Essays on Financial Integration and International Risk Sharing

by

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Acknowledgements

Contents

Chapter 1*

Introduction

1. Background

Since the early 1970s, international trade in financial assets has grown at an explosive pace. The most important forces behind this development are the widespread trend of deregulation of domestic financial markets and international capital flows, advances in information technology, and the creation of new financial products. As a result, we have today a degree of international capital mobility not seen since the beginning of the twentieth century.[1]

According to standard economic theory, the potential benefits of international financial trade are clear. First, it allows residents from different nations to pool country-specific shocks, thereby providing better scope for insurance than a purely domestic capital market would. Second, a country can borrow or lend abroad when facing temporary economic shocks. Third, an international capital market directs savings to the world's most productive investment opportunities. More generally, trade in international assets should improve welfare because it allows for smoother consumption paths and stimulates economic growth.

In practice, the advantages of free capital mobility are much more disputed than the standard theory suggests. Recent financial and economic crises (for example, in Asia

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[1] For documentation, see, for example, Bordo, Eichengreen and Kim (1998), Bordo, Eichengreen and Irwin (1999), and Obstfeld and Taylor (1998).

during 1997–98) have been explained partly in terms of destabilizing disturbances occurring in international asset markets, leading some economists to call for restraints on capital flows (e.g., Bhagwati, 1998; Krugman, 1998). Moreover, full integration into international capital markets restricts national governments' ability to pursue legitimate economic and social objectives. For instance, full capital mobility limits a nation's ability to use monetary policy and fiscal policy simultaneously for stabilization purposes, and it curtails the taxation of internationally mobile capital relative to more immobile factors such as labor.[2]

International capital mobility can thus have potentially important *costs* by making economies more prone to crisis and by reducing policy autonomy. To evaluate the desirability of free capital movements, one needs to know more about the potential *benefits* than is suggested by the general theoretical arguments expressed above. In particular, I believe the following two questions are highly relevant to economic policy.

1. How *large* are the potential benefits from cross-border asset trade?

2. To what extent are the potential benefits *realized* in practice?

In this thesis, I offer four essays, all of which are intended to contribute to our understanding of these issues.

In chapters 2 and 3, I examine the growth and welfare effects of international asset trade, using stochastic simple endogenous growth models. In chapter 4, which is co-authored with Øystein Thøgersen, we evaluate the degree of consumption risk-sharing among the Nordic countries. This chapter may be viewed as an assessment of whether the residents of these countries have traded among themselves their nation-specific consumption risks. In chapter 5, I analyze the international portfolio decisions of a hypothetical investor who has more information on the properties of domestic projects than of foreign projects. This chapter is an attempt to shed light on the implications of such information asymmetries, which may, in reality, be important for net asset positions across countries.

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[2] See Obstfeld (1998) for an assessment of these mechanisms.

2. Mini-survey of the Literature[3]

2.1 Research on the Potential Gains from International Asset Trade

Research into the welfare effects of cross-border asset trade can be divided into two main categories. One approach estimates gains from smoother consumption paths resulting from international diversification, assuming that output is exogenous. The state of this research agenda is nicely summarized and reviewed in van Wincoop (1999). The estimated gains from smoother consumption paths resulting from international risk-sharing vary from less than 0.5% of a permanent increase in (tradables) consumption (e.g., Backus *et al.*, 1992; Cole and Obstfeld, 1991) to 29% by the same measure (Shiller and Athanasoulis, 1995). However, van Wincoop (1999) points out that this difference stems mainly from different parameterization. He narrows down the likely values of the key parameters and finds that for OECD countries, the gain at a 50-year horizon is in the range of 1.1% to 3.5 %. The overall impression given by this line of research is that there are gains from smoother consumption paths, but they are not very large.

The second approach acknowledges that international asset trade may also affect long-term growth rates. Given that increased ability to diversify risk affects the optimal allocation of resources (such as portfolio allocation and saving rates), it may also have growth effects in certain types of models.[4] Chapters 2 and 3 of this thesis are contributions to this branch of research. Next, we give a brief summary of related research.

Devereux and Smith (1994) use a stochastic learning-by-doing growth model *à la* Romer (1986) to examine the growth and welfare effects of reduced technological uncertainty, which are presumably due to pooling opportunities in international financial markets. They show that reduced uncertainty can weaken the precautionary motive for saving, thereby lowering the growth rate. As the growth rate in these models is sub-optimal in the first place, this also implies that welfare may fall as a consequence of financial integration. This result is turned around in a model by Femminis (1999), where the accumulation of human capital is the result not of an externality, but, as in Lucas

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[3] For comprehensive surveys of research on the gains from international financial trade, see Obstfeld and Rogoff (1996), Lewis (1999), and van Wincoop (1999).

[4] Related research can also be found in the financial development literature; see Levine (1997) for a survey.

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(1988), of optimizing behavior on the part of agents in the economy. In this case, reduced technological uncertainty drives resources towards the 'growth-leading' educational sector, yielding an increased growth rate and higher welfare.

While these analyses are clearly relevant to evaluating the growth effects from financial integration, they are single-country models where reduced uncertainty is simply postulated. Obstfeld (1994a) develops a multi-country model with one low-return risk-free, and one high-return risky, linear technology in each country. International asset trade allows agents to lay off nation-specific risks in global markets, which provides incentives to increase risk-taking. In Obstfeld's complete markets model, this leads to huge gains in terms of both growth and welfare. Dumas and Uppal (1999) introduce goods-markets frictions in the same type of model. This reduces the gains from financial integration, but not very much.[5]

The results from this line of research suggest that there may be substantial positive growth effects and associated welfare gains from international asset trade.

2.2 Research on the Realization of Potential Gains

Assessments of whether the potential gains from asset trade are realized hinge on the measurement of the actual international mobility of capital. If capital markets, even without formal or legal barriers on capital mobility, appear disintegrated, the conditions for realizing the potential gains are not present. Several different approaches have been adopted to measure the degree of capital market integration. Three of these approaches are described below.

As already stated, a well-functioning international capital market should allocate investment to its most productive uses in the world. An indirect way of evaluating whether this occurs is to measure the association between national economies' saving and investment rates. This approach was adopted by Feldstein and Horioka (1980), who reasoned that, with highly mobile capital, a country's savings are free to flow to their most productive uses anywhere in the world. With integrated capital markets, there is no reason why changes in national savings should affect the national capital stock. Feldstein and Horioka showed that, across OECD countries, long-term averages of national saving

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[5] Devereux and Saito (1997) study the effects of asset trade that is restricted in the sense that agents can trade a non-contingent bond only. In this case, some countries may experience both higher growth and higher welfare under complete financial autarky.

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rates were highly correlated with similar measures of domestic investment rates. This is the 'Feldstein–Horioka puzzle': with open capital markets, why is there such a close relationship between domestic savings and investments? Although later studies indicate that the relationship has become looser since Feldstein and Horioka's article first appeared, and although numerous reasonable hypotheses have been proposed that may help explain the puzzle (see Obstfeld, 1995), the general opinion among economists seems to be that the puzzle remains. (See Obstfeld and Rogoff, 2000, for a recent assessment.)

Another line of research has taken a more direct approach in studying the world capital market's achievement in allocating consumption risk across nations. When residents from different nations engage in asset trade, they can diversify consumption risks that are systematic at a purely domestic level but idiosyncratic in an international setting. This implies that consumption growth should be closely synchronized across countries if there is extensive asset trade. In fact, under a set of restrictive but standard assumptions, the cross-country consumption growth correlations should be 1 (e.g., see Tesar, 1995). Much recent research is aimed at testing and relaxing the standard assumptions; chapter 4 of this thesis is one such contribution. A survey of this literature is given in Lewis (1999).

This research, including chapter 4 in this thesis, strongly suggests that cross-border consumption synchronization has increased over time, but still seems to be lower than would be expected with full financial integration. In turn, this suggests either that there are still unexploited gains to be made from consumption risk sharing, or that these gains are hard to realize in practice.

A third approach to analyzing the extent of risk sharing among countries is to directly examine international portfolio positions. All the studies of which I am aware conclude that there is a substantial 'home bias' in the portfolios of industrial-country investors (e.g., French and Poterba, 1991; Cooper and Kaplanis, 1994; Tesar and Werner, 1995), although the bias has lessened somewhat in recent years (Tesar and Werner, 1998). Again, see Lewis (1999) for a recent survey.

As for the low consumption growth correlations, this suggests that agents are not taking full advantage of the mutual risk-allocation gains that an international capital market facilitates. Why is this? A number of possible explanations of the home-bias puzzle have been put forward, of which the most common are non-traded income risk,

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non-traded consumption goods, asymmetric information across countries, and statistical measurement problems. Yet none of them seems entirely convincing, and so the puzzle remains (Lewis, 1999). In fact, some possible explanations may deepen the puzzle rather than explain it. Baxter and Jermann (1997), for instance, claim that introducing non-traded labor income risk in an international portfolio-allocation model implies, theoretically, that inhabitants from Germany, Japan, the UK, and the US should go short in their national stock markets and invest the proceeds abroad.[6] By the same token, the model that I develop in chapter 5 suggests that asymmetric information across countries need not imply that optimal portfolios are tilted towards the home economy. The information structure studied in that chapter may instead imply a 'foreign bias' relative to a situation with symmetrically distributed information.

The overall picture that emerges from this research is that, despite large gross cross-border asset flows, net asset positions across countries remain surprisingly small. Existing asset trade has gone only a small way towards realizing the potential gains from international risk sharing.

3. An Overview of the Thesis

In this section I briefly summarize and comment on the remaining chapters of this thesis.

3.1 Habit Persistence and Growth Effects from International Asset Trade

International financial markets facilitate the diversifying and pooling of nation-specific risks. Increased ability to diversify affects the optimal allocation of resources, and may therefore also influence the long-term growth rate of an economy. In chapter 2, I discuss the link between international asset trade, growth, and welfare, when agents exhibit habit formation in consumption. Following Obstfeld (1994a), individuals in each economy have access to a risk-free technology and a risky technology for the production of a single consumption good. The return on the investment in the risk-free alternative is assumed to be the same in all countries, while innovations to the risky technology are

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[6] However, Botazzi et al. (1996), who study the same issue with a different model, come to different conclusions. See also chapter 3 of this thesis.

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imperfectly correlated across countries. I derive optimal rules for consumption, savings, and portfolio allocation, as well as the growth rates of the economies under financial autarky and integrated financial markets.

Habit formation leads to lower welfare gains from financial integration than under the standard case of time-separable preferences. This is because of the common (across countries) risk-free technology and because habit formation implies higher risk-aversion than time-separable preferences. With high risk aversion, households would not be willing to reallocate much of their investments to the high-return risky technology as the nations integrate. Hence, households with habit formation would not experience the same growth stimulant as if they had time-separable preferences, and would gain less in terms of welfare. I also show that opening up to international asset trade increases the mean consumption growth rate *and* the volatility of the growth rate. This is so for both habit-forming and time-separable preferences. Moreover, the result of lower welfare gains with habit formation can be viewed as a corollary of this: highly risk-averse households perceive the increased variability as a large cost of obtaining higher average growth.

A preference specification with habit formation in consumption means that, as opposed to standard time-separable preferences, the well-being of economic agents is a function of how large their current consumption is relative to their past consumption level. Besides the intuitive appeal of habit formation, and the support for such preferences in the psychological and sociological literature, I have three reasons for including it in an analysis of asset trade and growth. First, the stochastic growth model used in chapter 2, which is closely related to Obstfeld (1994a),[7] is very similar in structure to the portfolio-selection and asset-pricing models from the finance literature. These latter models have recently been extended to encompass habit formation in consumption, since such preferences have been relatively successful in explaining different asset-market puzzles (see, for example, Constantinides, 1990; Campbell and Cochrane, 1999). Second, finance research has taught us that habit formation generates endogenous, time-varying attitudes towards risk. Thus, if habit formation is a good description of household behavior, asset trade can affect the equilibrium path of an economy both through changes in the available technology *and* through induced changes in agents' attitudes towards risk. Finally, and

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[7] It is also similar to the models used by Devereux and Saito (1997) and Dumas and Uppal (1999) in their analyses of financial integration and growth.

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relatedly, the high risk aversion in these models implies that consumers dislike fluctuations more than they do with standard preferences. The analysis can therefore also shed some additional light on the findings, in the research reviewed above, of small welfare gains from pure consumption smoothing.

The numerical simulations that I present in the paper show that the welfare gains from asset trade are about 30% of the implied gains under time-separable preferences. However, since these gains are very large (Obstfeld, 1994a), the habit-formation model also implies considerable advantages from financial integration. Moreover, the assumption of constant-returns technologies can bias the estimated gains downwards. This assumption implies that the expected risky-asset returns are constant, so the time-varying risk aversion in our model is transferred solely into time-varying portfolio shares. If asset returns were endogenous, expected returns would be higher in 'bad times' (when risk aversion in the model is high), since highly risk-averse agents demand higher expected returns to incur the costs of a risky investment. Accordingly, risky assets would not be as unattractive with high risk aversion as our model suggests. Endogenous asset returns could therefore lead to larger reallocation towards risky assets upon financial integration, and possibly also larger growth and welfare gains.

3.2 International Diversification, Growth, and Welfare with Non-traded Income Risk and Incomplete Markets

In chapter 3, I examine how the potential growth and welfare effects from financial integration are affected by the presence of non-traded income risk when markets are incomplete. There are two economies, with production side as in chapter 2, and preferences modeled in the standard time-separable way. To this standard setup I add an exogenous stochastic income component in each country. This component represents income from an asset that cannot be traded in financial markets. The non-traded income process may be imperfectly correlated with the returns of the risky assets in the two nations—in which case we have incomplete markets. That is, the non-traded income risk cannot be hedged in financial markets.

As opposed to the model of chapter 2 and the models by Obstfeld (1994a) and Dumas and Uppal (1999), this situation may imply that the average growth rate is lower when the two countries can trade their marketable assets than under financial autarky. There are two responses to asset trade opportunity that can create this: the hedging

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demand for the risky, high-return technology may be lower with asset trade, and precautionary savings can decrease as more of the non-traded income risk can be hedged under financial integration. However, welfare is higher under integration in this model, but the welfare gain may be very small. A key parameter for the size of the welfare gain is the correlation between the domestic non-traded income process and the returns on foreign assets. The higher this correlation is, the smaller the gains are. This is because the risk-return benefits from diversifying into foreign assets are counteracted by reduced hedging ability of the risky-asset portfolio when the correlation is high.

Non-traded assets make up a very large proportion of national wealth in most economies. By far the most important such asset is human capital, but a claim on the social security system is also an example of a large non-traded asset for inhabitants in developed economies. Since the existence of such assets has important effects on decisions concerning savings and portfolios, it seems important to explore the consequences for the effects on growth and welfare of financial integration, especially under incomplete markets.

A shortcoming of the model in chapter 3 is that it postulates an exogenous non-traded income process. It clearly would be more satisfactory to have a model where, for example, labor income was the result of the behavioral decisions of households. An extension of the model to encompass this would come at the cost of tractability, as it would be difficult to derive both the rules relating to consumption and portfolios on the one hand, and on the other hand the growth rates of the economies in an incomplete market setting.[8] However, models that can be solved by numerical methods would, of course, provide additional insight into the problem studied in chapter 3.

The result, that the correlation between shocks to non-traded income and foreign asset returns is decisive for the size of the gains from asset trade, warrants empirical research. Current estimates of the hedging ability of foreign assets for domestic labor income vary considerably. Baxter and Jermann (1997) conclude that labor income makes the case for international diversification even stronger, while Bottazzi et al. (1996) find that foreign assets generally are less attractive than domestic assets for hedging labor—

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[8] Svensson and Werner (1993) derive closed-form allocation rules in incomplete markets, but only under exponential utility specification. Such preferences do not allow for analytical solutions of the growth rates.

income uncertainty. However, as these papers use different data and different methods, additional estimates are needed.

3.3 Financial Integration and Consumption Co-Movements in the Nordic Countries

With extensive international asset trade, consumption growth across countries should be closely synchronized because consumption should respond primarily to global shocks. A standard model of international risk sharing (e.g., Tesar, 1995) implies that cross-country consumption growth correlations should be 1. In chapter 4, Øystein Thøgersen and I use a model setup developed by Obstfeld (1994b) and Bayoumi and MacDonald (1995) to test for aggregate risk sharing among Denmark, Finland, Norway, and Sweden.

The consumption growth correlations between these four countries over the period 1973–92 are much lower than predicted by the benchmark model. We try to rationalize this finding along two dimensions. First, we search for external shocks that may be uninsured and affect the Nordic nations in different ways. We have little success in identifying such shocks. Second, we consider the possibility that parts of the population in different countries may face liquidity constraints. Following Campbell and Mankiw (1991) and Bayoumi and MacDonald (1995), we derive a model specification that discriminates between effects of domestic credit-market imperfections and lack of financial integration. We find (weak) evidence of myopic consumption behavior for Denmark, Finland, and Sweden, and some indications of Nordic financial integration, taking this into account.

There are two main reasons we think an analysis of financial integration in the Nordic countries may be of general interest. First, a common culture and low language barriers imply closer relations between these countries than most other nations. The political climate is also very stable compared with many other regions. It thus seems fair to expect that the suggested barriers to asset trade in the form of non-familiarities with foreign markets, institutions, and firms (French and Poterba, 1991) are small within the Nordic region. Second, the small open economies in this region are strongly exposed to external shocks, which should give them clear incentives to engage in international diversification.

Nevertheless, our results suggest that the degree of financial integration among these nations was relatively limited over the 1973–92 period, and not significantly higher

than the level that has been found for other groups of countries. We also provide some results for the 1951–72 period. These estimates show much clearer evidence of myopic consumption behavior, and no indication of financial integration. Thus, although we find only limited evidence of integration in the Nordic region for our latter period, it certainly seems to be higher than for the period up to 1972.

3.4 On Asymmetric Information across Countries and the Home-Bias Puzzle

The last chapter of the thesis explores the consequences for portfolio allocation of asymmetric information across countries. We analyze the decision of a hypothetical investor who can allocate his resources to one domestic project and one foreign project. For each project, there is a manager who governs the expected return. The investor can observe the actions of the domestic manager, while those of the foreign manager are unobservable.

Compared with a situation with full and symmetric information, this setup implies that the investor's optimal allocation policy will be tilted. One reason is that the domestic manager will make higher effort per dollar invested in the domestic project than the effort made by the foreign manager for each dollar invested abroad. Another reason is that the expected value and variability of the investor's salary costs will be affected by his allocation decision when there is asymmetric information. While the first reason for tilting the portfolio unambiguously induces a bias towards the domestic project, the interaction between portfolio allocation and salaries implies an ambiguous effect on the allocation decision. Theoretically, the effect on salaries *may* more than counteract the effect on effort levels. Hence, in the model of chapter 5, asymmetric information can induce both a home-bias or a 'foreign-bias'.

It has been suggested by several researchers that investors are better informed about the characteristics and prospects of their domestic economy, and therefore rationally tilt their portfolios to it (Brennan and Cao, 1997; Gehrig, 1993; Gordon and Bovenberg, 1996). Hence, it has been argued that asymmetric information across countries provides a potential rationale for the home-bias puzzle. My model differs from the earlier research in that optimal contracts are at center-stage in the analysis, and that the investor, through these contracts, can influence the expected returns on his assets. In this case, which certainly could be realistic for large investors, the risk–return tradeoff

implies an ambiguous link between asymmetric information and the direction of the allocation distortion.

The model in chapter 5 is far too simplistic, however, to enable us to draw any firm conclusions on the effects of cross-national moral hazard. A particularly interesting extension, I think, would be to consider a two-country model with one investor in each nation owning a divisible project. What would be the resulting equilibrium prices and portfolio allocation when the two investors meet to trade, given that they know less about the actions of foreign managers? Such a model could also tell us something about the effects of foreign ownership, as it would be possible to compare expected output and managerial compensation of foreign and domestically owned projects.

4. Some Concluding Remarks

The picture that emerges from the research into financial integration, including this thesis, is that international asset trade can potentially yield substantial benefits, but that the existing trade pattern has gone a relatively small way towards realizing these gains.

A Norwegian might be a little puzzled by this conclusion. To see why, consider figure 1.1. It shows Norway's historical and projected current account balance as a percentage of GDP for the period 1970–2001. The current account is the difference between domestic saving and domestic investment. Hence, whenever the current account is negative, the nation borrows abroad to finance its domestic investments. Conversely, a positive current account means that some of the nation's savings are flowing into foreign assets.

Figure 1.1 illustrates that the Norwegian current account balance has varied widely during the last 30 years. In the mid–1970s the nation borrowed up to 14% of GDP to finance domestic investments, not least in offshore petroleum extraction.[9] Without access to the international capital market, these investments would have required a tremendous increase in the national saving rate with a corresponding depressing effect on

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[9] Obstfeld and Rogoff (1996) use Norway's international borrowing during the 1970s as an example of large current account imbalances stemming from intertemporal considerations.

domestic consumption. Investments in other sectors of the economy would also have suffered, since the required increase in savings would have pushed up interest rates.
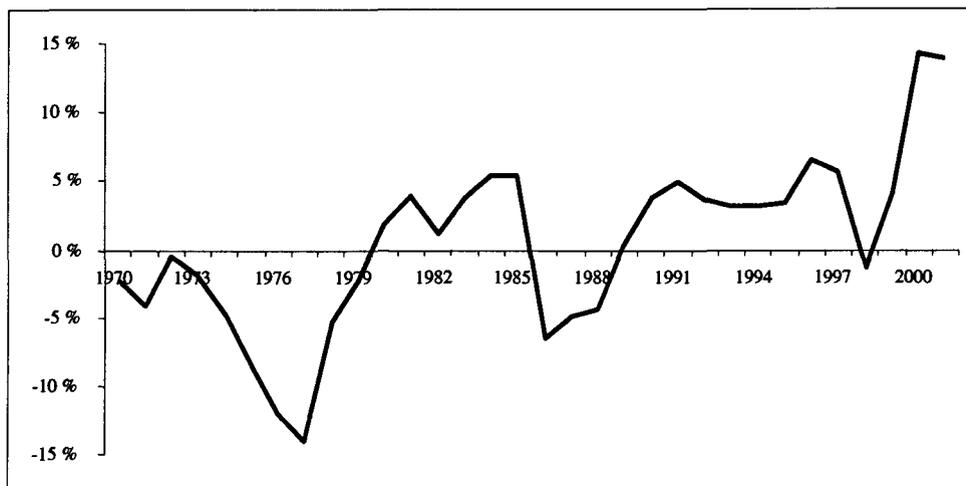
Figure 1.1: Norway's current account balance as a percentage of GDP. (*Source*: OECD)

At the other extreme, we see from figure 1.1 that the current account is expected to reach a surplus of 14% of GDP in 2000. This reflects the fact that oil production is approaching its peak, giving much higher disposable income than is currently absorbed domestically. In the management of this income in excess of domestic absorption, the international capital market is again extensively used. The surplus reflects mainly the accumulation of the public 'petroleum fund', which is invested solely in foreign financial assets. This strategy offers several advantages. First, it is unlikely that resources of this magnitude could have been invested effectively at home, given decreasing marginal productivity of (physical and human) capital. Second, the fund provides much-needed diversification of the Norwegian national wealth. Third, consuming substantially more of the oil revenues than we currently do could have very serious effects on the onshore Norwegian economy and would seem 'unfair' in an intergenerational perspective.

The Norwegian experience exemplifies the fact that, although net asset positions across countries are smaller than one would expect with full financial integration, international capital movements have provided, and do provide, substantial benefits. The challenge for policymakers is to encourage the realization of more of these benefits and simultaneously to develop institutions that are better capable of handling the destabilizing disturbances that occur from time to time in international financial markets.

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Chapter 2[*]

Habit Persistence and Growth Effects
from International Asset Trade

Abstract

We introduce habit formation in a model that studies the link between international trade in financial assets and economic growth. As with time separable preferences asset trade stimulates growth, but it also increases growth-volatility. We demonstrate that the welfare gain from asset trade is lower with habit persistence in consumption. This reflects that the habit-forming households perceive the higher growth-volatility as a higher cost to obtain increased average growth. The gains with time separable preferences are shown to be an upper bound on the gains with habit formation.

1. Introduction

An important function of international financial markets is to facilitate diversifying and pooling of nation-specific risks. This allows agents from different countries to obtain smoother consumption paths, holding expected growth rates fixed. In addition, the ability to diversify risk affects the optimal, temporal and intertemporal, allocation of resources (e.g., portfolio allocation and saving rates) which, in turn, can influence economic growth.[1] Thus, trade in financial assets may have important macroeconomic effects because it alters both the growth and volatility of national consumption paths.

The purpose of this paper is to study how the international financial system's ability to pool country-specific technological risk affects growth and welfare, when agents exhibit habit persistence in consumption. The motivation for introducing habit formation in an analysis of international risk sharing is threefold.

First, models incorporating habit persistence have been relatively successful in resolving the equity-premium/ risk-free rate puzzles of Mehra and Prescott (1985) and P. Weil (1989).[2] Since Campbell (1999) has documented the existence of these puzzles for several OECD-countries, it seems relevant to take them into account also in international settings. Still, existing models evaluating the link between international asset trade and economic growth - which are very similar in structure to asset-pricing/portfolio-selection models - do not attempt to include habit formation (Devereux and Smith, 1994; Obstfeld, 1994; Devereux and Saito, 1997; Dumas and Uppal, 1999).

Second, one of the main messages from the asset-pricing literature is that habit persistence generates endogenous, time-varying attitudes towards risk (e.g., Constantinides, 1990; Campbell and Cochrane, 1999). One of the determinants of risk-aversion in these models is the investment opportunity set that the agents face. Of course, this set will be altered by the opportunity to diversify risk internationally. Thus, with

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[1] See Levine (1997) for a recent survey on the relationship between financial development and economic growth. See also Acemoglu and Zilibotti (1997) on the link between the ability to diversify, risk-taking, and growth.

[2] Among others, Abel (1990), Constantinides (1990), Detemple and Zapatero (1991) and Campbell and Cochrane (1999) demonstrate the potential for explaining the puzzles within a habit formation model. For a different view, see Kocherlakota (1996).

habit formation in consumption, trade in financial assets can influence economic growth both through direct changes in the technology available and through induced changes in household behavior. A model with intertemporal dependent preferences could accordingly deepen our understanding of the link between the trade in financial assets and economic growth.

Finally, habit formation models capture that consumers dislike consumption fluctuations to a very large degree (Campbell and Cochrane, 1999). Consequently, it seems important to explore whether habit formation can alter the findings of small welfare gains from consumption smoothing (for a given mean growth rate) due to international diversification (e.g. Tesar, 1995; van Wincoop, 1999).

This paper considers a model where production takes place through linear technologies in which capital is the only factor of production. Since this implies constant returns to scale in the input, the resulting equilibrium is characterized by ongoing endogenously determined growth. As in Obstfeld (1994), the set of technologies consists of one risk-free and one risky type in each country. The risky technology has a higher expected return than the risk-free. Thus, the equilibrium growth rate depends on both total savings and on the allocation of investment between the technologies.

Our main finding is that habit formation, compared to the case of time separable utility, leads to lower welfare gains from financial integration. This result is due to a combination of high, time-varying risk-aversion and the presence of a common (across countries) risk-free technology: Risk-aversion is a function of the difference between current and past consumption (the habit level). When this difference approaches 0, risk-aversion goes to infinity. Then, households won't tolerate any fluctuations and will invest in the risk-free technology only. Since this technology is common across countries, there will not be any reallocation of resources upon financial integration, and welfare will be unchanged. At the other extreme, risk-aversion goes to its lower bound when the difference between current consumption and the habit level is very large. It turns out that this lower bound is equal to the (constant) level of risk-aversion with time separable utility. In this case, the optimal response to the asset-trade possibility is equal with or without habit formation, as is the welfare gain. In intermediate cases, the welfare gain from financial integration is positive, but smaller than with time separable utility.

We also show that opening up to international asset trade increases the expected consumption growth rate, but it also gives higher growth volatility. This is true for both

habit forming and time separable preferences and our main result above can be viewed as a corollary of this. Habit-forming households perceive the increased growth-variability as a higher cost to obtain higher average growth. Finally, we confirm that a setup with habit-persistence is better capable of explaining the large equity premium/ low risk-free interest rates observed in international data.

In addition to the asset market literature mentioned above, models using some sort of habit formation have been used quite extensively in consumption studies. Empirical work indicates that habit persistence may be necessary to explain various time-series features of consumption data (e.g., Deaton, 1992; Carroll and D. Weil, 1994). Use of such preferences in growth models has been relatively scarce though, with Ryder and Heal (1973) and Carroll et al. (1997) as notable exceptions.[3] Those papers discuss deterministic growth models in closed economy settings.

The rest of this paper is organized as follows: Section 2 explores the growth and welfare properties of a simple closed-economy, habit formation model. Section 3 extends the model to a symmetric multi-country world with free asset trade. The impact of asset trade on consumption growth, growth-volatility and welfare in the habit persistence model are compared to the time separable case. In section 4 we calibrate the model using stock-market data for Germany, Japan, the UK and the US. Section 5 provides a discussion of the findings and some possible extensions.

