on its production, despite energy capacity in its reservoir. This would be constraints on effect capacity. While some producers have full flexibility in the short run, other producers might be limited by the installed effect capacity. In the long run some producers are not able to shift production from one period to another, for example from one year to another. The reason is that they do not have reservoirs large enough to store water from one season to another and they are confined to produce more or less according to the current rainfall. The ability to store water may differ a lot from one producer to another. Irrespective of whether it is a short or long run constraint, we can apply our model to analyse the effects of asymmetries concerning such constraints.

In our model above we assumed that producer Y was located with its energy capacity only in region E confined to selling electricity in market 2; the low price market. What would happen if we allowed producer Y to be located in region W instead and selling electricity in the high price market? Producer X would then have no incentives to increase production in the high price region following the acquisition, simply because producer Y's capacity is located in that region. In contrast, we would expect increasing price differences as a result of the acquisition.

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Let us now introduce asymmetries not only on location, but also on storage. We model this feature by assuming that producer Y is located with energy capacity in region W and that the producer is lacking sufficient storage capacity. To highlight the effect of asymmetries on storage, we consider the extreme case where producer Y has no storage capacity at all. This means that producer Y's production is determined by the amount available in each period[18]. Using this assumption we can rewrite the two equilibrium conditions as follows:

$$S_{W1} = q_{W1}^X + \overline{q}_{W1}^Y + \overline{K}_1 \ , \quad S_2 = \overline{q}_E^X + q_{W2}^X + \overline{q}_{W2}^Y + \overline{q}_E^Z - \overline{K}_1 \tag{19}$$

We solve to find the expressions representing the price differences before and after the acquisition in the same manner as above. Doing so we can write the difference between the two price differences as follows:

$$\Delta p - \Delta \widehat{p} = \frac{1}{2} \frac{\overline{q}_{W2}^Y - (1 + 2b)\overline{q}_{W1}^Y}{1 + 2b} \tag{20}$$

For a given value of the parameter b in the equation above we observe that the price difference will be reduced if Y's energy capacity in market 2 (in period 2, region W) is sufficiently large compared to the capacity in region W in period 1. If $\overline{q}_{W2}^Y$ is sufficiently low compared to $\overline{q}_{W1}^Y$ the price difference will increase following the acquisition. Thus the changes in the price difference is dependent upon whether X earns the most by reducing production in market $W1$ or market 2. This again is directly linked to where producer Y has its production capacity distributed; between periods. Then we have shown the following result:

Proposition 5 *Let us assume that Y is located in region W, and $p_{W1} - p_2 > 0$. Then X acquiring Y would lead to a lower price difference if $\overline{q}_{W2}^Y > (1 + 2b)\overline{q}_{W1}^Y$.*

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[18]Alternatively we could have assumed that producer Y's production was effectively constrained on effect capacity in both periods both before and after the acquisition.

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3.4 Introducing thermal production

In most markets where hydropower is present we also have a significant share of thermal based electricity production. This is also the case within the Nordic electricity market. A natural extension of our model is to include thermal production in one of the two markets central to the analysis.

We assume that transmission capacity is fully utilized in period 1. In period 2 we have an integrated market. We assume that electricity flows from market 2 to market $W1$, making $W1$ the high price market. The linear inverse demand function in market $W1$ is exactly the same as before. We let the thermal producer be located in region E with just one production site. Now, the price in market 2 will depend on how much is produced (q_E^T) by the thermal producer.

$$p_2 = V - \frac{1}{1+2b}(\overline{q}_E^X + \overline{q}_W^X - q_{W1}^X + \overline{q}_2^Y + \overline{q}_E^Z + q_E^T - \overline{K}_1) \qquad (21)$$

We make two simplifications, $\overline{q}_E^X + \overline{q}_W^X = \overline{q}^X$ and $1 + 2b = h$. We assume that thermal production is based on one or more fossil fuel technologies (coal, oil or gas). We let the thermal producer's cost be represented by the following linear function:

$$C(q_E^T) = cq_E^T \qquad (22)$$

Thus, we have constant marginal cost (c) in thermal production of electricity. Now we let both producer X and producer T act strategically. Producer X chooses how to allocate water between the two markets given thermal production. The thermal producer chooses q_E^T given distribution of available water by producer X ($\overline{q}_E^X + \overline{q}_W^X - q_{W1}^X$ and q_{W1}^X). Hydro producer X has the same maximization problem as before with the change that production in market 2 now also include thermal production. The thermal producer maximizes:

$$\max_{q_E^T} p_2 q_E^T - cq_E^T \qquad (23)$$

We solve the system of first order conditions to find equilibrium quantities in the two markets. We can then solve for the difference in equilibrium prices between the two markets. The price difference prior to the acquisition is:

$$\Delta p \equiv p_{W1} - p_2 = -\frac{\begin{array}{c}(-2h^2 - 2h + 2\overline{K}_1 h^2 + \overline{q}^X h - Vh^2 - \overline{q}_2^Y h - q_E^Z h \\ + 3Kh + ch^2 + Vh - \overline{q}_2^Y - \overline{q}_E^Z + \overline{K}_1 + ch)\end{array}}{h(4h + 3)} \qquad (24)$$

In the same way as before this expression will be positive if there is enough capacity in region E relative to region W. The partial derivative of the expression above with respect to reservoir levels in region E ($\overline{q}_2^Y$, $\overline{q}_E^Z$, $\overline{q}_E^X$) are all positive. We assume these levels are high enough making the expression positive both before and after the acquisition.

We look at what will happen if producer X acquires producer Y's energy capacity, given that Y is located in region E. Through the same operations as described above we find the equilibrium values of q_{W1}^X and q_E^T. We then use these values to find the price difference after producer X has acquired control over producer Y's resources.

$$\Delta \widehat{p} \equiv \widehat{p}_{W1} - \widehat{p}_2 = -\frac{\begin{array}{c}(-2h^2 - 2h + 2\overline{K}_1 h^2 + \overline{q}^X h - Vh^2 + \overline{q}_2^Y h - q_E^Z h \\ + 3Kh + ch^2 + Vh - \overline{q}_E^Z + \overline{K}_1 + ch)\end{array}}{h(4h + 3)}$$

Assuming this price difference is still positive with electricity flowing from region E to W in period 1 equal to the capacity $\overline{K}_1$, we can solve for the change in price difference before and after the acquisition.

$$\Delta p - \Delta \widehat{p} = \frac{\overline{q}_E^Y(2h + 1)}{h(4h + 3)} > 0 \qquad (25)$$

We see that the existence of a thermal producer in market 2 does not alter the direction of the price change. We still have that the price difference becomes smaller

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due to a positive expression in equation (25). The intuition is identical to the one we provided in the previous analysis.

This is not the whole story. We assumed that all available water is used to produce energy, implying that total production based on hydro power is fixed through the analysis. Thermal production, on the other hand, can change. In order to see how the acquisition effects thermal production we simply compare equilibrium thermal production values before (q_E^T) and after $(\widehat{q}_E^T)$ the acquisition takes place.

$$\Delta q_E^T = \widehat{q}_E^T - q_E^T = \frac{\overline{q}_E^Y}{4h+3} \tag{26}$$

The expression above is positive, so we have that thermal production is increased following X's acquisition of producer Y. With a thermal producer located in region E we then have that total production in the two markets are increasing following an acquisition. All else equal, this is welfare improving. In addition, the acqusition leads to to a reduction in the price difference between the two markets. With a reduction in the price difference and increased total production we can safely state that welfare is increased.

Proposition 6 *Let us assume there is a thermal producer in region E, and and $p_{W1}-p_2 > 0$. If X acquires Y, which is located in region E, then the price difference becomes smaller and the thermal producer's production increases, and total welfare improves.*

This is an example where an acqusition leads to counter-intutive results: higher total production and reduction in price differences. It is straight forward to see that those results can be reversed. There can be instances where an acquistion leads to a reduction in the price difference and a lower total production, increase in price differences and higher production, and finally larger price differences and lower production.

4 Some concluding remarks

Hydropower producers are more flexible than any other energy producers. Is this a virtue or a problem seen from society's point of view? In the existing literature it has been shown that we should be concerned about a hydropower producer that is the marginal producer, since it has flexibility to withheld capacity in the period where other producers are constrained.

Our main point is that in an situation where such a producer does not have total dominance, the competitive effects of higher concentration is less clear-cut. The important feature in a hydropower system is that the producers must allocate their production between different sub-markets. If one producer withdraws production in one period, it must offer the withdrawn quantity at a later time period or export it to another region. In contrast to other producers, its total production is fixed unless it is able to spill water. Then the price effect of an increase in concentration depends on the location and flexibility of the hydropower producers. Asymmetries in location as well as asymmetries in the ability to store water (for example from one year to another) is decisive for whether an increase in concentration leads to larger or smaller price differences.

Our study has important implications of the evaluation of the competitive effect of an acqusition or merger in a hydropower system. Obviously, the comparison is between the market outcome before and after the merger (or acquisition).[19] As earlier studies have shown, it is of importance to check whether the merged firm becomes a marginal producer more often than what was the case before the merger. Given that this is not a very severe problem, our study suggests that it is important to evaluate any possible asymmetries between the merging parties. Are they located in different regions? Is one located in several regions, and another in only one region? Do they have the same flexibility concerning storage of water, or could it be that one of them

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[19]See for example Willig (1992), where such a comparison is recommended for merger analysis. For more recent examples, see Froeb and Werden (2002) and Epstein and Rubinfeld (2001).

has the ability to store water from one year to another and the other does not have such an option? How are the price differences before the merger or acquisition? Is the producer that a large firm acquires primarely active in a low price market, which can be regarded as a dumping market? These and similar questions must be answered in order to evaluate whether a merger or acquisition will increase or reduce existing price differences.

A n producers in both regions

In addition to producers Y and Z we now assume there are n other producers X_i (where $i = 1..n$) with energy capacity in both regions W and E. In the following we shall analyze the price difference between our two markets $W1$ and 2 before an acquisition by producer X_i. The analysis is analogous to the one in section 2, now with n flexible hydropower producer present in both regions.

Each producer X_i has $\overline{q}_W^{X_i}$ available for production in region W and $\overline{q}_E^{X_i}$ for production in region E. Production in market 2 by producer X_i can be expressed as; $q_2^{X_i} = \overline{q}_W^{X_i} + \overline{q}_E^{X_i} - q_{W1}^{X_i}$. In order to simplify we shall assume that all producers X_i have the same energy capacity available in both regions; $\overline{q}_W^{X_i} = \overline{q}_W^X$ and $\overline{q}_E^{X_i} = \overline{q}_E^X$. Thus, in the absents of any production costs these producers can be treated symmetrically.

The transmission line between regions W and E is only constrained in period 1, and electricity flows from region E to W with market $W1$ being the high price market. We can now write our two new inverse linear demand functions:

$$p_{W1} = 1 - \sum_i q_{W1}^{X_i} - \overline{K}_1 \tag{27}$$

$$p_2 = V - \frac{1}{1+2b}(n(\overline{q}_W^X + \overline{q}_E^X) - \sum_i q_{W1}^{X_i} + \overline{q}_E^Y + \overline{q}_E^Z - \overline{K}_1) \tag{28}$$

Producers Y and Z are only located with capacity in region E. The producers X_i, however, can choose how to distribute available capacity between the two markets. We assume that these suppliers act strategically according to a one shot Nash-Cournot strategy where they simultaneously determine the level of production in market $W1$. The maximization problem of producer X_i can be expressed as:

$$\max_{q_{W1}^X} = p_{W1}(q_{W1}^{X_i}) + p_2(q_2^{X_i})$$

subject to the constraints that apply for production in one region; $q_{W1}^{X_i} \leq \overline{q}_W^{X_i}$ and

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$q_2^{X_i} \leq \overline{q}_W^{X_i} + \overline{q}_E^{X_i}.$[20] In order to find the equilibrium before the acquisition we solve the n producers first order conditions simultaneously to find the optimal values of production in market $W1$. Since the producers are symmetric, we have that $\sum_i q_{W1}^{X_i} = nq_{W1}^{X_i}$. We then use these values to calculate the pre-acquisition price difference:

$$\Delta p \equiv p_{W1} - p_2 = \frac{-(\overline{K}_1 + V - 1)(1 + 2b) - \overline{K}_1 + \overline{q}_E^Y + \overline{q}_E^Z}{(1 + n)(1 + 2b)} \tag{29}$$

Let producer X_i acquire control over producer Y's energy capacity in region E. Because producer X_i now controls the production capacity of producer Y, X_i can no longer be treated symmetrically with the other producers X_j (where $j = 1..n$, $j \neq i$) having capacity in both markets. Now we have to solve for optimal production by producer X_i and one of the other $(n - 1)$ symmetric producers. Now we use the fact that $\sum_j q_{W1}^{X_j} = (n - 1)q_{W1}^{X_j}$ and solve for the optimal values of production in market $W1$. By substitution we can then write the new price difference as follows:

$$\Delta \widehat{p} \equiv \widehat{p}_{W1} - \widehat{p}_2 = \frac{-(\overline{K}_1 + V - 1)(1 + 2b) - \overline{K}_1 + \overline{q}_E^Z}{(1 + n)(1 + 2b)} \tag{30}$$

We assume this price difference to be positive also after the acquisition, meaning that electricity flows from region E to W in period 1. We look at how the acquisition effects the price difference; whether this difference becomes less or more positive. The change in price difference can be expressed by:

$$\Delta p - \Delta \widehat{p} = \frac{\overline{q}_E^Y}{(1 + n)(1 + 2b)} > 0 \tag{31}$$

This is the condition shown in proposition 4.[21]

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[20]With at least some production in both markets none of these constraints bind and we have an interior solution to the problem. As mentioned before the second constraint is irrelevant here because with higher prices in market $W1$ the producer will always have some production in this market. We discuss here the equilibrium price difference assuming an interior solution.

[21]Producer X_i's incentives to increase production in market $W1$ may be limited by constraints on production in region W. If producer X_i before the acquisition have used all the available capacity in region W, then the acquisition would not have any effect on the price difference. Similarly, the

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production constraint could constrain producer X_i from increasing production as much as wanted after the acquisition. In this case, the effect on the price difference would be lowered.

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Policy measures and storage in a hydropower system

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Abstract: In this paper we discuss how three different public policy measures
affect water storage controlled by hydropower producing firms. In particular we dis-
cuss measures to promote competition, increase transmission capacity and rationing.
The analysis is conducted within the framework of an oligopoly model where 2 hydro
producing firms engage in dynamic Bertrand competition. We extend this model to
be able to analyse how the three policy measures affect storage by hydropower pro-
ducing firms and focus especially on the probability of hydropower replacing thermal
production.

We find that competition, represented by the Bertrand-Nash solution leads to
lower storage compared to the monopoly solution. Furthermore, we find that in-
creased transmission capacity and rationing both lead to more fierce competition in
situations when water is plentiful and thus to a reduction in storage. These results
imply that increased competition, transmission capacity and rationing all contribute
to an increased probability of hydropower replacing thermal production.

1 Introduction

Deregulation of electricity markets around the world has been followed by a large number of analyses of competitive strategies in such markets[1]. Most of these studies focus on static analysis of competitive behavior in markets dominated by thermal power. In several electricity markets hydropower contributes a significant share of total production. This includes the Nordic market, New Zealand, South American markets like Chile, Colombia and Argentina, Switzerland and also to some extent markets in the US[2].

The problem facing a hydropower producer is to decide how to allocate a scarce renewable resource between different periods in time. As noted by Garcia et al. (2001), in markets where hydropower plays a significant role, analysis of dynamic pricing behavior is important in order to understand how firms act strategically in such markets.

Garcia et al. refer to an earlier version of a paper by Bushnell (2003) and another by Scott and Read (1996) as some of the few known exceptions focusing on dynamic strategic behavior. In addition we note that Crampes and Moreaux (2001), Johnsen (2001) and Mathiesen et al. (2004) also conduct an analysis of dynamic strategic behavior in electricity markets. These five papers analyze dynamic strategic behavior in a finite-horizon setting where quantity is the strategic variable. Also, only a couple of these papers (Johnsen (2001) and Mathiesen et al. (2004)) address the question of strategic behavior when there is uncertainty with regard to inflow.

Garcia et al (2001) analyze dynamic strategic behavior of hydropower producers in an infinite-horizon setting where two firms engage in dynamic Bertrand competition and where inflow is uncertain. They show that a tightening of the price cap on electricity in a market with significant hydropower production would reduce the alternative value of production in the current period and thus increase the competi-

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[1]This include for example Green and Newbery (1992), Green (1996), Newbery (1998), Borenstein and Bushnell (1999) and Borenstein, Bushnell and Wolak (2002).

[2]See Bushnell (2003).

tion between producers when water is plentiful. Furthermore, as an extension to their basic model they show that the price cap also affects the probability of hydropower replacing thermal production of electricity. If the price cap is sufficiently low, prices in situations with full reservoirs become so low that thermal production is eliminated and replaced by hydropower. If there is no inflow in the following peirod this might effect the reliability of the system.

Following the conclusion reached by Garcia et al (2001) it seems natural to ask whether there are any public measures that can be imposed to reduce the potential problem of hydropower replacing thermal production in situations where inflow is plentiful. Of cource, a high price cap would give producers incentives to store water for periods with little inflow. However, a high price cap might be politically undesirable as this would imply a high price for electricity. Thus it is relevant to look at the properties of other public measures. Motivated by the observed energy shortage during the winter 2002/2003 in Scandinavia and the following discussion, we look at three such public measures in this paper.

First, we look at measures to promote competition. Authorities can promote competition for instance by taking actions to prevent collusion or preventing mergers from taking place. We analyse the effect of promoting competition on the probability of hydropower replacing thermal production in the simplest possible way, by comparing the Bertrand-Nash outcome described by Garcia et al (2001) to the monopoly outcome. Even though Garcia et al (2001) describe the collusive outcome which can be identical to the monopoly solution, they make no explicit comparisons of the two outcomes. Our description of the monopoly outcome is also more suitable for comparisons with the Bertrand-Nash solution. We find that competition represented by the Bertrand-Nash outcome implies a higher probability of hydropower replacing thermal production than the monopoly solution.

Another public measure that has been proposed in order to reduce the problem of energy shortage in low inflow situations is increased transmission capacity. The idea is that increased transmission capacity would make it possible to increase pro-

duction through imports in situations with low inflow and thus reduce the problem of energy shortage. There are several ways this could be modelled. Here, we model transmission between two geographic areas with one hydropower company in each area. In this setting, we find that an increase in transmission capacity leads to a more fierce competition between the two hydro producing companies and thus increases the probability of hydropower replacing thermal production. This effect is similar to a reduction in the price cap as described by Garcia et al (2001).

Finally, we consider the effect on storage by rationing imposed by authorities. Rationing may be thought of as a measure to secure supply of electricity in situations with little or no inflow. We model rationing as an action by authorities to reduce demand in situations when the energy resource is belived to be scarce. Rationing will only affect profits directly in the periods where such rationing is imposed and also affect producers' storage levels. These effects are different from a reduction in the price cap analysed by Garcia et al. We find however, that the effect of rationing is similar to a reduction in the price cap. Increased rationing leads to a more fierce competition when water is plentiful and thus increases the probability of hydropower replacing thermal production of electricity.

In section 2 we restate the basic model developed by Garcia et al (2001) and compare the Bertrand-Nash solution to the monopoly solution. In section 3 we extend the model to include a situation with limited transmission capacity between two different geographic areas. In section 4 we change the model to include an element of rationing. In all three sections we discuss storage when thermal production is included. In section 5 we provide some concluding remarks.

