The Nonequivalence of the Earnings and Dividends Approaches to Equity Valuation

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Abstract

Accounting theory treats a wide class of equity valuation approaches as equivalent. For example, under clean surplus accounting, the earnings approach is viewed as identical to the discounted dividends approach. Empirical research, however, typically finds that the two valuation approaches do not predict market prices equally well.

This paper offers a theoretical explanation for this apparent anomaly: expectations of discounted infinite sums (incomes, cash flows, or dividends) are undefined unless some restrictive probabilistic conditions hold. Without the usual stationarity and ergodicity assumptions, it may still be possible to estimate upper and lower bounds on such sums, but these bounds need not coincide. In such a setting, earnings and discounted dividends yield intervals of justifiable valuations, which intersect but need not coincide.

Depending on the extent to which a firm is held by insiders, differences in the valuations that different formulae justify may not show up in market prices. This provides an explanation for two additional empirical puzzles. First, empirical studies detecting little incremental information in dividends over earnings may be predisposed toward this finding. Second, stronger apparent reactions to dividend omissions than to initiations may be an illusion.

Key Terms: Equity Valuation, Residual Income, Dividends.

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1 Introduction

This paper attempts to reconcile the widely-held view that the earnings and dividends approaches to equity valuation are logically equivalent with the empirical literature reporting different results under the two models. I argue that the theoretical equivalence of these approaches is overstated, and present a model where the equivalence breaks down. Thus the empirical research findings (e.g., Penman and Sougiannis (1998), Frankel and Lee (1998), and Francis, Olsson, and Oswald (2000)) are consistent with reasonable theoretical assumptions.

This view differs sharply from the conventional viewpoint, as stated in Lundholm and O'Keefe (2001a) and (2001b) and in Lys and Lo (2000). It is argued there that one should never get a different valuation using residual income from what one gets with discounted dividends. Differences in estimated valuations are attributed either to the invalidity of the linear information dynamics in the (related) models of Ohlson (1995) and Feltham and Ohlson (1995), or to estimation issues, such as failure to adjust for dirty surplus or for assumptions on terminal values and discount rates.

While the issues that Lundholm and O'Keefe and Lys and Lo raise may be present, they leave unexplained a systematic feature of the empirical results (e.g., in Penman and Sougiannis): there are circumstances under which the residual income model gives a better fit to market prices, and there are other settings—namely, when there is a greater importance of a terminal valuation—that lead the discounted dividends model to perform better. Estimation errors would seem likely either to produce a random pattern of one formula outperforming another, or to yield a general bias toward one formula. Hence the theoretical viewpoint provides an incomplete explanation for the empirical findings. By breaking the assumptions underlying the theoretical equivalence, this paper offers a more complete story.

The model here departs from the usual theoretical treatment by modifying assumptions about infinite discounted sums. Conventionally, the values of discounted sums are treated as known or at least of having known expectations. If earnings and dividends are not stationary and ergodic stochastic processes with known distributions, there is little reason to hope that the present value of expected dividends is even a meaningful construct. While it is often possible to find upper and lower bounds on discounted series, making these bounds converge requires a leap of faith.

Accordingly, I consider a setting where a firm's current owners issue an income statement or make a dividend announcement, while knowing only upper and lower bounds on both the earnings that will later be realized and on the market's premium over book value. I then contrast what potential investors can learn in different reporting regimes, in order to illustrate what the information content of earnings and of dividend declarations is, and how they can differ.

When both income and the premium over book value are known and can be stated exactly, the two approaches to valuation remain equivalent, as in their classical derivations. When information is approximate, however, the standard formulae only give upper and lower bounds on justifiable valuations. These bounds need not coincide, which reflects the fact that there are many ways to approximate the same limit.

Equilibrium prices are considered here, again across reporting regimes. The goal of comparing different regimes is to show the range of prices that could be estimated by a researcher using either the residual income or the discounted dividends model. While the equilibrium price intervals that emerge under each regime always overlap, they again will have different bounds, which is to say that researchers using each model could correctly get different estimated valuations.

It should be noted that the ranges of equilibrium prices can coincide under each regime, even when the information content in a dividend announcement is different from that in an income statement. The reason is that, to the extent that a firm is owned by insiders, the current owners' valuation determines the lower bound on equilibrium prices, while the potential buyerss valuation determines the upper bound. If the models differ only on the market's lower bound, and both these values are weakly below the current owner's private lower bound, then no difference will show up in the set of possible prices.

I find in particular that a dividend omission can refine the market's upper bound on valuation, and thus leads to observable effects on the set of possible prices. On the other hand, the introduction of a dividend or the payment of a planned dividend in general will not show up in market data. Thus two puzzles about dividends are resolved: first, the asymmetric market response to dividend omissions versus initiations (Healy and Palepu (1988) and Michaely, Thaler, and Womack (1995)) is a consequence of the omissions' lowering the market's upper bound on valuation. Secondly, the

apparent low information content of dividends (Watts (1973) and Watts (1976)) follows from the fact that dividend introductions affect the market's lower bounds.

The organization of the rest of this paper is as follows: the next section reviews the residual income and discounted dividends models. Section three presents a one-period, two-date model, where only bounds are available on earnings and on the premium over book value. This section compares valuations across reporting regimes. Section four discusses the economic implications of the results here. Section five concludes.

2   The Residual Income and Dividends Models

2.1   Derivation of the Models and the Equivalence Argument

The earnings approach to equity valuation goes back at least to the 1930s.[1] Preinreich (1936) presents a valuation model based on a company's original book value, its residual income, the market return on equity capital, and a final liquidating dividend, based on an infinite-horizon model in Preinreich (1932). He introduces this model by way of comparison to the model of Fisher (1907), which argues that the value of an equity is the present value of future dividends. Preinreich calls the earnings-based model "an alternative method of computation, which is equally correct." Elsewhere, Preinreich (1938) uses an analogous formula in the context of valuation of a productive asset, arguing that the asset's value is its book value plus discounted residual income, which he calls "excess profits."

The argument that the earnings and dividends valuation approaches are equivalent begins with an assumption of *clean surplus accounting*. This says that the ending book value of shareholders' equity equals its starting value plus income less dividends:

$$S_{t+1} = S_t + I_{t+1} - D_{t+1}.$$

Here S represents shareholders' equity, D represents dividends, and I represents income. Letting r represent the rate of return on equity capital, one can decompose income into *normal* income rS_t

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[1]Lys and Lo (2000) present evidence that the model is much older, as is implied in Preinreich (1936).

and *residual* income $I_{t+1} - rS_t \equiv \rho_{t+1}S_t$. The clean surplus equation can then be written as

$$S_{t+1} = (1 + r + \rho_{t+1})S_t - D_{t+1}. \tag{1}$$

The *discounted dividends* model says that, in period t, the value of the equity is the present value of the expected future dividend stream:

$$P_t = \sum_{\tau=1}^{\infty} \frac{D_{t+\tau}}{(1+r)^\tau} = \frac{D_{t+1}}{1+r} + \frac{P_{t+1}}{1+r}. \tag{2}$$

The earnings approach, or *residual income* model, says that the period t value of the equity is the book value of shareholders' equity plus the present value of the expected future residual income stream:

$$P_t = S_t + \sum_{\tau=1}^{\infty} \frac{\rho_{t+\tau}S_{t+\tau-1}}{(1+r)^\tau} = S_t + \frac{\rho_{t+1}S_t}{1+r} + \frac{P_{t+1} - S_{t+1}}{1+r}. \tag{3}$$

Subtracting the book value of shareholders' equity S_t from both sides of Equation 3 gives an interpretation of this model: the current premium of intrinsic value over book value equals the discounted value of next period's residual income plus the discounted value of next period's premium.