2. Individual Choice and Equilibrium in a Closed Economy

2.1 Model Structure and the Optimal Consumption and Asset Demand Functions

We start by considering a closed economy with a constant population (normalized to 1) of identical households that lives forever. There is a single physical good in the economy, which may be allocated to consumption or investment, and all values are expressed in terms of units of this good. As in the seminal paper of Cox et al. (1985),

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[3] In a sense, habit formation might be a more realistic assumption than the usual time separable presumption. Hicks (1965) argues that it is counter-intuitive to assume that marginal utility of consumption in one period is independent of the consumption in other periods, because there normally is a strong complementary between consumption of successive periods. Ryder and Heal (1973), Constantinides (1990), and Campbell and Cochrane (1999) give further arguments for introducing habit formation when modeling economic behavior.

production possibilities consist of a set of linear technologies in which capital is the only input. In the closed economy, the set is restricted to two types of technologies. One has a sure rate of return equal to $r\,dt$ over the period $[t,\ t + dt]$, while the other obeys the geometric diffusion process: $\alpha\,dt + \sigma\,dz_t$ over $[t,\ t + dt]$, where dz_t represents a standard wiener process with zero mean, and α and σ are constants. The constant returns associated with both types of technologies imply that the model is one of endogenous growth. The only source of uncertainty in the economy is the rate-of-return risk associated with the risky technology.

At time t, the representative household has capital W_t and faces the decisions of how much of it to save and how to allocate savings between the two technologies. To make the portfolio choice non-trivial, it is assumed that $\alpha > r$. By denoting ω_t as the time t fraction of wealth invested in the risky asset and the time t consumption by c_t, the instantaneous change in capital will be given by:

$$dW_t = [\omega_t\alpha + (1 - \omega_t)r]W_t\,dt + \omega_t\sigma W_t\,dz_t - c_t\,dt. \tag{1}$$

Capital per capita is equal to wealth per capita in this model, so equation (1) also describes the wealth dynamics in the closed economy.

At time 0, the representative household maximizes the intertemporal objective function

$$U_0 = E_0\left[\int_0^\infty u(c_t, x_t)e^{-\delta t}\,dt\right], \tag{2}$$

where E_0 is the conditional expectations operator and $\delta > 0$ is the subjective rate of time preference. The instantaneous utility of the households, $u(\bullet)$, depends on the prevailing consumption level as well as the habit level (x_t). The idea in the habit formation literature is that the utility derived from a given level of current consumption is lower, the higher the habit level. We assume that the instantaneous utility function is given by

$$u(c_t, x_t) = \frac{(c_t - x_t)^{1-\gamma}}{1 - \gamma}, \tag{3}$$

where $\gamma > 0$, $\neq 1$, is a utility curvature parameter. Later, we will find it convenient to capture the relation between consumption and habit by the state variable $s_t \equiv (c_t - x_t)/c_t$. As Campbell and Cochrane (1999) we refer to s_t as the surplus consumption ratio. By (3), this ratio is the fraction of consumption that is available to generate utility at each

point in time. The marginal utility of consumption goes to infinity as c_t approaches x_t, implying that the households will never permit consumption to fall below the habit level.

We follow Ryder and Heal (1973) in assuming that the habit level is a simple weighted average of past consumption:[4]

$$x_t = e^{-\beta t} x_0 + \beta \int_0^t c_\tau e^{-\beta(t-\tau)} d\tau,$$ (4)

where $t \geq \tau$ and $\beta \geq 0$ is a parameter that determines the relative weight of consumption in earlier time periods. The larger is β, the more important is consumption in the recent past. If $\beta = 0$, the habit level is equal to some predetermined standard $x_0 \geq 0$. The special case $\beta = x_0 = 0$ corresponds to time separable preferences. By equation (4), the habit level responds linearly to past consumption, evolving according to

$$dx_t = \beta(c_t - x_t) dt.$$ (5)

The representative household chooses c_t and ω_t to maximize (2), subject to (1), (5) and the initial period wealth endowment W_0. Following Sundaresan (1989) and Constantinides (1990), we show in the appendix that the value function

$$J(W_t, x_t) = \Theta(W_t - \frac{x_t}{r})^{1-\gamma},$$ (6)

where $\Theta = \dfrac{r^{1-\gamma}}{(\beta+r)^{1-\gamma}(1-\gamma)} \left[\dfrac{\gamma}{\delta - (1-\gamma)\left(r + \frac{(\alpha-r)^2}{2\gamma\sigma^2}\right)} \right]^{\gamma}$, solves this problem. The optimal

consumption policy is[5]

$$c_t = x_t + \left(W_t - \frac{x_t}{r} \right) \mu,$$ (7)

where $\mu \equiv \frac{r}{(\beta+r)} \eta$, and $\eta \equiv \frac{1}{\gamma}\left[\delta - (1-\gamma)\left(r + \frac{(\alpha-r)^2}{2\gamma\sigma^2}\right)\right] > 0$. Asset demand in equilibrium is given by

$$\omega_t = \lambda\left(1 - \frac{x_t}{rW_t}\right),$$ (8)

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[4] For a more complex specification of the habit evolution, see Campbell and Cochrane (1999).

[5] A sufficient condition for $c_t > x_t$, which must be the case by equation (2), is that $W_t > x_t/r$. In the appendix we show that $W_t - x_t/r$ is lognormally distributed, so that this will always be fulfilled provided that $W_0 > x_0/r$. This implicit assumption is not innocent. A risk-free real interest rate of, e.g., 2% requires wealth to be 5 times greater than the habit level. Higher interest rate requires an even higher initial wealth/habit ratio. Hindy et al. (1997) criticizes this feature of the habit formation model that we study.

where the constant $\lambda \equiv \frac{\alpha - r}{r\sigma^2}$.

We notice that time separable preferences (i.e. $\beta = x_0 = 0 \Rightarrow x_t = 0$) imply that the consumption policy and asset demand would be given by $c_t = \eta W_t$ and $\omega = \lambda$, respectively. As shown by Merton (1969); isoelastic time separable preferences imply that it is optimal to consume a constant fraction out of wealth and that the optimal fraction invested in the risky asset is constant over time. In the habit formation model, equations (7) and (8) show that both the consumption and investment policy depends on the lognormally distributed (see the appendix) difference $W_t - x_t/r$. Thus, both the consumption/wealth ratio and the optimal fraction of wealth invested in the risky asset vary over time.

Furthermore, the marginal propensity to consume out of wealth for habit forming households is $\eta[r/(r + \beta)]$, compared to η with time separable preferences. Thus, wealth shocks leads to smaller changes in consumption when preferences are characterized by habit persistence in consumption. Finally, equation (8) shows that the fraction of wealth invested in the risky technology is λ only if W_t approaches infinity while x_t is held fixed. This means that the portfolio share in the risky asset will be lower if investors are characterized by habit formation in consumption.

2.2 Equilibrium Consumption Growth and Volatility

This subsection derives the equilibrium consumption growth rate (following Constantinides, 1990) and provides a discussion of the model's growth properties. We impose the restriction that $0 \le \lambda \le 1$. Since rW_t will always be larger than x_t, this assumption implies that $0 \le \omega_t \le 1$, see equation (8). This ensures interior solutions characterized by a positive amount invested in both technologies.[6]

By substituting (7) and (8) into equation (1), the latter may be written as

$$dW_t = \left(W_t - \frac{x_t}{r} \right) \left[((\alpha - r)\lambda + r - \mu)dt + \sigma\lambda dz_t \right]. \tag{9}$$

Applying Ito's lemma to the term $W_t - x_t/r$ and inserting (5), (7) and (9) into the resulting expression, we obtain

$$\frac{d\left(W_t - \frac{x_t}{r} \right)}{W_t - \frac{x_t}{r}} = \left[(\alpha - r)\lambda + r - \mu\left(1 + \frac{\beta}{r} \right) \right] dt + \sigma\lambda dz_t.$$

Using the definitions of λ and μ, the term in the square brackets can be defined by the following parameter:

$$k \equiv \frac{r-\delta}{\gamma} + \frac{(1+\gamma)(\alpha-r)^2}{2\gamma^2\sigma^2}. \tag{10}$$

This implies

$$\frac{d\left(W_t - \frac{x_t}{r}\right)}{W_t - \frac{x_t}{r}} = kdt + \sigma\lambda dz_t.$$

By differentiating equation (7) and substituting the resulting expression into the left-hand side of this last expression, we obtain

$$dc_t - dx_t = (c_t - x_t)(kdt + \sigma\lambda dz_t).$$

We find the stochastic process for the per capita consumption by using equation (5):

$$dc_t = (c_t - x_t)\left[(k+\beta)dt + \sigma\lambda dz_t\right]. \tag{11}$$

We define the unconditional instantaneous expected growth rate of consumption as $\frac{E[dc_t/dt]}{c_t} \equiv g$. Recalling our definition of the surplus consumption ratio $s_t \equiv \frac{c_t - x_t}{c_t}$, $s_t \in (0,1]$, and using $E[dz_t] = 0$, it follows from equation (11) that

$$g = E[s](k+\beta) = (k+\beta)\int_0^1 s\pi_s(s)ds. \tag{12}$$

Here $\pi_s(s)$ is the probability density function of the surplus consumption ratio.

In the appendix, we show that s has a stationary distribution with density function

$$\pi_s(s) = Ms^{-2}\left(\frac{s}{1-s}\right)^{\frac{2k}{\lambda^2\sigma^2}}\exp\left(-\frac{2\beta}{\lambda^2\sigma^2}\frac{s}{1-s}\right), \tag{13}$$

$$0 < s \leq 1,$$

where it is assumed that $k - \lambda^2\sigma^2 > 0$. M is a constant equal to

$$M = \left(\frac{2\beta}{\lambda^2\sigma^2}\right)^{2k/\lambda^2\sigma^2 - 1} \bigg/ \Gamma\left[\frac{2k}{\lambda^2\sigma^2} - 1\right], \tag{14}$$

and $\Gamma[\bullet]$ is the gamma function. Noting that c has a stationary distribution as s, we can interpret g as the expected steady state growth rate of consumption. This growth rate is a function of the model parameters and of the state variable x_t, which appears in the surplus consumption ratio.

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[6] Interior solutions are necessary for the equilibrium to be consistent with a constant risk-free interest rate.

In the case of time separable preferences, the mean consumption growth rate would be constant and equal to k, see Obstfeld (1994).[7] Equation (12) reveals that this result is not valid if preferences are characterized by habit formation. If the mean surplus consumption ratio is very large - that is, if $E[s] \rightarrow 1$ - g will approach $k + \beta > k$. At the other extreme, $g \rightarrow 0$ if $E[s]$ is very small ($c_t \rightarrow x_t$). In the latter case, the consumer will only invest the amount necessary to sustain his established experience level of consumption.

The relationship between the surplus consumption ratio and growth is closely related to the effect of a change in s on risk-aversion, portfolio allocation and savings.[8] Relative risk-aversion is given by $R \equiv -WJ_{WW}/J_W$. From equation (6) we obtain

$$R(s_t) = \frac{\gamma}{1 - \frac{x_t}{rW_t}} = \gamma\left[1 + \frac{\mu}{r}\left(\frac{1-s_t}{s_t}\right)\right] \geq \gamma, \tag{15}$$

where the second equality follows from substitution of W_t from (7). We see that R_t is falling in s_t and approaches γ as $s_t \rightarrow 1$. As is well known, γ is the coefficient of relative risk-aversion in the case of isoelastic time separable expected utility preferences. This coefficient provides a lower bound on risk-aversion with habit formation. Generally, equation (15) tells us that risk-aversion is high when the surplus consumption ratio is low.

From equations (8) and (15), the optimal portfolio share in the risky technology can be written as

$$\omega_t = \frac{\lambda\gamma}{R(s_t)}. \tag{16}$$

A low s implies that a small fraction of wealth will be invested in the risky asset. If $c_t \rightarrow x_t$, risk-aversion goes to infinity and all investment will be in the risk-free asset. When s_t is very large, R_t approaches γ implying that $\omega_t = \lambda$, as in the time separable case.

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[7] Obstfeld focuses on non-expected utility preferences. As long as preferences are time separable, the growth rate will be constant in that case as well. The size of the constant could be different, however.

[8] Our preference setup does not allow for a separate treatment of risk-aversion and intertemporal substitution. By (2), the elasticity of intertemporal substitution is s_t/γ. Increased s lowers risk-aversion and increases the willingness to substitute consumption over time. We can not distinguish the effects on consumption growth. We choose to focus on risk-aversion, in this model of uncertainty. A possible extension could be to incorporate habit formation in a non-expected utility setup, using a similar procedure as Svensson (1989) does for time separable preferences.

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The first equality in (15) implies that $x_t/W_t = r(1 - \gamma/R_t)$, which can be used in equation (7) to derive the consumption/wealth ratio:

$$\frac{c_t}{W_t} = \frac{\gamma}{R(s_t)}(\mu - r) + r. \tag{17}$$

The impact of changes in the surplus consumption ratio on savings (as measured by c/W), depends on the difference $\mu - r$. For reasonable parameter values this difference will be negative, in which case low realizations of s_t (high R_t) give a larger consumption/wealth ratio. When the surplus consumption ratio is very large c/W will be equal to μ, which compares to $c/W = \eta > \mu$ in the time separable case. Thus, when $s_t \to 1$, the *allocation* of savings is identical with and without habit formation, but *total* savings are higher with habit formation. This explains why the growth rate is $k + \beta > k$ in this case.

When $s_t \to 0$ ($R_t \to \infty$), the consumption/wealth ratio is r. Appealing again to plausible parameter values, we have $r > \eta > \mu$, so that the consumption-wealth ratio is higher the smaller is s_t. It seems counterintuitive that savings are smaller when consumption is close to the habit level. This result is related to the degree of risk aversion and precautionary saving. When R_t is very large, the household invests in the risk-free asset only. Then it is optimal to consume the permanent income from the investment: rW. As the degree of relative risk-aversion falls, it becomes optimal to invest an increasing fraction of wealth in the risky asset. This implies that income becomes stochastic, which triggers precautionary saving and consequently increases total savings.

In order to derive the relationship between s and the growth we note that risk-aversion will be high if consumption is close to the habit level (a low s_t). The savings rate will be low (for plausible parameter values), and investment is done in the risk-free, low-productive technology. In our endogenous growth framework, both effects pull in the direction of a lower mean growth rate. Thus, the surplus consumption ratio is positively associated with economic growth.

The instantaneous variance of the steady state growth rate can be obtained from equations (11) and (12):

$$\frac{\text{var}[dc/c]}{dt} \equiv \sigma_c^2 = (\lambda\sigma)^2 \int_0^1 s^2 \pi_s(s)ds = (\lambda\sigma)^2 E[s^2]. \tag{18}$$

We notice that the consumption variance with time separable preferences would be $(\lambda\sigma)^2$. Habit formation implies a smoother consumption path. Ceteris paribus, the

fraction wealth invested in the risky asset will be lower in an economy characterized by habit persistence. Since return shocks are the only source of uncertainty, the economy is less exposed to disturbances.

2.3 Returns, Risk and Growth

To gain some insight into the relationship between international asset trade and growth, we will now take a closer look at how consumption growth and -volatility depends on the technological parameters α and σ. Shifts in these parameters change the growth rate because they affect the saving rate and the allocation of investment between the risky and the risk-free asset.

We begin with a review of the time separable case, analyzed in Obstfeld (1994) with non-expected utility preferences. Consider first the effects of a change in rate-of-return uncertainty, σ. It is clear from equation (10) that $\partial k/\partial \sigma < 0$, implying that consumption growth decreases in σ. In order to see why, we use equation (10) and the definitions of λ and η to rewrite k as

$$k = r + \lambda(\alpha - r) - \eta.$$

By the definition of λ, we see that $\partial \lambda/\partial \sigma < 0$. Hence, increased uncertainty lead to a portfolio shift towards the risk-free asset. The consumption/wealth ratio is η with time separable preferences. We find that $\partial \eta/\partial \sigma < 0$ when $\gamma > 1$ and > 0 if $\gamma < 1$. A change in σ has an ambiguous effect on savings. The intuition behind the negative relationship between σ and k must be as discussed by Obstfeld (1994): Increased uncertainty depresses risk-taking and the negative growth stimulus that follows will dominate the impact on savings (which may rise).

With time separable preferences the consumption growth variance is given by $(\lambda\sigma)^2$, and it follows that $\partial(\lambda\sigma)/\partial\sigma < 0$. The volatility of the growth rate is falling in σ, a result which is perhaps a bit surprising. If the portfolio allocation was unaffected (λ constant), increased uncertainty would increase the variance of the growth rate. However, resources are shifted towards the risk-free asset (captured by a lower λ) and this dominates the direct effect through a higher σ.

The impact of a shift in the expected rate of return on the risky technology, α, is parallel to σ but with opposite signs: An increased α leads to higher consumption growth, stimulated by a portfolio shift towards the risky asset which will swamp a possible fall in

the saving rate. Also, the consumption path will be more volatile due to the portfolio reallocation.

To sum up, the time separable model gives very clear predictions. A lower σ or a higher α stimulate risk taking. This spurs growth, despite potentially lower savings. In addition, higher risk taking gives a more volatile consumption growth path.

In the habit formation model, a marginal change in σ (and in any other parameter) will influence the mean growth rate through two channels. First, it will change the size of the constant k, an effect that is common with the time separable model. Second, it will affect the distribution of the state variable s_t including its mean value. From equation (12),

$$\frac{\partial g}{\partial \sigma} = \frac{\partial k}{\partial \sigma} E[s] + (k + \beta) \int_0^1 s \frac{\partial \pi_s(s;\sigma)}{\partial \sigma} ds. \tag{19}$$

It turns out to be analytically difficult to evaluate the sign of the last term in equation (19), but numerical calculations can provide some help.

For this purpose, it is convenient to consider four cases: (i) $r > \delta, \gamma > 1$, (ii) $r > \delta, \gamma < 1$, (iii) $r < \delta, \gamma > 1$ and (iv) $r = \delta, \gamma > 1$.[9] Figure 2.1 shows one example of $E[s]$ as a function of σ for each of the parametric subsets. We see that $E[s]$ is falling in all examples, a conclusion that turns out to be very robust for variations in the parameters. In fact, performing numerous simulations, we have not been able to find a single feasible parameter combination where $E[s]$ is increasing in σ. We therefore claim that the expected surplus consumption ratio is likely to be a decreasing function of σ.

This result implies that both terms on the right hand-side of (19) are negative. This means that the steady state growth rate is falling in the rate-of-return uncertainty. The variance of the growth rate is affected according to

$$\frac{\partial \sigma_c^2}{\partial \sigma} = \frac{\partial (\lambda \sigma)^2}{\partial \sigma} E[s^2] + (\lambda \sigma)^2 \int_0^1 s^2 \frac{\partial \pi_s(s;\sigma)}{\partial \sigma} ds,$$

and is thus also decreasing in σ if g is. It follows that both the growth rate and growth-volatility are affected by a change in uncertainty in the same manner as in the time separable model. We can conclude that the combined effects on savings and portfolio

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[9] The restriction $k - \lambda^2 \sigma^2 = \frac{r-\delta}{\gamma} + \frac{(\alpha-r)^2(\gamma-1)}{2\gamma^2\sigma^2} > 0$ implies that $\gamma < 1$, $\delta \geq r$ is a non-feasible combination.

Feasible parameter combinations must also fulfill $\lambda \leq 1$ and $\mu > 0$.
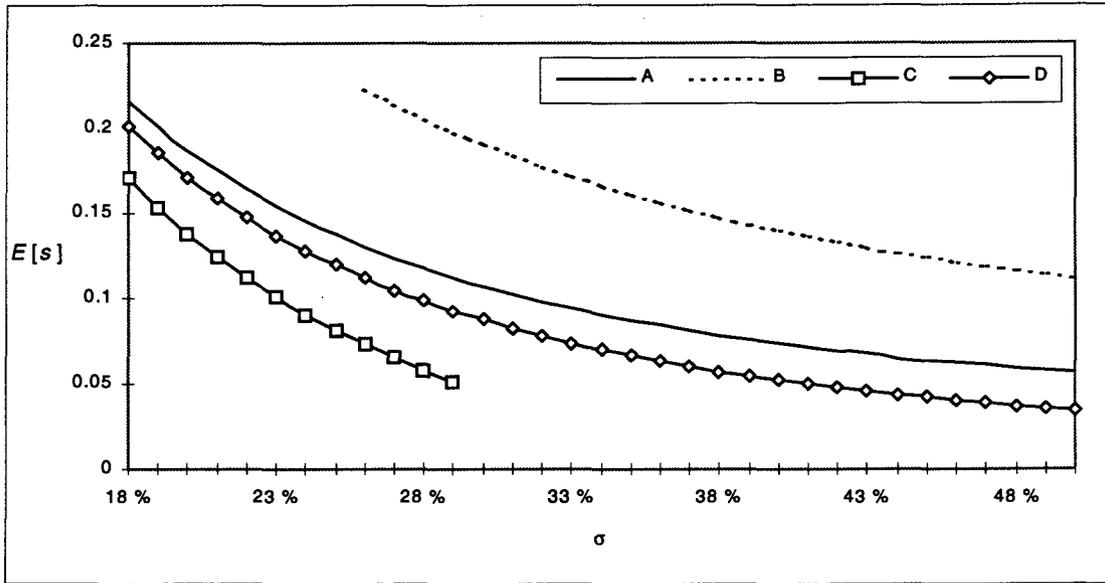
Figure 2.1: $E[s]$ as a function of σ for 4 different parameter sets:

A. $\alpha = 0.07$, $\beta = 0.1$, $r = 0.01 > \delta = 0.005$, and $\gamma = 2 > 1$.
B. $\alpha = 0.07$, $\beta = 0.1$, $r = 0.01 > \delta = 0.005$, and $\gamma = 0.9 < 1$.
C. $\alpha = 0.07$, $\beta = 0.1$, $r = 0.01 < \delta = 0.02$, and $\gamma = 2 > 1$.
D. $\alpha = 0.07$, $\beta = 0.1$, $r = 0.01 = \delta$, and $\gamma = 2 > 1$.
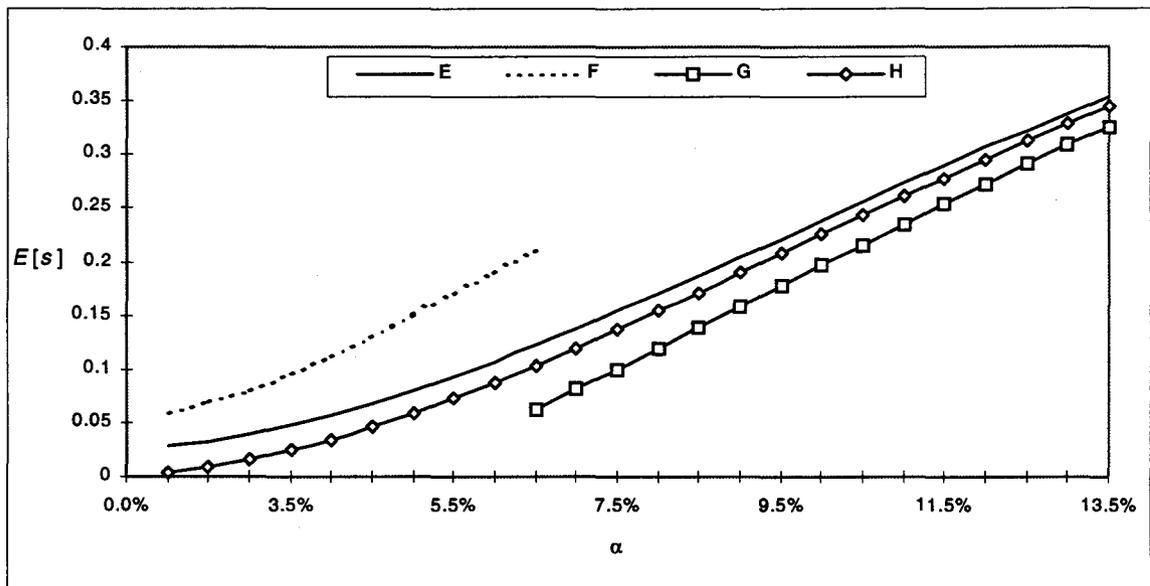
**Figure 2.2:** $E[s]$ as a function of α for 4 different parameter sets:

E. $\sigma = 0.25$, $\beta = 0.1$, $r = 0.01 > \delta = 0.005$, and $\gamma = 2 > 1$.
F. $\sigma = 0.25$, $\beta = 0.1$, $r = 0.01 > \delta = 0.005$, and $\gamma = 0.9 < 1$.
G. $\sigma = 0.25$, $\beta = 0.1$, $r = 0.01 < \delta = 0.02$, and $\gamma = 2 > 1$.
H. $\sigma = 0.25$, $\beta = 0.1$, $r = 0.01 = \delta$, and $\gamma = 2 > 1$.

composition, due to higher rate-of-return uncertainty, leads to lower growth and growth-volatility in both the time-separable and habit-persistence model.[10]

Figure 2.2 shows one example of $E[s]$ as a function of α for the parametric subsets (i)-(iv). $E[s]$ is an increasing function of α in all examples. Again, this turns out to be a robust pattern for variations in the other parameters. As in the case of σ, we have not been able to find any feasible parameter combination where $E[s]$ decreases in α. We thus assert that the expected surplus consumption ratio is likely to be increasing in α. By equations (12) and (18), this implies that both g and σ_c is higher the higher is the expected return on the risky asset. Again, this conclusion mirrors the one reached in the time separable case.

2.4 Welfare

Changes in the technology-parameters α and σ will affect welfare because it changes the consumption growth rate and the growth volatility. Lifetime utility is given by equation (6). This equation can be rewritten by observing that equation (10) implies that

$$r + \frac{(\alpha - r)^2}{2\gamma\sigma^2} = \frac{\delta + \gamma(k+r)}{1+\gamma}.$$

Substituting this expression into (6) we obtain:

$$J(W_t, x_t) = \frac{r^{1-\gamma}}{(\beta + r)^{1-\gamma}(1 - \gamma)}\left(\frac{1+\gamma}{2\delta - (1-\gamma)(k+r)}\right)^{\gamma}\left(W_t - \frac{x_t}{r}\right)^{1-\gamma}. \tag{21}$$

Shifts in α and σ influence lifetime utility (from time t and onward) only through their effect on k. Since J is increasing in k, higher α or lower σ will increase welfare. In an economy characterized by time separable preferences, we would have $\beta = x_t = 0$. It is then easy to see that lifetime utility would be

$$J(W_t) = \frac{1}{1-\gamma}\left(\frac{1+\gamma}{2\delta - (1-\gamma)(k+r)}\right)^{\gamma} W_t^{1-\gamma}. \tag{22}$$

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[10] Expressions for $E[\omega]$ and $E[c/W]$ are given section 4. Numerical calculations indicate that the mean fraction invested in the risky asset is decreasing in σ for feasible sets of the other parameters, while the average consumption/wealth ratio could be either increasing or decreasing. This also corresponds to the time separable model.

The welfare increase due to a rise in α or a fall in σ is common for both types of economies.

3. Habit Persistence and the Gains From International Risk Sharing

3.1 Multi-Country Equilibrium with Frictionless Trade in Financial Assets

In order to introduce habit formation in Obstfeld's (1994) multi-country model, we assume that the representative household in country i ($i = 1, 2,..., N$) has preferences specified by (2), (3) and (4). Preferences are nation specific, since country i has a rate of time preference δ_i, a habit smoothing constant β_i, and a utility curvature parameter γ_i. We assume that expectations are homogenous across consumers from all countries. Specifically, consumers from all countries perceive the risky asset return in country i to be governed by the diffusion process $\alpha_i dt + \sigma_i dz_{i,t}$, for $i = 1,...,N$, over the period $[t, t + dt]$. Thus, the expected return and risk associated with the risky technology in the different countries may be unequal. The cross-country correlation in the rates of return are represented by the structure $dz_i dz_j = \rho_{ij} dt$, with $V \equiv [\sigma_i \sigma_j \rho_{ij}]$ denoting the invertible $N \times N$ variance-covariance matrix. For simplicity we assume that the rate of return from the risk-free technology is common to all countries, equal to rdt over the period $[t, t + dt]$.

Following Obstfeld (1994), we make the important assumption that resources invested in one type of technology can be freely transformed into another type of technology. This implies that there will be no changes in the relative prices of assets when the economies are opened up to free trade. Accordingly, economic integration does not change any country's wealth. It turns out that this assumption greatly simplifies the welfare analysis.

With financial integration, households get access to several risky assets. Let $\mathbf{a}$, $d\mathbf{z}$, $\mathbf{I}$, and $\mathbf{w}_i$ all be $N \times 1$ vectors. The jth element is α_j in the first vector and dz_j in the second, while the third vector is the identity vector. The last one is the vector of country i portfolio weights of risky assets, meaning that the jth entry is country i's demand for the risky asset in country j. Wealth dynamics can now be written as[11]

$$dW_{i,t} = \mathbf{w'}_{i,t}(\mathbf{a} - r\mathbf{I})W_{i,t}dt + \mathbf{w'}_{i,t}\mathbf{V}W_{i,t}d\mathbf{z}_t + (rW_{i,t} - c_{i,t})dt, \quad \forall i. \tag{23}$$

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[11] See Merton (1971) for a detailed derivation of the wealth accumulation equation.