2 Market power and storage

In this section we repeat the basic features of a model developed by Garcia et al (2001). We then develop a monopoly solution and compare this to the basic Bertrand-Nash solution described by Garcia et al. (2001) in a situation with price taking thermal

producers present in the market. By doing so we are able to analyse in a simple way how market power affects storage in a hydropower system.

2.1 The Bertrand-Nash solution

The general framework of the Garcia et al (2001) infinite-horizon model is a situation with two hydropower producers, where each producer controls one storage facility. The two reservoirs are of equal size. At the beginning of each period both producers observe how much water that is available for production in the two storage facilities. Thus, there is complete information with regard to the history of the game. At each stage of the game, however, producer $i = 1, 2$ does not observe the other producer's action before the move is made. After observing storage levels the two producers set their prices simultaneously. If they set the same price, it is assumed for simplicity that one of the two producers serves the entire market. This produces asymmetry with regard to storage levels between the producers. However, as Garcia et al (2001) show, the results are unaffected by this assumption[3].

Without additional restrictions there would be a vast number of states (storage levels) visited by the two producers. Garcia et al (2001) solve this problem through a number of assumptions. First it is assumed that demand is equal to one unit in every period and perfectly inelastic. As noted by Garcia et al (2001) this assumption is consistent with the operation reality of many real-time wholesale electricity markets. In particular they argue with reference to Train and Shelting (2002) that the widespread use of fixed rate contracts in retail markets makes the responsiveness of market demand extremely limited.[4]

Second, it is assumed that the storage capacity is also equal to one unit. This implies that one producer is not able to store more water than just to cover demand in each period. Garcia et al (2001) also make the assumption that the maximum output capacity is equal to one unit in each period.

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[3]Se Garcia et al (2001) section 2.2.

[4]This is also noted by Borenstein et al. (2003) with regard to the electricity market in California.

Thirdly and finally it is assumed that water inflow w at the end of each period follows a simple binomial process, where $w = 1$ with probability q and $w = 0$ with probability $(1 - q)$. It either rains one unit or it does not. The initial assumption by Garcia et al. (2001) is that both players are identical with respect to the probability of inflow.

The three main assumptions above with regard to demand, storage capacity and water inflow imply that each producer either has one unit available for production or the reservoir is empty in which case production is zero. Thus, there are just two states of storage levels visited by each producer. When we combine the storage levels experienced by the two producers we get four different states, first when both producers have full reservoir, second and third when either producer 1 or 2 have full reservoir and fourth when neither of the two producers have water available for production. If we assume as Garcia et al (2001) that both producers have equal marginal costs normalized to zero, the two states where either producer 1 or 2 have a full reservoir would be identical.

The static game solution to a situation where both producers have empty reservoirs is simply that no production takes place. If one of the two producers has a full reservoir, then this producer is in fact a monopolist. The producer charges the maximum allowed price c^* for the one unit available for production. We interpret this price as the consumers' reservation price[5]. If both producers have full reservoirs, then because the firms are symmetric with respect to marginal costs we have the so-called "Bertrand paradox" where both firms charge a price of zero equal to the marginal cost.

In a dynamic game, actions taken today may affect payoff in future periods. It is assumed that current payoff is unaffected by storage levels in previous periods. It means that the action chosen by producer $i = 1, 2$ based on the current reservoir level would have been taken irrespective of storage levels experienced in previous periods.

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[5] Garcia et al (2001) interpret this price as either a reservation price or the price cap set by the regulator.

The Markov updating of the game makes it possible to express the payoff for producer i as a value function representing the payoff to producer i for the remainder of the game once a certain state of reservoir level has been reached. The value function consists of the current period payoff and the effect that the current action has on the probability of reaching a certain state in the next. The value function for producer i in the state (x, y) is given by V_{xy}, where $x \in \{0, 1\}$ is the reservoir level of producer i and $y \in \{0, 1\}$ is the rival's reservoir level. Future payoff is discounted by the factor $\beta \in (0, 1)$. Because both producers have marginal cost equal to zero and equal probability of inflow the value functions will be symmetric.

Garcia et al. (2001) define value functions for each of the four possible states that can be experienced by producer i. The most interesting state is where both producers have a full reservoir. In this state both producers are able to cover demand and have to decide whether to undercut the rival's price or store the water for future periods. Garcia et al. (2001) derive the following equilibrium price[6]:

$$p_{11}^* = \beta(1 - q)c^*. \tag{1}$$

The intuition here is that the equilibrium price would have to be equal to the alternative value associated with production in the next period. In the next period producer i would receive the reservation price c^* if there is no inflow at the end of the current period. This occurs with probability $(1 - q)$. Also, producer i would have to discount this expected payoff by the factor β. We observe that the equilibrium price is increasing in the reservation price c^* and in the discount factor β. Finally, a higher probability of inflow will reduce the equilibrium price.

When adding thermal production, Garcia et al. (2001) assume that demand in each period is equal to 2 units. Furthermore, it is assumed that the fringe thermal producers have a deterministic capacity in total of one unit and constant marginal cost equal to c^T. They then focus on the state where both hydro producing players have

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[6]See Garcia et al. (2001), equation 2.8.

one unit of water available for production, $(1,1)$ and find that hydropower replaces thermal production when the equilibrium price $p_{11}^* < c^T$.

If the equilibrium price is higher than the marginal cost in thermal production, one unit of water will be saved for production in the next period. Now, if both units of water are produced in the state $(1,1)$ and there is no inflow before the next period there will be to little capacity available to cover the demand of two units.

2.2 The monopoly solution

According to Garcia et al. (2001) a collusive agreement between the two Bertrand players is sustainable for any price $\widetilde{p}_{11}$, such that $c^* \geq \widetilde{p}_{11} \geq p_{11}^*$, if and only if $\beta \geq \frac{1}{1+q}$. Even though the collusive solution described by Garcia et al. (2001) is identical to the monopoly solution when $c^* = \widetilde{p}_{11}$, it is not straight forward to compare this solution to the Bertrand-Nash case when thermal production is introduced. When thermal production is introduced, this affects the extent to which a player can be punished for deviation from the collusive equilibrium. In order to avoid this problem we describe the monopoly solution.

We let the function V_x represent the monopolist's value for the state (x), where $x \in \{0,1,2\}$ denotes the monopolist's storage level. This value function represents the monopolist's value for all remaining periods once the state (x) is reached. We consider first the state where the storage facility is empty. In the current period there is no production. In the next period it either rains one unit with probability q or it does not with probability $(1-q)$. We want the monopoly case to compare to the two firm Bertrand-Nash outcome described in subsection 2.1 above. Thus we imagine the monopoly case to be the case where the two hydropower producers have merged into one company. Then, if it rains the monopolist would experience an inflow of two units $(w = 2)$ and receive the value V_2 for the remaining periods. Otherwise the producer faces the value function V_0. Future payoff is discounted with the factor $\beta \in (0,1)$. We can now state the monopolist's value function for the state $(x = 0)$ as follows:

$$V_0 = \beta[(1-q)V_0 + qV_2]. \tag{2}$$

The next state ($x = 1$) may occur as a result of a situation where the monopolist at the beginning of the previous period had 2 units available for production. In the previous period one unit was produced and there was no water inflow at the end of this period. With only one unit available for production the monopolist could choose between production in the current period at price c^* or save the water for future production. The monopolist would always choose to produce in the current period as long as future profit is discounted with a value less than 1 or the probability of inflow is higher than zero, $1 > \beta(1-q)$. In the following period the monopolist will either have zero or two units available for production depending on whether it rains or not.

$$V_1 = c^* + \beta[(1-q)V_0 + qV_2]. \tag{3}$$

The last possible state for the monopolist ($x = 2$), is a situation where the producer has more than enough water available to cover demand in the current period. This state is a result of inflow at the end of the previous period. Now, the monopolist would always produce one unit at price c^* during the current period and save one unit for possible production in the future. The choice is between producing now and receiving c^* and producing in two periods from now if there is no inflow in between. The discounted value of the latter option, $\beta^2(1-q)^2 c^*$ is always less than the certain payoff today (c^*) as long at either $\beta < 1$ or $q > 0$. Then, the monopolist's value function V_2 can be expressed in the following way[7]:

$$V_2 = c^* + \beta[(1-q)V_1 + qV_2]. \tag{4}$$

The monopolist sets the price equal to the reservation price in every period where the monopolist has at least one unit of water available for production.

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[7]When $c^* = \tilde{p}_{11}$, the monopoly solution is identical to the collusive solution described in Garcia et al. (2001). We have that $\frac{V_2}{2} = \tilde{V}_{1,1}$, where $\tilde{V}_{1,1}$ is defined as one players collusive value (see Garcia et al. equation (3.2)).

The monopoly outcome indicates a higher price in such situations compared to the case where producers compete in setting the price. We should also note that the actual production of electricity is the same in all states of total storage level $\{0, 1, 2\}$, regardless of the model of competition. This is due to the fact that demand is equal to one unit in each period and that the reservation price c^* is equal across periods. However, if we introduce thermal production also in the case of a monopoly producer this will change.

2.3 Introducing thermal production

We increase demand in each period to 2 units and introduce thermal production from a competitive fringe with a production capacity of one unit. The competitive fringe will always bid their marginal cost equal to c^T. Because demand has doubled this will not change the monopolist's value functions for states where the storage level is either 0 or 1. In the state where the reservoir is full however, the monopolist now must choose between an undercutting strategy receiving,

$$2c^T + \beta[(1 - q)V_0 + qV_2] \tag{5}$$

and charging a higher price than the fringe receiving

$$c^* + \beta[(1 - q)V_1 + qV_2]. \tag{6}$$

The undercutting strategy is preferred only when c^T is higher than the level making the monopolist indifferent between the two strategies:

$$c^T > \frac{c^* + \beta(1 - q)[V_1 - V_0]}{2}. \tag{7}$$

If we rearrange this equation and use the fact that $V_1 - V_0 = c^*$, we get the following condition for the monopolist to prefer the undercutting strategy:

$$c^T > \frac{c^*(1 + \beta(1 - q))}{2}. \tag{8}$$

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We see that the monopolist would prefer the undercutting strategy as long as he receives a price for his 2 units sold that is higher than the alternative value for this volume. The alternative is to receive c^* for one unit in the current period and the same price for one additional unit if it does not rain in the next period.

As shown by Garcia et al. (2001) hydropower will replace thermal production in the state $(1, 1)$ in the Bertrand-Nash case if $c^T > \beta(1 - q)c^*$. We can now state our proposition 1:

Proposition 1 *Assuming that* $0 < \beta < 1$, $0 < q < 1$ *and that the reservation price* $c^* > 0$. *Then,*

(i) if $c^T > \frac{c^*(1+\beta(1-q))}{2}$, *hydropower would replace thermal production in both cases.*

(ii) if $c^T < \beta(1 - q)c^*$, *hydropower would not replace thermal production in either cases.*

(iii) for intermediate values of c^T, $\frac{c^*(1+\beta(1-q))}{2} > c^T > \beta(1 - q)c^*$, *thermal production would only be replaced by hydropower in the case of Bertrand-Nash competition.*

The interesting case is for intermediate values of c^T where thermal production would only be replaced in the case of Bertrand-Nash competition. This means that as long as there is no inflow, then the reservoirs in the Bertrand competition case would be empty in the next period. In the monopoly case however, there will be enough water left in the reservoirs to serve demand. Thus, even though competition may lead to lower prices on electricity in periods with more than enough water, the downside is that less water may be available for periods with little or no inflow.

3 Introducing transmission capacity between two geographic areas

In this section we introduce transmission capacity between two geographic areas where one hydropower producer is located in each area. The objective is to analyse how a

change in the transmission capacity affect the possibility of hydropower replacing thermal production of electricity. We continue to assume that both producers experience inflow with the same probability. Furthermore, we assume that demand is equal to $\frac{1}{2}$ in both regions and that the maximum allowed price c^* is the same. Also, there is just one transmission line with capacity $k \in (0, \frac{1}{2})$ between two regions A and B. Electricity flows to the region with demand surplus (high price) until the transmission capacity is binding. The transmission line is operated by a grid operator behaving as a competitive arbitrage agent.

3.1 Bertrand-Nash solution

The restricted transmission capacity implies that we now have a set of new states of reservoir levels. The four original states where $(0,0)$, $(1,0)$, $(0,1)$ and $(1,1)$. Now, if both firms have full reservoirs at the beginning of the current period producer i could either produce $\frac{1}{2} + k$ or $\frac{1}{2} - k$ depending on whether the producer undercuts the rivals price or not. The rival would in both cases serve the residual demand equal to $\frac{1}{2} - k$ or $\frac{1}{2} + k$. In the following period both producers would have either $\frac{1}{2} - k$ or $\frac{1}{2} + k$ left in the reservoir if there is no inflow or one unit if inflow occur. Accordingly, we have the two additional states; $(\frac{1}{2} + k, \frac{1}{2} - k)$ and $(\frac{1}{2} - k, \frac{1}{2} + k)$.

In the state $(\frac{1}{2} + k, \frac{1}{2} - k)$ total production capacity is equal to total demand in both regions. In addition, the existing transmission capacity does not constrain producer i from producing all the available water in its storage facility. Thus, the dominant strategy for producer i is to set the reservation price c^* and sell $\frac{1}{2} + k$ units in the current period. The rival producer would also set the price c^* and sell the remaining $\frac{1}{2} - k$ in his reservoir. In the following period both reservoirs would either be empty or full depending on whether they experience inflow or not[8].

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[8]This implies that the states $(1,0)$ and $(0,1)$ would not be visited by the producers when both producers are symmetric with respect to inflow. The four new states are $(0,0)$, $(1,1)$, $(\frac{1}{2} + k, \frac{1}{2} - k)$ and $(\frac{1}{2} - k, \frac{1}{2} + k)$.

$$V_{(\frac{1}{2}+k),(\frac{1}{2}-k)} = c^*(\frac{1}{2} + k) + \beta[(1 - q)V_{0,0} + qV_{1,1}]. \tag{9}$$

In the state $(\frac{1}{2} - k, \frac{1}{2} + k)$ producer i's dominant strategy is to sell $\frac{1}{2} - k$ at price c^* and the rival producer sell $\frac{1}{2} + k$ at the same price.

$$V_{(\frac{1}{2}-k),(\frac{1}{2}+k)} = c^*(\frac{1}{2} - k) + \beta[(1 - q)V_{0,0} + qV_{1,1}]. \tag{10}$$

Thus, the difference in value function between the two states $V_{(\frac{1}{2}+k),(\frac{1}{2}-k)} - V_{(\frac{1}{2}-k),(\frac{1}{2}+k)} = c^*2k$. The difference between the two value functions reflects that when producer i charges a lower price than the rival in a state where both producers have full reservoirs, he then forsakes the alternative which is to sell $2k$ at price c^* in the following period if there is no inflow.

In the most interesting state $(1, 1)$ where water is plentiful, producer i faces two alternatives in competition with the other producer. First producer i may choose to undercut the price set by the other producer in which case he earns $p_{1,1}(\frac{1}{2} + k)$ in the current period. This would leave $(\frac{1}{2} - k)$ for production in future periods. The other producer will only produce $(\frac{1}{2} - k)$ in the current period and have $(\frac{1}{2} + k)$ left for production in the next period. Thus, the value function for producer i in the state $(1, 1)$ can be expressed as follows:

$$p_{1,1}(\frac{1}{2} + k) + \beta[(1 - q)V_{(\frac{1}{2}-k),(\frac{1}{2}+k)} + qV_{1,1}]. \tag{11}$$

The second alternative is to charge a price higher than the rival producer. This will result in full import to region A where producer i is located. However, since the transmission line is constrained, there is still positive residual demand facing producer i. Producer i would then charge the reservation price on the residual demand in region A in the current period and leave $(\frac{1}{2} + k)$ for production in the next period. The producer located in region B will only have $(\frac{1}{2} - k)$ left for production in the next period. The value function corresponding to this strategy is:

$$c^*(\frac{1}{2} - k) + \beta[(1-q)V_{(\frac{1}{2}+k),(\frac{1}{2}-k)} + qV_{1,1}]. \qquad (12)$$

The two alternatives above, (11) and (12), indicate an equilibrium price $p_{1,1} = \widehat{p}_{1,1}$ where both producers are indifferent between the undercutting strategy and charging a higher price than the rival firm.

$$\widehat{p}_{1,1} = \frac{c^*(\frac{1}{2} - k) + \beta(1-q)[V_{(\frac{1}{2}+k),(\frac{1}{2}-k)} - V_{(\frac{1}{2}-k),(\frac{1}{2}+k)}]}{(\frac{1}{2} + k)}. \qquad (13)$$

We observe that the equilibrium price depends on the difference in value between the states $(\frac{1}{2} + k, \frac{1}{2} - k)$ and $(\frac{1}{2} - k, \frac{1}{2} + k)$ which is equal to c^*2k.

We can now restate the equilibrium price in the state $(1, 1)$ as follows:

$$\widehat{p}_{1,1} = \frac{c^*(\frac{1}{2} - k) + \beta(1-q)c^*2k}{(\frac{1}{2} + k)}. \qquad (14)$$

We note that if $k = \frac{1}{2}$, then the equilibrium price would reduce to the equilibrium price in the case where all demand and production are located at the same node.

Proposition 2 *Assuming that* $0 < \beta < 1$, $0 < q < 1$ *and* $k \in (0, \frac{1}{2})$, *then the equilibrium price* $\widehat{p}_{1,1}$ *is decreasing in* k.

Proof. We have that $\frac{\partial \widehat{p}_{1,1}}{\partial k} = -4\frac{c^*(1-\beta+\beta q)}{(1+2k)^2}$. Furthermore, by assumption we have that $0 < \beta < 1$, $0 < q < 1$ and $k \in (0, \frac{1}{2})$. This implies that $(1 - \beta + \beta q) > 0$ and that k is positive. Accordingly, we have that $\frac{\partial \widehat{p}_{1,1}}{\partial k} < 0$. ∎

The intuition behind the result in proposition 2 is simple. As the transmission capacity increases, competition between our two producers becomes increasingly fierce in the current period because the payoff associated with the strategy of inducing a transmission constraint is reduced. In the extreme case where $k = \frac{1}{2}$, there will be no additional payoff from such a strategy at all.

Now, if we look more closely at the condition stated in equation (14) we can decompose the alternative value in two different components. First we have the value

of selling $(\frac{1}{2} - k)$ units of water at the reservation price c^* in the current period. This alternative value is equal to $\frac{c^*(\frac{1}{2}-k)}{(\frac{1}{2}+k)}$. This leaves us with $1 - \frac{1}{2} + k$ units for production in the following period. The second part of equation (14), $\frac{\beta(1-q)c^*2k}{(\frac{1}{2}+k)}$, represents the alternative value of selling k units in the other market in the next period and reducing import with the same amount, in total $2k$ units of water. This leaves us with $1 - \frac{1}{2} - k$ units that are not represented as part of the alternative value. The reason why this part of the reservoir is not represented in the alternative value is that this amount of water would have to be sold in the second period regardless of whether an undercutting strategy is followed or not. Thus, the payoff received for this part of the inflow is irrelevant when determining the equilibrium price. However, if we compare with other alternatives implying a production of 1 unit we should also account for the last $(\frac{1}{2} - k)$ units that are not accounted for in equation (14).

3.2 Adding thermal production

Again, recognizing the importance of the combination of hydro and thermal based electricity production in real world electricity markets, we add thermal production by a fringe producer in each market. We consider the case where both thermal producers offer their capacity of $\frac{1}{2}$ unit each at the same marginal cost c^T. Demand in each market region is increased to 1 unit in every period.

