Reflections about pseudo-dual prices in combinatorial auctions

Andreas Drexl, Kurt Jørnsten

Institut für Betriebswirtschaftslehre, Christian-Albrechts-Universität, Kiel, Germany, andreas.drexl@bwl.uni-kiel.de

Institute for Research in Economics and Business Administration, Norges Handelshøyskole, Bergen, Norway, kurt.jornsten@nhh.no

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Abstract

Combinatorial auctions permitting bids on bundles of items have been developed to remedy the exposure problem associated with single-item auctions. Given winning bundle prices a set of item prices is called market clearing or equilibrium if all the winning (losing) bids are greater (less) than or equal to the total price of the bundle items. However, the prices for individual items are not readily computed once the winner determination problem is solved. This is due to the duality gap of integer programming caused by the indivisibility of the items. In this paper we reflect on the calculation of approximate or pseudo-dual item prices. In particular, we present a novel scheme based on the aggregation of winning bids. Our analysis is illustrated by means of numerical examples.

Keywords: Combinatorial auctions, set packing, dual prices

1 Introduction

Combinatorial auctions are auctions where single bids on multiple distinct items are allowed. Single item auctions have been the topic of intensive research for many years and particular incentive compatible efficient auctions have been developed. In some markets, however, a participant's valuation of an item depends significantly on which other items the participant acquires. Items can be substitutes or complements, and the valuation of a particular bundle of items may not be equal to the sum of the valuations of the individual items, that is, valuations are not additive. In this setting, economic efficiency is increased by allowing bidders to bid on combinations of items, which is exactly what a combinatorial auction does. Due to this increased economic efficiency combinatorial auctions have become the focus of extensive research in recent years.

One major obstacle in the design of combinatorial auctions is the solution of the winner determination problem. Winner determination is equivalent to the weighted set packing problem which belongs to the class of NP-hard integer programs (for a detailed exposition of this issue see Rothkopf et al. 1998). In practice combinatorial auctions usually are applied in a multi-round setting. During each round, bidders submit bids on packages and then the auctioneer determines a provisional allocation of bundles to bidders. In this case approximate dual information may be useful for bidders as approximate marginal values that enable bidders to bid more efficiently in subsequent rounds.

When the linear programming relaxation of the winner determination problem for a combinatorial auction does not possess the integrality property there does not exist a linear price function that supports the optimal allocation of winning bundles.

In these situations an often used method is to adopt pseudo-dual prices, that is, prices that are in some sense close to the prices obtained for a pure linear program. The way these pseudo-dual prices are constructed are based on the following basic ideas:

1. The winning bundles should have reduced cost equal to zero. A standard requirement for a linear program based on linear programming duality theory is that a basic variables reduced cost should be equal to zero.

2. For the non-winning bids the item prices should ideally have the property that all non-winning bids are priced out, i.e. the reduced costs for these bids should be non-negative. However, in the general case when the linear programming relaxation does not yield an integral solution this is unachievable. The approximation made in these cases in order to obtain an approximate linear price function is to require that as many as possible of the non-winning bids are priced out or, alternatively, that the maximum deviation for a linear price to price out the non-winning bids is minimal.

3. As in linear programming it is often required that prices for constraints that have slack in the optimal solution yield an item price of zero.

All these requirements can be interpreted as requiring primal feasibility, primal complementary slackness, dual feasibility, and dual complementary slackness.

The exposition of our work is as follows: In section 2 we introduce the winner determination problem. In section 3 properties of dual prices are formally defined. In section 4 first we reflect on different alternatives for the construction of pseudo-dual prices and then we present a novel scheme based on the aggregation of winning bids. Some ideas for future work are provided in section 5.

2 Winner determination

One of the most important challanges with combinatorial auctions is solving the winner determination problem, a topic which has received most attention from researchers.

