Discussion paper

A Re-examination of Credit Rationing in the Stiglitz and Weiss Model

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Abstract

To explain the widely observed phenomenon of credit rationing, Stiglitz and Weiss (1981) propose a theory of random rationing under imperfect information. With a simple model plausibly expanding the Stiglitz and Weiss setting, we argue that, random rationing occurs only in some extreme cases and hence is not likely to be a prevalent phenomenon. We start by illustrating that the Stiglitz and Weiss (1981) model and hence random rationing are quite sensitive to the assumption of the ranking of projects. Given that the ranking is according to the Mean-preserving Spread, there is adverse selection but no moral hazard. In the absence of moral hazard, random rationing is almost impossible to occur. Then by presuming the coexistence of adverse selection and moral hazard, we derive two required conditions for the occurrence of random rationing. First, random rationing occurs only if collateral has an overall deadweight cost other than the negative adverse selection effect. As collateral is a widely observed debt feature in practice, such an overall deadweight cost should not be the case for the majority of borrowers. Second, the occurrence of random rationing entails that the potential negative effects of the loan rate, collateral, loan size and any restrictive debt covenant simultaneously overweigh their positive effects exactly at the current contracting level. In this case, the zero-profit curve of the lender degenerates to a single point and borrowers face a take-it-or-leave-it offer. We conjecture that such a required condition leaves little space for the significance of random rationing.

1 Introduction

Many borrowers cannot get the loan they demand even if they are willing to pay a higher interest rate than the lenders are asking. This is a prevalent phenomenon called credit rationing (Tirole (2006)). As a result in practice, some projects with positive NPV cannot be financed while the interest rate spread is remarkably low.[1] The observed credit rationing deviates from the standard neoclassical assumption which would predict that lenders can always increase the price (or the interest rate) of loans to clear the market and therefore there should be no space for the presence of excess demand. The literature resorts to some imperfections in the credit markets to explain the rationale behind credit rationing, e.g., asymmetric information between borrowers and lenders (e.g., Jaffee and Russell (1976), Stiglitz and Weiss (1981)).

The Stiglitz and Weiss (henceforth S-W) (1981) paper is among the most influential ones in the literature.[2] S-W proposes a random rationing framework in which the interest rate is excluded as a rationing device due to adverse selection and moral hazard effects, and then some *apparently identical* borrowers are randomly chosen to be creditly rationed. Of course, they do not argue that random rationing is always the case, but rather that the conditions for the occurrence of random rationing are easily to meet so that it is significant in the real world. This paper re-examines their work and derives some required conditions for the occurrence of random rationing. A main conclusion is that random rationing occurs only in some extreme cases and is unlikely to be a prevalent phenomenon, e.g., the widely observed credit rationing phenomenon in practice.

In the S-W (1981) model, the bank partitions its borrowers into groups in term of the expected return of projects. Within each group, borrowers' risk and actions are private information. For a group of apparently identical borrowers, the expected return received by

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[1]For example, Roberts and Sufi (2009) find that, for around 16,000 US loans in the Dealscan database between 1996 and 2005, the interest spread over LIBOR has a mean 2.06% and standard deviation 1.37%. Berger and Udell (1992) study the dataset from the Federal Reserve's Survey of Terms of Bank Lending. For over 1.1 million US commercial loans from 1977 to 1988, they report the mean and standard deviation of the interest spread are 2.47% and 2.59% respectively. Black and de Meza (1992) states that the interest spread is rarely over 3%-4% in UK.

[2]See Arnold and Riley (2009) for more discussions about the importance of the S-W 1981 paper in Financial Economics.

the lender does not increase monotonically with the interest rate due to adverse selection and moral hazard effects. From the adverse selection aspect, a higher interest rate tends to attract high-risk borrowers and hence leaves the bank with a "worse" customer pool; from the moral hazard aspect, a higher interest rate induces the borrower to choose more risky projects. In both situations, increasing the interest rate may reduce the bank's return resulting in a hump-shaped expected return function (see *Figure* 1). If excess demand exists at the bank-optimal interest rate, the interest rate will not be chosen as a rationing device. Among these apparently identical borrowers, some are randomly chosen to be creditly rationed even if they are willing to pay a higher interest rate. According to Stiglitz and Weiss, random rationing occurs at the turning point of the expected return function of the bank. *Figure* 1 illustrates three expected return curves respectively for three different borrower groups. In a competitive credit market, the zero-profit assumption of the bank means that only the group represented by the solid curve will be rationed. This rationed group is called the marginal group for which the bank breaks even only at the bank-optimal interest rate, $\widehat{R}$. The upper and lower curves respectively represent the not-rationed groups and groups that are entirely rationed out of the market.
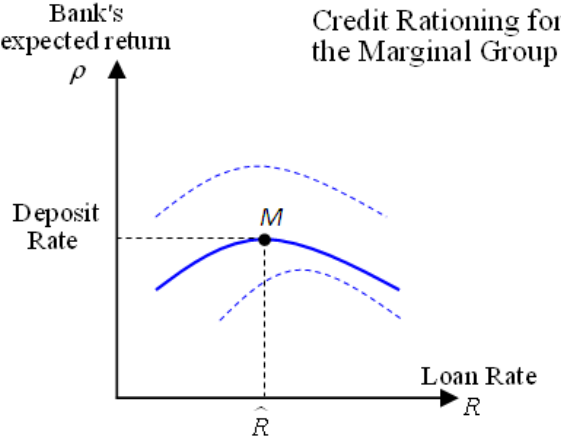

Figure 1: Credit Rationing in the sense of S-W (1981)

In the current paper, we construct a simple model following the S-W (1981) setting with some plausible expansions to re-examine the possibility of random rationing. The paper proceeds as follows.

