Implications of a Nested Stochastic/Deterministic Bio-Economic Model for a Pelagic Fishery

by

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Abstract

Use is made of an economically optimal feedback rule to determine optimal levels of exploitation of a pelagic fish species. Data from the southern bluefin tuna fishery for the years 1960 - 1996 are utilised to apply this rule to aggregated deterministic and stochastic models of population dynamics. Comparison of the rule-based results with historical records indicates that over much of the period the fishery was economically overexploited and a harvesting moratorium could have been imposed to improve economic returns from the fishery and to allow stock recovery.

key words: stochastic population dynamics model, feedback rule, management.

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1 Introduction

Fishery management strategies have often been directed at either maintaining existing fish stocks or at achieving maximum sustainable yield (Walters, 1986). Both of these objectives are difficult to achieve in practice. Both are vulnerable to errors in the measurement of fish stocks and, therefore, determination of suitable catches. Maintenance of the *status quo* has the added disadvantage, if successful, of preventing stock recovery after historical depletion or of foregoing potential yield by maintaining stocks at unnecessarily high (and counter-productive) levels. Additionally, attempting to achieve either of these objectives in practice can lead to catastrophic failure of the fish stock because of natural random fluctuations and as a result of inadequate attention to risk and uncertainty.

In order to overcome the problem of setting suitable harvest levels for a renewable resource that is subject to both significant random fluctuation and measurement error, feedback rules have been proposed. These are often based on the optimality principle of control theory and dynamic programming, and have been proposed as a means of achieving biological as well as economic objectives.

Feedback rules for adaptive management of fishery resources have been advocated for at least three decades (see, for example, Walters and Hilborn, 1976; Clark, 1990; and Walters, 1986). Such rules can be used by managers to regulate resource use for achieving defined objectives and may be used when evaluating management strategies. A deterministic feedback rule is used, for example, by Grafton *et al.* (2000) to examine Canada's northern cod fishery. In the present paper we examine a non-trivial feedback rule based on economic objectives for nested stochastic and deterministic bio-economic models of a fishery. This is akin to what Walters (1986) refers to as a passively adaptive policy. We then compare the derived optimal exploitation with the actual exploitation of a highly-migratory pelagic fish stock.

Data for the southern bluefin tuna (SBT) are used to illustrate the potential benefit of using optimal feedback rules to help achieve the objective of maximising the sum of discounted economic rents over time. The results reported below indicate that economic gains could have been realised from adoption of an optimal feedback rule of the type presented herein, as compared to the policy that gave rise to historical catches. The methods

and results displayed are of general interest not only because they demonstrate how optimal feedback rules can be used for both deterministic and stochastic bio-economic models, but also because they provide a benchmark relative to which one can compare the outcome of the true management strategy. It is also worth pointing out that the feedback rule we examine can be used as a practical tool by managers.

2 An Optimal Harvest Rule

A commonly-proposed fishery management objective, which we adopt here, is to maximise the flow of expected discounted net revenue from the fishery over time, subject to the constraint implied by fish stock dynamics. Net revenue is the total revenue from fish harvest minus operating costs. Operating costs are a decreasing function of fish biomass and are commonly believed to be an increasing function of harvest. That is:

$$\tilde{\Pi}(x,h) = p(h)h - c(x,h) \tag{1a}$$

$$\Pi(x,h) = \max\left[\tilde{\Pi}(x,h),0\right] \tag{1b}$$

where h is fish harvest, x is fish stock biomass, $p(\cdot)$ is the price function and $c(\cdot)$ is the instantaneous cost function.

The dynamics of the fish stock biomass, expressed in a straighforward way, can be captured by the stochastic differential equation:

$$dx = \left[f(x) - h\right]dt + \sigma(x)dw \tag{2}$$

where $f(x)$ is the natural growth in biomass, $\sigma(x)$ is the diffusion term (which is zero in the deterministic case), and dw is an increment of a standard Wiener process.

The management objective can be achieved in the context of a stochastic control problem using these equations. Analytical derivation of the optimal harvest feedback rule for this problem can be found in Sandal and Steinshamn (1997); here we concentrate on numerical solutions. The dynamic optimisation problem is:

$$V(y,s) = \max_h E\left\{ \int_s^\infty e^{-\delta t}\tilde{\Pi}(x(t),h(t))dt \right\},$$
$$x(s) = y$$
(3)

subject to the stock growth constraint given by equation 2 and appropriate boundary conditions. E is the expectation operator, δ is the discount rate and $V(\cdot)$ is the optimal value function.

