Some Comments on Free-Riding in Leontief Partnerships[1]

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Abstract

Holmström (1982) showed that free-riding is inevitable in partnerships where inputs are substitutes. Legros & Matthews (1993) and Vislie (1994) showed that when inputs are strict complements (Leontief technology), free-riding can be avoided with a linear sharing rule. This paper considers the robustness and some extensions of the positive result of LMV. First, I show that LMV's result is not robust to the introduction of participation constraints and limited liability. However, I construct a novel rule that mitigates that problem. Second, I perturb the (deterministic) model of LMV. It turns out that free-riding is avoidable with noise added to joint output, while free-riding is inevitable when noise is added to individual productivity.

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1. Introduction

A long tradition in economics compares the performance of different ownership arrangements of firms. While capitalist firms are defined through separating ownership and production by having an outside owner, partnership firms split the value of production in full between the partners. Building on Alchian & Demsetz (1972), the seminal paper by Holmström (1982) argues that partnership firms may suffer from free-riding problems. Holmström (1982) shows that efficient provision of effort is not consistent with Nash behavior in static partnership games where inputs are substitutes and actions are non-contractible. Capitalist firms, on the other hand, can mitigate the free-rider problems by a principal breaking the budget (i.e., keep some of the surplus for herself) whenever she observes a low joint output.

Motivated by the many examples of partnerships in the real world, a considerable literature has questioned the generality of Holmström's result.[3] The present paper considers a particular branch of that literature; works that explore effort taking in Leontief partnerships; partnerships where joint output is determined by the partner with the least effort.[4] Legros & Matthews (1993, section 3.1) and Vislie (1994), hereafter abbreviated LMV, find that if production is determined in such a

[3] There are two different modeling traditions in this literature. The first tradition, which the present paper belongs to, follows Holmström (1982) in modeling action and output spaces as continuous. This tradition includes Rasmusen (1987) dealing with the case of risk-averse partners, Legros & Matthews (1993), and Strausz (1999) which discusses a setup where the partners make their actions sequentially. The second tradition, which uses discrete models, includes Williams & Radner (1993) and Legros & Matsushima (1991). D'Aspremont and Gerard-Varet (1998) surveys the solution techniques available in the discrete case.

[4] The Leontief partnership model can shed light on surprisingly many interesting phenomena. First, in partnership projects where time is involved, the completion time of a project may be when the last agent in the partnership finishes his subtask. For example, the last author that completes his part determines the completion time of a book where several co-authors write a part each. Second, the Leontief model offers a continuous strategy space treatment of the stag hunt example used by Rousseau (1775/ 1993) to discuss the origins of the social contract (in the stage hunt example, hunting a stag is successful only if all hunters decide to do so). The stag hunt example later has become of large interest to game theorists and philosophers (see e.g., Fudenberg & Tirole 1991). In addition, Leontief type of models have been used within macroeconomics to study Keynesian demand failures (Cooper & John 1988), within

manner, there exists a linear sharing rule, denoted β^*, which implements the efficient provision of effort, and hence eliminates free-riding. The intuition for this result is that, given that the other agents stick to the efficient action, no agent can gain by providing more effort (since output does not change), and can be made to support the full decrease in output since his deviation is proportional to the change in output.

The present paper discusses the robustness of the implementation result of LMV in two different directions: by introducing participation constraints (and limited liability) on one hand, and by introducing noise in the production on the other hand.

Let me motivate why it is important to test for robustness along these two directions. First, the partnership literature has largely ignored whether a partnership can be expected to agree *ex-ante* on efficient sharing rules (when such rules exist). The interesting problem is that a partner with a strong bargaining power may wish to settle on a non-efficient rule, if that gives herself a greater return. Hence it is of importance to have sharing rules that make provision of effort incentive compatible under any distribution of participation constraints, to thereby make it possible to distribute a large share of surplus to agents with strong bargaining power.

Proposition 1 constructs a simple sharing rule that solves the participation problem. This sharing rule, which is inspired by the Groves' mechanism, has the attractive property of being able to implement the efficient provision of effort under any conceivable distribution of participation constraints.