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First, we illustrate that the S-W (1981) model is sensitive to its assumption regarding the ranking of considered projects. If borrower classification relies on the expected return of projects, i.e., all projects in the considered group have the same expected return, adverse selection is obvious because the lender and the borrower assign inverse rankings to the projects. Further if the ranking by the lender is according to the Mean-preserving Spread, moral hazard does not exist because in this special case, the ranking by the borrower is predetermined ex-ante. In the absence of ex-post risk-shifting, ex-ante risk-sorting through collateral (Bester (1985, 1987), Besanko and Thakor (1987a)) or the loan size (Besanko and Thakor (1987b), Milde and Riley (1988)) may be achieved to eliminate random rationing. Even if collateral (or equity finance) is not available, Arnold and Riley (2009) document that random rationing due to adverse selection occurs only under very extreme conditions.

Second, to justify the potential significance of random rationing, one must find cases that allow for the coexistence of adverse selection and moral hazard. Later models by S-W (1986, 1992) do change their 1981 model setting to introduce this coexistence. However, we argue that the crucial assumptions in these models are not intuitively plausible and require further clarifications.

Third, we derive two required conditions for the occurrence of random rationing under a plausibly more general setting than S-W. First, random rationing occurs only if collateral has an overall deadweight cost other than the negative adverse selection effect. As collateral is a widely observed debt feature in practice, such an overall deadweight cost should not be the case for large proportion of borrowers but only for some special group of borrowers, e.g., the poor-collateralized firms as observed in practice. However, the existing literature argues collateral is a binding constraint in finance for these firms (e.g., Bernanke and Gertler (1995), Holmstrom and Tirole (1997)). If this is true, only (many) borrowers who are bindingly constrained exactly at the current contracting level face random rationing because the ones with more collateral can avoid rationing by pledging more collateral.[3] This implies that random rationing should not be a general case. Second, the occurrence of random rationing entails that the potential negative effects of the loan rate, collateral, loan size and

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[3]This indicates a way to test the significance of random rationing by examining whether the creditly rationed borrowers are still rationed if they are willing to pledge more collateral. Probably due to data unavailability, such a test is difficult to undertake.

any restrictive debt covenant simultaneously overweigh their positive effects *exactly* at the current contracting level. In this case, the zero-profit curve of the lender degenerates to a single point and borrowers face a take-it-or-leave-it offer. We conjecture that such a required condition leaves little space for the significance of random rationing.

In the rest of the paper, section 2 constructs the model, section 3 analyzes the significance of random rationing and section 4 concludes.

2   The Model

2.1   Model Setting

We consider a competitive credit market under imperfect information.

Projects

There is a continuum of indivisible investment projects (technology) making up a set Ω. Each project is characterized by a risk-parameter θ and requires one unit of initial investment. Project θ has stochastic end-of-period payoff $x(\theta)$ with distribution function $F(x; \theta)$.

Lenders

Lenders in our model are risk-neutral financial intermediaries, e.g., banks, insurance companies and so on. They compete by offering standard debt contracts, (D, R, C) where D is the loan size, R is the gross loan rate and C is collateral.[4] There might be some other debt variables (e.g., debt covenants), but for simplicity they are ignored in our analysis. The loanable funds come from depositors. The aggregate supply of loanable funds is a function of the deposit interest rate δ. We assume all lenders are identical so that a representative lender will be analyzed. In a competitive credit market, the lender earns zero-profit. We fix the loan size D and write a contract as (R, C) for convenience reason. Note that endogenizing D has no conclusive influence on our results.

Borrowers

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[4]As S-W assumes, the value of collateral is fixed.

There are a large number of risk-neutral borrowers who are firms with limited liability. Borrower j is endowed with a set Ω_j of projects where $\Omega_j \subseteq \Omega$. Each borrower will choose one project to undertake if successfully obtaining the loan.

Information

The makeup of Ω and the return distribution of each project in Ω are common knowledge for all agents in the market. However, information concerning the projects in Ω_j is asymmetric between the lender and borrower j. First, Ω_j is only known to borrower j so that the borrower j's risk is hidden ex-ante. Second, the lender is also unaware of the exact project chosen by borrower j, that is, there might be hidden actions ex-post. Due to these two kinds of information asymmetry, the borrowers are *apparently identical.*

There are several important differences between the S-W model and ours. First, while in the S-W (1981) model the only changeable contract variable is the interest rate, we endogenize not only the interest rate but also the other contract variable, C. Second, the S-W 1981 paper models adverse selection and moral hazard separately, but here we combine both effects in a synthesized model. Third, in the S-W adverse selection model, each borrower is endowed with only one project. Since we try to address both adverse selection and moral hazard simultaneously, in our model the opportunity set for each borrower is allowed to include more than one project. It is worthy of notice that, except being more general, our model setting completely follows the S-W (1981) model. This makes it possible for us to re-examine random rationing.

