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The Propagation of Financial Extremes: An Application to Subprime Market Spillovers

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The Propagation of Financial Extremes: An Application to Subprime Market Spillovers

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Abstract

What drives extreme and rare economic events? Motivated by recent theory, and events in US subprime markets, we begin to open the black box of extremes. Specifically, we build a taxonomy of extremes, then extend standard economic analysis of extreme risk. First, we model the potentially relevant dimensions of dynamics and endogeneity. In characterizing individuals' endogenous propagation of extremes, we relate the latter to public goods. Second, using over a century of daily stock price data, we construct empirical probabilities of extremes. We document that extremes are relatively frequent and persistent. We find evidence that extremes are endogenous, raising the possibility that control of extremes is a public good.

Keywords: Extreme event; Subprime Market; Dynamics; Endogeneity; Public Good

JEL Classification: C10 E44, E51, H23, H41

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1 Introduction and Motivation

> For whoever knows the ways of Nature will more easily notice her deviations; and ... whoever knows her deviations will more accurately describe her ways.
> F. Bacon: *New Organum.*

In the spring and summer of 2007, the aftershock from the subprime market, a relatively small part of US financial markets, has reached over to touch hedge funds and international markets. In the US, credit spreads have widened ominously, even for safer debt, and the housing market reached record breaking levels. For example, as shown in Figure 1, the percentage change in the Case-Shiller index reached both its historical (20-year) maximum of nearly 16% in 2005 and its historical minimum of -4.52% in the third quarter of 2007. In Britain the interbank rate has reached its highest level in 9 years, as shown in Figure 2. Modern economies are repeatedly subject to such financial extremes, sometimes contemporaneously or in rapid succession, as in the contagion episodes in East Asia during the late 1990s. Extreme events often seem unpredictable, but are they? This paper begins to open the black box, and proposes a positive theory of extremes, based on externalities. By way of motivation, let us discuss two current puzzles.

A mortgage market puzzle: Recent events in the US subprime mortgage market are difficult to explain using standard economic analysis. The years leading up to 2007 featured a large demand for housing loans by US households of various credit levels. At the same time, lenders offered a large supply of low rate loans to prospective homeowners with extremely poor credit history, and high likelihood of default. According to standard information theory, the housing loan market features moral hazard and adverse selection, because borrowers know more about their ability to repay than do lenders. A standard solution to such information asymmetry involves credit rationing by lenders, or signalling quality by prospective borrowers (see Stiglitz and Weiss (1981) and Riley (2001)). However, during the period from 2000 to 2007, the opposite happened. As mentioned above, availability of credit to low credit history individuals increased, and potential borrowers did not have to signal.[1] Despite improvements in risk management by banks and regulatory authorities, such issues have recurred in recent years, in many developed economies. For example, a similar situation existed in Japan's Jusen loans, and in Norway during the 1990s. This

[1] For example, borrowers without collateral were encouraged to apply for loans, and therefore did not have to signal quality.

puzzle suggests that there might be other factors at work in addition to information issues, especially during extreme events.

A hedge fund spillover puzzle: A second puzzle relates to the recent hedge fund debacles. Two issues convey the essence of this puzzle. The first issue concerns spillovers from the subprime market to hedge funds and other domestic or international investors. There have been a number of instances, so we only consider one of the more outstanding. In July and August of 2007 hedge funds suffered such severe losses that Goldman Sachs had to infuse US$3 billion into one of its funds, Global Equity Opportunities. This fund lost 30 per cent of its value in the week between August 3 and August 10. This seems to be the first time that Goldman Sachs has assisted a hedge fund, especially in that magnitude. A major reason cited for the severe hedge fund losses was that the extremes that occurred in markets were '25 standard deviation' events (New York Times, August 13, 2007). These incidents are puzzling because hedge funds did not seem directly exposed to heavy enough risk to warrant such drops in value.[2] The second issue concerns extreme surprises. Most hedge funds and investment banks have risk management systems that are stress tested against extreme market events such as terrorism risk, banking crises, and interest rate changes. So what sort of event could surprise such respected hedge funds enough to lose as much as one-third of their value? A potential answer is that our approach to understanding "surprise" extreme events is incomplete. One source of incompleteness is that both information economics and current risk management are generally silent about time variation in the probability of extremes. Another issue is that they do not always account for endogenous spillover effects from one economic sector to others (such as mortgage market effects on hedge funds), especially in the face of extreme events.

A possible solution to both puzzles is to extend existing theory to include explicit, positive analysis of extremes. Existing theory acknowledges that individual agents' incentives or behavior can affect individual outcomes, for example, in insurance markets. This framework is usually restricted to individual agents or sectors, and typically requires asymmetric information between borrowers and lenders. The current issues, however, potentially affect numerous sectors and regions. Moreover, especially in the case of subprime mortgages, it is difficult to argue that lenders were oblivious to asymmetric information issues, and did not understand the potential for default when supplying loans to borrowers with poor credit history or no collateral. Therefore, current experience suggests that an extension of

[2]Moreover, the spillover effect on international markets was large enough to prompt unscheduled central bank interest rate cuts in the US and Europe.

existing approaches may be valuable, especially for analyzing extreme outcomes. In this paper we attempt to provide such an extension. In particular, we will illustrate that under some conditions, aggregate spillover effects can happen even in the absence of asymmetric information. A graphical depiction of our approach is in Table 1. This table shows that our view of endogenous probability is similar to that of moral hazard. The only difference is that we consider broader settings, where there may be spillovers and general information structures.

Discussions of extreme economic events often assume that extremes are generated exogenously by nature, and have a constant probability of occurrence.[3] But is the likelihood of extreme and rare events affected, at times, by our behavior? And do we sometimes observe spikes in the frequency of extremes? The answer to both questions is yes. Dynamic, endogenous extremes occur in economics and in nature, including the effect of human activity on both the likelihood of extreme financial events, and extreme climate changes.[4] Importantly, when human activity endogenously increases the likelihood of extremes, they may become less rare. In this paper, we explore a possible explanation for endogenous extremes, namely, externality effects. Externalities occur when one agent's actions directly affect the environment of other agents. Financial crises and extremes have externality features, since they affect many individuals in the national or global financial system, even though often precipitated by a small number of individuals. It is well known that externalities cause inefficiency of the price system.[5] Consequently, if extreme events are due to externalities, society may not pay the appropriate price for the extremes that it generates.

How does this formulation of extreme externalities help us? It does so in two ways. First, it allows us understand the origin of some extremes (the endogenous ones), thereby giving us insight into which we can plausibly try to avert. Second, it gives banks and regulatory authorities an additional set of tools from public finance–subsidies, property rights, and so on–that may help to address extreme events before and during their occurrence.

[3]See, for example, Barro (2006) and Friedman and Laibson (1989).

[4]See the cover story of Time, March 30, 2007; and Stern (2007); and Grossman (1988). The Economist's June 29 issue discusses climate change, where extremes become more normal over time.

[5]For textbook expositions of externalities, see Harris (2003), Chapter 9, Mas-Colell, Whinston, and Green (1995) and Varian (1992). For related economic work on aggregate effects of externalities, see Blanchard and Kiyotaki (1987).

1.1 Related Research

Our research is related to existing work on extreme events and liquidity. Regarding extreme and rare events, there are several recent, related papers. Barro (2006) constructs a Lucas (1978) model with rare extreme events. Upon calibrating the model to twentieth century data on extreme events, Barro (2006) finds that it allows him to address the equity premium and riskfree rate puzzles. Weitzman (2007) develops a Bayesian model of asset returns. He discovers that when agents consider the possibility of extremes, there is a reversal of all the major asset pricing puzzles. Chichilnisky and Wu (2006) present a model of endogenous uncertainty where increased financial innovation leads to greater likelihood of default. Chichilnisky (2007) shows that if by axiomatically extending expected utility to account for extreme responses to extreme events, we can overcome decision theory paradoxes, such as those due to Allais (1953) and Ellsberg (1961). Danielsson and Shin (2003) discuss a scenario where unanticipated coordination of agents' behavior leads to an endogenous increase in risk. The research of Bazerman and Watkins (2004) suggests that certain "surprise" events in modern society are predictable, since there may exist sufficient information to know that these events are imminent.[6] Regarding liquidity, Acharya and Schaefer (2005), on their page 7 discuss the notion that liquidity has regimes, which affect the prices of stocks. These authors also suggest that market liquidity and asset correlations are interrelated, due to large asset shocks. In a recent empirical study, Baele, Bekaert, and Inghelbrecht (2007) examine the comovement of stocks and bonds using a VAR approach. They first examine economic variables such as interest rates, inflation and risk aversion, which do not fully explain stock-bond correlations. However, they find that liquidity helps to explain the residual correlation that is unexplained by economic factors. Domowitz, Hansch, and Wang (2005) provide a model where liquidity comovements are determined by order types, and document that liquidity comovements are asymmetric, and much higher during extreme down markets. Herring and Wachter (2005) suggest that liquidity shocks and disaster myopia may play a role in deciding the pricking of real estate lending bubbles.

Our paper is similar to the above papers in that we discuss the importance of extreme events and liquidity in socioeconomic life. However, our paper is different in several ways. First, unlike previous research, we explicitly construct a taxonomy of extremes, then de-

[6] According to Bazerman and Watkins (2004), predictable surprises have six characteristics: leaders know about a problematic issue, which will not go away; the issue worsens over time; the issue is costly to fix now, and benefits would occur later; fixing the issue entails a certain cost, but uncertain reward; addressing the issue changes the status quo; and a small vocal minority benefits from lack of preventive action.

velop a simple model to explain the origins of endogenous extremes. Second, we derive the ”signature” of endogenous extremes, and relate it to liquidity spillovers. Third, we apply the insights from our model to US stock market data, providing evidence on the temporal nature and causes of market extremes. Finally, the model allows us to discuss new policy solutions to extreme events, using a standard public finance toolkit. The rest of the paper is organized in the following manner. Section 2 discusses general information on extreme events and proposes a taxonomy. Section 3 presents a simple, stylized, positive approach to analyzing dynamic, endogenous extremes. Section 4 outlines some policy implications for current financial markets. Section 5 discusses our empirical application, and Section 6 concludes.

2 Nature and Causes of Extremes

Knowing the origin of extremes is evidently valuable for investors and policymakers. In attempting to provide a glimpse of the origin, we now outline our positive approach to analyzing extremes. There are two aspects to this approach.

2.1 Temporal Nature of Extremes

The first aspect concerns dynamic behavior of extremes. In typical economic applications it is often implicitly assumed that the likelihood of extremes is constant over time. This assumption is useful for analytical tractability. Evidently economic and natural systems change and grow over time, which may affect the probability of extremes. There is some evidence that extreme probabilities change over time, such as record-breaking stock market levels in the 1990s, and increased numbers of Atlantic hurricanes since 2000. As shown in Figure 3, both the number of natural disasters and their impact seem to have varied over the past generation.[7] For stress testing in hedge funds, for example, the likelihood of large price deviations is very important to estimate. A mistaken assumption of constant likelihood of extreme price changes is clearly dangerous at many levels, to central bankers

[7] The definition of disaster by EM-DAT is ”A situation or event which overwhelms local capacity, necessitating a request to the national or international level for external assistance, or is recognized as such by a multilateral agency or by at least two sources, such as national, regional or international assistance groups and the media”. The definition of ”affected” encompasses individuals who were killed, injured, homeless or requiring immediate assistance (food, clothing etc) after a disaster. For more details, see page 16 of Below, Guha-Sapir, Hoyois, and Scheuren (2007).

as well as individual and institutional investors. Thus, we might allow the temporal nature of extremes to be static or dynamic. For static extremes, the likelihood of extreme events p_t is constant, and $p_t = p$ for all time periods. Dynamic extremes, by contrast, can be of two varieties, either random or with a discernible dynamic pattern. We shall discuss this in Section 3.

2.2 Causes of Extremes

The second aspect is an understanding of the distinction between exogenous and endogenous extremes, each of which has a different policy response.[8] Exogenous extremes arrive from outside the economic system and are truly acts of nature, from the perspective of the domestic economy. For example, in a crop-based economy, the probability p of extreme changes in crop value could depend on exogenous swings in weather.[9] Since weather is generally unpredictable beyond a few days, and exogenous to an individual farmer, we can represent the probability of extremes as essentially random. In order to obtain bounded probabilities, we may consider a random variable z_t and p_t that are related in the following manner:

$$\begin{cases} z_t = z_{t-1} + \varepsilon_t \\ p_t = \frac{\exp(z_t)}{1+\exp(z_t)}, \end{cases} \tag{1}$$

where $\varepsilon_t \sim i.i.d\ N(0, \gamma)$, with $\gamma > 0$, for example.

Endogenous extremes, by contrast, are generated and perhaps amplified within the economic system, by agents' activity and interaction. This activity persists because extremes have externality-like attributes, and therefore agents may 'over-produce' the amount of extremes in the system. For example, stock market crashes and banking panics may stem from excessive risk taking and borrowing of a segment of the economy (Fisher (1933)), excessive credit creation (Allen and Gale (2000)), and excessive reliance on computer-based trading (Grossman (1988))).[10] Since each agent has an incentive to borrow or risk too much

[8]In practice, there is likely to be a spectrum of extremes, with some being a mixture of exogenous and endogenous. The idea here is to give us tools to assess the dominant influence on extremes.

[9]Other causes of exogenous extremes may include foreign wars, natural catastrophes, and uncertainty about new technology.

[10]The above authors and other related researchers consider some form of extreme event or crisis, but vary in their emphasis on endogeneity. Some model a closed economy or a single sector, others an international setting. Therefore the applications differ, although endogeneity or externality issues are common to all. Our

from the social point of view, competition leads to overproduction of extremes. Hence, the probability of extremes may no longer be random as in (1). We will develop the relevant expression for this latter case in Section 3, after developing a concrete definition of extremes below.

2.3 A Simple Taxonomy

Why do we need definitions of extreme and rare events? The main reason is that extremes occur in many disciplines. Therefore, each has developed its own terminology, which may be incompatible with that of other disciplines. For example, the concept of rare event is used in at least four ways in decision-related sciences. First, in statistics and econometrics, rare refers to a record-breaking phenomenon, one that has never occurred before (de Haan and Sinha (1999)). Second, in political science, it denotes a low probability event with a high impact, which may have occurred before (King and Zeng (2001)). Third, in the theory of risky choice, it refers to a low probability event, which may have occurred before, but not necessarily with a high impact (Hertwig, Barron, Weber, and Erev (2005)). Fourth, in finance the closely related peso problem denotes an infrequent regime that is unobserved but anticipated by economic agents (Evans (1996)).

We therefore need to develop a common language to discuss extreme and rare events, since they arise in a wide variety of settings.[11] Possessing a common language, we can start to think about describing, forecasting and controlling extremes, a task that we begin to pursue in the next section. Based on previous research as well as what we feel to be intuitively appealing aspects of extremes, we now begin to develop a taxonomy. We will first provide a set of heuristic definitions of typical, extreme and rare events, in turn. Given the focus of this paper, we use definitions for quantitative data, such as security returns.

Typical events are those that are normal in some sense, or that we encounter frequently. In previous economic literature, typical events have been conceptualized in two ways. First, they are near the center of the distribution, for example, within 2 or 3 standard deviations. This intuitive definition is useful in the case of the normal distribution, where 3 standard

paper seems to be the first to use this framework explicitly in a general setting, in order to begin developing a positive theory.

[11] The study of extreme and rare events increasingly affects so many disciplines that it has the potential to be considered a field in its own right. Examples of some associated disciplines include astrophysics, chemistry, climatology, decision theory, finance, international relations, insurance, and statistics.

deviations around the mean capture 99.7% of the distribution.[12] Second, another way to think of typical events is in topology. In this sense, an event is typical if it fills up the space of events.[13] Rare or extreme events can be heuristically visualized as the complement of typical events, one in topology, the other in probability. Extreme events are 'far away' from the median, while rare events are 'small' in the set of all events, respectively. Armed with these heuristic descriptions, we suggest the following, simple taxonomy.

