A new approach of fitting biomass dynamics models to real data based on a linear total allowable catch (TAC) rule: An optimal control approach

Al-Amin M. Ussif[1], Leif K. Sandal [2] and Stein I. Steinshamn [3]

May 2000

Abstract

A non-traditional approach of fitting dynamic resource biomass models to data is developed in this paper. The adjoint technique is an optimal control or a variational method for parameter identification. It provides a novel and efficient procedure for combining all available information in the analysis of a resource system. Two alternative population dynamics models: the Schaefer logistic and the Gompertz model are proposed for estimating parameters by the method of constrained generalized least squares. A simplified feedback rule is used to tie the biology and economics of fishing. The R^2 statistic is used to evaluate the goodness of fit. Estimates of the parameters of the logistic and the Gompertz function are plausible and can be accepted. The main inference from the work is that the average fishing intensity rate is found to be significantly above the sustainable value.

[1]Corresponding author: Department of Finance and Management Science, Norwegian School of Economics and Business Administration, Bergen Norway. Tel. +4755959686, Fax. +4755959234, Email:ussif@hamilton.nhh.no.

[2]Department of Finance and Management Science, Norwegian School of Economics and Business Administration, Bergen Norway.

[3]Foundation for Research in Economics and Business Administration

1 Introduction

In spite of the growing criticisms of the biomass dynamics models or the surplus growth models (Clark, 1990; Schaefer, 1967), they remain the biological basis for most bioeconomic analysis. The trend in bioeconomic literature indicates that these models will continue to be in use for some time. Parameter estimation has been the most difficult aspect of application of biomass dynamics models in management schemes. The bulk of the research in this area has been done by fishery biologists in the past. Several methods have been developed for fitting these models to observed data. Three approaches have been commonly used to fit surplus production models to observations: effort averaging methods, process-error estimators, and observation-error estimators (see Polacheck et al., 1993). Polacheck et al. (1993) used real and simulated data to compare the approaches and concluded that the methods yield different interpretations of productivity. The method of effort-averaging, like many others, assumes that the stock is in equilibrium relative to effort. Ludwig et al. (1988) compared the method of total least squares and the approximate likelihood method. They found the two methods to be consistent with some significant differences. Least squares methods have also been used to estimate the Schaefer production model (Uhler, 1979).

In bioeconomics, identification of model input parameters has not been accorded the attention it so deserves. Simulations of these models have mostly been performed using hypothetical values of the model parameters. Useful qualitative insights have been gained in a more general setting. However, issues of quantitative and operational nature have largely been ignored. Of interest to managers of resource stocks such as fish are questions about the size of the standing stock, the sustainable yield, the net growth, etc. To better advise managers on these important issues, bioeconomists ought to develop techniques of improving and efficiently estimating the existing bioeconomic models.

In view of the above arguments, we introduce a novel and advanced approach of fitting biomass dynamics models to measurements. The technique in this paper is an optimal control (adjoint) method of model parameter estimation (Lawson et al., 1995; Smedstad and O'Brien, 1991). For recent applications of these techniques to biological and ecosystem models see (Lawson et al., 1995; Spitz et al., 1997; Matear,1995). The adjoint technique of data assimilation determines input parameters of a dynamical model using time series of observations of the state variables of the model dynamics. A least squares criterion is defined subject to the natural dynamic constraints governed by the simple generalized population dynamics models. The adjoint technique is then used together with a quasi-Newton algorithm (Gilbert and Lemarechal, 1991) to iteratively search for the minimum of the loss functional. The method is very powerful and efficient for parameter optimization. A major strength of the method is that it is highly suitable for high dimensional problems. It can also effectively handle nonlinear models. We also point out that this method does not require analytical forms of the functions estimated which distinguishes it from existing methods.

Two functional forms of the existing biomass dynamics models (Clark, 1990) in combination with a simple proportional exploitation rule will be used to estimate the biological and economic input parameters using real data for the Norwegian cod fishery (NCF) stock. The bioeconomics employed in this analysis is quite simple. It combines simplified surplus growth models with a simple linear yield or harvest function to analyze the data. The biological functions contain parameters that are very crucial in determining certain important quantities of interest to fisheries management and researchers. Estimates of parameters such as the intrinsic growth rate and the environmental carrying capacity are rare for some important fish stocks around the world. Accurate measurement of these parameters are in fact very difficult if not impossible. As a consequence, quantities of considerable importance to management such as the maximum sustainable yield (MSY) are unreliable.

The goals of this paper are to demonstrate the potentials of the variational adjoint technique in the analysis of natural resource systems, to apply the technique to the Norwegian cod fishery for the two different growth models and to make some inferences from the data. The paper is organized as follows. Section 2 is a discussion of the methodology used in the analysis. In section 3, we present the biological and economic submodels. The biology and economics are merged by the fishing intensity factor through a simplified yield function. In section 4 we present and discuss an empirical application of the model and conclude the paper.

