Strategic pricing of commodities

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ABSTRACT. In this paper we will consider a setting where a large number of agents are trading commodity bundles. Assuming that agents of the same type have a certain utility attached to each transaction, we construct a statistical equilibrium which in turn implies prices on the different commodities. Our basic question is then the following: Assume that some commodities come out with prices that are socially unacceptable. Is it possible to change these prices systematically if a new type of agents is paid to enter the market? In the paper we will consider explicit examples where this can be done.

Keywords: Agent preferences, efficient markets, statistical equilibria, commodity prices, arbitrageurs
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1. Introduction

The framework in this paper is based on a newly developed statistical framework, Jörnsten & Ubøe (2006), that can be used to quantify the effect of agent preferences. The framework is very versatile and roughly speaking it can be used in any setting where we have a large number of agents that have some utilities of doing some actions constrained by linear constraints. To be more specific we will in this paper consider commodity markets where the agents trade certain bundles of commodities. We assume that the market must clear in all commodities, that the agents can be divided into types, and that the total numbers of agents of each type are known. These are all linear constraint on the system.

If suboptimal solutions happens frequently, we would expect that authorities introduce legislation/incentives to avoid this. The basic idea in the Jörnsten & Ubøe (2006) approach is hence to assume that systems are benefit efficient in the sense that the probability of a macrostate increases with an increase in total utility. Under this assumption it is usually possible to construct a unique 1-parameter family of probability measures on the macrostates. This family contains all benefit efficient probability measures, i.e., if a probability measure on macrostates is outside this family, it is possible to find a pair of macrostates where the one with the lowest total utility occurs more frequently than the other. The parameter in these measures acts as a quantifier of the effect of utility. It simply corresponds to a choice of units for utility. Once such a choice has been made, we are usually left with a unique probability measure, and this measure we call the benefit efficient probability measure for the system.

The idea of cost efficiency has be used by regional scientists for a long period of time. In that setting the models reduces to a model of gravity type, see Erlander & Smith (1990), Erlander & Stewart (1990). The ideas of cost efficiency and strong cost efficiency developed in these papers are crucial to our approach. See also the seminal textbook on gravity modeling Sen & Smith (1995). If the agents are indifferent with respect to utility, our model reduces to a market model of entropy maximizing type, see Foley (1994). The model in Jørnsten & Ubøe (2006) is hence a unification and extension of well established economic theories. For further information and references, see Jørnsten & Ubøe (2006).

In the present paper we want to take the theory a step further and see how it can be used strategicly to influence prices in a market. Suppose that some market prices are undesirable from an administrative perspective. In our framework prices can be subject to change if we introduce a new type of agents in the market. These agents are initially unwilling to transact. The idea is then to use incentives/subsidies to increase their utility of transacting. We can think of such agents as arbitrageurs in the system. They transact only if they are paid sufficiently much for their "services". In return authorities might obtain socially more acceptable prices.

The paper is organized as follows. In Section 2 we give a full description of the Jörnsten & Ubøe (2006) framework. In Section 3 we consider an example where we initially have two types of agents trading bundles of 3 different commodities. The prices (shadow costs) of commodity 1 and 2 are positive, while the price on commodity 3 is negative. If we introduce a new type of agents, we can use incentives to steer the price to a target level. The target level cannot be arbitrary high, however. Due to the market clearing conditions, traders in commodity 1 and 2 will indirectly influence the price on commodity 2. We will consider cases where the target level of commodity 3 can be enhanced if we introduce new groups of arbitrageurs. The effect of such strategies might be quite difficult to predict, and we show some examples to illustrate the complexity of the problem. Finally in Section 4 we offer some concluding remarks.

2. Framework

In this paper we will consider a general setting where we have K types of commodities and T types of agents. We will assume that all agents of the same type have the same *offer set*, i.e., the same specification of what *transactions* they can perform. From a technical point of view a transaction is a K-dimensional vector specifying how much an agents buys (sells if negative) of each commodity. In the following we will introduce our basic notation through a series of examples. These examples are not chosen at random; we will later put these examples together to explain the basic features of our approach.

EXAMPLE 2.1

Assume that there are $K = 3$ different commodities. If an agent performs the transaction $x = (4, -2, 3)$, it means that he uses 4\$ to buy commodity 1, sells commodity 2 to receive 2\$, and 3\$ is used to buy commodity 3.

