Area Yield Futures and Options: Risk Management and Hedging.

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Suppose there exists a market for yield futures contracts as well as ordinary futures contracts for price. Intuitively one would think that a combined use of yield contracts and futures price contracts ought to provide a reasonable strategy for locking in revenue.

In the paper this is made precise - it is shown that revenue can be approximately locked in by a combined, dynamic use of these two markets. This procedure is *perfect* if the correlation between yield and price is zero. The relevant strategy is characterized: It depends only on *observable* price information in these two separate markets, not on the specification of parameters in utility functions of the agents involved.

Key words: Area yield options, futures, continuous time modelling, quantity and price securing, locking in a certain revenue, CBOT yield contracts

Introduction

In the farming industry as well as for many other primary commodity producers it is possible to effectively manage price risk by the use of futures price contracts and options on futures. However, in many of these industries there is still considerable uncertainty left when it comes to revenue, since quantities produced can be volatile, depending on many factors, such as e.g., weather conditions in the growing season. Until recently, similar market-based instruments for managing yield risk have not been available. Instead, federal agricultural support programs and subsidized crop yield insurance programs have served as alternatives. In an important development in 1995, the Chicago Board of Trade (CBOT) launched its Crop Yield Insurance (CYI) Futures and Options contracts. The first CYI contract that began trading on 2 June 1995 was Iowa Corn Yield Insurance Futures and Options. On 19 January 1996 the CBOT added a U.S. contract plus four additional state corn yield contracts for Illinois, Indiana, Ohio, and Nebraska. So far the trading volumes have been fairly modest. Regardless of the status of this particular market for the moment, we want to discuss such contracts from a principle point of view.

The CYI contracts are designed to provide a hedge for crop yield risk. For example, CYI futures users can lock in a certain crop yield several months into the future as a temporary substitute for a later yield-based commitment, or they can alternatively try to lock in the revenue of a given acreage by combining yield contracts with futures price contracts. How this can be done is the subject of the present paper.

In the following we abstract from production costs, and assume zero local price basis (i.e., local cash price equals futures price) and zero yield basis (i.e., individual farm yield equals index yield). Intuitively one would think that a combined use of yield contracts and futures price contracts ought to provide a "reasonable" strategy for locking in revenue. In the paper this intuition is made precise - in that it is shown that revenue can be approximately locked in by a combined, dynamic use of these two markets. This procedure is *perfect* if the correlation between yield and price is zero. Moreover, the relevant strategy is also characterized: We show what dynamic strategy can be used to lock in a certain revenue, when combining these two markets. Most importantly for practical implementations, this strategy depends only on observable price information in these two separate markets.

There is a large literature on non-market based risk management and insurance of crop yield, which we will not address here. Yield contracts have been dealt with from the perspective of hedging, using a mean variance approach by Vukina, Li and Holthausen 1996, while minimizing the variance of revenue was the objective in Li and Vukina 1998. In both these papers the yield contracts traded at CBOT are explained, so we need not elaborate on the market structure here. A more recent paper is Nayak and Turvey 2000, again using a simple mean-variance model. Two other papers try to examine this problem in the expected utility framework (see e.g., Hennessy, Babcock and Hayes 1997; Mahul and Wright 2000). The focus in this paper is rather different from these investigations, in that our results are not depending on any specific assumptions about utility functions: Our hedging ratios can be read straight from observed market prices.

There was another securitized insurance market at the CBOT centered around certain catastrophe indexes, these indexes playing a similar role to the yield index of the present paper; e.g. (Aase 1999, 2001). The analysis of such markets must typically differ from the model chosen in the present paper, since catastrophes can not be modelled well by a continuous stochastic process. However, we shall briefly discuss jump models in the last section of the paper.

The paper is organized as follows: In the first section we start by an illustration of the potential use of CYI contracts in order to manage risk. Then we proceed directly to the main results of the paper, demonstrating in Theorem 1 and Corollary 1 how a farmer can combine a pure yield contract with a pure futures price contract to lock in a revenue similar to what a farmer could ideally secure if a futures market on revenue were to exist.

