Volatility and price jumps in agricultural futures prices - evidence from wheat options

Steen Koekebakker
Agder University College and Agder Research
Servicebox 422
No-4604 Kristiansand
Norway

Gudbrand Lien
Norwegian Agricultural Economics Research Institute
P.O. Box 8024 Dep
No-0030 Oslo
Norway

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Abstract

Empirical evidence suggests that agricultural futures price movements have fat-tailed distributions and exhibit sudden and unexpected price jumps. There is also evidence that the volatility of futures prices is time-dependent. It varies both as a function of calendar-time (seasonal effect) and time to maturity (maturity effect). This paper extends Bates (1991) jump-diffusion option pricing model by including both seasonal and maturity effects in the volatility specification. An in-sample fit to market option prices on wheat futures shows that the suggested model outperforms previous models considered in the literature. A numerical example indicates the economic significance of our results for option valuation.

Key words: Option pricing, futures, time-dependent volatility, jump-diffusion, agricultural markets.

1 Introduction

Black (1976) derives a pricing model for European puts and calls on a commodity futures contract, assuming that the futures price follows a geometric Brownian motion (GBM). In the literature on agricultural futures markets, several empirical characteristics have been documented, indicating that the GBM assumption may be too simple. Research has detected leptokurtic returns in agricultural futures prices (e.g. Hudson et al. (1987) and Hall et al. (1989)), and the prices often exhibit sudden, unexpected and discontinuous changes. Price jumps will typically occur due to abrupt changes in supply and demand conditions, and such discontinuities in the price path of futures prices will affect the prices on options written on futures contracts. Hilliard and Reis (1999) used transactions data on soybean futures and futures options to test the Black (1976) diffusion model versus the jump-diffusion option pricing model of Bates (1991). Their results show that Bates' model performs considerably better than Black's model in both in-sample and out of sample tests.

A number of studies have demonstrated the presence of a volatility term structure in agricultural futures prices. Samuelson (1965) claimed that the volatility of futures price returns increases as time to maturity decreases. He argued that the most important information was revealed close to maturity of the contract. For example, the weather affecting demand or a temporary supply disruption will affect spot prices and hence short dated futures contracts. In the long-term, short-term price movements are not expected to persist rather revert back towards a normal level. This implies that long dated contracts will be less affected by spot price changes and experience lower volatility than short dated contracts. This maturity effect is sometimes referred to as the "Samuelson hypothesis". Other authors have argued that the volatility of futures prices depends on the distribution of underlying state variables. This is sometimes termed the "state variable hypothesis". For crop commodities one would typically expect the information flow to vary during the crop cycle. The most important information is revealed during the growth and harvest season, hence seasonality in the volatility of futures prices is expected. Empirical research has produced mixed evidence on the two effects. Milonas (1986) found strong support for the maturity effect after controlling for seasonality. Galloway and Kolb (1996) concluded that the maturity effect is present in markets where commodities experience seasonal demand and/or supply, but not in commodity markets where the cost-of-carry model works well. Anderson (1985) found support for the maturity effect, but claimed it is secondary to seasonality. Anderson (1985) also concluded that the pricing of options on futures contracts should allow for the regular pattern to the volatility of futures. Bessembinder et al. (1996) have reconciled much of the early evidence on the "Samuelson hypothesis". They have shown formally that, in markets where spot price changes include a temporary component so that investors expect some portion of a typical price change to revert in the future, the "Samuelson hypothesis" will hold. Mean reversion is more likely to occur in agricultural commodity markets than in markets for precious metals or financial assets (Bessembinder et al. (1995)), so we expect to see maturity effects in agricultural commodity markets.

Any regular pattern in the volatility is inconsistent with the underlying assumptions in Black (1976) and Bates (1991). Choi and Longstaff (1985) applied the formula of Cox and Ross (1976) for constant elasticity of variance option pricing in the presence of seasonal volatility. They found this superior to the model of Black (1976) for pricing options on soybeans futures. Myers and Hanson (1993) present option-pricing models when time-varying volatility and excess kurtosis in the underlying futures price are modelled as a GARCH process. Empirical results suggest that the GARCH option-pricing model outperforms the standard Black (1976) model. Fackler and Tian (1999) proposed a simple one-factor spot price model with mean reversion (in the log price) and seasonal volatility. They show that futures prices consistent with this spot price model have a volatility term structure exhibiting both seasonality and maturity effects. Their empirical results indicate that both phenomena are present in the soybean futures and option markets.

There are two basic approaches when it comes to valuation of commodity contingent claims valuation. The first concentrates on modelling the stochastic process of the spot price and other state variables such as the convenience yield (see for example Brennan and Schwartz (1985), Gibson and Schwartz (1990), Schwartz (1997) and Hilliard and Reis (1998). The main problem with spot price based models is that forward (or futures) prices are given endogenously from the spot price dynamics. As a result, theoretical forward prices will in general not be consistent with market observed forward prices. As a response to this, a line of research has focused on modelling the evolution of the whole forward curve using only a few stochastic factors taking the initial term structure as given. Examples of this research building on the modelling framework of Heath et al. (1992), are, among others, Cortazar and Schwartz (1994) (copper) and Clewlow and Strickland (2000) (crude oil).

In this paper we adopt the futures curve approach, and we assume that the futures price follows a jump-diffusion process. The diffusion term includes time-dependent volatility that captures (possibly) both seasonal and maturity effects. We derive a futures option pricing model given our specified forward curve dynamics. The model parameters are estimated from option prices written on the futures contract. Eleven years of futures and option data is collected from Chicago Board of Trade (CBOT). Parameters of our futures price model are estimated using non-linear least squares. The futures price dynamics of several models suggested previously in the literature are nested in our model specification, and we can use standard statistical tests to determine whether jumps and time-dependent volatility are present in the data. In accordance with the evidence presented in Fackler and Tian (1999), we find that both the maturity- and the seasonal effect is present in the wheat futures market. The estimated jump intensity is significantly different from zero. This result is in line with results found in the soybean futures option market reported in Hilliard and Reis (1999). The simpler models suggested previously are rejected in favour of our proposed model with jumps, seasonality- and maturity effects. A numerical example illustrates the economic significance of our results.