At the time of financial integration, the representative household in each country maximizes their intertemporal objective (2), subject to the evolution equations for habit and wealth, (5) and (23), and given their wealth endowment when integration occurs. Following the same steps as Merton (1971) we find that the equation of optimality for country i is:

$$\underset{\{c_{i,t},\mathbf{w}_{i,t}\}}{Max}\left\{\frac{(c_i-x_i)^{1-\gamma_i}}{1-\gamma_i}-J_i\delta_i+\frac{\partial J_i}{\partial W_i}[\mathbf{w}'_i(\mathbf{a}-r\mathbf{I})W_i-c_i+rW_i]+\frac{\partial J_i}{\partial x_i}\beta_i(c_i-x_i)+\frac{1}{2}\frac{\partial^2 J_i}{\partial W_i^2}(\mathbf{w}'_i\mathbf{V}\mathbf{w}_i)W_i^2\right\}=0.$$

First-order conditions are:

$$c_{i,t}=x_{i,t}+\left(W_{i,t}-\frac{x_{i,t}}{r}\right)\mu_i^* \qquad \forall i, \tag{24}$$

where $\mu_i^* \equiv \frac{r}{(\beta_i+r)\gamma_i}\left[\delta_i-(1-\gamma_i)(r+(\mathbf{a}-r\mathbf{I})'\,\mathbf{V}^{-1}(\mathbf{a}-r\mathbf{I}))\right]$, and

$$\mathbf{w}_{i,t}=\frac{\mathbf{V}^{-1}(\mathbf{a}-r\mathbf{I})}{\gamma_i}\left(1-\frac{x_{i,t}}{rW_{i,t}}\right) \qquad \forall i. \tag{25}$$

These equations are analogue to (7) and (8) in the closed economy, two-asset case, with the difference that (25) is a N x 1 vector. Absent time interdependence in the preferences, the demand for risky assets would be equal to $\mathbf{V}^{-1}(\mathbf{a}-r\mathbf{I})/\gamma_i$ and the consumption function would be $c_{i,t}=W_{i,t}\eta_i^*$ where $\eta_i^* \equiv \mu_i^*(\beta_i+r)/r$.

The fraction of wealth invested in risky assets by country i at time t is identified by the scalar $\mathbf{I}'\,\mathbf{V}^{-1}(\mathbf{a}-r\mathbf{I})\frac{1}{\gamma_i}\left(1-\frac{x_{i,t}}{rW_{i,t}}\right)$. To find the weight of each risky asset in the asset demand vector of country i we divide equation (25) by this expression, obtaining the following N x 1 weight vector:

$$\mathbf{q}\equiv\frac{\mathbf{V}^{-1}(\mathbf{a}-r\mathbf{I})}{\mathbf{I}'\,\mathbf{V}^{-1}(\mathbf{a}-r\mathbf{I})}. \tag{26}$$

This expression means that the mutual-fund theorem derived by Merton (1971) can be extended to the habit formation model: Every household wish to hold the same mutual fund of risky assets, independent of preferences and nationality. By implication, it also means that households will invest in the same mutual fund as with time separable preferences. Equation (25) shows that the representative households will invest a smaller *fraction* out of wealth in the mutual fund if habit persistence is relevant, while equation (26) tells us that the *composition* of the fund will be identical to the time separable case. Moreover, this composition will be constant since (26) is time independent. As with time

separable preferences (Obstfeld, 1994), we can thus proceed by studying one single global risky asset with mean return $\alpha^* = q'a$ and variance $\sigma^{2*} = q'Vq$.

It is not difficult to show that in country i the fraction of wealth invested in this risky asset is

$$\omega^*_{i,t} = \lambda^*_i \left(1 - \frac{x_{i,t}}{rW_{i,t}}\right),$$

where $\lambda^*_i \equiv \frac{\alpha^* - r}{\gamma_i (\sigma^*)^2}$. We assume that $0 \leq \lambda^*_i \leq 1$ for at least one i to ensure that there is some positive demand for the risk-free technology after the N autarkic economies open up to free asset trade. Thus, the relevant world interest rate is equal to r. Taking this modification into account, we can follow Obstfeld (1994) to describe the equilibrium with free asset trade. We let $L \leq N$ risky production technologies remain in operation after trade is opened,[12] available in the quantities $K_1, K_2, \ldots, K_L$. Letting a, V and q now referring to the L-dimensional subvectors and -matrix for mean returns, variance/covariance of returns and mutual fund weights, respectively, global equilibrium satisfies the conditions:

$$\frac{K_i}{\sum_{i=1}^{L} K_i} = q_i \quad \text{for all } i = 1, \ldots, L$$

$$\sum_{i=1}^{L} K_i = \sum_{i=1}^{N} \lambda^*_i \left(W_i - \frac{x_i}{r}\right),$$

where q_i refers to the ith element of q. With time-separable preferences, the last of these conditions would be $\sum_{i=1}^{L} K_i = \sum_{i=1}^{N} \lambda^*_i W_i$. The equilibrium conditions thus confirm that the global mutual fund demand will be lower if consumers are characterized by habit formation in consumption.

3.2 Consumption Growth and Volatility

We are now ready to analyze consumption growth and growth volatility with free asset trade. From equation (12) and the discussion in the preceding subsection, it follows that the mean consumption growth rate in the financially integrated equilibrium is given by

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[12] In general investors wish to go short in some of the countries' risky assets. This is not possible in the aggregate, so the associated production will shut down. The remaining L risky assets make up the "global market portfolio", composed as specified by (26). For further explanation, see Obstfeld (1994), p. 1317.

$$g_i^* = E[s_i^*](k_i^* + \beta_i),\qquad(27)$$

in country i. Here, $k_i^* \equiv \dfrac{r - \delta_i}{\gamma_i} + \dfrac{(1 + \gamma_i)(\alpha^* - r)^2}{2(\gamma_i \sigma^*)^2}$ and $E[s_i^*]$ is the mean surplus

consumption ratio prevailing in country i under financial integration. Other things equal, nations with a low mean surplus consumption will experience slower consumption growth than nations where the surplus consumption ratio is higher.

Comparing (12) and (27), we see that financial integration affects consumption growth through both k and the mean surplus consumption ratio. Two assumptions are crucial for the effect on growth with time separable preferences. First, wealth does not change in any country because the relative prices on different types of assets is unaffected by economic integration. Second, trade must enhance welfare since there are no distortions. When preferences are time separable, we see from equation (22) that welfare can only increase if k_i rises (as long as the risk-free interest rate is unaffected). Thus, as found by Obstfeld (1994), the growth rate that prevails with time separable preferences is higher with free asset trade than under autarky.

With habit persistence, the change in the mean surplus consumption ratio also affects the growth rate. In section 2, we argued that $E[s_i]$ is decreasing in σ and increasing in α. Thus, we can be certain that $E[s_i] \leq E[s_i^*]$ in countries where $\alpha_i \leq \alpha^*$ and $\sigma_i \geq \sigma^*$. In the cases where $\alpha_i \geq \alpha^*$, $\sigma_i \geq \sigma^*$ and $\alpha_i \leq \alpha^*$, $\sigma_i \leq \sigma^*$, no such simple argument can be used and analytical solutions are not attainable.[13] However, the increase in k that follows upon financial integration is qualitatively similar to a pure increase in α or a decrease in σ. We therefore conjecture that the effect from financial integration on $E[s_i]$ is similar to such shifts, so that the mean surplus consumption ratio increases.

Given that our conjecture is correct (this is supported by the calibration exercise in section 4), the effects on $E[s_i]$ and k_i in equation (27) both contribute to increased growth of international asset trade. The intuition is the same as in the time separable model of Obstfeld (1994): The opportunity to diversify idiosyncratic risk induces a shift in resources from technologies with (relatively) low return and low risk to riskier, high-return technologies.

The variance of the consumption growth rate in country i, given financial integration, can be written as

$$(\sigma^*_{c,i})^2 = (\lambda^*_i \sigma^*)^2 E[(s^*_i)^2],$$ (28)

compared to $(\lambda_i^* \sigma^*)^2$ with time separable preferences. The term $E[s_i^2]$ is larger with financial integration if the mean surplus consumption ratio is larger. In order to see what happens to the term $(\lambda_i \sigma_i)^2$ we rewrite the expression for k as

$$k_i = \frac{r-\delta_i}{\gamma_i} + \frac{1}{2}(1+\gamma_i)(\lambda_i \sigma_i)^2,$$

by using the definition of λ. Since k increases due to asset trade, so does $(\lambda_i \sigma_i)^2$. Thus, in our model setup, financial integration implies a more volatile consumption growth path than financial autarky. The case of time separable preferences illustrates this clearly. Increased opportunities to diversify could very well reduce the risk associated with holdings of risky assets (σ), but at the same time it would induce a portfolio shift towards risky assets so that λ increases. In this model, the portfolio shift will dominate. Correspondingly, the consumption volatility will increase. Although the model is very simple, this illustrates that it could be misleading to associate increased opportunities to share risk internationally with smoother consumption paths. In fact, the opposite could be true.

3.3 The Gains From International Risk Sharing

Turning to the welfare effects of financial integration, we first note that trade in financial assets affects welfare through its effect on consumption growth and volatility. We compare the present value of the welfare gains in economies with and without intertemporal dependence. A convenient measure is equivalent variation, i.e. the percentage increase in wealth in autarky that makes the households equally well off as with financial integration.

In the economy with time separable utility we wish to find EV_i, where EV_i is implicitly defined by

$$J_i[W_{i,t}(1+EV_i); k_i] = J_i^*[W_{i,t}; k_i^*].$$ (29)

In this expression, J_i and k_i denote lifetime utility measured at time t (the point in time when integration occurs) and the mean growth rate in autarky. The same quantities with financial integration are J_i^* and k_i^*. By substituting from equation (22) into (29), it is easy to show that

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[13] The case where $\alpha_i \leq \alpha^*$, $\sigma_i \geq \sigma^*$ is not considered, since this inconsistent with optimizing behavior.

$$EV_i = \left[\frac{2\delta_i - (1-\gamma_i)(k_i + r)}{2\delta_i - (1-\gamma_i)(k_i^* + r)}\right]^{\gamma_i/1-\gamma_i} - 1. \tag{30}$$

The first term on the right hand-side of (30) will always be > 1, confirming the positive welfare effect due to financial integration.

With habit formation, the equivalent variation is implicitly defined as

$$J_i\left\{[W_{i,t}(1 + EV_{i,t}^H), x_{i,t}]; k_i\right\} = J_i^*\left[(W_{i,t}, x_{i,t}); k_i^*\right], \tag{31}$$

where k_i must be interpreted as a parameter which is positively affected by financial integration. Substitution from equation (21) into (31) gives us

$$EV_{i,t}^H = \left\{\left[\frac{2\delta_i - (1-\gamma_i)(k_i + r)}{2\delta_i - (1-\gamma_i)(k_i^* + r)}\right]^{\gamma_i/1-\gamma_i} - 1\right\}\left(1 - \frac{x_{i,t}}{rW_{i,t}}\right). \tag{32}$$

By the first equality in equation (15), the last parenthesis in (32) is equal to $\gamma_i/R_{i,t}$. Hence, the welfare gain is lower if asset trade liberalization occurs when risk-aversion is high or, equivalently, consumption is close to the habit level (s is low). This seems counterintuitive; shouldn't international diversification be especially beneficial to agents who dislike risk? Not necessarily. If risk-aversion is high households dislike fluctuations to a large extent, and would not be willing to reallocate their portfolios towards risky assets in any significant degree. Thus, they will continue to have much of their portfolios invested in the risk-free asset. Accordingly, consumption growth and volatility would deviate little from the autarky case, and, as we saw above, the welfare effect will be small.

Equations (30) and (32) give us a simple relationship between the welfare gain with and without habit persistence:

$$EV_{i,t}^H = EV_i\left(\frac{\gamma_i}{R_{i,t}}\right). \tag{33}$$

When risk-aversion is close to its lower bound γ_i (a large surplus consumption ratio), households respond to the asset trade possibility in the same manner as with time separable preferences. Thus, the welfare gain will be equal in the two cases. When risk-aversion is very high, the optimal allocation of resources is the autarky-allocation. In this case, the opportunity to trade in financial assets does not change welfare. In intermediate cases, the welfare gain is positive, but smaller than with time separable preferences.

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How can we explain that households characterized by habit persistence obtain smaller gains from international risk sharing? This is related to the fact that habit-formation means greater reluctance towards fluctuations in marginal utility. In the model, consumption paths (and, hence, the paths for marginal utility) can be made smooth already in autarky by holding large fractions of wealth in the risk-free technology. Because households value this possibility to a larger extent than time separable preferences imply, they will respond less to the possibility of trading in risky assets. Underlying this result is the induced reallocation of portfolios that follows upon financial integration. In the asset demand function $\omega_{i,t} = \lambda_i \gamma_i / R_{i,t}$ [equation (16)], the size of λ_i shifts to λ_i^* when asset trade becomes possible. By utilizing the derivative $\partial \omega_{i,t}/\partial \lambda_i = \gamma_i / R_{i,t} \leq 1$ in (33),

$$EV_{i,t}^H = EV_i\left(\frac{\partial \omega_{i,t}}{\partial \lambda_i}\right),$$

it is easy to see that the magnitude of the welfare gain is related to the portfolio shift. When the optimal reallocation is small, the situation changes too little to give a significant welfare gain.[14] Further, if preferences are time separable $\partial \omega_i/\partial \lambda_i = 1$, illustrating that the more significant reallocation that occurs in this case is responsible for the larger implied welfare gains.

4. A Numerical Illustration of the Model

In this section, we will illustrate how the habit-formation model works by calibrating it to a four-country world, using stock market data for Germany, Japan, the UK and the US. We assume that these countries are imperfectly financially integrated and that stock market returns are a useful proxy for returns associated with the risky technology in the individual country. Together with the other parameters of the model, the stock market data allow us to generate consumption growth rates and variances in the habit formation model. These numbers provide the basis for calculating the theoretical welfare and growth gains due to perfect financial integration, and allow us to compare the gains to the corresponding ones in the model with time separable preferences.

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[14] This hinges on the assumption that the real risk-free interest rate is equal under free trade and autarky. If the risk-free interest rate rises, countries could experience higher growth rates without having to reallocate towards risky assets. This would probably enhance the welfare gains in the habit persistence model.

The illustration is similar to calibration example #2 in Obstfeld (1994), but we use updated data and include UK in the sample. Obstfeld considers time separable non-expected utility preferences. A drawback with that setup is that one must assume somewhat unrealistic parameter values to cope with the equity premium and risk-free rate puzzles. As mentioned in the introduction to this paper, and as will be shown below, habit formation models can be quite successful in explaining these puzzles. Hence, we believe that the numerical example given below is a worthwhile supplement to Obstfeld's analysis.

4.1 Data and Calibration

The annual return and variance on the Morgan Stanley Capital International stock market index for the four countries are used as proxies for the average return and variance on risky capital. All indices are in US-dollar, include reinvested dividends and ignore taxation. The index values we use covers the year-end quotes over the period 1969 - 96. Nominal returns are deflated by the US consumer price inflation to calculate the average annual real returns in dollars. Morgan Stanley provided us the data on the stock indices and the US consumer price index were collected from EcoWin. Table 2.1 reports the mean, standard deviations and correlation of risky capital dollar returns in our four countries.

Table 2.1: Mean (α), standard deviation (σ) and correlation matrix
of annual national stock market returns, 1970-96.

	Germany	Japan	UK	USA
α	9.3 %	13.8 %	11.2 %	7.2 %
σ	30.1 %	35.6 %	28.8 %	16.1 %
Correlation with:				
Japan	0.358			
UK	0.403	0.269		
USA	0.437	0.286	0.562	

Based on somewhat different time periods Campbell (1999) reports average annual real t-bill returns in several countries, including the four in our sample. These range from 0.85% in UK to 3.45% in Germany. We pick a crude average, setting the technological parameter $r = 0.015$ in all countries. Together with the stock market returns reported in table 2.1, this yields considerable equity premiums.

When choosing the behavioral parameters, we remember the restrictions that we imposed on our model in section 2. We required $k - \lambda^2\sigma^2 = \frac{r-\delta}{\gamma} + \frac{(\alpha-r)^2(\gamma-1)}{2\gamma^2\sigma^2} > 0$, implying that we can not simultaneously pick a very high δ and a very low γ. Given the technological parameters, the restriction that $0 \leq \lambda \leq 1$ also indicates that γ can not be set very low. We use $\delta = 0.02$ and $\gamma = 2.5$ as our benchmark values. The parameter β controls the "memory" of the households. Ryder and Heal (1973) discusses the size of this parameter in the context of a deterministic growth model, concluding that the values 0.1 - 0.3 span the likely β-values. We use $\beta = 0.15$, implying that consumption 10 years back weighs $\exp(-10\beta) \approx 22\%$ of current consumption in determining the habit stock. Subsection 4.4 presents some sensitivity analyses for the behavioral parameters.

4.2 Risk-Taking and Consumption Growth in the Pre-Integration Equilibrium

Based on the above parameter values, we can compute the mean consumption growth rate and it's variance in each country by numerical integration in equations (12) and (18). Table 2.2 reports these estimates.

It is also of interest to compute the implied mean values of R, ω and c/W. This is done as follows: In the appendix we have defined the stochastic variable $y \equiv (1 - s)/s$ and derived its steady state distribution and mean value. By equation (15), the results in the appendix imply that the coefficient of relative risk-aversion has a steady state distribution with an unconditional mean given by

$$E[R] = \gamma\left(1 + \frac{\mu}{r}\bar{y}\right) = \gamma\left[\left(1 + \frac{\mu}{r}\right)\frac{\beta}{k-\lambda^2\sigma^2}\right]. \tag{34}$$

By using the derivation in the appendix and equation (16), the mean fraction invested in the risky asset is

$$E[\omega] = \int_0^\lambda \omega\pi_\omega(\omega)d\omega = \lambda\int_0^\infty \left(1 + \frac{\mu}{r}y\right)^{-1}\pi_y(y)dy, \tag{35}$$

where $\pi_\omega(\omega)$ and $\pi_y(y)$ are density functions, with $\pi_y(y) = My^{-2k/\lambda^2\sigma^2}e^{-2\beta/y\lambda^2\sigma^2}$, $0 \leq y < \infty$. Given equation (17), we can apply the same procedure to derive the mean consumption/wealth ratio:

$$E[\tfrac{c}{W}] = \int_\mu^r \tfrac{c}{W}\pi_{\frac{c}{W}}(\tfrac{c}{W})d(\tfrac{c}{W}) = \int_0^\infty \left(\frac{\mu-r}{1+\frac{\mu}{r}y} + r\right)\pi_y(y)dy. \tag{36}$$

By using the parameter values assumed above, $E[R]$ can be calculated by plugging the assumed values into (34), while $E[\omega]$ and $E[c/W]$ can be computed numerically from equations (35) and (36), respectively. Table 2.2 reports these estimates. For the sake of comparison, all values in table 2.2 are also given for the time separable case.[15]

Table 2.2: Expected annual consumption growth rate per capita (g) with standard deviation (σ_c), and mean surplus consumption ratio (s), relative risk-aversion (R), fraction invested in the risky asset (ω) and consumption/wealth ratio (c/W).

	Habit formation model				Time separable model			
	Germany	Japan	UK	US	Germany	Japan	UK	US
g	1.1 %	2.2 %	2.1 %	2.3 %	1.7 %	3.1 %	3.0 %	3.3 %
σ_c	0.8 %	1.9 %	1.8 %	2.1 %	10.4 %	13.8 %	13.5 %	14.2 %
$E[s]$	0.07	0.12	0.12	0.13	1	1	1	1
$E[R]$	11.9	8.3	8.5	8.1	2.5	2.5	2.5	2.5
$E[\omega]$	0.11	0.15	0.18	0.35	0.34	0.39	0.47	0.88
$E[c/W]$	1.11 %	1.02 %	1.03 %	1.02 %	2.51 %	3.13 %	3.06 %	3.20 %

Note: Calculations are based on the numbers in table 2.1 and the assumptions that $r = 0.015$, $\delta = 0.02$, $\gamma = 2.5$, and $\beta = 0.15$

(with habit formation), in all countries. Procedures for calculations are described in the text.

The low fractions of wealth invested in the risky asset generates smoother steady state consumption paths with habit persistence, but it also implies lower mean growth rates despite higher savings in all countries.

The mean surplus consumption ratios reported in table 2.2 is of the same order of magnitude that Constantinides (1990) derives in his resolution of the US equity premium puzzle. The German combination of relatively low return and high variance on risky capital implies a high value of the steady state relative risk-aversion. However, we notice that it is the combined effect of α and σ that determines the willingness to take on risk. Japan has considerably larger variance on risky capital than the UK, but the higher expected return implies that mean risk-aversion is lower. Still, when it comes to the fraction of wealth invested in the risky asset the effect of higher σ dominates, leading to lower risk taking in Japan. Because the Germans invest relatively little in the risky technology in our model, their need for precautionary saving is lower and their consumption/wealth ratio higher (last row of table 2.2). In the time separable model,

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[15] As a reminder, time separable preferences imply: $g = k$, $\sigma_c = \lambda\sigma$, $E[s] = 1$, $E[R] = \gamma$, $E[\omega] = \lambda$ and $E[c/W] = \eta$.

savings is lowest for the US even though risk-taking and consumption volatility is the highest here.

How do the growth rates in table 2.2 compare to the actual numbers? In table 2.3, we have used the Penn World Table, Mark 5.2, to calculate the mean and standard deviation of private per capita consumption growth in the four countries, over the period 1970 - 92.[16]

Table 2.3: Annual mean growth rate (g) and standard deviation (σ_c) of actual consumption per capita, 1970 - 92.

	Germany	Japan	UK	US
g	2.5 %	3.4 %	2.4 %	1.8 %
σ_c	1.9 %	2.0 %	3.1 %	1.9 %

The implied growth rates seems quite reasonable in both versions of the model, although they are on the low side for Germany in the habit formation model[17] and too high for the US in the time separable model. The habit formation model also generates fairly reasonable steady state standard deviations, while they are much too high with time separable preferences. The reason is the equity premium puzzle. It is impossible to reconcile the large equity premiums implied by table 2.1 and the low consumption variance from table 2.3 without a much higher γ (and hence risk-aversion) in the time separable model. The phenomenon of habit formation generates high risk-aversion as a part of the equilibrium, leading to little risk taking and a smoother consumption path.[18]

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[16] An earlier version of the Penn World Table (PWT) is documented in Summers and Heston (1991). The consumption measure in PWT includes durable goods. The theory used in this paper implies that a consumption measure excluding these categories would be better, but comparable data are not available for all four countries.

[17] Germany is the country in our sample with lowest stock market capitalization relative to GDP (Campbell, 1999). Hence, stock market returns might be least suited as a proxy for risky capital returns here. In turn, this could explain the negative bias of the estimated German growth rate.

[18] A natural response would be is to experiment with higher values of risk-aversion and time separable preferences. This, however, would only create a new puzzle; the risk-free rate puzzle of P. Weil (1989). By (8) and (18), the constant steady state growth rate with time separable preferences may be written as $k = \frac{r-\delta}{\gamma} + \frac{1}{2}(1+\gamma)\sigma_c^2$. Solving for r, we obtain $r = \delta + \gamma k - \frac{1}{2}(1+\gamma)\sigma_c^2$. Forcing k and σ_c^2 equal to, e.g., the US numbers in table 2.3 and maintaining our assumption on δ, γ would have to be less than one to obtain any reasonable size of the real risk-free interest rate. A $\gamma = 5$ would for example imply $r \approx 10.9$ %.

4.3 Financially Integrated Equilibrium

The moments reported in table 2.1 provides the basis for computing the variance/covariance matrix of risky asset returns, and hence the equilibrium that would prevail in the four-country world after financial integration. Table 2.4 reports the portfolio shares in the four-country mutual fund of risky assets, calculated by using equation (26). Remember that this portfolio composition is constant and independent of national preferences.

Table 2.4: Equilibrium shares in the four-country mutual fund.

	Germany	Japan	UK	US
q_i	0.04	0.29	0.24	0.43

We notice that holdings of the German risky asset is very low in the integrated equilibrium, due to a combination of high risk and relatively high correlation with British and American capital (confer table 2.1).

The mean and standard deviation of the annual return of the portfolio in table 2.4 is reported in table 2.5. Together with the earlier stated assumptions on parameter values, this is sufficient to compute the common mean and standard deviation of the annual steady state per capita consumption growth with and without habit persistence in consumption. These numbers are also reported in table 2.5. Independent of model, all countries experience a tremendous increase in steady state consumption growth. They also experience a sharp increase in consumption variability regardless of intertemporal dependence in preferences. Given the parameters used in this example, households in all countries choose a mean surplus consumption ratio of 0.20 when they have the opportunity to diversify internationally. Also, mean risk-aversion is lower and the average fraction of wealth invested in risky assets higher, relative to the pre-integration equilibrium. By implication, consumption volatility increases in all countries, and this goes hand in hand with higher savings.

Table 2.5: Characteristics of the equilibrium under financial integration.

	α^*	σ^*	g^*	σ_c^*	$E[s^*]$	$E[R^*]$	$E[\omega^*]$	$E[(c/W)^*]$
Habit formation	10.2 %	19.0 %	4.1 %	4.0 %	0.20	6.6	0.45	0.98 %
Time separable	10.2 %	19.0 %	5.7 %	18.3 %	1	2.5	0.96	4.22 %

Table 2.5 clearly illustrates the differences in response to economic shocks that occurs with and without time interdependence in preferences. With time separable utility, financial integration changes the optimal allocation of resources only because it implies a shift in the technological parameters of the model. In the habit formation model, the technological shock also affects the distribution of the coefficient of relative risk-aversion. We thus have a second effect via changes in preferences it self. In our illustration of the model, financial integration imply increased willingness to bear risk in all countries, leading to a relatively larger increase in the fraction of wealth invested in risky capital compared to the case with time separable preferences.[19] In addition, households' find it optimal to simultaneously increase their savings, while the time separable model implies lower savings despite that consumption volatility increases.

The welfare gains from international financial integration are calculated from equations (30) and (32), and reported in table 2.6.[20]

Table 2.6: Welfare gains from international financial integration - evaluated by equivalent variation.

	Germany	Japan	UK	US
EV_i	138 %	64 %	70 %	58 %
EV_i^H	29 %	19 %	21 %	18 %

Note: Computations is done by equations (30) and (32) based on the same numbers as in table 2.1, and under the assumption that $R_{i,t} = E[R_i]$ $\forall$ i at the time of integration.

As shown in section 3, the gains are lower with habit persistence in consumption. The increase in per capita consumption growth comes through at the expense of a more volatile consumption path in this example. Habit formation implies that households dislike fluctuations to a larger extent than if preferences are time-separable and, hence, the welfare gain is lower with such preferences. The example we study here imply that the welfare gains in the habit formation model will be maximum 30 % of the corresponding gains with time separable preferences.

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[19] The relative increase in the portfolio share invested in the risky asset(s) range from just below 30 % for the US to more than a tripling for Germany, with habit persistence. In the time separable model, the increases range from 10 % in the US case to almost a doubling for Germany.

[20] With habit persistence, we assume that the coefficient of relative risk-aversion is equal to its pre-integration mean at the time of integration.

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Still, the gains from trade in financial assets are substantial. The EV_i^H are, for instance, typically 3-6 times larger than the gains reported by van Wincoop (1999) for models with exogenous growth rates. Hence, possible endogenous growth effects are very important in evaluating possible gains from asset trade, also with habit formation.

4.4 Sensitivity of Changes in the Behavioral Parameters

How sensitive are the above estimates for changes in the assumed behavioral parameters? This subsection presents a few calculations of the welfare gains, when β_i, δ_i and γ_i are given other values then assumed above. All computations follow the same procedure as in subsection 4.3.

We begin with the habit persistence parameter, β_i. Table 2.7 presents the estimated welfare gains from a transition to financial integration for different values of β_i, holding the other parameters fixed at the same values as earlier. The welfare gains under time separable preferences are included to provide comparison.

Table 2.7: The welfare gain from financial integration - measured by equivalent variation (EV_i^H) - under different values of β_i.

β_i	Germany	Japan	UK	US
0.5	27.5 %	18.5 %	19.8 %	17.1 %
0.3	27.9 %	18.7 %	20.1 %	17.4 %
0.15	29.0 %	19.4 %	20.8 %	17.9 %
0.05	33.0 %	21.7 %	23.3 %	20.1 %
0.01	52.0 %	31.8 %	34.3 %	29.3 %
0.0001	134.3 %	63.0 %	69.3 %	57.1 %
EV_i	138.0 %	64.1 %	70.5 %	58.0 %

Note: Other parameters than β are assumed to have the same values as in table 2.2.

The table illustrates that EV^H is a decreasing, convex function in β with maximum $= EV$ when $\beta \rightarrow 0$. That is, marginal differences in β do not affect the gains much as long as β is not very small. As $\beta \rightarrow 0$, the habit formation economy collapses into the time separable, and the welfare gains will be as in the latter case.

Table 2.8 reports the welfare gains for different values of the utility curvature parameter, γ_i. The upper line in each row shows the gain in the habit formation model, while the lower line refers to the time separable model. All other parameter values are as in subsection 4.3.

Table 2.8: The welfare gain from financial integration for different values of γ_i.

EV_i^H in the upper line of each row and EV_i in the lower.

γ_i	Germany	Japan	UK	US
2.5	29.0 %	19.4 %	20.8 %	17.9 %
	138.0 %	64.1 %	70.5 %	58.0 %
3	23.6 %	18.7 %	20.1 %	17.4 %
	113.2 %	54.3 %	59.5 %	49.3 %
5	13.0 %	19.4 %	20.8 %	17.9 %
	70.6 %	36.5 %	39.7 %	33.4 %
10	5.1 %	4.5 %	4.7 %	4.2 %
	39.1 %	21.9 %	23.6 %	20.1 %

Note: Other parameters than γ are assumed to have the same values as in table 2.2.

