

# Education and the Allocation of Talent\*

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## Abstract

This paper considers a two-sector education model with two novel features. First, contracts have an independent role in sorting workers into different sectors of the economy. Second, education improves workers' awareness of their abilities, and hence can improve the allocation of talent by making workers' choice of sector better informed. The implication is that the most able skip education, which stands in contrast to results from established theories of education. In the extension, we consider the case when education improves productivity directly, in addition to improving information. Using this extension, we compare the UK and the US undergraduate systems, and moreover analyze hybrid educational systems from Europe, that offer both UK and US types of undergraduate degrees.

## 1 Introduction

An important determinant of the prosperity of an economy is how well its labor markets allocate the pool of workers with heterogenous talent to appropriate sectors or jobs of

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the economy. One such market is the market for experienced workers, where workers change jobs according to their preferences and productive abilities. The purpose of this paper is to study how education shapes the allocation of talent. Surprisingly, established theories of education focus on reduced-form specifications or one-sector models that do not obviously allow such analysis.

To model the link between education and the allocation of talent, we add two novel, and we think realistic, features to a two-sector Spence (1974) type of education model. The structure of the paper is to first analyze the effects from introducing these two features, and then to extend the model to make it more applicable.

First we enrich the contractual space by allowing firms to offer credential contracts, where a worker's education level determines his pay (as in Spence), but also performance contracts, where performance in the job determines pay. Performance contracts imply that workers have the option to educate or not before entering a certain sector or job type. For example, for many jobs within business, an MBA degree may be commendable but is not required. Second, we explicitly take into consideration the old educator's argument that workers acquire information about their abilities through educating. For example, an MBA degree may learn an engineering graduate whether his talents lie within Project Management or within Finance.

Performance contracts affect the allocation of workers through giving workers incentive to choose the sector where they are most productive, given their available information. Education, on the other hand, affects the allocation of talent through making workers choice of sector better informed; a worker may change his opinion about which sector to work in after undertaking education. Education may also affect the allocation of talent through providing the worker with a signal that he belongs to an able cohort of workers. The model encompasses equilibria where the motive behind education is signaling, but the focus of the analysis will be on equilibria where the role of education is information acquisition.

In information acquisition equilibria, those with intermediate confidence educate (before choosing sector and contract), while the least and the most confident skip education. The intuition is that those with intermediate confidence have a higher valuation of education than those with low or high confidence, who are already quite sure who they

are. Those who are sure who they are skip education and start working in one of the sectors directly. Signaling equilibria are also consistent with the mediocre educating and the most able skipping education. The intuition here is that the most able skip education because those in the middle can imitate too cheaply, while those in the middle educate to distinguish themselves from the least able.

An extension considers the more realistic case when education has a direct effect on a worker's productivity. It is shown that the most able may still skip education, and conditions for when this result is reversed is considered. At a more applied level, we adapt the model to discuss the relative merit of the US and the UK undergraduate university education, and to analyze educational systems not uncommon in Europe, where US and UK type of undergraduate degrees co-exist.

Other realistic features omitted from the basic model, such that effort interacts with ability in determining production, and risk aversion, is also discussed in the extension.

Weiss (1983) extends the Spence (1973) signaling model to a setting where agents have superior, but imperfect, information about their own abilities, and moreover where students undergo a final test after educating (the result of the test is public information). Weiss (1983) focuses on the existence of separating equilibria where different belief types choose different length of education (there is a continuum of education levels). The model of Weiss has only one sector and one labor contract type. Therefore, Weiss (1983) is silent on the information acquisition role of education and its welfare properties, and also on the endogenous choice of labor contracts, which are main issues in the present paper. The same neglect hold for two recent surveys of the economics of education literature, Blaug (1992) and Weiss (1995).<sup>1</sup> To our knowledge, the only education model where education explicitly has a role in allocating talent occurs in Spence (1974). However, Spence (1974) does not consider the information acquisition role of education, and neither considers the possibility that contracts may serve to sort workers. Stiglitz (1975) and MacDonald (1983) mention the information acquisition role of education, but only *en passant*.

A parallel paper, Grossman (1999), also considers an adverse selection setting how

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<sup>1</sup>Fershtman, Murphy and Weiss (1996) consider the effect of education on the allocation of talent (and growth) when workers care about their relative status, in addition to their material payoff. Fershtman et al. focus on the productivity-augmenting role of education under perfect labor contracts, in contrast to our focus on the informational role of education under imperfect labor contracts.

contracts have an independent role in the allocation of talent. Along with the present paper, it seems that Grossman (1999) is the first work that studies a model where workers have private information about their abilities, and where the type of labor contracts employed is endogenously determined. While we focus on the interaction between contracts and education, Grossman (1999) focuses on the interaction between contracts and trade between nations, in a different type of model.<sup>2</sup> The individuals allocating themselves into different sectors in Grossman's model have no possible signals available, and moreover know exactly who they are, so there is no information acquisition possible either.

Starting with the seminal works Fama (1980) and Holmstrom (1982/1999), there is a large recent literature on career concerns and market learning about the abilities of managers. This literature considers learning about abilities under symmetric information models, with signaling motives hence excluded.<sup>3</sup> Prendergast and Stole (1996) and Avery and Chevalier (1999) are related to the present work in considering private learning. However, neither paper consider the contractual response by firms to workers' private information.<sup>4</sup> Hence the present paper can be seen as extending the theory of career concerns to a setting with private learning and with more realistic assumptions about contracts.

Section 2 outlines the model. Section 3 contains the basic results, Section 4 considers extensions, and Section 5 concludes. Some proofs are relegated to Appendices A and B.

## 2 The Model

*Production Technology.* There are two sectors in the economy, sector N and sector S. In each sector there are several risk-neutral firms, and wages are set competitively. There is a

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<sup>2</sup>The idea that contracts can affect the allocation of talent goes back at least to Murphy et al. (1991). For example Murphy et al. (1991) state on p. 513: 'In fact, differences in contracts between industries are as important or more important than physical diminishing returns to scale [for the allocation of talent]'.

<sup>3</sup>See e.g., Scharfstein and Stein (1990), Chevalier and Ellison (1999), Dewatripont et al. (1999a) and (1999b), Morris (2001), and Altonji and Pierret (2001).

<sup>4</sup>For both papers it is not obvious what equilibria would look like if contract employed emerged endogenously from the behavior of firms rather than being taken as given. Avery and Chevalier (1999) assume that managers maximize their reputation when choosing between alternative investment projects, while Prendergast and Stole (1996) assume that managers maximize a function that weighs both current period profits of the firm and the manager's reputation in the market.

continuum of risk-neutral workers of measure 1, where each worker has either low ability or high ability. The share of high ability workers equals  $\theta$ . Each worker is employed for one period of time in one of the sectors. In sector N, both types of workers have productivity  $\pi_N$ . In sector S, the low type has productivity  $\pi_L$ , whereas the high type has productivity  $\pi_H$ , where  $\pi_L < \pi_N < \pi_H$ . For simplicity, normalize  $\pi_L$  to zero. The case when productivity is determined by both productivity type and effort is considered in an extension.

*Compensation Contracts.* In the N sector, a worker's productivity is known to be  $\pi_N$ , so all workers are offered the wage  $\pi_N$ , independently of whether they are educated or not. In the S sector, worker productivity is unknown, and firms offer two types of contracts to attract able workers: performance contracts and credential contracts. A performance contract pays a worker according to a (possibly noisy, but unbiased) estimate of the production of the worker. The estimate costs  $m > 0$  per worker to obtain (referred to as the cost of monitoring). Let  $b$ , where  $b \in [0, 1]$ , be the belief of a worker, prior to educating, that he is the high type. Thus  $b$  is the *confidence level* of the worker. Hence the value of a performance contract in the S sector for a worker with belief  $b$  equals  $b\pi_H - m$ . A credential contract conditions wage upon the education level of a worker, and gives a worker with education level  $e$  the wage  $w(e)$ , where  $e \in \{0, 1, \dots, K\}$ .<sup>5</sup>

*Education.* For convenience, there are only two education levels, thus we set  $K = 1$ .<sup>6</sup> For simplicity, assume that firms only offer credential contracts to workers with education level  $e=1$ .<sup>7</sup> It is assumed that a worker who educates acquires information about his ability type through how hard he must work to complete the degree. Effort is costly to a worker, and the amount of effort required is assumed to be correlated with the worker's true type: if a worker's true productivity is low, then more likely he has to work hard to finish the degree, while if the worker's true productivity is high, then more likely he

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<sup>5</sup>A continuum of education levels would open up for signaling equilibria where each belief type choose a different education level, as in Weiss (1983). Since the focus of the paper is on equilibria where the motive behind education is learning (not signaling), however, not much is lost through taking education level to be a discrete variable.

<sup>6</sup>If agents can committ to a short or a long education at time 1, there can exist equilibria, qualitatively similar to those obtained, where agents with a long education is offered a more lucrative credential contract than workers with a short education. Cases where workers cannot precommit to an education length are studied by Nöldeke et al. (1990) and Swinkels (1999).

<sup>7</sup>This follows from an equilibrium argument, for  $\theta$  not too high.

can obtain the degree with less work.<sup>8</sup> Formally, the (non-pecuniary) cost of educating for an individual is an independent realization of a random variable  $X$ . For simplicity, it is assumed that  $X$  can take just two values,  $c_L$  and  $c_H$ , where  $c_H < c_L$ . Hence, the realization  $X = c_H$  can be interpreted as good news about ability for the worker, and the realization  $X = c_L$  can be interpreted as bad news about ability. If the worker has high ability, good news occurs with probability  $p$ , and bad news occurs with probability  $1 - p$ . But if the worker has low ability, then bad news occurs with probability  $p$ , and good news with probability  $1 - p$ , where  $\frac{1}{2} < p \leq 1$ . The larger  $p$ , the more informative is  $X$ . In the basic model, the only form of human capital acquired from education is information about abilities. An extension considers the case when education also augments the productivity of a worker.

