NHH

INSTITUTT FOR FORETAKSØKONOMI

DEPARTMENT OF FINANCE AND MANAGEMENT SCIENCE

FOR 25 2008

ISSN: 1500-4066

DECEMBER 2008

Discussion paper

The Propagation of Financial Extremes

BY

LORÁN CHOLLETE

Norges Handelshøyskole

NORWEGIAN SCHOOL OF ECONOMICS AND BUSINESS ADMINISTRATION

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Lorán Chollete *

Norwegian School of Economics and Business Administration (NHH), and Norwegian Central Bank

July 21, 2008

Abstract

What drives extreme economic events? Motivated by recent theory, and events in US subprime markets, we begin to open the black box of extremes. Specifically, we extend standard economic analysis of extreme risk, allowing for dynamics and endogeneity. We explain how endogenous extremes may arise in an economy of individuals who engage in resource transfers. Our model suggests that susceptibility to extremes depends on differences in marginal substitution rates. Using over a century of daily stock price data, we construct empirical probabilities of extremes, and document interesting dynamic behavior. We find evidence that extremes are endogenous. This latter finding raises the possibility that control of extremes is a public good, and that extreme events may be an important market failure for regulators and central banks to correct.

Keywords: Extreme Event; Subprime Market; Dynamics; Endogeneity; Public Good; Central Bank Policy

JEL Classification: C10, D62, E44, E51, G18, H23, H41

*This project is funded in part by grants from Småforsk and from Finansmarkedsfondet, administered by the Research Council of Norway. I am grateful for comments from Jonas Andersson, Max Bazerman, Olivier Blanchard, Ricardo Caballero, Øyvind Eitrheim, Jørgen Haug, Ingeborg Kleppe, Finn Kydland, Krisztina Molnár, Agnar Sandmo, Johannes Skjeltorp, Bent Vale, Elke Weber, Martin Weitzman and from seminar participants at Columbia's Center for Decision Sciences, Fordham University, MIT, NHH, Norwegian Central Bank, the Stern School (NYU), and Trondheim Business School. I thank Thomas Danhorn for graphics assistance. Address: NHH Department of Finance, Helleveien 30, Bergen N-5045, Norway. Email: Loran.Chollete@nhh.no; Phone: (47) 5595 9266; Fax: (47) 5595 9650.

1 Introduction and Literature Review

> For whoever knows the ways of Nature will more easily notice her deviations; and ... whoever knows her deviations will more accurately describe her ways.
> F. Bacon: *New Organum.*

1.1 Introduction

Extreme events often seem unpredictable, but are they? This paper proposes a positive theory of extremes, based on externalities. A major motivation for this work is our observation of two salient aspects of modern financial markets: dynamics and endogeneity in extremes. By dynamics, we refer to recurring episodes of 'surprise' extreme events. By endogeneity, we refer to the effect of economic agents on changing the likelihood of extremes.[1] The costs of extreme events can be prohibitive, including the risk of default, and an impaired trading process because prices are relatively uninformative. Extreme events also carry social and psychological costs, such as increased Knightian uncertainty in an unstable economy.[2]

Discussions of extreme economic events often assume that they are generated exogenously by nature, and have a constant probability of occurrence.[3] But do we sometimes observe spikes in the frequency of extremes? And is the likelihood of extreme events affected, at times, by our behavior? The answer to both questions is yes. Dynamic, endogenous extremes occur in economics and in nature, including the effect of human activity on the likelihood of extreme financial events, and on extreme climate changes.[4] In this paper, we explore a possible explanation for endogenous extremes, namely, externality effects. Externalities occur when one agent's actions directly affect the environment of other agents. Financial crises and extremes have externality features, since they affect many individuals in the national or global financial system, even though often precipitated by a small number

[1] Endogenous extremes could be due to agents' negligence, bounded rationality, excessive risk taking, or corruption.

[2] See Harris (2003), chapter 9; Caballero and Krishnamurthy (2007); and Weitzman (2007).

[3] See Barro (2006) and Friedman and Laibson (1989).

[4] For extremes in economics, see Fisher (1933) and Grossman (1988). For extremes in nature, see Below, Guha-Sapir, Hoyois, and Scheuren (2007); and Stern (2007).

of individuals. It is well known that externalities cause inefficiency of the price system.[5] Consequently, if extreme events are due to externalities, society may not pay the appropriate price for the extremes that it generates.

How does this formulation of extreme externalities help us? It does so in three ways. First, it allows us to understand the origin of some extremes (the endogenous ones), thereby giving us insight into which we can plausibly try to avert. Second, it gives banks and regulatory authorities an additional set of tools from public finance–subsidies, property rights, and so on–that may help to address extreme events before and during their occurrence. Finally, as noted by Allen and Gale (2007), contemporary economics does not show a specific market failure that central banks and regulators can correct by their intervention. Our formulation may provide a starting point for the role of government, since it emphasizes a clear market failure, namely the externality from agents' neglect to consider their impact on the likelihood of extremes.

In addition to being of academic and policy relevance, this paper may have immediate lessons for market participants, since financial markets have recently featured a large number of extreme events. In the spring and summer of 2007, the aftershock from the subprime market, a relatively small part of US financial markets, reached over to touch hedge funds and international markets. In the US, credit spreads widened ominously, even for safer debt, and the housing market reached record breaking levels. For example, as shown in Figure 1, the percentage change in the Case-Shiller index reached both its historical (20-year) maximum of nearly 16% in 2005 and its historical minimum of -4.52% in the third quarter of 2007. In Britain the interbank rate reached its highest level in 9 years, as shown in Figure 2. One of the more outstanding examples occurred in July and August of 2007, when hedge funds suffered such severe losses that Goldman Sachs, in a one-of-a-kind intervention, had to infuse US$3 billion into one of its funds, Global Equity Opportunities. This fund lost 30 per cent of its value in the week between August 3 and August 10. A major reason cited for the severe hedge fund losses was that the extremes occurring in markets were '25 standard deviation' events (New York Times, August 13, 2007). Such incidents are puzzling because hedge funds did not seem directly exposed to heavy enough risk to warrant such drops in value. Moreover, most large investors have risk management systems that are stress tested against extreme market events such as terrorism risk, banking crises, and interest rate changes. So what sort of event could surprise such respected investors

[5]For textbook expositions of externalities, see Harris (2003), Chapter 9; Mas-Colell, Whinston, and Green (1995); and Varian (1992). For related economic work on aggregate effects of externalities, see Blanchard and Kiyotaki (1987).

enough to lose as much as one-third of their value? A potential answer is that our approach to understanding surprise extreme events is incomplete. This incompleteness may stem from the fact that both information economics and current risk management are generally silent about time variation and endogeneity in the likelihood of extremes.

In light of the preceding observations, we extend existing theory to include explicit, positive analysis of extremes, which are dynamic and endogenous. It should be noted that a type of endogeneity is recognized in certain spheres of risk analysis. Information theory acknowledges that individual agents' behavior can affect individual outcomes in settings such as insurance markets. However, this framework is usually restricted to individual agents or sectors, and typically requires asymmetric information between borrowers and lenders.[6] In our model, we illustrate that under some conditions, endogenous risk effects can spill over to other sectors even in the absence of asymmetric information. A graphical depiction of our approach is in Table 1. This table shows that our view of endogenous probability is similar to that of moral hazard. The difference is that we consider broader settings, where there may be spillovers and general information structures.

1.2 Literature Review

Our research relates to existing work on extreme events and externalities. Regarding extreme events, there are several recent papers. Barro (2006) constructs a Lucas (1978) model with rare extreme events. Upon calibrating the model to twentieth century data, Barro (2006) finds that it allows him to address the equity premium and riskfree rate puzzles. Weitzman (2007) develops a Bayesian model of asset returns. He discovers that when agents consider the possibility of extremes, there is a reversal of all the major asset pricing puzzles. Chichilnisky and Wu (2006) present a model of endogenous uncertainty where increased financial innovation leads to greater likelihood of default. Chichilnisky (2007) shows that by axiomatically extending expected utility to account for extreme responses to extreme events, we can overcome decision theory paradoxes due to Allais (1953) and Ellsberg (1961). Montier (2002) discusses the notion that crashes and outliers are endogenous, perhaps due to a preponderance of sellers relative to buyers. Danielsson and Shin (2003) model a scenario where unanticipated coordination of agents' behavior leads to

[6] The current financial issues, however, potentially affect numerous sectors and regions. Moreover, especially in the case of subprime mortgages, it is difficult to argue that lenders were oblivious to asymmetric information issues, and did not understand the potential for default when supplying loans to borrowers with poor credit history or no collateral.

an endogenous increase in risk. The research of Bazerman and Watkins (2004) suggests that certain 'surprise' events in modern society are predictable, since there may exist sufficient information to know that these events are imminent. Gabaix, Gopikrishnan, Plerou, and Stanley (2006) develop a theory of stock volatility, where the driving force is trading by large investors, during illiquid markets. Regarding externality effects, two important strands of related work concern theories of corruption and tax evasion, and bubbles and crises. Andvig and Moene (1990) show that supply of corruption increases due to lower moral costs of taking bribes. Sandmo (2005) discusses the possibility, based on a 'social conscience' argument, that tax evasion for an individual taxpayer is less risky, the more other taxpayers are perceived as evading taxes. Allen and Gale (2007) discuss the notion that bubbles may be precipitated by incentive and limited liability issues, which reduce the costs of individual risk taking. The authors note that currently there is little theory guiding financial regulation. In chapter 7, Allen and Gale (2007) suggest that there is not always a clear market failure for regulators to correct, in the case of market instability.

Our paper is similar to the above papers in that we discuss the importance of extreme risk and externalities in socioeconomic life. However, our work differs in several respects. First, unlike previous research, we allow for extremes to be dynamic and endogenous. Specifically, we derive mathematical expressions to characterize the 'signature' of dynamic, endogenous extremes. Second, we develop a simple model explaining the propagation of endogenous extremes. Third, we apply the insights from our model to US stock market data, providing evidence on dynamics and endogeneity in market extremes. Finally, the model allows us to rationalize government intervention in the financial economy, and to discuss new policy solutions to extreme events, using a standard public finance toolkit. The remainder of the paper is organized in the following manner. Section 2 discusses dynamic extreme events. Section 3 presents a simple, stylized approach to analyzing dynamic, endogenous extremes. Section 4 reports the results of our empirical application, and Section 5 concludes.