We look at the state where both hydropower producing firms have 1 unit of water available for production. Now, they can choose to undercut the price set by the two thermal producers and sell 1 unit of power each in the current period at a price slightly below c^T. The alternative is to charge a higher price where we know that $(\frac{1}{2} + k)$ of the units available have an alternative value equal to $\frac{c^*(\frac{1}{2}-k)+\beta(1-q)c^*2k}{(\frac{1}{2}+k)}$, while the remaining $(\frac{1}{2} - k)$ units in the reservoir are sold in the second period at an alternative value equal to $\beta(1 - q)c^*$. The undercutting strategy is preferred if

$$c^T > v = (\frac{1}{2} + k)\frac{c^*(\frac{1}{2} - k) + \beta(1 - q)c^*2k}{(\frac{1}{2} + k)} + (\frac{1}{2} - k)\beta(1 - q)c^*. \qquad (15)$$

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Proposition 3 *Assuming that* $0 < \beta < 1$, $0 < q < 1$ *and* $k \in (0, \frac{1}{2})$, *then the alternative value* v *is decreasing in* k.

Proof. We have that $\frac{\partial v}{\partial k} = c^*(\beta - \beta q - 1)$. Furthermore, by assumption we have that $0 < \beta < 1$ and $0 < q < 1$. This implies that $\beta - \beta q - 1 < 0$. Accordingly, we have that $\frac{\partial v}{\partial k} < 0$. ∎

It follows from proposition 3 that the probability of observing a situation where hydropower replace thermal production increase when transmission capacity is increased. The intuition behind this result is most easily seen by looking at the two extreme cases where $k = 0$ and $k = \frac{1}{2}$. With $k = \frac{1}{2}$, we have that undercutting is preferred if $c^T > \beta(1 - q)c^*$. This is exactly the same as the condition derived in subsection 2.2 where there is only one integrated market. At the other extreme when $k = 0$ we have separate markets. In this situation undercutting becomes a strategy when $c^T > c^*\frac{1}{2} + \frac{1}{2}\beta(1 - q)c^*$. This is the same condition as derived for the monopoly case[9] described in subsection 2.3.

4 Rationing

The last public measure we consider in this paper is rationing. If the authorities introduce rationing, demand is set to R, where $0 \leq R \leq 1$. An increase in the level of rationing would in the same way as a reduction in the reservation price c^* lead to a reduction in profits for the two hydropower producing firms. However, in contrast to a change in the reservation price analysed by Garcia et al. (2001), rationing will also affect the producers' reservoir levels. Also, rationing will only affect profits directly in periods where such rationing is imposed while a change in the reservation price will affect profits in all periods.

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[9] The monopolist's value functions and alternative value are not affected by the introduction of a possible transmission constraint. Since inflow and demand are evenly distributed between the two regions, the monopolist is able to charge the reservation price in every period.

The level of rationing is set subsequent to the observation of the total reservoir level $s = x + y$, where x denotes the reservoir level of producer 1 and y denotes the reservoir level of producer 2.

Authorities have to follow a set of specific rules when they decide on the level of rationing. We assume no rationing to take place when authorities observe a total reservoir level $s > 1$. When they observe a total reservoir level of $s = 1$ the situation is considered critical and rationing is introduced. This is done by disconnecting some consumers from the network. If the total reservoir level is observed to be below 1 unit, $s < 1$, the authorities will impose rationing where demand is reduced to the available amount of water.

4.1 Bertrand-Nash solution

In the same way as described in the two previous sections, we let $V_{x,y}$ denote the value function of producer $i = 1, 2$ related to the state (x, y), where $x \in \{0, (1 - R), 1\}$ represents the reservoir level of producer i. The rival's reservoir level is described by the state variable $y \in \{0, (1 - R), 1\}$. Because both producers are symmetric with respect to costs and inflow, they will have identical value functions. Accordingly, it is sufficient to look at the value functions of only one of the two producers.

First, we look at the state where neither of the two producers have water in their reservoirs, $(0, 0)$. With no water in the reservoirs nothing is produced in the current period. Both producers will have filled up their reservoirs by the beginning of the next period with probability q. If so, they will receive the value $V_{1,1}$ for the remaining periods of the game. If no inflow occurs with probability $(1 - q)$, then none of the two producers will have water in their reservoir at the beginning of the next period. The value of this state is represented by $V_{0,0}$.

$$V_{0,0} = \beta[(1 - q)V_{0,0} + qV_{1,1}]. \tag{16}$$

The next state we consider, is where producer i has no water available while the

reservoir of the rival firm is full, $(0, 1)$. In this state the rival firm will be a monopolist in the current period. If no rationing is introduced, then the rival firm will produce all the available water in the current period and receive the reservation price c^*. However, as described above, the authorities observe that $s = 1$ and impose rationing where demand is reduced to the level R . Now, the rival firm will produce R in the current period and receive c^* for this production. Firm i produce nothing and receive no income in the current period. In the next period the rival firm will have $1 - R$ left in the reservoir if there is no inflow. If inflow occurs both firms will have 1 unit of water available for production.

$$V_{0,1} = \beta[(1 - q)V_{0,(1-R)} + qV_{1,1}].$$ (17)

We observe that $V_{0,0} - V_{0,1} = \beta(1 - q)[V_{0,0} - V_{0,1-R}]$. In the state $(0, 1 - R)$ the rival firm will be the only producer with water available for production. Because the total reservoir level is less than 1 unit, rationing will be imposed. The level of demand is set to the observed reservoir level $s = 1 - R$. The rival firm will in this state produce the remaining water in the reservoir at the reservation price c^*.

$$V_{0,1-R} = \beta[(1 - q)V_{0,0} + qV_{1,1}].$$ (18)

If we compare the value functions defined above we find that $V_{0,1-R} = V_{0,0} = V_{0,1}$. As long at firm i's reservoir is empty the value function is left unaffected by the rival's reservoir level.

The opposite situation is where only firm i has water in the reservoir. In that case the authorities will impose rationing restricting demand to R. Producer i will then produce R and receive the income Rc^* in the current period. If there is no inflow until the next period only firm i will have water in its reservoir. If inflow occurs, both firms will have 1 unit in their reservoirs.

$$V_{1,0} = Rc^* + \beta[(1 - q)V_{(1-R),0} + qV_{1,1}].$$ (19)

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In the state $(1 - R, 0)$ only producer i has water available for production. Because $s < 1$ the authorities impose rationing and set total demand equal to $s = 1 - R$. Producer i will produce the remaining water in the current period, receiving the reservation price. In the next period both producers will either have empty or full reservoirs.

$$V_{(1-R),0} = (1 - R)c^* + \beta[(1 - q)V_{0,0} + qV_{1,1}]. \tag{20}$$

By substitution from (16),(17), (18) and (19) we can rewrite the value function for the state $(1 - R, 0)$ as follows:

$$V_{1-R,0} = (1 - R)c^* + V_{0,0} = (1 - R)c^* + V_{0,1} = (1 - R)c^* + V_{0,1-R}. \tag{21}$$

Furthermore, by the use of equations (20) and (21) we can now express the value function $V_{1,0}$ in the following way:

$$V_{1,0} = Rc^* + \beta(1 - q)(1 - R)c^* + \beta[(1 - q)V_{0,0} + qV_{1,1}]. \tag{22}$$

If the authorities impose rationing, $0 < R < 1$, we can now express the difference between the value functions associated with the two states $(1, 0)$ and $(0, 1)$ as follows:

$$V_{1,0} - V_{0,1} = Rc^* + \beta(1 - q)(1 - R)c^*. \tag{23}$$

In state $(1, 0)$ only producer i has water available. When rationing is imposed producer i receives the reservation price c^* for the level of rationing imposed. If there is no inflow with probability $1 - q$, producer i will receive the reservation price c^* for the remaining water in the reservoir $1 - R$. This however, is future payoff that has to be discounted by the factor β.

The last state to consider is where both producers have 1 unit of water available, the state $(1, 1)$. In this state we assume that both producers will set the same price and serve the entire market with probability $\frac{1}{2}$. We look at producer i's optimal response to the price $p_{1,1}$ set by the rival firm. One strategy for producer i would now

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be to slightly undercut the rival and set the price $p_{1,1} - \varepsilon$. Knowing that no rationing is imposed when the authorities observe that $s > 1$ and assuming that $\varepsilon \to 0$, we can write the value of an undercutting strategy as follows:

$$p_{1,1} + \beta[(1 - q)V_{0,1} + qV_{1,1}]. \tag{24}$$

In the current period firm i captures the entire market of 1 unit. In the next period either both firms will have full reservoirs or only the rival firm will have water available.

The alternative strategy for firm i is to let the rival firm take the whole market in the current period. This will produce the following value for firm i:

$$\beta[(1 - q)V_{1,0} + qV_{1,1}]. \tag{25}$$

No rationing is imposed in the current period and the rival firm produces the entire water stock of 1 unit. If there is no inflow producer i will be the only producer in the next period. If on the other hand inflow occurs, then both firms will have full reservoirs at the beginning of the next period.

The discussion above indicates an equilibrium price $p_{1,1} = \overline{p}_{1,1}$ where both producers will be indifferent between the two strategies.

$$\overline{p}_{1,1} + \beta[(1 - q)V_{01} + qV_{11}] = \beta[(1 - q)V_{10} + qV_{11}]. \tag{26}$$

Knowing that $V_{10} - V_{01} = Rc^* + \beta(1 - q)(1 - R)c^*$, we can rewrite the equilibrium condition as follows:

$$\overline{p}_{1,1} = \beta(1 - q)[Rc^* + \beta(1 - q)(1 - R)c^*] \tag{27}$$

We can now state the following proposition:

Proposition 4 *Assuming that $0 < \beta < 1$, $0 < q < 1$, $0 < R < 1$ and that the reservation price $c^* > 0$, then we have that an increased (decreased) level of rationing given by a reduction (increase) in R will lead to a reduction (increase) in the equilibrium price $\overline{p}_{1,1}$.*

Proof. We have that $\frac{\partial \overline{p}_{1,1}}{\partial R} = (1-q)\beta(c - \beta(1-q)c)$. Furthermore, by assumption we have that $0 < \beta < 1$ and $0 < q < 1$. This implies that $(1-q)\beta(c - \beta(1-q)c) > 0$. Accordingly, we have that $\frac{\partial \overline{p}_{1,1}}{\partial R} > 0$. ∎

The intuition here is that the equilibrium price must be equal to the alternative value of producing in a future period. If the producers face rationing in the future this will reduce the value of production in the future. Accordingly, the producers would want to produce more in the current period.

4.2 Thermal production and rationing

We extend the model to include thermal production from a price taking producer (fringe) with a fixed capacity of 1 unit and marginal cost equal to c^T. Demand is increased to 2 units in every period.

In the situation where both hydropower producers have full reservoirs, there is an excess capacity equal to 1 unit. We know that the hydropower producing firm i would want to postpone production if the price received today is lower than the alternative value represented by $\beta(1-q)[Rc^* + \beta(1-q)(1-R)c^*]$. Thus, whether the hydropower producing firms choose to store their water for future production depends among other factors on the level of rationing imposed in situations where the water resource is considered to be scarce.

With demand equal to 2 units, the authorities impose rationing if they observe that the total level of available water resources is less than or equal to 2 units. If just one of the two hydro producing firms has a full water reservoir, then total available capacity is equal to 2 units and rationing is imposed. Demand is set to $1 + R$. If the observed available capacity is observed to be below 2 units, then demand is reduced to match the available capacity.

We look at the state where both hydro producing firms have full reservoirs. With demand equal to 2 units and thermal production present in the market the hydropower producing firms can choose to undercut the price set by the thermal producer and

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deplete their reservoir in the current period. They will choose to do so if $\widetilde{p}^*_{1,1} < c^T$. If the equilibrium price $\widetilde{p}^*_{1,1}$, decided by the alternative value, is higher than marginal cost in thermal production then the hydro producing firms will have a higher payoff if they store some of their water for future periods.

From proposition 4 we have that the equilibrium price is reduced when rationing is increased (reduction in R). This means that the probability of hydropower replacing thermal production in the state $(1,1)$ increases as the level of rationing is increased. Thus, by imposing rationing in order to secure water for future periods the authorities might achieve the opposite result. The intuition here is that rationing reduce the alternative value faced by hydropower producing firms. Accordingly, they find it relatively more profitable to produce in the current period when water resources are plentiful.

5 Concluding remarks

The starting point of this paper is an oligopoly model developed by Garcia et al (2001) where 2 hydropower producing firms engage in dynamic Bertrand competition. The basic features of their model was restated in section 2 of this paper.

On the basis of the model developed by Garcia et al (2001) we defined the monopoly outcome in subsection 2.3. We then compared the monopoly outcome to the Bertrand-Nash outcome as defined in the original model. We found that hydropower is less likely to replace thermal production in monopoly case. This result indicates that competition would not necessarily lead to the result that the water resources are allocated to the periods where they are most needed.

Furthermore, we extended the Garcia et al (2001) model to include transmission capacity between two price areas and rationing imposed by the authorities. Both of these extensions we also analyzed in the presence of thermal production.

With regard to transmission capacity described in section 3, we found that the Bertrand-Nash equilibrium price is reduced when transmission capacity is increased.

This happens because an increase in the transmission capacity makes competition more fierce. When we included thermal production, we found that an increase in transmission capacity would increase the probability of hydropower replacing thermal production. This indicates that increased transmission capacity may not be a good measure if the aim is to secure enough storage for periods with low inflow.

The same result holds with regard to the extensions made in section 4 where we looked at rationing as a possible measure for securing enough water in periods with low inflow. We found that an increase in the level of rationing would reduce the Bertrand-Nash equilibrium price and thus increase the probability of hydropower replacing thermal production.

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WATER WITH POWER:
Market power and supply shortage in dry years[1]

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Abstract:

The purpose of this paper is to analyse how market power may affect the allocation of production between seasons (summer and winter) in a hydro power system with reservoirs and where inflow in winter is uncertain. We find that even without market power we expect lower average prices during summer than during winter. Furthermore, we find that market power may in some situations lead to more sales during summer and in other situations to less sales during summer. Thus market power is found to have an ambiguous effect on the supply shortage in years with low inflow.

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1 Introduction

In some countries the electricity production is dominated by hydro power.[3] The hydro power production system is often quite complex, especially in those cases where water can be stored in reservoirs. In particular, there is uncertainty concerning the inflow of water. Storage in one period may lead to spill of water if there is a large inflow in the next period. In such cases, how do we expect producers with market power to behave? For example, one may ask whether exertion of market power may lead to a more severe shortage. Could it be that they produce early on, to have less water available later on?[4] If so, exertion of market power is expected to worsen situations with supply shortage in dry years and may lead to dramatic price hikes. With a few notable exceptions, there are no studies raising this question.[5] The purpose of this paper is to analyse whether market power can lead to a more severe shortage in periods with a limited supply (dry year) compared to a situation with perfect competition.

Our study is motivated by the observations in the Nordic power market 2002-03. The spot price in January 2003 was more than 80 øre/kWh, while the average price in a year with normal inflow is approximately 20 ø re/kWh. The producers claimed

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[3]In New Zealand 80% of production is from hydro, in Chile 70%, Brazil 97% and Norway close to 100%.

[4]A similar type of question has been analysed by Stiglitz (1976) with regard to exploitation of exhaustible natural resources. Assuming positive extraction cost and rate of interest and iso-elastic demand he finds that a monopolist is more conservation minded than what is socially optimal.

[5]There are some studies of the allocation of water between different time periods, see Førsund (1994), Bushnell (2003), Scott and Read (1996), Crampes and Moreaux (2001) and Skaar and Sørgard (2003). But none of these studies introduce uncertainty concerning water inflow.

The issue of water allocation between periods when there is uncertainty about inflow has been analysed by Mathiesen (1992). In this study however, producers are assumed to behave as price-takers only.

In a more recent study Garcia et al (2001) analyse strategic behaviour in an infinite horizon duopoly model where two hydro power producers can storage water and there is uncertainty concerning water inflow. The question of market power and storage when inflow is uncertain has also been analysed by Johnsen (2001). See below for more details concerning these two studies.

that the reason for the price hike was the low water inflow to the reservoirs in autumn 2002. Data seems to support this claim, saying that it was in fact an extraordinarily low rainfall in the late autumn of 2002.[6]

But is this the whole picture? Could it be that strategic producer behaviour also contributed to the price hike?[7] Victor D. Norman, the Minister responsible for competition policy, said the following at the outset of the period with supply shortage:

'A situation with low prices during summer and high prices during winter may indicate that there has been an abuse of market power'. (Dagbladet 13.11.2002)

The argument is that a producer with market power could benefit from producing a large amount during summer time, thereby limiting the supply in winter. By doing so the producer could earn a large profit from high prices during winter. In April 2003, when the supply shortage came to an end, the Norwegian Competition Authority suggested that one should consider to split the largest Norwegian hydro power producer, Statkraft, into several independent firms. One argument for doing so was the following:[8]

'.., The Competition Authority will not rule out the possibility that lack of competition may have increased the difficulties [supply shortage - our remark] we have experienced'.

We formulate a model where we are able to analyse how a producer with market power would distribute his sales between summer and winter. During autumn there will be either heavy rain or little rain. If there is heavy rain, the inflow is so large that some water may be spilled (reservoirs are full). Whether some water is spilled or not depends on the inflow and the size of the reservoirs. If there is little rain during autumn, all inflow can be stored in reservoirs and used for production in the winter season.

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[6]See, for example, Bye et al. (2003).

[7]We became recently aware that Førsund et al. (2003) has raised the same questions as we do, in a report for the Ministry of Oil and Energy.

[8]From a letter the Norwegian Competition Authority sent to the Ministry of Trade and Industry, the owner of Statkraft.

First, we show that even under perfect competition the average price during summer is lower than average price during winter. The reason is that a high inflow can lead to waste of water (reservoirs are full), and then it would have been better to sell a little more during summer at a low price than to wait and risk a spill of water if there is a large inflow. The implication is that one cannot conclude whether there has been an abuse of market power or not by just observing price differences between summer and winter. In contrast, when there is a zero probability of spill of water we find that absence of market power will lead to identical prices in summer and winter. In such a case a price difference between summer and winter would indicate exertion of market power.

Second, we find that exertion of market power has an ambiguous effect on the distribution of sales between summer and winter (storage). On the one hand, a producer with market power may sell a large quantity during summer in order to constrain his supply and obtain a high price during winter. Or he may choose to do the opposite, selling a low quantity during summer to achieve a higher summer price. In this latter case market power may lead to a more limited difference between prices summer and winter.

Our result contrasts with Garcia et al. (2001), who found that market power always leads to higher prices during summer. The driving force behind their result is the modeling of the demand side. They apply a rectangular demand function, where the price during winter time is exogenously given. Then a shift of production from summer to winter will have no effect the winter price. In contrast, in our model there is a trade off. A shift of production from summer to winter would lead to higher prices during summer and lower prices during winter. This explains why we found that market power in some instances can lead to a shift in production from summer to winter, and in other instances to a reallocation of production from winter to summer.

Johnsen (2001) analyses market power and storage in a situation with limited transmission capacity between two regions connected by a single radial transmission line. He applies a simple two-period model similar to the one used in our paper. A

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numerical example is provided to illustrate that a monopolist finds it profitable to increase production in the first period when inflow is certain. The monopolist does this to avoid the possibility of becoming export constrained in the second period if high inflow occurs. Thus, storage is concluded to be lower in the monopoly case than in the competitive case.

We abstract from the possibility of transmission constraints as we look only at allocation of water between periods within a single geographic area. Also, different from Johnsen (2001), we analyse situations where the size of the water reservoir may constrain production and situations where the energy constraint may not be binding. As mentioned above, we find that market power has an ambiguous effect on storage. This is in contrast to Johnsen (2001) who finds that storage is lower in the monopoly case.

