Patient allocations according to circumstances and preferences: Modelling based on the Norwegian patient list system.

Jostein Lillestøl[a,*], Jan Ubøe[a], Yngve Rønsen[b], Per Hjortdahl[b],

a The Norwegian School of Economics and Business Administration, Bergen, Norway

b. Institute of General Practice and Community Medicine, University of Oslo, Norway

Abstract:

In some countries every citizen has the right to obtain a designated general practitioner. However, each individual may have preferences that cannot be fulfilled due to shortages of some kind. The questions raised in this paper are: To what extent can we expect that preferences are fulfilled when the patients "compete" for entry on the lists of practitioners? What changes can we expect under changing conditions? A particular issue explored in the paper is when the majority of women prefer a female doctor and there is a shortage of female doctors. The analysis is done on the macro level by the so called gravity model and on the micro level by recent theories of benefit efficient population behaviour. The approach is quite general and can be applied in a variety of contexts.

Keywords: Patient allocations, shortages, preferences, benefit efficiency, gravity model.

* Corresponding author:

Address: NHH, Helleveien 30, N-5045 Bergen, Norway

E-mail: jostein.lillestol@nhh.no

NHH, December 20, 2006

Patient allocations according to circumstances and preferences: A model based on the Norwegian patient list system.

In some countries, like Norway, every citizen has the right to obtain a designated general practitioner. However, each individual may have preferences that cannot be fulfilled due to shortages of some kind. One example is the preference for having a general practitioner of the same sex, of the opposite sex, or be indifferent. Available data indicate that female patients have a stronger preference for doctor of the same sex than male patients. When female doctors are less frequent than male doctors, everybody cannot get a doctor according to their preference, unless there are unacceptable loads and unacceptable vacancies. To what extent can we expect that such preferences are fulfilled when the patients “compete” for entry on the lists? What changes in the distribution can we expect under changing conditions? In particular, in the case of many women with strong preference for a female doctor, how will the fraction of female patients assigned to female doctors expect to change when the fraction of available female doctors increases?

We have no ambitions to provide a dynamic analysis, describing changes at the micro level, i.e. how a patient having a doctor of the ”wrong sex” initiates the search and may find a doctor of the “correct sex”. A change will typically take place only at a vacancy at a doctor in the neighbourhood, and this varies not only between urban and rural areas, but also within certain regions. Initial assignment and decrement also complicates the matter, and it is probably futile to model a dynamic allocation process. A number of assumptions have to be made, that may be disputed and hard to verify empirically. Do we have alternatives that can provide insight?

On the macro level it is well known that behaviour in many populations, among plants, animals and humans have cost minimizing traits. For humans this was formulated by Zipf (1949). For localization problems we find the so called gravity model in the urban geography literature. This model is quite robust under varied circumstances, and has found applications in many fields, among them studies on travelling patterns and distribution of commodities in a network, and in tomography as well. Smith (1978) gave a formal statement of the Zipf principle, saying that patterns with lower total costs are at least as likely as those with higher costs. On this basis Erlander & Smith (1990) developed a general theory of efficient population behaviour, leading to a representation theorem for the feasible patterns. Among several special cases of this theory are situations leading to the gravity model. More recent contributions to this theory are given by Jørnsten et.al (2004).

The problem of distribution of patients among doctors may be formulated within the gravity model. The model has its limitations, and is not flexible enough to pick up interesting problems on the micro level. However, recent work by Jørnsten & Ubøe (2005) has provided opportunities for including more characteristics and restrictions of different nature. This is required if we think of using such models for planning purposes at the micro level.

New contribution

This paper contains new material to the medical and health profession on patient allocation according to circumstances and preferences, both with respect to modelling opportunities and qualitative results. The paper is in two parts, the first is on macro modelling based on the gravity model, well known in some areas outside medicine, and the second is on micro modelling based on new theory reported in Ubøe & Lillestøl (2006). Although the problem came out of the Norwegian patient list system, the modelling approach and qualitative results have general applicability. We also hope that this paper may create some discussion on which restrictions exist in practice, as input for further development of the ideas.

Macro analysis: The gravity model

What is the gravity model? Consider for example the travelling between residence and work, constituting the nodes in a network, where the distances between the nodes are given. The distances will influence the preferences for the locations, and therefore the travelling patterns between the nodes. Formalizing this we have a set I of ”departure nodes” and a set J of ”arrival nodes” and a distance function d(i,j) defined for all pairs (i,j) in I x J. The gravity model then writes the probability of ”travelling” from i to j as

$$P(i, j) = a_i \cdot b_j \cdot e^{-c\, d(i,j)}$$

where c is a coefficient expressing the sensitivity to distance. Sufficient assumptions on the marginals of P(i,j) leads to a unique solution of the gravity equation in terms of the coefficients a_i and b_j. These coefficients may be calculated by an easily programmed and fast converging iterative process. In practice the restrictions may be derived from observed travelling patterns. It is said that this model is implicit in software used by most public transportation departments

Early on this was just an attractive model to represent data, but later it turned out that assumptions on ”efficient population behaviour” lead to the conclusion that the travelling probabilities could be expressed this way. The expression can be obtained under different assumptions, in an equilibrium sense, both with utility analysis and maximum entropy considerations. In the general theory it is more convenient to talk in terms of costs rather than distance, and the theory allows different types of costs (in the wide sense), e.g. direct travelling cost and travelling time. The only difference is that this requires two constant terms in the exponent, each having a c-coefficient that can be estimated from data.

