

Segmentation and Pricing Behavior in a Market for Certification*

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Abstract

The paper offers a simple theory of pricing behavior in certification markets. The basis for the theory is that certifiers offer differentiated tests; for an object of given quality it may be more difficult to pass the test of certifier i than the test of certifier j . Given the test standards, certifiers compete for customers via their simultaneous pricing decisions. In equilibrium, each certifier attracts a connected segment of the market, and sellers of high quality products pay a higher price for certification than sellers of low quality products. Lemons may be certified in equilibrium, although the responsible certifier could have screened off the lemons by charging a higher price. The theory is applied to the US market for MBA education and finds support.

Keywords: Adverse Selection, Auditing, Certification, Investment Banking, Oligopoly theory, MBA, Signaling.

1 Introduction

To offset negative effects from asymmetries of information, different institutions can emerge. One example is warranties; if contracts can be written such that sellers of low-

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quality products are punished, such 'lemons' will be pulled from the market, and a more efficient level of trade can be realized. A different institution that can facilitate trade, particularly when contracts are difficult to write or to enforce, due to e.g., limited liability, is certification. Certifiers are third parties in the trading process with some ability to assess quality before trade takes place. Equipped with such assessment abilities, and a reputation for truth-telling, certifiers can make a business by charging a fee for testing objects and making their assessment known to potential buyers.

A common characteristic of certification markets is that different certifiers serve different segments of the market. For example, it is well known from the auditing industry that the 'Big 4' auditors attract mainly high quality firms; the equity offerings of highest quality are underwritten by the 'Big 8' investment banks; journals of higher rank generally publish papers of higher quality than lower rank journals, and top-ranked universities admit entry and award degrees to students of higher average ability than universities of lower rank.

The segmentation of sellers in certification markets implies that the value of a certificate can be highly dependent on which certifier issued it: A firm's value will be higher if KPMG finds its accounting practices in line with GAAP (Generally Accepted Accounting Principles) than if a regional auditing firm had formed the same conclusion, equity offerings by top investment banks are less underpriced than offerings by less prestigious investment banks, for an economist the value of a publication in *Econometrica* is higher than the value of a publication in lower-ranked journals, a student's job market prospects are better if he receives an MBA degree from Harvard than from most other universities, and, for marine vessels, a certificate from Lloyds or Veritas is a stronger indication of high quality (e.g., low risk of making environmental damage) than a certificate from one of the smaller agencies.¹

The paper proposes a simple theory of certification that has segmentation as a feature of equilibrium, and which moreover delivers testable hypotheses about the fees set by

¹There is ample evidence that the value of certificates strongly depend upon the identity of the certifier, controlling for the observable characteristics of the sellers. For auditing, Krishnan (2002) surveys the literature, for security and equity issues see e.g., Puri (1996), for the market for MBA education see Tracy & Waldfogel (1997).

different certifiers in equilibrium.² We can think of several possible explanations for segmentation. One would be that the certifier (or certifiers) with most reputation capital would give the most trustworthy reports, and therefore be the most attractive certifier for the high quality sellers. However, this approach seems inadequate to explain why the certifier with most reputation capital does not capture the whole market. In a similar vein, segmentation could occur because the high-quality sellers prefer to attend the certifier (or certifiers) with the most precise testing technology. However, in that case, it would be unclear why the medium-quality sellers would not also prefer the most precise test, and so on until only the lowest-quality sellers (if any) would attend a certifier with an imprecise test. In contrast, the present theory of segmentation is based on the assertion that different certifiers have tests that cannot be ranked in terms of precision, but can be ranked in terms of passing difficulty. For example, it demands more from a firm's accounting practices to have one of the Big 4 auditing firms to certify one's accounting than a local auditing firm, it is more difficult to gain admittance to a Harvard MBA than to a lower-ranked program, and it is more difficult to have a paper accepted in *Econometrica* than in most other journals.

The paper explores this intuition in a simple static model of an oligopolistic certification market. In the model, there are sellers and certifiers, and a competitive product market where the sellers' objects are traded. There is a continuum of possible product qualities, and initially only sellers know the quality of their product. Each certifier is endowed with a test that enables it to (imperfectly) distinguish objects with quality lower than a cutoff value from objects with quality higher than the cutoff value. Certifiers offer differentiated tests, in that it may be more difficult to pass the test of certifier i than the test of certifier j . Given the test structure, certifiers choose their testing fees simultaneously. Sellers then decide which certifier to attend (if any). Tests, which are binary,

²We are not aware of equilibrium theories of certification that are able to explain the segmentation in certification markets. For example, Biglaiser & Friedman (2001) builds on Biglaiser (1993) to consider a model with discrete product quality where symmetric certifiers compete for customers through their pricing rule. There is no sense of segmentation in their setting. Puri (1999) obtains segmentation of low and high quality security issues to different underwriters (commercial and investment banks which act as certifiers), but makes very restrictive assumptions about competition between underwriters; there is only one bank of each type and moreover pricing decisions are exogenous. The same type of criticism can be raised against the signaling model of Titman & Trueman (1986), which attempts to endogenize the choice of auditor quality, where auditors have a pure certification role.

are then performed, and the test results reported privately to sellers. Each seller then decides whether to make the report public or not. Finally the market bids for an object conditional on which certifier it attended (if any) and the report available about that object. Notice that since the tests are imperfect, the possibility of lemons being certified and traded cannot be excluded ex-ante. As we shall see, lemons can indeed be certified in equilibrium even if the responsible certifier(s) could screen off lemons by charging a higher price.

Denoting the number of active certifiers by n , equilibria are characterized by a price charged by each certifier and a sorting of sellers into $n + 1$ groups, one group for each certifier and one group that does not attend any certifier. Under a simple sufficient condition on the test structures, each of these groups are connected in equilibrium, i.e., segmentation of sellers obtains as an equilibrium outcome. For example, in the case of two active certifiers, sellers with the lowest quality skip certification, sellers with an intermediate quality attend one certifier, and the high-quality sellers attend the other certifier.

Turning to the prices charged for the certification test in equilibrium, it is shown that they will be monotonic: a certifier attracting sellers of higher quality will charge a higher price. While argued that the segmentation of sellers derived as a feature of equilibrium is consistent with one of the stylized facts from certification markets, I wish to assess the empirical validity of the price monotonicity result. To address that issue, a simple test is performed with data from the market for MBA education. The test, which takes as an article of faith that certification is indeed the most important function of MBA education and not e.g., human capital acquisition,³ gives support to the pricing hypothesis.

The structure of the paper is as follows. Section 2 sets up the model. Section 3 considers the monopoly certifier case, and Section 4 considers market structure under oligopoly. Section 5 considers the pricing (tuition fee) decisions for a sample of MBA programs in the US, and Section 6 concludes. Some proofs are relegated to the Appendix.

³Difficulties in entangling information from human capital effects is a well-known problem in the empirical education literature, see e.g., the overview by Weiss (1995).