Proposition 2.1 (Preinreich (1932))**.** *Given the clean surplus equation, the residual income model is derivable from the dividend model.*

Proof. Equation 2 can be rewritten as

$$P_t = \frac{D_{t+1}}{1+r} + \frac{S_{t+1} + P_{t+1} - S_{t+1}}{1+r}.$$

Plugging in the clean surplus equation (1) then gives

$$P_t = \frac{D_{t+1}}{1+r} + \frac{(1+r)S_t}{1+r} + \frac{\rho_{t+1}S_t}{1+r} - \frac{D_{t+1}}{1+r} + \frac{P_{t+1} - S_{t+1}}{1+r},$$

so that

$$P_t = S_t + \frac{\rho_{t+1}S_t}{1+r} + \frac{P_{t+1} - S_{t+1}}{1+r},$$

which is just the residual income model as stated in Equation 3. □

Conversely, it is straightforward to show the rest of the equivalence argument:

Proposition 2.2. *Given the clean surplus equation, the dividend model is derivable from the residual income model.*

Proof. Analogous to the proof of Proposition 2.1 □

2.2   Where the Equivalence of the Models Breaks Down

Both the residual income and discounted dividends models depend on expectations discounted infinite sums. For the residual income model, this requires having a unique probability distribution defined over purchases by customers who have not yet been born, and over product offerings by competitors that do not yet exist. For the dividends model, a unique distribution is needed about decisions of Boards of Directors that have not yet been appointed. Even given such a unique distribution, it is hard to argue that these distributions are necessarily stationary and ergodic. On the other hand, if state-space uncertainty or outcome-space uncertainty are acknowledged, it is known that the resulting Savage-style probability distribution is no longer unique (see Stecher (2005a) or Blume, Easley, and Halpern (2005)).

If a range of expectations is possible on both future incomes and premia over book values, the earnings and dividends valuation approaches are no longer simple algebraic transformations of each other. To see this, note that the clean surplus equation (1) says that ending book value of shareholders' equity equals starting book value plus income less distributions. Solving this for income gives

$$I_{t+1} = S_{t+1} - S_t + D_{t+1},$$

which says that income equals the change in book value plus distributions. If income is known, then clean surplus provides one equation in two unknowns (change in book value and distributions). As long as the Board of Directors has not chosen next period's dividend—i.e., as long as the next-period dividend is undetermined—knowledge of income does not provide any information on dividends or on the change in book value of equity. Thus, without violating clean surplus, there can be information available for use in the earnings approach without there being information for use in the dividends approach.

Conversely, the Board of Directors may set a dividend policy of paying a given amount to shareholders next period, irrespective of what income may be—for example, the Board may choose to set dividends to 0. Again, the clean surplus equation provides only one equation in the two unknowns: the dividend policy means that all income is retained, but does not provide any information on the change in book value or on earnings.

Thus, the firm's income stream and its dividend stream need not be estimable over the same horizons. Recognizing this difficulty, Preinreich (1932) makes the horizon an explicit part of the model. Beyond the horizon, all terms in the given sequence are viewed as undefined. The approach of Penman and Sougiannis is based on this idea: their interest is in valuation rules that do not depend on what happens after the finite horizon. Whether valuation rules should have this property of being insensitive to truncation underlies the debate between Penman (2001) and Lundholm and O'Keefe (2001a, 2001b). The next subsection discusses the mathematics of this issue in detail.

2.3   Choice Sequences and Brouwer's Continuity Principle

The argument in Penman and Sougiannis (1998) and in Penman (2001) is philosophically somewhat stronger than arguing against the existence of expectations of terms over a finite horizon. In a fundamental, metamathematical sense, the argument asserts that the remote terms in an infinite sequence may be at most partially defined. Thus the insensitivity of a valuation formula to truncation is a necessary property, because in some sense it is impossible to discuss what is being truncated.

This viewpoint is uncommon in economics and related fields, but there is an extensive mathematical literature on infinite sequences, whose terms are ill-defined (that is, defined at most up to some restrictions). For example, Borel (1912) considers sequences with terms chosen

> ... either entirely arbitrarily, or [by] imposing some restrictions which leave some arbitrariness.[2]

Such sequences, called *choice sequences* (because their terms have free choice), play an important role in the intuitionist approach to constructive mathematics (Brouwer 1921, Troelstra 1977, Spitters 2003).

Whether a function of a choice sequence ought to depend entirely on an initial segment, as Preinreich and Penman and Sougiannis seem to have in mind, depends on the nature of what is specified about

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[2]The translation of Borel's quotation is from Troelstra and van Dalen (1988).

the remote terms. For example, a company might have a rule written into its charter specifying a dividend policy (e.g., paying \$1 per share any time a dividend is declared). This does not state the value of any future dividend, but it does force the dividend stream to be a binary sequence. Given an interest rate r, this information is insufficient for deriving an exact present value by either the discounted dividends or earnings approach. Nevertheless, the dividends approach cannot assign a value to the share below 0 or above \$$1/r$. By contrast, the residual income formula cannot provide any bounds on the share's possible values from the above data: the firm needs more than just a charter for the residual income formula to assign a valuation. Returning to the truncation in Penman and Sougiannis (1998), in this example it would be valid to insist on earnings-based valuations only using data that could be estimated, but it would be inappropriately discarding information to do likewise with dividends-based valuations.

Equivalent restrictions need not bind with equal force. Consider the company in the last example, with one minor change: rather than having the dividend policy written into the company's charter, imagine that the Board of Directors has announced a policy of paying \$1 per share every time a dividend is declared. The sequences of payments for the company will be identical in each case, as long as the Board of Directors does not change its policy. However, the policy is only a provisional restriction, which a future Board of Directors might change. If at each date, the composition of the Board of Directors can only be known finitely far into the future, then the insensitivity to truncation that Penman and Sougiannis (1998) require seems not only justified but necessary for any sensible model.

Thus a somewhat weaker form of the argument in Penman and Sougiannis, called *Brouwer's continuity principle* (Brouwer 1918, Brouwer 1927) seems appropriate. In the present context, Brouwer's continuity principle says that any two sequences (say, of income streams) which have the same means on some initial (fully defined) segment and the same definitive restrictions on any subsequent terms should return the same valuation.