Consider a variable X with domain **X** $\subset R$. Define a relevant sample $X_s \subseteq$ **X**, comprising n realizations of this variable, $X_s = X_1, ...X_n$, with median $\bar{X}_s$, and standard deviation σ_s. If X_s is a time series, assume that the relevant sample data are covariance stationary. In the following, superscripts *T*, *R*, and *E* indicate 'typical', 'rare' and 'extreme', respectively.

Definition 1: A **typical event** $X^T \in X_s$ is in a range X^T_{range} that contains more than 1/2 of the observations in the relevant data sample:

$$\frac{\text{Number of } X_i \in X^T_{range}}{\text{Number of } X_i \in X_s} \geq \frac{1}{2}.$$

We now turn to rare events. The benchmark case for rare events is 1/5, to match the psychologically motivated definition of Hertwig, Barron, Weber, and Erev (2005). However, other researchers estimate different values for rare events.[14] Therefore, in order to give the researcher flexibility in deciding just how rare is rare, we index the definition by a multiplicative parameter $\delta \geq 1$, that reduces the size of the rare set accordingly.

Definition 2: A $\delta-$**Rare event** $X^{R(\delta)}$ is in a range X^R_{range} that contains less than $1/5\delta$ of the observations in the relevant sample, in the presence of another (nonoverlapping) range that occurs more frequently than itself:[15]

$$\frac{\text{Number of } X_i \in X^R_{range}}{\text{Number of } X_i \in X_S} \leq \frac{1}{5\delta}.$$

[12] For arbitrary non-normal distributions with finite variance σ^2, we can provide deviation bounds in a similar way, using Chebyshev's inequality.

[13] This may be expressed by saying that the typical events form an open dense set in the set of all events, as in Debreu (1970).

[14] For example, the empirical values all fall below 2% in the research of Barro (2006), Chollete, de la Pena, and Lu (2006), Jansen and de Vries (1991), and King and Zeng (2001). Weber (2007) explains that in experimental research on decisionmaking, the level 1/5 is the threshold at which the divergence between decisions based on experience and description becomes pronounced.

[15] We require there to be a more frequent event, since the notion of rare is relative. Typical and rare events are mutually exclusive but not exhaustive. An event can be neither typical nor rare, for example if its range has an empirical likelihood between $1/5$ and $1/2$.

Before defining extreme events, we observe that much research in finance, political science and statistics uses the terms extreme and rare interchangeably. This custom is misleading for at least two reasons. First is the possibility of extreme clusters, where extreme events occur relatively frequently. For example, during bubbles or periods of high financial market volatility, it is possible for the stock index to reach levels far from the recent median, routinely. Second, for highly skewed or heavy-tailed distributions, extremes can occur more frequently than central observations. Consequently, it is important to define extreme events in a way that does not assume, a priori, that they are either typical or rare. It is also helpful to employ a definition that is related to the current practice of using standard deviation or volatility. We therefore adopt the following definition.[16]

Definition 3: An ω**-Extreme event** $X^{E(\omega)}$ is an event that is at least $\omega \geq 1$ standard deviations away from $\bar{X}_s$, the relevant median:

$$|X^{E(\omega)} - \bar{X}_s| \geq \omega\sigma_s.$$

For financial time series, the benchmark median of the relevant dataset can easily be computed dynamically, to capture the notion that over time, what once was extreme may become commonplace.[17] We are now ready to implement a workable definition of the empirical probability of extremes, p_t, for later use.

Definition 4: The **Empirical probability** $p(\omega)$ of an ω-extreme event $X^{E(\omega)}$ measures the relative frequency of observations exceeding ω standard deviations from the relevant median $\bar{X}_s$:

$$p(\omega) = \frac{\text{Number of } [X_i \in X_s : |X_i - \bar{X}_s| \geq \omega\sigma_s]}{\text{Number of } X_i \in X_s}.$$

[16]This definition is related to that of extreme value theory, where extremes are usually phrased in terms of closeness to the maximum or minimum. The median is used instead of the mean or extrema because it is robust and achieves the highest possible breakdown value, see Casella and Berger (1990) page 230. Psychologically, people may take time to adjust their concept of normal, and the median embodies this more than the mean. Note that we choose a slightly different definition from extreme value theory because in finance we might worry about deviations from what is typical, even if they are not record-breaking events. For large ω, the definition will be identical to that of extreme value theory, by choosing ω such that $\omega\sigma_s = |X_{(1)} - \bar{X}_s|$, where $X_{(1)}$ is an extreme order statistic.

[17]For example, one could compute extremes relative to the previous quarter's benchmark median, to capture individuals' lag time in learning and computing the benchmark. The notation ω is chosen since it is often used in definitions of oscillation.

Evidently what is typical, extreme, or rare may change over time, and our definitions above are designed to reflect this notion.[18] We therefore emphasize that our definitions are conditional: we condition on the relevant data sample, which is chosen with the guidance of scientific theory and knowledge of the question at hand. This approach makes sense from a social science perspective, acknowledging that when the world changes, we take some time to recognize it. The conditional approach is a strength and a potential challenge. The strength is that it frees researchers in various disciplines or with different questions to choose their concept of rareness or extremeness, with alternative values of δ and ω. A challenge is potential lack of comparability across different studies. If comparability is an issue, one might compare extreme estimates using both the data sample suggested by scientific theory and the entire data available.

3 Dynamic, Endogenous Extremes

3.1 Dynamic Extremes

As mentioned in Section 2, dynamic extremes can be random or display patterns. The random case is represented by equation (1). Patterns may have many possible representations. For parsimony, and in order to relate our formulation to existing work in time series analysis, we consider a simple stationary pattern. One such model is a simple autoregressive representation, $p_t = \alpha + \sum_{j=1}^{J} \theta_j p_{t-j} + \varepsilon_t$. Although many lags are possible, we focus on the first order case:

$$p_t = \alpha + \theta_1 p_{t-1} + \varepsilon_t, \tag{2}$$

where $|\theta_1| < 1$. Expression (2) permits us to capture the potential clustering in extremes mentioned above.[19]

[18] Our definitions compare current events to past medians. The reason is that individuals' notions of extreme is often relative to what they have learned in previously. This can be motivated by psychology, where we take time to learn about rare events by experience (Hertwig, Barron, Weber, and Erev (2005)), or by disaster myopia (Herring and Wachter (2005)). It can also be motivated by econometric considerations, since we gather data at the end of the period before we can compute sample statistics.

[19] The focus of our discussion is on the *empirical* properties of p_t. Therefore, the regression residual ε in (2) must be compatible with bounded probabilities, because the p_t data used in our estimation will lie in the [0,1] interval. If we were interested in modeling the theoretical properties of the process, we could impose boundedness in a standard way by using some variant of a logistic function, as we illustrated in equation (1). We could also consider simple nonstationary models, for example a regime switching generalization.

What is the *signature* of dynamic extremes? According to equations (1) and (2), dynamic exogenous extremes have a frequency p_t that depends either on a random arrival ε_t, or else on some function of its own past values.

3.2 A Simple Model of Endogenous Extremes

Thus far, extreme probabilities are exogenous, and do not depend directly on variables under the control of economic agents. We now formalize the arguments of Section 2.2, and consider the possibility that economic agents and the economic environment influence the frequency of extremes. While exogenous extremes are statistically unrelated to the economic environment, endogenous extremes (since they are generated by economic agents) should be related to the optimizing or equilibrium behavior of agents.

More formally, consider an economy comprising a large number l of lenders, and a large number m of mortgage borrowers. Let $m = f \cdot l$, where f is some positive integer. Each lender deals with an equal number of borrowers, $f = \frac{m}{l}$. The m mortgages are drawn from the same distribution, and of similar term. Lender and borrower activity affects other agents in the financial system, including other banks, investment firms, hedge funds, and non-borrowing investors, domestically and internationally. We denote these other agents O, for other. In the following analysis we use the subscripts $0, 1$ and 2 to index variables pertaining to other, lenders, and borrowers, respectively. Borrowers and lenders are both in the market for borrowed funds. Effective supply of borrowing is b_1 and demand for borrowing is b_2. Investors and banks consider themselves small enough that their own borrowing and lending does not affect asset prices. As in the literature on credit cycles (Kiyotaki and Moore (1997)), the financial sector alternates between periods of easy and hard credit. The credit regime is denoted CR and varies continuously between 0 and $1,$ where $CR = 0$ denotes the hardest credit regime and $CR = 1$ denotes the easiest credit regime. In keeping with the spirit of credit cycle literature, we let the effective supply and demand for borrowed funds b_1 and b_2 depend continuously and positively on the availability of credit. That is, for each agent j, $b_1^j = b_1^j(CR)$, and $b_2^j = b_2^j(CR)$, with positive derivatives $b_1^{j\prime} > 0$ and $b_2^{j\prime} > 0$. In the following discussion we remove the j superscripts since we will be discussing an average agent.[20]

[20]Since borrowing depends on the credit cycle, our model has an important complementarity, delivering aggregate effects. Complementarity means that it is more attractive to borrow (or offer credit) if other agents are doing the same. Strategic complementarities tend to arise in situations of imperfect competition, costly search for trading partners and preference externalities (Cooper (1999)). These situations can plausibly exist

The framework is a two-period economy, where we indicate the first period as t and the second period as $t+1$ in order to distinguish the subscripts that refer to time and the subscripts that refer to agents. In the first period lenders and borrowers interact in the market for borrowed funds, and sell the securitized loans to the other sector. In the second period, lenders repay borrowers. If there is an endogenous extreme event in the second period, this increases the costs of other, lenders and borrowers by an additional c_0, c_1 and c_2, respectively. The timeline for decisions is shown in Figure 4. For simplicity, we assume that agents receive all their wealth and make all their repayments in the second period. Thus, the lender and borrower's wealth levels in the first period completely derive from borrowed funds: $W_{1,t} = -b_{1,t}(CR_t)$, and $W_{2,t} = b_{2,t}(CR_t)$, respectively. In the second period $t+1$, the lender and borrower have exogenous potentially unequal wealth levels $\bar{W}_1$ and $\bar{W}_2$, respectively.

We focus on a representative mortgage lender and borrower with utility functions u_1 and u_2, respectively. Both lender and borrower have locally nonsatiated preferences represented by concave differentiable utility functions with standard properties, depending continuously on wealth: $u_1 = u_1(W_1)$ and $u_2 = u_2(W_2)$. Moreover, both u_1 and u_2 are increasing in wealth, $u_1'(W_1) > 0$ and $u_2'(W_2) > 0$.[21] In order to control for contemporaneous costs, we consider utility to be net of current costs. Each agent knows there is a possibility of systemwide extreme events occurring, captured by the probability p, whose functional form is common knowledge. In the spirit of Fisher (1933) and Allen and Gale (2000), the probability of future extreme events increases with the average level of borrowed funds, $p = p(b_1, b_2)$, where $\partial p/\partial b_1 > 0$ and $\partial p/\partial b_2 > 0$.[22] As mentioned before, if an extreme event occurs in the future, agent i incurs a positive cost c_i, $i = 0, 1, 2$. This cost is financial, social and psychological discomfort suffered in an environment of extremes or financial instability.[23] There is no asymmetric information about the likelihood of extremes. Each

for lenders, borrowers and hedge funds, respectively. The credit regime summarizes for each agent how attractive other agents find it to engage in extensive borrowing or lending. Thus, when one agent borrows or lends, so do many others in the economy. Therefore an endogenous cause of extremes could be time varying incentives to offer and accept easy credit, perhaps related to the real estate cycle, as in Pavlov and Wachter (2006) and Pavlov and Wachter (2007).

[21]The assumption of increasing utility must hold in the relevant range, otherwise there is no interesting economic problem: borrowers and lenders would just be automatically prudent, and never in danger of over-borrowing or over-lending. This assumption helps convey the nature of the economic problem in the US subprime market.

[22]This summarizes the intuition that excessive borrowing is destabilizing, without emphasizing the particular channel of destabilization. Channels through which borrowing leads to increased likelihood of extremes are explored by a number of authors, including Fisher (1933) and Allen and Gale (2000).

[23]Two financial costs are the risk of default, and that trading suffers because prices are relatively uninformative, as in Harris (2003), Chapter 9). A social or psychological cost is increased Knightian uncertainty in

agent knows that this likelihood increases with average borrowing or lending.[24] We now consider the lender's problem, in an easy credit regime. Given a loan interest rate r, at period t the lender decides how much to lend this period by maximizing utility subject to the following wealth constraint, which accounts for the possibility of costly extreme events:

$$W_{1,t+1} \geq \bar{W}_1 + p_{t+1}(b_{1,t}, b_{2,t})[b_{1,t} \cdot (1+r) - c_{1,t+1}] + [1 - p_{t+1}(b_{1,t}, b_{2,t})][b_{1,t} \cdot (1+r)].$$

Given locally nonsatiated preferences, this constraint holds as an equality, which simplifies to $W_{1,t+1} = \bar{W}_1 + b_{1,t} \cdot (1+r) - p_{t+1}(b_{1,t}, b_{2,t}) \cdot c_{1,t+1}$. Thus, the lender's problem is:

$$\begin{gathered} \max_{b_1} u_1(W_{1,t}) + \beta u_1(W_{1,t+1}), \text{ s.t.} \\ W_{1,t} = -b_{1,t}(CR_t) \\ W_{1,t+1} = \bar{W}_1 + b_{1,t} \cdot (1+r) - p_{t+1}(b_{1,t}, b_{2,t}) \cdot c_{1,t+1}. \end{gathered}$$

After substituting the constraints into the utility arguments, first order conditions for an interior solution are $-u_1'(W_{1,t}) + \beta u_1'(W_{1,t+1})[(1+r) - \frac{\partial p_{t+1}(b_{1,t}, b_{2,t})}{\partial b_{1,t}} \cdot c_{1,t+1}] = 0$, which can be rewritten as

$$\frac{\partial p_{t+1}(b_{1,t}, b_{2,t})}{\partial b_{1,t}} = -\frac{u_1'(W_{1,t})}{\beta u_1'(W_{1,t+1}) \cdot c_{1,t+1}} + \frac{1+r}{c_{1,t+1}}. \tag{3}$$

Equation (3) says that optimally the (derivative of) extreme probability is related to the marginal rate of substitution for lending funds between periods t and $t+1$, discounted by expected costs. The actual sign of this expression is indeterminate, since expected costs can be negative or positive. Moreover, marginal utility can be positive or negative when agents

an unstable economy, see Caballero and Krishnamurthy (2007) and Weitzman (2007). Implicit in our work is the notion that this pattern of excess borrowing may recur because of time variation in not just financial but also moral and psychological costs of overborrowing (Agarwal, Driscoll, Gabaix, and Laibson (2007)). Learning may not occur, since different generations of individuals are involved, given the time of the asset cycle. For related ideas, see Kiyotaki and Moore (1997) and Minsky (1982). This framework parallels that in theories of corruption and tax evasion: Andvig and Moene (1990) show that supply of corruption increases due to lower moral costs of taking bribes; Sandmo (2005) discusses the possibility, based on a 'social conscience' argument, that tax evasion for an individual taxpayer is less risky, the more other taxpayers are perceived as evading taxes. In similar spirit, we suggest that the costs of over-borrowing for an individual may depend on the social attitude towards borrowing at the particular time. Thus, there is no a priori reason to rule out zero or even negative costs of borrowing during the upswing in real estate cycles, for example.

[24] Similar assumptions occur in many other economic contexts, such as the idea of price taking, competitive agents used in Arrow and Debreu (1954), Chichilnisky and Wu (2006) and Debreu (1959), even though the demand of each agent will affect the price to some extent. Such myopic behavior can be found in other rational settings–for example, investors with log utility decide their portfolios without reference to future investment opportunities, see Ingersoll (1987) Chapter 11.

are at a corner solution. An important result, since the first term of the right hand side of (3) depends on $b_{1,t}(CR_t)$ via the budget constraint, is that extreme probabilities respond to borrowing and to the credit regime. We will use this result to motivate our selection of instruments in the empirical application of Section 5.

