Public Stackelberg leadership in a mixed oligopoly with foreign firms*

Kenneth Fjell

Foundation for Research in Economics and Business Administration

and

Norwegian School of Economics and Business Administration

Helleveien 30, 5045 Bergen, Norway

Phone: +47-5595-9687 Fax: +47-5595-9650 Email: kenneth.fjell@snf.no

John S. Heywood

Department of Economics

University of Wisconsin - Milwaukee

and

Department of Commerce

University of Birmingham

* We are grateful to M. Bahmani, W.L. Holahan, J.H. Peoples, D. Pal, seminar participants at the Norwegian School of Economics and Business Administration and two referees for valuable comments.

Abstract

This is the first paper to consider a mixed oligopoly in which a public Stackelberg leader competes with both domestic and foreign private firms. The welfare maximizing leader is shown to always produce less than under previous Cournot conjectures. Introducing leadership also alters previous public pricing rules resulting in prices that may be either greater than or less than marginal cost depending on the relative number of domestic firms. Furthermore, entry of a foreign firm will increase welfare only when the relative number of domestic firms is small, but that share is shown to be larger than has been indicated without leadership. Unlike previous models, the influence on public profit of a foreign acquisition is ambiguous and is related to the relative number of domestic firms. Finally, the consequences of privatization are shown, for the first time, to depend on the relative number of domestic firms.

JEL: D43, L10, L13

Keywords: Stackelberg leadership, mixed oligopoly, foreign acquisition, privatization

1. Introduction

Earlier literature examines mixed oligopoly models in a domestic context (see, e.g., Merill and Schneider, 1966; Cremer, Marchand, and Thisse, 1989 and 1991; DeFraja and Delbono, 1989; Harris and Wiens, 1980; Sertel, 1988; Fershtman, 1990; DeFraja and Delbono, 1990) while some more recent literature includes foreign private firms (Fjell and Pal, 1996; and Pal and White, 1998). The introduction of foreign firms affects outcomes because the welfare maximizing public firm ignores the producer surplus of the foreign firms. For Cournot competition, the introduction results in a lower price and a different allocation of production (Fjell and Pal, 1996). However, the effects of public Stackelberg leadership, previously explored in a domestic oligopoly context (DeFraja and Delbono, 1989), have not been examined in the presence of foreign private firms.

This paper is motivated by the belief that Stackelberg leadership best describes certain critical industries. Industries such as telecommunications, electricity and, increasingly, the postal sector, are dominated by former public monopolies with a first mover advantage. These industries more closely resemble Stackelberg leadership than Cournot oligopoly and often fit the description of mixed oligopolies with foreign firms. Thus, Telenor, the traditional state monopoly in the Norwegian telecommunications industry, has faced increased competition since the market was fully opened in January 1998. Some of this competition is foreign owned including MCI from the United States and firms originally based in France, the Netherlands, Sweden and Ireland among others. Competitors have captured more than a quarter of the market in some products (Norwegian Post and Telecommunications Authority, 1999).[1] Even after a partial privatization in 2000, Telenor remains eighty percent owned by the state and retains a social obligation to provide universal service (Telenor Annual Report, 2000).

A similar case is provided by the postal services of many countries. Again, using Norway as an example, the postal service retains a monopoly only on addressed letters and light parcels, but is required to provide a broad range of services through "an efficient nationwide service for the distribution of mail in Norway at reasonable prices and offering good quality of service" (License for Norway Post). The postal service is a

[1] Indeed, there are currently more than 3 dozen firms competing against Telenor in various product lines in Norway (Norwegian Post and Telecommunications Authority, 26. August 2001, http://www.npt.no/english/E_fagomraader/off_telenett/lister/eng_registrliste1.htm).

fully state owned corporation competing with domestic and foreign firms in many markets. The range of products for which the postal service has a regulated monopoly was recently reduced as a consequence of Norway's commitment to implement the *EU Postal Services Directive of 15 December 1997*.[2] Further reductions are anticipated as the EU has signaled intentions to further open the market. Nonetheless, the original monopoly of Norway Post can be expected to retain a significant first mover advantage. Even more dramatically, the previous postal monopoly in Sweden has been fully eliminated since January 1993 (Swedish National Post and Telecom Agency) and in New Zealand since April 1998. Yet, despite this statutory change, the public postal firms retain dominant positions. Even in countries without such dramatic change, the general picture is similar. The Australian Post, for instance, has an expensive community service obligation yet faces increasing pressure from competitive tendering (Castro and Maddock, 1997). Thus, many postal industries have a combination of public ownership and service obligations, a historical monopoly position and increasing competition.

In this paper we explore the equilibrium effects of Stackelberg leadership by a public firm, discuss the effects of an open market policy allowing foreign and domestic firms to enter, and the effects of foreign acquisition of domestic firms. We explicitly model the consequences of privatization with the novel, but realistic, assumption that the public Stackelberg leader retains its first mover advantage when privatized.

The paper is organized in a series of short sections. Section 2 describes the model and its equilibrium solution. Section 3 compares the equilibrium with previous models. Section 4 identifies unique aspects of the pricing rule for the public leader. Section 5 examines the consequences of an open market with entry by either foreign or domestic firms. Section 6 considers the foreign acquisition of a domestic firm, and section 7 explores the consequences of privatization. Section 8 concludes.

2. Model and equilibrium

Consider an industry where one state-owned public firm is a Stackelberg leader, whereas m domestic private firms and n foreign private firms are Cournot followers taking other firms' output as given. The Stackelberg leader moves first, anticipating the reaction of the followers. All $(m+n+1)$ firms produce a homogeneous

[2] Norwegian Post and Telecommunications Authority, 2001, (http://www.npt.no/english/E_fagomraader/posttilsyn/postregelverk.html).

commodity and have identical technologies. Let the cost function of a firm be $C(q) = f + \frac{1}{2}kq^2$, that is, there is a fixed cost f and an increasing marginal cost (kq), where $k > 0$ is a constant.