Regardless of model, the gain is lower the higher the value of γ. We also notice that the cross-country differences in gains become small as γ increases, especially in the model with habit persistence. The reason is that financial integration induces very small portfolio shifts towards risky assets in this case, because risk-aversion is higher, the higher is γ.

Finally, table 2.9 consider the gains for different values of the rate of time preference, δ_i, with the other parameters set at the same values as in subsection 4.3.

Table 2.9: The welfare gain from financial integration for different values of δ_i.

EV_i^H in the upper line of each row and EV_i in the lower.

δ_i	Germany	Japan	UK	US
0	103.7 %	43.5 %	48.3 %	39.0 %
	218.2 %	88.9 %	98.9 %	79.6 %
0.01	58.3 %	29.5 %	32.3 %	26.9 %
	169.3 %	74.5 %	82.3 %	67.1 %
0.02	29.0 %	19.4 %	20.8 %	17.9 %
	138.0 %	64.1 %	70.5 %	58.0 %
0.03	8.4 %	11.6 %	12.0 %	11.0 %
	116.3 %	56.2 %	61.6 %	51.0 %

Note: Other parameters than δ are assumed to have the same values as in table 2.2.

The decreasing gain in δ is common to both models, but the table indicates that the difference across models is higher the more impatient are the households. The latter point is due to the relationship between risk-aversion and the time preference rate. In the time separable model the two concepts are disconnected, while there is a link between patience and attitudes towards risk in the habit formation economy. By equation (34), we can show that mean risk-aversion increase in δ. Thus, impatient countries will be less

willing to reallocate portfolios upon integration and therefore realizes smaller welfare gains.

5. Conclusions and Discussion

This paper has explored the growth and welfare consequences from international asset trade, under the assumption of habit persistence in consumption. In our linear-technology model, financial integration will spur both mean consumption growth and the variance of the growth rate. These qualitative results replicate the model with time separable preferences. More importantly, the welfare gain from asset trade is lower with habit-forming households, despite higher risk-aversion. The benchmark calibration in section 4 show that the gains in the case of habit formation is less than 30 % of the gains with time separable preferences. The analysis confirms that a setup with habit-persistence is better capable of explaining the large equity premium/ low risk-free interest rates observed in international data.

The result of lower gains from trade with habit formation hinges partly on the assumption of a common, global risk-free technology and that there always are some investment in this technology. Without these assumptions, the risk-free real interest rate would change upon integration, and this would have a different effect with habit formation than with time separable preferences. Potentially, our ranking of welfare gains in the two cases could be overturned.

Even though the gains from integration in our model are lower with habit persistence, the empirical application of the model illustrates that they could be substantial. The reported welfare gains should be interpreted with a bit of caution, however. Two potential problems stem from the no-adjustment-costs constant-returns-to-scale production technologies. The assumption of no adjustment costs is likely to bias the welfare estimates upwards. Obstfeld (1994) presents some rough calculations which indicate that capital adjustment costs could be important: Assume that the current annual welfare gain converges towards the long run gains in tables 6-9 at an instantaneous rate of κ % per year. Then, the actual capitalized gain amounts to a fraction $[\kappa/(r + \kappa)]$ of the numbers presented in the tables. Assuming, as Obstfeld, an annual rate of convergence of

2.2 %, a common annual risk-free interest rate of 1.5 % implies welfare gains of approximately 60 % of the gains in the respective tables.

On the other hand, and perhaps more importantly, the assumption on constant returns can bias the reported gains downwards. Constant returns imply that the expected returns on the risky assets are constant over time, so the time-varying risk aversion in our model is transferred solely into time-varying portfolio shares. If the distribution of asset returns were endogenous, we would have that expected returns would be higher in "bad times" (when s is low and R high) since households would demand higher returns when risk aversion is high. Accordingly, risky assets would not be as unattractive when R is high as our model suggests. With endogenous asset returns, it is thus possible that our model understates the investors reallocation towards risky assets and hence also growth and welfare effects of financial integration. This issue deserves further research, but abandoning the constant returns assumption would make the model very difficult to solve.

Anyway, we believe that this paper has demonstrated the following general point: When habit-formation is introduced the growth/stability trade-off is tilted in favor of stability, and so possible growth effects from financial integration become less important.

Appendix

The purpose of the appendix is to (i) derive the distribution of the difference $W_t - x_t/r$, (ii) show the existence of the lifetime utility (6) and (iii) show that the distribution of s is stationary and given by equation (13). Parallel derivations are given in Constantinides (1990), but we include them since our preference specification is slightly different.

A.1 W_t - x_t/r has a Lognormal Distribution

By defining the function $G = \ln[W_t - x_t/r]$, applying Ito's lemma and using the equation that follows immediately after (10), we find that

$$dG = d\left[\ln\left(W_t - \tfrac{x_t}{r}\right)\right] = \left(k - \frac{\lambda^2 \sigma^2}{2}\right)dt + \lambda \sigma dz_t.$$

Thus, the value of $W_t - x_t/r$ at time t may be written as

$$W_t - x_t/r = (W_0 - x_0/r)\exp[(k - \tfrac{1}{2}\lambda^2\sigma^2)t + \lambda\sigma(z_t - z_0)].$$ (A.1)

The change in the difference $W_t - x_t/r$ between time 0 and t is lognormally distributed with mean $(k - \tfrac{1}{2}\lambda^2\sigma^2)t$ and variance $\lambda\sigma t$. ■

A.2 The Maximized Value of the Intertemporal Objective (2)

By combining equations (7) and (A.1) together with the fact that $\text{var}[z_t - z_0] = t$, we obtain for $t \geq 0$:

$$e^{-\delta t} E_0\left[(c_t - x_t)^{1-\gamma}\right] = \mu^{1-\gamma}\left(W_0 - \tfrac{x_0}{r}\right)^{1-\gamma} \exp\left\{\left[-\delta + (1-\gamma)\left(k - \tfrac{\lambda^2\sigma^2}{2}\right) + \tfrac{\gamma^2\lambda^2\sigma^2}{2}\right]t\right\}.$$ (A.2)

The term in the last square bracket in (A.2) is equal to $-\eta = -[\mu(\beta+r)]/r$. Accordingly, we can find that maximized lifetime utility is given by:

$$J(W_0, x_0) = E_0 \int_0^\infty e^{-\delta t} \tfrac{1}{1-\gamma}(c_t - x_t)^{1-\gamma} dt$$

$$= \frac{\mu^{1-\gamma}}{1-\gamma}\left(W_0 - \frac{x_0}{r}\right)^{1-\gamma} \int_0^\infty \exp\left[-\frac{\mu(\beta+r)}{r}t\right]dt$$

$$= \frac{r}{(1-\gamma)(\beta+r)}\mu^{-\gamma}\left(W_0 - \frac{x_0}{r}\right)^{1-\gamma},$$

which is equal to equation (6) from period t and onward. ■

A.3 The Steady State Distributions of the State Variables s_t and y_t

Impose the assumption $k - \lambda^2\sigma^2 > 0$. By applying Ito's lemma to the definition of s and substituting from (5) and (11), we find that diffusion equation for s is

$$ds = \left[k - (k + \beta + \lambda^2\sigma^2)s + \lambda^2\sigma^2 s^2\right]sdt + s(1-s)\lambda\sigma dz.$$

Next, define the variable $y \equiv \tfrac{1-s}{s}$ which, after using Ito's lemma again, evolves according to

$$dy = \left[\beta - (k - \lambda^2\sigma^2)y\right]dt - y\lambda\sigma dz.$$

By following the same procedure as Constantinides (1990), it can be shown that the state variable y has a stationary distribution, $\pi_y(y_t; y_0, t)$, $0 < t < \infty$, which must fulfill the differential equation

$$\tfrac{1}{2}\lambda^2\sigma^2 y^2 \frac{\partial \pi_y}{\partial y} - (\beta - ky)\pi_y = 0.$$ (A.3)

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The solution of (A.3) is

$$\pi_y(y) = My^{-2k/\lambda^2\sigma^2}e^{-2\beta/y\lambda^2\sigma^2}, \quad 0 \le y < \infty. \tag{A.4}$$

Using the fact that $\int_0^\infty \pi_y(y)dy = 1$ we can solve for the constant M, obtaining equation (14) in section 2.

By integrating (A.4) by parts we find that

$$\tfrac{1}{2}\lambda^2\sigma^2\left[y^2\pi_y\right]_0^\infty + \left(k - \lambda^2\sigma^2\right)\int_0^\infty y\pi_y dy = \beta. \tag{A.5}$$

We see from equation (A.4) that the first term in (A.5) is equal to 0, giving us the unconditional mean value of y as $E[y] = \beta/(k - \lambda^2\sigma^2)$.

To derive the distribution of s we utilize that $s = 1/(1+y)$, implying that s is monotonically decreasing in y. Since y has a stationary distribution, so does s. The monotone relationship between s and y implies that the stationary distribution of s is given by

$$\pi_s(s) = \pi_y(y)\left|\frac{dy}{ds}\right| = s^{-2}\pi_y\left(\frac{1-s}{s}\right). \tag{A.6}$$

Substituting (A.4) into (A.6) confirms that the density of s is given by equation (14). ∎

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Chapter 3[*]

International Diversification, Growth, and Welfare with Non-Traded Income Risk and Incomplete Markets

Abstract

We ask how the potential benefits from cross-border asset trade are affected by the presence of non-traded income risk in incomplete markets. We show that the mean consumption growth may be lower with full integration than in financial autarky. This can occur because: the hedging demand for risky high-return projects may fall as the investment opportunity set increases, and precautionary savings may fall as the unhedgeable non-traded income variance decreases upon financial integration. We also show that international asset trade increases welfare if it increases the risk-adjusted growth rate. This is always the case in our model, but the effect may be close to negligible. The welfare gain is smaller the higher the correlation between the domestic non-traded income process and foreign asset returns.

1. Introduction

The turmoil in worldwide financial markets during 1997 and 1998 has lead to renewed discussions on the costs and benefits of free international capital mobility (see e.g. Obstfeld, 1998; Rogoff, 1999). Some economists point to the crisis as evidence on the risks of global financial trading, arguing for restraints on capital flows (Bhagwati, 1998; Krugman, 1998). On the other hand, economic theory predicts potential important advantages of international financial integration. The benefits are associated with consumption smoothing (across time and different states of nature) and the allocation of resources that the international financial markets facilitate. (See Obstfeld and Rogoff, 1996 for a comprehensive introduction.) The largest potential gains from international asset trade have been demonstrated in models where the ability to diversify risk increases the long-term growth rate by inducing producers to undertake riskier high-return projects (e.g. Obstfeld, 1994; Dumas and Uppal, 1999).[1]

This paper adds to the debate on capital mobility by asking how the growth and welfare effects of cross-border asset trade are affected by the presence of non-traded income risk. Earlier literature has ignored such income components despite its probable real-world importance. This would not be a problem if markets where complete, but some non-traded income risk (e.g. labor income) would be hard to diversify even under full capital mobility (Rodrik, 1997). Furthermore, income from non-traded assets will affect portfolio choice and savings decisions, exactly the channels through which financial integration may impact growth, growth-variability, and welfare.

Several recent papers have explored how free asset trade affects growth and welfare. Devereux and Smith (1994) show that increased ability to diversify risk can reduce the precautionary motive for saving, and this would lower growth with financial integration. While their analysis assume that agents have access to one technology only, Obstfeld (1994) analyses this issues in a model where investors can choose between a low-return risk-free and a high-return risky technology. Financial integration allows agents to lay off nation-specific risks in international markets, giving incentives to

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[1] See also Acemoglu and Zilibotti (1997) on the link between the ability to diversify, risk-taking, and growth. Related research can be found in the financial development literature; see Pagano (1993) and Levine (1997) for useful surveys.

increase risk-taking. In Obstfeld's complete markets model, this leads to huge gains in terms of both growth and welfare. Dumas and Uppal (1999) introduce goods-market frictions in the same type of model. They show that goods-market imperfections reduce the gains from free asset trade, but not by much.[2]

This paper extends this research to the case where some income risk can not be traded and financial markets are incomplete. We study a two-country simple linear continuos-time stochastic growth model, along the lines of Cox et al. (1985) akin to several of the papers referred to above. Each country has a set of traded assets associated with constant-returns-to-scale no-adjustment-cost production technologies. In addition, we follow Svensson and Werner (1993) and include stochastic income that corresponds to income from non-traded assets. Such assets could be due to asset-market imperfections like transaction costs, moral hazard, capital controls, etc. We do not attempt to model the reason(s) for the (partially) missing insurance markets, but treat this absence as an exogenous constraint. In our context we may think of the market for claims on GDP, proposed by Shiller (1993), as an example of such missing markets.

Throughout, we compare our results with the all-assets-tradable, complete-markets models of Obstfeld (1994) and Dumas and Uppal (1999). Several interesting differences from these models emerge from our analysis: First, growth rates may be different across countries with full financial integration, even if preferences are equal. This happens because different countries choose different resource allocations depending on the covariance of their non-traded income process with the set of internationally traded assets.

Second, the equilibrium consumption growth rate may be lower with free asset trade than under financial autarky. This occurs when extending the set of available marketable assets leads to lower hedging demand for risky high-return projects, and/or when the increase in the hedgeable non-traded income variance that follows from integration gives lower precautionary savings.

Third, the welfare gain from international asset trade is always positive, but can approach zero in our model. The positive welfare effect is common with the Obstfeld and Dumas-Uppal models. However, we show by the means of numerical simulations

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[2] Devereux and Saito (1997) study the effects of restricted asset trade, in the sense that agents can trade in non-contingent bonds only. In this case, some countries may experience both higher growth and welfare

that for certain parameter combinations, our model predicts a negligible welfare gain from cross-border asset trade while a model ignoring non-traded income risk would imply substantial gains. A key parameter in evaluating the size of the welfare gain is the correlation between the domestic non-traded income process and the return on foreign risky assets. The higher the correlation the lower is the welfare gain. This is because the risk-return benefits from diversifying into foreign assets are counteracted by reduced hedging ability of the risky asset portfolio when the correlation between domestic non-traded income and foreign assets is high.

The rest of this paper is organized in the following manner. In the next section we go through the basic model elements and assumptions that we build on. In section 3 we derive the equilibrium consumption growth rate, and its variance, for an economy in financial autarky. Section 4 contains the paper's central results on the link between asset trade, growth and welfare. Finally, in section 5 we present some numerical illustrations of the model, before concluding in section 6. The appendix contains the derivations of the optimal decision rules under autarky and integration.

2. Basic Model Elements

We will consider a world consisting of two countries, indexed by $i = H,F$. A large number of identical infinitely lived households populate each country. Time is continuous, and at time t the household in country i maximizes the intertemporal objective:

$$U_i(t) = E_t\left[(1-\gamma)^{-1}\int_t^\infty c_i(\tau)^{1-\gamma}e^{-\delta(\tau-t)}d\tau\right],$$ (1)

where E_t is the conditional expectations operator, $c_i(\tau)$ the consumption level prevailing in country i at time τ, and γ and δ are the common constant coefficients of relative risk aversion and rate of time preference, respectively. Notice that E_t is independent of nationality, so that we assume homogenous expectations across countries.

In each country, there are two distinct constant-returns-to-scale production technologies for the production of the single consumption/investment good. One of the technologies is assumed to be risky while the other is risk-free. Adjustments in the

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under complete financial autarky.

allocation of capital to the different technologies are costless and instantaneous. Let K_i and B_i denote the quantity of the good invested in country i's risky and risk-free technology, respectively. Geometric Brownian motions drives all investment processes:

$$\frac{dK_i(t)}{K_i(t)} = \alpha_i dt + \sigma_i dz_i(t), \quad i = H, F, \tag{2}$$

$$\frac{dB_i(t)}{B_i(t)} = r dt, \quad i = H, F. \tag{3}$$

Here, α_i and r are the constant instantaneous expected rates of return (it is assumed that $\alpha_i > r$, $i = H,F$), σ_i the constant instantaneous standard deviation of returns, and $dz_i(t)$ a standard wiener process. As can be seen from equation (3), we assume that the returns on investment in risk-free technologies are equal across countries. The cross-country correlation of technology shocks are represented by the structure $dz_H dz_F = \kappa dt$, where κ is the correlation coefficient.

In addition to the return on their portfolio of traded assets, households in both countries receive an exogenous stochastic income from a non-traded asset. We can interpret this as income from some production factor in fixed supply (e.g. labor or land). Non-traded income are driven by the processes:

$$\frac{dy_i(t)}{W_i(t)} = \mu_i dt + \sigma_{y,i} d\zeta_i(t), \quad i = H, F. \tag{4}$$

In (4) $W_i(t)$ is aggregate marketable wealth in country i at time t, μ_i a constant drift coefficient, $\sigma_{y,i}$ the constant instantaneous standard deviation and $d\zeta_i(t)$ another wiener process. (We will often refer to (4) as simply the non-traded income process. It implicitly understood that this refers to the process of non-traded income to marketable wealth.)

The assumption made in (4) implies that non-traded income is proportional to marketable wealth. It follows Losq (1978), whom adopts it to study consumption behavior in this setting. He does not, however, study the growth and welfare properties of the model. These properties are the focus of our paper. The analysis is greatly simplified by imposing this assumption and does not loose its illustrative power. Indeed, the process in (4) allows us to solve the model in closed form, given the assumed CRRA-preferences. Svensson and Werner (1993) demonstrate that the consumption/ asset-allocation problem considered below can be analytically solved in more general cases

with CARA-preferences, but such preferences would prevent us from deriving closed-form solutions of the growth rates that are very central to our analysis.[3]

3. Financial Autarky

Let us first imagine that the two economies can not trade its (marketable) assets with each other. This experiment provides a benchmark upon which we evaluate the gains from cross-border asset trade in the next section.

3.1 Individual Behavior

In the absence of international asset trade, the wealth of a representative household will evolve according to (country subscripts are ignored in this section):

$$dW(t) = [\omega(t)\alpha + (1 - \omega(t))r]W(t)dt + \omega(t)\sigma W(t)dz(t) + dy(t) - c(t)dt, \qquad (5)$$

where $\omega(t)$ denotes the fraction of wealth invested in the domestic risky technology at time t. We will assume that negative allocations are non-feasible; $0 \leq \omega(t) \leq 1$.[4] The change in wealth is determined by (i) the return on the investment in the risky asset, (ii) the return on the (composite) risk-free asset, (iii) exogenous income growth and (iv) the instantaneous consumption rate.

The representative household in the closed economy chooses a consumption path $\{c(\tau)\}_{\tau=t}^{\infty}$ and a portfolio path $\{\omega(\tau)\}_{\tau=t}^{\infty}$, to maximize (1) subject to (4), (5) and the current

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[3] Losq (1978) interprets the non-traded income process as dividends on human wealth. He argues that it is plausible to assume that the ratio of human wealth income to financial wealth stays relatively constant through time, in which case (4) is a reasonable representation of the labor income process.

[4] This restriction is imposed to ensure consistency with the assumption of a constant risk-free interest rate. As such, it is an innocuous restriction in the closed economy of this section. Since domestic agents are homogenous, we could easily have introduced a market for an instantaneous risk-free bond and showed that the endogenously determined interest rate on this bond would be constant in the closed economy equilibrium. In the open version of the model studied in the next section agents are heterogeneous across countries. Then, allowing non-positive allocations would imply that the equilibrium risk-free interest rate would be a time varying process, as would be the optimal consumption policy and the asset demand function. It is in general very difficult to calculate the equilibrium path with heterogeneous agents (Den Haan, 1994). We wish to make the model tractable and hence impose the above restrictions on the portfolio weights.

level of wealth. This problem is solved in the appendix.[5] Here, we summarize the solution as follows:

Lifetime utility evaluated at time t is given by (time indexes are ignored from now on when they are unnecessary):

$$J(W) = (1-\gamma)^{-1} A^{-\gamma} W^{1-\gamma},$$

(6)

where

$$A \equiv \frac{1}{\gamma}\left[\delta - (1-\gamma)\left(r + \mu + \omega(\alpha - r) - \frac{1}{2}\gamma(\omega^2\sigma^2 + 2\omega\sigma_{Ky} + \sigma_y^2)\right)\right]$$

(6a)

is a constant assumed to be positive. In (6a), σ_{Ky} is the instantaneous covariance between the ratio of non-traded income to marketable wealth and the traded risky asset. The consumption policy and asset demand of the household in the closed economy is

$$\frac{c}{W} = A,$$

(7)

and

$$\omega = \begin{cases} 0 & \text{if } \varpi < 0 \\ \varpi & \text{if } 0 \le \varpi \le 1 \\ 1 & \text{if } \varpi > 1, \end{cases}$$

(8)

respectively, where

$$\varpi = \frac{\alpha - r}{\gamma\sigma^2} - \frac{\sigma_{Ky}}{\sigma^2}.$$

(8a)

Equation (7) shows that the optimal consumption-wealth ratio is constant. For interior solutions, equation (8a) tells that the optimal portfolio is a combination of the tangency portfolio, corresponding to the first term on the right-hand-side of the equality, and a hedge portfolio given by the second term. Without non-traded income risk, the tangency portfolio would have been the only part of the household's asset demand. In our model, the household wishes to hedge against fluctuations in non-traded income and thus adjusts their portfolio holdings. The hedge portfolio is the portfolio that has the maximum negative correlation with non-traded income (see Ingersoll, 1987 for a general discussion of the tangency and hedge portfolios).

The result that both optimal consumption and asset-allocation are constant fractions of wealth replicates the standard model where all assets are marketable (Merton,

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[5] Losq (1978) also consider this problem, deriving equations (6) and (7) below.

1969). We have obtained this replication by using the special process in equation (4). Still non-traded income risk gives rise to important differences from the standard model. The following two observations underline this:

First, the existence of a non-traded income component affects the equilibrium, even if this income is uncorrelated with the return on the traded assets (i.e., $\sigma_{Ky} = 0$). The asset demand would be identical to the standard model, but both the consumption function and welfare are affected by the non-traded income component.

Second, non-traded income affects consumption and asset allocation decisions even if the domestic financial market is complete. Complete markets in this closed economy mean that the non-traded income risk is spanned by the risky technology. Formally, it implies that the unhedgeable variance of the ratio of non-traded income to marketable wealth conditional upon the set of traded assets, denoted $\sigma_{y|K}$, is 0. In the closed economy:

$$\sigma_{y|K} = \sigma_y^2 - \frac{\sigma_{Ky}^2}{\sigma^2} = \sigma_y^2(1 - \rho_{Ky}^2) = 0, \tag{9}$$

where ρ_{Ky} is the instantaneous correlation coefficient between returns on the risky technology and the growth rate of the ratio non-traded income to marketable wealth. Obviously, spanning in the closed economy requires $\left|\rho_{Ky}\right| = 1$. Still, equations (6)-(8) would be different from the corresponding ones without non-traded income. It may thus be misleading to ignore non-traded income components even if one believe that financial markets are complete.

3.2 Equilibrium

We have three possible equilibria in the closed economy: one where both types of technology are demanded, and one for each of the corner solutions with zero investment in either risk-free or risky capital.

3.2.1 Investments in Both Types of Technology

Consider first the case when the representative household wishes to incur positive investments in both assets. Notice that, since there are no adjustment costs, asset supply always accommodate the equilibrium asset demand, given by equation (8a). By

substituting equations (4) and (7) into (5), wealth accumulation in the closed economy can be written as

$$dW(t) = [\omega\alpha + (1-\omega)r + \mu - A]W(t)dt + (\omega\sigma dz(t) + \sigma_y d\zeta(t))W(t). \qquad (10)$$

Using this expression in (7), we obtain the stochastic process for per capita consumption:

$$dc(t) = [\omega\alpha + (1-\omega)r + \mu - A]c(t)dt + (\omega\sigma dz(t) + \sigma_y d\zeta(t))c(t). \qquad (11)$$

By defining g as the instantaneous expected per capita consumption growth rate, we find from equation (11):

$$\frac{E_t[dc(t)/dt]}{c(t)} \equiv g = \omega\alpha + (1-\omega)r + \mu - A. \qquad (12)$$

That is, g is endogenously determined as the expected return on the traded assets, plus the instantaneous expected non-traded income growth, minus the consumption-wealth ratio.

The growth rate can be expressed in closed-form by substituting for ω from (8a) in (6a) and (12):

$$g = m + n, \qquad (13)$$

where

$$m \equiv \frac{r - \delta}{\gamma} + \frac{(1+\gamma)(\alpha - r)^2}{2\gamma^2\sigma^2}, \qquad (13a)$$

and

$$n \equiv \frac{1}{\gamma}\left[\mu - (\alpha - r)\frac{\sigma_{Ky}}{\sigma^2} - \tfrac{1}{2}\gamma(1-\gamma)\sigma_{y|K}\right]. \qquad (13b)$$

We have split the components of (13) in two, because m is the consumption growth rate that would prevail if only the traded assets were present. With the non-traded income process (4) the growth rate needs to be adjusted by the term n. In (13b), we see that the growth adjustment is expected non-traded income plus the expected excess return on the hedge portfolio less the (risk-aversion weighted) unhedgeable variance $\sigma_{y|K}$. All this is multiplied by the elasticity of intertemporal substitution $(1/\gamma)$. We notice that non-traded income affects the growth rate of the economy also in the cases where there is spanning $(\sigma_{y|K} = 0)$ and when the return on the traded assets is uncorrelated with non-traded income $(\sigma_{Ky} = 0)$.

The instantaneous variance of the mean growth rate can be derived from equation (11):

$$\frac{\operatorname{var}[dc/c]}{dt} \equiv s^2 = \omega^2 \sigma^2 + 2\omega\sigma_{Ky} + \sigma_y^2 = \frac{(\alpha-r)^2}{\gamma^2\sigma^2} + \sigma_{y|K}, \tag{14}$$

where the last equality follows upon substitution from (8a). The consumption growth variance is simply the sum of the instantaneous variances of the return on the traded assets and the non-traded income. Notice that the first term after the last equality is the consumption variance that would prevail without the non-traded income component. Unless markets are complete, consumption growth becomes more volatile when we add non-traded income.

We can obtain insight into how international asset trade may affect the equilibrium growth rates of this model, by considering the growth impacts of a fall in σ. Imagine that the households hold both types of assets also after the parameter shift. In an economy without non-traded income risk, we would then have $\partial g/\partial\sigma = \partial n/\partial\sigma < 0$; lower rate-of-return risk stimulates consumption growth. This clear prediction arises because a lower σ unambiguously shift investments towards the high-productive, risky technology, dominating a possible fall in saving (Obstfeld 1994).

The direction of the portfolio shift is ambiguous when there is non-traded income risk, as can be seen from equation (8a). Resembling the economy where all income risks are traded, the fraction of wealth invested in the tangency portfolio will increase. However, lower rate-of-return risk has an ambiguous effect on the fraction of wealth invested in the hedge portfolio. This fraction can be written as $-\frac{\rho_{Ky}\sigma_y}{\sigma}$. Holding ρ_{Ky} fixed, a fall in σ will decrease the optimal investment in the hedge portfolio if $\rho_{Ky} > 0$, contributing to lower overall risk-taking. More precisely, it will increase further the absolute value of the negative amount invested in the hedge portfolio.[6] Whether the increase in the amount invested in the tangency portfolio will dominate this effect is theoretically undetermined. A similar ambiguity is present in the relationship between σ and the consumption-wealth ratio (equation (7)). Ultimately, this implies a theoretically

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[6] The effect is similar if we hold the covariance fixed. Higher ρ_{Ky} and lower σ both contribute to lower hedging demand (given that $\rho_{Ky} < 0$). I thank Diderik Lund for this point.

undetermined sign on the term $\partial m/\partial\sigma$ in equation (13), so there is an uncertain impact on the growth rate from lower rate-of-return risk.[7]

3.2.2 Investment in One Type Only

In the equilibrium where all investments are in the risky technology the instantaneous expected growth rate would be $g = \alpha + \mu - A$. By (6a), the consumption-wealth ratio is

$$A = \frac{1}{\gamma}\left[\delta - (1-\gamma)\left(\alpha + \mu - \tfrac{1}{2}\gamma(\sigma_K^2 + 2\sigma_{Ky} + \sigma_y^2)\right)\right]$$

when ω is forced to 1. Then the consumption growth rate can be written as

$$g = m' + n', \tag{15}$$

where

$$m' \equiv \frac{\alpha - \delta}{\gamma} - \tfrac{1}{2}(1-\gamma)\sigma_K^2, \tag{15a}$$

and

$$n' \equiv \frac{\mu}{\gamma} - \tfrac{1}{2}(1-\gamma)\left(2\sigma_{Ky} + \sigma_y^2\right). \tag{15b}$$

Here, m' is the growth rate that would prevail without non-traded income risk. Again, the equilibrium per capita consumption growth rate could be higher or lower than in an economy with only traded income risk, depending on the covariance between the traded asset return and non-traded income growth. The instantaneous variance of the growth rate is simply $\sigma + 2\sigma_{Ky} + \sigma_y$ when all marketable assets held are of the risky form.

The growth impact of lower technological uncertainty is ambiguous in this case as well, depending on the effect on σ_{Ky} and on whether γ is smaller than or larger than 1. The latter point is valid also if we ignore non-marketable assets, as can be seen from (15a) and as shown earlier by Devereux and Smith (1994) and Obstfeld (1994). Since the portfolio allocation is fixed in this case, the ambiguous effect on the growth rate from a fall in σ is due to a undetermined impact on the consumption-wealth ratio (confer equation (12)).

