Career Concerns, Multiple Tasks, and Short-Term Contracts*

Oddvar M. Kaarbøe† and Trond E. Olsen‡

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Abstract

We study optimal incentive contracts when commitments are limited, and agents have multiple tasks and career concerns. The agent career concerns are determined by the outside market. We show that the optimal compensation contract optimizes the combination of implicit incentives from both career concerns and ratchet effects. In contrast to existing results, implicit and explicit incentives might be complements, and the principal might want to give strongest explicit incentives for agents far from retirement to account for the fact that career concerns might induce behavior in conflict with the principal's preferences. Furthermore, we show that maximized welfare might be decreasing in the strength of the career concerns, and that optimal incentives might be both positively and negatively correlated with various measures of uncertainty.

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†Corresponding author. Programme for Health Economics, Department of Economics, University of Bergen. Fosswinckelsg. 6, N-5007 Bergen. Norway. Email: oddvar.kaarboe@econ.uib.no
URL http://www.econ.uib.no/stab/oddvar.html

‡Department of Finance and Management Science, Norwegian School of Economics and Business Administration, and The Norwegian Centre in Organization and Management (LOS Centre). Email: trond.olsen@nhh.no

1. Introduction

The purpose of this paper is to study optimal incentive contracts when commitments are limited, and agents have multiple tasks and career concerns - concerns about the effects of current performance on future compensation. An agent's career concerns are assumed determined by outside principals (or the market or the professional environment). As a result, agents' career concerns are determined by factors outside the principal's control.

The following example illustrates the type of situations we have in mind. Consider a physician's choice between treating more patients or spending more time on fewer patients within a fixed time-budget. While hospital management (the principal) might have a preference for treating more patients (due to e.g. DRG financing or waiting lists), the medical profession typically puts more weight on the quality of treatments. I.e. it prefers physicians to spend more time on each patient. Since the medical profession has some influence on employment decisions, physicians might allocate more time to each patient than hospital management prefer (to increase her chances of getting promoted).

Two questions that naturally arise are. How can the management, by offering agents explicit incentive contracts, induce behavior consistent with its preferences, and what are the implications for welfare? To analyze these questions we put forward two versions of a dynamic multitask models with both explicit and implicit incentives. The first version is a simple two-period model that mainly serves to introduce the issues. Implicit incentives are related only to career concerns in that version. The second version is an extension of the first to more than two periods, and implicit incentives are then seen to consist not only of career concerns, but also of ratchet effects (Weitzman, 1976).[1] In both cases we assume that commitment to long-term contracts is limited.

In the analysis we want to emphasize that career concerns are determined by factors outside the principal's control, and that the current principal has more information about the agent than prospective principals do. We therefore assume that career concerns are related to a signal which is not verifiable – and thus cannot be contracted upon – and that the inside principal observes an additional information signal.[2]

The general conclusions we obtain are firstly that optimal explicit incentives can be non-monotone or strongest earliest in agents' careers. The latter result resembles the fact often observed in government agencies where subordinates get paid overtime, while more senior officers are paid a fixed salary. Secondly, we find that career concern incentives might be harmful for welfare. Finally, we show that the presence of both ratchet effects and career effects produce incentives that can be highly non-monotone in observable measures of uncertainty. Consequently, we offer a possible explanation for the fact that empirical studies observe both a positive and negative correlation between risk and incentives.[3]

[1] The ratchet effect reflects the fact that future periods' performance standards depend on todays performance in a way such that better performance today implies a tougher standard tomorrow.

[2] Alternatively, the inside principal might learn the agent's ability faster than outside principals do, as in Waldman (1984) and Ricard i Costa (1988).

[3] Prendergast (2000a) gives an overview of the empirical literature on the tradeoff of risk and incentives. See also Prendergast (1999, 2000b).

Some of our results are at variance with findings in the existing literature, for instance that optimal explicit incentives are increasing over time (Gibbons and Murphy, 1992), and that career concern incentives have no effect on maximized welfare (Meyer and Vickers, 1997). The key to understand the difference in the results is to note that agents exert effort only on one task in both Gibbons and Murphy (1992) and Meyer and Vickers (1997). Thus, explicit incentives and career concern incentives are in these papers substitutes; higher career concerns reduce the required explicit incentives needed to induce a certain effort level. Since career concerns are strongest earliest in agents' careers, the required explicit incentives needed to induce a certain effort level are lower for agents far from retirement.

The substitutability effect is also the mechanism behind the welfare result in Meyer and Vickers (1997): Stronger career concerns reduce the need for explicit incentives and thus reduce the risk faced by (risk-averse) agents. Since maximized welfare is decreasing in the risk imposed on agents, stronger career concerns cannot lower welfare. In our model, however, explicit incentives and career concern incentives are complementary in the sense that higher career concerns (on one task) imply higher explicit incentives on the other task. Thus, stronger career concerns impose more risk on agents, and thus may lower welfare.

Our result that optimal incentives are non-monotone in various measures of uncertainty is also related to the fact that agents have multiple tasks. It is thus possible that both ratchet and career effects are present at the same time but working through different tasks. In other words, there might be a career effect present on one task, while, at the same time, ratchet effects influence the agent's effort choice through another task. Since optimal explicit incentives will balance the total effect of implicit incentives, the relative strengths of career and ratchet effects then influence how strong explicit incentives will be. Furthermore, the relative strength of ratchet and career effects varies with measures of uncertainty, such that optimal explicit incentives might vary non-monotonically with these measures.

The paper is organized as follows. In section 2, we present the two-period version with career concerns and explicit incentives. In section 3 we present the extended version in which implicit incentives consist not only of career concern incentives, but also of ratchet effects. Section 4 discusses the related literature. Finally, section 5 presents some concluding remarks.

2. Career Concerns and Explicit Incentives

There is one agent, two tasks (y and q), and two periods. It is assumed that the agent's career concerns are determined by the outside market (or outside principals or the professional environment). Career concerns are related to the $q-$signal. The agent's choices of effort generate two information signals, y_t, and q_t. Outside principals observe q_t, which is not verifiable. There is competition among these in period 2, and they (the market) offer the agent a reward based on the signal observed the previous period; $w_2^O(q_1)$. The inside principal observes q_t (not verifiable) and y_t which is sufficiently verifiable that contracts can be written on it. By this we mean the following. The signal can be verified, but only at a cost, and the parties know that whoever breaks the contract will have to pay the verification costs if the case is taken to court. Hence, no party will renege the contract if the verification cost is sufficiently high. The principal offers the agent (linear) payments $w_t = \alpha_t + \beta_t y_t$.[4] We further assume that only one-period contracts are feasible.

The agent privately chooses (e_t, a_t), where e_t (a_t) is effort supplied into the production of y_t (q_t). The private cost (in monetary units) is $C(e_t, a_t) = \frac{1}{2}(e_t + a_t - z)^2, e_t, a_t \geq 0$. Thus, efforts on the two tasks are perfect substitutes. Moreover, the cost function implies, in line with Holmström and Milgrom (1991), that the agent's ideal total effort is some positive level $z > 0$. The agent prefers to exert some effort rather than being totally idle at work.

Given the effort choices the two signals are

$$\begin{aligned} y_t &= \eta + e_t + \varepsilon_t, \\ q_t &= \eta + a_t + \varepsilon_t^q, \end{aligned}$$

where $\eta \sim N(m_0, \sigma_\eta^2)$, $\varepsilon_t \sim N(0, \sigma_y^2)$, $\varepsilon_t^q \sim N(0, \sigma_q^2)$. We assume that all error terms are independent of each other and of ability η.

The agent's utility function is exponential, and there is no discounting:

$$u(x_1, x_2) = -\exp\{-r \sum_{t=1}^{2} [w_t - C(a_t, e_t)]\},$$

where the coefficient $r \geq 0$ measures the agent's risk aversion. With linear compensation, exponential utility, and normal random variables, the agent's certainty equivalent is

$$CE = \sum_{t=1}^{2} \mathbb{E}\left[w_t - C(a_t, e_t)\right] - \frac{r}{2} var(w_1 + w_2),$$

where $\mathbb{E}$ is the expectation operator. Note that if the agent's incentives on the two tasks are not balanced, e.g. if there is a stronger (implicit) incentive on the $q-$task compared to the $y-$ task, then all effort will be concentrated on the high-incentive task; we would get $e_t = 0$ and $a_t = \beta_t^i + z$, where β_t^i is the incentive on the $q-$task.

