

Coordination of Limited Commercial Return

Jens Bengtsson
Department of Finance and Management Science
Norwegian School of Economics and Business Administration
Helleveien 30
N-5045 Bergen
Norway
jens.bengtsson@nhh
Tel: +47 55 95 94 25

Abstract

This paper analyses coordination in a supply chain consisting of a supplier and a retailer, where the retailer has the opportunity to return products at midlife and end-of-life. The paper examines particularly the coordination problem when the supplier has the opportunity to realise a limited amount of overstock items at a higher price than the retailer at midlife. In this paper return options are introduced in the channel, where an option gives the holder the right to return a product at midlife in exchange for a pre-specified amount of money. It is shown that the supplier must, to achieve coordination, determine one exercise price of the options and two return rebates, where the latter guarantee the retailer an amount of money for each product returned, at midlife and end-of-life, without a corresponding option. Conditions for pricing of return options and conditions for wholesale prices are derived as well. A numerical study shows that the supplier is better off when the number of return options increases. The numerical study also shows that the coordinating option price is relatively unaffected by the return rebate in the second period, but is more dependent on the exercise price and return rebate at midlife, which in turn are dependent on the production costs.

1 Introduction

This paper presents a model and principles, which coordinate a single supplier - single retailer channel where the supplier has the opportunity to extract higher values than the retailer from excess inventory. Specifically, the case when the supplier has two different recovery modes, one of higher value but limited and one of lower value but unlimited, is highlighted. In order to achieve coordination a return policy is used and return options are introduced since another degree of freedom is required. Return policies as such is nothing new but have been treated earlier as a potential way to e.g. achieve coordination, mitigate risk between supplier and retailer, and safeguard the brand (Pasternack 1985, Padmanabhan and Png 1995). During the last decade the issue of return has also become a matter at the management's agenda for another reason, namely due to environmental legislation and its being as an economic driver. These have also led to extensive activities both among researchers and practitioners, and many of the questions are collected under the reverse logistics subject. Following Thierry et al (1995), the recovery options available for returned products can be distinguished between, direct reuse (resale), product recovery management, and waste management. The most suitable option will be a question of priorities between aspects such as product condition and the product's life cycle stage. In businesses such as fashion or commercial electronics where life cycles are countered in months the alternative uses of returned products and the possibility to recover value are deteriorating in a rapid pace over time. This fact is highlighted in Souza et al (2003) and Blackburn et al (2004). Blackburn et al exemplify this by pointing at the fact that personal computers may lose value at rates excess of 1 % per week during its life cycle, and that the rate increases as the product nears the end of its product life cycle. Under such circumstances rapid return and recovery is a must in order to achieve profitability out of returned products.

The possibility to use e-commerce has significantly increased the flora of opportunities available to the manufacturers and suppliers to recover values from returned items. Products and components can for example, after have been checked, reconditioned or refurbished, either be sold through the company's own distribution channels or be sold to other external distributors, who may e.g. auction them on the Internet (see e.g. www.liquidation.com, www.overstock.com, www.qxl.com). Other alternatives are that components may be used as spare parts (see e.g. Fleischman et al 2003), or reused in new products. The direct economic gains from reverse logistics are less input materials, reduced costs, and so-called value added recovery (i.e. repair and remanufacture), see e.g. de Brito and Dekker (2004). From a management point view all this implies that there are opportunities, where the value of products and components can be recovered if it is shown that sales through the regular market channels has not been as successful as expected.

Because of the fact that many products do have short lifecycles and long production lead times, production takes place before the selling season begins. In a competitive environment with frequent introduction of new products and radical price reduction during a product's life cycle, it is critical to companies' profitability to quickly identify alternatives to recover value out of products that are expected to be unsold at their present location. It is also critical that the products return quickly to the supplier, or to another facility which handles return, where its alternative use may be decided upon. Moreover, to the supplier, and the supply chain as a whole, uncontrolled return may not be desirable because of high transportation costs and capacity constraints in handling and inventory. Higher costs and constraints result in lower profit margin and in order to avoid uncontrolled return incentives should be introduced in the relationship between the supplier and the retailer. In order to achieve an efficient return handling process it is desirable that the uncertainties regarding; point in time when returns arrive, quality of returned

products, and the quantity of return, are reduced. Guide (2000) gives evidence that companies control quantity, quality and timing.

Due to loss in value over time it is desirable to be as fast as possible in the recovery activities. However, capacity limitation in recovery activities may cause an unwanted loss in recovered value in that products may not be returned to its alternative market place, or use, as fast as wanted. In practice this will be a continuous process, but in this paper a simplified model is developed in order to analyse coordination. In this paper it is assumed that we have two different salvage values for return at midlife and one salvage value at end-of-life. The two different salvage values at midlife represent the higher value from the products recovered before the capacity hits its limitations and the second value represents the value of those products recovered after that capacity has become available again. The salvage value at end-of-life is assumed to be undifferentiated because of its position in the product life cycle where difference in recovered value is assumed to be insignificant. The return at midlife can be seen as a commercial return in that the products are still in their current production life cycle, whereas end-of-life returns may be a generation or more removed from the latest products, features and technologies, cf Souza et al (2003).

In order to control the return, with respect to point in time, quantity, quality, and to have the possibility to achieve coordination, options are introduced in the chain. An option gives the retailer the right to return a new product, or an as-good-as new, in exchange for a pre-specified amount of money, at a pre-specified point in time. Thus, there will also be clear financial incentive to return products, cf. Guide and van Wassenhove (2001)

In this paper two models have been developed, one where decisions in the centralised system are modelled, and one which models the decision in the decentralised system where the supplier is

the leader in a Stackelberg game. From there, conditions on wholesale prices, exercise prices, return rebates have been derived and a principle for determining option prices is presented, in order to achieve a coordinated decentralised chain under the condition with piece-wise linear salvage value.

The organisation of the paper is as follows. First there is a literature review on related works. Second, notations and assumptions are given. Third, the optimal decisions at midlife are treated with respect to the dynamics in demand and the differences in salvage values in the channel. Forth, the coordination issue is treated and conditions for coordination are derived. Fifth, a numerical analysis is performed and commented. Finally, there are a summary and ideas for further research.

2 Literature review

A model often used when analysing return policies and inventory decision is the extended newsvendor model. In the newsvendor model optimal order quantities are determined from a trade-off between sales price, cost of acquiring a product, inventory holding cost, penalty costs for not satisfying demand, and the salvage value of overstock items. Even though the basic model is simple it is widely used in order to highlight and analyse different aspects and perspectives of decisions concerning inventory and optimal order quantities, see e.g. Petruzzi and Dada (1999). In the literature on coordination it is often assumed that the supply contract is a model based on a Stackelberg game where the supplier is the leader. The supplier is thereby the party that determines the space of possible outcomes of the chain. Under these circumstances the retailer faces a newsvendor problem where the supplier is in charge of setting at least the wholesale price. Dependent on the types of clauses in the contract the supplier may also have the power to affect salvage values and this is the case when there is a return policy.

Return policies are not the only aspects that have been considered in order to achieve coordinating contract. Monahan (1984), Lee and Rosenblatt (1986), and Jeuland and Shugan (1983) analyse how quantity discounts can be used to achieve coordination. Moorthy (1987) suggests that a two-part tariff should be used instead of quantity discounts to achieve coordination since optimal decisions are separated from profit sharing. Later, Weng (1995) analyses coordination by means of quantity discount and a franchise fee. An extensive review of supply contract modelling is found in Tsay et al (1998).