2 The Model

There are sellers, certifiers, and a market for objects, all agents being risk-neutral. Each seller is equipped with an object with value q to the market and value 0 to himself. The value q is known to the seller, while the market merely knows that the distribution of objects follows the frequency function $h(q)$. I assume that $h(q)$ is a constant on the domain $Q = [a, 1]$, and moreover that $-\infty < a \leq -1$. Neither of these two assumptions are necessary for the analysis but captures in a simple manner that there will be no trade without certifiers present in the market.

There are finitely many (n) active certifiers in the certification market. This assumption can be justified both by the limited number of certifiers in real certification markets and by arguing for considerable fixed costs in acquiring expertise in testing products and in acquiring reputation for producing honest reports. The formal analysis is limited to the monopoly case ($n = 1$) and the duopoly ($n = 2$) case, but the duopoly results generalize to an arbitrary $n > 2$.

Certifier i operates a test grid with K levels, $\{I_i^1, I_i^2, \dots, I_i^K\}$, where $I_i^1 < I_i^2 < \dots < I_i^K \in \mathfrak{R}$. The test identifies which interval \hat{q}_i lies on, where \hat{q}_i is a noisy measure of q . Specifically, it is assumed that $\hat{q}_i = q + \varepsilon_i$, where ε_i is white noise with density function $f_i(x)$. There are followingly $K + 1$ possible test results, where the object obtains the test result m if $I_i^m < \hat{q}_i < I_i^{m+1}$, where $m \in \{1, 2, \dots, K + 1\}$. The analysis focuses on the special case of binary tests ($K = 1$). Hence there are only two possible test results, where an object obtains the test result 0 from certifier i if $\hat{q}_i < I_i$ and obtains the test result 1 if $\hat{q}_i > I_i$. For convenience, I label the 0-result by 'fail' and the 1-result by 'pass'.⁴ Although obviously a simplification, binary tests seem to be a good approximation to what goes on in several certifying markets, such as the market for auditing reports, MBA degrees, driving licenses, marine vessel certification, and industrial products certification. (GAAP standard or not, admit or not, fail or pass, ISO standard or not).⁵ The test

⁴This shorthand is slightly misleading as objects that fail may well be traded later on. Hence certifiers in the present model are not *gatekeepers*, as discussed by Choi (1996).

⁵Although credit rating agencies have an arsenal of possible grades, empirical studies show that the real difference in grade is between investment bond grade (BB and higher) and junk bond (B and lower), so one can argue that the model fits this situation pretty well, too. An interesting question is why reports are typically so coarse. One reason may be that a too rich 'language' gives too strong incentives for

standards $\{I_i\}_{i=1,\dots,n}$ are taken as exogenously given. It should be mentioned, however, that the results do not depend on any specific assumptions made on the location of the test standards $\{I_i\}_{i=1,\dots,n}$.

The timing is as follows. First, the certifiers compete in prices, taking the test standards as given. Certifier i 's price variable is labeled by $P_i \in \mathfrak{R}_+$. Sellers then decide simultaneously which certifier to attend (if any) after observing $\{P_i, I_i\}_{i=1,\dots,n}$. If a seller decides to attend certifier i , he pays the testing fee P_i to that certifier, and the test is performed. The test result is then reported privately to sellers, and sellers individually choose whether to hide the report or to make it public. Reports cannot be frauded, neither by certifiers nor by sellers.⁶ The product market is assumed to be competitive, meaning that a seller here gets a price for his product equal to its expected quality, conditional on which certifier attended to (if any), and the test report.

Equilibrium is a situation where each seller chooses certifier (if any) optimally, conditional on the pricing decision of the certifiers and his expectation of the behavior of other sellers. Moreover, each certifier sets a fee that is optimal given the fees charged by the other certifiers and the (expected) behavior of the sellers.

3 Monopoly

Let us begin by considering the case with only one active certifier in the market. I assume (and show later) that equilibrium has a connected structure: sellers on the interval $[a, q_1]$ do not to attend the certifier, and sellers on the interval $[q_1, 1]$ do attend the certifier, where the seller with quality q_1 is indifferent between attending the certifier or not.

Assuming that test results will be made public by sellers, the expected utility for an agent with quality q for attending a certifier with test standard I and price P , taking the cutoff q_1 as given, equals the expected market posterior of quality after the test has been

certifiers to misreport.

⁶Potentially, sellers and certifiers could have incentives to collude in frauding reports, as might occurred in the Anderson-Enron case. The present model does not consider that very interesting issue.

made public, subtracted the cost of certification,

$$U(q; q_1) = \Pr(\text{pass}|q, I)U(\text{pass}) + \Pr(\text{fail}|q, I)U(\text{fail}) - P \quad (1)$$

where,

$$\begin{aligned} \Pr(\text{pass}|q, I) &= \int_{I-q} f(x)dx \\ \Pr(\text{fail}|q, I) &= 1 - \Pr(\text{pass}|q, I) \end{aligned} \quad (2)$$

$U(\text{fail})$ and $U(\text{pass})$ denote the expected quality (and hence price obtained) for an object that passes and fails the test, respectively, given that all reports are made public in the market. Since there is a continuum of sellers, each seller takes $U(\text{fail})$ and $U(\text{pass})$ as constants (these constants are derived in the Appendix). The probability of passing the test increases in q , and therefore $U(\text{pass}) > U(\text{fail})$. It follows that $U'(q; \cdot) > 0$, $\forall q \in [q_1, 1]$ since $\Pr(\text{pass}|q, I)$ increases in q from (2).

It will be useful to define the (expected) gross utility for attending the certifier, $UU(q, \cdot)$ as,

Definition 3.1 $UU(q; q_1) = \Pr(\text{pass}|q, I)U(\text{pass}) + \Pr(\text{fail}|q, I)U(\text{fail})$

The $UU(q; \cdot)$ function gives the expected market posterior after the test result is revealed for an object with true value equal to q . Let us collect two useful properties of the $UU(q; \cdot)$ function.

Remark 1 *i)* $q_1 < UU(q; q_1) < 1$, $q \in [q_1, 1]$, and *ii)* $\frac{1}{2} < \frac{\partial UU(q_1; q_1)}{\partial q_1} < 1$.

Proof. *i)* follows directly from the test being imperfect and Bayesian updating by the market. To see that *ii)* holds, observe that a perfectly non-informative test has $\frac{\partial UU(q_1; q_1)}{\partial q_1} = \frac{\partial}{\partial q_1}(\frac{q_1 + 1}{2}) = \frac{1}{2}$, and a perfectly informative test has $\frac{\partial UU(q_1; q_1)}{\partial q} = 1$, with imperfectly informative tests lying in between. ■

The cutoff q_1 is determined by the marginal seller, i.e., the seller that is indifferent between attending the certifier and not. Since the utility for sellers that do not attend a

certifier must be zero (no trade),⁷ q_1 can be defined implicitly through the equation,

$$\Psi(P, q_1) = UU(q_1; q_1) - P = 0 \quad (3)$$

Since $\Psi(P, q_1)$ decreases strictly in P and increases strictly in q_1 from Remark 1, equation (3) defines q_1 implicitly as an increasing function of P . By the implicit function theorem, $\frac{dq_1}{dP}$ can be determined as,

$$\frac{dq_1}{dP} = -\frac{\Psi_P}{\Psi_{q_1}} = \frac{1}{\Psi_{q_1}} \quad (4)$$

where subscripts denote partial derivatives. This expression is greater than zero by Remark 1, since $\frac{1}{\Psi_{q_1}} = [\frac{\partial UU(q_1; q_1)}{\partial q_1}]^{-1} > 0$.