An offer set is hence a collection of K-dimensional vectors, i.e., a subset $O \subset \mathbb{R}^K$. If agents of type 1 have the offer set

$$O_1 = \{(4, -2, 3), (8, -4, 6), (0, 0, 1), (-1, 1, -2), (0, 0, 0)\} \tag{2.1}$$

it simply means that they can carry out the 5 different transactions $(4, -2, 3)$, $(8, -4, 6)$,$(0, 0, 1)$, $(-1, 1, -2)$, and $(0, 0, 0)$. All other transactions are void. Let N_t denote the number of agents of type t, and let $N = \sum_{t=1}^{T} N_t$ denote the total number of agents in the system. A *market transaction* is an $K \cdot N$ dimensional vector $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ specifying the transactions of all the N individual agents. Here each $x_i \in \mathbb{R}^K$ must be in the offer set of the corresponding agent. The agents can only choose transactions that are compatible with a clearing of the market, and we will call a market transaction *feasible* if it clears the market.

EXAMPLE 2.2

Assume that there are $K = 3$ different commodities, and $T = 2$ types of agents. Agents of type 1 has the offer set O_1 defined in (2.1) and agents of type 2 has the offer set

$$O_2 = \{(-1, 1, 0), (-3, 1, -5), (-8, 4, -7), (0, 0, -6), (2, -1, 1), (0, 0, 0)\} \tag{2.2}$$

An example of a feasible market transaction is then the following: One agent of type 1 does $(4, -2, 3)$, two agents of type 1 do $(0, 0, 1)$, one agent of type 2 does $(-1, 1, 0)$, one agent of type 2 does $(-3, 1, -5)$, and all the other agents choose the notransaction $(0, 0, 0)$. Obviously the market can be cleared in a large number of different ways. That, however, does not mean that all the feasible market transactions are equally likely. Assuming that we cannot distinguish agents of the same type, the same macrostate can be generated by a possibly large number of different microstates. If there are many agents of each type, we expect that certain macrostates are much more likely than the others. The next issue will then be to characterize these macrostates.

To get a suitable notation for macrostates, we will need an ordering of the various transactions. Assuming that $|O_t| = n_t$, i.e., that agents of type i can perform n_t different transactions, we first order the n_1 different transactions that agents of type 1 can do. We then let $f_i, t = 1, \ldots, n_1$ denote the total number of agents of type 1 that carries out transaction number i. Correspondingly we let $f_i, i = n_1 + 1, \ldots, n_1 + n_2$ denote the number of agents of type 2 that carries out the n_2 different transactions in the offer set for these agents. We continue like that until we have written down a single vector

$$\mathbf{f} = (f_1, f_2, \ldots, f_n) \qquad \text{where } n = \sum_{t=1}^{T} n_t \tag{2.3}$$

This vector completely describes the macrostate of the system. The market clearing conditions are all linear. Assuming that we know the total number of agents of each type, it is hence possible to find a $(T + K) \times n$ dimensional matrix A such that if

$$(2.4) \qquad A\mathbf{f}^{\perp} = (N_1, N_2, \ldots, N_T, \underbrace{0, \ldots, 0}_{K \text{ times}})^{\perp} \qquad (\perp \text{ denotes matrix transposition})$$

then $\mathbf{f}$ clears the market and has the correct number of agents of each type.

EXAMPLE 2.3

Assume that there are $K = 3$ different commodities, and $T = 2$ types of agents. If O_1 and O_2 are given by (2.1) and (2.2) above, then $n_1 = 5$ and $n_2 = 6$. If

$$(2.5) \qquad \mathbf{f} = (1\,500, 500, 11\,000, 1\,200, 800, 1\,000, 2\,200, 500, 500, 1\,400, 2\,400)$$

it means, e.g., that 1 500 agents of type 1 do $(4, -2, 3)$ that 1 000 agents of type 2 do $(-1, 1, 0)$ etc. In this case the matrix A in (2.4) can be written down as follows:

$$(2.6) \qquad A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 4 & 8 & 0 & -1 & 0 & -1 & -3 & -8 & 0 & 2 & 0 \\ -2 & -4 & 0 & 1 & 0 & 1 & 1 & 4 & 0 & -1 & 0 \\ 3 & 6 & 1 & -2 & 0 & 0 & -5 & -7 & -6 & 1 & 0 \end{bmatrix}$$

Computing $A\mathbf{f}^{\perp}$, we get

$$(2.7) \qquad A\mathbf{f}^{\perp} = (15\,000, 8\,000, 0, 0, 0)^{\perp}$$

Hence there are 15 000 agents of type 1, 8 000 agents of type 2, and the market clears in all 3 commodities.

