INTERNATIONAL COMPARISON OF INTEREST RATE GUARANTEES IN LIFE INSURANCE

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Abstract. Interest rate guarantees seem to be included in life insurance and pension products in most countries. The exact implementations of these guarantees vary from country to country and are often linked to different distribution of investment surplus mechanisms. In this paper we first attempt to model practice in Germany, the UK, Norway, and Denmark by constructing contracts intended to capture practice in each country. All these contracts include rather sophisticated investment surplus distribution mechanisms, although they exhibit subtle differences. Common for Germany, Denmark, and Norway is the existence of a bonus account, an account where investment surplus is set aside in years with good investment returns to be used to cover the annual guarantee in years when the investment return is lower than the guarantee. The UK contracts do not include annual bonus distribution, instead they include a potential bonus distribution at maturity of the contract.

These contracts are then compared with universal life insurance, a popular life product in the US market, which also includes annual guarantees and investment surplus distribution, but no bonus account. The contract parameters are calibrated for each contract so that all contracts have 'fair' prices, i.e., the theoretical market price of the contract equals the theoretical market price of all future insurance benefits at the inception of the contract.

For simplicity we ignore mortality factors and assume that the benefit is paid out as a lump sum in 30 years.

We compare the probability distribution of the future payoff from the contracts with the payoff from simply investing in the market index. Our results indicate that the payoffs from the Danish, German and UK contracts are surprisingly similar to the payoff from the market index. We are tempted to conclude that the presence of annual guarantees and sophisticated investment surplus distribution, annual or at maturity only, have virtually no impact on the probability distribution of the payoff. The Norwegian contract has lower risk than the mentioned contracts, whereas the universal life contract offers the lowest risk of all contracts. Our numerical analysis therefore indicates that the relative simple and more transparent US contract provides the insurance customer with a less risky future benefit than the more complex (and completely obscure?) European counterparts.

1. Introduction

Interest rate guarantees, or more precisely, annual minimum rate of return guarantees, seem to be included in life insurance products in most countries. Due to the recent rather low international interest rate level, such guarantees are of great practical concern. The exact implementations of these guarantees vary from country to country and is often linked to different distribution of investment surplus mechanisms.

In a companion paper Miltersen and Persson (2003) interest rate guarantees are analyzed together with a distribution of investment surplus mechanism. The current paper presents an extension to more realistically capture industry practice in several countries, including rather sophisticated investment surplus distribution mechanisms, although exhibiting subtle differences from country to country. Common for the German, Danish, and Norwegian systems is the existence of a bonus account, which can be visualized as a buffer account on the liability side of the insurer's balance sheet, where investment surplus is set aside in years with good investment performance to be used to cover the annual guarantee in years when the investment return is lower than the guarantee. We also include three versions of a UK contract which exhibits a different bonus mechanism. For these contracts bonus is paid at maturity of the contract only if the market value of the market index is higher than the accumulated premium reserve.

These contracts are compared with universal life insurance, a popular life product in the US market, which also include annual investment surplus distribution but no bonus account.

All contracts are valuated and the parameters calibrated using the standard no-arbitrage arguments from financial economics originating from Black and Scholes (1973) and Merton (1973).

The model does not take the life insurance specific factors *mortality* or *periodical premiums* into acount. Specifying mortality and periodical premiums would limit the analysis to only a specific product but could easily be done. The questions we address in this article are common for a wide range of life insurance and pensions products, and thus of interest even if mortality is not specified.

Our model does not include stochastic interest rates.

Only one risky investment opportunity is available and we refer to this as the market index. We attempt to implement the various bonus/surplus distribution mechanisms for each contract and simulate future benefits at the maturity of each contract. Based on 1 million simulations we plot the simulated densities of the future benefits. These densities of the future benefits are compared with the market index.