Let us now consider the monopolists profit maximization problem. Assuming zero fixed and variable costs of certification for convenience, and normalizing by setting $f(q) = 1$, the monopoly profits are,

$$\Pi = P(1 - q_1) \quad (5)$$

The optimal P solves the first order condition,

$$\frac{d\Pi}{dP} = 1 - q - \frac{dq_1}{dP}P = 0 \quad (6)$$

Since $\frac{dq_1}{dP} > 0$ from equation (4), equation (6) shows that the basic trade-off facing a certifier in the pricing decision is that a higher price brings a positive direct effect on profits but a negative indirect effect through a higher q_1 .

Denoting the equilibrium value of q_1 by q_1^* , the following can be noted.

Remark 2 *Monopoly equilibrium.* i) *The monopolist sets a price for certification such that $q_1^* > 0$. Hence no lemons will be certified in equilibrium, but some non-lemons, i.e., with $q \in [0, q_1^*]$, will not be certified.* ii) *The sellers will make the report public independent of the test result.*

⁷This holds because we assumed that $a \leq -1$. With $a > -1$ then the utility for not being certified can be greater than zero, since these objects may also be traded. Say for illustration that $a = 0$. Then the utility of not being certified equals $\frac{q_1}{2}$ (average quality of sellers that do not attend the certifier). Apart from that, the equilibrium will have the same qualitative features.

Proof. i) Clearly q_1^* must be on the interior of Q and hence it is sufficient to show that $\frac{d\Pi}{dP} > 0$ for $q_1 \leq 0$. For q_1 to equal zero, we can see from (3) that $P = UU(0, 0)$. Inserting into the expression for $\frac{d\Pi}{dP}$ in (4), we get,

$$\frac{d\Pi}{dP}_{q_1=0} = 1 - \frac{UU(0, 0)}{\Psi_{q_1}(0)} \quad (7)$$

By Remark 1, $\frac{1}{2} < \frac{\partial UU(q_1; \cdot)}{\partial q_1} < 1$ for any q_1 . Hence $\frac{1}{\Psi_{q_1}(0)} = \left[\frac{\partial UU(q_1; \cdot)}{\partial q_1}\right]_{q_1=0}^{-1} > 1$.

But, since $UU(0, 0) < 1$ by Remark 1, we must have that $\frac{UU(0, 0)}{\Psi_{q_1}(0)} < 1$. It follows that

$\frac{d\Pi}{dP}_{q_1=0} > 0$. By the same argument, it follows that $\frac{d\Pi}{dP}_{q_1 < 0} > 0$. Hence $\frac{d\Pi}{dP} > 0$ for $q_1 \leq 0$, and $q_1^* > 0$ follows. ii) After the sellers have received their reports, they are divided into three categories. Those with a pass report, those with a fail report, and those without a report. Clearly those with a pass report will have an incentive to make it public, since they have the highest quality. But conditional on the pass reports being made public, the sellers with fail reports will also make their reports public, since they have a higher quality than those that do not have a report. ■

The intuition for part i) can be understood in terms of standard monopoly theory. While the socially optimal cutoff is $q_1 = 0$, in which case no lemons ($q < 0$) will be traded and all non-lemons ($q > 0$) will be traded, monopoly maximizes profits by choosing a price that results in too low trade volume ($q_1^* > 0$), because it maximizes own revenue rather than social surplus.⁸ The second part follows from a standard unraveling argument.

To illustrate the solution of the monopolist pricing problem, let us consider an example.⁹

Example 1 *Let ε be normally distributed with mean zero and variance .1. $h(q) = 1$ for $q \in Q = [-1, 1]$. Then for $I = 0$, the equilibrium is $(P^*, q_1^*) = (.40, .06)$ with associated profits equal to .38. The average pass rate equals .76 .*

Let me make two comments on the equilibrium structure. First, the reason for why

⁸Having positive marginal costs of certification would not alter this insight, and would result in an even higher q_1^* . However, for $h(q)$ functions with a high $h(0)$, the result may not hold.

⁹The calculations for all the examples are performed in Maple V, and are available from the author.

the upper cutoff must equal 1 is that if the sellers on the top had incentives to deviate, then this must also be true for the sellers immediately below and unraveling would follow. So in any equilibrium with certification it must be true that the sellers attending the certifier is a connected set $[q_1^*, 1]$.¹⁰ To gain intuition for why q_1^* is unique, notice that for a given choice of P , the derived q_1 is the only one consistent with equilibrium, by equation (3). For example, for $P = .40$ in Example 1 then $q_1 = 0$ would imply that Ψ is negative and the persons with a low but positive q would get a negative utility from attending the certifier, and hence would have incentives to deviate. In this sense there must be *unraveling from the bottom* in equilibrium. For a candidate q_1^* higher than the solved for, Ψ is positive, and there will be sellers that can obtain a higher utility by attending the certifier rather than not attending the certifier.¹¹

4 Duopoly

Let us now consider equilibria in a market with n active certifiers, where n is taken to equal 2 for expositional clarity. Noting that equal test standards $I_1 = I_2$ would imply Bertrand competition between the certifiers and $P_1 = P_2 = 0$, we consider the case with unequal test standards $I_1 \neq I_2$, and apply the convention $I_1 < I_2$.

Assuming again that test results are made public, the expected utility from attending certifier i for an agent with quality q , denoted by $U_i(q; \cdot)$, equals,

$$U_i(q; \cdot) = \Pr(\text{pass}_i|q, I_i)U_i(\text{pass}) + \Pr(\text{fail}_i|q, I_i)U_i(\text{fail}) - P_i \quad (8)$$

where $U_i(\text{pass})$ is the average quality of the objects that pass test i , $U_i(\text{fail})$ is the average

¹⁰Imposing the intuitive criterion will eliminate the equilibrium where no agents attend the certifier because it fears that the market will ignore the information lying in the test result.

¹¹Setting $\sigma^2 = \frac{1}{30}$ instead of $\sigma = \frac{1}{10}$ in Example 1 gives $(P^*, q_1) = (.45, .16)$ with associated profits equal to .40. This result mirrors the finding from Lizzeri (1999), Theorem 1, where the monopolist certifier chooses an uninformative test in optimum. The intuition for the result in our setting is that when σ gets higher, setting a higher price will result only in a small change in q_1 , since $U'(q; \cdot)$ is close to zero, and hence the monopolist will charge a price close to 1/2 as σ tends to infinity, and take all the surplus in the market. Even if having a very imprecise test can be profitable in the monopoly case, in the oligopoly case such a test would make it too easy for the other certifiers (who has more informative tests) to steal sellers, and would not be optimal.

quality of the objects that fail test i , and

$$\Pr(\text{pass}_i|q, I_i) = \int_{I_i-q} f_i(x)dx = 1 - \Pr(\text{fail}_i|q, I_i)$$

In an equilibrium where both certifiers attract a positive measure of sellers, there must exist at least one value of q such that $U_1(q; \cdot) = U_2(q; \cdot)$. Define $\Phi(q; \cdot) = U_1(q; \cdot) - U_2(q; \cdot)$, and then define the set Q_2 as,

$$Q_2 = \{q : \Phi(q; \cdot) = 0\} \quad (10)$$

The set Q_2 contains the points of indifference between attending certifier 1 and certifier 2. I denote by \tilde{q} an arbitrary element in Q_2 , and consider equilibria with the following structure: for at least one \tilde{q} sellers with a q immediately below \tilde{q} prefer to attend certifier 1, and sellers with a q immediately above \tilde{q} prefer to attend certifier 2. This property is denoted by the 'crossing property'.