The Hamilton-Jacoby -Bellman (HJB) equation for this problem is:

$$V_t + \max_h\left\{e^{-\delta}\tilde{\Pi}(x,h) + V_x \cdot [f(x)-h]\right\} + \frac{1}{2}\sigma^2(x)V_{xx} = 0$$
(4)

where the subscripts denote derivatives. A solution in the form of a stationary-value function is:

$$V(x,t) = e^{-\delta}W(x) + \frac{1-e^{-\delta}}{\delta}P$$
(5)

which has the property:

$$\delta \to 0 \Rightarrow V(x,t) = W(x) + Pt$$

From (4) we get

$$\delta W = P + \max_h\left[\tilde{\Pi} + W'(f-h)\right] + \frac{1}{2}\sigma^2 W''$$
(6)

which results in the policy relation:

$$h^* = \begin{cases} 0, & W'(x) > \Pi_h(x,0) \ \ or \ \ \Pi(x,h) \leq 0 \\ h \geq 0, & W'(x) = \Pi_h(x,h) \ , \ \ \Pi(x,h) \geq 0 \end{cases}.$$
(7)

where W′ is the first and W″ is the second derivative with respect to x. This relation can be used to eliminate harvest from the equation and thereby obtain the ordinary differential equation for the value function in terms of the fish stock, x. Focusing on an interior solution ($h > 0$), that is where zero profits prompt closure of the fishery, we get

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$$h^*(x) = g(x, W') <=> \Pi_h(x,h) = W'. \qquad (8)$$

Then the HJB equation can be re-written as the nonlinear differential equation:

$$\delta \cdot W = P + \Pi(x, g(x, W')) + W' \cdot [f(x) - g(x, W')] + \frac{1}{2}\sigma(x)^2 W'' \qquad (9)$$

together with appropriate boundary conditions. In order to get the feedback policy, $y = W'$ must be determined.

Normally solving equation (9) is an immense task as it is not an initial-value problem. One should keep in mind that a solution of (9) must match the solution from the region with zero harvest (the high contact condition). Fortunately, in our case this problem can be overcome by using the following two requirements. For a sufficiently large stock we know the optimal harvest and the value function; namely the deterministic optimum. At the other end $\Pi = 0$ represents a lower bound on harvest.

3 The Southern Bluefin Tuna Fishery

The southern bluefin tuna (SBT) fishery spans the southern oceans between 30° S and 50° S. Most of the catch of this slow-growing highly-migratory species is taken in waters close to Australia. The SBT fishery is particularly interesting because of its spawning and migration patterns, as well as uncertainty regarding stock levels. Management of the fishery has not been effective in avoiding substantial stock depletion, largely because of the difficulties encountered in securing international agreement on appropriate exploitation levels. Further background information on this fishery can be found in Polacheck *et al*. (1999) and Cox *et al*. (1999).

The current status of the SBT stock is the subject of considerable disagreement among the countries making up the Commission for the Conservation of SBT (CCSBT). This has led to differing views on the level of harvest appropriate for achieving the stated objective of rebuilding the parental biomass to its 1980 level by the year 2020. In the present paper we examine the implications for catch quotas of an optimal feedback rule resulting from the

objective of maximising the discounted flow of economic returns. These optimal catch levels are then compared to historical experience.

The functional forms required for evaluating the optimal feedback rule for SBT were specified as:

$$f(x) = r(x - k)(1 - x/K) \tag{10}$$

$$p(h) = a - bh \tag{11}$$

$$c(x) = d/x \tag{12}$$

$$\Pi(x, h) = ah - bh^2 - \frac{d}{x} \tag{13}$$

where $r = 0.2246$ is an intrinsic growth rate; $K = 564\ 795$ is the environmental carrying capacity; and $a = 88.25$, b $= 0.0009$, $d = 1.633 * 10^{11}$, and $k = 71\ 581$ are scaling constants. These functional forms are similar to those reported by Kennedy and Pasternak (1991) and the parameter values used are calibrated with available data for the fishery.[1]

This results in

$$\Pi(x, g) = ah_B - bh_B^2 - \frac{1}{4b} y^2 - \frac{d}{x} \tag{14}$$

$$g = h_B - \frac{y}{2b} \tag{15}$$

where $h_B = \frac{a}{2b}$ is the static optimum in (13) which is the basic equation for the numerical approach.