Our results indicate that for both (i) low levels of volatility, and for (ii) high levels of volatility and high levels of the financial risk premium the German, Danish and the UK contracts behave just like the market index, i.e., the guaranteed rate and the investment surplus distribution have virtually no impact on the cashflow profile of the benefit. The Norwegian benefit has a a lower standard deviation than the mentioned contracts, whereas the benefit of the universal life contract offers the lowest standard deviation. Our numerical analysis therefore indicates that the relative simple US contract outperforms the more complex European counterparts if the objective is to provide the insurance customer a future benefit with low risk. By plotting the final

Insurer's balance sheet

PORTFOLIO OF ASSETS X	PREMIUM RESERVE -the customer's accounts A^1 with interest guarantee g_1 A^2 with interest guarantee g_2
	UNDISTRIBUTED RESERVE -bonus account B
	EQUITY -the insurer's account C

FIGURE 1. The balance sheet of the insurer.

balance of the equity of the insurance company, we observe (consistent with the observation above) that the insurance company is exposed to more financial risk by issuing the universal life or Norwegian contract than the others.

This paper is organized as follows: Section 2 describes the set-up including descriptions of the various contracts we would like to analyze. Section 3 explains the valuation principle used, section 4 presents the numerical results, and section 5 contains conclusions and suggestions for further research.

2. The model

Our model is based on an idealized picture of a life insurer's balance sheet depicted in Figure 1.

A fixed time horizon of T years is given and the initial point in time is denoted 0.

2.1. **The asset side.** We assume that all benefits are determined from an exogeneous and verifiable financial asset. Thereby we avoid moral hazard and adverse selection problems, in particular this assumption eliminates the insurance company's possibility to manipulate the amount of benefit to the customers. For simplicity we assume that this exogeneous and verifiable financial asset is identical to the market index, and we denote its market value by X_t. Furthermore, to keep things simple we assume that the investment portfolio of the insurance company is identical to the market index, even though in real life the investment portfolio may consist of various kinds of assets such as stocks, bonds, mortgages, or real estate.

We denote the logarithmic return in year t by δ_t. That is,

$$\delta_t = \ln\left(\frac{X_t}{X_{t-1}}\right),$$

where $\ln(a)$ represents the natural logarithm of a. Observe that $X_t = X_{t-1}e^{\delta_t}$.

2.2. **The liability side.** The premium reserve at each time t represents the insurer's liability to the customer(s) at time t. We split the premium reserve into two components. Let A_t^1 denote the balance of the first component of the premium reserve at time t. At this account interest accrues according to the guaranteed rate g_1, a constant. Let A_t^2 denote the balance of the second component of the premium reserve at time t. Two premium reserve accounts are included to facilitate a different guarantee on investment returns distributed to the customer throughout the contract period. The guaranteed rate on the second component is denoted by the constant g_2. Typically $g_2 < g_1$. In the case where there is no guarantee on the second component, this may be formally included in our model by letting g_2 approach $-\infty$. We sometimes refer to the two components just as A^1 and A^2, respectively.

We denote by G_t the *sum guaranteed* at time t, i.e.,

$$G_t = A_{t-1}^1(e^{g_1} - 1) + A_{t-1}^2(e^{g_2} - 1)$$

Let I_t denote the time t investment return after guarantees,i.e.,

$$I_t = X_{t-1}(e^{\delta_t} - 1) - G_t.$$

Observe that I_t can be both positive or negative, depending on whether the investment return $X_{t-1}(e^{\delta_t} - 1)$ is greater or less than, respectively, the sum guaranteed G_t. Therefore, we define

$$I_t^+ = (I_t)^+ = \max\{I_t, 0\}$$

as the *investment surplus* and

$$I_t^- = (I_t)^- = -\min\{I_t, 0\}$$

as the *investment deficit*.

We denote by B_t the balance of the bonus side at time t, and sometimes we refer to this account as account B. In years with a strictly positive investment surplus ($I_t^+ > 0$), a part of the investment surplus is set aside to the bonus account for potential future use in years with a strictly positive investment deficit. In principle, the balance of the bonus account belongs to the customer, and a positive terminal balance of this account is credited the customer.

Similarly, we denote by C_t the equity of the company at time t, and refer to this account as account C. In years with a strictly positive investment surplus ($I_t^+ > 0$), a part of the return is credit the insurer. These cashflows can be interpreted as a compensation for providing the annual guarantee and other embedded options of the contract. In our model these cashflows will be determined in a 'fair' way, i.e., the parameters of the model are calibrated so that the initial market value of the final balance of account C equals the initial market value of all future benefits of the contract. This is a mild requirement, if this was not fulfilled, the customer pay either more or less for the contract than it is worth, a rather unnatural situation in a otherwise stylized and frictionless model as ours.