Let us assume for simplicity that only one unit of each item is available. Two general models for winner determination in combinatorial auctions are known from literature. The first one (see Wurmann and Wellman 1999) assumes that every bidder submits a bid on every subset of items. Furthermore, the number of bundles a winner may win is limited to at most one. The second (see, e.g., DeMartini et al. 1999) allows multiple bundles per winner.

Let m denote the number of items and n the total number of bidders. Then the first model reads as follows:

$$\text{Maximize} \quad \sum_{i=1}^{n} \sum_{j=1}^{2^m-1} b_{ij} x_{ij} \tag{1}$$

$$\text{Subject to} \quad \sum_{i=1}^{n} \sum_{j=1}^{2^m-1} a_{hj} x_{ij} \leq 1 \qquad h = 1, \ldots, m \tag{2}$$

$$\sum_{j=1}^{2^m-1} x_{ij} \leq 1 \qquad i = 1, \ldots, n \tag{3}$$

$$x_{ij} \in \{0,1\} \qquad \begin{array}{l} i = 1, \ldots, n \\ j = 1, \ldots, 2^m - 1 \end{array} \tag{4}$$

b_{ij} is the bid price for bundle j from bidder i. x_{ij} is a binary variable indicating whether bidder i gets bundle j ($x_{ij} = 1$) or not ($x_{ij} = 0$). The binary parameter a_{hj} is equal to 1, if item h is contained in bundle j (0, otherwise). Constraint (2) indicates that at most one unit of each item is available for sale. Constraint (3) assures that each bidder gets at most one (over all $2^m - 1$ possibilities, ignoring the zero-valued) bundle.

The second model has no limits on how many bundles each bidder may obtain as long as the availability constraint is assured. For notational simplicity in the subsequent sections let $I = \{1, \ldots, m\}$ denote the set of items, and let $J = \{1, \ldots, n\}$ be the set of (bundles) bids. Then the second model reads as follows:

$$\left.\begin{array}{lll} \text{Maximize} & \sum_{j \in J} b_j x_j & \\ \text{Subject to} & \sum_{j \in J} a_{ij} x_j \leq 1 & \forall i \in I \\ & x_j \in \{0,1\} & \forall j \in J \end{array}\right\} \tag{5}$$

b_j is the bid price for bundle j from bidder i. x_j indicates whether bid j is accepted ($x_j = 1$) or not ($x_j = 0$). The parameter a_{ij} is equal to 1, if item i is contained in bid j or not ($a_{ij} = 0$). All bids submitted are covered by this formulation and, hence, more than one column may come from a particular bidder. Of course, this is equivalent to the case where each bundle is supposed to come from a unique bidder.

The first model has an advantage over (5) in describing XOR bids, i.e. bids sharing an "XOR" relation, where a bidder wants to get only one bundle out of a given set. When such bids are allowed a new onstraint has to be added to (5).

Model (5) is the most widely studied single-unit (each item is unique and there is only one unit for sale of each item), single-sided (one seller and multiple buyers) case and we will study it subsequently. It is the set packing problem, a well-known NP-complete optimization problem (Garey and Johnson 1979). Exact and heuristic algorithms for solving the set packing problem have been developed by, e.g., Borndörfer (1998), Delorme et al. (2004), Harche and Thompson (1994), Hoffmann and Padberg (1993) and Sandholm (2002).

A recent survey of combinatorial auctions is provided by de Vries and Vohra (2003). Combinatorial auctions can be useful in many environments and have been considered for problems including selling spectrum rights (McMillan 1994, Milgrom 2000), airport take-off & landing time slot allocation (Rassenti et al. 1982), railroad segments (Brewer 1999), and delivery routes (Caplice and Sheffi 2003). Other applications are surveyed in, for instance, Kwon et al. (2005).

