On the Micro-foundations of Money: The Capitol Hill Baby-Sitting Co-op*

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October 4, 2003

Abstract

This paper contributes to the micro-foundation of money in centralized markets with idiosyncratic uncertainty. It shows existence of stationary monetary equilibria and ensures that there is an optimum quantity of money. The rational solution of our model is compared with actual behavior in a laboratory experiment. The experiment gives support to the theoretical approach.

JEL classification: C73, C91, C92, E40, E41, E42.

Keywords: micro foundations of money, optimal. quantity of money

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*KRSH thanks Neil Wallace, B. Ravikumar, and seminar participants at Pennsylvania State University and University of Vienna. TH thanks seminar participants at the Universities of Bergen, Bern, Maastricht and Strasbourg. Financial support by the national center of competence in research "Financial Valuation and Risk Management" is gratefully acknowledged. The national centers in research are managed by the Swiss National Science Foundation on behalf of the federal authorities.

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1 Introduction

This paper studies a monetary economy with a perishable good that can be traded in a centralized market. There is a continuum $[0, 1]$ of agents, each has stochastic preferences. Agents cannot consume their own good but need to accept money to finance future consumption. The price is fixed, and trade is one-to-one exchange of money against one unit of the good. Market clearing is ensured by rationing.

The model is inspired by trade circles. It can be seen in particular as a formalization of the Great Capitol Hill Baby-Sitting Co-op, a trade circle whose fate is reported by Sweeney and Sweeney (1977).[1] Trade circles, which are increasingly common, provide good evidence for clinical studies on the role of money.[2] In a trade circle a set of participants can receive or deliver services at fixed prices. In return the supplier of a service receives some artificial money or coupon, which she can then spend to demand a service. Prices are fixed by fairness considerations and credit is limited.

Among economic theorists the Capitol Hill Baby-Sitting Co-op is perhaps the most well-known trade circle. This is because Krugman (1999)[3] illustrated the relevance of money to the everyday man/woman by means of this instructive example. In the Great Capitol Hill Baby-Sitting Co-op on average 150 couples tried to share baby-sitting fairly by introducing a coupon system. A coupon was the entitlement to receive one night of baby-sitting. Initially one coupon of baby-sitting was issued per couple. Supposing that coupons circulated, over time every couple would thus receive as many units of baby-sitting as it delivered. However, after a short while the system collapsed because there was insufficient demand of baby-sitting. Krugman (1999) attributes this breakdown to precautionary savings. The Co-op solved this problem simply by issuing more coupons. And having experienced that every couple was better off due to the increase in the number of coupons the Co-op issued more and more coupons so that the system broke down again. In the Great Capitol Hill Baby-Sitting Co-op fiat money has a positive value in exchange for services, it can lead to a Pareto-improvement over the situation without fiat money and there is an optimum quantity of money.

The model bears some similarities with Levine (1991) and Kehoe, Levine, and Woodford (1992). In our model, however, there is no possibility of lump sum subsidies, no money creation or destruction (i.e. no government and no interest), and prices are fixed.

In Wallace (2002)'s view, the model takes the extreme position of idiosyncratic uncertainty across agents and types, absence of any monitoring, and the largest possible market.

As a consequence, the first-best cannot be achieved in our model. Still using

[1]This model has become well-known ever since it was employed by Krugman (1999) to illustrate the relevance of money to the everyday man/woman.

[2]See, for example, www.tauschring-archiv.de and http://www.talent.ch.

[3]See also Sweeney and Sweeney (1977) for a more detailed report on the Great Capitol Hill Baby-Sitting Co-op.

the total utility of the economy as a welfare measure, we can show existence of an optimum amount of money.

The model also shares some features with the neo-Keynesian model as it was developed by Clower, Barro and Grossman, Benassy, Malinvaud and Dréze.[4] In both models prices are fixed and demand and supply is coordinated by quantity rationing. However, while the use of money in the standard neo-Keynesian model is justified by a temporary equilibrium approach, the microeconomic foundation of money that our model provides is based on complete rationality under rational expectations.

Last but not least, our model has some features in common with the famous Kiyotaki and Wright model. Starting from the seminal papers of Kiyotaki and Wright (1989, 1991, 1993), an impressive literature developed in which markets are no longer considered to be well organized but traders meet randomly in pairs, see also Boldrin, Kiyotaki, and Wright (1993), Trejos and Wright (1995). Our paper suggests a micro-foundation of money that is somewhat orthogonal to the standard search model that aroused out of the papers by Kiyotaki and Wright. On the one hand, we assume that markets are well-organized in the sense that every potential supplier and every potential demander of a service can meet traders of the other side of the market in every period. On the other hand, we assume that the market participants have stochastic preferences. Hence some day a trader would prefer to be a supplier of a service and some other day she prefers to demand the service. The role of money is to allow the traders to transfer income between supply and demand days. From a more general perspective our model is similar to the Kiyotaki and Wright model because in both approaches the need to hold money arises from the uncertainty of whether in the future one will be a demander or a supplier of a service. In the Kiyotaki and Wright model this uncertainty arises because one is possibly matched with a trading partner so that there is no double coincidence of wants. In our model this uncertainty arises from the stochastic preferences. Indeed, in the formal analysis of our model (the Bellman equations) we can reason analogously to Berentsen (2000, Lemma 1) – a result derived for the Kiyotaki and Wright model.