2 Dynamic Extremes

2.1 Definitions

Extreme events occur in many disciplines. Each has developed its own terminology, which may be incompatible with that of other disciplines.[7] We therefore require a common language for extremes, since they arise in a wide variety of settings. Possessing a common language, we can contemplate describing, forecasting and controlling extremes, a task that we begin to pursue in the next section. Based on previous research as well as what we feel to be intuitively sensible aspects of extremes, we now develop a taxonomy. Given the focus of this paper, we use definitions for quantitative data, such as security returns. Intuitively, extreme events are far away from what is normal. In the spirit of previous research such as de Haan and Ferreira (2006) and Friedman and Laibson (1989) we may say, heuristically, that extreme events are 'far away' from the median of the relevant dataset. Armed with this heuristic description, we suggest the following taxonomy.

Consider a variable X with domain **X** $\subset R$. Define a relevant sample $X_s \subseteq$ **X**, comprising n realizations of this variable, $X_s = X_1, ...X_n$, with median $\bar{X}_s$, and standard deviation σ_s. If X_s is a time series, assume that the relevant sample data are covariance stationary. Below, the superscript *E* indicates 'extreme'.

Definition 1: An ω**-Extreme event** $X^{E(\omega)}$ is an event that is at least $\omega \geq 1$ standard deviations away from $\bar{X}_s$, the relevant median:

$$|X^{E(\omega)} - \bar{X}_s| \geq \omega\sigma_s.$$

We can now implement a workable definition of the empirical probability of extremes, π.

[7]The concept of rare or extreme event is used in at least four ways in decision-related sciences. First, in statistics and econometrics, rare refers to a record-breaking phenomenon, one that has never occurred before (de Haan and Sinha (1999)). Second, in political science, it denotes a low probability event with a high impact, which may have occurred before (King and Zeng (2001)). Third, in the theory of risky choice, it refers to a low probability event, which may have occurred before, but not necessarily with a high impact (Hertwig, Barron, Weber, and Erev (2005)). Fourth, in finance the closely related peso problem denotes an infrequent regime that is unobserved but anticipated by economic agents (Evans (1996)).

Definition 2: The **Empirical probability** $\pi(\omega)$ of an ω-extreme event $X^{E(\omega)}$ measures the relative frequency of observations exceeding ω standard deviations from the relevant median $\bar{X}_s$:

$$\pi(\omega) = \frac{\text{Number of } [X_i \in X_s : |X_i - \bar{X}_s| \geq \omega\sigma_s]}{\text{Number of } X_i \in X_s}.$$

Our definitions are conditional in two respects. We condition on the degree of extremeness ω that we wish to consider. We also condition on the relevant data sample, which is chosen with the guidance of scientific theory and knowledge of the question at hand. Evidently what is extreme may change over time, and our definions capture this aspect. For financial time series, the benchmark median $\bar{X}_s$ can be computed dynamically, to capture the notion that over time, what once was extreme may become commonplace, and vice versa.[8] This approach makes sense from a social science perspective, acknowledging that when the world changes, individuals take some time to recognize and respond. The conditional approach is a potential challenge and strength. A challenge is possible lack of comparability across different studies.[9] The strength is that it frees researchers in various disciplines or with different questions to choose, clearly, their concept of extremeness, with alternative samples and values of ω.

Our taxonomy builds on and generalizes existing research. Our definitions compare current events to past medians because individuals' notions of extreme are often relative to what they learned previously. This builds on psychology research, where individuals take time to learn about rare events by experience (Hertwig, Barron, Weber, and Erev (2005) and Weber (2006)), or have disaster myopia (Herring and Wachter (2005)). It also builds on econometric considerations, since individuals gather data at the end of the period before they can compute sample statistics. Our definitions are related to extreme value theory, where extremes are usually phrased in terms of closeness to the maximum or minimum. We use the median instead of the mean or extrema for statistical and psychological reasons. Statistically speaking, the median is robust and achieves the highest possible breakdown value.[10] Psychologically, individuals may take time to adjust their reference points, and the median embodies this more than the mean.[11] We choose a slightly more general definition than in

[8] One could calculate extremes relative to the previous quarter's benchmark median, to capture individuals' lag time in learning and computing the benchmark. The notation ω is chosen since it is often used in definitions of oscillation.

[9] If comparability is an issue, one might compare extreme estimates using both the data sample suggested by scientific theory and the entire data available.

[10] See Casella and Berger (1990) page 230.

[11] For research on endogenous reference points for individuals, see Frydman and Goldberg (2007), chapter 9, and the references therein.

extreme value theory because economic agents might worry about events that deviate from what is typical, even if those events are not record-breaking. For large ω, Definition 1 will be identical to that of extreme value theory, by selecting ω such that $\omega\sigma_s = |X_{(1)} - \bar{X}_s|$, where $X_{(1)}$ is an extreme order statistic. Finally, our definition is flexible and does not assume extreme events are infrequent. This property is attractive because it permits the possibility of extreme clusters, where extreme events occur relatively frequently.[12]

2.2 Static versus Dynamic Extremes

Economic applications often implicitly assume a constant likelihood of extremes, which is useful for analytical tractability. Evidently economic systems change and grow over time, which may affect the probability of extremes. There is some evidence that extreme probabilities change over time, such as record-breaking stock market levels in the 1990s, and oil prices in 2007 to 2008. For economic agents the likelihood of large price deviations is very important to estimate. A mistaken assumption of constant likelihood of extreme price changes is clearly dangerous at many levels, to central bankers as well as individual and institutional investors. Thus, we allow the temporal nature of extremes to be static or dynamic. For static extremes, the likelihood π_t is constant, $\pi_t = \pi$ for all time periods t.

Dynamic extremes, by contrast, can be of two varieties, either random or with a discernible pattern. The random case is represented by equation (1) below. In order to obtain bounded probabilities, consider a random variable z_t and π_t, related in the following manner:

$$\begin{cases} z_t = z_{t-1} + \varepsilon_t \\ \pi_t = \frac{\exp(z_t)}{1+\exp(z_t)}, \end{cases} \tag{1}$$

where $\varepsilon_t \sim i.i.d\ N(0, \gamma)$, with $\gamma > 0$.

Patterns have many possible representations. Concretely, consider a simple stationary autoregressive representation, $\pi_t = \alpha + \sum_{j=1}^{J} \theta_j \pi_{t-j} + \varepsilon_t$. For parsimony, we focus on the first order case:

$$\pi_t = \alpha + \theta_1 \pi_{t-1} + \varepsilon_t, \tag{2}$$

[12] During bubbles or periods of high financial market volatility, stock and commodity indices may reach levels far from the recent median, routinely. For highly skewed or heavy-tailed distributions, extremes can occur more frequently than central observations.

where $|\theta_1| < 1$. Expression (2) permits us to capture potential clustering in extremes.[13]

What is the *signature* of dynamic extremes? According to equations (1) and (2), dynamic exogenous extremes have a frequency π_t that depends either on a random arrival ε_t, or else on some function of its own past values.

3 A Simple Model of Endogenous Extremes

3.1 Exogenous versus Endogenous Extremes

Extreme probabilities can be exogenous or endogenous, each with a different policy response.[14] Exogenous extremes arrive from outside the economic system and are truly acts of nature, from the perspective of the domestic economy. For example, in a crop-based economy, the probability π of extreme changes in crop value could depend on exogenous swings in weather.[15] Since weather is generally unpredictable beyond a few days, and exogenous to an individual farmer, we can represent the probability of extremes as essentially random, as in equation (1).

Endogenous extremes, by contrast, are generated and perhaps amplified within the economic system, by agents' activity and interaction. This activity persists because extremes have externality-like attributes, and therefore agents may 'over-produce' the amount of extremes in the system. For example, stock market crashes and banking panics may stem from excessive risk taking and borrowing of a segment of the economy (Fisher (1933)), excessive credit creation (Allen and Gale (2000)), and excessive reliance on computer-based trading (Grossman (1988)).[16] Since each agent has an incentive to borrow or risk too much

[13] The focus of our discussion is on *empirical* properties of π_t. Therefore, the regression residual ε in (2) must be compatible with bounded probabilities, because the π_t data used in our estimation will lie in the [0,1] interval. If modeling theoretical properties of the process, we could impose boundedness by using some variant of a logistic function, as in (1). We could also consider nonstationary models, such as regime switches.

[14] In practice, there is a spectrum of extremes, with some being a mixture of exogenous and endogenous. The tools developed herein help us to assess the dominant influence on extremes.

[15] Other causes of exogenous extremes include foreign wars, natural catastrophes, and uncertainty about new technology.

[16] The above authors consider some form of extreme event or crisis, but vary in their emphasis on endogeneity. Some model a closed economy or a single sector, others an international setting. The applications differ, although endogeneity or externality issues are common to all. Our paper seems to be the first to use this framework explicitly in a general setting.

from the social point of view, competition leads to overproduction of extremes. Hence, the probability of extremes may no longer be random as in (1). We develop the relevant expression for this latter case below.

While exogenous extremes are statistically unrelated to the economic environment, endogenous extremes (since they are generated by economic agents) should be related to the optimizing or equilibrium behavior of agents. We focus on a canonical form of economic interaction, namely transfer of resources.[17] The heart of the externality is as follows. A key feature of modern financial markets is that they enhance agents' ability to *transfer resources*, which involves either trading commodities and assets or moving assets across time. This transfer of resources can aid or harm other individuals not party to the transfer. For example, massive stock sales by some investors can decrease the stock price, thereby increasing market volatility and diminishing portfolio values of all other investors who own that stock.[18] In similar vein, excessive borrowing by a relatively small set of investors can increase the likelihood of a systemwide market crash.[19] Hence the behavior of individual agents may inherently affect the wellbeing of others without being reflected in a price–the definition of an externality. In sum, modern markets confer ability to transfer financial resources easily, but may bear hidden costs in the form of externalities.[20]

Therefore an important externality from resource transfers involves the likelihood π_t of extreme events or large price changes. We take this externality as the starting point of our paper.[21] Even though agents realize that their collective behavior raises the likelihood π_t of extremes, they may persist in that behavior, since they do not bear all the costs of extreme events.