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[7] A similar argument for the parameter α gives parallel conclusions. Without non-traded income risk, the equilibrium growth rate increases in α, while the link is theoretically ambiguous in the model with non-traded income.

The last possible equilibrium in the closed economy is one where there is investment in risk-free technology only. By (12), the consumption growth rate in this case is simply $g = r + \mu - A$. This can be expressed in closed form by observing that the definition of A simplifies to

$$A = \frac{1}{\gamma}\left[\delta - (1-\gamma)\left(r + \mu - \tfrac{1}{2}\gamma\sigma_y^2\right)\right],$$

whenever $\omega = 0$. Hence, the consumption growth rate is

$$g = \tfrac{1}{\gamma}(r + \mu - \delta) - \tfrac{1}{2}(1-\gamma)\sigma_y^2. \tag{16}$$

It is noteworthy that higher non-traded income variance increases saving (lowers the consumption-wealth ratio), and thus growth, only when the elasticity of intertemporal substitution ($1/\gamma$) is smaller than 1. This resembles the classic analysis of uncertainty and saving in Sandmo (1970).

4. Integrated Capital Markets

4.1 Trade in Marketable Assets

Assume now that the marketable assets can be traded internationally. Given the setup in section 2, wealth dynamics in the two countries are:

$$dW_i = \left[\left(\sum_{j=H}^{F}\omega_i^j(\alpha_j - r) + r + \mu_i\right)W_i - c_i\right]dt + \left(\sum_{j=H}^{F}\omega_i^j\sigma_j dz_j + \sigma_{y,i}d\zeta_i\right)W_i, \quad i = H,F \tag{17}$$

where ω_i^j is the fraction of country i's wealth invested in the risky asset of country j, $i,j = H,F$. To ensure consistency with the assumption of a constant risk-free interest rate we need to impose the short sale constraints: $0 \le \omega_i^j \le 1$ $i,j = H,F$, and $\sum_j \omega_i^j \le 1$, $i = H,F$. The problem solved by the representative households is as in the closed economy, with the budget constraint (17) replacing (5). We show how to proceed in the appendix.

Maximal utility is given by $J_i(W_i) = (1-\gamma)^{-1}(A_i^*)^{-\gamma}W_i$, where

$$A_i^* = \frac{1}{\gamma}\left[\delta - (1-\gamma)\left(r + \mu_i + \mathbf{w}'_i(\mathbf{a} - r\mathbf{1}) - \tfrac{1}{2}\gamma(\mathbf{w}'_i\Omega\mathbf{w}_i + 2\mathbf{w}'_i\mathbf{V}_i + \sigma_{y,i})\right)\right], \quad i = H,F. \tag{18}$$

In this expression $\mathbf{w}_i \equiv [\omega_i^H \ \omega_i^F]'$ is the portfolio weight vector, $\mathbf{a} \equiv [\alpha_H \ \alpha_F]'$, $\mathbf{1} \equiv [1 \ 1]'$, $\Omega \equiv [\sigma_H \ \sigma_F \ \kappa]$ is an invertible 2 x 2 variance-covariance matrix, and $\mathbf{V}_i \equiv [\sigma_{Hy,i} \ \sigma_{Fy,i}]'$ is the

vector of the covariance of each of the traded risky assets with the ratio of non-traded income. The optimal consumption policies are

$$c_i = A_i^* W_i, \quad i = H, F,$$ (19)

Due to the constraints on the portfolio weights, the asset allocation policy is somewhat complicated:

$$
\mathbf{w}_i' = \left[\omega_i^H \omega_i^F\right] \equiv
\begin{cases}
[0,0] & \text{if } \hat{\omega}_i^H < 0 \text{ and } \hat{\omega}_i^F < 0 \\[4pt]
\left[0, \hat{\omega}_i^F\right] & \text{if } \tilde{\omega}_i^H < 0 \text{ and } 0 \le \hat{\omega}_i^F \le 1 \\[4pt]
[0,1] & \text{if } \tilde{\omega}_i^H < 0 \text{ and } \hat{\omega}_i^F > 1 \\[4pt]
\left[\hat{\omega}_i^H, 0\right] & \text{if } 0 \le \hat{\omega}_i^H \le 1 \text{ and } \tilde{\omega}_i^F < 0 \\[4pt]
\left[\varpi_i^H, \varpi_i^F\right] & \text{if } 0 \le \varpi_i^H \le 1,\ 0 \le \varpi_i^H \le 1 \text{ and } \varpi_i^H + \varpi_i^F \le 1 \\[4pt]
\left[\frac{\varpi_i^H}{\varpi_i^H + \varpi_i^F}, \frac{\varpi_i^F}{\varpi_i^H + \varpi_i^F}\right] & \text{if } \varpi_i^H > 0,\ \varpi_i^F > 0 \text{ and } \varpi_H^H + \varpi_H^F > 1 \\[4pt]
[1,0] & \text{if } \hat{\omega}_i^H > 1 \text{ and } \tilde{\omega}_i^F < 0
\end{cases}
$$ (20)

$i = H, F$, where

$$\hat{\omega}_i^j \equiv \frac{\alpha_j - r}{\gamma \sigma_j^2} - \frac{\sigma_{jy,i}}{\sigma_j^2}, \quad i, j = H, F,$$

$$\tilde{\omega}_i^j = \frac{\alpha_j - r}{\gamma \sigma_j^2} - \frac{\sigma_{jy,i}}{\sigma_j^2} - \frac{\sigma_{HF}}{\sigma_j^2}, \quad i, j = H, F,$$

$$\tilde{\omega}_i^H \equiv \frac{\alpha_H - r}{\gamma \sigma_j^2} - \frac{(\alpha_F - r)\sigma_{HF}}{\gamma \sigma_H^2 \sigma_F^2} - \frac{\sigma_{Hy,i}}{\sigma_H^2} + \frac{\sigma_{Fy,i}\sigma_{FH}}{\sigma_H^2 \sigma_F^2}, \quad i = H, F,$$

$$\tilde{\omega}_i^F \equiv \frac{\alpha_F - r}{\gamma \sigma_F^2} - \frac{(\alpha_H - r)\sigma_{HF}}{\gamma \sigma_H^2 \sigma_F^2} - \frac{\sigma_{Fy,i}}{\sigma_F^2} + \frac{\sigma_{Hy,i}\sigma_{FH}}{\sigma_H^2 \sigma_F^2}, \quad i = H, F,$$

and

$$\varpi_i^j = \gamma^{-1} \sum_{k=H}^F v_{jk}(\alpha_k - r) - \sum_{k=H}^F v_{jk}\sigma_{jy,i}, \quad i, j = H, F,$$ (21)

In (21), v_{jk} are the elements of Ω^{-1}.

In comparing the asset allocation policy above to a world with only tradable assets, we restrict attention to the case where none of the short-sale constraints bind; that is, case 5 in equation (20). Equation (21) gives the asset demand functions in this case. It is instructive to rewrite it in matrix form:

$$\overline{\mathbf{w}}_i = \gamma^{-1}\Omega^{-1}(\mathbf{a} - r\mathbf{1}) - \Omega^{-1}\mathbf{V}_i, \quad i = H, F.$$ (22)

Define the scalars $D \equiv \gamma[\mathbf{1}'\Omega^{-1}(\mathbf{a} - r\mathbf{1})]$ and $H_i \equiv -\mathbf{1}'\Omega^{-1}\mathbf{V}_i$, so that (22) can be written as

$$\overline{w}_i = D_i t + H_i h_i, \qquad t \equiv \frac{\Omega^{-1}(a-r1)}{1'\Omega^{-1}(a-r1)}, \qquad h_i = \frac{\Omega^{-1}V_i}{1'\Omega^{-1}V_i}, \quad i = H, F. \qquad (23)$$

The two portfolios t and h_i are the tangency and hedge portfolio respectively (Ingersoll, 1987), and these are independent of preferences. A household from country i form a portfolio of risky assets by buying shares in the two mutual funds t and h_i. The construction of the tangency portfolio is identical across nations, while the composition of the hedge portfolio depends on the covariance between non-traded income in country i and the (global) set of traded risky assets. Hence, the portfolio of risky assets will be different across nations. This contrasts the case where all assets are marketable, in which all households would construct an identical mutual fund regardless of nationality, consisting of the tangency portfolio only (Obstfeld, 1994).

The optimal fraction of wealth invested in risky assets is given by the scalar $1'w_i$, while the composition of the risky asset portfolio can be found from the 2×1 vector $q_i \equiv w_i/1'w_i$, $i = H,F$.[8] Because of the short-sale constraints, the optimal asset allocation is time-invariant. This enables us to derive closed-form solutions of the mean growth rates, as will shown below.

4.2 Equilibrium

Let us now characterize the equilibrium in which the two economies above can trade marketable assets. The absence of adjustment costs has the convenient implication that the price of marketable assets relative to each other will be unchanged upon integration. We fix these relative prices at 1. Accordingly, given a, Ω, r, V_H and V_F, it is quantities that adjust to balance the demands given by (20). The equilibrium conditions are thus simply

$$K_i = \sum_{j=H}^{F} \omega_j^i W_j, \quad i = H, F.$$

To derive the mean consumption growth rates we can proceed as in subsection 3.2, obtaining

$$g_i^* = w'_i(a - r1) + r + \mu_i - A_i^*, \quad i = H, F. \qquad (24)$$

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[8] The composition is undetermined in the first case of equation (20) when the households hold no risky assets.

We thus have seven possible mean growth rates for each country, depending on the short-sale constraints (see equation (20)). Again we concentrate on the case where no constraint is binding (case 5 in equation (20)).[9] Using (22) in (18), we find that the consumption-wealth ratios are

$$A_i^* = \frac{1}{\gamma}\left[\delta - (1-\gamma)\left(r+\mu+\frac{(\mathbf{a}-r\mathbf{1})'\Omega^{-1}(\mathbf{a}-r\mathbf{1})}{2\gamma} - (\mathbf{a}-r\mathbf{1})'\Omega^{-1}\mathbf{V}_i\right)\right] + \frac{1}{2}(1-\gamma)V_i,$$

$i = H,F$, where $V_i = \sigma_{y,i}^2 - \mathbf{V}'_i\Omega^{-1}\mathbf{V}_i$ is the unhedgeable variance of the process (4) conditional upon the set of internationally traded assets. Using this in (24), the expected growth rate can be written as

$$g_i^* = m^* + n_i^*, \tag{25}$$

where

$$m^* = \frac{r-\delta}{\gamma} + \frac{(1+\gamma)}{2\gamma^2}(\mathbf{a}-r\mathbf{1})'\Omega^{-1}(\mathbf{a}-r\mathbf{1}), \tag{25a}$$

and

$$n_i^* = \frac{1}{\gamma}\left[\mu_i - (\mathbf{a}-r\mathbf{1})'\Omega^{-1}\mathbf{V}_i - \tfrac{1}{2}\gamma(1-\gamma)V_i\right]. \tag{25b}$$

A model without non-traded income would predict a common world growth rate, given by m^*, in a financially integrated equilibrium. This may no longer be the case when we include non-traded income, since different countries choose different resource allocations depending on the covariance of their non-traded income process with the set of traded assets. This is reflected in the growth adjustment term (25b) above.

Equation (25) corresponds to (13) in autarky. From Obstfeld's (1994) work we know that $m^* > m$, whenever there is investment in both risk-free and risky technologies both prior to and after trade has been opened. Such an unambiguous ranking is not present for n_i and n_i^*. The second term in the square brackets of (25b) is the expected excess return on the hedge portfolio under financial integration. This may be higher or lower than the corresponding return under financial autarky. As for last term in the square brackets, V_i is a decreasing function of available marketable assets and will accordingly decrease upon integration. Hence, when $\gamma > 1$ this term contributes to lower growth under integration, while the opposite is true for $\gamma < 1$. This reflects precautionary

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[9] We can derive the mean consumption growth rates for the other asset demand policies in equation (20) in the same manner as below.

savings behavior: With CRRA-utility lower unhedgeable income risk reduce (increase) savings when $\gamma > 1$ ($\gamma < 1$), slowing (spurring) growth in this model.

Now since n_i^* can be higher or lower than its autarky counterpart, we have an ambiguous effect on growth from financial integration. Underlying this indeterminacy is the uncertain effect that the increase in the investment opportunity set has on the hedging demand for the high-return risky technologies, and the ambiguous savings response to lower unhedgeable income risk.

The instantaneous variance of the consumption growth rates with financial integration are given by

$$(s_i^*)^2 = \mathbf{w}_i' \Omega \mathbf{w}_i + 2\mathbf{w}_i' \mathbf{V}_i + \sigma_{y,i}^2, \quad i = H, F.$$ (26)

This can be expressed in terms of the parameters of the model simply by plugging in the relevant optimal portfolio allocation from equation (20).

4.3 The Effect of Integration on Welfare

A convenient measure for evaluating the welfare effects of financial integration in this type of models is equivalent variation. This gives the percentage change in wealth under autarky necessary to make the households as well off as with integrated markets. That is, we wish to compute EV_i, which is implicitly defined as

$$A_i^{-\gamma}(1-\gamma)^{-1}\left[W_i(1+EV_i)\right]^{1-\gamma} = (A_i^*)^{-\gamma}(1-\gamma)^{-1}W_i^{1-\gamma}.$$ (27)

The left hand-side of (27) corresponds to (6), while the right hand-side is maximal utility with financial integration (equation (18)). Solving for EV_i, we obtain

$$EV_i = \left(\frac{A_i^*}{A_i}\right)^{\frac{\gamma}{\gamma-1}} - 1.$$ (28)

To interpret (28), it is instructive to notice that A_i can be written as

$$A_i = \delta - (1-\gamma)\left(g_i - \frac{1}{2}\gamma s_i^2\right), \quad i = H, F,$$ (29)

by substituting from (12) and (14) into (6a). The term in the last parenthesis of (29) is the *risk-adjusted* (or certainty equivalent) growth rate in country i. There is a similar expression for the consumption-wealth ratio under financial integration. Then, it follows from (28) that financial integration has a positive welfare effect for country i if, and only if, its risk-adjusted expected consumption growth rate is higher under financial integration than under financial autarky.

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By choosing the same resource allocations under both autarky and integration, the households can always obtain the same expected risk-adjusted growth rate. Utility-maximizing agents will never choose an allocation that implies lower welfare, so we can conclude that the risk-adjusted growth rate is non-decreasing upon integration and that financial integration improves welfare.

This qualitative result is common with the all-assets-tradable, complete-markets models of Obstfeld (1994) and Dumas and Uppal (1999). To investigate whether there may be significant quantitative differences between the welfare gains in those models and ours, we rewrite the autarky risk-adjusted growth rates in full:[10]

$$g_i - \tfrac{1}{2}\gamma s_i^2 = \tilde{m}_i + \tilde{n}_i,$$

$$\tilde{m}_i \equiv \frac{r-\delta}{\gamma} + \frac{(\alpha_i - r)^2}{2\gamma\sigma_i^2},$$

$$\tilde{n}_i \equiv \frac{\mu_i}{\gamma} - \frac{(\alpha_i - r)\sigma_{Ky}}{\gamma\sigma_i^2} - \tfrac{1}{2}\sigma_{y|K},$$

(30)

for $i = H,F$. The risk-adjusted growth rate that would prevail if we ignored non-traded income risk is given by $\tilde{m}_i$. Since welfare would be increasing upon financial integration in that case, this term must increase. This comes through as an increase in the expected excess return on the tangency portfolio (the last term in the definition of $\tilde{m}_i$). As explained earlier, the unhedgeable non-traded income variance is decreasing in available assets so that the last term in the definition of $\tilde{n}_i$ reinforces the welfare gain. What may counteract this, is the expected excess return on the hedge portfolio. That is, the second term in the definition of $\tilde{n}_i$ may be lower with asset trade than in autarky, contributing to lower welfare. This happens when the expected excess return on the optimal portfolio of risky assets is lower with full integration, and/or when the covariance between the non-traded income process and the portfolio of risky assets is higher with free asset trade. Although this can never dominate the combined effect of increased expected excess return on the tangency portfolio and lower unhedgeable income risk, the numerical examples constructed in the next section show that it could be important.

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[10] We discuss only the case when all short-sale constraints are slack both under autarky and integration. Equation (30) is derived by using (13) and (14).

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5. Numerical Illustrations

The preceding subsection has demonstrated that the growth-stimulus of international asset trade, implied by a complete market model, might be overturned when one introduces non-traded income components, while the positive welfare effect is retained. In this section we construct a few simple examples to demonstrate that non-traded income may significantly amplify the gains from cross-country asset trade in some cases, while it practically removes the gains in other instances.

Example 1: Consider a situation where $r = 0.02$, $\alpha_H = \alpha_F = 0.08$, $\sigma_H = \sigma_F = 0.20$, and $\kappa = 0.554$. These numbers are used by Dumas and Uppal (1999) in calibrating a friction-free version of their model, and are (roughly) based on stock-market data from the US and Germany presented by Obstfeld (1994). Let us also adapt Dumas and Uppal's preference parameters, setting $\delta = 0.02$ and $\gamma = 4$. To this, we add some imaginary parameters for the non-traded income processes. We assume that $\mu_H = \mu_F = 0.02$, $\sigma_{y,H} = \sigma_{y,F} = 0.05$, $\rho_{Hy,H} = \rho_{Fy,F} = 0.5$, and $\rho_{Hy,F} = \rho_{Fy,H} = 0$. That is, we start by using a fairly high domestic correlation between risky (marketable) asset return and non-traded income growth, while the domestic risky assets and foreign non-traded income growth are uncorrelated.

Under autarky, equation (8a) implies that both countries invest $\omega = 25$ % of their marketable wealth in the risky asset. This is the sum of investing long 37.5 % of wealth in the tangency portfolio and shorting 12.5 % of wealth in a hedge portfolio. Since there are investments in both types of technologies, we use equation (13) to find that the mean consumption growth rate is $g = m + n = 1.41$ % $+ 0.59$ % $= 2.00$ %. By equation (14) the instantaneous standard deviation of the growth rate is $s = 8.66$ %, giving a risk-adjusted mean growth rate of 0.5 %.

In the integrated equilibrium we use equations (20)-(22) to calculate that $1'w_H = 1'w_F = 0.402$. Risk-taking increases upon integration. This is the result of increasing the fraction of wealth invested in the tangency portfolio to 48.2 % and reducing the short hedge position to 8 % of wealth. In both countries, the portfolio of risky assets consists of 15 % invested in the domestic risky technology and 85 % in the foreign. This symmetric investment behavior occurs because we assume that the two countries are

identical. From (25) we find that the growth rate increases to $g^* = m^* + n^* = 1.81\% + 0.62\% = 2.43\%$ in both countries upon integration. The standard deviation of the consumption growth rates is also higher however. Equation (26) gives $s_i^* = 9.40\%$, $i = H,F$. Still, the risk-adjusted mean growth rates increase to 0.66 %.

By (28) we can then calculate that households in both nations requires an increase in marketable wealth of 18.9 % in autarky to obtain the same level of life-time utility as with financial integration. This is a large welfare gain; it is more than 40 % higher than Dumas and Uppal (1999) find in their frictions-free calibration. Hence, the covariance structure between marketable assets and non-traded income assumed above amplify the gains from international asset trade.

Example 2: Consider a second example where $\rho_{Hy,F} = \rho_{Fy,H} = 0.7$, while the other parameters are left unchanged. The foreign risky technology is less attractive in this example, leading to an increase in the fraction of wealth invested in risky assets to 29 % only. This allocation to tangency portfolio is still 48.2 %, but the non-traded income/risky assets covariance structure now imply a short hedge position of 19.3 % of wealth. The risky asset portfolio composition is 69.4 % in the domestic asset and 30.6 % in the foreign.

Since risk-taking increases less than in example 1 the impact on the mean growth rate is also smaller. It is still substantial though, increasing to $g^* = m^* + n^* = 1.81\% + 0.39\% = 2.20\%$ in both countries. We notice that the growth-adjustment term n contributes to lowering growth upon integration in this situation, but this is dominated by the increase in m. The instantaneous standard deviation of the growth rate increases to $s = 9.20\%$, giving a slightly higher expected risk-adjusted growth rate of 0.51 %. The implied welfare gain from integration is accordingly quite small, with $EV = 1.25\%$ in both nations. This demonstrates that the gain from international asset trade need not be very large, even though the positive impact on the growth rate is significant. An equivalent-variation gain of 1.25 % is less than a tenth of what this example would yield if we ignored non-traded income risk.

Examples 1 and 2 illustrate that the gains from asset trade are sensitive to the hedging ability of foreign marketable assets relative to the domestic ones. In figure 1 we represent the gains from trade for country H, varying $\rho_{Hy,F}$ between −0.3 and 0.9 (the other

parameters are fixed at the values of ex. 1). The lower the correlation between domestic non-traded income shocks and foreign risky assets return, the higher are hedging benefits from including foreign assets in the portfolio of risky assets, and the higher are the gains from trade. For high values of $\rho_{Hy,F}$, the risk-return benefits from diversifying into foreign assets are counteracted by the fact that this diversification reduces the hedging ability of the risky-assets portfolio. As shown in figure 1, the lower hedging potential can wipe out practically all gains from financial integration if $\rho_{Hy,F}$ is sufficiently high.
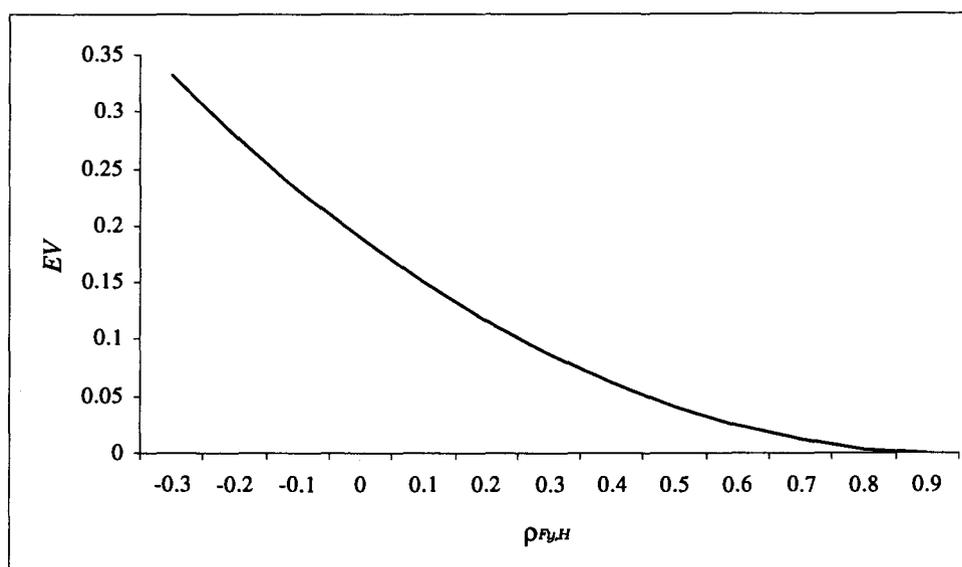
Figure 3.1: The welfare gain as a function of the correlation between the non-traded income process and foreign risky assets return.

Example 3: Consider finally an example where the two nations are asymmetric. We impose this asymmetry in the simplest possible manner, assuming that expected return on the risky technology in F is lower than in H. Specifically, we assume that $\alpha_F = 0.045$, while the rest of the parameters have the same values as in example 1.

We start by evaluating country H, which has autarky equilibrium as in example 1. The risky asset in F is less attractive than in that example since it now has a lower expected return. With integrated financial market, country H will bear only slightly more risk than in autarky, investing 26.1 % of wealth in the risky assets. The long position in the tangency portfolio is lower than in example 1 (now 34.1 % of wealth), but so is the short hedge position (which falls to 8.0 % of wealth) and the latter effect dominates. The portfolio of risky assets is heavily concentrated in the domestic technology; country H households invest 90.2 % of their risky-assets portfolio at home.

Even though there is a small increase in fraction of wealth invested in risky assets, the mean consumption growth rate falls to 1.97 % in this case. This arises because the expected return on the portfolio held with integrated markets gives a lower expected return than under autarky. Households choose a portfolio with lower expected return because it provides a better hedge against non-traded income risk.

The instantaneous standard deviation of consumption growth would decrease to s_H = 8.59 % in country H upon integration. This ensures a tiny increase in the risk-adjusted growth rate to 0.501 %. The welfare gain from financial integration is accordingly very small; the equivalent-variation gain is only 0.1 % of wealth. Even moderate trading costs would thus swamp the gains from trade in country H.

Turning to country F, we have an autarky equilibrium characterized by ω_F = 0.03, $g_F = m_F + n_F$ = 0.24 % + 0.70 % = 0.94 %, s_F = 5.34 %, and a risk-adjusted growth rate of 0.38 %. The low expected return on the risky asset in F implies little investment in this asset, and low consumption growth. With free asset trade between the two countries the short-sale constraints binds for country F. It wish to short its own risky technology and invest the proceeds in H's risky asset, since the latter provides both a better risk-return tradeoff and a better hedge against non-traded income risk. The short-sale constraints prohibit such allocations, however. By inspection it turns out that case 4 in equation (20) gives the optimal constrained asset allocation for country F, implying an investment of 37.5 % of wealth in the foreign risky asset while nothing is invested in the domestic counterpart. Using equations (24) and (26), we find that this allocation implies g_F^* = 2.84 % and s_F^* = 10.89 %, giving a risk-adjusted growth rate equal to 0.47 %. The equivalent variation measure of the welfare gain from financial integration is 11.9 % of initial wealth. Hence, country F derives large benefits from cross-country asset trade even with a binding short-sale constraint.

6. Conclusions

Our starting point was the debate on the costs and benefits of free capital mobility. This paper has dealt with the possible benefits, an asked how these might be affected by non-traded income risk when financial markets are incomplete.

We have shown that non-traded income risk can substantially alter the conclusions found in earlier research on the gains from cross-border asset trade. In our model, extending the set of available marketable assets may imply lower hedging demand for risky high-return projects, and lower precautionary savings as the unhedgeable non-traded income variance is reduced. These effects counteract the growth stimulus that all-assets-traded complete-markets models predict from international asset trade. It also implies that the welfare gains from financial integration may be negligible.

These results will typically occur if the non-traded income process has a higher correlation with foreign risky asset returns than the domestic equivalent. If, on the other hand, the non-traded income process has a lower correlation with the returns on foreign assets than with the domestic ones, the gains from asset trade would typically be amplified compared to a complete markets model.

Labor income fluctuations are probably the most important non-traded income risk in reality. Baxter and Jermann (1997) presents evidence from Germany, Japan, UK and US, where the pattern is that both (a synthetically computed) return on human capital and labor income growth is more highly correlated with domestic capital returns than foreign. This points to the conclusion that labor income risk strengthens the case for trade in financial assets. Bottazzi et al. (1996) concludes different, however. Using data for a large set of OECD-countries and taking into consideration redistributive shocks between capital and labor, they find that foreign assets generally are less attractive than domestic assets for hedging labor income uncertainty. Whether the presence of non-traded income risk strengthens or weakens the arguments in favor of international asset trade is therefore an empirical question that seems open.

Appendix

A.1 Individual Choice in the Closed Economy

We want to derive lifetime utility, asset demand, and the consumption policy of the households in the closed economy of section 2. The consumption path $\{c_\tau\}_{\tau=t}^\infty$ and portfolio path $\{\omega_\tau\}_{\tau=t}^\infty$ are chosen to maximize (1) subject to (4), (5), the current level of wealth $W(t)$. Let $J(W)$ denote the implied indirect utility function. The Hamilton-Jacobi-Bellman equation for this problem is

$$0 = \max_{\{c,\omega\}} \left\{ \frac{c^{1-\gamma}}{1-\gamma} - J\delta + J_W \left[W(\omega\alpha + (1-\omega)r + \mu) - c \right] + \tfrac{1}{2} J_{WW} W^2 \left[\omega^2\sigma^2 + 2\omega\sigma_{Ky} + \sigma_y^2 \right] \right\} \quad (A.1)$$

where subscripts on J denotes partial derivatives. The implied first order conditions with respect to the instantaneous consumption rate can be written as:

$$c^{-\gamma} = J_W . \quad (A.2)$$

Substituting this back into (A.1), we find that the differential equation for the value function J is

$$0 = \frac{\gamma}{1-\gamma} J_W^{\frac{\gamma-1}{\gamma}} - J\delta + J_W W \left[\omega\alpha + (1-\omega)r + \mu \right] + \tfrac{1}{2} J_{WW} W^2 \left[\omega^2\sigma^2 + 2\omega\sigma_{Ky} + \sigma_y^2 \right], \quad (A.3)$$

where the portfolio weights are optimized under the constraint that they must lie between 0 and 1:

$$\omega = \begin{cases} 0 & \text{if } \varpi < 0 \\ \varpi & \text{if } 0 \leq \varpi \leq 1 \\ 1 & \text{if } \varpi > 1, \end{cases} \quad (A.4)$$

where

$$\varpi = -\frac{J_W}{J_{WW} W} \frac{\alpha - r}{\sigma^2} - \frac{\sigma_{Ky}}{\sigma^2} . \quad (A.5)$$

Whenever there is an interior solution, (A.5) tells us that the optimal portfolio is a combination of the tangency portfolio, corresponding to the first term on the right-hand-side, and a hedge portfolio given by the second term.