Since our model is based on a rigorous idealization—the notion of stationary equilibria with rational expectations in a stochastic game—it is important to contrast the predictions of our model with actual play of possibly bounded rational participants in a laboratory experiment. To this end we set up a market in which six participants per market could repeatedly buy or sell an abstract commodity. Buying was forbidden if the participant had run out of artificial money, called coupons. And a payoff could only be attained if the participant was able to buy. In this case the per period payoff was the time-value (drawn randomly for each participant in every period). While the probability distribution of the random process was common knowledge, the time values remained private information to the participants. By and large the experiment gives evidence that the completely rational solution concept we have analyzed also

[4]For a comprehensive account of this theory see e.g. Malinvaud (1977) or Benassy (1982).

has descriptive merits. In all six markets, the behavior of the participants was exactly as our rational solution concept suggested. Even though each group forming a market consisted only of six participants, their behavior was conform with the best response to market averages which is well in line with our assumption that agents play the field. Moreover, the median maximal demand of money was exactly four units as the model predicted for those parameter values chosen in the experiment. We never observed the non-monetary equilibrium and money circulated fine in the experiment. It turned out that there is an optimum quantity of money, which was slightly higher in a setting with smaller expected time values, in which case there is a higher potential for a demand shortage.

2 The Model

In setting up the model, we will often refer to the Capitol Hill Baby-Sitting Co-op mentioned above which facilitates the intuition. This formal representation will enable us to derive falsifiable conclusions from explicit assumptions on rationality and characteristics of consumers. The abstraction will also help to set up a laboratory experiment to test the implications of the model.

Our formal model of a monetary economy embraces three essential properties. First, the good cannot be stored, is not divisible and can only be supplied in one unit. Each agent can either consume or produce the good but cannot consume the good he produces. We assume that production of the good incurs no costs. Second, the agent's decision problem is of intertemporal nature. She needs to transfer income across time, if she wants to consume in some period. The only possibility to transfer income is via the acceptance of money in exchange for his good because any form of private credit is ruled out by definition.[5] Third, we have to formalize the market structure in which all potential demanders meet all potential suppliers of the good. We assume that there is one market in every period. At the beginning of the period all agents announce their demand or supply position. Then the market coordinates demand and supply by some non-pricing mechanism because the price of the good is fixed to one unit of money. Since the good comes in a non-divisible unit, we model rationing in the way that the longer side is rationed by a mechanism that randomly selects as many agents from the longer side as there are on the shorter side. Any agent on the longer side is thus subject to a probability that she cannot carry out her planned transaction. The non-rationed agents are assigned randomly and the delivery of the good and the payments are made at the end of the period. An agent can only demand the good if she can show at least one unit of money.

The game consists of identical periods t ($t = 1, 2, 3, ...$). The agent's initial endowment is $m \geq 0$ units of money. In every period first a chance move determines the time value w: the probability of the high time value h is p_h and the probability of the low time value l is $p_l = 1 - p_h$. Then the agent

[5] In a non-monetary barter economy agents would need access to a (complete) market of contingent commodities because the future states of the individual preferences are unknown.

selects one of the three alternatives S, B or I. Alternative S corresponds to "sell baby-sitting," alternative B to "buy baby-sitting" and alternative I to "stay idle." Agents can only select alternative B if they have money (i.e. if the money holdings m in period t are greater than 0), which is due to the absence of credit. The choice of an agent is executed with probability p_S if she chooses alternative S and with probability p_B if she chooses alternative B.

The success probabilities are determined by rationing. Since at most one market side is rationed, one probability is always equal to one. These probabilities depend on the choices of the other agents. For the first part of our analysis (theoretical and experimental) we take these probabilities as exogenously given. The quantity of money of the agent and her utility changes depend on her choice. The changes are given as the vector of the changes (money, utility) in the figure. Money holdings decrease (increase) if the agent is successful in buying (selling), and utility equal to the current time-value is realized only if she is able to buy.

Agents discount future utility with an individual discounting rate $0 < \beta < 1$. For notational ease we do not use an index for the agent in this section.

A stationary policy $\psi(m, w, t) \in \{B, S, I\}$ of an agent determines the choice of an agent depending on her actual money holdings m and her actual time value w (high h or low l, where $0 < l < h$). The Bellman equation describes the continuation value $V(m)$ of the game if the money holdings are m, provided the agent is risk neutral and the parameters $0 < p_h < 1$, $0 < p_S \leq 1$ and $0 < p_B \leq 1$. One has

$$\begin{aligned} V(0) &= \beta \max\Big\{p_S\, V(1) + (1 - p_S)\, V(0), V(0)\Big\} \\ V(m) &= \beta \sum_{w=h,l} p_w \max\Big\{p_B\, [w/\beta + V(m-1)] + (1 - p_B)\, V(m), \\ &\qquad\qquad p_S\, V(m+1) + (1 - p_S)\, V(m), V(m)\Big\} \end{aligned} \tag{1}$$

Existence and uniqueness of a solution to the Bellman equation (1) is straightforward using Blackwell's sufficient condition for a contraction on the space of positive functions V with range $\mathbb{N}$. Denote this solution by $V^\star$.

Analogously to Berentsen (2000, Lemma 1) one can show that $V^\star$ has the following properties.

Lemma 2.1 *$V^\star$ is strictly positive, strictly increasing, strictly concave, and bounded by $h/(1-\beta)$.*

There also is a maximal amount of money, $\overline{m}$, an agent wants to accumulate. It is given by

$$\overline{m} := \min\left\{m \geq 1 \mid \frac{p_B\, l}{\beta} > p_S\, [V^\star(m+1) - V^\star(m)] + p_B\, [V^\star(m) - V^\star(m-1)]\right\}$$

This result is straightforward from the Bellman equation (1). Obviously, if $w = h$ then buy is optimal when the agent holds money. It thus remains to consider the case $w = l$. An agent (weakly) prefers to sell if and only if $p_B\,(l/\beta + V^\star(m-1)) + (1-p_B)\,V^\star(m) \leq p_S\,V^\star(m+1) + (1-p_S)\,V^\star(m)$. Buy is preferred when the inequality is reversed. This condition is equivalent to $p_B\,l/\beta \leq p_B(V^\star(m) - V^\star(m-1)) + p_S(V^\star(m+1) - V^\star(m))$. Concavity and boundedness of $V^\star$ immediately imply that $\overline{m}$ is well-defined and that $\psi^\star(m,l) = S$ for all $0 < m < \overline{m}$ and $\psi^\star(m,l) = B$ for all $m \geq \overline{m}$.