2 Data Assimilation Methods

According to Sasaki (1970), a variational inverse problem can be cast as a weak constraint inverse problem where the model is allowed to contain modeling errors or the strong constraint problem (Bennett, 1992; Evensen et al., 1998), where a perfect model is assumed. The weak constraint problem is a more general formulation with the strong constraint problem as a simple special case where the model weight is assumed to be infinitely large. It is a common practice among some researchers to assume a model that is perfect then vary some of the free parameters such as the initial conditions of the model in order to find the solution which best fit the data. Such a formulation is known as the strong constraint problem. In this paper, the adjoint technique will be employed to fit the dynamic resource models to the observations. Using the adjoint method the gradients of the cost functional with respect to the control variables are efficiently calculated through the use of the Lagrange multipliers. The gradients are then used to find the parameters of the model dynamics which best fit the data.

Data assimilation systems consist of three components: the forward model with a criterion function, the adjoint or backward model and an optimization procedure (Lawson et al., 1995). The forward model is our mathematical representation of the system we are interested in studying, e.g., an open access, a regulated open access or a sole owner fishery. The adjoint model consists of equations obtained by enforcing the dynamical constraints through Lagrange multipliers and provide a method of calculating the gradient of the cost function with respect to the control variables. The gradients are then used in a line search using standard optimization packages to find the minimum of the cost function. Most optimization routines are based on iterative schemes which require the correct computation of the gradient of the cost function with respect to the control variables. In the adjoint formulation, computation of the gradient is achieved through the adjoint equations forced by the model-data misfits. The model equations are run forward in time while the adjoint equations are run backward in time which are then used to calculate the gradient of the cost function.

An important step in data assimilation is the choice of the criterion function for the goodness of fit. The commonly used criterion is the generalized least squares criterion. It can be defined with no a priori information about the parameters or with prior information about the parameters incorporated as a penalty term in the criterion function. Some researchers argue that since some information about the parameters and their uncertainties are always available, adding the information is a plausible thing to do (Harmon and Challenor, 1997; Evensen et al., 1998).

2.1 Perfect dynamics

In this paper we will assume perfect dynamics and initial condition(s). This implies that we are neglecting modeling errors. The model dynamics will be governed by a simple ordinary differential equation given by

$$\begin{aligned} \frac{dx}{dt} &= g(\mathbf{p}; x) \\ x(0) &= u \qquad (1) \\ \mathbf{p} &= \mathbf{p}_0 + \hat{\mathbf{p}} \qquad (2) \end{aligned}$$

where $g(\mathbf{p}; x)$ is a nonlinear operator, $\mathbf{p}$ is a parameter(s) to be estimated and is assumed poorly known and u is the first or best guess initial condition of the model. The vector $\mathbf{p}_0$ is the first guess of the parameters and $\hat{\mathbf{p}}$ is a vector of random white noise term. Assume that we also have a set of observations of the state variable(s) which are related to the true state in this simple linear fashion

$$\mathbf{x}^{\mathbf{obs}} = \mathbf{x} + \mathbf{v} \qquad (3)$$

where $\mathbf{x^{obs}}$ and $\mathbf{x}$ are the observed and the model forecast vectors respectively, and $\mathbf{v}$ is the error vector in the observed values. The additive stochastic error term is quite general so far. In the subsections that follow, we will put some structure to the form of the noise term. Inverse methods combine the theoretical information contained in the model and the information about the true state from the data to optimally estimate the model parameters.

2.2 The estimator

One of the major components of data assimilation techniques is the choice of the estimator. Many estimators exist that are attractive in the literature. However, the least squares estimator has been the popular one among researchers partly because of its simplicity and mathematical convenience. The least squares fitting criterion is defined as

$$\mathcal{J} = (\mathbf{x} - \mathbf{x}^{obs})^T \mathbf{W} (\mathbf{x} - \mathbf{x}^{obs}) + (\mathbf{p} - \mathbf{p}_0)^T \mathbf{W}_p (\mathbf{p} - \mathbf{p}_0) \tag{4}$$

where $\mathbf{x}$ is the prediction of the model, $\mathbf{x}^{obs}$ is the observed or measured quantity. The $\mathbf{W}$ is the inverse measurement error covariance matrix, i.e., the weightng matrix and is assumed to be positive definite and symmetric and T denotes the transpose operator. Uncertainties in the parameters are represented by the symmetric positive definite covariance matrix $\mathbf{W}_p^{-1}$. The first term in the loss function is the sum of the square of the model-data misfits $\mathbf{v} = (\mathbf{x} - \mathbf{x}^{obs})$ and the second term is a penalty on the parameters. If the model parameters are poorly known then greater penalty is imposed, i.e., they are given less weight and vice versa. To simplify the calculations, we make the following assumptions about the errors and their uncertainties. The model-data and the parameter misfits are assumed to be Gaussian mean zero and constant variances. That is we have