Let q_0 be the output of the public firm, q_i^d be the output of the domestic private firm i $(i = 1,..., m)$ and q_j^f be the output of the foreign private firm j $(j = 1,..., n)$. For tractability, we assume that the inverse demand is linear and is given by $P = a - \left(q_0 + \sum_{i=1}^{m} q_i^d + \sum_{j=1}^{n} q_j^f \right)$. Consequently, the consumer surplus is given by $CS = \frac{1}{2}\left(q_0 + \sum_{i=1}^{m} q_i^d + \sum_{j=1}^{n} q_j^f \right)^2$. Each private firm's objective is to maximize its own profit given the output of the other firms. $p_i^d = Pq_i^d - f - (1/2)k(q_i^d)^2$ denotes the profit of domestic private firm i and $p_j^f = Pq_j^f - f - (1/2)k(q_j^f)^2$ denotes the profit of foreign private firm j.

The objective of the public firm is to maximize (domestic) welfare, W, which is defined as the sum of consumer surplus and total profits of the domestic firms:

$$(1) \qquad W = \frac{1}{2}\left(q_0 + \sum_{i=1}^{m} q_i^d + \sum_{j=1}^{n} q_j^f \right)^2 + p_0 + \sum_{i=1}^{m} p_i^d$$

where $p_0 = Pq_0 - f - (1/2)k(q_0)^2$ denotes the profit of the public firm. Since we assume that the profit of the foreign firms is transferred out of the public firm's home country, W does not include $\sum_{j=1}^{n} p_j^f$.

Solving the model yields the following equilibrium:

$$(2) \qquad q_0^* = \frac{[(1+k)(1+k+2n)+km]a}{1+2n+[4n+3(m+1)+(m+n)^2]k+[3+2(m+n)]k^2+k^3}$$

$$(3) \qquad q_i^{d*} = q_j^{f*} = \frac{(m+n+k+1)ak}{1+2n+[4n+3(m+1)+(m+n)^2]k+[3+2(m+n)]k^2+k^3}$$

$$(4) \qquad P^* = \frac{(1+k)(m+n+k+1)ak}{1+2n+[4n+3(m+1)+(m+n)^2]k+[3+2(m+n)]k^2+k^3}$$

(5) $$p_i^d * = p_j^f * = \frac{1}{2}\left(\frac{(k+2)(m+n+k+1)^2 a^2 k^2}{\left\{1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right\}^2} \right) - f$$

(6) $$p_0 * = \frac{1}{2}\left(\frac{\left[(1+k)^2+m(k+2)\right]\left[(k+1)(2n+k+1)+km\right]a^2 k}{\left\{1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right\}^2} \right) - f$$

(7) $$CS* = \frac{1}{2}\left(\frac{\left[(k+1)^2+k(m+n)(m+n+k)+(2m+3n)k+2n\right]^2 a^2}{\left\{1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right\}^2} \right)$$

(8) $$W* = \frac{1}{2}\left(\frac{\left[(k+1)(k+1+2n)+k(m+n)^2+km(k+3)\right]a^2}{1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3} \right) - f(m+1)$$

Note that if $n = 0$ (i.e. in the absence of foreign firms), we get back the results of DeFraja and Delbono (1989).

3. Comparison with previous models

A comparison of the Stackelberg equilibrium in (2) – (8) to one in which all followers are domestic (DeFraja and Delbono, 1989), shows that the presence of foreign private firms involves a lower price and a different allocation of production (superscript SD denotes Stackelberg equilibrium with only domestic firms and the equilibrium values for this case are in Appendix A):

(9) $$P* - P^{SD} = \frac{-akn\left[1+2m+(1+m)(4+m+n)k+(2+m)(3+m+n)k^2+(4+2m+n)k^3+k^4\right]}{\left\{1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right\}\left[(1+k)^2+nk+k(1+k+n)^2\right]} < 0$$

(10) $$q_i^d * - q_i^{SD} = \frac{-akn\left[1+2m+\left(3+n+3m+m^2+mn\right)k+(3+2m+n)k^2+k^3\right]}{\left\{1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right\}\left[(1+k)^2+nk+k(1+k+n)^2\right]} < 0$$

From (9) and (10) we can conclude that the presence of foreign firms results in higher output by the public leader, $q_0 * - q_0^{SD} > 0$. This follows as the combination of lower price and reduced follower output can only hold if the public leader's output has increased.

A comparison of the Stackelberg case with foreign firms to the Cournot case with foreign firms (Fjell and Pal 1996) shows that in the Stackelberg case the public firm's output is lower, the private firms' output is

higher, and price is higher (superscript CF denotes Cournot equilibrium in the presence of foreign private firms and the equilibrium values are in Appendix A):

$$q_0^* - q_0^{CF} = \frac{-ak\left[(m+n)^2 + kmn + nm^2 + m(k+1) + n^2(2m+k) + n^3\right]}{\left\{1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right\}\left[k(m+n+k+1)+n+k+1\right]} < 0 \tag{11}$$

$$q_i^{d*} - q^{CF} = \frac{ak\left(mn+m+n^2\right)}{\left\{1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right\}\left[k(m+n+k+1)+n+k+1\right]} > 0 \tag{12}$$

$$P^* - P^{CF} = \frac{ak\left[m(n+k+1)+kmn+n^2(k+1)\right]}{\left\{1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right\}\left[k(m+n+k+1)+n+k+1\right]} > 0 \tag{13}$$

The intuition behind this is that as a Stackelberg leader, the public firm is aware of the reaction functions of the private firms and uses this to move some of its production to the private firms (DeFraja and Delbono, 1989). As expected, welfare is higher under Stackelberg leadership and the proof is in Appendix A.