Te rest of this paper is organised as follows: In the next section we present the futures price dynamics and derive a futures option pricing formula. Section 3 describes the data. In section 4 the estimation procedure is described, and the results are presented in section 5. Section 6 concludes.

2 Model description

We assume that there exists an idealised futures market (liquid, frictionless, no taxes, limitless short selling allowed etc.). Let the forward market be represented by a continuous forward price function, where $F(t,T)$ denotes the forward price at date t for delivery of the commodity at time T. Throughout the paper we assume constant risk free interest rate, so that futures prices and forward prices with common maturity are identical (see Cox et al. (1981)). The futures price is assumed to be governed by the following dynamics under the equivalent martingale measure (EMM):

$$\frac{dF\,(t,T)}{F\,(t,T)} = -\lambda\overline{\kappa}dt + \sigma(t,T)dB(t) + \kappa dq,\;\; F(0,T)\;\forall\;T \tag{1}$$

where B is standard Brownian motion under the EMM and κ is the random percentage jump conditional upon a Poisson distributed event, q, occurring and $\overline{\kappa}$ is defined as the expected value of the jump size if it in fact occurs. The jump intensity is given by λ. The counting process q is independent of κ, with $Prob\,(dq = 1) = \lambda dt$ and $Prob\,(dq = 0) = 1 - \lambda dt$. Since the observed futures prices at time 0, $F(0,T)$, are given as initial conditions, our model is consistent with the observed futures curve by construction.

By standard no-arbitrage arguments we know that since it costs nothing to enter a futures contract, the expected return on holding the contract should be zero under the EMM. We can easily check that this is the case in our model: The Brownian motion has zero expectation. The expectation of κdq during a time increment dt is $E\,[\kappa dq] = E\,[\kappa]\,E\,[dq] = \overline{\kappa}\lambda dt$, thus $E\left[\frac{dF(t,T^*)}{F(t,T^*)}\right] = 0$. We now need to specify the jump distribution and the time-dependent volatility.[1] The inclusion of jumps in a model free of arbitrage raises some issues of market incompleteness. We give a brief discussion of this in the following subsection. We then describe a time-dependence volatility function that is able to capture both seasonal and maturity effects, and finally we provide analytical valuation expression for options written on futures contracts in our model.

[1] We present our modelling framework in a non-technical manner. Merton (1976) first introduced the jump-diffusion model of asset prices. The modern mathematical framework for modelling discontinuities in asset price is by the use of so called marked point processes, in which the Poisson distributed jump arrival process considered in this paper is one of many possible candidates. See Veredas (2000) for a nice, readable introduction on marked point processes. A very nice exposition of forward, futures and option pricing in a very general framework is given in Björk and Landén (2002). Since the focus of this paper is the empirical properties of a jump-diffusion model, we have omitted the technicalities.

2.1 Jump distribution and market incompleteness

We assume that $\ln(1+\kappa)$ is a normally distributed random variable with mean $\left(\gamma - \frac{1}{2}\nu^2\right)$ and variance ν^2. Consequently, the expected percentage jump size is $E[\kappa] \equiv \overline{\kappa} = e^{\gamma} - 1$. These distributional assumptions are equal to those stated in Merton (1976)[2] and Bates (1991), but other distributions might be considered.[3] Note that the jump parameters are constants, in particular they are independent of time to maturity. This means that if a jump occurs, a parallel shift in the term structure of futures prices will emerge. If we observe futures contracts with time to maturity spanning several years into the future, the assumption that the returns on all contracts jump with equal amounts may seem inadequate. If, for example, exceptional bad weather (such as a hurricane) partly destroys a harvest, then futures prices are likely to jump due to a negative shift of supply. But long term futures contracts will depend on future harvests, and so intuition suggest that long term contracts are less jumpy, compared to short term contracts. This behaviour can easily be incorporated in our model by imposing time dependence on the jump amplitude. Such an extension is ignored here since the maturity of the futures contracts analysed in the empirical part of this paper never exceed one year. Hence, in our data set, imposing parallel jumps may be a satisfactory assumption.

Merton (1976) assumed that jumps are symmetric (zero mean) and non-systematic. In a stock market model, this means that jumps are of no concern to an investor with a well-diversified portfolio, since jumps on average cancel out. Given such assumptions of firm specific jump risk, parameters concerning the jump part are equal under both the real world probability measure and the EMM. The assumption of non-systematic jump risk may be inappropriate in many settings, and this is also the case in commodity futures markets. If, for example, bad weather results in a poor harvest, futures prices may jump. However, the occurrence of such an event is likely to move all the commodity futures prices in the same direction, and so diversifying the jump risk within this market is impossible. In other words, jump risk is systematic. It is well known that the presence of systematic jump risk makes it incomplete in the Harrison and Pliska (1981) sense. This means that it is not possible to set up a dynamic hedging strategy in the underlying asset and a risk free asset that replicates a contingent claim due to the possibility of abrupt jumps in the underlying asset price. This essentially means that under the absence of arbitrage opportunities, there are many (infinite) equivalent martingale measures. Furthermore, without explicit assumptions on preferences and technologies, each martingale measure defines an admissible price of a contingent claim (see Harrison and Kreps (1979)).

Bates (1991) derives a unique martingale measure in a jump-diffusion setting

[2] Merton (1976) assumed zero mean jump size, hence $\gamma = 0$.