Preferences of the CRRA-form leads to the conjecture that the indirect utility function is of the form $J(W) = A^{-\gamma}(1-\gamma)^{-1}W^{1-\gamma}$ for some constant A. Plugging the conjectured function into (A.3) confirms that it is indeed correct, and that A is given by equation (6a) in the main text. Finally, using (6) in (A.2) and (A.5) gives the consumption policy (equation (7)) and asset demand (equation (8)) for the representative household in the closed economy.

A.2 Behavior with International Asset Trade

We can follow the same methodology, as above to derive the optimal policies when there is financial integration. Let us start by considering the problem without restrictions on the portfolio weights. The Hamilton-Jacobi-Bellman equations are

$$0 = \max_{\{c_i, \omega_i^H, \omega_i^F\}} \left\{ \begin{array}{l} \dfrac{c_i^{1-\gamma}}{1-\gamma} + J\delta + J_{W_i}\left[W_i\left(\sum_{j=H}^{F} \omega_i^j(\alpha_j - r) + r + \mu_i - c_i \right) \right] \\ + \tfrac{1}{2} J_{W_iW_i} W_i^2 \left[\sum_{j=H}^{F}\sum_{k=H}^{F} \omega_i^j\omega_i^k\sigma_{jk} + \sigma_{y,i}^2 + 2\sum_{j=H}^{F}\omega_i^j\sigma_{jy,i} \right] \end{array} \right\}, \quad i = H, F \qquad \text{(A.6)}$$

In (A.6) the ω_i^j's are the unconstrained portfolio weights, σ_{jk} is the instantaneous variance/covariance of risky asset returns, and $\sigma_{jy,i}$ is the instantaneous covariance between non-traded income in i and the return on the risky asset in country j, $j = H, F$. From this, we find that the optimal consumption policy is as in the closed economy:

$$c_i^{-\gamma} = J_{W_i}, \quad i = H, F.$$

The optimal unconstrained portfolio weights satisfy

$$0 = J_{W_i}W_i(\alpha_j - r) + J_{W_iW_i}W_i^2\left(\sum_{k=H}^{F}\omega_i^k\sigma_{jk} + \sigma_{jy,i} \right), \quad i, j = H, F. \qquad \text{(A.7)}$$

This condition can conveniently be rewritten in matrix form as

$$0 = J_{W_i}(\mathbf{a} - r\mathbf{1}) + J_{W_iW_i}W_i\left(\Omega\overline{\mathbf{w}}_i - \mathbf{V}_i \right), \quad i = H, F, \qquad \text{(A.8)}$$

where $\overline{\mathbf{w}}_i = [\omega_i^H \ \omega_i^F]'$. (The other notation is explained in the main text.) Solving for $\overline{\mathbf{w}}_i$, we obtain:

$$\overline{\mathbf{w}}_i = -\dfrac{J_{W_i}}{J_{W_iW_i}W_i}\Omega^{-1}(\mathbf{a} - r\mathbf{1}) - \Omega^{-1}\mathbf{V}_i, \quad i = H, F, \qquad \text{(A.9)}$$

which implies that

$$\omega_i^j = -\dfrac{J_{W_i}}{J_{W_iW_i}W_i}\sum_{k=H}^{F}v_{jk}(\alpha_k - r) - \sum_{k=H}^{F}v_{jk}\sigma_{jy,i}, \quad i, j = H, F. \qquad \text{(A.10)}$$

In (A.10), v_{jk} are the elements of Ω^{-1}. We notice that the tangency and hedge portfolio is given by the first and second term, respectively, on the right hand side of (A.9).

Taking into account the short sale constraints $0 \leq \omega_i^j \leq 1$, $i,j = H,F$, and $\sum_j \omega_i^j \leq 1$, $i = H,F$, the asset allocation policies are considerably more complicated. The Hamilton-Jacobi-Bellman equation is as (A.6), with the unconstrained portfolio weights ω_i^j replaced by the constrained ones ω_i^j. The first order conditions with respect to ω_i^H and ω_i^F are:

$$\omega_i^H = -\dfrac{J_{W_i}}{J_{W_iW_i}W_i}\dfrac{\alpha_H - r}{\sigma_H^2} - \dfrac{\sigma_{Hy,i}}{\sigma_H^2} - \omega_i^F\dfrac{\sigma_{HF}}{\sigma_H^2}, \quad i = H, F \qquad \text{(A.11)}$$

$$\omega_i^F = -\frac{J_{W_i}}{J_{W_iW_i}W_i}\frac{\alpha_F - r}{\sigma_F^2} - \frac{\sigma_{Fy,i}}{\sigma_F^2} - \omega_i^H\frac{\sigma_{HF}}{\sigma_F^2}, \quad i = H,F.$$ (A.12)

Next, define the following subsets of (A.11) and (A.12):

$$\hat{\omega}_i^j \equiv -\frac{J_{W_i}}{J_{W_iW_i}W_i}\frac{\alpha_j - r}{\sigma_j^2} - \frac{\sigma_{jy,i}}{\sigma_j^2}, \quad i,j = H,F,$$ (A.13)

$$\tilde{\omega}_i^j = -\frac{J_{W_i}}{J_{W_iW_i}W_i}\frac{\alpha_j - r}{\sigma_j^2} - \frac{\sigma_{jy,i}}{\sigma_j^2} - \frac{\sigma_{HF}}{\sigma_j^2}, \quad i,j = H,F,$$ (A.14)

and

$$\breve{\omega}_i^H \equiv -\frac{J_{W_i}}{J_{W_iW_i}W_i}\left(\frac{\alpha_H - r}{\sigma_j^2} - \frac{(\alpha_F - r)\sigma_{HF}}{\sigma_H^2\sigma_F^2}\right) - \frac{\sigma_{Hy,i}}{\sigma_H^2} + \frac{\sigma_{Fy,i}\sigma_{FH}}{\sigma_H^2\sigma_F^2}, \quad i = H,F,$$ (A.15)

where there is an analogous expression for $\breve{\omega}_i^F$. Together with (A.10) and (A.13)-(A.15), equations (A.11) and (A.12) gives us the following constrained asset allocation policies:

$$\mathbf{w}_i' = \left[\omega_i^H, \omega_i^F\right] \equiv \begin{cases} [0,0] & \text{if } \hat{\omega}_i^H < 0 \text{ and } \hat{\omega}_i^F < 0 \\ \left[0,\hat{\omega}_i^F\right] & \text{if } \tilde{\omega}_i^H < 0 \text{ and } 0 \le \hat{\omega}_i^F \le 1 \\ [0,1] & \text{if } \tilde{\omega}_i^H < 0 \text{ and } \hat{\omega}_i^F > 1 \\ \left[\hat{\omega}_i^H, 0\right] & \text{if } 0 \le \hat{\omega}_i^H \le 1 \text{ and } \breve{\omega}_i^F < 0 \\ \left[\varpi_i^H, \varpi_i^F\right] & \text{if } 0 \le \varpi_i^H \le 1, \ 0 \le \varpi_i^H \le 1 \text{ and } \varpi_i^H + \varpi_i^F \le 1 \\ \left[\frac{\varpi_i^H}{\varpi_i^H + \varpi_i^F}, \frac{\varpi_i^F}{\varpi_i^H + \varpi_i^F}\right] & \text{if } \varpi_i^H > 0, \ \varpi_i^H > 0 \text{ and } \varpi_H^H + \varpi_H^F > 1 \\ [1,0] & \text{if } \hat{\omega}_i^H > 1 \text{ and } \tilde{\omega}_i^F < 0 \end{cases}$$ (A.16)

for $i = H,F$.

Substituting the optimal consumption policy into the constrained Hamilton-Jacobi-Bellman equation gives the differential equations for the value functions $J_i(W_i)$:

$$0 = \tfrac{\gamma}{1-\gamma}J_{W_i}^{\frac{\gamma-1}{\gamma}} + J_i\delta + J_{W_i}W_i\left[\mathbf{w}'_i(\mathbf{a}-r\mathbf{1})+r+\mu_i\right]+\tfrac{1}{2}J_{W_iW_i}W_i^2\left[\mathbf{w}'_i\Omega\mathbf{w}_i + 2\mathbf{w}'_i\,\mathbf{V}_{y,i}+\sigma_{y,i}^2\right],$$

where $\mathbf{w}'_i$ are given by (A.16), and $i = H, F$. The functional form of the intertemporal indirect utility function does not change upon financial integration. For country i it is still $J_i(W_i) = (1-\gamma_i)^{-1}(A_i^*)^{-\gamma_i}W^{1-\gamma_i}$, for some constant A_i^*. Using this in (A.8) we find that A_i^* is given by (18) in the main text and that the optimal asset allocation is given by (20) and (21).

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Chapter 4[*]

Financial Integration and Consumption Co-movements in the Nordic Countries

Abstract

The cross-country correlations between annual per capita consumption growth in the Nordic countries (Denmark, Finland, Norway and Sweden) during the period 1973-1992 are much lower than predicted by the basic theory of international financial integration. Capturing that the consumption behavior of parts of the population may be myopic and that some external consumption risks may be uninsured, this paper attempts to shed light on this observation. We find evidence of myopic consumption behavior for Denmark, Finland, and Sweden, possibly due to liquidity constraints. Taking this into account, the financial mar   Finland, Norway, and Sweden seem to be fairly well integrated. It proves hard to identify uninsured external consumption risks at the aggregate level.

1. Introduction

In the OECD countries we have witnessed a deregulation of domestic capital and credit markets as well as the abolition of cross-border capital controls over the last decades. Hence, the idea of one global financial market seems more realistic than ever before, at least among industrialized countries. We would therefore expect a potentially more efficient allocation of capital and improved opportunities to smooth consumption over time and across states of nature. This depends partly on whether capital is perfectly mobile internationally and partly on whether international trade in contingent assets takes place. If agents engage in contingent asset trade in a global financial market, they may diversify consumption risks that appear to be systematic at a domestic level but are idiosyncratic in a global setting. This implies that consumption growth should tend to be more synchronized across countries. In fact, the cross-country consumption correlations should be equal to unity under a set of restrictive but nevertheless standard assumptions, see for example Tesar (1995).

Turning to the rather large amount of recent consumption-based studies of international financial integration, we find that the consumption correlations have increased over time, but they are still far below unity.[1] In addition, French and Poterba (1991) and Tesar and Werner (1995) have documented a considerable bias towards domestic assets in the portfolios of investors in the industrialized countries. Since there are no longer any institutional constraints on cross-border capital movements in main parts of OECD, there is an apparent international risk-sharing puzzle.

Possible explanations to these findings include the incompleteness of markets for contingent assets (Obstfeld, 1994) and behavioral explanations in the sense that investors according to French and Poterba (1991) find investments abroad more risky because they are less familiar with foreign markets, institutions and firms. Other explanations are

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[1] The literature on consumption-based studies of international financial integration includes Backus *et al.* (1992, 1993), Obstfeld (1994, 1995), Stockman and Tesar (1995) and Tesar (1995). Of course, this literature is closely related to the literature on savings-investment correlations stimulated by the well-known paper of Feldstein and Horioka (1980). Obstfeld (1995) and Obstfeld and Rogoff (2000) discuss the relationship between these literatures.

transaction costs and the existence of non-traded goods, see Tesar (1993, 1995).[2] Further, the forward-looking consumption model underlying most of the international financial integration literature may be criticized. Myopic consumption behavior caused, for example, by liquidity constraints, may explain part of the observed low consumption correlations (Bayoumi and MacDonald, 1995).

Adopting a model framework, which captures the potential effects of uninsured consumption risks and domestic credit market imperfections, this paper investigates the degree of financial integration and consumption co-movements in the four Nordic countries Denmark, Finland, Norway and Sweden.[3] To our best knowledge a coherent study of consumption co-movements in these countries has not yet been undertaken.[4] We believe that this sample of countries may be of particular general interest. A common cultural background and very similar languages (Finland is an exception in the latter respect) imply that the Nordic countries are closely related. The political climate is also stable compared to most other regions. We therefore conjecture that the suggested effects of investors' non-familiarities with foreign markets, institutions and firms are minimized within the Nordic countries. Further, the national income of the small, open Nordic economies is strongly exposed to external shocks. For example, the oil price drop in the winter of 1985-86 explains the main parts of the 25% reduction in Norway's terms-of-trade in the period 1984-87, see Steigum (1993). Correspondingly, the crisis of the Finnish economy was to a large extent related to declining Soviet trade and the drop in the world market prices of paper and pulp products, see Honkapohja *et al.* (1993). This sensitivity of national income to external shocks implies that the Nordic economies have a strong incentive to engage in international asset trade and diversification. While

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[2] Tesar and Werner (1995) and French and Poterba (1991) deem the effect of transaction costs as insignificant. The effect of non-traded goods is also controversial. Baxter *et al.* (1998) find that a home bias with respect to domestic traded-goods equities is never optimal. On the other hand, Lewis (1996) concludes that international risk sharing can not be rejected when non-separabilities between tradeables and non-tradeable leisure and goods are taken into account in countries facing no capital market restrictions.

[3] An earlier version of this paper included Iceland as well. It turned out, however, that the consumption behavior of Iceland was highly myopic and completely unrelated to the consumption patterns in the other Nordic countries.

[4] An analysis of the bilateral consumption co-movements between Denmark and Norway is included in the study of Thøgersen (1997).

international non-diversification and the domestic asset bias may be explained by small welfare gains which are dominated by even moderate transaction costs in large economies like the U.S. (see Tesar, 1995), this explanation seems less likely in the small Nordic economies.

The next section takes a first look at some striking observations of the consumption patterns in the Nordic countries. Section 3 presents our model framework, which extends a reference model of international financial integration in two directions. Firstly, we follow Obstfeld (1994) and derive the implications of uninsured consumption risks. Secondly, we consider the possibility that parts of the population in the different countries may face liquidity constraints. Following Campbell and Mankiw (1991) and Bayoumi and MacDonald (1995), this leads to a model specification that discriminates between the effects of domestic credit market imperfections and lack of international financial integration. Our empirical evidence from 1973-1992 is presented in section 4. The general impression is that the financial markets of Finland, Norway and Sweden – but to a less extent Denmark – are fairly well integrated. For Denmark, Finland and Sweden we also find evidence of myopic consumer behavior. It proves hard to identify uninsured external variables. In the final section 5, we offer some concluding remarks.

2. A First Look at Consumption Co-Movements and Variability in the Nordic Region

As a point of departure, we consider the cross-country correlations in the data. Table 4.1 shows the correlation coefficients for changes in the logarithms of annual per capita private consumption for various combinations of regions and individual countries in our sample in the periods 1951-1972 (first row) and 1973-1992 (second row). The consumption data underlying the calculations here and in the rest of the paper are obtained from Penn World Table (version 5.6).[5] Our calculations can be compared to

[5] All data are measured in 1985 international prices. The Penn World Table (PWT) data used in this paper have been obtained from internet, http:/datacenter.epas.utoronto.ca:5680/pwt/pwt.html. An early version of PWT is documented in Summers and Heston (1991). The consumption measure in PWT includes durable goods. The theory used in this paper implies that a consumption measure excluding these categories would be better, but comparable data are not available for all countries throughout the period of our interest.

Obstfeld's (1994) similar calculations for the G-7 countries in the periods 1951-1972 and 1973-1988. While Obstfeld finds that the consumption correlations have increased from the first to the second period in almost all cases, the evidence in table 4.1 is more mixed. In many cases the consumption correlations between individual countries have decreased to a very low level. If we look at the consumption correlations between each individual country and the rest of the OECD, we see that the correlations involving Denmark, Finland and Sweden have increased, while the correlation between Norway and the rest of the OECD has dropped significantly.[6] Recalling the very large Norwegian petroleum sector, a possible interpretation is that uninsured oil price risk accounts for parts of this low consumption correlation.

Table 4.1: Correlation coefficients for changes in the logarithms of annual per capita private consumption growth in 1951-1972 (first row) and 1973-1992 (second row).

Country (region) i:	Nordic Region	Denmark	Finland	Norway	Sweden
OECD – except	0.34	0.05	0.28	0.45	0.35
country i	0.55	0.50	0.42	0.14	0.46
Nordic Region -		0.53	0.30	0.61	0.56
except country i		0.57	0.44	0.17	0.46
Denmark			0.21	0.40	0.56
			0.40	0.55	0.29
Finland				0.42	0.22
				-0.04	0.57
Norway					0.52
					0.01

It is also interesting to look at the variability of each country's per capita consumption growth relative to the variability of the consumption growth in the OECD for the two periods, see table 4.2. Through international diversification it is in principle possible for each Nordic country to reduce its consumption variability relative to the variability of the average consumption growth in the whole OECD area. Table 4.2 reveals, however, that only Denmark has experienced significant reductions in their relative consumption variability.

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[6] Our definition of OECD includes all OECD members in 1990.

Table 4.2: Standard deviation of each individual country's private annual consumption growth (logs) relative to standard deviation of private annual consumption growth in OECD (logs) in 1951-1972 and 1973-1992.

	Denmark	Finland	Norway	Sweden	Nordic Region
1951-1972	2.45	3.68	1.64	1.36	1.49
1973-1992	2.09	3.57	3.31	1.56	1.62

Standard deviation of OECD private annual consumption growth is 1.10% in 1951-1972 and 1.18% in 1973-92.

We also observe that the consumption variability of Finland and Sweden is approximately constant during the two periods, while there is a remarkable increase in Norway's consumption variability. Compared to Obstfeld's calculations, this implies that Denmark is in the same league as Germany and Japan. Norway, on the other hand, has experienced the same development in consumption variability as Canada, the exception among the G-7 countries.

3. The Framework

3.1 A Reference Model of International Financial Integration

Following Obstfeld (1994), we consider a reference model based on complete asset markets, perfect capital mobility and forward-looking consumption behavior. In the initial period 0 a representative infinitely-lived individual in country i, $i = 1,2,...,N$ maximizes

$$U_0 = E\left[\sum_{t=0}^{\infty} (\beta_i)^t u(C_{i,t}, \theta_{i,t}) | s_0 \right], \qquad (1)$$

where β_i is the rate of time preference, $C_{i,t}$ is consumption of a single tradable consumption good, $\theta_{i,t}$ is a preference shock and the period utility function satisfies $u' > 0$ and $u'' < 0$. For each period there is a set of possible states of nature, and s_t is the realised state in period t. The probability that a given state is realized in period $t+1$ depends only on the value of s_t and possibly on time (i.e. a Markov structure). Hence, $E[\bullet | s_t]$ is expectation conditional on information observed up to period t. The maximization of (1)

is subject to feasibility constraints for each period and each state. National income in each country follows a stochastic process known by all individuals.

Assuming that the representative individual in country i and country j have rational expectations and face identical asset prices, we obtain first-order conditions which can be written as

$$\frac{\beta_i u'[C_i(s_{t+1}), \theta_{i,t+1}]}{u'[C_i(s_t), \theta_{i,t}]} = \frac{\beta_j u'[C_j(s_{t+1}), \theta_{j,t+1}]}{u'[C_j(s_t), \theta_{j,t}]}. \tag{2}$$

Here $C_i(s_t)$ is the period t consumption per capita in country i provided that state s_t occurs. This condition means that for all states of nature the ex-post marginal rate of intertemporal substitution is equalized between country i and country j.

We assume an isoelastic period utility function,

$$u(C_{i,t}, \theta_{i,t}) = \frac{1}{1-\gamma}(C_{i,t})^{1-\gamma} \cdot \exp(\theta_{i,t}), \tag{3}$$

where γ is a common coefficient of constant relative risk aversion. Defining $t = 0$ as the initial period and normalizing θ_i so that $\theta_{i,0} = 0$, this specification implies that the following time-series model can be derived from (2):

$$\log C_{i,t} = a + \log C_{j,t} + \frac{t}{\gamma} \cdot \log\left(\frac{\beta_i}{\beta_j}\right) + \frac{1}{\gamma}(\theta_{i,t} - \theta_{j,t}). \tag{4}$$

Here $a = \log(C_{i,0}/C_{j,0})$ is a constant term. We see from (4) that equal time preference rates ($\beta_i = \beta_j$) and identical preference shock ($\theta_{i,t} = \theta_{j,t}$) imply equal ex-post co-movements in $\log C_{i,t}$ and $\log C_{j,t}$. We will, however, take into account that country specific preference shocks and differences in the time preference rates break this complete ex-post synchronization of $\log C_{i,t}$ and $\log C_{j,t}$.

Since we consider financial integration in the Nordic region, equation (4) should hold for all combinations of the four countries. Consequently, we may define "country j" as the aggregate of these countries minus country i. This procedure is common in the literature, and as explained in the appendix, it limits a potential endogenous regressor problem in the empirical application. The appendix also demonstrates that we, based on (4), may derive the following link between the change in the log of per capita consumption in country i and in the rest of the countries in the Nordic region:

$$\Delta \log C_{i,t} = b + \Delta \log C_{No-i,t} + \varepsilon_{i,t}. \tag{5}$$

Here $\Delta logC_{i,t} \equiv logC_{i,t} - logC_{i,t-1}$, $b = \frac{1}{\gamma}[log\beta_i - log(\Sigma_j\beta_j)]$ is a constant, $C_{No-i,t}$ is the per capita consumption in the Nordic region except country i in period t and $\varepsilon_{i,t}$ is a stationary disturbance term which reflects preference shocks.

The representative individuals in each country smooth consumption over time and across future states of nature. As we see from (5), this leads to proportionality between per capita consumption growth in country i and in the rest of the Nordic region. The only effect of idiosyncratic income shocks is through their impact on the total Nordic consumption possibility set. In order to test these predictions, we may estimate the equations

$$\Delta \log C_{i,t} = b + \alpha_i \Delta \log C_{No-i,t} + \varepsilon_{i,t}, \quad \forall i. \tag{6}$$

The joint hypothesis of perfect financial integration and complete markets implies $\alpha_i = 1$. Correspondingly, we may cautiously interpret α_i-values close to 0 as an indication of a low degree of financial integration.[7]

3.2 Incomplete Asset Markets

As discussed by Obstfeld (1994) among others, the international financial markets do not offer a complete set of insurance contracts. Accordingly, it seems relevant to consider a model where the asset markets may be incomplete in the sense that contracts can be made contingent on only a subset of the possible future states of nature. Obstfeld (1994) provides such an extension of the reference model, and he proves that "the date $t+1$ ex post marginal rate of intertemporal substitution difference between any two countries i and j,

$$D_{i,j}(s_{t+1},s_t) = \frac{\beta_i u'[C_i(s_{t+1}),\theta_{i,t+1}]}{u'[C_i(s_t),\theta_{i,t}]} - \frac{\beta_j u'[C_j(s_{t+1}),\theta_{j,t+1}]}{u'[C_j(s_t),\theta_{j,t}]}, \tag{7}$$

is statistically uncorrelated with any random variable on which date $t+1$ contracts can be written, as well as any variable realized on date t or before."

In the case of the reference model, the complete markets assumption implies $D_{i,j}(s_{t+1},s_t) = 0$. Hence, equation (2) applies. When the markets are incomplete, $D_{i,j}(s_{t+1},s_t)$

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[7] If the hypothesis $\alpha_i = 0$ can not be rejected, this indicates no financial integration. A rejection of the $\alpha_i = 0$ hypothesis does not, however, exclude the possibility of no financial integration since common shocks (for example global technology shocks) may imply $\alpha_i > 0$ even if there is no integration of financial markets.

is, however, correlated with some period $t+1$ variables which reflect uninsured shocks. As carefully demonstrated by Obstfeld, this leads to the following simple modification of (5):

$$\Delta log C_{i,t} = b + \Delta log C_{No\text{-}i,t} + \eta_{i,t} + \varepsilon_{i,t}. \tag{8}$$

Here $\eta_{i,t}$ is a function of noninsurable risks facing the representative consumer in country i in period t. We observe that the model still predicts proportional movements in per capita consumption growth in the different nordic countries after we have controlled for uninsured variables and preference shocks.

In order to test for Nordic financial integration, we may then estimate

$$\Delta log C_{i,t} = b + \alpha_i \Delta log C_{No\text{-}i,t} + \pi_i \Delta log X_{i,t} + \varepsilon_{i,t}, \quad \forall i , \tag{9}$$

where $X_{i,t}$ is a vector of variables which reflects the uninsured risks (i.e. $\eta_{i,t}$). If X_i is correctly specified, financial market integration implies $\alpha_i = 1$ and π_i-coefficients which are significantly different from 0.

3.3 Myopic Consumption Behavior

So far we have relied on forward-looking consumption behavior and perfect domestic credit markets. This may be criticized since available empirical evidence indicates that the consumption behavior of a significant share of the population in many OECD countries has been myopic during the period we analyze (1973-1992), see for example Campbell and Mankiw (1991). As a final modification of the model framework, we will therefore include the consumption set-up of Campbell and Mankiw which assumes that a proportion λ_i of aggregate consumption is associated with myopic current income consumers and a proportion $1 - \lambda_i$ with forward-looking consumers. Basicly, this means that we combine the reference model above with Campbell and Mankiw's model along similar lines as in Bayoumi and MacDonald (1995). We may interpret the current income consumers as consumers who face liquidity constraints in an imperfect domestic credit market.

The consumption of the current income consumers is given by $\lambda_i Y_{i,t}$ where $Y_{i,t}$ is real disposable income. This implies that

$$\log C_{i,t} = \lambda_i \log Y_{i,t} + (1 - \lambda_i) \log C_{i,t}^{FL} \quad \forall i, \tag{10}$$

where $\log C_{i,t}^{FL}$ is the consumption of the forward-looking consumers (which is given by (4)). As before we want to consider each individual country i versus the rest of the Nordic

countries. Hence, we follow the procedure outlined in the appendix and substitute equation (A-2) (in the appendix) for $\log C_{i,t}^{FL}$ in (10). We may then derive the following estimation equation:

$$\Delta logC_{i,t} = b + \lambda_i \Delta logY_{i,t} + \omega_i \Delta logC_{No-i,t} - \phi_i \Delta logY_{No-i,t} + \varepsilon_{i,t} \quad \forall i. \tag{11}$$

Here $\omega_i = \frac{1-\lambda_i}{1-\lambda_{No-i}}$ and $\phi_i = \lambda_{No-i} \cdot \omega_i$. Estimates of λ_i which are significantly larger than 0, indicate that parts of the population is characterized by myopic consumption behavior, possibly caused by liquidity constraints. Furthermore, ω_i-coefficients significantly larger than 0 indicate financial integration between the Nordic countries after we have controlled for myopic behavior in parts of the population. We observe that $\lambda_{No-i} = \lambda_j$ implies $\omega_i = 1$ if the financial markets are completely integrated in the region.

4. Estimation Issues and Empirical Evidence

4.1 The Reference Model

Turning to our empirical analyses, we first consider the estimation of (6), i.e. the reference model which includes preference shocks but disregards uninsured risks and domestic credit market imperfections. Table 4.3 reports the results.

Table 4.3: Reference model - SUR estimation of equation (6), 1973-1992.

Country i:	Denmark	Finland	Norway	Sweden
α_i	**0.88**	**1.56**	**0.81**	0.47[*]
	(0.21)	(0.45)	(0.39)	(0.14)
R^2	0.34	0.20	0.03	0.20
LM het. test	0.35	1.86	0.20	0.15
DW	1.00	1.49	1.45	1.58

LM test for contemporaneous correlation across equations = 17.61

Note: Standard errors of coefficients are shown in parantheses. **Boldface** entries indicate coefficients that are significantly larger than 0 at the 5% level, asterisks indicate coefficients that are significantly different from 1 at the 5% level.

As mentioned above and explained further in the appendix, there is a potential endogenous regressor problem in the reference model, but this is mitigated by using the aggregate consumption growth for the whole region minus country i as the regressor. (See

the appendix for details.) The LM test for cross-country correlation in the error terms we report in table 4.3 imply that we can reject the hypothesis of uncorrelated error terms at the 5 % level. We accordingly choose to estimate the reference model using Zellner's seemingly unrelated regression (SUR) estimation. The Durbin-Watson (DW) tests that we report in table 4.3 indicate a first-order autocorrelation problem for the Danish equation. Employing a Cochrane-Orcutt procedure changes the results very little however, so we report only the SUR results.

We can reject the hypothesis that $\alpha_i = 1$ in the case of Sweden only. Further, the α_i-coefficients are significantly larger than 0 for all countries. These results indicate that the private consumption patterns of Denmark, Finland and Norway are consistent with full financial integration, while the consumption pattern of Sweden also indicates a significant degree of synchronization with the rest of the region. Compared to Obstfeld's (1994) results for the G-7 countries, which are obtained from a similar model specification, our results at this stage indicate that the degree of financial integration within the Nordic area is approximately at the same level as between the G-7 countries. Recalling the low consumption correlations reported in section 2, this implies that we (as well as Obstfeld) must attribute an important role to preference shocks.[8]

4.2 Uninsured Risks

As discussed in the introduction, the national income of each of the Nordic economies seem to be vulnerable to fluctuations in a small number of international commodity prices and interest rates abroad. Despite our results in table 4.3, which suggest a rather high degree of financial integration and risk sharing, we therefore use the regression equation (9) in order to investigate whether shocks in various external variables may reflect idiosyncratic consumption risks. Potentially important variables in this respect include the German interest rate and the prices for paper and pulp products as

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[8] The inclusion of unexplained preference shocks in the utility function is standard in the literature on consumption based studies of international financial integration. In addition to Obstfeld (1994), see Ubide (1994), Stockman and Tesar (1995) and Canova and Ravn (1996). As discussed by Obstfeld (1995), such preference shocks are fully plausible. Still, a more elaborate modelling of these shocks is necessary in order to judge whether their important role in the present literature is reasonable. This is a natural topic for future research.

well as oil. The German interest rate may reflect idiosyncratic risks to the extent that the ties to the German currency and financial markets differ among the Nordic countries. The commodity prices may capture idiosyncratic risks through the relatively large size of the paper and pulp industry in Finland and the petroleum sector in Norway. It turns out, however, that available price indexes for paper and pulp products do not enter significantly into the regression equation (9) for any country. Hence, we consider only the effects of oil prices and German interest rates in the following. In both cases there are evidence of error terms correlation across equations, and we thus continue to apply SUR estimation.