3 Properties of dual prices

The winner determination problem formulated above is an integer programming problem. In general solving the linear programming relaxation of the winner determination problem will result in a solution in which some of the variables have non-integral values. In such cases where the integer programming problem has a duality gap which is strictly greater than zero, we know from theory that there does not exist a linear price function that supports the optimal allocation of winning bundles. In this situation the use of an approximate linear price system has been advocated. To the best of our knowledge this idea of an approximate linear price system for use in a combinatorial auction setting was presented for the first time in the article by Rassenti et al. (1982) on the allocation of landing rights.

The approximate linear price system or the pseudo-dual prices are based on trying to replicate the properties of the dual price system that exist for a linear program.

Doing so we have to look at the linear programming relaxation of the winner determination problem, that is, the problem

$$\left.\begin{array}{lll} \text{Maximize} & \sum_{j \in J} b_j x_j & \\ \text{Subject to} & \sum_{j \in J} a_{ij} x_j \leq 1 & \forall i \in I \\ & x_j \geq 0 & \forall j \in J \end{array}\right\} \qquad (6)$$

and the corresponding dual

$$\left.\begin{array}{lll} \text{Minimize} & \sum\limits_{i\in I} u_i & \\ \text{Subject to} & \sum\limits_{i\in I} a_{ij}u_i \geq b_j & \forall\, j \in J \\ & u_i \geq 0 & \forall\, i \in I \end{array}\right\} \tag{7}$$

where $\mathbf{u} = (u_i)$ is the vector uf dual variables.

For the linear programming relaxation we know that an optimal primal solution $\bar{\mathbf{x}}^* = (\bar{x}_j^*)$ of (6) and the corresponding optimal dual solution $\mathbf{u}^* = (u_i^*)$ of (7) have the following properties:

Property 1 *(primal feasibility)*
An optimal primal solution $\bar{\mathbf{x}}^* = (\bar{x}_j^*)$ satisfies the constraints

$$\begin{array}{ll} \sum\limits_{j\in J} a_{ij}\bar{x}_j^* \leq 1 & \forall\, i \in I \\ \bar{x}_j^* \geq 0 & \forall\, j \in J \end{array}$$

and is said to be *primal feasible*. □

Property 2 *(dual feasibility)*
An optimal dual solution $\bar{\mathbf{u}}^* = (\bar{u}_i^*)$ satisfies the constraints

$$\begin{array}{ll} \sum\limits_{i\in I} a_{ij}u_i^* \geq b_j & \forall\, j \in J \\ u_i^* \geq 0 & \forall\, i \in I \end{array}$$

and is said to be *dual feasible*. □

Property 3 *(primal complementary slackness)*
If an optimal primal solution $(\bar{x}_j^*)$ and the corresponding optimal dual solution (u_i^*) satisfy the constraints

$$\bar{x}_j^* \left(\sum_{i\in I} a_{ij}u_i^* - b_j \right) = 0 \quad \forall\, j \in J,$$

then the *primal complementary slackness condition* is assured. □

Property 4 *(dual complementary slackness)*
If an optimal primal solution $(\bar{x}_j^*)$ and the corresponding optimal dual solution (u_i^*) satisfy the constraints

$$u_i^* \left(\sum_{j\in J} a_{ij}\bar{x}_j^* - 1 \right) = 0 \quad \forall\, i \in I,$$

then the *dual complementary slackness condition* is assured. □

4 Calculation of pseudo-dual prices

In the following first in section 4.1 we detail the underlying assumptions made by 'normal' approaches when constructing a set of approximate pseudo-dual prices. Then a new scheme based on the aggregation of winning bids is presented in section 4.2. Finally, we give in section 4.3 further insights into how the different schemes work using a nontrivial, meaningful instance.

4.1 Some basic characteristics

In a combinatorial auction the auctioneer is trying to get a good and hopefully optimal solution to the winner determination problem. Assume that the optimal integer solution $\mathbf{x}^* = (x_j^*)$ to the winner determination problem (5) has been found and that the linear programming relaxation (6) does not have the integrality property. We now know that there does not exist a linear price system that can be interpreted as an equilibrium market clearing mechanism.