Foley (1994) assumes that all transactions are equally likely. In the absence of information this might be a reasonable assumption. Often, however, some information is available, and we believe that it is better to assume that each agent has a certain utility attached to every transaction that he or she can do. Again we assume that agents of the same type attach the same utility to each transaction in their offer set. The various utilities can be ordered in the same manner as above. Hence if we write down a n-dimensional vector $\mathbf{U}$, this vector expresses the utility that is attached to each different transaction.

$$\mathbf{U} = (U_1, U_2, \ldots, U_n) \qquad \text{where } n = \sum_{t=1}^{T} n_t \tag{2.8}$$

Given such a utility vector, what macrostates can we expect to observe? A definite answer to this nontrivial problem can be found from the construction in Jörnsten and Ubøe (2006). Their basic result can be expressed as follows:

Theorem (Jörnsten & Ubøe (2006))

Assume that the system is benefit efficient in the sense that the probability of any particular market transaction is nondecreasing with an increase in total utility, and that there is a large number of agents of each type. Then there exists a constant $\beta \geq 0$*, and a set of real numbers* $\pi_1, \pi_2, \ldots, \pi_{T+K}$*, such that*

$$\mathbf{f} = \exp\left[-(\pi_1, \pi_2, \ldots, \pi_{T+K})A + \beta \mathbf{U}\right]^{\perp} \tag{2.9}$$

Here a function, like exp above, acts on each of the components of the vector. Given $\beta \geq 0$ and $\mathbf{U}$ an arbitrary utility vector, then $\pi_1, \pi_2, \ldots, \pi_{T+K}$ can be found solving the non-linear system

$$A \exp\left[-(\pi_1, \pi_2, \ldots, \pi_{T+K})A + \beta \mathbf{U}\right] = (N_1, N_2, \ldots, N_T, \underbrace{0, \ldots, 0}_{K \text{ times}})^{\perp} \tag{2.10}$$

Jörnsten & Ubøe (2006) prove that this system always has a solution, and that the solution is unique in all except a small set of degenerate cases. The reason for this is quite simply that $\mathbf{f}$ solves (2.9) and (2.10) if and only if it is a solution to the non-linear program

$$\begin{aligned} &\max_{\mathbf{f}} -\sum_{i=1}^{n} f_i \ln[\mathbf{f}_i] \\ &A\mathbf{f}^{\perp} = (N_1, N_2, \ldots, N_T, \underbrace{0, \ldots, 0}_{K \text{ times}})^{\perp} \\ &\sum_{i=1}^{n} U_i \mathbf{f}_i \geq \beta \end{aligned} \tag{2.11}$$

Hence $\pi_{T+1}, \pi_2, \ldots, \pi_{T+K}$ is nothing but the shadow costs of the K different commodities. From (2.9)-(2.10) or (2.11) we can see that β simply acts a a choice of units for the utility. If $\beta > 0$ and $\beta \neq 1$, we can change units of $\mathbf{U}$ such that

$$\beta \mathbf{U} \text{ (old units)} = \mathbf{U} \text{ (new units)} \tag{2.12}$$

The case $\beta = 0$ corresponds to the case where the agents are indifferent to utility, i.e., all transactions are equally likely, see Foley (1994). If we put $\beta = 1$, we can recover that particular case using $\mathbf{U} = 0$. Hence we can (and will) assume that $\beta = 1$ without loss of generality.

Although the system in (2.10) is non-linear, it can be solved quite efficiently by a variation of the Bregman balancing algorithm, see Bregman (1967). It can be shown, see Jörnsten & Ubøe (2006), that the i-th component of the lefthand side of (2.10) is a monotone function of variable i. Our numerical approach is to set some starting values for Π, fix all but one component 1, and find variable 1 such that the first component of (2.10) is satisfied. We then update variable 1, and fix all but variable 2. The next step is to compute variable 2 so that the second component of (2.10) is satisfied. We continue like that until we have updated all variables, and repeat everything from the start. This procedure quickly contracts to a numerical solution of (2.10).