Summarizing these findings, one can state:

Theorem 2.1 *Fix any values $0 < l < h$, $0 < p_h < 1$, $0 < p_S \geq 1$, $0 < p_B \geq 1$, and $0 < \beta < 1$. The optimal stationary policy $\psi^\star$ is given by*

- *Rule 1: $\psi^\star(0,w) = S$ for $w = h, l$;*
- *Rule 2: $\psi^\star(m,h) = B$ for all $m > 0$;*
- *Rule 3: $\psi^\star(m,l) = S$ for all $0 < m < \overline{m}$; and $\psi^\star(m,l) = B$ for all $m \geq \overline{m}$.*

The maximum money holdings $\overline{m}$ is defined above.

The intuition behind the optimal policy $\psi^\star$ is as follows. Alternative I, staying idle, is weakly dominated because the other alternatives may increase utility or money holdings.

An agent with no money is certainly better off choosing alternative S (sell), aiming to obtain money for future consumption. Alternative B (buy) is not available to her, thus rule 1 gives the optimal choice.

Rule 2 uses that the optimal choice of an agent with high time value h (and money) is alternative B (buy) because this yields maximal current utility she can obtain for one unit of money. Keeping the money would reduce her utility by at least the discounting rate.

Rule 3 states that an agent has a endogenous maximum quantity of money holdings. If her money holdings exceed or is equal to $\overline{m}$, she chooses alternative B even if her time value is low. The intuition is as follows. The current decision of an agent with money holdings $\overline{m}$ is irrelevant for the optimal continuation in the next $\overline{m}-1$ periods. An agent can use an additional unit of money she might obtain in the current period only after at least $\overline{m}-1$ periods. Rule 3 results from the observation that an agent should go for the lower time value now if discounted expected utility in the $\overline{m}-1$st period leads to a lower expected utility than the lower time value l (times the success probability). If the inequality holds in the other direction an agent should choose the additional unit of money.

A numerical solution of the Bellman equation gives $\overline{m} = 4$ for both success probabilities being 1, equal chances of high and low time value, i.e. $p_h = p_l = 0.5$, and a continuation probability $\beta = 95\%$.

The model requires agents to commit to an action without the opportunity to revise their decision if they are actually rationed. It would certainly increase the agent's utility being able to do so. However, this behavior is not rational in a stationary market equilibrium. The intuition is as follows. Suppose half of the rationed agents revise their decision and decide to offer their service (while they were on the demand side before). Then each single agent on the supply side has an incentive to switch back to the demand side, since her weight is negligible. Neither can randomization between revising or upholding the current decision circumvent this rationale. This free-riding on others' decision revision leads to a break down of any incentive-compatible switching behavior (as long as agents play the field).

3 The Market Equilibrium

We consider a market with a continuum of agents. Agents can be heterogenous with respect to their discounting rate. We allow for finitely many different types $i \in I$, each type with an individual discounting rate $0 < \beta^i < 1$. The relative number of agents of type i is denoted by $\lambda^i > 0$, the Lebesgue measure of the respective type.

In a stationary market equilibrium all factors that influence the market have to be "consistent." That is, the policy of each agent has to result from his optimization given the other factors, the distribution of money holdings has to be stationary (across types and quantities), the success probabilities given by the stationary distribution have to be the same as the ones taken for the determination of the policy, and the quantity of money has to be equal to average money holdings of agents. The formal definition is as follows.

Definition 3.1 *A* stationary equilibrium *is a tuple* $((\psi_i^\star)_{i\in I}, \mu^\star, p_S^\star, p_B^\star, M^\star)$, *consisting of a stationary policy, a distribution of types over money holdings, success probabilities, and a quantity of money, such that*

(i) given $p_S^\star$ *and* $p_B^\star$, $\psi_i^\star$ *is an optimal stationary policy for type* i *agents;*

(ii) given $(\psi_i^\star)_{i\in I}$, $p_S^\star$ *and* $p_B^\star$, $\mu^\star$ *is a stationary probability measure for the corresponding Markov chain on* $I \times \mathbb{N}$ *with* $\sum_m \mu_i^\star(m) = \lambda^i$ *for all* $i \in I$*;*

(iii) given $(\psi_i^\star)_{i\in I}$ *and* $\mu^\star$, *the probabilities* $p_S^\star$ *and* $p_B^\star$ *satisfy the rationing given by (2);*

(iv) the average money holdings are equal to the average money supply, i.e. $\sum_{m\in\mathbb{N}, i\in I} m\, \mu_i^\star(m) = M^\star$.

The rationing probabilities are given by[6]

$$\begin{aligned} p_S &= \min\left\{1, \frac{\sum_{m\in\mathbb{N},i\in I} \mu_i(m) \sum_{w=h,l} p_w \mathbf{1}_{\psi_i(m,w)}(B)}{\sum_{m\in\mathbb{N},i\in I} \mu_i(m) \sum_{w=h,l} p_w \mathbf{1}_{\psi_i(m,w)}(S)}\right\} \\ p_B &= \min\left\{1, \frac{\sum_{m\in\mathbb{N},i\in I} \mu_i(m) \sum_{w=h,l} p_w \mathbf{1}_{\psi_i(m,w)}(S)}{\sum_{m\in\mathbb{N},i\in I} \mu_i(m) \sum_{w=h,l} p_w \mathbf{1}_{\psi_i(m,w)}(B)}\right\} \end{aligned} \quad (2)$$

A stationary equilibrium is called monetary, *if the value function solving (1) is strictly positive on* $\mathbb{N}$ *for at least one agent type.*

[6] $\mathbf{1}_{\psi_i(m,w)}(S)$ is the indicator function which gives value 1 if $\mathbf{1}_{\psi_i(m,w)} = S$ and value zero otherwise.