Similarly, the borrower's problem is

$$\begin{gathered}\max_{b_2} u_2(W_{2,t}) + \beta u_2(W_{2,t+1}), \text{ s.t.} \\ W_{2,t} = b_{2,t}(CR_t) \\ W_{2,t+1} = \bar{W}_2 - b_{2,t} \cdot (1+r) - p_{t+1}(b_{1,t}, b_{2,t}) \cdot c_{2,t+1},\end{gathered}$$

which yields first order conditions that can be rewritten as

$$\frac{\partial p_{t+1}(b_{1,t}, b_{2,t})}{\partial b_{2,t}} = \frac{u_2'(W_{2,t})}{\beta u_2'(W_{2,t+1}) \cdot c_{2,t+1}} - \frac{1+r}{c_{2,t+1}}. \tag{4}$$

As in equation (3), the above expression implies that the future probability of extremes is dynamic, and depends positively on the current level of credit availability.

Equilibrium: In equilibrium, the demand and supply of borrowed funds will be equal, $b_1 = b_2 \equiv b$. For illustrative purposes, let us consider a symmetric equilibrium where lender and borrower have identical utility functions and costs, $u_1 = u_2 = u$, and $c_1 = c_2 = c$. Assume this symmetry, and equate the optimality conditions for the lender and borrower in 3 and 4: $-\frac{u'(W_{1,t})}{\beta u'(W_{1,t+1}) \cdot c_{t+1}} + \frac{1+r}{c_{t+1}} = \frac{u'(W_{2,t})}{\beta u'(W_{2,t+1}) \cdot c_{t+1}} - \frac{1+r}{c_{t+1}}$. This expression implies

$$1 + r = \frac{1}{2\beta}[\frac{u'(W_{1,t})}{u'(W_{1,t+1})} + \frac{u'(W_{2,t})}{u'(W_{2,t+1})}].$$

Substituting this in equation (4) and simplifying, we obtain that in this equilibrium, extreme probabilities p_{t+1} satisfy

$$\frac{dp_{t+1}}{db_t} = \frac{1}{2\beta c_{t+1}} \left[\frac{u'(W_{2,t})}{u'(W_{2,t+1})} - \frac{u'(W_{1,t})}{u'(W_{1,t+1})}\right] \tag{5}$$

Equation (5) constitutes the *signature* of endogenous extremes. The responsiveness of extreme probability to borrowing is proportional to the marginal rates of substitution of agents in the market for borrowed funds.[25] If extremes were truly exogenous, there would be no

[25] Our result is intuitive: agents affect extreme probability by their optimizing behavior over a certain variable with external effects. Therefore, optimally their marginal utility relates to the responsiveness of

statistical relation between extreme probability and b, and $\partial p(b)/\partial b = 0$. The difference between equations (1) and (5) gives a sense of the estimation error from assuming extremes are exogenous, when they are in reality endogenous. It is important to note that we have insufficient information to determine the sign of the sensitivity of extreme events to borrowing, in equation (5). The reason is that we do not know the sign of either expected costs of extremes $c_{1,t+1}$ or marginal utility $u_1'(b_t(CR_t))$. For example, marginal utility may be negative if wealth-constrained individuals who borrow for house purchase are temporarily at a corner solution. Similarly, expected costs may be perceived as close to zero or even negative during euphoric building boom periods, such as those experienced by the USA from the mid-1990s through the turn of the century, and depicted in Figure 1.

We are not just saying there is a link between over-borrowing and extremes. Instead, we are showing that even without asymmetric information, over-borrowing may arise as an equilibrium phenomenon. This phenomenon occurs due to the failure of *both* borrowers and lenders to internalize an important externality, the excessive probability of systemwide future financial crashes. An easy way to see that the probability of crashes is excessive is to consider what happens if the lender considers the effect of her lending on other agents O, namely, if she internalizes the costs $c_{0,t+1}$. Then, using similar logic to that before equation (3), her problem is identical, except that the second budget constraint becomes

$$W_{1,t+1} = \bar{W}_1 + b_{s,t} \cdot (1+r) - p_{t+1}(b_{s,t}, b_{d,t}) \cdot (c_{0,t+1} + c_{1,t+1}).$$

Solving the first order conditions and rewriting as before, we obtain the counterpart of equation (3) for a socially optimal level of extremes:

$$\frac{\partial p_{t+1}(b_{d,t}, b_{s,t})}{\partial b_{s,t}} = -\frac{u_1'(W_{1,t})}{\beta u_1'(W_{1,t+1}) \cdot (c_{0,t+1} + c_{1,t+1})} + \frac{1+r}{c_{0,t+1} + c_{1,t+1}}. \tag{6}$$

The quantities in equations (3) and (6) will differ in general. Thus, when the lender takes into account the future costs of other agents, optimal behavior involves a different extreme probability for a given level of borrowed funds. A similar logic exists for borrowers. It is in this sense that competitive markets may lead to endogenous, inefficient probability of

extreme probability to this variable. Since the marginal rate of substitution depends on the credit regime through the budget constraint, the expression also captures the notion that the easiness of credit affects the likelihood of the financial system's suffering future crashes.

crashes.[26] To fix ideas, suppose that the terms in equation (6) are all positive, which loosely speaking implies that the social optimum features relatively lower probability of extremes. Then there are two ways to express the situation described above. First, as before, we can recognize that overborrowing due to easy credit has a negative externality, and is therefore overproduced. Second, in language perhaps closer to regulators' concerns, we can say that financial system stability (control of extremes) is a public good, which suffers from classic underprovision.

We summarize the findings from equations (3) and (6) in the following Proposition:

Proposition 1. *In an economy with symmetric preferences and nonzero social costs of extremes, the equilibrium level of extreme probability is in general not socially optimal, and depends on equilibrium borrowing as well as the credit regime.*

The most important implications from equation (5) relate to the likelihood and persistence of extremes. For a given level of borrowing, the likelihood of future extremes reacts to the ease of credit, and based on our previous discussion, reacts to any instruments related to strategic complementarity, such as investor sentiment and consumer confidence. It decreases with expected social, financial and psychological costs of extremes. Persistence of extremes is higher when the marginal utility of borrowing and costs are persistent. This finding accords with the behavioral decision research such as Weber (2006), who documents that low-probability events generate insufficient worry (psychological costs) than appropriate before they occur.

3.2.1 Extreme Spillovers

The above formulation gives little intuition on spillovers, or the expected breadth of extremes. To tease out this information, one possibility is to observe that the breadth of spillovers depends on the strength of aggregation and complementarity. These effects may be assessed using indices of imperfect competition and search costs, for example (see Cooper (1999)). However, we feel a more realistic approach is to examine a situation where spillovers are typically very unlikely to begin with, and then ask what drives spillovers? In today's markets, spillovers are increasingly important to consider, since globalization and

[26] Note that optimality will not necessarily entail complete elimination of extreme events. Rather, the extreme probability level is adjusted to the point where the marginal benefit to lenders of an additional unit of the externality-generating activity, $u_1'(b)$, equals its marginal cost to other agents, $-u_0'(b)$.

financial innovation not only spread risk, but also *hide* risk–investors in one sector might unknowingly bear some part of the risk from agents in distant sectors and nations. These risks are diversified away in normal times, but may be significant in periods of correlated returns and dependent defaults. Until now, our model has suggested that spillovers happen to hedge funds and all other sectors automatically in the subsequent period. This can occur if other sectors are directly exposed to default risk by purchasing high risk debt from lenders. In modern financial markets, however, there are numerous ways of diversifying such risk, for example by securitizing debt into a new instrument. Consequently, even if several borrowers default, their risk will be spread over many buyers, and therefore have little impact on each buyer of the securitized asset. What could cause a spillover in this case? The main channel is a systematic comovement in defaults or selling, as in the LTCM case during summer of 1998, and in many US financial markets in spring through fall of 2007. Such comovement may result in sharply reduced value of even securitized assets. If this occurs, then securitized assets become highly undiversified, and may propagate the effects of extreme events. The question therefore becomes, what could cause a systematic comovement in defaults or selling? A compelling answer is liquidity. If economic agents face sharp, simultaneous liquidity drops in many asset classes, they may be forced to default (borrowers) or sell assets (lenders), which will simultaneously reduce the value of many securitized assets.[27] According to this logic, the incidence of extreme spillovers is determined at least partially by the extent of liquidity comovement.

More concretely, we present some evidence of the behavior of liquidity during extreme events, in Figures 5 to 7. The first figures, 5 and 6, show the liquidity measure of Pastor and Stambaugh (2003) during the stock market crash of 1987 and 1998. In both instances liquidity dropped sharply. A third incident in which liquidity might have been an issue concerns the internet bubble's bursting in 1999 and 2000, which could be considered an extreme event for internet stocks. Unfortunately we do not have specific data on the liquidity of IT stocks affected in the US. We do, however, have detailed liquidity data and IT indexes for the Norwegian stock market, which are presented in Figure 7. Again, the liquidity measures show sharp spikes during the period when the internet bubble was bursting. It is beyond the scope of our paper to prove a definitive link between liquidity and correlations. Therefore, in addition to the above graphs, we draw attention to several recent studies that explore such a link, which we became aware of after completing most of this paper. These liquidity papers are discussed in the literature review of Section 1.1. In sum,

[27] Since our focus is on extremes, for simplicity we model liquidity as exogenous. In practice liquidity may respond to changes other variables such as collateral prices (e.g. real estate) for securitized instruments.

even though individual extreme events may have different causes, there seem to be some common patterns to extremes, related to borrowing and liquidity.

We offer a simple, stylized formalization of the above arguments, describing a channel through which other economic agents experience increased extreme probability p. An important aspect of the economic environment is securitization. Loans are pooled into a diversified security S, like a CDO, then resold to the other sector at a competitive price equal to its discounted value. The reason for supply of this asset is evident, since lenders wish to diversify away their risk. Why does the other sector demand this asset? The reason is that S may dominate other risky assets, or provide diversification benefits.[28] As in spring and summer of 2007, extreme spillovers are hastened by liquidity demands. To meet these demands, hedge funds and other investors may sell off a liquid, unrelated tranche such as municipal bonds. This selloff inhibits liquidity in that tranche, which leads to further selloffs in one tranche after another. Therefore, the exogenous driving force behind extreme spillovers is liquidity comovement–rare but high impact contemporaneous drops in liquidity across various securities. These liquidity shocks increase asset correlations and default dependence, making them move 'in step', all selling or defaulting at the same time. In addition, an important endogenous source of spillover risk is excessive diversification, which implies the other sector is highly exposed during periods of correlated default.[29] This endogenous risk is amplified by inordinate, perhaps unknown, exposure to securitized assets like CDOs, by various market participants. Why is this endogenous risk left unchecked? There are three reasons. First, agents may not fully understand rare events. They have disaster myopia or otherwise underestimate the likelihood of rare events, as documented by (Herring and Wachter (2005)), Hertwig, Barron, Weber, and Erev (2005), and Pavlov and Wachter (2006). It is well known that myopic behavior can arise for even rational preferences, for example in the case of logarithmic utility. Other reasons why agents do not understand rare events include limited computational ability, or statistical issues–it is hard to estimate the probability of rare events with limited data.[30] Second, even if some

[28]In practice, CDOs are often bought by fixed income investors in search of high yields. CDOs will be at least as attractive as high yield bonds because the former are generally uncorrelated. In the present situation, sub-prime CDOs had relatively low risk when the real estate market was going up, since the collateral was extremely valuable. We are grateful to Arjun Jayaraman for discussions on this point.

[29]Paradoxically, therefore, diversification can increase risk during extreme periods, since investors have a wider network of obligations, and are thus more likely to affect others and be affected by an extreme event in any sector. This relates to work of (Chichilnisky (2004), Danielsson and Shin (2003), Dembo, Deuschel, and Duffie (2003) and Ibragimov and Walden (2007).

[30]Underweighting of rare events has been documented in several ways. First, agents may discount rare events when they estimate probabilities based on experience (Barron and Erev (2003), Hertwig, Barron, We-

individuals take rare events seriously, when liquidity is high, the ease of resale makes them more comfortable with elevated risk, since they feel they can resell if necessary. Third, during some stages of the asset cycle the collateral for securitized debt has a relatively high price. In the subprime market case the collateral is real estate, which featured rising prices around the turn of the century, as shown in Figure 1. Such a scenario can be another cause of complacency, since the value of collateral is high.

More formally, the mortgages are packaged into m securitized assets $S_1, ..., S_j..., S_m$, each with $1/m$ of the original securities, for maximal diversification. Each original mortgage i yields a stream of payments with liquidity-adjusted excess returns $r^e_{i,t}$. For simplicity, we assume symmetric correlations and standard deviations: $\rho_{ij,t} = \rho$, all $i \neq j$, and $\sigma_i = \sigma$ for all i. The return on each securitized asset is $r_{S,t}$, with mean $\bar{r}_{S,t} = \frac{1}{m}\sum_{i=1}^{m} r_{i,t} = \bar{r}_{i,t}$. The variance is denoted $\sigma^2_{S,t}$, and computed as[31]

$$\sigma^2_{S,t} = \frac{1}{m}(\sigma^2 + 2\rho). \tag{7}$$

We now define an extreme spillover, then discuss the role of liquidity.

Definition 5: An Extreme Spillover is an $\omega-$extreme event in S_j. That is, a situation where $|r_{S,t} - \bar{r}_{t-1}| \geq \omega\sigma_{S,t-1}$.

In order to implement Definition 5, we need explicit expressions for portfolio returns, which we now develop. Previous research has documented that the correlation of asset returns,

ber, and Erev (2005), and Rabin (2002)). Moreover, econometrically there is a bias to under-estimate rare events (King and Zeng (2001), and de Haan and Sinha (1999)). Finally, expected utility does not effectively incorporate low probabilities (Bhide (2000) and Chichilnisky (2000)). All of these effects may be compounded by the fact that every few years, a new generation of borrowers needs to learn how to compute the likelihood of rare events.

[31]To obtain equation (7), note that the security variance $\sigma^2_{S,t}$ is the variance of a sum of random variables,

$$Var\left[\sum_{i=1}^{m}\frac{1}{m}r^e_{i,t}\right] = \sum_{i=1}^{m}\frac{1}{m^2}Var(r^e_{i,t}) + 2\sum_{i\neq j}\sum_{i\neq j}\frac{1}{m^2}Cov(r^e_{i,t}, r^e_{j,t}).$$

This can be computed as

$$\frac{1}{m^2}\sum_{i}\sigma_i^2 + 2\sum_{i\neq j}\sum_{i\neq j}\frac{1}{m^2}\rho_{ij} = \frac{m\sigma_i^2}{m^2} + 2\frac{m\rho_{ij}}{m^2} = \frac{\sigma^2}{m} + \frac{2\rho}{m},$$

where the last equality invokes the identical variance and correlation assumption from above.

and correlation of defaults tend to increase together during extreme periods.[32] We formalize this empirical observation starkly by saying that individual mortgages are uncorrelated in general, but highly correlated a small fraction δ of the time. This is represented as a regime shift,

$$\rho = \begin{cases} \rho^{hi}, & \text{with probability } \delta \\ 0, & \text{with probability } 1-\delta, \end{cases} \tag{8}$$

where δ is close to zero. Given assumption (8), the securitized asset variance in equation (7) is nonlinear, equalling $\frac{\sigma^2}{m}$ most of the time, and equalling $\frac{\sigma^2}{m} + \frac{2\rho^{hi}}{m}$ a small fraction of the time. Thus, the benefits of diversification accrue to the securitized asset in typical times, when its variance is much smaller than the sum of the individual security variances. By contrast, during extreme times correlations become important for returns, and diversification benefits evaporate.

These extreme correlations are driven by liquidity. In particular, most of the time liquidity is plentiful in some markets. However, a small fraction δ of the time, liquidity dries up in most markets, which forces multi-market margin calls and flight to quality. Consequently, there is an increase in defaults and delinquent payments across many assets, and their returns enter a high correlation regime. We assume liquidity drives the correlation regimes in (8) directly.[33] Specifically, liquidity is an additive cost or benefit to gross returns $r_{i,t}$–mortgage securities have higher excess returns $r^e_{i,t}$ if idiosyncratic liquidity $L_{i,t}$ is positive, and a lower return if it is negative:

$$r^e_{i,t} = r_{i,t} + L_{i,t}.$$

Average gross returns are equal across the individual mortgages, and over time, $\frac{1}{m}\sum_{i=1}^{m}(r_{i,t} = \bar{r}_{i,t} = \bar{r}_{i,t-1}$. Thus the most important dynamics come from liquidity.[34] Every period there is an exogenous liquidity shock $L_{i,t}$ to the return on each mortgage. Most of the time

[32] See for example, Ang and Bekaert (2002), Cappiello, Engle, and Sheppard (2006), Dembo, Deuschel, and Duffie (2003), and Embrechts, Frey and McNeil (2005), page 331.