Pasternack (1985) analyses return policies from a channel coordinating perspective in a single period model. It is assumed that retailer and supplier have identical salvage values and that returns are frictionless, i.e. there are no costs associated with the return. Pasternack examines the pricing decision of the supplier, who is the Stackelberg leader, and analyses for which prices and returns rebates coordination is achieved. The retailer is assumed to be a price-taker since it has no opportunity to affect neither wholesale prices nor market prices. The main results are that i) unlimited return and full refund, i.e. rebate is equal to wholesale price, is suboptimal to the system in that coordination is not achieved, ii) not to offer any return at all is not optimal to the system, iii) unlimited return to partial rebate is optimal given a correct combination of wholesale price and rebates, iv) there exists a continuum of combination of return policies, i.e. combinations, of wholesale price and rebate, which coordinates the channel and are independent of the distribution of demand, v) coordination can be achieved in a single period model when returns are limited but a precondition is that the retailer and the supplier dispose the products at identical values. Kandel (1996) extend Pasternack to incorporate the effect of price-sensitive end customers, i.e. the retailer has influence on end customers' demand. Emmons and Gilbert (1998) also examine a single period model with possibility to return unsold products and where the retailer determines both optimal order quantity and price paid by the customer. Like Kandel they come to the conclusion that coordination cannot be achieved unless the supplier determines the

resale price. However, Emmons and Gilbert show that there exists a range of wholesale prices where both the supplier and the retailer are better off compared to the case of no policy, i.e. return is not allowed.

Donohue (2000) analyses incentive contracts for coordination in a two period setting but, however, under a single point demand occurrence. As well as in Pasternack, it is assumed that retailer and supplier are equally effective in liquidating overstock. Donohue's model considers a situation where demand information is upgraded during the time until demand is realised. Two production modes are available, the first is a cheaper mode with longer lead time and the second is more expensive but is on the other faster since lead time are shorter. The existence of the more expensive and faster mode allows for fine-tuning of ordered quantity as information regarding demand is updated. After that demand is realised there is also an opportunity to the retailer to return unsold products to the supplier. Donohue analyses coordination using three parameters, i.e. wholesale prices for the first and second production mode and return rebate, and shows like Pasternack that there will be several contracts that implies coordination.

Tsay (1999) models quantity flexibility contracts, which can be interpreted as a kind of return contracts. The retailer guarantees not to go below a given percent from the forecast given to the supplier. On the other hand the supplier promises to deliver products up to a given percent above the forecast. One of Tsay's objective is to describe how different control parameters affects the economic outcome but he also shows that under certain conditions this quantity flexible contract is enough to achieve coordination.

Barnes-Schuster et al. (2002) build a general framework in order to study options in supply contracts and establish sufficient conditions for channel coordination. A two-period model is examined and stochastic demand, unaffected of market prices, occurs at two points in time and

demand may be correlated. At the beginning of the horizon the retailer places firm orders, which are products to be delivered in period one and two. In addition the retailer also determines, at the same point in time, how many options to reorder a product that should be acquired. An option gives the holder the right to reorder an extra product at the same time as demand for the first period is realised. Barnes-Schuster et al. extend the work of Pasternack (1985) and Donohue (2000) since they allow for different salvage values between a retailer and a supplier, and generalise the results of Donohue and Pasternack to the two-period case with arbitrary demand correlation. They also show that if return is not frictionless then this will prevent arbitrary distribution of profit between agents in the channel, which in turn may result in violations of the individual rationality conditions. Barnes-Schuster et al find that, in general, channel coordination will only be achieved if the exercise price of the options are allowed to be piecewise linear. The reason for that is to be found in the fact that the supplier has two different production modes, one expensive and one cheap, which thus may result in different marginal costs for same type of product.

Taylor (2001) examines coordination of a channel in a two-period setting where there are possibilities to return products at midlife as well as end-of-life when salvage values are identical between supplier and retailer. In addition the case of return policies combined with price protection if market prices fall is considered. The model allows for unrestricted return and considers two order occasions, midlife and end-of-life. Thus, if the inventory level is too low replenishment is done and if it is too high products are returned. Taylor shows that if market prices are falling then, under proper wholesale prices and return rebates, mid- and end-life return achieve coordination but win-win is not guaranteed. However, if midlife and end-of-life returns and proper prices are used together with price protection then coordination as well as win-win is achieved. If market prices are constant over the periods, midlife and end-of-life returns with proper prices and rebates is enough to achieve channel coordination as well as win-win.

Compared to Barnes-Schuster et al (2002), this paper differs in that it is assumed that the produced quantity and order quantity will not differ and two occasions of product return are examined; midlife and end-of-life. Barnes-Schuster et al. consider reorder, and to some extent, midlife return, but this paper focus on coordination when the supplier has two different salvage values of midlife returns. Compared to Taylor (2001) reorders at midlife are not considered but Taylor's analysis is restricted to the case when the salvage values of the supplier and retailer are identical. No such assumptions are made in this paper but prices, costs, and salvage values are allowed to vary to the extent that the conditions allow.

3 Notations and assumptions

This paper considers a system consisting of a retailer and a supplier in a two-period model where returns of product are allowed at midlife and end-of-life. It is assumed that the supplier and the retailer act according to a Stackelberg game, where the supplier is the leader. In the general case, which allows for limited return possibilities, the supplier determines five different prices for each decision point in time, in the beginning of the horizon. The supplier determines the unit wholesale prices, w_i , for products to be delivered in period $i=1, 2$; the option price, Γ , for the right to return a product at midlife in exchange for the exercise price, x_1 ; the return rebate r_1 , for those returned at midlife without a corresponding option, and the return rebate, r_2 , for those which are returned at end-of-life. Then, from these prices, the retailer decides in the beginning of the horizon, how many products, q_i , to order for delivery in period i , and how many return options m_i to buy. This implies that the retailer faces a single buying occasion where order quantities as well as options are determined for the whole product life cycle of the product. The supplier is obliged to deliver the amount ordered by the retailer and the quantity produced equals the ordered quantity. The cost to the supplier of producing an item to be delivered in period i , is denoted c_i . s_1^h denotes the higher unit salvage value at midlife, and s_1^l denotes the corresponding

lower salvage value. s_2 denotes the salvage value at end-of-life. In practice, products subject to be returned at midlife can, in the first place, be netted against the quantity ordered for the second period. However, it is assumed that other costs will appear in those cases leaving no difference in cost between a physical return and netting. The retailer sells the product to consumers at a retail price p_i , in period i , and it is assumed that the retailer has no possibility to affect prices in such a way that it will affect the demand probability distribution. This assumption is well motivated if e.g. a highly competitive market is considered since under such conditions there will be no possibilities to any retailer to have impact on market prices, i.e. the retailers is a price taker. It is assumed that holding costs, h_i , and shortage costs, k_i , only affect the retailer. In this case it can be motivated by the fact that if the supplier makes to order no inventory is built at the location of the supplier.

Moreover it is assumed that demand not met in the first period is transferred and met in the second period, like in Barnes-Schuster et al (2002). However, demand, which cannot be met, is penalised with a cost of shortage and is sold at the prevailing price in the second period. Penalty cost of demand in the second period is assumed to be zero, $k_2=0$, and is motivated by the reason that it is in the end of a product life cycle thus causing a minor loss in goodwill.