Proposition 3.1 *As a Stackelberg leader, the public firm will always produce less than as a Cournot competitor when maximizing welfare in a mixed oligopoly whether foreign private firms are present or not.*

The proof of proposition 3.1 is established by (11), and by DeFraja and Delbono (1989) who show that public output is also less for the Stackelberg leader in the absence of foreign private firms. This result is contrary to that for a profit maximizing Stackelberg leader, which normally produces an output greater than if it were a Cournot competitor. The intuition is that as a Stackelberg leader, the public firm recognizes the inverse relationship between own output and that of the Cournot followers. Hence, it maximizes welfare in part by reducing its output relative to the public Cournot firm, realizing that some of the reduction will instead be produced by the private firms at a lower marginal cost.[3]

[3] Increasing marginal cost is thus critical to this result.

4. Pricing rules for the Stackelberg leader

Beato and Mas-Colell (1984) characterize duopoly conditions under which marginal cost pricing is the best simple rule for a welfare maximizing firm. DeFraja and Delbono (1989) show that in the context of a mixed Cournot oligopoly, the public firm does indeed choose an output such that its marginal cost equals price. However, they proceed to show that a public Stackelberg leader will choose a lower output for which marginal cost is less than price. Extending the Cournot analysis to include foreign private firms, Fjell and Pal (1996) find that the public Cournot firm chooses an output such that its marginal cost exceeds price if foreign firms are present. We find that in the presence of foreign private firms, a public Stackelberg leader chooses an output such that its marginal cost may be either greater than or less than price.

Proposition 4.1 *The relationship between marginal cost of the leader and price is such that:* $mc_0^* > P^*$ *as* $m < n(k+1)$ *and* $mc_0^* = P^*$ *as* $m = n(k+1)$ *and* $mc_0^* < P^*$ *as* $m > n(k+1)$.

Thus, the marginal cost of the public Stackelberg leader is greater (smaller) than the market price if the number of domestic firms is small (large) relative to the number of foreign firms. The proof of proposition 4.1 follows from comparing the market price (eq. 4) and the marginal cost of the public firm, kq_0^* (from eq. 2).

This result, new to the literature, shows that only when $m = n(k+1)$, will it be optimal for the public leader to price at marginal cost. The relationship between public output, marginal cost, price and additional foreign private firms is illustrated in figure 1. This figure captures the essence of the proposition and shows the relationship between the relative number of domestic firms and the output of the public leader. The leader produces the most when $m = n(k+1)$ which is associated with price equal to marginal cost.

The intuition behind the proposition is as follows. To maximize welfare, the public firm chooses an output such that the marginal (increase in) consumer surplus equals the marginal (decrease in) total domestic profits. When the share of domestic firms is relatively large (left hand side of figure 1), increases in the share of foreign firms results in an increasing share of profit transferred out of the country. The public leader responds by increasing output which adds to consumer surplus more than it reduces domestic profit. When the share of domestic firms is relatively small (right hand side of figure 1), increases in the share of foreign

firms and the increasing share of profit transferred out of the country elicits a different response. The public leader reduces output which increases domestic profit more than it reduces consumer surplus.

5. Effects of an open market policy

The next propositions consider the effects of an open market policy that allows foreign and domestic firms to enter a mixed oligopoly. This reflects the current status of many of the previously regulated monopolies in Western Europe.

Proposition 5.1 *The public firm's output decreases (increases) and welfare increases (decreases) with the entry of a foreign private firm if the relative number of domestic firms is small (large).*

These results are analogous to the case of the public Cournot competitor and follow from differentiation of q_0^* and W^* with respect to n. For public output, the effect of foreign entry is:

$$(14) \qquad \frac{\partial q_0^*}{\partial n} = \frac{-2ak(kn+n-m)(m+n+k+1)}{\left\{1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right\}^2}$$

This derivative can be shown to be negative only if $m < n(k+1)$, that is if the relative number of domestic private firms is small. The derivative is positive when the inequality reverses, that is, when the relative number of domestic private firms is large, public output will increase with an additional foreign private firm.

The welfare effects are similarly found:

$$(15) \qquad \frac{\partial W^*}{\partial n} = \frac{a^2k^2(kn+n-m)(m+n+k+1)}{\left\{1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right\}^2}$$

Thus, the entry of a foreign private firm will increase welfare only if $m < n(k+1)$, the same condition for public output to decrease.

The intuition behind Proposition 5.1 is as follows. Recall that for a given rival output, the public firm produces less if the output comes from domestic firms. The addition of a foreign firm increases consumer surplus and decreases domestic profit. When the relative number of domestic firms is large and the public

firm's marginal cost is low relative to price, it is optimal for the public firm to increase its output. The increase in consumer surplus outweighs the decrease in domestic profit. When the ratio of domestic to foreign private firms equals $(k+1)$, public marginal cost equals market price and this represents a turning point for the public firm's strategy. Beyond this point, further foreign entry will now be accommodated by reductions in public output as the reduction in consumer surplus is outweighed by the increase in domestic profit. However, in spite of the reduction in public output, public marginal cost will exceed market price (see figure 1).

Despite the similarities with the case of Cournot competition, the results are not identical.

Corollary 5.1 *The range for which public output decreases and welfare increases with the entry of a foreign private firm, is larger in the Stackelberg case than in the case of a public Cournot competitor.*

Compared to the Cournot oligopoly, welfare will increase over a greater range of domestic firms when the public firm is a Stackelberg leader, *ceteris paribus.* The equivalent Cournot condition for an increase in welfare following foreign private entry is $m < \frac{n(k+1)^2}{(1+k+n)}$ (Fjell and Pal, 1996). It can readily be verified that this inequality holds for a smaller range of m than is true for the Stackelberg case.

An implication of this corollary is that for a specific range in the number of private firms, foreign entry will increase welfare in the Stackelberg case but decrease welfare in the Cournot case. This difference emerges because the leader is better able to accommodate entry by altering its own output in a fashion that alters market output (this because it knows the reaction functions of the followers). Thus, a greater extent of entry can be accommodated in a fashion that increases welfare.