### *Information*

Two limit informational assumptions are considered. In the first case, all information received by a worker, both at the interim stage between birth and education, and at the education stage, is public. This will be referred to as the *public information* case. In the second limit case, all information received by a worker, both before and during education, is private to a worker. Hence in this *private information* case, firms only know  $\theta$ , the distribution of  $X$ , and whether a worker is educated or not.<sup>9</sup> Cases with partly private, partly public information resembles more the pure private information case and will be considered later.

Under private information, the education level of the worker is the only individual-specific information a credential contract can be conditioned on. In the public information case, where workers and firms are equally well informed, firms can offer a (fixed) wage conditional on the commonly known estimate  $b$  of a worker's ability. Since  $b$  is a sufficient statistic for ability, a worker's education level will not give independent information about his ability, and hence will not be contracted upon under public information. To demar-

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<sup>8</sup>There are many other ways education can make a worker learn about his abilities, e.g., through grades obtained and feedback from other students and teachers. I choose to view the feedback through the lens of the cost of education for tractability reasons; the results do not depend on it.

<sup>9</sup>We implicitly assume that firms cannot have better information than a worker about that worker's ability. As pointed out by a referee, a situation where firms know more than workers is not necessarily implausible. For example, financial firms employing physicists may know more about their future prospects in the finance industry than the physicists know themselves.

cate the contracts with fixed pay under public information from the private information credential contracts, for clarity label the former *fixed wage contracts*.

*Timing.* Figure 1 illustrates the sequence of events.

0	1	2	3			
Workers born with prior $\theta$	Workers receive info about their abilities	Firms offer employment contracts	Workers enter a sector directly, or choose to educate first	Education	Educated workers choose sector (and contract)	Production (Monitoring)

Figure 1: Timing of Events

At time 0, workers are born with a common prior  $\theta$ . Between time 0 and time 1, workers receive imperfect information about their abilities, on which they update their prior  $\theta$ , and form the belief  $b$ . The information received between time 0 and time 1 may be thought of as learning from compulsory education.<sup>10</sup> At time 1, firms offer employment contracts to workers. At time 2, workers choose whether to educate or not, given the offered contracts and their confidence level  $b$ . Workers that do not educate choose sector at time 2.<sup>11</sup> Workers that educate do so between time 2 and time 3, and receive information about their abilities when doing so. Such workers delay choosing sector and contract until time 3. At time 3, all workers are employed for one period, and then wages are paid, according to the contract.<sup>12</sup> All discounting factors are set to one. Notice that the equilibrium sorting at time 2 uniquely determines the allocation of workers, i.e., the fractions of workers that are employed in the different sectors at time 3 (and on which type of contract).<sup>13</sup>

<sup>10</sup>There are a variety of other possible interpretations of the information received between time 0 and time 1. For example, the learning may come from parental guidance. Alternatively, we may think of the model as analyzing the decision to undertake 'higher' education (like an MBA degree), and where the information received between time 0 and time 1 reflects learning from undergraduate work (and where all student start their undergraduate career with a common prior  $\theta$ ).

<sup>11</sup>It makes no difference to the results whether those without education decide which sector to work in at time 2 or at time 3. For ease of exposition, I choose the former.

<sup>12</sup>Hence workers that do not educate stand idle for one period. This assumption is meant to capture a situation where the duration of the period of work is (much) longer than the duration of education.

<sup>13</sup>Although the sorting choice at time 3 for an individual that chooses to educate is stochastic, the *fractions* (for each confidence level  $b$  at time 2) that choose the different alternatives after education are deterministic.

Since agents receive different information between time 0 and time 1, their confidence levels differ at the education decision at time 2; some agents will be underconfident (i.e., have a too low opinion about themselves), and some workers will be overconfident (i.e., have a too high opinion about themselves). While there is nothing suspicious about underconfidence and overconfidence at the individual level, an interesting question is whether imperfect beliefs at the aggregate level 'cancel out' at the population level, and, if yes, in which sense. This question will be addressed under 2.1.

## 2.1 Preliminaries

First we derive an individual worker's payoff under private information, for a given credential wage  $w(1)$ , written just  $w$ . The full expressions are relegated to Appendix B. At time 2, a worker has three different possible actions; to skip education and choose the N sector, to skip education and choose the S sector, and to educate. Since wages are competitive, the expected utility from choosing sector N equals  $\pi_N$ , and the expected utility from choosing a the S sector equals  $b \pi_H - m$ . The expected utility from educating depends on whether  $w > \pi_N$  or  $w < \pi_N$ . When  $w < \pi_N$ , signaling motives behind education are excluded, and the motive behind education can only be information acquisition: choose the N sector at time 3 if education gives bad news about ability, and choose a performance contract (in the S sector) at time 3 if education gives good news about ability. When  $w > \pi_N$ , a worker can educate to signal his favorable private information, i.e., educate and then choose a credential contract in the S sector independently of the information he gets from educating.<sup>14</sup> In that case, the expected utility from educating equals  $w - c(b)$ , where  $c(b)$  denotes the expected cost of education for a worker with self-confidence  $b$ .

An *equilibrium* includes firms' offer of  $w$ , and the beliefs supporting this offer. Denote the average productivity of those that accept a credential contract by  $\alpha$ . Holding  $\alpha$  constant at  $\bar{\alpha}$ , competitive wage setting implies  $w = \bar{\alpha}$ , i.e., a wage equal to average productivity. But clearly  $\alpha$  depends on  $w$  through some function  $\alpha(w; ..)$ , since changes in  $w$  affects the composition of the group that educates. A firm's decision about which

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<sup>14</sup>A different possibility when  $w > \pi_N$  is that a worker undertakes education *both* to acquire information and to signal, i.e., to choose a performance contract in the S sector if the news are good, and a credential contract in the S sector if the news are bad. This case is briefly discussed later.



$w$  to offer depends on its *conjecture* about  $\alpha(w; \cdot)$ , denoted  $\hat{\alpha}(w; \cdot)$ . The following is assumed about  $\hat{\alpha}(w; \cdot)$ : firms expect that a worker with belief  $\hat{b}$  after educating chooses the maximal element of  $\{w, \hat{b}\pi_H - m, \pi_N\}$ . For example, if a worker has the choice between  $w = 3$ ,  $\hat{b}\pi_H - m = 2$ , and  $\pi_N = 1$ , the firm believes that the worker would choose a credential contract, since  $w$  is the maximal element.<sup>15</sup> Together with firms' knowledge of the distribution of beliefs (recall that firms know the distribution function of  $X$ ), that criterion determines  $\hat{\alpha}(w; \cdot)$ , and hence  $w$ . The equilibrium definition ensures that there are unique equilibria in the model.<sup>16</sup>

It is convenient to divide the workers into three different categories, according to their sorting choice at time 2: those that educate (labeled E), those that choose the N sector and skip education (labeled N), and those that skip education and choose the S sector (labeled S). Equilibria where all three groups are present will be denoted *fully separating equilibria*, and will be our main focus.

It will be helpful to clarify whether fully separating equilibria can be 'unconnected', in the sense of a group being split into two or more disjoint parts on the unit interval. For example, the sorting {N,S,E}, where those with the lowest confidence level choose the N sector without educating, those with an intermediate confidence level choose the S sector (and a performance wage) without educating, and finally those with the highest confidence educate, is connected. In contrast, the sorting {N,E,S,E} is not connected, since the E group is split into two disjoint parts (both those with low intermediate confidence and those with the highest confidence educate). The following remark, which is proven using individual workers' payoff only, excludes non-connected equilibria.

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<sup>15</sup>Both in a signaling equilibrium (where  $w > \pi_N$ ), and in a non-signaling equilibrium (where  $w < \pi_N$ ), this requirement governs the off-equilibrium path beliefs of firms (in addition to the on-equilibrium path beliefs). The requirement is similar in the spirit to the Intuitive Criterion of Cho and Kreps, but not identical since  $w$  is endogenous. See Avery and Chevalier (1999) for a similar refinement.

<sup>16</sup>Assuming that there exist equilibria, I have not put enough structure on the distribution of beliefs to exclude more than one fixed point to the equation  $w = \alpha(w; \cdot)$  [with  $w > \pi_N$ ], so potentially there can exist several signaling equilibria. The only candidate signaling equilibrium, however, is the fixed point where the  $\alpha(w; \cdot)$  line crosses the 45 degree line, with the highest value of  $w$ . Why? Suppose there are two fixed points,  $w_1$  and  $w_2$ , where  $w_1 < w_2$ , and that  $w_1$  is an equilibrium wage level. But then a firm can offer a  $w$  in between  $w_1$  and  $w_2$ , and make a profit. Thus, if a signaling equilibrium exists, it is unique. This argument also ensures that there cannot exist a signaling and a non-signaling equilibrium simultaneously. In the case where a signaling equilibrium does not exist (i.e., there are no fixed points with  $w > \pi_N$ ), it is trivial, and hence omitted, to see that there exist a unique equilibrium.

**Remark 2.1** (i) Fully separating equilibria are of two possible types,  $\{N,E,S\}$  and  $\{N,S,E\}$ .  
(ii) In a  $\{N,S,E\}$  equilibrium, the motive behind educating is purely signaling.