[17] Resource transfers include such activities as borrowing funds, and trading commodities or securities.

[18] See Gabaix, Gopikrishnan, Plerou, and Stanley (2006).

[19] Allen and Gale (2007), Fisher (1933), Minsky (1982) and Montier (2002), discuss the fact that large asset price and output fluctuations for the entire economy may result from various forms of resource transfers within specific sectors–increased trading, increased desire to liquidate assets, and increased borrowing.

[20] The externality costs of large resource transfers for an individual depend on the dominant social attitude towards transfers at the particular time. Thus, there might be zero or even negative perceived costs of transferring resources during the upswing in asset cycles. There is also evidence of different attitudes by the same individuals at different stages of their life cycles, see Agarwal, Driscoll, Gabaix, and Laibson (2007). Learning may not occur, since different generations of individuals are involved. For related ideas, see Kiyotaki and Moore (1997) and Minsky (1982).

[21] We focus on the likelihood of extremes. For work on the structure of specific extreme events, see Abreu and Brunnermeier (2003) and Brunnermeier and Pedersen (2005). For work discussing rational individuals' perception of extreme risk, see Weitzman (2007).

3.2 Basic Framework

The purpose of the following example is solely to fix ideas. Consider an economy populated by 2 agents with differentiated resource or wealth endowments, who use the financial system to transfer resources between themselves. Suppose that Agents 1 and 2 transfer resources to each other in the amounts r_1 and r_2, recognizing that these transfers might raise the likelihood π of extreme events. Below, we show that, under fairly moderate assumptions, the likelihood of extremes in this economy is socially inefficient.

In order to model the reality that resource transfers enhance individual agents' wellbeing, let each agent i have a neoclassical utility function $u_i(r_i)$, with $u_i'(r_i) > 0$, for $i = 1, 2$.[22] The more agents engage in resource transfers such as excessive borrowing or investing in risky securities, the more likely it is that asset prices reach an extremely high level, affecting the entire system. Thus, let $\pi(r_1, r_2)$ be the likelihood of such extreme events, with $\partial\pi(r_1, r_2)/\partial r_i > 0$, $i = 1, 2$. Let c_1 and c_2 be the costs of extreme events, net of interest, for Agents 1 and 2, respectively. With probability $1 - \pi(r_1, r_2)$ there is no extreme event and each agent receives 0 net. Agents derive utility from transferring resources, but dislike the costs imposed on them by extreme events.[23] Agent $1'$s utility maximization problem is

$$\max_{r_1} u(r_1) - \pi(r_1, r_2)c_1.$$

The first order conditions are given by $u'(r_1) - \frac{\partial\pi(r_1,r_2)}{\partial r_1} \cdot c_1 = 0$, which can be rewritten as

$$\frac{\partial\pi(r_1, r_2)}{\partial r_1} = \frac{u'(r_1)}{c_1}. \tag{3}$$

This equation says that sensitivity of the probability of extreme events to increased resource transfers is proportional to Agent 1's marginal utility, and depends inversely on her costs during extreme events.

By contrast, from society's point of view, the relevant optimization problem has to account for *all* the costs to society, both c_1 and c_2. Therefore, the social problem is

$$\max_{r_1} u(r_1) - \pi(r_1, r_2)[c_1 + c_2].$$

[22]By neoclassical utility, we signify strictly concave and twice continuously differentiable utility, which represents locally nonsatiated preferences. This is similar to the usage in Allen and Gale (2007), chapter 3.

[23]Financial costs of extremes include risk of default and an impaired financial system. Social and psychological costs include increased Knightian uncertainty in an unstable economy.

Now the first order conditions are given by $u'(r_1) - \frac{\partial\pi(r_1,r_2)}{\partial r_1} \cdot (c_1 + c_2) = 0$, which can be rewritten as

$$\frac{\partial\pi(r_1, r_2)}{\partial r_1} = \frac{u'(r_1)}{c_1 + c_2}. \tag{4}$$

The numbers on the right hand side in equation (4) will be smaller than before, in (3), because the denominator is larger. Intuitively, by neglecting to account for the full cost of extreme events, individuals might choose excessive amounts of financial resource transfers. Hence, susceptibility of the economy to increased extremes will be excessively large.

3.3 A Dynamic Model

We now develop a more realistic model, where resources may also be transferred over time, instead of only from one agent to another. We also account for resource constraints. For concreteness, let the two main types of agents each conduct only one type of resource transfer–only selling and buying. We call these agents sellers and buyers, respectively. Consider an economy with a large number of buyers and a large number of sellers. The transfer of resources may affect other agents in the economy, including other buyers, sellers, banks and investors, domestically and internationally. We denote these other agents by O, for other. In the following analysis we use subscripts $0, 1$ and 2 to index variables pertaining to other, sellers and buyers, respectively.

Sellers and buyers are both in the market for transferring resources. Effective supply of resources by sellers is r_1 and demand for resources by buyers is r_2. The framework is a two-period economy, where the first period is t and the second period is $t+1$, in order to distinguish subscripts that refer to time from those that refer to agents. In the first period sellers and buyers interact and transfer resources. In the second period, sellers are repaid with interest $r_{1,t} \cdot (1+i)$, and buyers repay the resources, $r_{2,t} \cdot (1+i)$, where i is the prevailing interest rate. The timeline for decisions is shown in Figure 3. For simplicity, assume that agents receive all their wealth and make all their repayments in the second period.[24] Thus, the seller's and buyer's wealth levels in the first period completely derive from resource transfers: $w_{1,t} = -r_{1,t}$, and $w_{2,t} = r_{2,t}$, respectively. In the second period $t+1$, the buyer and seller receive exogenous wealth endowments $\bar{w}_1$ and $\bar{w}_2$, respectively.

We focus on representative sellers and buyers with neoclassical utility functions u_1 and u_2, respectively, which depend on wealth: $u_i = u_i(w_i)$, where $u_i'(w_i) > 0$, $i = 1, 2$. To control

[24]This timing allows us to model the feature of using financial markets to transfer wealth over time.

for contemporaneous costs, we consider utility to be net of current costs. Each agent knows there is a possibility of systemwide extreme events, captured by the probability π, whose functional form is common knowledge. There is no asymmetric information regarding the likelihood of extremes.[25] The probability of future extreme events increases with the average level of current resource transfers, $\pi_{t+1} = \pi_{t+1}(r_{1,t}, r_{2,t})$, where $\partial\pi_{t+1}/\partial r_{i,t} > 0$, $i = 1, 2$.[26] If an extreme event occurs in the future, agent i incurs a positive cost $c_{i,t+1}$, $i = 0, 1, 2$.[27]

Consider the seller's problem. Given an interest rate i, at period t the seller decides how much resources to transfer this period by maximizing utility subject to the following wealth constraint, which accounts for the possibility of costly extreme events:

$$w_{1,t+1} \geq \bar{w}_1 + \pi_{t+1}(r_{1,t}, r_{2,t})[r_{1,t} \cdot (1+i) - c_{1,t+1}] + [1 - \pi_{t+1}(r_{1,t}, r_{2,t})][r_{1,t} \cdot (1+i)].$$

Given locally nonsatiated preferences, this constraint holds as an equality, which simplifies to $w_{1,t+1} = \bar{w}_1 + r_{1,t} \cdot (1+i) - \pi_{t+1}(r_{1,t}, r_{2,t}) \cdot c_{1,t+1}$. Using β to denote the discount factor, the seller's problem is:

$$\begin{gathered} \max_{r_1} u_1(w_{1,t}) + \beta u_1(w_{1,t+1}), \text{ s.t.} \\ w_{1,t} = -r_{1,t} \\ w_{1,t+1} = \bar{w}_1 + r_{1,t} \cdot (1+i) - \pi_{t+1}(r_{1,t}, r_{2,t}) \cdot c_{1,t+1}. \end{gathered}$$

After substituting the constraints into the utility arguments, first order conditions for an interior solution are $-u_1'(w_{1,t}) + \beta u_1'(w_{1,t+1})[(1+i) - \frac{\partial\pi_{t+1}(r_{1,t}, r_{2,t})}{\partial r_{1,t}} \cdot c_{1,t+1}] = 0$, which can be rewritten as

$$\frac{\partial\pi_{t+1}(r_{1,t}, r_{2,t})}{\partial r_{1,t}} = -\frac{u_1'(w_{1,t})}{\beta u_1'(w_{1,t+1}) \cdot c_{1,t+1}} + \frac{1+i}{c_{1,t+1}}. \tag{5}$$

[25] Similar assumptions occur in many other economic contexts, such as the idea of price taking, competitive agents used in Arrow and Debreu (1954), Chichilnisky and Wu (2006) and Debreu (1959), even though the demand of each agent will affect the price to some extent. Such myopic behavior can be found in other rational settings: investors with log utility decide their portfolios without reference to future investment opportunities, see Ingersoll (1987), chapter 11.

[26] This summarizes the intuition that excessive resource transfers are destabilizing, without emphasizing the particular channel of destabilization. Channels through which resource transfers lead to increased likelihood of extremes are explored by a number of authors, including Fisher (1933) and Allen and Gale (2000).

[27] This cost is financial, social and psychological discomfort suffered in an environment of extremes or financial instability.

Equation (5) says that optimally the (derivative of) extreme probability is related to the marginal rate of substitution for transferring resources between periods t and $t+1$, discounted by expected costs. Since the first term of the right hand side of (5) depends on $r_{1,t}$ via the budget constraint, it follows that extreme probabilities respond to variables affecting the level of resource transfers. We will use this result to motivate our selection of instruments in the empirical application of Section 4.