Proposition 5.2 *Regardless of m and n, the public firm's output decreases and welfare increases with the entry of a domestic private firm.*

The proof follows from taking the derivatives of public output and welfare with respect to *m.*

(16) $$\frac{\partial q_0^*}{\partial m} = \frac{-ak\left\{2\left[1+m+(3+2m+2n)n\right]+\left[5+4m(n+1)+10n+m^2+3n^2\right]k+2\left[2(n+1)+m\right]k^2+k^3\right\}}{\left\{1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right\}^2} < 0$$

(17) $$\frac{\partial W^*}{\partial m} = \frac{a^2k^2\left(k^2+3k+3kn+km+4n+2\right)(m+n+k+1)}{2\left\{1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right\}^2} - f$$

Entry of a domestic firm always results in an accommodation in output by the public leader as it realizes that the new entrant can produce some of its output at a lower marginal cost.[4] The increased production at a lower marginal cost increases welfare provided the fixed cost is sufficiently low.

6. Effects of foreign acquisitions of domestic firms

The practice in many developing countries of banning or limiting foreign ownership of domestic firms has been relaxed in recent years making the issue of foreign acquisitions more relevant.

A useful benchmark for analyzing the effects of foreign acquisition is an oligopoly without a public firm. In this case, the purchase of a domestic private firm by foreign nationals does not change individual outputs since all private firms have identical objectives (of profit maximization) and technologies. As a result, total output, market price, profits of the private firms, and consumer surplus do not change. Welfare, however, decreases, as the foreign private firm is assumed to transfer its profit out of the home country. Thus, in the absence of a public firm, a foreign acquisition of domestic firms reduces total welfare although it does not affect either consumer surplus or profits of other private firms. This benchmark can be compared to the case of a public Cournot firm. In this case, output of the public firm increases, consumer surplus increases, welfare decreases and profit of the public firm decreases following a foreign acquisition (Fjell and Pal, 1996).

Although the presence of a public Stackelberg leader changes the analysis with public profit depending on the relative number of domestic firms, parts of the analysis remain analogous to the public Cournot case.

[4] This result depends critically on the assumption of increasing marginal cost.

Proposition 6.1 *Regardless of m and n, if a domestic firm is acquired by foreign nationals, the output of the public firm increases, the outputs of all private firms decrease, consumer surplus increases and welfare increases.*[5]

These results are similar to those from the Cournot case (Fjell and Pal, 1996) and the proof follows from comparing the equilibrium outcomes before and after acquisition. These results are in Appendix B.

Proposition 6.2 and Corollary 6.1 summarize the results with respect to public firm profit and are unique to the Stackelberg case.

Proposition 6.2 *If a domestic firm is acquired by foreign nationals, profit of the public firm will decrease when m < n.*

The proof of proposition 6.2 follows by comparison of the equilibrium outcomes before and after the acquisition. A foreign acquisition implies that n increases by one and m simultaneously decreases by one. Identifying the equilibrium value after a foreign acquisition by superscript *A*, we get the following equilibrium change in public profit:

$$(18) \qquad p_0^A - p_0^* = -\frac{1}{2}\left(\frac{(k+2)(m+n+k+1)a^2kD}{\left\{1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right\}^2 B^2} \right)$$

where *B* is positive and *D* may be positive or negative, and both are in Appendix B. Equation (18) can be shown to be negative whenever $m < n$.

In contrast to when the public firm is a Cournot competitor and its profit always decreases when foreign nationals acquire a domestic private firm, public profit can actually increase when the public firm is a Stackelberg leader.

[5] If fixed cost, f, is sufficiently high, welfare may increase as the fixed cost is now being paid by a foreign firm.

Corollary 6.1 *If a domestic firm is acquired by foreign nationals, the profit of the public leader can increase.*

A necessary, but not sufficient, condition is $m > n$. A numerical illustration serves as proof. If k=3, m=8, and n=0, a foreign acquisition increases leader profit $\left(p_0^A = \frac{85}{5618} a^2 - f > \frac{105}{6967} a^2 - f = p_0 * \right)$. However,, for an additional acquisition, i.e. when k=3, m=7, and n=1, leader profit is reduced $\left(p_0^A = \frac{1725}{116162} a^2 - f < \frac{85}{5618} a^2 - f = p_0 * \right)$.

The public firm always chooses an output such that the marginal (increase in) consumer surplus is equal to the marginal (decrease in) total domestic profits. When the relative number of domestic firms is large, the public leader tends to limit its output below its individual profit maximum to shield profits of domestic followers. A foreign acquisition results in increased leader output (and leader profit) as the increase in consumer surplus outweighs the decrease in domestic profits. It is this possibility that is unique to the current model. When the relative number of domestic firms is small, the public firm produces beyond its own profit maximizing output, and foreign acquisition causes it to produce even more (which further reduces its own profit) as the increase in consumer surplus continues to outweigh the decrease in domestic profits.

7. Privatization

As discussed in the introduction, many public monopolies in Europe have been forced to compete with both domestic and foreign competitors following partial or full deregulation. Parallel, or sometimes subsequent, to this, there has been privatization of the public incumbent. For example, in the Netherlands, the TNT Post Group[6] has only 34.9% state ownership, operates as a private enterprise,[7] and has been subject to a more rapid deregulation of its monopoly than required by EU regulation (OPTA Annual Report, 2000). Similarly, the Swedish public postal service was subject to full competition in 1993 and organized as a state-owned limited liability company known as Sweden Post in 1994. Although Sweden Post still has a universal service obligation, it determines the level of service and operates on the whole as a private for-profit company

[6] Holding company of the PTT Post.