**Proof.** See Appendix B. ■

Part (i) of the remark establishes the convenient fact that only connected sortings as  $\{N,E,S\}$  and  $\{N,S,E\}$  are consistent with fully separating equilibria. Part (ii) of the remark shows that in a  $\{N,S,E\}$  equilibrium, the motive behind educating is necessarily (purely) signaling. Intuitively, since those with confidence level close to 1 are pretty sure that they are the high type, their motive for educating cannot possibly be information acquisition, and hence must be signaling. Somewhat surprisingly, the converse result, that the sorting  $\{N,E,S\}$  implies an information acquisition motive for educating is false, as shown later.

Now to the question of whether under- and overconfidence at the individual level cancels out at the aggregate level, given that agents initially have a common prior.<sup>17</sup> Consider the following calibration condition for a distribution of beliefs, denoted condition (C).

**Definition 2.1** Condition (C). A distribution of beliefs is calibrated if the fraction of agents with belief  $b$  that are high, equals  $b$ , for all  $b \in [0, 1]$ .

For example, for a calibrated distribution of beliefs, the share of workers with belief  $\frac{3}{4}$  that are in fact high equals  $\frac{3}{4}$ .<sup>18</sup> The following lemma shows that beliefs will indeed be calibrated in the sense of (C).

**Lemma 1** With probability 1, the distribution of beliefs satisfies (C) at time 2 and at time 3.

**Proof.** See Appendix A. ■

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<sup>17</sup>Motivated by findings of overconfidence from Camerer & Lovallo (1999), Hvide (2001) discusses the case when learning about own abilities does not follow Bayesian principles.

<sup>18</sup>Let me state condition (C) formally. Let  $H(b)$  compute the frequency of high agents with belief  $b$ , and let  $L(b)$  compute the frequency of low agents with belief  $b$ . Then (C) states that,

$$\frac{\theta H(b)}{\theta L(b) + (1 - \theta)L(b)} = b, \forall b \in (0, 1).$$

The lemma says that although individual workers may be under- or overconfident, it follows from Bayesian learning that self-beliefs are calibrated at population level.<sup>19</sup> Lemma 1 follows from common priors and straightforward assumptions on the information acquisition prior to educating. In appendix A, the robustness of Lemma 1 is discussed. Notice that by condition (C), those with low confidence are on average of low ability, those with intermediate confidence are on average mediocre, and those with high confidence are on average of high ability. Therefore, we will interchangeably refer to those with  $b$  close to zero (one), as having low (high) confidence level and having low (high) ability level in the following.<sup>20</sup>

### 3 Equilibrium Sorting

We start out by considering equilibria where the role of education is information acquisition, and then consider signaling equilibria.

#### 3.1 Information Acquisition Equilibria

We now consider equilibria where the motive behind education is information acquisition.

**Definition 3.1** *An information acquisition equilibrium (IAE) is a fully separating equilibrium where the role of education is information acquisition.*

The definition of an IAE does not distinguish between the public and the private information case. Where necessary, we label an IAE under public information for a *public information IAE*, and an IAE under private information for a *private information IAE*. Uniqueness of an IAE follows directly from the derivation of individual payoffs in Appendix B. We now solve for the equilibrium sorting in an IAE.

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<sup>19</sup>Lemma 1 has some interest in its own right. First, an interesting task could be to compare Lemma 1 to findings of *overconfidence* in real life data (Asubel, 1991), and in experimental settings (Camerer and Lovo, 1999). Second, Lemma 1 seems useful in (yet undeveloped) multi-agent career concerns models, and moreover fits well into the framework of Benabou & Tirole (2000a,b).

<sup>20</sup>In a model with 3 productivity types, a belief of a worker would be a point in a simplex with 3 vertices, each of length unity. Intermediate confidence in such a generalized model can be understood to have a belief near the middle of the simplex. Such a person would be of intermediate ability in expected terms, provided that the difference between each adjoined type is not great. This argument can be generalized to the  $k$ -type case.

**Proposition 1** *In an IAE, the sorting of workers is  $\{N,E,S\}$ . In a private information IAE, the  $S$  group are employed on performance wages, while in a public information IAE, the  $S$  group are employed on fixed wage contracts.*

**Proof.** We start out with the second claim. First, contracts in the  $S$  sector under private information must be of the performance type (with monitoring), because of the adverse selection that occurs without monitoring. Under public, symmetric, information, it follows from competition arguments that, in equilibrium, firms offer the worker the fixed wage  $\tilde{b}\pi_H$  (with no monitoring cost involved), where  $\tilde{b}$  is the common belief about the worker's ability. A performance contract, on the other hand, gives the worker the utility  $\tilde{b}\pi_H - m$ . Since  $m > 0$ , all workers reject performance contracts, and hence only fixed wage contracts occur in a public information equilibrium. Now the first claim. Provided  $w < \pi_N$ , which is necessarily the case in an IAE, from Remark 2.1 it follows that the sorting must be  $\{N,E,S\}$  in a private information IAE. From a similar argument the same conclusion follows for the public information case follows. The conditions for  $w < \pi_N$  will be considered further down, under existence. ■

Thus if the motive behind education is information acquisition, the least able (in expected terms) choose the  $N$  sector, the mediocre educate, and the most able choose the  $S$  sector in a fully separating equilibrium. The intuition behind the result is that those at the extremes have a lower value of information than those in the middle, and hence if any workers educate, those in the middle must be included in that group.

A private information IAE and a public information IAE both have the sorting  $\{N,E,S\}$ , but equilibrium contracts are different in the two cases because of the presence of monitoring under private information. A natural question is whether the difference in contracts under public and private information implies different sortings under these two informational assumptions (holding the parameter values constant).

**Proposition 2** *i) When  $m = 0$ , the allocation of workers in the private information IAE and the public information IAE are identical. ii) The private information IAE has more able students the higher  $m$ .*

**Proof.** See Appendix B. ■

Recall that the utility of a fixed wage contract for a worker with confidence level  $\bar{b}$ , equals  $\bar{b}\pi_H$ , under *public information*, while it equals  $\bar{b}\pi_H - m$  under *private information*. When  $m$  approaches zero, the two values converge (the same convergence can be seen for the utility from educating), and the allocations of workers must also converge. The intuition for why students are more able in the private information IAE the higher  $m$  is the following. A private information IAE has two cutoff beliefs; the cutoff between N and E (denoted  $b_1$ ), and the cutoff between E and S (denoted  $b_2$ ). An increase in  $m$  reduces the payoff from educating (since a performance contract becomes less attractive later on), while the payoff from choosing the unskilled sector directly is not affected. Hence the cutoff  $b_1$ , which separates the N and the E group, increases with  $m$ . On the other hand, an increased  $m$  decreases the payoff from a performance contract directly, even more than it decreases the payoff from educating. Hence both cutoffs  $b_1$  and  $b_2$  are increasing in  $m$ , and it follows that the (average) ability of the educated group in the private information IAE increases with  $m$ .

Two questions are under which circumstances an IAE is produced, and whether an IAE is more likely to occur under private than under public information.

**Remark 3.1** *An IAE exists provided  $c_L$  not too low,  $c_H$  not too high,  $m$  not too high, and  $p$  sufficiently high. Furthermore, for identical parameter values, the conditions for existence of an IAE are more restrictive under private than under public information.*

**Proof.** Start out by considering the private information case, and for the moment assume  $w < \pi_N$ . An IAE is then characterized by a)  $U_N$ ,  $U_E^{IA}$  and  $U_S$  lines intersecting in an appropriate way, and b)  $w < \pi_N$ . For a), we have the following five conditions, which together are sufficient; (i)  $U_S(1) > U_E^{IA}(1)$ , (ii)  $U_S(0) < U_E^{IA}(0)$ , (iii)  $U_S(1) > U_N(1)$ , (iv)  $U_E^{IA}(0) < U_N(0)$ , and (v) The intersection between  $U_S(b)$  and  $U_E^{IA}(b)$  must occur above the  $U_N(b)$  line. As can easily be seen, (iv) is always satisfied, (iii) implies (i), and (v) is satisfied for  $\pi_N$  not too high and  $p$  not too low (if either of these do not hold, then education is dominated by either N or S). Hence, given  $\pi_N$  not too high and  $p$  not too low, there are only two conditions required for a) to be satisfied. From (ii)  $U_S(0) < U_E^{IA}(0)$ , we get,  $c_L + \frac{1-p}{p}c_H < \frac{m + \pi_N}{p}$ , i.e.,  $c_H$  and  $c_L$  not too high. And (iii)  $U_S(1) > U_N(1)$  implies  $m < \pi_H - \pi_N$ , i.e.,  $m$  sufficiently small. For b) to hold, any firm deviating with offering

$w' > \pi_N$  must run a deficit. Obviously, any sensible deviation must have  $w' \in [\pi_N, \pi_H)$ . It is now showed that for  $m$  sufficiently small and  $p$  sufficiently high, a simple unraveling argument ensures that there cannot exist a gaining deviation. Ex-post of education, a worker chooses a performance contract rather than a credential contract if  $\hat{b}\pi_H - m > w'$ , where  $\hat{b}$  is the worker's ex-post belief. A firm offering  $w'$  will thus attract all workers with  $\hat{b} \leq \frac{w'}{\pi_H - m}$ . When  $p$  is high, each educated worker has an ex-post belief  $\hat{b}$  either close to 0 (those that received bad news), or  $\hat{b}$  close to 1 (those that received good news). For  $m$  sufficiently low, then those that received good news will prefer a performance contract rather to a credential contract, and only those that received bad news choose a credential contract. But, since  $p$  is high, the productivity of the agents that received bad news is close to zero in the S sector. Consequently, for  $p$  high and  $m$  low, the deviating firm will run a deficit, and together with a), we then have obtained conditions for existence of a private information IAE, which together are sufficient.