2.2 The Expected Payoffs of the Contract Parties

With the signed contract, (R, C), the expected payoff of the chosen project by the borrower, will be split between the two contract parties. Let the partition to the borrower and lender be $\mu_B(x)$ and $\mu_L(x)$ respectively.

$$\mu_B(x) = \max(-C, x - R) \tag{1}$$

$$\mu_L(x) = \min(x + C, R) \tag{2}$$
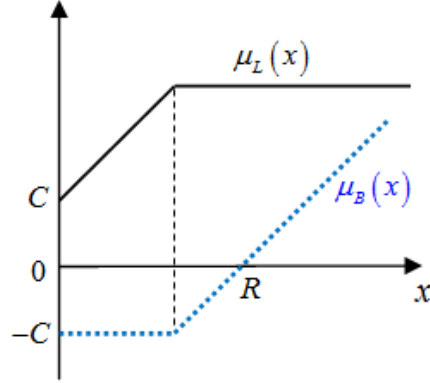
$$\mu_B(x) + \mu_L(x) = x \qquad (3)$$

Figure 2: Credit Rationing in the sense of S-W (1981)

Clearly, $\mu_B(x)$ is convex and $\mu_L(x)$ is concave. See *Figure* 2 for an illustration.

If borrower j signs contract (R, C) with her lender and chooses a project θ_j, the expected payoffs of borrower j and the lender from θ_j are respectively

$$\pi(R, C; \theta_j) = \int_0^\infty \mu_B(x) \, dF(x; \theta_j) = \int_\alpha^\infty x \, dF(x; \theta_j) + \alpha F(\alpha, \theta_j) - R \qquad (4)$$

$$\rho(R, C; \theta_j) = \int_0^\infty \mu_L(x) \, dF(x; \theta_j) - \delta = \int_0^\alpha x \, dF(x; \theta_j) - \alpha F(\alpha, \theta_j) + R - \delta \qquad (5)$$

where $\alpha = R - C$. Then with the signed contract (R, C), the lender's expected payoff from all her borrowers is

$$\Gamma(R, C) = \sum_j \rho(R, C; \theta_j) \qquad (6)$$

At (R, C), the slope of the indifference curve of the borrower $\pi = c$ where c is a positive constant is

$$\left. \frac{dR}{dC} \right|_{\pi=c} = -\frac{F(R - C; \theta)}{1 - F(R - C; \theta)} < 0 \qquad (7)$$

Similarly, it is easy to get from (7) that the slopes of the indifference curves of the lender (from an individual borrower or from all borrowers in the considered group) are also negative. Therefore, the interest rate and collateral can be considered as substitutes for every contract party.

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3 Re-examination of Random Rationing

Let's first identify an uninteresting case of random rationing. Suppose a mass of borrowers are *perfectly identical* so that they break even at the same contracting level. If the loan supply does not suffice to fill in the loan demand from this mass of borrowers, random rationing might occur in the sense that some of them are randomly chosen to be given credit but the others to be rationed. This is true not only for the credit market but also for any kind of markets (e.g., eggs or desks). By any means, it is not a case worthy of study. Given a continuum of projects and a large number of borrowers, we exclude the above case as a proof for the potential significance of random rationing.

3.1 Ranking of Projects and Sensitivity of the Model

In the model, since the project set Ω is exogenously given, the ranking of projects by the lender should also be exogenously determined. Moreover, it might be impossible to rank the projects by a simple rule, e.g., the first-order stochastic dominance (FOSD), given the exogenous distributions of project returns. However, to address incentive problems, it is required that lenders have specific preferences over the projects or, equivalently to say, the lender should assign a ranking of the projects (for any given contract). Therefore, we have to impose constraints on the set in order to rank the projects by a tractable rule proposed by the lender. Let's assume that, if two projects are not able to be ranked according to such a rule, the lender treats them identically and thus all projects in Ω can be ranked according to the rule.[5]

We first examine the case when the lenders only consider offering a single contract like the one in the S-W (1981) model.

Assumption 1 (A.1): *For the lender, the projects are ranked by a simple rule and the ranking according to this rule exhibits no inconsistency, i.e., the ranking is identical across contracts.*

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[5]In the whole paper, when we say a project is "more risky" than the other, we mean the former is preferred by a lender to the latter. That is, for the lender, "risky" is interchangeable with "bad".

Lemma 1: (A.1) implies the Second-order Stochastic Dominance (SOSD).

