Does Prospect Theory
Explain the Disposition Effect?[*]

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Abstract

The disposition effect is the observation that investors hold winning stocks too long and sell losing stocks too early. A standard explanation of the disposition effect refers to prospect theory and in particular to the asymmetric risk aversion according to which investors are risk averse when faced with gains and risk-seeking when faced with losses. We show that for reasonable parameter values the disposition effect can however not be explained by prospect theory as proposed by Kahneman and Tversky. The reason is that those investors who sell winning stocks and hold loosing assets would in the first place not have invested in stocks. That is to say the standard prospect theory argument is sound ex-post, assuming that the investment has taken place, but not ex-ante, requiring that the investment is made in the first place.

Keywords: Disposition effect, prospect theory, portfolio choice

1 Introduction

The disposition effect is the observation that investors tend to sell winning stocks while they have a disposition to keep losing stocks. This observation has been made by a series of papers, including Shefrin and Statman (1985), Odean (1998), Weber and Camerer (1998), Heath, Huddart, and Lang (1999), Locke and Mann (2001), Grinblatt and Keloharju (2000), Grinblatt and Keloharju (2001) and Ranguelova (2002). Of course, selling winners and keeping losers as such is perfectly compatible with complete rationality. A well known result is that an expected utility maximizer, with constant relative risk aversion, would rebalance a fixed-mix portfolio strategy in a setting where the investment opportunity set is constant.[1] Hence when prices rise (fall) he would sell (buy) the security. However, as Odean (1998) has shown investors are reluctant to sell losers even when controlling for rebalancing. Hence the disposition effect is the observation that investors show a more aggressive contrarian behavior than following the fixed-mix rule. As compared to the fixed-mix case, investors prone to the disposition effect hold winners too long and sell losers too early.

The disposition effect could result from a strong believe in mean-reversion of the asset returns. Following this argument the disposition effect would then stem from a misperception of the return process. An alternative behavioral finance explanation of the disposition effect refers to prospect theory and in particular to the asymmetric risk aversion. Under prospect theory, see Kahneman and Tversky (1979), investors evaluate outcomes relative to a reference point which in the case of stock investments is typically the price at which the stock was bought. The reference point divides outcomes into two regions: losses occur if the final wealth is below the reference point and gains occur if the final wealth is above the reference point.

In the behavioral finance literature the disposition effect is explained by two main features of prospect theory. First, decision-makers frame their choices in terms of potential gains and losses. Second, they behave as if evaluating the decision consequences on an S-shaped value function, which is concave for gains and convex for losses. This reflects risk aversion in the gain region and risk-seeking in the loss region. The disposition effect is seen as an important implication of extending the prospect theory of Kahneman and

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[1]See Samuelson (1969) and Merton (1969).

Tversky (1979) and Tversky and Kahneman (1992) to investment decisions and securities trading. The standard behavioral finance argument for the disposition effect is that a gain (loss) moves the investor to his risk averse (seeking) part of the value function so that he is leaned to reduce (increase) his position in the risky assets; or stated differently, he sells winners and holds losers.

However, in this standard argument, it is generally *assumed* that the investor has bought the risky stock and thus the issue whether the investor really will decide in this way is neglected. Hence this standard argument is in fact an ex-post argument that corresponds to a liquidation situation as analyzed by Kyle, Ou-Yang, and Xiong (1979). Similarly, Gomes (2005) analyzes the comparative statics of a one period portfolio decision. Berkelaar, Kouwenberg, and Post (2004) consider the dynamically optimal portfolio allocation of a loss averse agent investing in continuous time. They focus on the time diversification due to a change in the investment horizon. Our paper is in between these two approaches since we consider a repeated portfolio choice not requiring intertemporal optimization.

In our paper we consider a model with two consecutive portfolio choices in a stylized financial market where the investor's preferences are described by prospect theory as suggested by Kahneman and Tversky (1979) and Tversky and Kahneman (1992). We investigate the investor's risk-taking behavior following a rise, respectively a fall, in the price of the risky asset. In our analysis we use a more complete definition of the disposition behavior, i.e. besides requiring investors to sell winners and to hold losers, we require them explicitly to buy the stock in the first period.

In our framework, there is a financial market on which two assets are traded. A riskless asset, also called the bond, and a risky asset, the stock. The evolution of the stock prices is described by a binomial process. The preferences of the investor are based on changes in wealth and described by prospect theory. We assume that he owns an initial endowment and that he earns no other income. Since we want to model a small individual investor, we assume that no short selling is allowed. Further we assume that the investor acts myopically, in the sense that when taking his first decision he does not already anticipate his optimal second period decision, and that the reference point relative to which he measures his gains and losses is his initial wealth. The assumption of myopic behavior is also justified by the fact that we present a descriptive model for a small individual investor. Note that requiring dynamic optimization, i.e. integrating into today's decision

4

the correctly anticipated optimal future decisions, seems to be at odds with assuming reference point based behavior on the other hand. The investor would then be very rational and very behavioral at the same time. The investor's portfolio decision consists of allocating his wealth to the two assets traded in the financial market. For simplicity we restrict the fraction of wealth invested in the risky asset to be either zero or one, i.e. the agent chooses to invest fully or not to invest at all in the risky asset. Hence as soon as the stock appreciates this is seen as a gain. Moreover, earning the risk-free rate amounts to a gain and losses can only occur if the investor invests in stocks.

Our first point of interest is the second period behavior of the investor conditional on the stock price movement in the first period. In particular, we ask whether we can explain the behavior of an investor prone to the ex-post disposition effect. Assuming that the investor bought the stock in the first period, we call him a disposition investor if he sells the risky asset after a gain and keeps holding it after a loss[2]. We show how important aspects of prospect theory, in particular loss aversion and probability weighting, interact with asymmetric risk aversion. This analysis is of interest in itself but it also will lay the foundations for the inter-temporal argument. In the inter-temporal view we investigate the agent's behavior with a focus on the more complete definition of the disposition behavior. We show interactions between loss aversion, decision weighting and asymmetric risk-taking.

Our findings are that the inter-temporal disposition effect arises rather for lower coefficients of loss aversion and that whenever the agent can undo the first period loss by investing in the risky asset the same is true for the ex-post disposition effect. In the opposite case, the ex-post disposition effect arises rather for more loss averse investors. Furthermore investors are generally prone to the ex-post disposition effect, but hardly to the true disposition effect. The reason is that those investors who sell winning stocks too early and keep losing stocks too long would in the first place not have invested in stocks. So even when considering explicitly the asymmetric risk-taking behavior of the investor, a standard explication for the disposition behavior, investors are not prone to the disposition effect. We conclude that prospect theory can indeed explain the ex-post disposition behavior, but not the more

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[2]The opposite behavior to the disposition effect is the house money effect found by Thaler and Johnson (1990) according to which the investor sells the risky asset after a loss and keeps holding it after a gain.

complete inter-temporal definition of the disposition behavior.

The rest of the paper is organized as follows. In the next section we precisely describe the framework. In section 3 we analyze the ex-post behavior of a prospect theory investor and then we consider the ex-ante point of view. In the last two sections we offer further discussion of our results and conclude.

2 The Model

We present a two period model for portfolio choice in a stylized financial market with two assets where the investor's preferences are described by prospect theory as suggested by Kahneman and Tversky (1979) and Tversky and Kahneman (1992). After describing the financial market and the agent's preferences, we derive the investor's maximization problem and the conditions under which the disposition effect arises.

In our framework, there is a financial market on which two assets are traded. A riskless asset, also called the bond, and a risky asset, the stock. The evolution of the stock prices is described by a binomial process, so that at the end of the following period there are two possible states. If the stock price rises, we call the corresponding state the up-state; the other state is called the down state. In the up state, which realizes with probability p, the risky investment yields a gross return R_U. Note that $0 < p < 1$. In the down state, arising with probability $1 - p$, it yields R_D. The risk-free bond yields a sure gross return of R_f. We assume that the time value of money is positive, i.e. that interest rates are non-negative. Absence of arbitrage requires that $R_U > R_f > R_D$. For simplicity and without loss of generality we assume further that $R_D < 1$. To prevent negative stock prices we assume $R_D \geq 0$. These assumptions about the financial market are summarized in the following inequality: $R_U > R_f \geq 1 > R_D \geq 0$. All the parameters are assumed to be constant over time.