[3] Other jump distributions are considered in the financial literature. Duffie et al. (2000) assume that abrupt changes in volatility are caused by Pareto distributed jumps, and Kou (2000) investigates option pricing in the presence of double-exponentially distributed price jumps. The literature on jumps in financial agricultural prices, as far as we know, concentrates on the lognormal jump model. Investigating other jump distribution in agricultural markets is left for further research.

by considering a specific equilibrium model. He assumes that optimally invested wealth follows a jump-diffusion, and the representative consumer is equipped with time-separable power utility. Bates (1991) shows that both the diffusion term and the variance of the jumps in (1) are the same under both the EMM and the real world measures. But both the jump intensity and mean jump size is, in general, different under the two measures. Bates (1991) interprets λ as the cost per unit time of jump insurance. If the mean jump size is zero, and the representative investor is risk averse, he finds that $\lambda > \lambda_R$, where λ_R is the jump intensity under the real world probability measure. Mathematically this means that the probability of a jump occurring is greater under the risk neutral measure than under the objective measure. The economic intuition is that risk aversion among market participants increases the price of jump insurance. Bates (1991) also finds that the mean jump size will typically be downward biased under the equivalent martingale measure. The model suggested by Merton (1976) can be seen as a special case of Bates (1991) with a risk neutral agent and zero mean jump size. In this special case all jump parameters are equal under both probability measures. In the empirical part of this paper, we extract jump parameters from option prices. From the discussion above it is clear that these parameters are not equal to the parameters of the actual jump process governing futures prices under the objective measure. This must be kept in mind, when evaluating the parameters implicit in option prices.

2.2 Time-dependent volatility

We now proceed to specify the volatility dynamics of our model, but first we discuss some spot price models (without jumps) suggested previously in the literature. Consider the model proposed by Fackler and Tian (1999) for soybeans. They model the spot price as

$$\frac{dS(t)}{S(t)} = \delta\left(\mu(t) - \ln S(t)\right) dt + \sigma(t) dB(t) \tag{2}$$

where δ refers to the speed of adjustment and μ and σ are seasonal functions of time. They show that stochastic differerential equation governing the futures price in this model can be written as

$$\frac{dF(t,T)}{F(t,T)} = \sigma(t) e^{-\delta(T-t)} dB(t) \tag{3}$$

where the initial futures price $F(0,T)$ is a function of the spot price at time 0 and the parameters of the model.[4] We see from (3) that the time-dependent volatility of the futures price can be decomposed into two distinct parts: $\sigma(t)$ governs the changing volatility over the course of the year, and $e^{-\delta(T-t)}$ governs the maturity effect. If δ is high (strong mean reversion), the price movements for a futures contract with long time to maturity will be substantially smaller than

[4]Here we assume that the model is set up under the EMM. If (2) is specified under the real world measure, the initial futures price will also depend on the market price of risk.

the price movements for a contract with short time to maturity. The one-factor model in Schwartz (1997) appears when μ and σ are constants.

In some markets volatility raises sharply as contracts approaches maturity. This can be achieved by a high value of δ in (3). But this again implies that the futures contracts with long time to maturity gets very low, and a pure negative exponential maturity effect may have a hard time capturing the volatility of both short and long term contracts. For this reason we propose the following volatility function for the diffusion term in (1):[5]

$$\sigma(t,T) = \sigma(t)\left((1-\widetilde{\sigma})\, e^{-\delta(T-t)} + \widetilde{\sigma}\right) \tag{4}$$

with the seasonal part as a truncated Fourier series

$$\sigma(t) = \overline{\sigma} + \sum_{j=1}^{J}\left(\alpha_j \sin(2\pi jt) - \beta_j \cos(2\pi jt)\right) \tag{5}$$

The parameters are restricted in the following way: α_j and β_j are real constants, $\overline{\sigma}, \delta \geq 0$ and $0 \leq \widetilde{\sigma} \leq 1$. From (4) we see that as the contract approaches maturity, $T \to t$, the volatility function collapses to $\sigma(t,T) = \sigma(t)$. Since the spot price in this model is given implicitly as $S(t) = F(t,t)$, this means that $\sigma(t)$ governs the implied spot price volatility in our futures price model. The seasonal specification given with the truncated Fourier series has been applied previously in e.g. Fackler and Tian (1999). We can also investigate the dynamic properties of contracts with long time to maturity. In the limit, as T approaches infinity, we note that $\sigma(t,T) = \sigma(t)\widetilde{\sigma}$. Thus in our model specification the volatility of a futures contract is bounded within $[\sigma(t)\widetilde{\sigma}, \sigma(t)]$, where $\sigma(t)\widetilde{\sigma} \leq \sigma(t)$ since $0 \leq \widetilde{\sigma} \leq 1$. If, for example, $\widetilde{\sigma} = 0.5$, instantaneous volatility of long term contracts is half the volatility of the spot price. If the maturity effect is strong (high value of δ), the instantaneous volatility of the futures contract quickly approaches $\sigma(t)\widetilde{\sigma}$ as time to maturity increases.

In addition to the added flexibility of our volatility function, this particular specification nests the futures price volatility dynamics of several models suggested previously in the commodity contingent claims literature.[6] The parameter restrictions corresponding to earlier models are listed in table 1. In the empirical part of this paper we will estimate these constrained models along with our new unconstrained model.

[5] This particular form has been suggested by Strickland (2002) for modelling the forward curve in the energy market.

[6] Note that the models of Schwartz (1997) and Fackler and Tian (1999) are spot based models. This means that the spot price dynamics are given exogenously and, using arbitrage arguments, futures prices can be endogenously calculated from parameters governing spot price dynamics. Endogenously determined futures prices typically do not match (exactly) real world prices observed in the market place. Since we use futures prices as inputs in our option valuation model, we need a futures price based model to ensure consistency between the theoretical model and the data in the empirical part of this study. Hence, when we refer to the models as nested, we are actually refering to the nesting of futures price dynamics implied by the previously suggested spot based models.