Now consider the public information case, and start out by assuming  $w < \pi_N$ . In that case, the conditions for existence of an IAE are exactly the same as i)-v) above, except that  $m = 0$ , since fixed wage contracts are applied. Hence condition iii) always holds under public information. That makes an IAE more likely to exist under public information than under private information. It is now shown that  $w < \pi_N$  always holds under public information, which pulls in the same direction. Firms believe that by setting  $w > \pi_N$ , they will only attract workers with expected productivity lower than  $w$ , i.e.,  $\hat{b} \pi_H < w$ , since workers with expected productivity higher than  $w$  will choose a fixed wage contract (since  $m = 0$  under such contracts). Hence firms offering  $w > \pi_N$  will run a negative profit, and such credential contracts will not be offered (i.e.,  $w < \pi_N$  in equilibrium). In other words, there will never be credential contracts in equilibrium under public information. ■

Under private information, the important conditions for an IAE to exist is that  $p$  is sufficiently high and  $m$  sufficiently low, in which case unraveling excludes the existence of a signaling equilibrium. Since  $m$  is irrelevant under public information, and moreover signaling excluded, the conditions for existence of an IAE are less restrictive under public than under private information.

The results obtained so far points out that education can come about in equilibrium even if it does not alter a worker's human capital stock, and even if education is not signaling. The role of education is acquiring information capital that does not alter the productivity in a given job, but does increase the probability of a successful match with the right job later on.

To illustrate the content of the previous results, consider the following example of information acquisition equilibria with private information and varying  $m$ .

**Example 1** *For simplicity assume that  $p = 1$ . We then have the following payoffs at time 2, where  $U_N(b)$  refers to the payoff from choosing  $N$ ,  $U_S(b)$  refers to the payoff from choosing the  $S$  sector directly, and  $U_E^{IA}(b)$  refers to the payoff from educating.*

$$\begin{aligned} U_N(b) &= \pi_N \\ U_S(b) &= b\pi_H - m \\ U_E^{IA}(b) &= \pi_N - c_L + b(\pi_H - \pi_N + c_L - c_H) \end{aligned} \quad (1)$$

Define the cutoff between  $N$  and  $E$  as  $b_1$ , and the cutoff between  $E$  and  $S$  as  $b_2$ . Then, as can be easily calculated,

$$\begin{aligned} b_1 &= \frac{c_L}{\pi_H - m - \pi_N + c_L - c_H} \\ b_2 &= \frac{c_L - \pi_N - m}{c_L - \pi_N - m - c_H} \end{aligned} \quad (2)$$

Furthermore, define  $\pi_H = 3$ ,  $\pi_N = 2$ ,  $c_L = 1$ ,  $c_H = 1/3$ , to obtain the cutoffs as a function of  $m$  alone. As explained before, by the continuum of workers assumption, these cutoffs uniquely define the allocation of workers.

$$\begin{aligned} b_1 &= \frac{3}{5 - 3m} \\ b_2 &= \frac{3(1 + m)}{4 + 3m} \end{aligned} \quad (3)$$

For  $m = 0$ , the public and the private information cutoffs coincide and equal  $b_1^{m=0} = .6$ ,  $b_2^{m=0} = .75$ . Both  $b_1$  and  $b_2$  are increasing in  $m$ , and hence the average quality of the

educated group increases in  $m$ . These two points confirm Proposition 2. The condition  $b_1 < b_2$ , which is necessary for existence, is satisfied for  $m < .43$ . For a private information IAE to exist we also must have that no firm can profitably deviate by offering a credential contract. Clearly, if a credential contract offer only attracts those who received a negative signal (and hence are low ability since  $p = 1$ ) then that deviation runs a negative profit. Hence a gaining deviation must have,

$$w > U_S(1) = 3 - m \quad (4)$$

However, such a deviation cannot be profitable, because the average productivity of the educated group must fall short of the wage offer, unless  $m < .49$ . Hence an IAE exists for  $m < .43$ . That confirms Remark 3.1.

Since the model has formal similarities to the Spence education model, it should not be surprising that signaling equilibria can also exist under private information. These are considered in the next section.

## 3.2 Signaling Equilibria

This section considers signaling equilibria.

**Definition 3.2** *A signaling equilibrium is an equilibrium where the motive behind education is not information acquisition.*

While education in the Spence (1973) model always is socially harmful, Spence (1974) considers a two-sector signaling model where signaling/education has a social role in allocating talent. As in Spence (1973), signaling equilibria in Spence (1974) have the property that the most able educate, and are allocated to the sector with the highest return to talent. There are two main differences between Spence (1974) and the present model. First, we have workers with imperfect information about their own abilities, and second in our model performance contracts is an alternative sorting mechanism to schooling. The differences in informational assumptions in our model and in Spence (1974) can most easily be compared by assuming private information and excluding performance contracts, by letting  $m$  tend to infinity.



**Remark 3.2** *Provided  $m = \infty$ , separating equilibria must have the sorting  $\{N,E\}$  and the role of education must be signaling.*

**Proof.** Since performance contracts are not used, there is no point in gaining information about one's type. Therefore signaling must be the motive for education. Obviously those at the bottom cannot educate in a signaling equilibrium, and hence the sorting must be  $\{N,E\}$  in a separating equilibrium. ■

Hence the result from the present model, with  $m$  high, is that the workers on the top choose to educate, which is in line with Spence (1974). The next question is which impact the presence of performance contracts has on the sorting of signaling equilibria. Is the sorting of signaling equilibria confined to  $\{N,S,E\}$ , in the spirit of Spence, but contrary to the sorting in IAE? The following comment to Spence points out that even in signaling equilibria, the sorting can be  $\{N,E,S\}$  provided that performance contracts are feasible.

**Proposition 3** *For finite  $m$ , there exists two types of fully separating signaling equilibria. One type has the sorting  $\{N,S,E\}$  and the other type has the sorting  $\{N,E,S\}$ .*

**Proof.** See Appendix B. ■

Hence both the sorting  $\{N,S,E\}$  and the sorting  $\{N,E,S\}$  are consistent with signaling equilibrium. A  $\{N,S,E\}$  signaling equilibrium occurs when  $c_L$  is high compared to  $c_H$ , so that it is costly for those in the middle to imitate those at the top. For a  $\{N,E,S\}$  signaling equilibrium, on the other hand, the intuition is that  $c_L$  is in an intermediate range compared to  $c_H$ , so that those at the top choose S, to avoid being imitated by those in the middle.

Notice that the condition  $w > \pi_N$  does not guarantee that a signaling equilibrium is played, since it does not exclude that some agents have 'mixed' motives behind educating. Mixed motives equilibria are characterized by agents with a positive signal from education taking a performance wage in the S sector, and those with a negative signal taking a credential contract. Such equilibria are hard to characterize but may have some empirical plausibility; those that educate work in the same sector, but on different payment schemes. Those who do best in school choose jobs with relatively high-powered incentives, and those that are second in school choose jobs with relatively low-powered incentives.

A persistent finding from the equilibrium analysis is that the mediocre educate, while the most able skip education. Since established theories of education (see e.g., Borjas, 1996), predict that the most able educate, we should discuss that issue. Is the real world characterized by those in the middle educating, rather than those at the top? For several professions, like Medicine, education serves as a license, and individuals without a license are denied work. Here, there is no reason to believe that the prediction of the model should hold. In an area like business, however, a degree is not required. It is interesting to note that there do exist some evidence (admittedly rather casual) that some of the most able within business skip education. For example, Orzach and Tauman (1999) argue that surprisingly many on the 1996 Forbes 400 list, the 400 richest people in the US, do not have an academic degree (Bill Gates is a well-known example).<sup>21</sup> Another interesting finding is that regularly, MBA students from top schools drop out to work in new economy firms, like internet start-up companies.<sup>22</sup> This finding seems consistent with the best choosing a direct way of entering the job market, rather than educating first, as the basic model predicts.<sup>23</sup>

In Appendix C, we consider some welfare properties of the model. We now make the model more realistic by letting education have a direct productivity augmenting effect, in addition to the information acquisition effect.

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<sup>21</sup>Orzach and Tauman (1999) argue that this finding is consistent with equilibrium in a signaling model where students have the option to quit school after a short period of study. Feltovich, Harbaugh and To (1999) also argue that the mediocre invest in schooling to sort themselves from the least able, while the most able skip school (like Gates did). To construct such separating equilibria, Orzach and Tauman (1999) assume that ordinary individuals learn at an exceptional rate at the basic education level, and Feltovich, Harbaugh and To (1999) assume that firms know other individual-specific characteristics than education level.

<sup>22</sup>According to the Exec-Express Magazine issue of August 2000, top MBA programs as London Business School have experienced a substantial drop-out to internet start-up companies (it is also well-known that Stanford University has experienced substantial drop-out rates to the high-tech industry). As a compromise, some business schools allow MBA students to gain work experience in companies like Garage.com, which has a summer intern program designed to provide first year MBA students with experience of working in start-ups.

<sup>23</sup>Another practical example of the basic model is the entry draft for the National Basketball Association. High school players have the choice to enter the draft for the professional league directly or to enter college. In college, those players gain skills, but also learn more about their own abilities.

## 4 Extensions and Applications

We have emphasized that education and monitoring can play the role of producing information, but have ignored the fact that they can also increase productivity directly, through human capital growth and incentives. In this section, we consider the case when education implies an increase in general human capital, and moreover discuss the case when worker productivity is determined by effort in addition to type.