The preferences of the investor are based on changes in wealth and described by prospect theory. We assume that he owns an initial endowment, W_0, and that he earns no other income. Since we want to model a small individual investor, we assume that no short selling is allowed. Further we assume that the investor acts myopically [3] and that the reference point rel-

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[3]We think that assuming a myopic behavior for a small individual investor is appropri-

6

ative to which he measures his gains and losses is his initial wealth. [4]

The overall value of a prospect is given by the sum of the subjective values of the outcomes weighted by the agent's decision weights associated with the probability of the outcome. The overall value of a prospect yielding a gain x with probability p and a loss y with probability $1-p$ is given by : $V(x,p;y,1-p) = w(p)v(x) + w(1-p)v(y)$; where $x \geq 0 \geq y$. The decision weights w measure the impact of events on the desirability of prospects. Following Tversky and Kahneman (1992) the decision weights take the following form

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}}, \text{ for some } 0 \leq \gamma \leq 1. \tag{1}$$

The value function v assigns to each outcome x, edited as a gain or a loss, a number $v(x)$ which reflects the subjective value of that outcome. As a possible form of the value function Tversky and Kahneman (1992) proposed a two part power function. This function describes the experimental evidence the authors found. The key features of their theory are the coding of outcomes into gains and losses, that a loss hurts more than an equivalent gain and asymmetric risk-taking behavior

$$v(x) = \begin{cases} (x)^\alpha & \text{if } x \geq 0 \\ -\beta(-x)^\alpha & \text{if } x < 0 \end{cases}.$$

The function v assigns to each outcome x, edited as gain or a loss, a number $v(x)$ which reflects the subjective value of that outcome. The parameter β is the coefficient of loss aversion and reflects the fact that losses hurt more than equivalent gains, which is true for all $\beta > 1$. Using data from their experiments the authors estimated β to be equal to 2.25. The coefficient α measures the agent's risk aversion and takes on values between zero and one. Using data from their experiments the authors estimated α to be equal to 0.88. Observe that in the domain of gains, i.e. $x \geq 0$, the value function is concave, implying that the agent is risk averse, whereas for the domain of losses the function is convex, i.e. the investor is risk-seeking in this domain. We assume that all parameters are constant over time.

In the first period the investor's portfolio decision consists of allocating his initial wealth to one of the two assets traded in the financial market. He

ate for a descriptive model.

[4]Given the assumption that all the parameters are constant over time, an investor who measures his gains and losses relative to the last period's wealth faces in each period the same decision problem and hence makes always the same choice.
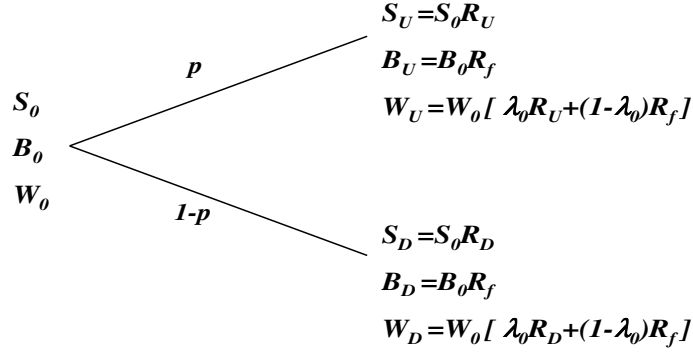
$$S_U = S_0 R_U$$
$$B_U = B_0 R_f$$
$$W_U = W_0 [\lambda_0 R_U + (1-\lambda_0) R_f]$$

$$S_D = S_0 R_D$$
$$B_D = B_0 R_f$$
$$W_D = W_0 [\lambda_0 R_D + (1-\lambda_0) R_f]$$

Figure 1: Binomial tree for the first period.

maximizes his utility in $t = 0$ by allocating a fraction λ_0 of his initial wealth in the risky asset and $1 - \lambda_0$ in the riskless asset. For simplicity we restrict the fraction of wealth invested in the risky asset to be either zero or one. [5] The situation he is confronted with at time zero is depicted in Figure 1.

In $t = 0$ the stock is worth S_0, the bond B_0 and the investor owns his initial wealth W_0. With probability p the stock price goes up and the good state realizes. In this case the stock is worth $S_U = S_0 R_U$, the bond price is worth $B_U = B_0 R_f$ and the investors wealth is W_U. Note that we skip the time index in $t = 1$ and index variables simply by the unambiguous short cut U, for the up state in $t = 1$, and D for the down state in $t = 1$. The investor's wealth position in the up state equals his initial wealth multiplied by the portfolio return, where λ_0 is the fraction of wealth invested in the risky asset. Under the above assumption W_U always exceeds the initial wealth, except for the trivial case where the risk-free rate is zero and the agent does not invest in the risky asset. Therefore the investor experiences a gain in the good state following either investment strategy.

The bad state realizes with probability $1 - p$ and the stock price depreciates. In this case it is worth $S_D = S_0 R_D$. The riskless bond yields the certain gross return of R_f and the agent's wealth position is W_D. Given the setting, the wealth in the down state can be greater, equal or smaller than the initial position. The performance depends on the returns offered by the

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[5]A possible interpretation is that the risky asset is a project that absorbs all the agent's wealth. If the agent decides not to invest in the project he simply keeps his wealth in a risk-free bank account.

traded securities and the portfolio choice of the investor. In the case where $\lambda = 0$ and as long as interest rates are positive, the agent makes a sure gain on his portfolio, implying that his wealth is bigger than his initial wealth, even in the bad state. Conversely if he invests all his wealth in the risky asset he will experience a loss in his wealth in the down state.

This yields the following maximization problem

$$max_{\lambda_0 \in \{0,1\}} \quad w(p)v\big(W_0(R_U\lambda_0 + R_f(1 - \lambda_0) - 1)\big)$$
$$+ w(1 - p)v\big(W_0(R_D\lambda_0 + R_f(1 - \lambda_0) - 1)\big),$$

$$\text{where } v(x) = \begin{cases} (x)^\alpha & \text{if } x \geq 0 \\ -\beta(-x)^\alpha & \text{if } x < 0 \end{cases} \tag{2}$$

$$\text{and } w(p) = \frac{p^\gamma}{(p^\gamma + (1 - p)^\gamma)^{\frac{1}{\gamma}}}.$$

When in $t = 0$ the expected utility from holding the risky asset exceeds the utility from investing in the risk free bond the agent will invest in stock. If this conditions is not satisfied, the agent prefers to invest his entire wealth in the risk-free bond, i.e. $\lambda_0 = 0$. Hence he invests his entire wealth in the risky asset whenever

$$w(p)(R_U - 1)^\alpha - w(1 - p)\beta(1 - R_D)^\alpha > (R_f - 1)^\alpha. \tag{3}$$

As we assume that in our model all the parameters are constant over time, the setting in the second period has the same structure as in the first period. After the investor has made his first period investment decision the state of nature in $t = 1$ realizes. The market parameters, the investment decision λ_0 and the realized state of nature determine the agent's wealth in $t = 1$. In the second period the investor allocates his first period wealth to the two assets traded in the financial market. The situation he is confronted with is shown in Figure 2.

We will continue to skip time indices and to label the nodes of the binomial tree with the short cuts $0, U, D, UU, UD, DU,$ and DD where $0, U, D$ are as in the first period, UU stands for the node after two up movements, UD for an up movement followed by a down movement, DU for a down movement followed by an up movement and DD for two consecutive down movements.
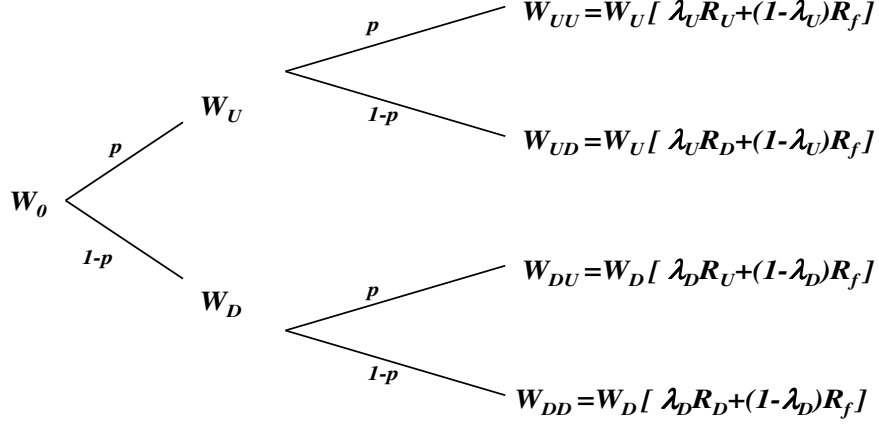
$W_{UU}=W_U[\ \lambda_U R_U+(1-\lambda_U)R_f]$

$W_{UD}=W_U[\ \lambda_U R_D+(1-\lambda_U)R_f]$

$W_{DU}=W_D[\ \lambda_D R_U+(1-\lambda_D)R_f]$

$W_{DD}=W_D[\ \lambda_D R_D+(1-\lambda_D)R_f]$

Figure 2: Binomial tree for two periods.