Regarding the statistical demand distributions at midlife and end-of-life, it is assumed that they follow lognormal distributions. This gives that two different approaches can be used in order to evaluate the cash-flow from the contract. In the first case it is assumed that a process representing demand information, D , evolves according to a geometric Brownian motion, and with an initial condition this can be expressed as

$$\begin{aligned} dD &= \beta D dt + \sigma D dW \\ D_0 &= d_0 \end{aligned} \tag{1}$$

D can be seen as representing a forecast for a period's demand, based on accumulated orders and forecast for the remaining time in the period. As time passes by during a period the uncertainty reduces and at the end of the period demand is known. In equation (1) β denotes the drift rate of the process, σ denotes the volatility rate and dW denotes the Wiener increment which follows a normal distribution with mean equal to zero and variance equal to the time increment dt . The geometric Brownian motion gives that the cumulative probability density function for D is given by a lognormal distribution. If it is then assumed that there exists a security or a portfolio or securities at the financial markets which perfectly tracks the stochastic movements in D , then we can use option pricing theory, i.e. adjusting the drift of D and discount the resulting expected cash flow using ρ , i.e. the risk free rate of return. The other approach is to assume that demand is distributed according to a lognormal distribution, without assuming that D follows a geometric Brownian motion, and assume that the retailer and the supplier are risk neutral. This implicates that cash flows are discounted using the risk free rate of return as well but the expected value is not affected, as it is in case of the option pricing approach. Throughout this paper all formulas are written as if D follows a geometric Brownian motion and as if it is possible to find a tracking portfolio. However, using the option pricing approach may require that the drift rate must be adjusted to account for a difference between the drift rate of the security portfolio and the drift rate of the parameter D . This issue is however beyond the scope of this paper and not further dealt with.

4 Optimal decision at midlife

4.1 Modelling the optimal exercise strategy

In this paper the decision at midlife basically concerns what level of inventory that should be kept in the beginning of the second period. The problem that the decision maker faces at midlife has similarities to the classical newsvendor problem but in this case the question concerns how many

options that should be exercised. This implies that the question is how the optimal exercise strategy looks like when there is a limited amount that can be returned at a higher value.

In this paper, only the case of return is of interest whereas the reorder is ruled out due to the fact that it is assumed that lead times are as long such that reordering is no option. In general, the present value of the second period can be written as

$$\pi_2(I) = s_1(I_1 - I) + e^{-\rho t} p_2 I + e^{-\rho t} (-p_2 - b + s_2) \int_0^I (I - d_2) f_D dD \quad (2)$$

when $I_1 \geq I$. I_1 and I denote the current and dispose-down-to level, respectively, and f denotes the probability density function of the demand distribution. Like the standard single period newsvendor model equation (2) is concave in I and differentiating (2) with respect to I and solving for the optimal dispose-down-to level gives that the optimal level I^* is determined from the cumulative density function of the demand distribution, F , and equation (3).

$$F(I^*) = \left(\frac{p_2 - s_1 e^{\rho t}}{p_2 + b - s_2} \right) \quad \text{where} \quad s_1 e^{\rho t} > s_2 - b \quad (3)$$

If $I_1 \leq I^*$, then the optimal action is to do nothing since there is no opportunity to reorder. Furthermore, in this paper the decision maker will face two different salvage values at midlife, the exercise price x , and the return rebate s_r . Lower salvage value implies higher inventory levels and vice versa and the optimal action is stated in theorem 1.

Theorem

Let $x > s$ and let the present value of the retailer $\pi(x)$ be a concave function such that

$$F(I^*) = \left(\frac{p_2 - x e^{\rho t}}{p_2 + b - s_2} \right)$$

solves the optimal dispose-down to level for salvage value equal to x . Then realising overstock at an exercise price x down to a level I^ maximises present value.*

Proof:

If $\pi(x, I)$ is concave in q , then differentiating $\pi(x, I)$ with respect to I , and solving I^* for $\partial\pi(x, I^*)/\partial I = 0$ gives that any $I \neq I^*$ implies $\pi(x, I^*) \geq \pi(x, I)$. Since $x > s$ then $\pi(x, I) > \pi(s, I)$ for any I . This implies $\pi(x, I^*) > \pi(s, I)$.

This implies that if return is unlimited then the retailer should return overstock according to what is optimal determined from the exercise price x , which is according to intuition. Since the number of return possibilities at a higher salvage value is limited it must be considered that its associated optimal level can not be reached in some cases. If the inventory level, after exercising all options lies between the two dispose-down-to inventory levels then nothing more should be done. If the inventory level, after exercising the options is above the dispose-down-to level determined from the lower salvage value, then one should continue to realise overstock items at the lower salvage value until the dispose-down-to level, determined from the lower salvage value, is reached. This type of problem shows similarities to the more common order-up-to and dispose-down-to levels, where items can be reordered at a higher price than the return rebate and where optimal actions are basically the same. Order-up-to and dispose-down-to are used and briefly described in e.g. Taylor (2001) and these issues can also be found in e.g. Zipkin (2000).

4.2 Conditional optimal inventory level at midlife

Because the demand variable is assumed to evolve according to a geometric Brownian motion the realisations of D_t , i.e. d_t follows a lognormal distribution for any t . The expectation and the conditional expectation of D can be written as

$$E[D_t] = D_0 e^{\rho t} \quad \text{and} \quad E[D_T | D_t = d_t] = d_t e^{\rho(T-t)} \quad T > t$$

For reasons of convenience the logarithm of D , which follows a normal distribution, is used. Let $y = \ln D$. Ito's formula gives

$$dy = \left(\rho - \frac{\sigma^2}{2} \right) dt + \sigma dW(t)$$

Similar to equation **Error! Reference source not found.**, the dispose-down-to level in case of fixed salvage prices, centralised system, and lognormal distribution can be expressed as

$$F_{Log}(I_{CS}^*) = \left(\frac{p_2 - s_1 e^{\rho t}}{p_2 + b - s_2} \right) \quad \text{where } s_1 e^{\rho t} > s_2 - b$$

where F_{Log} is the cumulative density function of a lognormal distribution and whose inverse determines the optimal dispose-down-to level. Adjusting the mean and the volatility levels accordingly the optimal level can be expressed in terms of the cumulative density function of the normal distribution, F , whose inverse gives the dispose-down-to level in terms of the logarithms of the absolute level, i.e.

$$F(\ln I_{CS}^*) = \left(\frac{p_2 - s_1 e^{\rho t}}{p_2 + b - s_2} \right) \quad \text{where } s_1 e^{\rho t} > s_2 - b$$

The nature of the process implies that the optimal dispose-down-to-level will be affected by the realisation of demand at midlife, which is unknown at the beginning of the horizon. Using the same approach as Barnes-Schuster et al (2002), but modified in order to work for stochastic variables driven by geometric Brownian motions, enables a desired separation between the realisation of future demand and optimal dispose-down-to levels. Let $\ln I_{CS}^* = \gamma$, then the optimal level γ can be written as

$$\gamma = \ln d + \left(\rho - \frac{\sigma^2}{2} \right) \tau + l^* \sigma \sqrt{\tau} \quad (5)$$

where

$$\Phi(l^*) = \left(\frac{p_2 - s_1 e^{\rho t}}{p_2 + b - s_2} \right) \quad \text{given } s_1 e^{\rho t} > s_2 - b \quad (6)$$

where $\Phi(l^*)$ is a standard normal distribution and τ is the time between two points in time when returns can take place. The first two terms in (6) is the conditional expectation of the logarithm of the demand if demand d is realised at midlife, and the last term is the level l^* and the conditional standard deviation $\sigma\sqrt{\tau}$. However, due to the fact that a geometric Brownian motion drives demand and that the logarithms of demand are considered there is no explicit relationship between the realised demand and the conditional standard deviation.

5 Coordination with return limitation at midlife

5.1 Centralised chain with return limitations

In this section the problem of coordination with return limitations at midlife is analysed. The introduction of limitations in the amount that the supplier can salvage at a higher value complicates the problem a bit compared to the unlimited case.