Equation 10 represents generalised logistic biomass growth that exhibits critical depensation at the level of $k = 71\ 581$ tonnes. In a deterministic model k is an unstable equilibrium point

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[1] Regression coefficients for equation 9 were obtained using catch data and a random choice of one of the stock series based on Virtual Population Analysis (VPA) used by Polacheck *et al.* (1998). These were estimated by indirect least squares from a = rk = - 16 077.9 (25, 931); b = r (1 + k/K) = 0.253 (0.099); c = r/k = - 0.398 * 10^{-6} (1.36 * 10^{-7}) where standard errors appear in parenthesis. These estimates are imprecise ($R^2 = 0.2$), underscoring the fact that the form of equation 9 is somewhat arbitrary and the estimates provide no clear evidence of critical depensation in SBT.

to the left of which the population is driven to extinction, and to the right of which the population recovers in the absence of fishing. In the stochastic case, random events can lead to critical depensation or compensation at expected stock levels other than k.

Equation 11 is the price (inverse demand) function, equation 12 is the fitted cost function and equation 13 is the net revenue function. Although spatially resolved catch and effort data for the SBT fishery are extensive for the period 1960-1996, economic data of similar precision and scope are not available. Hence, data limitations prevented specification of higher-order components in these functions, thus limiting the study to linear and quadratic terms only. Of particular note is the fitted cost function, for which harvest was not found to be a significantly explanatory variable. Cost data reported by McIlgorm and Campbell (1992) were used to fix the parameter in equation 12 at the point of the mean VPA stock estimate. Price data from Cox *et al.* were used to determine the co-efficients of equation 11 by regression.[2]

A number of important sources of uncertainty come to light in the evaluation of the SBT fishery using the model components above. A brief discussion of these is warranted because the empirical results we obtain are dependent on the model formulation and the data that we use. First, although the optimal-control formulation is uncontentious from the viewpoint of economists, there is considerable uncertainty about the bio-economic models used to represent the state of the fishery. The functional forms are themselves open to debate, as are some of the parameter values. Observational equivalence of possible competing models, combined with limitations on both the quality and the quantity of available data, give rise to this uncertainty. In the light of these uncertainties, we examined a number of different functional forms and have selected those most consistent with the data and previously-published research.

Paucity of market and business data, and limited knowledge of the lifecycle and migratory behaviour of SBT, as well as shortcomings in understanding many aspects of fleet behaviour, make it difficult to determine how closely our model matches the salient features of the SBT fishery. The existence of many different VPA stock-assessment series for SBT underscores the degree of uncertainty faced by researchers in evaluating both management and fish-population dynamics. In the present paper we introduce the issue of process

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[2] The estimates (standard errors in parenthesis) are a = 5913 (663.6); b = 0.061 (0.025); R^2 = 0.48.

uncertainty in the context of a stochastic model of SBT biomass dynamics; and leave other sources of uncertainty as the subject of further investigation in ongoing research by the authors.

Equations 7 - 14 were used to generate the optimal SBT harvest quotas (in tonnes) over the period 1960 - 1996. The results suggested by the deterministic case are presented in Figure 1 for several of the many discount rates examined by the authors. In this case $\sigma(x)$ is set to zero, reducing equation 9 to first order and simplifying its solution considerably. Historical data are also displayed in Figure 1 to allow easy comparison of the suggested optimal feedback quotas with actual harvests over the period covered by available data. The most striking feature of these results is that the optimal path for $\delta = 0.35$ tracks the historical harvests very closely: almost as if the resulting curve is a regression line. This is an extremely high discount rate and, notwithstanding the above-mentioned model uncertainty, indicates that the fishery has been operated as though the future matters little and that pursuit of immediate returns from fishing might have caused rapid stock depletion. As the discount rate is varied downwards model-generated optimal quotas decline for given stock levels; a result which is consistent with increased concern for future generations that lower discount rates imply.