In the same sense we will call λ_0 the fraction of wealth invested in the risky asset in $t = 0$, λ_U is the fraction of wealth invested in the risky asset in $t = 1$, given the stock went up in the first period and λ_D is the fraction of wealth invested in the risky asset in $t = 1$, given the stock went down in the first period. The asset prices in $t = 2$ are standard. The investors wealth position in $t = 2$ equals his position in $t = 1$ multiplied by the return of his portfolio in the second period.

The maximization problem for the second period writes

$$max_{\lambda_t \in \{0,1\}} \quad w(p)v\big(W_t(R_U \lambda_t + R_f(1 - \lambda_t)) - W_0\big)$$
$$+ w(1 - p)v\big(W_t(R_D \lambda_t + R_f(1 - \lambda_t)) - W_0\big),$$

$$\text{where } v(x) = \begin{cases} (x)^\alpha & \text{if } x \geq 0 \\ -\beta(-x)^\alpha & \text{if } x < 0 \end{cases},$$

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}}, \tag{4}$$

and $t = \{U, D\}$.

In $t = 1$ we have to distinguish different cases, which imply different possible portfolio performances, in terms of gains and losses, and therefore different valuations.

10

In the first case, where $R_U R_D > 1$ and $R_f R_D > 1$ [6], the agent, who invests in $t = U$ his entire wealth in the risky asset, experiences a gain in both states and he makes a sure gain, if he invests in the riskless bond. If the down state realized in the first period, the investor who bought the risky asset may make a gain, if after the bad state the good state realizes, or a loss, after the realization of two consecutive down states. If he chooses to put his wealth in the risk-free alternative, he makes a sure gain.

In the second case, where $R_U R_D > 1$ and $R_f R_D < 1$, the investor who in $t = U$ invests his entire wealth in the risky asset, experiences a gain in both states and he makes a sure gain, if he invests in the riskless bond. If the down state realizes in the first period and the investor invests in the risky asset, he experiences a gain and a loss. If the investor chooses to put all his wealth in the risk-free alternative, he makes a sure loss.

In the third case, where $R_U R_D < 1$ and $R_f R_D < 1$, the investor, who buys the risky asset in $t = U$, may make a gain, if after the up state the good state realizes, or a loss, if after the up state the down state realizes. He makes a sure gain, when investing in the risk-free bond. If the down state realizes and the agent invests in the risky asset, he experiences a loss independent of which state realizes in the second period. If the investor chooses to put all his wealth in the risk-free alternative, he makes a sure loss.

In the first two cases, i.e. when $R_U R_D > 1$ and $R_f R_D > 1$ and when $R_U R_D > 1$ and $R_f R_D < 1$, the condition that the agent invests in the risky asset after the stock price appreciated in the first period is

$$w(p)(R_U R_U - 1)^\alpha + w(1-p)(R_U R_D - 1)^\alpha > (R_U R_f - 1)^\alpha. \qquad (5)$$

In the third case, where $R_U R_D < 1$ and $R_f R_D < 1$ the agent prefers the risky asset to the risk-free bond whenever

$$w(p)(R_U R_U - 1)^\alpha - w(1-p)\beta(1 - R_U R_D)^\alpha > (R_U R_f - 1)^\alpha. \qquad (6)$$

Similarly, the condition that the agent invests in the risky asset after the stock price depreciated in the case where $R_U R_D > 1$ and $R_f R_D > 1$ is

$$w(p)(R_U R_D - 1)^\alpha - w(1-p)\beta(1 - R_D R_D)^\alpha > (R_f R_D - 1)^\alpha, \qquad (7)$$

in the case where $R_U R_D > 1$ and $R_f R_D < 1$ we get

$$w(p)(R_U R_D - 1)^\alpha - w(1-p)\beta(1 - R_D R_D)^\alpha > -\beta(1 - R_f R_D)^\alpha \qquad (8)$$

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[6]Note that it follows that $R_U R_U > 1, R_U R_f > 1$ and that $R_D R_D < 1$.

and in the case where $R_U R_D < 1$ and $R_f R_D < 1$ we get

$$w(p)(1 - R_U R_D)^\alpha + w(1-p)\beta(1 - R_D R_D)^\alpha < (1 - R_f R_D)^\alpha. \quad (9)$$

In the described setting the disposition effect is the situation, where the agent invests in the risky asset in $t = 0$, sells the asset after the price appreciated and keeps on holding the risky stock after its price went down. This means that we observe the disposition effect whenever $\lambda_0 = 1$, $\lambda_U = 0$ and $\lambda_D = 1$. Thus the conditions for the disposition effect to occur are[7]:

1. In the case, where $R_U R_D > 1$ and $R_f R_D > 1$:

$$w(p)(R_U - 1)^\alpha - w(1-p)\beta(1 - R_D)^\alpha \geq (R_f - 1)^\alpha,$$
$$w(p)(R_U R_U - 1)^\alpha + w(1-p)(R_U R_D - 1)^\alpha \leq (R_U R_f - 1)^\alpha \quad \text{and}$$
$$w(p)(R_U R_D - 1)^\alpha - w(1-p)\beta(1 - R_D R_D)^\alpha \geq (R_f R_D - 1)^\alpha.$$
$$(10)$$

2. In the case, where $R_U R_D > 1$ and $R_f R_D < 1$:

$$w(p)(R_U - 1)^\alpha - w(1-p)\beta(1 - R_D)^\alpha \geq (R_f - 1)^\alpha,$$
$$w(p)(R_U R_U - 1)^\alpha + w(1-p)(R_U R_D - 1)^\alpha \leq (R_U R_f - 1)^\alpha \quad \text{and}$$
$$w(p)(R_U R_D - 1)^\alpha - w(1-p)\beta(1 - R_D R_D)^\alpha \geq -\beta(1 - R_f R_D)^\alpha.$$
$$(11)$$

3. In the case, where $R_U R_D < 1$ and $R_f R_D < 1$:

$$w(p)(R_U - 1)^\alpha - w(1-p)\beta(1 - R_D)^\alpha \geq (R_f - 1)^\alpha,$$
$$w(p)(R_U R_U - 1)^\alpha - w(1-p)\beta(1 - R_U R_D)^\alpha \leq (R_U R_f - 1)^\alpha \quad \text{and}$$
$$w(p)(1 - R_U R_D)^\alpha + w(1-p)(1 - R_D R_D)^\alpha \leq (1 - R_f R_D)^\alpha.$$
$$(12)$$

In what follows, we investigate these conditions. First we analyze the ex-post condition for the disposition effect, i.e. the condition that the investor prefers simultaneously to invest in $t = U$ in the risk-free bond and in $t = D$ in the stock. Then we take an ex-ante perspective and require that the agent has to prefer the stock in $t = 0$, the bond in $t = U$ and the stock in $t = D$.

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[7]We assume that when the investor is indifferent between the risky and the riskless asset, he behaves like the disposition investor, i.e. he purchases the stock in $t = 0$ and $t = D$ and he invests in the bond in $t = U$.

3 Results

In this section we present the results of our model. First we discuss the relationship between the (ex-post) disposition effect and loss aversion. Next, we take on the traditional view, where it is implicitly assumed that the investor already owns the risky stock and analyze his behavior given the stock price movement. We show that in fact the ex-post disposition behavior is consistent with most of the parameter combinations. Then we take on an ex-ante view, and require for the disposition effect not only that the investor sells a winning asset and keeps a loosing asset, but also that the agent decides to buy the risky stock in $t = 0$. We show that the disposition effect arises very rarely.