A discussion of Brouwer's continuity principle is beyond the scope here, and is undertaken in detail in van Atten and van Dalen (2002). In the present context, it is worth noting that acceptance of Brouwer's continuity principle undermines even the strongest criticism of the empirical literature presenting horse-races between the two valuation approaches. The critique is that truncating the

discounted dividends model means ignoring the terminal price, whereas truncating the residual income model means ignoring a terminal premium over book value. If one accepts Brouwer's arguments, then the fact that one model is more sensitive to truncation than another is not properly interpreted as estimation error, but as a substantive reason for favoring one valuation approach over another.

3   An Example with One Trading Period

3.1   The Model

This section presents the basic model, where both income and a premium over book value do not have expectations, but can be given lower and upper bounds (interpreted as a non-ergodic case, where means can be estimated from above and below but need not converge). The model here is used throughout the rest of this paper.

There is a set $I = \{A, B\}$ of two agents. Agent A is thought of as the initial owner of a firm, while agent B is thought of as the market.

There are two dates, $T = \{0, 1\}$. Each agent has an initial endowment of one unit of the unique good at date 0, and nothing at date 1, when consumption takes place. Both agents have access to a public storage technology. An investment of S_0 at date 0 in the public technology gives the investor $S_0(1 + r)$ at date 1. The values of r and S_0 are common knowledge.

Agent A also has a private technology, on which the return is bounded. If A invests S_0 initially in the private technology, then at date 1, the investment yields $S_0(1 + r + \rho) + \gamma$. Depending on the reporting environment, if A reports accounting income, then $r + \rho S_0$ can be called income but γ cannot. Thus, ρS_0 is the residual income from the private technology, which is proportional to the size of the investment, while γ is thought of as the difference between the ending book value of equity and the liquidation value. There is no requirement that γ must vary proportionately to S_0.[3]

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[3]Adding such a requirement does not change the analysis, although it makes the form of the model similar to that in Lucas and Prescott (1971) and in Hayashi (1982). However, keeping γ as a lump-sum adjustment gives the model

At date 0, both agents have identical information about the private technology. Specifically, they can restrict ρ and γ to

$$\rho \in (\underline{\rho}, \overline{\rho}) \qquad \gamma \in (\underline{\gamma}, \overline{\gamma}).$$

One interpretation of these restrictions is that neither agent has enough experience with the private technology to distinguish investments for any values of ρ or γ within these intervals.

During period 0, A gains experience with the private technology and can therefore refines the bounds on ρ or γ (or both). When A refines the interval on ρ, I write the refined interval as (ρ', ρ''), where it is to be understood that $\underline{\rho} < \rho' < \rho'' < \overline{\rho}$. Refinements of the bounds on γ to (γ', γ'') are analogous.

After learning about the parameters, A issues a report to B. The report has one of three forms:

Income Statement A announces the accounting earnings. Since r and S_0 are common knowledge, I interpret the income statement as A stating a value for ρ.

Dividend Declaration A announces that, at date 1, a payment of D_1 is assured. The dividend declaration can have one of the following possible structures:

  Targeted A has precommitted to paying a specified dividend value $\widehat{D}$ if possible, and 0 otherwise.

  Proportionate A has precommitted to paying a divided as some fixed proportion $\lambda \in (0, 1]$ of the largest possible dividend that can be promised, and 0 if it is impossible to commit to a positive dividend.

  Unspecified A has not made any prior commitments, and simply announces a dividend.

Statement of Changes in Financial Position A announces changes in working capital—i.e., changes in the value of the investment that can be paid out at date 1. Income from ρ and changes in valuation from γ are not separated.

Remark: The targeted dividend policy matches the assumption in much of the dividends literature (Lintner 1956, Watts 1973, Watts 1976, Laub 1976). The proportionate dividend policy is also fre-

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the same general structure as the residual income model as presented in Equation 3.

quently studied (Dutta and Reichelstein 2004). Unspecified dividends are studied in generalizations of the residual income model, for example in Ohlson and Juettner-Nauroth (2003). The Statement of Changes in Financial Position is an announcement of the funds known to be available for distribution at date 1, and thus represents a maximally informative dividend announcement. Until 1987, this statement was one of the standard financial reports in the United States, after which it was replaced by the Statement of Cash Flows (APB 1971, FASB 1987). Stephens and Govindarajan (1990) describe these statements and related measures of funds available for distribution.

Once A issues the report, the market opens (still during period 0), and any trade that may occur takes place. After markets close, nothing happens until date 1, when ρ and γ are realized and the owner of the private technology receives the liquidating value.

All reporting rules obey the following convention: Agent A only reports amounts that are guaranteed. For example, if A were to learn nothing, the largest dividend that could be declared would be $\max\{S_0(1+r+\underline{\rho})+\underline{\gamma}, 0\}$, since anything higher is possibly more than A will be able to pay out. Similarly, the highest residual income rate that A could report in this case is $\underline{\rho}$. This matches the discussion of conservatism in Feltham and Ohlson (1995) (but see also Lys and Lo (2000)).

3.2   Reporting Under Full Information

This subsection analyzes the case where the firm A refines its information perfectly on both residual income and the premium over book value, prior to issuing its report. The purpose of evaluating this case is to restrict attention to the effects of the imprecision in the reporting language.

Initially, both agents have the same bounds on ρ and γ. The present values that can be assigned to the private technology at the start of date 0 are therefore:

$$P_0 = (S_0 + \frac{\underline{\rho}S_0 + \underline{\gamma}}{1+r}, S_0 + \frac{\overline{\rho}S_0 + \overline{\gamma}}{1+r}).$$

Since ρ and γ belong to open intervals $(\underline{\rho}, \overline{\rho})$ and $(\underline{\gamma}, \overline{\gamma})$, neither the upper bound nor lower bound of P_0 is attainable. The limits on this interval correspond to the calculation of the value under the residual income approach, using the bounds as worst and best case scenarios.

To use the discounted dividends approach, there would need to be some known yield at date 1.

The most that A could initially promise is $S_0(1 + r + \underline{\rho} + \underline{\gamma})$, leaving an ex dividend date 1 value in $(0, S_0(1 + r + \overline{\rho} - \underline{\rho}) + \overline{\gamma} - \underline{\gamma})$. It is clear that discounting these values leads to the same valuation interval under the dividends approach as is obtained above under the earnings approach. The reason is that there is no new information on either earnings or dividends.

Suppose that, prior to issuing any report, A observes ρ and γ. Then A knows that the present value of the investment of S_0 is

$$P_0^A(\rho, \gamma) = \{S_0 + \frac{\rho S_0 + \gamma}{1 + r}\}.$$

If A releases an income statement, B learns ρ and refines the interval of present values to

$$P_0^B(I) = (S_0 + \frac{\rho S_0 + \underline{\gamma}}{1 + r}, S_0 + \frac{\rho S_0 + \overline{\gamma}}{1 + r}).$$

The set of present values based on A's knowledge is a singleton, while those present values B can justify is an open interval. The reason is that the income statement does not provide any information about γ. In a sense, the information in the income statement is too specific: it resolves all ambiguity in ρ, but B would be better informed by seeing the aggregate resolution of ambiguity.