All principals are risk-neutral and receive an expected gross benefit of $B(e_t, a_t)$ where $B(.,.)$ is concave. We assume that principals have a preference for the effort being split among the tasks, i.e. $B(0, a_t) = B(e_t, 0) = -L$, with $L > 0$ large. For simplicity we assume $B(e_t, a_t) = \frac{1}{2}\mathbb{E}(y_t + q_t)$, for both $e_t > 0$, and $a_t > 0$. This formulation implies that any principal will provide balanced incentives for the agent. Given such (positive) balanced incentives the agent's total effort will exceed the 'whistle as you work' level $(a_t + e_t > z)$, and the agent will distribute this total effort on the tasks in any way the principal desires. Thus, balanced incentives are sufficient and necessary to avoid the

[4] The focus on linear contracts can be justified by appeal to a richer dynamic model in which linear payments are optimal (Holmström and Milgrom, 1987).

'disastrous' outcome that either task is left idle.[5] To simplify notation we renormalize effort such that $z = 0$ in what follows.

We further assume that, after an agent has worked for a principal, a special relationship is formed between the two, e.g. due to the agent learning specific ways to perform the tasks, resulting in an increased fixed benefit for this principal from keeping the agent in his service. The additional benefit is sufficiently large that the inside principal will always want to retain the agent, even if unfavorable signals are observed in the first period. This kind of assumption is in line with assumptions made in the existing literature (e.g. Gibbons and Murphy 1992; Meyer and Vickers 1997).

In the second period the agent may leave and seek outside employment. We assume that there is a (small) positive probability $p > 0$ that the agent must leave for exogenous reasons, such as a move triggered by a job change for the agent's spouse etc., and that an outside principal cannot observe whether the agent leaves voluntarily or due to such exogenous events. Competition among the outside principals will then ensure that the agent is offered a contract, $w_2^O(q_1)$, that earns zero expected profits for such a principal.[6] This will be an equilibrium because (a) the inside principal will in any case match this offer, hence (b) there is no reason for the agent to leave voluntarily (no self-selection), and (c) an outside principal cannot therefore deduce anything helpful about the agent's type from her behavior on the job market.

Since (i) the tasks are perfect substitutes in the agent's cost function, (ii) principals have a preference for the effort being split among both tasks, and (iii) q_t is not verifiable, the agent has no incentives to exert effort (beyond the 'whistle while you work' level, normalized to zero; $z = 0$) in the second period.[7] As a result, outside principals offer the agent a fixed payment equal to the expected benefit (profit) generated by this effort level, i.e.:

$$w_2^O(q_1) = \frac{1}{2}\mathbb{E}((y_2 + q_2) \mid q_1) = \mathbb{E}y_2 + r'_q(q_1 - \mathbb{E}q_1) = \frac{\sigma_q^2 m_0 + \sigma_\eta^2(q_1 - \widehat{a}_1)}{\sigma_q^2 + \sigma_\eta^2}.$$

where $\widehat{a}_1$ is their second-period conjecture about effort a_1 in period 1, and $r'_q = \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_q^2}$. The inside principal must offer the same payment to retain the agent. The agent is thus offered a contract which is dependent on—in fact equal to—the conditional mean of his ability given the observed first-period signal q_1; i.e. $w_2^O(q_1) = \mathbb{E}(\eta \mid q_1)$. Note that the agent receives this payment whether he stays with or leaves the inside principal in period 2.

[5] Our assumption is that a task left completely idle—or with only some minimal activity on it—exposes the principal to a significant risk of a large loss. This can be avoided by maintaining the minimal activity on the task. Balanced incentives guarantees that the minimal activity is maintained on both tasks, and thus avoids the large expected loss. On the other hand, errors or mistakes may occur on any active task–these are captured by the random variables in $y_t + q_t$–but the expected value of these are positive.

[6] We assume that outside principals offer relatively simple contracts and hence do not offer screening contracts.

[7] There are no career motives, since this is the last period, and hence all effort would be concentrated on the verifiable task if the inside principal provided incentives on that task. The inside principal prefers a balanced effort allocation, and hence does not provide such incentives.

Since the second period compensation depends positively on the first period signal, q_1, the agent has incentives to exert effort in the first period to increase his market value. Again, (i)-(iii) apply and optimal explicit incentives must equal career concern incentives, i.e. $\beta_1^* = w_2'(q_1) = \frac{\sigma_\eta^2}{\sigma_q^2+\sigma_\eta^2}$. Since $\beta_2^* = 0$ we have the following result.[8]

Proposition 1. *Explicit incentives from the optimal compensation contract are strongest early in the agent's career.*

This result is at variance with the predictions from the theoretical model in Gibbons and Murphy (1992), and is due to the fact that explicit and implicit incentives are complementary in the sense that higher career concern incentives (on one task) imply higher explicit incentives on the other task. Furthermore we note that this model—in which implicit incentives are related only to career effects—produces comparative statics results in line with those of Holmström (1982); optimal incentives are monotonically increasing (decreasing) in the ability variance, σ_η^2, (market noise, σ_q^2). These results are to be contrasted with those in the extended version where both career effects and ratchet effects are present—see Proposition 6-8.

The second result we get from this simple model is that welfare is non-monotone in the strength of the career concerns, which varies with σ_η^2 and σ_q^2. Specifically, career concerns are increasing (decreasing) in σ_η^2 (σ_q^2).

The total certainty equivalent for the agent and the principal is

$$TCE = \sum_{t=1}^{2} \left[\frac{1}{2}\mathbb{E}(y_t + q_t) - C(e_t + a_t) \right] - \frac{r}{2} var(w_1 + w_2)$$

Recall that the agent's second-period payment is independent of whether he stays with or must leave the inside principal. In appendix A we show that $var(w_1+w_2) = \beta_1^{*2}[4\sigma_\eta^2 + \sigma_q^2 + \sigma_y^2]$. Next, since $\beta_1^* = C'(e_1+a_1) = e_1 + a_1$, and $\frac{\partial \beta_1^*}{\partial \sigma_\eta^2} := \beta_1^{*\prime} = \frac{\sigma_q^2}{(\sigma_\eta^2+\sigma_q^2)^2} > 0$, and since moreover the production surplus (expected benefits minus effort costs) in period 2 does not depend on the variance σ_η^2, we get

$$\frac{\partial TCE}{\partial \sigma_\eta^2} = \beta_1^{*\prime} \beta_1^* \left[\frac{\sigma_q^2 - \sigma_\eta^2}{2\sigma_\eta^2} - r[6\sigma_\eta^2 + \sigma_q^2 + \sigma_y^2 + \frac{2(\sigma_\eta^2)^2}{\sigma_q^2}] \right]$$

which may be positive or negative, depending on the parameters. To sum up.

Proposition 2. *Expected welfare is non-monotone in the ability variance σ_η^2, and hence in the strength of the agent's career concerns. The more risk averse the agent is, the less beneficial are stronger career concerns.*

[8] Incentives are independent of risk aversion in this model; this stems from the invoked assumption that efforts are perfect substitutes for the agent. With less than perfect substitutes the principal could provide positive incentives on the verifiable task in period 2 and yet maintain a minimal effort level on both tasks. Risk aversion would then matter for incentives. It appears that such a model would yield similar results to those obtained in the simpler framework considered here.

When career concerns are low, (e.g., due to a low σ_η^2), the gain of more effort induced by a stronger career concern outweighs its costs in terms of effort costs plus risk costs. When career concerns are strong, (e.g., because σ_η^2 is high), optimal explicit incentives are high. Thus, the agent bears much risk, and higher career concerns reduce welfare.

The result also holds true when agents are risk-neutral ($r = 0$). In this case total welfare is decreasing in the strength of the career effect when $\sigma_\eta^2 > \sigma_q^2$. The intuition is that when $\sigma_\eta^2 > \sigma_q^2$, the career effects are so strong that the agent's cost of providing more effort outweights the associated increase in production value.[9] We see that risk costs add detrimental effects to career concern incentives.

3. Career Concerns, Ratchet Effects, and Explicit Incentives

We now analyze a three-period version of the model. In this setting implicit incentives may include not only career concerns, but also ratchet effects. Here we allow for the fact that the agent's working conditions may differ across principals. For example, consider two hospitals, one university hospital and one local hospital, and suppose the university hospital is better equipped for research than the local hospital. The agent's costs of providing effort for research relative to providing effort for clinical work are then lower in this hospital compared to the other. We represent this potential difference by two different agent cost functions, $C^O(e_t, a_t) = \frac{1}{2}(\gamma e_t + a_t)^2$ and $C^I(e_t, a_t) = \frac{1}{2}(e_t + a_t)^2$, $\gamma > 0$, for the outside and inside principals, respectively.