We first discuss the role of loss aversion: a first observation is that if the market parameters satisfy the condition $R_U R_D > 1$ and if the disposition effect arises for a $\beta_1 > 1$, then it arises for all β_2, where $\beta_1 > \beta_2 > 1$. The same statement is true for the ex-post disposition effect. The intuition is that an investor that is less loss averse more readily buys the risky stock in $t = 0$ and $t = D$. Note that since the agent does not face a loss in $t = U$ when investing in the risky asset, this condition is independent of loss aversion. If $R_U R_D < 1$ and if the ex-post disposition effect arises for a $\beta_1 > 1$, then it arises for all β_2, where $\beta_2 > \beta_1$. If $R_U R_D < 1$ then the agent makes a loss in $t = D$, independently of his investment decision, so that the investment decision in $t = D$ is independent of loss aversion. On the other hand, in $t = U$ the investor faces a gain and a loss, when buying the risky asset and therefore he prefers more the risk-free asset the more loss averse he is. Note that the effects of an increase in loss aversion go in opposite directions for the conditions in $t = 0$ and $t = U$. In absolute terms the effect is stronger in $t = 0$, so that if the disposition effect arises for a $\beta_1 > 1$, then it arises for all β_2, where $\beta_1 > \beta_2 > 1$. Again, a lower loss aversion implies that the investor more readily invests in the risky asset in the first period.

3.1 The Ex-post Disposition Effect

In this section we assume that the investor already owns the risky asset and analyze his portfolio decision given a stock price movement. A possible interpretation of this situation is that when the agent buys the risky asset in the first period the stock is very attractive. After the first period, an

external shock changes the characteristics of the asset so that the investor is faced with a liquidation decision. This liquidation decision corresponds to the ex-post view. If the risky asset is seen as an investment project, this liquidation decision corresponds to a situation where the agent is not allowed to liquidate the project in the first period, during which the project's characteristics, i.e. the returns and the probabilities, may change. Finally it could be the case where the project's characteristics are not observed when the project is initiated, but they are observed some time later.

The investment decision as described above depends on the parameters of the agent's preferences, α, β and γ, as well as the parameters of the financial market, i.e. the possible returns and the probabilities for the possible states. Since many different parameters are involved, we look first at different special cases in order to isolate the different effects of the parameters. As we have seen above, a lower loss aversion coefficient β favors the occurrence of the ex-post disposition effect whenever $R_U R_D > 1$ and it lowers it in the opposite case. In this section we focus on the impacts of the parameter of the decision weighting function γ and the coefficient of risk aversion α. We assume that the investor is loss averse.

To get more insights, we vary the two parameters in the following way: the parameter of the decision weighting function γ is either fixed at 1, so that the investor weights the outcomes with the objective probabilities or it is assumed to be between 0 and 1. When the coefficient of risk aversion α is fixed, it is kept constant either at 0, implying that the investor is quite risk-averse in the domain of gains and quite risk-seeking in the domain of losses, or at 1, where the agent is risk neutral. Otherwise it is assumed to be between 0 and 1. This yields six possible situations. The more restriction we impose on the preference parameters, the more tractable the inequalities describing the agents choices become. Allowing for more general parameter ranges often has the negative consequence that no analytical statements can be made, so that we have to provide numerical solutions.

Proposition 1 summarizes the results for the cases, where analytical statements can be made. The detailed proofs can be found in the appendix.

Proposition 1. *The ex-post disposition effect*

1. *An investor who weights outcomes with their objective probabilities and is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, i.e. $\gamma = 1$ and $\alpha = 0$, is prone to the ex-post disposition effect whenever $R_f R_D < 1$.*

14

2. *A risk neutral investor, who weights outcomes with their objective probabilities, i.e. $\gamma = 1$ and $\alpha = 1$, is prone to the ex-post disposition effect whenever $R_U R_D < 1$ and $\phi_4 \geq p \geq \phi_1$, where $\phi_4 = \frac{R_U R_f - 1 + \beta(1 - R_U R_D)}{R_U R_U - 1 + \beta(1 - R_U R_D)}$ and $\phi_1 = \frac{R_f - R_D}{R_U - R_D}$.*

3. *An investor who weights outcomes with the decision weights as proposed by Tversky and Kahneman (1992) and is who quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, i.e. $0 < \gamma < 1$ and $\alpha = 0$, is prone to the ex-post disposition effect whenever $R_f R_D < 1$.*

An investor who weighs outcomes with the objective probability and is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, i.e. $\gamma = 1$ and $\alpha = 0$, is prone to the ex-post disposition effect whenever $R_f R_D < 1$. The reason is that in $t = U$ the agent is in the gain zone and hence quite risk averse so that he never prefers the risky stock. Further, if $R_f R_D > 1$ the investor has the opportunity to realize a sure gain in $t = D$ and therefore prefers to invest in the risk free bond. However, if $R_f R_D < 1$, the investor is in the loss zone and is therefore quite risk-seeking, investing therefore in the risky asset. If he can undo the first period loss, i.e. if $R_U R_D > 1$, this behavior is consistent with the break even effect. Note that this is true even when the investor is not loss averse.

In absence of arbitrage, the risk neutral investor who weights the outcomes with their objective probabilities is prone to the ex-post disposition effect whenever after a first period loss, the agent cannot undo this loss, i.e. $R_U R_D < 1$ and hence $R_f R_D < 1$ and the probability of the occurrence of the good state is bounded by ϕ_4 from above and by ϕ_1 from below. This is the situation where the stock has a very high downside risk. We emphasize that even for a risk neutral agent the ex-post disposition effect arises. However only for restricted parameter values.

An investor who weights outcomes with the decision weights as proposed by Tversky and Kahneman (1992) and who is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, i.e. $0 < \gamma < 1$ and $\alpha = 0$, is prone to the ex-post disposition effect whenever $R_f R_D < 1$. The reason is that in $t = U$ the quite risk averse investor never prefers the stock. Further, if $R_f R_D > 1$ the investor has the opportunity to realize a sure gain in $t = D$ and prefers therefore to invest in the risk free bond. However, if $R_f R_D < 1$, the investor is in the loss zone and is therefore quite risk-seeking,

investing therefore in the risky asset. Note that this result is the same as in the situation where $\gamma = 1$.

For the other combinations of α and γ no unambiguous conclusions can be drawn. Therefore we provide a numerical analysis.

To illustrate the situation where $\gamma = 1$ and $0 < \alpha < 1$ we present Figure 3. It shows the parameter combinations for which the ex-post disposition effect arises for different returns of the risky asset, R_D and R_U. In the following graphics the value of the gross risk free rate, R_f, is kept constant at 1.1 and and the probability of the occurrence of the up-state, p is fixed at 0.5. The values of R_D vary between 0 and 1 and the R_U is varied between 1.1 and 2.1. For other values of p and R_f similar results are obtained. The loss aversion coefficient β is kept constant at 2.25 and the coefficient for risk aversion α equals 0.88. The parameter of the decision weights γ is fixed at 1. These values correspond to the empirical findings of Tversky and Kahneman (1992).[8] The parameter combinations, where the ex-post disposition effect occurs, are marked with black color, whereas the domains, where the conditions for the ex-post disposition effect are violated, are marked with grey color. In Figure 3 we see that the ex-post disposition effect occurs rarely, in about 12% of the cases (see Table 1 below). We observe it for moderate and low returns in the down state and high returns in the up-state. We can conclude that the ex-post disposition behavior for an agent that is described with parameters consistent with empirical findings of Tversky and Kahneman (1992) and $\gamma = 1$ is a special case and does not occur in general.

To illustrate the situation where $0 < \gamma < 1$ and $\alpha = 1$ we present Figure 4. It shows the parameter combinations for which the ex-post disposition effect arises for different returns of the risky asset, R_D and R_U. Except for α and γ the same parameter values as above are used. The parameter combinations, where the ex-post disposition effect occurs are marked with black color, whereas the domains, where the conditions for the ex-post disposition effect are violated are marked with grey color. In Figure 4 we see that the ex-post disposition effect occurs often, in about 50% of the cases (see Table 1 below). We observe it for moderate and low returns in the down state. We can conclude that the ex-post disposition behavior for an agent that is described with parameters consistent with empirical findings of Tversky and Kahneman (1992) and $\alpha = 1$ does occur in general for risky assets with a high downside risk.

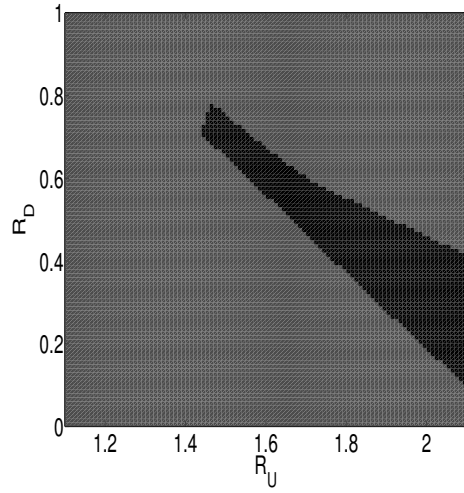[8]Again, for other parameter values similar results are obtained.