3.1 Monetary equilibria and the optimal quantity of money

Given the definition of a stationary equilibrium one can prove the existence of stationary equilibria. The existence of non-monetary equilibria is easy to understand: For any characteristic of an agent and any amount of money a non-monetary equilibrium exists. In this equilibrium $p_B^\star$ is zero. In this market everybody wants to buy and nobody wants to sell. It is also obvious that no agent improves by selling because with the additional unit of money received for selling she never can buy which is the only way to improve her utility.

It is more difficult to understand the existence of monetary equilibria for any characteristic of agents and any given success probabilities $p_B > 0$ and $p_S > 0$. The idea of this existence is analogous to the way we tried to subdivide the problem in the previous parts of this paper. Given the success probabilities an optimal policy is determined by solving the Bellman equation. Given this policy a stationary distribution on the money holdings is given which is "consistent" with the success probabilities taken for the determination of the optimal policy. The initial endowment with money is determined such that it is equal to the average money holdings.

Theorem 3.1 *For any characteristics of the agents and any prescribed success probabilities $p_B > 0$ and $p_S > 0$ (one of them being equal to one), there exists some quantity of money and a corresponding monetary stationary equilibrium such that $p_B^\star = p_B$, $p_S^\star = p_S$, and in which the optimal policy $\psi^\star$ is given by Theorem 2.1.*

Proof of Theorem 3.1. The proof relies on the results on the value function $V^\star$ and the optimal policy. Let $\psi_i^\star$ denote the optimal policy for agents of type i for given success probabilities $p_B > 0$ and $p_S > 0$. Denote the corresponding maximum money holdings by $\overline{m}_i$.

Fix a type i. Given the policy $\psi_i^\star$ and success probabilities $p_B > 0$ and $p_S > 0$, a Markov chain on the set of money holdings $X^i := \{0, ..., \overline{m}_i\}$ is defined. The transition probabilities $P_i(m, m')$, $m, m' \in X^i$ are as follows. Trade takes place at price one, therefore $P_i(m, m')$ can be strictly positive if and only if $|m - m'| \leq 1$. One has $P_i(0, 1) = p_S$ (because any agent without money prefers to sell), $P_i(\overline{m}_i, \overline{m}_i - 1) = p_B$ (because $\overline{m}_i$ is the maximal money holdings), and for all $0 < m < \overline{m}_i$, $P_i(m, m-1) = p_B\, p_h$ and $P_i(m, m+1) = p_S\, p_l$. All of these transition probabilities are strictly positive because $p_B > 0$, $p_S > 0$, and $0 < p_h < 1$. These findings imply that the Markov chain on X^i is irreducible. Denote the corresponding unique stationary distribution by $\tilde{\mu}^i$. We can now define a probability measure $\mu^\star$ on $I \times \mathbb{N}$ by $\mu_i^\star(m) := \lambda^i\, \tilde{\mu}^i(m)$, $i \in I$, and define by $M^\star := \sum_{i \in I, m \in \mathbb{N}} m\, \mu_i^\star(m)$ the corresponding quantity of money.

It remains to show that $((\psi_i^\star)_{i \in I}, \mu^\star, p_B^\star, p_S^\star, M^\star)$ actually is a stationary equilibrium, where $p_B^\star := p_B$ and $p_S^\star := p_S$. We check each condition in Definition 3.1 in turn. First, the stationary policies $\psi_i^\star$ are optimal by construction. Second, $\mu^\star$ is stationary and the marginal distribution over types is $\sum_m \mu_i^\star(m)$ $= \lambda^i \sum_m \tilde{\mu}^i(m) = \lambda^i$ also by construction. Third, we have to show that the actual success probabilities, cf. (2), are equal to the prescribed ones $p_B^\star$ and $p_S^\star$.

Let us consider the case $p_S^\star = 1$ (the case $p_B^\star = 1$ is analogous). Condition (iii) is satisfied if and only if

$$p_B^\star \sum_{m\in\mathbb{N}, i\in I, w=l,h} \mu_i^\star(m)\, p_w\, \mathbf{1}_{\psi_i^\star(m,w)}(B) = \sum_{m\in\mathbb{N}, i\in I, w=l,h} \mu_i^\star(m)\, p_w\, \mathbf{1}_{\psi_i^\star(m,w)}(S) \quad (3)$$

Stationarity of $\tilde{\mu}^i$ implies

$$p_B^\star \left(\tilde{\mu}^i(\bar{m}_i) + \sum_{0<m<\overline{m}_i} \tilde{\mu}^i(m) p_h \right) = \tilde{\mu}^i(0) + \sum_{0<m<\overline{m}_i} \tilde{\mu}^i(m) p_l$$

That is, for each fixed type of agent, the amount of agents whose money holdings decrease by one unit (due to actually buying the good) is equal to the amount of agents whose money holdings increase by one (due to actually selling the good). This holds by stationarity of $\tilde{\mu}^i$ for the corresponding Markov chain on X^i. Multiplying both sides of the last equation by λ^i and summing up over $i \in I$, we obtain (3), taking into account the optimal policy.

Finally, condition (iv) holds by our definition of $M^\star$. The equilibrium is monetary. □

Another interpretation of this result is that agents assume certain success probabilities (for example: $p_S = p_B = 1$, i.e. no rationing will occur) and determine their policy by optimization. In a market the quantity of money and the policy determine a dynamic process which determine the success probabilities in the market. These success probabilities have to be identical to the ones assumed for the optimization. The quantity of money is the parameter that allows to adjust the success probabilities of the process to the success probabilities taken for the determination of the policy. The quantity of money is chosen such that the average money holdings under the assumption the policy works are equal to this quantity of money. This adjusts the success probabilities.

The general existence result for monetary equilibria when the quantity of money can be chosen by an institution outside the model of our monetary economy raises the question on the optimum quantity of money. Obviously, any non-monetary equilibrium is Pareto-dominated by every monetary equilibrium. However, this does not provide a satisfactory answer in the case of a benevolent social planner who maximizes the welfare in the economy. The most simple measure is that of the number of trades. The number of trades is maximal if no rationing occurs ($p_B^\star = p_S^\star = 1$). If there is rationing, some agents cannot trade and thus obtain lower utility than in an equilibrium without rationing. While this criterion only measures individual but not social welfare it provides a simple benchmark for the social planner's policy.