Previous models	Parameter constraints
Black (1976)	$\lambda = \delta = \alpha_j = \beta_j = 0$, $\widetilde{\sigma} = 1$
Schwartz (1997)	$\lambda = \alpha_j = \beta_j = \widetilde{\sigma} = 0$
Fackler and Tian (1999)	$\lambda = \widetilde{\sigma} = 0$
Bates (1991)	$\delta = \alpha_j = \beta_j = 0$, $\widetilde{\sigma} = 1$

Table 1: Volatility dynamics of previously suggested models. The futures price volatility of previously suggested models in the literature can be recaptured in our model by constraining the parameters in (1), (4) and (5). The models are given in column one, and the corresponding parameter constraints are given in column two.

2.3 Valuation of futures options

Consider a European call option, C, with maturity T^* and strike K written on a futures contract with maturity T, where $T^* \leq T$. The value is given by

$$C\left(F\left(t,T\right),t,T^*\right) = e^{-r(T^*-t)} \sum_{n=0}^{\infty} P(n) \left(F\left(t,T\right) e^{b(n)(T^*-t)} \Phi\left(d_{1n}\right) - K\Phi\left(d_{2n}\right)\right) \tag{6}$$

where

$$P(n) = \frac{e^{-\lambda(T^*-t)}\left(\lambda(T^*-t)\right)^n}{n!}$$

$$b(n) = -\lambda\overline{\kappa} + \frac{n\gamma}{T^*-t}$$

$$d_{1n} = \frac{\ln\left(\frac{F(t,T)}{K}\right) + \frac{1}{2}\left(\omega^2 + n\nu^2\right) + b(n)(T^*-t)}{\sqrt{\omega^2 + n\nu^2}}$$

$$d_{2n} = d_{1n} - \sqrt{\omega^2 + n\nu^2}$$

$$\omega = \sqrt{\int_t^{T^*} \sigma\left(s,T\right)^2 ds}$$

and $\Phi\left(\bullet\right)$ denotes the cumulative standard normal distribution. This formula is a slight generalisation of the formula given in Bates (1991) and Merton (1976). A proof of the formula in a more general framework is given in Björk and Landén (2002). The formula can be understood intuitively as a sum of Black-Scholes (BS) type formulas with variance $\omega^2 + n\nu^2$ and a risk free rate $b(n)(T^*-t)$, with each BS formula weighted by the probability of n jumps occurring in the period $[t, T^*]$. Since there is no upper limit to the number of possible jumps occurring in this period, we are in fact summing over infinite BS formulas. In practise this is not a big problem, since, for reasonable jump parameters, very accurate prices can be obtained when truncating the infinite sum by setting

n rather low.[7] Put options can be calculated explicitly, or they can be found via the futures option put-call parity. In the empirical part of this paper, we use data on American futures options, and consequently, some modification of the above European option pricing model is required. Bates (1991) derives an approximation for an American option in the jump-diffusion framework. His approximation generalises the formula of Barone-Adesi and Whaley (1987) to a jump-diffusion model of the underlying asset. We use the same approximation as described in Bates (1991), replacing the constant volatility in his setting with the time-dependent volatility given by ω above. This model is called *New* in the empirical part of the paper.

3 Data description

We use price quotes on wheat futures and wheat futures options collected from CBOT to estimate the parameters of the futures price dynamics. Weekly data were obtained from January 1989 until December 1999. The total sample consists of fifty-five futures contracts. The futures contracts matures in March, May, July, September, and December. Each contract starts trading one year prior to maturity. At each point in time there are five contracts traded, and maximum time to maturity for a single contract is one year. The options written on the contracts can be exercised prior to maturity, hence they are of American type. The last trading day for the options is the first Friday preceding the first notice day for the underlying wheat futures contract. The expiration day of a wheat futures option is on the first Saturday following the last day of trading.

We applied several exclusion filters to construct the data sample. First, we did not use prices prior to 1989 since market prices then were likely to be affected by government programs in the United States (price floor of market prices and government-held stocks). Second, only trades on Wednesdays were considered, yielding a panel data set with weekly frequency. Weekly sampling is simply a matter of convenience. Daily sampling would place extreme demands on computer memory and time. The reason for choosing Wednesday is that this is the day of the week least affected by holidays. Third, only settlement (closing) prices were considered. Fourth, the last six trading days of each option contract were removed to avoid the expiration related price effects (these contracts may induce liquidity related biases). Fifth, to mitigate the impact of price discreteness on option valuation, price quotes lower than 2.5 cents/bu were deleted. Sixth, assuming that there is no arbitrage in this market, option prices lower or equal to their intrinsic values were removed. Three-month Treasury bill yields were used as a proxy for the risk free discount rate. The exogenous variables for each option in our data set are strike price, K, futures spot price,

[7] In our empirical investigation we set out with $n \approx \lambda T e^{\gamma}$. Then n is extended until additional terms do not increase accuracy. Following Bates (1991) we set $n = 1000$ at the maximum. There is a way of avoiding the truncation problem altogether. Zhu (1999) computes the characteristic function of the jump-diffusion and by inverting this using Fourier inversion technique, he propose an alternative formula without summation. This method could easily be applied in our model as well, but this is not done here.

F, today's date, t, the maturity date of the option contract, T^*, the maturity date of the futures contract, T, the instantaneous risk-free interest rate, r, and observed settlement option market price, C.

4 Estimation method

Besides the exogenous variables obtained from the data set, the option pricing formula requires some parameters as inputs. In the full model the maturity- and seasonal parameters $\left(\overline{\sigma},\ \widetilde{\sigma},\ \alpha_j,\ \beta_j,\text{ and }\delta\right)$ and the jump related parameters (γ, ν, and λ) must be estimated. There are two main approaches to estimate these parameters; from time series analysis of the underlying asset price, or by inferring them from option prices conditional upon postulated models (Bates (1995)). There are two main drawbacks of the former approach. First, very long time-series are necessary to correctly estimate jump parameters, at least if prices jump rarely. Second, parameters obtained from this procedure correspond to the actual distribution, and hence the parameters cannot be used in an option pricing formula, since the parameters needed for option pricing are given under the EMM. The latter approach is chosen here, and it has been used previously in e.g. Bates (1991, 1996 and 2000), Bakshi et al. (1997) and Hilliard and Reis (1999). Implicit parameter estimation is based on the fact that options, if rationally priced, contain information of the future probability distribution under the EMM.