Similarly, the buyer's problem is

$$\max_{r_2} u_2(w_{2,t}) + \beta u_2(w_{2,t+1}), \text{ s.t.}$$
$$w_{2,t} = r_{2,t}$$
$$w_{2,t+1} = \bar{w}_2 - r_{2,t} \cdot (1+i) - \pi_{t+1}(r_{1,t}, r_{2,t}) \cdot c_{2,t+1},$$

which yields first order conditions that can be rewritten as

$$\frac{\partial \pi_{t+1}(r_{1,t}, r_{2,t})}{\partial r_{2,t}} = \frac{u_2'(w_{2,t})}{\beta u_2'(w_{2,t+1}) \cdot c_{2,t+1}} - \frac{1+i}{c_{2,t+1}}. \tag{6}$$

As in equation (5), the above expression implies that the future probability of extremes is potentially dynamic, and depends on the current level of resource transfers.

Equilibrium: In equilibrium, the demand and supply of resource transfers will be equal, $r_1 = r_2 \equiv r$. For illustrative purposes, consider a symmetric equilibrium where buyers and sellers have identical utility functions and costs, $u_1 = u_2 = u$, and $c_1 = c_2 = c$. Assume this symmetry, and equate the optimality conditions for the seller and buyer in (5) and (6): $-\frac{u'(w_{1,t})}{\beta u'(w_{1,t+1}) \cdot c_{t+1}} + \frac{1+i}{c_{t+1}} = \frac{u'(w_{2,t})}{\beta u'(w_{2,t+1}) \cdot c_{t+1}} - \frac{1+i}{c_{t+1}}$. This implies

$$1 + i = \frac{1}{2\beta} \left[\frac{u'(w_{1,t})}{u'(w_{1,t+1})} + \frac{u'(w_{2,t})}{u'(w_{2,t+1})} \right].$$

Substituting this expression in equation (6) and simplifying, we obtain that in equilibrium, extreme probabilities π_{t+1} satisfy

$$\frac{\partial \pi_{t+1}}{\partial r_t} = \frac{1}{2\beta c_{t+1}} \left[\frac{u'(w_{2,t})}{u'(w_{2,t+1})} - \frac{u'(w_{1,t})}{u'(w_{1,t+1})} \right] \tag{7}$$

Equation (7) constitutes the *signature* of endogenous extremes. Somewhat surprisingly, the responsiveness of extreme probability to resource transfers is proportional to the differential in marginal rates of substitution for agents in the corresponding market. When there is a big difference in marginal rates of substitutions between borrowers and lenders, the susceptibility to extreme events is higher.[28] As before, the extreme probability is dynamic: it depends directly on the expected costs of extremes, and indirectly (with indeterminate sign) on the equilibrium level of resource transfers via the budget constraint. If extremes were truly exogenous, there would be no statistical relation between extreme probability and r, and $\frac{\partial \pi(r_{t+1})}{\partial r_t} = 0$. Thus, the distance of the right side of (7) from zero gives a sense of the error from assuming extremes are exogenous, when they are in reality endogenous.

Social Optimum. To see that the likelihood of extremes is excessive, we proceed as in section 3.2. Suppose the seller considers the effect of her selling on other agents O, and therefore internalizes the costs $c_{0,t+1}$. Her problem is similar to that preceding equation (5), except that the second budget constraint becomes

$$w_{1,t+1} = \bar{w}_1 + r_{1,t} \cdot (1+i) - \pi_{t+1}(r_{1,t}, r_{2,t}) \cdot (c_{0,t+1} + c_{1,t+1}).$$

Solving the first order conditions and rewriting as before, we obtain the counterpart of equation (5) for a socially optimal level of extremes:

$$\frac{\partial \pi_{t+1}(r_{1,t}, r_{2,t})}{\partial r_{1,t}} = -\frac{u_1'(w_{1,t})}{\beta u_1'(w_{1,t+1}) \cdot (c_{0,t+1} + c_{1,t+1})} + \frac{1+i}{c_{0,t+1} + c_{1,t+1}}. \tag{8}$$

The quantities in equations (5) and (8) will differ in general. Thus, when the resource seller takes into account the future costs of other agents, optimal behavior involves a different extreme probability for a given level of borrowed funds. It is in this sense that competitive markets may lead to endogenous, inefficient probability of crashes.[29] We are not just saying there is a link between excessive resource transfers and extremes. Instead, we are showing that even without asymmetric information, excess transfers may arise as an equilibrium phenomenon. This phenomenon occurs due to the failure of *both* resource sellers and buyers to internalize an important externality, the excessive probability of systemwide, future financial extremes.

[28] Intuition for our result is that agents inadvertently affect extreme probability by optimizing over a variable (r_i) with external effects. Therefore, optimally their marginal utility relates to the responsiveness of extreme probability to this variable. Since the marginal rate of substitution depends on resource transfers through the budget constraint, equation (7) also captures the notion that the easiness of effecting transfers (loose credit) affects the likelihood of the financial system's suffering future crashes.

[29] Note that optimality will not necessarily entail complete elimination of extreme events. Rather, the extreme probability level is adjusted to the point where the marginal benefit to sellers of an additional unit of the externality-generating activity, $u_1'(r)$, equals its marginal cost to other agents, $-u_0'(r)$.

To clarify our result, suppose that the terms in equation (8) are all positive, which implies that the social optimum features relatively lower probability of extremes. There are two ways to express this situation. First, we can recognize that excessive financial transfers have a negative externality, and are therefore overproduced. Second, in language perhaps closer to regulators' concerns, we can say that financial system stability (control of extremes) is a public good, which suffers from classic underprovision.

3.4 Model Summary and Implications

We summarize our results from equations (5), (7), and (8) in the following Propositions:

Proposition 1. *In an economy with symmetric preferences and nonzero social costs of extremes, the equilibrium level of extreme probability is in general not socially optimal.*

Proposition 2. *In an economy with symmetric preferences and nonzero social costs of extremes, extreme probabilities are potentially dynamic. The sensitivity of extreme probabilities depends indeterminately on equilibrium resource transfers; decreases with expected costs of extreme events; and increases with the divergence between agents' marginal rates of substitution.*

These results have implications for regulatory policy and risk management. Proposition 1 suggests, in principle, a role for regulators and central banks to intervene and prevent excessive financial extremes.[30] Proposition 2 cautions risk managers against the assumption that exposure to extreme events does not change over time. Further, Proposition 2 suggests possible warning signals for regulators and risk managers–low expected costs and a large gap between agents' desires to transfer resources over time.[31]

More tentatively, the results could also have relevance for current subprime issues. Proposition 1 may suggest that subprime spillovers can be explained as the result of an externality: the uninternalized effect of excessive resource transfers on future financial instability. According to Proposition 2, the increased extremes in today's markets could be driven to some

[30]Theoretically, regulators could tax 'excessive' transfers. This would require extensive monitoring of investors. A more realistic approach might involve reducing demand for excess borrowing by taxing or subsidizing certain large purchases. Another attractive alternative could be to increase education about costs of extreme events, and the role individual agents and institutions play in precipitating these costs. This latter approach is similar to education in recent years about human impact on extremes in the natural environment.

[31]More generally, Proposition 2 predicts that developments to enhance resource transferrals will, ceteris paribus, affect the likelihood of extremes. These developments include financial innovation and loose interest rates.

extent by low expected costs and a large gap between the marginal rates of substitution for borrowers and lenders of capital. After expected costs rise and marginal rates of substitution equalize, then the elevated extremes will begin to play out.

4 Empirical Application

An important pre-condition for the policy analysis described in the previous section is empirical documentation of the properties of extreme probabilities, to which we now turn. Since individual asset prices can be driven by idiosyncratic features, we focus on one series that summarizes aggregate security performance, and for which there is a relatively long time series of daily observations–the Dow Jones Industrial Average (DJIA). Our main series is the daily DJIA, from May 26, 1896 to September 28, 2007.

We use other variables to investigate endogeneity. From equation (7), extreme probabilities will respond to resource transfers, as well as variables that signal the prevailing socioeconomic attitude towards large resource transfers. In addition, the results of Gabaix, Gopikrishnan, Plerou, and Stanley (2006) suggest that illiquidity is relevant for predicting the probability of extremes. We therefore include the following data series: the degree of securitization in US financial markets (SEC), available from January 1989 to December 2006; the liquidity measure (LIQ) of Pastor and Stambaugh (2003) available from April 1962 to December 2006; the value of real estate loans in the US (REALLOAN) available from January 1947 to July 2007; and a measure of investor sentiment (DSENT1), used in the study of Baker and Wurgler (2007). The DSENT1 data is available from January 1966 to December 2005, and kindly provided at Professor Jeffrey Wurgler's website.[32] Unless otherwise noted, these data are of monthly frequency, obtained from WRDS and Datastream. There are three steps to our empirical approach. First, we examine whether our π series differ significantly from zero. Second, we explore dynamics by considering autoregressive models . Third, we analyze endogeneity by using logistic models. In all cases we carry out our estimation on several reference periods, imparting some degree of robustness.

[32] Since the empirical probability series are between 0 and 1, and SEC and REALLOAN are in billions of dollars, we use percentage changes in SEC and the ratio of real estate loans to total loans, in order to scale them down comparably.

4.1 Summary Statistics

The main series we compute is $\pi_t(\omega)$, according to Definition 2 in Section 2.1. Using the DJIA as our base series, we calculate the proportion of times each month that there is an observation more than ω standard deviations away from the median. Both the median and standard deviation are computed over the preceding k months, for $k = 12$, 24, 60 and 120. This procedure is done on a rolling basis.[33] The π_t series for the 12 month reference period is shown in Figure 4. As should be expected, the probability of extremes becomes much smaller as we move from 1 or 2 sigma events to 3-sigma and beyond. Evidently, the series move around quite a bit, so even from a visual perspective the series are not constant.

Figures 5 and 6 display histograms of extreme probabilities for 1 and 2 sigma events. In both cases there is a u-shaped pattern for all reference periods. Moreover, the longer reference periods tend to have more mass concentrated at 0 and 1. Thus, when economic agents actually compute extremes in this range (1 and 2 sigma events), their probability estimates will tend to be more volatile. Figures 7 to 9 show histograms for extreme probabilities of 3- to 5 sigma events. As we would expect, the distributions become more concentrated at zero, with the shortest horizon (12 months) being the last to have all probabilities at zero.