The analogy to the above for allocating doctors to patients, is that we have four types of patients, males and females preferring doctor of the same sex or not, i.e. four ”departure nodes”, while we have two ”arrival nodes” male and female doctors. Formally we can write I = {ff, fm, mf, mm}, where the first letter is the gender of the patient and the second letter is the preference for gender of assigned doctor. Furthermore J = {F, M} with letters representing the gender of the assigned doctor. The analogy to distance is the felt nuisance of being assigned contrary to preference. As distance function one may simply use d(i,j) = 0 or 1 according to whether the second letter in the ”departure node” corresponds to or deviates from ”the arrival node” j, for example d(mf,F) = 0 while d(mk,M) = 1. We have then implicitly assumed that the nuisance of having a doctor of ”unwanted sex” is the same for men as well as for women. If we believe that the women feel this nuisance stronger, we may set d(kk,M) = 2 rather than 1. A problem for practical application in the context of allocating patients to doctors, may of course be the specification of the differences and the strengths of the preferences.

The model has several possible applications

- estimate the parameters of the model from real data
- gain insight by considering hypothetical situations by specification of parameters

We will here consider the latter opportunity. Although the absolute figures in a solution should not be taken as definite at the outset, it is possible to make comparisons under different assumptions, which in some cases lead to approximately the same result. Moreover, we can study what is most likely to affect changes in the allocation pattern, e.g. how the fraction of females assigned to female doctors changes as the fraction of female doctors increases.

Even if we say that a certain fraction of patients would like to have doctor of a certain gender, the degree of nuisance by contrary allocation varies, both among the sexes and absolutely. Possible larger nuisance among the females than males, of having a doctor of the opposite gender contrary to preference, may be adjusted by enlarging the distance for this female group. Generally the strength of nuisance may be adjusted by a suitable weighing of the distance structure.

We will now illustrate the gravity model in this context by numerical examples. We stress that the choice of numbers was solely to illustrate the main features of the model. However, the qualitative results obtained, were partly a surprise to the profession, and initiated some rethinking. Throughout in our illustrations we assume a population with equal number of male and female patients. As our point of departure we take a situation where 70% of the female patients would like a female doctor the most, if available, while 30% of them would like a male doctor the most. Among the male patients we assume an even distribution of 50% for male doctor and 50% for female doctor. In the gravity model we specify the ”distance” equal to 0 for allocations in concordance with the preferences and ”distance” equal to 1 for discordance between preference and allocation.

First we will look at how the fractions of females vary among doctors of each gender for increasing fraction of female doctors. They are given in Table 1 for preference strength c=1.

Fraction F-doctors	10%	20%	30%	40%	50%	60%	70%	80%	90%
Fraction f among F	0.564	0.561	0.557	0.552	0.546	0.538	0.529	0.519	0.509
Fraction f among M	0.493	0.485	0.476	0.465	0.454	0.443	0.433	0.424	0.416

Table 1. Fraction of female patients among female and male doctors for increasing fraction of available female doctors.

We see that the fraction of female patients is decreasing as the fraction of female doctors increases. This is so for doctors of both genders. With few female doctors, the fraction of female patients among them will be approximately 56%, and only decrease slowly until there is a more even number of doctors of the two genders, thereafter the fraction of female patients decreases more rapidly towards 50% of female patients among them. With few female doctors we see that the fraction of female patients among the male doctors is slightly below 50% and have a similar decreasing pattern. We also see that if the fractions of female and male doctors are equal, the fraction of female patients among the female and male doctors will be 54.6% and 45.4% respectively.