The assumption that the inside principal derives some extra benefits from the agent is maintained, implying that this principal will in every period outbid the other principals in equilibrium. In this section we further assume that the probability p of the event that triggers a move by the agent is small. To simplify notation it will be ignored in the following, but it should be kept in mind that all results are conditional on this probability being sufficiently small.[10]

3.1. Equilibrium Contracts

Outside and inside principals offer in each period $t = 1, 2, 3$ contracts $w_t^O = \alpha_t^O + \beta_t^O y_t$, and $w_t = \alpha_t + \beta_t^* y_t$, respectively. The model is solved by backward induction, thus we first consider the last period.

Period 3: The agent has no incentives to exert effort since (i) the tasks are perfect substitutes in the agent's cost function, (ii) principals have a preference for the effort being split among both tasks, and (iii) q_t is not verifiable. Moreover, there is competition

[9] Holmström (1982) contains a similar result.

[10] In what follows, all expressions for the agent's and the inside principal's surpluses are conditional on the agent not being forced (by exogenous events) to quit the relationship, and thus should, strictly speaking, be multiplied by the probability of exit not occurring. Additional terms capturing the surpluses conditional on exit taking place should also be included, but these are insignificant for p small.

among the outside principals. and therefore[11]

$$w_3^O(q_1, q_2) = \alpha_3^O = \mathbb{E}(\frac{1}{2}(y_3 + q_3) \mid q_1, q_2) = \frac{\sigma_q^2 m_0 + \sigma_\eta^2 \sum_{t=1}^2 (q_t - \widehat{a}_t)}{\sigma_q^2 + 2\sigma_\eta^2},$$

where $\widehat{a}_t$ is outside principals' conjecture about $\widehat{a}_t$, $t = 1, 2$. The agent is thus offered the conditional expectation of his ability, given the observed signals q_1, q_2, and the market's conjectures about the prior effort levels, $\widehat{a}_1$, $\widehat{a}_2$.

The inside principal conditions the contract she offers the agent on all signals, y_1, y_2, q_1, q_2, and will adjust the contract such that the agent's third-period certainty equivalent equals the market contract's certainty equivalent. I.e. $CE_3^O(\alpha_3^O) \leq CE_3^I = \alpha_3 + \beta_3 \mathbb{E}(y_3 \mid y_1, y_2, q_1, q_2) - \frac{1}{2}(e_3 + a_3)^2 - \frac{r}{2} var(w_3^I \mid y_1, y_2, q_1, q_2)$. But, since, $e_3 = a_3 = 0$, and $\beta_3^* = var(w_3^I \mid y_1, y_2, q_1, q_2) = 0$, [12] we get $\alpha_3 = \alpha_3^O$. To sum up, both the inside principal and outside principals offer the agent the fixed payment $\alpha_3 = \alpha_3^O$.

Period 2: By using the fact that the agent has career incentives to exert effort a_2 in this period, (to increase his market value), that the inside principal prefers balanced effort, and hence most provide balanced incentives, we have

$$\beta_2^* = \frac{\partial \alpha_3(q_1, q_2)}{\partial q_2} = \frac{\sigma_\eta^2}{2\sigma_\eta^2 + \sigma_q^2}.$$

Outside principals offer a contract $w_2^O = \alpha_2^O + \beta_2^O y_2$. To balance incentives on the two tasks, the bonus offered by an outside principal must satisfy $\beta_2^O = \gamma \beta_2^*$. This is so because the marginal cost of the two activities satisfy $\frac{\partial C^O}{\partial e_t} = \gamma(\gamma e_t + a_t) = \gamma \frac{\partial C^O}{\partial a_t}$ if the agent works for an outside principal. Since there is competition between outside principals, they earn zero expected profit.

The precise payment schemes for outside and inside principals in period 2 are derived in appendix B. Here we are primarily interested in the implicit incentives that these schemes give rise to, and in the following we give an intuitive derivation of these incentives. We first show that the agent is exposed to implicit incentives on the $y-$task in period 1, and that these are given by

$$\beta_{1y}^i = (\gamma - 1)\beta_2^* R_y', \qquad \text{where} \qquad R_y' = \frac{\partial}{\partial y_1} \mathbb{E}(\eta | y_1, q_1) = \left[\frac{\sigma_\eta^2}{\sigma_\eta^2 \sigma_q^2 + \sigma_y^2 \sigma_\eta^2 + \sigma_y^2 \sigma_q^2} \right] \sigma_q^2$$

The last equality follows from well-known formulas for conditional expectations (see DeGroot (1970) and appendix A). To consider the implicit incentive, suppose the agent contemplates an (out-of-equilibrium) effort variation de_1 in period 1. He can then expect that the inside principal will adjust her estimate of the agent's ability by $R_y' de_1$. This higher ability implies that the value of the outside contract for the agent increases by $dw_2^O = \beta_2^O R_y' de_1$. The inside principal must match this offer by increasing the agent's fixed (non-performance based) period 2 payment. On the other hand, the increased

[11] See e.g. Gibbons and Murphy (1992).

[12] At the beginning of the third period, both q_1 and q_2 are known.

ability translates into an increased performance payment $\beta_2^* R_y' de_1$ if the agent continues to work for the inside principal, and the latter therefore only needs to adjust the non-performance based salary by the difference $(\beta_2^O - \beta_2^*) R_y' de_1$. Since $\beta_2^O = \gamma \beta_2^*$, it follows that the agent is faced with implicit first-period incentives on the $y-$task, and that these are given precisely by β_{1y}^i. Of course, the salary adjustments that give rise to these incentives are possible only because the parties are not bound by a long-term non-renegotiable contract.

Next we will argue that period-2 contract adjustments induce implicit first-period incentives on the q-task given by

$$\beta_{1q}^i = (\gamma - 1)\beta_2^* R_q' + (1 - \gamma \beta_2^*) r_q', \qquad \text{where}$$

$$R_q' = \frac{\partial}{\partial q_1} \mathbb{E}(\eta | y_1, q_1) = \left[\frac{\sigma_\eta^2}{\sigma_\eta^2 \sigma_q^2 + \sigma_y^2 \sigma_\eta^2 + \sigma_y^2 \sigma_q^2} \right] \sigma_y^2, \qquad r_q' = \frac{\partial}{\partial q_1} \mathbb{E}(\eta | q_1) = \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_q^2}$$

Again, the formulas involving conditional expectations are well known. To derive the implicit incentive, suppose the agent contemplates an effort variation da_1 in period 1. He can then expect that an outside principal, who observes only q_1, will adjust her estimate of the agent's ability by $r_q' da_1$, and on that basis will adjust her estimate of expected profits by $d\pi = (1 - \beta_2^0) r_q' da_1$. Competition implies that the agent's outside offer will increase by this amount.

The inside principal, who observes both y_1 and q_1, updates her estimate of the agent's ability by $R_q' da_1$, and consequently adjusts her estimate of the value of an outside contract for the agent by $dw_2^O = d\pi + \beta_2^O R_q' da_1$. She must match this higher offer, but since the higher ability will increase the agent's inside performance payment by $\beta_2^* R_q' da_1$, it is sufficient for the inside principal to adjust the non-performance based part by $dw_2^O - \beta_2^* R_q' da_1$. Substituting for dw_2^O and for $\beta_2^O = \gamma \beta_2^*$, we see that this adjustment equals $\beta_{1q}^i da_1$. Hence we have shown that contract adjustments in period 2 generate implicit first-period incentives on the $q-$task amounting to β_{1q}^i.

Note that, the more noise there is in the $y-$signal (the larger is σ_y^2), the more weight is put on q relative to y in estimating the agent's ability. If $\sigma_y^2 = 0$ (∞), the principal puts all (no) weight on the $y-$signal in estimating the ability. Similar considerations apply to σ_q^2. Finally we note that if $\sigma_y^2 > \sigma_q^2$, $(\sigma_y^2 < \sigma_q^2)$ the relative weight the inside principal puts on the $q-$signal ($y-$signal) increases (decreases) when σ_η^2 increases.

As we have seen, the fact that second-period compensation contracts depend on first-period signals, q_1 and y_1, induces implicit incentives that affect the agent's first-period effective (i.e. explicit plus implicit) incentives. These distortive effects can take the form of either career effects (which increase first-period effective incentives) or ratchet effects (which decrease first-period effective incentives). Consider first the $y-$task.