Finally we briefly discuss the possibilities to enlarge the analysis to models containing jumps of unpredictable sizes, thus extending our results to incomplete financial markets, and the final section concludes.

Area Yield Futures and Options

Introduction

Imagine a country, or another area, sectioned into regions which are uniform in terms of growing conditions for a certain crop, say corn. In each area there is a quantity index y_t, for time t running from 0 to T, where T is the time of sale and 0 is the time of sowing. As an example, for agricultural yield contracts in the USA traded at the CBOT the values of y are provided by the United States Department of Agriculture (USDA). One may think of y_t as a forecast at each time t of quantity, measured in bushels per acre, up for sale in this specific region at the final time T. On this index we assume it is possible to trade futures, and futures options contracts. In order to bring in the quantum uncertainty, we assume that this index can be modeled as a stochastic process. A farmer in this region may have production uncertainty that is well represented by this index, where the relevant number of contracts can be determined from each farmer's production area.

The idea is that if the producer can buy options on this quantity index or on its corresponding futures index, the farmer can lock in a prespecified quantity by buying an appropriate number of such contingent claims. This strategy is of course only 100% efficient if the farmer's yield uncertainty is perfectly represented by the index, an unlikely event, but a careful selection of homogeneous regions may make such markets useful for practical risk management purposes. Since agricultural agents are concerned about revenue rather than solely about yield, or about price, one may think that the yield market may be combined in an appropriate manner with the futures market for crop price to lock in a certain revenue. The conditions when this can be done is the topic of this paper.

A private insurance market giving the farmer insurance against quantity shortfall is of course difficult to establish, partly because of the adverse incentives this would create for the farmers, as the rich literature on this topic in agricultural economics journals show. The yield futures market may, however, avoid this difficulty, at least under some presumptions: The farmers do not engage in any kind of "collective moral hazard" which effects the yield index, and there is no moral hazard in the construction of the index. There is an implicit assumption that the farmer's actions do not influence the quantity index to any significant degree. Also the individuals in USDA constructing the index should have no economic interest attached to this market.

Market failures that seem caused by asymmetric information have been dealt with by several authors, among them Chambers 1989, Skees and Reed 1986, Nelson and Loeman 1987, Quiggins 1994 and Goodwin and Smith 1995. There seems to be a tradeoff between moral hazard and basis risk. There is a growing literature that examines this tradeoff (see e.g., Doherty (2000); Doherty and Mahul (2001)). CYI contracts, like cat options, does not seem to be widely used among farmers or insurers. One standard explanation is the existence of basis risk, i.e., individual losses are not sufficiently correlated with aggregate losses. In our approach we assume there is no problem with asymmetric information, and, as already mentioned, we assume a zero yield basis.

Ordinary crop quantity insurance would also exhibit another obvious difficulty, namely positive correlation between produced quantities from the different farmers in each area. This kind of insurance would thus be highly risky compared to ordinary "household insurance" of say cars, houses, etc., where one can consider the different insurance cases as statistically independent. This phenomenon is well understood in the general theory of insurance. In the agricultural economics literature the risk that is not diversifiable in this regard has been termed "systemic risk"; e.g. (Miranda and Glauber 1997). In our approach this positive correlation will not be any problem.

Crop Yield Insurance Futures Contracts

The mechanics of using yield futures can perhaps best be illustrated by an example.

Example 1. Consider a farm of 1000 acres in Iowa, in an area with expected crop $F_t^y = 130$ bushels per acre at time t. The futures price of corn is $F_t^q = \$2.50$ per bushel, also at time t, in both cases for contracts expiring at a future time T.