To analyze the dividends approach, imagine first that A releases a statement of changes in financial position. This tells B what A can commit to pay at date 1, which is $(1 + r + \rho)S_0 + \gamma$. B now can assign a unique valuation:

$$P_0^B(C) = \{S_0 + \frac{\rho S_0 + \gamma}{1 + r}\}.$$

Other forms of dividend declarations are less informative. If A did not initially specify any target dividend and announces a commitment to pay dividend $D_1 > 0$, then the announcement can be viewed as a dividend initiation. In this case, B can infer

$$(1 + r + \rho)S_0 + \gamma \geq D_1.$$

Because the dividend announcement does not identify earnings or the premium over book value, B cannot use the dividend to restrict the upper bound on possible valuations:

$$P_0^B(D_1 > 0|\text{no policy}) = [\frac{D_1}{1 + r}, S_0 + \frac{\overline{\rho} S_0 + \overline{\gamma}}{1 + r}).$$

The interval is closed on the left because B knows that A has full information. Note that this valuation says that the present value is the discounted value of the dividend plus the discounted value of the remaining liquidating value.

A dividend omission is only meaningful if A had precommitted to paying some target dividend, and cannot guarantee that amount. When no dividend policy is in place, the declaration of a zero dividend is simply uninformative:

$$P_0^B(D_1 = 0|\text{no policy}) = (S_0 + \frac{\rho S_0 + \gamma}{1+r}, S_0 + \frac{\overline{\rho} S_0 + \overline{\gamma}}{1+r}).$$

As discussed above, a prespecified dividend policy provides additional information, which can be used in the dividends approach to valuation. If A has a policy of targeting a specific dividend, say $\widehat{D} > 0$, then the dividend announcement states whether earnings and the premium over book value are at least $\widehat{D}$. Thus, if the dividend of $D_1 = \widehat{D}$ is declared, then

$$P_0^B(D_1 = \widehat{D}|\text{targeted policy}) = [\frac{\widehat{D}}{1+r}, S_0 + \frac{\overline{\rho} S_0 + \overline{\gamma}}{1+r}).$$

On the other hand, if the dividend is omitted, then A cannot guarantee $\widehat{D}$. This restricts the upper bound on B's possible valuations:

$$(1 + r + \rho)S_0 + \gamma < \widehat{D},$$

$$\Rightarrow P_0^B(D_1 = 0|\text{targeted policy}) = (S_0 + \frac{\rho S_0 + \gamma}{1+r}, \frac{\widehat{D}}{1+r}).$$

As long as the dividend target is ex ante feasible ($0 < \widehat{D} < (1+r+\overline{\rho})S_0+\overline{\gamma}$), this interval is smaller than the defensible valuations B had prior to the dividend announcement.

The targeted dividend policy hence partitions the range of possible payouts at the amount of the stated dividend. This is because of A's full information. It will be shown below that, once A receives only partial information on either income or the premium over book value, the valuations justified under an omission overlap with those justified when the dividend is paid.

A proportionate dividend policy is likewise informative in both the case when A can commit to a dividend and when A cannot. In the first case, for fixed $\lambda \in (0, 1]$, a positive dividend announcement tells B that

$$D_1 = \lambda((1 + r + \rho)S_0 + \gamma)$$

$$\Rightarrow \qquad P_0^B(D_1 > 0|\text{proportionate policy}) = \{S_0 + \frac{\rho S_0 + \gamma}{1+r}\} = \{\frac{D_1}{\lambda(1+r)}\}.$$

Dividing the dividend by the payout rate λ gives B the exact future value, so the report is equivalent to the statement of changes in financial position and is fully revealing.

If A does not declare a dividend when the proportionate policy is in place, then B knows $(1 + r + \rho)S_0 + \gamma) \leq 0$, so that

$$P_0^B(D_1 = 0|\text{proportionate policy}) = (S_0 + \frac{\rho S_0 + \underline{\gamma}}{1 + r}, 0].$$

Unless A's private technology is highly destrutive (i.e., unless $(1 + r + \overline{\rho})S_0 + \overline{\gamma} \leq 0$), the lack of a dividend is informative under the proportionate policy.

Theorem 3.1. *Assume that A learns income and the premium over book value perfectly. Then the income statement, the dividend announcement, and the statement of changes in financial position all yield different sets of justified present values. Moreover, the dividend announcement contains information in addition to that in the income statement under the following parameter restrictions:*

1. *For an unspecified dividend policy, if $D_1 > (1 + r + \rho)S_0 + \underline{\gamma}$, then a dividend initiation provides a greater lower bound on valuation than the income statement. A zero dividend with an unspecified policy is never informative.*

2. *For a targeted dividend policy, if $\widehat{D} > (1 + r + \rho)S_0 + \underline{\gamma}$, then a dividend declaration provides a greater lower bound on valuation than the income statement. If $\widehat{D} < (1 + r + \rho)S_0 + \overline{\gamma}$, then the dividend omission provides a smaller upper bound than the income statement.*

3. *For a proportionate dividend policy, a dividend declaration is fully revealing, while the income statement is not. If $(1 + r + \rho)S_0 + \overline{\gamma} > 0$, then the dividend omission provides a smaller upper bound than the income statement.*

The income statement can, conversely, contain information not in the dividend announcement. However, neither the income statement nor the dividend announcement contains information in addition to that contained in the statement of changes in financial position.

Consequently, both the earnings and the dividends approaches can yield more informative valuations, unless the firm issues a statement of changes in financial position. In that case, the dividends approach is always more informative.

The proofs of this and other main results are in the appendix.

3.3   Partial Information

In the case considered so far, agent A obtains perfect information about both earnings and the premium over book value, prior to having to issue a report. I now consider the case where A receives only partial information on at least one of these quantities.

3.3.1   Case 1: One quantity learned perfectly, the other refined

The situation where A learns ρ exactly and refines γ is analogous to an infinite horizon model, where the the mean of the discounted sum of remote earnings is undefined.

Conversely, if agent A learns the premium over book value exactly but only can learn income approximately, then the model is also analogous to an infinite horizon model where the earnings at each date form a choice sequence. The ability to learn γ perfectly may then be interpreted as a restriction on the discounted infinite sum of earnings, as for example in Ohlson (2003). In terms of Penman and Sougiannis (1998), this case corresponds to the setting where the terminal value estimate is more crucial, i.e., where the dividends model outperforms the residual income model.

The private valuations to agent A are symmetric in these cases. Upon observing ρ and refining γ to (γ', γ''), A's valuation interval is:

$$P_0^A(\rho, (\gamma', \gamma'')) = (S_0 + \frac{\rho S_0 + \gamma'}{1+r}, S_0 + \frac{\rho S_0 + \gamma''}{1+r}).$$

Conversely, upon observing γ and refines ρ to (ρ', ρ''), A's valuation interval is:

$$P_0^A((\rho', \rho''), \gamma) = (S_0 + \frac{\rho' S_0 + \gamma}{1+r}, S_0 + \frac{\rho'' S_0 + \gamma}{1+r}).$$

If agent A issues an income statement, then B can learn about ρ but not about γ. Consequently, in the case where A learns ρ perfectly,

$$P_0^B(I|A \text{ learns } \rho, (\gamma', \gamma'')) = (S_0 + \frac{\rho S_0 + \underline{\gamma}}{1+r}, S_0 + \frac{\rho S_0 + \overline{\gamma}}{1+r}).$$

Thus, the income statement refines both the upper and the lower bound on B's valuations, but does not fully disclose either of A's bounds on the valuation.