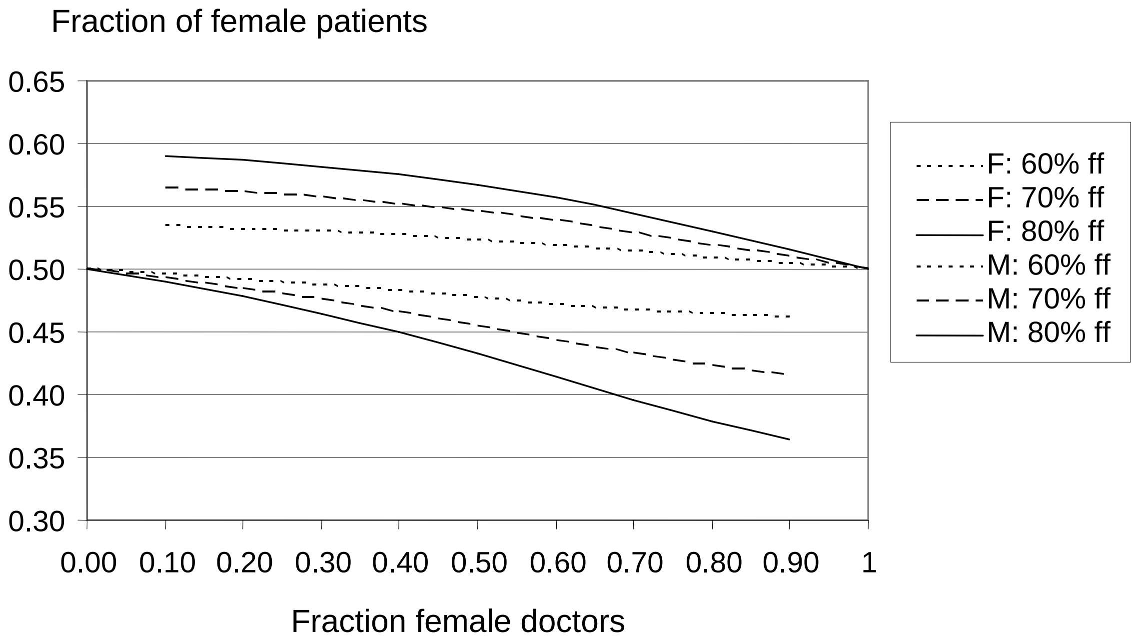

Figure 1. Fraction of female patients as function of fraction female doctors among female and male doctors for fraction females preferring female doctors (60%, 70%, 80%) all with male preferences 50%-50% (preference strength = 1)

Figure 1 shows for moderate weights (c=1) the fraction of female patients among female and male doctors respectively as function of the fraction of female doctors in three situations, respectively where 60% (dotted line), 70% (broken line) and 80% (solid line) of the female patients prefer female doctors and male preferences in all three situations are 50-50. The three upper curves are for female doctors, while the three lower ones are for male doctors. We see that all curves are monotonically decreasing, and that the decrease is slow until the fraction of female doctors is slightly above 50%, thereafter the decrease is more rapid.

We have performed similar computations for median/heavy weights (c=2), and for heavy weights (c=3) for the same distance structure. Results for the case of 70% females preferring a female doctor and the males are 50-50 are given in Table 2.

	Fraction of f among F			Fraction of f among M		
Fraction F-doctors	10%	30%	50%	10%	30%	50%
Preference weight c=1	0.564	0.557	0.546	0.593	0.476	0.454
Preference weight c=2	0.580	0.579	0.573	0.491	0.466	0.427
Preference weight c=3	0.583	0.583	0.581	0.491	0.465	0.419

Table 2. Influence of preference weights on the fraction of female patients among female and male doctors.

The corresponding graphs (not given here) for moderate weights (c=2) are similar to c=1, with the two bundles of curves further apart, but the difference is surprisingly not that much. Heavier preference weights (c=3) gives results that do not deviate much from c=2, and less so for small fraction of female doctors, where the fraction of female patients among female doctors is just increased from 56% to 58%, and eventually starts to decrease more rapid above 50% female doctors. It may come as a surprise that the changes towards a larger fraction of female patients among female doctors are moderate as the preference strength increases. This may be interpreted as "limits to change" in a system where the felt nuisance for mismatch is the same for both gender, even if there is a strong majority of females feeling nuisance of a mismatch. However, if we change the preference structure itself, we may obtain large differences. We will return to this later.

Before turning to our next question, we note that the fractions considered above are tied together by a simple formula: Let FF and FM be the fraction of female patients on the list of female and male doctors respectively, and let a be the fraction of female doctors. The specification of a, c and d determines FF and FM, and implicitly the corresponding fraction of male patients among the female and male doctors respectively as 1-FF and 1-FM. However, FF and FM are also tied together, since we in a population of even number of female and male patients have $a \cdot FF + (1-a) \cdot FM = \frac{1}{2}$, so that $FM = (1/2 - a \cdot FF)/(1-a)$.

Next we will look at the fraction of patients not allocated according to preferences. For short, we name these patients mismatched. We may study the fraction of mismatched patients among each gender of doctors and the fraction of mismatched patients of each gender of patients as function of the fraction of female doctors. Computations using the gravity model for preference strength c=1 gave the results in Table 3 and are illustrated in Figure 2 and 3.

Fraction F-doctors	10%	20%	30%	40%	50%	60%	70%	80%	90%
Mismatched Total	0.519	0.443	0.376	0.320	0.281	0.262	0.266	0.293	0.339
Mismatched f-patients	0.601	0.508	0.423	0.349	0.290	0.251	0.234	0.239	0.263
Mismatched m-patients	0.436	0.378	0.329	0.292	0.271	0.272	0.298	0.347	0.416
Mismatched at F-doctor	0.094	0.108	0.127	0.150	0.181	0.218	0.261	0.308	0.355
Mismatched at M-doctor	0.566	0.527	0.483	0.433	0.381	0.327	0.277	0.233	0.197

Table 3. Fraction of mismatched patients among each gender of patients and the fraction of mismatched patients of each gender of doctors as function of the fraction of female doctors.