Using the fact that demand and the dispose-down-to level can be expressed in terms of exponential functions gives that D_t and I can be expressed as

$$D_t = d_0 \exp\left(\left(\rho - \frac{\sigma^2}{2}\right)t + \sigma W(t)\right) \quad \text{and} \quad I = d_1 \exp\left(\left(\rho - \frac{\sigma^2}{2}\right)t + l^* \sigma \sqrt{t}\right).$$

In turn D_t can be expressed in terms of a stochastic variable Z , which is a normal distributed variable with mean $\mu = (\rho - \sigma^2 / 2)t$ and standard deviation $\bar{\sigma} = \sigma\sqrt{t}$ and expressed as

$$D_t = d_0 e^{Z(t)}$$

If the return possibilities are limited then Lagrange optimisation must be used in order to determine optimal order quantities on products and options to return. The constraints in the problem are

$$m \leq M, \quad m \leq \exp(Q_2), \quad 0 \leq m \quad (7)$$

The expression for the present value in terms of unconditional distributions of Z_1 and Z_2 , and the constraints above in (7) gives that the its associated Lagrangean for the centralised system with return limitations can be expressed as in (8). The parameter Q_1 denotes the logarithm of the number of products ordered to cover demand in the first period and Q_2 denotes the logarithm of the aggregate order quantity over both periods.

$$\begin{aligned} L^{CS}(Q_1, Q_2, m, \lambda) = & (p_1 e^{-\rho t} - c_1) \exp(Q_1) + e^{-\rho t} (-p_1 - b_1) \int_{-\infty}^{a_1} (\exp(Q_1) - d_0 \exp(z_{i1})) f_1 dZ_1 \\ & - e^{-\rho t} k_1 \int_{a_1}^{\infty} (d_0 \exp(z_{i1}) - \exp(Q_1)) f_1 dZ_1 + e^{-\rho t} s_1' \int_{-\infty}^{a_6} (\exp(Q_2) - I^{**} - d_0 \exp(z_{i1}) - m) f_1 dZ_1 \\ & + e^{-\rho t} s_1^b \int_{a_7}^{a_2} (\exp(Q_2) - I^* - d_0 \exp(z_{i1})) f_1 dZ_1 - e^{-\rho t} c_2 (\exp(Q_2) - \exp(Q_1)) + e^{-\rho t} s_1^b \int_{-\infty}^{a_7} m f_1 dZ_1 \\ & + e^{-2\rho t} p_2 \int_{a_2}^{a_3} (\exp(Q_2) - d_0 \exp(z_{i1})) f_1 dZ_1 + e^{-2\rho t} p_2 \int_{a_1}^{a_3} (d_0 \exp(z_{i1}) - \exp(Q_1)) f_1 dZ_1 \\ & + e^{-2\rho t} p_2 \int_{-\infty}^{a_6} I^{**} f_1 dZ_1 + e^{-2\rho t} p_2 \int_{a_6}^{a_7} (\exp(Q_2) - d_0 \exp(z_{i1}) - m) f_1 dZ_1 + e^{-2\rho t} p_2 \int_{a_7}^{a_2} I^* f_1 dZ_1 \\ & + p_2 \int_{a_3}^{\infty} (\exp(Q_2) - \exp(Q_1)) f_1 dZ_1 \\ & + e^{-2\rho t} (-p_2 - b + s_2) \left[\int_{a_2}^{a_3} \int_{-\infty}^{a_4} (\exp(Q_2) - d_0 \exp(z_{i1}) - d_0 \exp(z_{i1} + z_{i2})) f_2 dZ_2 f_1 dZ_1 \right. \\ & + \int_{a_7}^{a_2} \int_{-\infty}^{a_5} (I^* - d_0 \exp(z_{i1} + z_{i2})) f_2 dZ_2 f_1 dZ_1 + \int_{-\infty}^{a_6} \int_{-\infty}^{a_8} (I^{**} - d_0 \exp(z_{i1} + z_{i2})) f_2 dZ_2 f_1 dZ_1 \\ & \left. + \int_{a_6}^{a_7} \int_{-\infty}^{a_9} (\exp(Q_2) - d_0 \exp(z_{i1}) - d_0 \exp(z_{i1} + z_{i2}) - m) f_2 dZ_2 f_1 dZ_1 \right] + \\ & - \lambda_1 (m - M) - \lambda_2 (m - \exp(Q_2)) + \lambda_3 m \end{aligned} \quad (8)$$

where

$$\begin{aligned}
a_1 &= Q_1 - \ln(d_0) ; & a_2 &= Q_2 - \ln(\alpha^* d_0); & a_3 &= Q_2 - \ln(d_0) \\
a_4 &= \ln\left(\frac{\exp(Q_2) - d_0 \exp(z_{11})}{d_0 \exp(z_{11})}\right); & a_5 &= \left(\rho - \frac{\sigma^2}{2}\right)t + I^* \sigma \sqrt{t}; & a_6 &= \ln\left(\frac{\exp(Q_2) - m}{\alpha^{**} d_0}\right); \\
a_7 &= \ln\left(\frac{\exp(Q_2) - m}{\alpha^* d_0}\right); & a_8 &= \left(\rho - \frac{\sigma^2}{2}\right)t + \sigma I^{**} \sqrt{t}; & a_9 &= \ln\left(\frac{\exp(Q_2) - d_0 \exp(z_{11}) - m}{d_0 \exp(z_{11})}\right) \\
\alpha^* &= 1 + \exp(a_5); & \alpha^{**} &= 1 + \exp(a_8); & I^*(d_1) &= d_0 \exp(z_{11} + a_5); & I^{**}(d_1) &= d_0 \exp(z_{11} + a_8)
\end{aligned}$$

The two inventory levels that are of interest are determined from the following expression, which can be compared to equation (6). Adjusting for the assumption that penalty cost in the end of the second period is equal to zero the expression in case of the higher salvage value is

$$\Phi(I^*) = \left(\frac{p_2 - s_1^b e^{\rho t}}{p_2 + b - s_2} \right) \quad \text{given } s_1^b e^{\rho t} > s_2 - b$$

and in case of the lower salvage value the expression is given by

$$\Phi(I^{**}) = \left(\frac{p_2 - s_1^l e^{\rho t}}{p_2 + b - s_2} \right) \quad \text{given } s_1^l e^{\rho t} > s_2 - b$$

Before deriving the Kuhn-Tucker conditions to the problem in equation (8) it is necessary to highlight the issue of the existence of a maximum of the goal function. In order to assure that there exist at least a local optimum the first and second order derivative of the goal function with respect to Q_1 , Q_2 and m are derived. These are almost identical to the corresponding derivatives of the Lagrangean with respect to Q_1 , Q_2 and m , except from that all λ -terms are excluded. In order to check for quasiconcavity of the goal function the determinants of the bordered Hessian are numerically determined for different values of Q_1 , Q_2 and m , and for all checked values the determinants shows to have the features, which are necessary for a quasiconcave function. In addition the constraints are quasiconvex and thus the necessary conditions are fulfilled in order to use Kuhn-Tucker conditions to find a local maximum. Below the Kuhn-Tucker conditions determined from the Lagrangean in equation (8) are derived, and the second order derivatives of

the goal function, which are used when the determinants are determined, can be found in appendix.

