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On the Pricing of Performance Sensitive Debt*

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Abstract

Performance sensitive debt (PSD) contracts link a loan's interest rate to the borrower's measure of credit relevant firm performance, e.g., if the borrower becomes less creditworthy, the interest rate increases according to a predetermined schedule. PSD provisions are included in approximately 35% of all U.S. and Canadian corporate loans (1994 - 2009, Thomson Reuters Dealscan). Based on standard no-arbitrage theory and observed contractual specifications, we derive and empirically test a new pricing model for PSD contracts with a cash flow driven performance measure. Our sample consists of 270 PSD loans where the loan contractual terms are collected from Thomson Reuter's Dealscan database and the borrower information is collected from Compustat and CRSP. The theoretical market value of a PSD contract is on average 3.2% above par value at the time of issue. By considering the subsamples of only *interest increasing* and *interest decreasing* PSD contracts, respectively, we find that the former is overpriced by 7.7% on average, whereas the latter, on average, exhibits no significant mispricing. The empirically observed overpricing of interest increasing contracts is consistent with the signalling hypothesis of Manso, Strulovici & Tchistyi (2010) and the cost of moral hazard explanation of Asquith, Beatty & Weber (2005). The not significant mispricing of interest decreasing contracts is consistent with the idea that borrowers have increased bargaining power in the form of outside alternative financing options for these contracts.

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1 Introduction

Performance sensitive debt (PSD) contracts link the interest rate paid on a firm's loan to a measure of its credit relevant performance over time. The two most common categories of firm credit performance measures are cash flow ratios and credit ratings. Since the mid 1990's performance sensitive provisions in both private and public corporate loans are common. Using Thomson Reuter's Dealscan database for the years 1994 - 2009 we find that PSD loans constitute 11.2% of the total number of loans in the database and 35.1% of loans granted in the U.S. and Canada (see also Figure 1, Section 3). Market participants indicate that more than 50% of recently issued syndicated bank loans in Europe include such provisions. Based on standard no-arbitrage theory, using results from Mjøs & Persson (2010) and observed contractual specifications, we derive a closed-form valuation formula to price PSD contracts with a cash flow driven performance measure, and compare the theoretical market price of a PSD contract with the par value of the loan at the time of issue. We also study two important subclasses of PSD contracts; *interest increasing* and *interest decreasing* PSD contracts. The first category includes loan contracts where the borrower initially pays the lowest contractual interest rate, and where the interest rate increases if the borrower's credit quality deteriorates. The second category includes loan contracts where the borrower pays the highest contractual interest rate initially, and where the interest rate decreases if the borrower's credit quality improves.

1.1 Our Contribution

Our paper is related to the article by Manso, Strulovici & Tchistyi (2010). Using the framework of Leland (1994) they develop a general pricing model for a broad class of performance sensitive debt. They derive closed form solutions for the market value of infinite horizon, linear, asset-based PSD contracts and step-up PSD contracts, respectively. A linear PSD contract has interest payments of the form $C(x) = \beta_0 - \beta_1 x$, where x is some continuous credit relevant performance measure, β_0 and β_1 are constants, and $\beta_0 > 0$.

The essential property of a step-up/-down PSD contract, compared to a linear PSD contract, is that the interest rate is constant within a certain range of the credit performance measure. As such, step-up/step-down loans are easier to implement, and actually observed in markets in contrast to linear PSD contracts, which imply continuously changing interest rates for a continuously changing performance measures.

An important implementation issue for PSD contracts is the observability and verifiability of the underlying performance measure. As in Manso, Strulovici & Tchistyi (2010), our model assumes that the performance measure is continuously observable to all contracting parties. For accounting based performance measures the observability is determined by the borrower's external financial reporting frequency, i.e., typically a maximum of 4 (quarterly) observations per year. These reports also present a delayed measure of the borrower's credit quality. Publicly listed companies typically file their financial reports at least one month after the end of the reporting period[1], subject to listing requirements. Any reported profit, cash flow, or other flow measures represent averages over a discrete time period and not the most recent continuous rates. This limited observability of the credit performance measure implies that linear PSD contracts are more sensitive to the assumption of a continuously observable performance measure compared to step-up/-down contracts. Any valuation effects of these implementation issues are not included in our analysis.

Manso, Strulovici & Tchistyi (2010) show that with no other market imperfections than bankruptcy costs and tax benefits of debt, the use of PSD leads to earlier default and lower equity value compared to comparable fixed-rate debt, and therefore find the use of these contracts not optimal. Consequently, they develop a screening model where the company can choose to issue either performance sensitive debt or fixed rate debt. They find that the existence of PSD contracts can be explained by the contracts' ability to mitigate adverse selection problems for the borrower. This conclusion is supported by an empirical analysis which shows that firms using performance sensitive debt are more likely to get improved credit ratings in the future compared to firms that choose ordinary fixed-interest loans. They do not empirically test their pricing models for PSD contracts.

We focus on the pricing of PSD contracts, and our model of a step-up/-down PSD contract differs from Manso, Strulovici & Tchistyi (2010) in the following ways. We derive a closed form pricing formula for a contract with finite maturity. Many PSD contracts include a covenant specifying when

[1]Market participants inform us that borrowers tend to report observations of the performance measure early in case of interest decreasing contracts and late in case of interest increasing contracts, as would be expected.

the borrower defaults on the contract (although a contract default may not necessarily lead to a full liquidation of the borrowing company). We include this contractually specified default covenant. This contractual default is exogenous as opposed to the optimal endogenous company liquidation-trigger analyzed in Manso, Strulovici & Tchistyi (2010). We only analyze PSD contracts where the performance criterion is based on total firm cash flow. This assumption excludes, e.g., rating based contracts. Market evidence indicates that cash flow based performance measures, which our model covers, are the most common performance measures in such contracts (See Table 1, section 4.1). In addition, we separately consider both interest increasing PSD contracts, interest decreasing PSD contracts, and contracts with a mixture of both provisions.

Several papers have empirically tested the ability of structural debt models to produce correct prices and/or spreads (e.g, Eom, Helwege & Huang (2004), Huang & Huang (2003)). Our paper contributes to this literature by empirically testing the pricing model using 270 U.S. loan contracts obtained from Thomson Reuter's Dealscan database. Our sample includes both interest increasing PSD loans and interest decreasing PSD loans, as well as PSD loans including both provisions.

1.2 Decomposition of Interest Increasing and Interest Decreasing PSD Contracts

In order to interpret our empirical results we find it useful to decompose a PSD contract into a sum of a fixed rate loan and an option portfolio. From a lender's point of view an interest increasing PSD contract is equivalent to a fixed rate loan plus a portfolio of long put options. The put options give the lender a right to receive increased interest payments if the borrower's credit quality deteriorates. The lender has this right at every point interest payments are due in the contract period. Thus, we interpret this as a portfolio of options where the maturity of each option corresponds to an interest payment date. Denote the time 0 market value of the fixed rate loan and the put option portfolio by F_0^I and P_0, respectively. Thus, the time 0 market value of the interest increasing PSD loan is $V_{PSD}^I = F_0^I + P_0$.

An interest decreasing PSD contract is, from a lender's point of view, equivalent to a fixed rate loan plus a portfolio of short call options. The call options give the borrower a right to pay reduced interest rates, at any interest payment date, if its credit quality improves. Since this right is held by the borrower, the lender is short in these options. Denote the time 0 market value of the fixed rate loan and the call option portfolio by F_0^D and C_0, respectively. The market value of the interest decreasing PSD loan is $V_{PSD}^D = F_0^D - C_0$. The interest increasing/decreasing provisions are contractually determined and one could, thus, argue that the put and call options do not include the customary optionality at maturity included in regular options. However, any rational optionholder would exercise the options when they are in the money at maturity, so we can safely apply the option interpretation. Also since the size of the increase or decrease in interest rate payments is independent of the underlying performance measure within a certain range of the credit performance measure, the put and call options are of digital type[2]. Normalizing the par value of the PSD loan to 100, the theoretical correct time 0 price is $F_0^I + P_0 = 100$ and $F_0^D - C_0 = 100$ for the interest increasing and interest decreasing cases, respectively.

A PSD contract which has both interest increasing and interest decreasing provisions can be decomposed into a portfolio of a fixed rate loan F_0^B, with fixed interest rate equal to the initial interest rate of the PSD contract, a portfolio of short digital call options, and a portfolio of long digital put options. This contract's time 0 market value can be written as $F_0^B - C_0 + P_0 = 100$.

1.3 Initial Interpretation of Empirical Results

Our empirical findings suggest that the lenders, on average, overprice interest increasing PSD contracts at the time of issue, i.e., $F_0^I + P_0 > 100$, whereas there is no significant difference between our average theoretical price and par value for interest decreasing PSD contracts, i.e., $F_0^I - C_0 = 100$. Assuming that the fixed rate component of the loan is correctly priced, these observations are consistent with the idea that lenders systematically undervalue (and underpay for) the options which they buy, whereas the options they sell are valued correctly.

One possible explanation for the overpricing of interest increasing contracts is that the borrower would accept a low price of the option he sells if this is consistent with his private valuation of the

[2] A digital option is an option whose payout is fixed after the underlying asset exceeds a predetermined threshold or strike price. In the literature these options are also commonly referred to as 'cash-or-nothing' options, see, e.g., McDonald (2006).

option. This interpretation is in correspondance with the signalling hypothesis in Manso, Strulovici & Tchistyi (2010). They find that high quality borrowers facing information asymmetry signal their quality by using interest increasing PSD contracts.

The observed not incorrect pricing of interest decreasing contracts may reflect a borrower's improved outside financing options for improved credit quality, and, thus, gives the borrower more bargaining power. Our findings are consistent with the findings of Asquith, Beatty and Weber (2005), that interest decreasing provisions are more likely to be included in the loan contract the higher the borrower's probability of prepayment of the loan. Prepayment may be interpreted as non-contractual early payments, a likely indicator of the borrower's access to new sources of funding.

The observed overpricing of interest increasing PSD contracts may also be interpreted from the lenders' perspective. Asquith, Beatty and Weber (2005) find that lenders tend to prefer interest increasing contracts when moral hazard costs are likely to be high. The apparent overpricing of these contracts in our full information model may, thus, reflect lenders' more negative private information and risk assessment. The overpricing may also be explained by lenders' relative bargaining power. Lenders may have private information about the borrowing firm which limits the firm's ability to switch lenders and realize benefits from competition (Sharpe (1990), Rajan (1992), von Thadden (2004)). The borrowing firm, thus, risks a 'lock-in' situation since uninformed potential lenders face a winner's curse which limits competition.

In addition to the possible interpretations mentioned above, our results are consistent with other empirical tests of structural models of credit risk. Using *distance-to-default*, the normalized distance (measured in standard deviations) of a firm's time 0 asset value from its contractual default threshold, as a measure of credit-quality, our results confirm those of Huang & Huang (2003). They find that credit risk accounts for only a small fraction of observed yield spreads for loans with higher credit-quality (interest-increasing), but accounts for a larger share of yield spreads for loans with lower credit-quality. The distance-to-default measure have become widely used in the finance literature as a measure of credit quality[3].

[3]See, e.g., Sundaram & Yermack (2007) for a reference.

The remainder of the paper is structured as follows. Section 2 discusses related literature. Section 3 briefly presents the details of the theoretical pricing model. Section 4 provides a brief description of the market for PSD loans, as well as a description of the data we use. Section 5 includes an example of a PSD contract and calculates its theoretical price. In Section 6 we present and discuss empirical results for the whole sample. Section 7 concludes. Technical calculations, supplementary descriptive statistics and regression results are collected in three appendices.

2 Related Literature

Our paper is related to the literature on performance sensitive debt and to the literature of empirical tests of structural credit risk models.

The existing literature on performance sensitive debt can broadly be divided into two categories. One is the study of public corporate debt (i.e., bonds), and the other is the study of private corporate debt (predominantly bank loans). Typically, the focus has been on explaining the existence of these contracts. If one assumes positive bankruptcy costs, performance sensitive debt contracts, at least at a first glance, seem inefficient. Whilst increased interest payments in bad states of the world may have an *ex ante* disciplining effect, the *ex post* conditional probability of bankruptcy increases, and destroys, rather than adds, firm value. In the light of this intuition, existing research on PSD has mainly focused on the efficiency and existence of these contracts. Regarding the existence, the problem of potential information asymmetry as pointed out in the seminal work of Myers & Majluf (1984), and the problem of agency costs, identified by Jensen & Meckling (1976), both may be important explanations.