We now turn to summary statistics and formal tests. Table 2 shows that as the level of ω increases, both the mean and standard deviation decrease. However, both the t-test and nonparametric sign rank test generally have minute p-values until the level of $\omega = 5$. This suggests that the likelihood of extreme events beyond 2 standard deviations may be non-zero, regardless of agents' reference periods.

Is there a difference in extreme probabilities within reference periods? We test this hypothesis in Table 3. Except for some marginal significance between 4 and 5 sigma events, there is very little evidence of similarity between the various extreme probabilities within a given reference period. A different pattern emerges, however, when we examine tests for differences across reference periods, in Table 4. This latter table reports mixed evidence about the similarity of average extreme probability depending on the reference period. This result may be of practical relevance, if different investor groups have different time horizons when deciding whether a particular event is extreme.

[33] The series begins in May 1896. Thus, to compute $\pi_t(1)$ for a 12 month reference period, we count the number of times in June 1897 that the DJIA exceeded 1 standard deviation from the median, calculated from May 1896 to May 1897. We then do the same for July 1897, where the median and standard deviation are calculated from June 1896 to June 1897, and so on.

4.2 Evidence on Dynamics

The dynamic behavior of extreme probabilities is highly relevant for risk management and stress testing. We therefore examine the time series behavior of our $\pi_t(\omega)$ series. The results are displayed in Table 5.[34] Importantly, except for the very extreme 5-sigma events, the Q-test of white noise is rejected. This suggests that there are important dynamics in the likelihood of extreme events. The best-fitting models generally range from AR(1) to AR(3), although there are a few models with higher lags. Thus, our empirical probabilities seem to exhibit memory–extreme events cluster over time, regardless of our reference period.

4.3 Evidence on Endogeneity

Now we would like to assess whether π_t depends on plausible aspects of economic behavior, suggested by theory. We focus on ranges rather than point estimates of extreme probabilities, since the former are useful as financial 'warning signals' that indicate whether the economy is likely to be entering a regime with high levels of extreme events.[35] We divide the empirical probabilities π_t into three ranges, Low, Medium and High. Low corresponds to empirical probabilities less than 0.33, Medium to the range 0.33 to 0.67, and High to the range 0.67 to 1. We then estimate a cumulative logistic model for all reference periods.[36] The estimated model shows the effect of each explanatory variable on the likelihood of High levels of extreme probabilities. For example, the estimated coefficient on REALLOAN shows the relation between a one-unit increase in real estate borrowing and the likelihood of being in a period of High extreme probabilities. Based on the considerations of Gabaix, Gopikrishnan, Plerou, and Stanley (2006) and Montier (2002), we include dummy variables for low, medium and high levels of liquidity, namely, LIQ0, LIQ1 and LIQ2, respectively, as well as interaction terms between liquidity and real estate borrow-

[34] Some series had insufficient variance to compute time series models. This was particularly the case with the 5-sigma extremes, since much of that series consists of zeros.

[35] From a statistical viewpoint, the use of ranges is attractive for several reasons. Importantly, estimation of ranges is valuable for incorporating model uncertainty, as discussed by Granger, White, and Kamstra (1989) and Hansen (2006). Range based empirical methods have also been used successfully in financial economics, for example by Alizadeh, Brandt, and Diebold (2002).

[36] Logistic regression with k explanatory variables is based on the following empirical model: $g(p) = \alpha + \beta_1 X_1 + ... + \beta_k X_k$. The link function $g(p)$ is linearly related to the explanatory variables, and in the case of logistic regression, the link function is the logit, $g(p) = log(p/(1-p))$. Thus, in our application, we are estimating the effect of various explanatory variables on the (log of the) probability of high extremes divided by the probability of no high extremes. A similar methodology has been used in explaining crises, by Bordo, Eichengreen, Klingebiel, and Martinez-Peria (2001).

ing. The results are reported in Tables 6 to 9, which we now discuss.[37] We are primarily interested in the significance rather than sign of our explanatory variables, because we have no plausible variables for expected costs from equation (7), which could be either negative or positive. Since 3 sigma events exhibit less variation (mostly zeros and ones), the model fit for these events is not as good as for the other cases. With the exception of 2-sigma events in the two year reference period, the models generally work well for 1 and 2 sigma extremes, as documented by the small p-values for the LR, Score and Wald tests.[38]

The most striking finding is that for all reference periods, real estate borrowing is significantly related at the 5% level to high probability of extremes for at least one specification: 1-sigma events, 2-sigma events, or both. In Table 6, the estimated coefficient on REAL-LOAN is -21.52, with a p-value of 0.0052. Moreover, in the same table, a low level of liquidity (LIQ0) is significant for high extremes. This latter result holds for 1 and 2 sigma events, both individually and in interaction with real estate borrowing. The amount of securitization, SECPCT, is significant only for 2-sigma extremes at the 10 year horizon. The investor sentiment variable DSENT1, is never significant.[39]

To summarize our empirical exploration, the main findings on the dynamics of extremes are very encouraging: extreme probabilities are strongly dynamic and persistent, as documented in Tables 2 through 5. The evidence on endogenous extremes is somewhat encouraging. A cumulative logistic analysis shows that the level of real estate borrowing is related to high likelihood of extremes for at least one specification in all reference periods. Moreover, current illiquidity may interact with past real estate borrowing to affect the likelihood of extreme events. These latter findings corroborate the theoretical and anecdotal evidence of Allen and Gale (2000), Fisher (1933) and Gabaix, Gopikrishnan, Plerou, and Stanley (2006), and support the idea that extremes may be endogenous.

[37]The full set of explanatory variables is only available from 1989 to 2005. During this period the 4 and 5 sigma extremes featured only zeros for all reference periods. Therefore we can only estimate and report estimation results for 1- to 3-sigma extremes.

[38]These three statistics test the null hypothesis that all the explanatory variables have zero coefficients.

[39]The estimation reports two intercepts because cumulative logit computes it that way, one less than the number of categories in the dependent variable

5 Conclusions

Our paper develops a simple, positive approach to extreme events. We suggest that the probability of extremes may vary systematically over time, and might be explained and forecasted on the basis of economic theory. We have four main contributions. First, we distinguish exogenous from endogenous extremes, the latter of which can be understood in the framework of externalities. This distinction has immediate policy implications: for truly exogenous extremes, we must focus on ex post protection, while for endogenous extremes, we can use economic incentives to entice agents to reduce extremes themselves. Second, our approach suggests a role for central bank and regulatory intervention. In tackling issues related to economic instability, regulators have at their disposal a set of public finance tools, in addition to traditional interest rate solutions. Third, we show the 'signature' of different types of extremes, and provide insight on their incidence. According to equation (7), economies are more susceptible to extremes if expected costs are low and there is a large discrepancy between marginal rates of substitutions for resource borrowers and lenders. Finally, on the empirical side, we compute extreme probabilities, and discover that extremes often possess interesting dynamics. Extreme probabilities generally differ significantly from zero, and have strong autoregressive components, indicating memory in extremes. Regarding endogeneity, a logistic analysis shows that between 1989 to 2005, real estate borrowing and (to a smaller extent) market illiquidity can help to explain the likelihood of extremes. In light of our theoretical and empirical findings, it might be plausible to consider regulatory intervention in an effort to control extreme events.

While our paper describes a method for understanding patterns in the likelihood of extremes, it does not aim to predict all possible extreme events. The aim is to show that, far from being random, the probability of some extremes have similar dynamics, and relate to economic fundamentals. Our research may be seen as a first step towards incorporating dynamic, endogenous, extremes into standard economic analysis. Acknowledgement of dynamic extremes may be helpful for risk management. Regarding endogeneity, even if a channel differs from the one we focus on empirically (borrowing), the message remains: endogenous extreme events might be prevented using tools from public finance. Important extensions include identifying dynamic extremes in other assets, and exploring various channels of endogenous extremes encountered in practice. Such refinements present an exciting task for future research.

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Figure 1: Percentage Change in US House Prices.

The figure shows the percentage change in the Case-Shiller US House Price Index, relative to the previous year. Source: Standard and Poors.

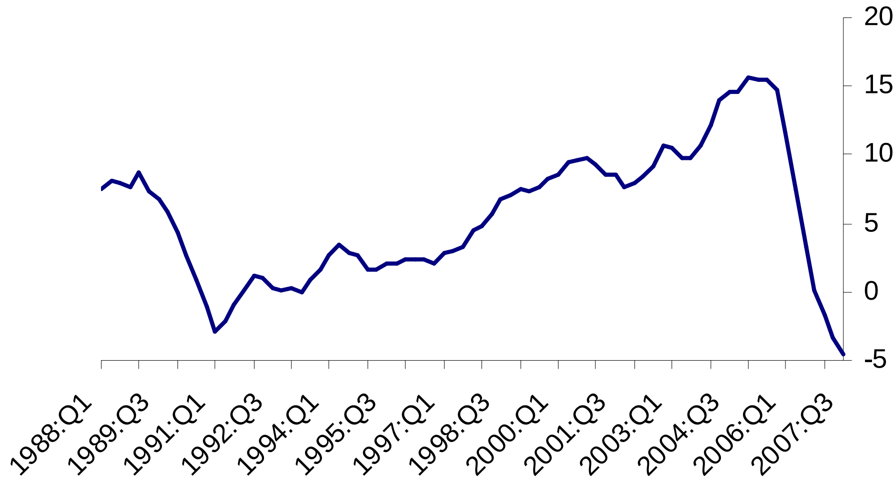

Figure 2: UK banks' price of borrowing.

The figure shows the price of interbank borrowing in the UK. The solid (red) line is the 3-month interbank rate and the dotted (green) line is the base rate. Source: DataStream.