Figure 3: Return combinations for which the ex-post disposition effect arises. The values of R_D vary between 0 and 1 and R_U is varied between 1.1 and 2.1. The value of the gross risk free rate, R_f, is kept constant at 1.1 and the probability of the occurrence of the up-state, p is fixed at 0.5. The loss aversion coefficient β is kept constant at 2.25 and the coefficient for risk aversion α equals 0.88. The parameter of the decision weights γ is fixed at 1. The parameter combinations, where the ex-post disposition effect occurs are marked with black color. The ex-post disposition effect occurs in about 12% of the cases.
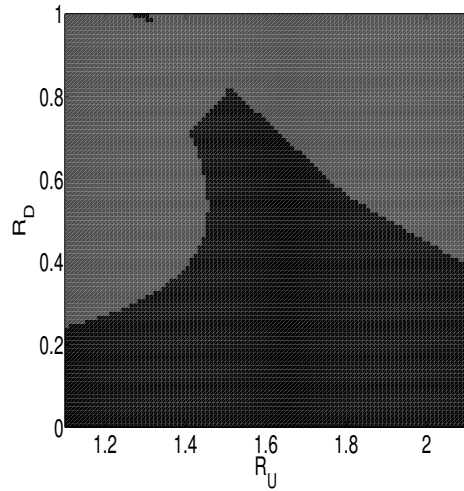
Figure 4: Return combinations for which the ex-post disposition effect arises. The values of R_D vary between 0 and 1 and R_U is varied between 1.1 and 2.1. The value of the gross risk free rate, R_f, is kept constant at 1.1 and the probability of the occurrence of the up-state, p is fixed at 0.5. The loss aversion coefficient β is kept constant at 2.25 and the coefficient of risk aversion α equals 1. The parameter of the decision weights γ is fixed at 0.65. The parameter combinations, where the ex-post disposition effect occurs are marked with black color. The ex-post disposition effect occurs in about 50% of the cases.

18

To illustrate the most general case, i.e. the situation where $0 < \gamma < 1$ and $0 < \alpha < 1$, we present Figure 5. It shows the parameter combinations for which the ex-post disposition effect arises for different returns of the risky asset, R_D and R_U. Except for α and γ the same parameter values as above are used. These values correspond to the empirical findings of Tversky and Kahneman (1992)[9] The parameter combinations, where the ex-post disposition effect occurs are marked with black color, whereas the domains, where the conditions for the ex-post disposition effect are violated are marked with grey color. In Figure 5 we see that the ex-post disposition effect occurs often, in about 59% of the cases. We observe it for moderate and low returns in the down state. We can conclude that the ex-post disposition behavior for an agent that is described with parameters consistent with empirical findings of Tversky and Kahneman (1992) does occur in general for risky assets with a high downside risk.

3.2   The True Disposition Effect

In this section we make one step backward in time and impose the additional condition that besides selling a winning stock and keeping a losing stock the investor has bought the stock in the first place. So that the disposition effect arises whenever the requirements to simultaneously prefer the stock in $t = 0$ and $t = D$ and to prefer the bond in $t = D$ are satisfied. This makes the definition of the disposition effect more consistent. Since the conditions for the disposition effect in $t = 1$ stay the same as for the ex-post disposition effect, in this section we focus on the ex-ante conditions.

The investment decision as described above depends on the parameters of the agent's preferences, α, β and γ, as well as the parameters of the financial market, i.e. the possible returns and the probabilities for the possible states. Since many different parameters are involved, we first look at different special cases in order to isolate the different effects of the parameters. As we have seen above, a lower loss aversion coefficient β favors the occurrence of the disposition effect. In this section we focus on the impacts of the parameter of the decision weighting function γ and the coefficient of risk aversion α. We assume that the investor is loss averse.

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[9]Tversky and Kahneman have estimated the value of γ to be 0.61 if gains are involved and 0.69 when losses are involved. For simplicity we take the same value for gains and losses and set $\gamma = 0.65$. Again, for other parameter values similar results are obtained.
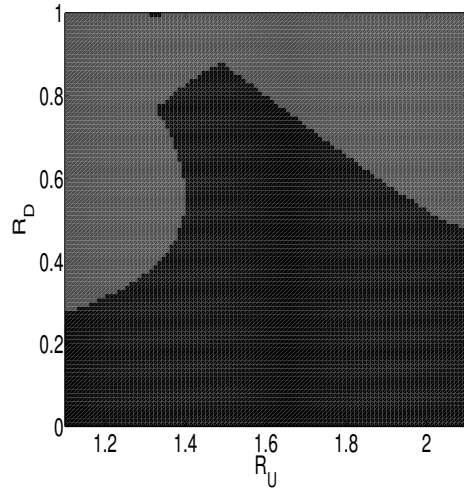
Figure 5: Return combinations for which the ex-post disposition effect arises. The values of R_D vary between 0 and 1 and the R_U is varied between 1.1 and 2.1. The value of the gross risk free rate, R_f, is kept constant at 1.1 and the probability of the occurrence of the up-state, p is fixed at 0.5. The loss aversion coefficient β is kept constant at 2.25 and the coefficient for risk aversion α equals 0.88. The parameter of the decision weights γ is fixed at 0.65. The parameter combinations, where the ex-post disposition effect occurs are marked with black color. The ex-post disposition effect occurs in about 59% of the cases.

To get more insights, we vary the two parameters in the following way: the parameter of the decision weighting function γ is either fixed at 1, so that the investor weights the outcomes with the objective probabilities or it is assumed to be between 0 and 1. When the coefficient of risk aversion α is fixed, it is kept constant either at 0, implying that the investor is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, or at 1, where the agent is risk neutral. Otherwise it is assumed to be between 0 and 1. This yields six possible situations. The more restriction we impose on the preference parameters, the more tractable the inequalities describing the agents choices become. Allowing for more general parameter ranges often has the negative consequence that no analytical statements can be made, so that we have to provide numerical solutions.

Proposition 2 summarizes the results for the cases where analytical statements can be made. The detailed proofs can be found in the appendix.

Proposition 2. *The true disposition effect*

1. *An investor who weights outcomes with their objective probabilities and is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, i.e. $\gamma = 1$ and $\alpha = 0$, never is prone to the disposition effect.*

2. *A risk neutral investor, who weights outcomes with their objective probabilities, i.e. $\gamma = 1$ and $\alpha = 1$, is never prone to the disposition effect.*

3. *An investor who weights outcomes with the decision weights as proposed by Tversky and Kahneman (1992) and is who quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, i.e. $0 < \gamma < 1$ and $\alpha = 0$, never is prone to the disposition effect.*

An investor who weights outcomes with their objective probabilities and who is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, never invests in the risky asset in $t = 0$ implying that he cannot be prone to the disposition effect.

A risk neutral investor, who weights outcomes with their objective probabilities never is prone to the disposition effect because he either does not prefer the stock in $t = 0$ or , if he invests in the risky asset in the first period, after a gain, he will prefer to hold the stock in the second period.

An investor who weights outcomes with the decision weights as proposed by Tversky and Kahneman (1992) and who is quite risk averse in the domain

of gains and quite risk-seeking in the domain of losses, never invests in the risky asset in $t = 0$ implying that he is not prone to the disposition effect.

For the other combinations of α and γ no unambiguous conclusions can be drawn. Therefore we provide a numerical analysis.

To illustrate the situation where $\gamma = 1$ and $0 < \alpha < 1$ we present Figure 6. It shows the parameter combinations for which the disposition effect arises for different returns of the risky asset, R_D and R_U. In the following graphics the value of the gross risk free rate, R_f, is kept constant at 1.1 and and the probability of the occurrence of the up-state, p is fixed at 0.5. The values of R_D vary between 0 and 1 and the R_U is varied between 1.1 and 2.1. For other values of p and R_f similar results are obtained. The loss aversion coefficient β is kept constant at 2.25 and the coefficient for risk aversion α equals 0.88. The parameter of the decision weights γ is fixed at 1. These values correspond to the empirical findings of Tversky and Kahneman (1992). [10] The parameter combinations, where the disposition effect occurs are marked with black color, whereas the domains, where the conditions for the disposition effect are violated are marked with grey color. In Figure 6 we see that the disposition effect almost never occurs, in fact overall it occurs in less than 0.5% of the cases (see Table 1 below).