Proposition 3. *When $\gamma < 1$ there is a ratchet effect associated with the $y-$signal. When $\gamma > 1$ there is a career effect associated with the $y-$signal. For $\gamma = 1$, the inside and outside principals offer the same wage contract in period two, and there is neither a ratchet nor a career effect associated with the $y-$signal.*

Recall that γ is the marginal rate of substitution between $a-$effort and $e-$effort in the agent's cost function if he works for an outside principal. In the case $\gamma < 1$, the lower γ is, the more the agent is punished for high expectations about second-period performance on y. The intuition is that when γ is low ($\gamma < 1$) outside principals offer low-powered incentives on the verifiable task (i.e. $\beta_2^O < \beta_2^*$). They do so because it is in this case relatively inexpensive for the agent to provide effort on that task. As a result, agents with high ability have less to gain by working for these principals. The inside principal's response is to lower the fixed part of the second-period salary. This is the **ratchet** effect on the $y-$signal.

In the case $\gamma > 1$ the agent is rewarded for high expectations by the inside principal about second-period performance on y. The intuition is that when $\gamma > 1$ 'good' agents would like to work for outside principals (since they offer a high bonus). The inside principal cannot give such high-powered incentives (since she prefer balanced incentives), and responds by offering a higher fixed (non-performance based) salary component. I.e. there is a **career** effect associated with the signal y_1 when $\gamma > 1$.

Finally, when $\gamma = 1$, inside and outside principals give the same explicit incentive β_2^*. As a result, the inside principal must also give the same fixed salary component to ensure that she offers a wage contract with the same certainty equivalent as the market. Thus there is neither a ratchet effect nor a career effect related to the $y-$signal in this case.

Consider next the $q-$task. If the agent increases his effort on q relative to the inside principal's conjecture by da_1, his second-period salary changes by $\beta_{1q}^i da_1 = [(1 - \gamma\beta_2^*)r_q' - \beta_2^*(1-\gamma)R_q']da_1 \gtrless 0$ depending on the values of γ. To further understand this result first note that when γ becomes high enough outside principals will lower their fixed salary component, α_2^O, in response to $dq_1 > 0$. More specifically this effect occurs when $\gamma > \frac{1}{\beta_2^*} > 1$. It reflects the fact that outside principals offer high powered incentives, i.e. $\beta_2^O > \beta_2^*$, and then reduce the fixed part of the salary to break even (the zero profit constraint). The inside principal cannot give such high powered incentives, and her response is to increase the fixed salary component for these values of γ. Similar considerations apply to 'low' values of γ, i.e. for $\gamma < \frac{1}{\beta_2^*}$. As a result, a change in γ has two opposite effects on the fixed salary components offered, and the total effect is determined by their relative size. Specifically we note that the change da_1 has a positive (negative) effect on the second-period salary when $\gamma \in [0, \Gamma)$ (Γ, ∞), for some $\Gamma > 1$ given $\beta_2^* > 0$. [13] To sum up.

Proposition 4. *When $\gamma < \Gamma := \frac{1+R_y' r_q'}{R_y' r_q'}$ there is a career effect associated with the $q-$signal. When $\gamma > \Gamma$ there is a ratchet effect associated with the $q-$signal.*

To analyze the total effect of changes in γ on first-period incentives, which in turn will determine the agent's choices of effort, we finally turn to period 1.

[13] Note that $(1-\gamma\beta_2^*)r_q' = \beta_2^*[1-(\gamma-1)r_q']$, and that $r_q' - R_q' = R_y' r_q' > 0$. In appendix A we give the exact expression for Γ.

Period 1: Working for the inside principal, the agent chooses effort according to

$$\max_{e_1,a_1}[\alpha_1+\beta_1 y_1 - C(e_1,a_1)+\overline{w}_2(e_1,a_1)+\overline{w}_3(a_1)+const],$$

where $\overline{w}_t(\cdot)$ is the expected payment in period $t>1$, given efforts in period 1. As we have seen, $\frac{\partial \overline{w}_t}{\partial e_1}=\beta_{1y}^i$ and $\frac{\partial \overline{w}_t}{\partial a_1}=\beta_{1q}^i$, and these represent the implicit first-period incentives generated by period-2 contracts. Moreover, effort on the $q-$task will also have implications for contracts in period 3, and we have $\frac{\partial \overline{w}_3}{\partial a_1}=\frac{\partial w_3}{\partial q_1}=\frac{\partial w_3}{\partial q_2}=\beta_2^*$. The first-order conditions for efforts in period 1 are thus

$$\begin{aligned} e_1 &: \frac{\partial C_1}{\partial e_1}=\beta_1+\beta_{1y}^i=\beta_1+\beta_2^*(\gamma-1)R_y' \\ a_1 &: \frac{\partial C_1}{\partial a_1}=\beta_{1q}^i+\beta_2^*=\beta_2^*(\gamma-1)R_q'-(\gamma\beta_2^*-1)r_q'+\beta_2^* \end{aligned}$$

Since efforts on the two tasks are perfect substitutes in the agent's cost function, and principals have a preference for effort being split among the tasks, the optimal first-period bonus (on the verifiable $y-$task) is given by

$$\beta_1^*=\beta_{1q}^i-\beta_{1y}^i+\beta_2^*=\beta_2^*\left[2-(\gamma-1)(R_y'-(R_q'-r_q'))\right].$$

where the last equality follows from $\beta_2^*=\frac{\sigma_\eta^2}{2\sigma_\eta^2+\sigma_q^2}$ and the expressions for β_{1q}^i and β_{1y}^i.

We first note that if $\gamma=1$, optimal first-period incentives are twice as high as second-period incentives. This is the case of a pure career effect. The last part of the expression for β_1^* reflects the effect of the additional information the inside principal has access to (through the $y-$signal). Since $R_y'-(R_q'-r_q')>0$ (see appendix A), it follows that this extra information leads the inside principal to further increase incentives if $\gamma<1$. In this case outside principals offer low powered incentives, and agents with high ability are less eager to work for these principals. There is then a ratchet effect associated with the $y-$signal, and the inside principal's optimal response is to raise first-period explicit incentives.

From the formula we see that the relationship between first- and second-period explicit incentives β_1^* and β_2^* depends on the relative magnitudes of the implicit incentives β_{1y}^i and β_{1q}^i. The latter are illustrated in Figure 1. They are equal for some $\bar{\gamma}$ in $(1,\Gamma)$—the exact value is $\overline{\gamma}=1+\frac{1}{R_y'-(R_q'-r_q')}$, see appendix A.

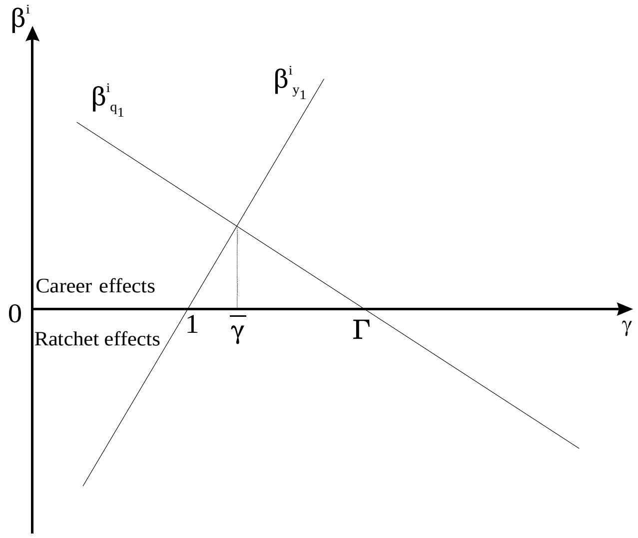

Figure 1: Implicit incentives

For $\gamma = \bar{\gamma}$ we thus have $\beta_1^* = \beta_2^*$, i.e. explicit incentives in periods 1 and 2 are then equal. In this case there are career effects (induced by period-2 contracts) on both tasks, but since they are of equal magnitude, explicit first-period incentives on the $y-$task need only match the career incentive (β_2^*) on the $q-$task stemming from period-3 contract adjustments. For $\gamma < \bar{\gamma}$ the implicit incentive β_{1y}^i on the $y-$task is weaker than that (β_{1q}^i) on the $q-$task (for $\gamma < 1$ the former is in fact negative), and the principal must compensate by increasing the explicit incentive β_1^*. In this region career incentives (on the non-verifiable $q-$task) and explicit incentives (on the $y-$task) are complementary in the sense that higher career incentives imply higher explicit incentives. For $\gamma > \bar{\gamma}$ the career effect on the $y-$task dominates, and it suffices for the principal to provide lower explicit incentives on that task. Explicit first-period incentives are thus lower than explicit second-period incentives ($\beta_1^* < \beta_2^*$) in this region. Since $\beta_3^* = 0$, we may summarize this discussion regarding the time profile of explicit incentives in the following result.