We have the following result. There exists some quantity of money $M^\star$ and a corresponding monetary stationary equilibrium such that no rationing occurs ($p_B^\star = p_S^\star = 1$). Every quantity of money which is different to $M^\star$ leads to a stationary equilibrium in which the number of trades is strictly less.

Theorem 3.1 is a general existence result for monetary equilibria when the quantity of money can be chosen by an institution outside the model of our monetary economy. This raises the question on the optimum quantity of money. Obviously, any non-monetary equilibrium is Pareto-dominated by every monetary equilibrium. However, this does not provide a satisfactory answer in the case of a benevolent social planner who maximizes the welfare in the economy. The most simple measure is that of the number of trades. If there is rationing, some agents cannot realize their plans and thus obtain lower utility than in an equilibrium without rationing. While this criterion only measures individual but not social welfare it provides a simple benchmark for the social planner's policy. We have the following result.

Corollary 3.1 *For any characteristics of the agent types there exists some quantity of money $M^\star$ and a corresponding monetary stationary equilibrium such that $p_B^\star = p_S^\star = 1$.*

Every quantity of money which is different to $M^\star$ leads to a monetary stationary equilibrium in which the number of trades is strictly less than with $M^\star$.

This Corollary can be shown as follows. The existence of an equilibrium with $p_B^\star = p_S^\star = 1$ has been shown above, Theorem 3.1. In the equilibrium without rationing and the quantity of money $M^\star$ the number of trades is maximal, because no rationing occurs. We have to compare the utility an agent obtains in this equilibrium with the utility she gets in any other equilibrium with different quantity of money M_1. In the second equilibrium rationing has to occur, because if $p_B^\star = p_S^\star = 1$ the policies of the agents would be the same, leading to the same Markov chain and the same quantity of money. If rationing occurs in an equilibrium the utility that a player obtains in this equilibrium is lower than in a non equilibrium policy combination without rationing in which the same policy is used, but the quantity of money M_2 is different. This non-equilibrium policy combination is dominated by the equilibrium policy combination with $p_B^\star = p_S^\star = 1$ and the quantity of money $M^\star$, because the equilibrium policy is the solution to the Bellman equations without rationing.

This shows that it is legitimate to call the quantity of money that is determined this way the *optimum quantity of money.*

4 The Experiment

We base our experimental design on the results of former experimental studies on fiat money concerning the transformation of theoretical models into the laboratory (Duffy and Ochs (1999)and McCabe (1989)). Duffy and Ochs test the Kiyotaki-Wright model. They conclude that transforming a model with a continuum of players and infinite time horizon with discounting into the laboratory with a finite number of players and a finite time horizon has only minor influence on the results. We also use this transformation.

McCabe (1989) shows in his paper that if the number of time periods (without discounting) is small (i.e. 6 periods) only experienced participants show equilibrium behavior which was a non monetary equilibrium in his experiment.

In our experiment we are also interested in the behaviour of experienced participants. We let all subjects play all types of games in a learning phase (which lasts several hours in total). After this we perform a strategy game in which experienced participants use their strategies. The participants could take as much time as they wanted for the strategy game.

Clearly our design differs in several aspects from the ones of Duffy and Ochs (1999) and McCabe (1989). As Duffy and Ochs test the Kiyotaki-Wright model traders meet pair wise, several goods can be produced and stored. Traders can be of three different types which are determined at the beginning of the experiment. In our setting traders meet in a market, only one good can be produced that cannot be stored. Players are not of different types in our experiment. A chance move determines their preference every period.

Compared with the experiment of Duffy and Ochs we tried to minimize complexity to avoid disturbances caused by too much complexity. Duffy and Ochs test for the question of equilibrium selection between a fundamental and a speculative equilibrium. The prediction of our model is only one equilibrium.

McCabe does not use discounting. Therefore his only equilibrium prediction is the non monetary Nash equilibrium. We use discounting and obtain a different equilibrium prediction which we test.

Since in the theoretical models individual behaviour is modeled it should be tested in the experiments. Duffy,Ochs and McCabe perform market experiments and analyze the behaviour in the market. In addition we perform individual decision making experiments to analyze which part of the behaviour of subjects is due to individual optimization and what is caused by the market. This procedure allows for a more detailed test of the theory.

Our goal in the experiment is to test the main predictions of the model. Predictions fall within two categories: individual behavior in an artificial resp. actual market and market dynamics. We shortly summarize our hypotheses and the implementation of the model in an experimental setting before giving a detailed description of the experimental procedure and experimental results.

The first prediction is that individual behavior should render the optimal policy. That is, (1) holding no money should imply choosing action S (sell); (2) if money holdings are non-zero observing a high time-value should result in playing B (buy) and (3) if money holdings are non-zero and below some certain quantity $\overline{m}$, observing a low time-value should imply S; if money holdings exceed $\overline{m}$, alternative B should be chosen.

The second hypothesis concerns our assumption that agents play the field and do not engage in strategic reasonings how to influence market averages. That is, the individual behavior described in the first hypothesis should be observed with or without actual markets. In particular a decrease in the probability of the high time-value should have a uniform impact regardless of the initial endowment of money.

The third hypothesis is derived from the result on the optimum quantity of money. In every market there is some quantity of money for which the average number of transactions is maximal. The optimal quantity depends on the fundamentals; in particular the less probable the high time-value, the lower the maximal money holdings. Agents may have different maximum money holdings due to individual perception of the actual continuation probability β. For instance, if agents are risk neutral and $p_h = p_l = .5$, the optimum quantity of money is equal to half of the average maximum money holdings across types (weighted according to their frequency) because of the symmetry of the stationary distribution of the Markov chain.