$$E\mathbf{v} = \mathbf{0}, \quad E\mathbf{v}\mathbf{v}^T = \mathbf{W}^{-1} = w^{-1}\mathbf{I} \tag{5}$$

$$E\hat{\mathbf{p}} = \mathbf{0}, \quad E\hat{\mathbf{p}}\hat{\mathbf{p}}^T = \mathbf{W}_p^{-1} = w_p^{-1}\mathbf{I}_p \tag{6}$$

where the capital letter E denotes mathematical expectation operator, $\mathbf{I}$'s are unit matrices and the scalar constants w^{-1} and w_p^{-1} are the variances of the random errors in the measurement and the parameters respectively. In view of the above assumptions, the loss function $\mathcal{J}$ can be identified with a normal probability distribution function. Thus, minimizing the cost function is equivalent to maximizing the likelihood, i.e., the best fit corresponds to the most likely outcome of the measurements.

2.3 Minimization technique

Minimization of the loss functional $\mathcal{J}$ subject to the dynamics is a constrained optimization problem (Luenberger, 1984; Bertsekas, 1992). An efficient technique for the minimization of the loss functional is the adjoint method. It consists of transforming the constrained problem into an unconstrained optimization problem via the use of the undetermined Lagrange multipliers. It is then possible to use a gradient search method to find model parameters that yield predictions which are as close as possible to the observations. To illustrate the numerical procedure, we use the discrete equivalent of the continuous model dynamics

$$x_{n+1} = x_n + g(\mathbf{p}; x_n)dt, \tag{7}$$

$$x_0 = u, \quad 0 \le n \le N-1 \tag{8}$$

where N is the number of observations and dt is the time step. The discretization scheme used is a simple forward difference scheme. The discrete form of the Lagrange functional is constructed as follows

$$\begin{aligned}\mathcal{L} = w\sum_{n=1}^{N}(x_n - x_n^{obs})^2 + w_p\sum_{i=1}^{N_p}(p_i - \hat{p}_i)^2 \\ +\sum_{n=1}^{N-1}\lambda_n(x_{n+1} - \{x_n + g(\mathbf{p}; x_n)dt\})\end{aligned} \tag{9}$$

where λ_n is the value of the multiplier at time step n and N_p is the number of model parameters which are the control variables of the problem. The extrema conditions for the problem are

$$\frac{\partial\mathcal{L}}{\partial\lambda_n} = 0 \tag{10}$$

$$\frac{\partial\mathcal{L}}{\partial x_n} = 0 \tag{11}$$

$$\frac{\partial\mathcal{L}}{\partial p_i} = 0 \tag{12}$$

From these equations, we obtain

$$x_{n+1} - \{x_n + g(\mathbf{p}; x_n)dt\} = 0 \tag{13}$$

$$\frac{\partial\mathcal{J}}{\partial x_n} - \lambda_n(1.0 + dt\frac{\partial g}{\partial x_n}) + \lambda_{n-1} = 0 \tag{14}$$

$$\Delta_{p_i}\mathcal{L} = \Delta_{p_i}\mathcal{J} - \sum_{n=1}^{N-1} \lambda_n dt \frac{\partial g}{\partial p_i} = 0 \tag{15}$$

where $\Delta_{p_i}\mathcal{L}$ is the derivative with respect to the i^{th} parameter and $\frac{\partial g}{\partial x_n}$ is the tangent linear operator. It is immediately seen that equation (13) recovers the model dynamics, i.e., the forward model, equation (14) gives the backward model forced by the model-data misfits and equation (15) is the gradient with respect to the parameters. To find the model parameters that give model forecasts that are as close as possible to the observations using the classical search algorithms, correct values of the gradients are required. Methods of verifying the correctness of the gradient are available both numerically and analytically where possible (see, Spitz et al., 1997; Smedstad and O'Brien, 1991). We have in this paper checked all gradient calculations to ensure reliable parameter estimates. The optimization procedure used for the minimization is the quasi-Newton procedure developed by Gilbert and Lemarechal. Implementation of the adjoint parameter algorithm is quite straightforward and involves the following steps.

- Choose the first guess for the control parameters.
- Integrate the forward model over the assimilation interval.
- Calculate the misfits and hence the cost function.
- Integrate the adjoint equation backward in time forced by the data misfits.
- Calculate the gradient of $\mathcal{J}$ with respect to the control variables.
- Use the gradient in a descent algorithm to find an improved estimate of the control parameters which make the cost function move towards a minimum.