On the other hand, if A observes γ and refines ρ to (ρ', ρ''), then the income statement is only informative about the lower bound on income. The information in the premium over book value is not part of the income statement, and the lower bound is the most income that A can guarantee. Therefore, in this case,

$$P_0^B(I | A \text{ learns } (\rho', \rho''), \gamma) = (S_0 + \frac{\rho' S_0 + \underline{\gamma}}{1+r}, S_0 + \frac{\overline{\rho} S_0 + \overline{\gamma}}{1+r}).$$

In each case, B's lower bound is strictly below A's lower bound and B's upper bound is strictly above A's upper bound.

The statement of changes in financial position, by combining the effects of income and the premium over book value, does give the lower bound exactly. Moreover, B can use the statement of changes in financial position to infer a refinement to the upper bound. In the case where A learns ρ exactly, B can use the fact that $\gamma' > \underline{\gamma}$ to deduce

$$\rho S_0 + \gamma' > \rho S_0 + \underline{\gamma} \quad \Rightarrow \quad \rho < \min\{\frac{(\rho S_0 + \gamma') - \underline{\gamma}}{S_0}, \overline{\rho}\}.$$

Similarly, if A learns γ exactly, then B can use the fact that $\rho' S_0 > \underline{\rho} S_0$ to deduce

$$\rho' S_0 + \gamma > \underline{\rho} S_0 + \gamma \quad \Rightarrow \quad \gamma < \min\{(\rho' S_0 + \gamma) - \underline{\rho} S_0, \overline{\gamma}\}.$$

Based on these refinements, B's valuation intervals are

$$P_0^B(C | A \text{ learns } \rho, (\gamma', \gamma'')) = (S_0 + \frac{\rho S_0 + \gamma'}{1+r}, S_0 + \min\{\frac{\rho S_0 + \gamma'}{1+r} + \frac{\overline{\gamma} - \underline{\gamma}}{1+r}, \frac{\overline{\rho} S_0 + \overline{\gamma}}{1+r}\}),$$

and

$$P_0^B(C | A \text{ learns } (\rho', \rho''), \gamma) = (S_0 + \frac{\rho' S_0 + \gamma}{1+r}, S_0 + \min\{\frac{\rho' S_0 + \gamma}{1+r} + \frac{(\overline{\rho} - \underline{\rho}) S_0}{1+r}, \frac{\overline{\rho} S_0 + \overline{\gamma}}{1+r}\}).$$

In sum, the following holds:

Proposition 3.1. *When agent A observes one parameter perfectly and the other imperfectly, the statement of changes in financial position fully reveals A's lower bound on valuation, while the income statement gives B a lower bound strictly below A's. Both statements give upper bounds to B above that of A, but B's upper bound from the income statement is strictly lower than B's upper bound from the statement of changes in financial position.*

Proof. Immediate from comparing P_0^A above with $P_0^B(I)$ and $P_0^B(C)$.   □

Remark: Proposition 3.1 says that the income statement limits the degree to which B can pay too much for the equity. Conversely, the statement of changes in financial position prevents B from missing opportunities, by keeping B from underestimating the lower bound on the equity's value. These effects are symmetric, but their observability in market prices is not: the information in the income statement restricts observable prices, while that in the statement of changes in financial position prevents non-trades.

If instead A initiates a dividend D_1 and had not previously specified a dividend policy, then B's valuation interval becomes

$$P_0^B(D_1 > 0 | \text{no policy}) = (\frac{D_1}{1+r}, S_0 + \frac{\overline{\rho}S_0 + \overline{\gamma}}{1+r}).$$

This is almost identical to the full information case, except now the left endpoint of this interval is excluded as a possible valuation. The reason is that A's valuation is an open interval, so A can only guarantee amounts strictly below the lower bound on A's private valuations. On the other hand, if there is no policy in place and A does not declare a dividend, then B receives no useful information. Thus, in the absence of a dividend policy, the information in dividends is nearly identical to the full information case, differing only in that the left endpoint of B's valuation interval is now excluded.

If there is a target dividend of $\widehat{D}$ and a dividend is declared, then B again can bound the valuation interval below at $\widehat{D}/(1+r)$. Conversely, a dividend omission bounds B's valuation interval above.

In the case where A learns earnings exactly and refines the premium over book value, not paying the target dividend $\widehat{D}$ tells B that

$$(1 + r + \rho)S_0 + \gamma' < \widehat{D}.$$

Since $\gamma' > \underline{\gamma}$, B can infer that

$$(1 + r + \rho)S_0 + \underline{\gamma} < \widehat{D} \quad \Rightarrow \quad (1 + r + \rho)S_0 + \overline{\gamma} < \widehat{D} + \overline{\gamma} - \underline{\gamma}.$$

Accordingly, the valuation B can assign from the dividend omission is:

$$P_0^B(D_1 = 0 | \text{targeted policy}; \rho, (\gamma', \gamma'')) = (S_0 + \frac{\rho S_0 + \underline{\gamma}}{1+r}, \min\{\frac{\widehat{D} + \overline{\gamma} - \underline{\gamma}}{1+r}, S_0 + \frac{\overline{\rho}S_0 + \overline{\gamma}}{1+r}\}),$$

and the valuation from paying the targeted dividend is

$$P_0^B(D_1 = \widehat{D}|\text{targeted policy}; \rho, (\gamma', \gamma'')) = (\frac{D_1}{1+r}, S_0 + \frac{\overline{\rho}S_0 + \overline{\gamma}}{1+r}).$$

When A instead learns the premium over book value exactly and refines earnings, the valuation interval for B is identical if the targeted dividend is paid, and changes in the symmetric way when the dividend is omitted:

$$P_0^B(D_1 = 0|\text{targeted policy}; (\rho', \rho''), \gamma) = (S_0 + \frac{\rho S_0 + \gamma}{1+r}, \min\{\frac{\widehat{D} + (\overline{\rho} - \rho)S_0}{1+r}, S_0 + \frac{\overline{\rho}S_0 + \overline{\gamma}}{1+r}\}).$$

Under the proportionate dividend policy, the information is the same as the information in the statement of changes in financial position whenever a dividend is declared. If A has a proportionate dividend policy in place and omits the dividend, then $(1 + r + \rho)S_0 + \gamma' \leq 0$ in the case where A learns income exactly, and $(1 + r + \rho')S_0 + \gamma \leq 0$ in the case where A learns the premium over book value exactly. This is identical to the case of an omission under the targeted policy, except that the target value $\widehat{D}$ is replaced by 0, since a proportionate dividend can be arbitrarily close to zero. By the above argument in the case of the targeted dividend policy, B's valuations in the cases of an omission are

$$P_0^B(D_1 = 0|\text{proportionate policy}; \rho, (\gamma', \gamma'')) = (S_0 + \frac{\rho S_0 + \gamma}{1+r}, \frac{\overline{\gamma} - \gamma}{1+r}),$$

and

$$P_0^B(D_1 = 0|\text{proportionate policy}; (\rho', \rho''), \gamma) = (S_0 + \frac{\rho S_0 + \gamma}{1+r}, \frac{(\overline{\rho} - \rho)S_0}{1+r}).$$

3.3.2 Case 2: Both learned partially

The last case to consider is where A refines both ρ and γ but continues to be incapable of distinguishing their true values from nearby values. To interpret this in the infinite horizon setting, imagine that earnings in every period are restricted, but that there are limits in how well earnings in any period can be measured.