Proposition 5. *For* $\overline{\gamma} = 1 + \frac{1}{R_y' - (R_q' - r_q')} > 1$ *we have:*
i) Suppose $\gamma \leq \overline{\gamma}$, *then* $\beta_1^* \geq \beta_2^* \geq \beta_3^* = 0$, *and explicit incentives from the optimal compensation contract are strongest early in the agent's career.*
ii) Suppose $\gamma > \overline{\gamma}$, *and* $\beta_2^* > 0$. *Then* $0 = \beta_3^* < \beta_1^* < \beta_2^*$, *and explicit incentives from the optimal compensation contract are non-monotone (inverse U-shaped) over the time periods.*

Remark 1. *In contrast, Gibbons and Murphy (1992) obtain the opposite result: Explicit incentives should be strongest for agents close to retirement. In their model agents only exert effort on one task. Thus, explicit incentives and career concern incentives are* substitutes *in the sense that higher career concerns incentives reduce the required explicit incentives needed to induce a certain effort level.*

3.2. Comparative Statics

We now analyze how optimal incentives and welfare vary with the different parameters, σ_η^2, σ_q^2, and σ_y^2. We will show that the presence of both ratchet effects and career effects produce incentives that can be highly non-monotone in observable measures of uncertainty. Consequently, the model offers a possible explanation for the fact that empirical studies observe both a positive and negative correlation between risk and incentives.[14]

We first analyze how optimal incentives vary with the uncertainty regarding ability, σ_η^2. To better understand why first-period incentives might be non-monotone in this variance, consider the following example. Suppose $\sigma_q^2 = 20$, $\sigma_y^2 = 1$, and $\gamma = 1.9$. Figure 2 shows a plot of optimal first-period incentives $\beta_1^* = \beta_1^*(\sigma_\eta^2)$ (thin line) and second-period incentives $\beta_2^* = \beta_2^*(\sigma_\eta^2)$ (thick line) for this example.[15]

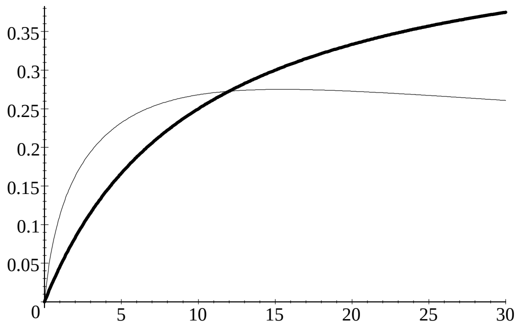

Figure 2: $\beta_t^*(\sigma_{\eta_1}^2)$

First note that the larger the uncertainty about ability, the easier it is for the agent, by increasing his effort, to influence the principals' estimate of his ability. This career concern effect increases second-period incentives, and ceteris paribus, first-period incentives. Secondly, when σ_η^2 increases, the relative weight the inside principal puts on the $y-$signal increases too (since $\sigma_q^2 > \sigma_y^2$ here). The strength of the ratchet effect is therefore increasing in σ_η^2 (in this example). An increase in σ_η^2 thus have two opposite effects on first-period incentives. When the former effect dominates the latter, first-period incentives are increasing in σ_η^2 and vice versa. More specifically we can prove the following proposition, which shows that β_1^* is non-monotone in σ_η^2 if and only if γ exceeds some $\hat{\gamma} > 1$.

Proposition 6. *i) For* $\gamma < 1 + \frac{1}{4}\left(1 + \frac{\sigma_y^4}{\sigma_q^2\left(\sigma_q^2 + 2\sigma_y^2\right)}\right) = \hat{\gamma}$, $\frac{\partial \beta_1^*}{\partial \sigma_\eta^2} > 0$ *for all* σ_η^2.

ii) For $\gamma > \hat{\gamma}$, $\frac{\partial \beta_1^*}{\partial \sigma_\eta^2}$ R 0 *for* σ_η^2 Q $\tilde{\sigma}_\eta^2$, *for some* $\tilde{\sigma}_\eta^2 > 0$.

Proof. Appendix A. ¤

[14]Prendergast (2000a) gives an overview of the empirical literature on the tradeoff of risk and incentives. See also Prendergast (1999, 2000b).

[15]The exact expression is $\beta_1^* = \frac{\sigma_\eta^2}{\sigma_\eta^2 + 10} \frac{3(\sigma_\eta^2)^2 + 260\sigma_\eta^2 + 800}{21(\sigma_\eta^2)^2 + 440\sigma_\eta^2 + 400}$.

Next, we consider how the noise in the market signal, σ_q^2, affects incentives.

If $\sigma_q^2 = 0$, then the principal puts no weight on the $y-$signal. Since both the inside principal and the market estimate the agent's ability on the same information, period t incentives are set to the level that equals the market's reward for a better estimate of the agent's ability, i.e. $\beta_1^* = 1$ and $\beta_2^* = \frac{1}{2}$. On the other hand, if $\sigma_q^2 = +\infty$, then $\beta_1^* = \beta_2^* = 0$: The $q-$signal is uninformative, and thus there are no career concern incentives. The principal's response is to set explicit incentives to zero as well.

Again the derivative shows that first-period incentives can be non-monotone in σ_q^2. We first illustrate this fact by an example. Let $\sigma_\eta^2 = 100$, $\sigma_y^2 = 20$, and $\gamma = \frac{1}{2}$. By plotting first-period (thin line) and second-period (thick line) incentives we get :[16]

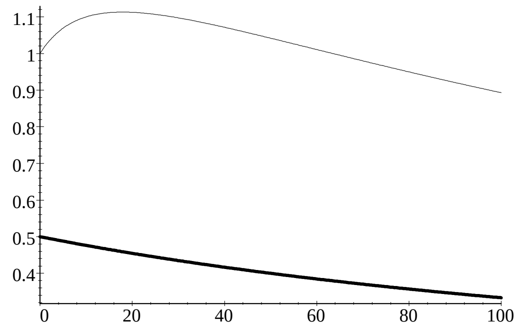

Figure 3: $\beta_t^*(\sigma_q^2)$

To understand this result, note that σ_q^2 influences first-period incentives in two ways. First, through the optimal second-period bonus β_2^*, which is decreasing in σ_q^2. Secondly, through the weight the principal puts on the $q-$signal relative to the $y-$signal. When σ_q^2 is low, the inside principal puts a relatively large weight on the market signal and the ratchet effect is weak. In addition, the effect of σ_q^2 on second-period incentives is low for small values of σ_q^2. Optimal first-period incentives are increasing in σ_q^2 for low values of σ_q^2. On the other hand, when σ_q^2 is large, the inside principal puts a relatively large weight on the $y-$signal implying that the ratchet effect is strong. Furthermore, second-period incentives are low. Now, optimal first-period incentives are decreasing in σ_q^2.

More formally we have the following proposition, which shows that β_1^* is non-monotone in σ_q^2 when γ is 'small' (and $\sigma_y^2 < 2\sigma_\eta^2$, case (ii)), or when γ is 'large' (case (iii)b).

Proposition 7. *i) For* $0 \leq 1 - \gamma < \frac{\sigma_y^2}{2\sigma_\eta^2}$, $\frac{\partial \beta_1^*}{\partial \sigma_q^2} < 0$ *for all* σ_q^2.

ii) For $\frac{\sigma_y^2}{2\sigma_\eta^2} < 1 - \gamma$, $\frac{\partial \beta_1^*}{\partial \sigma_q^2}$ R 0 *for* σ_q^2 Q $\hat{\sigma}_q^2$ *for some* $\hat{\sigma}_q^2 > 0$.

iii) For $\gamma > 1$, *(a)* $\frac{\partial \beta_1^*}{\partial \sigma_q^2} < 0$ *for* σ_q^2 *small and (b)* $\frac{\partial \beta_1^*}{\partial \sigma_q^2} > 0$ *for* σ_q^2 *large iff* $\gamma > 3 + 2\frac{\sigma_y^2}{\sigma_\eta^2}$

Proof. Appendix A. ¤

[16] The exact expression is $\beta_1^* = \frac{25}{200+\sigma_q^2} \frac{40000+33(\sigma_q^2)^2+3800\sigma_q^2}{5000+3(\sigma_q^2)^2+350\sigma_q^2}$.

Finally, comparative static results on optimal incentives due to changes in σ_y^2, are obtained.