The information asymmetry problem is the following: Assuming that lenders are unable to distinguish good firms from bad firms, good firms cannot raise funding on fair terms, and hence might withdraw from undertaking positive NPV projects. If firms with the best projects could credibly signal their quality to lenders, this could (partially) solve these problems. Performance sensitive debt contracts may work as such a signalling device. That is, raising such debt might be costly for a low quality firm since it will have a larger probability of increased coupon payments and hence also an increased probability of default. If these costs are high enough, the low quality firm chooses not to mimick the high quality firm. An important contribution by Manso, Strulovici & Tchistyi (2010) is the analysis of PSD contracts as a credible signalling device for borrowers.

With regards to agency costs, performance sensitive debt could mitigate such costs since the manager's incentives for asset substitution or risk-shifting[4] are reduced. More explicitly, if a manager increases risk, the probability of triggering an interest rate increase also increases, and, hence, the cost of capital and bankruptcy costs will be higher. In this spirit PSD contracts may have a disciplinary effect on the manager, reducing the incentives for asset substitution.

Tchistyi (2006) studies optimal security design in a dynamic setting where the agency problem arises from the assumption that a manager in charge of a project could divert cash flows for his own consumption. Allowing cash flows to be correlated over time, he finds that the optimal contract could be implemented using a credit line with performance sensitive provisions.

Tchistyi, Yermack & Yun (2010) examine the relation between CEO's equity incentives and their use of PSD debt. They find evidence that managers whose compensation is more sensitive to stock volatility choose steeper and more convex performance pricing schedules, while those where compensation is more sensitive to the actual stock price choose flatter, less convex pricing schedules. They conclude that PSD contracts provide a channel for managers to increase firms' financial risk in order to gain private benefits.

The literature on performance sensitive public debt has typically been focusing on rating-triggered bonds. Bhanot & Mello (2006) study such bonds and their ability to mitigate risk-shifting problems. They argue that rating-triggered bonds is not an attractive financing instrument. They find that this type of financing would increase incentives for asset substitution, as well as bankruptcy costs. The potential increase in tax benefits could not offset these costs. Contrary to this result, Koziol & Lawrenz (2009) find that rating-triggered bonds can be designed to mitigate asset substition

[4]Risk-shifting refers to the problem that arises when a company, post-financing, exchanges its low-risk assets for high-risk assets. This substitution transfers value from a firm's bondholders to its shareholders by increasing the option value of equity through increased asset volatility. See, e.g., Jensen & Meckling (1976).

or asymmetric information problems. They conclude that the optimal design and optimal use of step-up bonds are highly dependent on which of the two problems the bonds are intended to deal with. Using a reduced form model of credit risk, Lando & Mortensen (2004) examine the pricing of rating-triggered bonds in the European Telecom sector. They find that such bonds are traded at a discount relative to comparable fixed-coupon bonds, and, hence, that issuing rating-triggered bonds increase the cost of capital. Houweling, Mentink & Vorst (2004) find evidence that some rating-triggered bonds have lower volatility than comparable plain-vanilla bonds, and, hence, offer protection for investors in the form of reduced price movements.

Asquith, Beatty & Weber (2005) make an important contribution to the understanding of performance sensitive bank debt. In addition to information asymmetry and agency costs, they claim that the existence of renegotiation costs provides another rationale for using PSD loans. In a traditional bank loan, the lender only has the opportunity to adjust the interest rate charged when a borrower violates pre-specified covenants in the loan agreement. Thus, the lender does not receive a higher interest rate on the loan for small deteriorations in a borrower's credit quality. The only way the lender may obtain revised risk-adjusted interest rates is by costly renegotiations with the borrower. The argument also works the other way around. A borrower, whose credit quality improves after entering into a loan agreement, would argue for a lower interest rate on his loan. In a traditional bank loan this, again, only takes place if the borrower is willing to incur renegotiation costs. Performance sensitive debt may avoid costly renegotiations by automatically relating interest rates to the performance of the borrower. The authors find empirical evidence that performance sensitive debt is used when it has the largest net benefits, i.e., when moral hazard, adverse selection problems, or renegotiation costs are likely to be high. They also find that interest decreasing PSD is used when prepayment of the loan is more likely, i.e., when borrowers' relative bargaining power is assumed to be high. Finally, they find that including interest-increasing performance sensitive provisions in the debt contract has significant economic effects since, controlling for firm characteristics, borrowers are offered 26 basis points initial lower credit margins (over LIBOR) when these provisions are included in the contract. We do not find similar results in our analysis, see Table 18 in Appendix C.

Roberts & Sufi (2009) find evidence that performance sensitive provisions are included in the debt contract in order to shape the renegotiation game between borrower and lender, and not to mitigate high renegotiation costs. As an example they find that PSD contracts written on a measure based on the borrower's cash flow are more likely to experience a renegotiation leading to reductions in the amount of credit and an increase in the interest rate following an ex-post decline in cash flow. These findings lead them to suggest that the rationale for including pricing grids in debt contracts must be to allocate bargaining power between the contracting parties in different states of the world.

Existing literature mainly concentrate on agency cost and asymmetric information arguments in explaining why PSD contracts are so widely used in practice. In addition to Manso, Strulovici & Tchistyi (2010) our paper is the only paper focusing on the pricing of PSD contracts, cf. the discussion in the introduction.

Several papers have made empirical tests of different structural models of credit risk, focusing on the models' ability to replicate observed market prices and yield spreads. The evidence is mixed. Jones, Mason & Rosenfeld (1984) find that predicted prices are, on average, 4.5% too high, and that the pricing error is largest for speculative-grade firms. More recently, Eom, Helwege & Huang (2004) compare five different models, and find that predicted spreads from some are too high, whereas some models generate too low spreads. Huang & Huang (2003) also test several different models. They use a calibration approach based on historical data, and find that credit risk accounts for only a small fraction of observed corporate yield spreads for investment grade bonds, but accounts for a larger share of high-yield bond spreads. They also find that different structural models predict fairly similar yield spreads. Our results are consistent with these findings.

3 The model

3.1 General set-up

This section reviews the general set-up and the main results needed for our pricing model. A filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, Q)$ is given. We impose the standard frictionless, continuous time market assumptions, see, e.g., Duffie (2001).

We assume throughout that the time t firm cash flow rate ξ_t is given by a geometric Brownian motion under an equivalent martingale measure Q as

$$d\xi_t = \mu \xi_t dt + \sigma \xi_t dW_t, \tag{1}$$

where the initial value ξ_0 is a constant. Here the drift parameter μ and the diffusion parameter σ are constants, and W_t represents a standard Brownian motion under Q. Denoting the constant risk-free interest rate by r, the time t value of the firm's assets A_t equals the risk-adjusted expected discounted value of future cash flow

$$A_t = E_t^Q \left[\int_0^\infty e^{-r(s-t)} \xi_s ds \right] = \frac{\xi_t}{r-\mu}. \tag{2}$$

Hence, the market value of the firm's assets is given by the geometric Brownian motion

$$dA_t = \mu A_t dt + \sigma A_t dW_t, \tag{3}$$

where the initial value $A_0 = \frac{\xi_0}{r-\mu} = A$ is a constant. Assume that δ is a constant dividend rate paid to equityholders. The drift under the equivalent martingale measure Q is in this case equal to $\mu = r - \delta$.

Let T be the finite time horizon corresponding to the maturity of debt. Let the constant $C < A$ be an absorbing barrier, and define the stopping time τ (with respect to $\mathcal{F}_t$) as

$$\tau = \inf\{t \geq 0, A_t = C\}. \tag{4}$$

The constant C can be interpreted as the contractual default barrier, and τ as the time of default.

3.2 A Valuation Model of a PSD Contract

This section explains the general structure and the closed form valuation expression of a PSD contract. In addition to the contractual default barrier C, a PSD contract includes $n + m$ constant levels or non-absorbing barriers $B_1, \dots, B_{n+m}$ so that $B_1 > \cdots > B_{n+m} > C$. For notational simplicity only, we let $B_0 = \infty$ and $B_{n+m+1} = C$. We assume that $B_n > A > B_{n+1}$ (n barriers above A and m barriers below A). The contract specifies a sequence of interest rates, where c_{i+1} is paid when $B_i > A_t > B_{i+1}$, $i = 0, ..., n+m$. All c_is are assumed to be constants. An interest increasing contract is defined by $n = 0$ and $c_1 < c_2 < ... < c_m$, whilst an interest decreasing contract is defined by $m = 0$ and $c_1 < c_2 < ... < c_n$. See Figure 1 for an illustration. The time 0 market value of a PSD contract $L(A)$ is given by the expression

$$L(A) = V(A) + De^{-rT}P(A) + D(1-\kappa)H(A), \tag{5}$$

where

$$\begin{aligned} V(A) &= E\left[\int_0^{\tau\wedge T} \sum_{i=0}^{n} c_{i+1}e^{-rs}1\{B_i > A_s > B_{i+1}\}ds\right], \\ P(A) &= Q(\tau > T), \\ H(A) &= E^Q[e^{-r\tau}1\{\tau \leq T\}]. \end{aligned}$$

Here $V(A)$ represents the time 0 market value of the contractual interest rate payments. Also, $De^{-rT}P(A)$ is the time 0 market value of receiving the face value of debt (D) in the case of no default, and $D(1-\kappa)H(A)$ is the time 0 market value of the recovery amount in the case of default. The parameter κ, thus, captures any debtholders' loss proportional to the face value of debt in case of contractual default including, but not limited to, the loss in case of liquidation of the issuing company. The expressions for $P(A)$ and $H(A)$ are standard and can be found in Appendix A. Note carefully that in our model $H(A)$ depends on the contractually specified default barrier C, not an endogeneously determined (company) bankruptcy barrier. The expression for $V(A)$ is derived below.

The time 0 market value of the interest payments $V(A)$ can be seen as a portfolio of two different claims. The first claim pays interest rate c_i in the case where the asset process A_t is between the barriers B_{i-1} and B_i for $i = 1, ..., n$, and where the starting value of the process A is below the barrier B_n. Denote the time 0 market value of this claim by $V^B(A, B_1, ..., B_n, C)$. The second

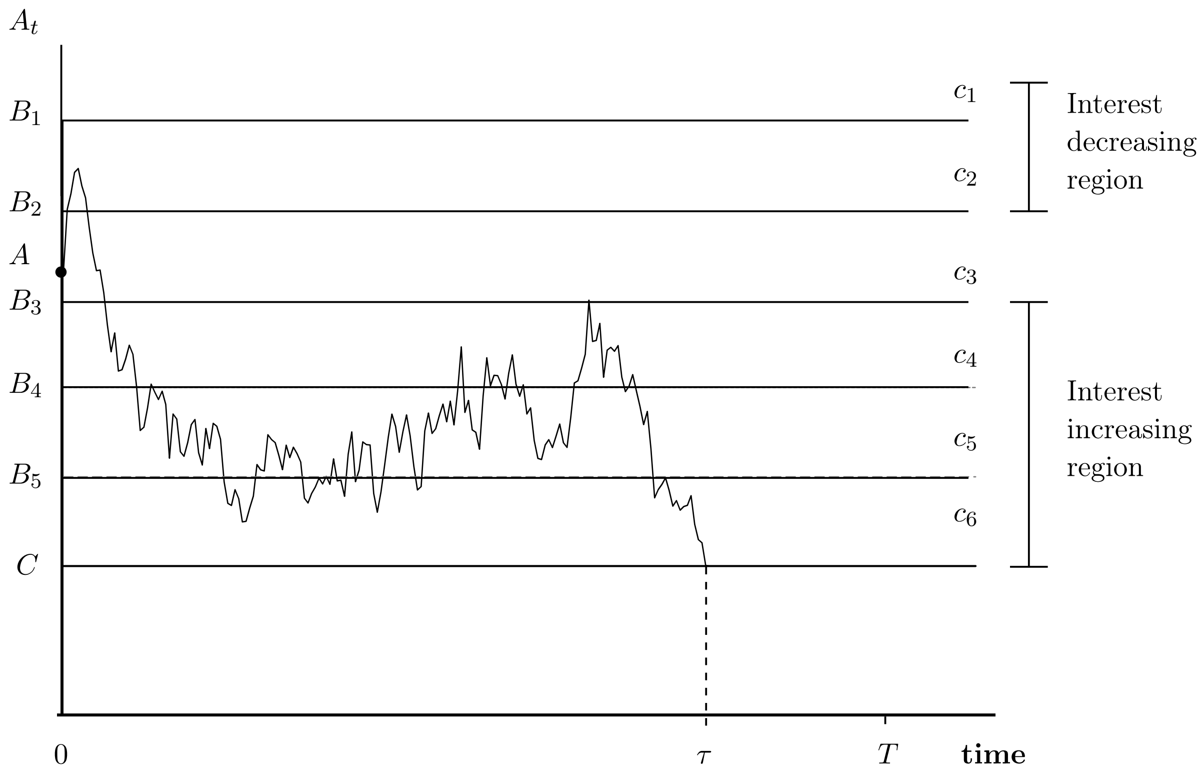

Figure 1: PSD interest rate payments for an arbitrary performance measure development. An illustration of the interest rate structure in a PSD contract. The graph contains an example of a path of the performance measure A_t and indicates in which regions the interets rates are c_1,..., c_6 respectively. Also, A, B_1,...,B_5, C, T, and τ are depicted. C denotes the contractual default barrier. The number of non-absorbing barriers above the starting level A is $n = 2$, and the number of non-absorbing barriers below A is $m = 3$. In order to make the two indicated regions interest decreasing and interest increasing, respectively, it is assumed that $c_1 < c_2 < ... < c_6$.

claim pays interest c_{n+i} in the case where the asset price A_t is between the barriers B_{n+i-1} and B_{n+i} for $i = 1, ..., m + 1$, and where the starting value of the process A is above the barrier B_{n+1}. Denote the time 0 market value of this claim by $V^A(A, B_n, B_{n+1}, ..., B_{n+m}, C)$.