Proof: If in the ranking, θ_1 is preferred by the lender to θ_2 given a contract (R, C), then

$$\rho(R, C; \theta_1) \geq \rho(R, C; \theta_2) \tag{8}$$

$$\iff \int_0^\alpha x \, dF(x; \theta_1) - \alpha F(\alpha, \theta_1) + R - \delta \geq \int_0^\alpha x \, dF(x; \theta_2) - \alpha F(\alpha, \theta_2) + R - \delta \tag{9}$$

$$\iff \int_0^\alpha x \, d[F(x; \theta_1) - F(x; \theta_2)] \geq \alpha[F(\alpha, \theta_1) - F(\alpha, \theta_2)] \tag{10}$$

$$\iff x \, [F(x, \theta_1) - F(x, \theta_2)]|_0^\alpha - \int_0^\alpha [F(x, \theta_1) - F(x, \theta_2)] \, dx \geq \alpha[F(\alpha, \theta_1) - F(\alpha, \theta_2)] \tag{11}$$

$$\int_0^\alpha [F(x, \theta_1) - F(x, \theta_2)] \, dx \leq 0 \tag{12}$$

Equation (12) holds for any contract (R, C), i.e., for all values of α. Obviously this is just the definition of the SOSD.   Q.E.D.

In the credit rationing literature, an exogenously given rule to rank the projects is common. For example, two special cases of the SOSD are discussed: the FOSD (De Meza and Webb (1987), Besanko and Thakor (1987a) and the Mean-preserving Spread in the sense of Rothschild and Stiglitz (1970) (S-W (1981)). Especially, (A.1) as an assumption generalizes the S-W 1981 setting. Therefore, concerning the relevance of our purpose, (A.1) is proper. Let's follow the literature to discuss the two special cases, the FOSD and the MPS.

Proposition 1 (P.1): *If the ranking is according to the First-order Stochastic Dominance, there is no adverse selection when increasing the loan rate or the collateral requirement.*

Proof: Recall (4) and (5). Given a project and a contract, the payoffs of both contract parties are non-decreasing on the end-of-period payoff of the project. Therefore, for any two projects, θ_1 and θ_2, where θ_1 first-order stochastic dominates θ_2, both the lender and the borrower prefer θ_1 to θ_2.[6] In other words, the rankings of the projects by the lender and the borrower are identical given the FOSD as the rule of ranking. With the same ranking,

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[6]By the definition of FOSD, any agent with increasing von-Neumann Morgenstern utility would prefer the first-order stochastic dominant project.

obviously there's no adverse selection.   Q.E.D.

If there's no adverse selection, increasing the loan rate can clear the market. De Meza and Webb (1987) assume the distributions of project returns are special cases that can be ranked by the FOSD and then prove that equilibrium must be market clearing. (P.1) generalizes their conclusion.

Proposition 2 (P.2): *If the ranking is according to the Mean-preserving Spread (MPS), there is adverse selection in the sense that some safer borrowers drop out when increasing the interest rate or the collateral requirement.*

Proof: With MPS, the projects in Ω have the same expected return that will be split between the two contract parties. With risk-neutrality, the "game" between the lender and the borrower is a "zero-sum game". Therefore, the lender and the borrower have inverse rankings of projects and there is adverse selection.   Q.E.D.

Proposition 2a (P.2a): *As long as all projects have the same expected return, there is adverse selection. This conclusion is independent of the rule of ranking.*

As a generalized version of (P.2), (P.2a) is intermediate given that the ranking by the lender is inverse to that by the borrower when all projects have the same expected return. Note that under the MPS, the considered borrower group in our model can be thought as the marginal group in the S-W (1981) model and thus (P.2a) generalizes their adverse selection model as well as the Wette (1983) model which addresses the negative adverse selection effect of collateral.

Figure 3 illustrates (P.1) and (P.2) in a case of two types of risk. Under the FOSD, it is the risky borrower who drops out first when increasing the loan rate or the collateral requirement and therefore there's no adverse selection. In contrast, under the MPS, the safe project drops out first resulting in adverse selection.

Proposition 3 (P.3): *In both cases, the Mean-preserving Spread and the First-order Stochastic Dominance, no moral hazard occurs.*
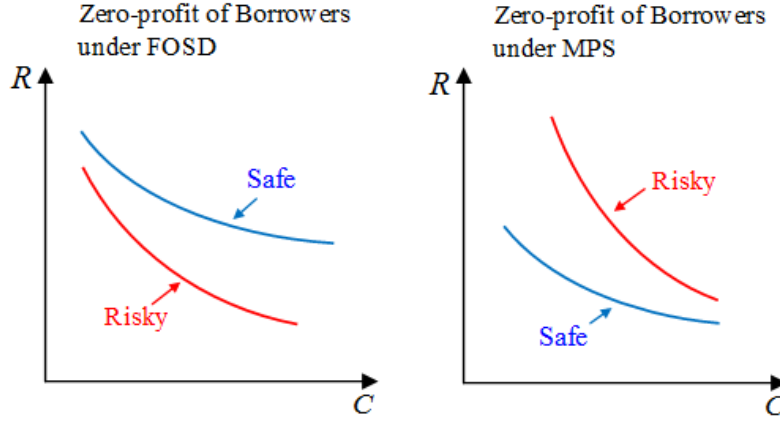
Figure 3: The Zero-profit Curves of Borrowers

Proof: According to (P.1) and (P.2), the ranking of projects by the borrower is identical to that by the lender under the FOSD and is inverse to that by the lender under the MPS. Given (A.1), the ranking by the lender is independent of contracts in both cases, so the ranking by the borrower is also independent of contracts. With predetermined rankings, borrowers have no incentive to shift risk of projects thus excluding the possibility of moral hazard.   Q.E.D.