- Check if the solution is found based on a certain criterion.

- If the criterion is not met, repeat the procedure until a satisfactory solution is found.

2.4 Goodness of fit measure

To examine the performance of the method we need a statistical measure of how the predicted and the observed variables covary in time. An appropriate parameter may be the correlation coefficient R. For the random vectors $\mathbf{x}$ and $\mathbf{x}^{obs}$, the correlation coefficient is given by

$$R = \frac{\sum_{n=1}^{N}(x_n - \bar{x})(x_n^{obs} - \bar{x}^{obs})}{[\sum_{n=1}^{N}(x_n - \bar{x})^2 \sum_{n=1}(x_n^{obs} - \bar{x}^{obs})^2]^{1/2}} \tag{16}$$

where the bars denote the means or expected values of the random variables and N is the number of observations. Notice that R is a dimensionless quantity and lies between -1 and +1 inclusive. From the R relation, another important quantity called the coefficient of determination R^2 can be calculated. The coefficient of determination is defined as $R^2 = SSR/SST$, where SSR is the variance explained and SST is the total variance (see Greene, 1997). The sign of the correlation is obtained from R whiles the joint variances are given by R^2.

3 The Dynamics of the Biomass

Management of many fisheries have often been based on the simplified population dynamics models of the Schaefer type (Sandal and Steinshamn, 1997; Clark, 1990). It is apparent that these models will continue to be used for some time in the management of some of the important commercial species around the world. While efforts are underway in the development of more complex models, it is appropriate to explore techniques of identifying the inputs of the existing models. A strong biological base is a key to good simulation and optimization analysis in renewable resource management. The surplus production models, though very simple, can be quite a good approximation of the complex dynamics. A continuous surplus biomass dynamics model is proposed for this analysis. The basic form of the mathematical equation is

$$\frac{dx}{dt} = g(x) - h \tag{17}$$

where $x(t)$ is the biomass at time t, $h(t)$ is the rate of depletion of the population due to human activities, e.g., commercial and recreational fishing, g is the natural additions to the biomass. Two functional forms of the net growth of the population will be investigated in this paper, i.e., the Schaefer logistic and the Gompertz functions will be used.

3.1 The net growth models

Two variants of the growth models are considered in this paper. Biological species grow by the gift of nature. The structure of their growth is quite complicated requiring sophisticated mathematical functions to adequately model them. Fortunately, there are simpler models that reasonably and approximately represent the intricate growth models. Two of the simplest parameterizations in fisheries management are

$$g(x) = \begin{cases} rx(1 - \frac{x}{K}) \\ rx\ln(\frac{K}{x}) \end{cases}$$

where x is as defined previously, r is the intrinsic growth rate, K is the maximum population level of the biological species. The first is the Schaefer logistic growth which is a special case of the modified logistic when the exponent is unity (Clark, 1990, Haakon, 1998) and the second is the Gompertz function.

The production function for a resource industry can be assumed to depend only on the stock biomass and the effort expended in fishing. The simplest form of the exploitation rate is the Gordon-Schaefer type of production function where the rate of removal of the stock is assumed to be linearly related to the effort and stock size. The coefficient of proportionality q in this case is called the catchability coefficient, i.e., $h = qex$, where e is the fishing effort. For the present purpose, this simple linear model will be employed. That is, we apply a proportional fishing criterion in order to analyze the fishery.

Let $f = qe$ be the fishing intensity rate, then the simple rule takes the form

$$h(x) = fx \tag{18}$$

which implies that at any given level of the population a fraction f will be removed. The formula explicitly assumes exploitation of the species to the last ton of fish. This is an oversimplification of the reality. However, it may serve as a good approximation of the complex system. For example, in the extreme situation where fishing becomes economically unprofitable or if on a purely ecological or social ground a moratorium is warranted, f is set to zero, i.e., the fishery is closed. The fishing mortality parameter f is a policy instrument for the management authorities. It is quite simple and easy to use formula. Once accurate and reliable methods of stock assessments are available, the rule can be used to set quotas appropriate for the objective of the fishery.

Using the relation for h in (18) and (2), the biology of the stock is tied to the economics by the fishing mortality f. In Figure 1 below, we show plots of the growth functions using arbitrary values of the parameters. The values of the parameters r and K are the same for both the functions. A straight line curve with a slope equal to 0.407 representing a linear in stock yield function is also shown.