The time 0 market value of the level dependent interest rate payments $V(A)$ with an arbitrary starting value $A \in [B_n, B_{n+1}]$ is

$$V(A) = V^B(A, B_1, ..., B_n, C) + V^A(A, B_n, B_{n+1}, ..., B_{n+m}, C), \tag{6}$$

where

$$V^{B}\left(A,B_{1},...,B_{n},C\right)=\frac{1}{r}\left[\sum_{i=1}^{n-1}(c_{i}-c_{i+1})\kappa_{i}+c_{n}\nu\right],$$
$$V^{A}\left(A,B_{n},B_{n+1},...,B_{n+m},C\right)=\frac{1}{r}\left[\left(c_{n}\right)\eta-\left(c_{n+m+1}\right)\zeta+\sum_{i=n}^{n+m}\left(c_{i}-c_{i+1}\right)\psi_{i}\right],$$

$$\kappa_{i}=\frac{\beta\left(\frac{A}{B_{i}}\right)^{\alpha}\left(1-Q_{lg}^{\alpha}\left(B_{i}\right)\right)-\beta\left(\frac{A}{C}\right)^{-\beta}\left(\frac{C}{B_{i}}\right)^{\alpha}\left(1-Q_{g}^{\beta}\right)+\alpha\left(\frac{A}{B_{i}}\right)^{-\beta}Q_{gg}^{\beta}\left(B_{i}\right)-Q_{gg}\left(B_{i}\right)e^{-rT}\left(\alpha+\beta\right)}{\left(\alpha+\beta\right)},$$
$$\nu=\frac{\beta\left[\left(\frac{A}{B_{n}}\right)^{\alpha}-\left(\frac{C}{B_{n}}\right)^{\alpha}\left(\frac{A}{C}\right)^{-\beta}\right]-\left[e^{-rT}Q_{g}-\left(\frac{A}{C}\right)^{-\beta}Q_{g}^{\beta}\right]\left(\alpha+\beta\right)}{\left(\alpha+\beta\right)},$$
$$\eta=1-\frac{\beta}{\alpha+\beta}\left[\left(\frac{A}{B_{n}}\right)^{\alpha}-\left(\frac{C}{B_{n}}\right)^{\alpha}\left(\frac{A}{C}\right)^{-\beta}\right],$$
$$\zeta=\left(\frac{A}{C}\right)^{-\beta}Q_{l}^{\beta}+e^{-rT}Q_{g},$$
$$\psi_{i}=\frac{1}{\alpha+\beta}\left[\alpha\left(\frac{A}{B_{i}}\right)^{-\beta}\left(Q_{gg}^{\beta}\left(B_{i}\right)-1\right)-\beta\left(\frac{A}{B_{i}}\right)^{\alpha}Q_{lg}^{\alpha}\left(B_{i}\right)-\beta\left(\frac{A}{C}\right)^{-\beta}\left(\frac{C}{B_{i}}\right)^{\alpha}Q_{l}^{\beta}\right],$$

$$\alpha=\frac{1}{\sigma^{2}}\left(\frac{1}{2}\sigma^{2}-\mu+\sqrt{\left(\frac{1}{2}\sigma^{2}-\mu\right)^{2}+2\sigma^{2}r}\right),$$

and

$$\beta=\frac{1}{\sigma^{2}}\left(\mu-\frac{1}{2}\sigma^{2}+\sqrt{\left(\frac{1}{2}\sigma^{2}-\mu\right)^{2}+2\sigma^{2}r}\right).$$

The proof and the probabilities $Q_{lg}^{\alpha}(B)=Q^{\alpha}(A_T<B,\tau>T)$, $Q_g^{\beta}=Q^{\beta}(\tau>T)$, $Q_{gg}^{\beta}(B)=Q^{\beta}(A_T>B,\tau>T)$, $Q_g=Q(\tau>T)$, $Q_{gg}(B)=Q(A_T>B,\tau>T)$, and $Q_l^{\beta}(B)=Q^{\beta}(A_T\leq B)$ are included in Appendix A.

For the special case $n=0$, $V(A)=V^{A}\left(A,B_0,B_1,...,B_m,C\right)$, where $B_0=\infty$, and for the special case $m=0$, $V(A)=V^{B}\left(A,B_1,...,B_n,C\right)+V^{A}\left(A,B_n,C\right)$.

4 Market and Data Description

This section gives an overview of the market for PSD loans and describes the data we use to empirically test our pricing model.

4.1 Overview and Descriptive Statistics

The tables and statistics in this section describe all PSD contracts in the Thomson Reuter's Dealscan database as of end 2009. This database contains detailed information about the global commercial loan market, focusing primarily on corporate bank debt with longer maturities. The database provides information for both publicly traded and privately held debt[5]. The PSD part of the database includes 16,864 deals for the period 1994 - 2009. The total outstanding pricipal of these PSD loans is USD 12,500 bn. One deal may consist of several facilities, usually referred to as tranches. Table 1 reports the use of different types of performance measures in PSD contracts. We see that total debt-to-cashflow and senior debt rating are the two most common performance measures in these contracts. In total, 44.8% of the PSD contracts are related to cash-flow (27.5% of loan amount), and could potentially be valued using our model.

Performance measure	Total number of deals	Total loan amount
Total debt-to-cashflow	41.6 %	24.9 %
Senior debt rating	28.6 %	52.7 %
Leverage	5.3 %	3.6 %
Maturity	5.0 %	6.3 %
Senior debt-to-cash flow	3.2 %	2.6 %
Outstandings	2.4 %	2.8 %
Fixed charge coverage	2.3 %	0.6 %
Debt to tang. net worth	2.0 %	0.4 %
Interest Coverage	1.9 %	1.1 %
Other	7.7 %	5.0 %

Table 1: This table shows the numbers of debt contracts with different types of pricing grids as a percentage of the total number of loans containing performance pricing provisions (N=16,864 loans), and the loan amount of debt contracts as a percentage of the total amount issued (measured in USD). Datasource: Thomson Reuter's Dealscan database for the years 1994-2009.

Table 2 shows the distribution of such debt contracts related to some broadly defined categories of

[5]See Carey et al. (1998) for a more detailed description of the database.

Purpose	Total number of loans	Total loan amount
Acquisition-related	21.8 %	34.0 %
Refinancing	23.1 %	17.7 %
Working Capital	20.8 %	10.4 %
Project Finance	1.4 %	1.5 %
All Others	32.9 %	36.4 %

Table 2: This table shows the purpose of a issued performance sensitive loan as a percentage of the total number of issued loans containing performance sensitive provisions (N=16,864), as well as a percentage of the total amount issued (measured in USD). Datasource: Thomson Reuter's Dealscan database for the years 1994-2009.

financing purposes. PSD contracts rely on verifiable criteria in order to trigger changes in interest rates. This fact may explain why such contracts are primarily issued at firm level where audited financial reports are available, and seldom used for project financing. Another interesting aspect is how the use of performance sensitive debt is related to debt ratings. Table 3 shows that 95% of all borrowers using performance sensitive loans are rated in the range A/A to B/B at issue[6]. Only 2.5% of PSD borrowers are rated above A/A, compared to 20.8% of other loans. Also, more than half of the borrowers using PSD are non-investment grade (51.9 %), compared to 37.2% for non-PSD loans. This observation supports the hypothesis of, e.g., Manso, Strulovici & Tchistyi (2010), that PSD loans are used to mitigate information asymmetry problems, which are less important for borrowers with higher credit ratings. In addition, we find that with negligible exceptions, all performance sensitive loans are senior (99.8%). Also, 52% of the loans are secured, whereas 28% are unsecured (information regarding security is not available for the remaining 20%.). Table 4 shows that USA and Canada alone accounts for almost 90% of these contracts which may be related to the historically high level of sophistication of the financial markets in this region. Market participants indicate that more than 50% of recently issued syndicated bank loans in Europe include such provisions.

Figure 2 shows the use of performance sensitive loan contracts, relative to all new loans, in the US and Canada during the last 15 years. We note that a substantial fraction of loans include performance sensitive provisions and that this fraction have been fairly stable during this period.

Appendix B includes additional tables showing how performance sensitive debt contracts are cate-

[6] The rating categories are S&P/Moody's senior debt rating, respectively.

Rating Category	PSD loans	Non-PSD loans
AAA/Aaa	0.5 %	8.6%
AA/Aa	2.0 %	12.2%
A/A	13.9 %	18.4%
BBB/Baa	31.7 %	23.6%
BB/Ba	25.9 %	16.7%
B/B	23.5 %	16.8%
CCC/Caa	2.2 %	3.2%
C /C	0.3 %	0.5%
Sum Investment Grade	48.1 %	62.8 %
Sum Non-Investment Grade	51.9 %	37.2 %

Table 3: This table shows the distribution of borrower ratings (S&P/Moody's senior debt ratings respectively) at issue for both PSD (N=16,684) and non-PSD loans (N=125,363). The numbers are calculated as the number of loans with a given borrower credit rating divided by the total number of loans for the given category. Datasource: Thomson Reuter's Dealscan database for the years 1994-2009.

Borrower region	Total number of loans
USA/Canada	89.2 %
Western Europe	6.0 %
Latin America/Caribbean	2.1 %
Asia-Pacific	1.4 %
Eastern Europe/Russia	0.7 %
Middle East	0.5 %
Africa	0.1 %

Table 4: This table shows the geographical distribution of issued performance sensitive loans as an equal-weighted percentage of the total number of such loans (N=16,864). Datasource: Thomson Reuter's Dealscan database for the years 1994-2009.

gorized by maturity, loan-amount, calendar year and broad borrower industry classes. We see that approximately 95 % of new deals have maturities of 7 years or shorter, although there are examples of loans with maturity longer than 10 years. We also see that the median loan amount is between USD 100-500 million, but the database includes loan amounts above USD 5 billion and below USD 1 million.

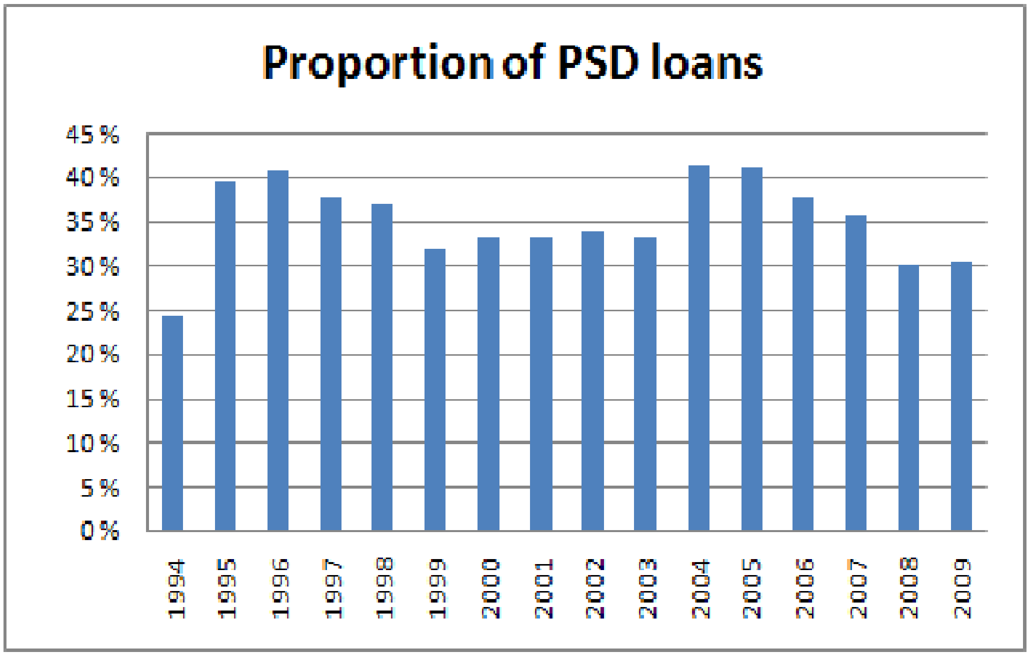

Figure 2: The histogram shows the annual proportion of performance sensitive loan issued in the US and Canada relative to the total number of new loans in this region. Numbers are based on data from Thomson Reuter's Dealscan database for the years 1994-2009.