We assume that initial distribution (A_0^1, A_0^2, B_0, C_0) between the four accounts on the liability side is given.

The distributions mechanisms between the different account at the liability side of the balance vary from country to country and they will therefore be explained for each case in the following subsections.

2.2.1. *The case of Norway.* The following description is meant to represent a stylized picture of practice in Norway for tradional life insurance products. This practice seems to be governed both by legislation, competition in the market, and established practice.

In 'good' years, when the investment surplus is positive ($I_t^+ > 0$), fractions of the investment surplus I_t^+ are distributed to the accounts A^2, B, and C, determined by the parameters α and β. For all contracts α can be interpreted as the share of the excess surplus which is credited directly to the customer, whereas β represents a 'cost' parameter.

In 'bad' years, when the investment deficit is positive ($I_t^- > 0$), it is subtracted from B_t. However, the maximum deduction from account B at time t is limited to the amount guaranteed at time t G_t. Any remaining deficit, i.e., $\max\{I_t^- - G_t, 0\}$, is subtracted from the equity C_t. The balance of the two premium reserve accounts are given by

$$A_t^1 = A_{t-1}^1 e^{g_1}, \tag{1}$$

and

$$A_t^2 = A_{t-1}^2 e^{g_2} + \alpha I_t^+. \tag{2}$$

The guaranteed rate g_1 determines the change of the balance of account A^1 every year. If g_1 is positive, the balance increases every year. Similarly, a positive g_2 contributes to an increase in the balance of A^2. In the case of a positive investment surplus, the balance of A^2 is further increased by a fraction α of the investment surplus. No deductions can be made from these accounts throughout the contract period.

The balance of the bonus account is given by

$$B_t = B_{t-1} + (1 - \alpha - \beta) I_t^+ - \min\{I_t^-, G_t\} \tag{3}$$

Also for account B, in the case of an investment surplus the balance is increased with a fraction $(1 - \alpha - \beta)$ (> 0 for realistic parameter values) of the investment surplus. In the case of a investment deficit, the balance of B is reduced by the investment deficit limited to the sum guaranteed.

The balance of the equity at time t is

$$C_t = C_{t-1} + \beta I_t^+ - \max\{I_t^- - G_t, 0\}. \tag{4}$$

Also for account C, in the case of an investment surplus the balance is increased with a fraction β of the investment surplus. In 'really bad' years, when the investment deficit is larger than the sum guaranteed ($I_t^- > G_t$), the insurer must cover the remaining investment deficit.

Currently, $g_1 = 3\%$ in Norway. There is no guarantee for A_2, so $g_2 = -\infty$. By legislation $1 - \beta > 65\%$. Practice suggests that α is around 50% and β is around 25%.

2.2.2. *Universal life.* Here we consider a popular product in the US market called *universal life*. Universal life is more flexible than traditional life insurance in that the buyer is permitted to skip premium payments and vary the amount of premium payments. This property is however not incorporated in our model. The product has currently a market share of 22% in the US.

Apparantly, the practice of reserving a part of the surplus in good years to prepare for future bad years is not common in the US industry. For an accurate description of this product we do not need to include the bonus account. Apart from this, the product is very similar to the Norwegian contract described in the previous section.

The balance of A^1 is for this product given by equation (1). Since there is no bonus account we rewrite expression (2) as

$$A_t^2 = A_{t-1}^2 e^{g_2} + (1-\beta) I_t^+,$$

eliminating the parameter α from this contract.

There is no equation similar to equation (3) above because of the missing bonus account. This fact implies that the insurer must cover the complete investment deficit in 'bad' years from the company's equity. The balance of account C is then

$$C_t = C_{t-1} + \beta I_t^+ - I_t^- \tag{5}$$

A more recent product in the US market is called *variable life* and includes the option to let the customer determine the asset allocation between bonds and typically, different mutual funds with different risk profiles.