The valuations A can justify at the time of issuing a report are now

$$P_0^A((\rho', \rho''); (\gamma', \gamma'')) = (S_0 + \frac{\rho' S_0 + \gamma'}{1+r}, S_0 + \frac{\rho'' S_0 + \gamma''}{1+r}).$$

The income statement can convey ρ' to B, but provides no information on ρ'' or on γ. Consequently,

$$P_0^B(I) = (S_0 + \frac{\rho' S_0 + \underline{\gamma}}{1 + r}, S_0 + \frac{\overline{\rho} S_0 + \overline{\gamma}}{1 + r}).$$

Thus, the income statement does not refine the upper bound on B's valuations, and does not fully reveal A's lower bound.

The statement of changes in financial position conveys the lower bound exactly, since $\rho' S_0 + \gamma'$ are combined. As this report says nothing about ρ'' or γ'', it provides no information for refining the upper bound; i.e.,

$$P_0^B(C) = (S_0 + \frac{\rho' S_0 + \gamma'}{1 + r}, S_0 + \frac{\overline{\rho} S_0 + \overline{\gamma}}{1 + r}).$$

This leads immediately to the following:

Proposition 3.2. *If A receives partial information on both income and the premium over book value, then the statement of changes in financial position is strictly more informative than the income statement.*

Proof. Compare $P_0^B(C)$ and $P_0^B(I)$ in this case: the statement of changes gives a higher lower bound on B's valuations, and neither refines the upper bound.   □

Any dividend policy must be based on ρ' and γ'. Thus, the following holds:

Proposition 3.3. *If A receives partial information on both income and the premium over book value, a dividend omission is uninformative.*

Proof. Since the dividend is zero, there is no refinement of the lower bound of possible valuations for B. On the other hand, the upper bound depends entirely on ρ'' and γ'', neither of which affects the dividend announcement.   □

This means the following result holds:

Corollary 3.1. *If A receives partial information on both income and the premium over book value, the targeted and unspecified dividend policies convey the same information. Also, under this assumption, the proportionate policy is either identical to the statement of changes in financial position in its information content (i.e., when a dividend is declared) or uninformative.*

3.4 Valuation with Truncated Calculations

Suppose now that B were to calculate the valuation models based on either the dividends alone or on the book value plus discounted residual income. That is, consider the truncated calculations

Dividends $V(D) \equiv D_1/(1+r)$, provided $D_1 > 0$, and undefined otherwise; and

Earnings $V(I) \equiv S_0 + \rho S_0/(1+r)$, where ρ is inferred from an income statement.

These two values correspond to the case where everything beyond the finite horizon is ignored.

The results from these calculations are as follow:

Proposition 3.4. *1. The value from the dividend calculation is the lower bound on B's valuation interval if the dividend policy is unspecified or targeted, or if there is a proportionate dividend policy and $\lambda = 1$.*

   2. The value from the dividend calculation is strictly below the lower bound on B's valuation interval if the dividend policy is proportionate and $\lambda < 1$.

   3. The value from the earnings calculation is an interior point of B's valuation interval if and only if $\underline{\gamma} < 0$ and either $\overline{\gamma} > 0$ or $\overline{\rho}$ is sufficiently large.

In other words, the valuation from the truncated residual income model is a justified valuation provided the difference between book value and market value can be positive or negative. By contrast, the valuation from the truncated dividend model is not justified whenever the dividend-based valuation is a left-open interval (i.e., in every case except when A has full information).

4   Economic Consequences

In all but the full information case, the valuations to both the firm's current owner A and the potential buyer B are open intervals. It is unclear why trade would occur in the full information case: if the current owner has a unique reservation price, then the potential buyer can never be

made better off by trade and has the possibility of being made worse off. Accordingly, this section focuses in trade when the firm has imperfect information.

Assume both A and B have preferences that are strictly increasing in their final outcomes—i.e., that a higher payment at date 1 is, other things being equal, more desirable for each agent than a lower payment. The problem is that the equity's valuation is restricted to an interval but otherwise indeterminate. Accordingly, I make the following assumption:

Assumption 4.1. *Let (x, y) and (x', y') be two ordered pairs in $\mathbb{Q} \times \mathbb{Q}$ or in $\mathbb{R} \times \mathbb{R}$. Assume $x < y$ and $x' < y'$. If $y \leq x'$, then both agents prefer (x', y') to (x, y), written*

$$(x, y) \prec (x', y').$$

For fixed $z \in \mathbb{R}$ and $(x, y) \in \mathbb{Q} \times \mathbb{Q}$ (or in $\mathbb{R} \times \mathbb{R}$), assume $(x, y) \prec \{z\} \equiv y \leq z$, and $\{z\} \prec (x, y) \equiv z \leq x$.

Remark: There is no loss of generality in limiting the intervals to those with rational endpoints, as they form the base of a topology on the real line; see Negri and Soravia (1999) for a discussion of convenient mathematical properties. One can also map preferences on rational intervals to interval-valued utilities, as in Stecher (2005b).

The relation $\prec$ is irreflexive and transitive. However, it is not negatively transitive:

Lemma 4.1. *Let $(x, y), (x', y'), (x'', y'')$ be ordered pairs in $\mathbb{Q} \times \mathbb{Q}$ or $\mathbb{R} \times \mathbb{R}$. Then*

1. *$\neg((x, y) \prec (x, y))$.*

2. *$(x, y) \prec (x', y')$ and $(x', y') \prec (x'', y'') \rightarrow (x, y) \prec (x'', y'')$.*

3. *$\neg((x, y) \prec (x', y'))$ and $\neg((x', y') \prec (x'', y''))$ does not imply $\neg((x, y) \prec (x'', y''))$.*

In other words, an increasing preference ordering on exact outcomes extends to a strict partial ordering on approximate information. The failure of negative transitivity makes the following immediate:

Corollary 4.1. *Let $(x, y) \approx (x', y') \equiv (\neg((x, y) \prec (x', y'))$ and $\neg((x', y') \prec (x, y)))$. Then $\approx$ is not an equivalence relation; in particular, it is intransitive.*

Because of Corollary 4.1, it seems sensible not to view $\approx$ as indifference, and to treat preferences as incomplete when facing approximate information.