$$\begin{aligned} \frac{\partial L_1^{CS}(Q_1^*)}{\partial Q_1} &= \exp(Q_1)(p_1 e^{-\rho t} - c_1 + k_1 e^{-\rho t} + c_2 e^{-\rho t} - p_2 e^{-\rho t}) + \\ & \exp(Q_1)(-p_1 e^{-\rho t} - b_1 e^{-\rho t} - k_1 e^{-\rho t} + p_2 e^{-2\rho t}) F_1(a_1) = 0 \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{\partial L^{CS}(Q_2^*, m^*, \lambda_2^*)}{\partial Q_2} &= -e^{-\rho t} c_2 \exp(Q_2) + e^{-2\rho t} p_2 \exp(Q_2) \left(\frac{\exp(a_8)}{\alpha^{**}} f_1(a_6) - \frac{\exp(a_5)}{\alpha^*} f_1(a_7) \right) + \\ & e^{-\rho t} s_1^l \exp(Q_2) F_1(a_6) + e^{-\rho t} s_1^h \exp(Q_2) (F_1(a_2) - F_1(a_7)) + \\ & e^{-2\rho t} p_2 \exp(Q_2) (1 - F_1(a_2)) + e^{-2\rho t} \exp(Q_2) (-p_2 - b + s_2) \int_{a_2}^{a_3} f_1 F_2(a_4) dZ_1 + \\ & e^{-2\rho t} \exp(Q_2) (-p_2 - b + s_2) \int_{a_6}^{a_7} f_1 F_2(a_9) dZ_1 + \lambda_2^{CS} \exp(Q_2) = 0 \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{\partial L^{CS}(Q_2^*, m^*, \lambda^*)}{\partial m} &= e^{-2\rho t} p_2 \left(\frac{\exp(a_5)}{\alpha^*} f_1(a_7) - \frac{\exp(a_8)}{\alpha^{**}} f_1(a_6) \right) + e^{-\rho t} s_1^h F_1(a_7) - \\ & e^{-\rho t} s_1^l F_1(a_6) - e^{-2\rho t} (-p_2 - b + s_2) \int_{a_6}^{a_7} f_1(Z_1) F_2(a_9) dZ_1 - \lambda_1^{CS} - \lambda_2^{CS} + \lambda_3^{CS} = 0 \end{aligned} \quad (11)$$

$$\lambda_1^{CS} (m - M) = 0, \lambda_2^{CS} (m - e^{-Q_2}) = 0, \lambda_3^{CS} m = 0, \lambda^{CS} \geq 0. \quad (12)$$

Considering the case when m is bounded by the total order quantity $\exp(Q_2)$, there should be no reason to acquire more opportunities to return products than the total order quantity. It should be noted that equation (9) is independent of Q_2 and m , equation (10) and (11) are independent of Q_1 . It can also be seen that the number of products ordered for the first period is independent of salvage values but is dependent on the value of the price and cost in the second period. The number of ordered items in the first period is unambiguously affected by the production cost of the second period giving that the number of items produced in the first period increases as the production cost of the second period increases and vice versa.

5.2 Coordination of the decentralised chain with return limitations

In case of decentralising the chain the problem shows great similarities to the problem described by (8) but since there are options to return products at midlife in the chain some additional term and parameters must be added. Since the retailer buys m options, at a price Γ , a term Γm must be introduced in order to account for the cost that might appear if the retailer determines to buy options. In addition, the salvage values s_1^b , s_1^l and s_2 must be exchanged for their respective counterpart, x_1 , r_1 and r_2 .

Since the decentralised retailer optimises its actions from the same control parameters as the decision maker in the centralised chain there is also great similarities between the Kuhn-Tucker conditions in the centralised and decentralised chain. Formulating the Lagrangean for the retailer in the decentralised system with return limitations (DS) , $L^{DS}(Q_1, Q_2, m, \lambda^{DS})$, and determining partial derivatives with respect to Q_1 , Q_2 and m gives

$$\begin{aligned} \frac{\partial L_1^{DS}(Q_1^*)}{\partial Q_1} &= \exp(Q_1) \left(p_1 e^{-\rho t} - w_1 + k_1 e^{-\rho t} + w_2 e^{-\rho t} - p_2 e^{-\rho t} \right) + \\ &\exp(Q_1) \left(-p_1 e^{-\rho t} - b_1 e^{-\rho t} - k_1 e^{-\rho t} + p_2 e^{-2\rho t} \right) F_1(a_1) = 0 \end{aligned} \quad (13)$$

Compared to the corresponding partial derivative in the centralised case it is almost identical but production costs are exchanged for the associated wholesale prices, and is, just like in the centralised case, independent on the total order quantity.

Considering the Kuhn-Tucker condition with respect to the total order quantity it can be seen that it is dependent on total order quantity and number of return options. In order to achieve coordination there must be internal consistencies between the exercise price and the return rebates to replicate the behaviour of the centralised chain.

$$\begin{aligned} \frac{\partial L^{DS}(Q_2^*, m^*, \lambda_2^*)}{\partial Q_2} &= -e^{-\rho t} w_2 \exp(Q_2) + e^{-2\rho t} p_2 \left(\frac{e^{a_8}}{\alpha^{**}} f_1(a_6) - \frac{e^{a_5}}{\alpha^*} f_1(a_7) \right) + \\ &e^{-\rho t} r_1 \exp(Q_2) F_1(a_6) + e^{-\rho t} x_1 \exp(Q_2) (F_1(a_2) - F_1(a_7)) + e^{-2\rho t} p_2 \exp(Q_2) (1 - F_1(a_2)) + \\ &e^{-2\rho t} \exp(Q_2) (-p_2 - b + r_2) \left(\int_{a_2}^{a_3} f_1 F_2(a_4) dZ_1 + \int_{a_6}^{a_7} f_1 F_2(a_9) dZ_1 \right) + \lambda_2^{DS} \exp(Q_2) = 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial L^{DS}(Q_2^*, m^*, \lambda^*)}{\partial m} &= -\Gamma + e^{-2\rho t} p_2 \left(\frac{\exp(a_5)}{\alpha^*} f_1(a_7) - \frac{\exp(a_8)}{\alpha^{**}} f_1(a_6) \right) + e^{-\rho t} x_1 F_1(a_7) - \\ &e^{-\rho t} r_1 F_1(a_6) - e^{-2\rho t} (-p_2 - b + r_2) \int_{a_6}^{a_7} f_1(Z_1) F_2(a_9) dZ_1 - \lambda_1^{DS} - \lambda_2^{DS} + \lambda_3^{DS} = 0 \end{aligned} \quad (14)$$

$$\lambda_1^{DS} (m - M) = 0, \lambda_2^{DS} (m - \exp(Q_2)) = 0, \lambda_3^{DS} m = 0, \lambda^{DS} \geq 0 \quad (15)$$

The Kuhn-Tucker condition for m in equation (14) gives that the optimal number of options to purchase is also dependent on the total order quantity which implies that these two must be considered at the same time when making the ordering decision.

Determining the optimal value for Q_1 is straightforward using equation (13) and the corresponding condition can be written as

$$F_1(a_1^*) = \frac{(p_1 e^{-\rho t} - w_1) - e^{-\rho t} (p_2 e^{-\rho t} - w_2) + k_1 e^{-\rho t}}{e^{-\rho t} (p_1 + b + k_1) - p_2 e^{-2\rho t}}$$

given that $b > w_2 - w_1 e^{\rho t}$ and that the nominator is bigger or equal to zero. This implies that increasing wholesale prices are ruled out for positive values of b . However, for those types of products that are of interest in this paper the situation is reversed, i.e. retail and wholesale prices are typically decreasing over time. Thus, this is not an important constraint in this setting.