3. Strategic pricing of commodities

In this section we will study pricing strategies of commodities using the framework from Jörnsten & Ubøe (2006). Commodity prices are assumed to be given via the shadow costs in (2.11), and we will consider extensions and strategic modifications of the examples in the previous section. More precisely we will consider the following:

Throughout this section we assume that $T = 2$, that $K = 3$ and that the offer sets of the agents are given by (2.1) and (2.2). Furthermore we assume that $\beta = 1$ and that the utilities are given by the vector

$$\mathbf{U} = (1, 2, 1, 1, 0, 1, 1, 2, 1, 1, 0) \tag{3.1}$$

Solving (2.10) and using this in (2.9), we find that the only benefit efficient distribution (with $\beta = 1$) is given by

$$\mathbf{f} = (2112, 3804, 7665, 246, 1173, 793, 1039, 4574, 9, 932, 653) \tag{3.2}$$

In the following our main interest will be shadow costs, hence we will not report any more values on the benefit efficient distributions. In this particular case the shadow costs of the 3 commodities are given by the price vector

$$\Pi = (2.33, 3.14, -0.88) \tag{3.3}$$

We see that the shadow cost of commodity 3 is negative. One reason for this is that we are dealing with a model with non-free disposal. If we relax the market clearing conditions to allow free disposal of scrap, we can increase the shadow cost of commodity 3 to 0. There is, however, much more subtle ways of doing this.

Assume that there is a third group of agents that are initially unwilling to participate. We will assume that this third group has the offer set

$$O_3 = \{(0, 0, 1), (0, 0, 0)\} \tag{3.4}$$

and that there are a total of 20 000 agents of this type. We can then consider a model extension to see how this affects prices. In the following we will assume that these agents have a utility V of doing $(0, 0, 1)$ and that the utility of doing the notransaction case is 0. Since this group is unwilling to participate, we can assume that V is negative and very large. Computing this case with $V = -10$, we obtain the commodity prices

$$\Pi = (2.33, 3.14, -0.88) \tag{3.5}$$

We see that a model extension of this type does not alter the prices. This is what we expect. But what if we pay these agents to participate. In that case we would expect that V increases. A numerical simulation confirms this. If $V = 10$, we obtain the prices

$$\Pi = (1.18, 2.50, 0.34) \tag{3.6}$$

We observe that all prices are now positive. To compare with the case with free disposal of scrap, we want to compute the minimum utility W giving nonnegative prices. In our framework, the prices are continuous functions of V. The computation is straightforward, and we get

$$V = 0.32 \ \Rightarrow \ \Pi_3 = 0 \tag{3.7}$$

From a management point of view, a case with 0 commodity price might be undesirable. In some cases we might wish to set a target price, say $\Pi_3 = 0.25$ and solve the inverse problem to find V replicating that value. We get

$$V = 2.32 \ \Rightarrow \ \Pi_3 = 0.25 \tag{3.8}$$

The plot in Figure 3 shows the development of the commodity price on commodity 3 as we continuously change V from -10 to 10.

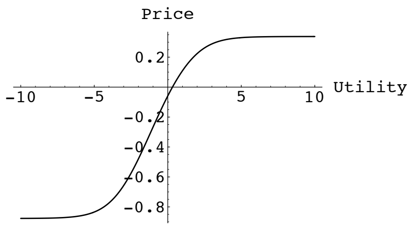


Figure 1: Price of commodity 3 as a function of the utility V

From the plot we see that there are clearcut limits to how much we can alter prices. No matter how much we increase V we cannot increase the price on commodity 3 beyond 0.34. Using a new extension, however, the price can be increased well above that point. To that end we introduce a 4th type of agents with offer set

$$O_4 = \{(0,1,0),(0,0,0)\} \tag{3.9}$$

and assume that there are 20 000 agents of that type. These are hence traders in commodity 2. If we let W denote the utility of doing $(0,1,0)$ and as before let 0 be the utility of doing the notransaction case, the price of commodity 3 will be a continuous function of V and W. A 3D-plot of this function is shown in Figure 2 .

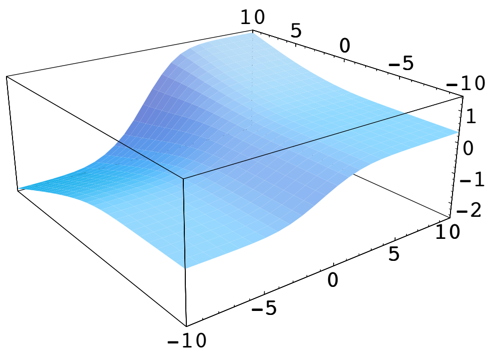
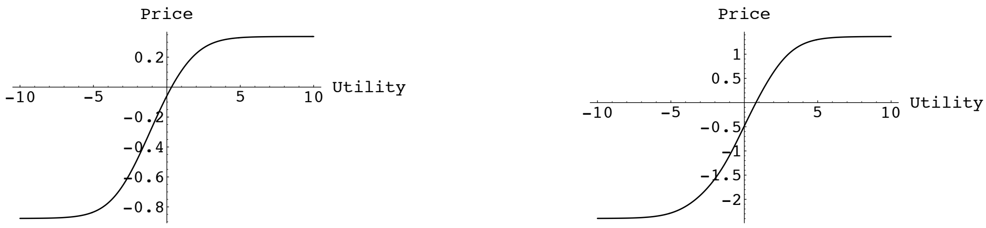