Consider a strategy that sells 130,000 corn futures and similarly sells 2,500 area yield futures, both at time t and these positions are held until maturity. The payoff at expiration for this strategy would be

$$(F_t^q - q_T^{obs})F_t^y \cdot 1000 + (F_t^y - y_T^{obs})F_t^q \cdot 1000.$$

Consider four scenarios:

(i) The observed price of corn at time T turns out to be $q_T^{obs} = \$2$ per bushel, the observed yield index y_T^{obs} ended up on 100 bushels per acre. This is the case of a situation the farmer would like to insure against. The payoff from this strategy would be \$140,000. Without futures contracts, the farmers would end up \$125,000 below the expectation, assuming a perfect correlation between the farm output and the yield index, and after the gain from the futures contracts are taken into consideration, the "net gain" would be \$15,000.

(ii) $q_T^{obs} = \$3$, $y_T^{obs} = 160$ bushels per acre. The payoff from the above strategy would be -\$140,000. Under the same simplifying assumptions as above, the farm would now end up with a result of \$155,000 higher than projected, in which case the "net gain" would also be \$15,000.

(iii) $q_T^{obs} = \$2$, $y_T^{obs} = 160$ bushels per acre. The payoff from the above strategy would be -\$10,000. Under the same simplifying assumptions as above, the farm would now end up with a result of \$5,000 below the expectation, in which case the "net loss" would be \$15,000.

(iv) $q_T^{obs} = \$3$, $y_T^{obs} = 100$ bushels per acre. The payoff from the above strategy would be \$10,000. Under the same simplifying assumptions as above, the farm would now end up with a result of \$25,000 below the expectation, in which case the "net loss" would also be \$15,000.

If these four cases were equally likely, the expected "net gain" would equal zero, so on average the insurance would then work.

When considering pure area yield contracts, one should notice that for yield futures contracts is used a multiplication factor of \$100 per bushel to convert production to income (here: in US \$), and also, the trading unit for corn futures is 5000 bushels. □

In Example 2 below we analyze strategies of this type, and investigate if a farmer could do better using other strategies. In general one must maintain that selling futures contracts is considered as risky business, and the farmers would usually be better off buying put options.

The market based yield contracts have the potential to satisfy various traditional users of yield insurance, as well as some not so traditional ones. The traditional users are companies tied to corn production contracts, who prefer to lock in certain rates of production. For bulk cargo goods, where the capacity in the transportation sector is determined largely by high, and fixed costs, a fall in production of, say corn, would typically lead to a shortage of goods to transport. This will in its turn lead to a fall in the freight rates, and consequently to a drop in the income also in the transportation sector. The yield futures options may therefore also be utilized by transporters for risk management purposes. Suppliers of capital and loans to the farming industry may indirectly profit from the general use of such contracts, since they are making the borrower's portfolios less risky.

Nontraditional users may typically be agricultural brokers, insurers and other intermediaries who whish to offer income guarantees to farmers. These agents can make direct use of these contracts to hedge the risks they are facing, or the risk they are carrying from insurance of their customers.

Potential users of yield contracts include, in addition to farmers, (1) companies offering crop insurance, (2) reinsurers/brokers, (3) large food processing companies, (4) wholesalers of corn, (5) cargo shipping companies, (6) railways (7) forage agents, (8) corn storage agents, (9) seed producers, (10) producers of agricultural equipment, (11) lenders/banks, (12) agricultural brokers, insurers and other intermediaries who offer income guarantees to farmers, (13) investors who want to diversify, (14) spread traders, (15) speculators.

Finally a few key words about yield futures.

Flexibility: Can help to reach a given investment, or risk management goal under almost any scenario.

Versatility: Even if one does not have a clue where the market is going, it is possible to gain from changes in volatility, reduced time to expiration, or change in some of the other distinguishing parameters of options.

Additional protection: The options offer revenue insurance without reducing the potential of gains.

Liquidity: Possibilities to quickly leave, or enter the market.

Offers reduced risk: The exposure of an option is limited to the premium paid. Options can be combined to obtain very little residual risk.

Traded at an exchange: This means *standardization*, *anonymity* and access to price and quantum expectations.

Finally, yield futures traded at an exchange may give the public access to weather forecasts that may sometimes outperform those that are otherwise accessible.