4.2 Data Description

4.2.1 Sample Construction

As stated before, we use Thomson Reuter's Dealscan[7] database to collect information from 16,864 PSD loan contracts issued in the period 1994 - 2009. We confine our analysis to contracts with interest rates linked to the company's total debt-to-cash-flow ratio (Debt/CF), a condition for the model in Section 3. This restriction reduces the number of loan contracts to 6,964. In addition to the performance sensitive feature, we, furthermore, require the existence of a total debt-to-cash-flow default covenant in the contract. This requirement reduces our sample to 3,705 loan contracts. We further restrict our sample to publicly listed borrowers with sufficent market and company information from the databases CRSP and Compustat prior to inception of the loan. Information from these databases is used to estimate the drift and volatility parameters of the borrowers' cash flow driven reference process in our pricing model. Hence, we require the borrowing company to be listed in the two latter databases when the loan is established, and to have a minimum of 3

[7] See www.loanpricing.com for more information on the database and how to access it.

years of historical data for parameter estimation. This last restriction reduces our sample to 275 loan contracts. We also remove 5 contracts where the initial Debt/CF value is lower than the value specified by the Debt/CF default covenant. Such observations imply immediate default, which both seems counterintuitive and falls outside our model. Our final sample consists of 270 loan contracts. The fairly strict restrictions we impose when selecting the sample, provide us with loan contracts where the input parameters of our model are readily available. Thus, we believe that our sample will give a precise test of the proposed pricing model[8]. The sample includes 86 interest decreasing contracts, 54 interest increasing contracts and 130 contracts containing both categories of performance sensitive interest rates. All loans are senior and secured, and are granted by banks in the time period 1999 - 2009.

4.2.2 Sample Presentation

Table 6 lists summary statistics of our sample. The loans have from 2 - 8 non-absorbing Debt/CF barriers with a mean of 4. The size of the loans also varies from USD 20m to USD 5.5bn, whilst the average loan amount is USD 290m. No PSD loan has maturity above 10 years, whilst the average maturity is 5.21 years. The average credit spread at issue, measured by the all-in-spread (AIS), is 229 basis points, with a sample standard deviation of 71 basis points. When analyzing the firms using PSD loans we find no indication that these loans are used by firms of any particular size. Measured in sales, PSD loans seem to be used by both small and large firms. The average borrower profitability, measured by the quarterly return on capital employed (ROCE), is 2.6%, with a median of 2.1% and a standard deviation of 4.8%. The average initial leverage, measured by the last reported observation of the borrower's debt to total book value of assets ratio prior to entering into the PSD deal, is 0.31. The corresponding median and standard deviation are 0.29 and 0.19, respectively. To measure the relative significance of the PSD loan in the borrowers' total leverage, we estimate the ratio of the PSD loan divided by the total debt (the sum of existing debt and the new PSD loan). The average share of the new PSD loan relative to borrower's total debt is 41%, with a median of 35% and a standard deviation of 25%. We do not know whether any of a borrower's other debt includes PSD provisions. All borrowers are rated in the range BBB/Baa to

[8]Eom, Helwege & Huang (2004) also choose their sample restrictively to gain precision. As a comparison their sample consists of 182 corporate bonds.

B-/B3[9]. The average and median of the estimated sample cash flow volatility are 20% and 18% respectively. These volatility estimates, however, are widely distributed ranging from 2% to 66% annually. The average distance-to-default, defined as the starting value of the CF/debt measure less the contractual default barrier and normalized by the borrower's asset volatility, is 0.63 with a standard deviation of 0.55. In Table 6 we also list the distance-to-default characteristics for interest increasing and interest decreasing loans, respectively. Table 6 shows, as we would expect, that interest increasing contracts have a greater distance-to-default compared to interest decreasing contracts. Summary statistics for each of the three subsamples (interest increasing, interest decreasing, both provisions) can be found in Tables 15, 16, and 17 in Appendix C. These tables show that borrowers using interest increasing PSD contracts are smaller, less profitable, have a higher cash flow volatility, use relatively more PSD debt and pay a higher initial all-in-spread compared to borrowers using interest decreasing PSD contracts. These results unanimously suggest that borrowers using interest increasing PSD loans seem to be of an overall lower 'quality', and are potentially more financially constrained than those using interest decreasing PSD loans.

To assess how representative our sample is we compare it to the population of PSD loans in the database. First of all, note that the sample borrower ratings correspond well with the observations in Table 3. For the entire database (our sample means in parenthesis) the average maturity is 4.5 (5.2) years, the average loan amount is USD 348m (USD 290m), the average borrower's sales size is USD 2,658m (USD 2,568m) and the average AIS is 194.6 (229) bp. These statistics, see Table 5, suggest that our sample is representative.

	Sample			**Population**		
Variable	**Mean**	**Median**	**Std. Dev.**	**Mean**	**Median**	**Std. Dev.**
Borrowers' Sales Size (MUSD)	2,568	1,266	3,247	2,658	581	9,866
Loan Maturity (Years)	5.2	5.0	1.38	4.5	5.8	1.94
Loan Amount (MUSD)	290	158	448	348	270	1,585
All In Spread (Basis points)	229.0	225.0	71.0	194.6	175.0	116.5
$N =$	270	270	270	16,864	16,864	16,864

Table 5: Table shows sample averages as well as population averages for available variables (Borrowers' sales, loan maturity, loan amount and All-in-spread). Population defined as all PSD loans in Thomson Reuter's Dealscan database as of end 2009.

[9]This is the general senior S&P/Moody's debt rating of the borrower, since the bank loans we analyze are not rated separately.

Variable	Mean	Median	Std. Dev.	Min.	Max.	N
Borrower Characteristics:						
Company Sales (MUSD)	2,568	1,266	3,247	86	21,978	270
ROCE (%,quarterly)	2.59	2.07	4.77	-9.50	47.00	270
Leverage (Debt/Total Assets)	0.31	0.29	0.19	0.00	0.86	270
PSD Loan/Total Debt	0.41	0.35	0.25	0.05	1.00	270
Drift of cash flow $(r-\delta)$	0.031	0.028	0.019	0.002	0.06	270
Volatility of cash flow (σ)	0.204	0.176	0.12	0.02	0.66	270
Loan Characteristics:						
Loan Amount (MUSD)	290	158	448	20	5,500	270
Maturity (Years)	5.21	5.00	1.38	0.92	9.08	270
All-In-Spread (Bp)	229	225	71	50	400	270
# of Barriers	4.42	5	1.488	2	8	270
Distance-to-default (Full Sample)	0.63	0.47	0.55	0.03	3.60	270
Distance-to-default (Interest increasing)	1.01	0.78	0.81	0.15	3.60	54
Distance-to-default (Interest decreasing)	0.43	0.35	0.33	0.03	1.48	86

Table 6: This table shows summary statistics for various model input parameters and firm characteristics for the final sample used in the paper. The loan contracts in the sample are issued in the period 1999 - 2009. Datasource: Thomson Reuter's Dealscan Database, Compustat and CRSP.

5 Pricing a PSD Contract

To illustrate the application of our pricing model we select the first loan contract in our sample (sorted alphabetically by borrower name), in order to show how the input parameters are estimated and how our model price is calculated. The results for the entire sample are included and analyzed in the next section.

5.1 Pricing of an Example Contract

Actuant Corporation[10] borrowed USD 100m in the year 2000 using a PSD contract. The main terms of this contract are given in Table 7. The performance measure in this contract links the interest paid on the loan to the performance of the company via the company's Debt/CF ratio as shown in Table 8[11]. We invert the Debt/CF ratio and assume that the borrower has a constant total debt level until the maturity of the loan. The performance measure is, thus, lognormally distributed as required in our theoretical model. Also note that this is an *interest decreasing* contract

[10] See www.actuant.com for more information on the company.

[11] The LIBOR (London Interbank Offered Rate) spread indicates the contractual spread over LIBOR 3 mth USD rate measured in basispoints. The total spread equals the sum of the LIBOR spread and a commitment fee. CF equals cash flow and is proxied in the loan contract, and in our analysis, by reported EBITDA (Earnings Before Interest, Tax, Depreciations and Amortizations).

Borrower	Actuant Corp.
Deal Active Date	31 Jul 2000
Amount	USD 100 m
Facility type	Term Loan
Seniority	Senior
Maturity	72 months
Distribution Method	Syndication
Lead Bank	Credit Suisse First Boston
Reference Rate	LIBOR 3 mth
Type of pricing grid	Interest decreasing
Initial CF/Debt ratio	0.24
Borrower Senior Debt Rating (S&P)	BB

Table 7: This table provides an overview of the main terms in the chosen example PSD contract.

		Performance Measure		**Interest Margins**		
Ranges	**Barriers**	**Debt/CF**	**CF/Debt**	**LIBOR Spread**	**Commitment Fee**	**Total Spread**
1	(B_1, B_0)	$[0, 1.75)$	$(0.57, \infty)$	150	25	175
2	$(B_2, B_1]$	$[1.75, 2)$	$(0.5, 0.57]$	162.5	37.5	200
3	$(B_3, B_2]$	$[2, 2.5)$	$(0.4, 0.5]$	187.5	37.5	225
4	$(B_4, B_3]$	$[2.5, 3)$	$(0.33, 0.4]$	212.5	37.5	250
5	$(B_5, B_4]$	$[3, 3.5)$	$(0.29, 0.33]$	225	50	275
6	$(B_6, B_5]$	$[3.5, 4.25)$	$(0.24, 0.29]$	250	50	300
7	$(\mathbf{C}, B_6]$	$[4.25, \mathbf{4.55})$	$(\mathbf{0.22}, 0.24]$	275	50	325

Table 8: Table shows how the interest rate is linked to company cash flow through the Debt/CF ratio and the CF/Debt ratio respectively. A Debt/CF ratio equal to 4.55, or equivalently a CF/debt ratio of 0.22, represents the maximum (minimum) ratio that is accepted by the contract terms, and hence is the exogenously given contractual default barrier.

since the starting level of the inverted performance measure (CF/debt) is 0.24, i.e, at the lowest non-absorbing barrier. In order to price this contract we estimate two crucial parameters, the drift and the volatility of the underlying asset process. Under the risk-adjusted probability measure Q we know that the drift in the asset process must equal the risk-free rate r if the company retains all of its earnings. However, if a company pays out a dividend rate δ proportional to earnings, the drift under Q is $r - \delta$. Collecting data on earnings per share (EPS) and dividends per share (DPS) from Compustat, we estimate δ by the historical average of (DPS/EPS) multiplied by the risk-free interest rate r. Data may indicate that a company has changed its payout policy, for example from never paying dividends to paying annual dividends. For such firms we base our estimate on the most recent dividend policy. Actuant Corp. did not pay any dividends for the years prior to the inception of this loan contract, so in this case $\delta = 0$ and $\mu = r$.

The volatility parameter under the probability measure Q is the same as under the original probability measure P. We therefore estimate σ using monthly stock return data from CRSP. The historical annual standard deviation, measured in the period $1988 - 2000$[12], for Actuant Corp's stock returns, denoted by $\hat{\sigma}_E$, is 36%. In the usual situation where the firm is financed by both debt and equity we need to estimate asset volatility from equity volatility. Obtaining historical balance sheet data on Actuant Corp., we calculate the estimated asset volatility $\hat{\sigma}_A$ as:

$$\hat{\sigma}_A = \hat{\sigma}_E \times \frac{\bar{E}}{D + E}.$$

where $\frac{\bar{E}}{E+D}$ is the firm's average fraction of book value of equity to total assets, based on quarterly market data for the period $1988 - 2000$. In the case of Actuant Corp., $\frac{\bar{E}}{D+E} = 0.38$, and the estimated annual volatility of the firm's assets is

$$\hat{\sigma} = 0.36 \times 0.38 = 0.1368.$$

This way of estimating asset volatility is arguably one of several possible ways to do this estimation. Furthermore, assuming debt is risk-free in the estimation is also clearly a simplification. Given access to more detailed data one might choose more sophisticated methods to estimate asset volatility, e.g., also using the observed volatility of a firm's debt claims, following Choi & Richardson (2010). To test the robustness of our results to different volatility estimates we calculate the loan prices with volatility estimates based on market data for the last 3 years prior to the loan issue, and also with the future realized volatility, during the loan contract period. Although the change in loan value is significant in the Actuant case, the results for the whole sample are not significantly changed using the different volatility estimates.