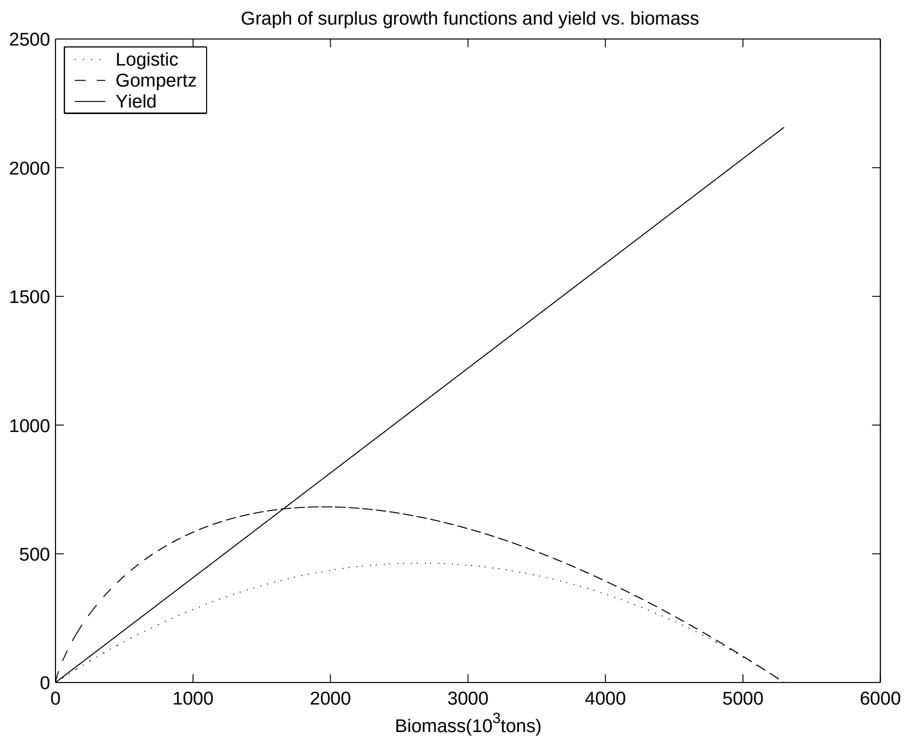

Figure 1: The growth models with r=.35, $K = 5300$.

The graph of the logistic is symmetric about one half the carrying capacity while the Gompertz is asymmetric and is skewed towards the left. For the same K, the former predicts lower MSY biomass (K/e), where $e \cong 2.7$ is the exponent operator and a corresponding higher MSY. In practical applications, the Gompertz function seems inappropriate for less resilient species. The combination of high MSY and low MSY biomass prescribed by this model can result in an unpardonable mistake on the side of management in case of recruitment failures.

4 An Application to NCF

The NCF is the most important demersal species along the coast of Norway and Northern Russia. This fishery has played an important economic role within the coastal communities for the past thousand years. The NCF has for the past half century experienced large variations which result in a corresponding variation in the annual harvest quantities. The stock size fell from its highest level in 1946 of 4.1 million tons to the lowest in 1981 of 0.75 million tons. A time series plot of the history of the stock indicated a sign of recovery from its worst state in the mid 90's but recent reports show that the fishery is again in deep trouble (see Figure 2 below).

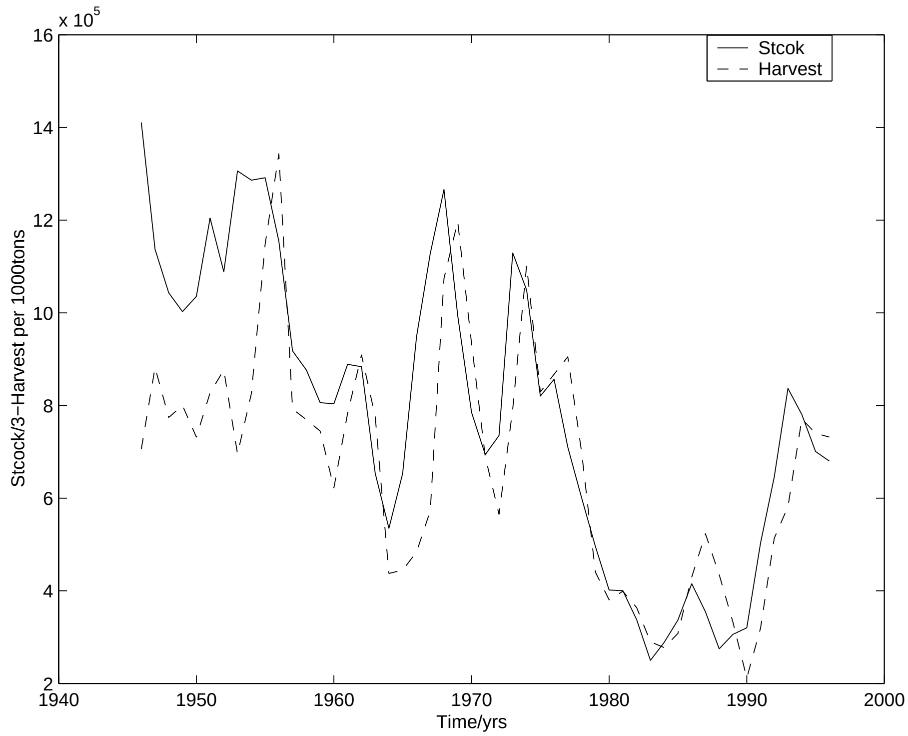

Figure 2: Graph of actual harvest and the stock biomass.