[33] Our liquidity cost is in similar spirit to Amihud and Mendelson (1986), and Jacoby, Fowler and Gottesman (2002), who model liquidity as a proportional cost relative to the security price.

[34] Alternatively, we can think of this as the gross returns being net of other effects, since we wish to concentrate on liquidity shocks.

these shocks are uncorrelated and zero-mean, $\frac{1}{m}\sum_{i=1}^{m} L_{i,t} = 0$. Somewhat rarely they are perfectly correlated, as in the following sort of structure:

$$L_{i,t} = \begin{cases} \sim i.i.d. \text{ Uniform}[-1,1], & \text{with probability } 1-\delta \\ L_t^-, & \text{with probability } \delta \end{cases} \tag{9}$$

where L_t^- is a large negative number.[35] This setup produces spillovers because, in the rare regimes, liquidity costs reduce the value of every component of the securitized assets S_j. That is, the S_j inherits the liquidity costs of all its component securities:

$$r_{S,t} = \begin{cases} \frac{1}{m}\sum_{i=1}^{m}(r_{i,t} + L_{i,t}) = \bar{r}_{i,t} + 0, & \text{with probability } 1-\delta \\ \frac{1}{m}\sum_{i=1}^{m}(r_{i,t} + L_{i,t}) = \bar{r}_{i,t} + L_t^-, & \text{with probability } \delta. \end{cases} \tag{10}$$

To see how this can lead to an extreme spillover, consider a correlated liquidity shock in period t (the $\delta-$ rare regime), while period $t-1$ features the typical uncorrelated shock. Recall from Definition 5 that an extreme spillover occurs when

$$|r_{S,t} - \bar{r}_{t-1}| \geq \omega\sigma_{S,t-1}. \tag{11}$$

From equation (10), the left side of (11) satisfies

$$|r_{S,t} - \bar{r}_{t-1}| = L^-. \tag{12}$$

The right hand side of (11), using equation (7), satisfies

$$\omega\sigma_{S,t-1} = \omega\frac{1}{m^{1/2}}[\sigma_{t-1}^2 + 2\rho_{t-1}]^{1/2} = \omega\frac{\sigma_{t-1}}{m^{1/2}}, \tag{13}$$

where the last equality uses the fact that $\rho_{t-1} = 0$ before the onset of the correlated liquidity shock. Combining (12) and (13) yields the condition for extreme spillover: $L_t^- \geq \omega\frac{\sigma_{t-1}}{m^{1/2}}$, or

$$\ln L_t^- \geq \ln\omega + \ln\sigma_{t-1} - \frac{1}{2}\ln m. \tag{14}$$

This expression is intuitive. In an environment where returns depend on liquidity shocks, extreme spillovers will happen if a liquidity shock is large enough relative to average return volatility σ_{t-1}. The term $-\ln m$ is also natural, since the larger the number of borrowers, the more sources of hidden risk in S and therefore the lower the liquidity shock needed to

[35] Alternatively, we could say there is a shift of the distribution, for example to $L \sim U[2L_t^-, 0]$. Since L_t^- is negative, the mean is now negative.

set off an extreme event.[36] This expression predicts that extreme events will tend to spill over and persist when liquidity shocks are large relative to average volatility of individual instruments. We will use this insight in our empirical exploration of Section 5, below.

A key question for investors and regulators is what determines the incidence and persistence of extreme spillovers? According to equation (14) this depends on the average number m of borrowers in a securitized debt instrument, and the amount of liquidity comovement.[37] Thus, the breadth and persistence of extreme spillovers depends on liquidity comovement and persistence of liquidity shocks. If we are interested in predicting the extent of endogenous versus exogenous extremes, note that exogenous extremes should be fairly contained, ceteris paribus. Endogenous extremes are only limited by the extent of liquidity comovement and diversification.

4 Potential Policy Implications

What well-defined question can this framework help us answer? While we have provided a simple analysis of economic extremes in general, we feel our approach is especially suited to address the puzzles discussed in the introduction. Our framework suggests that the subprime market and spillover puzzles can be understood as the result of an uninternalized externality, the effect of excessive borrowing on financial stability. This is not a simple externality, but a multilateral, public externality. It is generated by and affects many agents, for example mortgage lenders, borrowers, hedge funds, and even global investors. To solve this type of public externality the introduction of a standard sort of market will not be optimal, due to the free rider problem and the impact of liquidity.[38] We now discuss liquidity and externalities, in turn.

Our model suggests that liquidity is important and should be monitored carefully.[39] There are many aspects to liquidity's importance. Perhaps the most crucial is our implicit notion that liquidity has a dual role: high liquidity helps initiate extremes by providing

[36] Having a large enough L^- will reduce diversification benefits because it involves adding a highly *dependent* risk to a portfolio.

[37] Another influence will be the degree of diversification, that is, the average share CDOs held by individuals.

[38] Private externalities are depletable, while public externalities are non-depletable, and retain potency for all who are affected. For an exposition of multilateral externalities, see Mas-Colell, Whinston, and Green (1995), Chapter 11.

[39] For a discussion of what constitutes plausible liquidity measures, see Chollete, Nas, and Skjeltorp (2007).

a market for securitized debt; and correlated liquidity drives the subsequent spillovers. Regarding initiating extremes, the risk of counterparty default is unimportant if there is enough liquidity and innovation for a lender to repackage and resell her debt. Regardless of information structure, once individuals feel they have enough liquidity, they may take excessive risk. Thus, information asymmetry may become less relevant in an environment of plentiful liquidity. Regarding spillovers, in the face of correlated liquidity events, all diversification benefits might disappear, and even pooled securities can experience a sharp drop in value. Therefore, liquidity amplifies endogenous extremes, which aggregate from one sector to the larger economy because of complementarities due to the credit cycle, and externalities. Since control of these extremes has public externality aspects, it is unlikely that an individual agent will provide the public liquidity necessary to avert systemwide extremes.[40] Why do agents produce endogenous extremes in the first place? There are two main reasons, disaster myopia and externalities: the effects of excessive borrowing happen somewhat rarely, in the future, and mainly to others. Individuals may not fully internalize the cost of elevated future extremes because this probability is considered relatively low and distant, and individuals have a bias to underweight low probability events. These two reasons are reinforced for both borrowers and lenders by liquidity, as described above. Why do individual regulatory authorities allow excess extremes to happen, once started? An important reason is that it is difficult to predict extremes. Furthermore, there is a free riding problem at the domestic and international level. Finally, there is moral hazard– there is a knife edge aspect to central bank supplying liquidity to calm extremes, since liquidity is what can initiate extremes in the first place.

Understanding the externality aspect of extremes may enable us to address not just the effects but also the causes of extremes. The externality issue gives lenders and borrowers inadequate incentives, leading to an overproduction of extremes. In order to correct this, a standard public finance solution involves giving agents pecuniary or property right incentives to reach the optimality condition in (6). Given the diversity of agents and nations in the current situation, two further suggestions that we think plausible are to create an international institution responsible for monitoring global system stability, and to delegate responsibility reciprocally, for providing aggregate liquidity and control of extremes. The former would ensure information sharing and avoid costly international duplication of effort. The latter would be a global market maker, ensuring public provision of a global

[40]An exception is John Rockefeller in the Wall Street panic of 1907.

public good.[41] Let us discuss these three solutions. First, ex ante incentives include subsidizing housing and borrowing, and assigning or auctioning tradeable extreme production rights. A related incentive involves educating borrowers and lenders on the nature of liquidity shocks, and the consequences of endogenous, extreme price changes.[42] Second, a monitor of lending and borrowing activity, and diversification of portfolios, would have special tasks. It would assess adequate levels of borrowing and lending (given average liquidity), and provide indices of current and expected liquidity levels, and indices of average risk in all securitized assets that could be subject to hidden risk during extremes.[43] Third, a global market maker would be used as an ex post measure. Since systemwide liquidity is a public good, each actor has an incentive to free ride, therefore liquidity effects may be persistent, once extremes begin. Public provision of systemwide liquidity is thus necessary to disrupt persistence of liquidity shocks. Reciprocal delegation of the role of global market maker will allow for efficient provision, and remove the free rider issue at the individual, national and global level.[44] These three prescriptions together may reduce extremes, and ensure that financial markets pay an appropriate price for the extremes they produce.

5 Empirical Application

An important pre-condition for the policy analysis described in the previous section is empirical documentation of the properties of extreme probabilities, to which we now turn. Since each asset price series may have individual characteristics, we focus on one that summarizes aggregate security performance, and for which there is a relatively long time series of daily observations, the Dow Jones Industrial Average.

[41] For literature on delegated monitoring, see Diamond (1984) and Sheard (1994). This would involve coordination of international regulatory authorities, for example, each taking turn as delegated monitor. Regarding global market makers, a related suggestion has been made by Buiter and Siebert, at http://maverecon.blogspot.com/2007/08/central-banks-in-time-of-crisis.html. For a global market maker, there is of course a moral hazard problem.

[42] Similar to pollution permits, these would allow the market participants to engage in reasonable levels of borrowing or lending even during times of low aggregate liquidity.

[43] Such global monitoring of liquidity, hidden risk and extremes, should be delegated because of its public good aspects, and to avoid costly duplication (Diamond (1984), Sheard (1994)).

[44] This is similar to what is already done by individual regulator authorities, pumping in liquidity. The difference is that it would be coordinated internationally in a systematic way. More generally, our suggestions are related to the prescriptions of Sandmo (2003), who outlines issues in providing global public goods, and Weber (2006), who recommends generation of appropriate concern as a necessary condition for mitigating externality-driven rare events.

Our main data comprises the following series: daily Dow Jones Industrial Average series (DJIA), from May 26, 1896 to September 28, 2007; the degree of securitization in US financial markets (SEC), available from January 1989 to December 2006; the liquidity measure (LIQ) of Pastor and Stambaugh (2003) available from April 1962 to December 2006; the value of real estate loans in the US (REALLOAN) available from January 1947 to July 2007; and a measure of investor sentiment (DSENT1), used in the study of Baker and Wurgler (2007). The DSENT1 data is available from January 1966 to December 2005, and kindly provided at Professor Jeffrey Wurgler's website.[45] Unless otherwise noted, the data are all monthly frequency, and obtained from WRDS and Datastream.

There are three steps to our empirical approach. First, we examine whether our p series are significantly different from zero. Second, we examine their dynamics by considering autoregressive time series models . Third, we begin to analyze endogeneity by using simple VAR and logistic models. We discuss each of the above in turn.

5.1 Computing the p_t Series, and Summary statistics

The main series we compute is $p_t(\omega)$, according to Definition 4 in Section 2.3. Using the DJIA described above as our base series, we compute the proportion of times each month that there is an observation more than ω standard deviations away from the median. Both the median and standard deviation are computed over the preceding k months, where $k =$ 12, 24, 60 and 120. This procedure is done on a rolling basis.[46] An example of the p_t series for the 12 month reference period is shown in Figure 8. As should be expected, the probability of extremes becomes much less active as we move from 1 or 2 sigma events to 3-sigma and beyond. Evidently, the series move around quite a bit, so even from a visual perspective the series are not constant.

Figures 9 and 10 display histograms of our extreme probabilities for 1 and 2 sigma events. In both cases there is a u-shaped pattern for all reference periods. Moreover, the longer reference periods tend to have more mass concentrated at 0 and 1. Thus, when

[45] Since the empirical probability series are between 0 and 1, and SEC and REALLOAN are in billions of dollars, we use percentage changes in SEC and the ratio of real estate loans to total loans, in order to scale them down comparably.

[46] Recall that the series begins in May 1896. Thus, for example, to compute the $p_t(1)$ for a 12 month reference period, we count the number of times in June 1897 that the DJIA exceeded 1 standard deviation from the median, which were calculated from May 1896 to May 1897. We then do the same for July 1897, where the median and standard are calculated from June 1896 to June 1897, and so on.

economic agents actually compute extremes in this range (1 and 2 sigma events), their probability estimates will tend to be more volatile. Figures 11 to 13 show histograms for extreme probabilities of 3- to 5-sigma events in the DJIA. As we would expect, the distributions become more concentrated at zero, with the shortest horizon (12 months) being the last to have all probabilities at zero.

We now turn to summary statistics and more formal tests. Table 2 shows that as the level of ω increases, both the mean and standard deviation decrease. However both the t-test and nonparametric sign rank test generally have minute p-values until the level of $\omega = 5$. This suggests that the likelihood of extreme events beyond 2 standard deviations may be non-zero, regardless of agents' reference periods.

Is there a difference in extreme probabilities within reference periods? We test this hypothesis in Table 3. Except for some marginal significance between 4 and 5 sigma events, there is very little evidence of similarity between the various extreme probabilities within a given reference period. A different pattern emerges, however, when we examine tests for differences across reference periods, in Table 4. This latter table reports mixed evidence about the similarity of average extreme probability depending on the reference period. This result may be of practical relevance, if different investor groups have different time horizons when deciding whether a particular event is extreme.

5.2 Dynamics

As we discussed in Sections 1 and 3, the dynamic behavior of extreme probabilities has important impacts for risk management and stress testing. We therefore examine the time series behavior of our $p_t(\omega)$ series. The results are displayed in Table 5.[47] Importantly, except for the very extreme 5-sigma events, the Q-test of white noise is rejected. This suggests that there are important dynamics in the likelihood of extreme events. The best-fitting models generally range from AR(1) to AR(3), although there are a few models with higher lags. Thus, our empirical probabilities seem to exhibit memory–extreme events cluster over time, regardless of our reference period.

[47] Note that some series had insufficient variance to compute time series models. This was particularly the case with the 5-sigma extremes, since much of it consists of zeros.

5.3 Endogeneity

While extreme dynamics relate mainly to investment, extreme endogeneity also relates to policy analysis and financial stability. The results of this section are exploratory, since our theoretical guidance is quite general. Ideally, we would like to assess whether p_t depends on plausible aspects of economic behavior, suggested by theory. Based on our Section 3 discussion, some examples are borrowing, and investor sentiment. However, our ideas about endogeneity come from equations (5) and 14, which are not easily testable. For example, the empirical probability of extremes will depend on other factors besides borrowing behavior and liquidity. In addition, for simplicity we model the cause of period $t+1$'s extremes to occur in t, while in practice, there may be substantial lag affects. Without specific parametric assumptions on the utility function, we cannot test (5) directly. Moreover, as mentioned before, it is difficult to obtain estimates of the expected costs of overborrowing.[48] Therefore, we only feel comfortable testing a set of relatively simple hypotheses, namely that $\partial p(b)/\partial b = 0$.

We therefore use simple 3-variable vector autoregressions to examine both the effect of borrowing and liquidity on extreme probabilities. In light of our Section 3 discussion, we first examine whether borrowing and the general state of investor sentiment affect the probability of extreme events. Our proxy for borrowing is REALLOAN, and for sentiment is DSENT1, described above. The results from these tests are displayed in Table 6, which displays orthogonalized impulse responses for REALLOAN and DSENT1, with respect to extreme probabilities. For all reference periods the VAR is either first or second order. In general, the standard errors are large, therefore it is not easy to make strong statements. One possibility that emerges from this table is that the effects of a shock in REALLOAN may persist and even increase over time. For example, in Panel B, the effect of a shock in real estate loans on 4-σ event probabilities is around 0.0003 in the first month (k=1), and increases monotonically to 0.0004 by the twelfth month.

We also attempt to glean a preliminary sense of the spillover effect discussed in Section 3.2.1 above, by using a separate VAR. The results are in Table 7. Specifically, in the spirit of equation (14), we estimate a VAR that contains extreme probabilities, liquidity (LIQ) and percentage change in securitization (PCTSEC), as described above.[49] The liquidity estimates have large standard errors. However, securitization is marginally significant in

[48]Options markets may provide a simple estimate of expected costs, but this is a relatively small market, and not necessarily the same as the market for real estate mortgage borowers.