To illustrate the situation where $0 < \gamma < 1$ and $\alpha = 1$ we present Figure 7. It shows the parameter combinations for which the disposition effect arises for different returns of the risky asset, R_D and R_U. Except for α and γ the same parameter values as above are used. The parameter combinations, where the disposition effect occurs are marked with black color, whereas the domains, where the conditions for the disposition effect are violated are marked with grey color. In Figure 7 we see that the disposition effect occurs very rarely, in less than 0.5% of the cases. We observe it for very high returns in the down state and returns in the up-state of the order 1.3. We can conclude that the disposition behavior for an agent that is described with parameters consistent with the empirical findings of Tversky and Kahneman (1992) and $\alpha = 1$ is a very special case and does not occur in general.

To illustrate the general case, i.e the situation where $0 < \gamma < 1$ and $0 < \alpha < 1$, we present Figure 8. It shows the parameter combinations for which the disposition effect arises for different returns of the risky asset, R_D and R_U. Except for α and γ the same parameter values as above are used. The parameter combinations, where the disposition effect occurs are

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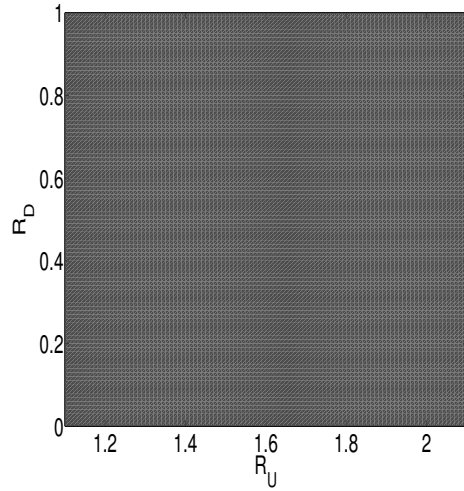[10] Again, for other parameter values similar results are obtained.

Figure 6: Parameter combinations for which the disposition effect arises for different returns of the risky asset, R_D and R_U. The values of R_D vary between 0 and 1 and the R_U is varied between 1.1 and 2.1. The value of the gross risk free rate, R_f, is kept constant at 1.1 and the probability of the occurrence of the up-state, p is fixed at 0.5. The loss aversion coefficient β is kept constant at 2.25 and the coefficient for risk aversion α equals 0.88. The parameter of the decision weights γ is fixed at 1. The parameter combinations, where the disposition effect occurs are marked with black color. The disposition effect occurs in less than 0.5% of the cases.
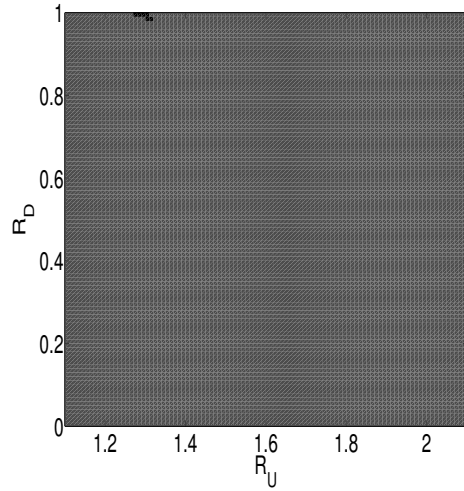
Figure 7: Parameter combinations for which the disposition effect arises for different returns of the risky asset, R_D and R_U. The values of R_D vary between 0 and 1 and the R_U is varied between 1.1 and 2.1. The value of the gross risk free rate, R_f, is kept constant at 1.1 and the probability of the occurrence of the up-state, p is fixed at 0.5. The loss aversion coefficient β is kept constant at 2.25 and the coefficient for risk aversion α equals 1. The parameter of the decision weights γ is fixed at 0.65. The parameter combinations, where the disposition effect occurs are marked with black color. The disposition effect occurs in less than 0.5% of the cases.
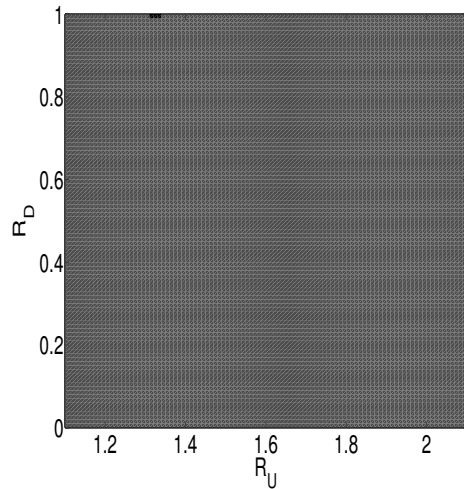
Figure 8: Parameter combinations for which the disposition effect arises for different returns of the risky asset, R_D and R_U. The values of R_D vary between 0 and 1 and the R_U is varied between 1.1 and 2.1. The value of the gross risk free rate, R_f, is kept constant at 1.1 and the probability of the occurrence of the up-state, p is fixed at 0.5. The loss aversion coefficient β is kept constant at 2.25 and the coefficient for risk aversion α equals 0.88. The parameter of the decision weights γ is fixed at 0.65. The parameter combinations, where the disposition effect occurs are marked with black color. The disposition effect occurs in less than 0.5% of the cases.

marked with black color, whereas the domains, where the conditions for the disposition effect are violated are marked with grey color. In Figure 8 we see that the disposition effect occurs very rarely, in less than 0.5% of the cases. We observe it for very high returns in the down state and returns in the up-state of the order 1.3. We can conclude that the disposition behavior for an agent that is described with parameters consistent with empirical findings of Tversky and Kahneman (1992) is a very special case and does not occur in general.

To gain more insight on the different drivers of the disposition effect we present Figure 9, where we take a preference oriented view. We present the cases where the disposition effect in the general case occurs in dependence of risk aversion α and loss aversion β; α ranges from 0 to 1 and β from
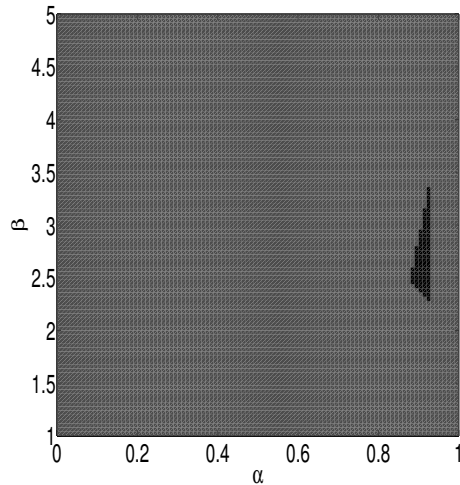
25

Figure 9: Parameter combinations for which the disposition effect arises in dependence of risk aversion α and loss aversion β; α ranges from 0 to 1 and β from 1 to 5. The market parameters are fixed for the case where we observed the disposition effect, i.e. $p = 0.5$, $R_U = 1.32$, $R_f = 1.1$, $R_D = 0.99$ and $\gamma = 0.65$.

1 to 5. The market parameters are fixed for the case where we observed the disposition effect, i.e. $p = 0.5$, $R_U = 1.32$, $R_f = 1.1$, $R_D = 0.99$ and $\gamma = 0.65$. Again we observe that the disposition effect occurs only for a very small part of the possible parameters and cannot be considered a systematic phenomenon.

4   Discussion

We have shown that the disposition effect arises rather for lower coefficients of loss aversion , i.e. lower β and that if $R_U R_D > 1$ the same is true for the ex-post disposition effect. If however, $R_U R_D < 1$, i.e. whenever the agent cannot undo the first period loss by investing in the risk free bond, the ex-post disposition effect arises rather for more loss averse investors.