### 4.1 Productivity augmenting effect of education

We model general human capital acquisition by assuming that education increases productivity by a factor  $h > 1$  in each job. Specifically, an educated low ability worker has productivities  $(h\pi_N, h\pi_L) = (h\pi_N, 0)$ , in the N and S job respectively, and an educated high worker person has productivities  $(h\pi_N, h\pi_H)$ , in the N and S job respectively.<sup>24</sup> The productivities of an uneducated worker is the same as before. We focus on the structure of IAE when education also adds to general human capital, and assume throughout the section that information is public.

**Proposition 4** *i) For  $h$  sufficiently close to 1, a fully separating equilibrium has the sorting  $\{N, E, S\}$ . ii) For larger  $h$ , the sorting must be  $\{N, S, E\}$ .*

**Proof.** The first, robustness, claim follows along the lines of the proof of Remark 2.1 and is skipped for brevity. For ii), observe that  $U_N$  and  $U_S$  are unaffected by  $h$ , while  $U_E$  is the same as in equation (1), except that  $\pi_N$  is replaced by  $h\pi_N$  and  $\pi_H$  is replaced by  $h\pi_H$ . Hence there must exist a value of  $h$ , denoted  $\hat{h}$ , such that for  $h > \hat{h}$  those at the top choose to educate. ■

While the first part of the claim follows from standard robustness arguments, the second claim provides some qualitatively new insight. The intuition for the result is that for  $h$  sufficiently high, those at the top will realize a high absolute productivity increase from educating. To provide an example of an  $\{N, S, E\}$  equilibrium, consider the following parameter values.

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<sup>24</sup>Other specifications of human capital acquisition, like an additive formulation, produces qualitatively the same type of results.

**Example 2** Let  $\pi_H=3$ ,  $\pi_N=1$ ,  $c_L=2$ ,  $c_H=1$ ,  $p=1$ , and  $h=\frac{3}{2}$ . As can easily be calculated, a fully separating equilibrium is characterized by the sorting  $\{N,S,E\}$ , with cutoffs  $\frac{1}{3}$  and  $\frac{1}{2}$ . Those in the middle will face a lesser productivity increase (and a higher cost of education) than those at the top, and will prefer to enter the  $S$  sector directly, without educating first.

## 4.2 Comparison of Bachelor's degrees

It may be the case that some types of education primarily have a productivity augmenting effect, while other types of education have an informational effect. For example, a large part of a US Bachelor's degree will typically consist of a mixture of courses across fields, which can be as distant as chemistry and philosophy. The specialization that occurs will consist of general knowledge within a field and will not necessarily make the student much more productive in any given job. However, the experimentation allowed the student will give her important information about which field she should later specialize in. In contrast, the UK Bachelor's degree is typically rather specialized, covering the same field for at least three years, and allowing some true specialization that will make the student more productive in given jobs. However, due to the specialization, the UK Bachelor's degree will not provide students with the same information about different fields as the corresponding US degree.

So the point is that the UK system will make students more productive than the US system, holding the job constant. However, the US system enable students to make better informed choices of which sector they should work in.<sup>25</sup> Depending on the relative magnitude of the two learning effects, one system may dominate the other system, given that they educate workers to the same type of society. One reason why different systems are preferred in the two countries can be that moving costs are lower in the US than in the UK (both sectorwise and geographically), so that information about abilities is more valuable for a person living in the US than for a person living in the UK.

In several European countries, the education system is a hybrid of the US and the UK system, meaning that special and general educations coexist at the undergraduate level.

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<sup>25</sup>We are ignoring other differences between the two systems, for example that the UK undergraduate education usually lasts for 3 years, while the US undergraduate education usually lasts for 4 years.

For example, in Norway both general and specific undergraduate degrees are available. The general degree (Cand. Mag.) is similar to the US Bachelor's degree, and the specific degrees are similar to the UK Bachelor's degree.

By extending the model, we can ask whether the general or the specific education tend to recruit the most able. We label by  $E^A$  the education which primarily leads to information acquisition, and by  $E^B$  the education that primarily leads to a productivity increase.<sup>26</sup> For simplicity it is assumed that  $E^A$  is perfectly revealing about type, i.e., that  $p^A=1$  and that  $E^A$  does not have a productivity increasing effect, i.e.,  $h^A=1$ . For  $E^B$ , it is assumed that  $p^B = \frac{1}{2}$  and that  $h^B$ , simply written  $h$ , is greater than unity. We then have the following result.

**Proposition 5** *In a fully separating equilibrium, the sorting is  $\{N, E^A, S, E^B\}$ .*

**Proof.** We have the following payoffs.

$$\begin{aligned}
U_N(b) &= \pi_N \\
U_S(b) &= b\pi_H \\
U_E^A(b) &= b[\pi_H - c_H] + (1-b)[\pi_N - c_L] = \pi_N - c_L + b(\pi_H - \pi_N + c_L - c_H) \\
U_E^B(b) &= h\pi_N - \bar{c} \text{ for } b < \tilde{b} \text{ and } hb\pi_H - \bar{c} \text{ for } b \geq \tilde{b}.
\end{aligned} \tag{5}$$

where  $\tilde{b} := \frac{\pi_N}{\pi_H}$ , i.e., the belief that makes a worker indifferent between N and S, and  $\bar{c} = \frac{c_L + c_H}{2}$ . i) observe that  $\frac{\partial U_E^B}{\partial b} = h\pi_H > \frac{\partial U_S}{\partial b} = \pi_H$  for  $b \in [\tilde{b}, 1]$ . Hence for a fully separating equilibrium to exist, those with the highest belief must prefer  $E^B$  to S. ii) observe that  $U_E^A(1) = \pi_H - c_H < U_S(1) = \pi_H$ . Hence S must dominate  $E^A$  for those with the highest beliefs. From i) and ii) it follows that the sorting at the top must be  $\{E^A, S, E^B\}$  for a fully separating equilibrium to exist. By the same argument as before, N must be preferred by those with lowest beliefs to be preferred by anyone, which occurs for  $\tilde{c}$  sufficiently high. Hence the sorting in a fully separating equilibrium must be  $\{N, E^A, S, E^B\}$ .

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<sup>26</sup>Different degrees are characterized by different degrees of rationing, which may reflect underlying differences in the cost for the educational institution of providing a degree. We abstract from such issues here.

The intuition for the result is that although the percentage productivity increase from  $h$  is uniform across agents, the absolute productivity increase is higher for those with high beliefs than for those with low beliefs. That leads the more able to prefer the B education rather than the A education. And, as before, the value of information is higher for those in the middle than those at the top. That leads those in the middle to prefer the A education to the B education. The intuition for why the sorting must be  $\{N, E^A, S\}$  below the  $E^B$  group, i.e., that S group consists of more able workers than  $E^A$ , is the same as before.<sup>27</sup>

It is interesting to notice that Proposition 6 is consistent with empirical evidence. In e.g., Norway, the most able (measured by high school grades) tend to be recruited to the professional educations (MD, BBA, BScint, and to a certain extent Law, Psychology and Economics), similar to the UK bachelor degrees, while the (on average) less able tend to undertake general university educations that are quite similar to the US bachelor degrees.<sup>28</sup>

### 4.3 Other issues

We now discuss the impact on the results to changes in the assumptions of the model.

We have only considered the polar cases where either all information received is private or where it is public. Let us consider intermediate cases. First the effect of letting the information prior to education be private, while the information during education be public would create two classes of educated, those with low grades and those with high grades. For information acquisition equilibria, this distinction would not be important, since both groups are offered the same type of contracts. There would, however, exist more sophisticated signaling equilibria than before, where workers with low (high) grades receive a low (high) credential wage. Since workers with lower (higher) beliefs are less (more) likely to receive high grades, such a distinction would tend to make those with lower (higher) beliefs less (more) motivated to educate, but would not have much additional impact.

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<sup>27</sup>The model does not take into account that different educations tend to recruit students to different occupations, or sectors. As long as those sectors do not have very different underlying returns to ability, such an extension would not alter the results in any significant way.

<sup>28</sup>It should be mentioned that the author holds a general university degree from Norway.

Second, letting the information prior to education be public, while the information during education be private, would destroy the possibility of signaling equilibria, since such equilibria relies on information that is private before education. In that case, IAE would be the only possible type of equilibrium.

The introduction of risk aversion would make the A education more attractive compared to B education and to S, because this option reduces risk. But for risk preferences without strong wealth-effects, the group of agents undertaking the A education would still be those in the middle, since their insurance motive would be the strongest. A negative wealth-effect on risk could lead those at the top to also have a strong insurance motive, and it is conceivable that the group of agents undertaking the A education would be unconnected (but in any case those at the very top would not choose A education).

We have only considered adverse selection effects - there has been no notion of workers choosing their level of effort once employed. If workers choose their level of effort, and effort is observable, contracts can be conditioned on the appropriate level of effort being expended, and the same type of results as before would follow.<sup>29</sup> If effort is unobservable, appropriate forcing contracts can be defined to implement the first best level of effort, as long as workers are risk neutral, and again the same type of results would follow. If effort is unobservable and workers are risk averse, second-best performance contracts, trading off risk and incentive effects, would be constructed. Those performance contracts would be less high-powered incentives than those in the present model, due to risk concerns. However, provided that risk aversion is not severe, we expect IAE in such an extended model to have qualitatively very similar properties to the IAE studied presently, since performance contracts are already used in such equilibria. For signaling equilibria, however, the introduction of effort and risk aversion would make the equilibrium credential contracts include performance elements, to induce effort. A simple linear contract could be one way for a firm to ensure that the worker both expends effort (through the bonus component) and would wish to sign for the firm (through the salary component). At any rate, we expect sorting to be very similar in such an extended model.

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<sup>29</sup>Prendergast (2000) argues that contracts that conditions on input rather than output are both common in practice, and models that builds on such contracting can explain important facts not captured by the standard risk-return incentive model.