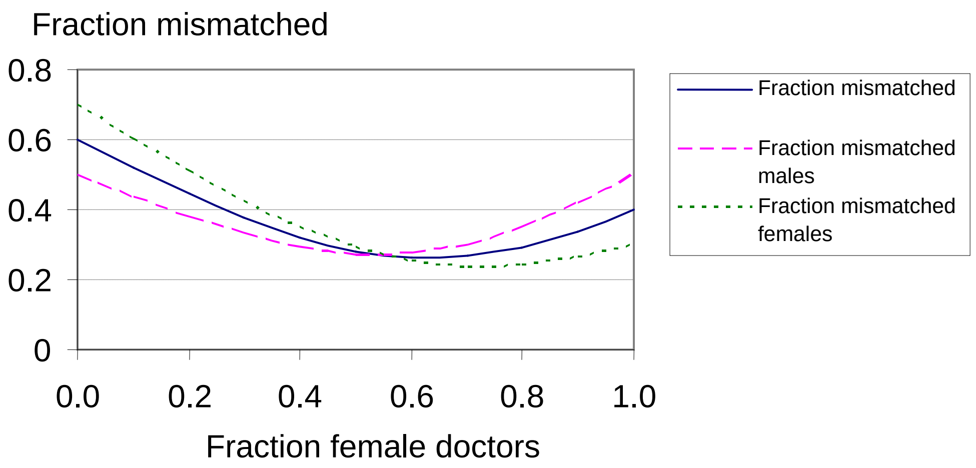

Figure 2. Fraction mismatched patients as function of fraction female doctors (Case 70% ff-50% mm, preference strength c=1)

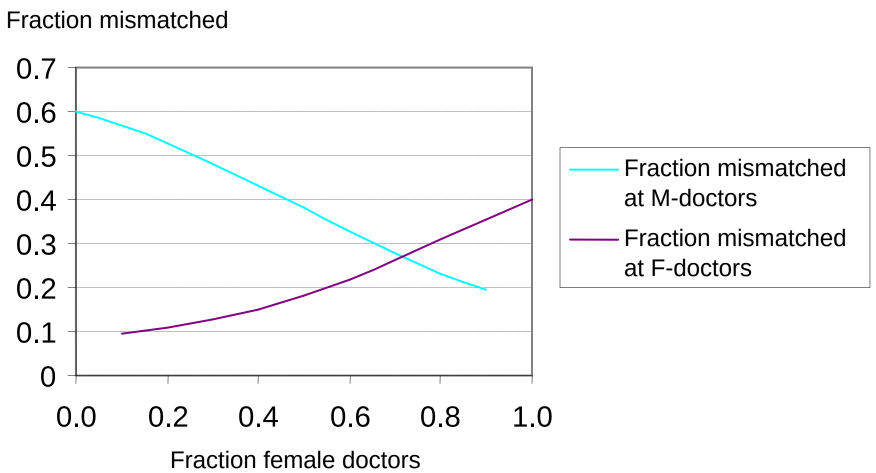

Figure 3. Fraction mismatched at female and male doctors (Case 70% ff-50% mm, preference strength c=1)

We see that the fraction of total mismatched is decreasing as the fraction of female doctors increases up to about the level where they are capable of accommodate the majority of female patients preferring a female doctor. Then the fraction of mismatched is increasing, since from then on more men are mismatched. A similar pattern is seen for both the female and male patients separately, except for the fact that the reversal for the males occurs at about 50%, as expected. Furthermore we see that the fraction mismatched at doctors of a give gender starts out low for the female doctors when they are few, and increases throughout as they become more abundant, ending with the situation where many of their patients are mismatched males. For male doctors the pattern is the opposite, starting out with a majority of male doctors with a large share of mismatched female patients, and ending up with few male doctors with mostly patients according to their preference.

Our calculations on macro level are based on the assumption that the patient lists of all doctors are filled up, which means that adjustments to increase the total satisfaction at the micro level can be achieved by exchanging patients only. Some doctors will in practice of course have vacancies on their lists, and will be able to accept new patients according to their stated preference. It is not obvious how this should be implemented in the model. One possibility is to define a fifth fictitious patient category, representing an empty list position. If we are indifferent whether this happens to a male as female doctor, we can represent this with a zero in the distance function. We have performed calculations according to this and with different fractions for total vacancy regardless of gender, and it turned out that the distribution of patient gender on the lists of both male and female doctors changed surprisingly little. However, the under-represented gender of doctors, in view of the preferences, will of course experience less vacancy on their lists.

Fraction F-doctor	10%	20%	30%	40%	50%	60%	70%	80%	90%
Vacant among F (10%)	0.064	0.070	0.076	0.083	0.090	0.095	0.100	0.102	0.102
Vacant among M (10%)	0.104	0.107	0.110	0.111	0.110	0.107	0.101	0.094	0.086
Vacant among F (20%)	0.134	0.146	0.158	0.171	0.182	0.192	0.199	0.203	0.203
Vacant among M (20%)	0.207	0.214	0.218	0.220	0.218	0.212	0.202	0.189	0.175

Table 4. Fraction of vacancies for doctors of each gender for 10% and 20% total vacancy.

Table 4 shows the fraction of vacancies for doctors of each gender for 10% and 20% total vacancy respectively for the situation above, where 70% of the female patients want a female doctor, and the male patients were distributed even.