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game theory to study the evolution of conventions (Crawford, 1993), and within experimental economics to study

The second part of the paper considers two alternative perturbations of the deterministic model of LMV; noise added to joint productivity, and noise added to individual productivity. It turns out, surprisingly, that these two perturbations give opposite conclusions. While implementation is possible with noise added to joint output, it is not possible with noise added to individual productivity. The intuition for the results is that while noise added to joint productivity leaves the non-differentiability of the partnership problem intact, noise added to individual productivity makes the (expected) production function differentiable, and a similar intuition to in Holmström (1982) applies.

The paper is structured as follows. Part 2 presents the basic, deterministic, model, and Part 3 discusses the role of participation constraints and limited liability in that model. Part 4 consider perturbed versions of the deterministic model, and Part 5 concludes.

2. The Model

Joint output, x, equals $f(\min [b_1e_1, \dots , b_ne_n])$; where b_i is a productivity parameter and $e_i \in [0, E_i]$ is agent i's choice of effort. The function f is differentiable, strictly increasing and concave with $f(0) = 0$. Cost of effort equals $v_i(e_i)$, where $v_i(..)$ is differentiable, strictly increasing and convex with $v'(0) = v(0) = 0$. Following the usual assumption in the literature, the utility of an agent is assumed to be additively separable in money and effort; $U_i := s_i(x) - v_i(e_i)$, where $s_i(..)$ is agent i's share of joint output. Sharing rules that satisfy $\Sigma_i s_i(x) = x$, $\forall x$, and $s_i(x) \geq 0$, $\forall i,x$ are considered.

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behavior in coordination games (Van Huyck, Battalio & Beil, 1990, Van Huyck, Battalio & Cook 1997).

While the first condition just means budget-balance, the defining feature of a partnership, the latter is a 'limited liability' assumption; a partner cannot be forced to pay a net transfer to the other partners. Apart from being empirically plausible (see e.g., Baker, Jensen, and Murphy, 1988), limited liability eliminates simplistic solutions with some troublesome properties. In particular, consider linear sharing rules of the form $b_i x + a_i$. As can easily be checked, by an appropriate choice of the vector a, one can rig a sharing rule of the form $\beta_i^* x + a_i$ [recall that β^* is LMV's solution to the partnership problem], where $a_j > 0$ for some agent j (and hence $a_k > 0$ for some other agent k) to satisfy both participation constraints and balancedness. Such a scheme violates limited liability (insert $x = 0$). Why not leave the limited liability assumption and accept such a sharing rule as a solution to the participation problem? Since the transfer a_j is independent of production level and thus independent of exerted effort of agent j, a_j can be interpreted as an up front transfer from the other partners to agent j. But then the optimal strategy for agent j is to receive a_j and then exert no effort to rather cash the outside option.[5] Realizing this time inconsistency problem, the partners with a low participation constraint will not accept to pay a transfer up front to agent j, and hence make efficient production impossible.

Define the efficient effort-vector, e^*, as the vector maximizing social surplus:

$$e^* := \arg\max_{e} [f(\min [b_1 e_1, ..., b_n e_n]) - \sum_i v_i (e_i)] \tag{1}$$

[5] This follows immediately from the fact that the size of the outside option exceeds the equilibrium utility from participating when $a_j = 0$. The idea is that agents choosing $e_i = 0$ have resources to capitalize the outside option (e.g., think of the outside option as another job). This argument assumes that the utilization of the outside option is non-verifiable to courts. If it is verifiable, contracts may be written that punishes any agent that cashes the outside option,

Observation 1 (Legros & Matthews 1993, Vislie 1994).

There exists a balanced linear sharing rule β that makes e* incentive compatible, where*

$$\beta_i^* \equiv \frac{v_i'(e_i^*)}{f'(b_i e_i^*) b_i}, \forall i.$$

Lemma 1.

Given limited liability, β is the unique incentive compatible linear sharing rule.*

The proof of Observation 1 can be found in LMV, and Lemma 1 follows immediately.

3. Participation Constraints

For illustration, consider an example where participation problems makes implementation of efficient provision of effort impossible with linear rules.[6]

Example 1.