[49]We exclude the 4-sigma events since their estimation generally does not converge.

some cases at the 6 and 12 month lags. For example, in Panel D, the relevant impulse response is significant for 2-σ events. In most cases, however, the standard errors are large, perhaps reflecting the persistence documented in Table (5), and the relatively short sample. This suggests the importance of finding better instruments for liquidity and securitization, with a longer sample.

One issue in the above results is the large autocorrelation of our empirical probabilities. From a practical point of view, these point estimates are less useful than early warning signals that indicate whether the economy is likely to be in a range corresponding to 'high' levels of extreme events. From a statistical viewpoint, the use of ranges is attractive for several reasons. Importantly, estimation of ranges is valuable for incorporating model uncertainty, as discussed by Granger, White, and Kamstra (1989) and Hansen (2006).[50] Range based empirical methods have also been used successfully in financial economics, for example by Alizadeh, Brandt, and Diebold (2002). In light of these considerations, we take another look at endogeneity, by dividing the empirical probabilities into three ranges, Low, Medium and High. Low corresponds to empirical probabilities less than 0.33, Medium to the range 0.33 to 0.67, and High to the range 0.67 to 1. We then estimate a cumulative logistic model for all the various reference periods.[51] The estimated model shows the effect of each explanatory variable on the likelihood of High levels of extreme probabilities. For example, the estimated coefficient on REALLOAN shows the relation between a one-unit increase in real estate borrowing and the likelihood of being in a period of High extreme probabilities. In light of equation (14), we include dummy variables for low, medium and high levels of liquidity, namely, LIQ0, LIQ1 and LIQ2, respectively, as well as interaction terms between liquidity and real estate borrowing. The results are reported in Tables 8 to 11, which we now discuss.[52] As mentioned before, we are primarily interested in the significance of our explanatory variables rather than their sign, because we have no plausible

[50] While using ranges can be argued to lose information in the current context, it allows us to test the important economic concept of whether there are some threshold effects, where extremes are triggered by excessive borrowing, or insufficient liquidity, for example. For an introduction to the benefit of using interval estimation, see Chapter 9 of Casella and Berger (1990).

[51] Logistic regression with k explanatory variables is based on the following empirical model: $g(p) = \alpha + \beta_1 X_1 + ... + \beta_k X_k$. The link function $g(p)$ is linearly related to the explanatory variables, and in the case of logistic regression, the link function is the logit or log-odds, $g(p) = log(p/(1-p))$. Thus, in our application, we are estimating the effect of various explanatory variables on the (log of the) probability of high extremes divided by the probability of no high extremes. A similar methodology has been used in explaining crises, by Bordo, Eichengreen, Klingebiel, and Martinez-Peria (2001).

[52] Note that the full set of explanatory variables is only available from 1989 to 2005. During this period the 4 and 5 sigma extremes featured only zeros for all reference periods. Therefore we can only estimate and report estimation results for 1- to 3-sigma extremes.

variables for expected costs from equation (5), which could be either negative or positive. Since three sigma events exhibit less variation (mostly zeros and ones), the model fit for these events is not as good as for the other cases. With the exception of 2-sigma events in the two year reference period, the models generally work well for 1 and 2 sigma extremes, as documented by the small p-values for the LR, Score and Wald tests.[53]

The most striking finding is that for all reference periods, real estate borrowing is significantly related at the 5% level to high probability of extremes for at least one specification: 1-sigma events, 2-sigma events, or both. For example, in Table 8, the estimated coefficient on REALLOAN is -21.52, with a p-value of 0.0052. Moreover, in the same table, a low level of liquidity (LIQ0) is significant for high extremes. This latter result holds for 1 and 2 sigma events, both individually and in interaction with real estate borrowing. The amount of securitization, SECPCT, is significant only for 2-sigma extremes at the 10 year horizon. The investor sentiment variable DSENT1, is never significant.[54]

To summarize our empirical exploration, The main findings on the nature of extremes is that extreme probabilities are dynamic and persistent, as documented in Tables 2 through 5. The evidence on the causes of extremes gives some support for endogeneity. A VAR analysis suggests that instruments related to borrowing and securitization may play a small role in the dynamics of extreme probabilities, although given the strong persistence in the raw probabilities, this analysis often delivers large standard errors. More encouragingly, a cumulative logistic analysis shows that the level of real estate borrowing is related to high likelihood of extremes for at least one specification in all reference periods. Moreover, we document that current illiquidity may interact with past real estate borrowing to affect the likelihood of extreme events. These latter empirical findings corroborate the theoretical and anecdotal evidence of Allen and Gale (2000) and Fisher (1933), and support the idea that extremes may be endogenous.

6 Conclusions

In light of recent developments in financial markets, our paper develops and tests a simple, positive approach to extreme events. This approach suggests that some extreme events may

[53] These three statistics test the null hypothesis that all the explanatory variables have zero coefficients.

[54] The estimation reports two intercepts because cumulative logit computes it that way, one less than the number of categories in the dependent variable

vary systematically over time, and might be explained and forecasted on the basis of economic theory. In addition to creating a taxonomy, we have four main contributions. First, we distinguish exogenous from endogenous extremes, the latter of which can be understood in the framework of externalities. This distinction is particularly important when there is the possibility of large spillovers, and has immediate policy implications: for truly exogenous extremes, we must often focus on ex post protection, while for endogenous extremes, we can use economic incentives. Therefore, in tackling subprime issues, or anywhere that individuals' actions spill over to harm economic stability, we have at our disposal a new set of public finance tools in addition to the traditional solutions of interest rates. Second, we show the 'signature' of different types of extremes. In relation to the signature of extremes, we provide some insight on their incidence and size. According to equation (5), they are more responsive to borrowing if expected costs are lower and credit is easier. According to equation (14), extremes have a higher incidence if correlated liquidity shocks dominate volatility, and if the degree of securitization m is large. Third, in light of our model and empirical findings that extremes have memory and may be endogenous, we propose coordination of global regulatory authorities in controlling extreme events. In addition to providing incentives for borrowers and lenders, this global coordination will involve monitoring and sharing information on liquidity and extremes, activities that have global public good qualities. Finally, on the empirical side, we have computed extreme probabilities for various reference periods, and have shown that in many cases extremes are neither rare nor constant. We carry out our estimation on several reference periods, which give us some degree of robustness. In most cases extreme probabilities are significantly different from zero, and have strong autoregressive components–there is memory in extremes. An exploratory vector autoregression analysis suggests that some instruments based on theory may have small effects on extreme probability that evolve over many months. More encouragingly, a cumulative logistic analysis shows that between 1989 to 2005, real estate borrowing and (to a smaller extent) market liquidity can help to explain the likelihood of experiencing extreme events.

It is important to bear in mind that while our paper begins to open the black box of extremes, we do not claim to predict all extremes. Instead, we wish to show that, far from being random, some extremes may have similar dynamic patterns, and may be related to economic fundamentals. Our central concepts, embodied in equations (5) and (14), are the externality argument, and the notion that information-based incentives are counteracted by liquidity and credit cycles before extreme events. These concepts yield quite new and unexpected implications for economic behavior and regulatory policy in securities markets.

A major implication is that if we can identify the signature of endogenous extremes, we may potentially counter them using other tools in addition to traditional interest rate management by central banks. Thus, it is not necessary to try to remove extremes ex post, agents may do it themselves, given the right ex ante incentives. Another implication is that anything driving the credit cycle will increase the likelihood of extremes, for example, innovation or loose interest rates. A final implication is that global coordination of regulatory bodies extends the idea of ’too big to fail’ to an international setting, where the institution in question is the entire global economy. In the future, such coordination may help ensure effective domestic and global diversification.

This paper may be seen as a first step on the road to incorporating extreme events into standard economic analysis. Even if the particular channel of endogeneity differs from the one we focus on (borrowing), the message remains: endogenous extreme events may be prevented using tools from public finance. By viewing extremes as the outcome of optimizing behavior, we can attempt to address the proximate cause, for example, reducing the demand for overborrowing by subsidizing certain house purchases. Our approach differs from previous work because we give a method for computing, detecting and predicting excessive extreme events. Another difference is that we suggest ex ante and ex post methods for dealing with extremes. Important extensions to this work include dynamic modelling, and identifying the various channels of endogenous extremes encountered in practice. Refinements of this approach are an exciting task for future research.

References

Acharya, V., and S. Schaefer, 2005, Understanding and managing correlation risk and liquidity risk, Working paper, London Business School.

Agarwal, S., J. Driscoll, X. Gabaix, and D. Laibson, 2007, The Age of Reason: Financial Decisions Over the Lifecycle, Working paper, Harvard.

Alizadeh, S., M. Brandt, and F. Diebold, 2002, Range-based estimation of stochastic volatility models, *Journal of Finance* LVII, 1047–1091.

Allais, M., 1953, Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'ecole americaine, *Econometrica* 21, 503–546.

Allen, F., and D. Gale, 2000, *Comparing Financial Systems*. (MIT Press MA).

Amihud, Y., and H. Mendelson, 1986, Asset Pricing and the bid-ask spread, *Journal of Financial Economics* 17, 223–249.

Andvig, J., and K. Moene, 1990, How corruption may corrupt, *Journal of Economic Behavior and Organization* 13, 63–76.

Ang, Andrew, and Geert Bekaert, 2002, International Asset Allocation with Regime Shifts, *Review of Financial Studies* 15, 1137–87.

Arrow, K., and G. Debreu, 1954, Existence of an equilibrium for a competitive economy, *Econometrica* 22, 265–290.

Baele, L., G. Bekaert, and K. Inghelbrecht, 2007, The determinants of stock and bond return comovements, Working paper, National Bank of Belgium.

Baker, M., and J. Wurgler, 2007, Investor sentiment in the stock market, *Journal of Economic Perspectives* 21, 129 – 151.

Barro, R., 2006, Rare Disasters and Asset Markets in the Twentieth Century, *Quarterly Journal of Economics* 121, 823–866.

Barron, G., and I. Erev, 2003, Small feedback-based decisions and their limited correspondence to description-based decisions, *Journal of Behavioral Decision Making* 16, 215–233.

Bazerman, M., and D. Watkins, 2004, *Predictable Surprises*. (Harvard Business School Press).

Below, R., D. Guha-Sapir, P. Hoyois, and J-M. Scheuren, 2007, *Annual Disaster Statistical Review: Numbers and Trends, 2006*. (Center for Research on the Epidemiology of Disasters (CRED)).

Bhide, A., 2000, Taking care: How concerns about prior knowledge affect the financing of novel projects, Working paper, Columbia Business School.

Blanchard, O., and N. Kiyotaki, 1987, Monopolistic competition and the effects of aggregate demand, *American Economic Review* 77, 647–666.

Bordo, M., B. Eichengreen, D. Klingebiel, and M. Martinez-Peria, 2001, Financial crises: Lessons from the last 120 years, *Economic Policy* April, 52–82.

Caballero, R., and A. Krishnamurthy, 2007, Collective risk management in a flight to quality episode, *Journal of Finance* Forthcoming.

Cappiello, L., R. F. Engle, and K. Sheppard, 2006, Asymmetric dynamics in the correlations of global equity and bond returns, *Journal of Financial Econometrics* 4, 537–572.

Casella, G., and R. Berger, 1990, *Statistical Inference*. (Duxbury Press).

Chichilnisky, G., 2000, An axiomatic approach to choice under uncertainty with catastrophic risks, *Resource and Energy Economics* 22, 221–231.

Chichilnisky, G., 2007, The Topology of Fear, Working paper, Columbia University.

Chichilnisky, G, and H. Wu, 2006, General equilibrium with endogenous uncertainty and default, *Journal of Mathematical Economics* 42.

Chollete, L., V. de la Pena, and C. Lu, 2006, Security comovement: Alternative measures, and implications for portfolio diversification, Working paper, Columbia University and NHH.

Chollete, L., R. Nas, and J. Skjeltorp, 2007, What Captures Liquidity Risk? A Comparison of Trade- and Order-Based Factors, Working paper, Norwegian Central Bank.

Cooper, R., 1999, *Coordination Games: Complementarities and Macroeconomics*. (Cambridge Press New York).

Danielsson, J., and H. Shin, 2003, Endogenous risk, Working paper, LSE.

de Haan, L., and A. Sinha, 1999, Estimating the probability of a rare event, *The Annals of Statistics* 27, 732–759.

Debreu, G., 1959, *Theory of Value: An Axiomatic Analysis of Economic Equilibrium*. (Yale University Press).

Debreu, G., 1970, Economies with a finite set of equilibria, *Econometrica* 38, 387–392.

Dembo, A., J. Deuschel, and D. Duffie, 2003, Large portfolio losses, Working paper, Stanford University.

Diamond, D., 1984, Financial intermediation and delegated monitoring, *Review of Economic Studies* 51, 393–414.

Domowitz, I., O. Hansch, and X. Wang, 2005, Liquidity commonality and return co-movement, *Journal of Financial Markets* 8, 351–376.

Ellsberg, D., 1961, Risk, ambiguity and the savage axioms, *Quarterly Journal of Economics* 76, 643–669.

Evans, M., 1996, Peso problems: Their theoretical and empirical implications, in G. Maddala, and C. Rao, eds.: *Handbook of Statistics, Vol 14* (Elsevier Science,).

Fisher, I., 1933, The Debt-deflation theory of great depressions, *Econometrica* 1, 337–357.

Friedman, B., and D. Laibson, 1989, Economic implications of extraordinary movements in stock prices, *Brookings Papers on Economic Activity* pp. 137–172.

Granger, C., H. White, and M. Kamstra, 1989, Interval forecasting: an analysis based upon ARCH-quantile estimators, *Journal of Econometrics* 40, 87–96.

Grossman, S., 1988, An analysis of the implications for stock and futures price volatility of program trading and dynamic hedging strategies, *Journal of Business* 61, 275–298.

Hansen, B., 2006, Interval forecasts and parameter uncertainty, *Journal of Econometrics* 135, 377–398.

Harris, L., 2003, *Trading and Exchanges. Market Microstructure for Practitioners*. (Oxford University Press).

Herring, V., and S Wachter, 2005, Bubbles in real estate markets, in W. Hunter, G. Kaufman, and M. Pomerleano, eds.: *Asset Price Bubbles* (MIT Press, MA).

Hertwig, R., G. Barron, E. Weber, and I Erev, 2005, Decisions from experience and the effect of rare events in risky choice, *Psychological Science* 15, 534–539.

Ibragimov, R., and J. Walden, 2007, The limits of diversification when losses may be large, *Journal of Banking and Finance* 31, 2551–2569.

Ingersoll, J., 1987, *Theory of Financial Decision Making*. (Rowman and Littlefield Publishers).

Jansen, D., and C. de Vries, 1991, On the frequency of large stock returns: Putting booms and busts into perspective, *The Review of Economics and Statistics* 73, 18–24.

King, G., and L. Zeng, 2001, Explaining rare events in international relations, *International Organization* 55, 693–715.

Kiyotaki, N., and J. Moore, 1997, Credit cycles, *Journal of Political Economy* 105, 211–248.

Lucas, R., 1978, Asset prices in an exchange economy, *Econometrica* 46, 1429–1445.

Mas-Colell, A., M. Whinston, and J. Green, 1995, *Microeconomic Theory*. (Oxford Press).

Minsky, H., 1982, *Can it Happen Again? Essays on Instability and Finance*. (M. E. Sharpe).

Pastor, Lubos, and Robert Stambaugh, 2003, Liquidity risk and expected stock returns, *Journal of Political Economy* 111, 642–685.

Pavlov, A., and S. Wachter, 2006, The inevitability of marketwide underpricing of mortgage default risk, *Real Estate Economics* 34, 479–496.

Pavlov, A., and S. Wachter, 2007, Underpriced lending and real estate markets, Working paper, The Wharton School, University of Pennsylvania.

Rabin, M., 2002, Inference by believers in the law of small numbers, *Quarterly Journal of Economics* 117, 775–816.

Riley, J., 2001, Silver signals: Twenty-five years of screening and signalling, *Journal of Economic Literature* XXXIX, 221–231.