(Swedish National Post and Telecom Agency). Past literature assumes that privatization results in the public Stackelberg leader becoming a simple private Cournot competitor (DeFraja and Delbono 1989: pp. 307). Yet, the public incumbent often retains a first mover advantage and the effect of privatization might better be characterized as creating a private Stackelberg leader. Certainly the incumbent postal companies in both the Netherlands and Sweden retain dominant positions in their markets. Examining the privatization of a Stackelberg leader is particularly interesting in the presence of foreign firms.

Proposition 7.1 *Privatization of a public Stackelberg leader reduces leader output, increases follower output, increases prices, and increases follower profit when* $m < n$. *Regardless of m and n, privatization always increases profit of the leader and decreases welfare.*

The comparison on quantity of the leader comes from subtracting the equilibrium output of a profit maximizing privatized leader (superscript P) from that of a welfare maximizing public leader:

$$(19) \qquad q_0^P - q_0^* = \frac{-\left(mnk^2 + n^2k^2 + nk^3 - m^2k + n^2k + 4nk^2 + k^3 + 5nk + 3k^2 + 2n + 3k + 1\right)a}{\left(mk + nk + k^2 + 3k + 2\right)\left(1 + 2n + \left[4n + 3(m+1) + (m+n)^2\right]k + \left[3 + 2(m+n)\right]k^2 + k^3\right)}$$

It can be confirmed that a sufficient, but not necessary, condition for public leader output to exceed private leader output, is that $m < n$.

In other words, if the number of foreign followers exceeds that of domestic followers, output of the leader declines following privatization. For a given amount of follower output, the public firm produces more as the share from foreign firms increases because foreign firms' profits do not enter the welfare function. Hence, a relatively large number of foreign followers induces the public leader to produce beyond its profit maximizing output, and thus privatization reduces output. Appendix C presents the equilibrium solution to the case of the privatized Stackelberg leader, and comparison with equations (2) - (8) constitutes the remainder of the proof of proposition 7.1.

---

[7] TPG Post Group, 21 September 2001 (http://www.tntpost-group.com/wwwenglish/investorrelations/index.html?inv_frequentquestions.html)

Corollary 7.1 *With a relatively large number of domestic firms, the privatization of a public Stackelberg leader can increase leader output, decrease follower output, decrease prices and decrease follower profit.*

A necessary, but not sufficient, condition for this corollary is $m > n$. A numerical illustration serves as proof. If $k=1$, $m=6$, and $n=1$, privatization increases leader output $\left(q_0^P = \frac{2}{13}a > \frac{14}{95}a = q_0 * \right)$. In all such cases in which the public Stackelberg leader produces less than the private leader, it can be readily confirmed that each of the results sensitive to m and n in proposition 7.1 are reversed. By way of illustration, if $k=5$ rather than $k=1$ the main results of proposition 7.1 return and quantity of the leader, in particular, decreases $\left(q_0^P = \frac{6}{77}a < \frac{6}{71}a = q_0 * \right)$.

As the proposition and corollary make clear, the interests of the followers depend dramatically upon their composition. When the followers are disproportionately foreign, the proposition shows that their profit will increase with privatization. When the followers are disproportionately domestic, the corollary indicates their profit can fall with privatization. Thus, even though foreign and domestic follower firms have identical interests, one would anticipate their political lobbying on the issue of privatization to vary with the relative number of domestic firms.[8]

8. Summary and conclusions

In this paper we consider a mixed oligopoly model, in which a state-owned public firm competes as a Stackelberg leader with domestic and foreign private firms. We have characterized the equilibrium and compared it to those from previous models. We find that regardless of the mix of foreign and domestic firms, the public leader produces less than under a Cournot conjecture.

A variety of results are shown for the first time to depend critically upon the relative number of domestic firms. First, we find that the public firm produces where marginal cost is greater (less) than price if the number of domestic firms is relatively small (large). Second, given an open market, entry of a foreign firm increases welfare only when the share of domestic firms is small, but that share is shown to be larger than

has been indicated in models without leadership. Third, foreign acquisitions can increase public profit, but only when the share of domestic firms is large. Fourth, privatization can decrease follower profit, but also only when the share of domestic firms is large. The possibility that foreign acquisitions will increase public profit and that privatization will decrease follower profit is unique to the Stackelberg mixed oligopoly.

Increasing privatization and open-door policies in Europe and many developing countries, makes further research in this area desirable. In this paper, as in most that preceded it, the analysis has been based on partial equilibrium. Recognizing the importance of feedback loops might alter some of the results. For instance, foreign acquisition of a domestic firm directly reduces welfare but it may increase foreign demand for domestic products and thus indirectly increase domestic welfare in a general equilibrium analysis. While beyond the scope of this article, such inquiry might be extremely fruitful.

[8] Note that after privatization the leader is indifferent to whether followers are foreign or domestic. As a consequence, the acquisition of a domestic firm by a foreign firm will not change the behavior of the private leader.

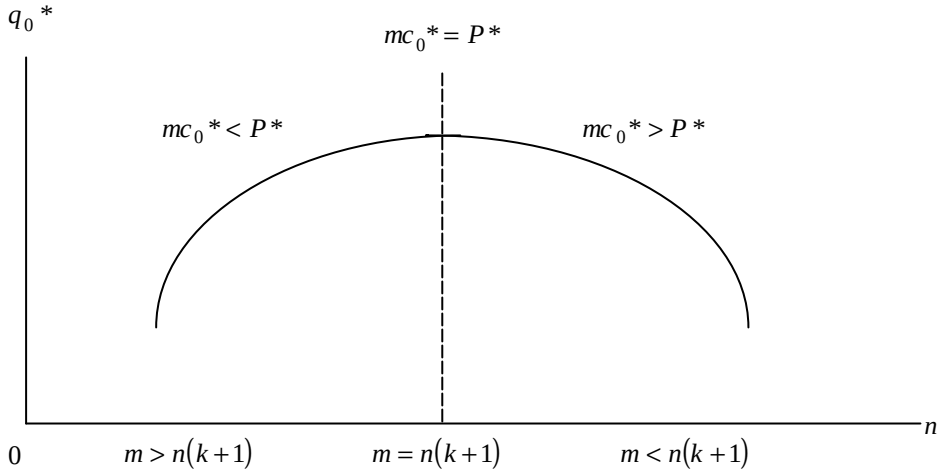

Figure 1. Public output versus additional foreign private firms.