Of primary interest in the present paper is the stock level at which a harvest moratorium is optimal, as defined by our objective function. Despite substantial uncertainty, this model-generated relationship between moratoria-triggering stock levels and discount rates is what one would expect from an economic perspective. As the discount rate falls, harvest moratoria are invoked at progressively higher stock levels. It is important to realise, however, that the moratoria are applicable at particular stock levels and so may be appropriate for single periods or multiple periods, depending on the observed stock dynamics. The plots in Figure 1 should not be confused as time series.

In order to emphasise the dynamic nature of possible moratorium switching, Figure 2 plots the time series of the ratio of optimal harvest to actual harvest. Clearly, overfishing is implied by values of less than unity but, although the values plotted are of interest in themselves, the main point is that optimal harvests may vary substantially over time, even relative to actual historical harvest levels. Figure 3, which displays the time series of optimal and actual harvests, is an alternative way of depicting this variation. Of particular interest from a management perspective is that the optimal deterministic feedback rule does not

necessarily yield constant quotas and that variation in quotas is necessary for optimality when the fish stock is variable.

The addition of stochastic elements to the stock dynamics sheds additional light on the effect of optimal economic feedback rules for managing the exploitation of migratory pelagic fish species for which assessment of population dynamics is subject to considerable uncertainty. Equation 9 is of second-order and can be solved uniquely using numerical techniques for a given pair of initial or terminal conditions. In order to obtain the economically-optimal solution we have adopted terminal conditions at an arbitrarily large stock level: these are the stock level at which asymptotic harvest level is reached for all chosen discount rates (stocks greater than 700,000 tonnes apply in the present example) and the zero slope condition that this implies. Figures 4 and 5 are included to display the results for this stochastic case with various diffusion terms.[3]

Figure 4 displays the quotas implied by the deterministic feedback rule along with the quotas implied by the stochastic rule for three different diffusion terms, all for a discount rate of 5%. It is interesting that inclusion of the stochastic term has a similar impact to increasing the discount rate in the deterministic case (a method commonly used to incorporate a risk premium). This serves as a warning not to conclude in haste that overfishing has been due solely to a lack of concern for stock recovery and future generations: it may also be the response to, among other things, a high degree of uncertainty that is a major cause of fishdown.

For comparison with the deterministic case, directly accounting for uncertainty in the way it has been done here may decrease optimal harvests at moderate to large stocks, but may increase optimal harvests at small stocks, thus reducing the stock level at which a moratorium is suggested. This result is counter-intuitive although, on reflection, makes sense from an economic point of view: because it reflects the possibility that fishing down the stock will occur at an increasing rate once the natural net mortality rate gives rise to critical depensation. In cases where the population falls below the level of critical depensation, the stock will be unable to recover and so it will be optimal economically to harvest as much of the stock as

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[3] The feedback rule that we use is consistent with a completely general function for $\sigma(x)$. Our specifications were chosen to demonstrate the impact of large-magnitude process error (with, by loose analogy, a coefficient of variation of up to 0.5). The form of this diffusion remains the subject of ongoing research.

possible, provided marginal revenue does not fall below marginal cost, before the remaining fish perish, especially if the natural variation in the size of the fish population dwarfs the effects of the fishery.

The time series of optimal harvest derived from the stochastic case, shown in Figure 5 for a 5% discount rate, shows marked departure from the deterministic case. Clearly the historical catches represent economic overfishing (compared to the stochastic model-generated optimum) for much of the time interval, although a moratorium appears much later than suggested by the optimal deterministic feedback rule.[4] It is important not to read too much into these results, however, because they are conditioned, *inter alia*, on the particular form adopted for process uncertainty. Despite this, the general pattern of results depicted here is consistent with several forms of the diffusion term that have been examined by the authors; particularly those which are of large magnitude relative to historical catches and stock estimates.

Concluding Remarks

Optimal feedback harvest rules have been enumerated using stock-assessment and industry data for the SBT fishery. Optimal harvests so obtained have then been compared to historical catches. In the deterministic case the temporal pattern of actual harvests is well matched by optimal quotas at very high discount rates. In the stochastic case a somewhat lower (but still relatively high) discount rate yields time series of optimal harvest quotas that are broadly consistent with historical harvest levels, although the size of the difference depends on the magnitude and form of the diffusion term which represents one of many sources of uncertainty. This points to the need for greater emphasis on process, model and measurement uncertainty in future work of this type.