We infer model-specific parameters from weekly option prices over an eleven years long time period. In previous studies, implicit parameters are inferred from option prices during very short time intervals, often daily (e.g., Bates (1991, 1996) and Hilliard and Reis (1999)). However, this method can be applied to data spanning any interval that has sufficient number of trades (Hilliard and Reis (1999)). Daily calibrations can fail to pick up longer horizon parameter instabilities (Bates (2000)). In this study we need the data to span several years in order to reveal any predictable seasonal patterns in volatility. American option prices, C_{is}, are assumed to consist of model prices plus a random additive disturbance term:

$$C_{is} = C\left(F_{is}, K_i, t, T, T^*, r, \gamma, \nu, \lambda, \overline{\sigma}, \widetilde{\sigma}, \sum_{j=1}^{J}\alpha_j, \sum_{j=1}^{J}\beta_j, \delta\right) + \varepsilon_{is} \tag{7}$$

The subscript i represents an index of transactions (calls of assorted strike prices and maturities), and the subscript s represent an index of weekly observations in the sample. Equation (7) can be estimated using non-linear regression. The unknown parameters γ, ν, λ, $\overline{\sigma}$, $\widetilde{\sigma}$, α_i, β_i, δ are estimated by minimising the sum of squared errors (SSE) for all options in the sample given by

$$SSE = \sum_{s=1}^{S}\sum_{i=1}^{I}\left[C_{is} - C(\bullet)\right]^2 = \sum_{s=1}^{S}\sum_{i=1}^{I}\left[\varepsilon_{is}\right]^2 \tag{8}$$

Many alternative criteria could be used to evaluate performance of option pricing models. The overall sum of squared errors (SSE) is used as a broad summary measure to determine how well each alternative option pricing model fits actual market prices. Assuming normality of the error term, nested models can be tested using F-tests. We will perform such tests in the next section.

5 Results

The following models were estimated (abbreviations used later in the paper are in parentheses). The diffusion model of Black (1976) with constant volatility (*Black76*), the one-factor model of Schwartz (1997) (*Schwartz97*), the jump-diffusion model of Bates (1991) (*Bates91*), the model suggested by Fackler and Tian (1999) with a seasonal and maturity dependent diffusion term (*Fackler99*) and our unrestricted model with both time dependence and jumps (*New*). Table 2 shows implicit parameter estimates for wheat futures call options. In the seasonal specification for both *Fackler99* and *New* we have set $J = 3$. Experimenting with higher order lags resulted in only marginally better fit, and the results are not reported here. As a result of forcing eleven years of data into one option pricing model with constant parameters, the SSE is quite large. From table 2 we also see indication that both time-dependent volatility and jumps are important. The unrestricted model (*New*) produces the lowest SSE for all contracts. This is not surprising, since more parameters necessarily means better fit.

We have formally tested the models against each other using F-statistics. The F-statistic is computed as $F[G, N-L] = \frac{(SSE_R - SSE_U)/G}{SSE_U/(N-L)}$ where SSE_U and SSE_R are sum of squared errors for the unrestricted and restricted models respectively, G is the number of restrictions, N is number of observations in the sample, and L is number of parameters in the unrestricted model. The test statistic is asymptotically F-distributed with G and $(N-L)$ degrees of freedom.[8] The appropriate restrictions for each model are in table 1. The results, given in table 3, shows that we can reject the null hypothesis of a pure lognormal model of *Black76* versus both the volatility time-dependent models of *Schwartz97* and *Fackler99* and the jump-diffusion model of *Bates91*. This last observation is in accordance with the conclusion in Hilliard and Reis (1999) - *Bates91* performs better than *Black76*. We also find that all these models are rejected in favour of the model *New* with both jumps and time-dependent volatility. In the following sub-sections we investigate these parameter estimates further.

5.1 A closer look at the time-dependent volatility

Recall the volatility dynamics in (1), (4) and (5). We have plotted the volatility time-dependence in figure 1, using the estimated parameters for the model *New*

[8] See for example chapter 5 in Davidson and MacKinnon (1993) for a description of different tests available in non-linear least squares regression. Since the test statistics is $F-$distributed only asymptotically, they term it a *pseudo-F* test.

Parms.	Black76	Schwartz97	Fackler99	Bates91	New
$\overline{\sigma}$	0.22	0.25	0.24	0.17	0.24
	(3247)	(3380)	(2027)	(758)	(242)
$\widetilde{\sigma}$	1			1	0.49
					(156)
γ				0.05	0.09
				(244)	(975)
ν				0.19	0.44
				(1039)	(1627)
λ				0.60	0.16
				(531)	(267)
δ		0.38	0.26		3.44
		(2931)	(205)		(325)
α_1			-0.001		-0.01
			(-11.2)		(-12.7)
β_1			-0.04		-0.05
			(-211)		(-40.6)
α_2			0.001		0.02
			(1.0)		(6.4)
β_2			0.01		0.005
			(9.3)		(2.9)
α_3			0.01		0.02
			(8.3)		(7.7)
β_3			-0.001		-0.005
			(-0.6)		(-1.5)
SSE	200570	195100	166930	188530	152990

Table 2: Implicit parameter estimates for various models. The table shows parameter estimates from non-linear least squares regressions on wheat futures call option prices. Estimations are made separately on weekly observations of all contracts in the period 1989-1999 using a toal of 18 831 observations. Five models are estimated: *Black76*, *Schwartz97*, *Bates91*, *Fackler99* and *New*. Three terms are used in the seasonal volatility specification of two latter models. The four former models are constrained versions of the latter (see table (1) for parameter constraints for each model.) Sum of squared errors (SSE) are reported for each model, and t-values are in parentheses.