Table 4.4: Oil price risk - SURE estimation of equation (9), 1973-1992,
$\Delta logOILP_t$ as a possibly uninsured variable.

Country i:	Denmark	Finland	Norway	Sweden
α_i	**0.74**	**1.52**	**0.85**	**0.51***
	(0.17)	(0.46)	(0.40)	(0.15)
π_i^{oilp}	**-0.03**	-0.01	0.01	0.01
	(0.01)	(0.02)	(0.02)	(0.01)
R^2 adjusted	0.57	0.16	-0.03	0.18
LM het. test	0.18	1.67	0.09	0.05
DW	0.86	1.45	1.42	1.69

LM test for contemporaneous correlation across equations = 20.93

Note: Standard errors of coefficients are shown in parantheses. Boldface entries indicate coefficients that are significantly larger than 0 at the 5% level, asterisks indicate α_i-coefficients that are significantly different from 1 at the 5% level.

Table 4.4 reports the results from the estimation of (9) when we include the change in the log of the real oil price ($\Delta logOILP_t$) as a possibly uninsured variable.[9] We observe that oil price changes have a significant impact on the private consumption patterns in Denmark only. In this case there is a negative relation between oil price changes and consumption growth which means that Denmark has not traded their idiosyncratic oil price risk to the other Nordic countries. (This conclusion is not affected by estimating the Danish equation alone, even if we correct for the autocorrelation that table 4.4 reveals for Denmark.) The negative relation probably indicates that the existence

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[9] We obtained our real oil price data series from Green et al. (1993) and OECD "Economic Outlook".

of large Danish petroleum resources was not recognized during most of the period 1973-92. Surprisingly, the results indicate that the Norwegian private consumption growth has not been significantly affected by oil price changes.[10] This is puzzling when we take the considerable size of the Norwegian petroleum sector and the high oil price volatility during the last decades into account, see for example Thøgersen (1995). From table 4.4 we also see that both the magnitudes and the significance of the α_i-coefficients are almost similar to the reference case (compare table 4.3).

Table 4.5: German interest rate risk - SUR estimation of equation (9), 1973-1992, $\Delta logGI_t$ as a possibly uninsured variable.

Country i:	Denmark	Finland	Norway	Sweden
α_i	**0.78**	**1.70**	0.73$^\#$	**0.53***
	(0.18)	(0.47)	(0.38)	(0.15)
π_i^{gi}	**-0.93**	0.73	-0.87	0.49
	(0.32)	(0.75)	(0.71)	(0.32)
R^2 adjusted	0.51	0.17	0.04	0.21
LM het. test	0.33	3.23	0.55	0.45
DW	1.60	1.68	1.60	1.68

LM test for contemporaneous correlation across equations = 14.31

Note: Standard errors of coefficients are shown in parentheses. Boldface entries indicate coefficients that are significantly larger than 0 at the 5% level, asterisks indicate α_i-coefficients that are significantly different from 1 at the 5% level.

Table 4.5 reports our results of testing the impact of changes in the log of German long-term interest rates $(\Delta logGI_t)$.[11] Once again our results indicate that Denmark has not traded all its idiosyncratic consumption risk to the other Nordic countries. We see that the German interest rate has a significant negative effect on the Danish private consumption growth. This probably reflects that the Danish capital market has been closely connected to the German capital market during most of the period 1973-92. Increases in German

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[10] To some extent this result contrasts with the results in Thøgersen (1997), which indicate that oil price fluctuations contribute to the explanations of low consumption correlations between Norway and respectively Denmark and Germany.

[11] The data series on the German interest rates (yield on long term German T-bonds) are collected from the EcoWin database and OECD "Economic outlook".

interest rates have therefore rapidly spilled over to Danish interest rates and depressed Danish consumption.

While Danish consumption seems to be exposed to fluctuations in both oil prices and German interest rates, it proves hard to identify uninsured private consumption risks at the aggregate level in the other Nordic countries. This may, of course, imply a rather high degree of risk sharing between the countries in the region. Still, we conjecture that our results to some extent also reflect that tax-transfer policies in the Nordic countries share consumption risks over time and between generations in a way which weakens the immediate responses in private consumption to external shocks.

4.3 Myopic Consumption Behavior

According to table 4.3 above, our reference model performs rather well. The results in table 4.3 may, however, be explained by other economic mechanism than forward-looking consumption behavior and a high degree of financial integration. The results may simply be the consequences of myopic consumption behavior and common income shock in economies which are not highly financially integrated. Based on the regression equation (11), we therefore investigate simultaneously the relevance of the forward-looking consumption model and the degree of financial integration.

For the 1973-92 period, equation (11) was estimated by using the Generalized Methods of Moments (GMM). Bayoumi and MacDonald (1995) elaborate why GMM is appropriate in these models. The primary argument is that the disturbances to domestic income contain information about future income and could accordingly be correlated with consumption. An instrumental variable technique, such as GMM, should therefore be applied. In addition, GMM is robust to heteroscedastisity and autocorrelation, and provides a direct test of orthogonality of the errors to the instruments.

As instruments, we use the second lag of the level of real consumption per capita and real disposable income per capita for both the home country and the rest of the region.[12] We have also experimented with lagged growth rates of consumption and disposable income as instruments. Based on the Wu-Hausman test for evaluation of

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[12] We use the second lag because both the inclusion of nondurables in the consumption measure and the time averaging of consumption data can induce a correlation between the error term and the first lag of consumption (see Campbell and Mankiw, 1989).

instruments (see e.g. Johnston and DiNardo, 1997, pp. 348-342), the level variables generally performed better. Thus, we only report the parameter estimates where the level variables are used as instruments. These results are given in table 4.6.[13]

Table 4.6: GMM estimation of equation (11), 1973-92.

Country i:	Denmark	Finland	Norway	Sweden
λ_i	1.16[#]	**1.26**	0.31	0.38[#]
	(0.60)	(0.36)	(0.45)	(0.21)
ω_i	-0.32	**0.84**	**0.90**	0.48[#]
	(0.66)	(0.42)	(0.45)	(0.25)
ϕ_i	1.04	-0.77	-0.40	0.44
	(0.75)	(0.76)	(0.40)	(0.54)
Overidentif. Restrictions (Sargan)	0.29	0.83	2.02	1.25

Note: Standard errors (robust to heteroscedasticity and first-order autocorrelation) are shown in parentheses.
Boldface (#) entries indicates coefficients that are significantly larger than 0 at the 5% (10%) level.
The instruments were the second lags of (per capita) domestic and external consumption, and domestic and external disposable income. Critical value of the Sargan test at the 5% level is 3.84 from $\chi_{(1)}$.

The last row in table 4.6 reports a Sargan test of whether the errors in equation (11) are orthogonal to the instruments. The reported values all imply that we can not reject the orthogonality hypothesis, so the model in equation (11) can be regarded as an adequate description of the data.

All coefficients on domestic disposable income (λ_i) have the expected positive sign, but it is significant at the 5 % level for Finland only (and close to for Denmark with a p-value on λ of 0.054). The ω_i-coefficients are significant at the 5 % level for Finland and Norway, and do also have the expected positive sign in the case of Sweden. For Denmark it is negative however, albeit clearly insignificant. None of the coefficients on external income (ϕ_i) are significantly different from 0.

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[13] For Finland, Norway and Sweden, we collected the data for per capita real private disposable income from the OECD national accounts. The full series is non-available for Denmark, so here we use real GDP as a proxy for disposable income.

Comparing our results to earlier studies, we first consider the λ-estimates and note that Bayoumi and MacDonald (1995), using the same sample period as us, report approximately similar estimates for Denmark and Finland as those presented in table 6a (Norway and Sweden are not included in that analysis). Further, our λ–estimate for Sweden is approximately similar to the estimate provided by Campbell and Mankiw (1991) for the period 1972:2-1988:1 (see table 2 of that paper) and also very close to the series of λ–estimates presented by Agell and Berg (1996). Finally, for Norway Boug *et al.* (1995) report insignificant λ-estimates for the period 1984:3-1994:4.

Bayoumi and MacDonald also estimate coefficients corresponding to our ω_i-coefficients. For Denmark, they report negative estimates both with respect to their broad OECD-sample and with respect to other EU-countries (members in 1991). This finding is strengthen by our table 4.6, which suggests that Denmark was not financially integrated with their Nordic neighbors once rule-of-thumb consumption behavior is taken into account. For Finland, Bayoumi and MacDonald's coefficient on external consumption is positive, but clearly insignificant. Because our coefficient on external (Nordic-) consumption is significant, this suggests that Finland's capital market is well integrated with the other Nordic countries, but imperfectly connected to other OECD nations. We suspect that Finland's close economic relations to particularly Sweden might be responsible, since Sweden is in our sample but not in Bayoumi and MacDonald's. (See also the high consumption growth correlation between Finland and Sweden in table 4.1.)

The overall impression from table 4.6 is that once myopic consumer behavior is taken into account, financial integration in the Nordic countries except Denmark seems fairly high. We can throw additional light on this issue by looking at similar regression equations for the period 1951-72. In table 4.7 we report the results from OLS estimation in this period.

We use OLS because we have great difficulties in identifying good instrument from our set of candidates (which was as for the 1973-92 period) for 1951-72. In all our attempts, the Wu-Hausman test is very far from rejecting that an eventual endogenity of Y_i has no effect on the consistency of the estimates. This indicates that OLS is more

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efficient than GMM in this case.[14] Since data on disposable income are unavailable for 1951-72 period, we rely on real GDP per capita as a proxy.

Table 4.7: OLS estimation of equation (11), 1951-72.

Country i:	Denmark	Finland	Norway	Sweden
λ_i	**0.47**	**0.79**	**0.50**	**0.52**
	(0.14)	(0.16)	(0.22)	(0.17)
ω_i	-0.11	0.68	0.47	0.28
	(0.40)	(0.74)	(0.37)	(0.20)
ϕ_i	0.61	-0.01	-0.03	-0.27
	(0.41)	(0.66)	(0.31)	(0.21)
Adjusted R^2	0.51	0.64	0.35	0.44
LM het. test	1.71	0.06	0.22	1.25
DW	2.39	1.87	2.12	2.07

LM test for contemporaneous correlation across equations = 12.51.

Note: Standard errors are shown in parentheses. **Boldface** entries indicate coefficients that are significantly larger than 0 at the 5% level.

We now observe significant λ_i-coefficients for all countries. In addition we see that none of the ω_i-coefficients are significant at the 5 % level. Comparing table 4.6 and 6b, we may conclude that the gradual reforms in domestic and global financial markets after 1970 have led to both increased consumption smoothing over time as well as increased real integration of financial markets in the Nordic region.

5. Final Remarks

This paper has analyzed the degree of financial integration in the Nordic countries based on their per capita private consumption patterns. The removal of formal restrictions on cross border capital flows during the last decades has led to a high degree of financial integration between financial *centers* in different countries. Our consumption based analysis sheds light on a more fundamental issue, however. The question is to what

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[14] We did run GMM for 1951-72, using the same instruments as in table 4.6. This exercise gives a significantly positive λ for Finland, while all other parameters are insignificant and extremely imprecise.

extent the liberalization of cross border capital flows as well as domestic credit markets has real implications for per capita consumption paths.

Our analysis indicates that the financial markets of Finland, Norway and Sweden – but to less extent Denmark - were fairly well integrated over the 1973-92 period. For Denmark, Finland, and Sweden we have also found evidence of myopic consumption behavior, in line with earlier research. This may be interpreted as the consequences of liquidity constraints in imperfect domestic credit markets. Finally, our analysis indicates that it is hard to identify uninsured external consumption risks.

We have seen that both the degree of financial integration and the relevance of forward-looking consumption behavior have increased in the Nordic region when we compare the period 1951-1972 with the period 1973-1992. Our choice to study the period 1973-1992 is clearly influenced by the studies of Obsfeld (1994) and Bayoumi and MacDonald (1995) which consider the same period. Still, it is tempting to ask whether we should have chosen a shorter and more recent period, e.g. after 1985, since we know that the financial markets were deregulated over a considerable time span and several restrictions in some of the Nordic countries prevailed into the early 1980s. In order to investigate whether a shorter time period may lead to stronger support to financial integration and forward-looking consumption behavior, we have experienced with varying sample periods in the different regressions. Our impression is that a shorter and more recent sample periods do not alter the results significantly, but low degrees of freedom makes it difficult to give strong conclusions.

Appendix

This appendix - which closely follows Obstfeld (1995) - briefly derives the relationship between equation (4) and equation (5) in the main text. If (4) is estimated directly for different combination of countries, it may cause econometric difficulties. A high realization of $\theta_{j,t}$ raises the marginal utility of country j's consumption in period t. Thus, country $j's$ consumption in (4) is likely to be positively correlated with $\theta_{j,t}$. This creates a potential endogenous-regressor problem. In order to reduce this problem, we

define $n_{i,t}$ as country i's share of the total Nordic population and $C_{No,t}$ as Nordic consumption per capita. This means that

$$C_{No,t} \equiv \sum_{j=1}^{5} n_{j,t} C_{j,t}. \tag{A.1}$$

Using (A.1), we rewrite equation (4) in the main text as

$$\log C_{i,t} = \log C_{No,t} + \log C_{i,0} + \frac{t}{\gamma} \cdot \log \beta_i$$
$$+ \left\{ \frac{\theta_{i,t}}{\gamma} - \log\left[\sum_j (\beta_j)^{\frac{t}{\gamma}} \cdot exp\left(\frac{\theta_{j,t}}{\gamma} \right) \cdot n_{j,t} \cdot C_{j,0} \right] \right\}. \tag{A.2}$$

Compared to the error term $\frac{1}{\gamma}(\theta_{i,t} - \theta_{j,t})$ in equation (4), it is more plausible that the composite error in the brackets in (A.2) is uncorrelated with $\log C_{No,t}$. This implies that the endogenous-regressor problem has been reduced. Taking first-differences in (A.2) yields equation (5) in the text when we remove country i from the aggregate consumption variable. If country i is not removed from the aggregate, we would probably face another endogenous-regressor problem since positive realizations of $\theta_{i,t}$ in many cases would be correlated with $C_{No,t}$ (particularly if country i is large).

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Chapter 5[*]

On Asymmetric Information across Countries

and the Home-Bias Puzzle

Abstract

This paper investigates the allocation decision of an investor who owns two projects, a domestic and a foreign one. A manager governs the expected return from each project, and the investor has less information on the actions of the foreign manager. The investor's portfolio will be tilted relative to a situation with full information. With asymmetric information, he generally achieves a better risk-return characteristic of his net terminal wealth with an allocation different from full diversification, because a "biased" allocation can be beneficial to the managers' efforts and/or risk properties of the optimal contracts. However, numerical simulations illustrate that, in general, the portfolio bias is small for plausible parameter values, and theoretically it may even be towards the foreign project. This weakens the case for asymmetric information as a prime reason for the observed home-bias in portfolio allocation.

1. Introduction

The bias towards domestic markets in international asset allocation, documented in e.g. French and Poterba (1991), Cooper and Kaplanis (1994), and Tesar and Werner (1995), has been one of the most extensively researched areas in international finance during the last 10-15 years. This behavior is labeled the "home-bias puzzle" since it is squarely at odds with the predictions of standard international portfolio selection models (e.g. Adler and Dumas, 1983), and the estimated gains in terms of the risk-return trade-off from international diversification risk appear to be substantial (Grauer and Håkansson, 1987).

A number of possible explanations of the home-bias puzzle have been discussed in the literature.[1] Among these is the hypothesis that there is asymmetric information across countries. Gehrig (1993) and Brennan and Cao (1997) explore the implications of investors, on average, being better informed about the risk-return characteristics of domestic stocks, and they show that this leads to home-bias in portfolio holdings. Gordon and Bovenberg (1996) analyze a "lemons" problem where foreigners systematically overpay for domestic firms - from the point of view of the domestic investors - because they cannot observe firm-specific shocks. Foreign investors may still gain from acquiring domestic firms since they face a lower cost of capital in this model, but capital mobility would be lower than with full information.

The present paper adds to this research by investigating a model where domestic investors have less information about the actions of managers in charge of their foreign projects, compared to the information available on domestic managers. The investors may still want to invest in foreign projects, however, because the return on the projects is partly a function of project specific shocks, which are imperfectly correlated. The extent to which they want to invest in foreign projects depends on the effort-level they can generate from foreign managers through optimal contracts.

Our paper differs from those above in that moral hazard creates the information problem, and optimal contracts are at the center stage of the analysis. We argue that hidden actions may be equally plausible as the information distortions analyzed in earlier research on the home-bias puzzle. Obstfeld and Rogoff (1996, p. 416) emphasize the importance of moral

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[1] Besides asymmetric information, the most common explanations are the existence of non-traded income risk, non-traded consumption goods, and statistical measurement problems. See Lewis (1999) for a recent survey.

hazard in explaining why friction-free models of international asset trade square so poorly with data, including portfolio allocation across countries.

It could be argued that foreign investors might avoid possible information disadvantages by investing in a diversified portfolio of publicly traded domestic firms. This would not eliminate the disadvantage, however, since only a subset of national firms list their shares on public exchanges and information available to domestic investors might possibly not be fully conveyed through market prices, due to noise in these prices. Another argument against the relevance of asymmetric information across countries is that foreign investors can hire local experts. Since those experts would be better informed than the investor, a serious asymmetric information problem would materialize in this case as well.[2] The model presented here is easiest interpreted as investments in individual firms rather than in diversified portfolios. A large fraction of purchases of foreign assets takes the form of direct investment and thus, studying such investment decisions in the context of the home-bias puzzle also seems relevant.

The model we develop draws on the dynamic principal-agent problems by Holmström and Milgrom (1987), Schättler and Sung (1993), Sung (1995), and Müller (1998). That is, we study a continuous-time model where output follows a Brownian motion and both the principal (the investor) and the agents (the managers) have constant absolute risk aversion. This is a natural point of departure since, unlike the static principal-agent model, the continuous-time version admits a simple closed-form solution: the optimal compensation schemes are linear in output. To this principal-agent problem, we add a portfolio problem on behalf of the principal who can invest his resources in a domestic and a foreign project. The managers control the drift rates of the Brownian motions governing output. It is easier for the investor to observe the actions of the domestic than those of the foreign manager. Specifically, domestic effort will be assumed to be perfectly observable while the effort put in by the foreign manager is unobservable.

Compared to a situation with full and symmetric information, the investor's allocation policy will be tilted. The mechanisms behind this can be explained by observing that the argument in the investor's (expected) utility function is $X_1 + X_1^* - (S + S^*)$, where X_1 [X_1^*] is output from his domestic [foreign] project and S [S^*] is the salary paid to the domestic [foreign] manager. Later we will refer to this expression as the investor's net terminal wealth.

[2] See the papers by Gehrig (1993), Gordon and Bovenberg (1996), and Brennan and Cao (1997) for further arguments in favor of the relevance of asymmetric information across countries.

In the first-best situation, the two managers will make the same effort for given invested resources in their respective projects. In the model, this implies that expected value of $W_1 \equiv X_1 + X_1^*$ is independent of the allocation of resources between the two projects. Moreover, both the expected value and standard deviation of total salary costs $(S + S^*)$ are independent of the allocation decision with full information. This leaves only the variability of W_1 as relevant for the allocation decision, and this is minimized by what we label 'full diversification'.

With asymmetric information, the domestic manager will make higher effort than the foreign manager for given invested resources. Hence, the expected value of W_1 can be increased by investing more resources at home than abroad. This will unambiguously contribute to a home-bias. However, with asymmetric information, the expected value and variability of salary costs $(S + S^*)$ are also affected by the allocation decision. For instance, the size of the constant amounts or what fraction of output the managers are to keep according to the salary contracts is in part determined by how the investor allocates his resources. The consequences for the allocation decision are ambiguous however. That is, it *may* be that allocating more resources to the domestic project make the optimal contracts less attractive as seen from the investor. If so, the effect on $(S + S^*)$ will counteract the home-bias induced by asymmetric information on the expected value of W_1. Indeed, it is theoretically possible that the optimal portfolio will be tilted towards the foreign project. But the opposite may also be the case; the effect on $(S + S^*)$ can strengthen the home-bias in portfolio allocation. Numerical simulations illustrate that, for plausible parameter values, the portfolio bias is likely to be towards the domestic project, but seems to be small in magnitude.

Section 2 presents the model and the solution to the first-best problem. The case of asymmetric information across countries is analyzed in section 3. In section 4 we present some numerical illustrations, and then conclude and discuss some possible extensions in section 5.

2. The Model and the Full Information Case

We investigate the principal-agent relationship on the time interval [0,1]. At time 0, the principal (the investor) decides how to allocate his initial resources W_0 to two projects, a domestic and a foreign project. The investment decisions are assumed to be irreversible; the

allocation is fixed until time 1.[3] The output from the projects is publicly observable and governed by the processes

$$dX_t = u_t X_0 dt + \sigma X_0 dz_t,$$

$$dX_t^* = u_t^* X_0^* dt + \sigma X_0^* dz_t^*,$$

for the domestic and foreign project, respectively. In these equations, X_0 $[X_0^*]$ is the amount invested in the domestic [foreign] project, so that $X_0 + X_0^* = W_0$. Furthermore, σ is the common diffusion parameter, while dz and dz^* are standard wiener processes representing shocks, which are considered as project-specific. The instantaneous correlation coefficient, ρ, of these shocks is obtained from $dz dz^* = \rho dt$, $\rho \in [-1,1)$. The drift variables u_t and u_t^* are controlled by a domestic and foreign agent (manager) respectively, and may or may not be observed by the investor. On basis of these assumptions, the investor's wealth accumulation equation can be written as

$$dW_t = \left[\omega(u_t - u_t^*) + u_t^* \right] W_0 dt + W_0 \mathbf{w}' \Sigma d\mathbf{B}_t, \qquad (1)$$

where

$$\Sigma = \begin{bmatrix} \sigma & 0 \\ \sigma\rho & \sigma\sqrt{1-\rho^2} \end{bmatrix},$$

ω is the fraction of initial wealth invested in the domestic project, $\mathbf{w}' = [\omega \; (1-\omega)]$, $d\mathbf{B} = [dz \; dh]'$, and dh a standard wiener process independent of dz.

At time 0, the investor and the mangers individually agree on sharing rules specifying payment from the investor to the managers at time 1. The sharing rules specify salaries S and S^* for the domestic and foreign manager, respectively, and are random via dependence on the outcome of the stochastic process for W. The managers' control variables, $u \geq 0$ and $u^* \geq 0$, can be revised continuously during the time interval $[0,1]$ and may depend on the history of W in $[0,t]$, but not on the future $(t,1]$. The managers incur costs for putting effort into the projects. For simplicity, these costs are assumed to be given by $\frac{1}{2}ku_t^2$ and $\frac{1}{2}k(u_t^*)^2$, respectively, where k is a constant. The important thing is that the effort costs are convex. Using the quadratic form makes the model easier to solve.

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[3] This assumption is imposed to obtain tractability, since allowing for continuous reallocation would introduce time-dependent drifts in the processes for X and X^*. Schättler and Sung (1997) show that introducing time-dependent drifts of the Brownian motions would destroy the result that sharing rules are linear in output, and thus also the tractability of the model.

Finally, both the investor and the managers have exponential time separable utility. The investor's constant coefficient of absolute risk aversion is R while the two managers are equally risk averse with a CARA-coefficient r.

For the sake of later comparison, let us first characterize the optimal sharing rules, effort levels, and resource allocation in the first-best setting; that is, when the managers' controls are observable and can be enforced at no cost. At time 0, the investor's first-best problem is

$$\max_{\{u,u^*\},S,S^*,\omega} E\left[-\exp\{-R(W_1 - S - S^*)\}\right],\tag{2}$$

subject to (1) and subject to the managers' participation constraints:

$$E\left\{-\exp\left[-r\left(S - \tfrac{1}{2}k\int_0^1 u_t^2 dt\right)\right]\right\} \geq -\exp\{-rU_0\},\tag{3}$$

$$E\left\{-\exp\left[-r\left(S^* - \tfrac{1}{2}k\int_0^1 (u_t^*)^2 dt\right)\right]\right\} \geq -\exp\{-rU_0\},\tag{4}$$

where U_0 is the managers' certainty equivalent at time 0, assumed to be identical for the domestic and the foreign manager. The solution to this problem is summed up in the first result.

Proposition 1: *Under full information, the salaries of the domestic and the foreign manager are equal, linear in combined output and given by*

$$S = S^* = K + \frac{R}{r + 2R}W_1,\tag{5}$$

where $K \equiv (r + 2R)^{-1}\left[\ln(\lambda r / R) - RW_0 + \tfrac{1}{2}rk\int_0^1 u^2 dt\right]$ *is a constant. Moreover, the effort levels are constant, equal across countries, and determined by the equality of marginal productivity of invested resources and marginal cost of effort*

$$\frac{\omega}{k}W_0 = u = u^* = \frac{(1-\omega)}{k}W_0.\tag{6}$$

Finally, the investor allocates equal amounts to the domestic and the foreign project,

$$\omega = \frac{u - u^*}{2W_0\sigma^2(1-\rho)R}\frac{(r+2R)}{r} + \frac{1}{2} = \frac{1}{2}.\tag{7}$$

The proof is in the appendix.

The optimal sharing rule given in (5) is very similar to the corresponding rule in the one-agent model of Müller (1998). One difference is that the coefficient before W_1 gives more weight to the principal's risk aversion, since he now shares the final output with two

agents. It also worth noting that the first-best sharing rules imply full risk sharing between the domestic and the foreign manager. They receive a fixed share of total output, independent of the relative output from the project of which they are in charge.

Constancy of u and u^* over time follows from the fact that ω is constant and that W_0 is given. Equal effort levels across countries follow by combing $ku = \omega W_0$ and $ku^* = (1-\omega)W_0$ with the expression on the right hand-side of the first equality in (7). In turn, this yields the result that the investor allocates equal amounts to the two projects.

The demand function on the right hand-side of the first equality in (7) warrants a comment. The first term here represents demand arising from potentially higher return on one of the projects. Relative to a standard CAPM (e.g. Adler and Dumas, 1983), this demand is adjusted by a factor $(r + 2R)/r$. The first term is 0 under full information because the optimal contract ensures equal effort levels across countries. The second term on the right hand-side of the first equality in (7) is the portfolio share that minimizes the variance of time 1 wealth. This is always equal to ½ in our case, because the instantaneous standard deviation σ is equal for the two projects. To sum up, with full information the investor chooses an allocation minimizing his wealth variance because the optimal contracts ensure equal expected returns for the two projects.

3. Moral Hazard in the Foreign Project

We now turn to the case where the investor cannot observe the actions of the foreign manager. Then, the investor faces an additional constraint in his problem:

$$u^* \in \underset{u^*}{\arg\max}\, E\left[-\exp\left\{ -r\left(S^* - \tfrac{1}{2}k\int_0^1 (u_t^*)^2 dt \right)\right\}\right].$$

This is the familiar incentive compatibility constraint, that is, the foreign manager chooses the u^* that is in his best interest. We follow Schättler and Sung (1993), and use the so-called first-order approach to solve the investor's problem. In this approach, the incentive compatibility constraint in the principal's problem is relaxed to the first-order necessary condition for optimality in the agent's problem.

We also make the simplifying assumption that the optimal salary of the foreign manager is contingent on his own output only. By introducing this assumption some generality is lost, but the model's tractability and its illustrative ability are preserved.[4]

Given these assumptions, the problem of the foreign manager is

$$\max_{u^*} E\left[-\exp\left\{-r\left(S^* - \tfrac{1}{2}k\int_0^1 (u_t^*)^2\,dt\right)\right\}\right],$$

subject to

$$dX_t^* = u_t^*(1-\omega)W_0\,dt + W_0\mathbf{w}^{*\,\prime}\Sigma d\mathbf{B}_t,$$

where $\mathbf{w}^{*\prime} = [0\ (1-\omega)]$. Schättler and Sung (1993) show that the solution to this problem implies the optimal sharing rule to be of the following form (using our notation):

$$S^* = U_0 + \tfrac{1}{2}k\int_0^1 (u_t^*)^2\,dt + k\int_0^1 u_t^*(1-\omega)^{-1}\mathbf{w}^{*\prime}\Sigma d\mathbf{B}_t + \tfrac{1}{2}r\int_0^1\left[ku_t^*(1-\omega)^{-1}\right]^2\mathbf{w}^{*\prime}\Sigma\Sigma'\mathbf{w}^*\,dt. \quad (8)$$

The first two terms in (8) provides the foreign manager with his certainty equivalent plus the compensation for the cost he actually incurs. The next term is the compensation error, arising because the investor's compensation is based on realized outcome rather than the manager's actual effort. Finally, to compensate the foreign manager for the risk he carries, a risk premium is paid, given by the last term in (8).[5]

The investor's relaxed problem can then be written as

$$\max_{u,u^*,S,\omega} E\left[-\exp\{-R(W_1 - S - S^*)\}\right],$$

subject to (1), (3) and (8). The solution to this problem is summarized in the second result.