Both the deterministic and stochastic feedback rules imply that harvest moratoria are optimal when the stock becomes sufficiently depleted. They also imply similar harvest quotas when the stock is sufficiently large. The deterministic rule performs quite well overall and can be used with confidence at low discount rates when little is known about the nature and magnitude of the stochastic component and when a precautionary approach is deemed to be

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[4] Given the consistent downward trend in historical catches, biological overfishing is also evident. The stock is therefore well below that required for maximum sustainable yield.

desirable. Interestingly, the stochastic feedback rule can lead to greater caution than its deterministic counterpart, especially at moderate to high stock levels, but can give rise to higher harvest quotas for low stock levels: such a result is surprising perhaps, but is consistent with feasible actions of a profit-maximiser when natural uncertainty is potentially more damaging to the stock than fishing.

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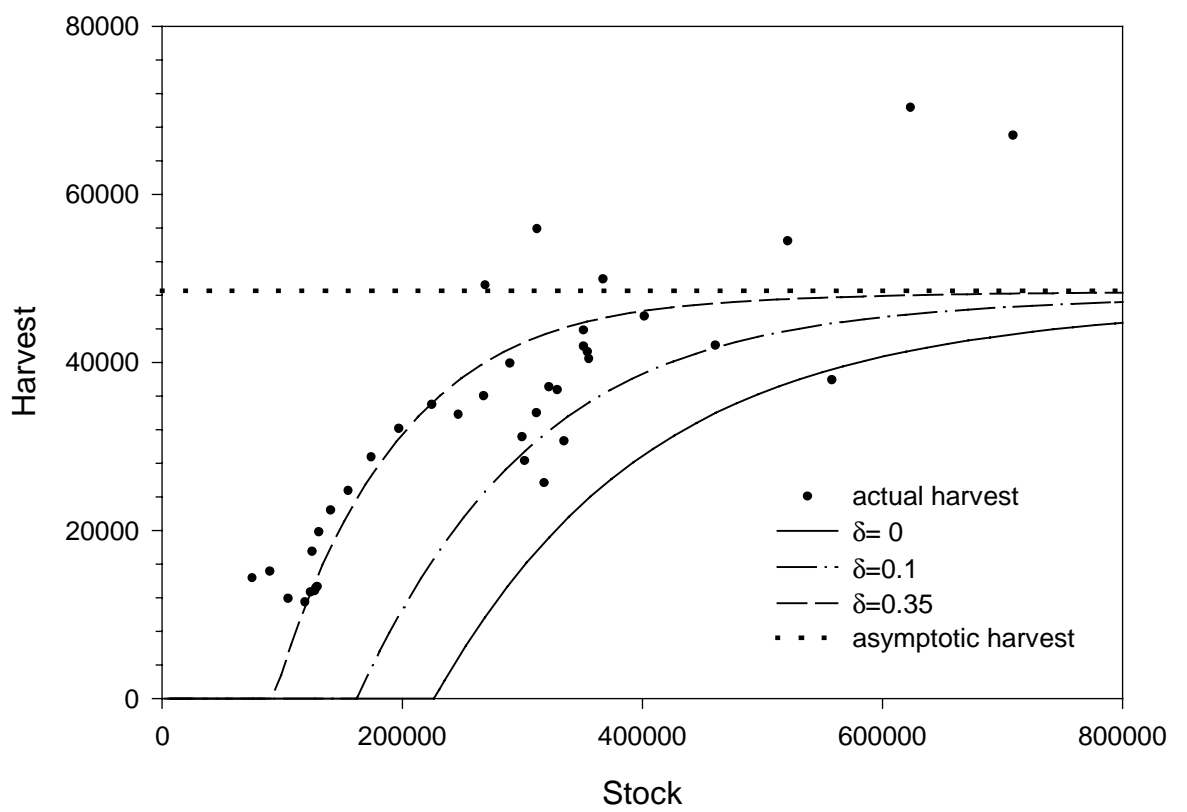Walters, C.J and Hilborn, R., 1976. Adaptive control of fishing systems. *J. Fish. Res. Board Can.* 33(1): 145-159.