The experiments are organized in two stages. First individual behavior is analyzed in games in which participants play against nature without markets. Subjects play these games repeatedly on computer terminals—referred to as the learning phase. After subjects have acquired experience in these games, a strategy game is carried out. In the strategy game subjects have to determine their individual strategies for each game. A form was supplied, but subjects were allowed to use a blank sheet of paper and did not need to fill in this form. Second we let subjects interact in a common market, facing the decisions of other human subjects. The procedure is the same as in the case discussed above. This approach is intended to reveal whether the notion of a stationary equilibrium in our model—which is based on complete rationality—is a realistic description of actual behavior in a situation with actual market-interaction.

In order to perform the experimental testing of our hypotheses in a laboratory, we have to operationalize our model. In the experiment the time-value corresponds to a potential monetary payoff. The discounting in the model is implemented as the break-off probability of the game. While the model postulates an infinite number of agents in the market, the number of subjects in the experiment has to be finite. A detailed description of the games follows.

Individual Behavior Game. Two games have been implemented to analyze the individual behavior in an artificial market. Each game consists of a maximum number of 100 periods. All periods are identical in both games. At the beginning of each period a random number generator determines the time-value of the subject. The probability of the two time-values is set to $p_h = p_l = .5$. After observing the time-value, agents have to choose one of the three alternatives S (selling), B (buying), or I (staying idle). Alternative B can only be chosen if money holdings are not zero. When the agent has made her choice, it is decided by a random draw whether the intention of the subject is put into execution.

In game N (no rationing), the success-probabilities are $p_B = p_S = 1$, i.e. no rationing takes place and each choice of agents is actually executed. In game R (rationing), the success-probabilities are $p_B = p_S = .8$, i.e. rationing takes place and each choice of the agents is only executed in 80% of all cases.

Depending on the agent's choice and on the outcome of the random draw determining whether this choice is put into action, the money holdings and the accumulated payoffs of the agent change. If S had been chosen then money holdings increase by one. If B had been chosen then money holdings decrease by one and the time-value is added to the payoff account. Nothing happens if choice I had been made. At the end of each period it is determined by a random draw whether the game ends. If the game ends, money holdings of agents become worthless and the accumulated payoffs turn into cash for the subject.

Market Game. The individual behavior in an actual market has also been analyzed in two games. Six participants interact in a market. The order of events is analogous to the game in an artificial market except that the rationing depends on the choices of the participants. In each period the longer side of the market is rationed according to an i.i.d. random draw such that the number of selected subjects matches the shorter side. All selected subjects on the longer side and all subjects on the shorter side actually trade. The time-value of each subject is also determined by a random draw that is i.i.d. across time and subjects.

In game M.5 the probability of the high time-value is set to $p_h = .5$; and in game M.2 we set $p_h = .2$. Each market consists of six participants. With a total of 36 participants who were students at the university of Zurich, we have six independent observations of the market game. The rational for game R in which participants are rationed regardless of the side of the market they choose is founded in this implementation of the market game. Suppose money holdings do not matter and strategies of all subjects are buy/sell when observing a high/low time-value. Then for $p_h = .5$ the probability of actually making a trade in the current period from the perspective of a subject (regardless of the particular action chosen) is 68.75%. That is, due to the fact that a small number of subjects trade in a common market, the average number of high (resp. low) time-value actually realized differs from the expected number in most cases. However, this should not influence the result, because if the success probabilities for selling or buying are the same ($p_B = p_S$) the same behavior should be observed as if both were 1.

Payment of each participant is based on his/her individual performance in the experiment. The experiments were conducted in the computer laboratory of the Institute for Empirical Research in Economics at the University of Zurich. No communication was allowed between participants and computer terminals were separated from one another. Total time for the experiment was about four hours. Each game was played for about one hour with instructions (see Appendix) given in the first 15 minutes.

Throughout the experiment we let $\beta = 0.95$ (i.e. break–off probability is 5% in each round), $h = 10$, and $l = 5$. Each unit is worth 0.05 SFr. ($\approx$ \$0.03). All parameter settings are common information. At the beginning of each game all participants were given an identical initial endowment of m ($m = 1, 3, 8$) units of money, referred to as coupons. The traded commodity was not termed. Each participant earned on average about 100 SFr. ($\approx$ \$70).

4.1 Results

The strategy game is analyzed to compare the experimental results with the hypotheses to be tested. This procedure aims at obtaining a clear description of experienced agents' behavior after they have passed through the "learning phase" in which the respective game is repeatedly played.

The main observation in the strategy game concerns the structure of the strategies specified by the participants. The strategies of all participants exhibit three phases: start, main and end. Start and end phase originate from the explicitly announced commitment that at least 20 and at most 100 periods are played. Since we are interested in the stationary behavior of subjects we analyze the main phase.

The most striking finding is that all strategies are of the form described in our first hypothesis. Table 3 in Appendix 4.1 gives a full description of the result of the strategy game by reporting the maximum money holdings of all agents. The median $\overline{m}$ across groups is also reported. The result is significant: 36 out of 36 observations are of this type.[7] Summarizing we can conclude that our first prediction is is not rejected by the data.

Our finding on the properties of the maximal number of coupons held by subjects can be summarized as follows. Table 3 shows that this quantity exhibits high variation across subjects within each game. The median $\overline{m}$ over all subjects is given by $\overline{m} = 4$ in the games N, R, and M.5 for all initial endowments $m = 1, 3, 8$ of coupons, except for game M.5 with $m = 1$ in which the median $\overline{m} = 3.5$. In game M.2 we find that $\overline{m} = 2.75$ for the initial endowment $m = 1$, and $\overline{m} = 3$ for the initial endowments $m = 3, 8$.

The theoretical model predicts (regardless of the initial money holdings) that any risk neutral agent with a correct perception of the discounting rate $\beta = .95$ has maximum holdings $\overline{m} = 4$ in games N and M.5, and $\overline{m} = 2$ in game M.2 under the assumption that success probabilities are equal to 1. In game R, one has $\overline{m} = 3$. These values are determined numerically using MATLAB scripts, employed also in all further simulations. The scripts are available on the web (Schenk-Hoppé 2003).