2.2.3. *The case of Denmark.* The following description is meant to capture practice for tradional life insurance products in Denmark. The Danish system is different from the two previous systems and is characterized by two special properties. See Grosen and Jørgensen (2000); Hansen and Miltersen (2002). First, insurance companies try to maintain the balance of the bonus account at a fixed predetermined ratio of the sum of the balances of the premium reserve and the equity. We denote this ratio by γ^D. The fraction $\frac{B_t}{A_t+C_t}$ should therefore, at least some time after inception, be close to the ratio γ^D. Second, the return on the insurance policy in year $t+1$, is determined at time t, i.e., independent of investment performance in year $t+1$. Only one A account is used to describe Danish practice (so superscripts are dropped to simplify notation).

These features are incorporated as follows in our general model. First, the dynamics of the sum of the A and the C account is modeled as

$$(A+C)_t = (A+C)_{t-1} e^{\max\{g, \ln(1+\alpha(\frac{B_{t-1}}{(A+C)_{t-1}} - \gamma^D))\}}, \tag{6}$$

where g denotes the annual guaranteed rate. Here α can be interpreted as the fraction of the bonus account in excess of the desired level γ^D which is credited the accounts A and C. Second, the dynamics of only the A account is given by

$$A_t = A_{t-1} e^{\max\{g, \ln(1+\alpha(\frac{B_{t-1}}{(A+C)_{t-1}} - \gamma^D))\} - \beta}. \tag{7}$$

The parameter β determines the deduction of the return credited account A and can, as in the previous cases, be interpreted as a *cost* parameter.

Third, the balance of the C account only is calculated as the difference between the balance of the sum of the A and the C account and the balance of the A account as

$$C_t = (A+C)_t - A_t. \tag{8}$$

Finally, the balance of the bonus account is determined residually as

$$B_t = B_{t-1} + [X_t - X_{t-1}] + [(A+C)_t - (A+C)_{t-1}]. \tag{9}$$

2.2.4. *The case of Germany.* The following description is based on Mertens (2000). For the German contract the account A_2 has some special properties. We therefore redefine the investment surplus as

$$I_t^+ = (X_{t-1}(e^{\delta_t} - 1) - A_{t-1}^1(e^{g_1} - 1))^+.$$

Observe that this expression is independent of A_t^2. The development of A_t^1 is as given in expression (1). The development of A_t^2 is

$$A_t^2 = A_{t-1}^2 + A_{t-1}^1 \min\{\frac{I_t^+}{A_{t-1}^1}, \gamma^G\} + H_t,$$

where

$$H_t = \max\{\alpha \max\{B_{t-1} - \frac{\Gamma_{t-1}^3}{3}, 0\}, \max\{B_{t-1} - \Gamma_{t-1}^3, 0\}\} 1\{t \geq 3\}$$

where Γ_t^3 represents the sum of the last three *positive* contributions to the bonus account at time t and $1\{t \geq 3\}$ is the usual indicator function, returning the value 1 of $t \geq 3$, and 0 otherwise. The indicator function implies that the final term H_t always equals zero the first years of the contract, i.e., at time 0, 1, and 2.

The second term of A_t^2 reflects the fact that the customer receives the complete investment surplus only if this represents less than γ^G percent return of A^1, otherwise γ^G percent of A^1 is credited A^2. The last term H_t of A^2 compares the previous years balance of the bonus account with, respectively, the average of the last three positive contributions to the bonus account and the sum of the last three positive contributions to the bonus account. The maximum of a fraction α of the difference between the bonus account and the average of the three last contributions and the difference between the bonus account and the sum of the three last contributions is credited the costumer.

For this contract

$$C_t = C_{t-1}e^{\delta} + I_t^+\beta,$$

the insurance company obtains the market return δ and a fraction β of the investment surplus.

Finally, the bonus account is determined residually as

$$B_t = B_{t-1} + (X_t - X_{t-1}) + (A_t^1 - A_{t-1}^1) + (A_t^2 - A_{t-1}^2) + (C_t - C_{t-1}).$$

2.2.5. *The case of UK.* We include three contracts from United Kingdom, as analyzed by Haberman Steven and Wang (2003). These contracts apply a different bonus mechanism. Bonus is not distributed annually, only at maturity of the contracts. The contracts only include one A account (so superscripts are dropped).