Figure 1. Asymptotic harvest, actual harvest and optimal feedback harvest (given sto[c])
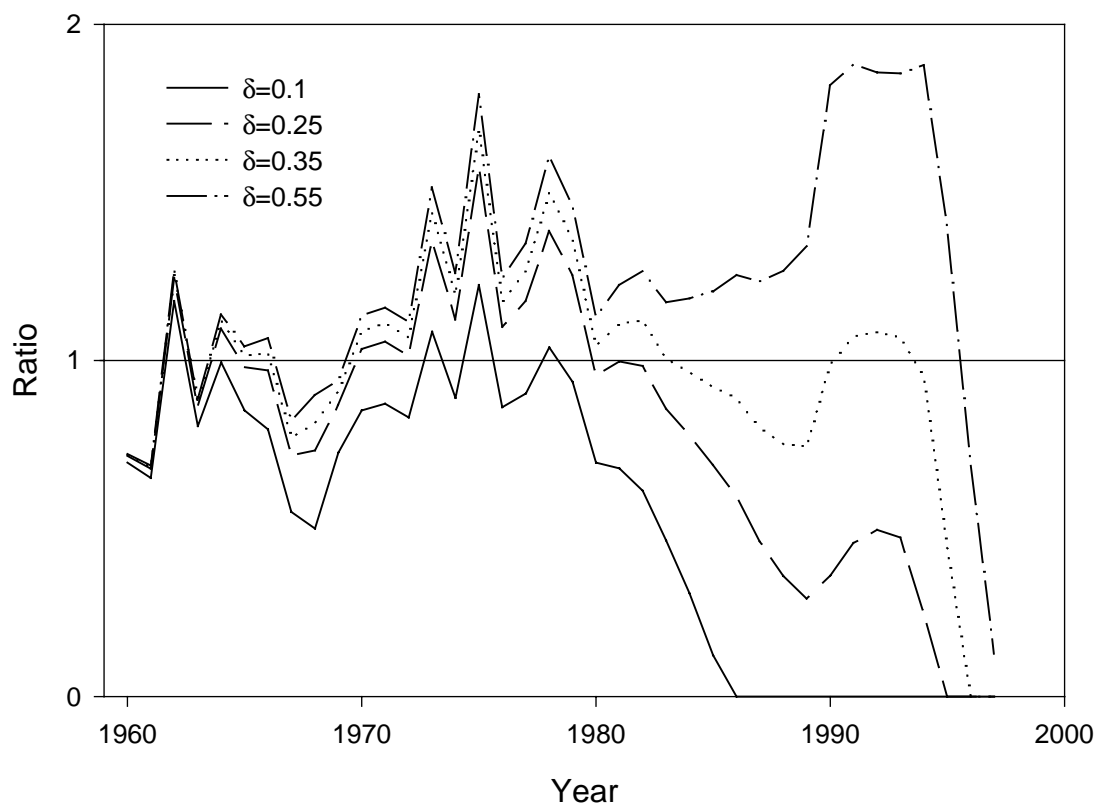for different discount rates.