Testing H_0 versus H_1	F-value	F-critical	Decision
Black76 vs. Schwartz97	527.9	254.3	Reject H_0
Black76 vs. Fackler99	541.9	3.2	Reject H_0
Black76 vs. Bates91	400.8	8.5	Reject H_0
Schwartz97 vs. New	518.0	2.5	Reject H_0
Fackler99 vs. New	428.7	5.6	Reject H_0
Bates91 vs. New	546.5	2.9	Reject H_0

Table 3: Model specification tests. The table reports the results from several hypothesis tests. The null hypothesis of constant volatility (H_0 =*Black76*) is tested separately against time-dependent volatility (H_1 =*Schwartz97* and H_1 =*Fackler99*) and the presence of jumps (H_1 =*Bates91*). The volatility time-dependent models (H_1 =*Schwartz97* and H_1 =*Fackler99*) and the pure jump model (H_0 =*Bates*) are tested against the full model (H_1 =*New*). The critical value of the F-tests are given for a confidence level of 95 per cent.

in table 2. For each contract the volatility dynamics spans one year and ends as the futures contract expires. In panel A we have plotted the overall (both maturity- and seasonal effect) instantaneous volatility of the March, May, July, September and December contracts. Both seasonal and maturity effect is clearly present, the latter effect is most clearly observed in the May, July and September contracts.

To study each effect separately, we have decomposed the overall volatility into the maturity effect in panel B and the seasonal effect in panel C. From panel B we see a strong maturity effect, and as the contract approaches maturity the volatility of the futures approaches the implied spot price volatility. And, having "turned off" the seasonal variation, the implied spot price volatility equals the yearly average of $\overline{\sigma} = 0.24$. The dashed line represents the long run volatility ($\overline{\sigma}\widetilde{\sigma}$) meaning the volatility of futures contracts with infinite time to maturity. We note that, due to the strong maturity effect, volatility of futures contract is essentially equal to the long run volatility when there is more than one year to maturity. In panel C we have "turned off" the maturity effect (setting $T = t$), to concentrate on the seasonal volatility of the implied spot price volatility. We see that volatility is high during summer and autumn months. This supports previous findings that the most important information is revealed during growing and harvest season (e.g. Choi and Longstaff (1985)). During winter the implied spot price volatility is considerably lower. The dashed line in panel C represents the the average implied spot price volatility $\overline{\sigma}$.

5.2 A closer look at the jump parameters

Futures prices characterised solely by deterministic time-dependent volatility are lognormally distributed. As a result the implied volatility from option prices will be constant across strike prices.[9] However, if jumps are likely to occur, implied

[9] The fact that we are dealing with American options, means that implied volatility is not necessarily constant across strikes. However, prices on American and European futures option

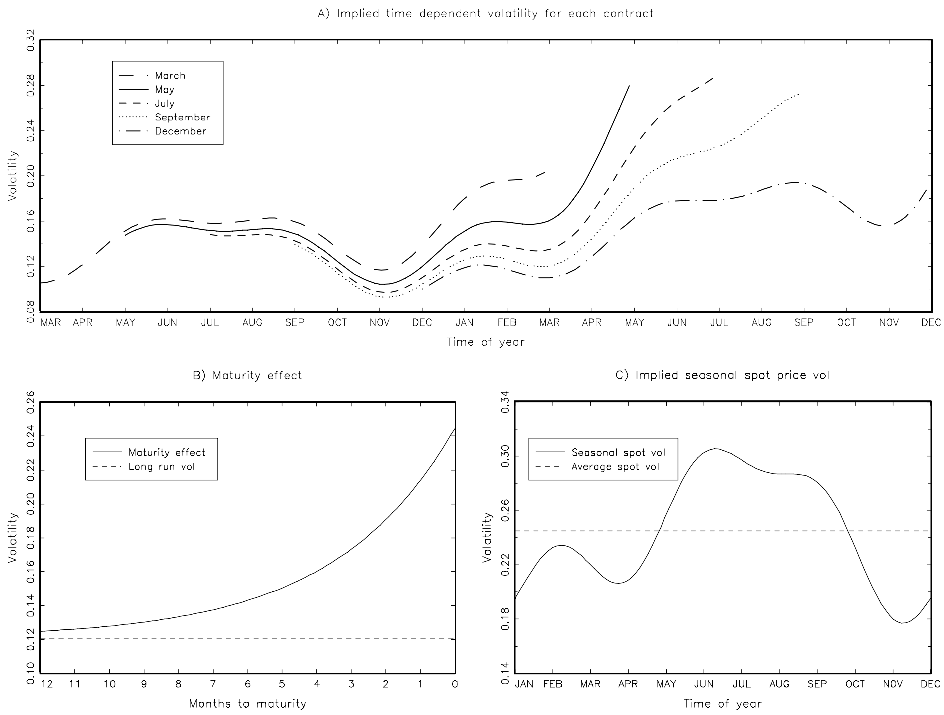

Figure 1: Time-dependent volatility of wheat futures contracts decomposed into maturity-and seasonality effects. Panel A plots the overall instantaneous volatility of the March, May, July, September and December contracts. The volatility function is given by $(\sigma(t,T) = \sigma(t)((1-\widetilde{\sigma})\,e^{-\delta(T-t)} + \widetilde{\sigma})$ and the parameters are those given in table 2. Panel B plots the maturity effect with seasonality "shut off" (setting $\sigma(t) = \overline{\sigma}$). The dashed line represent the long run volatility $lim(\sigma(t,T))_{T\to\infty} = \overline{\sigma}\widetilde{\sigma}$. Panel C plots the implied spot price volatility given by $\sigma(t) = \overline{\sigma} + \sum_{j=1}^{3}\left(\alpha_j \sin(2\pi jt) - \beta_j \cos(2\pi jt)\right)$. The dashed line is the average implied spot price volatility volatility $\overline{\sigma}$.