For the first contract the development of the premium reserve is given by

$$A_t = A_{t-1}e^{\max\{g_1, \ln(1+\frac{\alpha}{n}(\frac{X_t}{X_{t-1}}+\cdot\frac{X_{t-n+1}}{X_{t-n}}-n))\}}, \tag{10}$$

where n is given. The bonus is incorporated at the maturity by the following condition.

$$B_T = (1-\beta)\max\{X_T - A_T, 0\}. \tag{11}$$

For the second contract the development of the premium reserve is given by

$$A_t = A_{t-1}e^{\max\{g_1, \ln(1+\alpha(\sqrt[n]{\frac{X_t}{X_{t-n}}}-1))\}}.$$

The bonus is incorporated by an equation identical to equation (11).

The third UK contract is based on the concept of a smoothed asset share. Let $\hat{A}_0 = A_0$. Define the development of $\hat{A}$ by

$$\hat{A}_t = \hat{A}_{t-1}e^{\max\{g_1, \ln(1+\alpha(\frac{X_t}{X_{t-1}}-1))\}}.$$

The premium reserve is defined by

$$A_t = \gamma^U\hat{A}_t + (1-\gamma^U)A_{t-1}.$$

The bonus element is also for this contract incorporated by an equation identical to equation (11).

For the UK contracts account C is residually determined as

$$C_t = X_t - A_t, x = 0, \ldots, T-1$$

and $C_T = X_T - B_T - A_T$.

3. Valuation

Let Z_s be a general payoff payable at time s, dependent of the value of some underlying asset. Denote the market value at time $t \leq s$ of Z_s by $V_t(Z_s)$.

At maturity of the contract the customer receives the final balances of the two premium reserve accounts A_T^1 and A_T^2 and a potential positive balance of the bonus account denoted by B_T^+. From the customer's point of view a fair valuation is expressed by

$$V_0(A_T^1) + V_0(A_T^2) + V_0(B_T^+) = V_0(X_0) = X_0,$$

i.e., the initial market value of the future benefits equals the (market value of) the initial premium, to be thought of as a single premium or initial deposit.

From the insurance company's point of view a fair valuation is expressed by

$$V_0(C_T) = V_0(B_T^-),$$

the initial market value of the company's future cashflow equals the initial market value of the potential negative balance of the bonus account, a situation which may occur if the annual guarantee is consistently higher than the investment returns.

4. Numerical results

4.1. **The model.** For the numerical analysis we use the original Black and Scholes (1973) set-up. We assume that the market value of the market index at time t X_t is given by the following stochastic differential equation

$$dX_t = (r + \pi)X_t dt + \sigma X_t dW_t, \tag{12}$$

where the initial value of the process X_0 is given and W_t is a Brownian motion. The parameter σ is referred to as the volatility of the market index.

We assume that the continuously compounded riskfree rate of return r is constant. According to standard financial terminology π is interpreted as the (instantaneous) financial *risk premium*.

Equivalently, the logarithmic return in year t is

$$\delta_t = r + \pi - \frac{1}{2}\sigma^2 + \sigma(W_t - W_{t-1}).$$

For this specification of the model the market value operator takes the form

$$V_t(Z_s) = e^{-r(s-t)} E^Q[Z_s],$$

σ	Nor	Den	UL	Ger	UK1	UK2	UK3
5%	.1192	5.16e-05	.3658	.0062	.0020	.0031	.0020
15 %	.5925	.0048	.7166	.0753	.1849	.1861	.2762

TABLE 1. Numerically calibrated values of β to make the contracts fair.

where $E^Q[\cdot]$ denotes the expectation under the equivalent martingale measure and Z_s a future benefit payable at time s.

4.2. **Choice of parameters.** We assume that the time horizon of the contract is $T = 30$ and that the riskfree rate of return is constant and equal to $r = 0.05$. The initial deposit of the contract is normalized to $X_0 = 1$. The initial distribution between the different accounts at the liability side of the balance sheet is assumed to be $A_0^2 = B_0 = C_0 = 0$ and $A_0^1 = 1$. We assume that the parameter $\alpha = 0.25$ for all contracts (except for universal life which does not include this parameter). The parameter β is determined so that each contract is fair (determined numerically from 30 000 simulations). In Table 1 we present the estimated values of β for our assumed set of parameters and volatilities of 5% and 15%, respectively.