While the reported medians and the prediction of the model are in good agreement, there is considerable variation of the maximum amount $\overline{m}$ across individuals. This can possibly be explained by a misperception of the continuation probability β. For instance, in games N and M.5, $\overline{m} = 4$ if the discounting rate is between .931 and .957, $\overline{m} = 3$ if $.868 < \beta < .931$, and $\overline{m} = 2$ for $.666 < \beta < .868$.

Summarizing we can state that the experimental results on the maximal amount of money holdings are in good agreement with the theoretical model.

We now turn to a comparison of different games to examine our second hypothesis that subjects play the field and do not engage in strategic reasonings how to influence market averages. Table 1 evaluates the six independent observations (one for each group) on the changes of the median between different games. Except for the clear pattern in the change in the mean from game M.5 to M.2, Table 1 does not provide any strong evidence about the direction of change of the median between different games.

The hypothesis that subjects play the field can be tested empirically by the change from game R (or N) to game M.5. Our prediction is that there is no change in the median of $\overline{m}$. It is clear from Table 1 that there is no strong empirical evidence against our hypothesis. Even on a significance level of 30%

[7]The hypothesis that half of the population from which participants are drawn would pursue a different strategy is rejected in a one-sided binomial test on a significance level of 0.1%.

Direction of change	N to R (initial m)			N to M.5 (initial m)			R to M.5 (initial m)			M.5 to M.2 (initial m)		
	1	3	8	1	3	8	1	3	8	1	3	8
increased	2	1	1	1	1	2	0	1	2	0	0	0
unchanged	3	5	4	2	2	1	3	2	1	0	0	0
decreased	1	0	1	3	3	3	3	3	3	6	6	6

Table 1: Evaluation of the strategy game: Number of groups in which the median of $\overline{m}$ increases/remains constant/decreases between different games.

the hypothesis that $\overline{m}$ is constant is not rejected in a binomial test. However, the effect of a change in the initial endowment of coupons is not very strong. For instance in game M.5 the median $\overline{m}$ increases only slightly in merely 2 out of 6 groups when the initial endowment increases from $m = 1$ to $m = 8$. In game M.2 this effect is present in 3 groups. If subjects would fully take into account their own potential to effect market averages, an increase of at least about 5 coupons in the maximum money holdings should be observed. This is not the case here, see Table 3. The hypothesis that $\overline{m}$ increases more than one coupon is rejected in a one-sided binomial test on a significance level of 2%. Let us also compare the effect of a change in the probability p_h for different initial endowments. Table 1 shows that the direction of change of $\overline{m}$ from game M.5 to M.2 is significant. For each initial endowment $\overline{m}$ decreases in 6 out of 6 observations.[8] Thus changes in the fundamentals have the predicted effect and give a hint that subjects actually play the field.

Summarizing we may conclude that our second hypothesis is supported by the data.

We finally address the third hypothesis on the optimum quantity of money. We employ the strategies that subjects supplied in the strategy game in a numerical simulation of the markets M.5 and M.2. It is worth to point out that maximal money holdings vary with the initial endowment in 17 (resp. 15) out of 36 case in game M.5 (resp. M.2). Thus one cannot expect to obtain a clear-cut result.

The strategy of every subject within a group is taken from Table 3.

In order to obtain a most complete specification of the effect of different quantities of money, we linearly interpolate the maximal $\overline{m}$ of each subject over non-retrieved initial money holdings by using the available data for $m = 1, 3, 8$. For simplicity the results are rounded to integers.

Starting from a uniform initial distribution of money holdings m over subjects (thus m is the quantity of money), we simulate the market interaction for $10,000$ iterations (ignoring the break-off probability). From these data we calculate the average number of trades in a single period and the average time-values realized by the subjects (Schenk-Hoppé 2003). Table 2 details the numerical results.

[8]The hypothesis that half of the population would hold more money in game M.2 than in game M.5 is rejected in a one-sided binomial-test on a significance level of 2%.

Group		Game M.5 quantity of money 1	2	3	4	5	6	Game M.2 quantity of money 1	2	3	4
1	Trans.	1.85	2.05	2.09	2.08	1.81	0.50	0.85	1.19	1.37	2.25
	Value	3.03	3.31	3.29	3.18	2.58	0.62	1.30	1.74	1.93	2.64
2	Trans.	1.97	2.15	1.96	1.25	0.0	0.0	1.74	2.26	1.55	0.0
	Value	3.09	3.32	2.89	1.66	0.0	0.0	2.09	2.66	1.59	0.0
3	Trans.	1.89	2.14	1.99	0.91	0.0	0.0	1.13	1.77	2.22	0.0
	Value	3.09	3.35	2.94	1.16	0.0	0.0	1.57	2.27	2.34	0.0
4	Trans.	1.85	2.14	1.83	0.50	0.0	0.0	0.94	1.83	0.80	0.0
	Value	3.04	3.37	2.61	0.62	0.0	0.0	1.44	2.36	0.80	0.0
5	Trans.	2.10	2.14	1.97	0.93	0.0	0.0	1.02	1.16	1.99	0.80
	Value	3.20	3.25	2.87	1.20	0.0	0.0	1.51	1.68	2.46	0.81
6	Trans.	1.92	2.06	2.08	2.07	2.07	1.95	0.90	1.25	1.75	1.74
	Value	2.99	3.23	3.27	3.23	3.18	2.85	1.26	1.67	2.06	1.76

Table 2: Average number of transactions (Trans.) and average realized time-values (Value) per period in both market games for different quantities of money.

One further finds that trade breaks down in game M.5 (game M.2) for all groups if the quantity of money $M \geq 8$ ($M \geq 5$).

For all groups and market games we observe an increase of transactions from an initial endowment of 1 to an optimal endowment m_O ($1 < m_O < 8$) and a decrease for higher endowments. The hypothesis that the number of transactions does not depend on the initial endowment is rejected on a 2% significance level for all markets. This strongly supports our third hypothesis. For a further test of our predictions we compare the optimal initial endowments m_O with the predicted optimal quantities.