How to lock in a prespecified revenue

Introduction

Consider two futures markets, one where yield futures options are traded, and one where standard price futures options are traded. The quantity index $y(t)$ at time t is measured in bushels per acre, and spot price $q(t)$ at time t is measured in \$ per bushel. As earlier explained we abstract from production costs, and assume zero local price basis and zero yield basis. In this case we can define the revenue $R(t) = y(t)q(t)$. A yield futures option contract will specify a real function g so that the payoff from a yield futures option contract at the expiry time T is given by $g(y(T))$ bushels per acre, having yield futures price at time $t < T$ given by

$$F_t^{g(y)} = E_t^Q(g(y(T)) \cdot c). \tag{1}$$

To be specific, given is a filtered probability space $(\Omega, \mathcal{F}, \mathsf{F}, P)$, where Ω is the set of states with generic element ω, P is a probability measure, the "objective probability", $\mathcal{F}$ is the set of events in Ω given by a $\sigma-$algebra, $\mathsf{F} = \{\mathcal{F}_t, 0 \leq t \leq T\}$ is a filtration satisfying the usual conditions, where $\mathcal{F}_s \subseteq \mathcal{F}_t$ if $s \leq t$, $\mathcal{F}_t$ signifying the possible events that could happen by time t, or "the information available by time t". We assume $\mathcal{F}_0$ to be trivial, containing only events of probability zero or one, meaning roughly that there is no information available at time zero, and $\mathcal{F}_T = \mathcal{F}$, i.e., at time T all uncertainty is resolved. Here Q appearing in equation (1) is an equivalent martingale measure, assumed to exist, Q being equivalent to the given probability measure P (i.e., the measures P and Q coincide on the null sets). The symbol E_t in (1) is the conditional expectation operator given the "information" $\mathcal{F}_t$ possibly available by time t.

The constant c signifies a conversion factor measured in \$ per bushel, so that the futures price is measured in \$ per acre. For example, for the Iowa Corn Yield Insurance Futures (ticker symbol CA) the unit of trading is the Iowa yield estimate times \$100 (e.g., a yield of 140.3 bushels per acre gives a contract value of \$14,030). In this section we set this conversion factor equal to 1 without loss of generality.

Similarly an ordinary futures option contract on price will specify a real function h so that the corresponding payoff from a futures option contract at expiration is given by $h(q(T))$ \$ per bushel, and the associated futures price at any time t prior to T is given by $F_t^{h(q)} = E_t^Q(h(q(T))$ measured in \$ per bushel.

The main result

We now turn to the dynamics of the two processes q and y. We assume that the processes q and y are both defined on the given probability space as follows:

$$dy(t) = \mu_y(t)dt + \sigma_y(t)dB(t) \tag{2}$$

and

$$dq(t) = \mu_q(t)dt + \sigma_q(t)dB(t), \tag{3}$$

where $B(t) = (B_1(t), B_2(t))$ is a standard two dimensional Brownian motion, $\sigma_y(t) = (\sigma_{y,1}(t), \sigma_{y,2}(t))$ and $\sigma_q(t) = (\sigma_{q,1}(t), \sigma_{q,2}(t))$ are adapted volatility processes satisfying standard L^2-type integrability and regularity conditions. Similarly the drift terms $\mu_y(t)$ and $\mu_q(t)$ are adapted stochastic processes satisfying standard L^1-type integrability conditions.

Now we consider product contracts of the form $R^{gh}(t) = g(y(t))h(q(t))$. Our revenue process $R(t) = y(t)q(t)$ would then follow as a special case, when both g and h are the identity function. We want to investigate whether we can lock in a prespecified "revenue" $R^{gh}(t)$ at any time t prior to the expiration time T by dynamically trading in the two separate futures options markets described above. To this end imagine first that a separate market for this type of "revenue" were available. The futures price of this contract we denote by $F_t^{g(y)h(q)}$, and it must be given as follows under our assumptions:

$$F_t^{g(y)h(q)} = E_t^Q\{g(y(T)) \cdot h(q(T))\}, \qquad 0 \leq t \leq T. \tag{4}$$

Notice that this can be written

$$E_t^Q\{g(y(T)) \cdot h(q(T)) - F_t^{g(y)h(q)}\} = 0, \qquad 0 \leq t \leq T, \tag{5}$$

the usual starting point for analyzing futures contracts. Equation (5) implies that if the futures price $F_t^{g(y)h(q)}$ is agreed upon at time t, then nothing is actually paid at this time.