Sandmo, A., 2003, International aspects of public goods provision, in I. Kaul, P. Conceicao, K. Le Goulven, and R. Mendoza, eds.: *Providing Global Public Goods* (Oxford Press,).

Sandmo, A., 2005, The theory of tax evasion: A retrospective view, *National Tax Journal* LVIII, 643–663.

Sheard, P., 1994, Reciprocal delegated monitoring in the Japanese Main Bank system, *Journal of the Japanese and International Economies* 8, 1–21.

Stern, N., 2007, *The Economics of Climate Change: The Stern Review*. (Cambridge Press).

Stiglitz, J., and A. Weiss, 1981, Credit rationing in markets with imperfect information, *American Economic Review* 71, 393–410.

Varian, H., 1992, *Microeconomic Analysis*. (Norton and Company).

Weber, E., 2006, Experience-based and description-based perceptions of long-term risk: Why global warming does not scare us (yet), *Climatic Change* 77, 103–120.

Weber, E., 2007, Personal communication, October 18, 2007, .

Weitzman, M., 2007, Subjective expectations and asset return puzzles, *American Economic Review* Forthcoming.

Figure 1: Percentage Change in US House Prices.

The figure shows the percentage change in the Case-Shiller US House Price Index, relative to the previous year. Source: Standard and Poors.

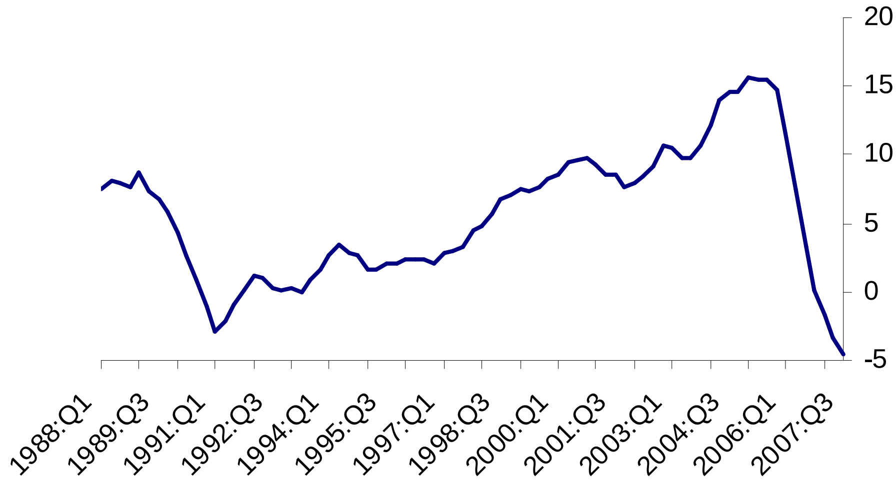


Figure 2: UK banks' price of borrowing.

The figure shows the price of interbank borrowing in the UK. The solid (red) line is the 3-month interbank rate and the dotted (green) line is the base rate. Source: DataStream.

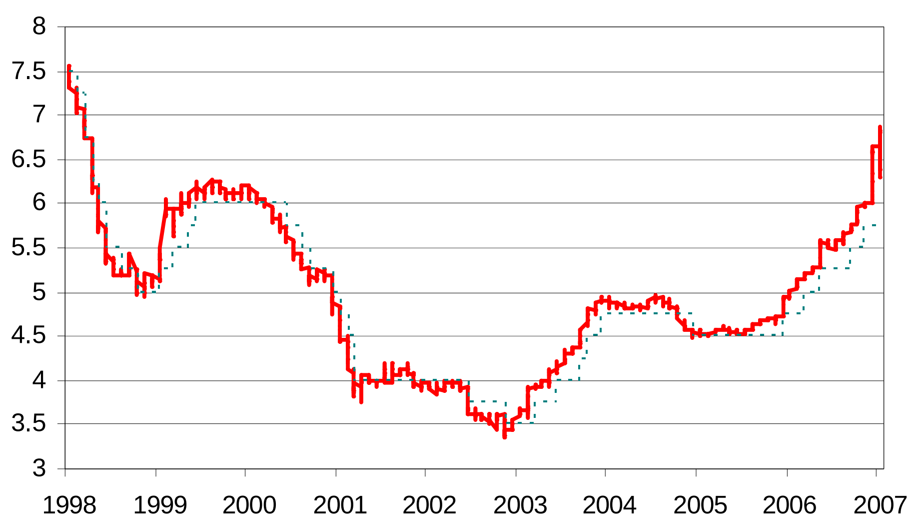


Table 1: Two Examples of Endogenous Probabilities

	Effects felt mainly in one market or transaction	*Spillover effects in many markets*
Asymmetric Information	**Moral Hazard**	
Symmetric or Asymmetric Information		**Endogenous Extremes**

Figure 3: Frequency and Impact of Extreme Events in Nature, 1974-2003.

Source: EM-DAT.

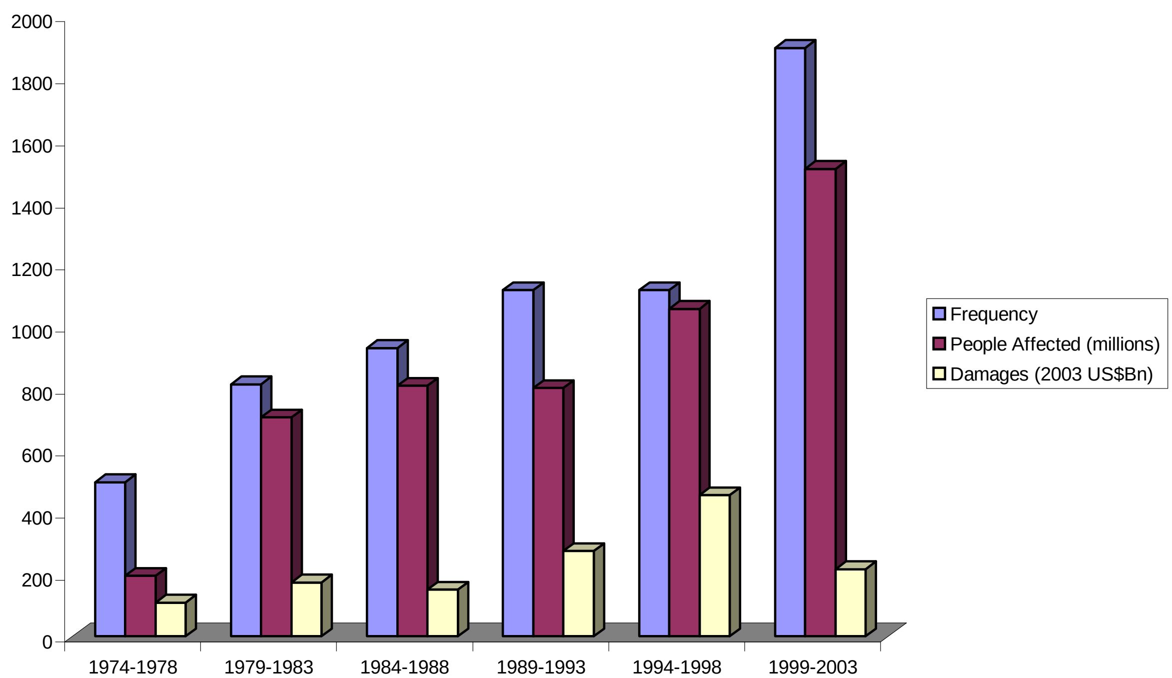


Figure 4: Sequence of Events.

The figure shows the timing of decisions in our stylized model. $\bar{p}$ is a threshold level of extreme probability where spillovers to other sectors begin. Endogenous extremes occur during easy credit regimes, exogenous extremes occur during hard credit regimes. The spillover costs c_0, c_1, c_2 include costs related to default, financial instability and aggregate uncertainty.

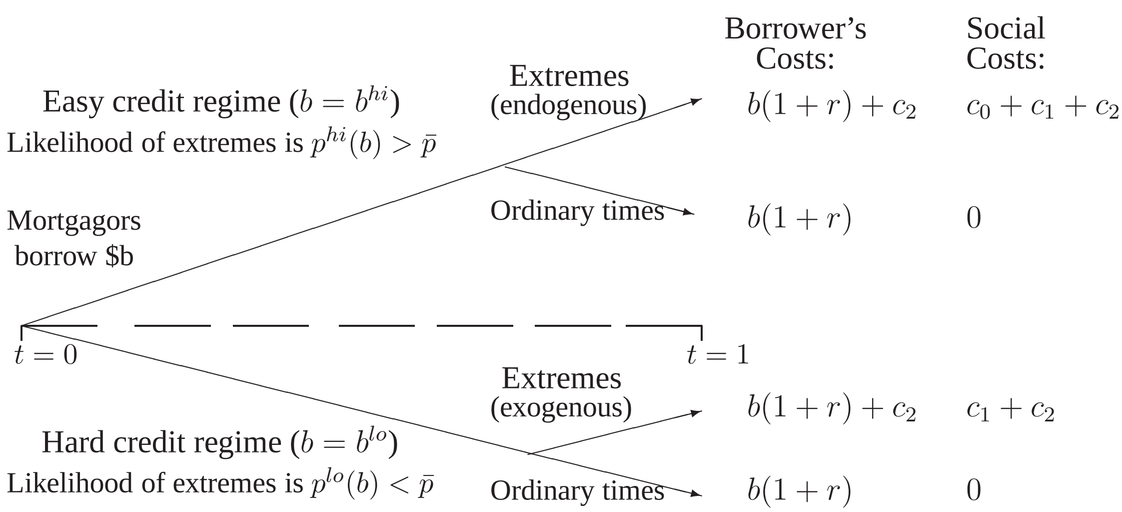


Figure 5: Marketwide Liquidity during the 1987 Crash.

The figure shows the level of the Pastor and Stambaugh (2003) liquidity measure in the period around the US stock market crash of 1987.

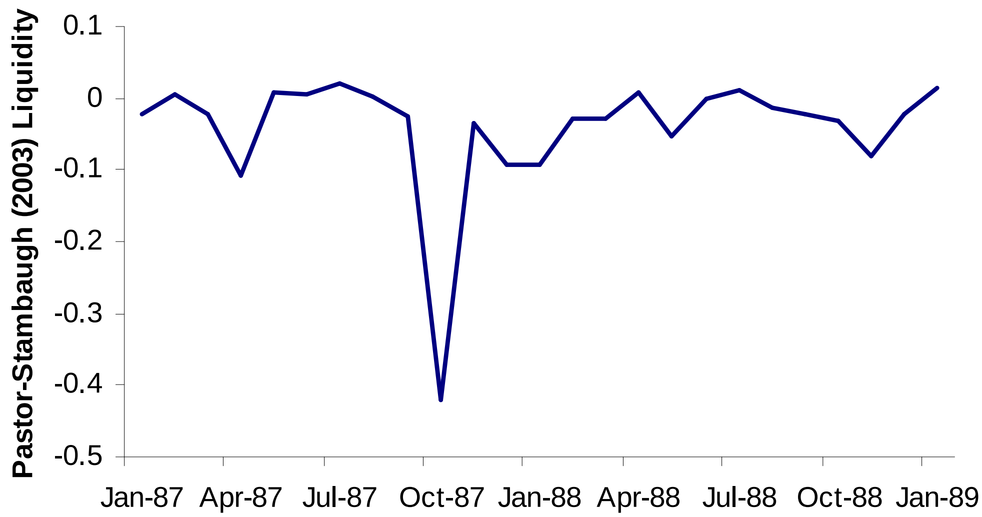


Figure 6: Marketwide Liquidity during the LTCM Event in 1998.

The figure shows the level of the Pastor and Stambaugh (2003) liquidity measure around the time of the LTCM events in summer 1998.

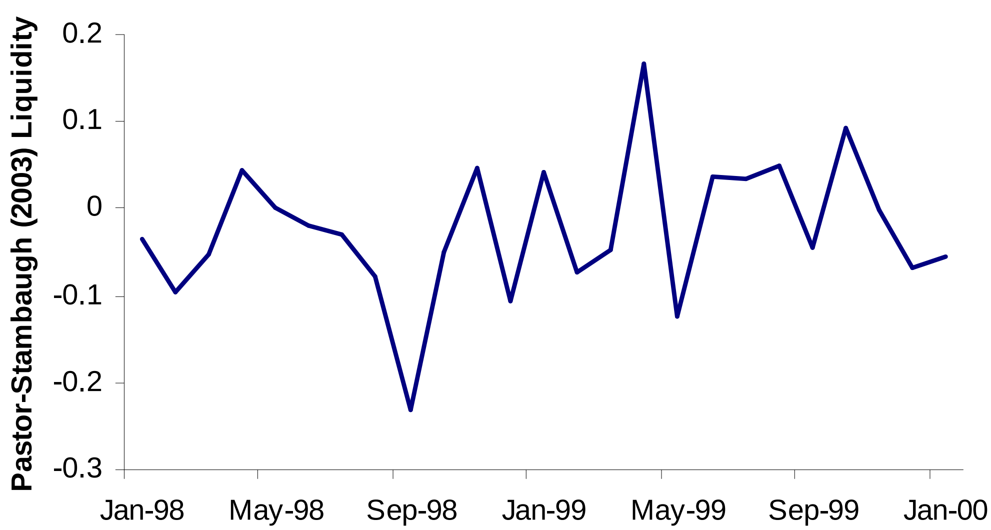
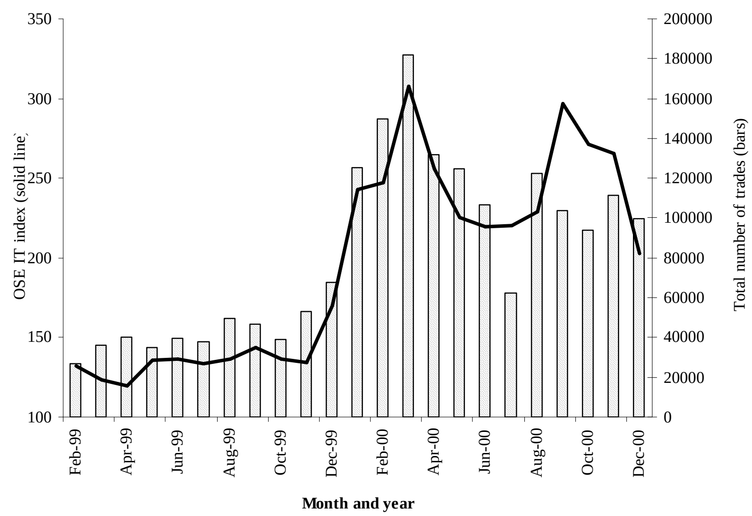


Figure 7: Marketwide Liquidity during the Burst of the Internet Bubble.

Figure (a) shows the level of the Oslo Stock Exchange IT index and the total number of trades each month. Figure (b) shows the average (across companies) quoted spread and depth at the Oslo Stock Exchange each month. Source: Chollete, Nas, and Skjeltorp (2007).

(a) Close price (index value) and trades



(b) Quoted spread and order book depth

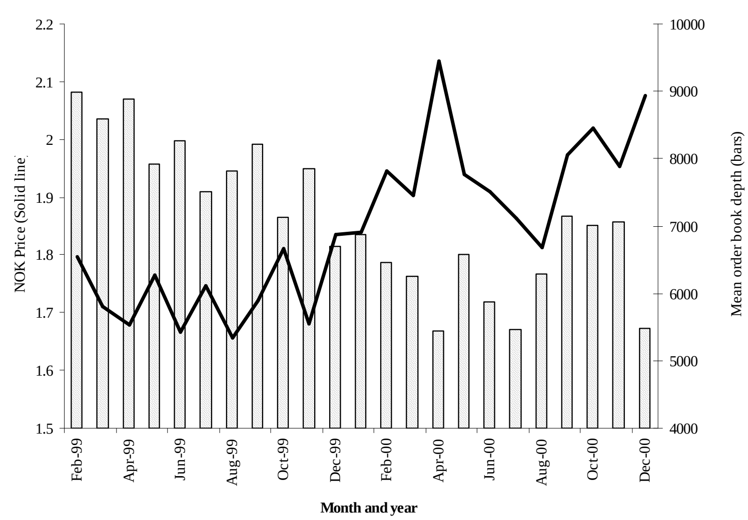


Figure 8: Time Series of Extremes

The figure shows a sample of the time series for various levels of extreme probabilities, from 1967 to 2007. The relevant reference period is 12 months.