First we note that second-period incentives are independent of σ_y^2. Secondly, while first-period incentives are non-monotone in both σ_η^2 and σ_q^2 the following calculation shows that β_1^* is monotonically decreasing (increasing) in σ_y^2 if $\gamma <$ $(>)$ 1.

Proposition 8. $\frac{\partial \beta_1^*}{\partial \sigma_y^2} = \beta_2^*(\gamma - 1)\frac{\sigma_\eta^2 \sigma_q^2 (2\sigma_\eta^2 + \sigma_q^2)}{\left(\sigma_\eta^2\sigma_q^2 + \sigma_y^2\sigma_\eta^2 + \sigma_y^2\sigma_q^2\right)^2} \gtrless 0$, *when* $\gamma \gtrless 1$.

Proof. Appendix A. ¤

The intuition is that when $\gamma > 1$, there is a career effect present on the $y-$signal (see Proposition 3). Thus, if the agent increases his effort on y relative to the inside principal's conjecture, his expected second-period salary increases, but at a decreasing rate as σ_y^2 increases. As a result, the agent's career incentives decrease as the $y-$signal becomes more noisy. The inside principal's response is to raise first-period incentives. Similarly, when $\gamma < 1$ there is a ratchet effect on the $y-$signal, and the negative effect on the fixed salary part of increased effort relative to the principal's conjecture decreases, when σ_y^2 increases. The inside principal's optimal response is, thus, to damped first-period incentives.

After characterizing optimal incentives, we now study how expected welfare depends on the strength of career concerns. The total certainty equivalent is

$$TCE = \sum_{t=1}^{3}\left[\frac{1}{2}\mathbb{E}(y_t + q_t) - \frac{1}{2}(e_t + a_t)^2\right] - \frac{r}{2}var(w_1 + w_2 + w_3).$$

In appendix C we show that

$$var(w_1 + w_2 + w_3) = (\beta_2^*)^2 V; \quad V := \left[4\sigma_\eta^2 (v+1)^2 + \left(v^2 + 1\right)\left(\sigma_q^2 + \sigma_y^2\right)\right],$$

where $v := \left[2 + (1-\gamma)R_y' r_q'\right] > 0$, for $\gamma < \frac{2 + R_y' r_q'}{R_y' r_q'}$. We also note that $R_y' r_q' > 0$ and that $\frac{\partial R_y' r_q'}{\partial \sigma_\eta^2} > 0$. Since $e_t + a_t = \beta_t^*$, $t = 1, 2, 3$, the total certainty equivalent is

$$TCE = 3m_0 + \frac{1}{2}\sum\nolimits_{t=1}^{2}\left[\beta_t^*(1 - \beta_t^*)\right] - \frac{r}{2}\left(\beta_2^*\right)^2 V.$$

To evaluate how changes in career concerns affect welfare note that the career effect is increasing in σ_η^2. Again let $\frac{\partial \beta_t^*}{\partial \sigma_\eta^2} := \beta_t^{*\prime}$, $t = 1, 2$. We get

$$\frac{\partial TCE}{\partial \sigma_\eta^2} = \frac{1}{2}\sum\nolimits_{t=1}^{2}\left(\beta_t^{*\prime}(1 - 2\beta_t^*)\right) - r\beta_2^*\left(\beta_2^{*\prime} V + \frac{r}{2}\beta_2^*\left[\begin{array}{c} 4(v+1)^2 \\ +v'\left(8\sigma_\eta^2(v+1) + 2v(\sigma_\eta^2 + \sigma_y^2)\right)\end{array}\right]\right),$$

which may be positive or negative since $\beta_1^{*\prime} \gtrless 0$, $\beta_2^{*\prime} > 0$, and $v' := \frac{\partial v}{\partial \sigma_\eta^2} = (1-\gamma)\frac{\partial r_q' R_y'}{\partial \sigma_\eta^2} \gtrless 0$. To sum up.

Proposition 9. *Expected welfare is non-monotone in the strength of career concerns.*

The intuition is the same as in the two-period model: When career concerns are low, (e.g., due to a low σ_η^2), the gain of more effort outweighs the welfare loss due to increased risk premium. When career concerns are strong, (e.g., because σ_η^2 is high), optimal explicit incentives are high. Thus, the agent bears much risk, and higher career concerns reduce welfare.

When agents are risk-neutral, ($r = 0$), total welfare might again be decreasing in the strength of the career effect. This happens when the career effects are so strong that the agent's cost of providing more effort outweight the associated increase in the production value.

We end this section by showing that expected welfare is non-monotone in $\gamma > 0$. I.e.

$$\frac{\partial TCE}{\partial \gamma} = \frac{\partial \beta_1^*}{\partial \gamma}(1 - 2\beta_1^*) - \frac{r}{2}(\beta_2^*)^2 \frac{\partial V}{\partial \gamma},$$

which may be positive or negative since $\frac{\partial V}{\partial \gamma} := -R_y' r_q' (8\sigma_\eta^2 (v + 1) + 2v(\sigma_\eta^2 + \sigma_y^2)) < 0$ and $\frac{\partial \beta_1^*}{\partial \gamma} < 0$. To understand this result first note that first-best is achieved for $\beta_t^* = \frac{1}{2}$, $t > 0$. Second, the variance is decreasing in $\gamma > 0$ (since $\frac{\partial \beta_1^*}{\partial \gamma} < 0$). Finally, β_1^* might be both to high and to low relative to first-best. Suppose now that β_1^* is too high relative to first-best, i.e. $\beta_1^* > 0$. In this case expected welfare is increasing in γ. This is due to the fact that an increase in γ both reduces the risk faces by risk-averse agents, and brings first period incentives closer to first-best. Next, suppose $\beta_1^* < \frac{1}{2}$ such that an increase in γ brings first-period incentives further away from the first-best incentives. Ceteris paribus this decreases total expected welfare. The increase in γ still reduces the variance of the first period salary. Thus the total effect on welfare depends on the relative strength of these two effects. To sum up.

Proposition 10. *Expected welfare is non-monotone in $\gamma > 0$. The more risk averse the agent is, the more beneficial is an increase in γ, since it reduces the risk premium.*

4. Related Literature

The fact that career concerns is a means to provide incentives for exerting effort was first discussed by Fama (1980) and Holmström (1982). Fama (1980) argued that incentives contract are not necessary since agents are disciplined by career concerns, while Holmström (1982) showed that career concerns incentives are not sufficient to induce efficient effort. Building on this fact, Gibbons and Murphy (1992) added explicit contracts to the Fama-Holmström model, and showed that an optimal compensation contract optimizes the combination of explicit and implicit incentives. None of these models discuss implicit incentives related to the ratchet effect—see also Dewatripont, Jewitt, and Tirole (1999).

Meyer and Vickers (1997) raised this question and showed that the influence of ratchet effects are more fundamental than career concerns in the sense that maximized welfare is decreasing in the strength of the former but unaffected by the latter. Roland and Sekkat (2000) analyze how career concerns may induce managers in state-owned enterprises to restructure their firms. Building on the model by Ickes and Samuelson (1987), they show that competition for managers eliminates the ratchet effect. All of these models assume that agents only exert effort on one single task.

Building on the work by Holmström and Milgrom (1991)—see also Itoh (1991, 1992, 1993)—Martimort (1993), Olsen and Torsvik (1993, 1995, 1998), and Meyer, Olsen, and Torsvik (1996) analyze how ratchet effects affect optimal explicit incentives and welfare in a multitask agency model. These models do however suppress career concerns and focus exclusively on ratchet effects.

5. Concluding Remarks

We have shown that some of the guidelines for the impact of career concern incentives that emerge from a single task analysis can be overturned in models where agents do several tasks. The key to understand this fact is to note that explicit incentives and career concern incentives are complementary in the multitask models we propose. As a result, higher career concern incentives (on one task) imply higher explicit incentives on the other task. Thus, stronger career concerns impose more risk on agents, and might lower maximized welfare.

Moreover, the interaction of implicit incentives in the form of both career concerns and ratchet effects produces non-standard comparative statics results with respect to explicit incentives. In particular, it was shown that more prior uncertainty about the agent's ability, or more noise in the available performance measures may lead to either higher or lower explicit incentives for the agent. The model thus offers a possible explanation for the fact that empirical studies observe both positive and negative correlations between risk and incentives.

An important, but realistic, assumption in this paper is that the inside principal has more information about the agent's type than outside principals. We have chosen to model this by assuming that the inside signal is sufficiently verifiable such that it can be contracted upon, but outside principals do not observe the signal. An alternative way of expressing the idea that the inside principal has more information about the agent is to follow Waldman (1984) and Ricard i Costa (1988) and assume that the current employer learns the agent's ability faster than prospective employers do. This resembles our model in the case outside principals' observation of the inside signal is so noisy that they base their conjectures of the agent's type solely on the market signal.