## 5 Conclusion

While the received education literature tends to focus on the accumulation of human capital through education and the implied dispersion of wages, we extend the literature by focusing on how education serves to allocate talent into different sectors of the economy. In particular, we focused on a two-sector setting where contracts are determined endogenously, and where education provides workers with information about their abilities. In contrast to established theories of education, the basic model obtained the result that those with intermediate ability educate, while those at the top skip education.

When the model was made more realistic, by including a direct productivity augmenting effect of education, this conclusion could be reversed, provided that the productivity augmenting effect is sufficiently strong. More interestingly, the extension could be applied to discuss the properties of different educational systems. For example, it was shown that in a system where general and specific educations co-exist, the most able will tend to undertake the specific education, while those in the middle will undertake general education, a finding that is consistent with empirical facts.

One extension of the present model could be to build a dynamic setting where agents can learn about their abilities through work experience, in addition to through educating. A realistic feature of such a model could be to include a cost of switching sectors (such a cost could be monetary as well as non-monetary). Included in the returns to education for an individual would then be a reduced switching cost later in the career. Since empirical studies on the returns to education typically take into account increases in wages from education, but not the benefit of a reduced switching costs, this argument suggests that the estimated returns to education found in empirical studies are biased downwards, due to the ignored allocation effect of education.

Another extension of the present work is to attempt to better understand the functioning of the education market when several educations co-exist. For example, while the present model assumes that the cost of education is essentially constant across educations, the cost of education, including tuition fees, could more realistically be seen as emerging from competition between different education institutions.



## 6 Appendix A: The Relation Between Common Priors and Condition (C)

It is proven that the distribution of beliefs satisfies condition (C) at time 2 and at time 3, with probability 1. Since it is trivial to see that condition (C) is satisfied at time 3, if it is satisfied at time 2, we save space by merely proving that the distribution of beliefs satisfies (C) at time 2.

At time 0, workers are born with a common prior  $\theta$ . Between time 0 and the education decision at time 2, each worker receives independent information about their abilities, and constructs a pre-education belief  $b$ . Formally, the information received by each worker between time 0 and time 1 is an independent realization of a random variable  $T$ , where, for simplicity,  $T$  has the support  $[0, 1]$ . If the true ability of a worker is high, then  $T$  follows the density  $d_H(t)$ , while if the worker's true ability is low then  $T$  follows  $d_L(t)$ . To avoid 'holes' in the distribution of beliefs at time 1, assume that  $d_L(\cdot)$  is continuous and strictly decreasing, and  $d_H(\cdot)$  is continuous and strictly increasing, with  $d_L(1), d_H(0) \geq 0$ , and  $d_L(0)$  and  $d_H(1)$  finite. Thus the higher realization of  $T$ , the better news for a worker.

Let  $h(b)$  be the fraction of agents that are high among those with belief  $b$  prior to education. Thus,

$$h(b) := \frac{\theta H(b)}{\theta L(b) + \theta H(b)}, \quad b \in (0, 1) \quad (\text{A1})$$

where  $H(b)$  is the frequency of high agents that have the belief  $b$ , and  $L(b)$  is the frequency of low agents that have the belief  $b$ . Notice that since the information received by each worker is stochastic,  $H(b)$  and  $L(b)$  are random variables, and hence  $h(b)$  is also a random variable. We wish to prove that,

**Lemma 1** *With probability 1,  $h(b) = b$  at time 2,  $\forall b \in (0, 1)$ .*

**Proof.** From Bayes' rule it follows that an individual who receives information  $T = t$ , has posterior  $b(t; \theta) = \frac{\theta d_H(t)}{\theta d_H(t) + (1 - \theta) d_L(t)}$ . Let  $N(b)$  denote the number of individuals with posterior  $b$ ,  $M(b)$  the number of high agents with posterior  $b$ , and the  $m(b)$  the *share* of high agents;  $m(b) = \frac{M(b)}{N(b)}$ . Since  $\Pr(H|b) = b$ , it follows that  $M$  is a random variable,

binomially distributed with parameters  $b$  and  $N$ , with  $E(M) = bN$ , and  $Var(M) = Nb(1 - b)$ . It follows that  $E(m|b, N) = b$ , and  $Var(m|b, N) = \frac{b(1 - b)}{N}$ . Since  $\frac{b(1 - b)}{N}$  converges to zero as  $N$  increases, the probability of  $m(b) \in (b - \epsilon, b + \epsilon)$  converges to 1 as  $N$  becomes large, for any  $\epsilon > 0$ . Since there is a continuum of agents at the outset, continuity and monotonicity of  $d_L(\cdot)$  and  $d_H(\cdot)$  ensures that there will be a continuum of agents for each posterior  $b \in (0, 1)$ ; hence  $N(b)$  goes to infinity. Thus condition (C) holds with probability 1 at time 2. ■

Lemma 1 shows that starting with a common prior, and assuming that each individual receives a private and independent signal about his ability, the distribution of beliefs prior to education will satisfy condition (C) with probability 1.

Lemma 1 will hold also in the case where the distributions of the information received by individual agents are independent, but not identical. [It follows that condition (C) also will hold at time 3]. Second, the distribution functions need not be independent for Lemma 1 to hold. By a slightly more elaborate argument it can be shown that Lemma 1 holds even if the information received by workers is (imperfectly) correlated, or if the information received by some agents is correlated, and by others not. Third, obviously a continuum of workers is needed to get convergence with probability 1. With a finite, but large, number of workers, the distribution of beliefs will be 'close' to (C) with a high probability. Thus Lemma 1 is fairly robust.

## 7 Appendix B: Proofs

### Expected utility for the three possible actions at time 2:

As explained in the text, the payoff from choosing the N sector directly and choosing a performance contract in the S sector directly equals,

$$\begin{aligned} U_N(b) & : = \pi_N \\ U_S(b) & : = b\pi_H - m \end{aligned} \tag{B0}$$

Denote the value of  $b$  where a worker is indifferent between these options for  $b_0$ . Hence,

$b_0 = \frac{\pi_N + m}{\pi_H}$ . Now the utility from educating. The cost of education,  $c(b)$ , equals,

$$\begin{aligned}
c(b) &= \text{Prob}(X = c_H|b)c_H + \text{Prob}(X = c_L|b)c_L \\
&= [bp + (1-p)(1-b)]c_H + [b(1-p) + (1-b)p]c_L \\
&= [c_H - c_H p + p c_L] - b[2p - 1][c_L - c_H]
\end{aligned} \tag{B1}$$

which is linearly decreasing in  $b$ . For  $w < \pi_N$ , the utility from educating can be split into three intervals, depending on the value of  $b$ . The first (third) interval consists of the values of  $b$  where N (S) is the optimal choice independently of the realization of  $X$ . The second interval consists of the values of  $b$  where the optimal choice of sector depends on the realization of  $X$ ; N if the realization is  $c_L$  and S if the realization is  $c_H$ . Denote the boundaries of this interval for  $b_L$  and  $b_H$ , where  $b_L < b_H$ . Then,

$$\begin{aligned}
U_E^{IA}(b) &: = \text{Pr}(X = c_H|b)[\text{Pr}(H|X = c_H, b)\pi_H - m] + \\
&[1 - \text{Pr}(X = c_H|b)]\pi_N - c(b), \quad b \in [b_L, b_H].
\end{aligned} \tag{B2}$$

where topscript  $IA$  stands for information acquisition. Rewriting this expression and adding the utilities off the interval  $[b_L, b_H]$ , we get,

$$U_E^{IA}(b) = \begin{cases} \pi_N - c(b) & \text{if } b < b_L \\ [bp + (1-p)(1-b)][-m - c_H] + \\ bp\pi_H + (b(1-p) + (1-b)p)(\pi_N - c_L) & \text{if } b \in [b_L, b_H] \\ b\pi_H - m - c(b) & \text{if } b > b_H \end{cases} \tag{B3}$$

First notice that  $U_E^{IA}(b)$  is piecewise linear. Since  $U_E^{IA}(b)$  consequently is the upper envelope of three linear components,  $U_E^{IA}(b)$  is convex. Second, as can be verified,  $b_L = \frac{b_0(1-p)}{b_0 + p - 2b_0p}$  and  $b_H = \frac{b_0p}{1-p - b_0 + 2b_0p}$ . Notice that off  $[b_L, b_H]$  education, and hence further down in the proof we just use the expression in (B2). When  $p$  equals 1,  $b_L$  equals 0,  $b_H$  equals 1, and  $U_E^{IA}(b)$  becomes linear and equal to the expression in the second line of (B3) inserted for  $p = 1$ , which equals the expression in equation (1).

When  $w > \pi_N$ , education can be both signaling (choose  $w$  independently of the value

of  $X$ ) and information acquisition (choose  $w$  if news are negative, and a performance contract if news are positive), and we label the corresponding utility for  $U_E^M(b)$ , where  $M$  stands for mixed motives. The expression for  $U_E^M(b)$  is identical to  $U_E^{IA}(b)$ , except that  $\pi_N$  is replaced by  $w$ , and is not reproduced. By the same argument as with  $U_E^{IA}(b)$ , it can be seen that  $U_E^M(b)$  is convex.

The pure signaling motive behind education is a special case of the mixed motives behind education, and occurs if  $w$  is preferred to a performance contract in the  $S$  sector independently of  $b$ . The expected utility for a pure signaling motive then equals,

$$U_E^S(b) = w - c(b) \tag{B4}$$

where topscript  $S$  stands for signaling. Notice that for all  $b$ ,  $U_E^M(b) \geq U_E^S(b)$  by construction.