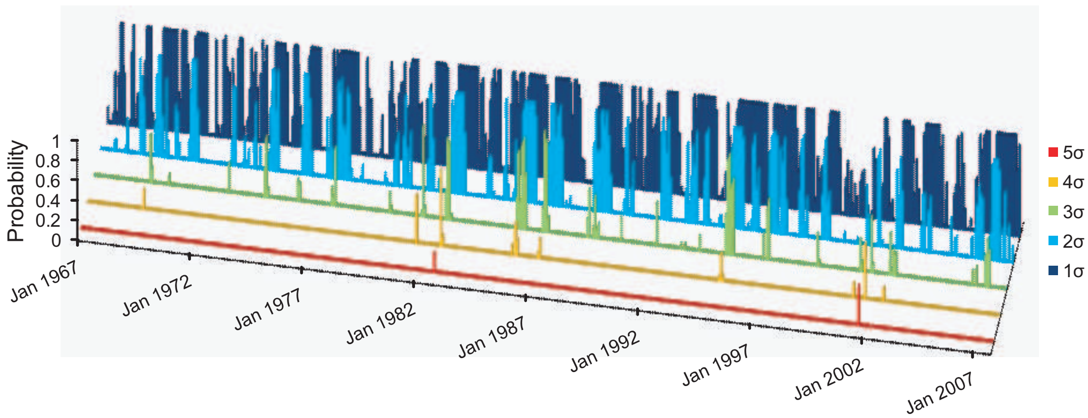


Figure 9: The Distribution of 1-sigma Events.

The histogram shows the empirical probability of Dow-Jones Industrial Average levels that exceed one standard deviation from the relevant median. The median is calculated over different reference samples, ranging from 12 months to 120 months.

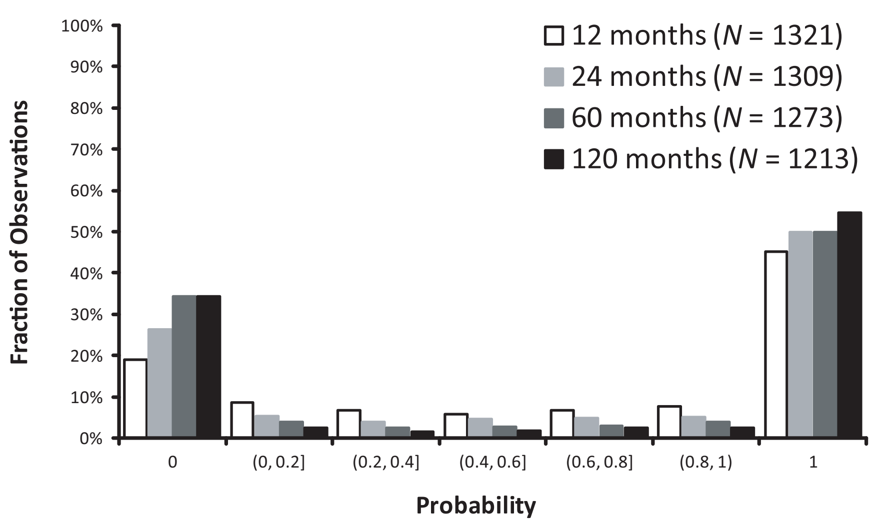


Figure 10: The Distribution of 2-sigma Events.

The histogram shows the empirical probability of Dow-Jones Industrial Average levels that exceed two standard deviations from the relevant median. The median is calculated over different reference samples, ranging from 12 months to 120 months.

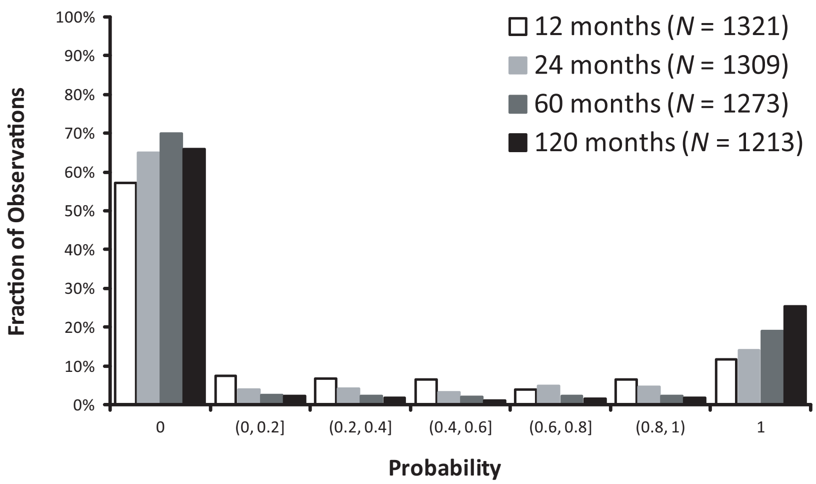


Figure 11: The Distribution of 3-sigma Events.

The histogram shows the empirical probability of Dow-Jones Industrial Average levels that exceed three standard deviations from the relevant median. The median is calculated over different reference samples, ranging from 12 months to 120 months.

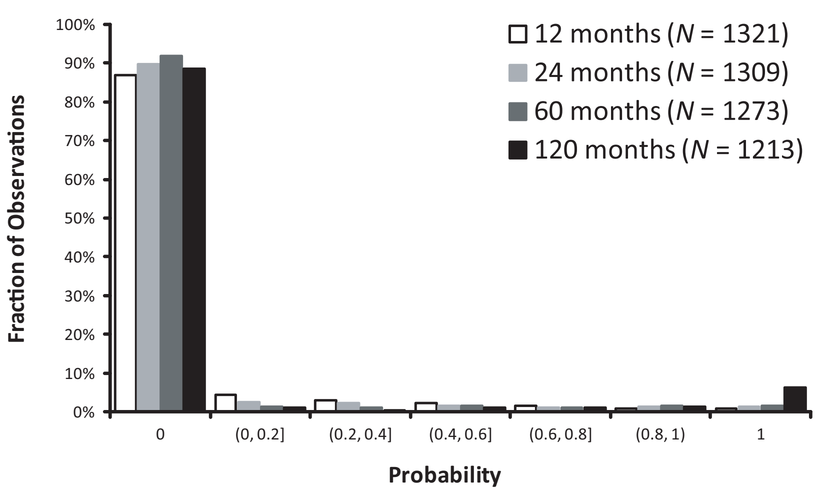


Figure 12: The Distribution of 4-sigma Events.

The histogram shows the empirical probability of Dow-Jones Industrial Average levels that exceed four standard deviations from the relevant median. The median is calculated over different reference samples, ranging from 12 months to 120 months.

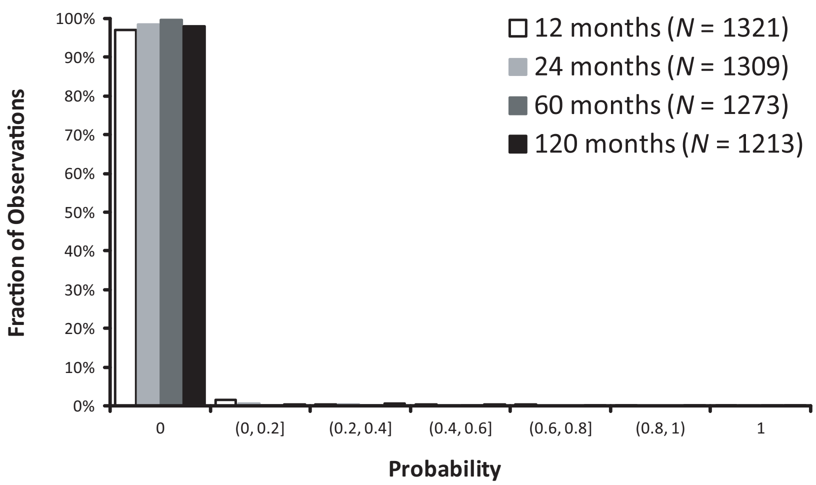


Figure 13: The Distribution of 5-sigma events.

The histogram shows the empirical probability of Dow-Jones Industrial Average levels that exceed five standard deviations from the relevant median. The median is calculated over different reference samples, ranging from 12 months to 120 months.

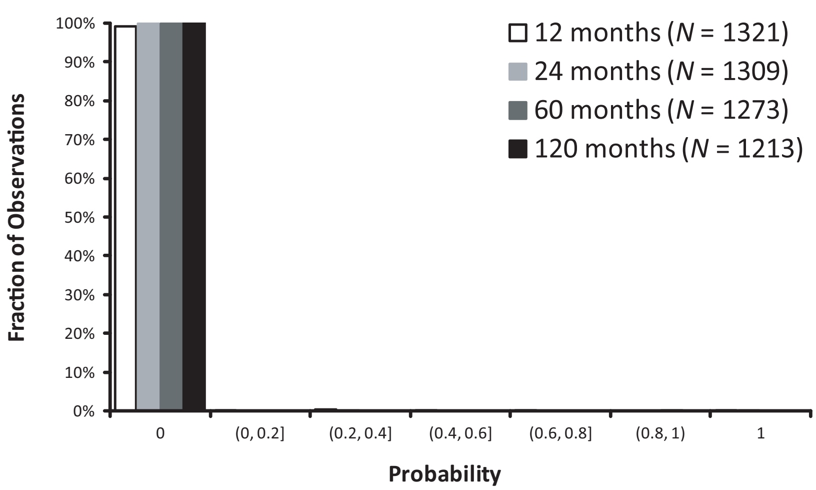


Table 2: Basic Properties of Extreme Probabilities $p_t(\omega)$

Panel A: 12-month reference period

	Mean	**Standard Deviation**	**P-value for t-test**	**P-value for sign rank test**
($\omega = 1$):	0.6288	0.4202	< 0.0001	<0.0001
($\omega = 2$):	0.2647	0.3806	< 0.0001	< 0.0001
($\omega = 3$):	0.0542	0.1768	< 0.0001	< 0.0001
($\omega = 4$):	0.0098	0.0739	< 0.0001	< 0.0001
($\omega = 5$):	0.0033	0.0437	0.0056	0.0020

Panel B: 24-month reference period

	Mean	**Standard Deviation**	**P-value for t-test**	**P-value for sign rank test**
($\omega = 1$):	0.6189	0.4439	< 0.0001	< 0.0001
($\omega = 2$):	0.2489	0.3906	< 0.0001	< 0.0001
($\omega = 3$):	0.0517	0.1844	< 0.0001	< 0.0001
($\omega = 4$):	0.0044	0.0502	0.0014	< 0.0001
($\omega = 5$):	0.0010	0.0291	0.2060	0.5000

Panel C: 60-month reference period

	Mean	**Standard Deviation**	**P-value for t-test**	**P-value for sign rank test**
($\omega = 1$):	0.5787	0.4680	< 0.0001	< 0.0001

($\omega = 2$):	0.2450	0.4083	< 0.0001	< 0.0001
($\omega = 3$):	0.0496	0.1899	< 0.0001	< 0.0001
($\omega = 4$):	0.0026	0.0445	0.0368	0.0313
($\omega = 5$):	0.0007	0.0259	0.3175	1.000

Panel D: 120-month reference period

	Mean	**Standard Deviation**	**P-value for t-test**	**P-value for sign rank test**
($\omega = 1$):	0.6038	0.4711	< 0.0001	< 0.0001
($\omega = 2$):	0.2961	0.4413	< 0.0001	< 0.0001
($\omega = 3$):	0.0908	0.2722	< 0.0001	< 0.0001
($\omega = 4$):	0.0086	0.0732	< 0.0001	< 0.0001
($\omega = 5$):	0.0000	0.0000	.	.

The table shows stylized facts for the time series of extreme probabilities $p_t(\omega)$. As in the text, ω denotes the number of standard deviations away from the relevant median. The t- and sign rank tests examine whether the mean differs significantly from zero.

Table 3: P-values for Test of Differences *Within* Reference Periods

	1-σ vs 2-σ	1-σ vs 3-σ	1-σ vs 4-σ	1-σ vs 5-σ	2-σ vs 3-σ	2-σ vs 4-σ	2-σ vs 5-σ	3-σ vs 4-σ	3-σ vs 5-σ	4-σ vs 5-σ
Panel A: 12-month reference period										
t-test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Sign test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
SR test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Panel B: 24-month reference period										
t-test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	0.0018
Sign test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
SR test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Panel C: 60-month reference period										
t-test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	<0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	0.0530
Sign test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	<0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	0.0313
SR test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	<0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	0.0313
Panel D: 120-month reference period										
t-test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	<0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Sign test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	<0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
SR test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	<0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001

The table shows the p-values from statistical tests for significant differences in the means of our $p_t(\omega)$ series, for various levels of extreme events. SR denotes the sign rank test.

Table 4: P-values for Test of Differences *Across* Reference Periods

Panel A: 1-σ Events

	12- vs 24-month	24- vs 60-month	60- vs 120-month	12- vs 120-month
t-test:	0.6024	0.0032	0.0408	0.1750
Sign test:	0.8438	0.0009	0.0049	0.4969
SR test:	0.6499	0.0028	0.0469	0.0710

Panel B: 2-σ Events

	12- vs 24-month	24- vs 60-month	60- vs 120-month	12- vs 120-month
t-test:	0.1012	0.8694	$<$0.0001	0.0417
Sign test:	0.1198	0.1902	$<$0.0001	0.6960
SR test:	0.0998	0.9772	$<$0.0001	0.0146

Panel C: 3-σ Events

	12- vs 24-month	24- vs 60-month	60- vs 120-month	12- vs 120-month
t-test:	0.9167	0.9404	$<$0.0001	$<$0.0001
Sign test:	0.3580	0.1679	$<$0.0001	0.9508
SR test:	0.8088	0.7975	$<$0.0001	$<$0.0001

Panel D: 4-σ Events

	12- vs 24-month	24- vs 60-month	60- vs 120-month	12- vs 120-month
t-test:	0.0073	0.3936	0.0187	0.8629
Sign test:	0.0026	0.0490	0.0023	0.4885
SR test:	0.0004	0.2112	0.0114	0.7465

Panel E: 5-σ Events

	12- vs 24-month	24- vs 60-month	60- vs 120-month	12- vs 120-month
t-test:	0.0203	0.2306	0.3175	0.0111
Sign test:	0.0156	0.5000	1.0000	0.0078
SR test:	0.0156	0.5000	1.0000	0.0078

The table shows the p-values from statistical tests for significant differences in the means of our series, for various levels of extreme events. SR denotes sign rank test.

Table 5: Time Series properties of Extreme Probabilities

Panel A: 12-month reference period					
	1-σ	**2-σ**	**3-σ**	**4-σ**	**5-σ**
Q-test, original series	$<$ 0.0001	$<$ 0.0001	$<$ 0.0001	$<$0.0001	0.4192
Selected Model:	AR(2)	AR(2)	AR(2)	AR(2)	AR(1)
Q-test of residuals	0.1729	0.0266	0.3694	0.9654	0.9982
Panel B: 24-month reference period					
Q-Test, original series	$<$ 0.0001	$<$ 0.0001	$<$ 0.0001	$<$0.0001	1.0000
Selected Model:	AR(2)	AR(2)	AR(2)	AR(4)	NA
Q-test of residuals	0.0952	0.0949	0.8880	0.2330	NA
Panel C: 60-month reference period					
Q-test, original series	$<$ 0.0001	$<$ 0.0001	$<$ 0.0001	$<$0.0001	1.0000
Selected Model:	AR(1)	AR(4)	AR(4)	AR(3)	NA
Q-test of residuals	0.4002	0.0575	0.0166	0.1698	NA
Panel D: 120-month reference period					
Q-test, original series	$<$ 0.0001	$<$ 0.0001	$<$ 0.0001	$<$0.0001	NA
Selected Model:	AR(2)	AR(1)	AR(3)	AR(5)	NA
Q-test of residuals	0.0327	0.0349	0.2157	0.5157	

The table shows the results of time series estimation of the $p_t(\omega)$ series, for different reference periods. NA denotes 'not applicable', where estimation did not converge.