In order to better understand what follows, let us recall the main features of a simple futures contract on, say, price. For the holder of one long contract, the payoff at expiration is

$$\int_t^T 1 \cdot dF_s = F_T - F_t = q_T - F_t \tag{6}$$

by the principle of convergence in the futures market, where F_t is the futures price of one contract at time t. If an agent holds θ_s futures contracts at time s in the time interval $(t, T]$, the resettlement gain at time T from this strategy would similarly be

$$\int_t^T \theta_s dF_s. \tag{7}$$

Now consider a strategy that holds $F_s^{g(y)}$ pure futures options on $h(q_T)$, and $F_s^{h(q)}$ pure futures options on $g(y_T)$ at each time s between t and T. The resettlement gain from this strategy is given by

$$\int_t^T F_s^{g(y)}\, dF_s^{h(q)} + \int_t^T F_s^{h(q)}\, dF_s^{g(y)} \tag{8}$$

Using stochastic integration by parts, this can be written

$$= g(y_T)h(q_T) - (F_t^{g(y)} F_t^{h(q)} + \int_t^T dF_s^{g(y)}\, dF_s^{h(q)}). \tag{9}$$

Consider the expression in (9) and compare it to (6) and (7). If it were the case that the term $(F_t^{g(y)} F_t^{h(q)} + \int_t^T dF_s^{g(y)}\, dF_s^{h(q)})$ is the futures price given in equation (4), it would be the case that the above combined strategy is equivalent to a futures contract on the product $h(q_T)g(y_T)$. Since a futures price at time t must be $\mathcal{F}_t$-measurable, this can not be strictly the case for the above strategy, since the integral naturally does not satisfy this information constraint. On the other hand, if we take the conditional expectation under Q, we do indeed get the futures price in question. Let us demonstrate this. In order to do so, we will need some technical conditions, which we relegate to Appendix 1. Assuming these, we use the following notation: The futures price processes $F_t^{h(q)}$ and $F_t^{g(y)}$ can both be written as smooth functions $a(q_t, t)$ and $b(y_t, t)$ respectively. Denote by $a_q(q, t)$ the partial derivative of the function $a(q, t)$ with respect to its first argument, and similarly for $b_y(y, t)$. We have the following result:

Theorem 1 *Consider the resettlement gain from the strategy given in (8).*

(i) Suppose $\sigma_q(s) \cdot \sigma_y(s) = 0$ *for all* $s \in (t, T]$ *, i.e., a zero correlation rate between yield and price. Then this strategy is equivalent to a futures contract on the product* $h(q_T)g(y_T)$ *having futures price at each time* t *given by* $F_t^{g(y)h(q)}$ *in equation (4).*

(ii) Consider the general case. The futures price (4) can always be written

$$F_t^{g(y)h(q)} = F_t^{g(y)} F_t^{h(q)} + E_t^Q \left(\int_t^T dF_s^{g(y)} dF_s^{h(q)} \right) \tag{10}$$

$$= F_t^{g(y)} F_t^{h(q)} + E_t^Q \left(\int_t^T a_q(q_s, s)(\sigma_q(s) \cdot \sigma_y(s)) b_y(y_s, s)\, ds \right).$$