Definition 4.1 *The crossing property (CP) holds if there exists $q \in Q_2$ such that i) $\Phi(q - \epsilon) > 0$, and ii) $\Phi(q + \epsilon) < 0$, for ϵ sufficiently close to zero.*

It will be shown that the local condition CP implies connectedness of equilibria, given that a sufficient condition on the test technologies hold. First a definition.

Definition 4.2 *Single crossing property (SCP) holds if for any $q \in Q_2$ and $\epsilon > 0$, then $\Phi(q - \epsilon) > 0$ and $\Phi(q + \epsilon) < 0$.*

If the global condition SCP holds, then there exists only one value of q that makes sellers indifferent between attending the two certifiers, and connectedness follows.

Assumption 1.

The likelihood ratio $f_1(I_1 - q)/f_2(I_2 - q)$ decreases in q , for all $q \in Q$.

A decreasing likelihood ratio (DRLP) implies that the higher q , the higher is the relative probability of passing the difficult test (compared to the easy test).¹² This is a

¹²Similar conditions to the decreasing likelihood ratio function are often assumed to hold in the moral hazard literature (see e.g., Holmstrom 1979), but we are not aware of such conditions being applied in the adverse selection literature.

natural requirement that is satisfied for a range of joint distributions. For example, let ε_1 and ε_2 be *iid* normally distributed with variance σ^2 , to obtain,

$$\begin{aligned} f_1(I_1 - q; \cdot) / f_2(I_2 - q; \cdot) &= \frac{1}{\sigma\sqrt{2\pi}} e^{-(I_1 - q)^2 / 2\sigma^2} / \frac{1}{\sigma\sqrt{2\pi}} e^{-(I_2 - q)^2 / 2\sigma^2} \\ &= e^{-\frac{(I_2 - I_1)(I_2 - 2q + I_1)}{2\sigma^2}} \end{aligned} \quad (11)$$

This expression decreases in q if $I_2 - I_1 > 0$, or in other words if $I_1 < I_2$, and hence DLRP is satisfied.

We now have the following lemma.

Lemma 1 *CP implies connectedness.*

Proof. Suppose that CP holds in the point \tilde{q} , i.e., $\tilde{q} \in Q_2$ and $\frac{\partial \Phi(q)}{\partial q} \Big|_{q=\tilde{q}} < 0$. Recall that $U_i(q; \cdot) = \Pr(\text{pass}_i | q, I_i) U_i(\text{pass}) + [1 - \Pr(\text{pass}_i | q, I_i)] U_i(\text{fail}) - P_i$, and observe that only the $\Pr(\text{pass}_i | q, I_i)$ terms in this expression depend on q . Further observe that $\frac{\partial \Pr(\text{pass}_i | q, I_i)}{\partial q} = f_i(I_i - q)$, and define $\Delta_i = U_i(\text{pass}) - U_i(\text{fail})$. We then have,

$$\begin{aligned} \frac{\partial \Phi(q; \cdot)}{\partial q} &= U_1'(q; \cdot) - U_2'(q; \cdot) \\ &= f_1(I_1 - q) \Delta_1 - f_2(I_2 - q) \Delta_2 \end{aligned} \quad (12)$$

For an arbitrary value of q , this expression is negative if,

$$\theta = \frac{f_1(I_1 - q) \Delta_1}{f_1(I_2 - q) \Delta_2} < 1 \quad (13)$$

Since $\frac{\Delta_1}{\Delta_2}$ is a constant, it is sufficient for SCP to hold that the likelihood ratio $f_1(I_1 - q) / f_2(I_2 - q)$ decreases in q for $q > \tilde{q}$, which is ensured by Assumption 1. Hence CP implies SCP and connectedness. ■

Assumption 1 ensures that if a seller with a given quality prefers test 2 to test 1 then a seller with a higher quality also prefers test 2 to test 1. It follows that equilibria must be connected, and a unique divide is obtained between the sellers that prefer to attend

certifier 1 and to attend certifier 2, respectively. This result is obtained without making any assumptions about pricing behavior.

In an equilibrium where both certifiers are active, there must also exist a seller that is indifferent between attending certifier 1 and not attending a certifier. This cutoff value, denoted by q_1 , can be defined implicitly through the equation,

$$UU_1(q_1, \cdot) - P_1 = 0 \tag{14}$$

As can be seen by the same type of argument as in the monopoly case, q_1 is uniquely determined for given values of (P_1, q_2) . Let us denote by q_1^* the equilibrium value of q_1 . Then the following holds.

Lemma 2 *i) In an equilibrium with two active certifiers, CP implies that $q_1^* < q_2^*$. ii) $q_1^* > a$.*

Proof. i) follows from straightforward manipulations, and is skipped. To prove ii), observe that $q_1^* = a$ would imply that the average quality of those that attend certifier 1 being negative, since $a \leq -1$ and certifier 2 attracts the upper end of the market by Lemma 1. But in that case certifier 1 must charge a negative price, which is clearly not consistent with equilibrium. ■

Note that part ii) means that sellers on $[a, q_1^*]$ do not attend a certifier in equilibrium. We now have the following.

Proposition 1 *Segmentation. In an equilibrium with two active certifiers, CP implies that sellers can be split into three connected segments. In increasing order of quality, the segments are: those that do not attend a certifier, those that attend certifier 1, and those that attend certifier 2.*

Proof. Follows from Lemma 1 and Lemma 2. ■

This is a key result, since it shows that the model produces equilibria where different certifiers capture different connected segments of the market, as the motivation for the paper called for.

After the testing, sellers will be separated into five groups: those that did not attend a certifier, those that attended certifier 1 and failed, those that attended certifier 1 and

passed, those that attended certifier 2 and failed, and finally those that attended certifier 2 and passed. These groups are of strictly increasing quality, and will therefore be traded at strictly increasing prices in the market (the reports will be revealed due to unraveling). An implication is that sellers that attend certifier 2 must (on average) be traded at a higher price than the sellers that attend certifier 1, consistent with one of the stylized facts posited in the Introduction.¹³

Consequently, we can rank certifiers in equilibrium according to the magnitude of the value increase due to attending that certifier: certifier 1 provides a lower value increase for sellers than certifier 2. A natural question is whether this ranking has any implications for the fees set by the certifiers. Will the top ranked certifier always charge a higher price than a lower ranked certifier? It turns out that the answer to this question is in the affirmative.