This article is organised as follows. In the next section we present the model, and in Section 3 we analyse the equilibrium outcomes in four different regimes. In Section 4 we apply a linear demand function, while in Section 4 we provide some concluding remarks. Proofs are presented in Appendices A and B.

2 The model

We consider an industry where electricity is generated from water, and we will use the Norwegian market as an illustration.

With reference to the graphs in Figure 1 a hydrological year may be described in terms of four periods (of somewhat unequal length). Starting in spring, (between week 16 and 21) there is a large inflow of water from snow melting. In summer there is little rain and typically consumption exceeds additional inflow in this period. The autumn is normally a rainy season where inflow again exceeds consumption, while in winter the precipitation comes mainly in the form of snow that is unavailable for electricity production until spring.[9] This seasonal pattern repeats. Inflow varies considerably

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[9]2002 was a dry year with little inflow during the autumn. This is illustrated in Figure 1, where
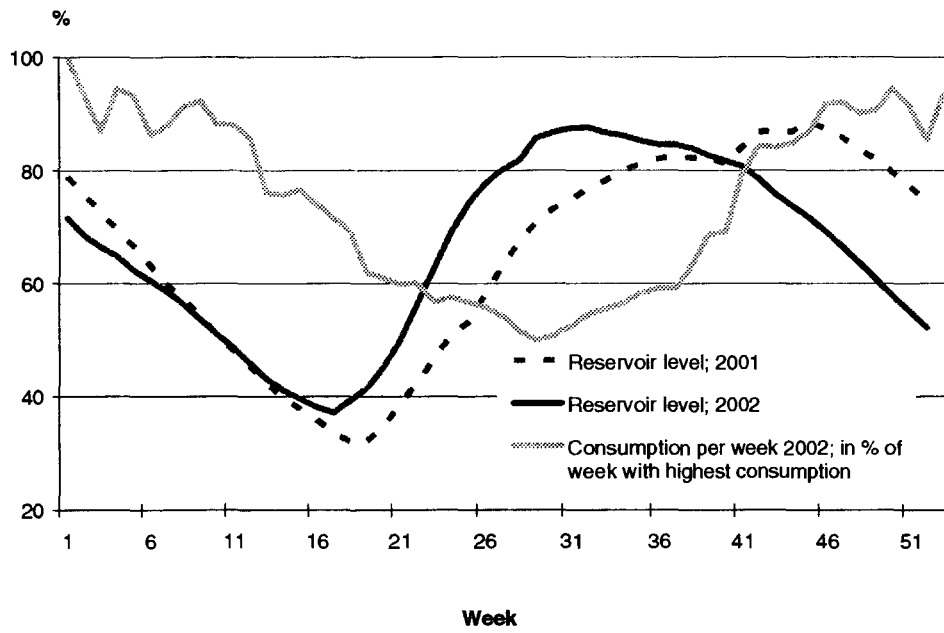
Figure 1: Seasonal pattern for reservoir level and consumption.
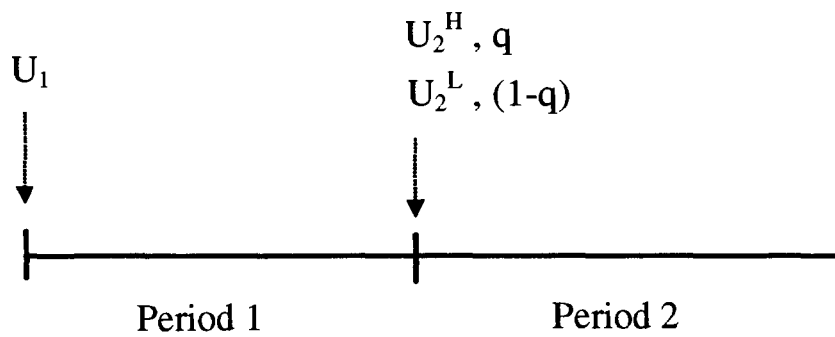

Figure 2: Illustration of the model.

over the year, while consumption changes less. Also, inflow in any period is highly uncertain, while consumption is considerably less uncertain. Thus we will treat inflow as uncertain and consumption as deterministic.

Although the graphs indicate four distinct periods, the analysis is conducted within a two-period model, interpreted as summer and winter. Our concern is the allocation of water between these two periods. In particular, how much water will be used for electricity production in summer and thereby, how much water will be stored for later use? This allocation is studied for two different modes of producer behaviour: A price taker and a price setter. Our interest lies with finding out who will use most water during summer and thus have less available for the winter season.

The model is illustrated in Figure 2. To simplify the analysis we disregard the multi-year dimension, i.e., the possibility to even out extreme inflows over several years. In our analysis, the hydrological year starts with the inventory U_1.[10] At the decision point in summer this inventory is known, while the autumn inflow U_2 is uncertain. Our analysis is one of decision making under uncertainty with respect to the autumn inflow. This inflow is thought of as materializing at the beginning of period 2. We assume that the inflow will be high (U_2^H) or low (U_2^L) with probabilities q and $(1 - q)$ respectively, and that $U_2^H > U_2^L$. Furthermore, we will assume that U_2^L is so low that reservoir capacity (R) always is sufficient, while U_2^H is so high compared to R, that some water may be spilled.

We want to compare the outcome of a competitive equilibrium with that of an industry of producers with market power. To represent the latter we consider the collusive outcome. In the subsequent presentation and discussion of the model we simplify by talking about one producer that may behave as a price taker or as a monopolist when making his decisions, i.e., production (in the two periods): x_1 respectively x_2^i, $i = H, L$. Furthermore, let w^i denote spill of water in state i, $i = H, L$.

Both kinds of producers seek to maximize profits, the difference between them

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the reservoir level during the autumn 2001 is considerably higher compared to the autumn 2002.

[10] U_1 may alternatively be thought of as an inflow of water at the beginning of period 1.

being how they regard the market prices. A monopolist assumes he can set prices, while a price taker by definition assumes that prices are given beyond his control. Let $p_t(.)$ and $D_t(.)$, $t = 1, 2$ denote inverse and direct demand respectively.

The objective for the monopolist can be formulated as[11]

$$\underset{x_1, x_2^L, x_2^H}{Max} p_1(x_1)x_1 + qp_2(x_2^H)x_2^H + (1-q)p_2(x_2^L)x_2^L.$$

The maximization of the sum of producers and consumers surpluses would generate the competitive equilibrium

$$\underset{x_1, x_2^L, x_2^H}{Max} \varphi_1(x_1)x_1 + q\varphi_2(x_2^H)x_2^H + (1-q)\varphi_2(x_2^L)x_2^L, \text{ where } \varphi = \int_0^x p(s)ds.$$

There are several constraints on decision variables x_1, x_2^H and x_2^L, some of which are obvious, like production has to be non-negative. In addition, we have the following restrictions:

(i) Period 1 production:

$$x_1 \leq min\{U_1, D_1(0)\}.$$

Production has to stay within the given amount of water (U_1), and the producer will never supply more than demand at a price of zero.

(ii) Available energy after autumn inflow:

$$Z_i \equiv U_1 - x_1 + U_2^i - w^i \leq R, \qquad i = H, L.$$

What is not consumed for production in period 1 $(U_1 - x_1)$ will be stored. With the addition of autumn inflow (U_2^i), stored water has to be within the reservoir capacity (R). This restriction may imply that some water (w^i) has to be spilled. Thus Z_i denotes the available energy for production in period 2.

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[11]Except for water values, variable production costs of electricity from hydro are small. In line with this, other variable costs than water values are by assumption set to zero in our analysis.

(iii) Period 2 production:

$$x_2^i \leq min\{Z_i, D_2(0)\}, \qquad i = H, L.$$

Production has to stay within the given amount of energy (Z_i).[12] Furthermore, the producer will never produce beyond demand at a price of zero.[13]

3 Equilibrium outcomes

Our concern is as mentioned above: Will a firm with market power store more or less water from summer to winter than a firm without market power? In order to highlight this issue, we make a few assumptions that further delineate the analysis.

For given demand, represented by $D_1(p_1)$ and $D_2(p_2^i)$ (or equivalently the willingness to pay $p_1(x_1)$ and $p_2(x_2^i)$), the model has four parameters R, U_1 and U_2^i, $i = H, L$. We now rule out parameter combinations that are of little relevance to our issue.

Assumption 1:

$$x_1 < U_1.$$

The initial inventory of water is sufficient for optimal production in any mode of producer behaviour in period 1. We are concerned with the consequences of shortage or surplus of energy in winter, not a shortage in summer.

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[12]If demand, $D_2(0)$, is lower than available energy, the surplus is spilled. This spill comes in addition to the spill of water caused by limited reservoir capacity, w_i.

[13]We assume that demand during period 2 is independent of the state of world with respect to inflow at the beginning of the period.

Assumption 2:

$$U_1 - x_1 + U_2^L < R.$$

Inflow in a dry autumn (U_2^L) is so small that the reservoir never becomes binding. Hence, $w^L = 0$. Assumption 2 also applies to any mode of behaviour. It is the high inflow state that may cause spill of water.

Assumption 3:

$$x_2^i = Z_i \text{ whenever } p_2(Z_i) > 0, i = H, L.$$

Assumption 3 relates to period 2 production in both states. With regard to state L, we qualify Assumption 3 further by considering only combinations of inflow and reservoir capacity resulting in a positive price and marginal revenue in period 1 and positive expected price and marginal revenue in state L. The implications are that all the available energy is used both in the competitive and collusive equilibrium if there is low inflow and that the realised price in this state is positive by definition.

In state H on the other hand, there may be so much water available that the monopoly producer would like to produce less than what is available in order to ensure a positive marginal revenue from sales in state H. We make the assumption that authorities can enforce production as long as there is demand at a positive price. In particular, we assume that the authorities can detect and prevent water from being spilled (in excess of w^H) as long as there is positive demand. This assumption applies to the monopolist, who in some situations in state H would otherwise spill water in order to increase the price.[14] We relax this assumption in Section 4.3 below.

Through assumptions $1 - 3$ the producers' maximization problem is reduced to a question of finding production levels in period 1 and in the high inflow state of the

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[14]The assumption that there is no spill of water is common in the literature, see for example Johnsen et al (1999) and Crampes and Moreaux (2001). Morover, The Norwegian Competition Authority also made such an assumption in an acquisition case in the Norwegian power market in 2001-02 (Statkraft acquiring Agder Energi).

world in period 2.

$$\underset{x_1, x_2^H}{Max} \quad \{p_1(x_1)x_1 + qp_2^H(x_2^H)x_2^H + (1 - q)p_2^L(U_2^L + U_1 - x_1)[U_2^L + U_1 - x_1]\}.$$

Subject to

$$x_2^H = \min\{Z^H, D_2(0)\}.$$

Next, we characterise different equilibrium outcomes (regimes) depending on the parameter values.

3.1 The four regimes

For a given demand, available energy and the reservoir capacity may constrain the solution. The various combinations of these parameters are illustrated in Table 1.

		Reservoir capacity, R	
		$R = x_2^H$	$R > x_2^H$
Energy, $U_1 + U_2^H$	$U_1 + U_2^H$ $= x_1 + x_2^H$	R4: R constrains; energy constrains; $p_2(x_2^H) > 0.$	R3: Energy constrains; $p_2(x_2^H) > 0.$
	$U_1 + U_2^H$ $> x_1 + x_2^H$	R2: R constrains; $p_2(x_2^H) > 0$; $w^H > 0.$	R1: $D_2(0)$ constrains; $p_2(x_2^H) = 0.$ $w^H = 0.$

Table 1: The four regimes

Regime 1.

The inflow in state H is high. In this regime, reservoir capacity is also large and further production in period 2 is constrained by demand. Hence, we have that $p_2(x_2^H) = 0$ and any additional water is spilled. In the low inflow state, and by assumption 3, the energy constraint is binding and $p_2(x_2^L) > 0$.

When considering the optimal allocation of water between periods, a price taking firm, at the margin, either sells one unit in period 1 at the price p_1 or stores it; with probability $(1 - q)$ the unit sells at price $p_2(x_2^L)$ or with probability q it sells at a price equal to zero. In a competitive equilibrium the price in period 1 equals the expected price of period 2,

$$p_1 = (1 - q)p_2(x_2^L) + qp_2(x_2^H) = (1 - q)p_2(x_2^L). \tag{1}$$

The monopoly firm considers its marginal revenue rather than the price it obtains when allocating water between periods. Thus the equilibrium in this regime has to satisfy

$$\begin{aligned} p_1[1 - \frac{1}{|e_1|}] &= (1 - q)p_2(x_2^L)[1 - \frac{1}{|e_2|}] + qp_2(x_2^H)[1 - \frac{1}{|e_2|}] \\ &= (1 - q)p_2(x_2^L)[1 - \frac{1}{|e_2|}], \end{aligned} \tag{2}$$

where e_1 and e_2 denote price elasticities of demand in period 1 and 2 respectively.

Regime 2.

In this regime, the reservoir capacity constrains the amount of available energy in state H. The producer is unable to satisfy all demand in period 2, also in a wet year. This implies that $p_2(x_2^H) > 0$. In summer the producer knows that a high inflow in autumn will lead to spill of water, which means that the marginal unit stored in summer does not make it to the winter.[15]

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[15]We might say that $p_2(x_2^H)$ is irrelevant because the marginal unit stored in summer never survives to period 2 state H.

If there is low inflow, however, the marginal unit stored will be used for production in winter. The equilibrium condition in a situation with no market power is as follows:

$$p_1 = (1 - q)p_2(x_2^L). \tag{3}$$

In the monopoly equilibrium the equivalent condition is

$$p_1[1 - \frac{1}{|e_1|}] = (1 - q)p_2(x_2^L)[1 - \frac{1}{|e_2|}]. \tag{4}$$

Regime 3.

In this regime, inflow in state H is low compared to reservoir capacity and demand, whereby $p_2(x_2^H) > 0$. Furthermore, because the reservoir capacity is non-binding, there is no spill of water and the marginal unit stored in period 1 has a positive value in state H.

Because of the non-binding reservoir, water is optimally allocated between periods. Thus in a competitive equilibrium the price of period 1 has to equal the expected price of period 2

$$p_1 = (1 - q)p_2(x_2^L) + qp_2(x_2^H). \tag{5}$$

Similarly, the equilibrium condition for the monopoly has to satisfy

$$p_1[1 - \frac{1}{|e_1|}] = (1 - q)p_2(x_2^L)[1 - \frac{1}{|e_2|}] + qp_2(x_2^H)[1 - \frac{1}{|e_2|}]. \tag{6}$$

Regime 4.

In this regime, both energy and reservoir constrain production. By definition, the limited energy implies that prices are positive in both states as in regime 3. The reservoir constraint further implies that the producer cannot freely allocate the scarce energy resource between periods, as in regime 3, whereby production in period 1 will be higher than optimal, and the price in period 1 will be lower than the expected

price of period 2.[16]

The competitive equilibrium of this regime is therefore characterized by

$$p_1 < (1-q)p_2(x_2^L) + qp_2(x_2^H) \tag{7}$$

Similarly, in the monopoly equilibrium, the marginal revenue of period 1 is lower than the expected marginal revenue of period 2

$$p_1[1 - \frac{1}{|e_1|}] < (1-q)p_2(x_2^L)[1 - \frac{1}{|e_2|}] + qp_2(x_2^H)[1 - \frac{1}{|e_2|}]. \tag{8}$$

With reference to the four regimes described above and by focusing on the competitive case, we can state the following proposition with respect to the expected price difference between period 1 and period 2:

Proposition 1 *Assume that* $0 < q < 1$. *If the reservoir constraint is binding in equilibrium, then the price in period 1 is lower than the expected price in period 2.*

Proof. When the reservoir constraint is binding in the high inflow state we have that $x_2^H = R$. The reservoir constraint is binding in regimes 2 and 4 and the competitive equilibrium in both these regimes is characterized by $p_1(x_1) < (1-q)p_2(x_2^L) +$ $qp_2(x_2^H)$. ∎

Proposition 1 tells us that even under perfect competition the price in period 1 may be lower than the expected price in period 2. Such price differences may very well be the result of an efficient allocation of resources between periods. The intuition here is that if we do not use the water to produce electricity in period 1, there is a probability that water may be lost due to the reservoir constraint. This result is important, because it means that when the price in summer proves to be lower than the price in winter year after year this is no proof of exertion of market power.

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[16]Assumption 1 and the infeasibility of moving water (storage) forward in time (from period 2 to period 1), rule out the possibility of having $p_1 > (1-q)p_2(x_2^L) + qp_2(x_2^H)$.

3.2 Equilibria in the same regime

Since we are unable to verify the existence of market power from observing price differences between periods, we proceed by comparing price differences between the competitive and the monopoly equilibrium. As an alternative to focusing directly on the price difference we look at period 1 production. If period 1 production is lower in one case than the other, this means that storage is higher and also that the price in period 2 will be lower.

Above we have characterized four regimes and conditions on equilibrium prices and marginal revenues with and without market power, respectively. For a given parameter set $(R, U_1, U_2^i, i = H, L)$ there is no guarantee that the competitive and monopoly equilibria fall into the same regime. In fact, numerical examples show that the competitive regimes do not overlap completely with the corresponding monopoly regimes. Moreover, regime 1 implies that marginal revenue will be negative. We deal with this more complex situation below.

In this section, we compare production (and storage) of the competitive and monopoly equilibria where by assumption both belong to the same regime. In addition, we assume that marginal revenue corresponding to competitive equilibrium production in state H is positive.[17] One implication is that regime 1 is ruled out of the analysis in this section.

Let $\widetilde{x}_1^v$ and $\widehat{x}_1^v$ represent equilibrium production in period 1 in regime v ($v = 1, 2, 3, 4$) under competition and monopoly, respectively. The equilibrium production levels are derived from the equilibrium conditions stated in equations (1) to (8). We consider the price elasticities in the competitive equilibrium as the benchmark. Let $|e_t|$ denote the absolute value of the price elasticity in the competitive equilibrium. We then ask whether the introduction of market power will lead to reallocation of

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[17]The assumption that price and marginal revenue are positive holds for iso-elastic demand where $|e_t| > 1$. However, since we we are interested in whether storage in the monopoly case is higher or lower than in the competitive case we only need to assume that marginal revenue is positive in state H at the level of competitive equilibrium output.

production.

Proposition 2 *Assume that $|e_t| > 1$ and that reservoir capacity and inflow are such that the same regime applies to both the competitive and the monopoly equilibrium. Then,*

i) in regime 2, $\widehat{x}_1^2 > \widetilde{x}_1^2$ if $|e_1| < |e_2|$.

ii) in regime 3, $\widehat{x}_1^3 > \widetilde{x}_1^3$ if $|e_1| < |e_2|$.

ii) while $\widehat{x}_1^4 = \widetilde{x}_1^4$ in regime 4.

Proof. See Appendix A. ∎

Proposition 2 tells us that as long as the competitive and the monopoly equilibrium are in the same regime, the price elasticity of demand is decisive for whether market power leads to more or less production in period 1. If, for example, demand is less price elastic in period 1 than in period 2, monopoly production in period 1 is lower than the competitive production. The reason is that a firm with market power will exploit differences in market characteristics. Note that the result in Proposition 2 does not depend on the probability of high inflow.

3.3 Equilibria in different regimes

As mentioned, the borderlines that define cut-off values between the four regimes are not identical for the two equilibria. For certain values of reservoir capacity and inflow $(R, U_1, U_2^i, i = H, L)$, the competitive equilibrium may for instance be in regime 4, while the monopoly equilibrium belongs to regime 3. Then the simple criteria we reported in Proposition 2 may no longer apply.