Proof: In order to prove (ii) first, according to (5) we have to show that

$$E_t^Q\left(g(y_T)h(q_T) - (F_t^{g(y)}F_t^{h(q)} + \int_t^T dF_s^{g(y)}\, dF_s^{h(q)})\right) = 0, \qquad t \leq T, \quad (11)$$

and this follows, since

$$E_t^Q\left(\int_t^T F_s^{g(y)}\, dF_s^{h(q)} + \int_t^T F_s^{h(q)}\, dF_s^{g(y)})\right) = 0 \qquad \text{for any} \qquad t \leq T \quad (12)$$

by the standard conditions (20) - (23) of Appendix 1; under these conditions it is known, essentially by Hølder's inequality, that the stochastic integrals in (8) both have zero conditional expectations under Q, since $F_t^{g(y)}$ and $F_t^{h(y)}$ are both Q-martingales. Thus we get the conclusion of (ii) from the expression for the futures price in equation (4), the fact that $F_t^{g(y)}$ and $F_t^{h(q)}$ are both $\mathcal{F}_t$-measurable, and from the representations (18) and (19) of Appendix 1 for the stochastic processes $a(q_t, t)$ and $b(y_t, t)$.

Now the conclusion in (i) follows from (ii) just proven, by the expression for the resettlement gain given in (9), since the integral

$$\int_t^T dF_s^{g(y)}\, dF_s^{h(q)} = 0$$

in this case, combined with the observations in (6) and (7). □

Before we comment on this theorem, we briefly describe the situation with pure futures contracts only. Consider a strategy that holds F_s^y futures contracts on price and F_s^q futures contracts on quantity at any time s, where $0 \leq t \leq s \leq T$, t signifying the present. The resettlement gain from this strategy is given by

$$\int_t^T F_s^y\, dF_s^q + \int_t^T F_s^q\, dF_s^y. \quad (13)$$

We then have the following corollary:

Corollary 1 *Consider the resettlement gain from the strategy that, for any time s between the present time t and the expiration time T, holds F_s^y futures contracts on price and F_s^q contracts on quantity, given in (13).*

(i) Suppose $\sigma_q(s) \cdot \sigma_y(s) = 0$, for all $s \in (t, T]$ i.e., a zero correlation rate between yield and price. Then this strategy is equivalent to a futures contract on revenue R_T having futures price at each time t given by $F_t^R := E_t^Q\{q(T)y(T)\}$.

(ii) Consider the general case. The futures price F_t^R can always be written

$$F_t^R = F_t^y F_t^q + E_t^Q\left(\int_t^T a_q(q_s, s)(\sigma_q(s) \cdot \sigma_y(s))b_y(y_s, s)\, ds\right).$$

Proof: Set $g(x) = x$ and $h(x) = x$ for all real x in Theorem 1. □

The above results show that, in the case with a zero correlation rate between yield y and price q, there is no need for a specialized futures market of, say, revenue $R = yq$ for someone who has access to the two separate markets for price and yield contracts. One can then, at least in principle, achieve exactly the same results in terms of risk management by simultaneous, dynamic trade in these two markets. Since a dynamic strategy is then needed, needless to say, we here abstract from transactions costs.

In the situation where the correlation $\sigma_{y,q}(s) := \sigma_q(s) \cdot \sigma_y(s)$ does not vanish for all $s \in (t, T]$, this strategy would not constitute an exact substitute of a market for revenue. The relevant "correction term" is given by

$$\left(E_t^Q \{ \int_t^T dF_s^y dF_s^q \} - \int_t^T dF_s^y dF_s^q \right), \tag{14}$$

with a similar expression for the futures options case in Theorem 1. The smaller the correction term given in (14) is, the better would the given strategy in (13) constitute a substitute to a futures market for revenue. This term has conditional expected value equal to zero under the measure Q, so it is not a bias in ordinary statistical language, at least not under Q. Obviously, even if the term $\sigma_{y,q} \neq 0$, this correction term may still be small. Clearly this term goes to zero as t approaches T. If $\sigma_{y,q} = 0$, the correction term vanishes.

Let us illustrate by a simple example.

Examples and discussion

Example 2. Consider the case of a zero cross-correlation rate $\sigma_q(t) \cdot \sigma_y(t)) = 0$ for all t. In this particular case, the strategy (F^y, F^q) duplicates exactly the pay-off $(y_T q_T - F_t^R)$ from one long "revenue"-contract: At the initiation time t the futures prices F_t^y and F_t^q are both set such that nothing is actually paid at this time.