volatility will not be constant across strike prices. An illustration of the effect the jump parameters have on implied volatility follows. Suppose that our model specification is correct; that both the time-dependent volatility and jumps are present in futures prices, and hence our option pricing formula calculates the true option price. First, fix the current date to October 1, and assume that, at this date, the May contract is trading at 300, hence $F(t,T) = 300$, $t = 9/12$ and $T - t = 7/12$ years. Second, assume that that the volatility dynamics are given by the estimated parameters from the *New* model in table 2 and that $r = 0.05$. Now consider American call options with 2,4 and 6 months to maturity ($T^* - t = 2/12$, $4/12$ and $6/12$), and 5 different strike prices ($K = 240$, 270, 300, 330 and 360) at each maturity. The resulting implied volatility curves are plotted in figure 2.

Note that for at-the-money (ATM) options ($K = 300$), implied volatility increase in time to maturity. This is mainly due to time-dependent volatility. Recall from panel A in figure 1 the volatility time-dependence for the May contract. The upward slope of the instantaneous volatility causes the average volatility for a short term call options to be lower than for an option with maturity closer to the maturity of the futures contract. We note that implied volatility is not constant across strike prices. This is known as the volatility "smile", and it is also evident that this "smile" gets more pronounced as option expiration gets closer. As we get close to option maturity, far out-of-the-money (OTM) calls in a lognormal model are worth relatively little, since an extreme upward price movement is very unlikely. In a jump-diffusion model, these options may end up in-the-money (ITM) if a positive jump occurs, and consequently, these options will be relatively more valuable in a jump-diffusion than in a lognormal world. ITM call options will be relatively more valuable in a jump diffusion model compared to a lognormal model, since a positive jump may push the option deeper into the money. When there is long time to option maturity, the jump component plays a less prominent part, and the smile flattens. This is due to the fact that for OTM options say, the diffusion term alone will be able to move the futures price so that the option will end up ITM, and the difference between the two models decreases.[10]

5.3 A numerical example

Finally, we provide a numerical example showing the potential economic significance of our findings. Suppose that the model suggested in this article is in fact correct. What kind of mispricing will take place if we use the model of Black (1976), Schwartz (1997), Fackler and Tian (1999) or Bates (1991) previously suggested in the literature? We consider the same May contract described in the previous sub-section. For each of the models *Black76*, *Schwartz97*, *Fack-*

differ very little (Bates (2000)), hence implied volatility from American futures options are close to horizontal in a lognormal model.

[10] Jump effects will in general be more visible in terms of implied volatility as time to expiration shortens (see Das and Sundaram (1999) for an investigation of term structure effects in a jump-diffusion model).

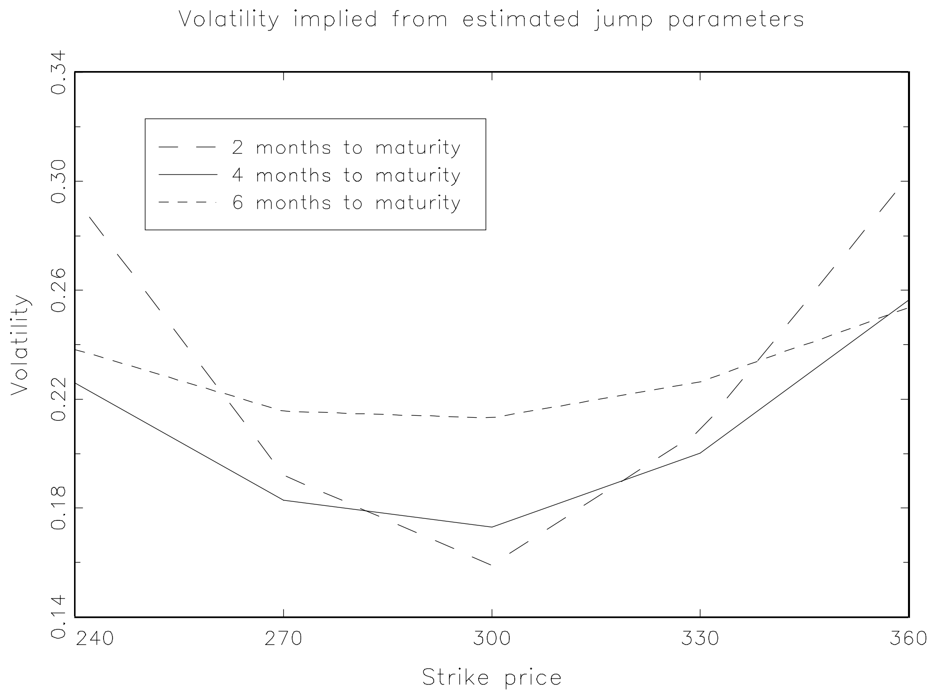

Figure 2: Implied volatility smiles from wheat call options. Parameters for the the model New reported in table 2 are used in the computations. The futures price is set to 300 for a futures contract with maturity 7 months from now $((T - t = 7/12))$, and the risk free rate is 5%. Option prices are computed using the formula in (6) adjusting for the early exercise feature as in Bates (1991) for different strikes ($K = 240, 270, 300, 330$ and 260) and option maturities ($T^* - t = 2/12, 4/12$ and $6/12$). To back out implied volatilities we use the Black (1976) model adjusted for early exercise premium of American options as described in Barone-Adesi and Whaley (1987).