**Proof.** of Remark 2.1.

We first show that fully separating equilibria must be connected, and then prove i) and ii). Notice first that the utility from education, labeled  $U_E(b)$  (which encompasses both cases  $w < \pi_N$  and  $w > \pi_N$ ) is increasing in  $b$ . Since  $U_N(b)$  is a constant, and both  $U_S(b)$  and  $U_E(b)$  are increasing in  $b$ ,  $U_N(b)$  cannot cross  $U_E(b)$  or  $U_S(b)$  more than once. To establish the claim, it is hence sufficient to show that  $U_E(b)$  and  $U_S(b)$  can cross at most once. Consider first the case when  $w < \pi_N$ , which implies that the motive behind education must be information acquisition. There are two cases, a)  $U_E^{IA}(1) > U_S(1)$  and b)  $U_E^{IA}(1) < U_S(1)$ . Case a) can only occur if  $\pi_N > \pi_H - m$ , in which case  $U_N(b) > U_S(b)$  for all  $b$ , and a fully separating equilibrium cannot exist. Now consider b)  $U_E^{IA}(1) < U_S(1)$ . But then, from the convexity of  $U_E^{IA}(b)$ , the lines  $U_E^{IA}(b)$  and  $U_S(b)$  can cross only once and connectedness follows. For the case  $w > \pi_N$  the argument is analogous. Again there are two cases, a)  $U_E^M(1) > U_S(1)$  and b)  $U_E^M(1) < U_S(1)$ . Case a) can only occur if  $w > \pi_H - m$ , in which case  $U_E^M(b) = U_E^S(b)$ .  $U_E^S(b)$  is linear, implying that also  $U_E^M(b)$  is linear under a), and connectedness follows immediately. Now consider b)  $U_E^M(1) < U_S(1)$ . But then, from the convexity of  $U_E^M(b)$ , the lines  $U_E^M(b)$  and  $U_S(b)$  can cross only once and connectedness follows. Hence fully separating equilibria must be connected. Since  $U_E^{IA}(b)$ ,  $U_E^M(b)$ , and  $U_S(b)$  are upward-sloping and  $U_N(b)$  a constant, it follows that i) fully

separating equilibria must be of the type  $\{N,E,S\}$  or the type  $\{N,S,E\}$ . To prove ii), observe that workers with  $b = 1$  have a zero value of information, and hence their motive behind education must be signaling. If signaling is the motive behind education for those with  $b = 1$ , it must also be the motive behind education for those with a lower  $b$ , by a simple revealed preference argument. Hence signaling is necessarily the motive behind education in an  $\{N,S,E\}$  equilibrium. ■

**Proof.** of Proposition 2.

Let  $b_1$  denote the cutoff between N and E, and  $b_2$  denote the cutoff between E and S in an IAE. The proof proceeds to show that for  $m = 0$ , then  $b_1$  and  $b_2$  are equal to the corresponding cutoffs under public information. Moreover, we show that both  $b_1$  and  $b_2$  are monotonically increasing in  $m$ . Solving for  $b_1$  and  $b_2$  (from the expressions of the payoffs at the beginning of this appendix, using the relevant expression for  $U_E^{IA}(b)$ , from (B2)) yields,

$$\begin{aligned} b_1 & : = \{b : U_N = U_E^{IA}(b)\} = \frac{(1-p)(c_L - \pi_N - c_H - m) - c_L}{\Psi} \\ b_2 & : = \{b : U_N = U_E^{IA}(b)\} = \frac{mp - c_H + c_H p + p\pi_N - pc_L}{\Psi + \pi_H} \end{aligned} \quad (\text{B5})$$

where  $\Psi := (2p-1)(\pi_N + c_H + m - c_L) - p\pi_H$ . Since  $b\pi_H - m$  (the value of a performance contract under private information) converges to  $b\pi_H$  (the value of a fixed wage contract under public information) when  $m$  goes to zero, it is immediate that the allocation of workers under private information converges to the allocation of workers under public information, when  $m$  approaches zero. Differentiating the cutoffs  $b_1$  and  $b_2$  with respect to  $m$  yields,

$$\begin{aligned} \frac{\partial b_1}{\partial m} & = \frac{(2p-1)c_L + p(1-p)\pi_H}{\Psi^2} > 0 \\ \frac{\partial b_2}{\partial m} & = \frac{(2p-1)c_H + p(1-p)\pi_H}{(\Psi + \pi_H)^2} > 0 \end{aligned} \quad (\text{B6})$$

It follows immediately that the average ability of those educating in a private information IAE increases with  $m$ . Hence the average ability of the educated group is higher under private information than under public information IAE. ■

**Proof.** of Proposition 3.

We prove existence of an  $\{N,E,S\}$  signaling equilibrium, and then prove the existence of an  $\{N,S,E\}$  signaling equilibrium. To make the example analytically tractable, assume throughout that  $p = 1$ ,  $\theta = \frac{1}{2}$  and that beliefs are uniformly distributed, i.e.,  $f(z) = 1$ ,  $\forall z \in [0, 1]$ . Then, by Lemma 1, we have that,  $f_L(z) = 2(1 - z)$  and  $f_H(z) = 2z$ . Denote the credential wage in an  $\{N, E, S\}$  signaling equilibrium for  $w^{middle}$ . Provided that an  $\{N,E,S\}$  signaling equilibrium exists, it is straightforward to see that the credential wage just equals the average productivity of the educated group, i.e.,

$$w^{middle} = \pi_H \frac{b_3 + b_4}{2} \quad (\text{B7})$$

where  $b_3$  is the cutoff between N and E, and  $b_4$  is the cutoff between E and S. For an  $\{N,E,S\}$  signaling equilibrium to exist, there are three conditions. First, a person with a high belief at stage 2 must prefer a performance wage to educating, i.e.,  $U_S(1) > U_E^S(1)$ , which is equivalent to  $\pi_H - m > w^{middle} - c_H$ . Second, a person that receives good news when educating ( $X = c_H$ ) must prefer the credential wage to the performance wage. Hence  $w^{middle} > \pi_H - m$ . Putting the first and the second condition together, we have that

$$\pi_H - m < w^{middle} < \pi_H - m + c_H \quad (\text{B8})$$

The first inequality ensures that a person with a high belief at stage 2 prefers a performance wage to educating, and the second inequality ensures that a person with a high belief *after educating* prefers the credential wage to the performance wage contract. Third, to ensure that the equilibrium is fully separating, we must have that,

$$0 < b_3 < b_4 < 1 \quad (\text{B9})$$

Using the expressions for payoffs at the beginning of the appendix, we get the following

system of equations determining  $b_3$ ,  $b_4$  and  $w^{middle}$ ,

$$\begin{aligned}
U_N(b_3) &= \pi_N = U_E^S(b_3) = w^{middle} - b_3 c_H - (1 - b_3) c_L \\
U_E^S(b_4) &= w^{middle} - b_4 c_H - (1 - b_4) c_L = U_S(b_4) = b_4 \pi_H - m \\
w^{middle} &= \pi_H \frac{b_3 + b_4}{2}
\end{aligned} \tag{B10}$$

Solving this system yields,

$$\begin{aligned}
b_3 &= \frac{2c_H \pi_N + 2c_H c_L - 2c_L \pi_N + \pi_N \pi_H - m \pi_H - 2c_L^2 + 2c_L \pi_H}{\pi_H^2 - 2c_H^2 + 4c_H c_L - 2c_L^2} \\
b_4 &= \frac{\pi_N \pi_H + m \pi_H + 2c_H c_L - 2c_H m - 2c_L^2 + 2c_L m}{\pi_H^2 - 2c_H^2 + 4c_H c_L - 2c_L^2}
\end{aligned} \tag{B11}$$

We can now construct numerical examples of an {N,E,S} signaling equilibrium. For example, let  $\pi_H = 4$ ,  $\pi_N = 1$ ,  $c_L = 3/2$ ,  $c_H = 1$ ,  $m = 2$ , and insert into (B11) to obtain  $b_3 \approx .35$ , and  $b_4 \approx .81$ , and  $w^{middle} = \pi_H \frac{b_3 + b_4}{2} \approx 2.32$ . As can easily be seen, (B8) is satisfied since,  $\pi_H - m = 2 < w^{middle} = 2.32 < \pi_H - m + c_H = 3$ .