As an approximation for the risk-free rate we use the quote of the 3-month risk-free rate in the

[12]Since we use monthly data we use the full available history of Actuant's stock returns to estimate the stock return volatility. An alternative, to exclude earlier and possibly less relevant observations, would be to use the last 3-5 years of data. Using 3 years the estimated volatility is 42% which would result in a loan price equal to 91.52, i.e., a reduction by 4.3%.

month prior to the the loan issuance date[13], i.e., in this case June 2000. The risk-free rate equals 5.83%, and implies a risk neutral drift equal to $r - \hat{\delta} = 0.0583 - 0 = 0.0583$. The LIBOR 3 month rate is used as the reference interest rate in all loan contracts in our sample. To find the correct interest rates to use throughout the loan period as our model input, we add the contractual spreads to the T-forward LIBOR rate, where T is the time to maturity. Forward LIBOR rates are only available up to one year maturity, and hence we proxy longer-term forward rates by swap rates obtained from the quoted swap-curve at the time of issue[14]. In this example, the maturity of the loan is 6 years, and we use the 6 year forward swap rate quoted in June 2000 as our reference rate. Adding the contractually determined spreads yields the interest rate. As the contractual default barrier C, we use the financial covenant stating that the maximum Debt/EBITDA ratio should not be above 4.5. This ratio corresponds to a value of C (i.e., CF/Debt) equal to $1/4.5 = 0.22$ in our model. The starting value of the asset process, i.e., the current value of CF/Debt is 0.24. As an approximation of the recovery rate (1 - κ) we use the estimated recovery rate for senior secured bank debt from Altman, Resti & Sironi (2004). This recovery rate[15] is 73%, implying that the liquidation cost parameter κ equals 27%. The liquidation cost parameter determines the loss in the case of default. The size of the loss depends on whether the default leads to a full liquidation or not. Practitioners argue that default occurs when all possible refinancing options are exhausted. A default on a single debt contract commonly leads to defaults on the company's other debt contracts by contractual cross-default. Based on these arguments it makes sense to apply the estimated liquidation cost parameter of 27%, even if we value a single debt contract and not necessarily a company's total debt obligations. Practitioners confirm the size of this parameter. The face value of debt is normalized to 100. Table 9 summarizes the values of the input parameters we use in the model valuation.

Given the parameters in Table 9 the market value of the PSD contract at issue is 95.65, calculated using Expression (5). Thus, the theoretical market price is below the par value of 100, and this contract is underpriced.

[13]These are collected from the Federal Reserve's official statistical releases (http://www.federalreserve.gov/releases/h15/data.htm). The website also contains descriptions of how these rates are measured.

[14]See also www.federalreserve.gov/releases/h15/data.htm for information on swap curves.

[15]The estimated recovery rate is based on recovery of principal 30 days after default.

Parameters	Values	Explanations
T	6	Maturity, in years
D	100	Face value of debt, normalized
B_1	0.57	Barrier 1 (CF/Debt)
B_2	0.50	Barrier 2 (CF/Debt)
B_3	0.40	Barrier 3 (CF/Debt)
B_4	0.33	Barrier 4 (CF/Debt)
B_5	0.29	Barrier 5 (CF/Debt)
B_6	0.24	Barrier 6 (CF/Debt)
C	0.22	Default Barrier (CF/Debt)
c_1	0.0892	Interest rate paid when $A_t \geq B_1$
c_2	0.0917	Interest rate paid when $B_1 > A_t \geq B_2$
c_3	0.0942	Interest rate paid when $B_2 > A_t \geq B_3$
c_4	0.0967	Interest rate paid when $B_3 > A_t \geq B_4$
c_5	0.0992	Interest rate paid when $B_4 > A_t \geq B_5$
c_6	0.1017	Interest rate paid when $B_5 > A_t \geq B_6$
c_7	0.1042	Interest rate paid when $B_6 > A_t > C$
A	0.24	Starting value of the CF/Debt process
μ	0.0583	Risk-neutral drift of the CF/Debt process
σ	0.1368	Volatility of the CF/Debt process
r	0.0583	Risk-free interest rate
κ	0.27	Liquidation cost parameter

Table 9: This table states the value of all relevant input parameters needed to estimate the price of the example PSD contract, as described in Tables 7 and 8.

In light of the interpretation given in Section 1.2 in the introduction, we wish to calculate the value of the portfolio of call options. This is done by using the theoretical value of a fixed interest rate loan from which the theoretical value of the PSD contract is subtracted. The time 0 price of the fixed rate loan, with an interest rate equal to the initial interest rate c_7 in Table 9, is 96.86. This price is calculated using a version of the pricing formula in Black & Cox (1976) modified to include finite maturity

$$F_0 = \frac{c_0}{r}\left[1 - e^{-rT}Q_g - (1 - Q_g^{\beta})\left(\frac{A}{C}\right)^{-\beta}\right] + De^{-rT}P(A) + D(1-\kappa)H(A), \tag{7}$$

where c_0 is the initial payment interest rate, $P(A)$ and $H(A)$ are given by expression (5), and the probabilities $Q_g = Q(\tau > T)$ and $Q_g^{\beta} = Q^{\beta}(\tau > T)$ are included in Appendix A. In this numerical example $c_0 = c_7$.

The time 0 market value of the call option portfolio can then be found by subtracting Expression (5) from Expression (7), i.e.,

$$C_0 = \frac{c_0}{r}\left[1 - e^{-rT}Q_g - (1 - Q_g^{\beta})\left(\frac{A}{C}\right)^{-\beta}\right] - V(A). \tag{8}$$

The time 0 market value of the call option portfolio is equal to $96.86 - 95.65 = 1.21$.

The time 0 price F_R of a comparable risk-free contract paying the same initial interest rate is 123.24. This price is calculated using

$$F_R = \frac{c_0}{r}\left(1 - e^{-rT}\right) + De^{-rT}. \tag{9}$$

The difference of 27.59 between the market value of the risk-free contract and the PSD contract may be decomposed into 26.38 due to default risk and 1.21 due to the reduction in interest rate contractually attributed to better performance.

5.2 Sensitivity Analysis

To understand how sensitive the theoretical loan price is to model input parameters we perform a simple sensitivity analysis using the example PSD contract from above. In Figure 3 we plot the theoretical price of the example interest decreasing PSD contract and the corresponding fixed interest rate loan F_0^D for varying levels of borrower's cash flow volatility. F_0^D is defined as the time 0 market value of a fixed interest rate loan paying the initial interest rate of the corresponding PSD loan contract, with the remainder terms identical. The value of the corresponding call option portfolio is defined as the market value of the fixed-rate loan less the market value of the PSD loan, and is also plotted in Figure 3. Observe that the price of both debt contracts are monotonically decreasing in volatility, cf. standard results for debt from Merton (1974). The market value of the option portfolio is also decreasing in volatility. The latter result is in line with the literature on vulnerable options, see, e.g., Johnson & Stulz (1987). The theoretical price of the PSD contract is a monotonically decreasing function of borrower's cashflow volatility. We define *implied volatility* as the volatility level which yields a theoretical loan price of 100 (par value). In this example the implied volatility is 11.8%, 1.9%-points lower than the estimated volatility of 13.68%.

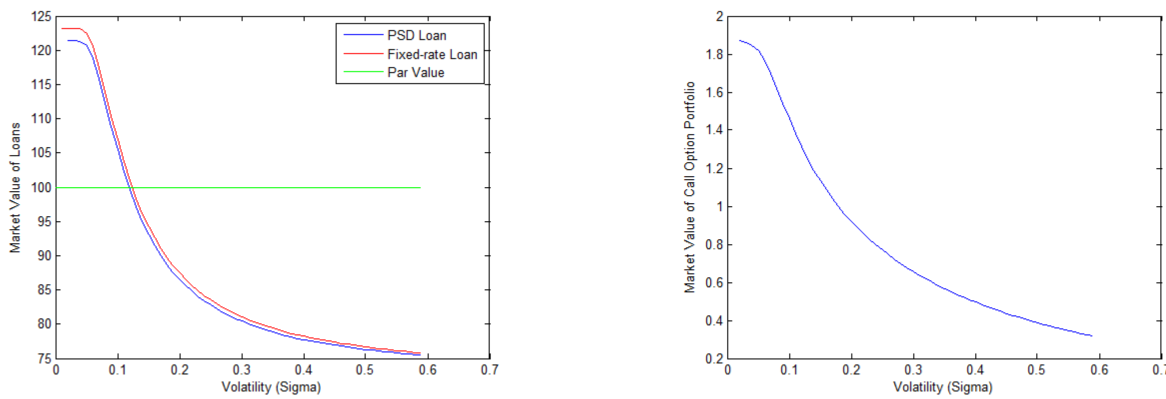

Figure 3: The market value of interest decreasing PSD contract consisting of the market value of a fixed-rate loan F_0^D less the value of a call option portfolio C_0. Left plot shows the theoretical price of the example PSD contract and the fixed rate part F_0^D plotted against **borrower's cash flow volatility** for the example PSD contract parameter values. Right plot shows the corresponding value of the call option portfolio as a function of borrower's cash flow volatility.

In Figure 4, 5, 6 and 7 we plot the theoretical price of the PSD contract and the fixed-rate part, together with the corresponding theoretical market value of the call option portfolio against levels of the risk-free rate r, the liquidation cost parameter κ, the time to maturity T, and the contractual default barrier C, respectively. All figures use the same valuation range on the Y-axis to facilitate comparisons. The time 0 market values of the debt contract and the call option portfolio are decreasing in r and C. The time 0 market value of the debt contracts are decreasing linearly in κ, whereas the market value of the call option portfolio is independent of this parameter. Finally, the prices of both debt contracts are convex functions of T, i.e., it is decreasing for maturities less than a year and increasing for longer maturities. The time 0 market price of the call option portfolio is monotonically increasing in T. In Figure 8 we also plot the value of the call option portfolio for various combinations of C and σ. The option value decreases when volatility increases, but the effect of increased volatility is larger for higher levels of C.

This sensitivity analysis is based on the example contract only and is partial in the usual *ceteris paribus* sense, only one factor is changed at the time.

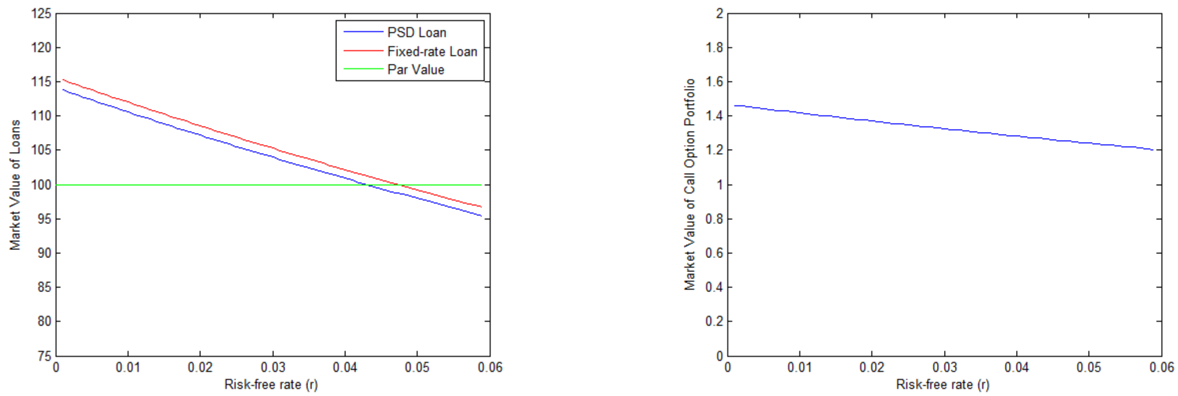

Figure 4: The market value of an interest decreasing PSD contract consists of the market value of a fixed-rate loan F_0^D less the value of a call option portfolio C_0. Left plot shows the theoretical price of the example PSD contract and the fixed rate part F_0^D plotted against **the risk-free rate** r for example contract parameter values. Right plot shows the corresponding value of the call option portfolio as a function of r.