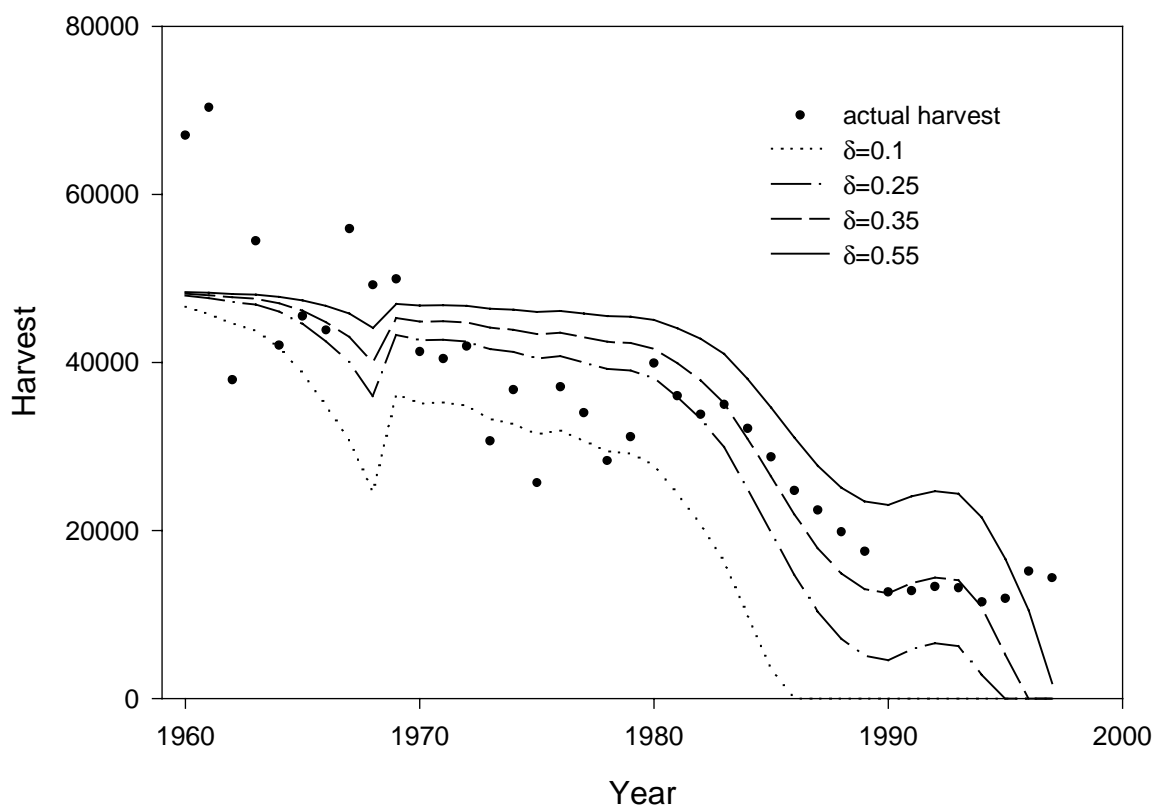Figure 2. Ratio of optimal and actual harvest for different discount rates.

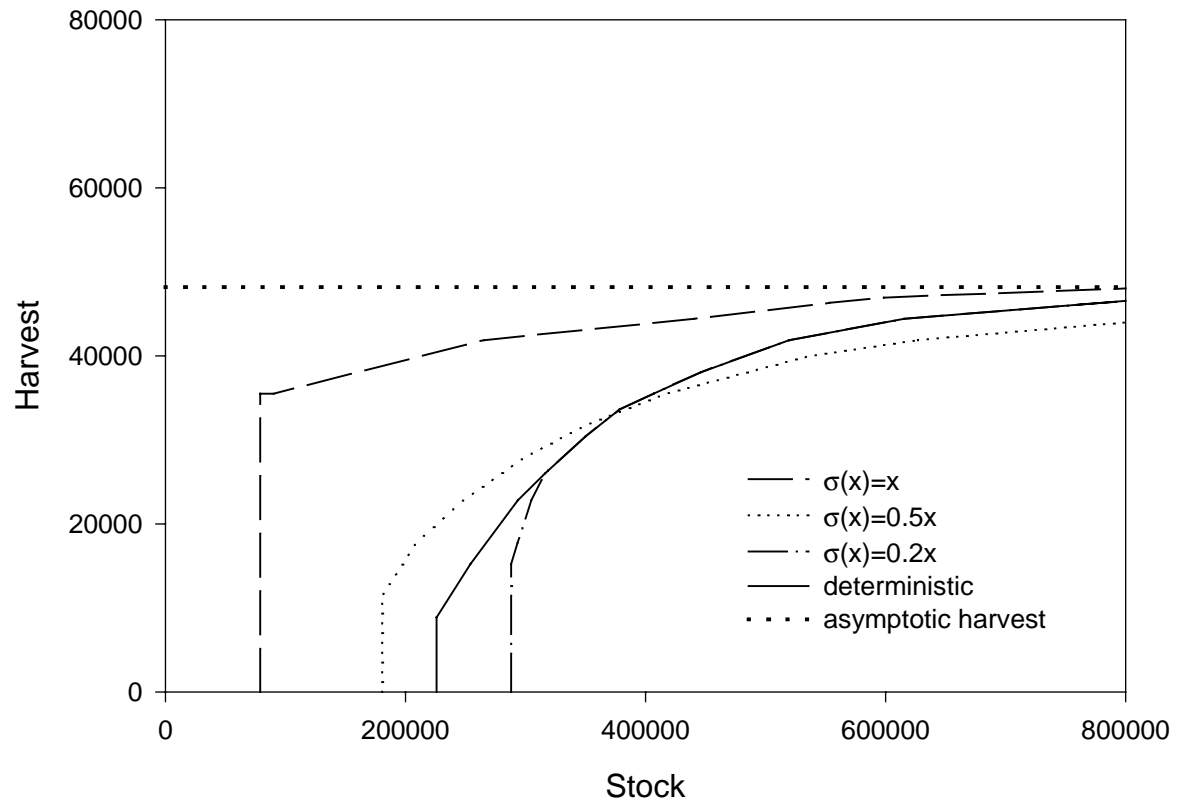Figure 3. Time plot of actual harvest and optimal feedback harvest for different discount rates.