Figure 2: Price of commodity 3 as a function of V and W

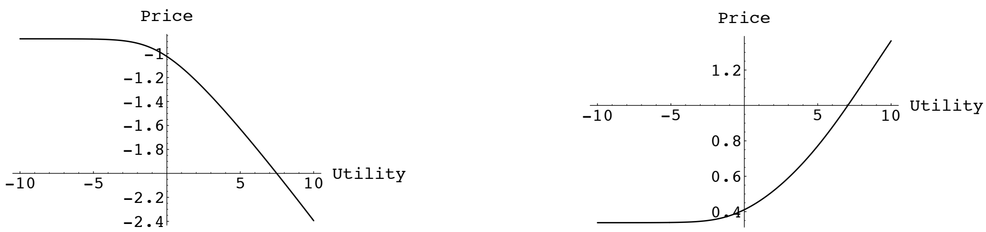
Of particular interest are the intersections with the planes $W = -10$ and $W = 10$. They are shown in Figure 3.



$W = -10$ $W = 10$

Figure 3: Price on commodity 3 as a function of V

If we compare the left graph to Figure 1, we see that the two graphs are identical. The graph to the righthand side of Figure 3 is different. In that graph, the price increases to a value $\Pi_3 = 1.37$ which is far above the value we got in the previous case. To examine this further, we can take a look at the intersections of the graph in Figure 2 with the planes $V = -10$ and $V = 10$.



$V = -10$ $V = 10$

Figure 4: Price on commodity 3 as a function of W

It is interesting to note that the introduction of a buyer of commodity 2 affects the price of commodity 3 in two completely different ways. In the graph to the left (where $V = -10$), we see that the price of commodity 3 falls (from the starting level $\Pi_3 = -0.88$) when agents of type 4 get an increased utility of buying commodity 2. In the graph to the right (where $V = 10$), the situation is reversed. In that case the price on commodity 3 is enhanced (to the final level 1.37) when agents of type 4 get an increased utility of buying commodity 2.

In the case above, we examined the effect of introducing what we could call pure buyers of commodities 2 and 3. Note that in this framework, agents can only do the transactions listed in their offer sets. They are not allowed to short their positions, and hence the signs are important. Transactions like $(0, 1, 0)$ and $(0, 0, 1)$ correspond to buying commodities. If we instead introduce a seller of commodity 2, the behavior of the system will be quite different. An example of this sort would be that case where we keep the agents of type 1,2, and 3 above, but replace the pure buyers of type 4 with 20 000 pure sellers, i.e., agents with offer set

$$O_4 = \{(0,-1,0),(0,0,0)\} \tag{3.10}$$

and with utility W of doing the transaction $(0,-1,0)$. The resulting prices on commodity 3 are shown in Figure 5.

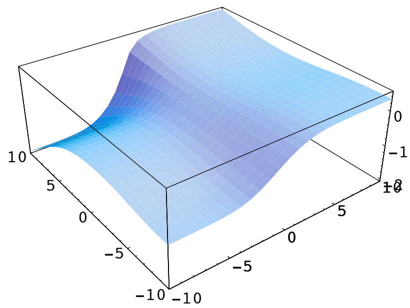
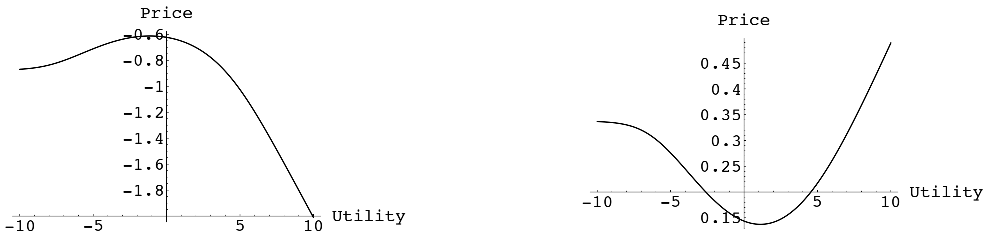


Figure 5: Price of commodity 3 as a function of V (pure buyers) and W (pure sellers)

If we examine the intersections with the planes $V = -10$ and $V = 10$ again, we see a complex interaction pattern.