Table 6: k-Period Impulse Responses: Sentiment and Borrowing

	1-σ		2-σ		3-σ		4-σ	
Panel A: 12-month reference period								
Impulse Response for DSENT1								
k=1	-0.05897	(0.04603)	0.02139	(0.04586)	0.06479	(0.04618)	0.07833	(0.04625)
k=6	-0.00196	(0.00278)	-0.00381	(0.00331)	-0.00328	(0.00245)	0.00001	(0.00015)
k=12	-0.00001	(0.00006)	-0.00003	(0.00012)	-0.00003	(0.00026)	0.00001	(0.00016)
Impulse Response for REALLOAN								
k=1	0.00008	(0.00011)	0.00012	(0.00011)	0.00014	(0.00011)	0.00017	(0.00011)
k=6	0.00002	(0.00017)	0.00008	(0.00018)	0.00035	(0.00017)	0.00020	(0.00012)
k=12	0.00002	(0.00018)	0.00008	(0.00019)	0.00037	(0.00018)	0.00020	(0.00013)
Panel B: 24-month reference period								
Impulse Response for DSENT1								
k=1	-0.00718	(0.04629)	0.07087	(0.04624)	0.03068	(0.03643)	0.02286	(0.04619)
k=6	-0.00969	(0.00783)	-0.00305	(0.00669)	0.00248	(0.00297)	0.00014	(0.00080)
k=12	-0.00175	(0.00159)	-0.00037	(0.00086)	0.00014	(0.00020)	-0.00001	(0.00027)
Impulse Response for REALLOAN								
k=1	0.00017	(0.00011)	0.00013	(0.00011)	0.00003	(0.00010)	0.00032	(0.00011)
k=6	0.00011	(0.00019)	0.00009	(0.00019)	0.00009	(0.00018)	0.00038	(0.00013)
k=12	0.00010	(0.00023)	0.00008	(0.00022)	0.00010	(0.00019)	0.00040	(0.00014)
Panel C: 60-month reference period								
Impulse Response for DSENT1								
k=1	-0.02476	(0.02344)	0.00608	(0.04595)	0.00997	(0.02964)	0.07863	(0.04503)
k=6	-0.01091	(0.00995)	-0.00619	(0.01102)	0.00248	(0.00734)	0.00241	(0.00228)
k=12	-0.00468	(0.00425)	-0.00294	(0.00524)	0.00053	(0.00159)	0.00003	(0.00006)
Impulse Response for REALLOAN								
k=1	-0.00006	(0.00009)	0.00007	(0.00011)	0.00002	(0.00009)	0.00002	(0.00011)
k=6	-0.00025	(0.00019)	-0.00011	(0.00018)	0.00007	(0.00020)	-0.00005	(0.00014)

k=12	-0.00036	(0.00026)	-0.00024	(0.00026)	0.00009	(0.00024)	-0.00005	(0.00014)
Panel D: 120-month reference period								
Impulse Response for DSENT1								
k=1	-0.03066	(0.02124)	0.00413	(0.01622)	-0.00825	(0.04619)	0.00757	(0.04618)
k=6	-0.01547	(0.01019)	0.00282	(0.01044)	0.00090	(0.01067)	-0.00376	(0.00486)
k=12	-0.00774	(0.00513)	0.00200	(0.00710)	0.00037	(0.00446)	-0.00035	(0.00056)
Impulse Response for REALLOAN								
k=1	-0.00007	(0.00009)	0.00002	(0.00008)	0.00002	(0.00011)	0.00018	(0.00011)
k=6	-0.00034	(0.00018)	0.00012	(0.00016)	0.00013	(0.00018)	0.00036	(0.00018)
k=12	-0.00052	(0.00026)	0.00021	(0.00026)	0.00019	(0.00026)	0.00041	(0.00021)

The table shows the results of VAR estimation, where the impulse responses are for our $p_t(\omega)$ series, from January 1967 to December 2005. DSENT1 is the investor sentiment measure of Baker and Wurgler (2007). REALLOAN is the ratio of real estate loans to total consumer loans in the US. Standard errors are in parentheses.

Table 7: k-Period Impulse Responses: Liquidity and Securitization

	1-σ		**2-σ**		**3-σ**	
Panel A: 12-month reference period						
Impulse Response for LIQ						
k=1	-0.00070	(0.00374)	0.00384	(0.00367)	-0.00057	(0.00359)
k=6	0.00013	(0.00204)	-0.00088	(0.00179)	0.00088	(0.00206)
k=12	0.00002	(0.00020)	0.00008	(0.00034)	-0.00006	(0.00037)
Impulse Response for SECPCT						
k=1	0.00021	(0.00149)	-0.00015	(0.00145)	0.00051	(0.00143)
k=6	0.00016	(0.00074)	0.00152	(0.00069)	0.00130	(0.00077)
k=12	0.00003	(0.00016)	0.00013	(0.00022)	0.00002	(0.00024)
Panel B: 24-month reference period						
Impulse Response for LIQ						
k=1	0.00460	(0.00365)	0.00433	(0.00366)	NA	
k=6	0.00040	(0.00164)	-0.00077	(0.00164)		
k=12	0.00007	(0.00041)	-0.00007	(0.00029)		
Impulse Response for SECPCT						
k=1	-0.00028	(0.00148)	-0.00175	(0.00143)		
k=6	0.00035	(0.00067)	0.00149	(0.00069)		
k=12	0.00009	(0.00018)	0.00018	(0.00024)		
Panel C: 60-month reference period						
Impulse Response for LIQ						
k=1	0.00203	(0.00361)	0.00701	(0.00368)	NA	
k=6	-0.00031	(0.00185)	0.00010	(0.00161)		
k=12	0.00100	(0.00130)	-0.00001	(0.00098)		
Impulse Response for SECPCT						
k=1	0.00069	(0.00148)	-0.00012	(0.00150)		
k=6	0.00117	(0.00090)	0.00043	(0.00065)		

k=12	0.00130	(0.00077)	0.00030	(0.00041)		
Panel D: 120-month reference period						
Impulse Response for LIQ						
k=1	NA		0.00104	(0.00364)	-0.00040	(0.00360)
k=6			-0.00128	(0.00113)	-0.00089	(0.00232)
k=12			-0.00106	(0.00096)	-0.00157	(0.00130)
Impulse Response for SECPCT						
k=1			0.00249	(0.00144)	0.00190	(0.00143)
k=6			0.00128	(0.00043)	-0.00045	(0.00091)
k=12			0.00102	(0.00038)	-0.00017	(0.00050)

The table shows the results of VAR estimation where the impulse responses are for our $p_t(\omega)$ series, from January 1989 to December 2006. LIQ is the liquidity measure of Pastor and Stambaugh (2003). SECPCT is the percentage change in the value of securitized loans in the US. NA denotes a model where estimation did not converge. Standard errors are in parentheses.

Table 8: Cumulative Logistic Estimation, 12-month Reference Period

Panel A: 1-σ events

	Intercept1	**Intercept2**	**REALLOAN**	**DSENT1**	**SECPCT**	**LIQ0**	**LIQ1**	**REALLOAN*LIQ0**	**REALLOAN*LIQ1**
Coefficient	5.3675	4.5980	-21.5218	0.1487	-2.3985	-5.7905	-1.5121	26.0566	9.5101
	(0.0011)	(0.0047)	(0.0052)	(0.2759)	(0.6978)	(0.0179)	(0.4771)	(0.0197)	(0.3332)
Tests of Overall Fit (p-values):		*LR*	0.0186						
		Score	0.0175						
		Wald	0.0409						

Panel B: 2-σ events

	Intercept1	**Intercept2**	**REALLOAN**	**DSENT1**	**SECPCT**	**LIQ0**	**LIQ1**	**REALLOAN*LIQ0**	**REALLOAN*LIQ1**
Coefficient	8.6457	7.9444	-45.0513	-0.0339	-9.3293	-8.0929	-2.9394	39.1506	15.3235
	(0.0112)	(0.0195)	(0.0091)	(0.8263)	(0.1551)	(0.0443)	(0.4729)	(0.0472)	(0.4558)
Tests of Overall Fit (p-values):		*LR*	0.0011						
		Score	0.0094						
		Wald	0.0275						

Panel C: 3-σ events

	Intercept1	**Intercept2**	**REALLOAN**	**DSENT1**	**SECPCT**	**LIQ0**	**LIQ1**	**REALLOAN*LIQ0**	**REALLOAN*LIQ1**
Coefficient	8.7419	7.6638	-57.7413	0.2965	-9.3413	-15.1531	-6.2688	68.5561	34.5645
	(0.3130)	(0.3765)	(0.2006)	(0.4033)	(0.4732)	(0.1723)	(0.5143)	(0.2016)	(0.4852)
Tests of Overall Fit (p-values):		*LR*	0.3148						
		Score	0.4609						
		Wald	0.5756						

The table shows the results of logistic regression estimation, from January 1989 to December 2005. The dependent, categorical variable is $p_t(\omega)$, which is ranked as Low (less than 0.33), Medium (between 0.33 and 0.67), and High (above 0.67). DSENT1 is the investor sentiment measure of Baker and Wurgler (2007). LIQ0 and LIQ1 correspond to low and medium levels of liquidity, SECPCT is the percentage change in securitized loans, REALLOAN is the ratio of real estate loans to other consumer loans. A chi square statistic is computed as the squared ratio of each parameter to its standard error, and the corresponding p-values are in parentheses.

Table 9: Cumulative Logistic Estimation, 24-month Reference Period

Panel A: 1-σ events

	Intercept1	Intercept2	REALLOAN	DSENT1	SECPCT	LIQ0	LIQ1	REALLOAN*LIQ0	REALLOAN*LIQ1
Coefficient	8.0943	7.6624	-32.8161	0.0229	0.2950	-2.7495	-1.8432	11.9173	11.2734
	(<.0001)	(<.0001)	(0.0003)	(0.8768)	(0.9685)	(0.3271)	(0.4599)	(0.3483)	(0.3182)
Tests of Overall Fit (p-values):		*LR*	<.0001						
		Score	<.0001						
		Wald	<.0001						

Panel B: 2-σ events

	Intercept1	Intercept2	REALLOAN	DSENT1	SECPCT	LIQ0	LIQ1	REALLOAN*LIQ0	REALLOAN*LIQ1
Coefficient	1.9688	1.6283	-15.3030	-0.0169	7.4385	1.5592	-0.1817	-5.0872	1.3811
	(0.3934)	(0.4802)	(0.1751)	(0.9135)	(0.2383)	(0.6483)	(0.9515)	(0.7540)	(0.9245)
Tests of Overall Fit (p-values):		*LR*	0.0851						
		Score	0.1339						
		Wald	0.1613						

Panel C: 3-σ events

	Intercept1	Intercept2	REALLOAN	DSENT1	SECPCT	LIQ0	LIQ1	REALLOAN*LIQ0	REALLOAN*LIQ1
Coefficient	-1.4408	-2.3552	-6.4250	0.1366	2.2496	-0.6735	0.3425	4.2006	-2.9245
	(0.6920)	(0.5189)	(0.7127)	(0.6338)	(0.8520)	(0.8974)	(0.9489)	(0.8624)	(0.9094)
Tests of Overall Fit (p-values):		*LR*	0.9911						
		Score	0.9930						
		Wald	0.9943						

The table shows the results of logistic regression estimation, from January 1989 to December 2005. The dependent, categorical variable is $p_t(\omega)$, which is ranked as Low (less than 0.33), Medium (between 0.33 and 0.67), and High (above 0.67). DSENT1 is the investor sentiment measure of Baker and Wurgler (2007). LIQ0 and LIQ1 correspond to low and medium levels of liquidity, SECPCT is the percentage change in securitized loans, REALLOAN is the ratio of real estate loans to other consumer loans. P-values based on chi-square tests are in parentheses.

Table 10: Cumulative Logistic Estimation, 60-month Reference Period

Panel A: 1-σ events

	Intercept1	**Intercept2**	**REALLOAN**	**DSENT1**	**SECPCT**	**LIQ0**	**LIQ1**	**REALLOAN*LIQ0**	**REALLOAN*LIQ1**
Coefficient	16.9786	16.5095	-70.4190	0.1522	-10.5182	-1.8429	2.0033	10.0758	-6.1854
	(<.0001)	(<.0001)	(<.0001)	(0.4344)	(0.3580)	(0.7002)	(0.6823)	(0.6377)	(0.7765)
Tests of Overall Fit (p-values):		*LR*	<.0001						
		Score	<.0001						
		Wald	<.0001						

Panel B: 2-σ events

Coefficient	2.9659	2.7202	-18.6782	-0.0187	-0.8247	2.2365	1.8791	-10.9524	-7.0953
	(0.1951)	(0.2345)	(0.0962)	(0.9068)	(0.8959)	(0.5649)	(0.5405)	(0.5557)	(0.6379)
Tests of Overall Fit (p-values):		*LR*	0.0032						
		Score	0.0099						
		Wald	0.0326						

Panel C: 3-σ events

Coefficient	0.4406	0.00661	-16.0284	0.1180	-6.6053	8.8289	2.9406	-41.4283	-12.1881
	(0.9290)	(0.9989)	(0.5133)	(0.7092)	(0.5846)	(0.3623)	(0.6712)	(0.3911)	(0.7251)
Tests of Overall Fit (p-values):		*LR*	0.4572						
		Score	0.6365						
		Wald	0.7028						

The table shows the results of logistic regression estimation, from January 1989 to December 2005. The dependent, categorical variable is $p_t(\omega)$, which is ranked as Low (less than 0.33), Medium (between 0.33 and 0.67), and High (above 0.67). DSENT1 is the investor sentiment measure of Baker and Wurgler (2007). LIQ0 and LIQ1 correspond to low and medium levels of liquidity, SECPCT is the percentage change in securitized loans, REALLOAN is the ratio of real estate loans to other consumer loans. P-values based on chi-square tests are in parentheses.

Table 11: Cumulative Logistic Estimation, 120-month Reference Period

Panel A: 1-σ events

	Intercept1	**Intercept2**	**REALLOAN**	**DSENT1**	**SECPCT**	**LIQ0**	**LIQ1**	**REALLOAN*LIQ0**	**REALLOAN*LIQ1**
Coefficient	88.9107	NA	-349.3	-0.2846	-86.9502	189.6	79.0265	-834.0	-352.6
	(0.4142)	NA	(0.4126)	(0.9322)	(0.8750)	(0.5211)	(0.6861)	(0.5032)	(0.6674)
Tests of Overall Fit (p-values):		*LR*	$<.0001$						
		Score	$<.0001$						
		Wald	0.8770						

Panel B: 2-σ events

Coefficient	4.0076	3.9309	-21.7267	0.0876	17.3339	6.0011	3.4082	-25.7645	-17.0596
	(0.0612)	(0.0662)	(0.0367)	(0.5570)	(0.0204)	(0.1275)	(0.3536)	(0.1638)	(0.3492)
Tests of Overall Fit (p-values):		*LR*	$<.0001$						
		Score	$<.0001$						
		Wald	$<.0001$						

Panel C: 3-σ events

Coefficient	0.0380	-0.2100	-9.2003	0.0761	0.0676	7.7500	3.1538	-37.1530	-15.2701
	(0.9885)	(0.9367)	(0.4716)	(0.7288)	(0.9933)	(0.1886)	(0.4639)	(0.1975)	(0.4732)
Tests of Overall Fit (p-values):		*LR*	0.2074						
		Score	0.4223						
		Wald	0.5305						

The table shows the results of logistic regression estimation, from January 1989 to December 2005. The dependent, categorical variable is $p_t(\omega)$, which is ranked as Low (less than 0.33), Medium (between 0.33 and 0.67), and High (above 0.67). DSENT is the investor sentiment measure of Baker and Wurgler (2007). LIQ0 and LIQ1 correspond to low and medium levels of liquidity, SECPCT is the percentage change in securitized loans, REALLOAN is the ratio of real estate loans to other consumer loans. P-values based on chi-square tests are in parentheses.