When we compare equilibria in different regimes, we need to define the cut-off values between the different regimes in our two cases. Each regime is defined for a certain range of inflow and reservoir values as illustrated in Table 1. In order to be able to describe the cutoff values between all 4 regimes, we make use of period 1 equilibrium production determined by the first order conditions applying to the

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different regimes. After having defined each regime under perfect competition and monopoly, we then compare period 1 production (and storage) in situations where the equilibria are in different regimes. Thus, we confine the discussion to the range of inflow and reservoir values where the regime related to the competitive equilibrium is different from the regime associated with the monopoly equilibrium. We continue to assume that marginal revenue is non-negative as described in the previous section. The situations where regimes are identical are covered by proposition 2.

We start by defining the cut-off values between the different regimes. The cut-off values are illustrated in Table 2.

	Cut-off value	
Regime	Competitive	Collusive
2 and 4	$U_1 + U_2^H = \widehat{x}_1^2 + R$	$U_1 + U_2^H = \widetilde{x}_1^2 + R$
3 and 4	$U_1 + U_2^H = \widehat{x}_1^3 + R$	$U_1 + U_2^H = \widetilde{x}_1^3 + R$

Table 2: Regime borders

If we look at the equilibrium conditions defined for regime 2 (equations (3) and (4)) and regime 3 (equations (5) and (6)) we find that $\widehat{x}_1^2 > \widehat{x}_1^3$ and $\widetilde{x}_1^2 > \widetilde{x}_1^3$. A special situation arises when $|e_1| = |e_2|$. Then we have that period 1 production is the same in the competitive and the monopoly case both in regime 2 and regime 3, $\widehat{x}_1^2 = \widetilde{x}_1^2$ and $\widehat{x}_1^3 = \widetilde{x}_1^3$. Accordingly, the cut-off values defining the borderlines between the regimes are identical and there is no combination of inflow and reservoir capacity resulting in different equilibria. If $|e_1| \neq |e_2|$ however,then for some values of inflow and reservoir capacity the competitive regime is different from the monopoly equilibrium regime and we can state the following proposition. As in Proposition 2, let us consider the price elasticity in the competitive equilibrium as the benchmark:

Proposition 3 *Assume that* $|e_t| > 1$ *and that reservoir capacity and inflow* $(R, U_1, U_2^i, i = H, L)$ *are such that different regimes apply to the competitive and the monopoly equilibrium. Then,*

i) if $|e_1| < |e_2|$*, period* 1 *production in the monopoly case is always lower (and storage higher) than in the competitive equilibrium.*

ii) if $|e_1| > |e_2|$*, period* 1 *production in the monopoly case is always higher (and storage lower) than in to the competitive equilibrium.*

Proof. See Appendix B. ∎

We see that period 1 production in the monopoly equilibrium is always lower or equal to period 1 equilibrium production in the competitive case if demand in period 1 is less price elastic than demand in period 2. This result holds irrespective of whether the competitive and the monopoly equilibrium belong to the same regime (Proposition 2) or not (Proposition 3). Put differently, we find that when $|e_1| < |e_2|$ in the competitive equilibrium, storage under perfect competition will never be higher than storage under monopoly. This is in line with the results we found in Proposition 2, and it shows that our basic result is quite robust.

This result however, rests on the assumption that price and marginal revenue in state H are both non-negative. Sufficient inflow in state H may lead to a situation where the monopolist would want to spill some of the available water. Also, in the competitive case the price in state H may be driven to zero if inflow is sufficiently large. According to Assumption 3, the monopolist may be forced to produce a quantity that implies a negative marginal revenue in state H. The only situations where authorities allow spill of water are when the price in state H is zero or the reservoir is full. We analyze these situations in the next subsection through an example with linear demand.

4 An example: Linear demand

Let us now introduce a linear demand function. Such a demand function implies that the price will become zero for a large enough production quantity. In contrast, with iso-elastic demand the price will never equal zero and marginal revenue to a monopolist

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will be positive (when demand is elastic).[18] Linear demand and Assumption 3 implies that regime 1 equilibria are possible and that marginal revenue may be negative in equilibrium. Since the first order conditions are identical under regimes 1 and 2, we let $\widehat{x}_1$ and $\widetilde{x}_1$ represent period 1 production in these regimes under competition respectively monopoly.

We employ the following inverse demand functions[19]:

$$p_1 = \alpha_1 - \beta_1 x_1, \tag{9}$$

$$p_2 = \alpha_2 - \beta_2 x_2^i, \text{ where } i = H, L. \tag{10}$$

Note that the parameter β captures the market size, while the parameter α captures the willingness to pay. This is easily seen from the monopoly price and quantity; $p = \alpha/2$ and $x = \alpha/2\beta$.

4.1 Equilibria in the same regime

Substituting the demand functions defined in (9) and (10) into the equilibrium conditions of regimes 1 to 4, we can state the following proposition:

Proposition 4 *Assume linear demand as specified in* (9) *and* (10), *and that reservoir capacity and inflow are such that the same regime applies to both the competitive and the monopoly equilibrium. Then,*

i) in regimes 1 and 2, $\widehat{x}_1 > \widetilde{x}_1$ *if* $\alpha_1 > (1 - q)\alpha_2$.

ii) in regime 3, $\widehat{x}_1^3 > \widetilde{x}_1^3$ *if* $\alpha_1 > \alpha_2$.

ii) in regime 4 $\widehat{x}_1^4 = \widetilde{x}_1^4$.

---

[18]When demand is inelastic there is no monopoly equilibrium. Hence, analysing monopoly equilibrium with iso-elastic demand by assumption rules out the case we want to characterise.

[19]As mentioned above (see footnote 10), demand parameters may differ between periods, but not over the states of the world. In period 2, when $x_2^H \neq x_2^L$, it follows that $p_2(x_2^H) \neq p_2(x_2^L)$.

Proof. See Appendix A. ∎

Proposition 4 shows that the parameter α is crucial for understanding how market power affects allocation of production. At equal prices in the two periods the absolute value of the price elasticity of demand is lower in the period with the highest α. A firm with market power would then find it optimal to reallocate production so that the price is highest in the period with the highest parameter α. If $\alpha_1 > \alpha_2$, reallocation leads to less production in period 1 in the monopoly equilibrium and higher storage $(U_1 + U_2^H - x_1)$. This result is in line with what we found in Proposition 2. In both cases production is increased in the period with the highest absolute value of price elasticity of demand.

We also note that in regime 1 and regime 2 equilibria the willingness to pay for electricity in period 1 only have to be higher than the expected willingness to pay in state L for storage to be higher in the monopoly equilibrium.

4.2 Equilibria in different regimes

The cut-off values between different regimes in the case of linear demand are reported in Table 3.

	Cut-off value	
Regime	Competitive	Monopoly*
1 and 2	$R = \frac{\alpha_2}{\beta_2}$	$R = \frac{\alpha_2}{\beta_2}$
1 and 3	$U_1 + U_2^H = \hat{x}_1^3 + \frac{\alpha_2}{\beta_2}$	$U_2^H = y(U_1, U_2^L, R = \frac{\alpha_2}{\beta_2})$
2 and 4	$U_1 + U_2^H = \tilde{x}_1 + R$	$U_1 + U_2^H = \tilde{x}_1 + R$
3 and 4	$U_1 + U_2^H = \hat{x}_1^3 + R$	$U_1 + U_2^H = \hat{x}_1^3 + R$
2 and 3		$U_2^H = y(U_1, U_2^L, R)$

Table 3: Cut off values assuming linear demand. * The cut off value between regimes 2 and 3 is defined below in equation (11).

As indicated by Table 3 there are regime 1 equilibria in the competitive case if

reservoir capacity and inflow are sufficiently high; $R > \frac{\alpha_2}{\beta_2}$ and $U_1 + U_2^H > \tilde{x}_1^3 + \frac{\alpha_2}{\beta_2}$. The two remaining cut-off values in the competitive case are identical to the values listed in Table 2 above.

In order to make a graphical representation of these regimes, we use a numerical example where we focus on variations in state H inflow (U_2^H) and reservoir capacity (R). Assume that $q = 0.5$, $U_1 = 0.6$, $U_2^L = 0$, $\beta_1 = \beta_2 = 1$ and $\alpha_1 = \alpha_2 = 1$. The border lines between the different regimes in the case of a competitive equilibrium are illustrated in Figure 3.

The cut-off values in the monopoly case listed in Table 3 depend on whether marginal revenue in state H is positive or negative. Marginal revenue in state H is non-negative as long as $x_2^H \leq \frac{1}{2}\frac{\alpha_2}{\beta_2}$. If $R < \frac{1}{2}\frac{\alpha_2}{\beta_2}$ marginal revenue in state H is never negative because of the reservoir constraint. In addition, regime 1 is infeasible since regime 1 equilibria require that $R > \frac{\alpha_2}{\beta_2}$; $p_2(\frac{\alpha_2}{\beta_2}) = 0$. Thus, there is no cut-off value between regime 1 and 2 or 1 and 3 as long as marginal revenue in state H is positive.

Furthermore, when marginal revenue is non-negative in state H it follows from the equilibrium conditions that $\tilde{x}_1^3 < \tilde{x}_1$. For intermediate values of inflow and reservoir capacity, $\tilde{x}_1 + R > U_1 + U_2^H > \tilde{x}_1^3 + R$, regime 4 equilibria apply.

Let us turn to the combinations of inflow and reservoir capacity where marginal revenue in state H is negative. We observe from the equilibrium conditions that in such cases $\tilde{x}_1^3 > \tilde{x}_1$. Instead of a range of inflow/reservoir values where regime 4 applies, there is now a range of values where it seems that both regime 2 and regime 3 apply. Recall that the equilibrium solution should be in regime 3 if $U_1 + U_2^H < \tilde{x}_1^3 + R$ and in regime 2 if $U_1 + U_2^H > \tilde{x}_1 + R$. Because $\tilde{x}_1^3 > \tilde{x}_1$, we know that we are in regime 2 if $U_1 + U_2^H > \tilde{x}_1^3 + R$ and in regime 3 if $U_1 + U_2^H < \tilde{x}_1 + R$. In situations where $\tilde{x}_1^3 + R > U_1 + U_2^H > \tilde{x}_1 + R$, the monopoly equilibrium is either in regime 3 or in regime 2 depending on the level of profit associated with the relevant regime. This is illustrated in Figure 4, where $\Delta\Pi$ indicate the direction of increased profits.

We calculate profits $\Pi(.)$ using regime 2 and 3 equilibrium output. Then we set the difference $\Pi(\tilde{x}_1) - \Pi(\tilde{x}_1^3) = 0$ and solve for inflow in state H.
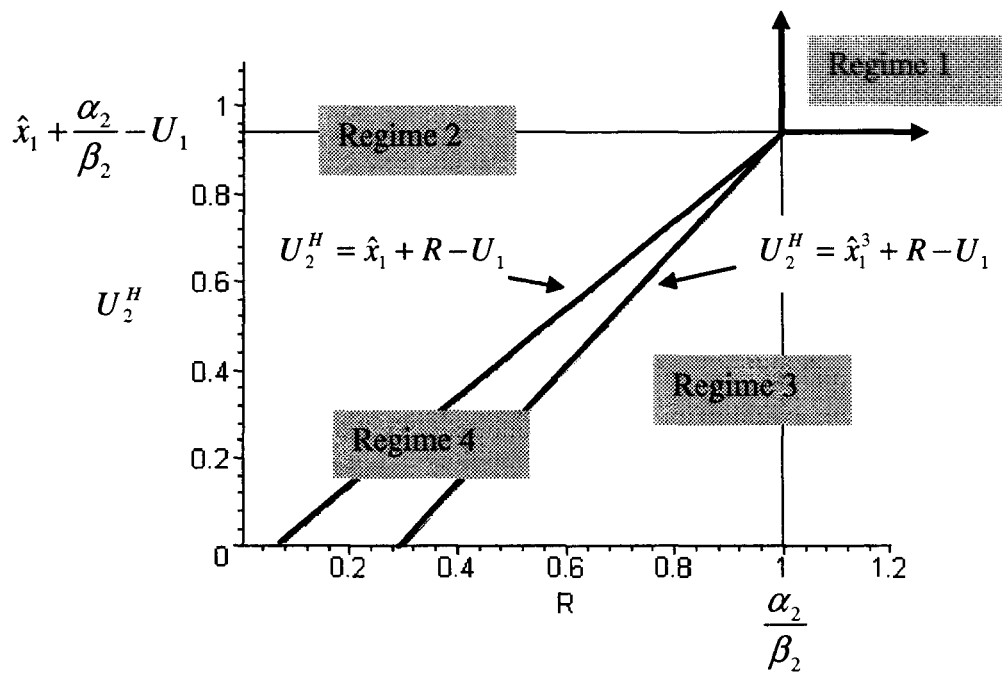
100

Figure 3: The border lines between the 4 different regimes in the competitive case.

1.4

1.2

$1.44-x_1=x_2{}^H$

Spill

1

$R=1, p_2(x_2{}^H)=0.$

$x_2{}^H$ 0.8

$\Delta\Pi$

0.6

Constrained
optimum

0.4

Unconstrained
optimum

0.2

0

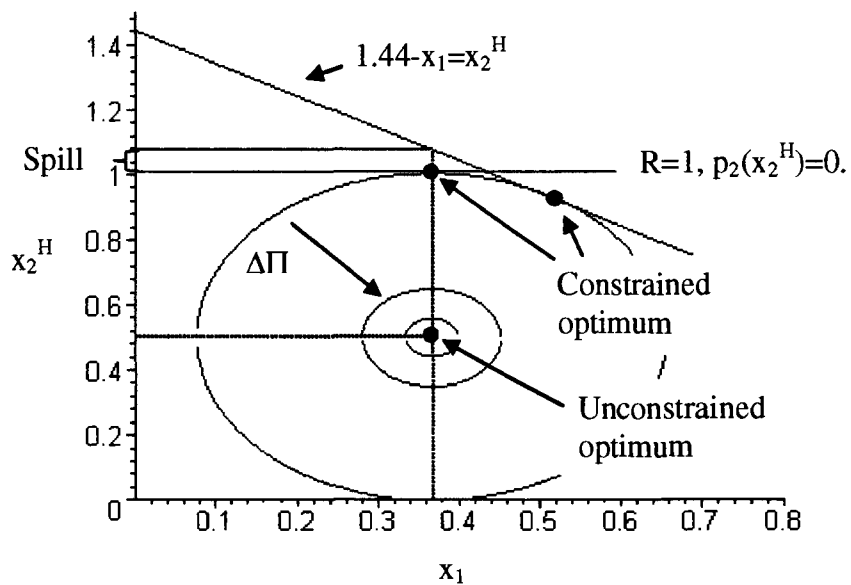0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8

x_1

Figure 4: The two constrained optimum points illustrate situations where the monopoly producer is indifferent between a regime 2 and regime 3 equilibrium output in period 1. In regime 2 the reservoir constraint is satisfied, while the energy constraint is satisfied in regime 3. The unconstrained optimum is the monopoly choice of equilibrium production when there is no limit to his ability to spill water.

We find that a monopoly producer would be indifferent between a regime 2 and 3 strategy (when marginal revenue in state H is negative) if

$$
\begin{aligned}
U_2^H &= \frac{1}{2} \frac{\beta_2 \alpha_1 - \beta_1 \alpha_2 + 2(1-q)(\beta_2)^2 U_2^L - 2\beta_1 \beta_2 U_1}{(\beta_1 + \beta_2 - q\beta_2)\beta_2} \\
&\quad + \frac{\sqrt{(2\beta_2 R - \alpha_2)^2 (\beta_1 + \beta_2)(\beta_1 + \beta_2 - q\beta_2)}}{(\beta_1 + \beta_2 - q\beta_2)\beta_2} \\
&= y(U_1, U_2^L, R).
\end{aligned}
\tag{11}
$$

If $U_2^H > y(U_1, U_2^L, R)$, then profit is higher in regime 2. If so, the monopolist would choose regime 2 equilibrium output in period 1 .

The cut-off value between regimes 1 and 3 is found by inserting $R = \frac{\alpha_2}{\beta_2}$ into equation (11). The cut-off value between regimes 1 and 2 is identical to the competitive case; $R = \frac{\alpha_2}{\beta_2}$. The border lines between the different regimes under monopoly are illustrated in Figure 5.

Comparing the regime border lines in the competitive case (Figure 3) with the borders applying to the monopoly case (Figure 5), we observe that the border lines do not overlap perfectly.[20] Thus, as Figure 3 and 5 show, we have equilibria in different regimes for some values of inflow and reservoir capacity.

Using the border lines defined in Table 3 together with our knowledge of period 1 production and storage related to the different regimes we state the following proposition:

Proposition 5 *Assume linear demand as specified in* (9) *and* (10). *Furthermore, assume that reservoir capacity and inflow are such that different regimes apply to the competitive and the collusive equilibria. Then,*

i) if $\alpha_1 > \alpha_2$, period 1 production in the monopoly equilibrium is always lower (and storage higher) than in the competitive equilibrium.

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[20]In our numerical example, the cut off value between regimes 3 and 4 however are identical in the competitive and collusive case.
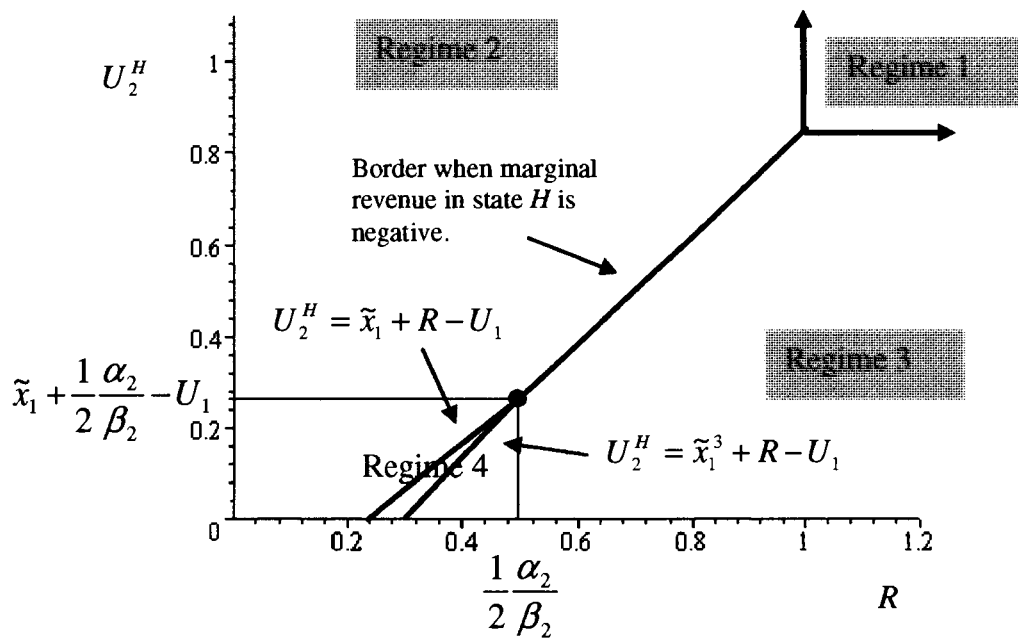
Figure 5: The border lines between the 4 regimes in the monopoly case.

ii) if $\alpha_1 < \alpha_2$ and $\alpha_1 > (1 - q)\alpha_2$, period 1 production in the monopoly equilibrium will be higher or lower than in the competitive equilibrium.

iii) if $\alpha_1 < (1 - q)\alpha_2$, period 1 production in the monopoly equilibrium is always higher (and storage lower) than in the competitive equilibrium.