Adam and Betty's joint payoff is given by $x = 2\min(e_A, e_B)$, where $v_i(e_i) = \frac{1}{2}e_i^2$. Consequently, we get, $e_A^* = e_B^* = 1$, $\beta_i^* = \frac{1}{2}$, and $U_i(e^*,\beta^*) = \frac{1}{2}$. When participation constraints are uniformly zero, e^* is clearly a Nash equilibrium under β*, since both incentive compatibility and individual rationality is satisfied. Suppose, however, that Betty has an outside option of ¾ (any number

and hence makes the implementation with a linear rule possible. However, other arguments supporting limited liability, like wealth-constraints, as in Dow & Gorton (1997), may then apply.

[6] It is assumed throughout that the formation of the partnership is efficient, i.e., that social surplus generated under e^* exceeds the sum of the outside options.

between ½ and 1 will work), while Adam's outside option is zero. In that case, Betty will not participate in the partnership under β*, since it gives her less than doing her own project, in spite of partnership formation being efficient. I now construct a novel sharing rule that mitigates the participation problem.

Let $g_i : \Re \rightarrow E_i$ for any *x, compute agent i's effort assuming that she did not waste effort.*[7] Furthermore, let the *quasi-surplus*, labeled *Q*, be determined by the function,

$$Q(x) := [x - \sum_i v_i(g_i(x))\,]. \tag{2}$$

For any *x*, the function *Q*(..) calculates the *maximum* social surplus for that output. For illustration, consider $n = 2$ with $b_1 = b_2 = 1$. On the efficiency locus, where $e_1 = e_2$, quasi-surplus equals social surplus, since no agent wastes effort. Off the efficiency locus, however, say $e = (1, 2)$, agent 2 wastes effort, and the quasi-surplus exceeds the social surplus. It follows that quasi-surplus is at least as high as the social surplus. Since social surplus increases along the efficiency locus, social surplus as well quasi-surplus is maximized at e^*. Now define the sharing rule σ* as,

$$\sigma_i^*(x) := v_i(g_i(x)) + k_i Q(x), \text{ with } k_i > 0 \text{ and } \sum_{i=1}^{n} k_i = 1. \tag{3}$$

For a given joint output, the first part of σ* compensates agent *i* for her cost of effort, assuming that she did not waste effort. The second term of σ* gives agent *i* a constant fraction k_i of quasi-surplus. I now show that, like the Groves mechanism, σ* makes the partners fully internalize the social costs and benefits of their effort choice. However, unlike the Groves mechanism, σ* is balanced both in and out of equilibrium, and also puts no strains on the liability of the agents.

<u>Proposition 1.</u>

The sharing rule σ(x) is balanced, satisfies limited liability, and makes e* satisfy both incentive compatibility and individual rationality.*

<u>Proof.</u>

Clearly $\sigma^*(x)$ is budget-balanced since,

$$\sum_{i=1}^{n} \sigma_i^*(x) = \sum_{i=1}^{n} v_i(g_i(x)) + \sum_{i=1}^{n} k_i Q(x) = \sum_{i=1}^{n} v_i(g_i(x)) + \sum_{i=1}^{n} k_i[x - \sum_{i=1}^{n} v_i(g_i(x))] =$$

$$\sum_{i=1}^{n} v_i(g_i(x)) + x - \sum_{i=1}^{n} v_i(g_i(x)) = x. \qquad (4)$$

For incentive compatibility, consider agent *j*'s best reply to $e_{-j}*$. Notice that the best reply of agent *j* lies on the interval $[0, e_j*]$, in which case x is a function of e_j alone. Consequently, $v_j(g_j(x(e_j, e_{-j}*))) = v_j(e_j)$ on the relevant interval, and we have that,

$$\arg\max_{e_j} \{v_j(g_j(x(e_j, e_{-j}*))) + k_j Q(x(e_j, e_{-j}*)) - v_j(e_j)\} = \arg\max_{e_j} k_j Q(x(e_j, e_{-j}*)) = e_j*. \qquad (5)$$

[7] Formally, $g_i(x) \equiv f^{-1}(x)/b_i$.

Hence e^* is incentive compatible under σ^*. That σ^* satisfies individual rationality follows since σ^* can be rigged to satisfy any participation constraint by a proper adjustment of k_j. Limited liability follows from $\sigma^*(x)$ being non-negative in any equilibrium. Q.E.D.

Since all partners get their highest possible equilibrium payoff in e^*, this choice is a Pareto-dominating equilibrium under σ^*. Returning to Example 1, consider σ^* with $k_A = 1/5$ and $k_B = 4/5$ [any $k_B \in (¾, 1)$ works]. Then Betty gets ¾ from doing her own project, and 4/5 in equilibrium from participating in the partnership. Hence σ^* solves the participation problem in Example 1.