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Appendices

A. Technicalities

In this appendix we provide more details regarding some of the calculations in this paper.

A.1. $var(w_1 + w_2)$

We first show that $var(w_1 + w_2) = \beta_1^{*2}\left[\sigma_y^2 + \sigma_q^2 + 4\sigma_\eta^2\right]$. Note that $\beta_1^* = \frac{\sigma_\eta^2}{\sigma_q^2+\sigma_\eta^2}$. Thus,

$$\begin{aligned} & var(w_1 + w_2) = var(\beta_1 y_1 + w_2(q_1)) \\ = \;& var(\beta_1^*(y_1 + q_1)) \\ = \;& (\beta_1^*)^2[var(y_1) + var(q_1) + 2cov(y_1, q_1)]. \end{aligned}$$

The covariance-matrix (η, q_1, q_2) is

	η	q_1	q_2
η	σ_η^2	σ_η^2	σ_η^2
q_1	σ_η^2	σ_q^2	σ_η^2
q_2	σ_η^2	σ_η^2	σ_q^2

.

Thus

$$var(w_1 + w_2) = \beta_1^{*2}\left[\sigma_y^2 + \sigma_q^2 + 4\sigma_\eta^2\right].$$

A.2. The Expressions for R'_y and R'_q.

Note that the covariance matrix (y_2, y_1, q_1) is

	y_2	y_1	q_1
y_2	$\sigma_\eta^2 + \sigma_y^2$	σ_η^2	σ_η^2
y_1	σ_η^2	$\sigma_\eta^2 + \sigma_y^2$	σ_η^2
q_1	σ_η^2	σ_η^2	$\sigma_\eta^2 + \sigma_q^2$

.

By inverting and applying well-known formulas (see e.g., DeGroot (1970)) we get

$$\begin{aligned} R'_y \;&:\; = -R_y = \left[\frac{\sigma_\eta^2}{\sigma_\eta^2\sigma_q^2 + \sigma_y^2\sigma_\eta^2 + \sigma_y^2\sigma_q^2}\right]\sigma_q^2, \text{ and} \\ R'_q \;&:\; = -R_q = \left[\frac{\sigma_\eta^2}{\sigma_\eta^2\sigma_q^2 + \sigma_y^2\sigma_\eta^2 + \sigma_y^2\sigma_q^2}\right]\sigma_y^2. \end{aligned}$$

A.3. The Value of Γ.

By using the expressions for R_q', r_q', and β_2^* we get

$$(1-\gamma\frac{\sigma_\eta^2}{2\sigma_\eta^2+\sigma_q^2})\frac{\sigma_\eta^2}{\sigma_\eta^2+\sigma_q^2}+\frac{\sigma_\eta^2}{2\sigma_\eta^2+\sigma_q^2}(\gamma-1)\left[\frac{\sigma_\eta^2}{\sigma_\eta^2\sigma_q^2+\sigma_y^2\sigma_\eta^2+\sigma_y^2\sigma_q^2}\right]\sigma_y^2.$$

By setting this expression equal to zero and solve for γ we obtain

$$\Gamma := \frac{1}{(\sigma_\eta^2)^2}\frac{2(\sigma_\eta^2)^2\sigma_q^2+(\sigma_\eta^2)^2\sigma_y^2+2\sigma_\eta^2\sigma_y^2\sigma_q^2+(\sigma_q^2)^2\sigma_\eta^2+(\sigma_q^2)^2\sigma_y^2}{\sigma_q^2}.$$

A.4. The Value of $\overline{\gamma}$.

By using the expressions for R_y' and R_q' we get

$$\beta_1^*=\beta_2^*\left[\frac{2\sigma_y^2[(\sigma_\eta^2)^2+2(\sigma_q^2)^2]+\sigma_\eta^2\sigma_q^2[(4-2\gamma)\sigma_\eta^2+(3-\gamma)\sigma_q^2+4\sigma_y^2]}{\sigma_y^2[(\sigma_\eta^2)^2+2(\sigma_q^2)^2]+\sigma_\eta^2\sigma_q^2[\sigma_\eta^2+\sigma_q^2+2\sigma_y^2]}\right].$$

From this expression it follows easily that i) it decreases in γ, and ii) that the expression is equal to one when

$$\gamma:=\overline{\gamma}=\frac{(\sigma_\eta^2)^2+2(\sigma_q^2)^2+2\sigma_\eta^2\sigma_q^2}{\sigma_\eta^2\sigma_q^2\left(2\sigma_\eta^2+\sigma_q^2\right)}\sigma_y^2+\frac{3\sigma_\eta^2+2\sigma_q^2}{2\sigma_\eta^2+\sigma_q^2}.$$

A.5. The Partial Derivative $\frac{\partial\beta_1^*}{\partial\sigma_\eta^2}$. (Proof of Proposition 6).

Substituting for β_2^* and R_y', r_q' in the expression for β_1^* we get

$$\begin{aligned}\beta_1^* &= \beta_2^*\left[2-(\gamma-1)R_y'(r_q'-1)\right]\\ &= \frac{2\sigma_\eta^2}{2\sigma_\eta^2+\sigma_q^2}+(1-\gamma)\frac{\sigma_\eta^2\sigma_q^2}{\sigma_\eta^2\sigma_q^2+\sigma_y^2\sigma_\eta^2+\sigma_y^2\sigma_q^2}(\frac{\sigma_\eta^2}{\sigma_\eta^2+\sigma_q^2})\\ &= \frac{2}{2+Q}+(1-\gamma)\frac{nQ}{nQ+y+yQ}\frac{1}{1+Q},\end{aligned}$$

where $Q=\frac{\sigma_q^2}{\sigma_\eta^2}, n=\sigma_\eta^2, y=\sigma_y^2$. Differentiation shows that

$$\frac{\partial}{\partial Q}\beta_1^*>0 \qquad iff \qquad f(Q)=2\frac{n+y}{n}\left(Q+\frac{y}{n+y}\right)^2\frac{(1+Q)^2}{(2+Q)^2}-(1-\gamma)\left(\frac{y}{n+y}-Q^2\right)<0.$$

We see that $f(0)<0$ iff $\frac{y}{2n}<(1-\gamma)$ and $f(\infty)<0$ iff $2(1+\frac{y}{n})<-(1-\gamma)$. Moreover $f(Q)>0$ for $Q=(\frac{y}{n+y})^{\frac{1}{2}}$, for any γ. The proposition then follows by noting that $f(Q)$ is monotone increasing for $\gamma\leq 1$.

A.6. The Partial Derivative $\frac{\partial \beta_1^*}{\partial \sigma_q^2}$. (Proof of Proposition 7).

We may write

$$\beta_1^* = \frac{2N}{2N+1} + (1-\gamma)\frac{N}{N+NY+Y}(\frac{N}{N+1}), \qquad where \qquad N = \frac{\sigma_\eta^2}{\sigma_q^2}, Y = \frac{\sigma_y^2}{\sigma_q^2}.$$

Differentiation shows that

$$\frac{\partial}{\partial N}\beta_1^* > 0 \qquad iff \qquad g(N) = \frac{2\left(N+1\right)^2}{\left(2N+1\right)^2} + (1-\gamma)N\frac{N+2NY+2Y}{\left(N+NY+Y\right)^2} > 0.$$

We see that $g(0) > 0$ and that $g(\infty) > 0$ iff $\frac{1}{4}\frac{(1+Y)^2}{1+2Y} > -(1-\gamma)$. Moreover, we have $g(N) > 0$ for $\gamma \leq 1$, and $g'(N) < 0$ for $\gamma > 1$ ($g'(N)$ has the same sign as $-2 + (1-\gamma)Y^2\frac{(2N+1)^3}{(N+NY+Y)^3}$). The statements in the proposition follow from these observations.