Random rationing pools different risks. In the absence of ex-post moral hazard, a branch of risk-sorting models based on the seminal work by Rothschild & Stiglitz (1976) introduce some self-selection mechanisms to obtain a separating equilibrium and hence eliminate random rationing, for example, collateral (Bester (1985, 1987), Besanko and Thakor (1987a, b)) and the loan size (Besanko and Thakor (1987b), Milde and Riley (1988)). In the general case of the MPS, the Mirlees-Spence single-crossing condition does not hold and hence perfect sorting cannot be achieved, but risk-sorting at least mitigates rationing in many plausible cases (e.g., the two-state case of Bester (1987)).

One may argue that collateral or net worth might be limited for some group of borrowers. Arnold and Riley (2009) find that, even if collateral is excluded, the hump-shaped expected return function in the S-W (1981) adverse selection model cannot be hump-shaped. The idea is straight-forward. In a single-contract case, when the lender increases the loan rate, some safer borrowers drop out resulting in a potential decrease of the expected return. If the lender continues to increase the loan rate until only one type of borrowers remains

and breaks even, the lender earns the entire surplus of the remaining projects that is the maximum expected return she can get. Therefore, the expected return function of the lender is impossible to be *purely* hump-shaped. Two cases are possible: monotonically increasing or first hump-shaped and then monotonically increasing. With this finding, Arnold and Riley (2009) further illustrate that random rationing occurs only under very extreme conditions. In sum, we get the following proposition.

Proposition 4 (P.4): *In the absence of moral hazard, random rationing only occurs under very restrictive assumptions.*

So far, (P.1) - (P.4) show that our model as well as the S-W (1981) model is very sensitive to the assumption of the ranking of projects. The following graph is a summary of these above propositions:

$$\text{Consistency of ranking} \Rightarrow \text{SOSD} \begin{cases} \left. \begin{array}{l} \text{FOSD} \Rightarrow \text{No Adverse Selection} \\ \text{MPS} \Rightarrow \text{Adverse Selection} \end{array} \right\} \Rightarrow \text{No Moral Hazard} \\ \text{Others} \end{cases}$$

Figure 4: Summary of the First Four Propositions

When the ranking is according to the FOSD or the MPS as discussed in the literature, there is no moral hazard. However, in the absence of moral hazard, random rationing only occurs under extreme conditions. It is not difficult to check that in the other cases of the SOSD than the FOSD and the MPS, the rankings by the borrower might depend on the given contract restoring the possibility of moral hazard when increasing the interest rate. In these cases, the information of both the expected return and the risk of projects are required to be asymmetric between the borrower and the lender. This deviates from the S-W 1981 setting that within a borrower group, all projects have observed identical return but hidden risk and that the ranking of these projects is according to the MPS. That is, lenders first classify borrowers into many borrower groups in term of the expected return of their projects, and then within each group, rank the projects (borrowers) by risk.[7] This assumption makes their

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[7]It is an empirical issue whether expected returns are observable and how the lender classifies its borrowers or loans. Although it is not clear how borrowers are classified in practice, risk rating seems to play an important rule. Banks usually have their internal risk rating system (English and Nelson 1998) to rate loans or borrowers.

model more tractable. Jaffee and Stiglitz (1990) emphasize the MPS as a general assumption to derive random rationing.

3.2   The Stiglitz and Weiss 1986/1992 Models

Assumption (A.1) in our model as well as the MPS assumption in the S-W (1981) model excludes the coexistence of adverse selection and moral hazard and hence the significance of random rationing. In contrast to this conclusion, two other papers by S-W (1986, 1992) change the original setting in their 1981 model to allow for the coexistence of adverse selection and moral hazard. These two papers are quite similar, so we consider the S-W 1992 model. The model has two key assumptions different from their 1981 model: first, borrowers are risk averse; second, more wealthy borrowers are less risk averse so that they take more risky projects.

If borrowers are risk averse, there is risk sharing benefit from the borrowing-lending relationship and hence the "game" is no longer a "zero-sum game" even under the MPS assumption. Let u be the borrowers' von-Neumann-Morgenstern utility function where $u' > 0$ and $u'' < 0$. With a given project θ, the expected utility of the borrower from contract (R, C) is