4. The Model under Uncertainty

This part investigates whether implementation is possible when the deterministic model is perturbed. To focus on incentive issues, agents are assumed to be risk-neutral.[8] For brevity, I sometimes write $f(..)$ instead of $f(\min [b_1e_1, ... , b_ne_n])$. First consider the case when individual productivity is deterministic but noise is added to joint output, and then consider the case with noise added to individual productivity.[9] In the first case we have,

$$x := f(..)\varepsilon \quad (6)$$

[8] I use multiplicative noise rather than additive noise since an additive formulation creates the awkward possibility of negative output.

[9] The case when noise is added to $\min(b_1e_1, \ldots, b_2e_2)$ directly is complex. In the case of constant returns to scale, as in LMV, it will be clear from the same logic as underlying Proposition 2 that the efficient effort vector is implementable, but the more general case is open; the joint (expected) production function is not differentiable but it is unclear whether there exists a sharing rule implementing e^*.

where the stochastic term ε has support $\Re^+$ with $E(\varepsilon) = 1$, where E is the expectation operator.

Proposition 2.

Given that the deterministic model is perturbed in the sense of (6), the efficient effort vector is implementable given sufficiently low participation constraints.

Proof.

Since $E[f(..)]\varepsilon$ simply equals $f(..)$, the efficient e of the perturbed problem equals the efficient e of the deterministic problem. Formally,

$$e^* := \arg\max_e E[f(..)\varepsilon - \textstyle\sum_i v_i(e_i)] = \arg\max_e [f(..) - \textstyle\sum_i v_i(e_i)] \qquad (7)$$

But, again since $E[f(..)\varepsilon] = f(..)$, if e_i^* is incentive compatible in the deterministic problem it is also incentive compatible in the perturbed problem; if e_i^* maximizes $\beta_i^* f(e_i, e_{-i}^*) - v_i(e_i)$, then e_i^* also maximizes $E[\beta_i^* f(e_i, e_{-i}^*)\varepsilon - v_i(e_i)]$. Q.E.D.

Notice that σ^* is not viable in solving the implementation problem of Proposition 2 since noise makes it impossible to invert the production function, while β^* does not rely on invertibility. In consequence we can solve the implementation problem if participation constraints are sufficiently low. Let me comment more specifically on what the distribution of participation constraints must be for implementation to be possible with β^*.

When agents are symmetric with respect to technologies (i.e., $b_i = b_j$, $\forall i,j$ and $v_i(..) = v_j(..)$ for $\forall i,j$), symmetric participation levels are sufficient to ensure efficiency with β*. However, when technologies are *asymmetric*, symmetric participation constraints are no longer sufficient for implementation, since asymmetric technologies imply that the partners get a different share of (net) surplus under β* (see Observation 1). It is of some interest to check the relation between asymmetries of technology and (asymmetry of) participation constraints to ensure efficiency with β*. In particular, a desirable property of β* would be that a more productive agent is given a higher share of surplus under β* than a less productive agent, since it can easily be argued that a higher productivity inside the partnership is associated with a higher productivity outside the partnership, and hence a higher outside option.[10] To illustrate that β* does *not* have this property generally, consider the following example.

<u>Example 2.</u>

Suppose that $v_i(e_i) = \frac{1}{2}e_i^2$, while $x = \min(b_1e_1, e_2)$, where $b_1 > 0$. Hence $b_1 < 1$ reflects a situation where agent 1 is more productive than agent 2, and $b_1 > 1$ reflects a situation where agent 1 is more productive than agent 2. As can easily be computed, $e^* = (\frac{b_1}{1+b_1^2}, \frac{b_1^2}{1+b_1^2})$, and consequently $\beta^* = (\frac{v_1'(e_1^*)}{b_1}, v_2'(e_2^*)) = (\frac{1}{1+b_1^2}, \frac{b_1^2}{1+b_1^2})$. We then get equilibrium utilities equal to,

[10] Legros & Newman (1996) considers partnerships in a general equilibrium framework.