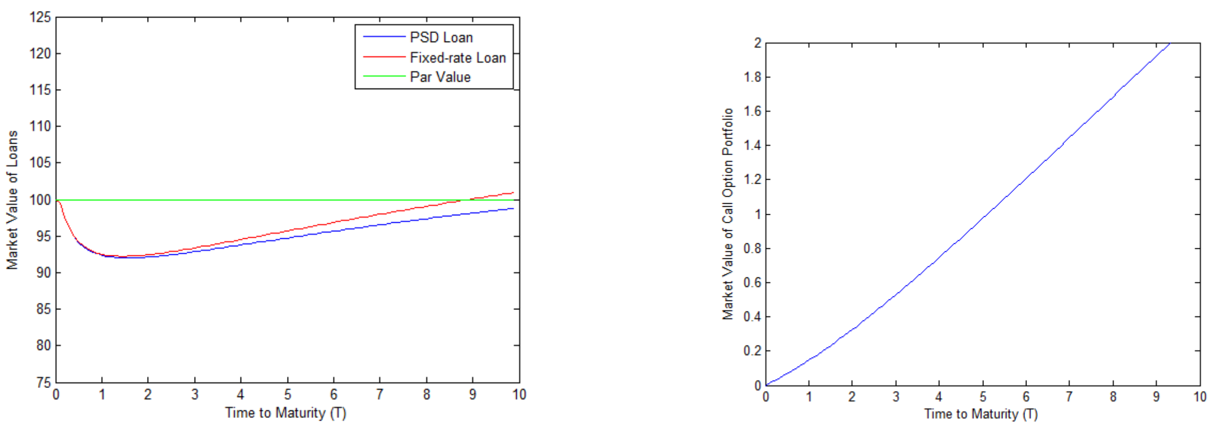

Figure 5: The market value of an interest decreasing PSD contract consists of the market value of a fixed-rate loan F_0^D less the value of a call option portfolio C_0. The left plot shows the theoretical price of the example PSD contract and the fixed rate part F_0^D plotted against **the time to maturity** T for example contract parameter values. The right plot shows the corresponding value of the call option portfolio as a function of T.

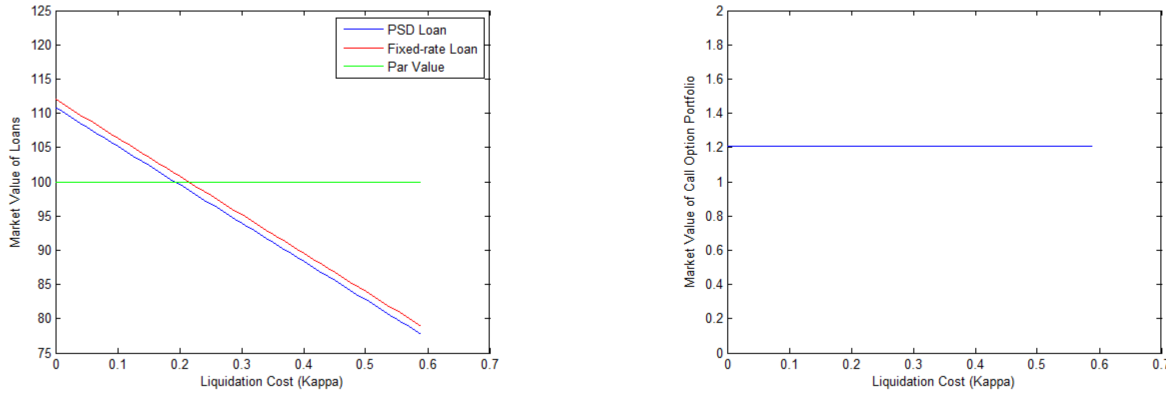

Figure 6: The market value of an interest decreasing PSD contract consists of the market value of a fixed-rate loan F_0^D less the value of a call option portfolio C_0. The left plot shows the theoretical price of the example PSD contract and the fixed rate part F_0^D plotted against **the liquidation cost parameter** κ for example contract parameter values. The right plot shows the corresponding value of the call option portfolio as a function of κ.

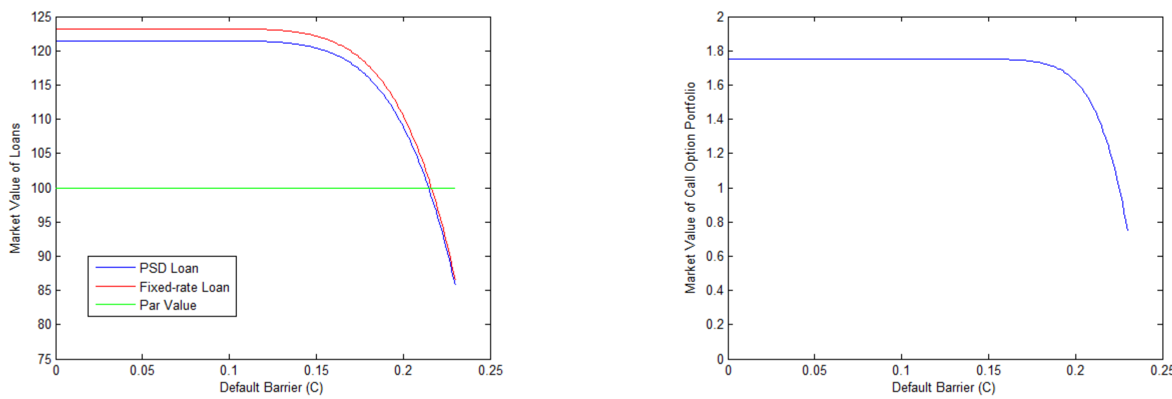

Figure 7: The market value of an interest decreasing PSD contract consists of the market value of a fixed-rate loan F_0^D less the value of a call option portfolio C_0. The left plot shows the theoretical price of the example PSD contract and the fixed rate part F_0^D plotted against **the contractual default barrier** C for example contract parameter values. The right plot shows the corresponding value of the call option portfolio as a function of C.

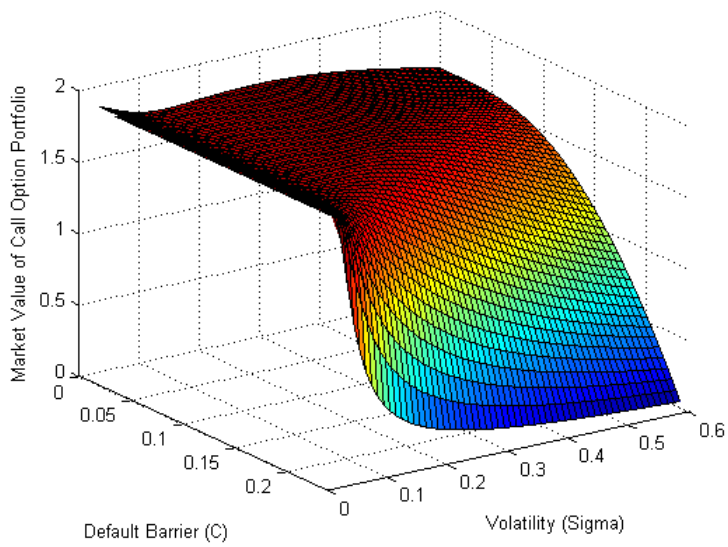

Figure 8: The market value of an interest decreasing PSD contract consists of the market value of a fixed-rate loan F_0^D less the value of a call option portfolio C_0. This plot shows the theoretical market value of the call option portfolio as a function of both **the contractual default barrier** C **and the borrower's cash flow volatility** σ, using the example contract parameter values.

6 Sample Analysis

We empirically test the average time 0 model value of the loan contracts in our sample and compare it to the par value. For all contracts we normalize the loan amount (par value) to 100. The average theoretical price of all contracts is 103.19 (standard deviation of 12.24). The median price is 102.91. The histograms in Figure 9 show how prices center around the par value for the full sample and two subsamples.

Splitting the sample into interest increasing, interest decreasing, and combination contracts, and using a standard t-test to test the null hypothesis that the average price is different from the par value, we find the following results. Interest increasing contracts are, on average, overpriced with a theoretical value equal to 107.70. Interest decreasing contracts are, on average, not significantly mispriced according to our model, with a theoretical value of 100.20. Contracts including a combination of both provisions are priced somewhere between the two other contract types, with an average theoretical market value equal to 103.29, these contracts are also overpriced. Table 10

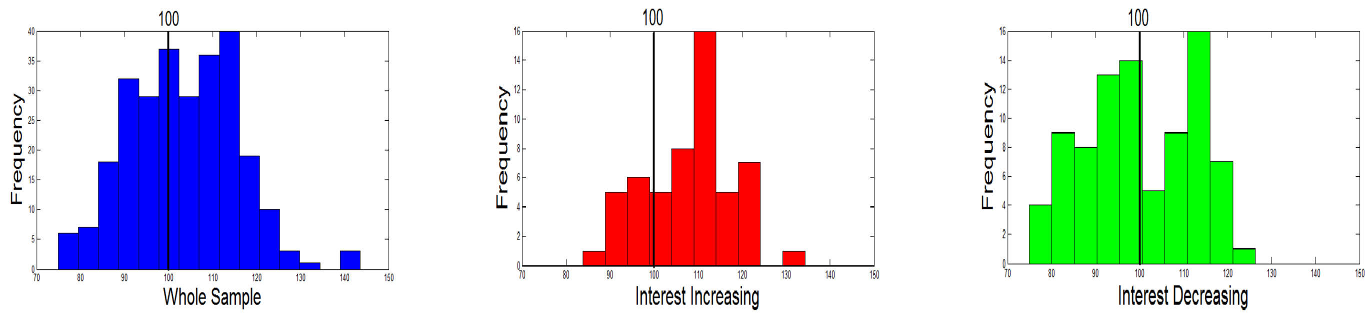

Figure 9: The histograms show the distribution of time 0 theoretical market values of the full sample of PSD contracts, interest increasing PSD contracts and interest decreasing PSD contracts, respectively.

gives summary statistics of the theoretical prices for the full sample and for the subsamples. The corresponding t-statistics testing the null hypothesis that the estimated average prices are equal to the par value are also reported. For the full sample, the interest increasing PSD contracts, and for PSD contracts containing both interest increasing and interest decreasing provisions we reject the null hypothesis on a 1% significance level. For the interest decreasing subsample we cannot reject the null hypothesis, i.e., we conclude that these contracts are, on average, not mispriced.

	Full Sample	**Subsamples**		
		Interest Increasing	**Interest Decreasing**	**Both**
Mean	103.19	107.70	100.20	103.29
Min	74.89	83.13	74.89	78.16
25-percentile	94.16	100.23	91.21	93.55
Median	102.91	109.25	99.30	102.81
75 - percentile	112.07	113.67	111.49	111.52
Max	143.57	134.29	126.35	143.57
St.dev	12.24	10.34	12.47	12.31
T-value	4.28**	5.47**	0.15	3.05**
N	270	54	86	130

Table 10: Table shows descriptive statistics of the theoretical prices obtained for the full sample, and when splitting the sample into contracts with interest-increasing provisions, interest-decreasing provisions, or both. A standard t-test statistic testing whether the average theoretical price is different from the par value of 100 is included. The superscript of two stars indicates a significance level of 1%.

For each PSD contract we are able to back out the implied cash flow volatility, i.e., the volatil-

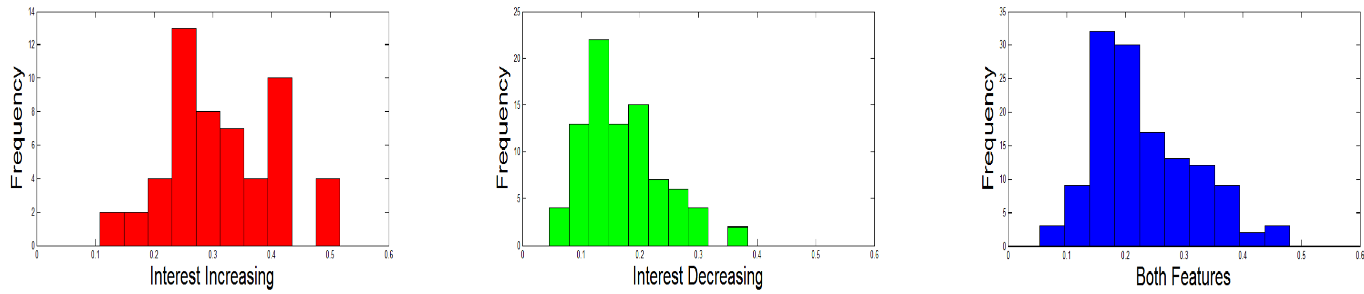

Figure 10: The histograms show the distribution of implied volatilities for interest increasing PSD contracts, interest decreasing PSD contracts, and contracts with both provisions, respectively.

ity that would price each loan at par value. The average implied volatility is 23% with a median of 21%. This is higher than the estimated volatility which has a mean of 20.4% and a median of 17.6%. Table 11 lists summary statistics for the implied volatility for the full sample as well as for the subsamples. Descriptive statistics for the volatility estimated using stock market returns are also reported for comparison. The results indicate that borrowers using interest increasing PSD contracts grant lenders the options of increased interest rates at a discount (the median implied volatility is 30.8% which is significantly higher than the median estimated volatility of 19.2%). Borrowers using interest decreasing PSD contracts pay a fair price for the options of paying lower interest rates (the median implied volatility is 15.5%, which is close to the median estimated volatility of 15.1%). Note that the variation in estimated volatility is less than the variation in implied volatility. Figure 10 shows the distribution of implied volatilities by subsamples.