The following assumption is made in order to discuss equilibrium under incomplete preferences:

Assumption 4.2. *Let $i \in \{A, B\}$. Let p be a given price. Then:*

1. *If $P_0^i \prec \{p\}$, then i is willing to sell and unwilling to buy the equity.*

2. *If $\{p\} \prec P_0^i$, then i is willing to buy and unwilling to sell the equity.*

3. *If $\{p\} \approx P_0^i$, then i may or may not be willing to trade the equity.*

The assumption means that equilibrium prices behave as follows:

Lemma 4.2. *Suppose P_0^A is an open interval. Then equilibrium prices are given by*

$$(\inf P_0^A, \sup P_0^B).$$

If P_0^A is a singleton or a left-closed interval, then the above is replaced with its closure on the left. If both P_0^A and P_0^B are singletons, then the equilibrium price is their intersection.

The lemmata justify the following result:

Theorem 4.1. *Suppose A does not learn income or the premium over book value perfectly, but refines at least one of these. Then the equilibrium prices are the same, whether A reports an income statement, a statement of changes in financial position, or a dividend announcement of any of the types considered.*

When A learns either quantity perfectly, Theorem 4.1 no longer holds. For example, the income statement and the statement of changes in financial position may yield different equilibrium prices.

Theorem 4.2. *Suppose A learns income perfectly and the premium over book value imperfectly.
Then the equilibrium prices that result from A reporting an income statement are a proper subset
of those resulting from A reporting a statement of changes in financial position.*

*Conversely, suppose A learns the premium over book value perfectly and income imperfectly. Then
the equilibrium prices that result from A reporting an income statement either coincide with or
are a superset of those resulting from A reporting a statement of changes in financial position. In
particular, if the initial bounds on income are sufficiently small relative to those on the premium
over book value, then the statement of changes in financial position induces a smaller set of possible
equilibrium prices.*

The next result gives the relationship between dividends and equilibrium prices.

Theorem 4.3.   *1. An unspecified dividend policy never restricts the set of equilibrium prices.*

  *2. A targeted dividend policy or a proportionate dividend policy can contain restrict the set of
      equilibrium prices beyond what the income statement or statement of changes in financial
      position can do.*

The above results explain why dividends may seem uninformative. A dividend initiation or a
payment in keeping with a dividend policy raises B's lower bound on valuation, but this is always
weakly lower than A's lower bound. Since the market prices are bounded below by A's lower bound
on valuation, the information in a dividend payment or initiation is likely to be unobservable. The
dividend prevents B from rejecting some trades, but has no effect on possible prices.

Similarly, Theorem 4.3 explains why there would appear to be a strong market reaction to dividend
omissions. When a dividend is omitted, agent B refines the upper bound on possible valuations, and
it is B's upper bound that restricts the supremum of possible market prices. Thus the apparently
stronger market reaction to dividend omissions is a consequence of the asymmetric ability to observe
trades versus non-trades.

5   Concluding Remarks

It is widely accepted that the dividends and earnings approaches to valuation are theoretically equivalent. This viewpoint rests on the notion that each model (under the assumption of clean surplus) can be derived from the other.

The derivability of the dividends and residual income models from each other depends, however, on more than clean surplus. Discounted infinite sums have expectations only if stationarity and ergodicity conditions hold. Given the nature of remote future incomes or dividends, there seems little a priori reason to expect these properties.

With these observations as motivation, this paper considers the residual income and discounted dividend models in a setting where only a lower and upper estimate of a future variable is available, in particular when these estimates do not necessarily converge. It turns out that, in this setting, there are cases where earnings are more informative than dividends, and there are cases where the converse holds. Morover, the pattern of which valuation formula is more informative matches the empirical findings in Penman and Sougiannis (1998).

The approximate nature of information in this setting gives both the firm's current owner (modeled as an insider) and a potential buyer an open interval of justifiable valuations, rather than a single point. Equilibrium market prices are also intervals, which may differ from the firm's or market's valuations. Thus, the information in reports and its effects on the firm or the market are not always apparent from observed prices.

The argument that dividends contain little or no information beyond what is in market prices seems to fail here, yet a reason emerges for why one would draw this inference. When dividends are positive, they do not restrict the set of equilibrium prices generated by the income statement and by the statement of changes in financial position (i.e., the statement of funds available for distributions). However, if a dividend is omitted when a dividend policy is in place, the dividend announcement does restrict equilibrium prices, by refining the upper bound on a potential buyer's valuation. Thus, the results here explain the market's apparent stronger reaction to dividend omissions than to initiations or payments, without relying on any form of bounded rationality.

Future work might explore the role of voluntary disclosure in providing upper bounds on valuation, and thus may add to the theoretical understanding of voluntary disclosure (Dye 1986, Gigler 1994, Verrecchia 1983). Recent empirical work has found evidence that firms whose earnings are relatively less correlated with abnormal returns are more likely than other firms to provide supplemental balance sheet information (Chen, DeFond, and Park 2002) or pro forma earnings statements (Lougee and Marquardt 2004). The former might be viewed as the firm providing the market with information on the premium over book value (more specifically, recording information that had previously not been recorded), while the latter might be viewed as the firm releasing an upper bound as well as a lower bound on income.

A Proofs

Proof of Theorem 3.1:

Proof. The differences in the information content of the dividends announcements, the income statement, and the statement of changes in financial position are immediate by comparing $P_0^B(\cdot)$ under the various reporting schemes.

The statement of changes in financial position is fully revealing, so none of the other statements can improve on this statement.

When the dividend policy is unspecified and $D_1 = 0$, there is no information in the announcement.

When the dividend is positive, either for the unspecified or the targeted dividend policy, the announcement provides a lower bound on P_0^B at the present value of the dividend. Because the dividend can be based on either ρS_0 or γ or their combination, these lower bounds can be higher than those under the income statement (which does not refine γ).

When there is a dividend policy in place, the announcement that $D_1 = 0$ informs B that $(1 + r + \rho)S_0 + \gamma$ is insufficient to pay a dividend under the policy. Thus, $D_1 = 0$ provides an upper bound on P_0^B that depends on both ρ and γ (and, in the case of the targeted dividend policy, on $\widehat{D}$), whereas the upper bound generated by the income statement contains no information on γ. □

Proof of Proposition 3.4:

Proof.   1. If the dividend policy is unspecified, then the lower bound on $P_0^B(D_1|D_1 > 0) = D_1/(1 + r)$, irrespective of what B knows about A's information structure. Similarly, for the targeted dividend policy, the lower bound on $P_0^B(D_1 > 0|\text{targeted policy})$ is always $\widehat{D}/(1+r)$. The proportionate dividend policy has a lower bound on $P_0^B(D_1 = 0|\text{proportionate policy}) = D_1/(\lambda(1 + r)) = D_1/(1 + r)$ when $\lambda = 1$. In each of these cases, the lower bound is $V(D)$.

   2. If the dividend policy is proportionate and $\lambda < 1$, then the lower bound on $P_0^B(D_1 > 0|\text{proportionate policy}) = D_1/(\lambda(1 + r)) > D_1/(1 + r) = V(D)$.