5.3 Necessary conditions for coordination in the decentralised system

From the Kuhn-Tucker conditions on the first period order quantity in the centralised and decentralised chain, i.e. equation (9) and (13), the necessary condition on the relationship between the wholesale prices and production costs can be achieved and is given by

$$w_2 - w_1 e^{-\rho t} = c_2 - c_1 e^{-\rho t}$$

In order for the decentralised retailer to replicate the decision of the centralised decision maker at midlife the retailer must face the identical critical fractile as the centralised decision maker. The condition which assures that the retailer adjusts inventory to the optimal level is given by

$$\left(\frac{p_2 - s_1^b e^{\rho t}}{p_2 + b - s_2} \right) = \left(\frac{p_2 - x_1 e^{\rho t}}{p_2 + b - r_2} \right) \text{ and } \left(\frac{p_2 - s_1^l e^{\rho t}}{p_2 + b - s_2} \right) = \left(\frac{p_2 - r_1 e^{\rho t}}{p_2 + b - r_2} \right) \quad (16)$$

If it should be the case that the number of return possibilities are smaller than the optimal total order quantity then (11) and (12) gives that the system benefits from getting as many return opportunities that it possibly can get, i.e. $m=M$. This has also implications to the pricing of the return options. In order to determine the value of the options price, Γ , it is possible to use the information that is given from the Lagrange multipliers, λ_1^{DS} , λ_2^{DS} and λ_3^{DS} . Regarding λ_2^{DS} it can be found out that it must be equal to zero since if its associated constraint is not binding then it is equal to zero, and if it is binding then λ_2^{DS} must be equal to zero as well. The latter is due to the fact that the value of having an extra opportunity to return a product is zero since no further products can be returned. Consider now equation (14) and (15) and the case when $M=0$, which gives that $m=0$ and two constraints are binding, i.e. $\lambda_1^{DS}, \lambda_3^{DS} \geq 0$. Due to the structure of the problem, and that $x_1 > r_1$, one of them will be equal to zero. In equation (14) the option price, Γ , will determine which one of them that is equal to zero and in order to achieve a feasible contract it must be the case that $\lambda_3^{DS} = 0$, if not, the option price is too high and that the retailer wants to “buy” a negative number of options. If the option price is too low this will give that $\lambda_1^{DS} > 0$ and

in turn that the retailer would gain from buying more options. However, at a specific price $\lambda_1^{DS} = 0$ and $\lambda_3^{DS} = 0$, and at this price the retailer gains nothing from buying or “selling” any option.

In case of $M > 0$ the supplier wants to induce the retailer to make the decision where $m = M$. In this case $\lambda_3^{DS} = 0$ is zero and only λ_1^{DS} needs to be considered. The same line of reasoning can be used in this case in that for a specific value of the option price, Γ , $\lambda_1^{DS} = 0$ and this option price will induce the retailer to buy $m = M$ options. Thus it is possible to use this information in that if the supplier wants to induce the retailer to buy M options then it should set the option prices such that all $\lambda^{DS} = 0$ for the M :th options. That will also imply that the option price is set such that the net present value of the last option bought is equal to zero, but that the other options add present values to the retailer.

6 Numerical illustrations and analysis

In this chapter a study is performed in order to illustrate the implications of the conditions that are derived previously in the paper. In the study the parameters, which are unchanged throughout the study, are $p_1 = p_2 = 1$, $c_1 = c_2 = 0.5$, $b = 0.02$, $k_1 = 0.1, k_2 = 0, s_1^b = 0.4, s_1^l = 0.3, s_2 = 0.2$, $\rho = 0.05$, and each period is set to be equal to $t = 0.5$ years. The starting value of the demand variable d_0 , is equal to 10000 and the volatility rate $\sigma = 25\%$.

In order to achieve coordination in the channel a number of conditions must be fulfilled and this is also shown in the previous chapter. In this section the relationships between prices are studied in order to take the analysis a step further. In figure 1 coordinating exercise and wholesale prices are illustrated when the return rebate in the second period is set to be equal to the wholesale price of the second period, which is determined in the beginning of the horizon. In detail the figure

shows the exercise price, which the retailer receives for up to M returned products, and also the return rebate for those items above M , which are returned. Since these are constrained by expression (16) and there are certain rules how these are set in order to achieve coordination.

Another observation is that the exercise price that implies coordination is higher than the wholesale price in the first and the second period. This might seem like there are arbitrage opportunities in the chain but this is not the case since all decisions are made in the beginning of the horizon, i.e. there is no opportunity for the retailer to reorder any products at midlife, return them and extract the exercise price. The only function that the exercise price and return rebates have at midlife is to certify that optimal decisions seen from the system-wide performance objective are made. This explanation can also be seen in equation (16), implying that the first period's exercise price and return rebate, respectively, must be higher than the return rebate in the second period minus the holding cost. In addition the retailer has to pay to get the option to return at the higher value.

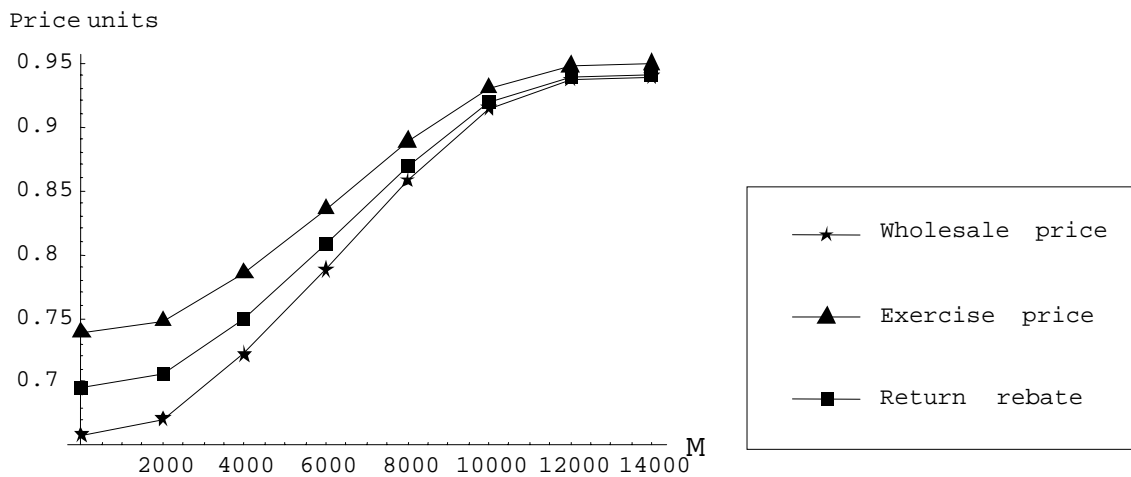


Figure 1: Coordinating w , x_1 and r , as a function of M .

Figure 2 presents the option prices, which implies that the number of options that the retailer finds it optimal to buy coincides with optimality seen from the system-wide performance objective. Considering the retailer's Kuhn-Tucker condition for m gives that at a certain option

price the present value increases to the retailer if another option is bought and this is the case until the M :th option is bought. This implies that acquiring an option has a positive net present value except for the last M :th option, which has a net present value equal to zero. So in case of $M=0$, i.e. there is no opportunity to the supplier to realise overstock at a higher value than the retailer, the option price for $M=0$ in figure 2 implies that the retailer finds that the net present value of acquiring an option to return is equal to zero, and no option is bought.

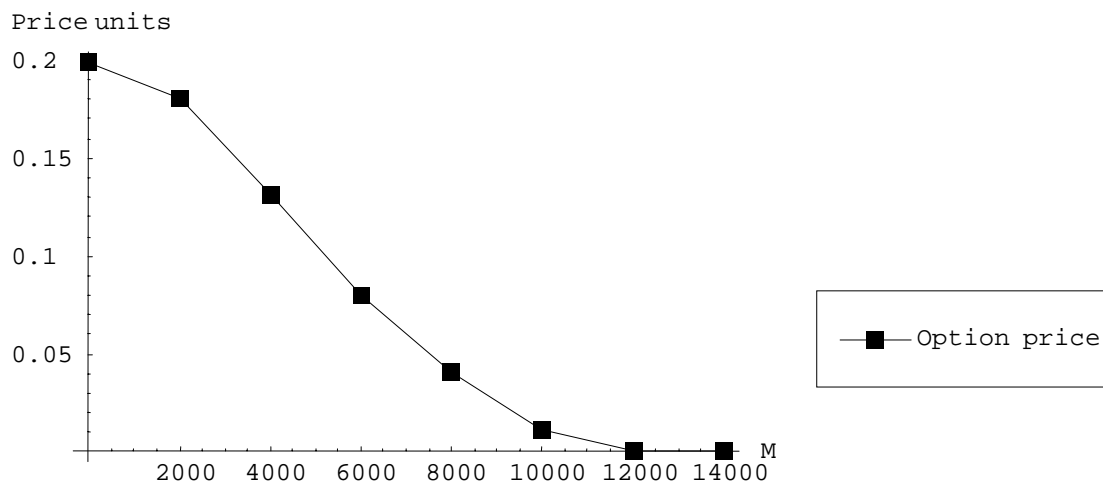


Figure 2: Coordinating option prices as a function of M when $r_2=0.6$.