In this study, a time series of observations from 1946 to 1996 is used. The adjoint method is used to fit the hypothesized dynamics to the observations. The NCF provides a good example to which the data assimilation method can be tested. To estimate the parameters, the intrinsic growth rate is assumed fairly known by fixing its value to 0.3499. The other parameters of the models are then estimated. The optimization was started by randomly generating reasonable initial guesses using a uniform random deviate intrinsic function. By seeding the generator, different initial guesses were used to check for the presence of local extrema. The performance of the algorithm is very impressive. Convergence was obtained in a few iterations in all the runs. The best fit parameters and the R^2 values are shown in the table 1 below.

Parameters	Logistic	Gompertz
r^*	0.3499	0.3499
K	5268.5	5499.99
f	0.4076	0.4964
R^2	0.550	0.529

Table 1: Model parameters for the biomass dynamics models.

The star in the table means those values were restricted. The Schaefer logistic and the Gompertz functions tend to give plausible estimates. The fit to the data is quite good for both models with the logistic model explaining about 55.0% of the data while the Gompertz function explains about 53% of the data. It is observed that the estimates for the latter model are relatively higher than the former.

Next, the growth functions are presented on the same graph with the actual harvest data. The goal is to show one of the findings of the paper. That is, the stock is exploited at an unsustainable rate leading to the alarming state of the fishery. Figures 3-4., show the plots of the actual harvest and growth curves against the biomass. The plus sign represents the actual harvest while the solid line represents the net growth curve. The logistic growth model predicts that the harvest rate has been persistently above the net growth curve see Figure 3 below. At the lower end of the graph, we notice that the actual harvest is close to the growth curve and is below it on a few occasions. One interesting observation is that several points tend to cluster around the maximum sustainable yield (MSY). This gives a more acceptable picture of the actual fishery.

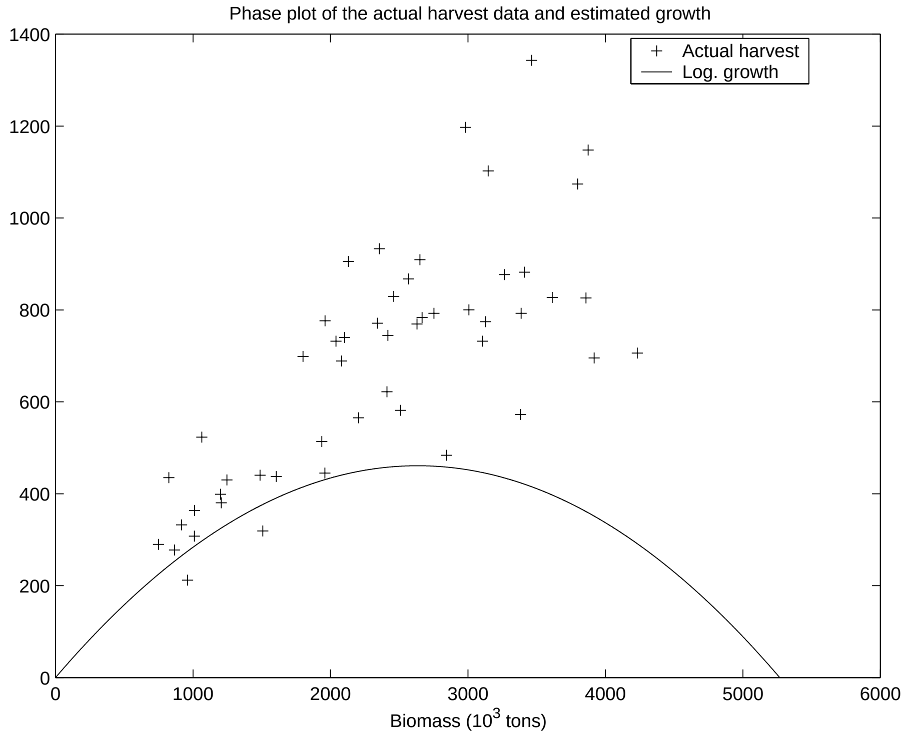

Figure 3: The logistic growth model

The forecasts of the latter model, i.e., the Gompertz model, is quite similar to the predictions of the logistic model but appears to point to other factors for the recent troubles of the fishery rather than excessive harvesting of the stock (Figure 4).