The underlying assumptions made when constructing a set of approximate pseudo-dual prices are:

(a) The solution $\mathbf{x}^* = (x_j^*)$ is primal feasible.

(b) At least one of the properties dual feasibility, primal complementary slackness or dual complementary slackness must be relaxed.

The 'normal' approach taken in the procedures that have been developed to construct pseudo-dual prices is that:

(i) Primal complementary slackness should be required. This means that we make sure that the winning bids for the different bundles of items all have reduced cost equal to zero.

(ii) Dual complementary slackness should be required. This means that the price for an unsold item should be equal to zero.

Hence the 'normal' relaxation used is to relax the requirement of dual feasibility leading to the fact that some of the loosing bids for a particular bundle of items will have a negative reduced cost when faced with the pseudo-dual price system making the agents that have submitted these bids suspicious and wondering why their bid has not been successful. This is the approach taken by Rassenti et al. (1982) and by DeMartini et al. (1999) among others.

In the following we will describe the approach by DeMartini et al. (1999) in more detail (an in-depth description of a couple of other approaches can be found in Bjørndal and Jørnsten 2002 and Xia et al. 2004).

Assume that the winner determination problem (5) has been solved to optimality and that (x_j^*) is the corresponding optimal integer solution. Let $J_0 := \{j \in J : x_j^* = 0\}$ and $J_1 := \{j \in J : x_j^* = 1\}$ denote the set of loosing and winning bids, respectively. Apparently, we have $J_0 \cap J_1 = \emptyset$ and $J_0 \cup J_1 = J$. Then the main component of the approach is to solve the linear program (8) to (13).

$$\text{Minimize} \quad z \tag{8}$$

$$\text{Subject to} \quad \sum_{j \in J} a_{ij} u_i + y_j \geq b_j \qquad \forall\, j \in J_0 \tag{9}$$

$$\sum_{j \in J} a_{ij} u_i = b_j \qquad \forall\, j \in J_1 \tag{10}$$

$$z \geq y_j \qquad \forall\, j \in J_0 \tag{11}$$

$$u_i \geq 0 \qquad \forall\, i \in I \tag{12}$$

$$y_j \geq 0 \qquad \forall\, j \in J_0 \tag{13}$$

At the prices (u_i) there may be some losing bids for which $\sum_{j \in J} a_{ij} u_i \leq b_j$, falsely signaling a possible winner, which is by virtue the nature of package bidding. Of course, such bids can be resubmitted if $(b_j - \sum_{j \in J} a_{ij} u_i)$ is 'large enough'. The objective (8) has been designed to minimize the number of such bids. If "ideal" prices exist, they will be the solution with $y_j = 0$ for all $j \in J_0$ and, hence z will be equal to zero. If the prices from (8) are not unique a sequence of iterations each of which requires to solve the linear program (8) to (13) is performed (for details see DeMartini et al. 1999).

If dual complementary slackness (see Property 4) is required, too, we add

$$u_i = 0 \quad \forall\, i \in I \;:\; \sum_{j \in J} a_{ij} \bar{x}_j^* < 1 \tag{14}$$

to the set of constraints (9) to (13).

Example 1 An example with 6 items and 21 bids taken from Parkes (2001) illustrates the idea. The bid prices (b_j) and the coeficient matrix (a_{ij}) are provided in Table 1. □

Table 3 provides the results of the solution of the integer program and of the linear programming relaxation of instance 1. Variables not given there have value 0. OFV abbreviates optimal objective function value.

solution of model (5)	solution of model (6)
$x_4 = x_{17} = 1$	$\bar{x}_4 = \bar{x}_{12} = \bar{x}_{20} = 0.5$
OFV = 275,000	OFV = 300,000