To check if there are other factors, apart from model input parameters, that affect our model pricing performance, we run an OLS regression testing whether borrower size, loan size, borrower profitability measured by quarterly Return On Capital Employed[16] (ROCE), borrower's book leverage (Debt/Total Assets) and the number of contractual barriers significantly affect loan prices. The output is included in the two columns under Model 1 in Table 19 in Appendix C. Variable descriptions and their corresponding means are included in Table 20, also in Appendix C. Controlling for maturity, volatility and distance-to-default, as well as dummy variables for the type of PSD contracts and calendar years and, we find that borrower profitability has significant negative effect on

[16]ROCE is calculated as the ratio of EBIT to total assets less current liabilities, using observations from the last reporting period before raising PSD debt.

	Full Sample		**Interest Increasing**		**Interest Decreasing**		**Both**	
Volatility	**Estimated**	**Implied**	**Estimated**	**Implied**	**Estimated**	**Implied**	**Estimated**	**Implied**
Mean	20.4	22.9	22.2	31.8	18.1	16.8	21.1	23.1
Median	17.6	20.6	19.2	30.8	15.1	15.5	18.4	20.6
St.dev	11.4	9.9	13.4	9.5	9.1	6.8	11.6	8.8
Max	66.5	51.7	66.5	51.7	41.1	38.4	66.4	48.0
Min	2.7	4.6	6.0	10.8	2.7	4.6	7.0	5.4
T-test	2.72**		4.29**		1.06		1.57	
N	270	270	54	54	86	86	130	130

Table 11: Table shows descriptive statistics of the estimated and implied borrower's cash flow volatilities for full sample of PSD contracts, as well as for the subsamples of interest increasing PSD contracts, interest decreasing PSD contracts and PSD contracts with both provisions. Implied volatility is defined as the volatility that produces a time 0 theoretical loan price equal to par value. The estimated volatility is based on borrower stock price volatility and leverage. A standard t-test for difference in means between the estimated and implied volatility is also reported. The superscript of two asterix means the null hypothesis of equal means between the estimated and implied volatilities is rejected at a 1% significance level. For interest increasing PSD contracts and contracts with both provisions we cannot reject the null hypothesis at a 5% significance level.

loan prices, i.e., more profitable firms get better loan deals, as expected. The regression results show that loan prices are increasing in maturity and distance-to-default, and decreasing in volatility. If we interpret the distance-to-default as a measure of credit-quality we see that our model tend to overprice loans (or equivalently produce too low yield spreads) to the most credit-worthy borrowers. These results essentially confirm the results of Huang & Huang (2003) that credit risk (which is the only source of risk in our model) accounts for only a small fraction of observed yield spreads for the loans given to borrowers with the highest credit-quality.

Models 2, 3, and 4 in Table 19 display the results from running separate regressions for each of the three subsamples (interest increasing, interest decreasing and both). Again, we see that borrower's cash flow volatility, distance-to-default and loan maturity are the most important parameters explaining loan prices. In particular, distance-to-default explains a large part of the value for interest increasing contracts, which are overpriced by our model. This is shown by the coefficients and the large mean value for this subsample. Interest increasing contracts have a mean distance-to-default of 1.01 compared to the mean distance-to-default of 0.43 for interest decreasing contracts. When we compare the standardized coefficients in the subsample regressions, the scale of the coefficient on distance-to-default dominates the other explanatory variables (except for volatility for interest increasing).

For the subsamples of interest increasing and interest decreasing loans the effect of profitability is not significant. It is interesting to note that neither borrower size, borrower leverage nor loan size have any significant pricing impact.

In Table 18 in Appendix C we calculate the theoretical correct fixed interest rate using Expression (7). The table shows that the initially observed market interest rates in the PSD contracts are not significantly different from the theoretical Black & Cox fixed interest rates, except from the subsamples of interest increasing and interest decreasing PSD contracts. For the latter category the difference is not significant if we remove three outliers. This observation lends credibility to the way we define the option part of the PSD contracts, see Section 1.2. Note that the initial interest rates in interest increasing PSD contracts are significantly higher than the corresponding Black &

Cox fixed interest rate. This observation is in contrast to the findings of Asquith, Beatty & Weber (2005) that borrowers are offered lower interest rates in order to compensate for the inclusion of interest increasing provisions in debt contracts.

7 Conclusion

Based on standard no arbitrage theory this paper derives a valuation model for PSD loan contracts with cash-flow based performance measures. Our model incorporates finite maturities and an exogenous contract-specific default covenant. We, furthermore, test model performance by comparing the average theoretical market value of the loan to its par value at the time of issue. The theoretical market value of a PSD contract is on average 3.2% above par value at the time of issue, indicating that our model performs reasonably well. In particular, interest decreasing contracts, where the borrower pays decreased interest rates for improved credit quality, are not priced significantly different from par value. This result is in line with the finding of Asquith, Beatty & Weber (2005) that, in these cases, borrowers are likely to have stronger relative bargaining power. However, interest increasing PSD contracts, where the borrower pays increased interest rates in case of reduced credit quality, are overpriced by 7.7% on average (significant on 1% level). PSD contracts represent, on average, 41% of the borrowers' total debt at issue, indicating its significant, but not dominant, role in financing the borrowing firms. There may be several explanations for the observed overpricing of interest increasing contracts:

- Under information asymmetry, high quality borrowers signal their true quality by using interest increasing PSD contracts (Manso, Strulovici & Tchistyi (2010)).
- Lenders exploit their bargaining power by systematically underpricing (underpaying for) the option they buy, i.e., the option to increase interest rates for deteriorating credit quality (cf., Table 11 and Figure 3). Lenders do not have the same bargaining power regarding the options they sell and these are, thus, priced correctly.
- Lenders charge an extra premium to make up for the potential higher costs of moral hazard. Asquith, Beatty & Weber (2005) find that interest increasing PSD contracts are more likely to be used when estimated moral hazard costs are high.

- Borrowers, conditional on reduced credit quality in the future, are locked-in to a relationship with its lending bank and have more limited access to alternative outside lenders. This argument is less relevant for interest decreasing contracts, incorporating improved credit quality.

- Empirical tests of structural debt models typically find that the model-predicted interest rates are too low if the borrower's initial distance-to-default is large (see, e.g., Huang & Huang (2003) and Eom, Helwege & Huang (2004)). Predicting too low interest rates is equivalent to overpricing the contract. Interest increasing PSD contracts typically have larger distance-to-default compared to interest decreasing PSD contracts (see Tables 6 and 19).

- The practice that borrowers have the discretion to time the reporting of performance measures to their benefit, and at the cost of lenders, suggests that our model price overstates the actual value of the contract. This effect is stronger for interest increasing contracts than for interest decreasing contracts.

Our empirical analysis suggests that our model price cash flow based PSD contracts reasonably well. Thus, although some variation is present, financial markets on average price PSD contracts in line with theory. We document an overpricing of interest increasing PSD contracts and suggest some possible explanations.

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Appendix A

Proof of Main Pricing Result

The value of the level dependent interest rate payments $V(A)$, given in Expression (6) is

$$V(A) = V^B(A, B_1, ..., B_n, C) + V^A(A, B_n, B_{n+1}, ..., B_{n+m}, C)$$

where

$$\begin{aligned} V^B(A, B_1, ..., B_n, C) &= \frac{1}{r}\left[\sum_{i=1}^{n-1}(c_i - c_{i+1})\kappa_i + c_n\nu\right], \\ V^A(A, B_n, B_{n+1}, ..., B_{n+m}, C) &= \frac{1}{r}\left[(c_n)\,\eta - (c_{n+m+1})\,\zeta + \sum_{i=n}^{n+m}(c_i - c_{i+1})\,\psi_i\right], \end{aligned}$$

where

$$
\begin{aligned}
\kappa_i &= \beta \left(\frac{A}{B_i}\right)^{\alpha} \left(1 - Q_{lg}^{\alpha}(B_i)\right) - \beta \left(\frac{A}{C}\right)^{-\beta} \left(\frac{C}{B_i}\right)^{\alpha} \left(1 - Q_g^{\beta}\right) + \alpha \left(\frac{A}{B_i}\right)^{-\beta} Q_{gg}^{\beta}(B_i) - Q_{gg}(B_i)\, e^{-rT} (\alpha + \beta), \\
\nu &= \beta \left[\left(\frac{A}{B_n}\right)^{\alpha} - \left(\frac{C}{B_n}\right)^{\alpha} \left(\frac{A}{C}\right)^{-\beta}\right] - \left[e^{-rT} Q_g - \left(\frac{A}{C}\right)^{-\beta} Q_g^{\beta}\right] (\alpha + \beta), \\
\eta &= 1 - \frac{\beta}{\alpha + \beta} \left[\left(\frac{A}{B_n}\right)^{\alpha} - \left(\frac{C}{B_n}\right)^{\alpha} \left(\frac{A}{C}\right)^{-\beta}\right], \\
\zeta &= \left(\frac{A}{C}\right)^{-\beta} Q_l^{\beta} + e^{-rT} Q_g, \\
\psi_i &= \frac{1}{\alpha + \beta} \left[\alpha \left(\frac{A}{B_i}\right)^{-\beta} \left(Q_{gg}^{\beta}(B_i) - 1\right) - \beta \left(\frac{A}{B_i}\right)^{\alpha} Q_{lg}^{\alpha}(B_i) - \beta \left(\frac{A}{C}\right)^{-\beta} \left(\frac{C}{B_i}\right)^{\alpha} Q_l^{\beta}\right],
\end{aligned}
$$

and

$$
\begin{aligned}
\alpha &= \frac{1}{\sigma^2} \left(\frac{1}{2}\sigma^2 - \mu + \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\sigma^2 r}\right), \\
\beta &= \frac{1}{\sigma^2} \left(\mu - \frac{1}{2}\sigma^2 + \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\sigma^2 r}\right).
\end{aligned}
$$

Proof. The time 0 market value V^A of the interest rate payments in the case where the starting value A_0 is above the barriers $B_{n+1}, ..., B_{n+m+1}$, is essentially the same as the one found in Expression (3) in Mjøs & Persson (2010), with the modification that $\frac{c_1}{r}$ is multiplied by η. The factor η stems from the fact that $B_0 \neq \infty$ in our case.

The proof for V^B is derived by similar arguments. The time 0 market value V^B of the interest rate payments in the case where the starting value A_0 is below the barriers $B_1, ..., B_n$, is $V^B = \sum_{i=1}^{n-1} \left(D_b(A, B_{i+1}) - D_b(A, B_i)\right) - M_T^{\infty}(A)$, where $D_b(A, B_i)$ is given by Expression (12) and $M_T^{\infty}(A)$ is given by Expression (22) in Mjøs & Persson (2010) respectively. The formula for V^B follows by direct calculations with $c = 1$ and $c_{n+1} = 0$. □

The survival probability $P(A)$ is given by

$$
P(A) = Q(\tau > T) = N(d_1) - \left(\frac{A}{C}\right)^{\alpha - \beta} N(-d_2), \tag{10}
$$

where

$$d_1 = \frac{\ln\left(\dfrac{A}{C}\right) + (\mu - \dfrac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

and

$$d_2 = \frac{\ln\left(\dfrac{A}{C}\right) - (\mu - \dfrac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}.$$

The time 0 market value V_0 under Q, of a security paying 1 when the default barrier is hit is given by

$$V_0 = E^Q[e^{-r\tau}1\{\tau \leq T\}] = e^{b(z-w)}N\left(\frac{b - wT}{\sqrt{T}}\right) + e^{b(z+w)}N\left(\frac{b + wT}{\sqrt{T}}\right),$$

where $z = (\mu - (1/2)\sigma^2)/\sigma$, $w = \sqrt{z^2 + 2r}$, and $b = \ln(C/A)/\sigma$, see Appendix B.2 in Lando (2004). Here $N(\cdot)$ denotes the cumulative standard normal distribution function.