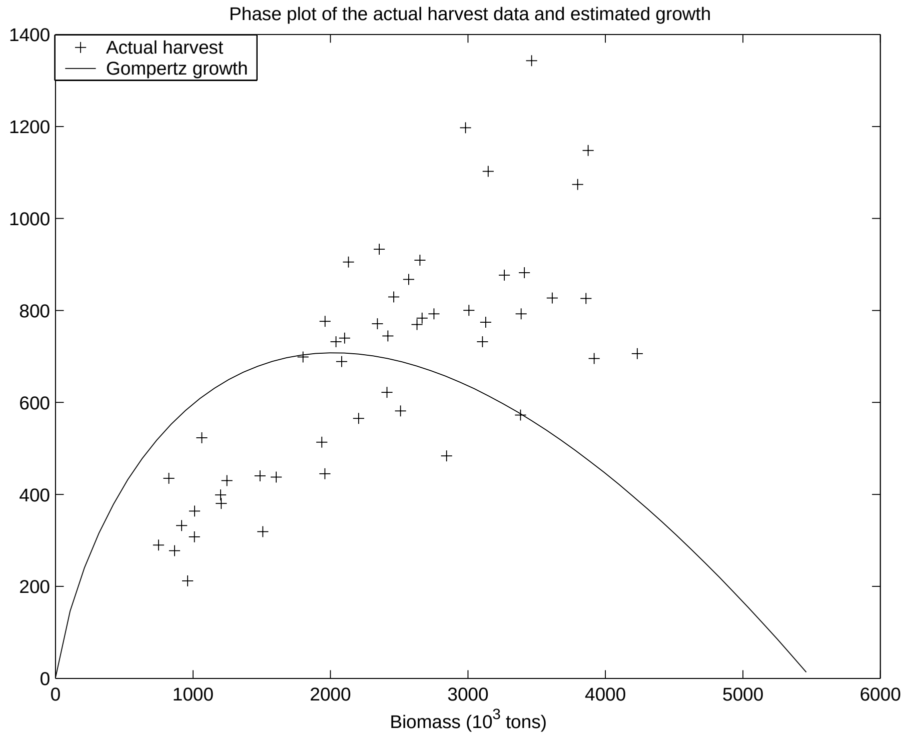

Figure 4: The Gompertz growth model.

To further discuss the results of the paper, we provide estimates that might be of considerable interest to managers of the NCF. An important caveat however is that, while these values have empirical significance, a direct translation to that fishery may not be advised.

The use of surplus growth functions implies there exist a certain level of biomass at which natural additions to the stock are greatest. This occurs at the turning point of the concave functions. For each model an f exists that will direct the stock to the sustainable level. In the case of the Schaefer logistic, a simple algebra yields optimal fishing mortality rate for an MSY policy equal to one half the intrinsic growth rate ($f = r/2$) if the population is below the sustainable biomass level. The table below shows some quantities of practical interest pertaining to the NCF.

Parameters	Logistic	Gompertz
r^*	0.3499	0.3499
K	5268.5	5499.99
x_{MSY}	2634.25	2023.33
MSY	460.9	707.96

Table 2: Sustainable parameters for the two biomass dynamics models.

Estimates of x_{MSY} and MSY quantities are shown in rows 3 and 4 of table 2. The Schaefer logistic model seems to out perform its counterpart, i.e., the Gompertz model. It gave the lowest MSY estimate but an inbetween value of optimum sustained biomass. These estimates are quite appealing and are more acceptable than the predictions of the Gompertz. The MSY for the Gompertz is around the values of TAC in the late 90's. The sustainable biomass level of around 2.0 million tons may be a bit low. However, it may not be advisable to completely discard the results from the Gompertz model since there are other important factors that may account for the troubles of the fishery. For instance, factors such as sea pollution and unfavorable weather conditions may be accountable for the recent sorry state of the NCF stock.

4.1 Conclusions

The NCF fishery is analyzed using an optimal control approach of dynamic model parameter estimation. Two alternative growth models are proposed and used in the analysis.