Table 3: Instance 1 – IP- and LP-solution

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
b_j	60	50	50	200	100	110	250	50	60	50	110	200	100	255	50	50	75	100	125	200	250
a_{1j}	1			1		1	1	1			1		1	1	1			1		1	1
a_{2j}		1		1	1		1		1		1	1		1		1		1	1		1
a_{3j}			1		1	1	1			1		1	1	1			1		1	1	1
a_{4j}	1	1	1	1	1	1	1														
a_{5j}								1	1	1	1	1	1	1							
a_{6j}															1	1	1	1	1	1	1

Table 1: Instance 1 – Parkes (2001)

j	1	2	3	5	6	7	8	9	10	11	12	13	14	15	16	18	19	20	21	22
b_j	60	50	50	100	110	250	50	60	50	110	200	100	255	50	50	100	125	200	250	275
a_{1j}	1				1	1	1			1		1	1	1		1		1	1	1
a_{2j}		1		1		1		1		1	1		1		1	1	1		1	1
a_{3j}			1	1	1	1			1		1	1	1				1	1	1	1
a_{4j}	1	1	1	1	1	1														1
a_{5j}							1	1	1	1	1	1	1							
a_{6j}														1	1	1	1	1	1	1

Table 2: Parameters of Instance 1′

Solving the linear program of DeMartini et al. (1999) produces the results shown in Table 4. Column one corresponds to model (9) to (13), that is the case without enforcing dual complementary slackness, and column two corresponds to model (9) to (14), that is the case with enforcing dual complementary slackness. By not requiring dual complementary slackness the auctioneer makes use of the fact that agent 2 does not get any bundle and hence prices out that agent resulting in non-anonymous item prices. On the other hand in case of requiring dual complementary slackness bids 7 and 20 have negative reduced cost.

solution of model (9) to (13)	solution of model (9) to (14)
$\mathbf{u} = (150{,}50{,}75{,}0{,}75{,}0)$	$\mathbf{u} = (100{,}100{,}75{,}0{,}0{,}0)$
$z = 0$	$z = 25$

Table 4: Instance 1 – Results for DeMartini et al. (1999)

An alternative to the above scheme would be to only require dual feasibility (see Property 2); this would off course lead to the fact that the linear programming prices are used as approximative prices. The negative effect of this is that the item prices in sum will be too high and, hence, some agents might be reluctant to rise their bids based on this price information. Also it must be assumed that the winning bids only are required to pay their bid price. For instance 1 this would give us the approximate prices $u_1 = 100$, $u_2 = 100$ and $u_3 = 100$.

4.2 Prices based on aggregation of winning bids

An alternative idea that so far to the best of our knowledge has not been suggested and evaluated is to require only that the winning bids are lump together into one single bid. Consequently, in the formulation for calculating pseudo-dual prices we only require that this winning aggregate bid has reduced cost equal to zero whereas the individual winning bundle bids might either have reduced cost negative, zero or positive.

First of all we need some definitions.

Definition 1 *(aggregate winning bid)*

Let (x_j^*) denote an optimal solution to the winner determination problem (5). Furthermore, recall $J_1 := \{j \in J : x_j^* = 1\}$. Then an *aggregate winning bid* $j = n+1$ is constructed as follows:

$$b_{n+1} = \sum_{j \in J_1} b_j x_j^*$$

$$a_{i,n+1} = \begin{cases} 1, & \text{if } \sum_{j \in J_1} a_{ij} x_j^* \geq 1, \\ 0, & \text{otherwise.} \end{cases} \qquad \forall\, i \in I \qquad \square$$

Definition 2 *(aggregate winning bid instance)*

Consider a particular instance of the winner determination problem (5) with I items and J bids. Let (x_j^*) denote an optimal solution for this instance and $J_1 := \{j \in J : x_j^* = 1\}$ the set of winning bids. Then an *aggregate winning bid instance* with the set of I items and the set

$$J' = J \setminus J_1 \cup \{n+1\}$$

of bids with b_{n+1} and $a_{i,n+1}$ $\forall\, i \in I$ according to Definition 1 is constructed. □

If we apply Definitions 1 and 2 to the example from Table 1 we get the modified instance 1′ provided in Table 2.