The benchmarks for the average number of trades and realized time-values are as follows. The maximum number of trades in any one period is 3, i.e. both sides of the market are of the same length for all periods in time. The expected time-value of each subject is 7.5 in game M.5 resp. 6 in game M.2. Assuming that buyers have an expected time-value that matches the overall expected time-value, a proxy for the average time-value is 3.75 in game M.5 resp. 3 in game M.2.

Given these benchmarks we can state that the monetary economy determined by the strategy game works surprisingly well. The average number of trades reaches 2.15 in game M.5 and 2.26 in game M.2; and the average realized time-value tops at 3.37 in game M.5 and 2.66 in game M.2. The median of the quantities of money for which trades resp. payoffs are maximal is $M = 2$ in game M.5 whereas in game M.2 the median is 3 for trades and 2.5 for payoffs. In game M.5, 4 out of 6 observations are conform with $M = 2$ being the optimum quantity of money. The hypothesis that the optimum quantity of money is not in the interval $[2, 3]$ is rejected on a significance level of 5%. In group 1 the behavior of subject 3 stands out because of a very high $\overline{m}$. This behavior stimulates trade within the respective group even for high quantities of money.

Beside the fact $\overline{m}$ exceeds the total amount of money in games M.5 and M.2 with $m = 1$, a reason for not being reluctant to hold so many coupons may be an extremely dissenting individual perception of the break-off probability.

For the sake of comparison we simulate an artificial model in which $\overline{m} = 4$ (resp. $\overline{m} = 3$) for all agents in game M.5 (resp. game M.2). It turns out that average trades and payoffs are maximal for $M = 2$ in game M.5. The average number of trades is 2.13 and the average payoff is 3.35. In game M.2, the quantity of money $M = 2$ is also optimal. The average number of trades is 2.08 and the average payoff is 2.57.

We can state that performance of subjects—which exhibit a high degree of heterogeneity—is very good compared to this artificial market in which agents are homogenous.

Summarizing the experimental results we can conclude that there is experimental evidence for the correctness of our theoretical model which is based on complete rationality of agents. In the median the behavior of the participants in the experiment coincided with the optimal behavior of the representative agent as predicted by the model. The concept of a stationary (monetary) market equilibrium which assumes that agents "play the field" was also confirmed by the experiments. Even though each market consisted only of six participants, the median of their behavior was conform with the best response to market averages. We observed that money circulated fine in the different markets and the average number of trades and realized time-values were surprisingly high. In each market game an optimum quantity of money could be determined.

Grp.	Part.	Game N (initial m) 1	3	8	Game R (initial m) 1	3	8	Game M.5 (initial m) 1	3	8	Game M.2 (initial m) 1	3	8
1	1	6	4	4	6	4	4	6	4	6	5	3	5
	2	6	6	6	6	6	6	8	8	8	6	6	6
	3	12	12	12	12	12	12	12	12	12	12	12	12
	4	5	5	5	5	5	5	5	5	5	2	4	5
	5	3	3	3	3	3	3	3	3	3	2	2	2
	6	4	4	4	3	3	3	2	3	4	2	2	2
median		5.5	4.5	4.5	5.5	4.5	4.5	5.5	4.5	5.5	3.5	3.5	5
2	7	3	3	3	3	3	3	4	4	4	1	4	1
	8	3	3	3	4	3	4	1	3	9	1	3	9
	9	4	4	4	4	4	4	2	4	3	2	4	4
	10	5	5	5	4	5	5	5	5	5	3	3	3
	11	3	3	3	3	3	3	4	4	4	3	3	3
	12	10	10	10	10	10	10	4	6	6	2	3	3
median		3.5	3.5	3.5	4	3.5	4	4	4	4.5	2	3	3
3	13	3	4	5	6	7	2	3	6	3	3	7	5
	14	3	3	3	2	2	2	3	3	3	2	2	2
	15	6	6	6	6	6	6	3	3	3	2	2	2
	16	7	7	7	7	7	7	7	7	7	5	5	5
	17	7	7	7	7	7	7	3	3	3	3	3	3
	18	4	4	4	4	4	4	4	4	8	1	2	2
median		5	5	5.5	6	6.5	5	3	3.5	3	2.5	2.5	2.5
4	19	4	4	4	5	5	5	3	4	4	2	2	2
	20	6	8	10	9	6	10	5	4	8	3	3	4
	21	5	5	5	5	5	5	5	5	5	5	5	5
	22	5	5	5	4	4	4	4	4	4	3	3	3
	23	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	6.5
	24	4	4	4	4	4	4	4	4	4	3	3	3
median		4.5	4.5	4.5	4.5	4.5	4.5	4	4	4	3	3	3.5
5	25	5	7	12	5	7	12	3	6	10	2	4	9
	26	4	4	4	4	4	4	4	7	7	3	3	3
	27	1	1	1	1	1	1	1	3	3	3	2	2
	28	2	3	1	2	3	1	2	3	1	2	2	2
	29	3	3	3	3	3	3	3	3	3	2	2	2
	30	3	3	3	3	3	3	3	3	3	11	11	11
median		3	3	3	3	3	3	3	3	3	2.5	2.5	2.5
6	31	15	15	15	15	15	15	6	11	14	6	9	13
	32	5	5	5	5	5	5	3	3	3	1	1	1
	33	4	4	4	3	3	3	1	3	3	1	1	1
	34	3	3	3	2	2	2	2	2	2	2	1	1
	35	16	16	16	16	16	16	16	17	17	7	7	7
	36	5	6	8	4	6	8	5	4	6	5	5	8
median		5	5.5	6.5	4.5	5.5	6.5	4	3.5	4.5	3.5	3	4

Table 3: Evaluation of the strategy game: Maximal number of coupons $\overline{m}$ that subjects wanted to hold detailed for all games and initial endowments of money.

(Subject 23 is indifferent between 2 and 3 as maximal money holdings and randomizes with probability one half.)

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