Proposition 2: *Suppose that the foreign manager's effort level cannot be observed and that his salary depends on his own output only. The salary of the foreign manager is linear in his own output and is given by*

$$S^* = \kappa^* + \frac{ku^*}{(1-\omega)W_0}X_1^*, \quad (9)$$

where $\kappa^* \equiv U_0 - ku^* - \tfrac{1}{2}ku^{*2} + \tfrac{1}{2}rk^2\sigma^2 u^{*2}$ *is a constant. The salary of the domestic manager depends on combined output:*

$$S = \kappa + \frac{R}{r+R}X_1 + \frac{R}{r+R}\left(1 - \frac{ku^*}{(1-\omega)W_0}\right)X_1^*, \quad (10)$$

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[4] Actually, the model is solvable if S^* is made a function of W_1 in this section also, but the central asset demand equation turns out to be a polynomial of degree 4, giving very little economic insight.

[5] See Holmström and Milgrom (1987) or Schättler and Sung (1993) for further discussion of the optimal sharing rule under asymmetric information.

where $\kappa \equiv (r + R)^{-1}[\ln(\lambda r/R) - RW_0 + \frac{1}{2}rku^2 - R\kappa^*]$ *is another constant. The optimal effort level for both the domestic and the foreign manager is constant over time and given by*

$$u = \frac{\omega}{k}W_0 \tag{11}$$

$$u^* = \frac{(1-\omega)}{k}W_0\frac{r+R+rR\sigma^2}{r+R+(r^2+2rR)\sigma^2} + \frac{\omega}{k}W_0\frac{rR\sigma^2\rho}{r+R+(r^2+2rR)\sigma^2}, \tag{12}$$

respectively. Finally, the investor allocates a fraction

$$\omega = \frac{u - u^*}{2W_0\sigma^2(1-\rho)R}\frac{r+R}{r} - \frac{ku^*}{2W_0} + \frac{1}{2} \tag{13}$$

of initial wealth to the domestic project.

The proof is in the appendix.

From equation (9), we notice that the investor's share of foreign final output depends on the choices of u^* and ω, while with full information, it is determined solely by the parameters r and R. For a given portfolio allocation, equation (12) implies that the effort level is lower than in the first-best situation (confer equation (6)). Finally, for given effort levels, the asset demand function (13) differs from the first-best by the second term on the right hand-side, which is the foreign manager's marginal cost of effort (divided by $2W_0$). The higher is the marginal cost, the more resources are allocated to the foreign project (given u and u^*), because a high marginal cost discourages effort, but the investor can counteract this by investing more resources in the foreign project, as can be seen from (12).

Equations (11)-(13) can solely be expressed in terms of the parameters of the model, R, r, k, σ, and ρ, and these solutions will be presented below. However, some interesting implications are apparent already at this stage.

Suppose for instance that, as under full information, the parameters imply that the investor chooses the same allocation as under full information, $\omega = \frac{1}{2}$. If so, equations (11) and (12) imply

$$\frac{u^*}{u} = \frac{r+R+(1+\rho)rR\sigma^2}{r+R+2rR\sigma^2+r^2\sigma^2} < 1.$$

With full diversification, the foreign manager puts in a lower effort than his domestic counterpart. As a second example, suppose that the parameters are such that the domestic and the foreign manager make the same effort. With $u = u^*$, equations (11) and (13) imply that

$\omega = 1/3$. Hence, to generate the same effort level from the two managers, the investor has to invest twice as much in the foreign project as in the domestic one.[6]

These examples illustrate that the investor can increase expected output by investing more in the domestic project than in the foreign one. More formally; from (11) we find that $\partial u/\partial \omega = W_0/k$, while $|\partial u^*/\partial \omega| < W_0/k$ from (12). Hence, by reallocating resources from the foreign to the domestic project, the effort made by the domestic manager increases more than the corresponding decrease in the foreign manager's effort. This clearly contributes to a home-bias under asymmetric information. However, the investor does not care about final output only, but also about how much of it he has to pay the managers according to the optimal contracts. Now, since these contracts are influenced by the allocation of resources, we must take this into account when the optimal allocation policy is derived.

Solving equations (11)-(13) for u, u^*, and ω, we obtain

$$u = (kN)^{-1}\left[(r+R)^2 + rR\sigma^2\left((r+R)(1-k\sigma^2(1-\rho))\right)\right]W_0 \qquad (14)$$

$$u^* = (kN)^{-1}\left[(r+R)^2 - kR^2\sigma^2(1-\rho) + rR\sigma^2\left((r+R)-k(1-\rho)-kR\sigma^2(1-\rho^2)\right)\right]W_0 \qquad (15)$$

$$\omega = (kN)^{-1}\left[(r+R)^2 + rR\sigma^2\left((r+R)(1-k\sigma^2(1-\rho))\right)\right], \qquad (16)$$

where

$$N = rR\sigma^2\left[r(4-\rho)+R(3-\rho)-k(1-\rho)-2kR\sigma^2\left(\tfrac{3}{2}-\rho-\tfrac{1}{2}\rho^2\right)-2kr\sigma^2(1-\rho)\right]$$
$$+2(r+R)^2 + r^3\sigma^2 - kR^2\sigma^2(1-\rho).$$

The third result follows immediately.

Proposition 3: If $r^3\sigma^2 - kR^2\sigma^2(1-\rho) < rR\sigma^2[kR\sigma^2(1-\rho^2) - r(2-\rho) - R(1-\rho)]$, *the allocation under asymmetric information is tilted towards the domestic project relative to the first-best allocation.*

Proof: Follows immediately by imposing $\omega > \frac{1}{2}$ in (16) and simplifying.

Interestingly, this result implies that we cannot rule out a "foreign-bias" where $\omega > \frac{1}{2}$. But in general, the condition in proposition 3 is too complex to give any general statements on when a home-bias is most likely to occur in the model. The reason for this ambiguity is that, as

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[6] There are two exceptions to these statements. If either σ or r approaches 0, the foreign effort level u^* will approach its first-best, see equation (12). In these cases we would have $u = u^*$ and $\omega = \frac{1}{2}$ even with asymmetric information. See also the numerical examples presented in the next section.

opposed to a standard portfolio selection model, the optimal allocation decision is the result of interactions with the optimal actions of the managers and the optimal compensation schemes. To get a hinge on these interactions and what conditions are most likely to create a home-bias, we next provide some numerical simulations.

4. Numerical Examples

4.1 A Baseline Example

The intuition in the following numerical illustrations is somewhat simplified by considering the case where the return shocks to the projects are uncorrelated, $\rho = 0$. Notice in particular that from (12), the optimal sharing rules under asymmetric information can be expressed as

$$S = \kappa + AX_1 + BX_1^*, \tag{17}$$

$$S^* = \kappa^* + CX_1^*. \tag{18}$$

where $A \equiv R/(r+R)$, $B \equiv rR\sigma^2/[r+R+(r^2+2rR)\sigma^2]$, and $C \equiv (r+R+rR\sigma^2)/[r+R+(r^2+2rR)\sigma^2]$ are all constants. Hence, a fraction $1 - A = r/(r+R)$ of domestic final output and a fraction $1 - B - C = r^2\sigma^2/[r+R+(r^2+2rR)\sigma^2]$ of foreign final output are retained by the investor.

Let us think of the [0,1] time interval as one year. Then, σ gives the annual standard deviation of returns of the two projects. $\sigma = 0.25$ is used as the baseline value. To obtain sensible annual expected returns, we furthermore set $k = 5$. In principal-agent problems, the agents are commonly assumed to be more risk-averse than the principal. In this baseline experiment, we stick to this assumption, using $r = 4$ and $R = 2$ for the coefficients of absolute risk-aversion. Finally, we assume a zero certainty equivalent ($U_0 = 0$) on behalf of the managers and that the investor enters the year with wealth $W_0 = 1$.

By proposition 1, the first-best situation in the baseline experiment is characterized by $u = u^* = 0.1$, and $\omega = 0.5$. Both managers receive $R/(r+2R) = 25\%$ of final total output plus/minus the constant amount $K = \ln\lambda/8 - 0.15$. The investor thus keeps half of the final output from the projects in addition to the constant amount $2K = 0.30 - \ln\lambda/4$. In turn, this implies the investor's expected net terminal wealth to be $E[W_1 - S - S^*] = 1.33 - \ln\lambda/4$, with a standard deviation of $SD[W_1 - S - S^*] = 0.118$.

When the foreign manager's effort level is unobservable, equations (14)-(18) can be used to characterize the solution. Table 5.1 reports the results.

u	u^*	ω	κ	κ^*	A	B	C	$E[W_1-S-S^*]$	$SD[W_1-S-S^*]$
0.111	0.071	0.557	$\ln\lambda/6-0.08$	-0.35	1/3	0.066	0.801	$0.92-\ln\lambda/6$	0.094

Note: Calculations are based on the parameter values: $r = 4$, $R = 2$, $k = 5$, $\sigma = 0.25$, $\rho = 0$.

The foreign manager's second-best effort level implies that the expected return in his project drops to 7.1%. Together with an increase in expected return from the domestic project to 11.1%, this creates a home-bias. The domestic manager's effort level is higher than in the first-best situation, even though the investor can observe his actions. This is due to the fact that the marginal productivity of invested resources in the domestic project, which is linear in the amount invested, is higher than in the first-best and thus, the marginal cost of effort should be higher, implying increased effort.

The foreign manager's salary is $S^* = -0.35 + 0.801X_1^*$ while the compensation to the domestic manager is $S = \ln\lambda/6 - 0.08 + 1/3X_1 + 0.066X_1^*$. Thus, the investor acquires only 12.3 % of the output from the foreign project, while he retains 2/3 of domestic output. In addition, the investor pays a constant amount $\kappa + \kappa^* = \ln\lambda/6 - 0.49$ to the managers. (Notice that the constant amount may very well be negative. That is, the managers may pay the investor to get a share of the final output). Compared to the first-best situation, the standard deviation of expected net terminal wealth falls, which contributes to explain why the investor diversifies less than with full information: Optimal contracts contribute to lower wealth variance, and his need for diversifying the asset portfolio is thus smaller.

4.2 Sensitivity Analysis

In this baseline experiment, asymmetric information across countries generates a small home-bias in portfolio allocation. How sensitive is this result to our parameter assumptions? In this subsection, we try to answer this question by performing some simple sensitivity analyses.

We start by considering changes in the underlying uncertainty of the projects, σ. If the effort levels were given, equation (13) shows that an increase in σ would mitigate the importance of the expected difference in return for the portfolio allocation decision. However, in our model, the expected difference in return itself is affected by a shift in σ. Moreover, the optimal salary contracts will also be affected, giving further bearings on the

asset allocation decision. Table 5.2 reports the characteristics of the asymmetric information solution for different values of σ.

Table 5.2: Sensitivity of the asymmetric information solution for changes in σ.

σ	u	u^*	ω	κ	κ^*	A	B	C	$E[W_1-S-S^*]$	$SD[W_1-S-S^*]$
$\to 0$	0.1	0.1	0.5	$\ln\lambda/6{-}0.03$	-0.53	1/3	0	1	$0.92{-}\ln\lambda/6$	0
0.1	0.101	0.095	0.507	$\ln\lambda/6{-}0.04$	-0.49	1/3	0.01	0.96	$0.92{-}\ln\lambda/6$	0.034
0.25	0.111	0.071	0.557	$\ln\lambda/6{-}0.08$	-0.35	1/3	0.07	0.80	$0.92{-}\ln\lambda/6$	0.094
0.35	0.143	0.040	0.716	$\ln\lambda/6{-}0.12$	-0.19	1/3	0.10	0.70	$0.91{-}\ln\lambda/6$	0.168

Note: Other parameters than σ have the same values as in table 5.1.

As intuition suggests, the solutions for u, u^* and ω approach first-best as σ goes to 0. For large values of σ, the expected difference in return, $u - u^*$, is indeed substantial, thereby leading to a significant home-bias. Notice also that the investor's fraction of foreign output ($1 - B - C$) increases with σ, while the constant amount he receives from the foreign manager, κ^*, decreases. Together with less diversification and the increase in σ itself, this contributes to increase the standard deviation of net terminal wealth.

Table 5.3: Sensitivity of the asymmetric information solution for changes in r.

r	u	u^*	ω	κ	κ^*	A	B	C	$E[W_1-S-S^*]$	$SD[W_1-S-S^*]$
0.01	0.1	0.1	0.5	$\ln\lambda/2.01{-}3.1$	-0.53	≈ 1	≈ 0	≈ 1	$3.5{-}\ln\lambda/2.01$	≈ 0
0.5	0.105	0.092	0.525	$\ln\lambda/2.5{-}0.97$	-0.48	0.8	0.03	0.97	$1.57{-}\ln\lambda/2.5$	0.026
1	0.108	0.087	0.541	$\ln\lambda/3{-}0.59$	-0.45	2/3	0.03	0.95	$1.27{-}\ln\lambda/3$	0.045
4	0.111	0.071	0.557	$\ln\lambda/6{-}0.08$	-0.35	1/3	0.07	0.80	$0.92{-}\ln\lambda/6$	0.094
6	0.108	0.068	0.541	$\ln\lambda/8{-}0.07$	-0.33	1/4	0.06	0.74	$0.95{-}\ln\lambda/8$	0.104
10	0.097	0.066	0.484	$\ln\lambda/12{-}0.04$	-0.31	1/6	0.06	0.64	$0.96{-}\ln\lambda/12$	0.108

Note: Other parameters than r have the same values as in table 5.1.

Table 5.3 reports on the solution with asymmetric information for different values of the managers' risk-aversion, r. In the same manner as in table 5.2, a small r gives solutions for u, u^* and ω close to the first-best. Moreover, the optimal contracts are such that managers carry the entire output risk, giving a certain terminal net wealth to the investor. For moderate values of r, the investor chooses to allocate more to his domestic project. This effect is reversed for larger values of r, and from table 5.3, it appears that $r = 10$ implies foreign bias in portfolio allocation. One of the mechanisms behind this is apparent from table 5.3. The

fraction of foreign output the domestic manager obtains, increases with r for low values of this parameter. As r increases, this fraction starts falling, however, making the foreign project more attractive for the investor. The standard deviation of the investor's net terminal wealth increases with r, as he must accept a larger part of the output-risk, the more risk-averse are the managers.

Table 5.4 reports the solution of the model for different values of R. The foreign manager's optimal effort level decreases with R as does the fraction of foreign output retained by the investor $(1 - B - C)$. Both effects make the foreign project less attractive. Thus, even though increasing risk-aversion should strengthen the case for diversification, the portfolio allocation implies a considerable home-bias for large values of R. We also notice that despite less diversification, the standard deviation of the net terminal wealth decreases with R, since the optimal contracts imply that the managers take more and the investor less output-risk, the higher is the investor's risk-aversion.

Table 5.4: Sensitivity of the asymmetric information solution for changes in R.

R	U	u^*	ω	κ	κ^*	A	B	C	$E[W_1-S-S^*]$	$SD[W_1-S-S^*]$
0.01	0.089	0.088	0.445	$\ln\lambda/4.01+1.5$	-0.44	≈ 0	≈ 0	0.79	$-0.47-\ln\lambda/4$	0.115
0.5	0.095	0.085	0.474	$\ln\lambda/4.5+0.42$	-0.42	$1/9$	0.02	0.80	$0.56-\ln\lambda/4.5$	0.108
1	0.101	0.080	0.503	$\ln\lambda/5+0.18$	-0.40	0.2	0.04	0.81	$0.75-\ln\lambda/5$	0.102
2	0.111	0.071	0.557	$\ln\lambda/6-0.08$	-0.35	$1/3$	0.06	0.81	$0.91-\ln\lambda/6$	0.094
10	0.172	0.023	0.862	$\ln\lambda/14-0.68$	-0.11	0.71	0.12	0.83	$1.09-\ln\lambda/14$	0.062

Note: Other parameters than R have the same values as in table 5.1.

5. Discussion and Concluding Comments

We have studied the allocation decision of an investor with a foreign and a domestic project. If he can observe the actions of the manager in charge of the domestic project, but not those of the foreign manager, his allocation will be tilted relative to the full information situation. In the model, the investor's allocation decision affects the managers' effort levels, and the form of the contracts. These effects are generally different under asymmetric information as compared to the first-best situation, and generally imply that the investor achieves a better risk-return characteristic of his net terminal wealth with an allocation different from full diversification.

Four different effects determines the desirability of a given allocation decision:

1. The effect on expected final output. This effect is due to the impact the allocation decision has on optimal effort levels from the two managers.

2. The effect on the standard deviation of final output. This is a direct effect of the portfolio allocation decision.

3. The effect on the fraction of final output retained by the investor. This is due both to the portfolio decision itself and to the indirect effect on the effort level of the foreign manager (see equation (9)).

4. The effect on the constant amounts that the investor receives from/pays to the managers. This effect is due to induced changes in optimal effort levels.

Under full information, effect 3 is irrelevant since the fraction of final output that investor retains is determined solely by the parameters r and R. Moreover, effects 1 and 4 are zero because $du^*/du = -1$ with full information (see equation (6)). That is, a certain reallocation of the portfolio from, e.g., the foreign to the domestic project decreases the optimal effort level of the foreign manager by exactly the same amount as the increase in the domestic manager's effort level. Then, expected final output will be independent of portfolio allocation, as will be the *total* constant amount that the investor pays/receives. This leaves only effect 2 as the relevant under full information. The standard deviation of final output is minimized by full diversification, which the investor accordingly chooses.

The story is a different one under asymmetric information, when all effects above come into play. Investing solely in the domestic project maximizes expected final output (effect 1). This can be seen from equations (11) and (12) which imply that $du^*/du < -1$. I.e., when the investor increases investment in the domestic project, the higher expected return from the domestic project more than outweighs the corresponding fall in expected returns from the foreign project. Effect 1 thus pull in the direction of a home-bias under asymmetric information. Effect 2 counteracts this, but can not offset it. The effect on the fraction of foreign output that is retained by the investor is ambiguously affected by the allocation decision, as is the effect on the total constant amount received/paid by the investor. Hence effects 3 and 4 may or may not counteract the home-bias induced by effect 1. In certain cases, the combined effect may be such that the investor tilts his portfolio towards the foreign project.

Although there is a theoretical possibility that asymmetric information may imply 'foreign-bias' in the model, the numerical illustrations presented in section 4 generate a home-

bias more often than the opposite. However, the home-bias is often small in magnitude, leading us to conclude that the information structure investigated in this model is unlikely to explain the bias towards domestic markets observed in the data.

This model is, in effect, a static portfolio allocation model where the investor can influence the expected returns from the assets through his allocation decisions. It would be interesting to explore whether our result of an ambiguous effect from introducing asymmetric information was to survive a more general setup. A first extension would be to allow for continuous reallocation on behalf of the investor. What would then be the effect on the optimal allocation decisions under asymmetric information? A second, and perhaps more interesting, extension is to consider a situation with one investor in each country owning a (divisible) domestic project. With imperfectly correlated shocks, what would be the resulting equilibrium prices and allocation when the investors meet to trade, given that they know less about the actions of foreign managers? What about managerial compensation and expected output compared to a situation where the domestic project has a domestic owner only? These and other questions are left open to future research.

Appendix

A.1 Proof of Proposition 1

We start by deriving the optimal sharing rules in terms of the optimal controls u and u^*, following Müller (1998). Define net compensation to the domestic and foreign manager as

$$y = S - \tfrac{1}{2}k\int_0^1 u_t^2 dt \quad \text{and} \quad y^* = S^* - \tfrac{1}{2}k\int_0^1 (u_t^*)^2 dt,$$

respectively. Then, integrating (1) and inserting the result in (2) imply that the investor's problem can be expressed as

$$\max_{u,u^*,y,y^*,\omega} E\left[-\exp\left\{ -R\left(\begin{array}{l} W_0 + W_0\mathbf{w}'\Sigma(\mathbf{B}_1 - \mathbf{B}_0) - y - y^* \\ + \int_0^1 [\omega W_0 u_t - \tfrac{1}{2}ku_t^2 + (1-\omega)W_0 - \tfrac{1}{2}k(u_t^*)^2]dt \end{array} \right) \right\} \right],$$

subject to (3) and (4). Pointwise maximization with respect to y and y^* gives the first-order conditions

$$y = \frac{1}{r+R}\ln\left(\frac{\lambda r}{R}\right) + \frac{R}{r+R}\left[W_1 - W_0 - \tfrac{1}{2}k\int_0^1 (u_t^2 + (u_t^*)^2)dt\right] - \frac{R}{r+R}y^*$$

$$y^* = \frac{1}{r+R}\ln\left(\frac{\lambda^* r}{R}\right) + \frac{R}{r+R}\left[W_1 - W_0 - \tfrac{1}{2}k\int_0^1 (u_t^2 + (u_t^*)^2)dt\right] - \frac{R}{r+R}y,$$

where λ and λ^* are the Lagrange-multipliers associated with (3) and (4), respectively. Solving these two equations for y and y^* and using (3) and (4) to demonstrate that $\lambda = \lambda^*$, we find that

$$y = y^* = \frac{1}{r+2R}\ln\left(\frac{\lambda r}{R}\right) + \frac{R}{r+2R}\left(W_1 - W_0 - \tfrac{1}{2}k\int_0^1[u_t^2 + (u_t^*)^2]dt\right). \tag{A.1}$$

The optimal sharing rules are $S = y + \tfrac{1}{2}k\int_0^1 u_t^2 dt$ and $S^* = y + \tfrac{1}{2}k\int_0^1 (u_t^*)^2 dt$, where y is given in (A.1).

By substituting the optimal salary functions into (2), the investor's problem can be simplified to

$$\max_{u,u^*,\omega} E\left[-\exp\left\{-a\left(W_1 - b - \tfrac{1}{2}k\int_0^1[u_t^2 + (u_t^*)^2]dt\right)\right\}\right]$$

subject to (1), where $a \equiv rR/(r+2R)$ and $b \equiv (r/2R)W_0 - (r/2)\ln(\lambda r/R)$ are constants. Let $V(t,W_t)$ be the investor's value function, giving the optimal remaining utility at time t. By Lemma A1 in Sung (1995), the value function solving the above problem satisfies the following dynamic programming equation:

$$0 \equiv \frac{\partial V}{\partial t} + \max_{u,u^*,\omega}\left\{\begin{array}{l} \dfrac{\partial V}{\partial W}[\omega u + (1-\omega)u^*]W_0 + \tfrac{1}{2}\dfrac{\partial^2 V}{\partial W^2}W_0^2\mathbf{w'\Sigma\Sigma'w} \\ +\tfrac{1}{2}ak[u^2 + (u^*)^2]V(t,W_t) \end{array}\right\}, \tag{A.2}$$

with the terminal condition being $V(t,W_t) = -\exp[-a(W_1 - b)]$. From (A.2), the first-order conditions with respect to u_t, u_t^*, and ω are

$$u_t = -\frac{\partial V/\partial W_t}{V(t,W_t)a}\frac{\omega}{k}W_0 \tag{A.3}$$

$$u_t^* = -\frac{\partial V/\partial W_t}{V(t,W_t)a}\frac{(1-\omega)}{k}W_0 \tag{A.4}$$

$$\omega = \frac{u^* - u}{2W_0\sigma^2(1-\rho)}\frac{\partial V/\partial W}{\partial^2 V/\partial W^2} + \frac{1}{2}. \tag{A.5}$$

Next, we guess that the value function has the form

$$V(t,W_t) = -\exp\left\{-a\left[\begin{array}{l} W_t + (1-t)\left(\omega W_0 u + (1-\omega)W_0 u^*\right) \\ +\tfrac{1}{2}\left(aW_0^2\mathbf{w'\Sigma\Sigma'w} - ku^2 - ku^{*2}\right) \end{array}\right]\right\}. \tag{A.6}$$

Using (A.6) in (A.3)-(A.5), we obtain equations (5)-(7). Finally, substituting (A.6) into (A.2) confirms that (A.6) solves the investor's dynamic problem.

A.2 Proof of Proposition 2

We can proceed as under full information to find the optimal sharing rule between the investor and the domestic manager in terms of the optimal control u. The first-order condition with respect to S for the investor's relaxed problem is thus

$$S = \frac{1}{r+R}\ln\left(\frac{\lambda r}{R}\right) + \frac{R}{r+R}(W_1 - W_0 - S^*) + \frac{r}{r+R}\tfrac{1}{2}k\int_0^1 u_t^2 dt. \tag{A.7}$$

Given the optimal sharing rules in (8) and (A.7), the investor's (stochastic) net terminal wealth can be expressed as

$$W_1 - S - S^* = \frac{r}{r+R}\left[W_1 - \frac{1}{r}\ln\left(\frac{\lambda r}{R}\right) - \frac{R}{r}W_0 - V_0 - \tfrac{1}{2}k\int_0^1[u_t^2 + (u_t^*)^2]dt \right.$$
$$\left. -k\int_0^1 u_t^*(1-\omega)^{-1}\mathbf{w}^{*}{}'\Sigma d\mathbf{B}_t - \tfrac{1}{2}rk\int_0^1[u_t^*(1-\omega)^{-1}]^2\mathbf{w}^{*}{}'\Sigma\Sigma'\mathbf{w}^* dt \right]$$

It follows that the investor's problem can be reduced to

$$\max_{u,u^*,\omega} E\left[-\exp\left\{-\alpha\left(\begin{matrix}W_1 - \beta - k\int_0^1 u_t^*(1-\omega)^{-1}\mathbf{w}^{*}{}'\Sigma d\mathbf{B}_t \\ -\tfrac{1}{2}k\int_0^1[u_t^2 + (u_t^*)^2 + \tfrac{1}{2}r[u_t^*(1-\omega)^{-1}]^2\mathbf{w}^{*}{}'\Sigma\Sigma'\mathbf{w}^*]dt\end{matrix}\right)\right\}\right],$$

subject to (1), and where $\alpha \equiv rR/(r+R)$ and $\beta \equiv (1/r)\ln(\lambda r/R) - (R/r)W_0 + U_0$ are constants.

The dynamic programming equation becomes

$$0 \equiv \frac{\partial V}{\partial t} + \max_{u,u^*,\omega}\left\{\begin{matrix}\frac{\partial V}{\partial W}[W_0(\omega u + (1-\omega)u^*) + \alpha k u^*(1-\omega)^{-1}W_0\mathbf{w}'\Sigma\Sigma'\mathbf{w}^*] \\ +\tfrac{1}{2}\frac{\partial^2 V}{\partial W^2}W_0^2\mathbf{w}'\Sigma\Sigma'\mathbf{w} \\ +\tfrac{1}{2}\alpha k V(t,W_t)[u^2 + u^{*2} + (r+\alpha)(u^*(1-\omega)^{-1})^2\mathbf{w}^{*}{}'\Sigma\Sigma'\mathbf{w}^*]\end{matrix}\right\}$$

with the terminal condition being $V(t,W_t) = -\exp[-\alpha(W_1 - \beta)]$. Writing out the matrices, the dynamic programming equation can be rewritten in a somewhat simpler form:

$$0 \equiv \frac{\partial V}{\partial t} + \max_{u,u^*,\omega}\left\{\begin{matrix}\frac{\partial V}{\partial W}W_0[\omega u + (1-\omega)u^* + \alpha k u^*(1-\omega(1-\rho)\sigma^2)] \\ +\frac{1}{2}\frac{\partial^2 V}{\partial W^2}W_0^2\sigma^2(1 + 2\omega^2(1-\rho) - 2\omega(1-\rho)) \\ +\tfrac{1}{2}V(\cdot)\alpha k[u^2 + u^{*2} + (r+\alpha)u^{*2}\sigma^2]\end{matrix}\right\}. \tag{A.8}$$

The first-order conditions with respect to u, u^*, and ω, respectively, read:

$$u = \frac{-\partial V/\partial W_t}{V(\cdot)\alpha}\frac{\omega}{k}W_0 \tag{A.9}$$

$$u^* = \frac{-\partial V/\partial W}{V(\cdot)\alpha}W_0\left(\frac{(1-\omega)}{k}\frac{1+\alpha\sigma^2}{1+(r+\alpha)\sigma^2} + \frac{\omega}{k}\frac{\alpha\sigma^2\rho}{1+(r+\alpha)\sigma^2}\right) \tag{A.10}$$

$$\omega = \frac{\partial V / \partial W}{2W_0 (\partial^2 V / \partial W^2)} \left(\frac{u^* - u}{\sigma^2 (1 - \rho)} - \alpha k u^* \right) + \frac{1}{2}. \qquad (A.11)$$

We use

$$V(t, W_t) = -\exp\left\{ -\alpha \left[W_t - \beta + (1 - t) \begin{pmatrix} W_0 \left[\omega u + (1 - \omega) u^* + \alpha k u^* (1 - \omega(1 - \rho)) \sigma^2 \right] \\ -\tfrac{1}{2} \alpha W_0^2 \sigma^2 \left[1 + 2\omega^2 (1 - \rho) - 2\omega(1 - \rho) \right] \\ -\tfrac{1}{2} k \left[u^2 + u^{*2} + (r + \alpha) u^{*2} \sigma^2 \right] \end{pmatrix} \right] \right\}$$

as a trial solution for the value function. Taking the appropriate derivatives and substituting into (A.9)-(A.11) gives (11)-(13). Equations (9) and (10) are obtained by combining (11) and (12) with (8) and (A.7) respectively. Substituting the trial solution into (A.8) confirms that it solves the dynamic programming equation.

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