Proposition 2 *Pricing behavior. CP implies that $P_2^* > P_1^*$ in equilibrium.*

Proof. Recall that the $UU_i(q; \cdot)$ functions give the expected market conception ex-post for an agent with ability q that attends certifier i . Since CP implies connectedness, by Remark 1, part i), it must be the case that $UU_1(q; \cdot) < UU_2(q; \cdot)$. In particular, for agent q_2^* , which is indifferent between which certifier to attend, it must be the case that $UU_1(q_2^*; \cdot) < UU_2(q_2^*; \cdot)$. But the indifference condition says that $UU_1(q_2^*; \cdot) - P_1^* = UU_2(q_2^*; \cdot) - P_2^*$. Combining these two expressions immediately yields that $UU_1(q_2^*; \cdot) - UU_2(q_2^*; \cdot) = P_1^* - P_2^* < 0$, and hence $P_2^* > P_1^*$ follows. ■

The proposition says that prices will be monotonic in the equilibrium rank of certifiers: the certifier who attracts the sellers of highest quality will charge a higher price. The intuition for the result is that a seller knows that if he takes the simple test, the market will believe that his object is of lower quality than if he takes the difficult test (observe that this holds independently of the test outcome). Given this drawback of attending

¹³The increase in market value for a seller from attending a certifier depends on which certifier he attended and on whether he passes the test or not. The relative magnitude of these two effects will depend on the informativeness of the tests: if the tests are relatively uninformative (high variance of the σ_i 's) then the difference in market value for attending different certifiers (and, say, passing) will be much larger than the difference in market value from passing or failing a given test. On the other hand, if the tests are relatively informative, then the difference in market value from passing or failing a given test can be almost as large as the difference in market value for passing different tests.

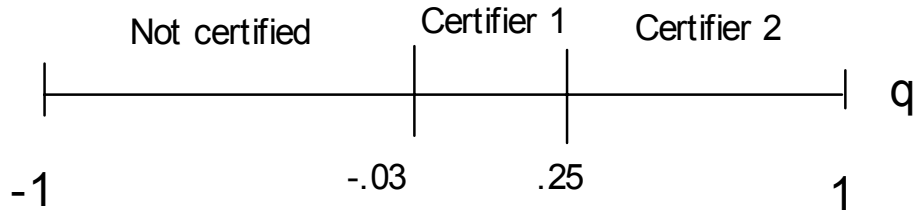
certifier 1, the price for attending certifier 1 must be lower than the price for attending certifier 2 for an indifferent seller to exist.

This result is very useful in that it gives a concrete testable hypothesis from the model. In the next section I discuss how one can test this hypothesis, and perform a simple test with data from the market for MBA education.¹⁴

Let us now consider a numerical example, to illustrate the equilibrium structure.

Example 2 Let ε_i be normally and independently distributed with mean zero and variance .35. $h(q) = 1$ for $q \in Q = [-1, 1]$. Then for $I_1 = 0$ and $I_2 = .35$ we get $(P_1^*, P_2^*, q_1^*, q_2^*) = (.09, .43, -.03, .25)$ with associated profits, $\Pi_1^*(.) \approx .04$, $\Pi_2^*(.) \approx .32$. The average pass rate is .75 for test 1, and .83 for test 2.

The equilibrium can be illustrated with a figure.



Equilibrium separation

The sellers between -1 and -.03 do not attend a certifier, the sellers between -.03 and .25 attend certifier 1, and the sellers between .25 and 1 attend certifier 2. After the test results have been made public, the market will hold the following belief about the

¹⁴Let us here make two comments on the uniqueness properties of the model.

First, the crossing property may seem like an obvious property of equilibrium, but in fact there can exist equilibria with the reverse structure of that considered, namely that the middle group of sellers attends the certifier with the highest I_i and the upper group of sellers attends the certifier with the lowest I_i . By the same type of argument as in Lemma 1, it can be shown that such equilibria will also be connected, and hence that the equivalent of Proposition 1 and Proposition 2 will also hold.

Second, SCP does not exclude the possibility of a multiplicity of equilibria. The reason for possible multiplicity is that the $\Phi(.)$ function has q_1 as a free variable, and hence it is possible that more than one value of q_1 (and hence q_2) is consistent with equilibrium. Therefore our main results, Proposition 1 and Proposition 2, apply to *every* equilibrium in the equilibrium set, and does not hinge on uniqueness of equilibria.

The underlying reason for the potential multiplicity of equilibria is the *social interaction* aspect of the model: which test a seller wishes to attend depends on the behavior of other sellers, because their behavior determines $U_i(\text{pass})$ and $U_i(\text{fail})$. This aspect of the model is in contrast to related models of product differentiation, see footnote 18.

(average) quality of the five groups: $(-.48,.07,.13,.38,.68)$, and hence only objects that did not attend a certifier will not be traded in equilibrium.¹⁵

Since objects between $-.03$ and 0 attend a certifier and are traded in equilibrium, the example shows that lemons may be certified in equilibrium. Let us state that as a remark.

Remark 3 *In a duopoly, lemons may be certified.*

Let us explain the intuition for this result in some detail. The bottom certifier attracts the group $[q_1^*, q_2^*]$. If q_2 were independent of P_1 then $q_1^* > 0$, by the same argument as in the monopoly case. However, since q_2 depends on P_1 , then decreasing P_1 to the point where $q_1^* < 0$ may be profitable for certifier 1, if q_2 increases in P_1 . In a different phrasing, certifier 1 does not internalize the negative externality imposed on certifier 2 from decreasing the price. Although the effect of having q_1^* below 0 in isolation decreases the certifier 1 profits, the positive effect on profits from increasing q_2^* outweighs this effect, and we get an inefficient equilibrium where some lemons are certified. The result seems to be quite general, in that I have been unable to generate examples with $q_1^* > 0$.

To sum up, I have shown that the model produces equilibria where different certifiers attract different, connected, seller segments. Moreover, a certifier attracting a segment with a higher quality will charge a higher price than a certifier attracting a lower quality segment in equilibrium. Let me now discuss some points on robustness of the results obtained.

The introduction of costs of testing for the certifier would have no effect on the basic segmentation and price monotonicity result, as can readily be seen from the proofs of Lemma 1 and Proposition 1; such costs would not affect the proofs. Similarly, from the same type of argument as in Lemma 1, DLRP is sufficient to get connected equilibria also in a setting where there are arbitrary many active certifiers. Hence the price monotonicity result would hold also hold in such a generalized oligopoly setting.¹⁶

¹⁵The example suggests that the profits for the upper certifier is higher than the profits for the bottom certifier. We have been unable to generate counterexamples to this assertion, but have also been unable to prove it.

¹⁶The generalized Assumption 1 would be that $f_i(I_i - q)/f_j(I_j - q)$ decreases in q for all i, j such that $I_i < I_j$.