In the situation with 30% female doctors and 10% total vacancy, that we have 7.6.% vacancy among the female doctors and 11.0% vacancy among the male. With total vacancy of 20%, the numbers are respectively 15.8% and 21.8%. It may be of interest to regulate this such that the vacancies of both genders are about the same. For the case of lack of female doctors and where the males get the larger fraction of vacancies, we may change the "distance" between the categories "empty list position" and female doctor from zero to a positive number.

With vacancies, the quantities are tied together as follows: Let t be the fraction of total vacancies, and TF and TM be the fraction of empty list positions among female and male doctors respectively. Let as before a be the fraction of female doctors, and FF and FM be the fraction of female patients among female and male doctors respectively, but now taken to be among the non-vacant entries. The specification of t, a, c and d determines TF, TM, FF and FM, and implicitly the corresponding fractions of male patients among the female and male doctors as 1-FF and 1-FM respectively. TF, TM, FF and FM are, however, linked together since we in a population of equal number of men and women have that $a\cdot(1-TK)\cdot FF + (1-a)\cdot(1-TM)\cdot FM = \frac{1}{2}\cdot(1-t)$. Given three of the quantities with capital letters, the forth is determined as well.

In the examples above we have assumed that more women (70%) than men (50%) feel some nuisance with a doctor of the opposite sex, while the felt nuisance is about equal for both genders. We have seen that this difference between the genders do not put appreciable pressure towards skew distribution of patient gender for the doctors. Only when this nuisance is felt stronger among women than men can we expect larger changes. If we again take the situation above, where 70% of the female patients will prefer a female doctor, and the male patients were distributed evenly, and where as before d(mm, F) =1, but take d(ff,M) = 1, 2, and 3 respectively, we get the results of Table 5 for the cases of 0%, 10% and 20% total vacancy and 30% female doctors (note that the fractions are with respect to the non-vacant entries)

Fraction of vacancies	0%			10%			20%		
Relative preference strength	1	2	3	1	2	3	1	2	3
F-doctors: Fraction f-patients	0.557	0.684	0.794	0.557	0.679	0.781	0.556	0.673	0.767
F-doctors: Fraction vacant	0.000	0.000	0.000	0.076	0.053	0.035	0.158	0.115	0.081
M-doctors: Fraction f-patients	0.476	0.421	0.374	0.475	0.418	0.367	0.474	0.414	0.359
M-doctors: Fraction vacant	0.000	0.000	0.000	0.110	0.120	0.128	0.218	0.236	0.251

Table 5. Fraction of female patients and vacancies for doctors of each gender for 0%. 10% and 20% total vacancy.

We see how the increased relative preference strength between females and males forces the fraction of females among female doctors to increase, while it is reduced among the males. We see, with increased fraction of vacancies, that the change caused by increased felt nuisance will be somewhat more rapid with respect to the move away from male doctors, while the increase among the female ones is slower.

The fraction within each patient gender which does not get their primary wish fulfilled is also of interest and may be tabulated. However, some information can be read from the table above, when we know that at the outset 70% of the female patients preferred a female doctor and just 50% of the male patients. For instance if we look at the situation in the rightmost column of Table 5, we have 76.7% female patients among female doctors, and 35.9% female patients among male doctors. However, in both groups we have some who have fulfilled their primary wish, and some not. We have to increase the relative preference strength considerably in order for the fractions to approach 100% among female doctors and 0% among the male ones. The latter is in general not possible, but in the case of 20% total vacancy, we may in principle absorb all females according to their primary wish.

Micro analysis: The benefit efficiency model

In the examples above we have studied the problem at the macro level, with a large and undefined number of patients and doctors, where the focus quantity is fractions. We have limited ourselves to situation with sufficient capacity to cover the demand for a doctor of some gender. In reality a limited number of doctors are available in the neighbourhood. Some of them may be fully booked, and we can imagine situations with waiting lists. We may also have additional characteristics separating both the patients and the doctors. Recent theory based on the idea of ”efficient behaviour” offers an opportunity to analyze various situations on the micro level, see Jørnsten & Ubøe (2005) for the general theory and Ubøe & Lillestøl (2006) for theory in the current context.

We will here give some qualitative results coming out of this theory, see the appendix for a brief account of the main result on which the computations are based. As before we look at the four patient categories mm, mf, fm, ff. Suppose that the categories mm and ff have moderate preferences for a doctor of the same gender, while fm and mf feel some, but not particularly strong nuisance by a doctor of the opposite gender. However, all categories feel a stronger nuisance by not being on the patient list of any doctor, i.e. being on a waiting list. For a start we assume that doctors are not penalized by not having their patient lists filled up. We will now find it more convenient to express the preferences in terms of ”utility”, taking both positive and negative values. A possible representation of the described situation by utility numbers is given in Table 6.

Group	mm-p	mf-p	fm-p	ff-p	mm-w	mf-w	fm-w	ff-w	vacancy
M-doctor	1	0	0	-1	-2	-2	-2	-2	0
F-doctor	-1	0	0	1	-2	-2	-2	-2	0

Table 6. Utility numbers for assignments to doctor patient/waiting list and vacancy.