Solving the linear programming relaxation of instance 1′ yields the optimal objective function value 275,000 and the solution $\bar{x}_{22} = 1$ ($\bar{x}_j = 0$, otherwise) turns out to be integral.

Since the linear programming relaxation has the integrality property a linear price system that clears the market has been found with $u_1 = 75$, $u_2 = 75$ and $u_3 = 125$.

With this price system all non winning bids have non-negative reduced cost whereas one of the winnig bids, bid 17 has a positive reduced cost of 50 and the other winning bid 4 has a negative reduced cost of 50 and, hence, in total for all winning bids the reduced cost is zero.

The idea could equally well have been formulated in terms of the linear program (15) to (20) for calculating pseudo-dual prices

$$\text{Minimize} \quad z \tag{15}$$

$$\text{Subject to} \quad \sum_{j \in J} a_{ij} u_i + y_j \geq b_j \qquad \forall\, j \in J_0 \tag{16}$$

$$\sum_{i \in I} \left(\max_{j \in J_1} a_{ij} \right) u_i = \sum_{j \in J_1} b_j \tag{17}$$

$$z \geq y_j \qquad \forall\, j \in J_0 \tag{18}$$

$$u_i \geq 0 \qquad \forall\, i \in I \tag{19}$$

$$y_j \geq 0 \qquad \forall\, j \in J_0 \tag{20}$$

and additionally constraint (14) if dual complementary slackness is also required.

What are the pros and cons of these approximate pseudo-dual prices? First, with this approach it is more likely that many more of the loosing bids will have reduced costs that are non-negative since the approximate prices are less restricted. The winning bidders get to know if complementary bids do exist that make them winners. Although some of the winning bids might appear to be very profitable with a hugh negative reduced cost the bidder should be aware of the fact that it must mean that their complementary player has an equally big loss at the current

Table 5: Instance 2 – CATS [11]

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b_j	693	4,924	81	420	2,791	761	2,806	858	2,763	4,040	3,876	5,852	3,147	2,279	5,158	1,598	4,798	701	4,850	2,788
a_{1j}					1		1	1			1	1	1	1	1		1			1
a_{2j}				1					1	1		1	1	1		1	1		1	
a_{3j}	1	1							1		1	1			1		1		1	1
a_{4j}			1				1			1	1	1			1	1	1		1	1
a_{5j}		1			1				1	1	1	1			1					
a_{6j}		1				1			1	1		1			1		1			1
a_{7j}		1					1					1	1		1	1	1		1	
a_{8j}		1			1				1	1		1	1	1	1	1		1	1	

Table 6: Parameters of Instance 2′

j	1	2	3	4	5	7	9	10	11	12	13	14	15	16	17	18	20	21
b_j	693	4,924	81	420	2,791	2,806	2,763	4,040	3,876	5,852	3,147	2,279	5,158	1,598	4,798	701	2,788	6,469
a_{1j}					1	1				1	1	1	1	1	1		1	1
a_{2j}				1			1	1		1	1	1		1	1			1
a_{3j}	1	1					1		1	1			1		1		1	1
a_{4j}			1			1		1	1	1			1	1	1		1	1
a_{5j}		1			1		1	1	1	1			1					
a_{6j}		1						1	1		1			1	1		1	1
a_{7j}		1				1				1	1		1	1	1			1
a_{8j}		1			1		1	1		1	1	1	1	1		1		1

prices. However, if the potential winners are told what they have to pay as their bid price and if the auction rules stipulate how potential winning bids can be updated it is very likely that these approximate pseudo-dual bids are better than the pseudo-dual prices used so far.

4.3 Further insights

We will illustrate the various alternatives on a slightly more complicated auction.