Appendix A

Solving for the equilibrium in a standard fashion, we begin by deriving the reaction function of the Cournot followers. The expanded objective function of a representative domestic private firm, k, is:

$$\text{(A1)} \qquad p_k^d = \left(a - q_0 - \sum_{i=1}^{m-1} q_i^d - \sum_{j=1}^{n} q_j^f - q_k^d\right) q_k^d - f - \frac{1}{2}k\left(q_k^d\right)^2$$

Maximizing the firm's profit with respect to q_k^d, taking other outputs as given, and equating to zero, we obtain:

$$\text{(A2)} \qquad a - q_0 - \sum_{i=1}^{m-1} q_i^d - \sum_{j=1}^{n} q_j^f - 2q_k^d - k\left(q_k^d\right) = 0$$

Due to symmetry, the optimal outputs are identical for all private firms. Equating all private outputs in (A2) to q and solving, we get the reaction function of a representative private firm as a function of public output:[9]

$$\text{(A3)} \qquad q = \frac{a - q_0}{m + n + k + 1} \qquad \text{where } q = q_i^d = q_j^f$$

Expanding (1) using (A3) we obtain the public firm's objective function in terms of own output:

$$\text{(A4)} \qquad \begin{aligned} W = {} & \frac{1}{2}\left[q_0 + (m+n)\left(\frac{a - q_0}{m + n + k + 1}\right)\right]^2 + \left[a - q_0 - (m+n)\left(\frac{a - q_0}{m + n + k + 1}\right)\right] q_0 - f - \frac{1}{2}k(q_0)^2 \\ & + m\left\{\left[a - q_0 - (m+n)\left(\frac{a - q_0}{m + n + k + 1}\right)\right]\left(\frac{a - q_0}{m + n + k + 1}\right) - f - \frac{1}{2}k\left(\frac{a - q_0}{m + n + k + 1}\right)^2\right\} \end{aligned}$$

Maximizing (A4) with respect q_0 yields the equilibrium output for the public firm; equation (2) in the paper. The FOC and SOC for the public firm are:

$$\text{(A5)} \qquad \begin{aligned} & \frac{\partial W}{\partial q_0} = 0 \\ & \frac{a\left[(1+k)(1+k+2n) + km\right] - \left\{1 + 2n + \left[4n + 3(m+1)(m+n)^2\right]k + \left[3 + 2(m+n)\right]k^2 + k^3\right\} q_0}{(m+n+k+1)^2} = 0 \end{aligned}$$

[9] Concavity is satisfied by $\dfrac{\partial^2 p_k^d}{\partial q_k^2} = -(k+2)$

(A6) $$\frac{\partial^2 W}{\partial q_0^{\,2}} = \frac{-\left\{1+2n+\left[4n+3(m+1)(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right\}}{(m+n+k+1)^2} < 0 \qquad \text{concave}$$

The equilibrium when the public firm is Stackelberg leader and all $M=(m+n)$ private firms are domestically owned is (DeFraja and Delbono, 1989):

(A7) $$q_0^{SD} = \frac{a\left[(1+k)^2+Mk\right]}{(1+k)^2+Mk+k(1+k+M)^2}$$

(A8) $$q^{SD} = \frac{ak(1+k+M)}{(1+k)^2+Mk+k(1+k+M)^2}$$

(A9) $$P^{SD} = \frac{ak(1+k)(1+k+M)}{(1+k)^2+Mk+k(1+k+M)^2}$$

(A10) $$p_0^{SD} = \frac{a^2k\left[(1+k+M)^2(1+k)^2-M^2\right]}{2\left[(1+k)^2+Mk+k(1+k+M)^2\right]^2} - f$$

(A11) $$p^{SD} = \frac{a^2k^2(1+k+M)^2}{\left[(1+k)^2+Mk+k(1+k+M)^2\right]^2}\left(1+\frac{k}{2}\right) - f$$

(A12) $$W^{SD} = \frac{(1+k)^2+Mk+kM(2+k+M)}{2\left[(1+k)^2+Mk+k(1+k+M)^2\right]}a^2 - (M+1)f$$

The equilibrium under Cournot-Nash conjectures with foreign private firms is (Fjell and Pal, 1996):

(A13) $$q_0^{CF} = \frac{a(n+k+1)}{k(m+n+k+1)+n+k+1}$$

(A14) $$q^{CF} = \frac{ak}{k(m+n+k+1)+n+k+1}$$

(A15) $$P^{CF} = \frac{ak(k+1)}{k(m+n+k+1)+n+k+1}$$

(A16) $$p_0^{FC} = \frac{a^2k\left[(k+1)^2-n^2\right]}{2\left\{k(m+n+k+1)+n+k+1\right\}^2} - f$$

(A17) $$p^{FC} = \frac{\left(ak^2\right)(k+2)}{2\left\{k(m+n+k+1)+n+k+1\right\}^2} - f$$

(A18) $$W^{FC} = \frac{k\left[(k+1)^2-n^2\right]+mk^2(k+2)+\left[n+k+1+k(m+n)\right]^2}{2\left\{k(m+n+k+1)+n+k+1\right\}^2} - (m+1)f$$

Proof that welfare is higher under Stackelberg leadership than under Cournot competition:

$$W - W^{FC} = \frac{\left(mn + n^2 + m\right)a^2k^2}{2\{k(m+n+k+1)+n+k+1\}^2\{1+2n+[4n+3(m+1)+(m+n)^2]k+[3+2(m+n)]k^2+k^3\}} \quad \text{(A19)}$$