We will here consider situations with different number of available doctors of each gender. Assume first that all doctors have the same list length, and that we have the same number of patients in each of the four categories. It turns out that in the case of deficit of doctors, the expected number of patients in each of the four patient categories will differ between male and female doctors, but be the same for all doctors of the same gender. However, the expected number and the distribution of patient categories on the waiting list will be the same for all doctors irrespective of gender. If we let the list lengths vary among doctors, but in a way so that the total numbers of patient entries are the same as above within each gender, it turns out that the fractions in each of the four patient categories are unchanged, while the number and distribution on the waiting list are unchanged. The relative distribution among the four patient categories on the patient list is therefore common for doctors of the same gender, so that the expected number is given by multiplication of the list length. Consider, as before, a situation with equal number of patients in each of the four patient groups, but so few that there is a deficit of patients. Now it turns out that all doctors of the same gender have the same share of vacancy on their lists, but different between the genders. Let us look at some specific examples:

Example

Given a population of 16 000 persons in an area served by 7 doctors, 4 male and 3 female, all with list lengths 2000, represented by the vector (2000,2000,2000,2000,*2000,2000,2000*). Taking 4000 patients in each of the four patient categories, the expected distributions of the 16 000 patients on the 14 000 patient entries or the waiting lists are given in Table 7, for each of the male and female doctors (table sums deviating from marginals are due to rounding errors):

Group	mm-p	mf-p	fm-p	ff-p	mm-w	mf-w	fm-w	ff-w	vacancy
M-doctor	835	509	509	147	51	84	84	67	1
F-doctor	102	458	458	981	51	84	84	67	1

Table 7. Expected distribution of patients on patient list and waiting list for doctors of each gender (Doctors: 4 male, 3 female, Patients: 16 000, Entries: 14 000).

If we introduce varying list lengths (1000,2000,2000,3000,*1000,2000,3000*), i.e. the total number of entries for male and female doctors are as above, the number of patients for doctor 2, 3 and 6 are unchanged, while doctor 1 and 5 have cut their number of patients in half, and doctor 3 and 7 get their numbers multiplied by the factor 1.5. The waiting list numbers are the same for all seven doctors.

Now suppose we have 12 000 patients served by the 7 doctors, having 14 000 entries, but with varying list lengths as above, so that we have 14.3% total vacancy. Again assuming equal number of patients in the four categories, this time 3000, the model gives the allocation in Table 8, where we have lumped together the four waiting list groups, since they are all empty.

We see that all male doctors have 14.9% expected vacancy, while the female doctors have 13.5% expected vacancy.

Group	mm-p	mf-p	fm-p	ff-p	Waiting	vacancy
M-doctor 1 (1000)	343	223	223	62	0	149
M-doctor 2 (2000)	686	446	446	124	0	298
M-doctor 3 (2000)	686	446	446	124	0	298
M-doctor 4 (3000)	1029	669	669	186	0	447
F-doctor 1 (1000)	42	203	203	417	0	135
F-doctor 2 (2000)	84	405	405	834	0	271
F-doctor 3 (3000)	127	608	608	1251	0	406

Table 8. Expected distribution of patients and vacancies for each of 7 doctors with varying list lengths (Patients: 12 000, Entries: 14 000).

We are now ready to study what happens when the preferences are changed. First, if we change all disutilities for being on the waiting list from -2 to -3 (or even -5), there will be no change in the table, telling that the message is picked up already at -2 for the case of lots of vacancies and no loss of vacancy. If we introduce a loss for the doctor for vacancy, e.g. replace 0 by -1, we also get the same result as above. To create a difference, we must have a difference between genders with respect to felt loss. If the female doctors feel this stronger than the male doctors, they will have less vacancies. Some of this is of course fairly obvious, but shows that the model gives meaningful results throughout. For instance, if we change the losses from -1 to -2 for the female doctors, the expected vacancies among female doctors are reduced to 7.6%, while they among the male doctors are increased to 19.3.%. By taking -5 instead, the vacancies are changed to about 0.6% for female doctors and 24.6% for male doctors. One may ponder on the kind of administrative means required for such a transfer of welfare.

An interesting question is whether there are preference structures, where some doctors have vacancy, while there is a lack of doctors in the system as a whole. There are! One example is when the ff-group prefers to be on the waiting list of a female doctor, instead of being assigned to a male doctor.

In the discussion and examples for the micro-model, we have assumed an equal number of patients in the four patient categories. In our main macro-example of the preceding section we had 70% of the female patients favouring a doctor of the same gender, but 50% of the males. However, both models are general and accommodate any configuration. We can also extend the macro-model to include waiting lists.