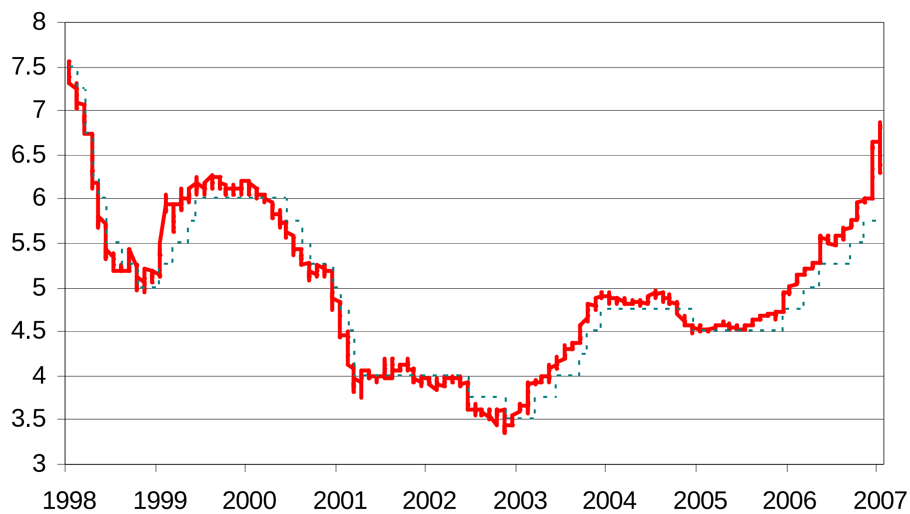

Table 1: Two Examples of Endogenous Probabilities

	Effects felt mainly in one market or transaction	*Spillover effects in many markets*
Asymmetric Information	**Moral Hazard**	
Symmetric or Asymmetric Information		**Endogenous Extremes**

Figure 3: Sequence of Events.

The figure shows the timing of decisions in our stylized model. $\bar{r}$ and $\bar{\pi}$ are threshold levels of resource transfers and extreme probability where spillovers to other sectors begin. Endogenous extremes occur during 'easy' regimes–when it is simple to transfer resources. Exogenous extremes occur during 'hard' regimes. c_0, c_1, c_2 include costs related to default, financial instability and aggregate uncertainty.

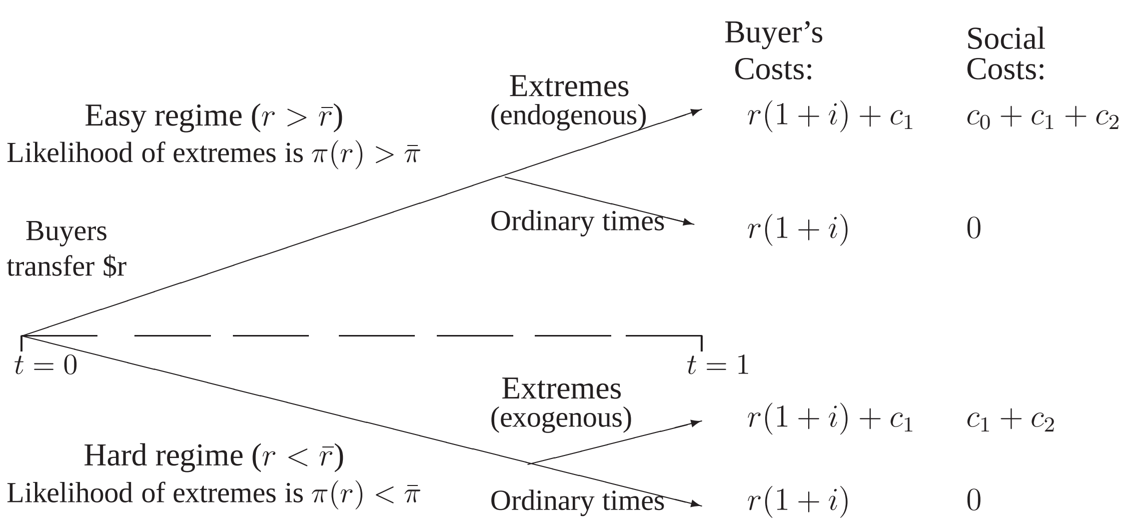

Figure 4: Time Series of Extremes

The figure shows a sample of the time series for various levels of extreme probabilities, from 1967 to 2007. The relevant reference period is 12 months.

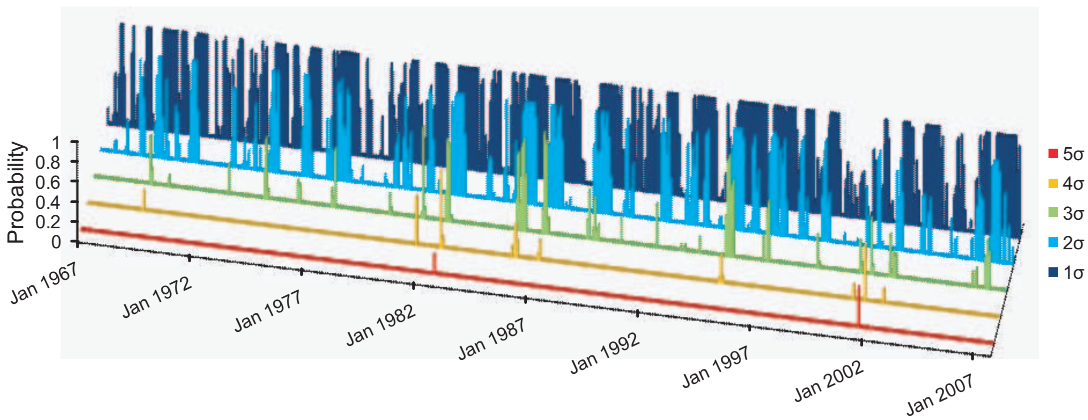

Figure 5: The Distribution of 1-sigma Events.

The histogram shows the empirical probability of Dow-Jones Industrial Average levels that exceed one standard deviation from the relevant median. The median is calculated over different reference samples, ranging from 12 months to 120 months.

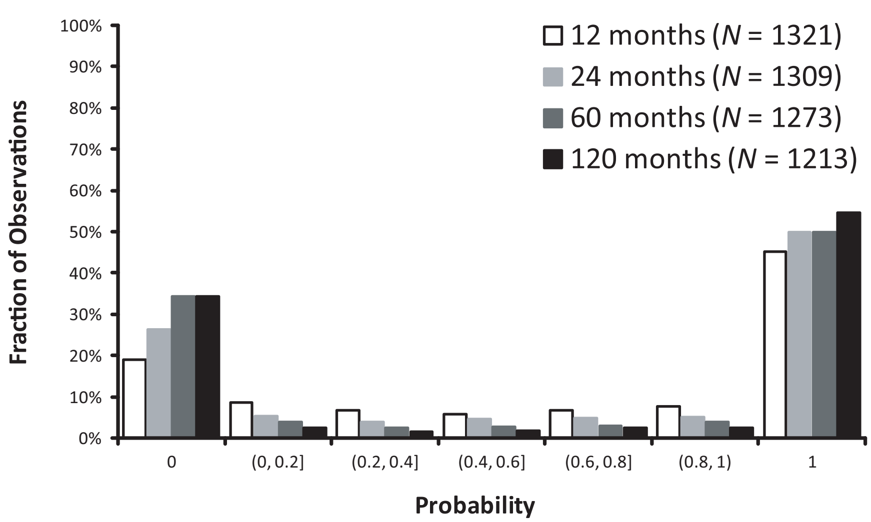

Figure 6: The Distribution of 2-sigma Events.

The histogram shows the empirical probability of Dow-Jones Industrial Average levels that exceed two standard deviations from the relevant median. The median is calculated over different reference samples, ranging from 12 months to 120 months.

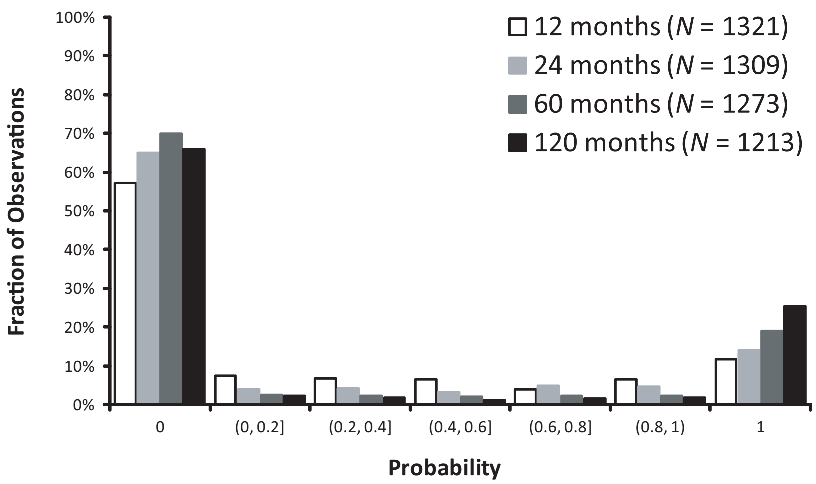

Figure 7: The Distribution of 3-sigma Events.

The histogram shows the empirical probability of Dow-Jones Industrial Average levels that exceed three standard deviations from the relevant median. The median is calculated over different reference samples, ranging from 12 months to 120 months.

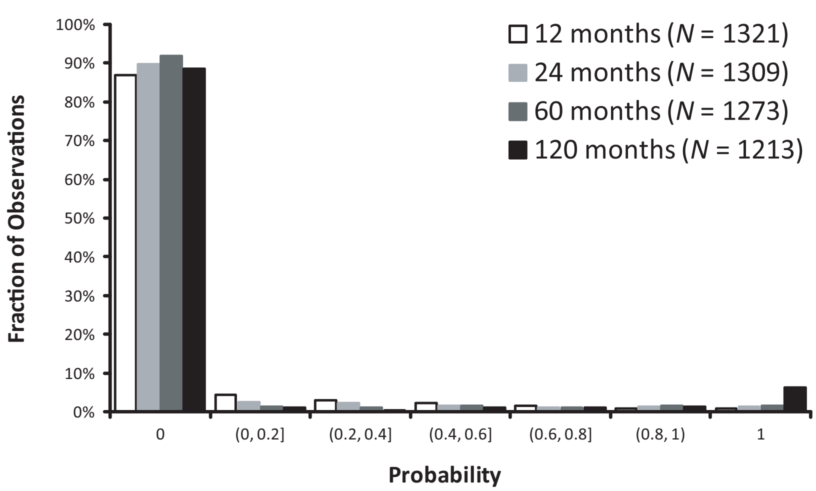

Figure 8: The Distribution of 4-sigma Events.