A.7. The Partial Derivative $\frac{\partial \beta_1^*}{\partial \sigma_y^2}$. (Proof of Proposition 8)

Note that $\frac{\partial \beta_1^*}{\partial \sigma_y^2} = \beta_2^*(\gamma-1)\frac{\partial\left(R_q' - R_y'\right)}{\partial \sigma_y^2}$. We get

$$\frac{\partial \beta_1^*}{\partial \sigma_y^2} = \beta_2^*(\gamma-1)\frac{\sigma_\eta^2\sigma_q^2(2\sigma_\eta^2+\sigma_q^2)}{(\sigma_\eta^2\sigma_q^2+\sigma_y^2\sigma_\eta^2+\sigma_y^2\sigma_q^2)^2} \gtreqless 0, \text{ when } \gamma \lesseqgtr 1.$$

B. The Wage Contracts in Period 2

Period 2: By using the fact that the agent has career incentives to exert effort a_2 in this period, (to increase his market value), that the inside principal prefers balanced effort, and hence most provide balanced incentives, we have

$$\beta_2^* = \frac{\partial \alpha_3(q_1, q_2)}{\partial q_2} = \frac{\sigma_\eta^2}{2\sigma_\eta^2 + \sigma_q^2}.$$

Outside principals offer a contract $w_2^O = \alpha_2^O + \beta_2^O y_2$. To balance incentives on the two tasks, the bonus offered by an outside principal must satisfy $\beta_2^O = \gamma \beta_2^*$. This is so because the marginal cost of the two activities satisfy $\frac{\partial C^O}{\partial e_t} = \gamma(\gamma e_t + a_t) = \gamma \frac{\partial C^O}{\partial a_t}$ if the agent works for an outside principal. Since there is competition between outside principals, they earn zero expected profit. The expected wage payment from such a principal, had she hired the agent, would be $\mathbb{E}(w_2 \mid q_1) = \alpha_2^O + \beta_2^O[\mathbb{E}(y_2 \mid q_1)]$. Competition implies that this must be equal to the expected benefit $\frac{1}{2}[\mathbb{E}(y_2 + q_2 \mid q_1)]$. The zero profit condition is then, $\alpha_2^O + \beta_2^O[\mathbb{E}(y_2 \mid q_1)] = \mathbb{E}\left[\eta + \widehat{a}_2 \mid q_1\right]$. Thus the contract offered by an outside principal in period 2 is

$$w_2^O = (1 - \beta_2^O)[\frac{\sigma_q^2 m_0 + \sigma_\eta^2(q_1 - \widehat{a}_1)}{\sigma_\eta^2 + \sigma_q^2} + \widehat{a}_2] + \beta_2^O y_2.$$

For the agent, this schedule has certainty equivalent $CE_2^O = \alpha_2^O + \beta_2^O \mathbb{E}(y_2 \mid q_1, y_1) - \frac{1}{2}(\gamma e_2 + a_2)^2 - \frac{r}{2} var(w_2^O \mid q_1, y_1)$, where efforts are given by $(\gamma e_2 + a_2)\,\gamma = \gamma\beta_2^* = \beta_2^O$.

The inside principal offers $w_2 = \alpha_2 + \beta_2^* y_2$, and conditions the contract on both signals: q_1, y_1. To ensure that the agent continues to work for the inside principal, she must offer the agent at least as high reservation utility (or certainty equivalent) as the market. I.e. $CE_2^O \leq CE_2^I = \alpha_2 + \beta_2^* \mathbb{E}(y_2 \mid q_1, y_1) - \frac{1}{2}(e_2 + a_2)^2 - \frac{r}{2} var(w_2 \mid q_1, y_1)$. Hence, $\alpha_2 = (1 - \gamma\beta_2^*)\mathbb{E}(y_2 \mid q_1) + (\gamma - 1)\beta_2^* \mathbb{E}(y_2 \mid q_1, y_1) + const$, where the constant is independent of q_1, y_1. (It represents effort costs, risk premium etc.) The contract offered by the inside principal is thus

$$w_2 = (1 - \gamma\beta_2^*)\mathbb{E}(y_2 \mid q_1) + \beta_2^* \left[y_2 - (1 - \gamma)\mathbb{E}(y_2 \mid q_1, y_1)\right] + const.$$

C. Welfare in the 3-Period Model

In this appendix we show that

$$var(w_1 + w_2 + w_3) = (\beta_2^*)^2 \left[4\sigma_\eta^2 (v+1)^2 + \left(v^2+1\right)\left(\sigma_q^2 + \sigma_y^2\right)\right],$$

where $v := \left[2 + (1-\gamma) R_y' r_q'\right]$. We have,

$$\begin{aligned} w_1 &= const + \beta_1^* y_1, \\ w_2 &= const + (1-\gamma\beta_2^*)\mathbb{E}[y_2 \mid q_1] + \beta_2^* \left[y_2 - (1-\gamma)\mathbb{E}[y_2 \mid y_1, q_1]\right] \\ w_3 &= \frac{\sigma_q^2 m_0 + \sigma_\eta^2 \sum_{t=1}^2 (q_t - \widehat{a}_t)}{\sigma_q^2 + 2\sigma_\eta^2}, \quad \text{where} \\ \mathbb{E}[y_2 \mid q_1] &= \mathbb{E}y_2 + r_q'(q_1 - \mathbb{E}q_1); \quad r_q' = \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_q^2}, \quad \text{and} \\ \mathbb{E}[y_2 \mid y_1, q_1] &= \mathbb{E}y_2 + R_q'(q_1 - \mathbb{E}q_1) + R_y'(y_1 - \mathbb{E}y_1). \end{aligned}$$

Thus,

$$\begin{aligned} & var(w_1 + w_2 + w_3) \\ = \;& var\left\{\beta_1^* y_1 + (1-\gamma\beta_2^*) r_q' q_1 + \beta_2^* y_2 + \beta_2^*(\gamma-1)[R_q' q_1 + R_y' y_1] + \beta_2^*(q_1+q_2)\right\} \\ = \;& var\left\{\left[\beta_1^* + \beta_2^*(\gamma-1)R_y'\right] y_1 + \beta_2^* y_2 + \left[\beta_2^*(\gamma-1)R_q' + \beta_2^* + (1-\gamma\beta_2^*) r_q'\right] q_1 + \beta_2^* q_2\right\}. \end{aligned}$$

Note that $(\gamma\beta_2^* - 1) r_q' = [(\gamma-2) r_q' - (1-r_q')]\beta_2^*$ such that

$$\beta_1^* + \beta_2^*(\gamma-1)R_y' = \beta_2^*(\gamma-1)R_q' + \beta_2^* + (1-\gamma\beta_2^*) r_q' = \beta_2^* \left[2 + (1-\gamma)(r_q' - R_q')\right].$$

Let

$$v := \left[2 + (\gamma-1)(R_q' - r_q')\right] = \left[2 + (1-\gamma) R_y' r_q'\right]$$

Then

$$var(w_1+w_2+w_3) = (\beta_2^*)^2 \left\{ \begin{array}{c} v^2\left[var(y_1) + var(q_1)\right] + var(y_2) + var(q_2) + \\ 2\left[\begin{array}{c} v\left[cov(y_1,y_2) + cov(y_1,q_2) + cov(q_1,y_2) + cov(q_1,q_2)\right] \\ + v^2 cov(y_1,q_1) + cov(y_2,q_2) \end{array}\right] \end{array} \right\}.$$

The expressions for the variances and covariances are:

$$\begin{aligned} var(y_1) &= var(y_2) = \sigma_\eta^2 + \sigma_y^2, \\ var(q_1) &= var(q_2) = \sigma_\eta^2 + \sigma_q^2, \qquad \text{and,} \\ cov(y_1,y_2) &= cov(y_1,q_1) = cov(y_1,q_2) = cov(y_2,q_1) = cov(y_2,q_2) = cov(q_1,q_2) = \sigma_\eta^2. \end{aligned}$$

Thus

$$var(w_1 + w_2 + w_3) = (\beta_2^*)^2 \left[4\sigma_\eta^2 (v+1)^2 + \left(v^2+1\right)\left(\sigma_q^2 + \sigma_y^2\right)\right].$$

We also note that

$$\begin{aligned} R_y' r_q' &= \frac{(\sigma_\eta^2)^2 \sigma_q^2}{(\sigma_\eta^2 + \sigma_q^2)(\sigma_\eta^2\sigma_q^2 + \sigma_y^2\sigma_\eta^2 + \sigma_y^2\sigma_q^2)} > 0, \quad \text{and} \\ \frac{\partial r_q' R_y'}{\partial \sigma_\eta^2} &= \frac{\sigma_\eta^2 \left(\sigma_q^2\right)^2 (2\sigma_y^2(\sigma_\eta^2 + \sigma_q^2) + \sigma_\eta^2\sigma_q^2)}{(\sigma_\eta^2 + \sigma_q^2)^2(\sigma_\eta^2\sigma_q^2 + \sigma_y^2\sigma_\eta^2 + \sigma_y^2\sigma_q^2)^2} > 0. \end{aligned}$$