$$EU_1(\beta^*,e^*) = E(\beta_1^* e_1^* \, b_1\varepsilon) - v_1(e_1^*) = \frac{1}{1+b_1^2}\frac{b_1}{1+b_1^2} - \frac{1}{2}\left(\frac{b_1}{1+b_1^2}\right)^2 = \frac{1}{2}\left(\frac{b_1}{1+b_1^2}\right)^2, \text{ while} \qquad (8)$$

$$EU_2(\beta^*,e^*) = E(\beta_2^* e_2^* \varepsilon) - v_2(e_2^*) = \frac{b_1^2}{1+b_1^2}\frac{b_1^2}{1+b_1^2} - \frac{1}{2}\left(\frac{b_1^2}{1+b_1^2}\right)^2 = \frac{1}{2}\left(\frac{b_1^2}{1+b_1^2}\right)^2.$$

From (8) it follows that the equilibrium utility of agent 1 from participating in the partnership is *decreasing* in b_1 (in fact his utility goes to zero as b_1 increases). Intuitively, when b_1 increases, the efficient action for agent 2 increases, and for this increase to come about, β_2^* increases, and hence β_1^* must decrease, with a resulting utility loss for agent 1.[11]

Now consider the case when noise is added to individual productivity. Let joint productivity be given by $f(A)$, where $A := \min[A_1, \ldots, A_n]$. Moreover,

$$A_i := b_i e_i \varepsilon_i. \qquad (9)$$

The stochastic term ε_i is *iid* with support $\Re^+$. Let $G_i(z)$ denote ε_i's distribution function, assumed to be twice differentiable, and let $g_i(z)$ denote the density. Realized surplus, $H(e,\varepsilon)$, equals,

$$H(e,\varepsilon) := f(A) - \sum_i v_i(e_i), \qquad (10)$$

and the ex-ante efficient vector, e^*, equals,

[11] In general, there are two effects on $EU_1(\beta^*,e^*)$ from increasing b_1. β_1^* decreases, which goes in the direction of lower utility of agent 1, and x increases (while e_1^* decreases) which goes in the direction of higher utility for agent 1. While the first effect dominates the second effect in the constructed example, the general case is open.

$$e^* := \arg\max_{e \in \mathrm{E}} E[H(e,\varepsilon)] \tag{11}$$

It is natural to assume, by free disposal type of arguments, that optimal sharing rules must be monotonic when output is stochastic (see e.g., Innes, 1990). I therefore confine attention to sharing rules that are increasing in each of its elements, i.e., satisfy $s_i(x) \geq s_i(x')$ for $x \geq x'$. Notice that the sharing rules considered so far have been monotonic. We then have the following result.[12]

Proposition 3.

Given that the deterministic model is perturbed in the sense of (9), there does not exist a sharing rule that implements the efficient effort vector.

Proof.

To avoid tedious notation, confine attention to the case n = 2. Let $M_i(a;e_i)$ be A_i's distribution function conditional on e_i. Then,

$$M_i(a;e_i) := \mathrm{Prob}(A_i \leq a|e_i) = \mathrm{Prob}(b_i e_i \varepsilon_i \leq a) = \mathrm{Prob}(\varepsilon_i \leq a/b_i e_i) = \int_0^{a/b_i e_i} g_i(z)dz = G_i(a/b_i e_i) \tag{12}$$

[12] Puzzlingly, if disposal were impossible, and hence non-monotonic sharing rules are feasible, it is not straightforward to derive a negative result.

Thus for $e_i > 0$, A_i's density function, $m_i(a; e_i)$ equals $\frac{g_i(a/b_ie_i)}{b_ie_i}$. Let $M(a;e)$ be the distribution function of A, i.e., the distribution function of $\min(A_1, A_2)$, and $m(a;e)$ the corresponding density. Then, by independence,

$$M(a;e) = \text{Prob}(A_1 \leq a \text{ or } A_2 \leq a) = 1 - \text{Prob}(A_1 > a \text{ and } A_2 > a) =$$

$$1 - [1-M_1(a;e_1)][1-M_2(a;e_2)] = M_1(a;e_1) + M_2(a;e_2) - M_1(a;e_1)M_2(a;e_2) \quad (13)$$

$$m(a;e) = m_1(a;e_1)[1 - M_2(a;e_2)] + m_2(a;e_2)[1 - M_1(a;e_1)] =$$

$$\frac{g_1(a/b_1e_1)}{b_1e_1}[1 - G_2(a/b_2e_2)] + \frac{g_2(a/b_2e_2)}{b_2e_2}[1 - G_1(a/b_1e_1)] \quad (14)$$