Instead of the resettlement strategy in (13) let us consider a more "laid back" strategy that buys F_t^q quantity contracts, priced at F_t^y at time $t \leq T$, and holds this position until maturity, and buys F_t^y price contracts, priced at F_t^q at time $t \leq T$, and holds this position till maturity as well. This is similar to the strategy of Example 1, except that there the contracts where sold.

The payoff at expiration for the hypothetical contract on revenue would be $(y_T q_T - F_t^R)$, for a farmer buying one such contract. On the other hand, the combined contracts described above would yield the following payoff:

$$(q_T - F_t^q) F_t^y + (y_T - F_t^y) F_t^q,$$

where the first term is the payoff of F_t^y long futures contracts on price q, and the second term is the corresponding payoff of F_t^q long contracts on quantity y.

This latter sum can be seen to be equal to

$$(y_T q_T - F_t^R) + (F_t^y - y_T)(q_T - F_t^q), \tag{15}$$

in the situation where $F_t^R = F_t^q F_t^y$, e.g., when the cross-correlation rate is zero.

Since F_t^y can be considered as an "economic forecast" of y_T at time t, and similarly is F_t^q an economic forecast of q_T, the remainder term in (15) should be "small of second order" (it goes to zero faster than the first term in (15) as t approaches T), in which case this strategy may function reasonably close to a hypothetical futures market for revenue. Of course, this latter ideal market does not exist, so this "laid back" arrangement of combining existing markets for quantity and price separately may be a reasonable substitute. The strategy described in equation (13) constitutes, on the other hand, a perfect substitute in this situation. $\square$

Finally, we remark the following: The results of Theorem 1 and Corollary 1 do not depend on any specific assumptions about utility functions of the agents (except from some obvious axioms of the preferences in situations like these, like agents prefer more to less). The result is that a futures market for revenue can approximately be obtained through the combination of the two markets for yield and price futures. There exists a dynamic replication strategy in quantity futures and price futures which is, under certain conditions, equivalent to a futures contract on revenue. Moreover, this strategy can be obtained directly from futures price information in these two separate markets. There are no parameters to estimate, no assumptions about the relative risk aversion, or the subjective interest rate, or anything like that. Thus this result should obviously be of great practical interest.

More Dramatic Uncertainty Revelation

In a companion paper we give an example of a model for the price of the crop, q_t, and the index y_t, as well as a valuation model where market prices can be found for a large class of relevant financial contracts. In other words, we give an example how to construct an equivalent martingale measure Q for yield contracts. Since we have chosen an Ito process framework, this is done by judiciously transforming to two *price* processes (y is not a price process), and then it is natural to choose a complete model, in which case we have to solve a linear system of equations, and use Girsanov's theorem to establish a unique market-price-of-risk process (see Aase 2002).

Suppose instead that we imagine agricultural yields are exposed to natural disasters and thus it seems natural to include more dramatic changes than continuous ones in the process dynamics for y and q. Consider the following dynamics

$$dy(t) = \mu_y(t)dt + \sigma_y(t)dB(t) + \int_R \gamma_y(t,z)\tilde{N}_y(dt,dz) \qquad (16)$$

and

$$dq(t) = \mu_q(t)dt + \sigma_q(t)dB(t) + \int_R \gamma_q(t,z)\tilde{N}_q(dt,dz). \qquad (17)$$

Here $\tilde{N}_i(dt,dz) = N_i(dt,dz) - \nu_i(dz)dt$ signify two independent, compensated Poisson random measures of an underlying Levy process, $i = y, q$, also independent of the two dimensional Brownian motion B. Here $\nu_i(dz)$ is the Levy measure of process i. The idea is that jumps of random sizes $\gamma_i(t,z)$ occur at unpredictable time points of a two-dimensional Levy process in the respective processes y and q. If a jump happens to take place at time t, and the underlying jump sizes of the two Levy-processes is $(Z_y = z_y, Z_q = z_q)$, then the jump size in the quantity index y is $\gamma_y(t,(z_y,z_q))$. Without going into further technical details, we note the following: This class of models is obviously very general, and can be made to fit well most observed time series of data one can imagine. There is an *integration by parts formula* also for the type of processes given in the equations (16) and (17) above. Since this is an essential part of the proof or Theorem 1, our results in Theorem 1 and Corollary 1 can still be shown to go through.