$$\pi(R, C; \theta) = \int_0^\infty u(\mu_B(x)) \, dF(x; \theta) \tag{13}$$

In (13), $\mu_B(x)$ is convex but u is concave. There's no conclusive property concerning the concavity (or convexity) of $u(\mu_B(x))$ on x. Given that the lender has an identical ranking of projects in Ω across contracts, is the ranking by the borrowers also identical across contracts (i.e., is the ranking independent of the signed contract)? The answer is "no". To give a simple illustration, let us fix the collateral requirement and think of changing the loan rate only. Assume there are only two types of projects for simplicity, high-risk and low-risk. On one hand, $u(\mu_B(x))$ is concave for some contract − when the loan rate is low enough, $\mu_B(x)$ is close to be "flat" and $u(\mu_B(x))$ is concave. In this case, the borrower and the lender have the same ranking. The borrower earns lower expected return but higher expected utility from a low-risk project, so she still prefers the low-risk project. On the other hand, the high-

risk project may be chosen for some other contracts — when the loan rate is high enough, the borrower earns nothing from a low-risk project but still positive utility from a high-risk project. In sum, for any given collateral level, there is a critical value of the loan rate below which the borrower prefers a low-risk project to a high-risk one and over which the borrower prefers a high-risk project to a low-risk one.[8] This means that increasing the loan rate might induce moral hazard effects. Therefore, the assumption of risk-averse borrowers allows for the coexistence of adverse selection and moral hazard.

The risk-sorting models (e.g., Bester (1985, 1987)) cannot solve the rationing problem if adverse selection and moral hazard coexist. In this sense, some papers by Stiglitz (e.g., S-W (1992), Stiglitz (2001)) argue that the risk-sorting models are *special* or *wrong* . In addition, the Arnold and Riley (2009) conclusion is valid also in the absence of moral hazard. Therefore, the coexistence of both adverse selection and moral hazard makes random rationing potentially significant. However, if borrowers are risk averse, why is the standard debt contract chosen as the offered contract by the lender? For risk-averse borrowers and risk-neutral lenders, the optimal contract should allocate a fixed return to borrowers leaving lenders to bear all the risk. Therefore, more clarifications are required in the S-W model to reconcile the assumption of risk-averse borrowers with the chosen standard debt contract.

In addition, if more wealthy borrowers are less risk-averse so that they take more risky projects as S-W (1992) assume, why do lenders not classify borrowers according to their wealth? It is plausible to assume that information concerning the risk of projects is asymmetric because the projects are implemented after the contract is signed so that the estimation of risk by the lender is based on the borrower's description of the project. However, wealth is a current state variable. In most cases wealth of a borrower can be observed by the lender. If wealth is a strong indicator of risk attitude of borrowers, borrower classification according to the level of wealth should be profitable for the lender.

To sum up, the S-W (1992) model is based on two assumptions that need more reasonable clarifications. When relaxing these assumptions, the model cannot justify random rationing

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[8]It is intuitive that the critical value is increasing on the collateral level because increasing the collateral requirement does reduce the incentive of borrowers to take risk. This conclusion in the Stiglitz and Weiss (1992) model is summarized by a "switch line" along which the borrower is indifferent between the high-risk and the low-risk, below which the low-risk is chosen and over which the high-risk is chosen.

like their 1981 model. In this sense, the conclusions we draw in subsection 3.1 are not undermined by the S-W 1986/1992 models.

3.3   The Required Conditions for Random Rationing

To justify the potential significance of random rationing, one needs to address cases that allow for the coexistence of adverse selection and moral hazard. In this section, we derive two required conditions for the occurrence of random rationing presuming this coexistence.

Let $(\widehat{R}, \widehat{C})$ denote the equilibrium contract with rationing. To derive the first required condition for random rationing, define a contracting set around $(\widehat{R}, \widehat{C})$ in the $C - R$ space, $\Phi = \{(a, b) : a \in [\widehat{R} - \epsilon_1, \widehat{R} + \epsilon_1], b \in [\widehat{C} - \epsilon_2, \widehat{C} + \epsilon_2]\}$ where $\epsilon_1 > 0$ and $\epsilon_2 > 0$ are arbitrarily small real numbers.

Assumption 2 (A.2): *at any point $(R, C) \in \Phi$, the higher is the risk of a project according to the ranking by the lender, the steeper the indifference curve of the borrower who undertakes the project* (see *Figure* 5).
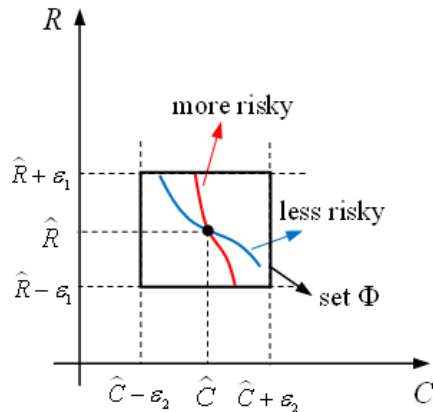

Figure 5: Assumption 2

Note that (A.2) imposes more constraints on the return distributions of projects in Φ. It is more strict than (A.1) inside set Φ, but it says nothing outside set Φ. The intuition is