Notice that from (13), it can easily be verified that $\frac{\partial M(a;e)}{\partial e_i} < 0$, i.e., effort increase of one of the agents induces an FOSD distribution of output. Given (14), we get expected utility for i,

$$E[s_i(f(a(e)))] = \int_0^\infty s_i(f(a))m(a;e)da \quad (15)$$

Importantly, $E[s_i(f(a(e)))]$ is differentiable with respect to e_i since $m(a;e)$ is differentiable with respect to e_i from (14). From (15) and budget-balance, efficiency implies:

$$\frac{\partial \int_0^\infty s_i(f(a))m(a)da}{\partial e_i} + \frac{\partial \int_0^\infty s_j(f(a))m(a;e)da}{\partial e_i} - v_i'(e_i) = 0 \qquad (16)$$

However, in a Nash equilibrium, $\frac{\partial \int_0^\infty s_i(f(a))m(a;e)da}{\partial e_i} - v_i'(e_i) = 0$. Consistency with (16) requires, $\frac{\partial \int_0^\infty s_j(f(a))m(a;e)da}{\partial e_i} = 0$. But since $s_j(..)$ is increasing, and since an increase in e_i induces a FOSD distribution of output, it is trivial to show that $\frac{\partial \int_0^\infty s_j(f(a))m(a;e)da}{\partial e_i} > 0$, unless s_j is identically equal to zero, in which case there is a contradiction for agent *j*. Q.E.D.

Intuitively, noise imposed on individual productivity smoothens the kink in the production function, which implies that the free-riding intuition of Holmström (1982) holds true:[13] Nash equilibrium implies that the agents do not take into account the positive externality when inducing effort, in contrast to their behavior in social optimum.[14]

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[13] To understand why differentiability makes such a difference, consider the necessary conditions for social optimum in the differentiable and the non-differentiable case. With differentiability, the necessary condition for optimum is $x' - v_i'(e_i) = 0$, while in the non-differentiable case the necessary condition is $x' - v_i'(e_i) - v_j'(e_i) = 0$. The point is that in the second case, it is not necessary to give agent *i* full incentives to make him perform the efficient action. This leeway, which makes implementation possible, does not exist in the first case.

[14] Notice that the specification of noise in (9) is quite general since it allows for any (differentiable) distribution of the ε_i's. Thus noise at individual level can be made arbitrarily 'small' and still implementation is impossible. However,

5. Conclusion

This paper has considered the robustness of a positive implementation result for Leontief partnerships obtained by Legros & Matthews (1993) and Vislie (1994). I first showed that implementation with their sharing rule is non-robust to the introduction of participation constraints and limited liability. However, Proposition 1 mitigated this problem by the construction of σ^*, a sharing rule reminiscent of the Groves mechanism, which satisfies both incentive compatibility, individual rationality, and limited liability.

The positive results of LMV and of the present paper are obtained in a deterministic model. Part 4 considered whether the positive results are robust to perturbing the model. First, noise was added to joint output. The interpretation of this perturbation is that the partners know each others' productivity, but are uncertain about how much production that will emerge from their joint efforts. Proposition 2 showed that, in that case, implementation is still possible. The next robustness test added noise to individual productivity. Then the joint production function becomes differentiable, and by a similar argument to Holmström (1982), implementation was shown to be impossible. Thus free-riding in Leontief partnerships can be mitigated if uncertainty is added to the aggregate level, while uncertainty added to the individual level makes free-riding unavoidable.

An interesting interpretation of the second type of perturbation is that the stochastic component in individual output reflects a partner's uncertainty about the other partner's production technology.

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the continuity of the problem suggests that for sufficiently low level of noise, e^* is implementable in an ε-equilibrium. See Legros & Matthews (1993), Section 5, for a related discussion.

In that case, which model is more appropriate, the deterministic model or the stochastic model, depends on whether the partners have learned each other's production technology. Since a negative conclusion follows in the case where agents have not learned their technologies, the findings of the present paper suggest that partnerships are most likely to be formed by individuals with complementary skills, and moreover that know each other well.

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