Figure 4. Actual harvest and optimal stochastic feedback harvest
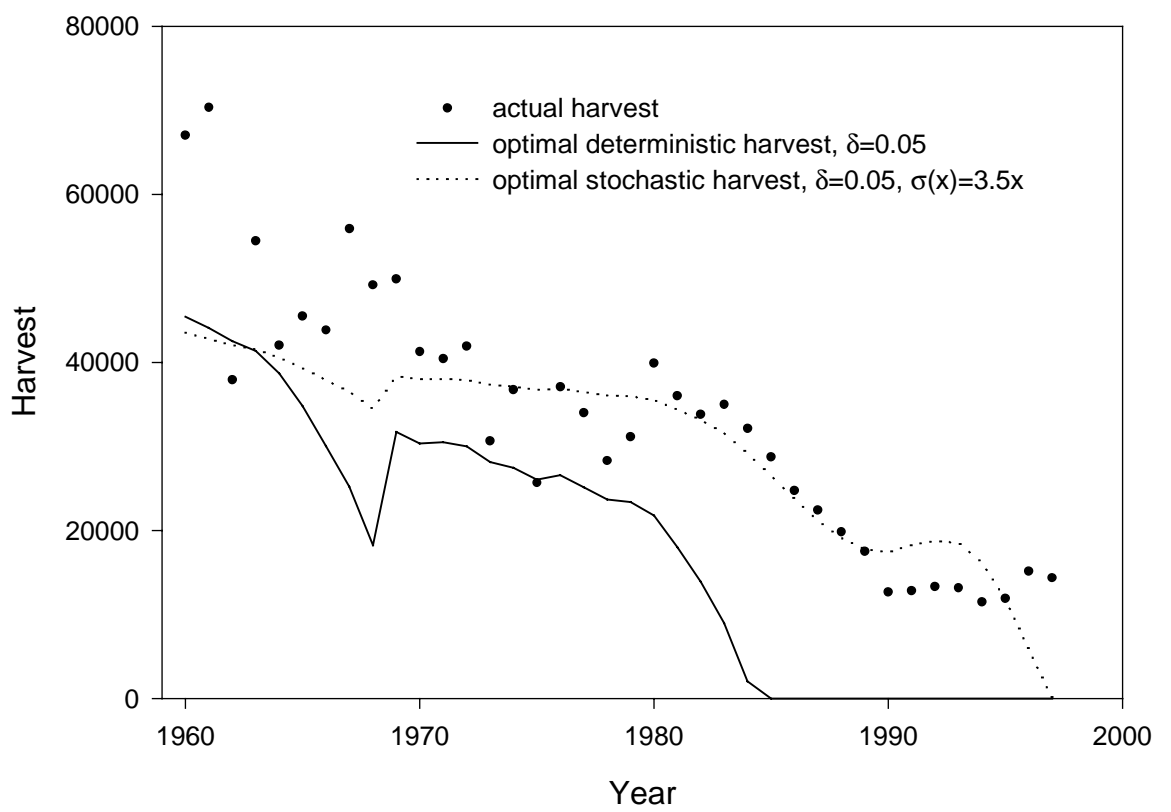(given stock) for different diffusion terms.

Figure 5. Time series of harvest, optimal determisitc harvest and optimal stochastic harvest.