We have considered a one-shot game where sellers can attend only one certifier. In some certification markets sellers can attend several subsequent certifiers, which could give sellers incentives to hide negative reports. For example, in the market for MBA degrees and in the market for publication of scientific papers, acts of burning rejection slips are not directly observable. However, such hiding can be indirectly observable; being enrolled in a low-ranked MBA program is a pretty strong signal that (at least some) higher-ranked programs declined entry (the case with scientific papers is analogous). In other markets, such as auditing, to hide reports is simply illegal. Examples of certification markets where the possibility of hiding negative reports makes an important difference seems to be quite limited.¹⁷

Moreover, one can imagine an extended analysis taking the test standards as endogenous, in the spirit of the product differentiation literature,¹⁸ or for each certifier to be endowed with more than one test standard ($K > 1$). For anything but extreme cost structures, there will be incentives for certifiers to differentiate their tests, to create market power, and it is conjectured that segmentation would occur also in such generalized settings. To illustrate that idea, let us consider an example of a setting where $K = 1$, but where the test standards are chosen endogenously. To make the example computationally tractable, I focus on the Stackelberg game where the leader chooses I_2 , and the follower chooses an I_1 after observing the choice of I_2 , where $I_i \in \{-1, -\frac{3}{4}, -\frac{1}{2}, 0, \frac{1}{2}, \frac{3}{4}, 1\}$.¹⁹ The cost of choosing I_i is assumed to be uniformly zero on this set. After observing the choices of $\{I_i\}_{i=1,2}$, the certifiers choose $\{P_i\}_{i=1,2}$ simultaneously.

¹⁷The market for money-lending by commercial banks could be one such market. Broecker (1990) constructs a model where the possibility of borrowers hiding negative 'reports' (declined applications for loan) can have an impact on equilibrium interest rates. In Broecker's theory banks have identical credit test technology and hence that theory cannot explain segmentation.

¹⁸In product differentiation models, firms first decide on product characteristics and then compete for customers through their pricing decision. In models of vertical product differentiation (e.g., Shaked & Sutton, 1982), firms offer products of different quality, and sellers differ in their willingness to pay for quality. In models of horizontal product differentiation (e.g., Salop 1979), customers have different tastes over products of the same quality. There are several differences between the present model and those models in the product differentiation literature, perhaps the most important being that here, the test product that an agent (seller) wishes to purchase depends not only on properties of the test itself, but also on behavior of the other agents, as discussed in footnote 14.

¹⁹The case where the two certifiers choose I_i simultaneously would involve (symmetric) equilibria in mixed strategies in the choice of I_i , and are computationally very complex, but should lead to the same type of results.

Example 3 Let ε_i be normally and independently distributed with mean zero and variance .40. $h(q) = 1$ for $q \in Q = [-1, 1]$. We then get $(I_1^*, I_2^*, P_1^*, P_2^*, q_1^*, q_2^*) \approx (-\frac{3}{4}, 1, .08, .53, -.06, .21)$ with associated profits, $\Pi_1^*(.) \approx .02$, $\Pi_2^*(.) \approx .41$.

The profits of the upper certifier are much higher than the profit for the lower certifier, which creates incentives for the first entering certifier to choose a tough standard ($I_2^* = 1$). The second entering certifier avoids stiff competition by choosing a soft standard I_1^* , in safe distance from the choice of I_2^* .²⁰ Interestingly, this simple example captures some of the dynamics of the market for business school degrees, where the oldest business schools have the most demanding standards, attract the most able students, charge the most presumptuous fees, and presumably makes the highest profits.²¹

5 Example: The Market for MBA Programs

In this section I wish to corroborate the insights of the model by testing the price monotonicity result in the market for MBA degrees. The market for MBA education is chosen as an application of the model for two reasons. The first reason is that certification presumably plays an important role in this market. The second reason is that relevant data, e.g., on pre-MBA and post-MBA salaries, and the costs of tuition, is easily available.²²

Certification can take different forms in the MBA market. The most obvious examples are that MBA institutions certify students through their admittance decisions and through the grading of individual students. A third possible certification effect from participating in an MBA program is that of establishing a network: getting a 'pass' can then be

²⁰There are conditions under which the first entering certifier could enter at the bottom, for example costs for setting I_i that are increasing in the location I_i .

²¹Price data are more covert in other certification markets, but similar comments can probably be made about several of them, such as the market for auditing reports and the market for underwriting services.

²²The present paper is to our knowledge the first one with an empirical analysis of certification based on an equilibrium model, and furthermore the first that takes into account both the market value increase and the cost of being certified. For example, Megginson & Weiss (1991) tests whether venture capitalists fill a certification role when backing IPOs, and Puri (1996) investigates the certification role of investment banks and commercial banks when underwriting security issues (before the Glass-Steagall Act), where both papers consider only the market value increase from attending a certifier, presumably due to data limitations.

interpreted as being accepted into an important student group, rather than as a high grade. In sum, there are several sources of certification effects in MBA programs.²³

Let me now describe the data, discuss the test strategy, and then describe the empirical results. The primary data source is the Financial Times 2001 ranking of MBA programs worldwide based on the 1998 class (FT 2001). FT 2001 includes information on (average) student characteristics at each program, such as salaries before and 3 years after the MBA, percentage of international students, alumni networks, etc. FT 2001 also includes data on program characteristics such as faculty research output, and faculty Ph.D. ratio, sex ratio, etc. In addition to the FT ranking I use the *Official MBA Guide* for information on GMAT scores and costs of tuition for the programs. I confine the analysis to the two-year US programs in the FT 2001 that provides tuition fees and (average) GMAT scores in the Official MBA guide, which gives a sample size of 48 programs.²⁴

Suppose that we rank certifiers according to how much they (on average) add to the market value of the objects that are certified. Then the main empirical implication of the model is that certifiers with higher rank charging a higher price (i.e., tuition costs).²⁵ If MBA students were observationally equivalent in the market ex-ante, their salaries would be the same before the MBA, and I could rank programs according to the post-MBA salaries they generate. In that case, the pricing hypothesis could be tested simply by evaluating the correlation between post-MBA salaries and the costs of tuition.

However, different programs attract students with different *observable* characteristics (in contrast to in the model), and therefore post-MBA salaries is not a proper measure of program rank. To control for ex-ante observable student heterogeneity when testing

²³Consequently, there are several interpretations of the test standard of a certifier being lower than the test standard of a different certifier; i)it is easier to be admitted into a lower ranked MBA program, ii)it is easier to obtain good grades in a lower ranked program, and finally iii)it is easier to be a member of an important peer group in a lower ranked program. Since there are only small fees associated with applying for a program, ii) and iii) more easily fits with a literal interpretation of the model than i) does.

²⁴100 programs are ranked in FT 2001. All salary figures are indexed to the 2000 level. Since the cost of tuition is relatively stable over time, we have used the easily available cost of tuition for year 2000 figures, rather than indexing the 1995 figures. The average salary across programs before the program starts equals \$63.008, the average salary after the end of a two-year program equals \$107.104, the average tuition cost equals \$40.417 (per year), and the average GMAT score equals 653.