The fact that a certain relationship between the exercise price and return rebates in the first and second period must be preserved, in order to achieve coordination, gives implications to the price of the options. The option prices in figure 2 are based on a return rebate in the second period, which is equal to 0.6. However, performing the same numerical analyses, in case of return rebates equal to 0.3 and 0.9, reveal that the option prices only differ to a small amount, i.e. the sizes of the deviations are in the range between 0 and 0.008, and lower return rebates result in slightly higher option prices.

In figure 3, the aggregated present value of the coordinated chain and how it is divided between the supplier and the retailer in a coordinated decentralised chain is shown. Figure 3 illustrates the

values under the condition that the wholesale price in the second period is equal to the return rebate in the second period.

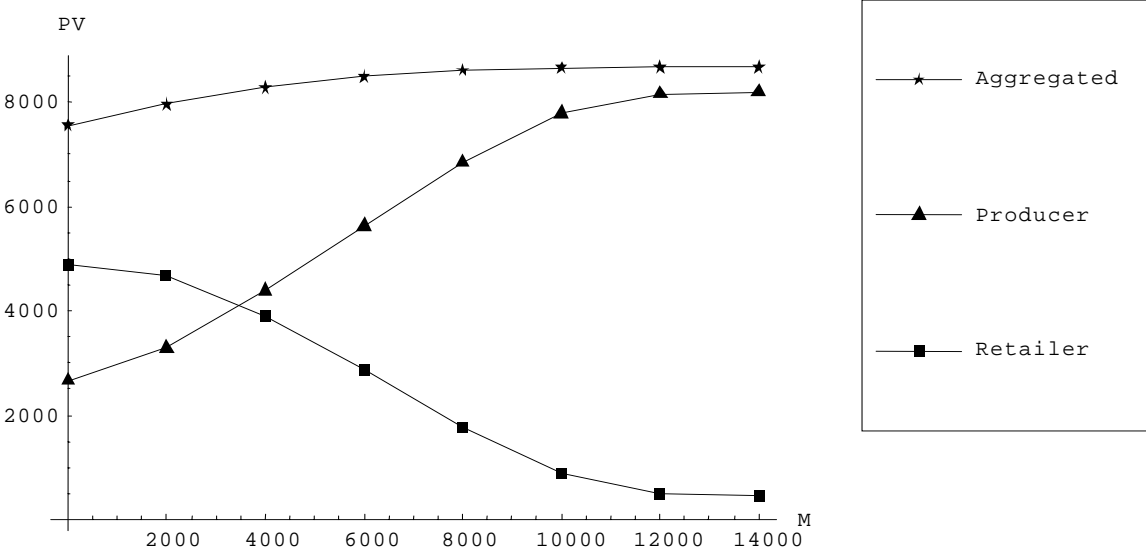


Figure 3: Present values of coordinated chains and the retailer’s and supplier’s share of the contract for different values of M .

As one could expect the present value of the chain increases as the number of possible return increases, since this implies that the number of products that can be disposed at a higher value increases. The fact that the supplier’s present value increases as M increases can be explained by considering figure 1, which shows that the coordinating wholesale prices increase, thus increasing the positive cash-flow of the supplier. The exercise price and return rebates increase as well, but since the number of return most likely is smaller than the order quantity the net effect on the present value of the supplier is positive.

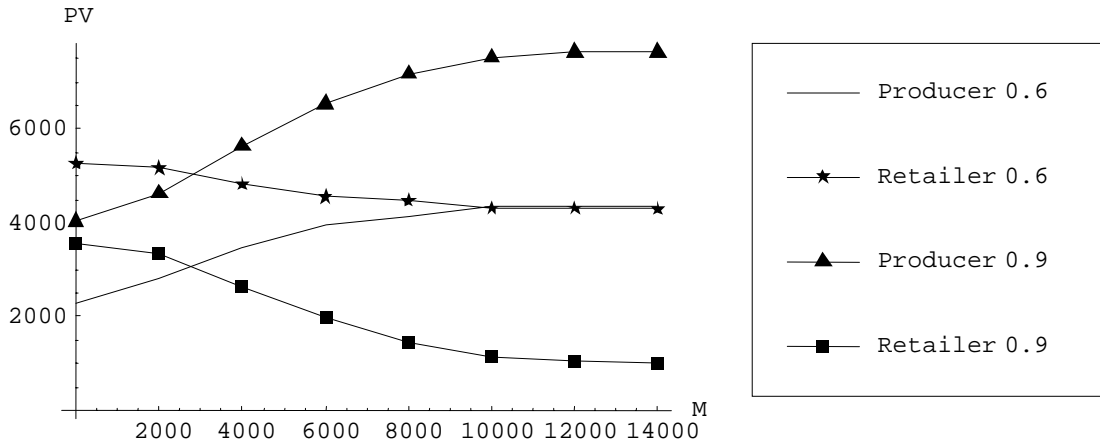


Figure 4: Present values of the retailer's and supplier's share of the contracts as a function of M and for $r_2=0.6$ and $r_2=0.9$, respectively.

Figure 4 presents the individual present values of the retailer and the supplier when the condition that the return rebate in the second period must be equal to the corresponding wholesale price is relaxed. As shown, the present value of the supplier increases as the exercise price increases and as the number of possible return at higher salvage value increases. A reason for the increase in the present value of the supplier can be seen in figure 5, which illustrates the wholesale prices, which implies coordination, for different values of the return rebate in the second period and for different values of M .

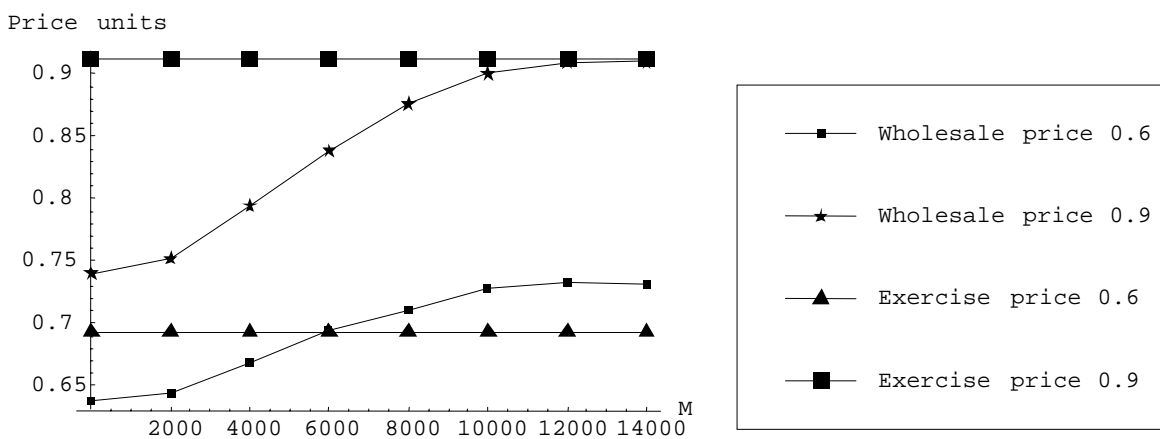


Figure 5: Coordinating w_1 and x_1 as a function of M , and for $r_2=0.6$ and $r_2=0.9$, respectively.