Example 2 An example with 8 items and 20 bids taken from the combinatorial auction test suite (CATS; see [11]) illustrates the various pricing schemes presented above. The bid prices (b_j) and the coeficient matrix (a_{ij}) of this instance are provided in Table 5. □

Table 7 provides the results of the solution of the integer program and of the linear programming relaxation of instance 2. Again, variables not given there have value 0 and OFV is an abbreviation for optimal objective function value.

solution of model (5)	solution of model (6)
$x_6 = x_8 = x_{19} = 1$ OFV = 6,469	$\bar{x}_2=\bar{x}_5=\bar{x}_7=\bar{x}_{11}=\bar{x}_{19}$=0.33, $\bar{x}_4=\bar{x}_6$=0.66 OFV = 7,203

Table 7: Instance 2 – Results

If we apply Definitions 1 and 2 to the example from Table 5 we get the modified instance 2′ provided in Table 6. The optimal solution of the linear programming relaxation of this modified instance with one winning bundle is $\bar{x}_2 = \bar{x}_4 = 0.4$ and $\bar{x}_5 = \bar{x}_7 = \bar{x}_{10} = \bar{x}_{11} = \bar{x}_{17} = \bar{x}_{21} = 0.2$ (0, otherwise) with 7093.6 as objective function value.

For this example the various approximate prices are:

1. Linear programming prices:
   $\mathbf{u}$ = (858; 420; 1283; 1001; 734; 761; 947; 1,199)
2. Pseudo-dual prices without dual complementary slackness:
   $z = 0$, $\mathbf{u}$ = (858; 420; 822; 358; 1,849; 761; 1,590; 1,671)

   With these prices we can see how the auctioneer might use unsold items and price them up in order to achieve dual feasibility.
3. Pseudo-dual prices with dual complementary slackness inforced:
   $z = 408.7143$; $\mathbf{u}$ = (858; 11.2857; 1,524.8571; 1,084.4285; 0; 761; 454.857; 1,774.57); eight of the loosing bids have negative reduced cost.

4. For the aggregate formulation based on only requiring that the winning bundles in total have reduced cost zero we get the following approximate prices: $z = 347$; $\mathbf{u} = (426; 73; 1{,}710; 1{,}393; 0; 209; 640; 2{,}018)$; for these prices seven non winning bids have negative reduced cost and for the three winning bids two have positive reduced cost and one winning bid a negative reduced cost.

Another interesting aspect that is worth a more thorough study regarding the use of pseudo-dual prices is the effect a bid from one of the winning bundle bidders on an unsold item has on the pseudo-dual prices.

Here we let bidders 6, 8 and 19 extend their bid so as to also include item 5, one at a time; the resulting pseudo-dual prices then change and are displayed in Table 8.

bid	z	$\mathbf{u}$
6	378.2	(858; 41.8; 1,402.8; 1,023.4; 213.6; 547.4; 546.4; 18,359)
8	367	(712; 53; 1,504; 1,147; 146; 761, 580; 1,566)
19	307	(858; 113; 1,118; 881; 712; 761; 760; 1,266)

Table 8: New Dual Prices

Doing the same for the aggregate winning bundle case results in $z = 275.6$ and $\mathbf{u} = (1{,}419.2; 144.4; 417.4; 257; 1{,}506.8; 1{,}416.4; 868; 439.8)$.

5 Summary and future work

We have in this note suggested that there is a need to broaden our understanding of the use of approximate pseudo-dual prices in combinatorial auction. So far the most common assumption made when constructing pseudo-dual prices is that primal complementary slackness should always be required. We have here presented a relaxed version of this assumption requiring only aggregate primal complementary slackness. This leads to an alternative approximate price system with different characteristics. It would be interesting to conduct some experiments in which the different alternative approximate prices are presented by different groups of agents in order to find out how the differences affect the bidding in an iterative combinatorial auction setting.

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