²⁵The opportunity cost of time spent on the program is a very important component of the total costs of attending an MBA program. These costs are hard to estimate in a precise manner, and are not included in the analysis. Notice, however, that not including such costs makes the pricing hypothesis less obvious, since higher-ranked programs have on average students with higher opportunity costs.

the pricing hypothesis, I therefore regress post-MBA salary on the cost of tuition, controlling for pre-MBA wage and GMAT differences.²⁶ A positive and significant estimated coefficient on the relation between post-MBA salary and cost of tuition is interpreted as a confirmation of the pricing hypothesis.

Ideally, I should have decomposed the effect of attending an MBA program into two parts, the value added stemming from increases in human capital, and the value added that stems from identification of ability. Given the data limitations, such a decomposition cannot be performed, and I will therefore assume that the increases in value due to human capital acquisition are either negligible or roughly constant across programs (in the latter case human capital acquisition would only have an impact upon the intercepts of the regressions).

The following estimates were obtained (standard deviations in parentheses).²⁷

²⁶ Although GMAT score is probably less observable than salary in the market ex-ante, including GMAT is a simple way of correcting bias due to omitted observable variables such as occupational level (affecting e.g., expected career path) and geographical variations in employment (affecting take home value of salary).

²⁷ We include GMAT² as a right hand side variable to accommodate non-linearities. The regressions were performed in MINITAB. The data files and regression procedures are available from the author.

Table 1	Relation between post-MBA wage and tuition cost				
		MBA 2001		MBA 2000	
Variable	(I)	(II)	(III)	(IV)	
Intercept	1079581 (1235704)	48964 (12659)	-76227 (1354858)	55284 (14036)	
Salary before	0,593 (0,388)	...	0,489 (0,361)	...	
GMAT	-3623 (3762)	...	-115 (4116)	...	
GMAT ²	310 (287)	...	0,47 (3,137)	...	
Tuition cost	0,809 (0,273)	1,44 (0,303)	0,667 (0,278)	1,29 (0,329)	
# of observations	48	48	41	41	
R^2	64%	33%	62%	28%	

Considering first column (II), there is a strong positive raw correlation between post-MBA wage and tuition cost (TC). Since a higher cost of tuition may reflect both higher unobservable *and* observable student characteristics, column (I) controls for observable differences in student quality by including pre-MBA wage and GMAT scores on the right hand side in the regression. The relation between post-MBA wage and TC is now weaker, as expected, but is still positive and highly significant.²⁸ Notice also that the R^2 of regression (I) is significantly higher than in regression (II). The regression indicates that a 1\$ increase in yearly tuition cost leads to a \$0.81 expected increase in salary after the program ends. For example, a \$5.000 increase in tuition cost increases the expected salary after the program by \$4.500. To check the robustness of this finding, I performed the same regressions on the FT 2000 data and obtained very similar results, see (III) and (IV).²⁹

²⁸For FT 2001, the t -value for the coefficient on tuition cost is 2,97, with corresponding p -value equal to 0,005.

²⁹The similar findings can partly be explained by the same cost and GMAT data being used for the two years, and we therefore performed an alternative test. In this alternative test, we first generated a rank of schools by using the procedure of Tracy & Waldfogel (1997). This procedure regresses post-MBA wage on student characteristics, and ranks schools according to the magnitude of the residuals from this

Hence the support of the price monotonicity hypothesis is rather strong in the data.

6 Conclusion

The paper has proposed a simple theory of pricing behavior in an oligopolistic market for certification. The usefulness of the theory is twofold. First, it clarifies the relation between signaling and certification. In signaling models, separation between different types is created through an exogenously imposed cost for taking certain actions, as in Spence (1974).³⁰ In contrast, I obtain a signaling type of equilibrium where the costs for different actions (which certifier to attend) is made endogenous through the certifiers' pricing decisions. Hence one can view the present approach as providing a possible microfoundation for models of signaling.³¹ Second, at a more practical level, it was shown that equilibria of the model accommodates several stylized facts from certification markets, the most important being that different certifiers capture different segments of the market in equilibrium. In addition, I derived the testable implication that the price for being certified should increase in the rank of a certifier. The validity of this prediction was tested using the US MBA market as an example, and received empirical support. However, the data is too aggregated to make very definitive conclusions; future studies attempting to e.g., disentangle information from human capital acquisition effects in the MBA market would clearly be of interest.

Another possible extension would be to consider a model where certifiers fill additional roles to certification. For example, auditing firms are divided into an accounting (certification) part and a consulting part, where the latter part offers advice based on the insights generated by the first part. It would be of interest to investigate what the

regression. To test the pricing hypothesis, we then regressed the residuals on TC, and obtained the same type of results here as those expressed in Table 1 (the induced ranking for the FT 2001 data became: 1. Columbia, 2. Wharton, 3. Chicago, 4. Harvard, 5. Cornell, 6. Emory, 7. Georgetown, 8. Duke, 9. Vanderbilt, and 10. Southern Methodist).

³⁰There are numerous other examples of such models, two of the more relevant to the present context being Puri's (1999) signaling model of investment banking and Titman & Trueman's (1986) signaling model of auditing. Both models have exogenous prices.

³¹In addition to the examples of the previous footnote, Weiss (1983) creates separation into different lengths of education through a differential probability of passing a test for different types. In the model of Weiss (1983), however, there is only one school in the market, with an exogenous test standard and tuition fee, while our tuition fee is endogenous.

implications of this dual role can have for price setting and information revelation by auditors. In which way would the dual role affect the price competition for auditing services, and how would it affect certifiers' incentives to reveal bad information? In light of recent corporate scandals such as the weak reporting by Arthur Andersen in the Enron case, these questions seem worthwhile to pursue.

7 Appendix

First $U(pass)$ and $U(fail)$ are derived. Recall that in general we have $E(q|A) = \frac{\int_A g_A(q)q dq}{Pr ob(A)}$ where $g_A(\cdot)$ is the conditional density function and A is some event. By the law of large numbers, the fraction of sellers in the point q that passes the test is deterministic and equals $Pr(pass|q, I)$. The conditional density, i.e., the density of those that pass, is just equal to this entity, and the probability of passing test I for a random seller on $[q_1, q_2]$ equals $\int_{q_1}^{q_2} Pr(pass|q, I) dq$. Hence we have that,

$$\begin{aligned} U(pass) &= \frac{\int_{q_1}^{q_2} Pr(pass|q, I) q dq}{\int_{q_1}^{q_2} Pr(pass|q, I) dq} \\ U(fail) &= \frac{\int_{q_1}^{q_2} Pr(fail|q, I) q dq}{\int_{q_1}^{q_2} Pr(fail|q, I) dq} \end{aligned} \quad (A1)$$

For an agent with ability q who attends a certifier whose customers lie on the interval $[q_1, q_2]$ we can therefore write,

$$U(q; q_1, q_2) = Pr(pass|q, I) \frac{\int_{q_1}^{q_2} Pr(pass|q, I) q dq}{\int_{q_1}^{q_2} Pr(pass|q, I) dq} + Pr(fail|q, I) \frac{\int_{q_1}^{q_2} Pr(fail|q, I) q dq}{\int_{q_1}^{q_2} Pr(fail|q, I) dq} - P \quad (A2)$$