In order to compare the results from the micro-model above with the macro-model, we may put heavy disutilities on waiting, so that this is ruled out. We will here consider cases where the number of entries exactly matches the number of patients, and put heavy disutility to wipe out vacancy as well. Consider therefore 14 000 patients to be assigned to 7 doctors, each with 2000 available entries totalling 14 000, and where 3 out of 7 doctors are female i.e. 42.8%. Assume first the same utilities for both gender, as we did for the macro-model (there measured by “distance”), and take the case of 70% of the female patients favouring a doctor of the same gender and 50% of the males. The distribution of the four categories (mm, mf, fm, ff) is (3500, 3500, 2100, 4900) and we take utilities -2 on waiting list and -2 for vacancy. We obtain the results given in Table 9.

Group	mm-p	mf-p	fm-p	ff-p	waiting	vacancy
M-doctors	725	348	435	487	5	5
F-doctors	198	700	119	980	5	4

Table 9. Expected distribution of patients for each of 7 doctors with equal list lengths 2000 and patients in categories (mm, mf, fm, ff) = (3500, 3500, 2100, 4900), for comparison with macro example.

By changing the disutility for waiting and vacancy from -2 to -4 there will be none waiting and no vacancies. This gives the fraction of female doctors assigned to male doctors 46.2% and to female doctors 55.1%, which is close to the numbers obtained for the macro-model.

On the other hand, if we take the situation of stronger affection among the females mentioned above, we may specify the utilities as in Table 10.

Group	mm-p	mf-p	fm-p	ff-p	mm-w	mf-w	fm-w	ff-w	vacancy
M-doctor	1	0	0	-2	-4	-4	-4	-4	-4
F-doctor	-1	0	0	2	-4	-4	-4	-4	-4

Table 10. Utilities to reflect stronger dislike of vacancy and patients on waiting list.

The results for the micro-model and the macro-model coincides, and the computation gave the results in Table 11, for some group fractions of females preferring female doctors.

% ff vs fm	10-90	20-80	30-70	40-60	50-50
f among M-doctors	0.303	0.273	0.240	0.206	0.180
f among F-doctors	0.763	0.802	0.846	0.890	0.923

Table 11. Computations for comparison with the macro model.

At this point we are reminded that we can obtain major differences for the macro-model as well, it all depends on differences in the preference structure of the gender, and not so much on the number in each category and the absolute preference strength. In order to illustrate the sensitivity to change we take a macro example (with no vacancies and no waiting list), with equal number of patients in the four patient categories and utility structure as in Table 12.

Group	mm-p	mf-p	fm-p	ff-p
M-doctor	1	0	0	-x
F-doctor	-1	0	0	x

Table 12. Utility structure to explore the sensitivity to change in female preferences for female doctor.

In Figure 4 we plot, as function of x of Table 12, the fraction of female patients among female doctors (top three curves) and among male doctors (bottom three curves) for three fractions of female doctors 20%, 30% and 40% (in this order from top). Equal preferences correspond to x=1, and the most relevant part of the curves is to the right of this. We see that the disparity of utility affects the female doctors more than the male doctors in this region, and that there is not much change beyond x=3.

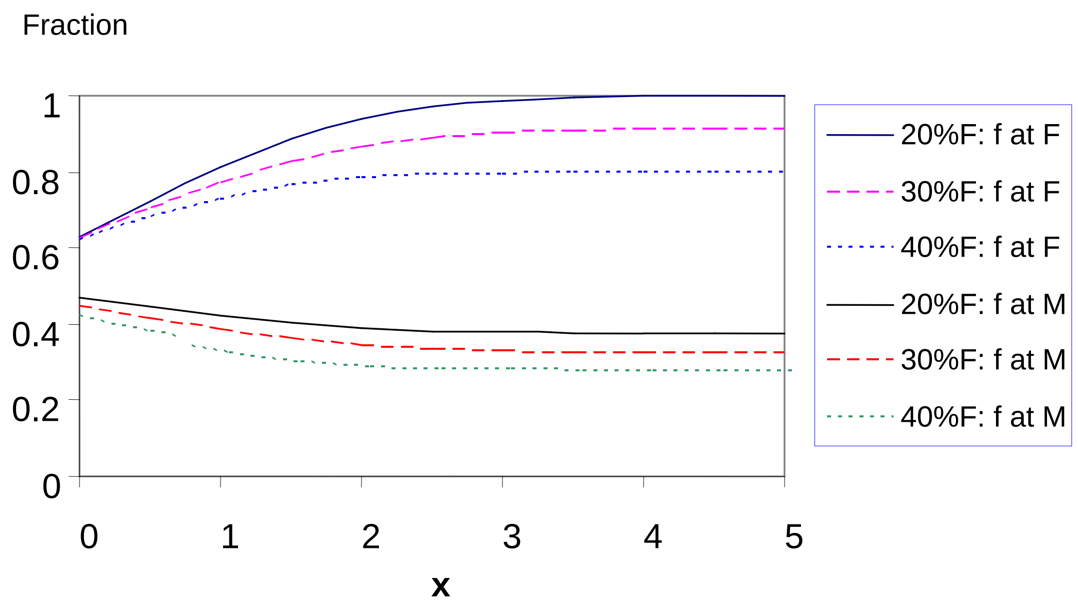

Figure 4. Fraction of female patients among female and male doctors as function of disparate utility parameter x for different fractions of female doctors (20%, 30%, 40%).