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that, given contract $(\widehat{R}, \widehat{C})$, a risky borrower (i.e., the borrower with a more risky project) should be more reluctant than a safe one to obtain a reduction of the loan rate by pledging an increment of the same amount of collateral. Although (A.2) does not hold generally given the SOSD as the ranking rule, it is reasonable if set Φ is arbitrarily small.[9]
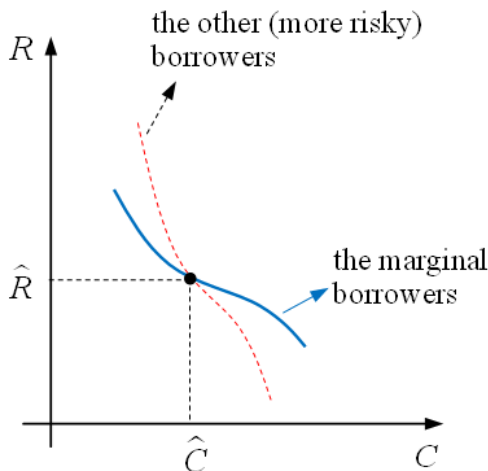

Figure 6: Proposition 5

Proposition 5 (P.5): *Random Rationing occurs only if collateral has a significant* dead-weight cost at the current contracting level.

Proof: At the equilibrium contract with rationing, say $(\widehat{R}, \widehat{C})$, increasing the loan rate results in adverse selection, so there must be one type of borrowers who exactly breaks even. We call them the marginal type of borrowers. In Figure 5, the solid curve denotes the zero-profit curve of the marginal type. Let's consider moving the current contract $(\widehat{R}, \widehat{C})$ along this curve to the right-down side, i.e., decreasing the loan rate and increasing the collateral requirement. On one hand, from (A.1), it is clear that no adverse selection occurs due to this movement because the zero-profit curves of the other (more risky) types lie above that of the marginal type. On the other hand, there is no moral hazard because a decrease of the loan rate and increase of collateral both induce borrowers to reduce risk. Therefore, under the movement, neither adverse selection nor moral hazard occurs. Moreover, according to (A.2),

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[9]This is consistent with Bester (1985) in the sense that more risky borrowers prefer less collateral.

the indifference curves of the other types should be steeper than that of the marginal type at $(\widehat{R}, \widehat{C})$ because the marginal type of borrowers is the safest remaining loan applicants. Therefore, under the movement the lender earns constant profit from the marginal type (i.e., the total surplus of the marginal type) and increasing profit from the other types as long as the movement does not reduce the end-of-period payoff of the projects or, equivalently to say, as long as collateral does not incur a deadweight cost. Given that the end-of-period payoff of the projects is given, the movement increases the lender's profit in total. This contradicts with $(\widehat{R}, \widehat{C})$ being the equilibrium contract. Thus, collateral must have an overall deadweight cost to justify random rationing. This deadweight cost should be significant enough to offset the larger profit of the lender.   Q.E.D.

Collateral does incur some cost, for example, the inflexibility cost for the borrower and deadweight loss from the inefficient transfer of collateral in the presence of default when the lender and the borrower have divergent valuations of the pledged collateral (e.g., Barro (1976)). However, it is well documented in the literature that collateral mitigates ex-post moral hazard by extracting efforts, reducing risk-taking behavior and other moral hazard concerns (e.g., Holmstrom and Tirole (1997), Aghion and Bolton (1997)), reducing enforcement cost (e.g., Banerjee and Newman (1993)) and state verification cost (e.g., Townsend (1979), Gale and Hellwig (1985)). That is, collateral also has positive effects. In practice, collateral is a widely observed debt feature and, for small and medium businesses (SMEs), the collateral-debt ratio is near to one.[10] This stylized fact shows that an overall deadweight cost of collateral at the offered contracting level is unlikely to be significant across many groups of borrowers. One interpretation is that an overall negative effect of collateral is the case only for some special group of borrowers, e.g., poor-collateralized firms as observed in practice, so is random rationing. However, another interpretation is widely accepted by the literature that collateral is a binding constraint in finance for the poor-collateralized firms (e.g., Bernanke and Gertler (1995), Holmstrom and Tirole (1997)). Are the poor-collateralized firms creditly rationed because lenders deny their application even if they are willing to pledge more collateral (due to an overall negative effect of collateral) or because

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[10]Binks, Ennew and Reed (1988) report that, for 85% of UK business loans, the ratio of collateral provided to the size of the loan exceeded unity. According to Berger and Udell (1990), in U.S. domestic bank lending, nearly 70% of all commercial and industrial loans are made on a secured basis.

collateral is a binding constraint in finance for them? Note the two possibilities are virtually different concerning the consequent rationing forms. The first results in random rationing, while the second does not. If collateral is a binding constraint, only (many) borrowers who are bindingly constrained exactly at the current contracting level are randomly chosen to be rationed. This is similar with the uninteresting case that we discussed at the beginning of section 3 and which is not the S-W random rationing. Therefore, one way to test the significance of random rationing is to examine whether the creditly rationed borrowers are still rationed if they are willing to pledge more collateral. To our knowledge, so far no research concerning this test has been done probably because of data unavailability.