The production relation for the fishery is assumed to be linear in the biomass and constitute a simple feedback rule. A quite restrictive assumption of constant fishing mortality is made which yields a proportional fishing policy. The model dynamic equation is nonlinear in the parameters and quadratic in the stock. A least squares criterion measuring the discrepancy between the data and its model equivalent was minimized subject to a dynamic constraint. The adjoint method is used to efficiently estimate the parameters. Parameter estimates from the Schaefer logistic and the Gompertz models are plausible. Both models have about the same explanatory power $R^2 = .55$. This seems quite reasonable since the models were able to capture the trend in the data but failed to capture the periodic oscillations. It is obvious that the models are not sophisticated enough to explain the random events inherent in the system. Ecosystem effects and environmental variability are very important variables and ought to be included in the model. Predictions from these models are consistent with many recent experiences in fisheries and other natural resource stocks. Both the stock biomass and the amount harvested have been declining while fishing intensity is increasing due to technical innovations. More powerful boats are being developed and other advanced fishing equipments are available making the population more vulnerable to exploitation.

This paper has demonstrated the utility of the data assimilation methods in dynamic parameter estimation for two alternative resource models. It exposes the strengths and weaknesses of the simplified biomass dynamics models and provides model parameters that are in close agreement with the observations. The methods have numerous additional capabilities that are worth exploring in the future. Bioeconomists may find these methods indispensable if questions that interest managers most have to be answered and if more realistic models become readily available.

References

- Bennett, A.F. 1992. Inverse Methods in Physical Oceanography. Cambridge University Press, Cambridge.
- Bertsekas, D.P. 1982. Constrained and Lagrange Multiplier Methods. Computer Science and Applied Mathematics.
- Clark, W. 1990. Mathematical Bioeconomics, New York: Wiley and Sons.
- Evensen, G., Dee, D.P. and Schroeter, J. 1998. Parameter Estimation in Dynamical Models. NATO ASI, Ocean Modeling and Parameterizations edited by E. P. Chassignet and J. Verron, 1998.
- Gilbert, J. C. and Lemarechal, C. 1991. Some Numerical Experiments with Variable-storage Quasi-newton Algorithms. Mathematical programming, 45, 405-435.
- Greene, W.H. 1997. Econometric Analysis. Third edition.
- Harmon, R. and Challenor, P. 1997. Markov Chain Monte Carlo Method for Estimation and Assimilation into Models. Ecological Modeling. 101, 41-59.
- Haakon, E.,1998. Bioeconomic Analysis and Management, The case of Fisheries. Environmental Resource Economics. 11(3-4): 399-411.
- Lawson, L. M., Spitz, H. Y., Hofmann, E. E. and Long, R. B. 1995. A Data Assimilation Technique Applied to Predator-Prey Model. Bulletin of Mathematical Biology, 57, 593-617.

- Luenberger, D. C. 1984. Linear and Nonlinear Programming. Reading: Addison-Wesley.
- Matear, R. J. 1995. Parameter Optimization and Analysis of Ecosystem Models Using Simulated Annealing: a Case Study at Station P. J. Mar. Res. 53, 571-607.
- Polacheck, T., Hilborn, R. and Punt, A.E. 1993. Fitting Surplus Production Models: Comparing Methods and Measuring Uncertainty. Can. J. Fish. Aquat. Sci. vol. 50: 2579-2607.
- Sandal, L. K. and S. I. Steinshamn, 1997a, A Stochastic Feedback Model for the Optimal Management of Renewable Resources, Natural Resource Modeling, vol. 10(1), 31-52.
- Sandal, L. K. and S. I. Steinshamn, 1997b. A Feedback Model for the Optimal Management of Renewable Natural Capital Stocks, Canadian Journal of Fisheries and Aquatic Sciences, 54, 2475-2482.
- Sandal, L. K. and S. I. Steinshamn, 1997c. Optimal Steady States and Effects of Discounting. Marine Resource Economics, vol.,12, 95-105.
- Sasaki, Y. 1970. Some Basic Formulation in Numerical Variational Analysis, Mon. Weather Rev., 98, 875-883.
- Schaefer, M. B. 1964. Some Aspects of the Dynamics of Populations Important to the Management of Commercial Marine Fisheries. Bulletin of the Inter-American Tropical Tuna Commission. 1, 25-56.

- Schaefer, M. B. 1967. Fisheries Dynamics and the Present Status of the Yellow Fin Tuna Population of the Eastern Pacific Ocean. Bulletin of the Inter-American Tropical Tuna commission 1, 25-56.

- Smedstad, O. M., and O'Brien,J. J. 1991. Variational Data Assimilation and Parameter Estimation in an Equatorial Pacific Ocean Model. Progr. Oceanogr. 26(10), 179-241.

- Spitz, H. Y., Moisan, J. R., Abbott, M. R. and Richman, J. G. 1998. Data Assimilation and a Pelagic Ecosystem Model: Parameterization Using Time Series Observations, J. Mar. Syst., in press.

- Uhler, R.S. 1979. Least Squares Regression Estimates of the Schaefer Production Model: Some Monte Carlo Simulation Results. Can. J. Fish. Aquat. Sci. vol. 37: 1284-1294.