The histogram shows the empirical probability of Dow-Jones Industrial Average levels that exceed four standard deviations from the relevant median. The median is calculated over different reference samples, ranging from 12 months to 120 months.

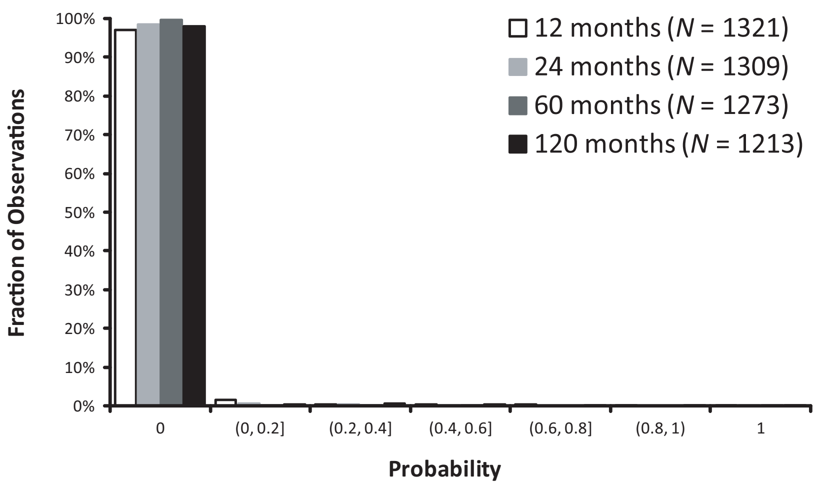

Figure 9: The Distribution of 5-sigma events.

The histogram shows the empirical probability of Dow-Jones Industrial Average levels that exceed five standard deviations from the relevant median. The median is calculated over different reference samples, ranging from 12 months to 120 months.

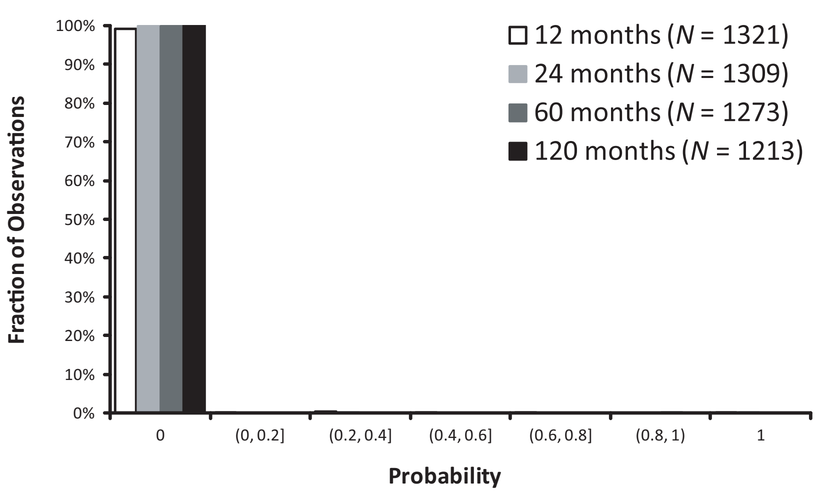

Table 2: Basic Properties of Extreme Probabilities $\pi_t(\omega)$

Panel A: 12-month reference period

	Mean	**Standard Deviation**	**P-value for t-test**	**P-value for sign rank test**
$(\omega = 1)$:	0.6288	0.4202	< 0.0001	<0.0001
$(\omega = 2)$:	0.2647	0.3806	< 0.0001	< 0.0001
$(\omega = 3)$:	0.0542	0.1768	< 0.0001	< 0.0001
$(\omega = 4)$:	0.0098	0.0739	< 0.0001	< 0.0001
$(\omega = 5)$:	0.0033	0.0437	0.0056	0.0020

Panel B: 24-month reference period

	Mean	**Standard Deviation**	**P-value for t-test**	**P-value for sign rank test**
$(\omega = 1)$:	0.6189	0.4439	< 0.0001	< 0.0001
$(\omega = 2)$:	0.2489	0.3906	< 0.0001	< 0.0001
$(\omega = 3)$:	0.0517	0.1844	< 0.0001	< 0.0001
$(\omega = 4)$:	0.0044	0.0502	0.0014	< 0.0001
$(\omega = 5)$:	0.0010	0.0291	0.2060	0.5000

Panel C: 60-month reference period

	Mean	**Standard Deviation**	**P-value for t-test**	**P-value for sign rank test**
$(\omega = 1)$:	0.5787	0.4680	< 0.0001	< 0.0001

($\omega = 2$):	0.2450	0.4083	< 0.0001	< 0.0001
($\omega = 3$):	0.0496	0.1899	< 0.0001	< 0.0001
($\omega = 4$):	0.0026	0.0445	0.0368	0.0313
($\omega = 5$):	0.0007	0.0259	0.3175	1.000

Panel D: 120-month reference period

	Mean	**Standard Deviation**	**P-value for t-test**	**P-value for sign rank test**
($\omega = 1$):	0.6038	0.4711	< 0.0001	< 0.0001
($\omega = 2$):	0.2961	0.4413	< 0.0001	< 0.0001
($\omega = 3$):	0.0908	0.2722	< 0.0001	< 0.0001
($\omega = 4$):	0.0086	0.0732	< 0.0001	< 0.0001
($\omega = 5$):	0.0000	0.0000	.	.

The table shows stylized facts for the time series of extreme probabilities $\pi_t(\omega)$. As in the text, ω denotes the number of standard deviations away from the relevant median. The t- and sign rank tests examine whether the mean differs significantly from zero.

Table 3: P-values for Test of Differences *Within* Reference Periods

	1-σ vs 2-σ	**1-σ vs 3-σ**	**1-σ vs 4-σ**	**1-σ vs 5-σ**	**2-σ vs 3-σ**	**2-σ vs 4-σ**	**2-σ vs 5-σ**	**3-σ vs 4-σ**	**3-σ vs 5-σ**	**4-σ vs 5-σ**
Panel A: 12-month reference period										
t-test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Sign test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
SR test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Panel B: 24-month reference period										
t-test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	0.0018
Sign test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
SR test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Panel C: 60-month reference period										
t-test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	$<$0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	0.0530
Sign test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	$<$0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	0.0313
SR test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	$<$0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	0.0313
Panel D: 120-month reference period										
t-test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	$<$0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Sign test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	$<$0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
SR test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	$<$0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001

The table shows the p-values from statistical tests for significant differences in the means of our $\pi_t(\omega)$ series, for various levels of extreme events. SR denotes the sign rank test.

Table 4: P-values for Test of Differences *Across* Reference Periods

	12- vs 24-month	24- vs 60-month	60- vs 120-month	12- vs 120-month
Panel A: 1-σ Events				
t-test:	0.6024	0.0032	0.0408	0.1750
Sign test:	0.8438	0.0009	0.0049	0.4969
SR test:	0.6499	0.0028	0.0469	0.0710
Panel B: 2-σ Events				
t-test:	0.1012	0.8694	$<$0.0001	0.0417
Sign test:	0.1198	0.1902	$<$0.0001	0.6960
SR test:	0.0998	0.9772	$<$0.0001	0.0146
Panel C: 3-σ Events				
t-test:	0.9167	0.9404	$<$0.0001	$<$0.0001
Sign test:	0.3580	0.1679	$<$0.0001	0.9508
SR test:	0.8088	0.7975	$<$0.0001	$<$0.0001
Panel D: 4-σ Events				
t-test:	0.0073	0.3936	0.0187	0.8629
Sign test:	0.0026	0.0490	0.0023	0.4885
SR test:	0.0004	0.2112	0.0114	0.7465
Panel E: 5-σ Events				
t-test:	0.0203	0.2306	0.3175	0.0111
Sign test:	0.0156	0.5000	1.0000	0.0078
SR test:	0.0156	0.5000	1.0000	0.0078

The table shows the p-values from statistical tests for significant differences in the means of our series, for various levels of extreme events. SR denotes sign rank test.

Table 5: Time Series properties of Extreme Probabilities

	1-σ	**2-σ**	**3-σ**	**4-σ**	**5-σ**
Panel A: 12-month reference period					
Q-test, original series	$<$ 0.0001	$<$ 0.0001	$<$ 0.0001	$<$0.0001	0.4192
Selected Model:	AR(2)	AR(2)	AR(2)	AR(2)	AR(1)
Q-test of residuals	0.1729	0.0266	0.3694	0.9654	0.9982
Panel B: 24-month reference period					
Q-Test, original series	$<$ 0.0001	$<$ 0.0001	$<$ 0.0001	$<$0.0001	1.0000
Selected Model:	AR(2)	AR(2)	AR(2)	AR(4)	NA
Q-test of residuals	0.0952	0.0949	0.8880	0.2330	NA
Panel C: 60-month reference period					
Q-test, original series	$<$ 0.0001	$<$ 0.0001	$<$ 0.0001	$<$0.0001	1.0000
Selected Model:	AR(1)	AR(4)	AR(4)	AR(3)	NA
Q-test of residuals	0.4002	0.0575	0.0166	0.1698	NA
Panel D: 120-month reference period					
Q-test, original series	$<$ 0.0001	$<$ 0.0001	$<$ 0.0001	$<$0.0001	NA
Selected Model:	AR(2)	AR(1)	AR(3)	AR(5)	NA
Q-test of residuals	0.0327	0.0349	0.2157	0.5157	

The table shows the results of time series estimation of the $\pi_t(\omega)$ series, for different reference periods. NA denotes 'not applicable', where estimation did not converge.