Proof. See Appendix B. ∎

Propositions 4 and 5 both show that storage in the monopoly case is always higher than storage in the competitive case if the willingness to pay for electricity in period 1 is higher than the willingness to pay for electricity in period 2; $\alpha_1 > \alpha_2$. This result holds irrespective of whether the monopoly and competitive equilibrium belong to the same regime or not. The intuition behind this result is simply that under monopoly, the producer want to reduce sales in the period with the lowest price elasticity. Thus, if we have the opposite situation where the price elasticity in period 1 is higher than in period 2 (also when there is overflow in state H) storage in the monopoly case will always be lower than storage in the competitive case.

From proposition 4 we know that if $(1 - q)\alpha_2 < \alpha_1 < \alpha_2$, there would be higher storage under regimes 1 and 2 in the monopoly case and lower under regime 3. Thus, whether there is higher or lower storage in the monopoly case depends on the combination of inflow and reservoir values. This is also the case when we focus on combinations of different regimes. For some combinations of different regimes storage will be higher and other combinations will result in lower storage in the monopoly case.

4.3 Relaxing Assumption 3

Above we have assumed that the authorities will force producers to produce electricity in state H as long as the price is positive (Assumption 3). Now, we relax this assumption.

As long as marginal revenue in state H in the monopoly case is positive, relaxing Assumption 3 adds no new insight to the problem of determining whether storage is

higher or lower in the monopoly case than in the competitive case. This situation is covered in Propositions 4 and 5.

However, if we look at reservoir and inflow values where $R > \frac{1}{2}\frac{\alpha_2}{\beta_2}$ and $U_1 + U_2^H > \tilde{x}_1 + \frac{1}{2}\frac{\alpha_2}{\beta_2}$, the situation is different. When we relax Assumption 3 and inflow and reservoir capacity are sufficiently high, the monopolist would produce a fixed amount equal to $\frac{1}{2}\frac{\alpha_2}{\beta_2}$ in state H. At this production level marginal revenue in state H is zero.

When marginal revenue in state H is zero, allocation of water between period 1 and period 2 state L is determined by:

$$p_1[1 - \frac{1}{|e_1|}] = (1 - q)p_2(x_1^L)[1 - \frac{1}{|e_2|}]. \tag{12}$$

Using the linear demand functions from (9) and (10) we get that period 1 production under monopoly is equal and fixed to $\tilde{x}_1$ when $R > \frac{1}{2}\frac{\alpha_2}{\beta_2}$ and $U_1 + U_2^H > \tilde{x}_1 + \frac{1}{2}\frac{\alpha_2}{\beta_2}$.

Now, we compare the monopoly equilibrium to the competitive equilibrium for different values of inflow and reservoir capacity. As long the competitive equilibrium is in regime 1 or 2, we have that period 1 production is equal to $\hat{x}_1$. Thus, we find as shown in Prpositions 4 and 5 that storage is higher in the monopoly case $(\tilde{x}_1 < \hat{x}_1)$ if $\alpha_1 > (1 - q)\alpha_2$.

When we look at regimes 3 and 4 competitive equilibria the condition that $\alpha_1 > (1 - q)\alpha_2$ is no longer sufficient to conclude that storage is higher in the monopoly case. As long as $\alpha_1 > (1 - q)\alpha_2$, this implies that $\tilde{x}_1 < \hat{x}_1$. However, because regime 3 and 4 competitive equilibria imply less production in period 1 than in regimes 1 and 2 we also have that $\hat{x}_1^3 < \hat{x}_1$ and $\hat{x}_1^4 < \hat{x}_1$. Thus, we are unable to conclude that storage is higher in the monopoly case simply by looking at the condition $\alpha_1 > (1 - q)\alpha_2$. If the opposite is true, $\alpha_1 < (1 - q)\alpha_2$, then storage in the competitive case will always be higher than in the monopoly case.

5 Some concluding remarks

The main question of this paper is whether market power would lead to higher or lower storage from summer to winter. We compare the monopoly equilibrium to the competitive equilibrium in a simple two period model with uncertainty concerning water inflow. We analyze situations where storage may or may not be constrained by the existing reservoir capacity, and where inflow is so high that the energy constraint may or may no longer be binding. We find as a general result that market power, represented by the collusive equilibrium, would not lead to lower storage if demand is more price elastic in the winter period. If on the other hand, demand is less price elastic in winter compared to summer, storage would not be lower in the competitive case.

Whether demand is more or less price elastic in winter than in summer is an empirical question. To our knowledge there is no empirical evidence available at present. Thus, it is not possible to conclude that market power lead to less storage during summer and thereby increases the probability of a supply shortage in dry years.

A Equilibria in the same regime

Here we provide proof of proposition 2 and 4 where we have equilibria in the same regime.

A.1 Proof of Proposition 2

We have that period 1 production in competitive equilibrium $(\hat{x}_1)$ under regime 2 is determined by

$$p_1 = (1 - q)p_2(x_1^L).$$

If we divide through by p_2 we find that $\frac{p_1}{p_2} = 1 - q$.

Period 1 collusive production $(\tilde{x}_1)$ under regime 2 solves

$$p_1[1 - \frac{1}{|e_1|}] = (1 - q)p_2(x_1^L)[1 - \frac{1}{|e_2|}].$$

We find that $\frac{p_1}{p_2} = (1 - q)\frac{[1 - \frac{1}{|e_2|}]}{[1 - \frac{1}{|e_1|}]}$. Period 1 competitive production $(\hat{x}_1)$ is the higher if $\frac{[1 - \frac{1}{|e_2|}]}{[1 - \frac{1}{|e_1|}]} > 1$ or $|e_1| < |e_2|$.

Under regime 3, period 1 production in the competitive equilibrium $(\hat{x}_1^3)$ is determined by

$$p_1 = (1 - q)p_2(x_1^L) + qp_2(x_1^H).$$

In the case of collusion we have that period 1 production under regime 3 $(\tilde{x}_1^3)$ solves

$$p_1[1 - \frac{1}{|e_1|}] = (1 - q)p_2(x_1^L)[1 - \frac{1}{|e_2|}] + qp_2(x_1^H)[1 - \frac{1}{|e_2|}].$$

We have that the price in competitive regime 3 equilibrium is lower and period 1 production $(\hat{x}_1^3)$ is higher compared to monopoly if

$$(1-q)p_2(x_1^L) + qp_2(x_1^H) < \left[(1-q)p_2(x_1^L) + qp_2(x_1^H)\right] \frac{[1 - \frac{1}{|e_2|}]}{[1 - \frac{1}{|e_1|}]},$$

or $|e_1| < |e_2|$.

Under regime 4 we have that $x_2^H = R$ and that the energy constraint is binding both under competitive and collusive equilibrium. Thus $\hat{x}_1^4 = \tilde{x}_1^4 = U_1 + U_2^H - R$ in both cases and they are identical.

A.2 Proof of Proposition 4.

We substitute the inverse linear demand functions defined in equation (9) and (10) into the equilibrium conditions applying to regimes 1 to 4. Under regime 1 and 2, period 1 production in competitive equilibrium is equal to

$$\hat{x}_1 = \frac{\alpha_1 - (1-q)\alpha_2 + \beta_2(1-q)(U_1 + U_2^L)}{\beta_1 + (1-q)\beta_2}$$

and period 1 collusive production under the same two regimes is given by

$$\tilde{x}_1 = \frac{1}{2} \frac{\alpha_1 - (1-q)\alpha_2 + 2\beta_2(1-q)(U_1 + U_2^L)}{\beta_1 + (1-q)\beta_2}.$$

Period 1 competitive production $(\hat{x}_1)$ is higher than the collusive equilibrium production $(\tilde{x}_1)$ if $\alpha_1 > (1-q)\alpha_2$.

Under regime 3, period 1 production in the competitive equilibrium is determined by

$$\hat{x}_1^3 = \frac{\alpha_1 - \alpha_2 + \beta_2(U_1 + qU_2^H) + \beta_2(1-q)U_2^L}{\beta_1 + \beta_2}$$

while in the case of collusion we have that period 1 production is equal to

$$\tilde{x}_1^3 = \frac{1}{2} \frac{\alpha_1 - \alpha_2 + 2\beta_2(U_1 + qU_2^H) + 2\beta_2(1-q)U_2^L}{\beta_1 + \beta_2}.$$

The difference $(\widehat{x}_1^3 - \widetilde{x}_1^3)$ is equal to $\frac{\frac{1}{2}(\alpha_1 - \alpha_2)}{\beta_1 + \beta_2}$, where competitive period 1 production $(\widehat{x}_1^3)$ is higher than collusive period 1 production $(\widetilde{x}_1)$ if $\alpha_1 > \alpha_2$.

For proof of the regime 4 condition see subsection $A.1$.

B Equilibria in different regimes

Here we provide proof of Proposition 3 and 5 where we assume that inflow and reservoir capacity is such that different equilibria apply to the competitive and monopoly equilibrium.

B.1 Proof of Proposition 3.

Because we assume that marginal revenue corresponding to the competitive equilibrium is positive, we can rule out any regime 1 from the analysis. The only regimes we have to consider is 2, 3 and 4.

The possible combinations of equilibria in different regimes are illustrated in Table 4.

		Competitive regime		
		2	3	4
Collusive	2		**2,3**	**2,4**
regime	3	**3,2**		**3,4**
	4	**4,2**	**4,3**	

Table 4: Combinations of equilibria in different regimes; possible combinations in bold.

With reference to Table 4 consider first a combination of inflow and reservoir capacity where we are in regime 3 in the competitive case and either in regime 2 or 4 in the collusive case. In this situation we can not have less storage under the collusive outcome compared to the competitive case. Under regime 2 there is overflow if high inflow occurs and some of the available energy is lost before the second period. In regime 4 the reservoir constraint is met. In contrast,.under regime 3 in state H there is no overflow and the reservoir constraint is not met. Regime 3 implies more period 1 production and less storage than any of the two other regimes.

Second, we consider situations where we have a regime 2 competitive equilibrium. In addition to a regime 2 equilibrium we could now also have a regime 3 or 4 equilibrium in the collusive case. If so, period 1 production is higher in the latter case.

Finally, in a situation where we have a regime 4 solution in the competitive case, we can either have a regime 2 or a regime 3 solution in the collusive case. Period 1 production would be lower in the collusive case if we have a regime 2 equilibrium and higher if we have a regime 3 equilibrium.

This leaves us with the following combinations of different regimes, collusive and competitive respectively, where storage is higher in the collusive case: $\{(2,3), (2,4), (4,3)\}$. On the other hand, storage is higher in the competitive case for the following combinations of regimes: $\{(3,2), (4,2), (3,4)\}$. The combination of regimes resulting in higher storage under competition is exactly the opposite to the ones defined for the collusive case.

Now, can we observe a situation where we have the regime combination $(4,3)$ for some values of inflow and reservoir capacity and the combination $(3,4)$ for others? The cut-off value between regime 4 and 3 in the competitive case is given by $U_1 + U_2^H = \widehat{x}_1^3 + R$, while in the collusive case the corresponding cut-off value is defined by $U_1 + U_2^H = \widetilde{x}_1^3 + R$. If inflow $(U_1 + U_2^H)$ is higher in any of the two cases we have a regime 4 solution. We observe that if $\widehat{x}_1^3 > \widetilde{x}_1^3$, then for some combinations of inflow and reservoir capacity we can have the regime combination $(4,3)$. The difference between $\widehat{x}_1^3$ and $\widetilde{x}_1^3$ is not affected by inflow or reservoir capacity. We have that $\widehat{x}_1^3 > \widetilde{x}_1^3$ if $|e_1| < |e_2|$. Thus, if we observe the regime combination $(4,3)$ for some values of inflow and reservoir capacity we can not have the opposite situation $(3,4)$ for other values of inflow and reservoir capacity. The same result holds if we compare the other two remaining combinations of regimes where storage is higher in collusive equilibrium.

In the competitive case we are in regime 3 if

$$U_1 + U_2^H - R - \widehat{x}_1^3 < 0.$$

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In the collusive case, we are in regime 3 if

$$U_1 + U_2^H - R - \tilde{x}_1^3 < 0.$$

When $|e_1| < |e_2|$ we have that $U_1 + U_2^H - R - \hat{x}_1^3 > U_1 + U_2^H - R - \tilde{x}_1^3$. This implies that there are more combinations of inflow and reservoir capacity resulting in regime 3 equilibrium output in the competitive case. For these combinations of inflow and reservoir capacity we have either a regime 2 or 4 equilibrium in the collusive case.

Also, because $|e_1| < |e_2|$ we have that

$$U_1 + U_2^H - R - \hat{x}_1^2 > U_1 + U_2^H - R - \tilde{x}_1^2.$$

This implies that there are less combinations of inflow and reservoir capacity resulting in regime 2 outcomes in the competitive case. Recall that we we have a regime 2 equilibrium if $U_1 + U_2^H - R - \hat{x}_1^2 > 0$. If inflow and reservoir capacity is such that we have a regime 2 equilibrium in the collusive case but not in the competitive case, then we either have a regime 3 or 4 equilibrium in the competitive case. Thus, when the equilibrium is in different regimes and $|e_1| < |e_2|$, storage is higher in the collusive case.

B.2 Proof of Proposition 5.

With linear demand as defined in equation (9) and (10), then if inflow and reservoir capacity is sufficiently large we may have regime 1 equilibria.

The possible combinations of equilibria in different regimes are illustrated in table 5.

If we have a regime 1 competitive or monopoly equilibrium we know that $R > \frac{\alpha_2}{\beta_2}$. This implies that is is not possible to have either a regime 2 or 4 equilibrium as this requires that $R < \frac{\alpha_2}{\beta_2}$. Thus, as indicated in Table 5 the regime combinations $\{(1,2),(1,4)\}$ and $\{(2,1),(4,1)\}$ are not possible. For a discussion of the remaining regime combinations we refer to the proof of Proposition 3 above.

		Competitive regime			
		1	2	3	4
Monopoly	1		1,2	**1,3**	1,4
	2	2,1		**2,3**	**2,4**
regime	3	**3,1**	**3,2**		**3,4**
	4	4,1	**4,2**	**4,3**	

Table 5: Combinations of equilibria in different regimes; possible combinations in bold.

This leaves us with the following combinations of different regimes where storage is higher in the monopoly case: $\{(1,3),\ (2,3),\ (2,4),\ (4,3)\}$. On the other hand, storage is higher in the competitive case for the following combinations of regimes: $\{(3,1),\ (3,2),\ (4,2),\ (3,4)\}$. As illustrated in subsection $B.1$, if we observe the regime combinations $\{(1,3),\ (2,3),\ (2,4),\ (4,3)\}$ for some values of inflow and reservoir capacity, we can not observe the regime combinations $\{(3,1),\ (3,2),\ (4,2),\ (3,4)\}$ for other combinations of inflow and reservoir capacity values.

In the competitive case we are in regime 3 if

$$U_1 + U_2^H - R - \frac{\beta_2(U_1 + qU_2^H) + \beta_2(1-q)U_2^L}{\beta_1 + \beta_2} < \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$$

for values of $0 < R < \frac{\alpha_2}{\beta_2}$ and

$$U_1 + U_2^H - \frac{\alpha_2}{\beta_2} - \frac{\beta_2(U_1 + qU_2^H) + \beta_2(1-q)U_2^L}{\beta_1 + \beta_2} < \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$$

for values of $R > \frac{\alpha_2}{\beta_2}$.

In the monopoly case, we are in regime 3 if

$$U_1 + U_2^H - R - \frac{1}{2}\frac{2\beta_2(U_1 + qU_2^H) + 2\beta_2(1-q)U_2^L}{\beta_1 + \beta_2} < \frac{1}{2}\frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$$

for values of $0 < R < \frac{1}{2}\frac{\alpha_2}{\beta_2}$,

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$$U_2^H - y(U_1, U_2^L, R) < 0$$

for values of $\frac{1}{2}\frac{\alpha_2}{\beta_2} < R < \frac{\alpha_2}{\beta_2}$ and

$$U_2^H - y(U_1, U_2^L, R = \frac{\alpha_2}{\beta_2}) < 0$$

for values of $R > \frac{\alpha_2}{\beta_2}$.

When $\alpha_1 > \alpha_2$ we have that $\frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2} > \frac{1}{2}\frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$. It implies that there are more combinations of inflow and reservoir capacity resulting in regime 3 equilibrium output in the competitive case for values of $0 < R < \frac{1}{2}\frac{\alpha_2}{\beta_2}$. For these combinations of inflow and reservoir capacity we have either a regime 1, 2 or 4 equilibrium in the monopoly case.

Also, because $\alpha_1 > \alpha_2$ we have that $\frac{1}{2}\frac{\alpha_1 - (1-q)\alpha_2}{\beta_1 + (1-q)\beta_2} < \frac{\alpha_1 - (1-q)\alpha_2}{\beta_1 + (1-q)\beta_2}$. This implies that there are less combinations of inflow and reservoir capacity resulting in regime 2 outcomes in the competitive case when $0 < R < \frac{1}{2}\frac{\alpha_2}{\beta_2}$. In the collusive case we are in regime 2 if

$$U_1 + U_2^H - R - \frac{1}{2}\frac{2\beta_2(1-q)(U_1 + U_2^L)}{\beta_1 + (1-q)\beta_2} > \frac{1}{2}\frac{\alpha_1 - (1-q)\alpha_2}{\beta_1 + (1-q)\beta_2}$$

In the competitive case, we are in regime 2 if

$$U_1 + U_2^H - R - \frac{\beta_2(1-q)(U_1 + U_2^L)}{\beta_1 + (1-q)\beta_2} > \frac{\alpha_1 - (1-q)\alpha_2}{\beta_1 + (1-q)\beta_2}.$$

When $\frac{1}{2}\frac{\alpha_2}{\beta_2} < R < \frac{\alpha_2}{\beta_2}$ there is no regime 4 equilibria in the monopoly case because $\tilde{x}_1^3 > \tilde{x}_1$. We know that we are in regime 2 if $U_1 + U_2^H > \tilde{x}_1^3 + R$ and in regime 3 if $U_1 + U_2^H < \tilde{x}_1 + R$. In situations where $\tilde{x}_1^3 + R - U_1 > U_2^H > \tilde{x}_1 + R - U_1$, the monopoly equilibrium is either in regime 3 or in regime 2. The cut-off value defined in equation (11) imply that $\tilde{x}_1^3 + R - U_1 > y(U_1, U_2^L, R) > \tilde{x}_1 + R - U_1$. If $R = \frac{1}{2}\frac{\alpha_2}{\beta_2}$ and $U_1 + U_2^H = \tilde{x}_1 + \frac{1}{2}\frac{\alpha_2}{\beta_2}$, we have that $\tilde{x}_1 = \tilde{x}_1^3$ and $\tilde{x}_1^3 + R - U_1 = y(U_1, U_2^L, R) = \tilde{x}_1 + R - U_1$ as illustrated in figure 4.

When $\alpha_1 > \alpha_2$ we have that $\widehat{x}_1^3 + R - U_1 > \widetilde{x}_1^3 + R - U_1 > y(U_1, U_2^L, R)$ implying there are more combinations of inflow and reservoir capacity resulting in regime 3 equilibrium output in the competitive case when $\frac{1}{2}\frac{\alpha_2}{\beta_2} < R < \frac{\alpha_2}{\beta_2}$. Furthermore, we have that $\widehat{x}_1 + R - U_1 > \widetilde{x}_1^3 + R - U_1$ implying there are less combinations of inflow and reservoir capacity resulting in regime 2 outcomes in the competitive case when $\frac{1}{2}\frac{\alpha_2}{\beta_2} < R < \frac{\alpha_2}{\beta_2}$.