Let us now consider the pricing game in a **duopoly**. The profits are,

$$\begin{aligned} \Pi_1 &= P_1(q_2 - q_1) \\ \Pi_2 &= P_2(1 - q_2) \end{aligned} \quad (A3)$$

For the lower cutoff q_1 we have the same condition as in the monopoly case,

$$\Psi_1(P_1, P_2, q_1, q_2) = UU_1(q_1; q_1, q_2) - P_1 = 0 \quad (\text{A4})$$

For the upper cutoff q_2 we have the condition,

$$\Psi_2(P_1, P_2, q_1, q_2) = UU_2(q_2; q_2) - P_2 - UU_1(q_2; q_1, q_2) + P_1 = 0 \quad (\text{A5})$$

The first order conditions for profit maximization are,

$$\begin{aligned} \frac{d\Pi_1}{dP_1} &= q_2 - q_1 + \left(\frac{\partial q_2}{\partial P_1} - \frac{\partial q_1}{\partial P_1} \right) P_1 = 0 \\ \frac{d\Pi_2}{dP_2} &= 1 - q_2 - \frac{\partial q_2}{\partial P_2} P_2 = 0 \end{aligned} \quad (\text{A6})$$

I use the implicit function theorem to determine $\frac{\partial q_1}{\partial P_1}$ as,

$$\frac{\partial q_1}{\partial P_1} = -\frac{\Psi_{1P_1}}{\Psi_{1q_1}} = \frac{1}{\Psi_{1q_1}} \quad (\text{A7})$$

and,

$$\begin{aligned} \frac{\partial q_2}{\partial P_1} &= -\frac{\Psi_{2P_1}}{\Psi_{2q_2}} = -\frac{1}{\Psi_{2q_2}} \\ \frac{\partial q_2}{\partial P_2} &= -\frac{\Psi_{2P_2}}{\Psi_{2q_2}} = \frac{1}{\Psi_{2q_2}} \end{aligned} \quad (\text{A8})$$

We then have the following four equations determining the four endogenous variables $(P_1^*, P_2^*, q_1^*, q_2^*)$,

$$\begin{aligned} \frac{d\Pi_1}{dP_1} &= q_2 - q_1 - P_1 \left[\frac{1}{\Psi_{2q_1}} + \frac{1}{\Psi_{1q_2}} \right] = 0 \\ \frac{d\Pi_2}{dP_2} &= 1 - q_1 + \frac{P_2}{\Psi_{2q_2}} = 0 \\ \Psi_1(P_1^*, P_2^*, q_1, q_2) &= 0 \\ \Psi_2(P_1^*, P_2^*, q_1, q_2) &= 0 \end{aligned} \quad (\text{A9})$$

For certifier 2 the second order condition for optimum equals,

$$\frac{\partial^2 \Pi_2}{\partial P_2^2} = \frac{1}{\Psi_{2q_2}} [P_2 \frac{\Psi_{2q_2 q_2}}{\Psi_{2q_2}^2} - 2] = \frac{1}{\Psi_{2q_2}} [(1 - q_2) \Psi_{2q_2} \frac{\Psi_{2q_2 q_2}}{\Psi_{2q_2}^2} - 2] = \frac{1}{\Psi_{2q_2}} [(1 - q_1) \frac{\Psi_{2q_2 q_2}}{\Psi_{2q_2}} - 2] < 0 \quad (\text{A10})$$

For certifier 1, the SOC is slightly more involved,

$$\begin{aligned} \frac{\partial^2 \Pi_1}{\partial P_1^2} &= \frac{\partial q_2}{\partial P_1} - \frac{\partial q_2}{\partial P_1} - \frac{1}{\Psi_{1q_1}} - \frac{1}{\Psi_{2q_2}} + \frac{\partial q_1}{\partial P_1} P_1 \frac{\Psi_{2q_1 q_1}}{\Psi_{2q_1}^2} + \frac{\partial q_2}{\partial P_1} P_1 \frac{\Psi_{1q_2 q_2}}{\Psi_{1q_2}^2}. \\ &= -2 \left[\frac{1}{\Psi_{1q_1}} + \frac{1}{\Psi_{2q_2}} \right] - (1 - q) (\Psi_{2q_1} + \Psi_{1q_2}) \frac{\Psi_{2q_1 q_1}}{\Psi_{2q_1}^3} - (1 - q) (\Psi_{2q_1} + \Psi_{1q_2}) \frac{\Psi_{1q_2 q_2}}{\Psi_{1q_2}^3}. \end{aligned} \quad (\text{A11})$$

In the numerical analysis, (A9) was used to compute equilibria, and the second order conditions (A10) and (A11) were confirmed to hold.

8 References

Biglaiser, G. (1993). Middlemen as Experts. *Rand Journal of Economics*, 24, 212-23.

Biglaiser, G. & J. W. Friedman (1999). Adverse Selection With Competitive Inspection. *Journal of Economics and Management Strategy*, 8, 2-33.

Broecker, T. (1990). Credit-Worthiness Tests and Interbank Competition. *Econometrica*, 58, 429-52.

Choi, S. (1996). Certification Intermediaries. Draft, University of Chicago.

Dunbar, C. G. (1997). Factors Affecting Investment Bank Initial Public Offering Market Share. Working paper, Katz Graduate School of Business, University of Pittsburgh.

Grossman, S. J. (1981). The Informational Role of Warranties and Private Disclosure about Product Quality. *Journal of Law and Economics*, 24, 461-83.

Holmstrom, B. (1979). Moral Hazard and Observability. *Bell Journal of Economics*, 10, 74-91.

Krishnan, G. V. (2002). Audit Quality and the Pricing of Discretionary Accruals. Available at <http://www.ssrn.com>.

Lizzeri, A. (1999). Information Revelation and Certification Intermediaries. *Rand Journal of Economics*, 30, 214-31.

- Meggison, W. L. & K. A. Weiss (1991). Venture Capitalist Certification in Initial Public Offerings. *Journal of Finance*, 46, 879-903.
- Puri, M. (1996). Commercial Banks in Investment Banking: Conflict of Interest or Certification Role? *Journal of Financial Economics*, 40, 373-401.
- Puri, M. (1999). Commercial Banks as Underwriters: Implications for the Going Public Process. *Journal of Financial Economics*, 54, 133-63.
- Salop, S. (1979). Monopolistic Competition with an Outside Good. *Bell Journal of Economics*, 10, 141-56.
- Shaked, A. & J. Sutton (1982). Relaxing Price Competition through Product Differentiation. *Review of Economic Studies*, 49, 3-13.
- Spence, A. M. (1974). Market Signaling. Cambridge, Mass.: *Harvard University Press*.
- Titman, S & Trueman, B. (1986). Information Quality and the Valuation of New Issues. *Journal of Accounting and Economics*, 8, 159-72.
- Tracy, J. & J. Waldfogel (1997). The Best Business Schools: A Market-Based Approach. *Journal of Business*, 70, 1-31.
- Weiss, A. (1983). A Sorting-cum-Learning Model of Education. *Journal of Political Economy*, 91, 420-442.
- Weiss, A. (1995). Human Capital versus Signalling Explanation of Wages. *Journal of Economic Perspectives*, 9, 133-54.