3. Since ρ comes from the income statement, it is the lower bound on what A can report for income, while no information is provided on γ. Consequently, the lower bound on $P_0^B(I)$ is $S_0 + \rho S_0/(1+r) + \underline{\gamma}/(1+r)$, which is strictly less than $V(I)$ iff $\underline{\gamma}/(1+r) < 0 \Rightarrow \underline{\gamma} < 0$.

If B can determine that the value of ρ from the income statement is exact, then the upper bound on $P_0^B(I)$ is $S_0 + \rho S_0/(1+r) + \overline{\gamma}/(1+r)$, which exceeds $V(I)$ iff $\overline{\gamma} > 0$. If B cannot bound ρ above, then the upper bound on $P_0^B(I)$ is $S_0 + \overline{\rho}S_0/(1+r) + \overline{\gamma}/(1+r)$, which exceeds $V(I)$ by $((\overline{\rho} - \rho)S_0 + \overline{\gamma})/(1+r)$. This is positive if either $\overline{\gamma} > 0$ or, given S_0, if $\overline{\rho}$ is sufficiently larger than ρ.

$\square$

Proof of Lemma 4.1:

Proof.   1. If $(x, y) \prec (x', y')$, then $x < y$; i.e., the interval must be nondegenerate, by the definition of $\prec$. On the other hand, $(x, y) \prec (x, y)$ requires $y \leq x$. So $\prec$ is irreflexive.

2. If $(x, y) \prec (x', y')$, then $y \leq x' < y'$. If $(x', y') \prec (x'', y'')$, then $y' \leq x''$, and therefore $y < x''$. So $\prec$ is transitive.

3. Observe $\neg((0, 2) \prec (1, 3))$ and $\neg((1, 3) \prec (2, 4))$, but $(0, 2) \prec (2, 4)$.

$\square$

Proof of Lemma 4.2:

Proof. A will not sell the equity for any price below $\inf P_0^A$, while B will not pay more than $\sup P_0^B$. For any price in this interval, at least one party may or may not be willing to trade, so there is not necessarily incentive for A to raise the asking price or B to lower the bid at any given interior point. $\square$

Proof of Theorem 4.1:

Proof. Consider the case where A learns $\rho \in (\rho', \rho'')$ and $\gamma \in (\gamma', \gamma'')$; other cases are similar. Here

$$P_0^A((\rho', \rho''); (\gamma', \gamma'')) = (S_0 + \frac{\rho' S_0 + \gamma'}{1 + r}, S_0 + \frac{\rho'' S_0 + \gamma''}{1 + r}),$$

so the lower bound on the equilibrium prices is $S_0 + (\rho' S_0 + \gamma')/(1 + r)$.

1. $P_0^B(I) = (S_0 + (\rho' S_0 + \underline{\gamma})/(1 + r), S_0 + (\bar{\rho} S_0 + \bar{\gamma})/(1 + r))$, because the income statement does not tell B what the upper bounds on ρ or γ are. Thus, the upper bound on the equilibrium prices is $S_0 + (\bar{\rho} S_0 + \bar{\gamma})/(1 + r)$, giving prices of

$$(S_0 + \frac{\rho' S_0 + \gamma'}{1 + r}, S_0 + \frac{\bar{\rho} S_0 + \bar{\gamma}}{1 + r}).$$

2. $P_0^B(C) = (S_0 + (\rho' S_0 + \gamma')/(1 + r), S_0 + (\bar{\rho} S_0 + \bar{\gamma})/(1 + r))$, by a similar argument. Thus, the upper bound on $P_0^B(C)$ is the same as under $P_0^B(I)$.

3. Suppose $D_1 > 0$, under any of the dividend policies. Then the dividend refines the lower bound on P_0^B but does not refine the upper bound.

4. Suppose $D_1 = 0$, and the dividend policy is unspecified or proportionate. Then the announcement does not refine the upper bound on P_0^B.

5. Suppose $D_1 = 0$, and there is a targeted dividend policy. Then, as discussed above, the dividend restricts the values for ρ' and γ', but does not provide any information on ρ'' or on γ''. Consequently, the upper bound on P_0^B is not refined.

$\square$

Proof of Theorem 4.2:

Proof. Recall $P_0^A(\rho, (\gamma', \gamma'')) = (S_0 + (\rho S_0 + \gamma')/(1 + r), S_0 + (\rho S_0 + \gamma'')/(1 + r))$. So the left endpoint on the equilibrium price interval is $S_0 + (\rho S_0 + \gamma')/(1 + r)$. From the income statement, B infers ρ but does not learn about γ, so the upper bound on γ remains $\bar{\gamma}$. Therefore, the equilibrium price intervals when A reports an income statement is

$$(S_0 + \frac{\rho S_0 + \gamma'}{1 + r}, S_0 + \frac{\rho S_0 + \bar{\gamma}}{1 + r}).$$

If A instead reports a statement of changes in financial position, then the upper bound of $P_0^B(C)$ was seen above to be $S_0 + (\rho S_0 + \gamma' + (\overline{\gamma} - \underline{\gamma}))/(1 + r)$. Since $\gamma' - \underline{\gamma} > 0$, it follows that

$$\overline{\gamma} + \gamma' - \underline{\gamma} > \overline{\gamma} \;\Rightarrow\; \sup P_0^B(C) > \sup P_0^B(I).$$

Hence, the statement of changes in financial position induces a higher upper bound on the prices B might be willing to accept as a buyer.

When A learns γ exactly and ρ imperfectly, the income statement does not refine the upper bound on P_0^B. Thus, the equilibrium prices in this case are

$$(S_0 + \frac{\rho' S_0 + \gamma}{1 + r}, S_0 + \frac{\overline{\rho} S_0 + \overline{\gamma}}{1 + r}).$$

The upper bound on $P_0^B(C)$ was seen above to be $S_0 + ((\overline{\rho} - \rho)S_0 + \rho' S_0 + \gamma)/(1 + r)$. Thus, the statement of changes in financial position yields a lower upper bound on equilibrium prices iff

$$\overline{\rho} S_0 + \overline{\gamma} > (\overline{\rho} - \rho' - \underline{\rho})S_0 + \gamma \;\Rightarrow\; \overline{\gamma} - \gamma > (\rho' - \underline{\rho})S_0.$$

$\square$

Proof of Theorem 4.3:

Proof.   1. When $D_1 > 0$, the dividend bounds P_0^B below. Since the equilibrium prices are bounded below by $\inf P_0^A$, the information in the dividend has no effect on the observed equilibrium prices. On the other hand, if $D_1 = 0$ and there is no specified policy, the announcement does not restrict P_0^B at all.

2. Suppose A learns ρ precisely and γ imperfectly or not at all. Then

$$\sup P_0^B(D_1 = 0|\text{proportionate policy}) = (\widehat{D} + \overline{\gamma} - \underline{\gamma})/(1 + r),$$

which for sufficiently low $\widehat{D}$ will be below the upper bounds on $P_0^B(I)$ or $P_0^B(C)$. Analogous results hold when A learns γ precisely and ρ imperfectly or not at all.

3. Analogous to the previous item, with $\widehat{D}$ replaced by 0.

$\square$

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