The proof related to values of $R > \frac{\alpha_2}{\beta_2}$ is similar. When $\alpha_1 > \alpha_2$ we have that $\widehat{x}_1^3 + \frac{\alpha_2}{\beta_2} - U_1 > \widetilde{x}_1^3 + \frac{\alpha_2}{\beta_2} - U_1 > y(U_1, U_2^L, R = \frac{\alpha_2}{\beta_2})$ and $\widehat{x}_1 + \frac{\alpha_2}{\beta_2} - U_1 = \widetilde{x}_1^3 + \frac{\alpha_2}{\beta_2} - U_1$. If $R = \frac{\alpha_2}{\beta_2}$ and $U_1 + U_2^H = \widehat{x}_1 + \frac{1}{2}\frac{\alpha_2}{\beta_2}$, we have that $\widehat{x}_1 = \widetilde{x}_1^3$ as illustrated in figure 2.

Thus, when $\alpha_1 > \alpha_2$, we can only observe combinations of different regimes where storage is higher in the monopoly case: $\{(1,3),(2,3),(2,4),(4,3)\}$.

The proof related to the situation where $\alpha_1 < (1-q)\alpha_2$ is similar to the one shown for the case where $\alpha_1 > \alpha_2$. When $\alpha_1 < (1-q)\alpha_2$ we have that $\widehat{x}_1 < \widetilde{x}_1$ and $\widehat{x}_1^3 < \widetilde{x}_1^3$.

For the intermediate situation where $\alpha_1 > (1-q)\alpha_2$ and $\alpha_1 < \alpha_2$ we have that $\widehat{x}_1 > \widetilde{x}_1$ and $\widehat{x}_1^3 < \widetilde{x}_1^3$. For simplicity, we restrict the discussion to values of $R < \frac{1}{2}\frac{\alpha_2}{\beta_2}$. Because $\widehat{x}_1 + R > \widetilde{x}_1 + R$ we have some combinations of inflow and reservoir capacity resulting in regime 2 solutions in the monopoly case and either regime 4 or 3 in the competitive case. In these situations storage is higher in the monopoly case. However, because $\widehat{x}_1^3 + R < \widetilde{x}_1^3 + R$ there are some combinations of inflow and reservoir capacity resulting in regime 3 equilibrium in the monopoly case while we are either in regime 2 or 4 in the competitive case. Thus, when $\alpha_1 > (1-q)\alpha_2$ and $\alpha_1 < \alpha_2$ storage can both be higher and lower in the monopoly case depending on the actual combination of inflow and reservoir capacity.

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Supply Function Equilibria in a Hydropower Market

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Abstract: The purpose of this paper is to study how energy constraints affect the performance of the Supply Function Equilibria (SFE) model. We start by developing the standard SFE model with two symmetric players. We then develop a simple numerical example to illustrate the effects of production constraints both related to installed effect capacity and to energy capacity. We illustrate through a simple numerical example that binding constraints on energy production reduce the number of allowable supply functions. Thus, if the constraint on energy produced is not taken into account when the SFE model is used to analyse competition in electricity markets, the welfare effects of market power might be exaggerated.

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1   Introduction

The purpose of this paper is to study how energy constraints affect the performance of the Supply Function Equilibria (SFE) model. In markets dominated by thermal production of electricity, production in one period has limited effect on production in other periods. However, power production in one period is constrained by the installed production capacity. The problem of modelling production constraints within the SFE model framework has been studied in several papers.[1] For hydropower producers the problem is different. Normally, the installed production capacity is so large that production is not constrained by the limit on production capacity. The problem facing hydropower producers is to allocate scarce water resources between different periods. Thus, when we use the SFE model to analyse competition in an electricity market dominated by hydropower the effects of energy constraints should be included in the analysis.[2]

The idea of competition in supply functions origins from the debate on whether firms choose prices or quantities as strategic variables. The idea first outlined by Grossman (1981) was that firms may not be able to set a price or a given quantity for every possible state of the market in advance of trade taking place. Instead, firms may resort to specifying supply functions relating quantity to price. Grossman (1981) studied supply function equilibria in the absence of uncertainty. According to Klemperer and Meyer (1989), this approach leads to a vast number of possible Nash equilibria in supply functions. In addition, without uncertainty, there is no reason to choose a more general supply function because firms can maximize profits either by fixing price or quantity.

Klemperer and Meyer (1989) introduced exogenous uncertainty into the supply function framework. They prove that under these conditions it is more profitable for

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[1]See for instance Baldick and Hogan (2001).

[2]This paper is motivated by a report from the Nordic competition authorities (2003) where a SFE model developed by the Danish system operator Eltra was used to analyse mergers and acquisitions in the Nordic electricity market.

firms to rely on supply functions rather than fixing price or quantity. With uncertainty, a supply function provides valuable flexibility to the firm. Furthermore, they also show that with uncertainty in demand, the number of possible Nash equilibria is dramatically reduced.

The supply function equilibria (SFE) concept developed by Klemperer and Meyer seems to fit quite closely to competition in several markets where firms must commit to bids in advance, including electricity spot markets. Thus, not surprisingly, several papers[3] used this approach in order to analyze electricity market competition. These papers typically focus on competition in a thermal based electricity market. In such a market the focus is on production constraints at a particular time, not on constraints on energy produced over a time period.

This paper is divided in three parts. In the first part we set up the Supply Function Equilibria (SFE) model based on the analysis by Baldick and Hogan (2001) and Green and Newbery (1992). On the basis of this model we develop a simple numerical example. In the second part of the paper we use this example to illustrate how competition in supply functions may be affected both by constraints on power produced at a particular moment in time and constraints related to available energy resources. We illustrate that binding constraints on energy production reduce the number of allowable supply functions. Thus, if the constraint on energy produced is not taken into account when the SFE model is used to analyse competition in electricity markets, the welfare effects of market power might be exaggerated.

2   The SFE model

In order to describe the SFE model we use a standard approach closely related to the presentation made by Baldick and Hogan (2001). First we discuss demand, generation costs and capacities. In our discussion on generation costs and capacities we will also

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[3]This includes Green and Newbery (1992), Green (1996), Newbery (1998), Baldick et al (2000) and Baldick and Hogan (2001).

address the problem of defining costs in the present of hydropower production. Then we discuss price, the assumptions on the form of the supply functions, profit and equilibrium conditions.

2.1 Demand

Baldick and Hogan (2001) use the following definition of the demand function Q:

$$\forall p \in R_+, \forall t \in [0, 1], \qquad Q(p, t) = a(t) - bp. \tag{1}$$

Where,

p is the price,

t is the (normalized) time,

$a : [0, 1] \to R_+$ is the load-duration characteristic and

$b \in R_+$ is the slope of the demand curve.

This definition implies that demand is additively separable in its two variables, price and time. Furthermore, the time variable is normalized to be between 0 and 1. The time variable describes the share of clearing periods or hours below the peak demand period. The load-duration characteristic is assumed to be non-increasing so that $t = 0$ corresponds to peak demand while $t = 1$ corresponds to minimum demand. The load-duration curve gives the time (number of hours) that demand exceeds a given level, so at $t = 0$ we would only have the highest demand period left.

This approach resembles the approach made by Klemperer and Meyer (1989) where they let demand at a particular time be subject to an exogenous shock. Here, instead of a shock to demand at a particular time we look at the variations in demand facing a supplier of electricity in the spot market. Producers are assumed to know the shape of the load duration curve. However, they face uncertainty with regard to the actual level of demand realized at a particular time.

2.2 Generation costs and capacities

A standard assumption is to let the total variable cost function be represented by a quadratic function of the form:

$$\forall i, \ C_i(q_i) = \frac{1}{2} c_i q_i^2 + v_i q_i. \tag{2}$$

The firms are labeled $i = 1..n$, with $n \geq 2$. q_i represents power produced by firm i and c_i and v_i are positive constants. $c_i \geq 0$ satisfies Klemperer and Meyer's condition that costs should be non-decreasing. If we differentiate the cost function, we get the firms' marginal cost[4].

$$\forall i, \ C_i'(q_i) = c_i q_i + v_i. \tag{3}$$

Furthermore, we assume that all the firms are able to produce down to zero output. Thus, the minimum capacity constraint is 0 for all firms. Baldick and Hogan (2001) note that firms may also face a maximum production capacity constraint, $\overline{q_i}$. The capacity constraint then becomes

$$\forall i, \ 0 \leq q_i \leq \overline{q_i}. \tag{4}$$

With hydropower production there is an additional constraint related to the amount of energy produced over time and the capacity constraint related to production of electricity at one point in time may not be as relevant as it is to thermal production. In the following we shall refer to the first type of constraint as the power constraint, while the second type of constraint is referred to as the energy constraint. We let $k = 1..n$ represent the number of load duration periods associated with the producer's planning horizon. Furthermore, we let $\overline{W}_i$ be the amount of energy available for hydropower

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[4]The trick used by Klemperer and Meyer here was to let $C_i'(0) = 0$, so if $C_i'(0) = v_i > 0$ then the supply functions would be expressed in terms of $\tilde{p} = p + v_i$. The marginal cost curve for symmetric producers where normalized to start at 0.

firm i over all k load duration periods. This implies that the following constraint must hold,

$$\sum_k \int_{t=0}^1 q_{ikt} dt \leq \overline{W}_i. \tag{5}$$

Using this constraint we implicitly assume there is a fixed amount of energy (water) available for production over all k load duration periods.

2.3 Feasible and allowable supply functions

Following Green and Newbery (1992) and Baldick and Hogan (2001) we assume that each firm bid a supply function into the market. The supply function represents the amount of power the firm is willing to produce at a specified price per unit of electricity.

Formally, a supply function S_i is a function that maps the level of prices into levels of output. In Klemperer and Meyer (1989) the functions are required to be defined for every price in the interval $[0, \infty]$. This implies that all firms will produce at prices in this interval. Furthermore, in order for a supply function to be feasible they only require that the function is contained in the quantity interval $[-\infty, +\infty]$. The lower boundary on quantities implying negative production has no meaning in real markets. Nor has it any implications for the result, so following Newbery (1998) we let this interval range from 0 to $+\infty$. The supply function in this case would be $S_i : [0, \infty] \rightarrow [0, \infty]$. Also, we shall require that the bid curves are monotonically increasing, or as Baldick and Hogan (2001) put it, that the supply functions must be non-decreasing.

If there are capacity constraints related to power production at a particular time, only supply functions mapping prices into the interval $[0, \overline{q}_i]$ would be feasible. Furthermore, following Baldick and Hogan (2001) (if the marginal costs are not normalized to 0) no firm would be willing to submit any bids at prices below operating costs corresponding to 0 output. A good candidate for the price minimum would be $\underline{p} = v_i$.

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In addition, with linear demand we would have a price $\bar{p}$ corresponding to $D(\hat{p}, t) = 0$ at $t = 0$. With our definition of demand this price is $\bar{p} = N(0)/b$, called the "choke price". The supply function does not have to be defined for higher prices. Baldick and Hogan (2001) also discuss price caps and bid caps. This problem will be omitted here.

$\underline{p} = v_i$ may be a good candidate for the minimum price in a thermal system, but will a hydro-producer be satisfied with this price? If the energy constraint is not effective, then the producer might want to produce at this price, but if the energy constraint is binding then receiving v_i would not make the firm produce. The reason is that production has a positive alternative value in production at other times. This might be seen as lost income or alternative cost at the time in question. If the number of load duration periods is large, then production within one particular load duration period has a small effect on this alternative value. We assume that we can neglect this effect and let λ denote the alternative value which is exogenous when looking at a particular load duration period. The relevant minimum price to a hydro-producer would have to cover operating cost corresponding to 0 output and the opportunity cost of producing in other periods, $\underline{p} = v_i + \lambda_i$.

Following these requirements, a feasible and allowable supply function for firm i is a function $S_i : [\underline{p}, \bar{p}] \rightarrow [0, \bar{q_i}]$.

2.4 Price, profit and equilibrium conditions

We need a market clearing condition. We assume that at each time $t \in [0, 1]$, the dispatcher chooses the lowest price $p(t)$ that clears the market. That is the price determined by

$$a(t) - bp = \sum_i S_i(p), \qquad (6)$$

provided that such a price exists. This is a uniform price auction where all firms receive the marginal clearing price for their supply. We assume that the solution to

our market clearing condition corresponds to prices within the range $[\underline{p}, \overline{p}]$ and do not consider prices outside this range.

The profit at time t for firm i is

$$\pi_{it} = S_i(p(t))p(t) - C_i(S_i(p(t))). \tag{7}$$

The profit over the whole load duration period is

$$\pi_i(S_i, S_{-i}) = \int_0^1 S_i(p(t))p(t) - C_i(S_i(p(t)))\, dt, \tag{8}$$

where $S_{-i} = S_{j \neq i}$.

Green and Newbery (1992) maximize the contributed profit per unit time as defined in (7) and use the assumption[5] made by Klemperer and Meyer (1989) to justify that the resulting first order condition (13) would also maximize profit over the whole time horizon. To see how this is possible we follow the argumentation laid out by Baldick and Hogan (2001).

The first point is to note that for each firm i we consider that all the other j firms have commited to differentiable supply functions, S_j. This ensures a solution. Now, if we say that firm i is committed to supply the residual demand at any given price then we have that

$$\forall t \in [0, 1], q_{it} = Q(p(t), t) - \sum_{j \neq i} S_j(p(t)). \tag{9}$$

We can now rearrange equation (7) as follows:

$$\pi_{it} = p(t)[Q(p(t), t) - \sum_{j \neq i} S_j(p(t))] - C_i[Q(p(t), t) - \sum_{j \neq i} S_j(p(t))]. \tag{10}$$

Since the supply functions S_j are assumed to be differentiable, we can derive the necessary conditions for maximizing the profit per unit time π_{it} at each time t over

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[5]The assumption is that firm i's residual demand at any price is the difference between demand and the quantity that other producers are willing to supply at that price.

choices of $p(t)$.

$$\frac{\partial \pi_{it}}{\partial p(t)} = [Q(p(t), t) - \sum_{j \neq i} S_j(p(t))] + [p(t) - C_i''][\frac{\partial Q(p(t), t)}{\partial p(t)} - \sum_{j \neq i} \frac{\partial S_j(p(t))}{\partial p(t)}]. \quad (11)$$

Because we know that firm i will produce the difference between the supply by other firms and total demand, we can find the price output pair optimizing firm i's production at time t by solving the first order condition with respect to q_{it}. Now, we assume that the relationship between price and quantity is monotonically non-decreasing over the time period. That is, a higher price means higher quantity and this quantity is unique. Then we can define a non-decreasing supply function for firm i that is infinitely differentiable.

$$S_i(p(t)) = q_{it}. \quad (12)$$

This function[6] also maximizes the integrated profit over the time horizon and can be calculated without reference to the load-duration characteristic.

$$S_i(p) = [p - C_i''][Q_p - \sum_{j \neq i} S_j'(p)], \quad (13)$$

where $Q_p = \frac{\partial Q(p(t), t)}{\partial p(t)}$ and $S_j'(p) = \frac{\partial S_j(p(t))}{\partial p(t)}$.

In the next subsection we develop a simple numerical example to be used as a benchmark throughout our analysis.

2.5 Numerical example: Two firms with marginal costs normalized to 0

Let us consider the solution to the SFE model in a situation with two symmetric firms, linear demand and constant marginal cost normalized to 0. This gives us the

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[6]The solution to this first order differential equation is shown in Appendix A.

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following coefficient $(u(p))$ and term $(w(p))$ related to the differential equation derived in Appendix A, equation (25):

$$u(p) = -\frac{1}{p}, \qquad w(p) = -b.$$

The supply function is

$$q(p) = e^{-\int \left(-\frac{1}{p}\right)dp} \left[A + \int -be^{\int \left(-\frac{1}{p}\right)dp}dp\right]. \qquad (14)$$

Rearranging (14) by $\int \frac{1}{x}dx = \ln x + c$ and $\int -f(x)dx = -\int f(x)dx$ we get:

$$q(p) = e^{\ln p + c} \left[A - b\int e^{-\ln p + c}dp\right].$$

Furthermore, by $e^{-\ln x} = \frac{1}{x}$, $\int \frac{1}{x}dx = \ln x + c$ and including the constant c in the term A the supply function becomes

$$q(p) = pA - pb\ln p. \qquad (15)$$

According to Klemperer and Meyer (1989) any SFE where the firms do not know the uncertainty is intermediate between Cournot and Bertrand equilibrium levels. Thus, we know that the intersection between the supply curve and the demand curve would have to lie below the Cournot level and above the Bertrand level. However, in order to define the SFE we need a boundary solution. Here, we follow Green and Newbery (1992) and use the Cournot equilibrium as the boundary solution in order to define the SFE. The resulting supply schedule is called the Cournot supply schedule.

With linear demand, $Q = a(t) - bp$, we then write the first order Nash-Cournot solution for two symmetric players $i = 1, 2$ where the marginal cost has been normalized to 0. For maximum demand this solution will form an upper boundary on the supply function.

$$q_i = \frac{a}{3} \text{ and } p = \frac{a}{3b}. \qquad (16)$$

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We use this result to calculate the value of A by inserting the solution into the supply function (16).

$$\left(\frac{a}{3}\right) = \left(\frac{a}{3b}\right)A - \left(\frac{a}{3b}\right)b\ln\left(\frac{a}{3b}\right),$$

rearranging,

$$A = b + b\ln\left(\frac{a}{3b}\right),$$

furthermore, by $\ln\left(\frac{a}{b}\right) = -\ln\left(\frac{b}{a}\right)$,

$$A = b[1 - \ln\left(\frac{3b}{a}\right)], \tag{17}$$

and replacing A in the supply function (16) we get

$$q(p) = pb[1 - \ln\left(\frac{3b}{a}\right)] - pb\ln p. \tag{18}$$

We let $a = 56$ at peak demand when $t = 0$, $b = 0.3$ and $Q = 2q$, where $q_1 = q_2$[7]. We can now calculate the (Nash-Cournot) upper boundary for the symmetric duopoly, $(Q, p) = (37, 62)$. We observe that the aggregated supply function should pass through zero $(p = 0)$ and that $\frac{dq}{dp} = 0$ at the boundary solution. The solution is shown in Figure 1 along with the downward sloping linear demand curve corresponding to the hour with highest demand $(t = 0)$.

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[7]The numerical values chosen here compare to the values used in an example in Newbery (1998). Below we use this example as a basis for analysing how an energy constraint would affect the firms' supply functions.
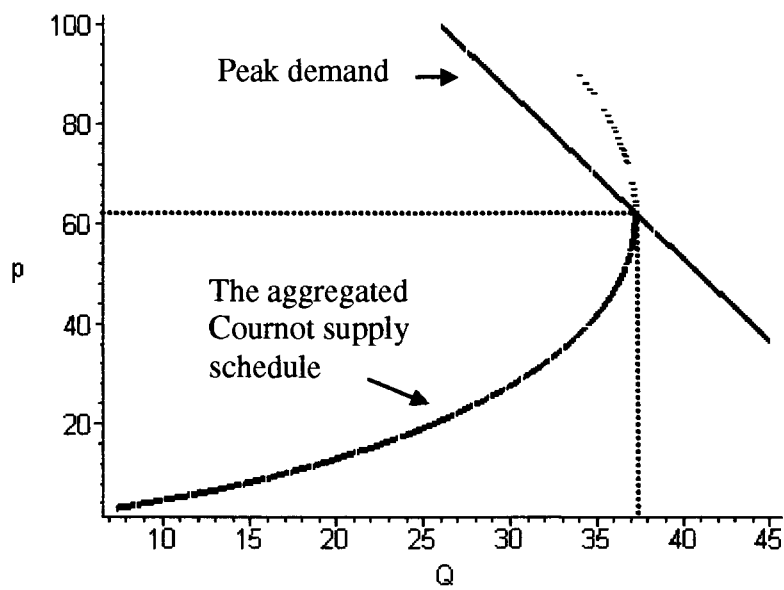
Figure 1: The figure shows the aggregated supply curve resulting from an SFE equilibrium with two symmetric firms, zero marginal cost and assuming a Cournot solution at the period of highest demand.