Concerning the impact of α and γ on the ex-post disposition effect we found the following results. An investor who weights outcomes with the

objective probability and is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, i.e. $\gamma = 1$ and $\alpha = 0$, is prone to the ex-post disposition effect whenever $R_f R_D < 1$. The reason is that in $t = U$ the agent is in the gain zone and hence quite risk averse so that he never prefers the risky stock. Further, if $R_f R_D > 1$ the investor has the opportunity to realize a sure gain in $t = D$ and therefore prefers to invest in the risk free bond. However, if $R_f R_D < 1$, the investor is in the loss zone and consequently he is quite risk seeking and hence he buys the risky asset. If he can undo the first period loss, i.e. if $R_U R_D > 1$, the respective behavior is called get-even-itis. Note that this is true even when the investor is not loss averse. In absence of arbitrage, the risk neutral investor who weights the outcomes with their objective probabilities is prone to the ex-post disposition effect whenever after a first period loss the agent cannot undo this loss, i.e. $R_U R_D < 1$ and hence $R_f R_D < 1$, and the probability of the occurrence of the good state is bounded by ϕ_4 from above and by ϕ_1 from bellow. This is the situation where the stock has a very high downside risk. We emphasize that even for a risk neutral agent the ex-post disposition effect arises, however only for restricted parameter values. We found that for the investor who weights outcomes with the objective probability and is characterized by $0 < \alpha < 1$, the ex-post disposition effect occurs rarely, i.e. in about 12% of the cases (see Table 1 below). We observe it for moderate and low returns in the down state and high returns in the up-state. An investor who weights outcomes with the decision weights as proposed by Tversky and Kahneman (1992) and who is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, i.e. $0 < \gamma < 1$ and $\alpha = 0$, is prone to the ex-post disposition effect whenever $R_f R_D < 1$. The reason is that in $t = U$ the quite risk averse investor never prefers the stock. Further, if $R_f R_D > 1$ the investor has the opportunity to realize a sure gain in $t = D$ and therefore prefers to invest in the risk free bond. However, if $R_f R_D < 1$ the investor is in the loss zone and is there for quite risk-seeking, investing therefore in the risky asset. Note that this result is the same as in the situation where $\gamma = 1$. For the investor characterized by $0 < \gamma < 1$ and $\alpha = 1$ we present numerical solutions. We observe the ex-post disposition effect for moderate and low returns in the down state, in about 50% of the cases. For the investor characterized by $0 < \gamma < 1$ and $0 < \alpha < 1$ we observe the ex-post disposition effect for moderate and low returns in the down state in about 59% of the cases. We can conclude that the ex-post disposition behavior for an agent that is described with parameters consistent with empirical findings of Tversky and

27

Kahneman (1992) does in general occur for risky assets with a high downside risk.

The impacts of α and γ on the occurrence of the disposition effect are summarized in the following paragraph. An investor who weights outcomes with their objective probabilities and is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, never invests in the risky asset in $t = 0$ implying that he cannot be prone to the disposition effect. A risk neutral investor, who weights outcomes with their objective probabilities also never is prone to the disposition effect because he either does not prefer the stock in $t = 0$ or , if he invests in the risky asset in the first period, after a gain, he will prefer to hold the stock in the second period. For the investor characterized by $\gamma = 1$ and $0 < \alpha < 1$ we observe that the disposition effect never occurs. An investor who weights outcomes with the decision weights as proposed by Tversky and Kahneman (1992) and who is quite risk averse in the domain of gains and quie risk-seeking in the domain of losses never invests in the risky asset in $t = 0$ implying that he is not prone to the disposition effect. For the investor characterized by $0 < \gamma < 1$ and $\alpha = 1$ we observe that the disposition effect occurs very rarely, i.e in less than 0.5% of the cases (see Table 1 below). For the investor characterized by $0 < \gamma < 1$ and $0 < \alpha < 1$, we also observe that the disposition effect occurs very rarely, so that we can conclude that the disposition behavior for an agent that is described with parameters consistent with empirical findings of Tversky and Kahneman (1992) is a very special case and does not occur in general.

We summarize these results in Table 1. We quantify the occurrence of the (ex-post) disposition effect for the following parameter values: $p = 0.5$, $R_U \in [1.1, 2.1]$ $R_f = 1.1$ and $R_D \in [0, 1]$. If no other parameter values are assumed, then $\alpha = 0.88$, $\beta = 2.25$, and $\gamma = 0.65$.

The agent being quite risk averse in the domain of gains and quite risk-seeking in the domain of losses is prone to the ex-post disposition effect whenever he cannot undo his first period loss by investing in the risk free bond. This result is very intuitive and is independent of the value of γ. However, this investor, because he is quite risk avers in the domain of gains and has the possibility to make a sure gain in $t = 0$ never invests in the risky asset, indecently of his loss aversion. From this it follows that he cannot be prone to the disposition effect.

The risk neutral investor, who weights outcomes with their objective probabilities, although being prone to the ex-post disposition effect when being in the loss zone, is never prone to the disposition effect. The reason is that

		Disposition Effect	Ex-Post Disposition Effect
$\gamma = 1$,	$\alpha = 0$	Never	If $R_f R_D < 1$, (90%)
$\gamma = 1$,	$\alpha = 1$	Never	If $R_U R_D < 1$, (6%)
$\gamma = 1$,	$0 < \alpha < 1$	< 0.5%	13%
$0 < \gamma < 1$,	$\alpha = 0$	Never	If $R_f R_D < 1$, (90%)
$0 < \gamma < 1$,	$\alpha = 1$	< 0.5%	50%
$0 < \gamma < 1$,	$0 < \alpha < 1$	< 0.5%	59%

Table 1: Summary of Results. We quantify the occurrence of the (ex-post) disposition effect for the following parameter values: $p = 0.5$, $R_U \in [1.1, 2.1]$ $R_f = 1.1$ and $R_D \in [0, 1]$. If no other parameter values are assumed, then $\alpha = 0.88$, $\beta = 2.25$, and $\gamma = 0.65$.

he either does not prefer the stock in $t = 0$ or , if he invests in the risky asset in the first period, after a gain, he will prefer to hold the stock in the second period. Note that this statement does not hold for an agent that is not loss averse.

For the other investors, we observe very similar results: they are generally prone to the ex-post disposition effect, but hardly to the ex-ante disposition effect, independently from the parameter values of γ and α. These results are confirmed in Figure 9, where we take a preference parameter oriented view and observe that the disposition effect occurs only in very restricted areas of the α-β room.

Other numerical analyses, we do not show here, confirm that the ex-post conditions are satisfied more often than conditions for the disposition effect and that the differences can be quite substantial. Further, the ex-post disposition effect occurs more often for low γ, i.e. the stronger the departure from the weighting by objective probabilities is. The conditions to sell a winning stock is satisfied more often for lower β, since a lower loss aversion implies higher risk-taking in the first period and because in $t = U$ the decision often is independent of loss aversion. We find no systematic influence of γ and α on this ex-ante condition. The condition to keep on holding a losing stock is in general more often satisfied for attractive stocks, i.e. when the probability of the up-state is high and the risky stock offers high returns. Further it is satisfied more often for low loss and risk aversion.

The condition is more often satisfied for higher values of γ. This shows that the effects of the different variables work often in different directions in the divers conditions that have to be satisfied simultaneously.

Similar results are obtained for other forms of value functions, as e.g. the piece-wise exponential function. For preference parameter values that approximate best the empirical evidence found by Tversky and Kahneman[11] and market parameter parameters as used above, we found that the ex-post disposition effect occurs in 59% of the cases, whereas the true disposition effect occurs in less than 0.5%. Note that if the investor did decide his investment decision with a fair coin then we would observe the ex-post disposition effect in 25% of the cases, while the true disposition effect would occur in 12.5% of the cases. Hence comparing to this benchmark, our results shows a tendency for the ex-post disposition effect but against the true disposition effect.

Moreover, introducing editing rules of prospect theory, as e.g. segregation, does not change the results substantially. For the parameter values used above, we found that the ex-post disposition effect occurs in 65% of the cases and the true disposition effect in less than 0.5%. Finally, requiring dynamic instead of myopic optimization makes the risky asset more attractive in the first period because one anticipates to optimally react to the future course of events. However, whenever the agent prefers to invest in the risky asset in the first period, he prefers to keep it after its price appreciated. In this case the true disposition effect also occurs in less than 0.5% of the parameter combinations.

Hence we have shown that various approaches, incorporating different types of value functions and editing rules, have difficulties to model the disposition effect. This suggests that in order to explain the disposition effect one must depart from the traditional forward looking optimization paradigm in a more radical way than replacing the von Neumann-Morgenstern utility function in the expected utility paradigm by the value function of prospect theory.

A possible alternative explanation could be to model the disposition effect as a consequence of a backward looking optimization. Given the past investment decision, the agent transforms the outcome such that he gets the highest utility: if the investment decision is successful, the agent realizes his

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[11] For a discussion and the concrete parameter values we refer the reader to DeGiorgi, Hens, and Levy (2005).