Appendix B Effects of Foreign Acquisition

A foreign acquisition implies that n increases by one and m simultaneously decreases by one. Identifying the equilibrium value after a foreign acquisition by superscript *A*, and subtracting from initial equilibrium values, we get:

(B1) $$q_0^A - q_0^* = \frac{ak(k+2)(m+n+k+1)^2}{\{1+2n+[4n+3(m+1)+(m+n)^2]k+[3+2(m+n)]k^2+k^3\}B} > 0$$

(B2) $$p^A - p^* = -\frac{1}{2}\left(\frac{\{4(n+1)+[7+6m+8n+2(m+n)^2]k+[6+4(m+n)]k^2+2k^3\}(k+2)^2(m+n+k+1)^2a^2k^2}{\{1+2n+[4n+3(m+1)+(m+n)^2]k+[3+2(m+n)]k^2+k^3\}^2B^2}\right) < 0$$

(B3) $$CS^A - CS^* = \frac{1}{2}\left(\frac{(k+1)(k+2)(m+n+k+1)a^2kC}{\{1+2n+[4n+3(m+1)+(m+n)^2]k+[3+2(m+n)]k^2+k^3\}^2B^2}\right) > 0$$

(B4) $$W^A - W^* = -\frac{1}{2}\left(\frac{(k+2)(m+n+k+1)^2a^2k^2}{\{1+2n+[4n+3(m+1)+(m+n)^2]k+[3+2(m+n)]k^2+k^3\}B}\right) + f < 0$$

provided f is sufficiently low.

where *B* is positive and equal to: $B = 3+2n+[4(n+1)+3m+(m+n)^2]k+[3+2(m+n)]k^2+k^3$

and *C* is positive and equal to:

$$\begin{aligned}C = {} & 2k^5m+2k^5n+6k^4m^2+12k^4mn+6k^4n^2+6k^3m^3+18k^3m^2n+18k^3mn^2+6k^3n^3+2k^2m^4\\ & +8k^2m^3n+12k^2m^2n^2+8k^2mn^3+2k^2n^4+2k^5+14k^4m+16k^4n+22k^3m^2+50k^3mn\\ & +28k^3n^2+10k^2m^3+34k^2m^2n+38k^2mn^2+14k^2n^3+11k^4+35k^3m+47k^3n+24k^2m^2\\ & +70k^2mn+48k^2n^2+8km^2n+16kmn^2+8kn^3+26k^3+41k^2m+69k^2n+8km^2+36kmn\\ & +36kn^2+33k^2+20km+52kn+8n^2+22k+16n+6\end{aligned}$$

(B5) Referring to equation (18), *D* may be positive or negative. A sufficient, but not necessary, condition for *D* to be positive (and equation (18) to be negative) is $m < n$:

$$\begin{aligned}D = {} & 2k^6n+6k^5n^2+6k^5nm+6k^4n^3+12k^4n^2m+6k^4nm^2+2k^3n^4+6k^3n^3m+6k^3n^2m^2\\ & +2k^3nm^3+3k^6+21k^5n+7k^5m+37k^4n^2+38k^4nm+3k^4m^2+21k^3n^3+37k^3n^2m+13k^3nm^2\\ & -3k^3m^3+2k^2n^4+4k^2n^3m-4k^2nm^3-2k^2m^4+19k^5+78k^4n+34k^4m+87k^3n^2+86k^3nm\\ & +11k^3m^2+24k^2n^3+36k^2n^2m+8k^2nm^2-4k^2m^3+51k^4+143k^3n+61k^3m+96k^2n^2\\ & +80k^2nm+10k^2m^2+8kn^3+8kn^2m+75k^3+142k^2n+48k^2m+48kn^2+24knm+64k^2+74kn\\ & +14km+8n^2+30k+16n+6\end{aligned}$$

Appendix C Effects of Privatization

Using the reaction function of a follower derived in Appendix A, and using simple profit maximization as the objective function of the privatized leader, we derive the following equilibrium solution (identified by superscript *P*):

(C1) $$q_o^P = \frac{(1+k)a}{2(1+k)+k(m+n+k+1)}$$

(C2) $$q^P = \frac{[1+k+(m+n+k+1)k]a}{[2(1+k)+(m+n+k+1)k](m+n+k+1)}$$

(C3) $$P^P = \frac{[1+k+(m+n+k+1)k]a}{[2(1+k)+(m+n+k+1)k](m+n+k+1)}$$

(C4) $$p_o^P = \frac{1}{2}\left(\frac{(1+k)^2 a^2}{[2(1+k)+(m+n+k+1)k](m+n+k+1)}\right) - f$$

(C5) $$p^P = \frac{1}{2}\left(\frac{(k+2)(1+2k+km+kn+k^2)^2 a^2}{[2(1+k)+(m+n+k+1)k]^2(m+n+k+1)^2}\right) - f$$

(C6) $$W^P = \frac{1}{2}\left(\frac{Ea^2}{[2(1+k)+(m+n+k+1)k]^2(m+n+k+1)^2}\right) - f(m+1)$$

where *E* is positive and equal to:

$$\begin{aligned}E = {} & m^4k^2 + 3m^3k^3 + 4m^3k^2n + 3m^2k^4 + 8m^2k^3n + 6m^2k^2n^2 + mk^5 + 4mk^4n + 7mk^3n^2 \\ & + 4mk^2n^3 + k^4n^2 + 2k^3n^3 + k^2n^4 + 8m^3k^2 + 17m^2k^3 + 22m^2k^2n + 10mk^4 + 26mk^3n + 20mk^2n^2 \\ & + k^5 + 4k^4n + 9k^3n^2 + 6k^2n^3 + 4m^3k + 29m^2k^2 + 12m^2kn + 32mk^3 + 48mk^2n + 12mkn^2 + 7k^4 \\ & + 18k^3n + 19k^2n^2 + 4kn^3 + 19m^2k + 46mk^2 + 34mkn + 18k^3 + 30k^2n + 15kn^2 + 4m^2 + 31mk \\ & + 8mn + 22k^2 + 22kn + 4n^2 + 8m + 13k + 6n + 3\end{aligned}$$