Probabilities

In this section we state the expressions for the probabilities needed to calculate the theoretical price of a PSD contract. More details are found in Appendix B of Mjøs & Persson (2010).

$$Q^{\alpha}_{lg}(B) = Q^{\alpha}(A_T < B, \tau > T) = N\left(d^{\alpha}_1\right) - N\left(d^{\alpha}_3\right) + \left(\frac{A}{C}\right)^{-(\alpha+\beta)}\left(N\left(-d^{\alpha}_4\right) - N\left(-d^{\alpha}_2\right)\right), \tag{11}$$

where

$$\begin{aligned}
d^{\alpha}_1 &= \frac{\ln\left(\frac{A}{C}\right) + \left(\mu + \sigma^2\alpha - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \\
d^{\alpha}_2 &= \frac{\ln\left(\frac{A}{C}\right) - \left(\mu + \sigma^2\alpha - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \\
d^{\alpha}_3 &= \frac{\ln\left(\frac{A}{B}\right) + \left(\mu + \sigma^2\alpha - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \\
d^{\alpha}_4 &= \frac{\ln\left(\frac{A}{C}\right) + \ln\left(\frac{B}{C}\right) - \left(\mu + \sigma^2\alpha - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}.
\end{aligned}$$

$$Q^{\beta}_{g} = Q^{\beta}(\tau > T) = N\left(d^{\beta}_1\right) - \left(\frac{A}{C}\right)^{\alpha+\beta} N\left(-d^{\beta}_2\right), \tag{12}$$

where

$$d_1^\beta = \frac{\ln\left(\frac{A}{C}\right) + \left(\mu - \sigma^2\beta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$
$$d_2^\beta = \frac{\ln\left(\frac{A}{C}\right) - \left(\mu - \sigma^2\beta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}.$$

Note that $Q_l^\beta(B) = Q^\beta(A_T \leq B) = 1 - Q_g^\beta$.

$$Q_{gg}^\beta(B) = Q^\beta(A_T > B, \tau > T) = N\left(d_3^\beta\right) - \left(\frac{A}{C}\right)^{\alpha+\beta} N\left(-d_4^\beta\right), \tag{13}$$

where

$$d_3^\beta = \frac{\ln\left(\frac{A}{B}\right) + \left(\mu - \sigma^2\beta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$
$$d_4^\beta = \frac{\ln\left(\frac{A}{C}\right) + \ln\left(\frac{B}{C}\right) - \left(\mu - \sigma^2\beta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}.$$

$$Q_{gg}(B) = Q(A_T > B, \tau > T) = N\left(d_3\right) - \left(\frac{A}{C}\right)^{\alpha-\beta} N\left(-d_4\right), \tag{14}$$

where

$$d_3 = \frac{\ln\left(\frac{A}{B}\right) + \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$
$$d_4 = \frac{\ln\left(\frac{A}{C}\right) + \ln\left(\frac{B}{C}\right) - \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}.$$

$$Q_g = Q(\tau > T) = N\left(d_1\right) - \left(\frac{A}{C}\right)^{\alpha-\beta} N\left(-d_2\right), \tag{15}$$

where

$$d_1 = \frac{\ln\left(\frac{A}{C}\right) + \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$
$$d_2 = \frac{\ln\left(\frac{A}{C}\right) - \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}.$$

Appendix B

Industry	% of total
Banks	0.8 %
Corporates	77.8 %
Government	0.3 %
Media/Communications	9.9 %
Non-bank Financial Inst.	5.6 %
Utilities	5.6 %

Table 12: This table shows the distribution of issuers of performance sensitive debt across broad industry classes. Datasource: Thomson Reuter's Dealscan database for the years 1994-2009 (N=16,864).

Maturity (Years)	# Deals	Split	Cumulative
0-1	2,524	15.0%	15.0%
1-2	2,056	12.2%	27.2%
2-3	679	4.0%	31.2%
3-4	2,945	17.5%	48.7%
4-5	885	5.2%	53.9%
5-6	5,228	31.0%	84.9%
6-7	1,039	6.2%	91.1%
7-8	786	4.7%	95.8%
8-9	380	2.2%	98.0%
>9	342	2.0%	100.00%
Total	16,864	100.0%	100.0%

Table 13: This table shows the number of loans issued, on deal level, and their respective maturities. Datasource: Thomson Reuter's Dealscan database for the years 1994-2009.

Deal Amount	# Deals	Split	Cumulative
1-10	316	1.9%	1.9%
10-50	2,392	14.0%	15.9%
50-100	2,681	15.7%	31.6%
100-500	7,481	43.8%	75.4%
500-1,000	2,319	13.5%	88.9%
1,000-5,000	1,768	10.3%	99.2%
>5,000	139	0.8%	100.0%
Total	16,864	100.0%	100.0%

Table 14: This table shows the number of deals issued and the deal size (amount in USD). Datasource: Thomson Reuter's Dealscan database for the years 1994-2009.

Appendix C

Variable	Mean	Median	Std. Dev.	Min.	Max.	N
Borrower Characteristics:						
Company Sales (MUSD)	2,310	980	2,856	86	10,800	54
ROCE (%,quarterly)	2.26	2.12	3.08	-9.50	13.34	54
Leverage (Debt/Total Assets)	0.28	0.24	0.21	0.00	0.86	54
PSD Loan/Total Debt	0.48	0.41	0.26	0.07	1.00	54
Drift of cash flow	0.032	0.029	0.020	0.009	0.06	54
Volatility of cash flow	0.222	0.192	0.13	0.11	0.66	54
Loan Characteristics:						
Loan Amount (MUSD)	325	228	352	25	1,800	54
Maturity (Years)	5.49	5.00	1.54	2.25	8.67	54
All-In-Spread (Bp)	245	256	82	50	400	54
# of Barriers	3.63	3	1.67	2	7	54
Distance-to-default	1.01	0.78	0.81	0.15	3.60	54

Table 15: This table shows summary statistics for various model input parameters and firm characteristics for the **interest increasing PSD contracts** in the sample used in the paper. The loan contracts are granted in the period 1999 - 2009. Datasource: Thomson Reuter's Dealscan Database, Compustat and CRSP.

Variable	Mean	Median	Std. Dev.	Min.	Max.	N
Borrower Characteristics:						
Company Sales (MUSD)	3,092	2,185	3,176	141	17,608	86
ROCE (%,quarterly)	2.36	2.99	2.33	-5.50	6.20	86
Leverage (Debt/Total Assets)	0.30	0.27	0.18	0.00	0.75	86
PSD Loan/Total Debt	0.39	0.32	0.26	0.06	1.00	86
Drift of cash flow	0.032	0.030	0.018	0.010	0.06	86
Volatility of cash flow	0.181	0.151	0.09	0.03	0.41	86
Loan Characteristics:						
Loan Amount (MUSD)	271	180	291	25	1,800	86
Maturity (Years)	5.38	5.00	1.26	1	8	86
All-In-Spread (Bp)	221	225	57	75	400	86
# of Barriers	3.97	4	1.48	2	7	86
Distance-to-default	0.43	0.35	0.33	0.03	1.48	86

Table 16: This table shows summary statistics for various model input parameters and firm characteristics for the **interest decreasing PSD contracts** in the sample used in the paper. The loan contracts are granted in the period 1999 - 2009. Datasource: Thomson Reuter's Dealscan Database, Compustat and CRSP.

Variable	Mean	Median	Std. Dev.	Min.	Max.	N
Borrower Characteristics:						
Company Sales (MUSD)	2,324	1,089	3,427	86	21,978	130
ROCE (%,quarterly)	2.86	1.69	6.28	-9.50	47.00	130
Leverage (Debt/Total Assets)	0.32	0.32	0.20	0.00	0.82	130
PSD Loan/Total Debt	0.39	0.35	0.24	0.05	1.00	130
Drift of cash flow	0.029	0.023	0.018	0.002	0.06	130
Volatility of cash flow	0.211	0.184	0.116	0.07	0.66	130
Loan Characteristics:						
Loan Amount (MUSD)	288	148	558	20	5,500	130
Maturity (Years)	4.98	5.00	1.36	0.92	9.08	130
All-In-Spread (Bp)	227	213	74	75	400	130
# of Barriers	5.04	5	1.11	3	8	130
Distance-to-default	0.61	0.49	0.46	0.07	2.82	130

Table 17: This table shows summary statistics for various model input parameters and firm characteristics for sample **PSD contracts containing both interest increasing and interest decreasing provisions**. The loan contracts are granted in the period 1999 - 2009. Datasource: Thomson Reuter's Dealscan Database, Compustat and CRSP.

	Full Sample	**Interest Increasing**	**Interest Decreasing**		**Both Provisions**
			All	**Excluding outliers**	
Mean	0.71	-1.54	1.96	0.74	0.22
Median	-0.94	-2.48	-0.22	-0.29	-0.85
St.dev	7.61	2.68	7.36	4.51	5.43
Max	42.26	6.29	42.16	12.60	34.43
Min	-6.49	-6.03	-4.52	-4.52	-6.49
T-test	1.54	4.23**	2.47**	1.52	0.45
N	270	54	86	83	130

Table 18: This table shows the difference between the fair fixed interest rate (calculated using the standard Black & Cox (1976)) formula in equation (7) and the initial interest rate paid on the PSD loan, measured in percentage of the normalized face value. A standard t-test statistic testing whether the difference in the two means are significantly different from zero is included. A superscript of two stars marks a significance level of 1%.

	Full Sample		Interest Increasing		Interest Decreasing		Both	
	Model 1		Model 2		Model 3		Model 4	
	Coeff.	Stand. Coeff	Coeff.	Stand. Coeff	Coeff.	Stand. Coeff	Coeff.	Stand. Coeff
Log of Borrower Sales	-0.457	-0.045	-1.054	-0.132	-0.370	-0.032	-0.142	-0.014
	(0.486)		(1.193)		(0.914)		(0.698)	
Cash-flow Volatility	-57.612***	-0.535	-62.09***	-0.805	-58.261***	-0.426	-50.362***	-0.476
	(4.323)		(13.893)		(9.676)		(7.411)	
ROCE	-55.690***	-0.216	-31.289	-0.092	-3.491	-0.006	-62.439***	-0.319
	(13.460)		(54.261)		(43.440)		(14.401)	
Initial Leverage	-0.075	-0.005	0.069	0.008	0.608	0.028	1.105	0.059
	(0.641)		(0.917)		(0.967)		(1.606)	
Log of Loan Amount	-1.240*	-0.430	-0.039	0.0493	-0.351	-0.026	-1.696	-0.137
	(0.582)		(1.625)		(1.026)		(0.917)	
Maturity	2.220***	0.249	1.526*	0.228	1.396**	0.141	2.947***	0.326
	(0.443)		(0.608)		(0.585)		(0.727)	
Barriers	0.212	0.026	-0.672	-0.109	-0.186	-0.022	1.468	0.132
	(0.331)		(0.840)		(0.550)		(0.797)	
Interest Increasing	9.034***	0.296						
	(2.074)							
Interest Decreasing	-5.516**	-0.210						
	(1.872)							
Distance-to-Default	13.401***	0.607	5.433**	0.426	21.543***	0.568	15.893***	0.597
	(2.002)		(1.892)		(3.349)		(2.425)	
Distance-to-Default×Increasing	-9.433***	0.416						
	(2.138)							
Distance-to-Default×Decreasing	9.387**	0.208						
	(3.306)							
Constant	102.601***		114.598***		99.691***		98.231***	
	(4.624)		(12.041)		(7.892)		(5.222)	
N	270		54		86		130	
R-squared	0.754		0.761		0.825		0.763	

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 19: Estimation results from an OLS regression of modeled loan prices at the time of issue on log of sales, log of loan amount, maturity, number of barriers, dummy variables indicating whether the loan has interest increasing or interest decreasing provisions, borrower's asset volatility, return on capital employed (ROCE), borrower's initial leverage, borrower's credit-worthiness (measured by the distance-to-default), calendar year dummy variables and two interaction variables, interacting distance to default with the interest increasing/decreasing dummies. The regression is run for the full sample as well as the three subsamples. We report conventional OLS coefficients and standardized (beta) coefficients.

Variable	**Description**	**Means**			
		Full Sample	Increasing	Decreasing	Both
Log of Borrower Sales	Logarithm of borrowers sales (in MUSD)	7.18	7.00	7.52	7.02
Borrower Sales	*Borrower sales (in MUSD)*	*2,568*	*2,310*	*3,092*	*2,324*
Cash flow Volatility	Estimated annual volatility of borrowers cash flow	0.204	0.222	0.181	0.211
ROCE	Borrower's Quarterly Return on Capital Employed Based on the last quarterly report prior to the loan issuance	2.59%	2.26%	2.36%	2.86%
Initial Leverage	Borrower's leverage prior to loan issue	0.31	0.28	0.30	0.32
PSD Loan/Total Debt	Ratio of PSD loan amount to the borrower's total debt (book value) at the time of loan issuance	0.41	0.48	0.39	0.39
Log of Loan Amount	Logarithm of loan amount (in MUSD)	5.15	5.35	5.19	5.05
Loan Amount	*Loan amount (in MUSD)*	*290*	*325*	*271*	*288*
Maturity	Maturity of the PSD Loan (in years)	5.21	5.49	5.38	4.98
Barriers	Number of non-absorbing contractual barriers	4.42	3.63	3.97	5.04
Distance-to-Default	The distance between borrower's initial CF/Debt ratio and the contractually specified default barrier (C), normalized by borrower's cash flow volatility	0.63	1.01	0.43	0.61
Interest Increasing	Dummy variable equal to one if the loan is of interest increasing type				
Interest Decreasing	Dummy variable equal to one if the loan is of interest decreasing type				
Distance-to-Default×Increasing	Interaction between Distance-to-Default and the dummy for interest increasing	1.01			
Distance-to-Default×Decreasing	Interaction between Distance-to-Default and the dummy for interest decreasing	0.43			

Table 20: Table describes variables used in the OLS regressions, and displays their respective means.