30

gain, i.e. he transforms the outcome to a realized gain. If he incurs a loss, he keeps the outcome as a paper loss, i.e. he keeps holding the asset. One could model such a behavior using two mental accounts, one for realized gains and losses and the other for paper gains and losses. Clearly in such a model the positions in the paper account have less weight than the ones in the realized account: paper losses hurt less than realized losses and realized gains give more utility than paper gains. Hence behavior consistent with the disposition effect makes the best out of a given investment decision. Note that for this argument neither loss aversion nor asymmetric risk aversion is needed since it is sufficient to assume that the utility of a gain is positive while that of a loss is negative. However this behavior is not forward looking because the resulting asset allocation may not be optimal in the future. This explanation corresponds to the story told by Gross (1982), page 150: *Investors who accept losses can no longer prattle to their loved ones, "Honey, it's only a paper loss."*

5 Conclusions

In the literature the disposition effect is explained by two main features of prospect theory, namely that decision-makers frame their choices in terms of potential gains and losses and that they maximize an S-shaped value function, which is concave for gains and convex for losses. The argument is often made without considering loss aversion. As we have shown, the assumption of no loss aversion favors the occurrence of the disposition effect. However, even for investors that are not loss averse, the disposition behavior is rather a rare result. Further, in the standard argument, it is generally *assumed* that the investor has bought the risky stock in the first place. Therefore, the issue whether the investor really will decide in this way is neglected. This implies that the standard argument is in fact an ex-post argument. Our model shows that the inter-temporal disposition behavior occurs only for very restricted parameter values. In general, the model predicts that those investors who sell winning stocks too early and keep losing stocks too long would in the first place not have invested in stocks. We conclude that prospect theory can indeed explain the ex-post disposition behavior, but not the more complete and inter-temporal definition of the disposition behavior. Possible alternative explanations for the disposition effect could include mental accounting combined with backward looking optimization.

A Appendix

A.1 Proof of Proposition 1

1. We analyze the two conditions for $t = 1$ for the parameter combination $\gamma = 1$ and $\alpha = 0$. In the first case, where $R_f R_D > 1$ and the second case, where $R_U R_D > 1$ and $R_f R_D < 1$, the condition to sell the asset after a gain yields

$$p + (1 - p) \leq 1, \tag{13}$$

which is satisfied for all $0 < p < 1$. The condition for the investor to prefer the risky asset in $t = D$ in the first case yields

$$-(1 - p)\beta \geq 1 - p, \tag{14}$$

which yields a contradiction for all $\beta \geq 1$ and $0 < p < 1$, so that no ex-post disposition effect occurs. In the second case the condition yields

$$p \geq -p\beta, \tag{15}$$

which is satisfied for all $\beta \geq 1$ and $0 < p < 1$, so that the ex-post disposition effect does arise. In the third case, where $R_U R_D < 1$, the condition to sell the winning stock yields

$$-(1 - p)\beta \leq 1 - p, \tag{16}$$

which is satisfied for all $\beta \geq 1$ and $0 < p < 1$, and to hold a loosing stock yields

$$p + (1 - p) \leq 1, \tag{17}$$

which is satisfied for all $0 < p < 1$, so that the ex-post disposition effect does arise.

2. For the first case, where $R_f R_D > 1$, the ex-post condition is satisfied whenever

$$\phi_1 \geq p \geq \phi_2$$
$$\text{where} \quad \phi_1 = \frac{R_f - R_D}{R_U - R_D},$$
$$\phi_2 = \frac{R_f R_D - 1 + \beta(1 - R_D R_D)}{R_U R_D - 1 + \beta(1 - R_D R_D)}.$$
(18)

In absence of arbitrage and for all $\beta > 1$ it follows that $\phi_2 > \phi_1$, so that this conditions is never satisfied.[12] For the case, where $R_U R_D > 1$ and $R_f R_D < 1$, the ex-post disposition effect arises whenever

$$\phi_1 \geq p \geq \phi_3$$
$$\text{where} \quad \phi_3 = \frac{\beta R_D (R_f - R_D)}{R_U R_D - 1 + \beta(1 - R_D R_D)}.$$
(19)

Note that in absence of arbitrage and for all $\beta > 1$ it follows that $\phi_3 > \phi_1$, so that this condition is never satisfied. For the case, where $R_U R_D < 1$, the ex-post disposition effect arises whenever

$$\phi_4 \geq p \geq \phi_1$$
$$\text{where} \quad \phi_4 = \frac{R_U R_f - 1 + \beta(1 - R_U R_D)}{R_U R_U - 1 + \beta(1 - R_U R_D)}.$$
(20)

Note that in absence of arbitrage and for all $\beta > 1$ $\phi_3 > \phi_1$.

3. In the first case, where $R_f R_D > 1$ the agent prefers to invest his wealth in $t = U$ in the risk free asset if

$$w(p) + w(1 - p) \leq 1,$$
(21)

which is true for all $0 < \gamma < 1$ and $0 < p < 1$. The condition to prefer to invest in the risky asset in $t = D$ yields

$$-w(1 - p)\beta \geq 1 - w(p),$$
(22)

---

[12]Note that for an investor that is not loss avers, i.e. $\beta = 1$, $\phi_2 = \phi_1$ for all parameters, so that the investor is prone to the ex-post disposition effect in the special case where $p = \phi_2 = \phi_1$.

which yields a contradiction for all $\beta \geq 1$ and $0 < w(x) < 1$. So that no ex-post disposition effect occurs. In the second case, where $R_U R_D > 1$ and $R_f R_D < 1$, the agent prefers to invest his wealth in $t = U$ in the risk free asset if

$$w(p) + w(1-p) \leq 1, \tag{23}$$

which is true for all $0 < \gamma < 1$ and $0 < p < 1$. The condition to prefer to invest in the risky asset in $t = D$ yields

$$w(p) \geq \big(w(1-p) - 1\big)\beta, \tag{24}$$

which is satisfied for all $\beta \geq 1$ and $0 < w(x) < 1$. So that the ex-post disposition effect occurs in this case. In the third case, where $R_U R_D < 1$, the agent prefers to invest his wealth in $t = U$ in the risk free asset if

$$-w(1-p)\beta \leq 1 - w(p), \tag{25}$$

which is true for all $\beta \geq 1$ and $0 < w(x) < 1$. The condition to prefer to invest in the risky asset in $t = D$ yields

$$w(p) + w(1-p) \leq 1, \tag{26}$$

which is true for all $0 < \gamma < 1$ and $0 < p < 1$. So that the investor behaves from an ex-post perspective as a disposition investor whenever the investor makes a sure loss investing in the risk free asset in $t = D$.

$$\square$$

A.2   Proof of Proposition 2

1. For the parameter combination $\gamma = 1$ and $\alpha = 0$ the condition to invest in the risky asset $t = 0$ writes:

$$-(1-p)\beta \geq 1 - p, \tag{27}$$

which is a contradiction for all $0 < p < 1$ and $\beta \geq 1$, since the left hand side is negative. Therefore the quite risk averse investor who weights outcomes with their objective probability never invests in the risky asset in $t = 0$ implying that he cannot be prone to the disposition effect.

2. For the parameter combination $\gamma = 1$ and $\alpha = 1$ in the first case, where $R_f R_D > 1$, the condition that the investor buys the stock in the first period and sells it after a gain yields

$$
\begin{aligned}
p(R_U - 1) - (1-p)\beta(1 - R_D) - R_f + 1 \geq 0, \\
p(R_U R_U - 1) + (1-p)(R_U R_D - 1) - R_U R_f + 1 \leq 0.
\end{aligned}
\tag{28}
$$

These conditions cannot be satisfied simultaneously since combining them yields $(1-p)(\beta - 1)(R_D - 1) \geq 0$ which is a contradiction for all $0 < p < 1, \beta > 1$ and $R_D < 1$[13]. For the case, where $R_U R_D > 1$ and $R_f R_D < 1$ the conditions for the investor to buy the risky asset in $t = 0$ and to sell it after a gain, are the same as in the case, where $R_U R_D > 1$ and $R_f R_D > 1$.

For the case, where $R_U R_D < 1$ and $R_f R_D < 1$, the condition that the investor buys the stock in the first period and sells it after a gain yields $(1-p)(\beta - 1) \leq 0$ which is a contradiction for all $0 < p < 1$ and $\beta > 1$.

3. For the parameter combination $0 < \gamma < 1$ and $\alpha = 0$ the condition for $t = 0$ writes:

$$
-w(1-p)\beta \geq 1 - w(p),
\tag{29}
$$

which is a contradiction for all $0 < w(p) < 1$ and $\beta \geq 1$. So that the quite risk averse investor never invests in the risky asset in $t = 0$ implying that he is not prone to the disposition effect.

$\square$

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[13]Note that an investor who is not loss averse, i.e. $\beta = 1$, would buy the stock in the first period and sell it after a gain.

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