Furthermore, collateral and the interest rate are substitutes. If collateral has a significant deadweight cost for a relatively large proportion of borrowers, loan contracts in practice should exhibit high loan rates but low collateral requirements. This is inconsistent with the stylized fact that collateral is widely used in loan contracts but the interest rate spread (margin) is remarkably low (e.g., Binks, Ennew and Reed (1988), Berger and Udell (1990, 1992), Roberts and Sufi (2009)). The remarkably low interest spread in the absence of government constraints such as usury laws or interest ceiling together with that high collateral requirement is strong evidence for the prevalence of credit rationing,[11] but this also indicates that an overall deadweight cost of collateral is unlikely to be similarly prevalent.

(P.5) is derived under (A.1) and (A.2). It is still possible that these assumptions result in the inconsistency between the potential prevalence of random rationing and the stylized facts in practice. What we have in mind is that, in some cases of the SOSD that have not been discussed in the literature, adverse selection and moral hazard might coexist to allow for the occurrence of random rationing in relatively general cases. Now let release (A.1) and (A.2) to get another required condition for random rationing. Obviously, to exclude the loan rate as a rationing device, the expected return function of the bank should not be always increasing on the loan rate. In the literature, it seems to be a common belief that this non-monotonic expected return function on the loan rate is also a sufficient condition for the occurrence of random rationing.[12] This is not true. If the lender can choose any

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[11]A possible alternative explanation is that all projects in practice are quite safe.

[12]Much evidence comes from that many papers only find a hump-shaped expected return function of the lender on the loan rate (e.g., due to bankruptcy cost) and then claim the occurrence of random rationing.

instrument (e.g., contract variable) other than the loan rate to clear the market, random rationing cannot occur. Therefore, random rationing does require every contract term has an overall negative effect at the current contracting level.

Proposition 6 (P.6): *Random rationing occurs only if the potential negative effects of the loan rate, collateral, loan size (or self-finance) and any restrictive debt covenant simultaneously overweigh their positive effects exactly at the current contracting level.*

Only if excess demand still exists after lenders exhaust all the instruments (contract variables) to ration credit, random rationing is possible. In this case, the zero-profit curve of the lender degenerates to a single point that is the maximum point of the expected return function of the lender. For the considered borrowing group with random rationing, the lender breaks even only at the lender-optimal contract that is not only the single choice of the lender but also a take-it-or-leave-it offer for borrowers in the considered group.[13] S-W (1992) also proposes three required conditions for the occurrence of random rationing, among which the second one is *"the adverse selection/ adverse incentive effects of changing interest rates or the non-price terms of the contract must be sufficiently strong (at some values of the relevant variables) that it is not optimal for the lender to use these instruments fully to allocate credit"*. This required condition is consistent to the one proposed by (P.6). Note that (P.6) is derived under very general setting. Such a required condition, even if being logically possible, is extremely strict leaving little space for the significance of random rationing.

De Meza and Webb (2006) find that random rationing implies infinite marginal cost of funds to the borrower, so the borrower has an overwhelming incentive to cut their loans by a dollar and avoid rationing. Their model endogenizes the loan size by assuming that borrowers are able to access some self-finance through reducing current consumption, delaying the project to collect internal funds, etc. Implicitly in their model lenders can use the loan size (or self-finance) to clear the market, i.e., decreasing the loan size (or increasing self-finance) at the pooling contract benefits the lender. From (P.6), we can see that this assumption itself has already excluded the possibility of random rationing.

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[13]Note that here for convenience we exclude the case in which there is a continuum of local maximum points.

4   Concluding Remarks

Why are many projects with positive NPV not able to get financed while the interest rate spread is remarkably low or why can't borrowers with these projects obtain the loan they need by increasing the loan rate? This issue is of importance not only because of its practical significance but also because of its implication for the money transmission merchanism. Stiglitz and Weiss (1981) in their seminal paper give the first explanation of the "true" credit rationing based on adverse selection and moral hazard due to imperfect information. In this paper, we reexamine their model, derive two required conditions for the occurrence of the S-W random credit rationing and conclude that, even if being logically possible, random rationing occurs under extremely strict conditions and hence is not likely to be a widely observed phenomenon.

Some empirical research does not find evidence in favor of the significance of credit rationing (e.g., Berger and Udell (1992)), which is consistent with the finding of Riley (1987), Arnold and Riley (2009) and this paper. A firm denied credit by one institution will simply go to another one and eventually obtain a loan (Gale and Hellwig (1985)). While the Stiglitz and Weiss adverse selection model (1981) is based on ex-ante asymmetric information, many other models assume information is symmetric ex ante and focus on illustrating how ex-post agency problems stemming from a specific borrowing-lending relationship induce credit rationing, e.g., costly state verification (Williamson (1987)), money diversion (Hart & Moore (1994)), hidden effort (Holmstrom & Tirole (1997)) and limited enforcement (Krasa and Villamil (2000)). Hart and Moore (1994, 1998) and Holmstrom and Tirole (1997) illustrate how poor-collateralized firms can be credit rationed because collateral (or net worth) is a binding constraint in corporate finance. Essentially, the rationale in these models is different from the S-W random rationing. This leaves a possible way to test the significance of random rationing by examining whether the credit rationed borrowers are still rationed if they are willing to pledge more collateral, to reduce the loan size and to add restrictive covenants.

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