Now {N,S,E} signaling equilibria. Denote the credential wage in an {N,S,E} signaling equilibrium for  $w^{top}$ , the cutoff between N and S for  $b_5$ , and the cutoff between S and E for  $b_6$ . Provided that an {N,S,E} signaling equilibrium exists,  $w^{top} = \pi_H \frac{b_6 + 1}{2}$ . For an {N,S,E} signaling equilibrium to exist, there are two conditions, which together are necessary and sufficient. First,  $U_E^S(1) > U_S(1)$ , which is equivalent to,

$$w^{top} - c_H > \pi_H - m \tag{B12}$$

The second condition is that,

$$0 < b_5 < b_6 < 1 \tag{B13}$$

By simple calculations, we find that  $b_5 = \frac{\pi_N + m}{\pi_H} > 0$ , and  $b_6 = \frac{2m + \pi_H - 2c_L}{2c_H + \pi_H - 2c_L}$ . Since  $m > c_H$ , the condition  $b_6 < 1$  from (B13) implies that  $c_L > \frac{\pi_H}{2}$ . Finally, for  $b_5 < b_6$  from (B13), we have that  $m < \frac{-\pi_N \pi_H + 2\pi_N c_H - 2\pi_N c_L - \pi_H^2 + 2\pi_H c_L}{\pi_H + 2c_L - 2c_H}$ . Since  $m > 0$ ,

this implies that (i)  $c_L$  large and (ii)  $\pi_H > 2\pi_N c_L > \frac{\pi_H^2 - \pi_N \pi_H - 2\pi_N c_H}{2(\pi_H - \pi_N)} > c_H$  [where the latter inequality implies  $\pi_H > 2\pi_N$ ], and moreover that  $m$  is on the appropriate interval, is necessary and sufficient for existence. From these conditions it is simple to construct examples of {N,S,E} signaling equilibria. For instance, let  $\pi_H = 2$ ,  $\pi_N = 1$ ,  $c_L = 4$ ,  $c_H = 0$ ,  $m = 1/10$ , to obtain  $b_5 = .55$ , and  $b_6 = \frac{29}{30}$ . (B10) is satisfied because,  $w^{top} - c_H = \frac{29}{15} = \frac{58}{30} > \pi_H - m = \frac{19}{10} = \frac{57}{30}$ . ■

## 8 Appendix C: Welfare

Education has a productive role in the model by improving workers' information, and hence their choice of sector. There are two reasons for why education may be at least partly counter-productive. The first is signaling, with associated information ignorance and free-riding problems. The other reason is that in information acquisition equilibria, aggregate cost of monitoring becomes high, because in such equilibria performance contracts are chosen. It can thus be conceivable that welfare would be higher if workers instead of educating to acquire information would educate to signal their abilities, and hence choose credential contracts later on (in which case the aggregate cost of monitoring would be lower). Hence a counter-productive effect of education may occur because the those who educate have the *wrong motive* behind education.<sup>30</sup>

Define the desirability, or *welfare*, of an allocation of workers simply as the sum of production, subtracted the cost of educating and the cost of monitoring.<sup>31</sup> To assess the forces above, suppose that the social planner may prohibit performance contracts, to induce a signaling equilibrium. Surprisingly, it turns that the planner may wish to do so under certain conditions, to switch the economy from an IAE to a signaling equilibrium.

It is assumed that the social planner has the same information about workers as firms do (i.e.,  $\theta$  and the distribution of  $X$ ). With a *global welfare optimum*, it is meant a situation where government intervention can only harm welfare. With a *local welfare optimum*, it is meant a situation where a 'small' government intervention (not large enough to shift

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<sup>30</sup>By the same argument, possibly welfare could increase if the S group chose to educate to signal.

<sup>31</sup>Formally,  $W := \Pi - M - C$ , where  $\Pi$  is aggregate production,  $M$  is the aggregate monitoring cost, and  $C$  is the aggregate cost of education.



the economy into a different type of equilibrium) can only harm welfare. By construction, a global welfare optimum is also a local welfare optimum.

**Proposition 6** *i) A public information IAE is always a global welfare optimum. ii) A private information IAE is always a local welfare optimum, and for  $m$  sufficiently low it is also a global welfare optimum. However, iii) For  $m$  sufficiently high, an IAE need not be a global welfare optimum. iv) A signaling equilibrium is never a local welfare optimum.*

**Proof.** i) When a worker chooses whether to educate, and which sector to work in, in a public information IAE, the social costs and benefits of the various alternatives are fully internalized: A social planner knowing that the distribution of beliefs is calibrated in the sense of (C) has the same valuation to the different alternatives for an worker with belief  $b$  as the worker with belief  $b$  has himself. Hence a public information IAE is a global welfare optimum. ii) By the same argument as under i), a private information IAE is local welfare optimum. If a played IAE is not a global optimum, then a social planner would prefer an equilibrium where at least some agents are signaling, since an IAE is a local welfare optimum. We show that provided  $m < c_{HP} + (1 - p)c_L$ , a social planner does not wish to switch the economy from an IAE to an equilibrium where some agents are signaling. First, consider agents in the S group of the played IAE. Recall that all agents in this group would also choose the S sector if they were to educate and then choose a credential contract. So in considering whether some of the agents in the S group should choose to educate and then choose credential contract in the S sector, the social planner compares the aggregate cost of monitoring in an IAE with the cost of education in a signaling equilibrium. But, since the cost of education is decreasing in  $b$ , and since  $m < c(1) = c_{HP} + (1 - p)c_L$ , the social planner clearly prefers that all agents in the S group choose a performance contract rather than undertake education and choose a credential contract. Now the E group, for completeness. From the same argument, a social planner prefers all members of this group to choose a performance contract rather than to choose a credential contract, if they were to choose anything different from information acquisition. So, denoting the welfare generated by the E group for  $W_E$ , we have in shorthand notation that  $W_E(S) > W_E(\text{Signaling})$ . But, since an IAE is a local optimum, we have that  $W_E(E) > W_E(S)$ . It follows that  $W_E(E) > W_E(\text{Signaling})$ , and hence the social surplus

cannot be increased by constructing an equilibrium where some in the E group uses education as a signal rather than as information acquisition. To show that a social planner cannot increase social surplus by constructing an equilibrium where (some of the) agents in the N group is signaling, follows from the same type of argument, and is omitted. Finally, it follows from an IAE being a local optimum that a social planner does not wish to move any of the agents in the E or S group into the N group. Thus we can conclude that an IAE is a global welfare optimum for  $m < c_{Hp} + (1 - p)c_L$ . iii) To show that an IAE is not necessarily a global optimum for  $m > c_{Hp} + (1 - p)c_L$ , an example of an IAE is constructed where welfare can be improved by prohibiting individual contracts (and hence make signaling the motive behind education). The calculation of the example is relegated to a separate worksheet, which is available from the author. Here a sketch is given. Let beliefs be distributed uniformly (see the proof of Proposition 1 in this appendix), and let  $c_H = \frac{1}{10}$ ,  $c_L = 2.5$ ,  $\pi_H = 5$ ,  $\pi_N = 2$ ,  $p = 1$ , and  $m = \frac{3}{2}$ . In that case,  $m = \frac{3}{2} > c_{Hp} + (1 - p)c_L = \frac{1}{10}$ , so the sufficient condition for global optimum is violated. Furthermore, there exists an IAE  $b_1 \approx .64$  and  $b_2 \approx .91$ , with welfare equal to  $W_* \approx 2.26$ . (In the worksheet, it is shown that a firm deviating with offering a  $w' > \pi_N = 2$ , would run a deficit). Now, with the same parameter values, suppose that the social planner prohibits performance contracts (i.e., sets  $m = \infty$ ). In that case (calculations are relegated to the worksheet), there exists a signaling equilibrium where those with belief lower than (approximately) .41 choose N, and those with belief higher than .41 choose education as a signal. The welfare level of this signaling equilibrium is  $W_{signal} \approx 2.42$ . Thus, in the constructed IAE, a social planner prohibits performance contracts and increases welfare from 2.26 to 2.42. iv) Denote the belief of the marginal worker undertaking education in an {N,E} or in an {N,S,E} signaling equilibrium by  $\tilde{b}$ . Furthermore, let  $P(b)$  denote the fraction of *high* type workers among those workers with beliefs on the interval  $[b, 1]$ . Then,

$$P(b) = \frac{\theta_H(1 - F_H(b))}{\theta_H(1 - F_H(b)) + \theta_L(1 - F_L(b))} \quad (C1)$$

where  $F_i(\cdot)$  is the cumulative frequency of workers with beliefs  $b$  that are in fact of type  $i$ . Notice that the shape of the distribution of beliefs enters  $P(b)$  through  $F_L(\cdot)$  and  $F_H(\cdot)$  (where one of them is redundant given (C)). Since wages are set competitively, the wage

for educated workers in a signaling equilibrium, where those at the top educate equals  $P(\tilde{b})\pi_H$ . As can easily be seen, in an equilibrium where  $P(\tilde{b}) > \tilde{b}\pi_H$ , welfare would be increased if the marginal workers had skipped education and instead chosen N directly. To see that  $P(\tilde{b}) > \tilde{b}$  is indeed the case in a signaling equilibrium, use (C) and insert into (C1) to obtain,

$$P(\tilde{b}) = \frac{\int_{\tilde{b}}^1 f_H(z) dz}{\int_{\tilde{b}}^1 \frac{1}{z} f_H(z) dz} \quad (\text{C2})$$

The rest of the proof of (ii) follows from subtracting  $\tilde{b}$  and observing that  $P(\tilde{b}) - \tilde{b} > 0$ . Thus, due to free-riding, there is too much education in equilibria where those at the top signal. We now show that there is an inefficient amount of education in an {N,E,S} signaling equilibrium. An individual chooses a performance contract rather than education at time 2, if  $b\pi_H - m \geq w - c(b)$ . However, maximization of welfare implies that a worker should choose a performance contract rather than educating if  $b\pi_H - m \geq b\pi_H - c(b)$ . Since  $b\pi_H < w$ , it follows directly that too few of those with high confidence educate in {N,E,S} equilibria. From the same argument as above it follows that too many of those with low confidence choose E in a signaling equilibrium. ■

The intuition for i) is that under public information, a government intervention will create distortions in the economy which can only harm the allocation of workers from a welfare perspective. The intuition for ii) is that under public information, an IAE is always a global welfare optimum. And since a private information IAE is close to a public information IAE when  $m$  is small, a private information IAE is also a global welfare optimum for  $m$  small, because the aggregate cost of monitoring is small. This result illustrates that if information acquisition is an important function of education, then private information does not have the dramatic effect on welfare it has e.g., in the Spence (1973) model. The intuition for iii), which is more surprising, can be shed in terms of externalities. In information acquisition equilibria, workers that educate can incur a negative externality on the workers below them on the confidence scale, because those workers would prefer the former group to choose a credential contract (and hence signal rather than acquire information) for them to free-ride on, rather than a performance contract. Hence, when the social planner prohibits performance contracts, that is a crude

measure to neutralize this externality.

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