Subtracting the results in (C1)-(C6) from the corresponding results for the public Stackelberg equilibrium (2)-(8), we get the effect of privatization of the public Stackelberg leader: [10]

(C7) $$q_0^P - q_0^* = \frac{-(mnk^2 + n^2k^2 + nk^3 - m^2k + n^2k + 4nk^2 + k^3 + 5nk + 3k^2 + 2n + 3k + 1)a}{(mk + nk + k^2 + 3k + 2)(1 + 2n + [4n + 3(m+1) + (m+n)^2]k + [3 + 2(m+n)]k^2 + k^3)}$$

(C8) $$q^P - q^* = \frac{(mnk^2 + n^2k^2 + nk^3 - m^2k + n^2k + 4nk^2 + k^3 + 5nk + 3k^2 + 2n + 3k + 1)a}{(mk + nk + k^2 + 3k + 2)(1 + 2n + [4n + 3(m+1) + (m+n)^2]k + [3 + 2(m+n)]k^2 + k^3)(m+n+k+1)}$$

[10] Omitting direct comparison of consumer surplus as this follows indirectly from comparison of price.

$$P^P - P^* =$$

(C9)
$$\frac{(1+k)\left(mnk^2+n^2k^2+nk^3-m^2k+n^2k+4nk^2+k^3+5nk+3k^2+2n+3k+1\right)a}{\left(mk+nk+k^2+3k+2\right)\left(1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right)(m+n+k+1)}$$

$$p_0^P - p_o^* =$$

(C10)
$$\frac{\left[\left(mnk^2+n^2k^2+nk^3-m^2k+n^2k+4nk^2+k^3+5nk+3k^2+2n+3k+1\right)a\right]^2}{2\left(mk+nk+k^2+3k+2\right)\left(1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right)^2(m+n+k+1)}$$

$$p^P - p^* =$$

(C11)
$$\frac{(2+k)\left(mnk^2+n^2k^2+nk^3-m^2k+n^2k+4nk^2+k^3+5nk+3k^2+2n+3k+1\right)Fa^2}{2\left\{\left(mk+nk+k^2+3k+2\right)\left(1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right)(m+n+k+1)\right\}^2}$$

where F is positive and equal to:

$$\begin{aligned}F = {} & 2m^3k^2+6m^2nk^2+6m^2k^3+6mn^2k^2+12mnk^3+6mk^4+2n^3k^2+6n^2k^3+6nk^4+2k^5 \\ & +10m^2k^2+21mnk^2+20mk^3+11n^2k^2+21nk^3+10k^4+3m^2k+8mnk+22mk^2+5n^2k \\ & +26nk^2+19k^3+8mk+13nk+17k^2+2n+7k+1\end{aligned}$$

$$W_0^P - W_o^* =$$

(C12)
$$\frac{-\left[\left(mnk^2+n^2k^2+nk^3-m^2k+n^2k+4nk^2+k^3+5nk+3k^2+2n+3k+1\right)a\right]^2}{2\left(mk+nk+k^2+3k+2\right)^2\left(1+2n+\left[4n+3(m+1)+(m+n)^2\right]k+\left[3+2(m+n)\right]k^2+k^3\right)(m+n+k+1)^2}$$

References

Beato, P., and A. Mas-Colell, 1984, The marginal cost pricing as a regulation mechanism in mixed markets, In *The Performance of Public Enterprises*, ed. M. Maarchand, P. Pestieau, and H. Tulkens (Amsterdam: North-Holland).

Castro, M. and R. Maddock, 1997, The universal service obligation for post: Some Australian calculations, in eds. M. Crew and P. Kleindorfer, *Managing Change in the Postal and Delivery Industries*, Kluwer Academic Press: Norwell Massachusetts.

Cremer, H., M. Marchand and J. Thisse, 1989, The public firm as an instrument for regulating an oligopolistic market, *Oxford Economic Papers*, 41, 283-301.

Cremer, H., M. Marchand and J. Thisse, 1991, Mixed oligopoly with differentiated products, *International Journal of Industrial Organization*, 9, 43-53.

DeFraja, G., and F. Delbono, 1989, Alternative strategies of a public enterprise in oligopoly, *Oxford Economic Papers*, 41, 302-11.

DeFraja, G., and F. Delbono, 1990, Game theoretic models of mixed oligopoly, *Journal of Economic Surveys*, 4, 1-17.

Fershtman, C., 1990, The interdependence between ownership status and market structure: the case of privatization, *Economica*, 57, 319-28.

Fjell, K. and Pal, D., 1996, A mixed oligopoly in the presence of foreign private firms, *Canadian Journal of Economics*, 29, 737-43.

Harris, R.G., and E.G. Wiens, 1980, Government enterprise: An instrument for internal regulation of industry, *Canadian Journal of Economics*, 13, 125-32.

Merrill, W.C. and N. Schneider, 1966, Government firms in oligopoly industries: a short run analysis, *Quarterly Journal of Economics*, 80, 400-12.

Norwegian Post and Telecommunications Authority, 1999, "The Norwegian Telecommunications Market – Statistics and Analysis 1999" (http://www.npt.no/english/E_publikasjoner/statistikk99/den_norske_telemarknaden3_2_5.html)

OPTA Annual Report, 2000, (http://www.opta.nl/download/annual_report_2000.pdf)

Pal, Debashis, and Mark D. White, 1998, Mixed oligopoly, privatization, and strategic trade policy, *Southern Economic Journal*, 65 (2), 264-81.

Sertel, M., 1988, Regulation by participation, *Journal of Economics*, 48, 114-34.

Telenor Annual Report, 2000, (htttp://www.telenor.com/reports/2000/main.html)