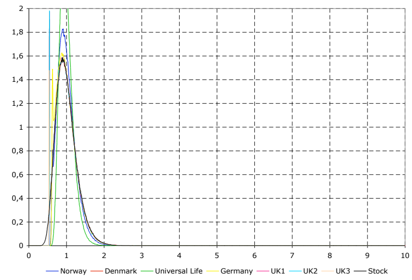
We assume that the guarantees are $g_1 = g_2 = 3\%$ (so there is no distinction between the two A accounts).

The additional Danish parameter $\gamma^D = 0.15$, which is supposed to be a realistic value of this parameter. The additional German parameter $\gamma^G = 0.015$. The additional UK parameter $\gamma^U = \frac{1}{2}$. We assume that $n = 3$ for the first two UK contracts.

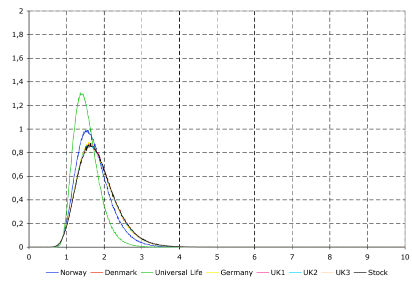
The following graphs show the probability distribution of the benefit. These graphs are based on 1 000 000 simulations.

4.3. **Preliminary numerical results.** In the absence of mortality the different plots represent the present value of the benefit payable in 30 years from investing one unit in the different contracts. The plots are probability distributions implying that the total area under the each plot is one. All plots are single peaked, the higher this peak is, the more the probability mass is concentrated, and implies less uncertainty about the future payoff. Some of the plots contain spikes. These spikes are due to the embedded guarantees. The spikes are pronounced for high levels of volatility and risk premium, whereas not visible for low levels of volatility and risk premiums.

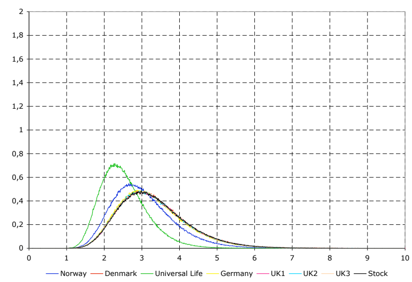
We have plotted risk premiums of 0%, 2%, and 4% for 2 levels of volatility (5% and 15%). For all plots universal life has the highest peak, the Norwegian contract has the second highest peak, whereas the Danish, German and UK contracts are similar to the market index and they have all lower peaks than universal life and the Norwegian



(a) $\pi = 0$.

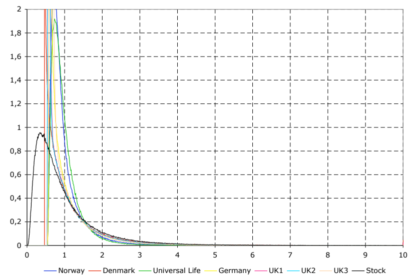


(b) $\pi = 0.02$.

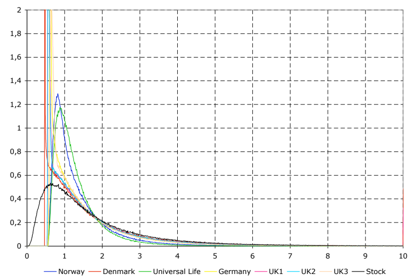


(c) $\pi = 0.04$.

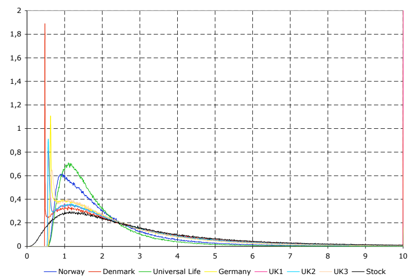
FIGURE 2. Simulated probability distributions of the amount of benefit for the Norwegian, Danish, universal life, German contracts, UK, and the market index for levels of the risk premium of 0 to 5% and low volatility, $\sigma = 5\%$.



(a) $\pi = 0$.



(b) $\pi = 0.02$.



(c) $\pi = 0.04$.