As can be seen the wholesale prices in the first period increase for increasing values on the return rebate and the number of possible returns. In turn, the value of the wholesale price in the second

period will also increase since the conditions on the relationship between the wholesale prices and the actual production cost specify how large the difference between the two wholesale prices is allowed to be. An increasing wholesale price in the first period must always be followed by an increase in the wholesale price in the second period in order to preserve coordination. In figure 5 it can also be seen that M does not affect the exercise prices, but these are only affected by return rebates. This must also be the case in order to determine optimal dispose-down-to levels.

7 Summary and future research

This paper deals with the coordination problem in a two-period setting. The problem analysed is when the supplier has the opportunity to realise a higher salvage value than the retailer, and especially when this opportunity is limited, due to e.g. capacity constraints. Conditions on wholesale prices, exercise price, and return rebates are derived in order to achieve coordination. In addition, conditions and an explanation how option prices can be determined in order to achieve coordination in the chain are also presented. It is shown that the supplier is better off if the return rebate in the second period increases and if the number of return option increases. This support and extend the finding of Pasternack (1985), which shows that the supplier is better off if the return rebates are increased.

The paper also shows that the option price is almost unaffected by changes in the exercise price and return rebate. This derives from the fact that in order to preserve coordination a certain relationship between exercise price, return rebates and salvage values must exist. This implies that the supplier must introduce a return rebate, which is paid to the retailer for those items, which are returned even though all return options are utilised. The relationship between the exercise price on those items that are returned against the option contract and those, which are returned at the lower return rebate, is affecting the price of the option. The difference between these tends

to be almost unchanged for the same value of M , which implies that the option price is almost unaffected.

This paper deals with limitation in return possibilities at midlife and a natural extension of the problem would be to analyse the effect of limited return in at the end-of-life decisions. In this paper it is assumed that returns at end-of-life are an unlimited opportunity and this enables the inventory decision at midlife to be analysed in a general newsvendor framework. Pricing of options, and coordination in case of limited return at both midlife and end-of-life requires more complex models in order to determine optimal actions at midlife. Another issue for further research is concerning the effect that differences in market prices and production costs between periods cause on coordinating wholesale prices, exercise price, return rebates, and option prices. The models developed and the conditions derived in this paper allow for such analysis to some extent.

Appendix – Second order derivatives

Let $G(Q_1, Q_2, m)$ denote the goal function which is subject to be maximised. In this appendix the second order derivatives of G with respect to Q_1, Q_2 and m are derived, in order to be able to determine the bordered Hessian and analyse quasiconcavity of G .

$$\frac{\partial^2 G}{\partial Q_1^2} = \exp(Q_1) \left(p_1 e^{-\rho t} - w_1 + k_1 e^{-\rho t} + w_2 e^{-\rho t} - p_2 e^{-\rho t} \right) + \exp(Q_1) \left(-p_1 e^{-\rho t} - b_1 e^{-\rho t} - k_1 e^{-\rho t} + p_2 e^{-2\rho t} \right) \left(F_1(a_1) + f_1(a_1) \right)$$

$$\begin{aligned} \frac{\partial^2 G}{\partial Q_2^2} = & \frac{\partial G}{\partial Q_2} + \exp(Q_2) \left[e^{-\rho t} r_1 \frac{1}{\exp(Q_2) - m} f_1(a_6) + e^{-\rho t} x_1 \left(f_1(a_2) - \frac{f_1(a_7)}{\exp(Q_2) - m} \right) - \right. \\ & e^{-2\rho t} p_2 f_1(a_2) + e^{-2\rho t} p_2 \frac{1}{\sigma^3 \sqrt{\pi} (\exp(Q_2) - m)} \left(\frac{\exp(a_5)}{\alpha^*} (a_7 - \mu) f_1(a_7) - \right. \\ & \left. \left. \frac{\exp(a_8)}{\alpha^{**}} (a_6 - \mu) f_1(a_6) \right) \right] + \\ & e^{-2\rho t} \left(-p_2 - b_2 + r_2 \right) \int_{a_2}^{a_3} \frac{f_1 f_2(a_4(z_1))}{\exp(Q_2) - d_0 \exp(z_1)} dZ_1 + \\ & e^{-2\rho t} \left(-p_2 - b_2 + r_2 \right) \int_{a_6}^{a_7} \frac{f_1 f_2(a_9(Z_1))}{\exp(Q_2) - d_0 \exp(z_1) - m} dZ_1 + \\ & e^{-2\rho t} \frac{1}{\exp(Q_2) - m} \left(-p_2 - b_2 + r_2 \right) f_1(a_7) F_2(a_5) - \\ & \left. e^{-2\rho t} \frac{1}{\exp(Q_2) - m} \left(-p_2 - b_2 + r_2 \right) f_1(a_6) F_2(a_8) \right] \end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 G}{\partial m^2} &= e^{-\rho t} x_1 \frac{f_1(a_7)}{\exp(Q_2) - m} + e^{-\rho t} r_1 \frac{f_1(a_6)}{\exp(Q_2) - m} + \\
&e^{-2\rho t} p_2 \frac{1}{\sigma^3 \sqrt{\pi} (\exp(Q_2) - m)} \left(\frac{\exp(a_8)}{\alpha^{**}} (a_6 - \mu) f(a_6) - \frac{\exp(a_5)}{\alpha^*} (a_7 - \mu) f(a_7) \right) - \\
&e^{-2\rho t} (-p_2 - h_2 + r_2) \int_{a_6}^{a_7} \frac{f_1 f_2(a_9(z_1))}{\exp(Q_2) - d_0 \exp(z_1) - m} dZ_1 + \\
&e^{-2\rho t} \frac{1}{\exp(Q_2) - m} (-p_2 - h_2 + r_2) f_1(a_7) F_2(a_9(a_7)) - \\
&e^{-2\rho t} \frac{1}{\exp(Q_2) - m} (-p_2 - h_2 + r_2) f_1(a_6) F_2(a_9(a_6))
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 G}{\partial m \partial Q_2} &= \exp(Q_2) \left[e^{-\rho t} x_1 \frac{f_1(a_7)}{\exp(Q_2) - m} - e^{-\rho t} r_1 \frac{f_1(a_6)}{\exp(Q_2) - m} - \right. \\
&e^{-2\rho t} p_2 \frac{1}{\sigma^3 \sqrt{\pi} (\exp(Q_2) - m)} \left(\frac{\exp(a_5)}{\alpha^*} (a_7 - \mu) f(a_7) - \right. \\
&\qquad \qquad \qquad \left. \left. \frac{\exp(a_8)}{\alpha^{**}} (a_6 - \mu) f(a_6) \right) + \right. \\
&e^{-2\rho t} (-p_2 - h_2 + r_2) \int_{a_6}^{a_7} \frac{f_1 f_2(a_9(z_1))}{\exp(Q_2) - d_0 \exp(z_1) - m} dZ_1 - \\
&e^{-2\rho t} \frac{1}{\exp(Q_2) - m} (-p_2 - h_2 + r_2) f_1(a_7) F_2(a_9(a_7)) + \\
&\left. e^{-2\rho t} \frac{1}{\exp(Q_2) - m} (-p_2 - h_2 + r_2) f_1(a_6) F_2(a_9(a_6)) \right]
\end{aligned}$$

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