Further research: The inverse problem

The theory and examples above have given valuable insights to qualitative issues, e.g. what affects the level and changes of allocations. As such it may be of value to decision makers at the general policy level. For applications in the more local setting, we need to reveal the preferences of patients in some sense, and be able to translate this into utilities in the sense used above. This may be done by a suitably designed questionnaire, and is a research challenge in itself. Another approach would be to use observed allocations, and from this infer the utility structure. This may be called the inverse problem, and is also a research challenge in itself, mainly because a given allocation does not uniquely determine the utilities. In technical terms, we have an identification problem. There are several solutions to this, now under investigation.

The Norwegian system by which every inhabitant has the opportunity of having a designated general practitioner was introduced in the year 2001 and is monitored by the authorities. Detailed information on availability of doctors and list composition and vacancies are available, and may serve as a laboratory for both research and applications. To give an idea of the kind of data available: At the end of the year 2004 the fraction of doctors with open lists was about 55%, 44% among female doctors and 59% among males in the country at large, while the corresponding numbers in the capital Oslo were 76%, 61% and 85%. Such numbers are available also regionally and locally. Movements over time are noticeable, and may indicate that the preference for a doctor of the same gender have increased since the system was introduced. However, part of this may be due to a tendency for most newborn to be assigned to the doctor of their mother. This raises an additional challenge, both for revealing real preferences and interpreting allocation data. It is possible that the solution to this is to combine the two approaches mentioned above.

Acknowledgement

We want to thank Kurt Jørnsten who pointed out the possibility to model patient allocations by gravity models.

Appendix

Assume p patient categories and q doctors (named by j=1,2,…,q) of r different types (named by k=1,2,…,r). Each doctor has a patient list with a given number of available entries and in addition a waiting list. To each doctor we associate 2p+1 categories, first the p patient categories for registered patients (named by i=1,2,…,p), then the same p categories for the patients on the waiting list (named in the same order by i=p+1, p+2,…,2p), and finally the category i=2p+1 for registration of possible vacant entries. Let for k=1,2,…,r

u(i,k) = Utility for patients of type i assigned to a doctor of type k (for i=1,2,…,p)
u(i,k) = Utility for patients of type i on the waiting list of a doctor of type k (for i=p+1,…,2p)
u(i,k) = (Dis)utility per vacant entry of a doctor of type k (for i=2p+1)

An assignment of all the patients to doctors and waiting lists, as well as vacancies, is judged by their total utility obtained by adding utilities over all patients. Let P(i,j,k) denote the probability of a patient/vacancy of type i (i=1,2,…,2p+1) belonging to doctor no. j (j=1,2,…,q) who is of type k. The assumption of ”efficient system behaviour” amounts to saying that for two allocations, the one with the higher total utility is more probable. From this assumption it follows that the allocation probabilities can be written on the following form, see Ubøe & Lillestøl (2006):

$$P(i,j,k) = \begin{cases} a_i \cdot b_j \cdot e^{cu(i,k)} & i = 1,2,\ldots.,p \\ a_{i-p} \cdot e^{cu(i,k)} & i = s+1,2,\ldots.,2p \\ b_j \cdot e^{cu(i,k)} & i = 2p+1 \end{cases}$$

where the a’s and b’s are coefficients determined by the restrictions in the situation, among others the list length of each doctor, and typically also that it is an equal number of patients of each gender to be assigned. As for the gravity model c is the weight put on the differences in the assigned utilities. Note that a constant added to all utility numbers have no effect, since this is absorbed in the multiplicative constants a and b. Note also that if we multiply all utilities with the same positive number, we get the same solution, since the c-coefficient becomes rescaled as well, while the products are the same. Thus c may be taken as “numeraire”.

Once we have specified c and the utilities u(i,k) we have an equation system that can be uniquely solved numerically, for instance by extensions of the Bregman balancing algorithm, see Bregman (1967) and Jørnsten & Ubøe (2005).

References

Bregman l. M. (1967) The relaxation method of finding the common point of convex sets and its applications to the solution of problems in convex programming, USSR Computational Mathematics and Mathematical Physics, vol.7, 200-217.

Erlander S. & Smith T.E. (1990) General representation theorems for efficient population behaviour, Applied Mathematics and Computation, 36, 173-217.

Jørnsten K, Thorsen I. & Ubøe J. (2004) Replication/prediction problems in the journey to work, Environment and Planning A, vol.36, 347-364.

Jørnsten, K. & Ubøe, J. (2005) Efficient statistical equilibria in markets. Discussion paper 2005/2 , The Norwegian School of Economics and Business Administration.

Smith T.E. (1978) A cost-efficiency principle for spatial behaviour, Regional Science and Urban Economics, 8, 313-337.

Ubøe J. (2004) Aggregation of gravity models for journeys to work, Environment and Planning A, vol. 36, 715-729.

Ubøe J. & Lillestøl J. (2006) Benefit efficient statistical distributions on patient lists,. Discussion paper 2006/3, The Norwegian School of Economics and Business Administration. To appear in Journal of Health Economics

Zipf G.K. (1949) Human behaviour and the principle of least effort, Addison-Wesley, Reading, MA.