FIGURE 3. Simulated probability distributions of the amount of benefit for the Norwegian, Danish, universal life, German, UK contracts, and the market index for levels of the risk premium of 0 to 5% and high volatility, $\sigma = 15\%$.

contract. In terms of variability or uncertainty this means that universal life gives the customer the least uncertain benefit. If we order the rest of the contracts according to this principle, the Norwegian contract comes second, before the remaining contracts, which all are hardly indistinguishable. The Danish, German, and UK contracts are also surprisingly similar to the market index, but for low level of risk premiums and volatility, spikes, representing the effect of the guarantees, are visible. The spikes illustrate the downside protection of the benefit due to the embedded guarantees.

The spikes are visible for a risk premiums of 0% for 5% volatility, and for all presented risk premiums for 15 % volatility.

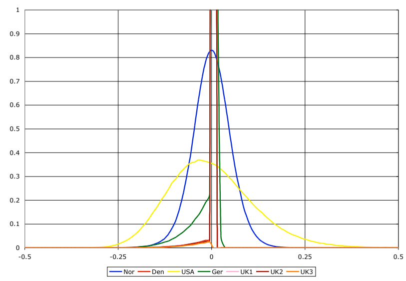
By increasing the risk premium the probability distributions are stretched to the right, reflecting increased probabilities for higher benefits. This effect naturally reduces the peaks because the area under the curves must equal one for all plots.

Increasing volatility leads to more pronounced spikes, and lower peaks. The first effect is due to the embedded guarantees as mentioned, the second effect is due to increased uncertainty.

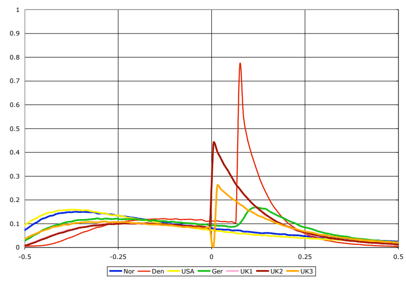
Finally, in figure 4 we plotted the final balance of the C account, i.e., the final value of the insurance company's equity from issuing these products. The low volatility graphs for the Danish, German, and UK contracts all have spikes around zero and low mass elsewhere. This reflects the low risk, from the insurance company's point of view, from issuing these products. This observation is consistent with the previous figures where these contracts are shown to be similar to the market index, i.e., the insurance company essentially passes all financial risk to the customer. The mass of the Norwegian and the universal life contracts are more spread out which indicates that the insurance companies take more of the financial risk by issuing these products. All contracts are fair so the insurance company may in some realizations experience a positive profit from thee contract, and a deficit in other realizations. The situation is not that clear cut in the case of high volatility, but the Norwegian and the universal life contracts do still have the most dispersed mass, indicating more financial risk taking by the company.

5. Conclusions and further research

Our conclusions are drawn from visual inspections of the graphs. We checked for any stochastic dominance between the various contracts, but found no stochastic dominance of first or second order by pairwise comparisons of the contracts. Based on our numerical results we can conclude that the universal life contract has the least uncertain benefit. The Norwegian contract provides a less uncertain benefit than the remaining contracts. This ranking seems to be independent of the



(a) $\sigma = 0.05$.



(b) $\sigma = 0.15$.

FIGURE 4. Simulated probability distributions of the final balance of the C account (the insurance company's equity) for the Norwegian, Danish, universal life, German contracts and UK contracts for a risk premium of zero $\pi = 0\%$.

levels of the financial risk premiums and volatility. We find it surprising that the simple universal life contract, which does not include any bonus mechanism, so clearly outperforms the more complex European counterparts, in terms of providing a future low risk benefit.

It is also surprising that the Danish, German and the UK contracts, which include sophisticated smoothing mechanisms, do not perform different from the market index. Furthermore, there does not seem to be any difference between annual bonuses (Denmark, Germany) and maturity bonuses (UK contracts). The effect of the guarantees are pronounced for high volatilities and high risk premiums, indicating that these contracts provide a downside protection compared to just investing in the market index.

One question which one may raise (at least for financial markets with low volatility and/or high risk premium) is whether a rational investor should invest in an insurance contract like the Danish, German or the UK contracts, or just the market index. The current analysis can not be used to justify the choice of any of these insurance contracts instead of just buying the market index. However, we feel that to more realistically address this question, we somehow have to include intertemporal issues such as the possibility to withdraw or rebalance throughout the investment period. In this analysis we have implicitly assumed that the contracts are fixed for 30 years. We leave this question for future research.

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