$V = -10$ $\qquad\qquad$ $V = 10$

Figure 6: Price on commodity 3 as a function of W

If $V = -10$, the price Π_3 is first enhanced (from a starting level of -0.88) as W increases, but after a while it starts to drop. If $V = 10$, the price Π_3 starts to fall (from a starting level of 0.34) as W increases, but after a while it starts to increase. Obviously there are forces pulling in different directions, and it is the total balance that determines the direction of the effect. This shows us that interaction patterns in commodity bundles might be surprisingly complex. Unless we are able to quantify these effects, it is more or less impossible to guess the final outcome or even the directions of change.

Stimulating original groups

In the cases above we have used arbitrageurs to control the prices. But what if we instead introduce incentives to the original groups? To examine this in detail, we return to the case with two types of agents. Agents of type 1 have the offer set (same as (2.1))

$$O_1 = \{(4, -2, 3), (8, -4, 6), (0, 0, 1), (-1, 1, -2), (0, 0, 0)\} \tag{3.11}$$

Agents of type 2, however, have extended their offer sets to

$$O_2 = \{(-1, 1, 0), (-3, 1, -5), (-8, 4, -7), (0, 0, -6), (2, -1, 1), (0, 0, 1), (0, 0, 0)\} \tag{3.12}$$

i.e., these agents can carry out pure trades in commodity 3. We let V denote the utility of the new transaction $(0, 0, 1)$ for agents of type 2. All the other transactions keep their original values. The total numbers of agents of each type is the same as before. If $V = -10$, we get the price vector

$$\Pi = (2.33, 3.14, -0.88) \tag{3.13}$$

This is of course the same as in (3.3). As V increases, the price of commodity 3 gradually changes. The price development is shown in Figure 7.

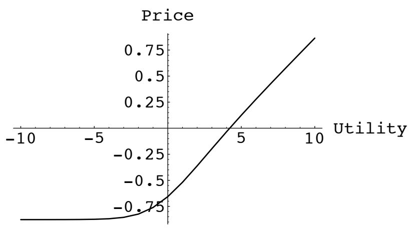


Figure 7: Price on commodity 3 as a function of V

This is hence another regime to provide socially acceptable prices. This regime is different from the cases above. For a direct comparison with the previous cases, we extended the number of agents of type 2 by 20 000. The resulting prices are shown in Figure 8.

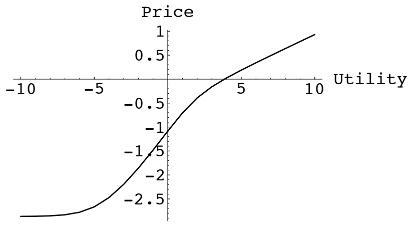


Figure 8: Price on commodity 3 as a function of V

If we compare Figure 1 and Figure 8, we see that the two graphs are quite different. The total number of agents is the same in the two cases, and the agents can carry out exactly the same transactions. Hence we see that giving incentives to the original groups does not produce the same result as giving the same incentives to a group of arbitrageurs.

4. Concluding remarks

In this paper we have used the framework in Jörnsten & Ubøe (2006) to see how incentives can be used to steer prices to target levels. The basic assumption in the paper is that a statistical equilibrium in a market will occur at benefit efficient states, i.e., states characterized by a benefit efficient probability measure.

In the paper we have studied the effects of “pure buyers” and ”pure sellers” of a commodity in a market where the other agents typically trades commodity bundles. If we give incentives to pure buyers of a commodity we expect that these agents get an increased utility of buying that particular commodity. As a result the price on the commodity increases. Correspondingly, we have seen how increased utility for “pure sellers” typically reduces the price. Our examples show, however, that the price cannot change arbitrary much.

When agents trade commodity bundles, a “pure buyer” or ”pure seller” of a commodity will implicity change the prices on the other commodities. Our examples show that such interactions might be surpricingly complex. Several forces are working in opposite directions, and it is the total balance that determines the direction of change. Increasing the utility of pure trade in one commodity may first lead to reduced prices on another commodity, while it may well happen that a further increase might reverse the sign of the effect.

The Jörnsten & Ubøe (2006) framework explains how to quantify the effects above, and our examples show that without proper quantifications it is very hard to form an opinion of the final outcome or even the directions of change.

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