3 Constraints on power and energy produced

In this section we look at how constraints on power and energy produced, limit the number of possible SFE. The first issue is described in Newbery (1998) and in Baldick and Hogan (2001). The second issue mentioned above, is more relevant to a hydropower based electricity system. First, we briefly restate the model used by Newbery (1998). In subsection 3.2 we discuss constraints on energy produced.

3.1 Power constraints

We look at the symmetric duopoly case and introduce the power constraint from equation (4) above. Neither of the two firms can supply more than a specific capacity $q = \overline{q}$.

With linear demand $Q = a(t) - bp$ and constant marginal cost c, the maximization problem becomes:

$$\pi_i = (p - C(q_i))q_i + \mu(\overline{q} - q_i).$$

We then find the Nash-Cournot solution for two symmetric players assuming positive output values. For maximum demand this solution will form an upper boundary on the supply function,

$$\frac{a - b(c + \mu)}{3} = q_i \text{ and } \frac{a + 2b(c + \mu)}{3b} = p.$$

If we assume a positive shadow value on the constraint, then we have that $q_1 = q_2 = \overline{q}$ and the price $p = (a - 2\overline{q})/b$. We use this result to calculate the value of A.

$$\overline{q} = (\frac{a - 2\overline{q}}{b} - c)A - (\frac{a - 2\overline{q}}{b} - c)b\ln(\frac{a - 2\overline{q}}{b} - c),$$

rearranging,

$$A = b[\frac{\overline{q}}{a - 2\overline{q} - bc} + \ln(\frac{a - 2\overline{q} - bc}{b})]. \tag{19}$$
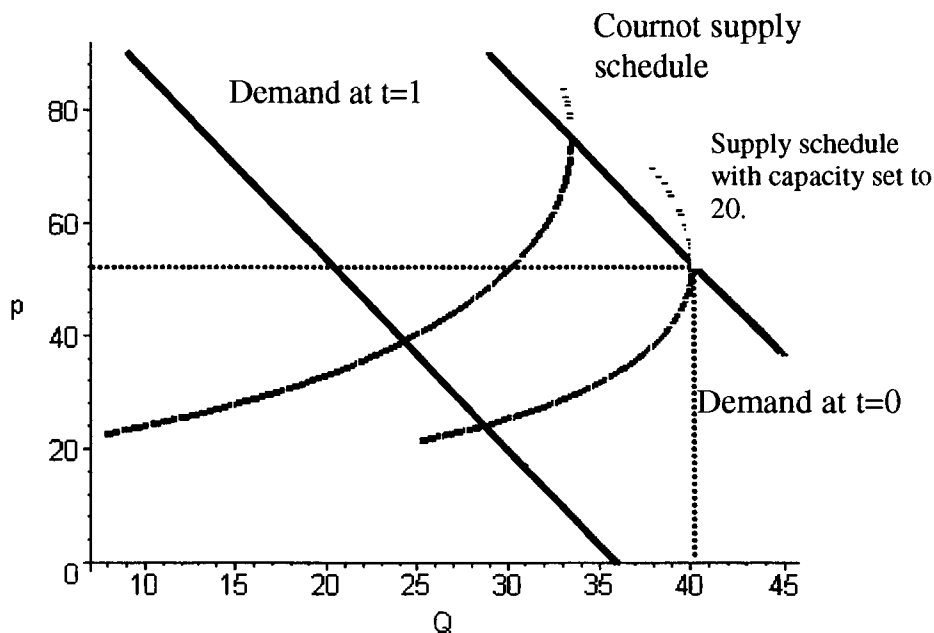
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Figure 2: Here we compare the Cournot supply schedule to the supply schedule where producers are facing a binding production constraint. Any supply schedule below the supply schedule at $\bar{q} = 20$ would violate the production constraint and is thus not feasible.

By replacing A from (20) in the supply function (16) we get that

$$q(p) = (p - c')b[\frac{\bar{q}}{a - 2\bar{q} - bc} + \ln(\frac{a - 2\bar{q} - bc}{b})] - (p - c)b\ln(p - c). \qquad (20)$$

If we let $a = 56$, $b = 0.3$, $c = 19$ and $\bar{q} = 20$, we can calculate the upper boundary $(Q, p) = (40, 53.3)$ corresponding to the intersection between the capacity limit and the demand curve. The solution is shown in Figure 2 along with the downward sloping linear demand curves for the lower and upper support and together with the supply function where the upper boundary is set to the Nash-Cournot solution.

The example shown in Figure 2 is similar to the one shown in Figure 4 in Newbery

(1998). Newbery finds that the effect of a capacity constraint on power is to reduce the number of feasible supply function equilibria. The allowable supply functions have to lie between the Cournot supply schedule and the supply schedule which intersects the capacity constraint on power in the hour of highest demand. Thus, more competitive bidding strategies are ruled out.

3.2 Constraints on energy produced

In this subsection we shall look at the competition between two symmetric hydropower producers and introduce a constraint on the amount of energy produced. It is assumed that each of the firms have a fixed amount of water $(\overline{W}_i)$ available for production over k load duration periods and that marginal cost in production is equal to zero. In equilibrium we must have that $\sum_k \int_{t=0}^{1} q_{ikt} dt \leq \overline{W}_i$ as stated in equation (5).We discuss the effects of this energy constraint through a numerical example.

A hydropower producing firm has allocate the limited amount of water between the k different load duration periods and also between t periods within the load duration period itself. First, we consider just one load duration period k and assume that production in this period does not affect bidding behaviour in other load duration periods. Thus, the alternative value of water produced in period k is fixed. Because the firms are symmetric, the alternative value of water (or water value) be identical. Let λ represent the firms water value. The problem for each of the two hydro-producing firms is then almost identical to the symmetric two-firm problem described in subsection 2.1. The only difference is that marginal cost now is represented by the firms'water value, λ. The supply function can then be expressed as

$$q(p) = (p - \lambda)A - (p - \lambda)b\ln(p - \lambda).$$ (21)

Let us use the same numerical values as in the example of the previous section, namely $a(t; t = 0) = a_0 = 56$, $b = 0.3$ and $\lambda = 19$ and calculate the (Nash-Cournot) upper boundary, $(Q, p)=(33.53, 74.8)$. With these numerical values, the aggregated

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supply function for the hydro-producing firms will be identical to the one described in figure 2.

Through a simple example we can measure exactly how much energy is produced within the load duration period k. We simplify to three periods t where $a_0 = 56$, $a_{0.5} = 46$ and $a_1 = 36$ and assume that each of the three periods has a duration of one hour. If the producers submit Cournot supply schedules we have already seen that the total production of energy in period 0 is 33.5 GWh. To find the price and quantities in the two other periods $t = 0.5$ and $t = 1$, we set the aggregated supply function equal to demand,

$$(p - \lambda)b[1 - \ln(\frac{3b}{a_0 - b\lambda})] - (p - \lambda)b\ln(p - \lambda) = a_1 - bp \qquad (22)$$

This gives us the values of price and total energy in period $t = 0.5$, $(Q, p) = (30.31, 52.3)$. For the last period with the lowest demand, the quantity price pair is $(Q, p) = (24.3, 39)$. In total, the amount of energy produced during the load duration period k under this strategy is approximately 88 GWh.

Now, we let each of the producers have k times 44 GWh available for production during all k load duration periods, $\overline{W}_i = k(44 \text{ GWh})$. Furthermore, we assume all load duration periods to be identical with respect to demand. Then if the two firms choose to submit Cournot supply schedules, the existing energy capacity and corresponding water value will not constrain their bids.

However, more competitive supply schedules would violate the energy capacity constraint. Assuming a fixed alternative value of water ($\lambda = 19$) and that the energy constraint is just binding in the case of two symmetric hydro power producers submitting Cournot supply schedules, then any pair of symmetric supply schedules below the Cournot schedules (more competitive) would violate the energy constraint and would thus not be feasible.

More competitive supply schedules would imply that the realized price corresponding to realized demand would lie below the price resulting from Cournot supply schedules. Since demand is downward sloping this implies that quantity is higher for all

realizations of demand. Thus, with more power produced throughout the load duration period, more energy is produced as well and the energy constraint is violated, $\sum_k \int_{t=0}^1 q_{ikt} dt > \overline{W}_i$.

3.3 Introducing a competitive supply schedule

With a fixed alternative value of water, we saw in the previous subsection that any supply schedule based on a more competitive solution than the Cournot equilibrium at the period of highest demand would violate the energy constraint. This does not mean that a more competitive strategy is ruled out, just that the producer's water value is too low. With reference to the numerical example in the previous subsection, if a more competitive strategy is followed, then there would not be enough water for production in all k load duration periods. The combination of lower prices and higher supply compared to the Cournot supply schedule would leave water for production in only some of the k periods in question. Thus, when firms bid more competitive supply schedules, they would take this into account and place a higher alternative value on the use of water resources.

We use the same numerical example as in the previous subsection and compare the Cournot supply schedule to the most competitive schedule possible where price equals marginal cost. This is shown in Figure 3.

As can be seen from Figure 3, the water value or marginal cost in the case of a competitive supply schedule would have to be as high as 55.5 if the energy constraint is not to be violated. This implies that prices are higher and supply is lower at periods of low demand while prices are lower and supply is higher at periods of high demand, compared to the Cournot supply schedule.

Recognising that all possible SFE are intermediate between the Cournot supply schedule and a supply schedule corresponding to Bertrand competition at the boundary solution, a binding energy constraint will reduce the number of possible SFE.

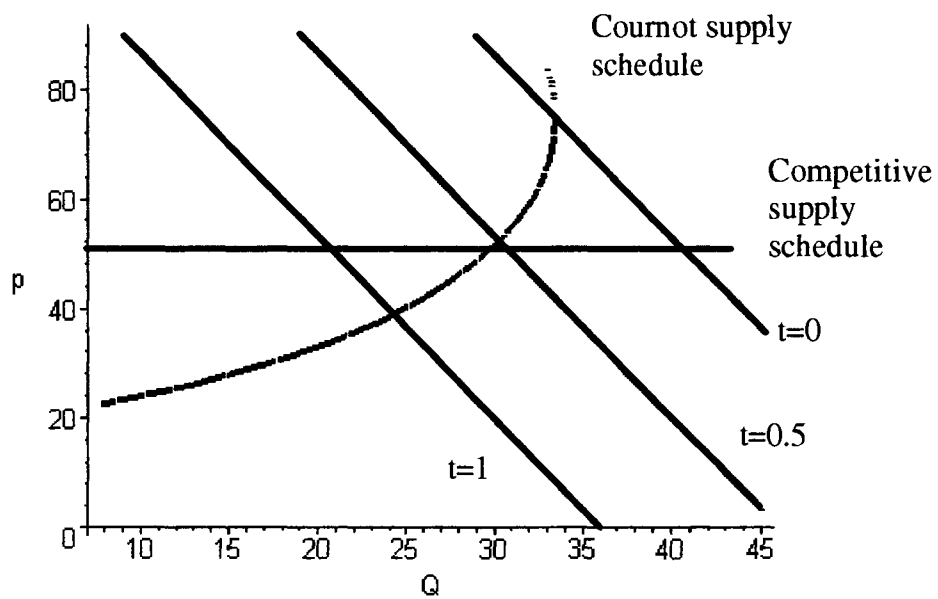The numerical example above illustrates that the welfare effect of market power

Figure 3: The Cournot supply schedule and the competitive supply schedule.

in an electricity market might be exaggerated if the energy constraint is not properly taken into account.[8] The intuition behind this result may be illustrated simply by comparing the competitive outcome to the monopoly outcome in a situation where firms face no uncertainty with regard to demand and firms maximises profit with regard to quantities produced in each period t within one load duration period k. We assume that the energy constraint is binding in both cases. In the competitive case production is allocated between the periods t such that the price in each period is identical. In the monopoly case production is allocated such that marginal revenue in each period t is identical. If we compare the two situations, we have the standard result that the monopoly producer reduces production in periods with low price elasticity and increases production in periods with high price elasticity. In the case of linear demand as in the example above, the monopoly producer will reduce production in periods with high willingness to pay (a is high) and incease production in periods with a low willingness to pay if the marginal revenue from production should be equal in each period.

4   Concluding remarks

The idea that firms compete in supply functions and the SFE model developed by Klemperer and Meyer (1989) fits closely to how firms compete in electricity markets. Firms face uncertainty with regard to demand at the time of trade, but are nevertheless required to deliver binding supply schedules before trading actually takes place.

Thus, in spite of the technical difficulties associated with the SFE framework,

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[8]The SFE model was used in a report by the Nordic competition authorities (2003) to analyse the effects of mergers and acquisitions in the Nordic electricity market. The simulations presented in this report fail to take into account the fact that hydro power producers allocate water between different periods subject to an energy constraint. Thus, the results may exaggerate the effects of market power on welfare.

several authors have tried to use this model as a basis for analyzing strategic behavior in electricity markets. In several papers R. J. Green and D. M. Newbery have used the SFE model in order to analyze electricity market competition in England and Wales. The success of their application of the SFE model relates to a large extent to the market structure of the England and Wales electricity market. Following deregulation, this market consisted for a significant period of time of only two rather symmetric firms. In this setting the SFE model is relatively easy to apply.

One central problem associated with the SFE model is that the model only predicts a range of possible equilibria depending on the boundary solution. Following the argumentation of Klemperer and Meyer (1989) and Green and Newbery (1992) we define an upper boundary where Cournot quantities and prices are realized at the period of highest demand. The lower bound corresponds to Bertrand prices and quantities at the same period. This leaves a rather wide range of possible SFE equilibria.

However, as shown by Newbery (1998) the number of possible SFE equilibria is reduced when we take into account that firms face production constraints. Constraints on energy produced have not been considered, as this is a special feature of hydropower production.

In this paper we have, by the use of a simple numerical example, illustrated that constraints on energy produced may contribute to a further reduction in the number of possible SFE equilibria. In particular, we looked at a case where the energy constraint was just binding when the SFE equilibrium was characterized by Cournot quantities and price at the upper boundary. Furthermore, we illustrated that a more competitive schedule would imply higher water values with more energy produced during periods of high demand and less during periods of low demand.

The implication of this result is that any attempt to model competition by the use of the SFE model in markets where hydropower plays a significant role should take into account the effect of energy constraints. If not, the welfare effect of market power might be exaggerated.

A possible extension of the analysis provided through a simple numerical example

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in this paper would be to model Supply Function Equilibria with energy constraints more formally. However, the problem of including energy constraints adds to the existing methodical problems related to the use of SFE models of competition. Thus, as a consequence, other models of competition might be preferred in analyses of competition in electricity markets with significant hydropower production.

A   Solving the differential equation

Here we derive the first order conditions for a SFE, first in the case with two symmetric firms and second in the case with n symmetric firms in the market. We start with the case of two symmetric firms. Firm i's (the same for firm j) profit is

$$\pi_i(p) = p[Q(p,t) - q_j(p)] - C\{Q(p,t) - q_j(p)\},$$

where $i \neq j$ and we assume that there are constant marginal costs in production (c). The residual demand facing producer i is

$$q_i = Q(p,t) - q_j(p).$$

The first order condition is

$$\frac{\partial \pi_i}{\partial p} = q_i + [p - c][\partial Q/\partial p - \frac{\partial q_j}{\partial p}].$$

Knowing that $q_i = q_j$ we only need to solve one equation,

$$\frac{dq}{dp} - \frac{q}{p - c} + Q_p = 0.$$

This is a differential equation. The equation may be written in the following form:

$$\frac{dq(p)}{dp} - u(p)q = w(p).$$

where $u(p)$ is the coefficient and $w(p)$ is the term of the differential equation. The equation may be described as a first order linear differential equation with variable coefficient,

$$u(p) = \frac{1}{p - c}$$

and constant term where we assume that $Q_{pt} = 0$,

$$w(p) = \frac{dQ}{dp}.$$

In order to solve this equation we need first to find out if the equation is exact, we form a new equation

$$dq + dp(uq - w) = 0.$$

If we let $M = df/dq$ and $N = df/dp$ we can see that the equation is not exact. That is

$$\frac{\partial M}{\partial p} \neq \frac{\partial N}{\partial q}.$$

Thus, we need to multiply the equation with a factor that makes the equation exact. The factor is

$$e^{\int u\, dp},$$

resulting in

$$e^{\int u\, dp}dq + e^{\int u\, dp}(uq - w)dp = 0. \tag{23}$$

Now the equation is exact

$$\frac{\partial M}{\partial p} = ue^{\int u\, dp}, \quad \frac{\partial N}{\partial q} = ue^{\int u\, dp}.$$

In order to solve the equation (24) we integrate the first part and add on a second element representing the other part.

$$F(q,p) = \int e^{\int u\, dp}dq + \psi(p).$$

(by $\int k\, dx = k(x + d)$ where we omit the constant of integration d). We can rewrite this as

$$F(q,p) = qe^{\int u\, dp} + \psi(p). \tag{24}$$

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In order to find the value of the second term, we differentiate the function above (25) with respect to the variable p

$$\frac{\partial F}{\partial p} = uqe^{\int u\,dp} + \psi'(p)$$

Since we know the value of $N = e^{\int u\,dp}(uq - w)$ and that this equals the partial differentiate of F with respect to p, we can write the equation as follows:

$$e^{\int u\,dp}(uq - w) = uqe^{\int u\,dp} + \psi'(p).$$

and we can find $\psi'(p)$

$$\psi'(p) = -we^{\int u\,dp}.$$

Now the solution to $F(q,p) = d$ becomes

$$uqe^{\int u\,dp} - \int we^{\int u\,dp}dp = A.$$

And if we solve for q we get (here $d = A$)

$$q(p) = e^{-\int u\,dp}\left[A + \int we^{\int u\,dp}dp\right]. \tag{25}$$

This is the general solution to a first order differential equation of the first degree.

Next, we look at how to reach the general solution in the case of n symmetric firms. Each of the n firms faces the same problem

$$\pi_i(p) = p[Q(p,t) - \sum_j q_j(p)] - C\{Q(p,t) - \sum_j q_j(p)\},$$

where $i \neq j$ and as before we assume that there are constant marginal costs in production (c). The residual demand facing producer i is:

$$q_i = Q(p,t) - \sum_j q_j(p)$$

Knowing that the firms are symmetric we have that $\sum_j \frac{\partial q_j}{\partial p} = (n-1)\frac{\partial q_j}{\partial p}$. The first order condition then becomes

$$\frac{\partial \pi_i}{\partial p} = q_i + [p - c][\partial q/\partial p - (n-1)\frac{\partial q_j}{\partial p}].$$

Rearranging for the symmetric case, we have that

$$\frac{\partial q}{\partial p} = (\frac{1}{n-1})(\frac{q}{p-c} + Q_p).$$

We see that this is a differential equation similar to the one discussed above for the symmetric two-firm case

$$\frac{dq(p)}{dp} - u(p)q = w(p),$$

where the coefficient $u(p)$ and the term $w(p)$ in this case is

$$u(p) = -\frac{1}{(p-c)(n-1)}, \text{ and } w(p) = D_p\frac{1}{n-1}.$$

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