Table 6: Cumulative Logistic Estimation, 12-month Reference Period

Panel A: 1-σ events

	Intercept1	**Intercept2**	**REALLOAN**	**DSENT1**	**SECPCT**	**LIQ0**	**LIQ1**	**REALLOAN*LIQ0**	**REALLOAN*LIQ1**
Coefficient	5.3675	4.5980	-21.5218	0.1487	-2.3985	-5.7905	-1.5121	26.0566	9.5101
	(0.0011)	(0.0047)	(0.0052)	(0.2759)	(0.6978)	(0.0179)	(0.4771)	(0.0197)	(0.3332)
Tests of Overall Fit (p-values):		*LR*	0.0186						
		Score	0.0175						
		Wald	0.0409						

Panel B: 2-σ events

	Intercept1	**Intercept2**	**REALLOAN**	**DSENT1**	**SECPCT**	**LIQ0**	**LIQ1**	**REALLOAN*LIQ0**	**REALLOAN*LIQ1**
Coefficient	8.6457	7.9444	-45.0513	-0.0339	-9.3293	-8.0929	-2.9394	39.1506	15.3235
	(0.0112)	(0.0195)	(0.0091)	(0.8263)	(0.1551)	(0.0443)	(0.4729)	(0.0472)	(0.4558)
Tests of Overall Fit (p-values):		*LR*	0.0011						
		Score	0.0094						
		Wald	0.0275						

Panel C: 3-σ events

	Intercept1	**Intercept2**	**REALLOAN**	**DSENT1**	**SECPCT**	**LIQ0**	**LIQ1**	**REALLOAN*LIQ0**	**REALLOAN*LIQ1**
Coefficient	8.7419	7.6638	-57.7413	0.2965	-9.3413	-15.1531	-6.2688	68.5561	34.5645
	(0.3130)	(0.3765)	(0.2006)	(0.4033)	(0.4732)	(0.1723)	(0.5143)	(0.2016)	(0.4852)
Tests of Overall Fit (p-values):		*LR*	0.3148						
		Score	0.4609						
		Wald	0.5756						

The table shows the results of logistic regression estimation, from January 1989 to December 2005. The dependent, categorical variable is $\pi_t(\omega)$, which is ranked as Low (less than 0.33), Medium (between 0.33 and 0.67), and High (above 0.67). DSENT1 is the investor sentiment measure of Baker and Wurgler (2007). LIQ0 and LIQ1 correspond to low and medium levels of liquidity, SECPCT is the percentage change in securitized loans, REALLOAN is the ratio of real estate loans to other consumer loans. A chi square statistic is computed as the squared ratio of each parameter to its standard error, and the corresponding p-values are in parentheses.

Table 7: Cumulative Logistic Estimation, 24-month Reference Period

Panel A: 1-σ events

	Intercept1	**Intercept2**	**REALLOAN**	**DSENT1**	**SECPCT**	**LIQ0**	**LIQ1**	**REALLOAN*LIQ0**	**REALLOAN*LIQ1**
Coefficient	8.0943	7.6624	-32.8161	0.0229	0.2950	-2.7495	-1.8432	11.9173	11.2734
	(<.0001)	(<.0001)	(0.0003)	(0.8768)	(0.9685)	(0.3271)	(0.4599)	(0.3483)	(0.3182)
Tests of Overall Fit (p-values):		*LR*	<.0001						
		Score	<.0001						
		Wald	<.0001						

Panel B: 2-σ events

	Intercept1	**Intercept2**	**REALLOAN**	**DSENT1**	**SECPCT**	**LIQ0**	**LIQ1**	**REALLOAN*LIQ0**	**REALLOAN*LIQ1**
Coefficient	1.9688	1.6283	-15.3030	-0.0169	7.4385	1.5592	-0.1817	-5.0872	1.3811
	(0.3934)	(0.4802)	(0.1751)	(0.9135)	(0.2383)	(0.6483)	(0.9515)	(0.7540)	(0.9245)
Tests of Overall Fit (p-values):		*LR*	0.0851						
		Score	0.1339						
		Wald	0.1613						

Panel C: 3-σ events

	Intercept1	**Intercept2**	**REALLOAN**	**DSENT1**	**SECPCT**	**LIQ0**	**LIQ1**	**REALLOAN*LIQ0**	**REALLOAN*LIQ1**
Coefficient	-1.4408	-2.3552	-6.4250	0.1366	2.2496	-0.6735	0.3425	4.2006	-2.9245
	(0.6920)	(0.5189)	(0.7127)	(0.6338)	(0.8520)	(0.8974)	(0.9489)	(0.8624)	(0.9094)
Tests of Overall Fit (p-values):		*LR*	0.9911						
		Score	0.9930						
		Wald	0.9943						

The table shows the results of logistic regression estimation, from January 1989 to December 2005. The dependent, categorical variable is $\pi_t(\omega)$, which is ranked as Low (less than 0.33), Medium (between 0.33 and 0.67), and High (above 0.67). DSENT1 is the investor sentiment measure of Baker and Wurgler (2007). LIQ0 and LIQ1 correspond to low and medium levels of liquidity, SECPCT is the percentage change in securitized loans, REALLOAN is the ratio of real estate loans to other consumer loans. P-values based on chi-square tests are in parentheses.

Table 8: Cumulative Logistic Estimation, 60-month Reference Period

Panel A: 1-σ events

	Intercept1	**Intercept2**	**REALLOAN**	**DSENT1**	**SECPCT**	**LIQ0**	**LIQ1**	**REALLOAN*LIQ0**	**REALLOAN*LIQ1**
Coefficient	16.9786	16.5095	-70.4190	0.1522	-10.5182	-1.8429	2.0033	10.0758	-6.1854
	(<.0001)	(<.0001)	(<.0001)	(0.4344)	(0.3580)	(0.7002)	(0.6823)	(0.6377)	(0.7765)
Tests of Overall Fit (p-values):		*LR*	<.0001						
		Score	<.0001						
		Wald	<.0001						

Panel B: 2-σ events

	Intercept1	**Intercept2**	**REALLOAN**	**DSENT1**	**SECPCT**	**LIQ0**	**LIQ1**	**REALLOAN*LIQ0**	**REALLOAN*LIQ1**
Coefficient	2.9659	2.7202	-18.6782	-0.0187	-0.8247	2.2365	1.8791	-10.9524	-7.0953
	(0.1951)	(0.2345)	(0.0962)	(0.9068)	(0.8959)	(0.5649)	(0.5405)	(0.5557)	(0.6379)
Tests of Overall Fit (p-values):		*LR*	0.0032						
		Score	0.0099						
		Wald	0.0326						

Panel C: 3-σ events

	Intercept1	**Intercept2**	**REALLOAN**	**DSENT1**	**SECPCT**	**LIQ0**	**LIQ1**	**REALLOAN*LIQ0**	**REALLOAN*LIQ1**
Coefficient	0.4406	0.00661	-16.0284	0.1180	-6.6053	8.8289	2.9406	-41.4283	-12.1881
	(0.9290)	(0.9989)	(0.5133)	(0.7092)	(0.5846)	(0.3623)	(0.6712)	(0.3911)	(0.7251)
Tests of Overall Fit (p-values):		*LR*	0.4572						
		Score	0.6365						
		Wald	0.7028						

The table shows the results of logistic regression estimation, from January 1989 to December 2005. The dependent, categorical variable is $\pi_t(\omega)$, which is ranked as Low (less than 0.33), Medium (between 0.33 and 0.67), and High (above 0.67). DSENT1 is the investor sentiment measure of Baker and Wurgler (2007). LIQ0 and LIQ1 correspond to low and medium levels of liquidity, SECPCT is the percentage change in securitized loans, REALLOAN is the ratio of real estate loans to other consumer loans. P-values based on chi-square tests are in parentheses.

Table 9: Cumulative Logistic Estimation, 120-month Reference Period

Panel A: 1-σ events

	Intercept1	Intercept2	REALLOAN	DSENT1	SECPCT	LIQ0	LIQ1	REALLOAN*LIQ0	REALLOAN*LIQ1
Coefficient	88.9107	NA	-349.3	-0.2846	-86.9502	189.6	79.0265	-834.0	-352.6
	(0.4142)	NA	(0.4126)	(0.9322)	(0.8750)	(0.5211)	(0.6861)	(0.5032)	(0.6674)
Tests of Overall Fit (p-values):		*LR*	$<.0001$						
		Score	$<.0001$						
		Wald	0.8770						

Panel B: 2-σ events

	Intercept1	Intercept2	REALLOAN	DSENT1	SECPCT	LIQ0	LIQ1	REALLOAN*LIQ0	REALLOAN*LIQ1
Coefficient	4.0076	3.9309	-21.7267	0.0876	17.3339	6.0011	3.4082	-25.7645	-17.0596
	(0.0612)	(0.0662)	(0.0367)	(0.5570)	(0.0204)	(0.1275)	(0.3536)	(0.1638)	(0.3492)
Tests of Overall Fit (p-values):		*LR*	$<.0001$						
		Score	$<.0001$						
		Wald	$<.0001$						

Panel C: 3-σ events

	Intercept1	Intercept2	REALLOAN	DSENT1	SECPCT	LIQ0	LIQ1	REALLOAN*LIQ0	REALLOAN*LIQ1
Coefficient	0.0380	-0.2100	-9.2003	0.0761	0.0676	7.7500	3.1538	-37.1530	-15.2701
	(0.9885)	(0.9367)	(0.4716)	(0.7288)	(0.9933)	(0.1886)	(0.4639)	(0.1975)	(0.4732)
Tests of Overall Fit (p-values):		*LR*	0.2074						
		Score	0.4223						
		Wald	0.5305						

The table shows the results of logistic regression estimation, from January 1989 to December 2005. The dependent, categorical variable is $\pi_t(\omega)$, which is ranked as Low (less than 0.33), Medium (between 0.33 and 0.67), and High (above 0.67). DSENT is the investor sentiment measure of Baker and Wurgler (2007). LIQ0 and LIQ1 correspond to low and medium levels of liquidity, SECPCT is the percentage change in securitized loans, REALLOAN is the ratio of real estate loans to other consumer loans. P-values based on chi-square tests are in parentheses.