Option maturities	Strike	%-Differences in option prices			
		Black76 vs. New	Schwartz97 vs. New	Fackler99 vs. New	Bates91 vs. New
	260	0.7	0.4	0.1	0.3
$T^* - t = 2/12$	300	22.1	14.2	5.9	21.1
	340	-69.4	-77.0	-83.8	-24.0
	260	1.8	1.3	0.3	1.1
$T^* - t = 4/12$	300	7.1	3.6	-5.0	10.9
	340	-49.8	-54.5	-65.3	-21.5
	260	-0.8	-0.8	-0.7	-1.1
$T^* - t = 6/12$	300	-10.5	-10.5	-10.2	-5.7
	340	-46.0	-45.9	-45.5	-26.0

Table 4: Comparison of American wheat futures option pricing models. The table reports percentage differences between the model *New*, and the models *Black76*, *Schwartz97*, *Fackler99* and *Bates91*. Option prices are calculated using (6) and adjusting for the early exercise premium of American options as in Bates (1991). For each model, the parameter estimates reported in table 2 are used in the computations. Additional inputs are: $F(t,T) = 300$, $t = 9/12$, $T-t = 7/12$ and $r = 0.05$. Prices are computed for strikes $K = 260$, 300, and 340 and maturities $T^* - t = 2/12, 4/12$ and $6/12$.

ler99, *Bates91* and *New* we pick parameters from table 2. We then compute option prices for American calls with the same parameters as in the example above for different strikes and maturities. The results are given in table 4. We report the percentage differences between each of the previously suggested models and our proposed model.

Prices for ITM options ($K = 260$) are more or less the same for all three models for all maturities. This is due to the fact that the intrinsic value dominates the value of an option when deep ITM, and hence most models would produce quite similar results. Notice from panel A in figure 1 the volatility time-dependence for the May contract. The instantaneous volatility for the May contract from November to April shows a very strong maturity effect. The seasonality effect is less pronounced. The fact that the volatility of futures contract increases as maturity approaches, means that using an average value for the volatility will produce too high option prices for short maturity options and too low prices for long maturity options. We observe this pattern for at-the-money (ATM) options of the *Black76* and *Bates91* models. For the shortest option maturity ($T^* = 2/12$) the prices of *Black76* and *Bates91* ATM options ($K = 300$) are just over 20% higher than *New*. This number is down to about $7 - 11\%$ for the next maturity ($T^* = 4/12$). At the maturity closest to the maturity of the futures contract ($T^* = 6/12$), we see that ATM option prices from *Black76* and *Bates91* produce prices $6 - 10\%$ lower than *New*. The evidence for ATM options in the *Schwartz97* and *Fackler99* models is somehow mixed. The overpricing for short end options are less dramatic than *Black76* and *Bates91,* and for the middle maturity *Schwartz97* produce a higher price and *Fackler99* a lower price than *New.* This discrepancy is due to the seasonality adjustment in *Fackler99* uncounted for in *Schwartz97.* Both models underprice ATM options for the long maturity. The explanation for this can be found in table 2. In *Fackler99* and *Schwartz97* the parameter δ governs the maturity effect of volatility, whereas in the model *New* both δ and $\widetilde{\sigma}$ governs this effect. We observe that the estimated parameter for δ in model *New* is much higher than the corresponding estimates in *Fackler99* and *Schwartz97.* In these models the estimated $\delta' s$ seem to be a compromise between short- and long term volatility, producing a less pronounced maturity effect. This also explains the overpricing of short term options. Finally, all alternative models produce significantly lower price for OTM calls ($K = 340$) than *New* for all maturities. For the *Black76, Schwartz97* and *Fackler99* models, this fact is not surprising since OTM calls are more valuable when prices are allowed to jump. The difference between *Bates91* and *New* deserves some remarks. We see from table 2 that the parameters governing the jump dynamics estimated for *Bates91* and *New* are very different. This is because, as the volatility time-dependence is restricted to be flat in *Bates91*, the jump parameters will influence both the volatility across strikes - the "smile" - and the overall level of volatility. In other words, if option prices with different maturities are generated by the model of *Bates91* and these prices are turned into implied volatility by the model of *Black76*, we would observe implied volatility curves with increasing level of overall volatility as option maturities increase. For this reason, parameter estimates in a pure jump model is likely to be biased of the correct specified model exhibits time-dependent volatility.[11] In the model *New*, the parameters governing the maturity- and seasonality effects can take care of the price level, and the jump parameters can "concentrate" on "smile" effects. Hence the parameters in *Bates91*, through the estimation method, emerge as a compromise of the two effects.

The results reported here might be important in other valuation contexts. For example, Hilliard and Reis (1999) argue that average based Asian options are popular in commodity over-the-counter (OTC) markets. They show that Asian option prices in the *Black76* versus *Bates91* differ even more than is the case for European/American options prices. The very pronounced volatility time-dependence suggests that Asian options are more expensive when the averaging period is in the summer than is the case the rest of the year. Also, if the futures contract itself is the underlying asset, an averaging period close to maturity will typically be more volatile and resulting in higher option prices than the case would be if the average period occurs with long time to maturity of the futures

[11] This fact may partly explain the observation reported in Hilliard and Reis (1999) that parameter values are not stable over time. In their estimation procedure, they calibrate the model each day. Using their procedure, *Bates91* will be able to replicate *New* as long as we are only considering options with one maturity date. When either the option or futures maturity change, the parameters in *Bates91* must change to capture the volatility time-dependence. Hence we would expect unstable parameters in the analysis of Hilliard and Reis (1999) if, in fact, there exist volatility time-dependence in the underlying futures price dynamics.

contract.

6 Conclusions

In this paper we develop an option pricing model that incorporates several stylised facts reported in the literature on commodity futures price dynamics. The volatility is allowed to depend on both calendar-time and time to maturity. Furthermore, futures prices may exhibit sudden, discontinuous jumps. We estimate the parameters of the futures price dynamics by fitting our model to eleven years of wheat options data using non-linear least squares. Volatility dynamics of several models suggested previously in the literature are nested within our model, and they all gave significantly poorer fit compared to the full model. In a numerical example we show that ignoring volatility time-dependence and jump effects in futures prices might lead to severe mispricing of options.

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