In the above model we are not able to construct a unique market-price-of-risk process as in the above mentioned companion paper, so there will exist many equivalent martingale measures Q (indeed, uncountable many) that will do for pricing purposes, even if no arbitrage prevails. All these measures will coincide on the marketed subspace $M \subset L^2$ containing all the random payoffs of the type that can be generated by portfolio formation of two different, correlated assets with pricing processes like the one in (17). However, for contingent claims with components in the orthogonal complement $M^\perp$ of M (here $L^2 = M \oplus M^\perp$), these components can not be hedged by the existing financial instruments. As a consequence we do not have a good pricing theory for this part, and the measures Q will normally not coincide on $M^\perp$. Thus our resulting model is incomplete.

But this is of no concern to the present results. The farmer can still observe the prices in the two separate futures markets, construct the dynamic strategy as time goes, and replicate the payoff of a futures (or a futures option) contract on revenue to the degree that we have explained above. Thus our results are robust to the modelling of uncertainty - more interesting models than Ito-processes can be used, in principal there are no major restrictions (other than technical ones).

Conclusions

We have presented a simple model for the analysis of futures contracts on quantity and futures contracts on price in separate markets for such contracts, in order to construct futures contracts on revenue.

The analysis is motivated from the actual creation of certain futures markets for agricultural quantity products, originally set up by the CBOT.

It is shown how a farmer can lock in a certain revenue by a combined trade in futures price and futures yield contracts, abstracting from production costs. This can be done perfectly if the correlation between yield and price is zero, otherwise the procedure is only approximately correct, where a correction term is identified. The main result is not depending upon the particular choice of model for the random dynamics. Thus the result is rather robust.

Furthermore, our results do not depend upon any specific assumptions about utility functions, relative risk aversions, subjective discount rates, and the like. Thus our results are practically implementable.

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Appendix 1

Here we present the technical conditions needed for Theorem 1:

The futures price processes $F_t^{h(q)}$ and $F_t^{g(y)}$ can be both be written as some $C^{2,1}(\mathbb{R}^2 \times [0,T))$-functions $a(q_t,t)$ and $b(y_t,t)$, say. Since they are both Q-martingales, by Itô's lemma

$$da(q_t,t) = a_q(q_t,t)\sigma_q(t)d\tilde{B}(t) \tag{18}$$

$$db(y_t,t) = b_y(y_t,t)\sigma_y(t)d\tilde{B}(t), \tag{19}$$

where $a_q(q,t)$ means the partial derivative of the function $a(q,t)$ with respect to its first argument, and similarly for $b_y(y_t,t)$, and where $\tilde{B}$ is a standard two dimensional Brownian motion with respect to the measure Q. We will now need the following technical conditions: We suppose the processes $a(q_t,t)$ and $b(y_t,t)$ satisfy the following:

$$\int_0^T \left(b(y_t,t)a_q(q_t,t)\right)^2 (\sigma_{q,1}(t)^2 + \sigma_{q,2}(t)^2)dt < \infty \qquad a.s. \tag{20}$$

$$E\left(\int_0^T \left(b(y_t,t)a_q(q_t,t)\right)^2 (\sigma_{q,1}(t)^2 + \sigma_{q,2}(t)^2)dt\right) < \infty \tag{21}$$

and

$$\int_0^T \left(a(q_t,t)b_y(y_t,t)\right)^2 (\sigma_{y,1}(t)^2 + \sigma_{y,2}(t)^2)dt < \infty \qquad a.s. \tag{22}$$

$$E\left(\int_0^T \left(a(q_t,t)b_y(y_t,t)\right)^2 (\sigma_{y,1}(t)^2 + \sigma_{y,2}(t)^2)dt\right) < \infty. \tag{23}$$

Notes